9 TIME, SPEED AND DISTANCE

It is sure and certain that it is one of the most important chapter, which includes various fundamental and logical concepts and therefore most of the problems are complex in their nature. Every year 4-6 problems are generally asked in CAT. Besides CAT almost every aptitude exam contains the questions pertaining to the concepts of TSD.

This chapter includes the following :

(a) Motion in a straight line
(b) Circular motion and races
(c) Problems based on trains, boats, river and clocks etc.

CONCEPT OF MOTION

When a body moves from a point A to another point B at a distance of D, then it requires some time (T) to cover a distance (D) with a particular speed (S).

The relation between T, S and D is as follows:

$$T \times S = D$$

i.e., Time × Speed = Distance

Therefore, when D is constant,

$$T \propto \frac{1}{S}$$

and when T is constant,

$$D \propto S$$

and when S is constant,

$$D \propto T$$

NOTE This relation of proportionality is very important.

Formulae : Distance = Speed × Time

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

NOTE To solve the problem, all the units involved in the calculation must be uniform *i.e.*, either all of them be in metres and seconds or in kilometres and hours etc.

Conversion of unit:

$$1 \text{ km/h} = \frac{5}{18} \text{ m/s}$$

$$1 \text{ m/s} = \frac{18}{5} \text{ km/h}$$

[1 km = 1000 m, 1 h = 60 min, 1 min = 60 s]

HINT Try to find these relations by unitary method.

$$1 \text{ mile} = 1609.30 \text{ m} = 1.6093 \text{ km}$$

and $1 \text{ km} = 0.621 \text{ mile}$

$$1 \text{ yard} = 0.9144 \text{ m}$$

and $1 \text{ m} = 1.0936 \text{ yards}$

and $1 \text{ m} = 39.4 \text{ inches}$

$$\textbf{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

When distances are equal :

$$\text{Average speed} = \frac{2xy}{x+y} \quad \text{(speeds } x, y\text{)}$$

and $$\text{Average speed} = \frac{3xyz}{xy+yz+zx} \quad \text{(speeds } x, y, z\text{)}$$

EXAMPLE 1 *Abhishek drives his bike at the speed of 150 km/h. What is the distance covered by him in 3 hours.*

SOLUTION $D = S \times T = 150 \times 3 = 450$ km

EXAMPLE 2 *Udai travels half of his journey by train at the speed of 120 km/h and rest half by car at 80 km/h. What is the average speed?*

SOLUTION Let the total distance be $2D$ km, then

Total time = Time taken by train + Time taken by car

$$= \frac{D}{120} + \frac{D}{80}$$

$$\therefore \quad \text{Average speed} = \frac{2D}{\frac{D}{120} + \frac{D}{80}}$$

$$\frac{2D}{D\left(\frac{1}{120}+\frac{1}{80}\right)} = \frac{2}{\left(\frac{2+3}{240}\right)} = 96 \text{ km/h}$$

Alternatively: Consider the distance in numerals (since it is independent of the distance) as the LCM of the speeds.

Then, the total distance = LCM of 120 and 80 = 240

Therefore, time taken by train $= \frac{120}{120} = 1$ h

and time taken by car $= \frac{120}{80} = \frac{3}{2}$ h

$\therefore$ Average speed $= \frac{\text{Total distance}}{\text{Total time taken}}$

$$= \frac{240}{1+\frac{3}{2}} = 96 \text{ km/h}$$

Alternatively: Average speed $= \frac{2(xy)}{(x+y)}$

where x and y are the speeds.

$\therefore$ Average speed $= \frac{2\times120\times80}{120+80} = 96$ km/h

NOTE This formula is applicable only when the distances travelled at the speed of x km/h and y km/h are same.

Alternatively: Solve through alligations.

EXAMPLE 3 *Speed of Karan is 40 km/h and speed of Arjun is 60 km/h. What is the ratio of time taken by each to cover the same distance?*

SOLUTION

	Karan		Arjun
Speed	40	:	60
$\Rightarrow$	2	:	3
$\therefore$ Time	3	:	2

Alternatively: Time taken $= \frac{1}{40} : \frac{1}{60} = 3 : 2$

NOTE For the constant distance covered, the ratio of time taken by each person is the ratio of the reciprocal of their speeds and *vice-versa*.

If the ratio of speeds of A, B, C, D be $a : b : c : d$, then the ratio of time taken by A, B, C and D to cover the same distance

$$= \frac{1}{a} : \frac{1}{b} : \frac{1}{c} : \frac{1}{d}$$

and if the time taken by A, B, C and D be $p : q : r : s$, then the ratio of their speeds

$$= \frac{1}{p} : \frac{1}{q} : \frac{1}{r} : \frac{1}{s}$$

EXAMPLE 4 *A car takes half of the time taken by truck to go from Lucknow to Bombay. A truck takes 20 hours to go for the same journey. What is the speed of truck, if the speed of car be 120 km/h.*

SOLUTION

	Car		Truck
Time $\rightarrow$	1	:	2
Speed $\rightarrow$	2	:	1

Therefore, speed of the truck = 60 km/h

EXAMPLE 5 *A cycle covers 75 km distance in 5 hours. What is the distance covered by the cycle in 6 hours?*

SOLUTION

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$\therefore$ $$\text{Speed} = \frac{75}{5} = 15 \text{ km/h}$$

Now, distance covered in 6 hours at the speed of 15 km/h

$$= 15 \times 6 = 90 \text{ km}$$

EXAMPLE 6 *The distance of the college and the home of Rajeev is 80 km. One day he was late by 1 hour than the normal time to leave for the college, so he increased his speed by 4 km/h and thus he reached to college at the normal time. What is the changed (or increased) speed of Rajeev?*

(a) 28 km/h *(b) 30 km/h*
(c) 40 km/h *(d) 20 km/h*

SOLUTION Let the normal speed be x km/h, then

$$\frac{80}{x} - \frac{80}{(x+4)} = 1$$

$\Rightarrow$ $$x^2 + 4x - 320 = 0$$

$\Rightarrow$ $$x(x+20) - 16(x+20) = 0$$

$$(x+20)(x-16) = 0$$

$$x = 16 \text{ km/h}$$

$\therefore$ $$(x+4) = 20 \text{ km/h}$$

Therefore increased speed = 20 km/h

Alternatively: If t_1 and t_2 be the original and changed time and S_1 and S_2 be the original and changed speeds, then

$$S_1 \times S_2 = \frac{\text{Distance} \times (S_1 \sim S_2)}{(t_1 \sim t_2)}$$

Also, $$\boxed{t_1 \times t_2 = \frac{\text{Distance} \times (t_1 \sim t_2)}{(S_1 \sim S_2)}}$$

$\therefore$ $$S_1 \times S_2 = \frac{80}{1} \times 4 = 320$$

$\therefore$ $$S_1 \times S_2 = 320$$

or $$S_1 \times (S_1 + 4) = 320$$

$\Rightarrow$ $$S_1 = 16 \quad \text{and} \quad S_2 = 20$$

NOTE Here you need not to solve necessarily quadratic equation. You can just try and find two factors of 320 in such a way that (in this particular problem) one factor must be greater than the other factor by 4.

Alternatively: Go through options. Consider 20

$$t_2 = \frac{80}{20} = 4$$

$$t_1 = 4 + 1 = 5$$

$\therefore$ $$S_1 = \frac{80}{5} = 16$$

Since, S_1 is 4 km/h less than S_2. Hence, option (d) is correct.

Alternatively: If you find yourself comfortable then factorise 80 in two factors and check the correct combinations.

$$80 = \text{(-1)}\begin{bmatrix} 1\times80 \\ 2\times40 \\ 4\times20 \\ 5\times16 \\ 8\times10 \end{bmatrix}\text{(+4)}$$

So, the increased speed = 20 km/h.

EXAMPLE 7 *Shweta when increases her speed from 24 km/h to 30 km/h she takes one hour less than the usual time to cover a certain distance. What is the distance usually covered by Shweta?*
(a) 160 km (b) 240 km (c) 120 km (d) 90 km

SOLUTION Let the original time be t hours, then

$$24t = 30 \times (t-1) = D \quad \text{(distance)}$$

then $t = 5$

$\therefore$ Distance $= 24 \times 5 = 120$ km

Alternatively: Go through options.

$$\frac{120}{24} = 5\text{ h}$$
$$\frac{120}{30} = 4\text{ h} \quad \Big\} \text{ 1 hour less}$$

Hence, the option (c) is correct.

Alternatively: Since distance (D) is constant.

Therefore, $D = S_1 \times t_1 = S_2 \times t_2$

It means here we can apply product constancy

Speed	Time	
$\frac{1}{4}\uparrow$	$\frac{1}{5}\downarrow$	$= 1$ hour

$\therefore$ Original time taken $= 5 \times 1 = 5$ hours

Therefore, Distance = Original speed × Original time

$= 24 \times 5 = 120$ km

NOTE In the given exercise or in the whole chapter you have to solve maximum problems through product constancy concept described in the chapter of ratio proportion and variation. Solving through product constancy gives faster results.

Alternatively: Let the distance be D, then

$$\frac{D}{24} - \frac{D}{30} = 1$$

$\Rightarrow$ $D = 120$ km

EXAMPLE 8 *Kriplani goes to school at 20 km/h and reaches the school 4 minutes late. Next time, she goes at 25 km/h and reaches the school 2 minutes earlier than the scheduled time. What is the distance of her school?*

SOLUTION Increase in speed = 5 km/h

Decrease in time = 6 min (4 + 2)

By product constancy :

Speed	Time	
$\frac{1}{4}\uparrow$	$\frac{1}{5}\downarrow$	$= 6$ min

It means original time = 30 min $\left(\because \frac{x}{5} = 6 \Rightarrow x = 30\right)$

$\therefore$ Total distance = Original speed × Original time

$$= 20 \times \frac{30}{60} = 10 \text{ km}$$

EXAMPLE 9 *Amit covers a certain distance with his own speed, but when he reduces his speed by 10 km/h his time duration for the journey increases by 40 hours, while if he increases his speed by 5 km/h from his original speed he takes 10 hours less than the original time taken. Find the distance covered by him.*

SOLUTION

$$\left.\begin{matrix} S & & T \\ -10 & \times & +40 \end{matrix}\right\} \quad 40S - 10T = |-400| \quad \text{...(i)}$$

$$\left.\begin{matrix} S & & T \\ +5 & \times & -10 \end{matrix}\right\} \quad -10S + 5T = |-50| \quad \text{...(ii)}$$

Solving eq. (i) and (ii), we get

$S = 25$ and $T = 60$

$\therefore$ Distance $(D) = S \times T$

$= 25 \times 60 = 1500$ km

where $D \rightarrow$ Distance, $S \rightarrow$ Speed, $T \rightarrow$ Time

'+' means increase in value.

and '–' means decrease in value.

Alternatively: Let distance be x km and usual speed be y km/h.

$$\frac{x}{(y-10)} - \frac{x}{y} = 40$$

$$\Rightarrow \quad x\left[\frac{10}{y(y-10)}\right] = 40$$

$$\Rightarrow \quad x = 4y\,(y-10) \quad \text{...(i)}$$

and $$\frac{x}{y} - \frac{x}{(y+5)} = 10$$

$$\Rightarrow \quad x = 2y(y+5) \quad \text{...(ii)}$$

from eq. (i) and (ii)

$$4y(y-10) = 2y(y+5)$$
$$2y - 20 = y + 5$$
$$y = 25 \text{ km/h}$$

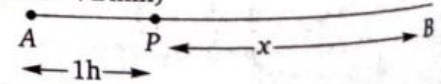
$\therefore$ $x = 1500$ km

EXAMPLE 10 *A train met with an accident 60 km away from Anantpur station. It completed the remaining journey at $\frac{5}{6}$th of the previous speed and reached the Baramula station 1 hour 12 min late. Had the accident taken place 60 km further, it would have been only 1 hour late.*
(a) What is the normal speed of the train?
(b) What is the distance between Anantpur and Baramula?

SOLUTION **Case I.** Since the speed is decreased by $\frac{1}{6}$. So, the time will be increased by 1/5, which is equal to 1 hour 12 minutes.

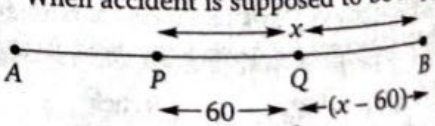
It means the normal time required for this remaining part (x) of the journey is 5×72 min = 360 min = 6 h.

($\because$ 1 h 12 min = 72 min)



P is the place of accident.

Case II. When accident is supposed to be happened at Q



Since, the speed is decreased by $\frac{1}{6}$, hence, the time will be increased by $\frac{1}{5}$, which is equal to 1 hour, hence the normal time required for this remaining part $(x - 60)$ of journey $= 5 \times 1 = 5$ hours

Thus, it is clear that when the train runs 60 km of its normal speed it takes 1 hour less, which implies that in 1 hour the train can run 60 km with its normal speed. Thus, the normal speed of the train is 60 km/h.

(b) Since the train requires 6 hours at its normal speed of 60 km/h for the x km. Hence,

$$x = 6 \times 60 = 360 \text{ km}$$

Thus, the total distance = Distance travelled before accident + Distance travelled after accident

$$= 60 \times 1 + 60 \times 6 = 420 \text{ km}$$

EXAMPLE 11 *A and B started simultaneously towards each other from P and Q respectively. The distance between P and Q is 600 km and the ratio of speeds of A is to B is 5 : 7. If they meet at a point M:*

(i) Find the ratio of PM : QM.

(ii) Find the distance PM.

SOLUTION (i) Since the time is constant so, the distance covered by A and B is directly proportional to the speeds of A and B.

Hence, $PM : QM = 5 : 7$

(ii) Since, the ratio of their speeds (or their distances covered) is 5 : 7. Hence, A will cover $\frac{5}{12}$ of the total length.

$$\therefore \quad PM = \frac{5}{12} \times 600 = 250 \text{ km}$$

(Similarly $QM = \frac{7}{12} \times 600 = 350$ km)

RELATIVE MOTION WITH TWO OR MORE BODIES

(i) When two bodies move in the same direction: If the speeds of the two bodies A and B be S_A and S_B, then their relative speed = $S_A - S_B$ or $S_B - S_A$ *i.e.*, in the same direction the relative speed or effective speed between two bodies is the difference of their speeds. (The difference is always considered as positive)

(ii) When two bodies move in the opposite direction: If the speeds of the two bodies A and B be S_A and S_B, then their relative speed = $S_A + S_B$.

i.e., in the opposite direction the relative speed or effective speed between two bodies is the sum of their speeds.

EXAMPLE 12 *The distance between two places P and Q is 700 km. Two persons A and B started towards Q and P from P and Q simultaneously. The speed of A is 30 km/h and speed of B is 40 km/h. They meet at a point M which lies on the way from P to Q.*

(i) How long will they take to meet each other at M?

(ii) What is the ratio of PM : MQ?

(iii) What is the distance MQ?

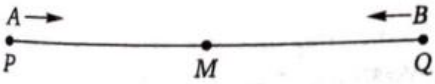
(iv) What is the extra time needed by A to reach at Q than to reach at P by B?

(v) What is the ratio of time taken by A and B to reach their respective destinations after meeting at M?

(vi) In how many hours will they be separated by only 560 km before meeting each other.

(vii) How long will it take to separate them by 280 km from each other when they cross M (time to be considered after their meeting)?

SOLUTION



(i) Since, they are coming towards each other from opposite ends, therefore the relative speed will be the sum of their speeds = 30 + 40 = 70 km/h.

Thus, the required time to meet at M

= Time required to cover 700 km (combined)

$$= \frac{700}{70} = 10 \text{ h}$$

Thus in 10 hours they will meet each other at M.

(ii) The ratio of their distances covered to meet at M

= Ratio of their speeds = 3 : 4

(Since, time is constant *i. e.*, same for each)

Thus $PM : MQ = 3 : 4$

(iii) $MQ = \frac{4}{7} \times 700 = 400$ km

(iv) Time required by A to reach at $Q = \frac{700}{30} = \frac{70}{3}$ h

Time required by B to reach at $P = \frac{700}{40} = \frac{70}{4}$ h

$\therefore$ Extra time required by $A = \frac{70}{3} - \frac{70}{4}$

$$= 70 \times \frac{1}{12} = 5 \text{ h } 50 \text{ min}$$

(v) Time required by A to cover $MQ = \frac{400}{30}$

and time required by B to cover $MP = \frac{300}{40}$

$\therefore$ Required ratio $= \frac{400/30}{300/40} = \frac{16}{9}$

Remember: If speed of A is S_A and speed of B is S_B and A takes t_A time to cover MQ and B takes t_B time to cover MP, then

$$\frac{S_A}{S_B} = \sqrt{\frac{t_B}{t_A}}$$

(vi) It means they have to cover (700 – 560) = 140 km. Thus, the required time to cover 140 km distance

$$= \frac{140}{70} = 2 \text{ h}$$

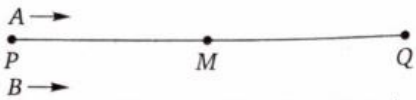
(vii) Since in each hour they separate by 70 km from each other. Hence, to separate by 280 km, time required

$$= \frac{280}{70} = 4 \text{ h}$$

EXAMPLE 13 *Two places P and Q are 800 km apart from each other. Two persons start from P towards Q at an interval of 2 hours. Whereas A leaves P for Q before B. The speeds of A and B are 40 km/h and 60 km/h respectively. B overtakes (or catches or meets) A at M, which is on the way from P to Q.*

(i) *How long will B take to overtake?*
(ii) *What is the distance from P, where B overtakes A (i.e., PM)?*
(iii) *What is the ratio of time taken by A and B to meet at M?*
(iv) *What is the extra time required by A to reach at Q?*
(v) *How many hours late A will reach at Q than that of B?*
(vi) *After how many hours A and B will be separated by 50 km before M, when both are moving?*
(vii) *How many hours does B require to advance himself, by 100 km, in comparison to A?*

SOLUTION



(i) Since A moves 2 hours earlier than B at the speed of 40 km/h, so in 2 hours A will cover 80 km. Thus A will be 80 km away from B.
Now, since they are moving in the same direction therefore their relative speed will be $(60 - 40) = 20$ km/h *i.e.*, with respect to A, B moves at 20 km/h. It means either they reduce (or create) the difference of 20 km in each hour between themselves.
Thus, the required time to overtake (or reduce the difference upto zero from 80 km)

$$= \frac{\text{Distance advanced}}{\text{Relative speed}} = \frac{80}{20} = 4\text{ h}$$

(ii) The distance between P and M
= (Time required to overtake × Speed of the faster body)
$= 4 \times 60 = 240$ km
Since, B has to move for 4 hours at 60 km/h. Hence, distance covered = 240 km.
(iii) Time taken by B to reach at $M = 4$ h
Time taken by A to reach at $M = (4 + 2) = 6$ h
Thus, the ratio of time taken by A and $B = 6 : 4 = 3 : 2$
(Since, A has left 2 hours earlier)
(iv) Time taken by A to reach at $Q = \frac{800}{40} = 20$ h
Time taken by B to reach at $Q = \frac{800}{60} = 13\text{ h } 20\text{ min}$
Hence, A takes 6 hrs 40 min extra to reach at Q.
(v) Since, A leaves 2 hrs earlier, thus he will reach at Q only 4 hours 40 min late.
Since, A takes 6 hrs 40 min extra to reach at Q.
(vi) When B starts to move towards Q the difference between A and B is 80 km.
The required difference between A and $B = 50$ km
Hence, they have to reduce it by 30 km.
Thus, the required time $= \frac{(80 - 50)}{20} = 1.5\text{ h}$
$= 1\text{ h } 30\text{ min}$
Thus, after 3/2 hrs A will be only 50 metres ahead of B.
(vii) When B starts to follow A (towards Q) A was 80 km ahead of B. Also B wants to overtake A and further go ahead of A by 100 km. Thus, the net difference (required) = 180 km.
∴ Required time

$$= \frac{(\text{Distance advanced} + \text{Required difference})}{\text{Relative speed}}$$

$$= \frac{80 + 100}{20} = \frac{180}{20} = 9\text{ h}$$

Thus, after 9 hrs (when B starts moving) B will be 100 km ahead of A *i. e.*, they will be separated by 100 km from each other after crossing M.

