

2

AVERAGES

In MAT, CET, FMS and other entrance exams the questions are directly asked from this chapter and they are very moderate, but in CAT this chapter constitutes a very low proportion of questions i.e., they are rarely asked based on direct formula. As in last 10-12 years on an average 1 question per year has appeared in CAT and each question was almost of applied nature and blended with the logical reasoning.

So, when we said that on an average there was 1 question was asked per year in the last decade (i.e., 10 years), it means we can conclude that almost 10 questions were asked by CAT in the last 10 years.

In general average is the central value of the given data. For example if the heights of three persons A , B and C be

90 cm, 110 cm and 115 cm respectively, then the average height of A , B and C together will be $\frac{90+110+115}{3} = 105$ cm.

So we can say that the height of each person viz. A , B and C is near about 105 cm. Thus in layman's language, we can be said that every one is almost 105 cm tall.

Basically the average is the arithmetic mean of the given data. For example if the $x_1, x_2, x_3, x_4 \dots x_n$ be any n quantities (i.e., data), then the average (or arithmetic mean) of these n quantities

$$= \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

PROPERTIES OF AVERAGES

1. The average of any two or more quantities (or data) necessarily lies between the lowest and highest values of the given data. i.e., if x_l and x_h be the lowest and highest (or greatest) values of the given data $(x_1, x_2, \dots x_l, \dots x_h, \dots x_n)$ then $x_l < \text{Average} < x_h$; $x_l \neq x_h$

$$\text{i.e., } x_l < \frac{(x_1 + x_2 + x_3 + x_l + \dots + x_h + \dots + x_n)}{n} < x_h$$

EXAMPLE 1 The average of 8, 9, 12, 13, 15, 9 is :

- (a) 11 (b) 6 (c) 16 (d) 18

SOLUTION From the above mentioned property (1) we can say that options (b), (c) and (d) are invalid since 6, 16, and 18 are out of range i.e., either below 8 (which is the least value) or above 15 (which is the highest value of the data).

So, option (a) can be acceptable only.

Alternatively :

$$\text{Average} = \frac{8 + 9 + 12 + 13 + 15 + 9}{6} = \frac{66}{6} = 11$$

EXAMPLE 2 A has 8 pencils, B has 10 pencils and C has 15 pencils, then the average number of pencils with them :

- (a) 8 (b) 10 (c) 15 (d) lies between 9 and 15

SOLUTION Average number of pencils $= \frac{8 + 10 + 15}{3} = 11$

So, option (d) is correct.

2. If each quantity is increased by a certain value 'K' the the new average is increased by K .

EXAMPLE 3 A, B, C, D, E are the five electronic shops in the market, which have 20, 30, 60, 80 and 50. T.V. sets with them respectively, then the average number of T.V. sets in each shop is :

- (a) 24 (b) 48 (c) 50 (d) 60

SOLUTION Average number of T.V. sets

$$= \frac{20 + 30 + 60 + 80 + 50}{5} = 48$$

So option (b) is correct.

EXAMPLE 4 In the previous example, if each of A, B, C, D and E have imported 12 new T.V. sets, then the average number of T.V. sets is :

- (a) 50 (b) 56 (c) 60 (d) 144

SOLUTION Average number of T.V. sets

$$= \frac{(20 + 12) + (30 + 12) + (60 + 12) + (80 + 12) + (50 + 12)}{5} = \frac{300}{5} = 60$$

and the total age of remaining 24 teachers when just the principal has retired
 $= 1275 - 60 = 1215$ years
 1 year later (i.e., 3 years ago from present) total age of 24 teachers (just before the recruitment of new principal)
 $= 1215 + (1 \times 24) = 1239$ years
 and the total age of 25 teachers including new principal just after the recruitment
 $= 1239 + 54 = 1293$ years

Thus the present age of all the 25 teachers
 $= 1293 + (3 \times 25) = 1368$ years

Hence, the present average age of the 25 teachers
 $= \frac{1368}{25} = 54 \frac{18}{25}$ years

Alternatively : 10 years ago, the average age of 25 teachers

$= 45$ years
 Let us assume that the principal has not retired from her post then the present average age of all the 25 teachers,
 $= 45 + 10 = 55$ years

Thus the total age of all the 25 teachers
 $= 55 \times 25 = 1375$ years

Now assume that the new principal has replaced the old principal 4 years ago instead of 3 years ago, when the age of new principal would have been 53 years. Thus the age of new principal was 7 years less than the age of old principal, which results in the reduction of total age of the group of 25 teachers by 7 years.

Thus the actual total age of the 25 teachers (presently)

$$= 1375 - 7 = 1368$$

Hence, the present average age of the 25 teachers

$$= \frac{1368}{25} = 54 \frac{18}{25}$$

EXAMPLE 19 The ratio of the ages of the father and the daughter at present is 3 : 1. Four years ago the ratio was 4 : 1. The average age of the father and daughter 2 years hence will be :

- (a) 24 (b) 26 (c) 25 (d) 36

SOLUTION Let the present ages of father be $3x$ and daughter be x .

So the 4 years ago father's age and daughter's age was $(3x - 4)$ and $(x - 4)$

$$\text{Therefore } \frac{3x - 4}{x - 4} = \frac{4}{1}$$

$$\Rightarrow x = 12 \text{ years and } 3x = 36 \text{ years}$$

Hence, the present average age of father and daughter = 24 years and the average age 2 years hence will be 26 years.

9. Problems based on income/salary

$$\text{Income} = \text{Expenditure} + \text{Savings}$$

EXAMPLE 20 The average salary of Rajesh, Bahadur and Amir is Rs. 8000 per month. The average expenditure of the Rajesh, Bahadur and Amir per month is Rs. 5000. The average savings of all the 3 persons per month is :

- (a) Rs. 3000 (b) Rs. 5000
 (c) Rs. 2500 (d) Rs. 9000

SOLUTION Average saving

$$\begin{aligned} &= \text{Average Income} - \text{Average Expenditure} \\ &= 8000 - 5000 = 3000 \end{aligned}$$

EXAMPLE 21 The average salary of A, B and C is Rs. 10,000 and average expenditure of A is Rs. 6000 then the average savings of B and C is :
 (a) Rs. 5500 (b) Rs. 4500
 (c) Rs. 4000 (d) can't be determined

SOLUTION Total income = Total Expenditure + Total savings
 We can not find the average savings of B and C, since data is insufficient.

EXAMPLE 22 The average salary of A, B is Rs. 6000 and that of C, D and E is Rs. 8000. The average salary of all the 5 people is :
 (a) Rs. 7200 (b) Rs. 7000
 (c) Rs. 7500 (d) can't be determined

SOLUTION Required average salary =

$$\frac{6000 \times 2 + 8000 \times 3}{(2 + 3)} = \frac{36000}{5} = \text{Rs. 7200}$$

EXAMPLE 23 The average salary of all the 60 employees in an office is Rs. 12,000 per month. If the number of executives is twice the number of non executive employees, then the average salary of all the non executive employees is :

- (a) Rs. 9000 (b) Rs. 8000
 (c) Rs. 6000 (d) can't be determined

SOLUTION $\frac{\text{No. of executives}}{\text{No. of non-executives}} = \frac{2}{1}$

Therefore number of executives = 40
 and number of non-executive employees = 20

Now, go through the options

Total salary = $40 \times \text{salary of executive} + 20 \times \text{salary of non-executive}$
 $60 \times 12000 = 40 \times k + 20 \times l$, here k, l are unknowns

So we can't determine the required average salary.

EXAMPLE 24 In the above problem, if the average salary of non-executives be $\frac{2}{5}$ th of the average salary of executives, then the average salary of non-executive employees is :
 (a) Rs. 9000 (b) Rs. 8000
 (c) Rs. 6000 (d) data insufficient

SOLUTION By options : $60 \times 12000 = 20 \times 6000 + 40 \times 15000$
 Hence option (c) is correct.

Alternatively : $60 \times 12000 = 20 \times x + 40 \times \frac{5}{2}x$
 $\Rightarrow x = 6000$

10. Problems based on speed, time and distance :

Case 1: When the distance travelled in different time slots or parts is same i.e., if a person or vehicle moves x km at a speed of u km/hr and further he goes or comes back the same distance x km at a speed of v km/hr. Then the average speed = $\frac{2uv}{(u+v)}$.

If there are 3 parts of distance x km travelled with 3 different speeds i.e., if a person goes first x km @ speed of u km/hr and next x km @ v km/hr and the last x km @ w km/hr. Then the average speed = $\frac{3uvw}{(uv + vw + wu)}$.

Averages

SOLUTION Let $A, B, C, D, E, F, G, H, I, J, K$ be the 11 players in the order of increasing weight then,

$$A + B + C + D + E + F = 49 \times 6 = 294$$

$$F + G + H + I + J + K = 52 \times 6 = 312$$

$$\text{and } A + B + C + D + E + F + G + H + I + J + K = 50 \times 11 = 550$$

$$\begin{aligned} \text{Therefore } F &= (A + B + C + D + E + F) \\ &+ (F + G + H + I + J + K) - (A + B + C + D + \dots + J + K) \\ &= 294 + 312 - 550 = 56 \end{aligned}$$

Hence the average weight of $F = 56$ kg.

EXAMPLE 30 The average presence of students in a class on Monday, Tuesday and Wednesday is 30 and on the Wednesday, Thursday, Friday and Saturday is 28 then number of students who attended the class on Wednesday is, if the average number of students on all the six days is 27 :

- (a) 24 (b) 25 (c) 20 (d) 40

SOLUTION Since $W = (M + T + W) + (W + Th + F + S) - (M + T + W + Th + F + S)$
 $= (30 \times 3) + (28 \times 4) - (27 \times 6)$
 $= 202 - 162 = 40$

EXAMPLE 31 The average age of A, B, C, D and E is 40 years. The average age of A and B is 35 years and the average age of C and D is 42 years. The average age of E is :

- (a) 46 (b) 48
(c) 32 (d) none of these

SOLUTION $A + B + C + D + E = 40 \times 5 = 200$

$$A + B = 35 \times 2 = 70$$

$$C + D = 42 \times 2 = 84$$

$$\begin{aligned} \text{Therefore } E &= (A + B + C + D + E) - [(A + B) + (C + D)] \\ &= 200 - (70 + 84) = 46 \end{aligned}$$

Thus the average age of $E = 46$ years.

EXAMPLE 32 The average temperature on Monday, Tuesday and Wednesday is 38°C . The average temperature on Tuesday, Wednesday and Thursday is 43°C . If the average temperature on Monday and Thursday is 18.5°C . The average temperature on Monday is :

- (a) 11°C (b) 21°C (c) 35°C (d) 27°C

SOLUTION $(M + T + W) = 38 \times 3 = 114$... (i)
 $(T + W + Th) = 43 \times 3 = 129$... (ii)

$$\begin{aligned} \text{Therefore } Th - M &= 15 & [(ii) - (i)] \\ \text{and } Th + M &= 37 \end{aligned}$$

Thus the temperature on Monday is 11°C and on Thursday is 26°C .

12. Average of Some Important Series of Numbers :

- Average of first ' n ' natural numbers = $\frac{n+1}{2}$
- Average of first ' n ' even numbers = $(n+1)$
- Average of first ' n ' odd numbers = n
- If there are $(p+q)$ elements in a set or group but the average of p elements is r and the average of q elements is s , then the average of all the elements of the set (or group) is $\frac{(pr+sq)}{(p+q)}$.

EXERCISE

LEVEL (1)

- The average weight of a class of 20 students is 45 kgs. A new student whose weight is 40 kgs replaces an old student of this class. Hence, the average weight of the whole class decreases by 1 kg. The weight of the replaced student is :
 - 55 kgs
 - 50 kgs
 - 60 kgs
 - none of these
- The average length of first 3 fingers is 3 inches and the average length of the other 2 fingers i.e., thumb and the index finger is 2.8 inches. If the length of the index fingers is 3 inches then the length of the thumb is :
 - 2 inches
 - 2.6 inches
 - 3 inches
 - none of these
- Three types of rice whose rates are Rs. 38, Rs. 43 and Rs. 49 per kg are blended together to make a 15 kg of new blend of rice in which there are 8 kgs, 4 kgs 3 kgs of the respective types of rice. The average price of the new blend of rice is :
 - Rs. 41.53
 - Rs. 43
 - Rs. 40
 - Rs. 43.3
- Pankaj went to the post-office at the speed of 60 km/hr while returning for his home he covered the half of the distance at the speed of 10 km/hr, but suddenly he realized that he was getting late so he increased the speed and reached the home by covering rest half of the distance at the speed of 30 km/hr. The average speed of the Pankaj in the whole length of journey is :
 - 5.67 km/hr
 - 24 km/hr
 - 22.88 km/hr
 - 5.45 km/hr
- 123 typists typed 984 papers in $1/15$ hour. The number of papers typed per minute by an average typist is :
 - 1
 - 2
 - 3
 - 5
- The cost of the Red, Green and Blue colours per kg is Rs. 20, Rs. 15 and Rs. 18 respectively. Rang Mahal is a renowned building in which these three colours are being used in the ratio of 3 : 2 : 4. The average cost of all the three colours used per kg is :
 - 18
 - 20
 - 17.66
 - can't be determined
- Rajeev earns $3/2$ times in January, April, July and October than his average earning of Rs. 600 per month in the rest of the months. So his savings in the January, April, July and October goes to $5/4$ times that of the rest months saving of Rs. 400 per month in the year. The average expenditure of per month is :
 - Rs. 266.66
 - Rs. 250
 - Rs. 233.33
 - Rs. 433.33

- The average of 3 prime numbers lying between 47 and 74 is $191/3$. The greatest possible difference between any two of the 3 prime numbers is :
 - 12
 - 14
 - 18
 - can't be determined
- The average of 9 numbers is 11. If each of these 9 numbers is multiplied by 5 and then 5 is added to each of these resultant numbers, then the new average is :
 - 20
 - 30
 - 60
 - 50
- The average score of Dhoni after 48 innings is 48 and in the 49th innings Dhoni scores 97 runs. In the 50th innings the minimum number of runs required to increase his average score by 2 than it was before the 50th innings :
 - 99
 - 149
 - 151
 - can't be determined
- The average age of Sachin and Ganguli is 35 years. If Kaif replaces Sachin, the average age becomes 32 years and if Kaif replaces Ganguli, then the average age becomes 38 years. If the average age of Dhoni and Irfan be half of the average age of Sachin, Ganguli and Kaif, then the average age of all the five people is :
 - 28 years
 - 32 years
 - 25 years
 - none of these
- Out of these five people (in question no. 11) whose age is the greatest?
 - Sachin
 - Ganguli
 - Kaif
 - can't be determined
- In a village the average age of n people is 42 years. But after the verification it was found that the age of a person had been considered 20 years less than the actual age, so the new average, after the correction, increased by 1. The value of n is :
 - 21
 - 20
 - 22
 - none of these
- The average rainfall in the months of January and February is 6 cm and in the months of March to June is 5 cm and July to October is 10 cm and in the November and December, it is 6 cm. The average rainfall for the whole year is :
 - 7
 - 5.5
 - 7.5
 - none of these
- On an average 300 people watch the movie in Sahu Cinema hall on Monday, Tuesday and Wednesday and the average number of visitors on Thursday and Friday is 250. If the average number of visitors per day in the week be 400, then the average number of people who watch the movie in the week (i.e., on Saturday and Sunday) is :
 - 300
 - 350
 - 400
 - 450

16. The average weight of 11 players of Indian cricket team is increased by 1 kg, when one player of the team weighing 55 kg replaced by a new player. The weight of the new player is :
 (a) 55 kg (b) 64 kg (c) 66 kg (d) none of these
17. The average age of a family of 6 members 4 years ago was 25 years. Mean while a child was born in this family and still the average age of the whole family is same today. The present age of the child is :
 (a) 2 years (b) $1\frac{1}{2}$ years (c) 1 year (d) data insufficient
18. Amitabh's average expenditure for the January to June is Rs. 4200 and he spends Rs. 1200 in January and Rs 1500 in July. The average expenditure for the months of February to July is :
 (a) Rs. 4250 (b) Rs. 4520 (c) Rs. 4060 (d) none of these
19. The average of $a, 11, 23, 17$ is 15 and the average of $a, b, 12, 25$ is 16. The value of $\frac{a}{b}$ is :
 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$
20. The average salary is being paid to all its employees by the Biotech corporation is Rs. 15,500. The average salary of the senior employees is Rs. 18000 per month and the average salary of the junior employees is Rs. 12,000 per month. If there are only two levels of employees viz junior and senior level, then what fraction of the total employees is the junior level employees are :
 (a) $\frac{7}{10}$ (b) $\frac{5}{12}$ (c) $\frac{5}{10}$ (d) none of these
21. The average of any 5 consecutive odd numbers a, b, c, d, e is :
 (a) $\frac{(abcde)}{5}$ (b) $\frac{bd}{3}$ (c) $\frac{(a+c+e)}{5}$ (d) none of these
22. The average age of 30 students of a class is 30 years. When the average age of class teacher is also included, the average age of the whole class increases by 1 year. The age of the class teacher is :
 (a) 31 years (b) 60 years (c) 61 years (d) none of these
23. There were five sections in MAT paper. The average score of Pooja in first 3 sections was 83 and the average in the last 3 sections was 97 and the average of all the sections (i.e., whole paper) was 92, then her score in the third section was :
 (a) 85 (b) 92 (c) 88 (d) none of these
24. The average age of 18 pupils of Dronacharya was 25 years. If the age of Dronacharya was also included, the average age of 19 people becomes 26 years. The average age of the Dronacharya at that time was :
 (a) 33 (b) 44 (c) 50 (d) 51
25. The average of 7 consecutive odd numbers if the smallest of those numbers is denoted by k :
 (a) $k+4$ (b) $k+7$ (c) $k+6$ (d) $7k$
26. If the average marks of $\frac{1}{4}$ th class is 85% and that of $\frac{1}{3}$ rd class is 70% and the average marks of the rest class is 56%, then the average of the whole class is (for the given subjects) :
 (a) 67.916% (b) 72.33% (c) 69.165% (d) can't be determined
27. The average length of any four fingers of my left hand is 600 mm. Then the average length of all the five fingers of my left hand is :
 (a) 800 mm (b) 750 mm (c) 480 mm (d) can't be determined
28. The average of $4\frac{3}{5}, 2\frac{2}{3}, 6\frac{8}{9}, 7\frac{7}{15}, 3\frac{5}{9}$ is :
 (a) $5\frac{3}{225}$ (b) $5\frac{8}{225}$ (c) $6\frac{3}{45}$ (d) $25\frac{8}{45}$
29. The average of 1000.0001, 100.001, 10.01, 1.1 is :
 (a) 277.777 (b) 322.222 (c) 11.11 (d) 233.333
30. The average of 7 consecutive numbers which are positive integers is 10. The average of lowest and highest such numbers is :
 (a) 7 (b) 10 (c) 15 (d) data insufficient
31. The average of first 100 natural number is :
 (a) 100 (b) 50 (c) 50.50 (d) 55
32. The average of first 50 odd natural numbers is :
 (a) 50 (b) 55 (c) 51 (d) 101
33. The average of first 99 even numbers is :
 (a) 9999 (b) 100 (c) 9801 (d) 9009
34. The average of a, b and c is 79 and the average of a and c is also 79. Then the value of b is :
 (a) 0 (b) 79 (c) -79 (d) none of these
35. The average value of property of Mittal, Ambani and Singhania is Rs. 11111 crore. The property of Singhania is as less as the property of Mittal is greater than the average property of both the Singhania and Mittal. The value of property of Ambani is :
 (a) Rs. 111 crore (b) Rs. 11111 crore (c) Rs. 3703.7 crore (d) can't be determined
36. I went to Delhi @ speed of 200 km/hr but suddenly I returned to the same place @ speed of 600 km/hr. What is my average speed :
 (a) 300 km/hr (b) 400 km/hr (c) 366.66 km/hr (d) none of these
37. The average of A and B is 400 and the average of C and D is 600 the average of A, B, C and D is :
 (a) 500 (b) 450 (c) 525 (d) 625

38. The average weight of liquid in 100 bottles is 500 gm. The total weight of all the bottles is 20 kg. The average weight of a bottle with liquid is :
 (a) 0.65 kg (b) 0.7 kg
 (c) 70 ml (d) none of these
39. The average score of Sehwag in 10 innings was 77 runs. In the 11th innings he had scored zero runs. The overall average score of Sehwag in all the 11 innings was :
 (a) 77 (b) 7.7
 (c) 11 (d) none of these
40. The average age of 3 children of Arijant Singh is 12 years and their ratio of ages is 3 : 4 : 5. The average age of the youngest and eldest child is if he had only 3 children :
 (a) 12 (b) 21
 (c) 8 (d) 9
41. The average income of all the Infosys employees is Rs. 20,000 per month. Recently the company announced the increment of Rs. 2,000 per month for all the employees. The new average of all the employees is :
 (a) Rs. 22,000 (b) Rs. 40,000
 (c) Rs. 22,00 (d) data insufficient
42. The average age of 10 students in a class is 20 years, if a new student is also included, then the new average age of all the students increases by 1 year. The age of the new student is :
 (a) 21 years (b) 30 years
 (c) 31 years (d) none of these
43. The average weight of 20, four wheelers is 180 kg. If an old car is removed from this group of four wheelers, the new average weight decreases by 2 kg. The weight of the removed car is :
 (a) 220 (b) 218
 (c) 182 (d) none of these
44. The average price of 3 diamonds of same weights is Rs. 5 crore, where the average price of the two costliest diamonds is double the price of the cheapest diamond. The price of the cheapest diamond is :
 (a) 3 crore (b) 5 crore
 (c) 1.66 crore (d) can't be determined
45. In the previous question, the price of the costliest diamond is :
 (a) 5 crore (b) 6 crore
 (c) 8 crore (d) can't be determined
46. Praveen gets 40 marks out of 50 in Computer Science, 70 out of 100 in Manufacturing science and 142 out of 150 in professional communication. The average marks of Praveen (in percent) in all the three subjects is :
 (a) 84% (b) 76%
 (c) 71% (d) 60%
47. The average of all the prime and composite numbers upto 100 is :
 (a) 51 (b) 49.50
 (c) 50.50 (d) 55
48. The average of all the perfect squares upto 100 is :
 (a) 38.5 (b) 1000
 (c) 100 (d) 385
49. The average of all the non-negative integers upto 99 is :
 (a) 50.49 (b) 49.50
 (c) 50.50 (d) 99

50. The average of 7, 14, 21, 28 ... 77 is :
 (b) 11
 (a) 7
 (c) 42
51. The average weight of A, B, C and D is 40 kg. A new person E is also included in the group, then the average weight of the group is increased by 1 kg. Again a new person F replaces A, then the new average of 5 persons becomes 42. The average weight of B, C, D, F is :
 (b) 41.25
 (a) 42
 (c) 42.5
 (d) none of these
52. The average of 3 consecutive natural numbers (which are in increasing order) is k . If two more consecutive numbers, just next the first set of numbers, is added, then the new average becomes :
 (b) $k + 1$
 (a) $k + 2$
 (c) $\frac{2k + 1}{2}$
 (d) $2k - 1$
53. The average of any 5 consecutive odd natural numbers is k . If two more such numbers, just next to the previous 5 numbers are added, the new average becomes :
 (b) $2k - 3$
 (a) $\frac{2}{7}(k + 1)$
 (c) $2k + 1$
 (d) $k + 2$
54. The average weight of the 5 officers of a regiment is 42 kg. If a senior officer was replaced by a new officer and thus the average increased by 500 gm, the weight of the new officer is :
 (a) 44.5 kg (b) 45 kg
 (c) 42.5 kg (d) none of these
55. The average age of 6 servants in my farm house is 28 years. A new and young servant replaces an old servant, then the new average reduces by 1 year, the age of the new servant is :
 (a) 26 years (b) 22 years
 (c) 35 years (d) can't be determined
56. In the above question (no. 55) if the age of the replaced servant was 31 years, then the age of the new servant is :
 (a) 25 years (b) 35 years
 (c) 24 years (d) none of these
57. The average income of A, B and C is Rs. 12,000 per month and the average income of B, C and D is Rs. 15,000 per month. If the average salary of D be twice that of A, then the average salary of B and C is (in Rs.) :
 (a) 8,000 (b) 18,000
 (c) 13,500 (d) 9,000
58. The average price of 80 computers in an electronic shop is Rs. 30,000. If the highest and lowest price computers are sold out then the average price of the remaining 78 computers is Rs. 29,500. The cost of the highest price computer is Rs. 80,000. The cost of lowest price computer is :
 (a) Rs. 19,000 (b) Rs. 20,000
 (c) Rs. 29,000 (d) can't be determined
59. A has 50 coins of 10 paise denominations. While B has 10 coins of 50 paise denominations. C has 20 coins of 25 paise denominations while D has 25 coins of 20 paise denominations. The average number of paise per person is :
 (a) 450 paise (b) 500 paise
 (c) 600 paise (d) can't be determined

60. A travel agency has three types of vehicles viz. four seater, autorickshaw, 10 seater maxi cab and 20 seater minibus. The rate of each passenger (irrespective of its age or weight or seniority) for the auto rickshaw is Rs. 12 and for the maxicab is Rs. 15 and for the minibus is Rs. 8 for the one round. The

LEVEL (2)

1. A train normally covers a certain distance at a speed of 60 km/hr. However, if it were to halt for a fixed time interval in each hour its average speed reduced to 50 km/hr. What is the time interval for which the train halt in each hour?

(a) 10 minutes (b) 20 minutes
(c) 6 minutes (d) 12 minutes

2. If p, q, r be three positive numbers such that $p > q > r$ when the smallest number is added to the difference of the rest two numbers, then the average of the resultant number and the original numbers except to the smallest number is 21 more than the average of all the three original numbers. The value of $(p - q)$ is :

(a) 7 (b) 14
(c) 63 (d) 42

3. Progressive express left for New Delhi, increasing its speed in each hour. It started its journey from Lucknow, but after four hours of its journey it met with accident. Its speed in the fourth hour was $\frac{7}{5}$ times that of the third hour and the speed in the third hour was $\frac{10}{7}$ times that of the second hour and in the second hour it was $\frac{7}{5}$ times that of the first hour. If it would have been travelled with the half of the speed that of the third hour, then it would have gone 160 km less in the same time (i. e., in four hours). The average speed of the train during the journey of 4 hours was :

(a) 50 km/hr (b) 90 km/hr
(c) 80 km/hr (d) can't be determined

4. In a Mock CAT 123 students appeared and the average score obtained was 120. But later it was found that the top three students were repeaters, so their score has been eliminated and then the new average score was found to be decreased by 1.5. Also, it is known that all the students obtained the marks in integers and the scores of the toppers were distinct. If the second highest topper has scored more than 185 marks, then the highest possible score of the third highest topper is :

(a) 166 (b) 167
(c) 168 (d) 170

5. The average age of all the 100 employees in an office is 29 years, where $\frac{2}{5}$ employees are ladies and the ratio of average age of men to women is 5 : 7. The average age of female employees is :

(a) 18 years (b) 35 years
(c) 25 years (d) none of these

6. There are two houses in Parliament. One is Lok Sabha and the other one is Rajya Sabha and the member of Parliaments (MPs) in both the houses is 300 and 200 respectively. The

average occupancy of the seats is 100%, 80% and 75% respectively. If he has only one vehicle of each kind, then the average earning for one round of each vehicle is :

average age of the members of Lok Sabha and Rajya Sabha is 40 years and 50 years respectively. A member of the Rajya Sabha when elected for the Lok Sabha also, he left the Rajya Sabha and becomes the member of the Lok Sabha. Thus the average age of both the houses increases. Which one of the following statements is true?