TO AND FRO MOTION IN A STRAIGHT LINE

This concept is just the extension of the previous concepts of relative motion between more than one dynamic (or moving) bodies.

(a) When two bodies start moving towards each other

(i) To meet each other they cover the distances in the ratio of their individual speeds.
(ii) If the initial distance (or gaping) between two bodies A and B is D, then A and B together have to cover D unit of distance for the first meeting.
(iii) For the next number of meeting (*e.g.*, second, third, fourth meeting and so on) both A and B together have to cover $2D$ distance more from the previous meeting *i.e.*, to meet for the fourth time they have to cover together $D + (3 \times 2D) = 7D$ unit of distances. Similarly for seventh meeting they have to cover together $D + (6 \times 2D) = 13D$ units of distance. Thus for each subsequent meeting they have to cover $2D$ distance extra from the previous one.

NOTE Individually they will cover the distances in the ratio of their speeds for any number of meeting.
Thus, the total distance covered for the n^{th} meeting = $(2n - 1)D$

(b) When two bodies start moving towards the same direction

(i) For the first meeting after they start to move they have to cover $2D$ distance, if the distance between two particular points (or places) be D unit.
Since, the faster body reaches the next (or opposite) end first than the slower body and the faster body starts returning before the slower body reaches the same opposite end and thus the two bodies meet somewhere between the two ends covering individually the distances in their respective speeds.
(ii) For every subsequent meeting they have to cover together $2D$ unit distance more from the previous meeting.
Thus, for n^{th} meeting they have to cover together $(n \times 2D)$ unit of distance.
(iii) At any point of time the distances covered by the bodies will be equal to the ratio of their speeds.

EXAMPLE 14 *The distance between two points P and Q is 400 m and the speeds of A and B are 20 m/s and 30 m/s respectively. Initially A and B are at P. They move between P and Q. Calculate:*

(i) the distance covered by A in first meeting.
(ii) the time required for the second meeting.
(iii) the distance covered by B to meet for the third time.
(iv) the ratio of distances covered by A and B till the fourth meeting.
(v) the distance between P and the place of fifth meeting.
(vi) the distance between Q and the point of third meeting.

SOLUTION

Meeting	Total distance covered by *A* and *B* together	Distance covered by *A*	Distance covered by *B*	Point of meeting from *P*	Point of meeting from *Q*	Time (in second)
1st	200 m	80 m	120 m	80 m	20 m	4
2nd	400 m	160 m	240 m	40 m	60 m	8
3rd	600 m	240 m	360 m	40 m	60 m	12
4th	800 m	320 m	480 m	80 m	20 m	16
5th	1000 m	400 m	600 m	0 m (at *P*)	100 m	20

(i) The distance covered by A in the first meeting = 80 m

(ii) Time required for the second meeting $= \frac{400}{50} = 8$ s

(iii) The distance covered by B for the third time meeting $= 12 \times 30 = 360$ m

(iv) It is always (for any moment, after the starting of movement) will be in the ratio of their respective speeds. So, the required ratio of distances covered by A and $B = 20 : 30 = 2 : 3$ *i. e.*,

$$A : B = 20 : 30 = 2 : 3$$

(v) Since for the fifth meeting they have to cover $50 \times 20 = 1000$ m.

(vi) 60 m.

EXAMPLE 15 *The distance between two points P and Q is 100 m. A is initially at P and B is at Q. The speeds of A and B is 20 m/s and 30 m/s. They move between P and Q to and fro :*

(i) Find the time required for the first meeting.
(ii) Distance covered by A till the third meeting.
(iii) Distance covered by B till the fifth meeting.
(iv) The distance between P and the place of fourth meeting.
(v) The distance between Q and the place of fifth meeting.
(vi) The ratio of distances covered by each one till the third meeting.

SOLUTION

No. of meeting	Distance covered by *A* and *B* together	Time (in second)	Distance covered by *A*	Distance covered by *B*	Distance between *P* and point of meeting	Distance between *Q* and point of meeting
1st	100 m	2	40 m	60 m	40 m	60 m
2nd	300 m	6	120 m	180 m	80 m	20 m
3rd	500 m	10	200 m	300 m	at *P*	at *P*
4th	700 m	14	280 m	420 m	80 m	20 m
5th	900 m	18	360 m	540 m	40 m	60 m

(i) 2 second $\left(\text{Time} = \frac{100}{50} = 2\right)$

(ii) 200 m (Distance = Speed × Time = $20 \times 10 = 200$ m)

(iii) 540 m ($D = 18 \times 30 = 540$ m)

(iv) 80 m

(v) 60 m

(vi) 2 : 3 always equal to the ratio of respective speeds.

EXAMPLE 16 *A and B are two friends. A lives at a place P and B lives at another place Q. Everyday A goes to Q to meet B at 120 km/h. Thus, he takes 3 hours. On a particular day B started to meet A so he moved towards P. On that day A took only 2 hours to meet B on the way instead to Q.*

(i) What is the ratio of speeds of A is to B? *(ii) What is the speed of B?*

SOLUTION Distance between P and $Q = 120 \times 3 = 360$ km

Let the speed of B be S_B, then

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$2 = \frac{360}{(120 + S_B)}$$

$$\Rightarrow \quad S_B = 60 \text{ km/h}$$

Here, A and B are moving towards each other. So, the relative speed will be the sum of the speeds of A and B both.

Therefore, ratio of speeds of $A : B = 2 : 1$.

(i) 2 : 1 (ii) 60 km/h

NOTE In this case ratio of speeds of

$$A : B = \frac{\text{Actual time required when } B \text{ is also moving}}{\text{Time difference when } B \text{ is also moving}}$$

$$= \frac{(3 - 1)}{1} = \frac{2}{1}$$

EXAMPLE 17 *A lives at P and B lives at Q. A usually goes to meet B at Q. He covers the distance in 3 hours at 150 km/h. On a particular day B started moving away from A. While A was moving towards Q thus A took total 5 hours to meet B.*

(i) What is the speed of B?

(ii) What is the ratio of speeds of $A : B$?

SOLUTION Distance $= 3 \times 150 = 450$ km

$$\text{Time} = \frac{\text{Distance}}{\text{Relative speed}}$$

$$5 = \frac{450}{(150 - S_B)} \qquad S_B \rightarrow \text{Speed of } B$$

$$\Rightarrow \quad S_B = 60 \text{ km/h}$$

Ratio of speeds of $A : B = \frac{150}{60} = \frac{5}{2} = 5 : 2$

(i) 60 km/h (ii) 5 : 2

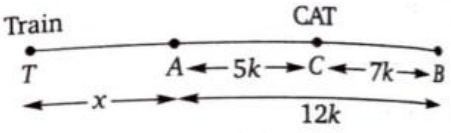
In this case ratio of speeds of

$$A : B = \frac{\text{Actual time required when } B \text{ is also moving}}{\text{Time difference}}$$

$$= \frac{5}{2}$$

EXAMPLE 18 *A train approaches a tunnel AB. Inside the tunnel a cat located at a point i. e., $\frac{5}{12}$ of the distance AB measured from the entrance A. When the train whistles, the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat at exactly the exit. The speed of the train is greater than the speed of the cat by what order?*

SOLUTION



Let the speed of train be u and the speed of cat be v and train whistles at a point T, x km away from A, then

$$\frac{u}{v} = \frac{x}{5k} = \frac{x + 12k}{7k}$$

$$\Rightarrow \quad 7x = 5(x + 12k)$$

$$\Rightarrow \quad \frac{x}{k} = \frac{30}{1}$$

$$\therefore \quad \frac{u}{v} = \frac{30}{5 \times 1} = \frac{6}{1}$$

Alternatively: $\frac{u}{v} = \frac{7k + 5k}{7k - 5k} = \frac{6}{1}$

NOTE Since time is constant, therefore distances covered by train and cat will be in the ratio of their respective speeds.

CONCEPT BASED ON TRAINS

(i) When two trains (or bodies) are moving in opposite direction, their relative speed will be equal to the sum of their individual speeds.

(ii) When two trains are moving in the same direction their relative speed will be equal to the difference of their speeds.

(iii) Distance to be covered to cross each other is always equal to the sum of their individual lengths.

(iv) Distance to be covered such as bridge, platform etc., is always equal to the sum of the length of train and the length of the particular object such as bridge, platform etc.

(v) Distance to be covered such as pole, man, tree etc. is always equal to the length of the train only.

(vi) If a man is travelling in a train, then this man has to cover the distance to cross another train is equal to the length of the train which is passing or crossing him. In this case the relative speed of both the trains will be considered.

EXAMPLE 19 *A train crosses a tree in 10 seconds. If the length of the train be 150 m, then find the speed of the train.*

SOLUTION Distance = Length of train

= Speed of train × Time

$150 = \text{Speed} \times 10$

$$\Rightarrow \quad \text{Speed} = 15 \text{ m/s}$$

$$\text{Speed} = 15 \times \frac{18}{5} = 54 \text{ km/h}$$

NOTE A train starts to cross a stationary thin object (of inconsiderable thickness) when the engine of the train meets the object and completes the crossing when the last wagon (or backend of the train just crosses the object.

EXAMPLE 20 *A train crosses a man coming from the opposite direction in 7.5 seconds. If the speed of man be 10 m/s and speed of train is 20 m/s, find the length of the train.*

SOLUTION Length of train = Time × Relative speed

$= 7.5 \times (10 + 20) = 7.5 \times 30 = 225$ m

EXAMPLE 21 *A train of length 250 m crosses a bridge of length 150 m in 20 seconds. What is the speed of train?*

SOLUTION (Length of train + Length of bridge)
= Speed of train × Time

$$(250 + 150) = 20 \times \text{Speed}$$

$$\text{Speed} = \frac{400}{20} = 20 \text{ m/s} = 72 \text{ km/h}$$

EXAMPLE 22 *Two trains coming from the opposite sides crosses each other in 10 seconds if the lengths of first train and second train be 125 m and 175 m respectively, also the speed of first train be 36 km/h, find the speed of second train.*

SOLUTION Speed of first train = 36 km/h = 10 m/s

Now, $\text{Time} = \dfrac{\text{Sum of length of the two trains}}{\text{Sum of their speeds}}$

$$10 = \frac{125 + 175}{(10 + x)}$$

$$\Rightarrow \quad x = 20 \text{ m/s} = 72 \text{ km/h}$$

EXAMPLE 23 *A fast moving superfast express crosses another passenger train in 20 seconds. The speed of faster train is 72 km/hr and speeds of slower train is 27 km/h. Also the length of faster train is 100 m, then find the length of the slower train if they are moving in the same direction.*

SOLUTION $\text{Time} = \dfrac{\text{Sum of length of the two train}}{\text{Difference in speeds}}$

$$20 = \frac{(100 + x)}{25/2}$$

$$\Rightarrow \quad x = 150 \text{ m}$$

NOTE Relative speed = (72 − 27) = 45 km/h

$$= 45 \times \frac{5}{18} = \frac{25}{2} \text{ m/s}$$

CONCEPT BASED ON BOATS AND RIVERS (OR STREAMS)

(i) When the boat and stream (or current) of river move in the same direction, then the relative speed of the boat is the sum of the individual speeds of boat and river. It is known as **downstream speed**.

(ii) When the boat moves against the current of the river (*i.e.*, in opposite direction), then the relative speed of the boat is the difference of the speeds of the boat and stream (of the river). It is known as **upstream speed**.

Let the speed of boat in still water be B and speed of current of river be C then,

$$\begin{cases} \text{Downstream speed} = (B + C) \\ \text{Upstream speed} = (B - C) \end{cases} ; \qquad B > C$$

$$\begin{cases} \text{Speed of the boat in still water} = \dfrac{(D + U)}{2} \\ \text{Speed of current (or stream)} = \dfrac{(D - U)}{2} \end{cases}$$

where $D \rightarrow$ downstream speed of the boat

and $U \rightarrow$ upstream speed of the boat

When the distance covered by boat in downstream (*i.e.*, with the flow of water) is same as the distance covered by boat in upstream (against the flow of the water) then,

$$\frac{\text{Time taken by boat in DS}}{\text{Time taken by boat in US}} = \frac{\text{Upstream speed}}{\text{Downstream speed}}$$

DS → Downstream, US → Upstream

EXAMPLE 24 *A boat can move at 5 km/h in still water (i.e., when water is not flowing). The speed of stream of the river is 1 km/h. A boat takes 80 minutes to go from a point A to another point B and return to the same point.*

(i) What is the distance between the two points?

(ii) What is the ratio of downstream speed and upstream speed?

(iii) What is the ratio of time taken in downstream speed to the upstream speed?

SOLUTION Downstream speed of boat = (5 + 1) = 6 km/h

Upstream speed of boat = (5 − 1) = 4 km/h

Therefore, $\dfrac{\text{Downstream speed}}{\text{Upstream speed}} = \dfrac{\text{Upstream time}}{\text{Downstream time}}$

$$\frac{6}{4} = \frac{3}{2} = \frac{\text{Time taken in upstream direction}}{\text{Time taken in downstream direction}}$$

$\therefore$ Time taken in downstream $= \dfrac{2}{5} \times 80 = 32$ min

and time taken in upstream direction $= \dfrac{3}{5} \times 80 = 48$ min

$\therefore$ Distance between two points = DS speed × DS time
= US speed × US time

where DS → Downstream and US → Upstream

$$D = \frac{6 \times 32}{60} = 3.2 \text{ km}$$

or $$D = 4 \times \frac{48}{60} = 3.2 \text{ km}$$

(i) 3.2 km (ii) 3 : 2 (iii) 2 : 3

EXAMPLE 25 *A man can row 9 km/h in still water. It takes him twice as long as to row up as to row down. Find the rate of stream of the river.*

SOLUTION $\dfrac{\text{Time taken in upstream}}{\text{Time taken in downstream}} = \dfrac{2}{1}$

$\therefore \quad \dfrac{\text{Downstream speed}}{\text{Upstream speed}} = \dfrac{2}{1}$ where $\dfrac{B + R}{B - R} = \dfrac{2}{1}$

$B \rightarrow$ Speed of boat in still water

$R \rightarrow$ Speed of current

$\Rightarrow \quad \dfrac{B}{R} = \dfrac{3}{1}$ (By componendo and dividendo)

$\Rightarrow \quad \dfrac{9}{R} = \dfrac{3}{1} \Rightarrow R = 3 \text{ km/h}$

RACES

Terminology

(i) **Startup or headstart:** When a runner allows to another runner to stay ahead in the same race, then it is said that there is a startup in the race.
For example if A allows B to go ahead before starting the race, then it is said that A gives startup to B and B has the startup. If before starting the race B goes ahead of x metre, then we can say A gives x metre startup to B or B has startup (or headstart) of x metre.

(ii) **Dead heat:** When the runners reach the finishing line (or the final post) then it is said that these runners finish (or end) the race in dead heat.

SOME MORE USEFUL CONCEPTS

(i) When it is said that A can give B a start of x metre race, then it means in y metre race B runs metre less than A in the same time.

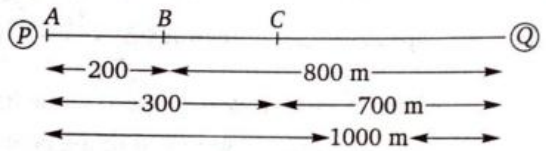
NOTE There is a great difference between 'can' and 'gives'.

(ii) When A beats B by t seconds in a race of y metre it means B is the loser and A is the winner and when A reaches the finishing line B is still some distance back to A. Thus B takes t seconds to cover the remaining distance. Hence, we can calculate the speed of loser (B).

(iii) Throughout the race there is always a certain relationship among runners *i.e.*, they always maintain the ratio of speeds. I think this is the main of all the problems pertaining to the races. Go ahead.

EXAMPLE 1 *A can give B a 200 m startup and C a 300 m startup in a race of 1 km. How many metres startup can B gives to C in a 1 km race.*

SOLUTION



Ratio of speeds of $A : B = 1000 : 800 = 5 : 4$

Ratio of speeds of $A : C = 1000 : 700 = 10 : 7$

Ratio of speeds of $B : C = 800 : 700 = 8 : 7$

Since, when B moves 8 m, C moves 7 metre. Therefore, when B moves 1000 m, C moves 875 metre. Thus, B can give C a start of $1000 - 875 = 125$ m.

Alternatively: Since C is 12.5% slower than B. So, C will cover 12.5% less distance than B in the same time.

(Since when time is constant, they cover the distances in the ratio of their speeds.)