- (a) The age of this member is greater than 50 years
 - (b) The age of this member is less than 40 years
 - (c) The age of this member is greater than 40 but less than 50 years
 - (d) none of these

4 **Directions for questions 7, 8 and 9 :** Eight years ago there were 5 members in the Arthur's family and then the average age of the family was 36 years. Mean while Arthur got married and gave birth to a child. Still the average age of his family is same now.

7. The present age of his wife is :
(a) 25 years (b) 26 years
(c) 32 years (d) data insufficient

8. The age of his wife at the time of his child's birth was. If the difference between the age of her child and herself was 26 years :
(a) 25 years (b) 26 years
(c) 20 years (d) can't be determined

9. The age of Arthur at the time of his marriage was :
(a) 22 years (b) 23 years
(c) 26 years (d) can't be determined

10. Eleven years earlier the average age of a family of 4 members was 28 years. Now the age of the same family with six members is yet the same, even when 2 children were born in this period. If they belong to the same parents and the age of the first child at the time of the birth of the younger child was same as there were total family members just after the birth of the youngest members of this family, then the present age of the youngest member of the family is :
(a) 3 years (b) 5 years
(c) 6 years (d) none of these

11. Mr. Patel walked 6 km to reach the station from his house then he boarded a train whose average speed was 60 km/hr and thus he reached his destination. In this way he took total of 3 hours. If the average speed of the entire journey was 30 km/hr then the average speed of walking is :
(a) 3 km/hr (b) 4.5 km/hr
(c) 4 km/hr (d) none of these

Q. Directions for question numbers 12 and 13: During the winter session all the women which constitutes $\frac{1}{5}$ th strength of the house left the house (i.e., parliament) due to the

rejection of their demand. Actually they were asking for the 50% reservation of seats for the women in the Lok Sabha. Thus the average age of the remaining members of the house (i.e., the Lok Sabha) increases by $\frac{1}{4}$ th than it was earlier when all the members (i.e., men and women) were present.

- Averages

(b) 2
(d) data insufficient

(a) 1
(c) 3

21. The average age of all the 20 students of a class is 24. The minimum age of a student is 18 and the maximum age of another student in the same class is 30 years. When the two students whose average age was 26 years resticated from the class but later on one of the resticated student was readmitted. Now the average age of the class is :
(a) 23.89 years
(b) 28.39 years
(c) 25 years
(d) can't be determined

22. In a set of prime and composite numbers, the composite numbers are twice the number of prime numbers and the average of all the numbers of the set is 9. If the number of prime numbers and composite numbers are exchanged then the average of the set of numbers is increased by 2. If during the exchange of the numbers the average of the prime numbers and composite numbers individually remained constant, then the ratio of the average of composite numbers to the average of prime numbers (initially) was :
(a) $\frac{7}{13}$
(b) $\frac{13}{7}$
(c) $9/11$
(d) none of these

23. The total age of all the guests in the party was 540 years. If a South Indian couple (guests) left the party, then the average of the remaining guests still remained unchanged, where the age of both the husband and wife (the South Indian couple) was same, then the average age of this couple and the total number of guests in the party, respectively, can be :
(a) 18, 27
(b) 20, 27
(c) 15, 38
(d) can't be determined

24. In the command hospital Lucknow the sum of the ages of all the 29 people i. e., physicians, surgeons and nurses is 696. If the age of each physician, each surgeon and each nurse be 1 year, 6 years and 3 years more, then the average age of the whole staff would have been 3 years more. If the number of surgeon is a square root of a two digit number which is also a perfect cube, then the number of nurses in the hospital is :
(a) 12
(b) 15
(c) 16
(d) none of these

25. The average expenditure of Sarvesh for the January to June is Rs. 4200 and he spends Rs. 1200 in January and Rs. 1500 in July. The average expenditure for the months of February to July is :
(a) 4250
(b) 4520
(c) 4060
(d) none of these

26. The average marks of Sameer decreased by 1, when he replaced the subject in which he has scored 40 marks by the other two subjects in which he has just scored 23 and 25 marks respectively. later he has also included 57 marks of Computer Science, then the average marks increased by 1. How many subjects were there initially?
(a) 6
(b) 12
(c) 15
(d) can't be determined

27. In a combined family the average age of 4 males and 7 females is 42 and 20 years respectively. If two persons whose average age is 13 years have left the family and other three

Averages

ages
people joined the family whose respective ages are 11, 15 and 28 years, then the average age of the new family is increased

Directions for question numbers 40, 41, 42, 43 and 44

44: Bhartiya Idol is a talent search programme launched and run by TV Tarana. In this programme each participating candidate has to appear for the audition in such a manner that a candidate will play the Antakchhary with its aspiring opponent until he fails, otherwise he can continue and become winner. If a candidate fails on his part, he will have to leave

the contest and in place of it another candidate will start off with the same existing opponent. The number of points a candidate scores is equal to the number of times he responds correctly to his opponents. The candidate who finishes the game (or who wins over the last opponent) will be declared as a winner, even if he/she has scored less points and started as the last participant. The points scored individually by all the 10 candidates are as shown below. Besides if a person loses the contest only if he/she responds incorrectly.

Participant	Points
1. Rajesh	8
2. Radhe	10
3. Harish	7
4. Prajakta	12
5. Aditya	25
6. Rahul	24
7. Shekhar	18
8. Amit Tandon	23
9. Amit Sana	29
10. Abhijeet	30

40. The average score of all the 10 participants is :
 (a) 21.5 (b) 18.6
 (c) 18.3 (d) none of these
41. A candidate can face maximum 'n' opponents. The value of 'n' is :
 (a) 3 (b) 4
 (c) 1 (d) can't be determined
42. The average score of five participants who started earlier :
 (a) 23 (b) 12
 (c) 19.5 (d) can't be determined
43. If Abhijeet has not started off earlier and he is the winner then the minimum number of participants who has lost the game before he has started off :
 (a) 4 (b) 6
 (c) 5 (d) none of these
44. For the above question the maximum possible average of the opponents of Abhijeet could be :
 (a) 13.5 (b) 9.25
 (c) 13.0 (d) 17.5
45. In an NGO, the daily average wages of 20 illiterate employees is decreased from Rs. 25 to Rs. 10, thus the average salary of

- all the literate educated and illiterate employees is decreased by Rs. 10 per day. The number of educated employees working in the NGO is :
 (b) 20
 (a) 15 (d) data insufficient
 (c) 10
46. Mr. Tyagi while going from Meerut to Saharanpur covered half the distance by train at the speed of 96 km/hr then he covered half of the rest distance by his scooter at the speed of 60 km/hr and finally he covered the rest distance at the speed of 40 km/hr by car. The average speed at which Mr. Tyagi completed his journey is :
 (a) 64 km/hr (b) 56 km/hr
 (c) 60 km/hr (d) 36 km/hr
47. There are four types of candidates in our coaching preparing for the CAT. The number of students of Engineering, Science, Commerce and Humanities is 40, 60, 50 and 30 respectively and the respective percentage of students who qualified the CAT is 80%, 75%, 60% and 50% respectively the overall percentage of successful candidates in our institute is :
 (a) 67.77% (b) 66.66%
 (c) 68.5% (d) none of these
48. Mr. Manmohan calculated the average of 10, 'three digit numbers'. But due to mistake he reversed the digits of a number and thus his average increased by 19.8. The difference between the unit digit and hundreds digit of that number is :
 (a) 8 (b) 4
 (c) 2 (d) can't be determined
49. Once my peon went to the office of SKYLINE COURIER with 4 different envelopes. The clerk in the office measured the weights in all possible pairs. The weights obtained are 59 gm, 61 gm, 62 gm, 63 gm, 64 gm and 66 gm. The weight of the heaviest envelope is :
 (a) 35 gm (b) 36 gm
 (c) 34 gm (d) can't be determined
50. The average expenditure of the hotel when there are 10 guests is Rs. 60 per guests and the average expenditure is Rs. 40 when there are 20 guests. If it is known that there are some fixed expenses irrespective of the number of guests then the average expenditure per guest when there are 40 guests in the hotel :
 (a) Rs. 30 (b) Rs. 25
 (c) Rs. 20 (d) can't be determined



Final Round

Directions for question numbers 1 to 15: There are 3 sets of natural numbers 1 to 100. Set A contains all the natural numbers which are prime, upto 100. Set B contains all the non-prime even natural numbers upto 100. Set C contains all the non-prime odd natural numbers upto 100 i.e.,
 A = {2, 3, 5, 7, 11, ..., 89, 97}
 B = {4, 6, 8, 10, 12, ..., 98, 100}
 C = {1, 9, 15, 21, 25, 27, 33, ..., 95, 99}

1. The average of all the elements of A, B and C is :
 (a) 49.50 (b) 50.50
 (c) 55 (d) none of these

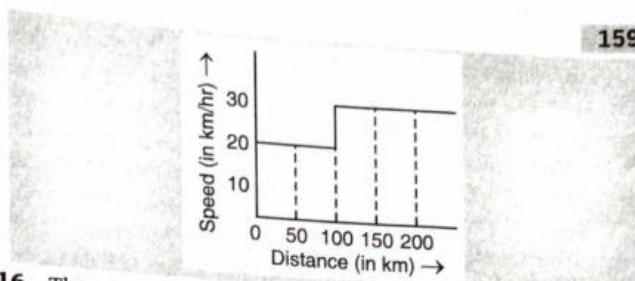
2. The average of all the elements of B is :
 (a) 52 (b) 48
 (c) 49 (d) none of these
3. If the average (factually correct) of the set A is 42.46, then the average of the Set C is :
 (a) 52 (b) 49.87
 (c) 55.46 (d) can't be determined
4. The average of the elements of the Set A and C combined is :
 (a) 49.0588 (b) 49.0372
 (c) 50 (d) none of these

5. If an element less than 50 belongs to Set A is transferred to Set B, then the average of Set B :
- increases
 - decreases
 - remains constant
 - can't say
6. If any two elements, greater than 50, belong to Set A are transferred to Set C, then the average of Set C :
- remains constant
 - decreases
 - increases
 - can't say
7. Any 10 elements of the Set A are transferred to the Set B, then the average of Set B :
- increases
 - decreases
 - remains constant
 - can't say
8. If a least and a greatest element of Set C are transferred from Set C to Set A then the average of Set A :
- increases
 - decreases
 - remains constant
 - can't be determined
9. If a smallest and a greatest element of the Set B is transferred to Set A, then the averages of A, B, C respectively :
- decreases, decreases, increases
 - decreases, constant and increased
 - increases, constant, constant
 - can't be determined
10. If an element 2 is also included in the Set B, then the average of B :
- decreases by 3
 - increases by 2
 - decreases by 1
 - can't be determined
11. The average of all the perfect squares of the Set C is :
- 35
 - 33
 - 30.5
 - none of these

Directions for question number 12, 13, 14: 5 elements below 25 from the Set A are transferred to Set B, and 10 elements lying between 25 and 50 from the Set B are transferred to Set C and 15 elements above 50 from the Set C are transferred to Set A.

12. The overall average of all the elements of A, B and C is :
- 39.8
 - 50.50
 - 71.2
 - can't be determined
13. The maximum increase in the average of set :
- A
 - B
 - C
 - can't say
14. When we intended to minimise the loss in average of Set B then the new average of Set B is :
- 54.4772
 - 45.74
 - 51.84
 - can't be determined
15. If 10-10 elements are transferred from Set A to Set B, then Set B to Set C and then from Set C to Set A but the received elements can not be transferred to the next set. e.g. the elements obtained from Set A can not be transferred to C through Set B. The average of which Set is maximum :
- A
 - B
 - C
 - can't say

Directions for question numbers 16 and 17: In the following graph the relation between speed and distance is given :



16. The average speed for the first 200 kms. (in km/hr)
- 24
 - 25
 - 26
 - 40
17. The average speed for the first 150 km :
- 15
 - 18
 - 25
 - none of these
18. Abhay working in Tele Bharti as a salesperson. His monthly salary is just Rs. 200. But he gets the bonus as per the given rule. If he sells the simcards of Rs. x then his bonus will be Rs. $\left[\left(\frac{x}{100} \right)^2 + 10 \right]$. In the first quarter of the year his average sale was Rs. 3000 per month. In the next 5 months his average sale was Rs. 5000 per month. In the last four months his average sale was Rs. 8000 per month. What is the average earning per month for the whole year?
- Rs. 3350
 - Rs. 3610
 - Rs. 3560
 - none of these
19. The price of Shirts at Sahara Ganj is defined as Rs. $(100 + 10x^2)$, where x is the number of shirts. Mallika purchased 5 shirts from the above shop. The average price of a shirt is :
- Rs. 70
 - Rs. 50
 - Rs. 75
 - none of these
20. In the above question if the sister of Mallika purchased k shirts but the average price was same as that of Mallika. The value of k could be :
- 6
 - 4
 - 2
 - none of these
21. There are 10 compartments in passenger train which carries on an average 20 passengers per compartment. If atleast 12 passengers were sitting in each compartment and no any compartment has equal number of passengers then maximum how many passengers can be accomodated in any compartment :
- 64
 - 45
 - 56
 - none of these
22. There are twice the number of two wheelers as there are three wheelers and the number of 4 wheelers are equal to the number of two wheelers. The average number of wheel per vehicle is :
- 3
 - 4
 - 5
 - none of these
23. Sone Lal has ' n ' magical eggs whose average weight is ' k ' gm. Each of the ' n ' eggs produces ' n ' eggs next day such that the average weight of ' n ' eggs produced is same as that of the parental (previous generation) egg for each ' n ' groups individually i.e., each egg produces ' n ' eggs in its next generation and the average weight of all the ' n ' eggs of next generation is same as the weight of the mother egg. This process is continued without any change in pattern. What is the total weight of all the eggs of r^{th} generation, where the initial number of eggs with Sone Lal are considered as the eggs of first generation :
- rnk
 - n^rk
 - nk^r
 - $n^{r+1}k$

Directions for question numbers 24 to 27 : A CAT training institute was established on January 1, 2004 with 3, 4, 5 and 6 faculty members in the Logical Reasoning (LR), Data Interpretation (DI), English Language and Quantitative Analysis (QA) areas respectively, to start with. No faculty member retired or joined the institute in the first three months, of the year 2004. In the next four years, the institute recruited one faculty member in each of the four areas. All these new faculty members, who joined the institute subsequently over the years were 25 years old at the time of their joining the institute. All of them joined the institute on April 1. During these four years, one of the faculty members retired at the age of 60. The following diagram gives the area-wise average age (in terms of number of completed years) of faculty members as on April 1 of 2004, 2005, 2006 and 2007.

Faculty	2004	2005	2006	2007
LR	49.33	44	45	46
DI	50.5	51.5	52.5	47.8
English	50.2	49	45	46
Quants	45	43	44	45

24. In which year did the new faculty member join as the faculty of English?
(a) 2004 (b) 2005
(c) 2006 (d) 2007

25. What was the age of the new faculty member, who joined the faculty of QA, as on April 1, 2007?
(a) 25 (b) 26
(c) 27 (d) 28

26. From which area did the faculty member retire?
(a) English (b) LR
(c) DI (d) Quants

27. Professors Sarvesh and Manish, two faculty members in the LR area, who have been with the Institute since its inception, share a birthday, which falls on 30th November. One was born in 1951 and the other one in 1954. On April 1, 2009 what was the age of the third faculty member, who has been in the same area since inception?
(a) 47 (b) 50
(c) 51 (d) 52

Answers

LEVEL-1

1. (c)	2. (b)	3. (a)	4. (b)	5. (b)	6. (a)	7. (a)	8. (c)	9. (c)	10. (b)
11. (a)	12. (a)	13. (b)	14. (a)	15. (c)	16. (c)	17. (c)	18. (a)	19. (b)	20. (b)
21. (d)	22. (c)	23. (d)	24. (b)	25. (c)	26. (a)	27. (b)	28. (b)	29. (a)	30. (b)
31. (c)	32. (a)	33. (b)	34. (b)	35. (b)	36. (a)	37. (a)	38. (b)	39. (d)	40. (a)
41. (a)	42. (c)	43. (b)	44. (a)	45. (d)	46. (a)	47. (a)	48. (a)	49. (b)	50. (c)
51. (b)	52. (b)	53. (d)	54. (d)	55. (d)	56. (a)	57. (c)	58. (a)	59. (b)	60. (a)

LEVEL-2

1. (a)	2. (c)	3. (b)	4. (b)	5. (b)	6. (c)	7. (d)	8. (b)	9. (d)	10. (a)
11. (c)	12. (d)	13. (d)	14. (a)	15. (c)	16. (b)	17. (a)	18. (d)	19. (a)	20. (b)
21. (d)	22. (a)	23. (b)	24. (d)	25. (a)	26. (c)	27. (d)	28. (d)	29. (d)	30. (a)
31. (b)	32. (a)	33. (a)	34. (a)	35. (c)	36. (b)	37. (a)	38. (a)	39. (b)	40. (b)
41. (b)	42. (d)	43. (c)	44. (a)	45. (c)	46. (a)	47. (a)	48. (c)	49. (c)	50. (a)

FINAL ROUND

1. (b)	2. (a)	3. (c)	4. (a)	5. (b)	6. (c)	7. (d)	8. (a)	9. (c)	10. (c)
11. (b)	12. (b)	13. (a)	14. (c)	15. (d)	16. (a)	17. (d)	18. (b)	19. (a)	20. (c)
21. (c)	22. (a)	23. (b)	24. (c)	25. (c)	26. (a)	27. (d)			


LEVEL (1)

1. Initially the total weight = $20 \times 45 = 900$

Now, the total weight when a student has been replaced = $20 \times 44 = 880$

It means the weight of the new student is 20 kgs less than the replaced student. Hence the weight of the replaced student = $40 + 20 = 60$ kgs

Alternatively: Since you know that there are total 20 students and when their average weight decreases by 1, it means on an average 1 kg weight is reduced from each of the students. Thus there is 20 kg weight loss, in total. Again this happens due to the student whose weight is 40 kg replaces an old student. Thus we can say that the weight of the old (or existing) student of the class was 60 kg, which is reduced by a 40 kg student.

2. The only useful data is that the average length of the thumb and index finger is 2.8 and the length of index finger is 3 inches.

Now the total length of index finger and thumb = $2.8 \times 2 = 5.6$ inches

Thus the length of thumb is $(5.6 - 3) = 2.6$ inches.

3. The average price of the new mixture

$$= \frac{(38 \times 8) + (43 \times 4) + (49 \times 3)}{15} = 41.53$$

4. Average speed when Pankaj was returning

$$= \frac{2 \times 10 \times 30}{40} = 15 \text{ km/hr}$$

Now the average speed of the whole journey

$$= \frac{2 \times 15 \times 60}{75} = 24 \text{ km/hr}$$

5. Use unitary method :

In $\frac{1}{15}$ hours, 123 typists can type 984 papers

In 1 minute, 123 typists can type $\frac{984}{4} = 246$ papers

In 1 minute, 1 typist can type $\frac{246}{123} = 2$ papers

(Hint : $\frac{1}{15}$ hours = 4 minutes)

6. Average cost = $\frac{20 \times 3x + 15 \times 2x + 18 \times 4x}{9x} = 18$

7. Earning in the 8 months = $600 \times 8 = 4800$

Earning in the 4 months = $\left(600 \times \frac{3}{2}\right) \times 4 = 3600$

Total earning = Rs. 8400

Saving in 8 months = $400 \times 8 = 3200$

Saving in 4 months = $\left(400 \times \frac{5}{4}\right) \times 4 = 2000$

Total savings = 5200

Total expenditure for 12 months

$$= 8400 - 5200 = 3200$$

Therefore average saving per month = $\frac{3200}{12} = 266.66$

8. There are 6 prime numbers between 47 and 74 as given below:

53, 59, 61, 67, 71, 73

There are only two combinations whose average is $\frac{191}{3}$ whose total sum is 191.

These are, {53, 67, 71} and {59, 61, 71}

We observe that in the first set the difference between greatest and smallest prime number is 18. Hence choice (c) is correct.

9. Go back to the fundamentals and see the property number 4 and 2 of averages.

Hence required average = $11 \times 5 + 5 = 60$

10. The average score after 48th innings = 48

The average score after 49th innings = 49

$$\text{Since } \frac{(48 \times 48 + 97)}{49} = 49$$

Now the requirement of runs = $49 + (50 \times 2) = 149$

$$\text{or } (50 \times 51) - (49 \times 49) = 149$$

- 11.

	Average	Total
$S + G$	→ 35	70
$K + G$	→ 32	64
$S + K$	→ 38	76
$S + K + G$	→ 35	105
$D + I$	→ $\frac{35}{2}$	35
	2	

$$\therefore S + K + G + D + I = \frac{105 + 35}{5} = 28$$

- 12.

Kaif → 35

Sachin → 41

Ganguli → 29

Dhoni < 35 and Irfan < 35

Hence Sachin is the senior person

13. It is the same as a person with 20 years more age replaces an existing person of the group (or village)

Since the total age of the village having n persons, is being increased by 20 years and the average age of village is being increased by 1 year, hence there are total 20 people in the village.

Alternatively : $(n \times 42) + 20 = (n \times 43)$
 $\Rightarrow n = 20$

14. Average rainfall = $\frac{2 \times 6 + 4 \times 5 + 4 \times 10 + 2 \times 6}{12} = 7$

15. $400 \times 7 = (300 \times 3) + (250 \times 2) + (n \times 2) \Rightarrow 700$

16. The new player must be 11 kg heavier than the replaced one. Hence the weight of the new person = $55 + 11 = 66$ kg.

Averages	No. of family members	Average age	Total age
17.	6	25	150
4 years ago	6	29	174

But the no. of family members (presently) = 7
and average age (presently) = 25

Therefore the total age = $25 \times 7 = 175$

Hence, the age of child = $175 - 174 = 1$ year

18. Amitabh's total expenditure for Jan-June = $4200 \times 6 = 25200$
Expenditure for February-June = $25200 - 1200 = 24000$
Expenditure for the months of February-July

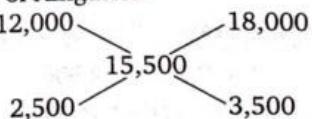
$$= 24,000 + 1500 = 25,500$$

The average expenditure = $\frac{25,500}{6} = 4250$

19. Total value of $a + 11 + 23 + 17 = 60 \Rightarrow a = 9$
again $a + b + 12 + 25 = 64 \Rightarrow b = 18$

therefore $\frac{a}{b} = \frac{1}{2}$

20. By the method of Alligation



Therefore the ratio of number of Jun. level employee to the senior level employee = 5 : 7

Hence the required fraction = $\frac{5}{(5+7)} = \frac{5}{12}$

Alternatively: Go through options

$$\frac{J}{J+S} = \frac{5x}{12x}$$

$$\Rightarrow \frac{J}{S} = \frac{5x}{7x}$$

Therefore $15,500 \times 12x = 12,000 \times 5x + 18,000 \times 7x$

L.H.S. = R.H.S., Hence correct

21. None of a, b, c is correct.

The correct answer can be $\frac{a+b+c+d+e}{5}$

or $\frac{a+e}{2} = \frac{b+d}{2} = c$

where a, b, c, d, e are consecutive odd numbers.

22. $31 \times 31 - 30 \times 30 = 61$ years

23. $a+b+c+d+e = 5 \times 92 = 460$

$$a+b+c = 3 \times 83 = 249$$

$$c+d+e = 3 \times 97 = 291$$

$$\therefore c = (a+b+c) + (c+d+e) - (a+b+c+d+e)$$

$$\text{or } c = 540 - 460 \text{ or } c = 80$$

24. $19 \times 26 - 18 \times 25 = 44$ years

25. $k+k+2+k+4+k+6+k+8+k+10+k+12 = k+6$

$$26. \frac{x}{4} \times \frac{85}{100} + \frac{x}{3} \times \frac{70}{100} + \frac{5x}{12} \times \frac{56}{100} \times 100 = 67.916$$

Here, x is the total number of students in the whole class.

Compiled By Jasjeet

27. Since there are five possibilities or combinations (as ${}^5C_4 = 5$) therefore average = $\frac{5 \times 600}{4} = 750$ mm

$$28. \frac{\frac{23}{5} + \frac{8}{3} + \frac{62}{9} + \frac{112}{15} + \frac{32}{9}}{5} = 5 \frac{8}{225}$$

$$29. \frac{1000.0001 + 100.001 + 10.01 + 1.1}{4} = \frac{1111.1111}{4} = 277.777$$

$$30. \frac{7+8+9+10+11+12+13}{7} = 10$$

$$\Rightarrow \frac{7+13}{2} = 10$$

$$31. \left(\frac{1+2+3+\dots+100}{100} \right) = \left(\frac{\frac{100 \times 101}{2}}{100} \right) = 50.50$$

$$32. \left(\frac{1+3+5+7+\dots+99}{50} \right) = \left(\frac{\frac{50 \times 50}{2}}{50} \right) = 50$$

$$33. \frac{2+4+6+\dots+198}{99} = \frac{99 \times 100}{99} = 100$$

$$34. a+b+c = 237$$

$$a+c = 158$$

$$\Rightarrow b = 79$$

$$35. \frac{M+A+S}{3} = 11111$$

$$\text{also } \frac{S+M}{2} = 11111 \quad (\because M - 11111 = 11111 - S)$$

$$\Rightarrow A = 11111$$

$$36. \frac{2 \times 200 \times 600}{800} = 300 \text{ km/hr}$$

$$37. \frac{A+B+C+D}{4} = \frac{400 \times 2 + 600 \times 2}{4} = 500$$

$$38. \text{Average weight} = \frac{20+50}{100} = 0.7 \text{ kg} \quad (\because 1 \text{ kg} = 1000 \text{ gm})$$

$$39. \text{Average} = \frac{770}{11} = 70$$

$$40. \frac{3x+4x+5x}{3} = 12 \Rightarrow x = 3$$

∴ The ages are 9, 12 and 15 years

and the required average = $\frac{9+15}{2} = 12$

41. Since the salary of each employee is being increased therefore the average salary will also increased by Rs. 2000. Thus the required average = 22,000.

$$42. 11 \times 21 - 10 \times 20 = 31$$

$$43. 20 \times 180 - 19 \times 178 = 218$$

44. Let the price of A > B > C

$$\text{then } \frac{A+B}{2} = 2C$$

$$A+B = 4C$$

$$A+B+C = 5 \times 3 = 15 \text{ crore}$$

$$\text{Now, } 5C = 15 \text{ crore}$$

$$\Rightarrow C = 3 \text{ crore}$$

45. Since we don't know about the price of B.

164

46. $\left(\frac{40 + 70 + 142}{50 + 100 + 150} \right) \times 100 = 84\% \quad \text{Averages} \quad \text{and } (k+3)$

47. Since 1 is neither prime nor composite number. Thus there are only 99 numbers viz. 2, 3, 4, 5, 6, ... 99, 100.

Hence $\left(\frac{2+3+4+5+6+\dots+100}{99} \right)$
 $= \left(\frac{(1+2+3+4+\dots+100)-1}{99} \right)$
 $= \frac{5050-1}{99} = \frac{5049}{99} = 51$

48. $\frac{1+4+9+16+\dots+100}{10}$

$$= \left(\frac{10 \times 11 \times 21}{6 \times 10} \right)$$

$$\left[\because 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \right]$$

$$= 38.5$$

49. The non-negative integers upto 99 are 0, 1, 2, 3, ... 99.

Therefore average = $\left(\frac{0+1+2+3+\dots+99}{100} \right)$
 $= \left(\frac{99 \times \frac{100}{2}}{100} \right) \quad \left[\because 1+2+3+\dots+n = \frac{n(n+1)}{2} \right]$
 $= 49.5$
50. $\frac{7+14+21+\dots+77}{11} = \frac{7(1+2+\dots+11)}{11}$
 $= \frac{7 \times 11 \times 12}{11 \times 2} = 42 \quad \left[\because 1+2+3+\dots+n = \frac{n(n+1)}{2} \right]$

Alternatively: Since all the numbers are in A.P. Further there are odd number of numbers (i.e., 11) in the sequence. Thus the middle most term is the average of the sequence, which is 42.