Thus, in 1000 m (or 1 km) when B runs 1000 m, C will run 125 m less than B.

Hence, B can give a start of 125 m to C in a 1 km race.

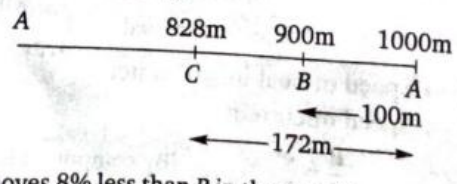
EXAMPLE 2 *In a one km race A gives B a start of 100 m and in a one km race B gives a start of 80 m to C. In a 1 km race who will win and by how much distance from the worst performer between two losers?*

SOLUTION Ratio of speeds of $A : B = 1000 : 900 = 100 : 90$

Ratio of speeds of $B : C = 1000 : 920 = 100 : 92$

Therefore, when A moves 1000 m, B moves 900 m and when B moves 900 m, C moves 828 m.

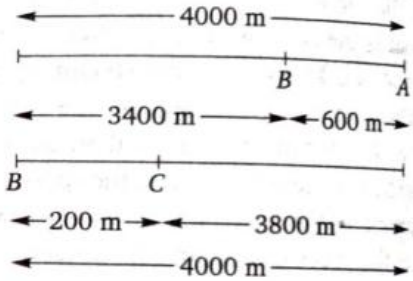
Thus,



Since, C moves 8% less than B in the same time. Thus, C is the worst performer and A will win by him by 172 m.

EXAMPLE 3 *In a 4 km race A wins by 600 m over B. B can give start of 200 m to C in a 4 km race. By how much distance C gets start up so that the race between A and C ends in dead heat in the same race of 4 km?*

SOLUTION



Ratio of speeds of $A : B = 20 : 17$

and Ratio of speeds of $B : C = 20 : 19$

$\therefore$ Ratio of speeds of $A : B : C = 400 : 340 : 323$

Therefore, in 4000 m race A run 4000 m, B run 3400 m and run 3230 m. Thus C can get 770 m start up from A.

EXAMPLE 4 *In a 1500 m race A wins over B by 350 m and in 1500 m race C can give a startup of 250 m to B. By how much distance to give start up to C, so that A beats C by 50 metres?*

SOLUTION Ratio of speeds of $A : B = 30 : 23$

Ratio of speeds of $B : C = 5 : 6$

$\therefore$ Ratio of speeds of $A : B : C = 150 : 115 : 138$

So, when A moves 1500 m, B moves 1150 m and C moves 1380 m. Thus C moves 120 m less than A. To win A just by 50 over C, A should give $120 - 50 = 70$ m startup to C.

EXAMPLE 5 *In a race of 2500 m, A beats B by 500 m and in a race of 2000 m, B beats C by 800 m. By what distance A gives startup to so that they will end up with dead heat in 3 km race. Also find by what distance A will win over C in a 1 km race?*

SOLUTION Ratio of speeds of $A : B = 5 : 4$

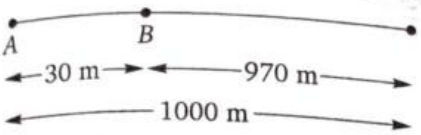
Ratio of speeds of $B : C = 5 : 3$

$\therefore$ Ratio of speeds of $A : B : C = 25 : 20 : 12$

In 3 km race A run 3000 m, B run 3400 m, C run 1440 m, so to end up the race in dead heat A should give C the startup of 1560 m and therefore in 1 km the same will be 520 m.

EXAMPLE 6 *A gives B, a start of 30 m or 10 seconds and end up the race of 1 km in dead heat. What is the ratio of speeds of A and B?*

SOLUTION



Since, either B has the startup of 30 m or 10 seconds. It means B runs 30 m in 10 seconds. Hence, the speed of B is 3 m/s.

NOTE Don't be confused that A's speed = 3 m/s.

Also, it is very simple when A moves 1000 m, B moves 970 m. Since, the ratio of speeds is equal to the ratio of distances covered by A and B in the same time. Thus, the speed of

$$A : B = 1000 : 970 = 100 : 97$$

EXAMPLE 7 *In a 1 km race A wins over B by 80 m or 20 seconds. B can give a start of 100 m to C in 1 km race. Find out that by how much time A will win over C? Also, find the ratio of speeds of B and C.*

SOLUTION Ratio of speeds of $A : B = 100 : 92$

Ratio of speeds of $B : C = 10 : 9$

$\therefore$ Ratio of speeds of $A : B : C = 1000 : 920 : 828$

Also, $\quad$ Speed of $B = \dfrac{80}{20} = 4$ m/s

Therefore, $\quad$ Speed of $C = 3.6$ m/s

Now, C has to cover 172 m distance in extra time. So, the time taken by C to cover the remaining distance $= \dfrac{172}{3.6} = 47.77$ s.

Ratio of speeds of $B : C$ = Ratio of distances covered by $B : C$

$$= 1000 : 900 = 10 : 9$$

$$\therefore \quad \frac{\text{Speed of } B}{\text{Speed of } C} = \frac{10}{9}$$

EXAMPLE 8 *A can win B by 250 m in a 2 km race. What should be the change in distance of startup? So, that B must cover 20% less distance than that by A in the same time.*

SOLUTION

$$\frac{\text{Distance covered by } A}{\text{Distance covered by } B} = \frac{5}{4}$$

$$\therefore \quad \frac{\text{Speed of } A}{\text{Speed of } B} = \frac{5}{4}$$

So, when A moves 2000 m, B should move 1600 m. But since initially B moves 1750 m. Therefore the new startup will be increased by 150 m.

EXAMPLE 9 *The ratio of speeds of A and B is 4 : 7 and A loses the race by 270 m, then what is the length of the race course?*

SOLUTION When B moves 7 m, A moves only 4 m. Hence, A loses the race by 3 m.

Now, since B loses by 3 m in the race of 7 m.

$\therefore$ B will lose 270 m in the race of 630 m.

EXAMPLE 10 *The ratio of time taken to run a certain distance by Pythagorus and Hawkins is 4 : 3 and thus Hawkins wins the race by 360 m. What is the distance of race course?*

SOLUTION

$$\frac{\text{Time taken by Pythagorus}}{\text{Time taken by Hawkins}} = \frac{4}{3}$$

$$\frac{\text{Speed of Pythagorus}}{\text{Speed of Hawkins}} = \frac{3}{4}$$

$$= \frac{\text{Distance covered by P}}{\text{Distance covered by H}}$$

Now, when Hawkins runs 4 m, Pythagorus runs 3 m and thus Hawkins wins by 1 m.

So, when Hawkins wins the race by 1 m, race course is 4 m. when Hawkins wins the race by 360 m, race course is

$$360 \times 4 = 1440 \text{ m}$$

Alternatively : 360 = 25% of the total length of race.

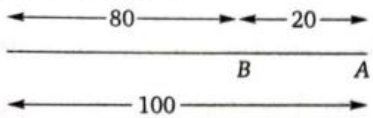
$\therefore$ length of race course = 1440 m.

EXAMPLE 11 *In a race of D km, A wins over B by 0.2 D and in the same length of race B wins over C by 0.25 D. What should be the head- start to C, So that A and C finish the race at the same time.*

SOLUTION **Note:** Instead of solving the problem using 'D' as a distance convert it into 100 *i. e.*, suppose $D = 100$.

(It is just for your convenience.)

Now,



Now, $\quad$ Speed of $A : B = 100 : 80 = 5 : 4$

Speed of $B : C = 4 : 3$

$\therefore$ $\quad$ Speed of $A : B : C = 20 : 16 : 12$

$$= \underbrace{100 : 80 : 60}_{\{40\}}$$

So, A can give 'C' a start of 40 m in 100 m race. Therefore, A can give 0.4 D. Start to 'C' in the race of D unit.

EXAMPLE 12 *In a 3 km race the speeds of A and B are in the ratio of 6 : 5 and A wins by 10 seconds. What is the time taken by B to finish the race. Also, to end the race in dead heat what per cent of total distance A should give the startup to B?*

SOLUTION Ratio of distances covered by A and B when A just reaches the finishing line = 6 : 5.

Thus, B has to cover 500 m distance in extra time and this 500 m distance is covered by B in 10 seconds. So, the speed of B = 50 m/s.

Thus, the total time required by B to complete the 3 km race

$$= \frac{3000}{50} = 60 \text{ s} = 1 \text{ min}$$

Since, B coveres 16.66% distance less than A covers in the same time, so A should give 16.66% of the total distance as a startup to B.

EXAMPLE 13 *In a 1 km race A gives B a startup of 5 seconds and still wins over B by 15 seconds. The ratio of speeds of A and B is 2 : 1. Find the time taken by A to finish 2.5 km race.*

SOLUTION

$$\frac{\text{Speed of } A}{\text{Speed of } B} = \frac{2}{1}$$

$$\frac{\text{Time taken by } A}{\text{Time taken by } B} = \frac{1}{2}$$

$$\frac{t}{(t+20)} = \frac{1}{2}$$

$$\Rightarrow \quad t = 20 \text{ s}$$

Thus, A needs 20 seconds to cover 1 km. Thus, to cover 2.5 km race he needs $20 \times 2.5 = 50$ seconds.

EXAMPLE 14 *X can beat Y by 200 m in a race of 2000 m. Y can beat Z by 100 m in a race of 2500 m. By how many metres can X beat Z in a race of 1000 m.*

SOLUTION Ratio of speeds of X and $Y = 10 : 9$ (2000 : 1800)

Ratio of speeds of Y and $Z = 25 : 24$ (2500 : 2400)

Ratio of speeds of X, Y and $Z = 250 : 225 : 216$

Since in a race of 250 m, X beats Z by 34 m.

So, in a race of 1000 m, X will beat Z by 136 m.

EXAMPLE 15 *In a 6 km race B has 250 m headstart and C has 500 m headstart by A, still A beats C and B by 235 m and 350 m respectively. How many metres startup can B give to C so as to end up the race at the same time with C in the race of 6 km. Also find the ratio of speeds of $A : B : C$.*

SOLUTION When A runs 6000 m, B runs 5400 m only and C runs 5265 m only.

So, when B runs 6000 m, C will run 5850 m. So, B can give 150 m startup to C.

Ratio of speeds of $A : B : C = 6000 : 5400 : 5265$

$= 1200 : 1080 : 1053$

$= 400 : 360 : 351$

EXAMPLE 16 *A can run 1 km in 2 min 20 second and B can run the same distance in 3 min. What is the distance travelled by B in the same time as A travels, when they start simultaneously in the race of 4.5 km.*

SOLUTION

$$\frac{\text{Time taken by } A}{\text{Time taken by } B} = \frac{140 \text{ s}}{180 \text{ s}} = \frac{7}{9}$$

$$\therefore \quad \frac{\text{Speed of } A}{\text{Speed of } B} = \frac{\text{Distance travelled by } A}{\text{Distance travelled by } B} = \frac{9}{7}$$

Therefore, B travels $\frac{7}{9} \times 4.5 = 3.5$ km.

EXAMPLE 17 *Shahrukh takes 4 min to cover the same distance for which Urmila takes 6 min 30 sec. What is the ratio of distances covered by Shahrukh and Urmila in the race of 2.6 km and by what distance Shahrukh wins over Urmila?*

SOLUTION

$$\frac{\text{Time taken by Shahrukh}}{\text{Time taken by Urmila}} = \frac{240}{390} = \frac{8}{13}$$

$$\frac{\text{Distance covered by Shahrukh}}{\text{Distance covered by Urmila}} = \frac{13}{8}$$

Hence, Shahrukh will win the race by 1 km.

EXAMPLE 18 *A runs at the speed of $\frac{7}{4}$ times the speed of B and in a calculation B finds that she has to run 300 m after A reaches to the winning post. What is the total distance of race?*

SOLUTION

$$\frac{\text{Speed of } A}{\text{Speed of } B} = \frac{7}{4}$$

$$\frac{\text{Distance of } A}{\text{Distance of } B} = \frac{7}{4}$$

So, the total distance of race = 700 m.

$(\because 7x - 4x = 300 \Rightarrow 7x = 700)$

EXAMPLE 19 *Time taken by A is 5/7 of B's time for the same length of race. The speed of A is 84 m/s and A beats B by 240 m. What is the length of race course?*

SOLUTION

$$\frac{\text{Speed of } A}{\text{Speed of } B} = \frac{\text{Time taken by } B}{\text{Time taken by } A} = \frac{7}{5}$$

Now since, $7x - 5x = 240$

$$\Rightarrow \quad 7x = 840 \text{ m}$$

EXAMPLE 20 *In a race of 3000 m, Michal beats Nicholas by 600 m and Nicholas in a race of 2000 m beats Oscar by 600 m. In a 1 km race by how much distance Michal beats Oscar?*

SOLUTION Ratio of speeds of $M : N : O = 100 : 80 : 56$

So, Michal (M) beats Oscar (O) by 440 m in 1 km race.

EXAMPLE 21 *In a race of 500 m, President runs at 5 m/s. If president gives chairman a start of 20 m and still beats him by 20 seconds, what is the chairman's speed?*

SOLUTION Chairman has to cover only 480 m. He takes total

$$100 + 20 = 120 \text{ s}$$

Therefore, speed of chairman $= \frac{480}{120} = 4$ m/s

EXAMPLE 22 *In a race of 800 m Dholakiya gives Preetam a start of 200 m and then loses the race by 20 seconds. What is the speed of Preetam and Dholakiya respectively? If the ratio of respective speeds be 3 : 2.*

SOLUTION Let the speed of Dholakiya be S_D and speed of Preetam be S_P and let the time taken by Preetam be t second then.

$$\frac{S_D}{S_P} = \frac{\frac{800}{(t+20)}}{\frac{600}{t}} = \frac{2}{3}$$

$$\frac{800}{600} \times \frac{t}{(t+20)} = \frac{2}{3}$$

$$t = 20 \text{ s}$$

$$\therefore \quad \text{Speed of Preetam} = \frac{600}{20} = 30 \text{ m/s}$$

and $\quad \text{speed of Dholakiya} = \frac{800}{40} = 20 \text{ m/s}$

EXAMPLE 23 *In race of 1 km Sahara gives Birla a start of 250 m still Sahara wins by 150 m. What is the ratio of speeds of Sahara and Birla?*

SOLUTION When Sahara covers 1000 m, Birla covers only 600 m.
So, the ratio of speeds of Sahara and Birla
$= 1000 : 600 = 5 : 3$

EXAMPLE 24 *Priyambda wins the race over Kokilaben by 150 m in a race of 1 km but when she gives a startup of 5 seconds to Kokilaben she wins by 65 m. Find the speed of Kokilaben.*

SOLUTION In 5 seconds Kokilaben runs 85 m.
So, the speed of Kokilaben = 17 m/s.

CIRCULAR MOTION

(i) When the bodies are moving in the opposite direction, their relative speed becomes equal to the sum of their individual speed.

(ii) When two bodies are moving in the same direction their relative speed becomes equal to the difference of the individual speeds.

First Meeting

(i) Let A and B be two runners. Time taken by them to meet for the first time

$$= \frac{\text{Length of the circular track}}{\text{Relative speed}}$$

(ii) When there are more than two runners, then suppose A is the fastest runner and A meets B for the first time in t_{AB} seconds/hours and A meets C for the first time in t_{AC} seconds/hours and A meets D for the first time in t_{AD} seconds/hours and so on. Then time taken by all of them to meet for the first time is the LCM of t_{AB}, t_{AC}, t_{AD} etc.

First Meeting at the Starting Point

Let A takes t_A time to complete one round and B takes t_B time and C takes t_C time and so on, then the time taken to meet for the first time at the starting point = LCM of t_A, t_B, t_C etc.

EXAMPLE 1 *Arjun and Bhishma are running on a circular track of length 600 m (i.e., circumference of the track). Speed of Arjun is 30 m/s and that of Bhishma is 20 m/s. They start from the same point at the same time in the same direction. When will they meet again for the first time?*

SOLUTION
$$\text{Time} = \frac{\text{Circumference}}{\text{Relative speed}}$$
$$= \frac{600}{10} = 60 \text{ s}$$

Actually Arjun (faster one) has to make a lead of 600 m, because when Arjun will be 600 m ahead (or extra distance) of Bhishma, they will be together again as a person when completes the total length (or circumference) it starts retracing the same path and thus Arjun and Bhishma can be together again.

Since, they make a difference (or Arjun makes a lead) of 10 m in 1 second. So, he will create 600 m difference in 60 second.

EXAMPLE 2 *In the previous problem if they move in opposite direction, then what is the time taken by them to meet again for the first time?*

SOLUTION
$$\text{Time} = \frac{\text{Circumference}}{\text{Relative speed}}$$
$$= \frac{600}{50} = 12 \text{ s} \qquad (30 + 20 = 50)$$

EXAMPLE 3 *Arjun and Bhishma run at the speed 30 m/s and 20 m/s respectively on the circular track of 600 m, as its circumference, when would the Arjun and Bhishma meet for the first time at the starting point if they start simultaneously from the same point?*

SOLUTION Time taken by Arjun to complete one round
$$= \frac{600}{30} = 20 \text{ s}$$
Time taken by Bhishma to complete one round $= \frac{600}{20} = 30$ s

Hence, after every 20 second, Arjun would be at the starting point and after every 30 second. Bhishma would be at the starting point. Thus the time taken by both to be at the starting point again for the first time

$$= \text{LCM of 20 and 30} = 60 \text{ s}$$

Thus, every 60 seconds they would be together at the starting point.

NOTE The required time for the bodies to meet for the first time at the starting point is immaterial to the direction of bodies *i.e.*, whether they move in the same direction or in opposite direction.

EXAMPLE 4 *Arjun, Bhishma and Nakul run on the circular path at the speed of 20 m/s, 30 m/s and 50 m/s respectivley in the same direction. The circumference of the track (or path) is 600 m.*
(i) When will they be together again for the first time?
(ii) When will they be together again for the first time at the starting point?

SOLUTION (i) Nakul meets Arjun after every $= \frac{600}{(50 - 20)} = 20$ s

Nakul meets Bhishma after every $= \frac{600}{(50 - 30)} = 30$ s

Therefore, all of the three would meet after every 60 seconds. (60 = LCM of 20 and 30).
Hence, they would all meet for the first time after 60 seconds.

(ii) Arjun takes $\frac{600}{20} = 30$ s to complete one round

Bhisma takes $\frac{600}{30} = 20$ s to complete one round

and Nakul takes $\frac{600}{50} = 12$ s to complete one round

Hence, they would meet for the first time at the starting point after 60 seconds.

HINT 60 = LCM of 30, 20 and 12.

EXAMPLE 5 *A tortoise can complete a full round on the circular track in 8 min, while the rabbit can do the same on the same circular track in 5 min. A, B, C and D are the four consecutive points on the circular track which are equidistant from each other. A is opposite to C and B is opposite to D.*

(i) After how many minutes will they meet together for the first time, when both have started simultaneously from the same point in the same direction?

(ii) After how many minutes will they meet together for the first time at the starting point, when both have started simultaneously from the same point in the same direction?

(iii) If they start from a point A, simultaneously in the same direction then after how many minutes will they meet at C which is just opposite to A?

(iv) If tortoise has a lead of 5 min, when would they meet for the first time, where they have started from the same point in the same direction?

SOLUTION (i) Ratio of time of rabbit and tortoise = 5 : 8

$\therefore$ Ratio of the speed of rabbit and tortoise = 8 : 5

Let the length of the circular track be 40 m, then the speed of rabbit = 8 m/min and speed of tortoise = 5 m/min.

(40 is the LCM of 8 and 5, taken just for conveninence in calculation)

Now, the time when they will be together for the first time

$$= \frac{\text{Circumference}}{\text{Relative speed}} = \frac{40}{3} \text{ min}$$

(ii) Time taken by rabbit to complete one round = 5 min

Time taken by tortoise to complete one round = 8 min

$\therefore$ They will meet at the starting point, for the first time

= LCM of 5 and 8 = 40 min

(iii) Rabbit reaches point C in $\frac{20}{8} = 2.5$ min.

Rabbit reaches the same point C in 7.5, 12.5, 17.5, 22.5 min etc. Tortoise reaches after 4 min, 12 min, 20 min, ... etc. They will never meet at C since time cannot coincide as time taken by rabbit is the decimal number and time taken by tortoise is a natural number.

(iv) In 5 min tortoise travels 25 m. Now, rabbit starts chasing tortoise @ 8 m/min.

$\therefore$ Time taken by rabbit to overtake tortoise

$$= \frac{25}{(8-5)} = \frac{25}{3} \text{ min}$$

EXAMPLE 6 *Alfred and Bernard run on the circular track of 600 m. Speeds of Alfred and Bernard are 30 m/s and 20 m/s respectively. Initially they are diametrically opposite to each other.*

(i) When will they meet for the first time if both move in the same direction?

(ii) If both of them move in opposite directions, when will they meet for the second time?

SOLUTION (i) Relative speed of Alfred = 30 − 20 = 10 m/s

Bernard is 300 m ahead in the race.

So, time taken by Alfred to catch Bernard $= \frac{300}{10} = 30$ s

(ii) To meet for the second time they have to cover

= 300 + 600 = 900 m

there relative speed = 30 + 20 = 50 m/s

(since direction is opposite)

$\therefore$ Time taken to meet for the second time $= \frac{900}{50} = 18$ s

EXAMPLE 7 *A, B and C run on a circular track of 800 m. Speeds of A, B and C are 20 m/s, 26 m/s and 33 m/s.*

(i) When will they meet for the first time?

(ii) What is the ratio of distances covered by each one to meet for the first time.

SOLUTION (i) Time taken by C to meet A for the first time

$$= \frac{800}{33-20} = \frac{800}{13} \text{ s}$$

Time taken by C to meet B for the first time

$$= \frac{800}{33-26} = \frac{800}{7} \text{ s}$$

LCM of $\frac{800}{13}$ and $\frac{800}{7}$ = 800 s.

Therefore, all of them (i. e., A, B and C) meet after 800 seconds = 13 min 20 sec.

(ii) The ratio of distances = ratio of speeds

= 20 : 26 : 33

INTRODUCTORY EXERCISE-9.1

1. P and Q starts running on the circular track in opposite directions. They start from a point A and meet for the first, second and third time at B, C and A respectively. What is the ratio of speeds of P and Q?

(a) 2 : 1 (b) 3 : 1

(c) 4 : 1 (d) none of these

2. P and Q run on a circular track of 100 m. Speed of P is 250 m/s and that of Q is 400 m/s. In how many points on a circular track will they meet if they start from the same point, simultaneously in the same direction?

(a) 1 (b) 2

(c) 3 (d) 4

3. A, B and C start to walk around a circular track. A completes one round in 91/6 seconds, B completes in 65/4 seconds and C completes in 56/3 seconds respectively. After how many seconds will they meet each other at the starting point again?

(a) 3460 s (b) 1820 s

(c) 3640 s (d) none of these

CLOCKS

Actually the movement of hour-hand and minute-hand follows the relative motion. The dial of the clock behaves like a circular track and where minute-hand is a faster runner and hour-hand is a slower one.

For better understanding with the clocks, assume 60 minutes shown on the dial as 60 points. Here we give an arbitrary new unit of distance as 'point'.

So, minute-hand (MH) runs on the circular track of 60 points at 60 points per hour and hour-hand (HH) runs at 5 points per hour. Now we become familiar with the relative motion of two hands of a clock.

Here, 1 point = 6° and 60 point = 360°

also 1 point = 6° = 1 min

EXAMPLE 1 *How many times minute-hand coincides with hour-hand in 12 hours?*

SOLUTION Time taken by minute-hand to meet hour-hand for the first time $= \frac{60}{55} = \frac{12}{11}$ hours.

(Assume initially both hands are at 12 *i.e.*, 12 O' clock is shown by them.)

Therefore, after every 12/11 hours-minute hand and hour-hand meet each other (or coincide or overtake).

Now, since in $\frac{12}{11}$ hours they coincide 1 time

So, in 12 hours they will coincide

$$= \frac{12}{12/11} = \frac{12}{12} \times 11 = 11 \text{ times}$$

Remember: Between 11 O'clock and 1 O'clock, two hands coincide only one time, that's why they coincide with each other only 11 times in 12 hours.

NOTE In every $\frac{12}{11}$ hours (or $65\frac{5}{11}$ min) two hands of a clock coincide.

EXAMPLE 2 *In 12 hours how many times the two hands of clock will be just opposite to each other i.e., they make a straight line having the difference of 180° between them?*

SOLUTION 180° = 30 points

For the first time minute-hand and hour-hand will be separated in $\frac{30}{55}$ h $\left(\text{Time} = \frac{\text{Required distance}}{\text{Relative speed}}\right)$ and for every next time they will take $\frac{60}{55}$ minutes more to occur as opposite to each other.

(Since in 60/55 hours they complete one round of clock's dial *i.e.,* 60 points undergoing the relative motion, we have total 12 hours.)

Thus, $\frac{30}{55} + \frac{60}{55} + \frac{60}{55} + \frac{60}{55} + \ldots = 12$

$$\Rightarrow \quad \frac{30}{55} + \frac{60}{55} + \frac{60}{55} + \ldots = \frac{12 \times 55}{55}$$

$$\Rightarrow \quad 6 + 12 + 12 + \ldots = 12 \times 11 = 132$$

$$\Rightarrow \quad 6 + 12n = 132$$

$$12n = 126 \quad \Rightarrow \quad n = 10$$

only integral value is admissible.

Thus, total 10 + 1 = 11 times both hands of a clock will be opposite to each other.

Remember: Between 5 O'clock and 7 O'clock the two hands make 180° angle only one time, that's why they make 180° angle only 11 times in 12 hours, *i.e.*, at exactly 6 O'clock they are 180° apart.

EXAMPLE 3 *In 12 hours how many times a minute-hand and hour-hand of a clock makes 90° between them or becomes perpendicular to each other?*

SOLUTION 90° = 15 points

This problem can be solved in two parts.

(i) When minute-hand goes ahead of hour-hand.

(ii) When hour-hand goes ahead of minute-hand.

(i) For the first time minute-hand and hour-hand will make 90° (or 15 points) difference in $\frac{15}{55}$ hours.

$\left(\text{time} = \frac{\text{distance}}{\text{relative speed}}\right)$

Now, after every $\frac{60}{55}$ hours they will occur at 90°. Since, in every $\frac{60}{55}$ hours they create a difference of 360° or 60 points (as the circumference of dial).

Now, we have 12 hours,

$$\therefore \quad \frac{15}{55} + \frac{60}{55} + \frac{60}{55} + \ldots = 12\text{h} = 12 \times \frac{55}{55}$$

$$\Rightarrow \quad \frac{3}{11} + \frac{12}{11} + \frac{12}{11} + \ldots = \frac{12 \times 11}{11}$$

$$\Rightarrow \quad \frac{3}{11} + \left(\frac{12}{11}n\right) = \frac{12 \times 11}{11}$$

$$\Rightarrow \quad 3 + 12n = 132$$

$$\Rightarrow \quad n = \frac{129}{12} = 10$$

(only integral value of n is acceptable)

Therefore, 10 + 1 = 11 times in 12 hours minute-hand makes 90° angle between the two hands, but when minute hand is ahead of hour-hand.

(ii) For the first time minute-hand and hour-hand will make 90° (or 15 points) difference in $\frac{60-15}{55} = \frac{45}{55}$ hours.

Since, in this case minute-hand goes till it appears to be 15 points behind of hour-hand (consider initially they are showing 12 O'clock) then you will see that at 12 : 49 : 05 two hand are making 90° angle between them, while it appears to be hour-hand is ahead of minute-hand.

Now, for every next time after $\frac{60}{55}$ hours they will show the same situation.

we have 12 hours.

So, $\frac{45}{55} + \frac{60}{55} + \frac{60}{55} + \frac{60}{55} + \ldots = 12\text{h} = 12 \times \frac{55}{55}$

$$\Rightarrow \quad \frac{3}{11}+\frac{4}{11}+\frac{4}{11}\ldots=\frac{4\times 11}{11}$$

$$\Rightarrow \quad 3+4+4+\ldots=44$$

$$\Rightarrow \quad 3+4n=144$$

$\Rightarrow n=\frac{41}{4}=10$, consider only integral value.

Thus, total $10+1=11$ times they will make 90° angle.

Hence, in 12 hours both hands make 90° angle $(11+11)=22$ times in different positions.

Remember: At 3 O'clock and 9 O'clock they are at right angled. Since, except between 2–4 O'clock and 8–10 O'clock in each hour both hands make 90° angle 4 times while in the 2–4 O'clock and 8–10 O'clock two hands makes three-three times in every two hour.

EXAMPLE 4 *Between 2 O'clock and 3 O'clock when two hands of a clock overlap each other?*

SOLUTION To overlap or overtake minute-hand has to reduce the gap of 10 points. Since at 2 O'clock two hands are 10 point apart.

$$\therefore \quad \text{Time}=\frac{\text{Distance advanced}}{\text{Relative speed}}$$

$$=\frac{10}{55}\text{h}=\frac{10}{55}\times 60\text{ min}$$

$$=10\text{ min }54\text{ s}$$

Thus, at 2 : 10 : 54 both hands of a clock coincide.

EXAMPLE 5 *Between 6 am and 7 am when the two hands of a clock coincide.*

SOLUTION $\text{Time}=\frac{30}{55}\text{h}=\frac{30}{55}\times 60\text{ min}=32\text{ min }43\text{ s}$

(Distance advanced $=6\times 5=30$ points)

Thus at 6 : 32 : 43 two hands of a clock coincide.

EXAMPLE 6 *Between 11 O'clock and 12 O'clock when will they coincide.*

SOLUTION Distance advanced at 11 O'clock = 55 points

Relative speed = 55 point/h

$$\therefore \quad \text{Time}=\frac{55}{55}=1\text{ h}$$

Hence, they will coincide at $(11+1)=12$ O'clock.

EXAMPLE 7 *Between 3 O'clock and 4 O'clock when will the two hands make 36° angle between them:*

(i) when hour-hand is ahead of minute-hand.

(ii) when minute-hand is ahead of hour-hand.

SOLUTION (i) 36° = 6 points

Now, at 3 O'clock two hands are separated by exactly 15 points to which we have to reduce upto 6 points. Thus, we have to reduce $15-6=9$ points distance, with the relative speed of 55 point/h.

$$\therefore \quad \text{Time required}=\frac{9}{55}\text{h}=\frac{9}{55}\times 60$$

$$=9\text{ min }49\text{ s}$$

Thus at 3 : 09 : 49 they are 36° apart from each other.

(ii) At 3 O'clock both hands are 15 points apart so to make them 6 points apart minute-hand has to move for $(15+6)=21$ points, since minute-hand has to go 6 points ahead of hour hand after crossing the hour-hand.

$$\therefore \quad \text{Time}=\frac{21}{55}\text{h}=\frac{21}{55}\times 60\text{ min}=22\text{ min }54\text{ s}$$

Thust at 3 : 22 : 54, both hands will be 6 points (or 36°) apart from each other.

Did you notice something?

The same angle can be formed in two situations, one when hour-hand is ahead of minute-hand and when minute-hand is ahead of hour-hand.

Thus, you can find the required time by dividing the required difference of points (which you have to either create or reduce) by the relative speed.

EXAMPLE 8 *What is the angle between the two hands at 3 : 10 am?*

SOLUTION Assume 60th point (*i.e.*, when it is 12 O'clock) as the origin.

Step 1. Find the distance of minute-hand from the origin.

Step 2. Find the distance of hour-hand from the origin.

Step 3. Take the difference between two values obtained in step 1 and step 2.

∴ **Step 1.** 10 point = 60°

Step 2. 90° + 5° = 95° (In 10 min hour-hand moves 5°)

Step 3. 95° − 60° = 35°

Thus, at 3 : 10 am two hands are 35° apart.

EXAMPLE 9 *What is the angle between two hands of a clock at 7 : 35?*

SOLUTION **Step 1.** At 7 : 35, minute-hand is $35\times 6=210°$ away from origin.

Step 2. At 7 : 35 hour-hand is $7\times 30+35\times\frac{1}{2}=210+17.5$ $=227.5°$ away from the origin.

Step 3. $227.5-210=17.5°$

Thus at 7 : 35, both hands make 17.5° angle between them.

NOTE A minute-hand moves 6° in one minute while a hour-hand moves $\left(\frac{1}{2}\right)°$ in one minute.

EXERCISE

LEVEL 1

1. 'A' goes 10 km distance with average speed of 6 km/h while rest 20 km he travels with an average speed of 15 km/h. What is the average speed of 'A' **during** the whole journey?
(a) 10 km/h (b) 12 km/h
(c) 13 km/h (d) 14.5 km/h

2. A covers half of his distance with 20 km/h and rest with 30 km/h. What is the average speed during the whole journey?
(a) 20 km/h (b) 24 km/h
(c) 25 km/h (d) 26 km/h

3. A covers 1/3rd of his journey at the speed of 10 km/h and half of the rest at the speed of 20 km/h and rest at the speed of 30 km/h. What is the average speed of A?
(a) $6\frac{2}{11}$ km/h (b) $16\frac{4}{11}$ km/h
(c) $16\frac{4}{11}$ m/s (d) none of these

4. A covers 1/4th of his journey at 20 km/h and 1/3rd of the rest at 25 km/h and half of the rest at 30 km/h and rest at the speed of 40 km/h. What is the average speed of A?
(a) $13\frac{78}{89}$ km/h (b) 12 km/h
(c) $26\frac{86}{89}$ km/h (d) 28 km/h

5. A covered half of his journey at 20 km/h and rest at x km/h, then his average speed is 24 km/h. What is the value of x?
(a) 30 (b) 32
(c) 36 (d) 40

6. A man covered half of the distance at $3x$ km/h and rest at $5x$ km/h. What is the average speed of the man?
(a) $4x$ km/h (b) $3.5x$ km/h
(c) $3.75x$ km/h (d) none of these

7. A person goes to his office at 1/3rd of the speed at which he returns from his office. If the average speed during the whole trip (i. e., one round) is 12 km/h. What is the speed of the person while he was going to his office?
(a) 10 (b) 6
(c) 8 (d) can't be determined

8. A person X starts from Lucknow and another persons Y starts from Kanpur to meet each other. Speed of X is 25 km/h, while speed of Y is 35 km/h. If the distance between Lucknow and Kanpur be 120 km and both X and Y start their journey at the same time, when will they meet?
(a) 1 h later (b) 2 h later
(c) $\frac{1}{2}$ h later (d) 3 h later

9. In the above question (no. 8), what is the distance from Lucknow where they meet?
(a) 50 km (b) 60 km
(c) 100 km (d) 80 km

10. Two persons A and B started from two different places towards each other. If the ratio of their speeds be 3 : 5, then what is the ratio of distance covered by A and B respectively till the point of meeting?
(a) 1 : 2 (b) 3 : 4
(c) 3 : 5 (d) 5 : 3

Directions for question number 11 to 16: *A person P is at X and another person Q is at Y. The distance between X and Y is 100 km. The speed of P is 20 km/h. While the speed of Q is 60 km/h?*

11. If they first time meet at point Z somewhere between X and Y then the distance between X and Z is :
(a) 20 km (b) 40 km
(c) 25 km (d) 30 km

12. If they continue to move to and fro between X and Y then what is the distance covered by P when they meet second time?
(a) 105 km (b) 100 km
(c) 80 km (d) 75 km

13. If they continue to move to and fro between X and Y then what is the distance travelled by Q, when they meet each other for the third time?
(a) 375 km (b) 225 km
(c) 350 km (d) 445 km

14. If P and Q continue to move between X and Y in the given manner and if they meet for the fourth time at a place M somewhere between X and Y, then the distance between X and M is :
(a) 10 km (b) 90 km
(c) 75 km (d) 25 km

15. If P and Q continue to move between X and Y, then the ratio of distances covered by P and Q when they meet for the 5th time?
(a) 1 : 4 (b) 1 : 3
(c) 2 : 3 (d) 3 : 4

16. If P and Q continue to move between X and Y, then the distance covered by P and Q together between any two consecutive meeting?
(a) 100 (b) 300
(c) 200 (d) can't be determined

Directions for questions number 17, 18 and 19: *A persons P starts his journey from A and another person Q starts his journey from B, towards each other. The speeds of P and Q are 16 km/h and 25 km/h respectively and they meet at point M somewhere between A and B when they start their journey simultaneously.*