	Average	Total
A, B, C, D	40	160
A, B, C, D, E	41	205
F, B, C, D, E	42	210

Thus $E = 205 - 160 = 45$

Hence $F, B, C, D = 210 - 45 = 165$

Therefore average of $F, B, C, D = 41.25$

52. Three consecutive natural numbers whose average is k are $(k-1), (k), (k+1)$

The next two numbers will be $(k+2)$ and $(k+3)$

LEVEL (2)

1. For this type of questions take the LCM of speeds and assume the LCM as the distance

then the time taken @ speed of 60 km/hr = $\frac{300}{60} = 5 \text{ hrs}$

again the time taken @ speed of 50 km/hr = $\frac{300}{50} = 6 \text{ hrs}$

Therefore average of $(k-1), (k), (k+1), (k+2), (k+3)$ is $(k+1)$

Alternatively: Consider any 3 consecutive natural numbers e.g. 4, 5, 6 the average is 5

Again 4, 5, 6, 7, 8 the average is 6.

To verify it consider some different numbers e.g. 10, 11, 12, 13, 14 average = 12

Hence, proved that the average is increased by 1.

53. The 5 consecutive odd numbers whose average is k are $(k-4), (k-2), k, (k+2), (k+4)$

Again the average of $(k-4), (k-2), (k), (k+2), (k+4)$, $(k+6), (k+8)$ is $(k+2)$

Alternatively: Consider some appropriate numbers.

54. The increase in weight = $(5 \times 42.5) - (5 \times 42) = 2.5 \text{ kg}$
But we don't know the weight of the replaced officer. So we can't determine.

55. The decrease in age = $6 \times (28 - 27) = 6 \text{ years}$
But we don't know the age of the old servant which is being replaced. So we cannot determine the average age of new servant.

56. The age of new servant = $31 - 6 = 25 \text{ years}$

57. $A + B + C = 12,000 \times 3$

$B + C + D = 15,000 \times 3$

$\Rightarrow D - A = 3000 \times 3$

$D - A = 9000$

also $D = 2A$

$\Rightarrow D = 18,000 \text{ and } A = 9000$

Therefore average salary of B and $C = \frac{(45,000 - 18,000)}{2} = 13,500$

58. The price of the costliest and cheapest computer

$= (80 \times 30,000) - (78 \times 29,500) = 99,000$

Therefore the price of the cheapest computer

$= 99,000 - 80,000 = 19,000$

59. $\frac{10 \times 50 + 50 \times 10 + 20 \times 25 + 25 \times 20}{4} = 500$

	Auto Rickshaw	Maxi. cab	Mini. Bus
No. of seats	4	10	20
No. of seats occupied	4	8	15
Rate per seat	12	15	8
Total amount (in Rs.)	48	120	120

Therefore average earning = $\frac{48 + 120 + 120}{3} = 96$

Thus we see that in place of 5 hrs train takes 6 hours. It means the train takes 1 hour extra and this one hour is stopping period in the total time of 6 hours. Thus in 6 hour train halts for 1 hour. So in 1 hour train will stop for $\frac{1}{6}$ hours or 10 minutes.

Alternatively : (short cut) yoursmahboob.wordpress.com

$$\text{Halting (or stopping) time} = \left(1 - \frac{\text{slower speed}}{\text{faster speed}}\right) \text{ hours}$$

$$= 1 - \frac{50}{60} = \frac{10}{60} = \frac{1}{6} \text{ hours}$$

$$= 10 \text{ minutes}$$

($\because 1 \text{ hour} = 60 \text{ minutes}$)

$$\frac{[r + (p - q)] + p + q}{3} = 21 + \frac{p + q + r}{3}$$

$$\frac{2p + r}{3} - 21 = \frac{p + q + r}{3}$$

$$\frac{p - q}{3} = 21$$

$$p - q = 63$$

3. Let the speed for the first hour be $x \text{ km/hr}$

then the speed for the second hour be $\frac{7}{5}x \text{ km/hr}$

then the speed for the third hour be $\frac{10}{7} \times \frac{7}{5}x = 2x \text{ km/hr}$

then the speed for the fourth hour be $2x \times \frac{7}{5} = \frac{14x}{5} \text{ km/hr}$

$$\text{Therefore total distance in four hours} = x + \frac{7}{5}x + 2x + \frac{14x}{5}$$

$$= \frac{36x}{5} \text{ km}$$

$$\therefore \text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{\left(\frac{36x}{5}\right)}{4} = \frac{9x}{5} \text{ km/hr}$$

Again the distance in 4 hours @ speed of $x \text{ km/hr}$

which is half of the third hour's speed is $4x \text{ km}$

$$\text{Hence } \frac{36x}{5} - 4x = 160 \text{ km}$$

$$\Rightarrow x = 50$$

$$\text{Hence, the average speed} = \frac{9 \times 50}{5} = 90 \text{ km/hr}$$

4. The total score of 3 toppers = $123 \times 120 - 120 \times 118.5 = 540$

The highest possible score of the third highest topper is possible when the score of other two toppers was minimum

So,

$$\begin{aligned} 1^{\text{st}} \text{ rankers score} &= 187 \text{ (minimum)} \\ 2^{\text{nd}} \text{ rankers score} &= 186 \text{ (minimum)} \\ 3^{\text{rd}} \text{ rankers score} &= 167 \text{ (maximum)} \end{aligned} \quad \left. \begin{aligned} & \\ & \\ & \end{aligned} \right\} 540$$

5. Go through options : $40 \times 35 + 60 \times 25 = 29 \times 100$

Since there are 40 ladies and 60 gents

$$\text{Alternatively: } 40 \times 7x + 60 \times 5x = 29 \times 100$$

$$\Rightarrow x = 5 \quad \therefore 7x = 35$$

	Lok Sabha	Rajya Sabha
No. of MPs	300	200
Ave. Age	40	50

Since when a member of Rajya Sabha joins the Lok Sabha and the average age of both the houses increases, it means the average age of this member must lie between 40 and 50. When the age of this member is greater than 40, then the average age of the Lok Sabha increases.

Again when the age of this member is less than 50, then after leaving it, the average age of the Rajya Sabha increases.

4. Solutions for question numbers 7, 8 and 9:

	No. of family members	Average	Total
8 years ago \rightarrow	5	36	180
Presently \rightarrow	(if) 5	$(36 + 8) = 44$	220
	7	36	252

7. From the above explanation we have no any clue about his wife's age.

8. Since we know that the difference between the age of any two persons remains always constant, while the ratio of their ages gets changed as the time changes.

So, if the age of his child be x (presently)

Then the age of wife be $x + 26$ (presently)

Thus, the total age = $x + (x + 26) = 32$ [$\because 252 - 220 = 32$]

$$\Rightarrow x = 3$$

Therefore the age of her child is 3 years and her self is 29 years. Hence her age at the time of the birth of her child was 26 years.

Alternatively: As we have mentioned above that the age difference remains always constant. Therefore her age at the time of her child's birth was 26 years.

9. Since there is no clue. So we can't determine.

	No. of family members	Average	Total
Eleven years earlier	4	28	112
Presently	if 4 6	39 28	156 168

Since it is obvious that just after the birth of the youngest member (i.e., child) was 6 family members in the family. Therefore at the time of the birth of the youngest child the elder child's age was 6 years.

Now the sum of their ages = $x + (x + 6) = 12 = (168 - 156)$

$$\Rightarrow x = 3 \quad \text{and} \quad (x + 3) = 9$$

11. Go through options

$$\frac{6}{4} = 1.5 \text{ hour, when he was walking}$$

Therefore time of journey by train = $3 - 1.5 = 1.5$

Now, the distance travelled by train = $1.5 \times 60 = 90$ km

Hence correct.

Alternatively: Total distance = $32 \times 3 = 6 + 60 \times x$

$$\Rightarrow x = 1.5 \text{ hours}$$

Thus, the speed of walking = $\frac{6}{1.5} = 4 \text{ km/hr}$

12. Let the number of total MPs = n and their average age be x then

$$n \times x = \frac{4}{5} n \times \frac{5}{4} x + \frac{n}{5} \times y$$

$$\Rightarrow \text{either } n = 0 \text{ or } y = 0$$

[Since there are only 80% MPs remained in the house which

is equal to $\frac{4}{5}n$ and the increase in average age = $20\% = \frac{5}{4}x$

Thus, there cannot be any possible value of n .

13. $nx = \frac{4}{5}n \times \frac{5}{4}x + \frac{n}{5} \times 30$

$$\Rightarrow n = 0 \quad \text{which is impossible}$$

So, there is no any woman MP in the Lok Sabha

14. Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{6 \times 20}{9} = 13.33 \text{ m/s}$$



Hexagonal path has six sides

15. Average of 26, 29, 35 and 43 is 33.25. Also the average of 26, 29, n , 35 and 43 lies between 25 and 35 i.e.,

$$25 < \frac{26 + 29 + n + 35 + 43}{5} < 35$$

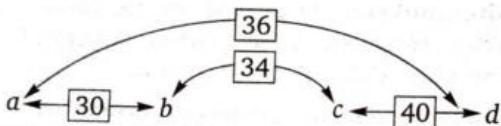
$$\Rightarrow 125 < 26 + 29 + n + 35 + 43 < 175$$

$$\Rightarrow 125 < 133 + n < 175$$

$$\Rightarrow n < 42$$

Since the value of n is an integer and greater than 33.25 then $33 < n < 42$; for every integer n .

16. Given that $a < b < c < d$



The only possible prime number between 30 and 34 is 31

Hence $b = 31$, therefore $a = 29$

Similarly $c = 37$ and $d = 43$

Therefore $d - a = 43 - 29 = 14$

17. $9261000 = 2^3 \times 3^3 \times 5^3 \times 7^3$

$$= (2 \times 3 \times 5) \times (2 \times 3 \times 7) \times (2 \times 5 \times 7) \times (3 \times 5 \times 7)$$

$$= 30 \times 42 \times 70 \times 105$$

Therefore the average of 30, 42, 70 and 105 is 61.75.

18. Since the number is as below 270 as its multiple is as above 270. It means these two numbers are equidistant from 270. Hence their average is 270.

Alternatively : Let the number be x then.

$$270 - x = 8x - 270$$

$$\Rightarrow x = 60 \quad \text{and} \quad 8x = 480$$

Therefore the average of 60 and 480 is 270.

19. The average of 60 and 270 is 165

20. Number of days in a week = 7

Average number of visitors = 40

Total visitors = 280 ($= 7 \times 40$)

Now, if n be the number of holidays in a week then

$$(7 - n) \times 56 = 280 \quad (40 + 16 = 56)$$

$$\Rightarrow n = 2$$

The rest data is redundant or useless, since our problem is solved here without using it.

Alternatively : Go through options

$$40 \times 7 = 280 = 56 \times 5 \text{ hence proved.}$$

21. Since we don't know their ages individually so we cannot calculate the average of the class when a student of unknown age readmitted in the class.

22. Let the average of prime numbers be P and average of composite numbers be C . Again the number of prime numbers be x , then the number of composite numbers be $2x$.

$$\text{Then} \quad \frac{Px + 2Cx}{3x} = 9 \Rightarrow P + 2C = 27$$

$$\text{and} \quad \frac{2Px + Cx}{3x} = 11$$

$$\Rightarrow 2P + C = 33$$

$$\text{on adding eq. (1) and (2)}$$

$$\text{we get} \quad P + C = 20$$

$$\text{and on subtracting eq. (1) from (2)}$$

$$\text{we get} \quad P - C = 6$$

$$\text{Therefore} \quad P = 13 \quad \text{and} \quad C = 7$$

$$\text{Thus} \quad \frac{C}{P} = \frac{7}{13}$$

23. Solve through option $\frac{540}{27} = 20$

$$\frac{540 - (2 \times 20)}{25} = \frac{500}{25} = 20$$

Hence option (b) is correct.

24. Number of members in the staff \times average age = Total age

$$29 \times 24 = 696$$

$$29 \times 27 = 783$$

hence change in total age = 87

	Physician	Surgeon	Nurse
No. of members	$(21 - x)$	8	x
Increase in average age	1	6	3
Increase in total age	$(21 - x)$	48	$3x$

Since $P + S + N = 29$

therefore if there would be x nurses then there must be $(21 - x)$ physicians.

Again total change (or increase) in age

$$= (21 - x) + 48 + 3x = 87$$

$$\Rightarrow x = 9$$

25. Total exp. Jan-June = $4200 \times 6 = \text{Rs. 25200}$

Total exp. Feb-June = $25200 - 1200 = \text{Rs. 24000}$

Total exp. Feb-July = $24000 + 1500 = \text{Rs. 25500}$

The average expenditure Feb-July = $\frac{25500}{6} = 4250$

26. Let the number of subjects be n and average marks be x , then total marks = nx

$$\text{Again} \quad (n + 1)(x - 1) = (nx - 40) + (23 + 25)$$

$$\Rightarrow x - n = 9$$

$$\text{Further} \quad (n + 2)(x + 1) = (nx - 40) + (23 + 25) + 57$$

$$\Rightarrow nx + 2x + n + 2 = nx + 65$$

$$\Rightarrow 2x + n = 63$$

Solving equations (1) and (2), we get

$$n = 15 \quad \text{and} \quad x = 24$$

27. Initially, the total age of family = $4 \times 42 + 7 \times 20 = 308$

$$\text{and} \quad \text{the average age} = \frac{308}{11} = 28$$

Now, the total age of family = $308 + 28 = 336$

$$= 308 + 28 = 336$$

Now, the new average of the family = $\frac{336}{12} = 28$

Since the average age of the original family and that of new family is same (i.e., 28)

Hence, the average age of the new family is increased by 0 year.

28. Let there be n number and he missed a number k , then the average (which he has calculated) = $\frac{\frac{n(n+1)}{2} - k}{n} = 15$

$$n^2 - 29n = 2k$$

$$n(n-29) = 2k$$

Thus at $n = 29$ or $n < 29$, the expression is invalid since the value of k is neither zero nor negative, which is actually a natural number. So for the least possible value of $n = 30$

$$k = 15$$

$$n = 31, k = 31$$

Again for $n > 31$, k is beyond the range i.e., greater than n . Since k can not be greater than n . Hence there are only two values of k . So there is no unique value of n .

29. Let there be n people (initially) in the group, then the total earning of the group = $n \times 50$

$$\text{Again } n \times 50 = (n-2) \times 49 + (2x + 45)$$

$$\Rightarrow n = 2x - 53; \text{ where } x \text{ is the lowest earning of any person.}$$

Now, since $42 < x < 47$ and $n \in \text{prime numbers}$

Then the only possible value of $n = 37$ for $x = 45$

30. Let the number of students who got the jobs of A , B and C categories is a , b and c respectively,

$$\text{then the total salary} = \frac{26(a+b) + 44(b+c) + 34(c+a)}{2(a+b+c)}$$

$$= \frac{60a + 70b + 78c}{2(a+b+c)}$$

$$= \frac{30(a+b+c) + (5b+9c)}{a+b+c}$$

$$= 30 + \text{some positive value}$$

So the minimum salary must be Rs. 30 lakh and the maximum salary can not exceed 44, which is the highest of the three.

$$D - C - A \quad \dots(1)$$

$$D > B > C \quad \dots(2)$$

from (1) and (2)

$$D > B > C > A \quad \dots(3)$$

Again but $B > A$, from (3)

$$E - B - A$$

So $E > D > B > C > A$ Since B is the average of E and A so it is equidistant from both E and A .

32. Let Donald be denoted by H (Husband)
His wife be denoted by W (Wife)
His daughter be denoted by D (Daughter)
His son be denoted by S (Son)

The average age of 4 persons = $\frac{(H+W+D+S)}{4} = 23$

\Rightarrow

Again

$$H = W + 4$$

So At the time when daughter is born
At the time when son is born

H	W	D	S
28	$\xleftarrow{(+4)} 24$	0	\times
\downarrow			
32	$\xrightarrow{(-4)} 28$	4	0

So at the time of birth of his son, total age of his family = 64 years $(32 + 28 + 4 + 0 = 64)$

and presently the total age of his family = 92 years
It means total increase in age of the whole family = 28 years

$$\text{Thus the average increase in age} = \frac{28}{4} = 7 \text{ years}$$

It means the age of Donald = 39 years

and age of his daughter = 11 years

Therefore the average age of Donald and his daughter is 25 years.

33. Aman Wife Son EI. D Yg. D

$$5x \xleftarrow{\quad} x \quad 5z \xleftarrow{\quad} 4z$$

$$8y \xleftarrow{\quad} 3y$$

$$\Rightarrow 40K \quad (W) \quad 8K \quad 15K \quad 12K \quad (K = x, y)$$

$$\text{Again since } Yg. D + W = A + S$$

$$\Rightarrow 12K + W = 40K + 8K \quad W \rightarrow \text{Age of wife}$$

$$\Rightarrow W = 36K$$

Thus 4 years ago

$$(36K - 4) = 8(8K - 4)$$

$$\Rightarrow 28K = 28 \Rightarrow K = 1$$

Therefore, the age of Aman = 40

$$\text{Wife} = 36$$

$$\text{Son} = 8$$

$$\text{Elder daughter} = 15$$

$$\text{Younger daughter} = 12$$

Hence, the average age of the family = $\frac{111}{5} = 22.22$ years

34. Let there be n politicians (initially) in the party and their average weight be x kg, then

$$nx + 209 = \frac{5}{4} n(x-1)$$

$$\frac{n}{4}(x-5) = 209$$

$$x = \frac{209 \times 4}{n} + 5$$

$$x = \frac{4 \times 11 \times 19}{n} + 5$$

So the possible value of n is 76 ($= 19 \times 4$)

$$\text{Thus } x = 16$$

Therefore the average weight of all the politicians is 15 kg.

35. Total distance covered by all the wheels = 4×160

$$\text{Number of wheels used} = 5$$

Therefore average distance covered by each wheel

$$= \frac{4 \times 160}{5} = 128 \text{ km}$$

168

36. Let the numbers be $(a-5), (a-3), (a-1), (a+1), (a+3), (a+5)$

then their average

$$= \frac{(a-5) + (a-3) + (a-1) + (a+1) + (a+3) + (a+5)}{6} = a$$

Again the value of 'a' can be found by using the last statement

i. e., $(a-5)^2 + (a+5)^2 = 178$

$$\Rightarrow a^2 = 64 \Rightarrow a = 8$$

37.

	No. of Directors	Average Age	Total Age
Just before death and resignation	10	48	480
Just after death and resignation	9		$\{480 - (53 + x) + 34\}$
One year later	9	46	414

So one year later, after the incident

$$\text{total age} = \{480 - (53 + x) + 34\} + 9 \times 1 = 414$$

$$\Rightarrow x = 56 \text{ years}$$

where x is the age of the dead person at the time of his death.

38. Go through options $= \frac{24 \times 60 + 12 \times 54}{(24 + 12)}$

= 58, which is a whole number.

39.

	Year/Time	No. of Nurses	Average Age	Total Age
	1982	100	50	5000
Just before retirement	1984	100	52	5200
Just after retirement	1984	80	50	$(5200 - 20 \times 60) = 4000$
Just before recruitment	1987	80	53	4240
Just after recruitment	1987	$(80 + 40) = 120$	48	$(4240 + 38 \times 40) = 5760$
	1990	120	51	6120

40. Average score

$$= \frac{8 + 10 + 7 + 12 + 25 + 24 + 18 + 23 + 29 + 30}{10} = \frac{186}{10} = 18.6$$

41. The candidate who has scored maximum (means continuing for long time and number of responses were maximum) can be such a required person.

Now, consider Abhijeet, whose score was 30.

Again consider Rajesh, Radhe, Harish and one more candidate since $8 + 10 + 7 < 30 \Rightarrow 25 < 30$

So there are exactly 3 persons which can be full time opponents. Further there are 5 ($= 30 - 25$) more chances. So these chances can be utilised by any other candidate but he or she must be in the beginning or in the ending of the Abhijeet, since the score of all the candidates is greater than 5. Hence there can be maximum $3 + 1 = 4$ opponents of Abhijeet.

42. There is no such an information.

43. The minimum number of participants who have lost the game before Abhijeet started will be possible only when the number of participant with Abhijeet be maximum, which is 4. Hence excluding these 4 players (opponents of Abhijeet) and Abhijeet himself there are 5 people left. So minimum 5

44. The opponents of Abhijeet per the requirement are Rajesh, Radhe, Harish and Amit Sana

$$\text{So, the average score} = \frac{8 + 10 + 7 + 29}{4} = 13.5$$

45. Go through options

$$\text{or Total employs} = \frac{(25 - 10) \times 20}{10} = 30$$

Hence number of educated employees $= 30 - 20 = 10$

46. Average speed of the later half journey

$$= \frac{2 \times 40 \times 60}{100} = 48 \text{ km/hr}$$

Now the average speed of the whole journey

$$= \frac{2 \times 48 \times 96}{144} = 64 \text{ km/hr}$$

$$47. \frac{40 \times 0.8 + 60 \times 0.75 + 50 \times 0.6 + 30 \times 0.5}{180} \times 100$$

$$= \frac{122}{180} \times 100 = 67.777 \dots \%$$

abc

- cba

$$48. \text{Remember } \frac{99(a-c)}{99(a-c)}$$

where abc and cba are the three digit numbers and $(a, c) \neq 0$

Again since the difference in average $= 19.8$

Therefore the difference in total $= 19.8 \times 10 = 198$

$$\text{Thus, } 99 \times (a-c) = 198$$

$$(a-c) = 2$$

49. If the highest weight be 35 gm, then the second highest weight will be 31 gm. Again if the second highest will be 31, then the third highest will be 33 which is inadmissible, since $35 + 33 = 68$ which is not the greatest possible combination. Hence wrong.

Similarly 36 (i. e., option b) is also invalid

$$\begin{array}{lll} \text{Highest} & \text{Sec. Highest} & \text{Third Highest} \\ 36 & 30 & 34 \end{array}$$

Thus $36 + 34 = 70 > 66$, hence wrong.

The greatest possible combination can not be greater than 66.

Now, consider option (c)

$$\begin{array}{lll} \text{Highest} & \text{Sec. Highest} & \text{Third Highest} \\ 34 & 32 & 32 \end{array}$$

(since weights are different)

$$\begin{array}{lll} 32 & 31 & 31 \end{array}$$

(since 65 is not a combination)

$$\begin{array}{lll} 32 & 30 & \checkmark \end{array}$$

So, the highest weight 34

Sec. highest weight 32

Third highest weight 30

Lowest weight 29

Since, all the weights obtained give all the 6 different combinations, hence 34 is the highest possible weight of an envelope.

50. Let the fixed expenditure of the hotel be Rs. x and the variable expenditure (which is dependent on the guest) is y , then

$$x + 10y = 600$$

$$\Rightarrow x + 20y = 800$$

$$10y = 200$$

$$y = \text{Rs. } 20 \text{ and } x = 400$$

hence the total expenditure when there are 40 guests

$$= 400 + 40 \times 20 = 1200$$

$$\text{Therefore, average expenditure} = \frac{1200}{40} = \text{Rs. } 30$$



1. Since all the total 100 elements of Sets A, B, C are the natural numbers upto. Thus the average of these first 100 natural numbers is the required average.
- $$\text{Average} = \frac{1+2+3+4 \dots 100}{100} = \frac{100 \times 101}{2 \times 100} = 50.50$$

2. Except to 2 there are all the even numbers upto 100
So, the required average = $\frac{(2+4+6+\dots+100)-2}{49}$
 $= \frac{50 \times 51 - 2}{49} = \frac{2548}{49} = 52$

NOTE There are only 49 elements in the Set B. Apply the formula of sum of first even numbers. Also use the property of AP.

3. The total value of all the 25 elements of the Set A
 $= 25 \times 42.4 = 1060$

Since, there are 25 prime numbers upto 100 in the Set A
Again in the Set A and C there are 50 odd numbers and one even number. So the sum of all the elements of A and C
 $= (1+3+5+7+\dots+99)+2$
 $= (50)^2 + 2 = 2502$

Therefore the sum of all the elements of Set C
 $= 2502 - 1060 = 1442$

Hence, the average of the Set C = $\frac{1442}{26} = 55.4615$

4. The average of all the elements of the Set A and C
 $= \frac{2502}{51} = 49.0588$

Solutions for question number 5 to 15 :

Set	No. of elements	Average	Least element	Greatest element
A	25	42.4	2	97
B	49	52	4	100
C	26	55.46	1	99

5. Since the value of element which is transferred to Set B is less than 50, which in turn less than the average of Set B, hence the average of Set B decreases.

NOTE If a quantity which is less than the average of the group introduces from outside then the new average of the group decreases.

6. The least possible numbers of Set A which are greater than 50 are 53 and 59 whose average is always greater than the average of C. Hence the average of C will necessarily increases.

7. Can't say, since we don't know which 10 numbers are being transferred. Whether their average is greater, less or equal to the average of B.

8. Definitely increases, since the average of those numbers (viz. 1 and 99) is 50 which is greater than the average of Set A.

9. The average of those numbers (viz. 4 and 100) is 52. Hence average of A will increase and average of B will remain constant and the average of C remains unaffected because Set C is not involved.

NOTE If an element or average of some elements is equal to the average of the group then this element (or subset of elements) does not change the average of the group when it joins the group or leaves the group.

Compiled

10. After the insertion of new element viz. 2 in the Set B the new average = $\frac{2+4+6+\dots+100}{50} = 51$

Hence, the new average of Set B decreases by 1.

11. The perfect square number of the Set C are 1, 9, 25, 49, 81
Hence, the average of these number = $\frac{165}{5} = 33$

12. Since there is no net change (i. e., all the elements even after being transferred are same). Hence their average is also same as in question no. 1.

13. Obviously A. Since the average of all those 15 elements which are joining the Set A is greater than the average of all those 5 elements which are leaving the Set A and this difference in average is largest in comparison to Set B or Set C. Even in Set C there is decrease in average.

14. To minimize the loss in average of Set B, we have to transfer the least possible values of the given range and have to bring the highest possible values from the Set A to the Set B.

Thus the absolute decrease in Set B
 $= (26+28+30+32 \dots 44) - (23+19+17+13+11)$
 $= 350 - 83 = 267$

Hence, the decrease in total value of Set B
 $= 2548 - 267 = 2281$

Therefore, new average = $\frac{2281}{44} = 51.84$

NOTE Now there are only 44 elements in Set B.