17. What is the ratio of time taken by P and Q to reach at M?
(a) 1 : 4 (b) 1 : 1
(c) 4 : 5 (d) 16 : 25

18. What is the ratio of time taken by P and Q to reach B and A respectively?
(a) 16 : 25 (b) 1 : 1
(c) 25 : 16 (d) 4 : 5

19. What is the ratio of time taken by P and Q after meeting each other at M to reach B and A respectively?
(a) 25 : 16 (b) 625 : 256
(c) 16 : 25 (d) 4 : 5

20. The speeds of Vimal and Kamal are 30 km/h and 40 km/h. Initially Kamal is at a place L and Vimal is at a place M. The distance between L and M is 650 km. Vimal started his journey 3 hours earlier than Kamal to meet each other. If they meet each other at a place P somewhere between L and M, then the distance between P and M is :
(a) 220 km (b) 250 km
(c) 330 km (d) 320 km

21. In the above question (no. 20) what is the distance between L and P ?
(a) 220 km (b) 320 km
(c) 330 km (d) none of these

Directions for questions number 22 to 27: *There are two places X and Y, 200 km apart from each other. Initially two persons P and Q both are at 'X'. The speed of P is 20 km/h and speed of Q is 30 km/h. Later on they starts to move to and fro between X and Y.*

22. If they starts to move between X and Y, then for the first time when they will meet each other?
(a) after 12 hours (b) after 24 hours
(c) after 30 hours (d) after 8 hours

23. If they meet first time at a point M somewhere between X and Y, then what is the distance travelled by P?
(a) 160 km (b) 150 km
(c) 200 km (d) 210 km

24. If they meet second time each other at a point N somewhere between X and Y, then the distance travelled by Q is :
(a) 240 km (b) 480 km
(c) 360 km (d) none of these

25. If they meet third time each other at a point C, somewhere between X and Y, then the ratio of distances CX and CY is :
(a) 3 : 2 (b) 1 : 3
(c) 2 : 3 (d) 2 : 5

26. If they meet fourth time each other at a point D somewhere between X and Y, then what is the distance between D and X?
(a) 75 (b) 80
(c) 150 (d) 160

27. After starting their race, they meet each other for the n^{th} time at point X, then what is the minimum possible value of n?
(a) 1 (b) 2
(c) 3 (d) 5

Directions for questions number 28 to 33: *A person X started 3 hours earlier at 40 km/h from a place P, then another person Y followed him at 60 km/h, started his journey at 3 O'clock, afternoon.*

28. At what time will they meet to each other (or at what time Y will overtake X)?
(a) 4:30 pm (b) 5 pm
(c) 6 pm (d) 9 pm

29. At what time the difference between X and Y was 30 km, before Y overtakes X?
(a) 6:30 pm (b) 7:30 pm
(c) 8:75 pm (d) none of these

30. At what time Y will be 30 km ahead of X, after overtaking it?
(a) 6:45 pm (b) 7:30 pm
(c) 10:30 pm (d) 8 pm

31. What is the distance travelled by Y to overtake X?
(a) 180 km (b) 420 km
(c) 320 km (d) 360 km

32. What distance should Y cover so that he may reach 360km ahead of X?
(a) 1440 km (b) 1200 km
(c) 920 km (d) 750 km

33. What is difference in time when X was 30 km ahead of Y and when Y was 30 km ahead of X?
(a) 2 (b) 3
(c) 3.5 (d) 4.25

34. A postman goes with a speed of 36 km/h what is the speed of postman in m/s?
(a) 4.5 m/s (b) 6 m/s
(c) 10 m/s (d) can't be determined

35. In the above question (no. 34) what is the speed in m/min?
(a) 325 (b) 432
(c) 360 (d) 600

36. In the above question (no. 34) what is the speed of postman in mile/h?
(a) 22.37 (b) 30.08
(c) 28.30 (d) 38.12

37. A train goes with a speed of 20 m/s. What is the speed of train in km/h?
(a) 57 km/h (b) 72 km/h
(c) 80 km/h (d) 120 km/h

38. As per the question (no. 37) what is the speed of train in km/min?
(a) 1.2 (b) 12
(c) 1200 (d) 120

39. A is twice fast as B and B is thrice as fast as C. The journey covered by C in 78 minutes will be covered by A in :
(a) 12 min (b) 13 min
(c) 15.5 min (d) none of these

40. The ratio of speeds of A is to B is 2 : 3 and therefore A takes 20 minutes less time than B takes. What is the ratio of time taken by A and B?
(a) 2 : 3 (b) 2 : 5
(c) 3 : 2 (d) 3 : 5

41. What is the time taken by A (in the above question)?
(a) 1 h (b) 1.2 h
(c) 0.6 h (d) 30 min

42. A certain distance is covered at a certain speed. If half of this distance is covered in double the time, the ratio of the two speeds is :
(a) 1 : 16 (b) 4 : 1
(c) 2 : 1 (d) 2 : 8

43. Two runner start runing together for a certain distance, one at 5 km/h and another at 3 km/h. The former arrives one and half an hour before the latter. The distance (in km) is :
(a) 12 (b) 20
(c) 25 (d) 36

44. The ratio between the rates of walking of A and B is 2 : 3. If the time taken by B to cover a certain distance is 48 minutes, the time taken (in minutes) by A to cover the distance is :
(a) 52 min (b) 68 min
(c) 72 min (d) 32 min

45. Two trains starting at the same time from two stations 300 km apart and going in opposite directions, cross each other at a distance of 160 km from one of them. The ratio of their speeds is :
(a) 7 : 9 (b) 16 : 20
(c) 8 : 7 (d) 8 : 12

46. A and B travel the same distance at 9 km/h and 10 km/h respectively. If A takes 20 minutes longer than B, the distance travelled by each is :
(a) 16 (b) 20
(c) 30 (d) none of these

47. Abhinav leaves Mumbai at 6 am and reaches Bangalore at 10 am Praveen leaves Bangalore at 8 am and reaches Mumbai at 11:30 am. At what time do they cross each other?
(a) 10 am (b) 8:32 am
(c) 8:56 am (d) 9:20 am

48. Two persons, Alif and Laila start at the same time from Allahabad and Lucknow and proceed towards each other at 45 km/h and 54 km/h respectively. When they meet, it is found that one of them has travelled 72 km more than the other. The distance between the places (in km) is :
(a) 729 (b) 792
(c) 540 (d) none of these

49. Walking at 4/5 of his normal speed, Dewang is 15 minutes late in reaching his club. What is the usual time taken by him to cover the distance?
(a) 1 h (b) 1 h 20 min
(c) 45 min (d) none of these

50. Walking at 3/4 of her normal speed Malleshwari takes 2 hours more than the normal time. What is the normal time?
(a) 4 h (b) 5 h
(c) 6 h (d) 8 h

51. Walking at 3/2 of his normal speed Shekhawat takes 40 minutes less than the usual time? What is the changed (new) time taken by Shekhawat?
(a) 1 h (b) 1.2 h
(c) 3 h (d) 2 h

52. A man reduces his speed from 20 km/h to 18 km/h. So, he takes 10 minutes more than the normal time. What is the distance travelled by him?
(a) 30 km (b) 25 km
(c) 50 km (d) 36 km

53. Osaka walks from his house at 5 km/h and reaches his office 10 minutes late. If this speed had been 6 km/h he would have reached 15 minutes early. The distance of his office from his house is :
(a) 15 km (b) 12.5 km
(c) 10.5 km (d) 18 km

54. Sachin and Mongiya starting from the same place walk at the rate of 7.5 km/h and 9 km/h respectively. What time will they take to be 7.5 km apart, if they walk in the same direction?
(a) 3 h (b) 4 h
(c) 5 h (d) none of these

55. Two aeroplanes start from the same place in opposite directions. One goes towards east at 320 km/h and the other goes towards west at 400 km/h what time will they take to be 720 km apart?
(a) 4 h (b) 3 h
(c) 1 h (d) 1.5 h

56. A man covers a certain distance by his own car. Had he moved 6 km/h faster he would have taken 4 hours less. If he had moved 4 km/h slower, he would have taken 4 hours more. The distance (in km) is :
(a) 240 km (b) 640 km
(c) 480 km (d) none of these

57. If Deepesh had walked 20 km/h faster he would have saved 1 hour in the distance of 600 km. What is the usual speed of Deepesh?
(a) 100 (b) 120
(c) 150 (d) none of these

58. Harsha takes 3 hours more than Ashok, who drives his car 5 km/h faster than Harsha drives, to cover 180 km distance. What is the speed of Harsha?
(a) 12 km/h (b) 15 km/h
(c) 30 km/h (d) 40 km/h

59. A minibus takes 6 hours less to cover 1680 km distance, if its speed is increased by 14 km/h? What is the usual time taken by minibus?
(a) 15 h (b) 24 h
(c) 25 h (d) 30 h

60. The driver of an ambulance sees a school bus 40 m ahead of him. After 20 second, the school bus is 60 metre behind. If the speed of the ambulance is 30 km/h, what is the speed of the school bus?
(a) 10 km/h (b) 12 km/h
(c) 15 km/h (d) 22 km/h

61. A postman riding a bicycle at 15 km/h can reach a village in 4 hours. If he is delayed by 1 hour at the start, then in order to reach his destination in time, he should ride with a speed of:
(a) 20 km/h (b) 16 km/h
(c) 14 km/h (d) 12 km/h

62. What is the time required by a train of length of 350 m to cross an electric pole with a speed of 70 m/s?
(a) 3 s (b) 3.5 s
(c) 5 s (d) 10 s

63. A train 270 m long is running over a bridge of length of 130 m with a speed of 40 m/s. What is the time taken by this train to cross the bridge?

(a) 6 s (b) 16 s
(c) 10 s (d) none of these

64. If a train 225 m long passes a telegraphic pole in 9 seconds, then the time taken (in seconds) by it to cross a tunnel 450 m long is :
(a) 8 s (b) 10 s
(c) 27 s (d) none of these

65. A train 350 m long is running at the speed of 36 km/h. If it crosses a tunnel in 1 minute, then the length of the tunnel (in metres) is :
(a) 200 m (b) 250 m
(c) 150 m (d) none of these

66. If a 250 m long train crosses a platform of the same length as that of the train in 25 seconds, then the speed of the train is :
(a) 150 m/s (b) 200 m/s
(c) 20 km/h (d) 72 km/h

67. Sabarmati express takes 18 seconds to pass completely through a station 162 m long and 15 seconds through another station 120 m long. The length of the Sabarmati express is :
(a) 132 m (b) 100 m
(c) 80 m (d) 90 m

68. A train 200 m long travels at the speed of 72 km/h. A man is running at 3.6 km/h in the same direction in which the train is going. The train will pass the man in :
(a) 10 s (b) $12\frac{3}{19}$ s
(c) $10\frac{10}{19}$ s (d) none of these

69. A train 350 m long is moving at the speed of 20 km/h. It will cross a man coming from the opposite direction at the speed of 1 km/h in :
(a) 27 s (b) 35 s
(c) 45 s (d) 60 s

70. The length of Lucknow mail is 120 m and that of Punjab mail is 80 m. These two trains are running in the same direction with velocities of 40 km/h and 50 km/h respectively. The time taken by them to cross each other is :
(a) 8 s (b) 72 s
(c) 11.5 s (d) 12.5 s

71. In the above question if the trains are running in opposite directions. The time taken by them to cross each other is :
(a) 8 s (b) 72 s
(c) 12.5 s (d) none of these

72. A train passes an electric pole in 10 seconds and a platform 120 m long in 18 seconds. Its length in metres is :
(a) 150 m (b) 130 m
(c) 240 m (d) 180 m

73. A 175 m long train crosses a man walking at a speed of 9 km/h in the opposite direction in 10 sec. The speed of the train (in km/h) is :
(a) 45 (b) 54
(c) 72 (d) 68

74. A train of length 100 m takes 1/6 minute to pass over another train 150 m long coming from the opposite direction. If the speed of first train is 60 km/h, the speed of the second train is :
(a) 45 km/h (b) 28 km/h
(c) 30 km/h (d) none of these

75. A train overtakes two girls who are walking in the same direction in which the train is going at the rate of 3 km/h and 6 km/h and passes them completely in 36 seconds and 40 seconds respectively. The length of the train (in metres) is :
(a) 120 m (b) 150 m
(c) 125 m (d) none of these

76. A coolie standing on a railway platform observes that a train going in one direction takes 4 seconds to pass him. A train of same length going in opposite direction takes 5 seconds to pass him. The time taken (in seconds) by the trains to cross each other will be :
(a) 35 (b) 36.5
(c) $\frac{40}{9}$ (d) none of these

77. Pushpak express leaves Lucknow at 6 am and two hours later another train Bhopal express leaves Lucknow. Both trains arrive Bhopal at 4 pm on the same day. If the difference between their speeds be 10 km/h, what is the average speed of both the trains over entire route :
(a) 40 km/h (b) $44\frac{4}{9}$ km/h
(c) $42\frac{3}{5}$ km/h (d) none of these

Directions for question number 78, 79: *Two trains leave Meerut at the difference of 4 hours. The first train leaves at 8 am at 40 km/h and the faster train leaves later at 60 km/h in the same direction.*

78. When the faster train will overtake the slower train?
(a) 4 pm (b) 2 pm
(c) 8 pm (d) 6:30 pm

79. What is the distance from Meerut, where one train overtakes another train?
(a) 480 km (b) 420 km
(c) 360 km (d) 250 km

80. The distance between Lucknow and Delhi is 700 km. Rajdhani express starts from Delhi for the Lucknow at 60 km/h. 50 minutes later Lucknow express leaves Lucknow for Delhi on the parallel tracks at 70 km/h. How far from Lucknow will they cross each other?
(a) 250 km (b) 360 km
(c) 350 km (d) 475 km

81. Patna express travels first 560 km in 7 hours and rest 360 km in 9 hours. What is the average speed of the train?
(a) 39 km/h (b) 43 km/h
(c) 63 km/h (d) 57.5 km/h

82. Jammutavi express leaves Jammu for Kanya Kumari at 120 km/h and returns to Jammu at 80 km/h. What is the average speed of the train during the whole journey?
(a) 47.5 km/h (b) 96 km/h
(c) 38 km/h (d) 57.5 km/h

83. In the above question if the total time taken by the train 25 hours, what is the distance between these two places?
(a) 1365.5 km (b) 1369 km
(c) 1200 km (d) can't be determined

64. Roorkee express normally reaches its destination at 50 km/h in 30 hours. Find the speed at which it travels to reduce the time by 10 hours?
(a) 38 km/h (b) 76 km/h
(c) 75 km/h (d) 60 km/h

65. Two trains A and B start simultaneously in the opposite direction from two points P and Q and arrive at their destinations 16 and 9 hours respectively after their meeting each other. At what speed does the second train B travel if the first train travels at 120 km/h per hour :
(a) 90 km/h (b) 160 km/h
(c) 67.5 km/h (d) none of these

66. There are two trains running on two parallel tracks. Length of each train is 120 m. When they are running in opposite directions, they cross each other in 4 seconds and when they are running in the same direction they cross in 12 seconds. What is the speed of the faster train?
(a) 80 km/h (b) 72 km/h
(c) 120 km/h (d) 144 km/h

67. Two trains are travelling in the same direction at 22.5 km/h and 7.5 km/h respectively. The faster train crosses a man in the slower train in 18 seconds. What is length of the faster train?
(a) 87.5 m (b) 75 m
(c) 122.5 m (d) none of these

68. A train covers a certain distance moving at a speed of 60 km/h. However if it were to halt for a fixed time every hour, its average speed comes out to be 50 km/h. For how much time does the train halt for every hour?
(a) 6 min (b) 10 min
(c) 12 min (d) none of these

69. Two horses start trotting towards each other, one from A to B and another from B to A. They cross each other after one hour and the first horse reaches B, 5/6 hour before the second horse reaches A. If the distance between A and B is 50 km. What is the speed of the slower horse?
(a) 30 km/h (b) 15 km/h
(c) 25 km/h (d) 20 km/h

70. Pankaj walked at 5 km/h for certain part of the journey and then he took an auto for the remaining part of the journey travelling at 25 km/h. If he took 10 hours for the entire journey. What part of journey did he travelled by auto if the average speeed of the entire journey be 17 km/h :
(a) 750 km (b) 100 km
(c) 150 km (d) 200 km

71. A car travelled first 36 km at 6 km/h faster than the usual speed, but it returned the same distance at 6 km/h slower than the usual speed. If the total time taken by car is 8 hours, for how many hours does it travelled at the faster speed?
(a) 4 (b) 3
(c) 2 (d) 1

72. A dog starts chasing to a cat 2 hours later. It takes 2 hours to dog to catch the cat. If the speed of the dog is 30 km/h, what is the speed of cat?
(a) 10 km/h (b) 15 km/h
(c) 20 km/h (d) can't be determined

93. Prachi starts from Barabanki at 6 am at constant speed of 60 km/h. She halts at Lucknow for half an hour and then drives at 40 km/h. If she reaches Kanpur at 9 : 30 am, which is 160 km from Barabanki, how far is Barabanki from Lucknow?
(a) 75 km (b) 80 km
(c) 100 km (d) 120 km

94. Two trains whose respective lengths are 200 m and 250 m cross each other in 18 s, when they are travelling in opposite direction and in 1 min, when they are travelling in the same direction. What is the speed of the faster train (in km/h)?
(a) 38.5 (b) 48.5
(c) 54 (d) 58.5

95. Abhinav started for the station half a km from his home walking at 1 km/h to catch the train in time. After 3 minutes he realised that he had forgotten a document at home and returned with increased, but constant speed to get it succeded in catching the train. Find his latter speed in km/h :
(a) 1.25 (b) 1.1
(c) $\frac{11}{9}$ (d) 2

Directions for question number 96-97: *The ratio of speeds at which Anil and Mukesh walk is 3 : 4. Anil takes 30 minutes more than the time taken by Mukesh in reaching the destination.*

96. If Anil drives the car at twice the speed of his walking then the time required to reach his destination by car is :
(a) 45 min (b) 60 min
(c) 1.5 h (d) 1 h 20 min

97. What is the total distance travelled by each of them, if the average of speeds of Anil and Mukesh is 28 km/h?
(a) 48 (b) 60
(c) 17 km (d) 70

98. Train X starts from point A for point B at the same time that train Y starts from B to A. Point A and B are 300 km apart. The trains are moving at a constant speed atleast at 25 km/h. The trains meet eachother 3 hours after they start. If the faster train takes atleast 2 more hours to reach the destination. By which time will the slower train have definitely reached its destination? (Ignoring the length of trains in crossing)
(a) 4 hours after the start
(b) 7.5 hours after the start
(c) 6 hours after the start
(d) none of the above

99. In reaching the Purnagiri a man took half as long again to climb the second third as he did to climb the first third and a quarter as long again for the last third as for the second third. He took altogether 5 hs 50 minutes. Find the time he spent on the first third of the journey?
(a) 72 min (b) 80 min
(c) 81 min (d) 88 min

100. Walking at four fifth of his usual speed Vijay Malya reaches his office 15 minutes late on a particular day. The next day, he walked at 5/4 of his usual speed. How early would he be to the office when compared to the previous day?
(a) 27 min (b) 32 min
(c) 30 min (d) none of these

101. Abdul starts in a car from Ahmedabad towards Bangalore. After sometime he realises that he will cover only 75% of the distance in the scheduled time and he therefore doubles his speed immediately and thus manages to reach Bangalore exactly on time. Find the time after which Abdul changed his speed, given that he could have been late by 3 hours if he had not changed his speed :

(a) 3 h (b) 4 h
(c) 5 h (d) 6 h

102. A man travels the first part of his journey at 20 km/h and the next at 70 km/h, covering the entire journey at an average speed of 50 km/h. What is the ratio of the distance that he covered at 20 km/h to that he covered at 70 km/h?

(a) 4 : 21 (b) 3 : 22
(c) 1 : 4 (d) 3 : 5

103. Anjali fires two bullets from the same place at an interval of 6 minutes but Bhagwat sitting in a car approaching the place of firing hears the second fire 5 minute 32 seconds after the first firing. What is the speed of car, if the speed of sound is 332 m/s?

(a) 56 m/s (b) 102 m/s
(c) 28 m/s (d) 32 m/s

104. A car crosses a man walking at 6 km/h. The man can see the things upto 450 m only in one direction due to fog. He sees the car which was going in the same direction for 4.5 minutes. What is the speed of the car?

(a) 9 km/h (b) 12 km/h
(c) 12.5 km/h (d) 15 km/h

105. A man takes 4 h 20 minutes in walking to a certain place and riding back. If he walk on both sides he loses 1 h. The time he would take by riding both ways is :

(a) 2 h 20 min (b) 3 h 20 min
(c) 2 h (d) 4 h 40 min

106. A train met with an accident 120 km from station *A*. It completed the remaining journey at 5/6 of its previous speed and reached 2 hours late at station *B*. Had the accident taken place 300 km further, it would have been only 1 hour late? What is the speed of the train?

(a) 100 km/h (b) 120 km/h
(c) 60 km/h (d) 50 km/h

107. For the above question what is the total distance between *A* and *B* ?

(a) 480 km (b) 520 km
(c) 600 km (d) 720 km

108. The wheel of an engine of 300 cm in circumference makes 10 revolutions in 6 seconds. What is the speed of the wheel (in km/h)?

(a) 18 (b) 20
(c) 27 (d) 36

109. A man can row downstream at 12 km/h and upstream at 8 km/h. What is the speed of man in still water?

(a) 12 km/h (b) 10 km/h
(c) 8 km/h (d) 9 km/h

110. A man can row upstream at 15 km/h and downstream at 21 km/h. The speed of water current of the river is :

(a) 8 km/h (b) 6 km/h
(c) 3 km/h (d) 5 km/h

111. A boat moves downstream at 1 km in 5 minutes and upstream at 1 km in 12 minutes. What is the speed of current?

(a) 4.5 km/h (b) 3.5 km/h
(c) 2 km/h (d) 2.5 km/h

112. A man rows downstream 60 km and upstream 36 km, taking 4 hours each time. The speed of the man is :

(a) 15 km/h (b) 16 km/h
(c) 8 km/h (d) 12 km/h

113. A man can row 5 km/h in still water. If the rate of current is 1 km/h, it takes $\frac{5}{4}$ hours to row to a place and back. How far is the place?

(a) 2 km (b) 2.5 km
(c) 3 km (d) 4 km

114. A man can swim 5 km/h in still water. If the speed of current be 3 km/h, the time taken by him to swim to a place 16 km upstream and back is :

(a) 8 h (b) 7.5 h
(c) 6.66 h (d) 10 h

115. A boat covers 48 km upstream and 72 km downstream in 12 hours, while it covers 72 km upstream and 48 km downstream in 13 hours. The speed of stream is :

(a) 2 km/h (b) 2.2 km/h
(c) 2.5 km/h (d) 4 km/h

116. A motor boat takes 2 hours to travel a distance of 9 km downstream and it takes 6 hours to travel the same distance against the current. The speed of the boat in still water and that of the current (in km/h) respectively are :

(a) 6, 5 (b) 3, 1.5
(c) 8, 5 (d) 9, 3

117. A man can row 15 km/h in still water and he finds that it takes him twice as much time to row up than as to row down the same distance in the river. The speed of the current (in km/h) is :

(a) 6 km/h (b) 6.5 km/h
(c) 4.5 km/h (d) 5 km/h

118. A motor boat takes 12 hours to go downstream and it takes 24 hours to return the same distance. What is the time taken by boat in still water?

(a) 15 h (b) 16 h
(c) 8 h (d) 20 h

119. The speed of a boat in upstream is 2/3 that of downstream. Find the ratio of speed of boat in still water and the average speed of boat in downstream and upstream?

(a) $\frac{24}{25}$ (b) $\frac{25}{24}$
(c) $\frac{5}{12}$ (d) none of these

120. The difference between downstream speed and upstream speed is 3 km/h and the total time taken during upstream and downstream is 3 hours. What is the downstream speed if the downstream and upstream distance are 3 km each?

(a) 2.5 km/h (b) 4.33 km/h
(c) 4 km/h (d) 3.3 km/h

121. A boat which sails at 10 km/h in still water starts chasing, from 10 km behind, another one which sails at 4 km/h in the upstream direction. After how long will it catchup if the stream is flowing at 2 km/h :
(a) 4 h (b) 2.5 h
(c) 2 h (d) 3.5 h

Directions for question number 122 and 123 : *A motor boat went downstream for 120 km and immediately returned. It took the boat 15 hours to complete the round trip. If the speed of the river were twice as high, the trip downstream and back would take 24 hours.*

122. What is the speed of the boat in still water?
(a) 20 km/h (b) 18 km/h
(c) 15 km/h (d) 16 km/h

123. What is the speed of the stream?
(a) 3.5 km/h (b) 4 km/h
(c) 6 km/h (d) 8 km/h

124. A boat sails 15 km of a river towards upstream in 5 hours. How long will it take to cover the same distance downstream, if the speed of current is one-fourth the speed of the boat in still water :
(a) 1.8 h (b) 3 h
(c) 4 h (d) 5 h

125. A boat takes 5 hours more while going back in upstream than in downstream. If the distance between two places is 24 km and the speed of boat in still water be 5.5 km/h. What must be the speed of boat in still water so that it can row donwstream, 24 km, in 4 hours?
(a) 1.5 km/h (b) 3.5 km/h
(c) 4.5 km/h (d) 3 km/h

126. A boat takes 7 hours to go from P to R, through a midpoint Q, but it takes 8 hours to go from P to Q and then return from Q to P. How long it would take to go from R to P?
(a) 7 h (b) 8 h
(c) 9 h (d) none of these

127. Mallah can row 40 km upstream and 55 km downstream in 13 h and 30 km upstream and 44 km downstream in 10 hours. What is the speed of Mallah in still water?
(a) 6 km/h (b) 12 km/h
(c) 3 km/h (d) 8 km/h

Directions for question number 128-129 : *In a kilometre race, A can give B a start of 20 m and also in a half kilometre race C beats A by 50 m.*

128. If A, B and C run a 2 km race, what is the difference between the distances covered by the two losers, when the winner finishes the race?
(a) 64 m (b) 36 m
(c) 32 m (d) 58 m

129. B and C run a half km race, who should give a start to the slower runner and of how many metres so that they both finish the race at the same time?
(a) C, 59 m (b) B, 34 m
(c) C, 48 m (d) B, 56 m

130. In a 1600 m race, A beats B by 80 m and C by 60 m. If they run at the same time then by what distance will C beat B in a 400 m race?
(a) $5\frac{15}{77}$ m (b) $5\frac{20}{76}$ m
(c) $15\frac{5}{77}$ m (d) none of these

131. Aman can run a distance in 190 seconds and Shakti can run the same distance in 200 seconds. If they start together, by what distance Aman can beat Shakti in 1 km race?
(a) 48 m (b) 25 m
(c) 24 m (d) 50 m

132. A runs $\frac{7}{4}$ times as fast as B. If A gives B a start of 300 m, how far must the winning post be if both A and B have to end the race at same time?
(a) 1400 m (b) 700 m
(c) 350 m (d) 210 m

133. A beats B by 100 m in a race of 1200 m and B beats C by 200 m in a race of 1600 m. Approximately by how many metres can A beat C in a race of 9600 m?
(a) 1600 m (b) 1800 m
(c) 1900 m (d) 2400 m

134. In a 1000 m race Ameesha gives a headstart of 100 m to Bipasha and beats her by 200 m. In the same race Ameesha gives a headstart of 100 m to Celina and beats her by 300 m. By how many metres would Bipasha beat Celina in a 50 m race?
(a) 6.66 m (b) 7.143 m
(c) 8 m (d) none of these

135. In a 1000 metres race Ravi gives Vinod a start of 40 m and beats him by 19 seconds. If Ravi gives a start of 30 seconds then Vinod beats Ravi by 40 m. What is the ratio of speed of Ravi to that of Vinod?
(a) 4 : 5 (b) 6 : 5
(c) 3 : 8 (d) 5 : 4

136. In a race, the man who came two places ahead of the last man finished one place ahead of the man who came three places behind the man just ahead of the one who stood second. How many men finished the race?
(a) 6 (b) 5
(c) 4 (d) 8

137. Vinay and Versha run a race with their speeds in the ratio of 5 : 3. They prefer to run on a circular track of circumference 1.5 km. What is the distance covered by Vinay when he passes Versha for the seventh time?
(a) 25.25 km (b) 26.25 km
(c) 132 m (d) none of these

138. A gives both B and C a start of 60 m in a 1500 m race. However, while B finishes with him, C is 15 m behind them when A and B cross the finishing line. How much start can B give C for the 1500 m race course?
(a) $7\frac{6}{23}$ m (b) $15\frac{5}{8}$ m
(c) $7\frac{11}{16}$ m (d) $5\frac{5}{24}$ m

139. In a 600 m race Prabhat has a start of 200 m and the ratio of speeds of Prabhat and Nishith is 4 : 5, then the distance by which Prabhat wins by :
(a) 100 m (b) 80 m
(c) 120 m (d) none of these

140. In the game of billiards, A can give B, 20 points in 80 and B can give C, 16 points in 80. How many points can A give C in a game of 200?
(a) 64 (b) 72
(c) 80 (d) none of these

141. In a day how many times the minute and hour-hands coincide (or overlap or come together)?
(a) 12 (b) 20
(c) 22 (d) 24

142. In a day how many times the minute-hand and hour-hand make right angle between them?
(a) 12 (b) 20
(c) 22 (d) 44

143. In a day how many times the minute-hand and hour-hand are opposite to each other?
(a) 12 (b) 24
(c) 11 (d) 22

144. The relative speed of minute-hand with respect to hour-hand is :
(a) $\left(5\frac{1}{2}\right)^\circ$ per minute
(b) $\left(\frac{11}{12}\right)$ minute per minute
(c) 6° per minute
(d) both (a) and (b)

145. At what time the hands of a clock coincide between 5 and 6?
(a) 5 : 30 (b) 5 : 27 : 16
(c) 5 : 32 : 16 (d) 5 : 28 : 56

146. What is the angle between hour-hand and minute-hand at 2 : 25 pm?
(a) 77.5° (b) 68°
(c) 67.5° (d) none of these

147. When do the two hands of a clock of just after 3 pm make right angle between them?
(a) 3 : 15 : 00 (b) 3 : 10 : 54
(c) 3 : 01 : 59 (d) none of these

148. At what time are the hands of clock together between 7 pm and 8 pm?
(a) 7 : 45 : 54 (b) 7 : 36 : 27
(c) 7 : 37 : 49 (d) 7 : 38 : 11

149. What is the angle between the hands of a clock when the time is 15 minutes past 6?
(a) 97.5° (b) 87.5°
(c) 77° (d) none of these

150. What are the possible times when a clock shows 35° angle between two hands between 3 pm and 4 pm?
(a) 20 min 25 sec (b) 25 min 20 sec
(c) 22 min 43 sec (d) none of these

LEVEL 2

Directions for question number 1-4 : *Aishwarya is going to cover a distance of 360 km from Ambala to Khandala. The first one-third of the distance she covers on a cycle. The second one-third she covers by an auto-rickshaw and the remaining distance she travels by car. The average speed of the journey by a car is 5 times the average speed by cycle and 20 km/h more than the average speed by auto-rickshaw, but she took 1 hour more by auto-rickshaw than by car.*

1. What is the average speed of the whole journey?
(a) 15 km/h (b) 24 km/h
(c) 20 km/h (d) none of these

2. What is the time taken in the whole journey?
(a) 10 h (b) 12 h
(c) 15 h (d) none of these

3. What is the distance covered by her in last five hours of her journey?
(a) 250 km (b) 240 km
(c) 200 km (d) can't be determined

4. Instead of travelling the first one-third by cycle if she travels by same auto-rickshaw with the same average speed, then what is the percentage decrease/increase in time taken during the entire journey?
(a) 46.66% (b) 33.33%
(c) 50% (d) 25%

5. Bipasha and Mallika leave towns Kolkata and Ambala at 6 am and travel towards Ambala and Kolkata respectively. Speed of Bipasha is 60 km/h and speed of Mallika is 120 km/h. Rani leaves Kolkata for Ambala sometime later and travels at a speed of 90 km/h. If the distance between Kolkata and Ambala is 1080 km and all three meet at the same point on the way, at same time, then at what time did Rani leave Kolkata?
(a) 7 am (b) 8 am
(c) 7:30 am (d) 10 am

6. A passenger sitting in a train of length l m, which is running with speed of 60 km/h passing through two bridges, notices that it crosses the first bridge and the second bridge in time intervals which are in the ratio of 7 : 4 respectively. If the length of first bridge be 280 m, then the length of second bridge is :
(a) 490 m (b) 220 m
(c) 160 m (d) can't be determined

7. Pathik and Rahi started from two places Andheri and Bhavnagar towards Bhavnagar and Andheri respectively at 8 : 30 am. The speeds of Pathik and Rahi are in the ratio of 4 : 5. They meet at Chandni Chowk, somewhere between Andheri and Bhavnagar, spent some-time together enjoyed coffee and burger and then both started towards their destination at 9 : 27 am. If Pathik reaches Bhavnagar at 10 : 32 am, how much time did they spend together?
(a) 8 min (b) 12 min
(c) 15 min (d) can't be determined

8. A train with 120 wagons crosses Arjun who is going in the same direction, in 36 seconds. It travels for half an hour from the time it starts overtaking the Arjun (he is riding on the horse) before it starts overtaking Srikrishna (who is also riding on his horse) coming from the opposite direction in 30 seconds. In how much time (in seconds) after the train has crossed the Srikrishna do the Arjun meets to Srikrishna?
(a) 3560 sec (b) 3600 sec
(c) 3576 sec (d) can't be determined

9. Kareena and Shahid start from Kurla and Worli towards Worli and Kurla respectively, at the same time. After they meet at Shantakruz on the way from Kurla to Worli, Kareena reduces her speed by 33.33% and returns back to Kurla and Shahid increases his speed by 33.33% and returns back to Worli. If Kareena takes 2 hours for the entire journey, what is the time taken by Shahid for the entire journey?
(a) 96 min (b) 84 min
(c) 168 min (d) can't be determined

10. Due to the technical snag in the signal system two trains start approaching each other on the same rail track from two different stations, 240 km away from each other. When the train starts a bird also starts moving to and fro between the two trains at 60 km/h touching each time each train. The bird is initially sitting on the top of the engine of one of the trains and it moves so till these trains collide. If these trains collide one and a half hour after the start, then how many kilometres bird travells till the time of collision of trains?
(a) 90 km (b) 130 km
(c) 120 km (d) none of these

11. Einstein walks on an escalator at a rate of 5 steps per second and reaches the other end in 10 seconds. While coming back, walking at the same speed he reaches the starting point in 40 seconds. What is the number of steps on the escalator?
(a) 40 (b) 60
(c) 120 (d) 80

12. A girl while walking diametrically across a semicircular playground, takes 3 minutes less than if she had kept walking round the circular path from A to B. If she walks 60 metres a minute, what is the diameter of the play ground :
(a) 60 m (b) 48 m
(c) 84 m (d) 315 m

13. Two trains start simultaneously from two stations Howrah and Bandra, respectively towards each other on the same track. The distance between the two stations is 560 km and the speed of trains are 30 and 40 km/h.
Simultaneously with the trains, a sparrow sitting on the top of one of the train starts towards the other and reverses its direction on reaching the other train and so on. If the speed of sparrow is 80 km/h then the distance that the sparrow flies before being crushed between the train is :
(a) 70 km (b) 560 km
(c) 640 km (d) 650 km

14. A surveillance plane is moving between two fixed places Pukhwara and Kargil at 120 km/h. The distance between two places is 600 km. After 18 hour what will be the distance between the Kargil and its position if it is starts moving from Pukhwara?
(a) 360 km (b) 300 km
(c) 240 km (d) none of these

15. The speed of a car during the second hour of its journey is thrice that in the first hour. Also its third hours speed is the average speed of the first two hours. Had the car travelled at the second hours speed during all the first three hours, then it would have travelled 150 km more. Find the percentage reduction in time in the second case for the first three hours :
(a) $33\frac{1}{3}\%$ (b) 40%
(c) 25% (d) 50%

16. There are three runners Tom, Dick and Harry with their respective speeds of 10 km/h, 20 km/h and 30 km/h. They are initially at P and they have to run between the two points P and Q which are 10 km apart from each other. They start their race at 6 am and end at 6 pm on the same day. If they run between P and Q without any break, then how many times they will be together either at P and Q during the given time period?
(a) 5 (b) 7
(c) 4 (d) 12

Directions for question number 17 and 18 : *Arjun and Srikrishna go by chariot from Mathura to Kurukshetra which is on the way to Hastinapur. Abhimanyu goes from Hastinapur to Kurukshetra. The distance between Mathura to Hastinapur is 700 km and the distance between Hastinapur and Kurukshetra is 300 km. Speed of Arjun and Srikrishna's chariot is 25 km/h and speed of Abhimanyu is 10 km/h. All the three persons start their journey at 10 am. After travelling some miles Srikrishna sees Duryodhan going (by riding on his horse) at 20 km/h to Kurukshetra. Arjun and Srikrishna go ahead meet Abhimanyu and pick him up. Then they return immediately to Kurukshetra and thus all the four reach at the same time.*

17. What is the total distance travelled by Arjun?
(a) 400 (b) 500
(c) 600 (d) can't be determined

18. What is the total time taken to reach Kurukshetra?
(a) 10 h (b) 15 h
(c) 18 h (d) 24 h

19. Priyanka, Akshay and Salman started out on a journey to watch the newly released movie "Mujhse Shaadi Karogi", which was being shown at wave cine-multiplex. The multiplex was 120 km away from their starting point of journey. Priyanka and Salman went by car at the speed of 50 km/h, while Akshay travelled by Tonga (horse cart) at 10 km/h. After a certain distance Salman got off and travelled the rest distance by another Tonga at 10 km/h, while Priyanka went back for Akshay and reached the destination at the same time that Salman arrived. The number of hours required for the trip was :
(a) 4 h (b) 5 h
(c) 4.8 h (d) can't be determined

Directions for question number 20 and 21 : *Ajai and Kajol start towards each other at the same time from Barabanki and Fatehpur for their destinations Fatehpur and Barabanki respectively which are 300 km apart. They meet each other 120 km away from Barabanki.*

20. Shahrukh starts from Barabanki to Fatehpur, 1 hour after Ajai starts. Shahrukh meets Kajol 1.5 hours after Shahrukh starts. If the speed of Shahrukh is atleast 20 km/h faster than the speed of Kajol.
Which of the following statements is true?