15. There is no relevant information regarding the numbers which are being transferred from one set to another set.

16. Average speed = $\frac{\text{Total distance}}{\text{Total time}}$
 $= \frac{200}{\left(5 + \frac{10}{3}\right)} = \frac{200 \times 3}{25} = 24 \text{ km/hr}$

Since for the first 100 km time required is $\frac{100}{20} = 5 \text{ hrs}$ and for

the last 100 km time required = $\frac{100}{30} = \frac{10}{3} \text{ hours}$

17. The average speed = $\frac{150}{5 + \frac{5}{3}} = \frac{150 \times 3}{20} = 22.5 \text{ km/hr}$

18. Average bonus for the first 3 months = $\left(\frac{3000}{100}\right)^2 + 10 = 910$

Average bonus for the next 5 months = $\left(\frac{5000}{100}\right)^2 + 10 = 2510$

Average bonus for the last 4 months = $\left(\frac{8000}{100}\right)^2 + 10 = 6410$

His average bonus for the whole year
 $= \frac{910 \times 3 + 2510 \times 5 + 6410 \times 4}{12} = \text{Rs. 3410}$

Hence his average earning per month
 $= 3410 + 200 = \text{Rs. 3610}$

19. Total price of 5 shirts = $\text{Rs. } [100 + 10 \times (5)^2] = \text{Rs. } 350$

Hence, the average price = $\frac{350}{5} = \text{Rs. } 70$

By Jasjeet

20. Check the option (c).

$$\text{Total price} = 100 + 10 \times (2)^2 = \text{Rs. } 140$$

$$\text{Average price} = \frac{140}{2} = \text{Rs. } 70$$

Hence, the average price is same as that of Mallika.

21. Total number of passengers $= 10 \times 20 = 200$

In the 9 compartments the total number of passengers $= 144 (= 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20)$
So the no. of passengers in the 10th coach $= 200 - 144 = 56$

22.

	No. of 2 wheelers	No. of 3 wheelers	No. of 4 wheelers
No. of wheels	$2x$ $2 \times 2x = 4x$	x $3 \times x = 3x$	$2x$ $2x \times 4 = 8x$

$$\text{Therefore average number of wheels} = \frac{4x + 3x + 8x}{5x} = 3$$

23. The average weight of eggs of first generation is k gm and the no. of eggs is ' n '.

Let $a_1, a_2, a_3, \dots, a_n$ be the weights of n eggs of the first generation

$$\therefore k = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

$$\therefore nk = a_1 + a_2 + a_3 + \dots + a_n \quad \dots(1)$$

where a_1 is the average weight of its ' n ' child eggs, a_2 is the average weight of its own ' n ' child eggs and so on. child egg is referred to the egg of next generation produced by its mother egg.

$$a_1 = \frac{a_1 + b_1 + c_1 + \dots + n_1}{n}$$

$$a_2 = \frac{a_2 + b_2 + c_2 + \dots + n_2}{n}$$

$$a_3 = \frac{a_3 + b_3 + c_3 + \dots + n_3}{n} \text{ etc.}$$

Substituting the values of a_1, a_2, a_3, \dots in equation ... (1)

$$nk = \left(\frac{a_1 + b_1 + c_1 + \dots}{n} \right) + \left(\frac{a_2 + b_2 + c_2 + \dots}{n} \right) + \left(\frac{a_3 + b_3 + c_3 + \dots}{n} \right) + \dots + \left(\frac{a_n + b_n + c_n + \dots}{n} \right)$$

$$\therefore nk = (a_1 + b_1 + c_1) + (a_2 + b_2 + c_2 + \dots) + (a_3 + b_3 + c_3 + \dots)$$

\Rightarrow Therefore n^2k is the total weight of all the eggs of second generation.

Similarly, each of $a_1, b_1, c_1, a_2, b_2, c_2, \dots$ are the average weights of n eggs of their child eggs. Hence in the third generation total weight will be n^3k . Thus the total weight of all the eggs of r^{th} generation is n^rk .

Solutions for question number 24 to 27 : Before going for the final solution we need to look for the fundamental concept of averages i.e., if a person of higher age than the average age of the group leaves the group, then the average age of the group

decreases. Also if a person of less age than the average age of the existing group joins the group, then the average age of the group decreases.

Besides it we also know that the average age of the same group after k years increases by K years.

Faculty of LR :

Year	No. of faculty	Average age	Total age
2004	3	49.33	148
2005	4	44	$176 = 148 + 3 + 25$
2006	4	45	$180 = 176 + 4$
2007	4	46	$184 = 180 + 4$

$176 = 148 + 3 + 25$, implies that due to 3 existing professors their total age will be increased by 3 years after one year time period and 25 years age will be added due to a new entrant in the faculty of LR.

Faculty of DI :

Year	No. of faculty	Average age	Total age
2004	4	50.5	202
2005	4	51.5	$206 = 202 + 4$
2006	4	52.5	$210 = 206 + 4$
2007	5	47.8	$239 = 210 + 4 + 25$

Faculty of English :

Year	No. of faculty	Average age	Total age
2004	5	50.2	251
2005	4	49	$196 = 251 + 5 - 60$
2006	5	45	$225 = 196 + 4 + 25$
2007	5	46	$230 = 225 + 5$

Faculty of Quants :

Year	No. of faculty	Average age	Total age
2004	6	45	270
2005	7	43	$301 = 270 + 6 + 25$
2006	7	44	$308 = 301 + 7$
2007	7	45	$315 = 308 + 7$

24. In the year 2006, a new faculty member joined the English faculty.
25. The new faculty member who joined on April 1, 2005 became 27 years old on April 1, 2007.
26. From the faculty of English a professor retired on April 1, 2005.
27. Age of Sarvesh on April 1, 2004 = 52 years + 4 months \approx 52 years
Similarly age of Manish on April 1, 2004
 $= 49$ years + 4 months \approx 49 years
 \therefore Age of the third professor on April 1, 2004
 $= 148 - (52 + 49) = 47$ years
Hence the age of the third professor on April 1, 2009
 $= 47 + 5 = 52$ years

3

ALLIGATIONS

As we have studied the normal average in the previous chapter, so here we study particularly weighted average. In this chapter we will learn to find the average of two different groups with different number of elements which is known as weighted average. So alligation method is the simplified technique to solve the complex average problems. In this chapter, various kinds of problems will be discussed, since this chapter works as a means not as an end for.

Basically in the entrance tests or competitive exams like

EXAMPLE 1 The average weight of a class of 40 students is 30 and the average weight of a class of 20 students is 15. Find the average weight of both the combined classes :

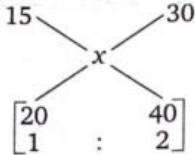
- (a) 20 (b) 25 (c) 17.5 (d) 15

SOLUTION To find the solution of this problem, we can use two widely used techniques :

1. Weighted average method

$$\text{The required average} = \frac{40 \times 30 + 20 \times 15}{(40 + 20)} = 25$$

2. Alligation method :



In this graphical representation of the solution, x is the weighted average where 15 and 30 are the averages of different class. So remember that the weighted average is always written in middle and the individual averages are written at the top, the smaller one in the left and the greater one in the right just for convenience and the number of elements (or the fixed quantities) are written below correspondingly.

Now, the difference between the two averages is divided in the inverse ratio of the quantities written below. As in the above problem, the difference 15 (= 30 - 15) will be divided in the ratio of 2:1 (as 40:20) but not in the ratio of 1:2, which must be clear from the arrows indicating cross proportion.

$$\text{So, the value of } x = 15 + \frac{2}{3} \times (30 - 15) = 25$$

CAT, the problems are not directly asked from this chapter, rather the techniques used in this chapter are applied for the higher level problems of CAT. As the alligation plays a vital role as a time saving method in solving the problems of Ratio Proportion, S.I., C.I. Profit Loss and some other topics, so we need to emphasize on the concept of Alligation and its applications extensively applied.

Remember, in some exams like MAT, IRMA, CET etc., the problems of this chapter are directly asked which you need to solve within seconds, even orally.

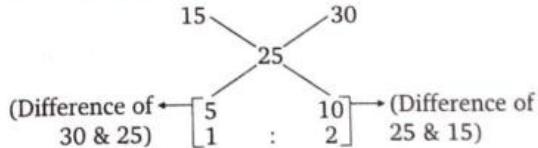
$$\text{or } x = 30 - \frac{1}{3} \times (30 - 15) = 25$$

Therefore, the average weight of both the classes is 25.

EXAMPLE 2 If the average weight of a class of students is 15 and the average weight of another class of students is 30, then find the ratio of the students of the first class to the another class of 30 students when the average weight of both the classes is 25 :

- (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 4

SOLUTION Let the ratio of the students with 15 students to 30 students be $x : y$, then



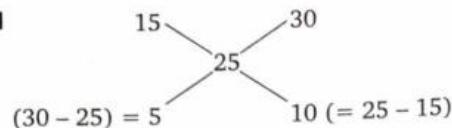
Here we know that the averages of individual classes is 15 and 30. Again we know that the averages of both the combined classes is 25. So the difference between 25 and 15 i.e., 10 and 30 & 25, i.e., 5 is written diagonally opposite.

$$\text{Thus the required ratio} = \frac{5}{10} = \frac{1}{2}$$

EXAMPLE 3 The average weight of girls is 15 and the average weight of boys is 30 and the average weight of boys and girls both is 25. If the number of boys are 12, then the number of girls is :

- (a) 4 (b) 6 (c) 10 (d) 18

SOLUTION



$$\text{Since the ratio of girls to boys is } \frac{5}{10} = \frac{1}{2}$$

Hence if there are 12 boys, so there will be 6 girls.

EXAMPLE 4 The ratio of number of girls to number of boys is 1 : 2. If the average weight of the boys is 30 kg and the average weight of both the boys and girls be 25 kg, then the average weight of the girls is :

- (a) 15 kg (b) 20 kg (c) 35 kg (d) 40 kg

SOLUTION

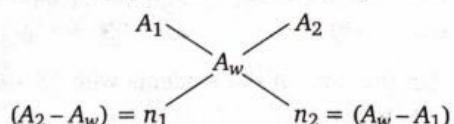
$$\begin{aligned} & \begin{array}{c} G \diagup \quad 30 \\ \diagdown 25 \end{array} \\ (30 - 25) &= x \quad 2x = (25 - G) \\ \Rightarrow & \left(\frac{30 - 25}{25 - G} = \frac{x}{2x} \right) \\ \text{So} & \frac{(30 - 25)}{(25 - G)} = \frac{1}{2} \Rightarrow G = 15 \end{aligned}$$

Therefore, the average weight of the girls is 15 kg. In general, if the average of group 1 be A_1 and the number of the elements be n_1 and the average of the group 2 be A_2 & the number of element be n_2 then the weighted average

$$\begin{aligned} A_w &= \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2} \\ \Rightarrow & n_1 A_w + n_2 A_w = n_1 A_1 + n_2 A_2 \\ \Rightarrow & n_1 (A_w - A_1) = n_2 (A_2 - A_w) \\ \Rightarrow & \frac{n_1}{n_2} = \frac{(A_2 - A_w)}{(A_w - A_1)} \end{aligned}$$

(known as Alligation Equation)

So this can be represented in the graphical form as given below



So all these problems concerned to the topic can be solved either by the formula of weighted average or Alligation equation or graphical representation method, as discussed above.

EXAMPLE 5 Two varieties of soda water with different prices is mixed in the ratio of 2 : 3. The price of first soda water is Rs. 10 per litre while the price of second soda water is Rs. 15 per litre, respectively. The average price of the mixture (per litre) is :

- (a) Rs. 12 (b) Rs. 13 (c) Rs. 14 (d) Rs. 15

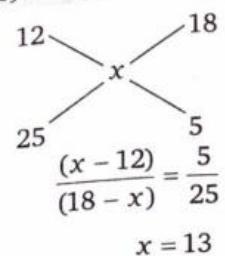
SOLUTION

$$\begin{aligned} & \begin{array}{c} 10 \diagup \quad 15 \\ \diagdown 2 \quad 3 \\ x \end{array} \\ \Rightarrow & \frac{(x - 10)}{(15 - x)} = \frac{3}{2} \\ \Rightarrow & 2(x - 10) = 3(15 - x) \\ \Rightarrow & x = 13 \end{aligned}$$

EXAMPLE 6 5 kg of superior quality of sugar is mixed with 25 kg of inferior quality sugar. The price of superior quality and inferior quality sugar is Rs. 18 and Rs. 12 respectively. The average price per kg of the mixture is :

- (a) Rs. 13 (b) Rs. 15 (c) Rs. 18 (d) Rs. 21

SOLUTION



⇒ The corresponding ratio is 25 : 5 = 5 : 1

Now reverse the ratio, which is 1 : 5

Now divide the differences of 12 and 18 in the ratio of 1 : 5, i.e., divide 6 into two parts in the ratio of 1 : 5, then the average price of mixture is $12 + 1 = 13$

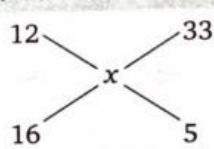
or

$$18 - 5 = 13$$

EXAMPLE 7 16 litres of kerosene is mixed with 5 litres of petrol. The price of kerosene is Rs. 12 per litre and the price of petrol is Rs. 33 per litre. The average price of the mixture per litre is :

- (a) Rs. 15 (b) Rs. 17 (c) Rs. 23 (d) Rs. 27

SOLUTION



Now divide the difference of 12 and 33 in the ratio of 5 : 16 (not in the ratio of 16 : 5)

Here, the difference of 33 and 12 = 21

Again on dividing 21 in the ratio of 5 : 16, we get the first part = 5 and second part = 16

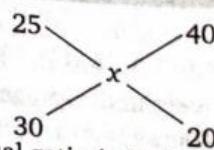
$$\begin{aligned} \text{So,} & \quad \text{the average price} = 12 + 5 = 17 \\ \text{or} & \quad = 33 - 16 = 17 \end{aligned}$$

NOTE This cross among the numbers written above shows that the difference between x and 12 is 5 and the difference between 33 and x is 16.

EXAMPLE 8 Amit travels 30 minutes at the speed of 25 km/hr. Further he travels 20 minutes at the speed of 40 km/hr. Find his average speed.

- (a) 25 km/hr (b) 30 km/hr
(c) 31 km/hr (d) none of these

SOLUTION



Since the actual ratio is 3 : 2, so reverse it, i.e., the required ratio is 2 : 3.

Now divide the difference of both the speeds i.e., 25 and 40 in the ratio of 2 : 3.

$$\text{i.e., } (40 - 25) \times \frac{2}{5} = 6 \quad \text{and} \quad (40 - 25) \times \frac{3}{5} = 9$$

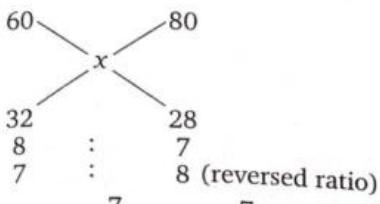
So the average speed is $25 + 6 = 31$

$$40 - 9 = 31$$

Alligations

EXAMPLE 9 A milkman has two types of milk. In the first container the percentage of milk is 80% and in the second container the percentage of milk is 60%. If he mixes 28 litres of milk of the first container to the 32 litres of milk of the second container, then the percentage of milk in the mixture is :

- (a) 63.99 (b) 69.33 (c) 72.5 (d) 75.2

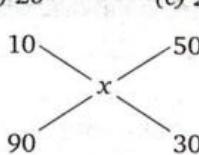
SOLUTION


$$\text{So } (80 - 60) \times \frac{7}{7+8} = 20 \times \frac{7}{15} = 9.33$$

Thus the required percentage = $60 + 9.33 = 69.33$

EXAMPLE 10 Modern electronic shop sold the 30% hardware at the profit of 50% and 90% software at the profit of 10%. The average profit percent of the Modern electronic shop is, if it sells only these two kinds of things :

- (a) 15 (b) 20 (c) 25 (d) 45

SOLUTION


The actual ratio is 3 : 1.

So the reversed ratio is 1 : 3.

Now divide the difference of 10 and 50 in the ratio of 1 : 3 then add the first part to the 10 or subtract the second part from the 50.

$$\text{i.e., Required average} = 10 + (50 - 10) \times \frac{1}{4}$$

$$= 10 + 10 = 20$$

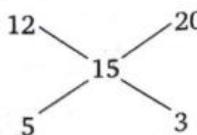
$$\text{or } 50 - (50 - 10) \times \frac{3}{4} = 20$$

NOTE All the problems discussed above can be solved in any way, i.e., either by using the formula of weighted average or Alligation Equation. But we have emphasised on the graphical (cross proportion) method. Since this technique is quite handy, i.e., sometimes you need not to calculate on paper.

EXAMPLE 11 Avinash covered 150 km distance in 10 hours. The first part of his journey he covered by car, then he hired a rickshaw. The speed of car and rickshaw is 20 km/hr and 12 km/hr respectively. The ratio of distances covered by car and the rickshaw respectively are :

- (a) 2 : 3 (b) 4 : 5 (c) 1 : 1 (d) none of these

SOLUTION The average speed of Avinash = $\frac{150}{10} = 15 \text{ km/hr}$



It means the rickshaw took $\frac{5}{8}$ and car took $\frac{3}{8}$ of the total time i.e., the ratio of time taken by rickshaw to car is 5 : 3

So the ratio of distances covered by rickshaw to car is

$$5 \times 12 : 3 \times 20 \Rightarrow 1 : 1 \text{ Compiled By Jasjeet}$$

NOTE

1. In this calculation (i.e., in alligation method) distance never involves directly. Only time and speeds are involved.
2. Since we have to find generally the average speed (not the average time) so speeds are written on the top and corresponding time taken is written below.

EXAMPLE 12 A mixture of rice is sold at Rs. 3.00 per kg. This mixture is formed by mixing the rice of Rs. 2.10 and Rs. 2.52 per kg. What is the ratio of price of cheaper to the costlier quality in the mixture if the profit of 25% is being earned.

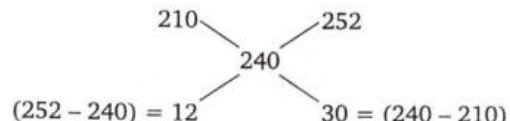
- (a) 5 : 2 (b) 2 : 7 (c) 2 : 5 (d) 15 : 8

SOLUTION Let the cost price of the mixture be Rs. x per kg, then selling price = $x + \frac{25 \times x}{100} = 3$

$$\Rightarrow 1.25x = 3$$

$$\Rightarrow x = 2.4$$

Now the average cost price of mixture = Rs. 2.4



Therefore, the ratio of cheaper to costlier rice is 12 : 30, i.e., 2 : 5.

NOTE Re 1 = 100 paise. There is no change in the ratio when we change the rupees into paise, just for our convenience in calculation.

EXAMPLE 13 A milkman has 20 litres of milk. If he mixes 5 litres of water, which is freely available, in 20 litres of pure milk. If the cost of pure milk is Rs. 18 per litre, then the profit of the milkman, when he sells all the mixture at cost price, is :

- (a) 20% (b) 25% (c) 33.33% (d) 18%

SOLUTION When the water is freely available and all the water is sold at the price of the milk, then the water gives the profit on the cost of 20 litres of milk.

$$\text{Therefore, profit percentage} = \frac{5}{20} \times 100 = 25\%$$

$$\text{Since, } \text{the profit \%} = \frac{\text{profit}}{\text{cost price}} \times 100$$

NOTE Here the milkman cheats his customer by false practise of creating illusion that instead of 20 litres milk, there is 25 litres of milk. So this extra 5 litres of milk (actually water) is the part of profit.

EXAMPLE 14 In what ratio should water and wine be mixed so that after selling the mixture at the cost price a profit of 33.33% is made?

- (a) 1 : 4 (b) 1 : 3 (c) 2 : 3 (d) 3 : 4

SOLUTION 33.33% profit means there is one part water and 3 part is pure wine. So the required ratio of water and wine in the mixture is 1 : 3.

NOTE The above problem (No. 14) and the previous problem are basically related to the topics of ratio and profit-loss.

EXAMPLE 15 In what ratio should freely available water be mixed with the wine worth Rs. 60 per litre so that after selling the mixture at Rs. 50 per litre, the profit will be 25%?

- (a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5

Alligations

EXAMPLE 20 Some amount out of Rs. 6000 was lent out at 10% per annum and the rest amount @ at 20% per annum and thus in 4 years the total interest from both the amounts collected was Rs. 3400. What is the amount which was lent out @ 10% per annum?

- (a) Rs. 2500 (b) Rs. 2800
 (c) Rs. 3200 (d) Rs. 3500

SOLUTION The total interest of one year

$$= \text{Rs. } 850 = \left(\frac{3400}{4} \right)$$

Therefore, the average rate of interest = $14 \frac{1}{6}\% = \frac{85}{6}\%$

$$\text{Thus, } \left(\frac{10 \times 6}{6} \right) \quad \left(\frac{20 \times 6}{6} \right)$$

$$\quad \quad \quad \left(\frac{85}{6} \right) \quad \quad \quad \left(\frac{35}{6} \right) \quad \quad \quad \left(\frac{25}{6} \right)$$

Hence the ratio of amount which is lent @ 10% per annum to the amount lent @ 20% per annum is 35 : 25, i.e., 7 : 5.

Therefore, the amount which is lent out @ 10% per annum is Rs. 3500.

EXAMPLE 21 From the 50 litres of milk, 5 litres of milk is taken out and after it 5 litres of water is added to the rest amount of milk. Again 5 litres of mixture of milk and water is drawn out and it was replaced by 5 litres of water. If this process is continued similarly for the third time, the amount of milk left after the third replacement:

- (a) 45 L (b) 36.45 L (c) 40.5 L (d) 42.5 L

SOLUTION

Milk	Water	
50 L	0 L	50 litre (initially)
- 5 L	- 0 L	withdrawn amount
45 L	0 L	50 litre (after first replacement)
+ 0 L	+ 5 L	
- 4.5 L	- 0.5 L	withdrawn amount
40.5 L	4.5 L	50 litre (after second replacement)
+ 0 L	+ 5 L	
- 4.05 L	- 0.95 L	withdrawn amount
36.45 L	8.55 L	50 litre (after third replacement)
+ 5 L		

$$\text{SHORTCUT} \quad 50 \times \left(1 - \frac{5}{50} \right) \times \left(1 - \frac{5}{50} \right) \times \left(1 - \frac{5}{50} \right)$$

$$= 50 \times \frac{45}{50} \times \frac{45}{50} \times \frac{45}{50} = 50 \times \left(\frac{45}{50} \right)^3 = 50 \times \left(\frac{9}{10} \right)^3$$

$$= 36.45 \text{ L}$$

GENERAL FORMULA

Final or reduced concentration
 = Initial concentration

$$\left(1 - \frac{\text{amount being replaced in each operation}}{\text{total amount}} \right)^n$$

where n is the number of times the same operation is being repeated. The "amount being replaced" could be pure or mixture as per the case. Similarly, "total amount" could also be either pure or mixture. Here amount being replaced denotes the quantity which is to be withdrawn in each time.

Compiled By Jasjeet

EXAMPLE 22 From a tank of petrol, which contains 200 litres of petrol, the seller replaces each time with kerosene when he sells 40 litres of petrol (or its mixture). Everytime he sells out only 40 litres of petrol (pure or impure). After replacing the petrol with kerosene 4th time, the total amount of kerosene in the mixture is :

- (a) 81.92 l (b) 96 l
 (c) 118.08 l (d) none of these

SOLUTION The amount of petrol left after 4 operations

$$= 200 \times \left(1 - \frac{40}{200} \right)^4 = 200 \times \left(\frac{4}{5} \right)^4$$

$$= 200 \times \frac{256}{625} = 81.92 \text{ litres}$$

Hence the amount of kerosene = $200 - 81.92 = 118.08$ litres

EXAMPLE 23 From a container of wine, a thief has stolen 15 litres of wine and replaced it with same quantity of water. He again repeated the same process. Thus in three attempts the ratio of wine and water became 343 : 169. The initial amount of wine in the container was :

- (a) 75 litres (b) 100 litres
 (c) 150 litres (d) 120 litres

SOLUTION $\frac{\text{wine (left)}}{\text{water (added)}} = \frac{343}{169}$

It means $\frac{\text{wine (left)}}{\text{wine (initial amount)}} = \frac{343}{512} \quad (\because 343 + 169 = 512)$

$$\text{Thus, } 343x = 512x \left(1 - \frac{15}{K} \right)^3$$

$$\Rightarrow \frac{343}{512} = \left(\frac{7}{8} \right)^3 = \left(1 - \frac{15}{K} \right)^3$$

$$\Rightarrow \left(1 - \frac{15}{K} \right) = \frac{7}{8} = \left(1 - \frac{1}{8} \right)$$

$$\Rightarrow K = 120$$

Thus the initial amount of wine was 120 litres.

EXAMPLE 24 A jar was full with honey. A person used to draw out 20% of the honey from the jar and replaced it with sugar solution. He has repeated the same process 4 times and thus there was only 512 gm of honey left in the jar, the rest part of the jar was filled with the sugar solution. The initial amount of honey in the jar was :

- (a) 1.25 kg (b) 1 kg
 (c) 1.5 kg (d) none of these

SOLUTION Let the initial amount of honey in the jar was K , then

$$512 = K \left(1 - \frac{1}{5} \right)^4 \quad \left(\because 20\% = \frac{20}{100} = \frac{1}{5} \right)$$

$$\text{or} \quad 512 = K \left(\frac{4}{5} \right)^4$$

$$\Rightarrow K = \frac{512 \times 625}{256}$$

$$\therefore K = 1250$$

Hence initially the honey in the jar = 1.25 kg.

Exercise

Answers

1. (a)	2. (c)	3. (a)	4. (c)	5. (d)	6. (c)	7. (b)	8. (a)	9. (b)	10. (b)
11. (b)	12. (d)	13. (a)	14. (d)	15. (d)	16. (d)	17. (a)	18. (b)	19. (b)	20. (b)
21. (b)	22. (b)	23. (d)	24. (b)	25. (b)	26. (a)	27. (b)	28. (b)	29. (a)	



Hints & Solutions

1. Let x litre pepsi is required.

$$\begin{array}{ccc}
 & 6 & 10 \\
 & \diagdown & \diagup \\
 x & 9 & \\
 & \diagup & \diagdown \\
 (10 - 9) = 1 & : & 3 = (9 - 6) \\
 \text{Therefore} & \frac{x}{15} = \frac{1}{3} & \\
 \Rightarrow & x = 5 \text{ litre} &
 \end{array}$$

Alternatively : Go through options.

2. Go through options :

$$90 \times 2 + 85 \times 4 = 520$$

If 2 wheelers be 90 then the four wheelers will be

$$85 = (175 - 90)$$

Alternatively : The average number of wheels per vehicle

$$= \frac{520}{175}$$

$$\begin{array}{ccc}
 2 \times \frac{175}{175} & & 4 \times \frac{175}{175} \\
 \diagdown & & \diagup \\
 \frac{520}{175} & & \\
 \diagup & & \diagdown \\
 \frac{180}{175} & : & \frac{170}{175} \\
 \Rightarrow & 180 & : 170 \\
 \Rightarrow & 90 & : 85 \\
 \Rightarrow & 18 & : 17
 \end{array}$$

Therefore the ratio of two wheelers to four wheelers is $18 : 17$.

Hence there are 90 "two wheelers".

3. Go through options :

$$30 \times 50 + 50 \times 20 = 2500 \text{ paise}$$

Alternatively : Since the average price of a coin

$$= \frac{2500}{80} = 31.25 \text{ paise}$$

$$\begin{array}{ccc}
 20 & & 50 \\
 \diagdown & & \diagup \\
 31.25 & & \\
 \diagup & & \diagdown \\
 18.75 & & 11.25
 \end{array}$$

So the ratio of no. of 20 paise coins to the no. of 50 paise coins

$$= 18.75 : 11.25$$

$$= 75 : 45 = 5 : 3$$

Therefore, the no. of coins of the denominations of 50 paise is 30.

4. Go through option :

$$24 \times 4 + 36 \times 2 = 168$$

Alternatively :

$$\begin{array}{ccc}
 2 & & 4 \\
 & \diagdown & \diagup \\
 & 2.8 & \\
 1.2 & & 0.8 \\
 \diagup & & \diagdown \\
 3 & : & 2
 \end{array}$$

Therefore, the ratio of men and sheep is $3 : 2$

Alternatively : Suppose there are only men, then the no. of legs $= 60 \times 2 = 120$. Now since there are $48 = (168 - 120)$ legs extra, it means there are $24 = \left(\frac{48}{2}\right)$ sheep, since a sheep has 2 extra legs than a man has.

5. Total quantity of mixture = 75 litre

$$\begin{array}{ccc}
 \text{Therefore} & \text{Milk} & \text{Water} \\
 & 4 & 1 \\
 & \diagdown & \diagup \\
 60 \text{ L} & & 15 \text{ L} \\
 & \diagup & \diagdown \\
 & 60 \text{ L} & 20 \text{ L} \\
 & 3 & 1
 \end{array}$$

6. Since the ratio of no. of female and male employees is $4 : 7$, the total no. of employees must be the multiples of 11. Hence the possible answer is 231.

$$\begin{array}{ccc}
 21 & & 32 \\
 & \diagdown & \diagup \\
 & 28 & \\
 & \diagup & \diagdown \\
 4 & & 7
 \end{array}$$

7. Since the ratio of cars sold at profit of 9% to the 36% is $19 : 3$, hence the no. of cars sold at 36% profit is 32.

$$\begin{array}{ccc}
 9 & & 36 \\
 & \diagdown & \diagup \\
 & 17 & \\
 & \diagup & \diagdown \\
 19 & & 8
 \end{array}$$

8. Here each girl receives 50 paise and each boy receives 100 paise and the average receiving of each student

$$= \frac{6900}{115} = 60 \text{ paise}$$

$$\begin{array}{ccc}
 50 & & 100 \\
 & \diagdown & \diagup \\
 & 60 & \\
 & \diagup & \diagdown \\
 40 & & 10
 \end{array}$$

$$\Rightarrow (G) 4 : (B) 1$$

Thus the no. of girls = 92

[Number of boys = 23]

$$9. \text{ Profit} = 12.5\% = \frac{1}{8}$$

Hence the ratio of water to spirit is $1 : 8$

since profit % = $\frac{\text{profit}}{\text{cost}} \times 100$.

$$\begin{array}{ccc}
 20 & & 50 \\
 & \diagdown & \diagup \\
 & 30 & \\
 & \diagup & \diagdown \\
 20 & : & 10
 \end{array}$$

Since the ratio of 20% wine to 50% wine is $2 : 1$, it means there is $\frac{2}{3}$ wine which is replaced with wine in which the concentration of spirit is 20%.

Alligations

11. $16 \xrightarrow{24} 19 \xrightarrow{3} 5$
 Thus the cost price of Indian factory is Rs. 45 crore.
 Therefore, the selling price of Indian factory is
 $= 45 + \frac{45 \times 16}{100} = 52.2$ crore

12. Milk Water
 80% 20%
 4 : 1
 20 L 5 L
 20 L 180 L $\boxed{+ 175 L}$
 10% 90%
 1 : 9

13. Profit (%) = $9.09\% = \frac{1}{11}$

Since the ratio of water and milk is 1 : 11,
 Therefore the ratio of water is to mixture = 1 : 12
 Thus the quantity of water in mixture of 1 litre

$$= 1000 \times \frac{1}{12} = 83.33 \text{ mL}$$

14. The selling price of mixture = Rs. 75

∴ The cost price of mixture = Rs. 60

Now we know that if he mixes the spirit (worth Rs. 40) with petrol (worth Rs. 60), the cost price of mixture must be less than Rs. 60, which is impossible. Hence there is no spirit with the petrol.

15. $-6 \xrightarrow{9} 15$
 $6 \xrightarrow{2} 15$
 $\Rightarrow 2 : 5$

Thus the ratio of B/W TV sets to the no. of colour TV sets
 $= 2 : 5$

Therefore, no. of B/W TV sets = 90

16. Since we do not know either the average weight of the whole class or the ratio of no. of boys to girls.

17. The S.P. of Desi Chai = Rs. 18

The S.P. of Videshi Chai = Rs. 30

The C.P. of Desi Chai = Rs. 20

The C.P. of Videshi Chai = Rs. 25

The S.P. of mixture = Rs. 27.5

The C.P. of mixture = Rs. 22

$20 \xrightarrow{22} 25$

$3 \xrightarrow{2} 2$

Therefore, the ratio of Desi Chai is to Videshi Chai is 3 : 2

18. $16.66 \xrightarrow{17.5} 18.75$
 (Boys) x (Girls) y

$\Rightarrow \frac{50}{3} \times \frac{4}{4} \xrightarrow{x} \frac{35}{2} \times \frac{6}{6} \xrightarrow{y} \frac{75}{4} \times \frac{3}{3}$

$\Rightarrow \frac{200}{12} \xrightarrow{210} \frac{225}{12} \xrightarrow{\frac{15}{12} \quad \frac{10}{12}}$
 $\Rightarrow 3 : 2$

Thus the no. of girls = 16 and no. of boys = 24

19. $W_1 : A_1 = 2 : 3$ $W_2 : A_2 = 4 : 5$ $W_N : A_N = 5 : 7$
 $\frac{W_1}{W_1 + A_1} = \frac{2}{5}$ $\frac{W_2}{W_2 + A_2} = \frac{4}{9}$ $\frac{W_N}{W_N + A_N} = \frac{5}{12}$
 $= \frac{72}{180}$ $= \frac{80}{180}$ $= \frac{75}{180}$

$\frac{72}{180} \xrightarrow{75} \frac{80}{180}$
 $\frac{5}{180} \xrightarrow{75} \frac{3}{180}$
 $\Rightarrow 5 : 3$

Therefore, the ratio is 5 : 3.

20. Since the average marks of sections B and C together are equal the average marks of all the four sections (i.e., A, B, C and D), therefore the average marks of the remaining two sections A and D together will also be equal i.e., 60%.

$45 \xrightarrow{60} 80$
 $20 \xrightarrow{4} 15$

Hence, the required ratio is 4 : 3.

21. Wine Water
 8 L 32 L
 1 : 4
 20% 80% (original ratio)
 30% 70% (required ratio)

In this case, the percentage of water being reduced when the mixture is being replaced with wine.

So the ratio of **left quantity** to the initial quantity is 7 : 8.

Therefore $\frac{7}{8} = \left[1 - \frac{K}{40} \right] \Rightarrow \frac{7}{8} = \left[\frac{40 - K}{40} \right]$

$\Rightarrow K = 5$ litre

Alternatively : Go through options.

22. Therefore no. of boys : Number of girls = 20 : 30

$20 \xrightarrow{23.25} 30$
 $6.75 \xrightarrow{27} 3.25 \xrightarrow{13}$
 $20 : 30$

23. Since there is insufficient data.

24. Milk Water

74% 26% (initially)

76% 24% (after replacement)

Left amount = Initial amount $\left(1 - \frac{\text{replaced amount}}{\text{total amount}}\right)$

$$24 = 26 \left(1 - \frac{7}{K}\right)$$

$$\Rightarrow \frac{12}{13} = \left(1 - \frac{7}{K}\right)$$

$$\Rightarrow \frac{1}{13} = \frac{7}{K}$$

$$\Rightarrow K = 91 \text{ litre}$$

NOTE In case of replacement, the initial (pure) amount is equal to the amount of final mixture.

25.

$$\begin{array}{c} 6 \quad \quad \quad x \\ \diagup \quad \diagdown \\ 15 \quad \quad \quad 3k = 9 \end{array}$$

$$6 = 2k \quad \quad \quad 3k = 9$$

Therefore $x = 21\%$

26. Copper in $4 \text{ kg} = \frac{4}{5} \text{ kg}$

and Zinc in $4 \text{ kg} = 4 \times \frac{4}{5} = \frac{16}{5} \text{ kg}$

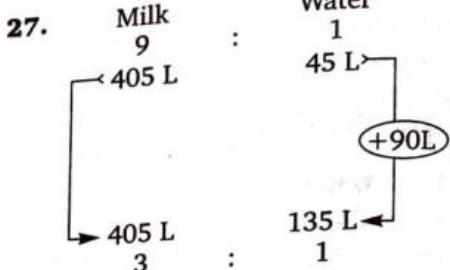
Copper in $5 \text{ kg} = 5 \times \frac{1}{6} = \frac{5}{6} \text{ kg}$

Zinc in $5 \text{ kg} = 5 \times \frac{5}{6} = \frac{25}{6} \text{ kg}$

Therefore, Copper in mixture $= \frac{4}{5} + \frac{5}{6} = \frac{49}{30} \text{ kg}$

and Zinc in the mixture $= \frac{16}{5} + \frac{25}{6} = \frac{221}{30} \text{ kg}$

Therefore, the required ratio $= 49 : 221$



28. Petrol : Kerosene

3 : 2 (initially)

2 : 3 (after replacement)

$\frac{\text{Remaining (or left) quantity}}{\text{Initial quantity}} = \left(1 - \frac{\text{replaced quantity}}{\text{total quantity}}\right)$

(for petrol) $\frac{2}{3} = \left(1 - \frac{10}{K}\right)$

$$\Rightarrow \frac{1}{3} = \frac{10}{K}$$

$$\Rightarrow K = 30 \text{ litre}$$

Therefore, the total quantity of the mixture in the container is 30 litre.

29.

$$\frac{9}{25} = \left(1 - \frac{6}{K}\right)^2$$

$$\Rightarrow \frac{3}{5} = \left(1 - \frac{6}{K}\right)$$

$$\Rightarrow K = 15 \text{ litre}$$

4

RATIO, PROPORTION & VARIATION

The basic applications of the concepts involved in this chapter are comparisons of two or more quantities and changes in their magnitudes, e.g., comparison of the ages, weights, income, savings, heights, volume, density, temperature etc.

So this chapter is very useful in solving the problems of Data interpretation. Also each and every year one or two problems from this chapter is/are asked in CAT, either directly or application based for QA section. Last but not the least, the concepts of ratio, proportion and variations are very useful in solving the maximum arithmetic problems. That's why these problems are usually asked in most of the competitive exams like FMS, IIFT, MAT, SYMBIOSIS etc.

RATIO

The comparison between two quantities in terms of magnitude is called the ratio, i.e., it tells us that the one quantity is how many times the other quantity.

For example, Amit has 5 pens and Sarita has 3 pens. It means the ratio of number of pens between Amit and Sarita is 5 to 3. It can be expressed as '5 : 3'.

NOTE It should be noted that in a ratio, the order of the terms is very important. For example, in the above illustration the required ratio is 5 : 3 while 3 : 5 is wrong.

So the ratio of any two quantities is expressed as $\frac{a}{b}$ or $a : b$.

The numerator 'a' is called the **antecedent** and denominator 'b' is called as **consequent**.

Rule of Ratio

The comparison of two quantities is meaningless if they are not of the same kind or in the same units (of length, volume or currency etc). We do not compare 8 boys and 6 cows or 15 litres and 5 toys or 5 metres and 25 centimetres. Therefore, to find the ratio of two quantities (of the same kind), it is necessary to express them in same units.

NOTE

1. We do not compare 8 boys and 6 cows, but we can compare the number (8) of boys and number (6) of cows. Similarly, we cannot compare the number (15) of litres and the number (5) of toys etc.
2. Ratio has no units.

PROPERTIES OF RATIOS

1. The value of a ratio does not change when the numerator and denominator both are multiplied by same quantities i.e.,

$$\frac{a}{b} = \frac{ka}{kb} = \frac{la}{lb} = \frac{ma}{mb} \text{ etc.}$$

e.g., $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} \dots$ etc. have the same ratio.

2. The value of a ratio does not alter (or change) when the numerator and denominator both are divided by same quantities i.e.,

$$\frac{a}{b} = \frac{a/k}{b/k} = \frac{a/l}{b/l} = \frac{a/m}{b/m} \text{ etc.}$$

e.g., $\frac{3}{4} = \frac{3/2}{4/2} = \frac{3/3}{4/3} = \frac{3/4}{4/4} \dots$ etc are same ratio.

3. The ratio of two fractions can be expressed in ratio of integers. e.g.,

$$\frac{3/4}{5/4} = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$$

Since $\frac{a/b}{c/d} = \frac{a}{c} \times \frac{d}{b}$ (Refer to the **fractions** in **Fundamentals**)

4. When two or more than two ratios are multiplied with each other, then it is called as compounded ratio e.g., $\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{16}{35}$ is the compounded ratio of $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}$

So, $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} \dots = \frac{k}{l}$ (compounded ratio)

5. When the ratio is compounded with itself, it is called as duplicate, triplicate ratios etc. e.g.,

$\frac{a}{b} \times \frac{a}{b} = \frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$ is called as duplicate ratio of $\frac{a}{b}$

and $\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \left(\frac{a}{b}\right)^3$ is called as triplicate ratio of $\frac{a}{b}$

Similarly, $\sqrt{\left(\frac{a}{b}\right)} = \left(\frac{a}{b}\right)^{1/2}$ is called as sub-duplicate ratio and $\sqrt[3]{\left(\frac{a}{b}\right)} = \left(\frac{a}{b}\right)^{1/3}$ is called as sub-triplicate ratio of $\frac{a}{b}$.

6. $\frac{a}{b} = \frac{c+am}{d+bm}$ if and only if $\frac{c}{d} = \frac{a}{b}$

i.e., $\frac{30}{40} = \frac{30+3m}{40+4m} = \frac{33}{44} = \frac{36}{48}$... etc at $m=1, 2, \dots$

This property is very useful when we compare two fractions

e.g., to compare between $\frac{10}{20}$ and $\frac{12}{22}$

We see that $\frac{10}{20} = \frac{1}{2}$

Now if $\frac{10}{20}$ and $\frac{12}{22}$ are equal then $\frac{12}{22} = \frac{10+m}{22+2m}$

Now putting $m=2$, we don't get the required fraction.

So $\frac{12}{22} \neq \frac{10}{20}$ for any value of m , $\frac{12}{22} \neq \frac{10}{20}$ (in terms of ratio)

7. $\frac{a+k}{b+k} < \frac{a}{b}$ if for every positive k , $\frac{a}{b} > 1$

and

$$\frac{a-k}{b-k} > \frac{a}{b}$$

EXAMPLE 1 Find the ratio of 25 to 40.

SOLUTION

$$\frac{25}{40} = \frac{5 \times 5}{5 \times 8} = \frac{5}{8}$$

NOTE To get the ratio, we rationalize the fractions by cancelling out the common factors of numerator and denominator.

EXAMPLE 2 Find the ratio of 90 cm to 1.5 m.

SOLUTION 1.5 m = 150 cm (units must be same)

So, the required ratio = $\frac{90}{150} = \frac{3}{5}$

EXAMPLE 3 The number of boys and girls in a school are 576 and 480 respectively. Express the ratio of the number of boys to that of girls in the simplest form.

$$\text{Required ratio} = \frac{576}{480} = \frac{6}{5} \quad \left(\because \frac{576}{480} = \frac{96 \times 6}{96 \times 5} \right)$$

EXAMPLE 4 Shukla earns Rs. 14,000 per month and Mishra earns Rs. 18,000 per month. Find the ratio of Shukla's salary to Mishra's salary.

$$\text{SOLUTION} \quad \text{Required ratio} = \frac{14,000}{18,000} = \frac{7}{9}$$

8. $\frac{a+k}{b+k} > \frac{a}{b}$ if for every positive k , $\frac{a}{b} < 1$ and $\frac{a-k}{b-k} < \frac{a}{b}$

9. $\frac{a+c}{b+d} > \frac{a}{b}$ if $\frac{c}{d} > \frac{a}{b}$

10. $\frac{a+c}{b+d} < \frac{a}{b}$ if $\frac{c}{d} < \frac{a}{b}$

11. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = k$
then $\frac{a+c+e+g+\dots}{b+d+f+h+\dots} = k$

12. Let $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}, \frac{g}{h}, \dots$ be some different ratios, then the value of $\left(\frac{a+c+e+g+\dots}{b+d+f+h+\dots} \right)$ must lies between the lowest and highest ratios.

13. If $a:b$ and $b:c$ are given, then

$$a:b:c = \frac{a}{b} : \frac{b}{c} = (a \cdot b) : (b \cdot b) : (b \cdot c)$$

14. If the ratios between $a:b, b:c, c:d, d:e$ are given individually, then the combined ratio of $a:b:c:d:e$ is

$$\begin{array}{ccccccccc} a : b & a & b & b & b & b & b & b & b \\ b : c & b & b & c & c & c & c & c & c \\ c : d & c & c & c & c & c & c & c & c \\ d : e & d & d & d & d & d & d & d & d \end{array} : : : : : : : :$$

$$\text{i.e., } a:b:c:d:e = (a \cdot b \cdot c \cdot d) : (b \cdot b \cdot c \cdot d) : (b \cdot b \cdot c \cdot c \cdot d) : (b \cdot c \cdot c \cdot d) : (b \cdot c \cdot d \cdot d) : (b \cdot c \cdot d \cdot c \cdot d)$$

EXAMPLE 5 Out of 144 persons working in an office, 56 are men and the remaining are women. Find the ratio of number of women to number of men.

SOLUTION Ratio = $\frac{88}{56} = \frac{11}{7}$ (\because number of women = 144 - 56)

EXAMPLE 6 In a club having 100 members, 20 play carrom, 24 play table-tennis and 16 play cricket and the remaining members do not play any game. No member plays more than one game. Find the ratio of the number of members who play.

- Carrom to the number of those who play table-tennis.
- Cricket to the number of those who play carrom.
- Cricket to the number of those who do not play any game.
- Table-tennis to the number of those who do not play any game.
- Some game to the number of those who do not play any game.

SOLUTION Total members = 100
Carrom = 20
Table-tennis = 24
Cricket = 16
No any game = 40

$$(a) \frac{20}{24} = \frac{5}{6}$$

$$(c) \frac{16}{40} = \frac{2}{5}$$

$$(e) \frac{60}{40} = \frac{3}{2}$$

$$(b) \frac{16}{20} = \frac{4}{5}$$

$$(d) \frac{24}{40} = \frac{3}{5}$$

EXAMPLE 7 A person earns Rs. 1200 per day and spends Rs. 800. Find the ratio of his savings to expenditure.

SOLUTION $\frac{400}{800} = \frac{1}{2}$ (Savings = Income - Expenditure)
 $= 1 : 2$

EXAMPLE 8 Simplify the following ratios :

$$(a) \frac{1}{6} : \frac{1}{8} \quad (b) 2 \frac{1}{3} : 3 \frac{1}{2} \quad (c) \frac{5}{6} : \frac{3}{8} : 3 \frac{3}{4}$$

$$\text{SOLUTION} \quad (a) \frac{1}{6} : \frac{1}{8} = \frac{1/6}{1/8} = \frac{1}{6} \times \frac{8}{1} = \frac{4}{3} \text{ or } 4 : 3$$

$$\text{Alternatively: } \frac{1}{6} : \frac{1}{8} = \frac{48}{6} : \frac{48}{8} \quad \left(\text{or } \frac{24}{6} : \frac{24}{8} \right)$$

$$= 8 : 6 = 4 : 3$$

NOTE In case of fractions, convert them to whole numbers by multiplying each term by the L.C.M. of their denominators.

$$(b) 2 \frac{1}{3} : 3 \frac{1}{2} = \frac{7}{3} : \frac{7}{2} = \frac{7/3}{7/2} = \frac{2}{3} \text{ or } 2 : 3$$

NOTE In case of two fractions if numerators are same, then the required ratio is the inverse ratio of the fractions.

$$(c) \frac{5}{6} : \frac{3}{8} : 3 \frac{3}{4} = \frac{5}{6} : \frac{3}{8} : \frac{15}{4} = \frac{5}{6} \times 24 : \frac{3}{8} \times 24 : \frac{15}{4} \times 24$$

$$= 20 : 9 : 90 \quad \{ \because \text{LCM of 6, 8 and 4 is 24} \}$$

EXAMPLE 9 Divide 14 toffees among Ankita and Anshul in the ratio 5 : 2

SOLUTION According to the question, if there are 7 toffees then Ankita will have 5 and Anshul will have 2 toffees but since there are 14 toffees, which is twice of 7. So Ankita will have 10 toffees and Anshul will have 4 toffees.

SHORTCUT	Ankita	Anshul
	5	2
	$14 \times \frac{5}{(5+2)}$	$14 \times \frac{2}{(5+2)}$
	10	4

EXAMPLE 10 Three boys are aged 2 years, 4 years and 8 years. They want to divide seventy rupees in the ratio of their ages. How much money would each get?

SOLUTION The ratio of their ages = $A : B : C = 2 : 4 : 8 = 1 : 2 : 4$

$$\begin{array}{ccc} A & : & B & : & C \\ 1 & : & 2 & : & 4 \\ 70 \times \frac{1}{1+2+4} & : & 70 \times \frac{2}{1+2+4} & : & 70 \times \frac{4}{1+2+4} \\ 10 & : & 20 & : & 40 \end{array}$$

Compiled By Jasjeet

So A gets Rs. 10, B gets Rs. 20 and C gets Rs. 40 if the ratio of their ages is 2 : 4 : 8.

EXAMPLE 11 An amount of Rs. 100 is being divided among two persons in the ratio $\frac{1}{10} : \frac{1}{15}$. How much money does each get?

SOLUTION $\frac{1}{10} : \frac{1}{15} = \frac{1}{10} \times 30 : \frac{1}{15} \times 30 = 3 : 2$ (here 30 is the LCM of 10 and 15)
 So, the ratio of amount of money = $\frac{3}{5} \times 100 : \frac{2}{5} \times 100$
 $= 60 \text{ and } 40$

EXAMPLE 12 The lengths of sides of a triangle are in the ratio 2 : 3 : 4. If the perimeter of the triangle is 63 cm, find the lengths of the sides of the triangle.

SOLUTION Let the sides of triangle be $2x$, $3x$ and $4x$, then $2x + 3x + 4x = 63$

$$\Rightarrow 9x = 63 \Rightarrow x = 7$$

∴ The sides of triangle = $2x$, $3x$, $4x = 14, 21, 28$

Alternatively:

$$63 \times \frac{2}{(2+3+4)} \text{ and } 63 \times \frac{3}{(2+3+4)} \text{ and } 63 \times \frac{4}{(2+3+4)}$$

$$= 14, 21 \text{ and } 28.$$

EXAMPLE 13 Divide 1224 into three parts such that first part be double that of second part and second part be $\frac{1}{3}$ of the third part.

SOLUTION Let A, B, C be three parts respectively, then

$$A : B = 2 : 1$$

$$\text{and } B : C = 1 : 3$$

$$\therefore A : B : C = 2 : 1 : 3$$

$$A = 1224 \times \frac{2}{6} = 408$$

$$B = 1224 \times \frac{1}{6} = 204$$

$$C = 1224 \times \frac{3}{6} = 612$$

EXAMPLE 14 If $A : B = 3 : 4$, $B : C = 5 : 2$ then find the value of $A : B : C$.

$$\text{SOLUTION} \quad A : B = 3 : 4$$

$$\text{and } B : C = 5 : 2$$

$$A : B : C = (3 \times 5) : (4 \times 5) : (4 \times 2)$$

$$\text{or } A : B : C = 15 : 20 : 8$$



EXAMPLE 15 The ratio of $A : B = 1 : 3$, $B : C = 2 : 5$, $C : D = 2 : 3$. Find the value of $A : B : C : D$.

$$\text{SOLUTION} \quad A : B = 1 : 3$$

$$B : C = 2 : 5$$

$$C : D = 2 : 3$$

$$A : B : C : D = (1 \times 2 \times 2) : (3 \times 2 \times 2) : (3 \times 5 \times 2) : (3 \times 5 \times 3)$$

$$A : B : C : D = 4 : 12 : 30 : 45$$

EXAMPLE 16 There are two types of mixtures of milk and water. In the first mixture, out of 12 litres of mixture, 5 litre is milk only and in the second mixture, 6 litre is milk and 12 litre is water. Which one mixture is better in terms of milk's strength?

SOLUTION First Mixture Second Mixture

$\frac{5}{12}$	$\frac{6}{18}$	$(6 + 12 = 18)$
$\frac{5}{12} \times 36$	$\frac{6}{18} \times 36$	(milk + water)
15	12	= mixture

So the first mixture has more milk in comparison to water.

EXAMPLE 17 The ratio of salary of $A : B = 1 : 2$, $B : C = 3 : 4$, $C : D = 5 : 6$ and $D : E = 7 : 8$. What is the ratio of salary of A and E ?

SOLUTION	$A : B$	1 : 2
	$B : C$	3 : 4
	$C : D$	5 : 6
	$D : E$	7 : 8

$$A : B : C : D : E = (1 \times 3 \times 5 \times 7) : (2 \times 3 \times 5 \times 7) : (2 \times 4 \times 5 \times 7) : (2 \times 4 \times 6 \times 7) : (2 \times 4 \times 6 \times 8)$$

$$\text{or } A : B : C : D : E = 105 : 210 : 280 : 336 : 384$$

So the ratio of salary of $A : E = 105 : 384 = 35 : 128$

NOTE In every next step, we leave the left term and adopt right term.

EXAMPLE 18 If $\frac{a}{b} = \frac{3}{4}$, then find the value of $7a - 4b : 3a + b$.

- (a) 7 : 1 (b) 5 : 13
 (c) 12 : 1 (d) none of these

SOLUTION Simply substitute the value of a and b as 3 and 4 in the given algebraic ratio

$$\text{as } (7 \times 3 - 4 \times 4) : (3 \times 3 + 4) = 5 : 13$$

EXAMPLE 19 If $a : b = 3 : 2$ and $b : c = 6 : 5$ then $a : b : c$ is equal to

- (a) 9 : 6 : 5 (b) 9 : 6 : 10
 (c) 3 : 3 : 5 (d) 3 : 6 : 5

SOLUTION

$$\begin{aligned} a : b &= 3 : 2 \\ b : c &= 6 : 5 \quad \downarrow \quad \downarrow \\ a : b : c &= (3 \times 6) : (2 \times 6) : (2 \times 5) \\ &= 18 : 12 : 10 = 9 : 6 : 5 \end{aligned}$$

EXAMPLE 20 The sum of two natural numbers is 64. Which of the following can not be the ratio of these two numbers?

- (a) 3 : 5 (b) 1 : 3 (c) 7 : 9 (d) 3 : 4

SOLUTION Let the numbers be $3x$ and $5x$, then

$$3x + 5x = 64 \Rightarrow 8x = 64$$

$\Rightarrow x = 8$, which is possible

The numbers are 24, 40.

Take another option:

$$x + 3x = 64 \Rightarrow x = 16$$

The numbers are 16 and 48.