(a) The minimum possible speed of Ajai is 45 km/h
(b) The maximum possible speed of Ajai is 45 km/h
(c) The minimum possible speed of Kajol is 60 km/h
(d) The maximum possible speed of Kajol is 60 km/h

21. What is the minimum speed of Shahrukh to overtake Ajai, before he meets Kajol? (Use the data from previous question, if necessary)
(a) 30 (b) 40
(c) 60 (d) none of these

Directions for question number 22-24 : *Raghupati goes at a speed of 60 km/h. Raghav goes at a speed of 36 km/h. Raja Ram can go from Azamgarh to Barelley in 2 hours. The distance between Azamgarh to Barelley is equal to the distance between Azamgarh to Chandoli. Raghav takes the same time travelling from Barelley to Azamgarh as from Barelley to Chandoli at his regular speed which is twice the speed of Raja Ram.*

22. What is the distance between Azamgarh and Chandoli?
(a) 60 km (b) 27 km
(c) 36 km (d) 18 km

23. How much time will Raghupati take to complete a round trip of the three cities?
(a) 1 h 12 min (b) 1 h 48 min
(c) 1 h 30 min (d) 1 h 36 min

24. If Raghupati and Raja Ram travel towards each other from Barelley and Chandoli respectively, how far from Barelley will they meet each other?
(a) $\frac{60}{13}$ (b) $27\frac{9}{13}$ km
(c) $37\frac{9}{13}$ (d) $\frac{360}{9}$

Directions for question number 25-26 : *Mohan, Namit and Pranav travel from Shantipur to Hulchulpur. They have a two seeter bike which can be driven by only Mohan. It is known that due to very stringent traffic rules only two persons can ride at a time. Hulchulpur is 180 km away from Shantipur. All of them can walk at 6 km/h, but reach to Hulchulpur simultaneously also they started their journey simultaneously.*

25. If the speed of the bike is 36 km/h, then what is the total distance that the bike travels?
(a) 400 km (b) 380 km
(c) 200 km (d) 320 km

26. If the speed of the bike is 42 km/h, then what is the shortest possible time in which all three of them can complete the journey?
(a) $7\frac{1}{3}$ h (b) $9\frac{4}{7}$ h
(c) $9\frac{3}{7}$ h (d) can't be determined

27. While walking down on the pavements of New York city. I notice that every 20 minute there is a city bus coming in the opposite direction and every 30 minute there is a city bus overtaking me from behind. What is the time gap between one city bus passing a stationary point known as Local Bus Stop beside the route and the immediately next city bus in the same direction passing the same stationary point?
(a) 27 min (b) 24 min
(c) 25 min (d) can't determined

28. Abhinav and Brijesh start from Allahabad and Barabanki respectively with uniform velocities. Abhinav is headed towards Barabanki and Brijesh towards Allahabad and both cities are 600 km apart. Abhinav rests whenever Brijesh is on the move and Brijesh rest whenever Abhinav is on the move. Abhinav's speed is 25 km/h and Brijesh's speed is 30 km/h. If Abhinav starts first and reaches Barabanki in 36 hours, then find the least time that Brijesh would take to reach his destination after Abhinav makes a start :
(a) 20 h (b) 36 h
(c) 44 h (d) none of these

29. A man can cross a downstream river by steamer in 45 minutes and same by boat in 1 hour. If the time of crossing the river in upstream direction by steamer is 50% more than downstream time by the steamer and the time required by boat to cross the same river by boat in upstream is 50% more than the time required in downstream by boat. What is the time taken for the man to cross the river downstream by steamer and then return to same place by boat half the way and by steamer the rest of the way?
(a) 85 min (b) 115 min
(c) 120 min (d) 125 min

Directions for question number 30 and 31 : *Awadh express and Bokaro express start simultaneously from Lucknow and Jamshedpur towards each other and continuously shuttle between these two places. Every time these trains meet each other, they turn back after exchanging their respective speeds. the initial ratio of their speeds is 2 : 1.*

30. What is the number of distinct places at which they will meet?
(a) 1 (b) 2
(c) 5 (d) none of these

31. Let these two trains first time meet at Patna, then what is the ratio of distances covered by Awadh express and Bokaro express till they meet for the third time at the same place Patna :
(a) 1 : 1 (b) 14 : 13
(c) 10 : 11 (d) none of these

32. Mahindra starts a journey for his office, which is in the north east of his home. An hour after starting meets with a minor accident. He takes one hour in resuming his journey. After that he proceeds at 5/6th of his former speed and arrives at the office 1 hour 36 minutes late than the scheduled time. Had the accident occurred 80 kms further from the actual place of accident, he would have arrived 1 hour 20 minutes beyond the scheduled time. What is the distance between his office and his home?
(a) 180 km (b) 240 km
(c) 250 km (d) 300 km

33. A soldier fired two bullets at an interval of 335 seconds moving at a uniform speed v_1. A terrorist who was running ahead of the soldier in the same direction, hears the two shots at an interval of 330 seconds? If the speed of sound is 1188 km/h, then who is the faster and by how much?

(a) Soldier, 22 km/h (b) Soldier, 25 km/h
(c) Soldier, 18 km/h (d) Terrorist, 20 km/h

34. A hunter fired two shots from the branch of a tree at an interval of 76 seconds. A tiger separating too fast hears the two shots at an interval of 83 seconds. If the velocity of the sound is 1195.2 km/h, then find the speed of tiger?
(a) 112.8 km/h (b) 100.8 km/h
(c) 80.16 km/h (d) none of these

35. A man goes to the fair in Funcity with his son and faithful dog. Unfortunately man misses his son which he realises 20 minutes later. The son comes back towards his home at the speed of 20 m/min and man follows him at 40 m/min. The dog runs to the son (child) and comes back to the man (father) to show him the direction of his son. It keeps moving to and fro at 60 m/min between son and father, till the man meets the son. What is the distance travelled by the dog in the direction of the son?
(a) 800 m (b) 1675 m
(c) 848 m (d) 1000 m

36. Amarnath express left Amritsar for Gorakhpur. Two hours later Gorakhnath express left from Amritsar to Gorakhpur. Both trains reached Gorakhpur simultaneously. If Amarnath express had started from Amritsar and Gorakhnath express had started from Gorakhpur at the same time and travelled towards each other they would meet in 1 h 20 min. Find the time taken by Amarnath express to travel from Amritsar to Gorakhpur (in hours) :
(a) 2 (b) 4
(c) 5 (d) 6

37. Akbar and Birbal setout at the same time to walk towards eachother respectively from Agra and Banaras 144 km apart. Akbar walks at the constant speed of 8 km/h, while Birbal walks 4 km in the first hour, 5 km in the second hour, 6 km in the third hour and so on. Then the Akbar and Birbal will meet :
(a) in 6 h
(b) in 8 h
(c) midway between Agra and Banaras
(d) 80 km away from Banaras

38. A tiger is 50 of its own leaps behind a deer. The tiger takes 5 leaps per minute to the deer's 4. If the tiger and the deer cover 8 m and 5 m per leap respectively, what distance will the tiger have to run before it catches the deer?
(a) 600 m (b) 700 m
(c) 800 m (d) 1000 m

39. Soniya and Priyanka started from Amethi and Bellari for Bellari and Amethi, which are 645 km apart. They meet after 15 hours. After their meeting, Sonia increased her speed by 3 km/h and Priyanka reduced her speed by 3 km/h, they arrived at Bellari and Amethi respectively at the same time. What is their initial speeds?
(a) 24 km/h and 30 km/h (b) 25 km/h and 18 km/h
(c) 18 km/h and 21 km/h (d) 20 km/h and 23 km/h

40. Den Bosch and Eastbourne are two famous cities 300 km apart. Maradona starts from Den Bosch at 8 : 24 am. An hour later Pele starts from Den Bosch. After travelling for 1 hour, Pele reaches Nottingham that Maradona had passed 40 minutes earlier. Nottingham falls on the way from Den Bosch to Eastbourne. If Pele and Maradona just reaches Eastbourne at the same time, what are the speeds of the Maradona and Pele respectively?
(a) 100 km/h, 125 km/h (b) 60 km/h, 80 km/h
(c) 60 km/h, 75 km/h (d) 75 km/h, 100 km/h

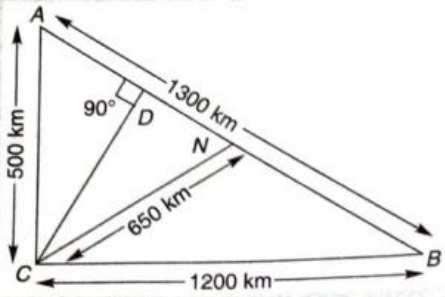
41. A thief sees a jeep at a distance of 250 m, coming towards him at 36 km/h. Thief takes 5 seconds to realise that there is nothing but the police is approaching him by the jeep and start running away from police at 54 km/h. But police realise after 10 seconds, when the thief starts running away, that he is actually a thief and gives chase at 72 km/h. How long after thief saw police did police catchup with him and what is the distance police had to travel to do so?
(a) 50 s, 1000 m (b) 65 s, 1150 m
(c) 65 s, 1300 m (d) 45 s, 1050 m

42. Inspired by the 'Golden quadrilateral project' UP Government recently accomplished a diamond triangular project. Under this project the State Government laid down 6 lane roads connecting three cities Ayodhya, Banaras and Chitrakoot, which are equally separated from each other *i. e.*, in terms of geometry they form an equilateral triangle. Angad and Bajrang start simultaneously from Ayodhya and Banaras respectively, towards Chitrakoot. When Angad covers 100 kms, Bajrang covers such a distance that the distance betweeen Angad and Bajrang makes 90° angle with the road joining Banaras and Chitrakoot. When Bajrang reaches Chitrakoot, Angad is still 150 km away from Chitrakoot. What is the distance between Ayodhya and Banaras?
(a) 250 km (b) 450 km
(c) 300 km (d) none of these

43. Two trains Ajanta express and Barouni express simultaneously started on two parallet tracks from Meerut to Nagpur, which are 390 km apart. The ratio of the speed of Ajanta express and Barouni express is 6 : 7. After how long (in kms) travelling, Barouni express exchanges the speed with Ajanta express so that both the trains reach at their destination simultaneously :
(a) 150 km (b) 190 km
(c) 210 km (d) can't be determined

44. In a circus there was a leopard and a tiger walking in the two different rings of same radii. There I observed that when leopard moved 3 steps, tiger moved 5 steps in the same time, but the distance traversed by leopard in 5 steps is equal to the distance traversed by tiger in 4 steps. What is the number of rounds that a leopard made when tiger completed 100 rounds?
(a) 120 (b) 48
(c) 75 (d) none of these

Directions for question number 45-48 : *In the following figure the route is shown which is followed by Professor Jai and Professor Jaya, who are visting faculty at IIM-A and IIM-B respectively. A, B denote IIM-A and IIM-B respectively and C denotes the residence of Prof. Jai and Prof. Jaya. They leave home for classes at the same time and their driving speeds are* $\frac{500}{13}$ *km/h and* $\frac{1200}{13}$ *km/h respectively. Also they finish the classes at the same time to reach home.*



The path adopted by Jai and Jaya is CADC and CBDC respectively. Prof. Jai and Prof. Jaya are husband and wife respectively.

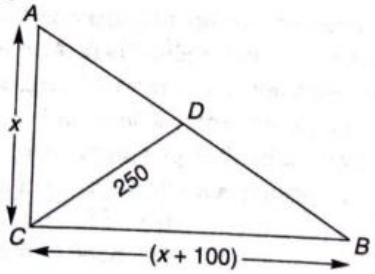
45. If both of them start and finish the classes at the same time, then who returned home earlier than other, if no one of them halts for anywhere in the route and they just leave the institution as soon as they finish the lectures?
(a) Prof. Jai
(b) Prof. Jaya
(c) Return at the same time
(d) Can't be determined

46. In the shown figure N and D denotes Noida and Delhi respectively, who returned home late and by how much time, if Jaya turned from Noida instead of Delhi :
(a) Jai, 9 h 10 min (b) Jaya, 9 h 50 min
(c) Jai, 2 h 55 min (d) Jai, 16 h 10 min

47. In the above question how many per cent time Jaya saved in going via Noida of the total time taken previously :
(a) 10% (b) 25%
(c) 50% (d) 17%

48. If Mrs. Jaya wants to watch the premier show of a movie at Wave Cinema in Noida while returning from institute through BNC. When will she return home given that she spends total time 3 hours at wave cinema?
(a) at the same time as normal
(b) 5 min late than her husband
(c) at the same time when her husband returns
(d) can't be determined

49. Preetam and Devi start running a race on the given track as shown in figure.



Where AC and BC are mutually perpendicular and CD is the median of triangular paths ABC. BC is 100 km longer than that of AC, again CD is 250 km. The speeds of Preetam and Devi are 30 km/h and 40 km/h, initially and their respective paths of running are $CADC$ and $CBDC$. After how much time they reverse their speeds so that they return C at the same time?
(a) $\frac{50}{7}$ h (b) $\frac{120}{7}$ h
(c) $\frac{80}{11}$ h (d) none of these

Directions for question number 50, 51, and 52 : *The markets $M_1, M_2, M_3, \ldots$ etc of Vyapaar city are lying besides the circular paths $P_1, P_2, P_3, \ldots$ etc. All the circular paths are concentric at centre 'O' and their distances are 1 km, 2 km, 3 km ... etc from the centre 'O' respectively. At the centre 'O' there is "Khoob Khao" tiffin agency which supplies the tiffins to all the markets $M_1, M_2, M_3, \ldots$ etc. A tiffin carrier starts from 'O' goes directly east of the shop and then on reaching the circular path moved 1 km in counter clock direction on it. After completing 1 km distance on P_1 the carrier moves to P_2 in the radial direction. Then it goes 2 km on P_2. Similarly 3 km on P_3 and 4 km on P_4 etc in counter clock direction moving radially from P_2 to P_3 and P_3 to P_4 etc and motion of the carrier continued in this manner till it reaches exactly in the east direction.*

50. After reaching east of the shop it can't move on further than the given distance on the current path. For how many markets can it supply its tiffins directly?
(a) 4 (b) 5
(c) 7 (d) can't be determined

51. The total distance covered by the carriers in providing the tiffins from centre 'O' to the last point in one way only is :
(a) 30 km (b) 28 km
(c) 35 km (d) none of these

52. The ratio of distances covered on the circular path P_2 to that on the last path, where the carrier reaches directly eastward of its shop is :
(a) 1 : 1 (b) 2 : 7
(c) 2 : π (d) none of these

Directions for question number 53-57 : *A train enters into a tunnel AB at A and exits at B. A jackal is sitting at O in another by passing tunnel AOB, which is connected to AB at A and B where OA is perpendicular to OB. A cat is sitting at P inside the tunnel AB making the shortest possible distance between O and P, such that $AO : PB = 30 : 32$. When a train before entering into the tunnel AB makes a whistle (or siren) somewhere before A, the jackal and cat run towards A, they meet with accident (with the train) at the entrance A. Further if the cat moves towards B instead of A it again meets with accident at the exit of the tunnel by the same train coming from the same direction.*

53. What is the ratio of speeds of jackal and cat?
(a) 4 : 3 (b) 5 : 3
(c) 1 : 1 (d) can't be determined

54. The ratio of speeds of jackal is to train is :
(a) 5 : 1 (b) 3 : 5
(c) 1 : 5 (d) can't be determined

55. If jackal moves towards OPA, it will meet with train at M_1 then AM_1 is :
(a) 20 km (b) 16 km
(c) 10 km (d) can't be determined

56. If jackal moves towards OPB and cat moves towards POA who will not meet with accident with the train?
(a) Jackal (b) Cat
(c) Both (a) and (b) (d) Can't be determined

57. The ratio of time taken by cat and jackal in moving $OAPO$ and $PBOP$ respectively given that they do not meet with accident
(a) 1 : 1 (b) 3 : 4
(c) 5 : 4 (d) none of these

58. A candle of 6 cm long burns at the rate of 5 cm in 5 h and another candle of 8 cm long burns at the rate of 6 cm in 4 h. What is the time required by each candle to remain of equal lengths after burning for some hours, when they start to burn simultaneously with uniform rate of burning?
(a) 1 h (b) 1.5 h
(c) 2 h (d) none of these