Check for option (c):

$$7x + 9x = 64 \Rightarrow 16x = 64$$

$$x = 4$$

The numbers are 28 and 36.

Check for option (d)

$$3x + 4x = 64 \Rightarrow 7x = 64$$

$$x = \frac{64}{7}$$

Numbers are $\frac{3 \times 64}{7}$ and $\frac{4 \times 64}{7}$ or $\frac{192}{7}$ and $\frac{256}{7}$

which are not the natural numbers.

Hence option (d) is the required answer.

EXAMPLE 21 Monthly incomes of A and B are in the ratio of 4 : 3 and their savings are in the ratio of 3 : 2. If the expenditure of each will be Rs. 600, then the monthly incomes of each are :

- (a) 1800, 2400 (b) 2400, 1600
 (c) 2400, 1800 (d) 1600, 1200

SOLUTION $\text{Income} = \text{Exp.} + \text{Savings}$

$$A \rightarrow 4x = 3y + 600$$

$$B \rightarrow 3x = 2y + 600$$

$$\text{Therefore, } 4x - 3y = 600 \text{ and } 3x - 2y = 600$$

$$\Rightarrow 4x - 3y = 3x - 2y \Rightarrow x = y$$

$$\therefore 4x - 3x = 600 \Rightarrow x = 600$$

$$\text{Then, the income of } A = 4 \times 600 = 2400$$

$$\text{and income of } B = 3 \times 600 = 1800$$

Alternatively: Check the options. Consider (c)

Income	A	B
	2400	1800
	(4)	(3) \Rightarrow correct
Again,	2400	1800
	<u>- 600</u>	<u>- 600</u>
Savings	1800	1200
	(3)	(2) \Rightarrow correct

Hence, option (c) is correct. If you check other options, the ratio will not match.

EXAMPLE 22 A, B and C have 40, x and y balls with them respectively. If B gives 20 balls to A , he is left with half as many balls as C . If together they had 100 more balls, each of them would have had 100 balls on an average. What is value of $x : y$?

- (a) 3 : 2 (b) 4 : 6 (c) 2 : 1 (d) 3 : 4

SOLUTION From the last statement

$$\frac{100 + x + y}{3} = 100$$

$$\Rightarrow x + y = 200$$

$$\text{Again from first statement, } \frac{x - 20}{y} = \frac{1}{2}$$

$$\Rightarrow 2x - y = 40$$

Solving equations (1) and (2), we get

$$x = 80 \text{ and } y = 120$$

Therefore, required ratio of $x : y = 2 : 3$

Hence, option (b) is correct.

Alternatively: After forming the equation (1), we can go through the options. Let us assume option (b).

$$x + y = 200$$

$$x : y = 4 : 6$$

$$x = 80 \text{ and } y = 120$$

Now from the first statement, $\frac{x-20}{y} = \frac{1}{2}$

$$\frac{80-20}{120} = \frac{1}{2}$$

So $\frac{80-20}{120} = \frac{1}{2}$ (verified)

hence option (b) is correct.

EXAMPLE 23 The incomes of A, B, C are in the ratio of 12 : 9 : 7 and their spendings are in the ratio 15 : 9 : 8. If A saves 25% of his income. What is the ratio of the savings of A, B and C?

SOLUTION Income = Expenditure + Saving

$$\begin{aligned} A &\rightarrow 12x = 15y + 3x & (3x = 25\% \text{ of } 12x) \\ B &\rightarrow 9x = 9y + (9x - 9y) \\ C &\rightarrow 7x = 8y + (7x - 8y) \\ \text{Therefore, } & 12x - 3x = 15y \\ &\frac{x}{y} = \frac{5}{3} \\ \Rightarrow & y = \frac{3x}{5} \end{aligned}$$

Therefore, savings = (income - expenditure)

$$\begin{aligned} A &= 12x - 9x = 3x \\ B &= 9x - \frac{27}{5}x = \frac{18}{5}x \\ C &= 7x - \frac{24}{5}x = \frac{11}{5}x \end{aligned}$$

$$\begin{aligned} \text{i.e., the ratio of savings of } A : B : C &= 3x : \frac{18}{5}x : \frac{11}{5}x \\ &= 15x : 18x : 11x \\ &= 15 : 18 : 11 \end{aligned}$$

EXAMPLE 24 There are total 100 coins consisting of 20 paise, 50 paise and Re 1 in the ratio of 7 : 8 : 5. What is the no. of coins of 50 paise if the difference between the amount yielded by 20 paise and Re 1 coin is 18?

- (a) 32 (b) 40 (c) 26 (d) 56

SOLUTION $(20 \times 7x) - (100 \times 5x) = 1800$ (Rs. 18 = 1800 paise)
 $(140x) - (500x) = 1800$

PROPORTION

An equality of two ratios is called a proportion and we say that the four numbers are in proportion.

i.e., if $\frac{a}{b} = \frac{c}{d}$, or $a : b = c : d$, then we say that a, b, c and d are in proportions and written as $a : b :: c : d$, where the symbol $::$ indicates proportion and it is read as 'a is to b as c is to d'.

Here a and d are called "extremes" (or extreme terms) and b and c are called as "means" (or middle terms). Thus four numbers are said to be in proportion, if the ratio of the first to the second number is equal to the ratio of the third to the fourth number. e.g., $2 : 3 :: 4 : 6$. Some important results are derived from the discussion of proportion, which are very important for clear understanding.

$$\begin{aligned} \Rightarrow & 360x = 1800 \\ \Rightarrow & x = 5 \end{aligned}$$

Therefore, number of coins of 50 paise = $8 \times 5 = 40$

Alternatively: Let there be 40 coins of 50 paise denomination, then

The no. of coins of $A, B, C = 7x : 8x : 5x = 35 : 40 : 25$
 Therefore, the amount from 20 paise coins = $35 \times 20 = \text{Rs. } 7$
 and amount from Re 1 coins = $1 \times 25 = \text{Rs. } 25$
 Hence difference = $\text{Rs. } 18 (25 - 7)$
 Thus the presumed option is correct.

EXAMPLE 25 There are 43800 students in 4 schools of a city. If half of the first, two-third of the second, three-fourth of the third and four-fifth of the fourth are the same number of students, then find the ratio of number of students of A and D if A, B, C and D be the first, second, third and fourth schools respectively.

SOLUTION $\frac{A}{2} = \frac{2B}{3} = \frac{3C}{4} = \frac{4D}{5}$

$$\text{Therefore, } \frac{A}{B} = \frac{4}{3} \text{ and } \frac{B}{C} = \frac{9}{8} \text{ and } \frac{C}{D} = \frac{16}{15}$$

$$A : B = 4 : 3$$

$$B : C = 9 : 8$$

$$C : D = 16 : 15$$

$$\begin{aligned} A : B : C : D &= (4 \times 9 \times 16) : (3 \times 9 \times 16) \\ &: (3 \times 8 \times 16) : (3 \times 8 \times 15) \end{aligned}$$

$$A : B : C : D = 576 : 432 : 384 : 360$$

Therefore, the ratio of number of students of A and D = $576 : 360 = 8 : 5$

$$\text{Alternatively: } \frac{A}{2} = \frac{2}{3} B = \frac{3}{4} C = \frac{4}{5} D = k$$

$$\text{then } A : B : C : D = 2k : \frac{3}{2}k : \frac{4}{3}k : \frac{5}{4}k$$

$$= 24k : 18k : 16k : 15k$$

$$A : B : C : D = 24 : 18 : 16 : 15$$

$$A : D = 24 : 15 = 8 : 5$$

Properties of Proportion

- If four numbers (quantities) are in proportion then product of the extremes is equal to the product of the means and if these are not in proportion, then product of extremes is not equal to the product of the means.

i.e., if $a : b :: c : d$,
 then $a \times d = b \times c$

Thus it is clear that if three out of four terms of a proportion are given, then we can find the fourth term by dividing the product of means (or extremes) by the remaining term.

- If a, b and c are three numbers such that $a : b = b : c$ then these numbers a, b, c are said to be in continued proportion or simply in proportion.

$$\text{i.e., } a:b = b:c \Rightarrow b^2 = ac$$

Here b is said to be the **mean proportional** between a and c , and c is said to be the **third proportional** to a and b . e.g., $3:9::9:27$

Here $9 \times 9 = 3 \times 27$ which are in continued proportion.

NOTE Sometimes the above idea is also expressed by saying that the three numbers are in the ratio $3:9:27$. Thus if three quantities are proportionals, the first is to the third as the duplicate ratio of the first to the second.

i.e., if
then

$$a:b::b:c$$

$$a:c = a^2:b^2$$

$$\left[\because \frac{a}{b} = \frac{b}{c} \Rightarrow \frac{a^2}{b^2} = \frac{ab}{bc} \Rightarrow \frac{a^2}{b^2} = \frac{a}{c} \Rightarrow a:c = a^2:b^2 \right]$$

Some Facts about Proportion

(a) Invertendo: If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

EXAMPLE 1 The first, second and fourth terms of a proportion are 5, 15 and 90 respectively. Find the third term.

SOLUTION Let the third term be x , then 5, 15, x , 90 are in proportion i.e., $5:15::x:90$

$$\begin{aligned} \Rightarrow 5 \times 90 &= 15 \times x && \text{(by the 1st property)} \\ \Rightarrow x &= 30 \end{aligned}$$

EXAMPLE 2 The ratio of length to width of a rectangular sheet of paper is $5:3$. If the width of the sheet is 18 cm, find its length.

SOLUTION Let the length of sheet of paper be x cm. Then the ratio of length to width = $x:18$

$$\begin{aligned} \text{Thus } x:18 &= 5:3 \\ \Rightarrow x \times 3 &= 18 \times 5 \\ \Rightarrow x &= 30 \end{aligned}$$

Hence the length of paper = 30 cm.

EXAMPLE 3 If $81, x, x, 256$ are in proportion, find x .

SOLUTION $81:x::x:256$

$$\Rightarrow 81 \times 256 = x \times x \Rightarrow x = 144$$

EXAMPLE 4 The ratio between the number of men and women in an office is $5:7$. If the number of women working in the office is 56, find the number of men working in the office.

SOLUTION $5:7 = x:56$ (suppose number of men = x)

$$\Rightarrow x = 40 \quad \text{(by the first property)}$$

Therefore, number of men in the office = 40

EXAMPLE 5 The age of Chandi and Radhika are in the ratio $5:3$. If Chandi's age is 20 years, find the age of Radhika.

SOLUTION $5:3 = 20:x$

$$\Rightarrow x = \frac{3 \times 20}{5} \quad \text{(by the first property)}$$

$$\Rightarrow x = 12$$

Hence the age of Radhika is 12 years.

(b) Alternando: If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$

NOTE Each of the two results (a) and (b) can be obtained by cross product.

(c) Componendo: If $\frac{a}{b} = \frac{c}{d} \Rightarrow \left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$

(d) Dividendo: If $\frac{a}{b} = \frac{c}{d} \Rightarrow \left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right)$

(e) Componendo and Dividendo: If $\frac{a}{b} = \frac{c}{d}$

$$\Rightarrow \left(\frac{a+b}{a-b}\right) = \left(\frac{c+d}{c-d}\right)$$

NOTE The result (e) can be obtained by dividing result (c) by result (d).

EXAMPLE 6 The ratio of the number of boys to that of girls in a school is $9:11$. If the number of girls in the school is 2035, find:

- (a) number of boys in school
(b) number of students in school.

SOLUTION (a) $9:11 = x:2035$

$$\Rightarrow x = \frac{9 \times 2035}{11}$$

$$x = 1665 \quad \text{(number of boys)}$$

(b) number of students

$$\text{number of boys} + \text{number of girls} = 3700$$

EXAMPLE 7 What is the least possible number which must be subtracted from 16, 19 and 23 so that the resulting numbers are in continued proportion?

- (a) 2 (b) 4 (c) 6 (d) 7

SOLUTION Going through options, we find option (d) is correct.
Alternatively:

$$\begin{aligned} (16-x):(19-x)::(19-x):(23-x) \\ \Rightarrow (19-x)^2 = (16-x)(23-x) \end{aligned}$$

By solving the above equation, we get $x = 7$

EXAMPLE 8 If $(a+b):(a-b) = 15:1$, then the value of $a^2 - b^2$ is:

- (a) 56 (b) 15 (c) 112 (d) 8

SOLUTION $\frac{(a+b)}{(a-b)} = \frac{15}{1}$

$$\Rightarrow \frac{a}{b} = \frac{8}{7} \quad \text{(by componendo and dividendo)}$$

therefore $a^2 - b^2 = 64 - 49 = 15$

EXAMPLE 9 The mean proportional between 8 and 98 is:

- (a) 16 (b) 53 (c) 112 (d) 28

SOLUTION $8:x::x:98$

$$\Rightarrow x^2 = 8 \times 98$$

$$x = 28$$

Hence (d) is correct.

Ratio, Proportion and Variation

EXAMPLE 10 The students in three classes are in the ratio of 2:3:4. If 40 students are added in each class, the ratio becomes 4:5:6. Find the total number of students in all the three classes is :

(a) 270 (b) 180 (c) 126 (d) 135

SOLUTION

$$\begin{aligned} 2x + 40 &= 4y & \dots (i) \\ 3x + 40 &= 5y & \dots (ii) \\ 4x + 40 &= 6y & \dots (iii) \\ 2x &= 40 \\ x &= 20 \end{aligned}$$

Therefore

$$\begin{aligned} \Rightarrow \text{Hence, total number of students} &= 2x + 3x + 4x = 9x \\ &= 9 \times 20 = 180 \end{aligned}$$

Alternatively: It can be solved through options also.

EXAMPLE 11 The dimensions of a photograph are 4 and 1.8 cms. If the breadth of the enlarged photo is 4.5 cm and it was enlarged proportionally then what is the new length of new photograph?

(a) 6 (b) 5.4 (c) 10 (d) 9

SOLUTION

$$4 : 1.8 = x : 4.5$$

$$\Rightarrow x = 10$$

Thus, the length of new photograph is 10 cm.

EXAMPLE 12 Two equal containers are filled with the mixture of milk and water. The concentration of milk in each of the containers is 20% and 25% respectively. What is the ratio of water in both the containers respectively?

(a) 15:16 (b) 16:15 (c) 4:5 (d) 5:4

SOLUTION

Milk	20%	25%
Water	80%	75%

$$\text{Thereofre, required ratio} = \frac{80}{75} = \frac{16}{15} \text{ or } 16:15$$

UNITARY METHOD

A method in which the value of a quantity is first obtained to find the value of any required quantity is called unitary method.

In solving problems based on unitary method.

Direct Proportion

Two quantities are said to be directly proportional if the increase (or decrease) in one quantity causes the increase (or decrease) in the other quantity by same proportion. e.g.,

(i) The cost of articles varies directly to the number of articles. More articles more cost, less articles less cost.

EXAMPLE 1 If 6 note books cost Rs. 45, how much would 8 notebooks cost?

SOLUTION More note books more cost; less note books, less cost

Note books	Cost
6	45
1	$45/6$
8	$\frac{8 \times 45}{6} = 60$

Hence 8 note books cost Rs. 60.

Compiled By Jasjeet

EXAMPLE 13 A cat takes 7 steps for every 5 steps of a dog, but 5 steps of a dog are equal to 6 steps of cat. What is the ratio of speed of cat to that of dog?

- (a) 24:25 (b) 42:25 (c) 24:19 (d) 25:42

SOLUTION

CAT	DOG
Given speed 7 steps	5 steps but the length of 5 steps of dog = length of 6 steps of cat

it means the ratio of length covered by dog is to cat = $\frac{6}{5}$

Therefore, in each step a dog will cover $\frac{6}{5}$ times distance than that of a cat.

Thus the ratio of actual speed of cat is to dog

$$= 7 : 5 \times \frac{6}{5} = 7 : 6$$

SHORTCUT Actual speed of A : B

$$= \frac{\text{Given speed of A}}{\text{no. of steps of A in terms of length}} :$$

$$\frac{\text{Given speed of B}}{\text{no. of steps of B in terms of length}}$$

$$= \text{CAT : DOG}$$

$$= \frac{7}{6} : \frac{5}{5} = 7 : 6$$

EXAMPLE 14 A camel pursue an elephant and takes 5 leaps for every 7 leaps of the elephant, but 5 leaps of elephant are equal to 3 leaps of camel. What is the ratio of speeds of camel and elephant?

SOLUTION Ratio of speed of camel and elephant

$$\begin{aligned} &= \frac{5}{3} : \frac{7}{5} = \frac{5}{3} \times 15 : \frac{7}{5} \times 15 \\ &= 25 : 21 \end{aligned}$$

(ii) The work done varies directly to the number of men (work force) at work. More men at work, more work done in same time. Less men, less work done in same time.

Inverse Proportion

Two quantities are said to vary inversely if the increase (or decrease) in one quantity causes the decrease (or increase) in the other quantity by same proportion e.g., The time taken to finish a work varies inversely to the number of men at work.

More men at work, less time taken to finish the same work. Less men at work, more time taken to finish the same work.

EXAMPLE 2 If 45 students can consume a stock of food in 2 months, find for how many days the same stock of food will last for 27 students?

SOLUTION More students, less days; less students, more days

Students	Days
45	60
1	60×45
27	$\frac{60 \times 45}{27} = 100$ days

188

EXAMPLE 3 A man working 8 hours a day takes 5 days to complete a project. How many hours a day must he work to complete it in 4 days?

SOLUTION 'More days, less hours; less days, more hours'

VARIATION

When two or more quantities are dependent upon each other and then if any one of them is changed, the other (dependent) quantity is also changed.

For example :

- When the salary of a person increases, then its savings/expenditure increases.
- When the number of guests in a hotel/number of students in a hostel/number of employees changes, their respective expenses increases.

Basically, as it happens in direct proportion and inverse proportion, there are two types of variation :

- Direct variation (ii) Inverse variation

Direct Variation

A quantity A is said to vary directly if the increase (or decrease) in B yields increase (or decrease) in A but not in proportion. It is expressed as

$$A \propto B \Rightarrow A = KB,$$

where K is called proportionality constant

$$\Rightarrow K = \frac{A}{B}$$

EXAMPLE 1 A varies directly as B and inversely as C . A is 12 when B is 6 and C is 2. What is the value of A when B is 12 and C is 3?

SOLUTION $A \propto B$ and $A \propto \frac{1}{C}$
 $\Rightarrow A \propto \frac{B}{C} \Rightarrow A = K \frac{B}{C}$

when $A = 12$, $B = 6$, $C = 2$, then

$$12 = K \frac{6}{2} \Rightarrow K = 4$$

Again $A = K \frac{B}{C} = 4 \times \frac{12}{3} = 16$

$$\Rightarrow A = 16$$

EXAMPLE 2 The value of a coin varies directly to the square of its radius, when its thickness is constant. The radius of a coin is 1.5 cm and its value is Rs. 2. What will be the radius of a coin if its value is Rs. 5?

PROBLEMS BASED ON AGES

This article is very suitable as an appendix of Ratio-proportion, since most of the questions based on ages involve the concept of ratio-proportion. e.g., the age of Ravi is

Ratio, Proportion and Variation		
Days	Hours	
5	8	
1	5×8	
4	5×8	
	$\frac{4}{4} = 10$ hours a day	

Inverse Variation

A quantity A is said to vary inversely if the increase (or decrease) in B yields decrease (or increase) in A but not in proportion. It is expressed as

$$A \propto \frac{1}{B} \Rightarrow A = \frac{K}{B}$$

or $K = AB$, K is called as proportionality constant.

NOTE

- If it is not mentioned that a particular quantity is inversely variable, then it means the given quantity is directly variable.
- A quantity sometimes vary jointly i.e., directly on any quantity and inversely on another quantity.

e.g., $A \propto B$ and $A \propto \frac{1}{C}$

It means $A \propto \frac{B}{C} \Rightarrow A = \frac{KB}{C}$

Here A varies directly as B but inversely as C .

Also it can vary as only directly or inversely as more than one quantity.

e.g., $A \propto BC \Rightarrow A = KBC$

and $A \propto \frac{1}{BC} \Rightarrow A = \frac{K}{BC}$

SOLUTION

$$\Rightarrow V \propto r^2$$

$$V = Kr^2$$

$$2 = K \times (1.5)^2$$

$$K = \frac{2}{2.25}$$

$$K = \frac{8}{9}$$

Again

$$5 = \frac{8}{9} \times r^2$$

$$\Rightarrow r^2 = \frac{5 \times 9}{8}$$

$$r = \frac{3}{2} \times \sqrt{\frac{5}{2}}$$

$$= 1.5 \times \sqrt{2.5} = 1.5 \times 1.6$$

$$r \approx 2.4 \text{ cm}$$

Hence, required radius = 2.4 cm

EXAMPLE 1 The ratio of ages of Krishna and Balram is 3 : 4. Four years earlier the ratio was 5 : 7. Find the present ages of Krishna and Balram :

(a) 15 years, 20 years (b) 24 years, 32 years
 (c) 16 years, 20 years (d) 32 years, 24 years

SOLUTION Let the present age of Krishna and Balram be $3x$ and $4x$, then

four years ago their ages be $(3x - 4)$ and $(4x - 4)$

$$\text{So } \frac{(3x - 4)}{(4x - 4)} = \frac{5}{7}$$

$$\Rightarrow 7(3x - 4) = 5(4x - 4)$$

$$\Rightarrow 21x - 28 = 20x - 20$$

$$\Rightarrow x = 8$$

$$\therefore \text{Present age of Krishna} = 3x = 24 \text{ years}$$

$$\text{and age of Balram} = 4x = 32 \text{ years}$$

EXAMPLE 2 The ratio of age of Aman and her mother is 3 : 11. The difference of their ages is 24 years. What will be the ratio of their ages after 3 years?

SOLUTION Let the present age of Aman and her mother be $3x$ and $11x$ then 3 years later their ages will be $(3x + 3)$ and $(11x + 3)$ respectively.

$$\text{Again } 11x - 3x = 24$$

PARTNERSHIP

When two or more than two people run a business jointly by investing their money/resources, then it is called a joint venture or the business in partnership.

All these people, who have invested their resources, are called as Partners.

Partners are basically of two types

- (i) **Working partner:** A partner who is directly involved with day-to-day activities of business is called as working partner.
- (ii) **Sleeping partner:** A partner who just invests his or her money is called as sleeping partner.

General rules of partnership :

- (i) If the partners invest different amounts for the same

EXAMPLE 1 Bhanu and Shafeeq started a business by investing Rs. 36,000 and Rs. 63,000. Find the share of each, out of an annual profit of Rs. 5,500.

SOLUTION Ratio of shares of Bhanu and Shafeeq

$$= 36000 : 63000 = 4 : 7$$

$$\therefore \text{Share of Bhanu} = 5500 \times \frac{4}{11} = \text{Rs. 2000}$$

$$\text{and Share of Shafeeq} = 5500 \times \frac{7}{11} = 3500$$

EXAMPLE 2 A starts some business with Rs. 50,000. After 3 months B joins him with Rs. 70,000. At the end of the year, in what ratio should they share the profits?

Compiled By Jasjeet

$$\Rightarrow 8x = 24 \Rightarrow x = 3$$

Therefore the ratio of their ages 3 years after

$$= \frac{3x + 3}{11x + 3} = \frac{12}{36} = \frac{1}{3}$$

EXAMPLE 3 The age of Sachin is 4 times that of his son. Five years present age of the Sachin is :

- (a) 25 years (b) 36 years
 (c) 32 years (d) 48 years

SOLUTION Let the age of son is x years, then the age of Sachin will be $4x$ years.

$$\therefore (4x - 5) = 9(x - 5) \Leftrightarrow x = 8$$

\therefore Age of Sachin is 32 years.

EXAMPLE 4 The ratio of Varun's age and his mother's age is 5 : 11. The difference of their ages is 18 years. The ratio of their ages after 5 years will be :

- (a) 19 : 59 (b) 2 : 3 (c) 37 : 75 (d) 10 : 19

SOLUTION Let their ages be $5x$ and $11x$.

$$11x - 5x = 18 \Leftrightarrow x = 3$$

So their present ages are 15 and 33 years. Therefore, ratio of their ages after 5 years = 20 : 38 = 10 : 19.

period of time, then the profits of all the partners are shared in the ratio of their investments.

- (ii) If the partners invest same amount for the different time periods, then the profits of all the partners are shared in the ratio of time periods for which their amounts were invested.
 - (iii) If the partners invest different amounts for different time periods, then their profits are shared in the ratio of products of respective investments with the time period for each partner, individually.
- Thus gain or loss is divided in the ratio of 'money-time' capitals.

NOTE Sometimes different problems are solved on the basis of partnership to find the expenses.

SOLUTION Ratio of amount of A and B = 50,000 : 70,000

Ratio of time periods for A and B = 12 : 9

\therefore Ratio of their money-time capital

$$\text{Investments} = 50000 \times 12 : 70000 \times 9 = 20 : 21$$

EXAMPLE 3 Harsh Vardhan started a business by investing Rs. 36,000. After 4 months Gyan Vardhan joined him with some investment. At the end of the year, the total profit was divided between them in the ratio of 9 : 7. How much capital was invested by Gyan Vardhan in the business?

SOLUTION

$$\frac{36000 \times 12}{x \times 8} = \frac{9}{7}$$

$$x = 42000$$

\Rightarrow Gyan Vardhan invested Rs. 42,000 for 8 months only.

EXAMPLE 4 A started some business with Rs. 26,000. After 3 months B joined him with Rs. 16,000. After some more time C joined them with Rs. 25,000. At the end of the year, out of a total profit of Rs. 15,453, C gets Rs. 3825 as his share. How many months after B joined the business did C join?

SOLUTION Ratio (of share) of profits

$$= 26,000 \times 12 : 16000 \times 9 : 25000 \times C$$

$$\text{Now } C's \text{ share} = \frac{25C}{456 + 25C} = \frac{3825}{15453}$$

↑ $C = 6$

Therefore C joined 3 months later than B joined.

EXAMPLE 5 A, B and C started a business with their investments in the ratio $1 : 2 : 4$. After 6 months A invested the half amount more as before and B invested twice the amount as before while C withdrew $\frac{1}{4}$ th of the their investments. Find the ratio of their profits at the end of the year.

SOLUTION Let us assume their initial investments were x , $2x$ and $4x$ respectively.

Therefore, ratio of their investments during the whole year

$$= (x \times 6 + \frac{3x}{2} \times 6) : (2x \times 6 + 4x \times 6) : (4x \times 6 + 3x \times 6)$$

$$= 15x : 36x : 42x$$

$$= 5x : 12x : 14x$$

$$= 5:12:14$$

∴ Ratio of their profits = 5: 12: 14

UNITARY METHOD

EXAMPLE 6 A started a business with Rs. 52,000 and after 5 months B joined him with Rs. 39,000. At the end of the year, out of the total profits B received total Rs. 20,000 including 25% of the profits as commission for managing the business. What amount did A receive?

A receiver profit's share of *A* and *B*

$$= 52,000 \times 12 : 39,000 \times 8 = 2 : 1$$

Let the profit be Rs. x , then B receives 25% as commission for managing business, the remaining 75% of the total profit is shared between A and B in the ratio 2 : 1. Hence B will get $\frac{1}{3}$ rd part of this in addition to his commission. Hence his total earning

$$= 0.25x + \frac{1}{3} \times 0.75x$$

$$= 0.5x = 20,000$$

$$x = 40,000$$

So, the remaining profit goes to A , hence the profit of A is $\text{₹}20,000$.

EXAMPLE 7 A working partner gets 20% as his commission of the profit after his commission is paid. If the working partner's commission is Rs. 8000, then what is the total profit in the business?

SOLUTION Let the total profit be Rs. x . The remaining profit after paying 20% working partner's commission = $(x - 8000)$. Again since 20% of this is working partner's commission,

$$\text{therefore } \frac{20}{100} \times (x - 8000) = 8000$$

$\Rightarrow x = 48,000$

∴ The total profit in the business is Rs. 48,000.

INTRODUCTORY EXERCISE-4.1

10. 15 men take 42 days of 4 hours each to do a piece of work. How many days of 6 hours each would 21 women take if 3 women do as much work as 2 men?
- (a) 15
(b) 22
(c) 25
(d) 30

11. 10 engines consume 80 litres diesel when each is running 9 hours a day. How much diesel will be required for 10 engines, each running 15 hours a day, whereas 6 engines of the former type consume as much as 5

engines of latter type?

- (a) 160 litre
(b) 75 litre
(c) 80 litre
(d) $111\frac{1}{9}$ litre

12. If $\frac{3}{5}$ th of a cistern is filled in 30 minutes, how much more time will be required to fill the rest of it?
- (a) 50 minutes
(b) 20 minutes
(c) 15 minutes
(d) 45 minutes

AGE RELATED PROBLEMS

1. Amit is as much younger to Barkha as he is older to Chaman. If the sum of the ages of Barkha and Chaman is 48 years, what is the present age of Amit?
- (a) 18 years
(b) 36 years
(c) 24 years
(d) 28 years

2. Bipin is 6 times old as Alok. Bipin's age will be twice of Chandan's age after 10 years. If Chandan's 7th birthday was celebrated 3 years ago, what is Alok's present age?
- (a) 15 years
(b) 12 years
(c) 5 years
(d) none of these

3. Renuka got married 8 years ago. Today her age is $1\frac{1}{3}$ times her age at the time of marriage. Her daughter's age is $\frac{1}{8}$ times her age. Her daughter's age is :

INTRODUCTORY EXERCISE-4.2

- (a) 3 years
(b) 4 years
(c) 6 years
(d) 8 years
4. Ten years ago B was twice of A in age. If the ratio of their present ages is 4 : 3, what is the sum of their present ages?
- (a) 25 years
(b) 30 years
(c) 40 years
(d) 35 years
5. The sum of the ages of Aryabhatta and Shridhar is 45 years. Five years ago the product of their ages was 4 times the Aryabhatta's age at that time. The present ages of Aryabhatta and Shridhar respectively are :
- (a) 25 and 20
(b) 35 and 10
(c) 36 and 9
(d) 40 and 5

PARTNERSHIP

INTRODUCTORY EXERCISE-4.3

1. A company make a profit of Rs. 9,00,000, 20% of which is paid as taxes. If the rest is divided among the partners P, Q and R in the ratio of $1:1\frac{1}{2}:2$, then the shares of P, Q and R are respectively :
- (a) 2,40,000; 3,20,000; 1,60,000
(b) 3,20,000; 2,40,000; 1,60,000
(c) 1,60,000; 3,20,000; 2,40,000
(d) 1,60,000; 2,40,000; 3,20,000

2. We have to divide a sum of Rs. 13,950 among three persons A, B and C. B must get the double of A's share and C must get Rs. 50 less than the double of B's share. The share of A will be :
- (a) Rs. 1950
(b) Rs. 1981.25
(c) Rs. 2000
(d) Rs. 2007.75

3. A started business with Rs. 45,000 and B joined afterward with Rs. 30,000. If the profits at the end of one year were divided in the ratio 2 : 1 respectively, then B would have joined A for business after :

- (a) 1 month
(b) 2 months
(c) 3 months
(d) 4 months
4. A and B are partners in a business. They invest in the ratio 5 : 6, at the end of 8 months A withdraws. If they receive profits in the ratio of 5 : 9, find how long B's investment was used?
- (a) 12 months
(b) 10 months
(c) 15 months
(d) 14 months
5. Four milkmen rented a pasture. A put to graze 16 cows for 3 months, B 20 cows for 4 months, C 18 cows for 6 months and D 42 cows for 2 months. If A's share of rent be Rs. 2400, the rent paid by C is :
- (a) Rs. 3200
(b) Rs. 4200
(c) Rs. 4000
(d) Rs. 5400
6. A, B and C subscribe Rs. 47000 for a business. If A subscribes Rs. 7000 more than B and B Rs. 5000 more than C, then out of total profit of Rs. 4700, C receives :
- (a) Rs. 1200
(b) Rs. 4500
(c) Rs. 1000
(d) none of these

- If $A:B = 4:5$, $B:C = 3:4$, $C:D = 7:11$, then $A:D$ is :
 - $3:4$
 - $21:55$
 - $21:44$
 - $7:5$
- Mean proportional between 17 and 68 is :
 - 51
 - 24
 - 4
 - 34
- Third proportional between 16 and 36 is :
 - 64
 - 144
 - 81
 - 49
- If $a:b = 2:3$, then $(5a+b):(3a+2b)$ is :
 - $13:12$
 - $15:17$
 - $12:13$
 - $13:11$
- $a = 2b = 3c = 4d$, then $a:b:c:d$ is :
 - $12:3:6:4$
 - $3:4:6:12$
 - $6:12:4:3$
 - $12:6:4:3$
- The fourth proportional to 4, 7 and 20 is :
 - 28
 - 21
 - 18
 - 35
- If $\sqrt{2}:(1+\sqrt{3})::\sqrt{6}:x$, then x is equal to :
 - $1+\sqrt{3}$
 - $\sqrt{3}-1$
 - $\sqrt{3}+3$
 - $2\sqrt{3}$
- If $\frac{a}{3} = \frac{b}{4} = \frac{c}{5}$ then $\frac{a+b+c}{b} = ?$
 - 2
 - 3
 - 4
 - 5
- If $\frac{a}{b} = \frac{c}{d}$, then :

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$	$\frac{a+b}{a^2} = \frac{c+d}{d^2}$
$\frac{a+b}{a^2} = \frac{c+d}{c^2}$	$ac = bd$
- If $a:b = b:c = c:d$ then $\frac{a}{b}, \frac{b}{c}, \frac{c}{d}$ are :
 - in AP
 - in continued proportion
 - in GP
 - both (b) and (c)
- If $(a+b):(a-b) = 3:2$, then $(a^2 - b^2):(a^2 + b^2)$ equals :
 - $5:13$
 - $12:13$
 - $9:4$
 - none of these
- Two whole numbers, whose sum is 64, can not be in the ratio :
 - $1:7$
 - $3:5$
 - $5:11$
 - $1:2$
- Two numbers are in the ratio 3:4. The difference between their squares is 28. Find the greater number :
 - 12
 - 8
 - 24
 - 16

- If a, b, c, d, e, f, g are in continued proportion, then the value of $a.b.c.e.f.g$ is :
 - d^3
 - d^6
 - d^9
 - none of these
- If A and B shared Rs. 1300 in the ratio 1:12, how much did A get?
 - 120
 - 1200
 - 100
 - 1000
- Rs. 3960 are divided among A , B and C so that half of A 's part, one-third of B 's part and one-sixth of C 's part are equal. Then C 's part is :
 - 720
 - 2160
 - 1080
 - 810
- A sum of Rs. 21000 is divided among A , B , C such that shares of A and B are in the ratio of 2:3 and those of B and C are in the ratio 4:5. The amount received by A is :
 - Rs. 6000
 - Rs. 4500
 - Rs. 4800
 - Rs. 8400
- A certain amount was divided between A and B in the ratio 7:9. If B 's share was Rs. 7200, the total amount was :
 - Rs. 1280
 - Rs. 6300
 - Rs. 5600
 - Rs. 12800
- Rs. 11250 are divided among A , B and C so that A may receive one-half as much as B and C together receive and B receives one-fourth of what A and C together receive. The share of A is more than that of B by :
 - Rs. 2500
 - Rs. 1500
 - Rs. 1800
 - Rs. 650
- A girl 1.2 metre tall casts a shadow 1.1 m at the time when a building casts a shadow 6.6 m long. The height of the building is :
 - 2.7 m
 - 7.2 m
 - 6.0 m
 - 5.5 m
- The prices of Bajaj Scooter and Bajaj Pulser are in the ratio of 4:9. If the Bajaj Pulser costs Rs. 30,000 more than a Bajaj Scooter, the price of Bajaj Pulser is :
 - Rs. 63,000
 - Rs. 45,000
 - Rs. 54,000
 - Rs. 60,000
- What is the ratio whose terms differ by 40 and the measure of which is 2/7?
 - 16:56
 - 14:49
 - 15:36
 - 16:72
- Two numbers are in the ratio 3:5. If 9 be subtracted from each, then they are in the ratio 12:23. The second number is :
 - 53
 - 55
 - 54
 - 52
- Consider the following statements :
 - If both the terms of a ratio are multiplied or divided by the same natural number, then the ratio remains unaltered.

- (2) A statement which states that two ratios are equivalent is called proportion.
- (3) If 4 quantities are in proportion, the product of extremes is not equal to the means.
- (4) The mean proportion between any two numbers is equal to the square root of their product. The wrong one statement is/are :
- (a) 1 (b) 3
(c) 3 and 4 (d) 1 and 4
25. In a mixture of 120 litres, the ratio of milk and water is 2:1. If the ratio of milk and water is 1:2, then the amount of water (in litres) is required to be added is :
(a) 20 (b) 40
(c) 80 (d) 120
26. A quantity x varies inversely as the square of y . Given that $x = 4$, when $y = 3$, the value of x when $y = 6$ is :
(a) 1 (b) 2
(c) 3 (d) 4

27. Suppose y varies as the sum of two quantities of which one varies directly as x and the other inversely as x . If $y = 6$ when $x = 4$ and $y = 3 \frac{1}{3}$ when $x = 3$, then the relation between x and y is :
(a) $x = y + 4$ (b) $y = 2x + \frac{8}{x}$
(c) $y = 2x - \frac{8}{x}$ (d) $y = 2x - \frac{4}{x}$
28. The time period of a pendulum is proportional to the square root of the length of the pendulum. Consider the following statements :
(1) If the length of the pendulum is doubled, then the time period is also doubled.
(2) If the length is halved, then time period becomes one-fourth of the original time period.
The correct assertions are :
(a) 1 (b) 2
(c) neither 1 nor 2 (d) both 1 and 2

EXERCISE

LEVEL 1

1. Four numbers are in proportion. The sum of the squares of the four numbers is 50 and the sum of the means is 5. The ratio of first two terms is 1 : 3. What is the average of the four numbers?
 (a) 2 (b) 3
 (c) 5 (d) 6

2. A naughty student breaks the pencil in such a way that the ratio of two broken parts is same as that of the original length of the pencil to one of the larger part of the pencil. The ratio of the other part to the original length of pencil is :
 (a) $1 : 2\sqrt{5}$ (b) $2 : (3 + \sqrt{5})$
 (c) $2 : \sqrt{5}$ (d) can't be determined

3. If $\frac{x+y}{x-y} = \frac{5}{3}$ and $\frac{x}{(y+2)} = 2$, the value of (x, y) is :
 (a) (4, 1) (b) (2, 8)
 (c) (1, 4) (d) (8, 2)

4. If $a^3 + b^3 : a^3 - b^3 = 133 : 117$; find $a : b$:
 (a) 2 : 3 (b) 5 : 4
 (c) 5 : 2 (d) none of these

5. A student obtained equal marks in History and Sociology. The ratio of marks in Sociology and Geography is 2 : 3 and the ratio of marks in History and Philosophy is 1 : 2. If he has scored an aggregate of 55% marks. The maximum marks in each subject is same. In how many subjects did he score equal to or greater than 60% marks?
 (a) 1 (b) 2
 (c) 3 (d) none of these

6. The ratio of income of Anil and Mukesh is 2 : 3. The sum of their expenditure is Rs. 8000 and the amount of savings of Anil is equal to the amount of expenditure of Mukesh. What is the sum of their savings?
 (a) 22,000 (b) 4,000
 (c) 16,000 (d) 12,000

7. Hutch and Essar entered into a partnership just 5 months ago. The ratio of profit claimed by Hutch and Essar is 6 : 17. If Essar had just started his business 12 months ago with Rs. 1275, what is the amount contributed by Hutch?
 (a) Rs. 980 (b) Rs. 1080
 (c) Rs. 1200 (d) Rs. 998

8. A child has three different kinds of chocolates costing Rs. 2, Rs. 5 and Rs. 10. He spends total Rs. 120 on the chocolates. What is the minimum possible number of chocolates, he can buy, if there must be atleast one chocolate of each kind?
 (a) 22 (b) 19
 (c) 17 (d) 15

9. In the following, which is the minimum possible no. of chocolates?
 (a) 52 (b) 53
 (c) 55 (d) 60

10. Mr. Teremere and Mr. Meretere have 5 chocolates and 3 chocolates with them respectively. Meanwhile Mr. Khabbu Singh joined them and all 8 chocolates were distributed equally among all these three people. In turn Khabbu Singh gave Rs. 16 to Mr. Teremere and Mr. Meretere, since Khabbu Singh did not have any chocolate. What is the difference of amounts received by Teremere and Meretere? Given that the amount was shared in proportion of chocolates received by Khabbu Singh.
 (a) Rs. 8 (b) Rs. 12
 (c) Rs. 14 (d) Rs. 15

11. Rs. 4536 is divided among 4 men, 5 women and 2 boys. The ratio of share of a man, a woman and a boy is 7 : 4 : 3. What is the share of a woman?
 (a) Rs. 336 (b) Rs. 498
 (c) Rs. 166 (d) Rs. 256

12. The concentration of petrol in three different mixtures (petrol and kerosene) is $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{4}{5}$ respectively. If 2 litres, 3 litres and 1 litre are taken from these three different vessels and mixed. What is the ratio of petrol and Kerosene in the new mixture?
 (a) 4 : 5 (b) 3 : 2
 (c) 3 : 5 (d) 2 : 3

13. Time period (T) of pendulum is directly proportional to the square root of length of string by which bob is attached to a fixed point and inversely proportional to the square root of gravitational constant ' g '. Time period of a bob is 3 seconds when the gravitational constant g is 4 m/sec² and length of string is 9 metre, what is the time period of a bob having a string of length 64 metre and gravitational constant 16 m/sec²?
 (a) 4 seconds (b) 12 seconds
 (c) 16 seconds (d) 10 seconds

14. In a milk shoppe there are three varieties of milk, 'Pure', 'Cure' and 'Lure'. The 'Pure' milk has 100% concentration of milk. The ratio of milk is to water in the 'Cure' is 2 : 5 and in the 'Lure' it is 3 : 8 respectively. Sonali purchased 14 litres of Cure and 22 litres of Lure milk and mixed them. If she wanted to make 50%. How many litres of 'Pure' milk she is needed?
 (a) 6 litres (b) 8 litres
 (c) 16 litres (d) 18 litres

15. In the squad Mig and Jaguar together is 5:7 and the ratio of Jaguar is to Sukhoi and Mig together is 1:2. Find the ratio of Sukhoi and Mig :
 (a) 2:7
 (b) 3:5
 (c) 3:1
 (d) 5:3

16. During our campaign against child labour we have found that in three glass making factories A, B and C there were total 33 children aged below 18 were involved. The ratio of male to female in A, B and C was 4:3, 3:2 and 5:4 respectively. If the no. of female children working in the factories B and C be equal then find the no. of female children working in factory A :
 (a) 5
 (b) 2
 (c) 8
 (d) 6

17. The value of a diamond is directly proportional to the square of its weight. A diamond unfortunately breaks into three pieces with weights in the ratio of 3:4:5 thus a loss of Rs. 9.4 lakh is incurred. What is the actual value of diamond :
 (a) 28.8 lakh
 (b) 13.5 lakh
 (c) 14.4 lakh
 (d) 18.8 lakh

18. In the Ruchika's wallet there are only Rs. 16, consisting of 10 paise, 20 paise and Re. 1 coins. The ratio of no. of coins of 10 paise and 20 paise is 6:1. The minimum no. of Re 1 coin is :
 (a) 5
 (b) 12
 (c) 4
 (d) 8

19. There are two vessels containing the mixture of milk and water. In the first vessel the water is $\frac{2}{3}$ of the milk and in the second vessel water is just 40% of the milk. In what ratio these are required to mix to make 24 litres mixture in which the ratio of water is to milk is 1:2?
 (a) 4:3
 (b) 5:7
 (c) 5:2
 (d) 7:5

20. Nehru Ji had 'n' chocolates. He distributed them among 4 children in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{5} : \frac{1}{8}$. If he gave them each one a complete chocolate, the minimum no. of chocolates that he had :
 (a) 139
 (b) 240
 (c) 278
 (d) none of these

21. The ratio of working efficiency of A and B is 5:3 and the ratio of efficiency of B and C is 5:8. Who is the most efficient ?
 (a) A
 (b) B
 (c) C
 (d) can't be determined

22. If $4A = 5B$ and $3A = 2C$, the ratio of B:C is :
 (a) 4:3
 (b) 5:8
 (c) 8:15
 (d) 10:15

23. Equal quantities of three mixtures of milk and water are mixed in the ratio of 1:2, 2:3 and 3:4. The ratio of water and milk in the mixture is :
 (a) 193:122
 (b) 122:193
 (c) 61:97
 (d) 137:178

24. The ratio of age of A and B is 8:9 and the age of B is $\frac{2}{3}$ of C's age and age of C is $\frac{9}{13}$ times the age of D. If the age of B is 18 years then the age of C is :
 (a) 36 years
 (b) 39 years
 (c) 27 years
 (d) 54 years

25. A milk man has a mixture of milk in which ratio of milk and water is 5:3. He sells 40 litres of milk i.e., mixture then he adds up 15 litres of pure water. Now the ratio of milk and water is 5:4. What is the new quantity of mixture?
 (a) 72 litres
 (b) 270 litres
 (c) 135 litres
 (d) data insufficient

26. A and B are two alloys of copper and tin prepared by mixing the respective metals in the ratio of 5:3 and 5:11 respectively. If the alloys A and B are mixed to form a third alloy C with an equal proportion of copper and tin, what is the ratio of alloys A and B in the new alloy C?
 (a) 3:5
 (b) 4:5
 (c) 3:2
 (d) 2:3

27. A hotel incurs two types of expenses, one which is fixed and others depend upon no. of guests. When there are 10 guests, total expenses of hotel are Rs. 6000. Also when there are 25 guests average expenses per guests are Rs. 360? What is the total expenses of hotel when there are 40 guests?
 (a) Rs. 8,000
 (b) Rs. 12,000
 (c) Rs. 15,500
 (d) none of these

28. The ratio of third proportional to 21 and 42 and mean proportional to 16 and 49 is :
 (a) 3:1
 (b) 2:3
 (c) 4:3
 (d) 1:3

29. The period of the pendulum is directly proportional to the square root of the length of the string. The period of such a pendulum with string of length 16 cm is 52 seconds. Find the length of the string if the period is 65 seconds :
 (a) 4.5 cm
 (b) 5 cm
 (c) 6 cm
 (d) none of these

30. For any two numbers m, n ; $(m+n):(m-n):mn = 7:1:60$
 Find the value of $\frac{1}{m} : \frac{1}{n}$
 (a) 4:3
 (b) 8:6
 (c) 3:4
 (d) 7:8

31. Rs. 960 were distributed among A, B, C and D in such a way that C and D together gets half of what A and B together gets and C gets one-third amount of B. Also D gets $\frac{5}{3}$ times as much as C. What is the amount of A?
 (a) Rs. 240
 (b) Rs. 280
 (c) Rs. 320
 (d) data insufficient

32. Find the value of $\frac{p^2 + q^2}{r^2 + s^2}$, if $p:q::r:s$.
 (a) $\frac{1}{4}$
 (b) $\frac{1}{9}$
 (c) $\frac{ps}{rq}$
 (d) $\left(\frac{p-q}{r-s}\right)^2$

33. Find the value of $\frac{p^2 + q^2}{r^2 + s^2}$, if $p:q::r:s$.
 (a) $\frac{1}{4}$
 (b) $\frac{pq}{rs}$
 (c) $\left(\frac{p+q}{r+s}\right)^2$
 (d) both (b) and (c)

34. The speeds of rickshaw, car and scooter are in the ratio of 3:5:6. What is the ratio of time taken by each one of them for the same distance?
 (a) 6:5:3
 (b) 10:6:5
 (c) data insufficient

- the age of Varsha and Vikram will be 2 : 3. Find the ratio of ages of Vinay and Veera :

 - 7 : 6
 - 5 : 8
 - 6 : 7
 - 8 : 9

11. A container is filled with the mixture of milk and water. The ratio of milk and water is same. Bobby and Sunny increases the concentration to 60%. Bobby makes it by adding the milk and Sunny makes it by replacing the mixture with milk. What is the percentage of milk added by Bobby to that of milk replaced by Sunny :

 - 100%
 - 120%
 - 133.33%
 - none of these

12. There are two vessels A and B containing 25 litres each of pure milk and pure water respectively. 5 litres of milk from A is taken and poured into B , then 6 litres of mixture from B is taken and poured in A . What is the ratio of water in A and B respectively :

 - 4 : 5
 - 1 : 4
 - 5 : 4
 - 2 : 3

13. The ratio of age between A and B is 6 : 5 and the age of each C and D is $\frac{9}{10}$ times that of B . Age of F is less than A but greater than B . The ratio of ages between B and E is 2 : 3 also age of A is 3 years less than E . What is the ratio of ages of A and F if all the ages are in integers?

 - 12 : 11
 - 9 : 7
 - 24 : 19
 - 12 : 13

14. The ratio of students in a coaching preparing for B.Tech and MBA is 4 : 5. the ratio of fees collected from each of B. Tech and MBA student is 25 : 16. If the total amount collected from all the students is 1.62 lakh, what is the total amount collected from only MBA aspirants?

 - Rs. 62,000
 - Rs. 72,000
 - Rs. 80,000
 - none of these

15. The cost of the marble varies directly with square of its weight. Marble is broken into 3 parts whose weights are in the ratio 3 : 4 : 5. If marble had been broken into three equal parts by weight then there would have been a further loss of Rs. 1800. What is the actual cost of the original (or unbroken) marble?

 - Rs. 3600
 - Rs. 10,800
 - Rs. 2160
 - none of these

Directions for 16 and 17: Four friends A , B , C and D have some money among them one day they decided to equate the money, so first A gave B what B had initially, then B gave C what C had initially. Again C gave D what D had initially and finally D had doubled the money of A . Thus each of them had equal sum of Rs. 48.

16. What was the initial amount of B ?

 - Rs. 36
 - Rs. 54
 - Rs. 45
 - Rs. 42

17. What was the amount with C after second transaction?

 - Rs. 45
 - Rs. 69
 - Rs. 72
 - Rs. 84

18. Hari and Murli have 24 cows and 30 cows respectively. Both of them together hired a grazing field for the whole month of November. The cost of the grazing field is Rs. 1500. Hari has to pay for the remaining days of the month Murli's cows grazed it. If Hari has paid Rs. 3500 and Murli has paid Rs. 5000 for grazing then for how many days Hari used the grazing field :

 - 14
 - 16
 - 21
 - 20

19. The speeds of scooter, car and train are in the ratio of 1 : 4 : 15. If all of them covers equal distance then the ratio of time taken/velocity for the each of the vehicle is :

 - 256 : 16 : 1
 - 1 : 4 : 16
 - 16 : 4 : 1
 - 16 : 1 : 4

20. Radhika purchased one dozen bangles. One day she slipped on the floor fell down. What can not be the ratio of broken to unbroken bangles :

 - 1 : 2
 - 1 : 3
 - 2 : 3
 - 1 : 5

21. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then the value of $\left(\frac{a^n p + c^n q + e^n r}{b^n p + d^n q + f^n r} \right)^{1/n}$:

 - $\frac{ad}{bc}$
 - $\frac{af}{be}$
 - $\frac{ck}{dk}$
 - none of these

22. At a casino in Mumbai, there are 3 tables A , B and C . The pay-offs at A is 10 : 1, at B is 20 : 1 and at C is 30 : 1. If a man bets Rs. 200 at each table and win at two of the tables, what is the maximum and minimum difference between his earnings can be?

 - Rs. 2500
 - Rs. 2000
 - Rs. 4000
 - none of these

23. If $m = \frac{4pq}{p+q}$, then the value of $\frac{m+2p}{m-2p} + \frac{m+2q}{m-2q}$:

 - 2
 - 4
 - $\frac{2mpq}{(p+q)}$
 - none of these

24. The ratio of volumes of two cubes is 8 : 27. What is the ratio of surface area of these cubes respectively?

 - 2 : 3
 - 4 : 9
 - 8 : 19
 - 9 : 4

25. Duderli Lal has two jars. Jar A is completely filled with milk and another jar B is totally empty. Before selling the milk in a town he transferred some milk in to the empty jar B then he fill the jar A with water. Once again he transferred the mixture of milk from A to B so that B is completely filled. Which one of the following is correct?

 - Concentration of milk in B cannot be less than 75%
 - Concentration of milk in B cannot be greater than 75%
 - Concentration of milk in B is always 75%
 - none of the above

26. In ABC corporation there are some management trainees. These trainees are divided into 3 groups A , B and C for 3 different projects in the ratio of 3 : 4 : 5 respectively, where P , Q , R are the projects-in-charge of A , B , C respectively. The difference between the no. of trainees in A and C is 10. Also P , Q , R belongs to the group of trainees. The no. of assistant of Q is less than the no. of assistance of R .

(a) 33.33%

(d) 16.66%

(c) 25%

If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \frac{1}{3}$, find the value of

$$\frac{(a+b+c+d+e)}{(b+c+d+e+f)}:$$

$$(a) \frac{1}{81}$$

$$(b) \frac{1}{27}$$

$$(c) \frac{1}{3}$$

$$(d) 1$$

28. An engine can move at the speed of $\frac{20}{3}$ m/s without any wagon attached. Reduction in the speed of the train is directly proportional to the square root of the no. of wagons attached to the engine. When there are only four wagons attached its speed is $\left(\frac{50}{9}\right)$ m/s. The greatest no. of wagons with which the engine can move is :

(a) 144

(b) 143

(c) 12

(d) none of these

29. Mrs. Annapurna per day sells exactly four quintal sugar at Rs. 2000 per quintal getting the profit of Rs. 25%. Since she mixes two varieties of sugar, one costs Rs. 14 per kg and another costs Rs. 22 per kg. One day due to huge demand in market she had only 3 quintal of the required mixture so she purchased the sugar costs Rs. 17 per kg at Rs. 18 per kg from the wholeseller on that day and then she mixed 300 kg mixture with 100 kg sugar costing Rs. 18 per kg to fulfill the demand of the market selling at the same price. How much percent less does she gain that she would have gained, if she had sufficient quantity of usual mixture of sugar?

(a) 12.5%

(b) 18.18%

(c) 62.5%

(d) can't be determined

30. Two vessels P and Q contain 'a' litres of petrol and 'b' litres of Kerosene respectively. 'c' litres of petrol and same quantity of Kerosene is taken out and then transferred to Q and P respectively. This process is repeated several times. If after the first operation the quantity of petrol or Kerosene in either of P and Q does not change. What is the value of 'c'?

$$(a) \frac{ab}{(a-b)}$$

$$(b) \frac{2ab}{(a+b)}$$

$$(c) \frac{ab}{(a+b)}$$

$$(d) \left(\frac{a}{b}\right)^2$$

31. Arvind Singh purchased a 40 seater bus. He started his services on route no. 2 (from Terhipuliya to Charbagh with route length of 50 km). His profit (P) from the bus depends upon the no. of passengers over a certain minimum number of passengers 'n' and upon the distance travelled by bus. His profit is Rs. 3600 with 29 passengers in the bus for a journey of 36 km and Rs. 6300 when there are 36 passengers travelled for 42 km. What is the minimum no. of passengers are required so that he will not suffer any loss?

(a) 12

(b) 20

(c) 18

(d) 15

32. Three cats are roaming in a zoo in such a way that A takes 5 steps, B takes 6 steps and cat C takes 7 steps. But the 6

steps of A are equal to the 7 steps of B and 8 steps of C . What is the ratio of their speeds :

(a) 140 : 144 : 147

(b) 40 : 44 : 47

(c) 15 : 21 : 28

(d) 252 : 245 : 240

33. In a family there were n people. The expenditure of rice per month in this family is directly proportional to the 5 times the square of no. of people in the family. If the elder son left the family to study in USA there was decrease in consumption of 95 kg rice per month. What is the value of n ?

(a) 5

(b) 12

(c) 9

(d) 10

34. One day in summer I wanted to chill me out, I went to a cool corner. I gave him a note of Rs. 10 and asked for a coke costing Rs. 5 per jar and he did so, but he returned me Rs. 5, in the denomination of Re. 1, 50 paise and 25 paise. What could be the ratio of no. of coins of Re. 1, 50 paise and 25 paise respectively :

(a) 2 : 3 : 1

(b) 1 : 7 : 2

(c) 6 : 1 : 3

(d) 2 : 1 : 2

35. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$ and $a+b+c \neq 0$ then the value of $\frac{b}{a+b+c}$ is :

$$(a) \frac{1}{2}$$

$$(b) \frac{1}{3}$$

$$(c) \frac{1}{4}$$

$$(d) 1$$

36. The ratio of the density of 3 kinds of petrol P_1 , P_2 and P_3 is 9 : 7 : 5. The density of P_1 is 18 gm/cc and P_1 , P_2 , P_3 are mixed in the ratio of 6 : 5 : 4 by weight. If a litre of P_3 cost Rs. 40, then find the cost of P_3 in 450 kg of mixture of P_1 , P_2 and P_3 :

(a) Rs. 380

(b) Rs. 480

(c) Rs. 355

(d) Rs. 448

37. Three persons Amar, Akbar and Anthony agree to pay their hotel bills in the ratio of 3 : 4 : 5. Amar pays the first day's bill which amounts to Rs. 26.65, Akbar pays the second day's bill which amounts to Rs. 42.75 and C pays the third day's bill which amounts to Rs. 53.00. When they settle their accounts, which of the following happens?

(a) Amar gives Rs. 3 to C

(b) Akbar gives Rs. 2 to Amar

(c) Amar gives Akbar Rs. 1.95 and Rs. 2 to Anthony

(d) none of the above

38. Find the value of x if $(14x - 4) : (8x - 1) = (3x + 8) : (9x + 5)$:

(a) 1

(b) 1/2

(c) 3/4

(d) none of these

39. Pooja, Shipra and Monika are three sisters. Pooja and Shipra are twins. The ratio of sum of the ages of Pooja and Shipra is same as that of Monika alone. Three years earlier the ratio of age of Pooja and Monika was 2 : 7. What will be the age of Shipra 3 years hence?

(a) 21 years

(b) 16 years

(c) 8 years

(d) 12 years

40. A couple got married 9 years ago when the age of wife was 20% less than her husband. 6 years from now the age of wife will be only 12.5% less than her husband. Now they have six

- children including single, twins and triplets and the ratio of their ages is 2:3:4 respectively. What can be the maximum possible value for the present age of this family?
- (a) 110 years (b) 103 years
(c) 105 years (d) 83 years
41. The price of a necklace varies directly as the no. of pearls in it. Also, it varies directly as the square root of radius of a pearl. The price of a necklace was Rs. 150. When it had 75 pearls each of radius 1 cm. Find the radius of the pearl of a necklace having 100 pearls whose cost is Rs. 600.
- (a) 2 (b) 9
(c) 3 (d) 4
42. The price of a book varies directly as the no. of pages in it and inversely as the time periods in years that have elapsed since the date of purchasing. Two books cost the same, however, the no. of pages in the first book is triple of the second book. If the first book is sold on 18 years ago, how long ago was the second book sold?
- (a) 54 years (b) 9 years
(c) 6 years (d) 3 years
43. Akbar and Birbal who purchased the shares for the cost of their basic salaries which are in the ratio of 5:6. later on company gave them 40 additional shares to each, due to which the ratio changed to 7:8. If the worth of each share is Rs. 75, what is the basic salary of the person who got less shares?
- (a) Rs. 10500 (b) Rs. 7500
(c) Rs. 8800 (d) Rs. 9000
44. Distance covered by a train is directly proportional to the time taken and also it varies directly as the square root of fuel used and varies inversely as the no. of wagons attached to it. A train covers 192 km journey in 20 hours when there are 10 wagons attached to it and total fuel consumption was 256 litre of diesel. Find the consumption of fuel per km when a train goes 200 km in 25 hours with 15 wagons attached to it:
- (a) 1.5 l/km (b) 2 l/km
(c) 2.8 l/km (d) 20 l/km
45. At Sahara shopping centre, a person can purchase as much articles at a time as his or her age that is a person of n years age can purchase only n similar articles at a time. Amisha is younger to her elder brother who has just entered into his twenties. One day Amisha went to the Sahara shopping centre, she purchased same toffees at a particular rate on the ground floor. But when she reached on third floor she found
- that she could purchase double the no. of toffees with the same amount as she had spent on the ground floor. Also to purchase the same no. of toffees on the third floor she had to spend Rs. 2 less than that of on the first floor. How many toffees did she buy?
- (a) 6 (b) 12
(c) 18 (d) 15
46. A contractor deployed some men to plant 1800 trees in a certain no. of days. But in $\frac{1}{3}$ rd of the planned time 120 plants could be less planted so to fulfill the target for the rest of the days every day 20 more plants were planted. Thus it saved one day out of the initially planned no. of days. How many plants he planned to plant each day initially?
- (a) 180 (b) 100
(c) 120 (d) 160
47. A and B have to write 810 and 900 pages respectively in the same time period. But A completes his work 3 days ahead of time and B completes 6 days ahead of time. How many pages did A write per hour if B wrote 21 pages more in each hour?
- (a) 45 (b) 72
(c) 54 (d) 100
48. Three friends A, B and C decided to share the soda water with D, who had no soda water. A contributed 2 tumblers more than that of B and B contributed 1 tumbler more than that of C and then all of them had equal amount of soda water. In turn D paid money, which was divided among A, B and C in the ratio of their contribution to D. Thus A had gotten thrice as much money as B had gotten. The price of each tumbler of soda water was Rs. 15 and each transaction was integral in numbers either the sharing of money or contribution of soda water. What was the sum of money that B had gotten?
- (a) Rs. 15 (b) Rs. 18
(c) Rs. 22.5 (d) none of these
49. In Maa Yatri Temple every devotee offers fruits to the orphans. Thus every orphan receives bananas, oranges and grapes in the ratio of 3:2:7 in terms of dozen. But the weight of a grape is 24 gm and weight of a banana and an orange are in the ratio of 4:5, while the weight of an orange is 150 gm. Find the ratio of all the three fruits in terms of weight, that an orphan gets :
- (a) 90:75:42 (b) 180:150:82
(c) 75:42:90 (d) none of these



Final Round

1. In two alloys the ratio of Iron and copper is 4:3 and 6:1 respectively. If 14 kg of the first alloy and 42 kg of the second alloy are mixed together to form a new alloy, then what will be the ratio of copper to iron in the new alloy :
- (a) 11:3 (b) 11:8
(c) 8:11 (d) none of these
2. In a zoo, there are rabbits and pigeons. If heads are counted, there are 340 heads and if legs are counted there are 1060 legs. How many pigeons are there?

Compiled By Jasjeet

- (a) 120 (b) 150
(c) 180 (d) 170
3. 6 litre is taken out from a vessel full of Kerosene and substituted by pure petrol. This process is repeated two more times. Finally the ratio of petrol and Kerosene in the mixture becomes 1701:27. Find the volume of the original solution:
- (a) 14 litre (b) 16 litre
(c) 8 litre (d) 42 litre



Answers

INTRODUCTORY EXERCISE-4.1

1. (c)	2. (b)	3. (d)	4. (c)	5. (a)	6. (b)	7. (d)	8. (b)	9. (c)	10. (d)
11. (a)	12. (b)								

INTRODUCTORY EXERCISE-4.2

1. (c)	2. (c)	3. (b)	4. (d)	5. (c)					
--------	--------	--------	--------	--------	--	--	--	--	--

INTRODUCTORY EXERCISE-4.3

1. (d)	2. (c)	3. (c)	4. (a)	5. (d)	6. (c)				
--------	--------	--------	--------	--------	--------	--	--	--	--

INTRODUCTORY EXERCISE-4.4

1. (b)	2. (d)	3. (c)	4. (a)	5. (d)	6. (d)	7. (c)	8. (b)	9. (a)	10. (d)
11. (b)	12. (d)	13. (b)	14. (c)	15. (c)	16. (b)	17. (c)	18. (d)	19. (b)	20. (b)
21. (c)	22. (a)	23. (c)	24. (b)	25. (d)	26. (a)	27. (c)	28. (c)		

LEVEL-1

1. (b)	2. (b)	3. (d)	4. (c)	5. (b)	6. (d)	7. (b)	8. (c)	9. (b)	10. (b)
11. (a)	12. (b)	13. (a)	14. (c)	15. (d)	16. (d)	17. (c)	18. (c)	19. (b)	20. (a)
21. (a)	22. (c)	23. (a)	24. (c)	25. (c)	26. (c)	27. (b)	28. (a)	29. (d)	30. (c)
31. (b)	32. (d)	33. (d)	34. (b)	35. (c)	36. (c)	37. (b)	38. (c)	39. (d)	40. (c)
41. (a)	42. (c)	43. (b)	44. (c)	45. (d)	46. (d)	47. (c)	48. (b)	49. (c)	50. (b)
51. (d)	52. (d)	53. (a)	54. (d)	55. (a)	56. (c)	57. (c)	58. (b)	59. (c)	60. (c)
61. (d)									

LEVEL-2

1. (a)	2. (c)	3. (b)	4. (c)	5. (c)	6. (d)	7. (d)	8. (a)	9. (c)	10. (c)
11. (d)	12. (b)	13. (a)	14. (b)	15. (d)	16. (c)	17. (d)	18. (a)	19. (a)	20. (c)
21. (c)	22. (c)	23. (a)	24. (b)	25. (a)	26. (c)	27. (c)	28. (b)	29. (a)	30. (c)
31. (d)	32. (a)	33. (d)	34. (b)	35. (b)	36. (b)	37. (c)	38. (b)	39. (c)	40. (b)
41. (b)	42. (c)	43. (b)	44. (b)	45. (b)	46. (b)	47. (c)	48. (a)	49. (a)	

FINAL ROUND

1. (d)	2. (b)	3. (c)	4. (a)	5. (b)	6. (b)	7. (d)	8. (b)	9. (d)	10. (b)
11. (c)	12. (c)	13. (c)	14. (b)	15. (b)	16. (d)				

Hints & Solutions

LEVEL 1

$a : b :: c : d$

1. $a^2 + b^2 + c^2 + d^2 = 50$

$b + c = 5$

$a : b = 1 : 3$

and

If consider $a : b = 1 : 3$ as it is, then

$c = 2$

($5 - 3 = 2$)

and

$d = 6$

($\because a : b :: c : d$)

$a^2 + b^2 + c^2 + d^2 = 1^2 + 3^2 + 2^2 + 6^2 = 50$

Hence, the presumed values are correct.

Thus, the average of a, b, c and $d = \frac{a+b+c+d}{4}$

$$= \frac{1+3+2+6}{4} = 3$$

Hence (b) is correct.

Alternatively: Assume option (b)

$$\frac{a+b+c+d}{4} = 3$$

$\Rightarrow a+b+c+d = 12$

Now $\therefore b+c = 5$

$\Rightarrow a+d = 7$

Again $a : b = 1 : 3$

$\Rightarrow c : d = 2 : 6$

Now verify that $a^2 + b^2 + c^2 + d^2 = 50$. Since it is correct.

Hence option (b) is correct.

NOTE It can also be solved by forming quadratic equations.

$$\begin{aligned} 2. \quad & \frac{a}{b} = \frac{a+b}{a} \\ \Rightarrow & a^2 = ab + b^2 \\ \Rightarrow & a^2 - b^2 - ab = 0 \end{aligned}$$

$\overbrace{\hspace{10em}}$

Let $b = 1$, then $a : b = a : 1$ (by putting $b = 1$)

$$\therefore a^2 - a - 1 = 0$$

$$\Rightarrow a = \frac{1 \pm \sqrt{5}}{2} \quad \begin{array}{l} \text{(Solving quadratic equation)} \\ \text{by Sridharacharya's formula} \end{array}$$

$$\therefore a = \frac{1 + \sqrt{5}}{2} \quad \begin{array}{l} \text{(negative value can't be considered)} \\ \text{Chances of solution to quadratic equation} \end{array}$$

$$a : b = \frac{1 + \sqrt{5}}{2} : 1$$

or

$$\text{Therefore, } \frac{b}{a+b} = \frac{2}{1 + \sqrt{5} + 2} = \frac{2}{3 + \sqrt{5}}$$

Hence option (b) is correct.

3. The best way is to go through options

$$\frac{x+y}{x-y} = \frac{8+2}{8-2} = \frac{10}{6} = \frac{5}{3}$$

(verified)

Again $\frac{x}{y+2} = \frac{8}{2+2} = 2$

(verified)

Hence option (d) is correct.

4. Best way is to go through options

and $\frac{a^3 + b^3}{a^3 - b^3} = \frac{5^3 + 2^3}{5^3 - 2^3} = \frac{125 + 8}{125 - 8} = \frac{133}{117}$

Hence option (c) is correct.

5. $H : S = 1 : 1$

and $S : G = 2 : 3$

$H : P = 1 : 2$

$$\therefore H : S : G : P = 2 : 2 : 3 : 4 \\ = 2x : 2x : 3x : 4x$$

Therefore $\frac{2x + 2x + 3x + 4x}{4} = \frac{11x}{4} = 55$

$\Rightarrow x = 20$

Marks in History = 40

Sociology = 40

Geography = 60

Philosophy = 80

Hence, only in two subjects he scored 60% or above.
Hence option (b) is correct.

6. Let the incomes of A and M is $2x$ and $3x$

Let the savings of A be K , then the expenditure of M be K

Also expenditure of $A = 2x - K$

Given $(2x - K) + K = 8000 \Rightarrow x = 4000$

\therefore Total income of

A and $B = 2x + 3x = 5x = 5 \times 4000 = 20000$

\therefore Total savings of A and $B = 20,000 - 8000 = \text{Rs. } 12,000$

7. $\frac{\text{Profit of Hutch}}{\text{Profit of Essar}} = \frac{\text{time period} \times \text{amount of Hutch invested}}{\text{time period} \times \text{amount of Essar invested}}$

$$\frac{6}{17} = \frac{5 \times K}{12 \times 1275}$$

$$\Rightarrow K = \frac{6 \times 12 \times 1275}{17 \times 5} = 1080$$

8. Minimum number of chocolates are possible when he purchases maximum number of costliest chocolates.

Thus $2 \times 5 + 5 \times 2 = \text{Rs. } 20$

Now Rs. 100 must be spent on 10 chocolates as $100 = 10 \times 10$.

Thus minimum number of chocolates = $5 + 2 + 10 = 17$

9. Maximum number of chocolates are possible only when he purchases minimum number of costlier chocolates and maximum number of cheaper chocolates

$\therefore 2 \times 5 + 1 \times 10 = \text{Rs. } 20$

Now Rs. 100 must be spent on 50 chocolates as $100 = 2 \times 50$.

Thus maximum number of possible chocolates

$$= 2 + 1 + 50 = 53$$

10. Meretere \rightarrow M, Teremere \rightarrow T and Khabbu Singh \rightarrow K
 $\frac{5+3}{3} = 2.66$ (since all the 8 chocolates were shared by 3)

It means M has given $5 - 2.66 = 2.33$ chocolates to K
 and T has given $3 - 2.66 = 0.33$ chocolates to K
 Thus M and T will receive the amount in the ratio of
 donations (i.e., share of chocolates)

$$\begin{array}{rcl} M & : & T \\ 2.33 & : & 0.33 \\ 2\frac{1}{3} & : & \frac{1}{3} \\ 7 & : & 1 \\ 3 & : & 3 \\ 7 & : & 1 \end{array}$$

So the M receives Rs. 14 and T receives Rs. 2

Thus the difference = Rs. 12

11. Share of a man, a woman and a boy = $7x$, $4x$ and $3x$
 then the share of 4 men = $4 \times 7x = 28x$
 then the share of 5 women = $5 \times 4x = 20x$
 then the share of 2 boys = $2 \times 3x = 6x$

$$\begin{aligned} \text{Now, the share of all women} &= \frac{20x}{(28x + 20x + 6x)} \times 4536 \\ &= \frac{20}{54} \times 4536 = \text{Rs. } 1680 \end{aligned}$$

$$\text{Hence, the share of one woman} = \frac{1680}{5} = 336$$

12. Concentration of petrol in

$$\begin{array}{ccc} A & B & C \\ \frac{1}{2} & \frac{3}{5} & \frac{4}{5} \end{array}$$

Quantity of petrol taken from A = 1 litre out of 2 litre
 Quantity of petrol taken from B = 1.8 litre out of 3 litre
 Quantity of petrol taken from C = 0.8 litre out of 1 litre
 Therefore total petrol taken out from

$$A, B \text{ and } C = 1 + 1.8 + 0.8 = 3.6 \text{ litre}$$

So, the quantity of Kerosene = $(2 + 3 + 1) - (3.6) = 2.4$ litre

$$\text{Thus, the ratio of petrol to Kerosene} = \frac{3.6}{2.4} = \frac{3}{2}$$

$$13. T \propto \frac{\sqrt{l}}{\sqrt{g}} \Rightarrow T = k \sqrt{\frac{l}{g}}$$

$$\text{Therefore, } 3 = K \sqrt{\frac{9}{4}} \text{ or } 3 = K \frac{3}{2} \Rightarrow K = 2$$

$$\text{Again } T = K \sqrt{\frac{l}{g}} = 2 \times \sqrt{\frac{64}{16}} = 4$$

$$T = 4 \text{ seconds}$$

Pure	Cure	Lure
100%	40%	37.5%
$\frac{1}{1}$	$\frac{2}{5}$	$\frac{3}{8}$

	Milk	Water	Mixture
Cure	4l	10l	14l
Lure	6l	16l	22l
→ New mixture	Milk	Water	
	10l	26l	
	16l		
	26l		
Required mixture	1	1	

Ratio, Proportion and Variation
 Since in the required mixture the ratio of milk and water is
 1 : 1 so she has to add up 16 litre of more milk (pure) to get it
 for the fixed quantity of water.

$$S : (M + J) = 5 : 7 \Rightarrow 7S = 5M + 5J$$

$$J : (S + M) = 1 : 2 \Rightarrow 2J = S + M$$

By solving equations (1) and (2), we get

$$S : M : J = 5 : 3 : 4$$

$$S : M = 5 : 3$$

so

$$\begin{array}{ll} \text{Male} & \text{Female} \\ A & 4x : 3x \\ B & 3y : 2y \\ C & 5z : 4z \end{array}$$

$$2y = 4z$$

but

$$\begin{array}{ll} \text{Male} & \text{Female} \\ A & 4x : 3x \\ B & 6z : 4z \\ C & 5z : 4z \end{array}$$

$$7x + 19z = 33$$

Therefore, but z can assume only one value i.e., $z = 1$

$$\text{Hence, } 7x + 19 = 33 \Rightarrow x = 2$$

Thus the no. of female children in factory $A = 3x = 6$.

17. The ratio of broken parts (by weight) = $3x : 4x : 5x$
 Therefore value of broken parts of diamond

$$= (3x)^2 + (4x)^2 + (5x)^2 = 50x^2$$

The value of original diamond = $(3x + 4x + 5x)^2 = 144x^2$

Therefore, loss in value = $144x^2 - 50x^2 = 9.4$ lakh

$$\Rightarrow 94x^2 = 9.4 \text{ lakh}$$

$$\Rightarrow 94x^2 = 940000$$

$$\Rightarrow x^2 = 10000$$

Hence, the actual value of the diamond = $144x^2$

$$= 144 \times 10000$$

$$= 14.4 \text{ lakh}$$

$$18. 10x + 20y + 100z = 1600$$

Again since

$$x : y = 6 : 1$$

$$60y + 20y + 100z = 1600$$

$$\Rightarrow 80y + 100z = 1600$$

$$\Rightarrow 4y + 5z = 80$$

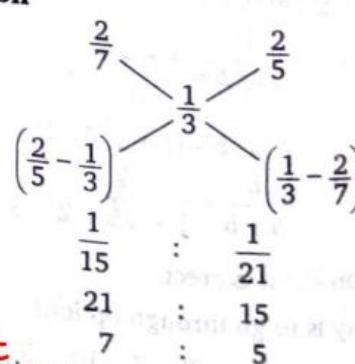
Putting $z = 1, 2, 3, 4, 5, \dots$, we get at $z = 4, y = 15$ (an integer)

Hence, min. 4 coins of Re. 1 will be there

19. Concentration of water in first vessel = $\frac{2}{5} = 40\%$

concentration of water in second vessel = $\frac{2}{7} = 28.57\%$

By alligation



Therefore,

$$\begin{aligned} \frac{1}{2} \times 120 &= 60 \\ \frac{1}{3} \times 120 &= 40 \\ \frac{1}{5} \times 120 &= 24 \\ \frac{1}{8} \times 120 &= 15 \end{aligned}$$

Therefore, minimum number of chocolates
 $= 60 + 40 + 24 + 15 = 139$

$$A : B = 5 : 3$$

$$B : C = 5 : 8$$

$$A : B : C = 25 : 15 : 24$$

∴ A is the most efficient.

$$A : B = 5 : 4 \Rightarrow 10 : 8$$

$$A : C = 2 : 3 \Rightarrow 10 : 15$$

$$A : B : C = 10 : 8 : 15$$

$$B : C = 8 : 15$$

$$\begin{array}{c} \text{Proportion of milk} \\ \text{in mixture} \end{array} \quad \begin{array}{ccc} A & B & C \\ \frac{1}{3} & : & \frac{2}{5} : \frac{3}{7} \\ \frac{35}{105} & : & \frac{42}{105} : \frac{45}{105} \end{array}$$

$$\text{quantity of milk in new mixture} = 35 + 42 + 45 = 122 \text{ l}$$

$$\text{quantity of water in new mixture} = (105 \times 3) = 122 = 193 \text{ l}$$

Therefore, ratio of water is to milk = 193 : 122

$$24. \quad A : B = 8 : 9$$

$$B : C = 2 : 3$$

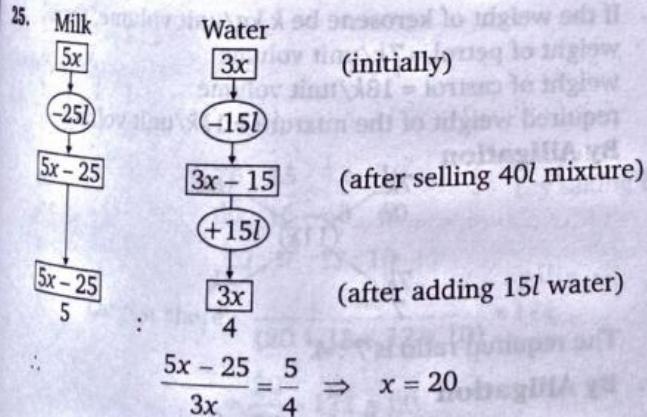
$$C : D = 9 : 13$$

$$A : B : C : D = 144x : 162x : 243x : 351x$$

But we need not to solve this, since we already know that

$$B : C = 2x : 3x$$

$$2x : 3x :: 18 : k \Rightarrow k = 27 \text{ years}$$

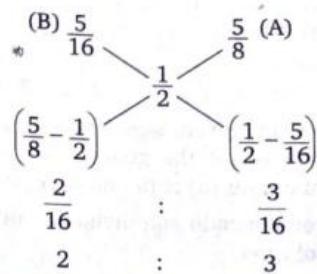


$$\begin{array}{ll} A & B \\ C : T & C : T \\ 5 : 3 & 5 : 11 \end{array}$$

$$\text{Concentration of copper in } A = \frac{5}{8}$$

$$\text{Concentration of copper in } B = \frac{5}{16}$$

By alligation



So, the required ratio of $A : B = 3 : 2$ (Since $B : A = 2 : 3$)

$$\begin{aligned} 27. \quad k + 10x &= 6000 \\ k + 25x &= 9000 \quad (\because 25 \times 360 = 9000) \\ \Rightarrow 15x &= 3000 \\ \Rightarrow x &= 200 \text{ and } k = 4000 \\ \therefore k + 40x &= 4000 + 40 \times 200 = 12,000 \end{aligned}$$

where k is the fixed expenditure.

28. Third proportional of 21 and 42 is 84 and mean proportional to 16 and 49 is 28.

$$\text{Therefore required ratio} = \frac{84}{28} = \frac{3}{1} \text{ or } 3 : 1$$

$$\begin{aligned} 29. \quad P \propto \sqrt{l} \Rightarrow P = k \sqrt{l} \\ \therefore 52 &= k \sqrt{16} \\ \Rightarrow k &= 13 \\ \text{Again,} \quad P &= k \sqrt{l} \\ 65 &= 13 \sqrt{l} \\ \Rightarrow l &= 25 \text{ cm} \end{aligned}$$

$$30. \quad \frac{m+n}{m-n} = \frac{7x}{x} \Rightarrow \frac{m}{n} = \frac{4x}{3x}$$

$$\text{Again } mn = 4x \times 3x = 12x^2$$

$$\text{and } mn = 60x$$

$$\text{So } 60x = 12x^2$$

$$\Rightarrow x = 5$$

$$\therefore m = 20 \text{ and } n = 15$$

$$\text{Hence, } \frac{1}{m} : \frac{1}{n} = \frac{1}{20} : \frac{1}{15} = 3 : 4$$

$$\text{Alternatively: } \frac{m}{n} = \frac{4x}{3x}$$

$$\frac{1}{m} : \frac{1}{n} = 3 : 4$$

$$31. \quad \underbrace{A+B}_2 : \underbrace{C+D}_1$$

$$\text{and } \frac{B}{3} : \frac{C}{1} \text{ and } \frac{C}{3} : \frac{D}{5}$$

$$\Rightarrow B : C : D = 9 : 3 : 5$$

$$\text{Again } A + B + C + D$$

$$\underbrace{7+9}_{16} : \underbrace{3+5}_{8}$$

$$\text{Thus } A + B = 16 \Rightarrow A = 7 \text{ when } B = 9$$

$$\therefore \text{Therefore share of } A = \frac{7}{24} \times 9600 = \text{Rs. 280}$$