59. Two boats start at the same instant to cross a river W metre wide. The faster boat reaches the other bank and returns back immediately. What are the distances travelled by them when they meet, where the speeds of these boats are b_1 & b_2?
(a) $\frac{2W}{(b_1 + b_2)}, \frac{2W}{(b_1 - b_2)}$
(b) $\frac{2W}{(b_1 + b_2)} b_1$ and $\frac{2W}{(b_1 + b_2)} b_2$
(c) $\frac{W}{(b_1 + b_2)} b_1, \frac{W}{(b_1 + b_2)} b_2$
(d) data insufficient

60. Mariya was travelling in her boat when the wind blew her hat off and the hat started floating back downstream. The boat continued to travel upstream for twelve more minutes before Mariya realized that her hat had fallen off and turned back downstream. She caught up with that as soon as it reached the starting point. Find the speed of river if Mariya's hat flew off exactly 3 km from where she started :
(a) 5 km/h (b) 6 km/h
(c) 7.5 km/h (d) can't be determined

61. Akbar, Birbal and Chanakya run around a circular track of length 500 m. Akbar and Birbal run with the speeds of 15 m/s and 20 m/s in the same direction respectively and Chanakya being very intelligent run in the opposite direction with a speed of 25 m/s. If all three of them start at the same time, then :
(a) Akbar meets Chanakya more frequently than Birbal does
(b) Akbar and Chanakya meets as frequently as Birbal and Chanakya
(c) Akbar meets Birbal least frequently
(d) Nothing can be concluded

62. Arun and Barun run with the speeds of 30 m/s and 20 m/s around a circular track of 600 m. They participate in a 3000 m race. What is the distance covered by Arun when he passes Barun for the 5th time?
(a) 2200 m (b) 2250 m
(c) 2850 m (d) none of these

63. Akkal and Bakkal are running on a circular track of radius 175 metres. Akkal can complete a round in 100 seconds and the speed of Bakkal is twice the speed of Akkal. They started simultaneously towards each other from two points 350 metres diametrically opposite on the circular path. If they first meet at a point they called it love point, which is between the two points P and Q from where they have started their race, after how much time from the start do they meet at love point for the third time?
(a) $218\frac{2}{5}$ s (b) $216\frac{2}{3}$ s
(c) 221 s (d) none of these

64. Arti and Barkha start swimming towards each other from the deep end and shallow end respectively of a swimming pool in Funcity. They start their swimming simultaneously in the length of 300 m pool. The ratio of their speeds is 1 : 2 respectively. Each swimmer rests for 6 seconds once she reaches the other end and starts swimming back. Where will they meet for the second time in the still water of swimming pool?
(a) 30 m from the shallow end
(b) at the shallow end
(c) at the deepend
(d) can't be determined

65. A and B runs around a circular track. A beats B by one round or 10 minutes. In this race, they had completed 4 rounds. If the race was only of one round, find the A's time over the course :
(a) 8 min (b) 7.5 min
(c) 12.5 min (d) 12 min

66. A, B and C participated in a race. A covers the same distance in 49 steps, as B covers in 50 steps and C in 51 steps. A takes 10 steps in the same time as B takes 9 steps and C takes 8 steps. Who is the winner of the race?
(a) A (b) B
(c) C (d) can't be determined

67. Shambhu drives his car very fast at 360 m/s. Moving ahead for some hours he finds some problem in headlights of the car. So he takes 20 seconds in changing the bulb of the headlight by stopping the car. Mean while he notices that another car which was 400 m back is now 200 m ahead of his car. What is the speed of this car?
(a) 100 km/h (b) 92 km/h
(c) 108 km/h (d) 300 km/h

68. Two persons start from the opposite ends of a 90 km straight track and run to and fro between the two ends. The speed of first person is 30 m/s and the speed of other is 125/6 m/s. They continue their motion for 10 hours. How many times they pass each other?
(a) 10 (b) 9
(c) 12 (d) none of these

69. At what time after 3 : 10 am, the acute angle made by the minute and hour-hand is double to that of at 3 : 10 am, for the first time?
(a) 4 h 43 min (b) 3 h 48 min
(c) $3\text{ h }\frac{320}{11}$ min (d) none of these

70. If the two incorrect watches are set at 12 : 00 noon at correct time, when will both the watches show the correct time for the first time given that the first watch gains 1 min in 1 hour and second watch loses 4 min in 2 hours :
(a) 6 pm, 25 days later
(b) 12 : 00 noon, 30 days later
(c) 12 noon, 15 days later
(d) 6 am 45 days later

71. Rajeev and Sanjeev are too close friends Rajeev's watch gains 1 minute in an hour and Sanjeev's watch loses 2 minutes in an hour. Once they set both the watches at 12 : 00 noon, with my correct watch. When will the two incorrect watches of Rajeev and Sanjeev show the same time together?
(a) 8 days later (b) 10 days later
(c) 6 days later (d) can't be determined

72. At a railway station a 24 hour watch loses 3 minutes in 4 hours. If it is set correctly on Sunday noon when will the watch show the correct time?
(a) 6 pm after 40 days (b) 12 noon after 75 days
(c) 12 pm after 100 days (d) 12 noon after 80 days

73. A swiss watch is being shown in a museum which has a very peculiar property. It gains as much in the day as it loses during night between 8 pm to 8 am. In a week how many times will the clock show the correct time?
(a) 6 times (b) 14 times
(c) 7 times (d) 8 times

74. A wrist watch which is running 12 minutes late on a Sunday noon is 16 minutes ahead of the correct time at 12 noon on the next Sunday. When is the clock 8 minutes ahead of time?
(a) Thursday 10 am (b) Friday noon
(c) Friday 8 pm (d) Tuesday noon

75. A clock loses 2 minutes in an hour and another clock gains 2 minutes in every 2 hours. Both these clocks are set correctly at a certain time on Sunday and both the clocks stop simultaneously on the next day with the time shown being 9 am and 10 : 06 am. What is the correct time at which they stopped?
(a) 9 : 54 am (b) 9 : 44 pm
(c) 9 : 46 am (d) 9 : 44 am

76. David sets his watch at 6 : 10 am on Sunday, which gains 12 minutes in a day. On Wednesday if this watch is showing 2 : 50 pm. What is the correct time?
(a) 1 : 50 pm (b) 2 : 10 pm
(c) 2 : 30 pm (d) 3 : 30 pm

77. Ramu purchased a second hand Swiss watch which is very costly. In this watch the minute-hand and hour hand coincide after every $65\frac{5}{11}$ minutes. How much time does the watch lose or gain per day?
(a) 4 min (b) 5 min
(c) 4 min, 20 sec (d) none of these

78. My watch was 8 minutes behind at 8 pm on Sunday but within a week at 8 pm on Wednesday it was 7 minutes ahead of time. During this period at which time this watch has shown the correct time :
(a) Tuesday 10 : 24 am
(b) Wednesday 9 : 16 pm
(c) It cannot show the correct time during this period
(d) None of the above

79. Out of the following four choices which does not show the coinciding of the hour hand and minute-hand :
(a) 3 : 16 : 2 (b) 6 : 32 : 43
(c) 9 : 59 : 05 (d) 5 : 27 : 16

80. Kumbhakarna starts sleeping between 1 am and 2 am and he wakes up when his watch shows such a time that the two hands (*i.e.*, hour-hand and minute-hand) interchange their respective places. He wakes up between 2 am and 3 am on the same night. How long does he sleep?
(a) $55\frac{5}{13}$ min (b) $110\frac{10}{13}$ min
(c) $54\frac{6}{13}$ min (d) none of these

SPEED TEST (TSD)

Directions for question number 1 to 3: *Arun took part in a triathlon, an athletic event. He had to swim, run and bicycle to 10 km, 24 km and 30 km, respectively and return the same way. Arun's average speed for the triathelon is 4 km/h. He took a total of 4 min for swimming and 20 min for bicycling in the triathelon.*

1. His speed of running is:
(a) 5 km/min (b) 6 km/min
(c) 7.5 km/min (d) $\frac{8}{3}$ km/min

2. He finishes race in:
(a) 16 min (b) 23 min
(c) 32 min (d) 35 min

3. How much time he took while returning for the bicycle if the time taken for the return of each phase (*i. e.*, each event) is 50% greater than that of taken initially:
(a) 12 min (b) 11 min
(c) $11\frac{1}{9}$ min (d) none of these

Directions for question number 4 to 5: *A, B and C start running a race from the same point P in the same direction. A runs around a path which is an equilateral triangle PQX and B runs around a square path PQYZ and C runs on the regular hexagonal path PQRSTU. Where the one side PQ of each path is common.*

4. If all of them complete one equal round at the same time then which of the following is true?
(a) Speed of *C* is twice that of *B*
(b) Speed of *A* is half that of *C*
(c) Speed of *B* is 50% more than that of *A*
(d) none of the above

5. If each of them run at a same constant speed, what is the maximum number of rounds anyone of them would have completed when they meet for the first time?
(a) 1 (b) 2 (c) 3 (d) 4

Directions for the question number 6 to 9: *City X and City Y are connected to a straight road. A and B start moving simultaneously towards each other. After travelling some distance, B takes a 60° turn to his left. Two hours later after B turns, A takes a 60° turn to his right. A travels 60 km after turning, before he meets B. A and B meet 10 hours after they start their journey. A and B together travel 200 km before turning and they arrive at the meeting point simultaneously.*

6. How many hours after turning does *A* meet *B*?
(a) 1 (b) 3 (c) 4 (d) 5

7. What distance does *A* travel before turning 60 degree to his right?
(a) 140 km (b) 150 km
(c) 160 km (d) 170 km

8. If *A* and *B* had not turned, after how many hours would they have met?
(a) $7\frac{1}{8}$ h (b) $9\frac{1}{8}$ h (c) $6\frac{1}{8}$ h (d) $8\frac{1}{8}$ h

9. What is *B*'s speed?
(a) 10 km/h (b) 12.5 km/h
(c) 12 km/h (d) 15 km/h

10. Chetak and Ashwa, two horses, start galloping from Patna to Ranchi which are 80 km apart. The speed of Chetak is 160 km/h and that of Ashwa is 150 km/h. They start galloping simultaneously, from Patna to Ranchi. Chetak reached to Ranchi and returned to Patna but Ashwa returned from Jamshedpur (which is somewhere between Patna and Ranchi) to Patna at the same time. What is the ratio of distances between Patna and Jamshedpur and Jamshedpur and Ranchi?
(a) 15 : 1 (b) 3 : 25
(c) 15 : 2 (d) none of these

INTRODUCTORY EXERCISE-9.1

1 (a)	2. (c)	3. (c)							

LEVEL-1

1 (a)	2. (b)	3. (b)	4. (c)	5. (a)	6. (c)	7. (c)	8. (b)	9. (a)	10. (c)
11. (c)	12. (d)	13. (a)	14. (d)	15. (b)	16. (c)	17. (b)	18. (c)	19. (b)	20. (c)
21. (b)	22. (d)	23. (a)	24. (b)	25. (c)	26. (d)	27. (d)	28. (d)	29. (b)	30. (c)
31. (d)	32. (a)	33. (b)	34. (c)	35. (d)	36. (a)	37. (b)	38. (a)	39. (b)	40. (c)
41. (a)	42. (b)	43. (b)	44. (c)	45. (c)	46. (c)	47. (c)	48. (b)	49. (a)	50. (b)
51. (b)	52. (a)	53. (b)	54. (c)	55. (c)	56. (c)	57. (a)	58. (b)	59. (d)	60. (b)
61. (a)	62. (c)	63. (c)	64. (c)	65. (b)	66. (d)	67. (d)	68. (c)	69. (d)	70. (a)
71. (a)	72. (a)	73. (b)	74. (c)	75. (b)	76. (c)	77. (b)	78. (c)	79. (a)	80. (c)
81. (d)	82. (b)	83. (c)	84. (c)	85. (b)	86. (d)	87. (b)	88. (b)	89. (d)	90. (c)
91. (c)	92. (b)	93. (d)	94. (d)	95. (c)	96. (b)	97. (a)	98. (b)	99. (b)	100. (a)
101. (d)	102. (a)	103. (c)	104. (b)	105. (b)	106. (c)	107. (d)	108. (a)	109. (b)	110. (c)
111. (b)	112. (d)	113. (c)	114. (d)	115. (a)	116. (b)	117. (d)	118. (b)	119. (b)	120. (b)
121. (b)	122. (b)	123. (c)	124. (b)	125. (b)	126. (c)	127. (d)	128. (b)	129. (a)	130. (a)
131. (d)	132. (b)	133. (c)	134. (b)	135. (b)	136. (b)	137. (b)	138. (b)	139. (a)	140. (c)
141. (c)	142. (d)	143. (d)	144. (d)	145. (b)	146. (a)	147. (b)	148. (d)	149. (a)	150. (c)

LEVEL-2

1 (b)	2. (c)	3. (b)	4. (a)	5. (b)	6. (c)	7. (c)	8. (c)	9. (b)	10. (a)
11. (d)	12. (d)	13. (c)	14. (c)	15. (a)	16. (b)	17. (c)	18. (d)	19. (c)	20. (b)
21. (d)	22. (c)	23. (b)	24. (b)	25. (b)	26. (c)	27. (b)	28. (c)	29. (b)	30. (b)
31. (c)	32. (b)	33. (c)	34. (b)	35. (d)	36. (b)	37. (c)	38. (c)	39. (d)	40. (d)
41. (b)	42. (c)	43. (c)	44. (b)	45. (c)	46. (c)	47. (d)	48. (b)	49. (b)	50. (c)
51. (a)	52. (a)	53. (b)	54. (c)	55. (c)	56. (b)	57. (c)	58. (d)	59. (b)	60. (c)
61. (c)	62. (d)	63. (b)	64. (b)	65. (b)	66. (a)	67. (c)	68. (c)	69. (c)	70. (b)
71. (b)	72. (d)	73. (d)	74. (b)	75. (d)	76. (b)	77. (a)	78. (a)	79. (c)	80. (a)

SPEED TEST-1

1 (b)	2. (c)	3. (b)	4. (b)	5. (d)	6. (b)	7. (a)	8. (d)	9. (c)	10. (a)

LEVEL 1

1. Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{10+20}{\frac{10}{6}+\frac{20}{15}}$

$$= \frac{30}{\left(\frac{90}{30}\right)} = \frac{30}{3} = 10 \text{ km/h}$$

2. Average speed (when distances are same) $= \dfrac{2uv}{u+v}$

$$= \frac{2\times 20\times 30}{(20+30)} = 24 \text{ km/h}$$

where u, v are the different speeds.
(or use the general formula–Total distance/Total time)

3. Average speed $= \dfrac{3uvw}{uv+vw+uw}$

$$= \frac{3\times 10\times 20\times 30}{200+600+300} = \frac{18000}{1100}$$

$$= 16\frac{4}{11} \text{ km/h}$$

Alternatively: Suppose the total distance 3 times the LCM of the given speeds, then solve by general formula.

Total distance = 180 km (say)

then Total time $= \dfrac{60}{10}+\dfrac{60}{20}+\dfrac{60}{30}$

$= 6+3+2 = 11$ h

$\therefore$ Average speed $= \dfrac{180}{11} = 16\dfrac{4}{11}$ km/h

Alternatively: Suppose the total distance equals to $3x$ km then solve as above.

4. Suppose the total distance equals to 4 times the LCM of the speeds.

$\therefore$ Total distance = 2400 km

$\therefore$ Total time $= \dfrac{600}{20}+\dfrac{600}{25}+\dfrac{600}{30}+\dfrac{600}{40}$

$= 30+24+20+15 = 89$ h

$\therefore$ Average speed $= \dfrac{2400}{89} = 26\dfrac{86}{89}$ km/h

5. $\dfrac{2\times 20\times x}{(20+x)} = 24$

$\Rightarrow$ $x = 30$ km/h

Alternatively: Go through options.

6. Average speed $= \dfrac{2uv}{u+v}$

$$= \frac{2\times 3x\times 5x}{(3x+5x)} = 3.75x \text{ km/h}$$

7. $u = k, \quad v = 3k$

$\therefore$ $\dfrac{2uv}{u+v} \Rightarrow \dfrac{2\times k\times 3k}{(k+3k)} = 12$

$\Rightarrow$ $1.5k = 12$

$\Rightarrow$ $k = 8$ km/h

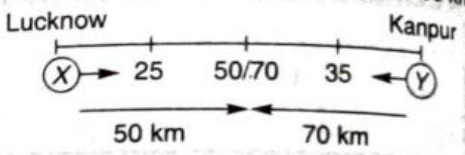
Alternatively: Go through options.
Alternatively: Solve through alligations.

8. Effective speed = 25 + 35 = 60 km/h

Total distance to be covered = 120 km

$\therefore$ Time taken $= \dfrac{120}{60} = 2$ h

HINT Since in each hour X and Y together covers 60 km.



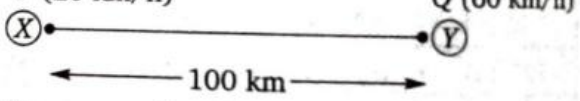
9. Since they take 2 hours to meet each other. Thus in 2 h X 50 km away from Lucknow.

Alternatively: Both X and Y will cover the respecti distances in the ratio of their speeds.

So, distance from Lucknow $= \dfrac{5}{12}\times 120 = 50$ km

10. When time is constant the distances covered by A and B wi be in the ratio of their speeds, respectively.

11.



To meet each other they have to cover 100 km distan together and the ratio of distances covered by each one directly proportional to the ratio of their speeds, respectiv Since the time taken by each one is same (i. e., constant).

Hence, $\dfrac{XZ}{YZ} = \dfrac{20}{60} = \dfrac{1}{3}$

$\therefore$ $XZ = \dfrac{1}{4}\times 100 = 25$ km

12. To meet the second time they have to cover 300 km distan together [for n^{th} time distance $= (2n-1)d$]
Time taken by them to meet each other, for the second ti

$$= \frac{300}{80} = 3\frac{3}{4} \text{ h}$$

Distance covered by $P = 20\times 3\dfrac{3}{4} = 75$ km

Alternatively: The ratio of distance covered = Ratio of their speeds

$\therefore$ Distance covered by $P = \dfrac{1}{4}\times 300 = 75$ km

13. The ratio of distances covered by each of them
= Ratio of their respective speeds

$\therefore$ Distance covered by Q for the third meeting $= \frac{3}{4} \times 500$

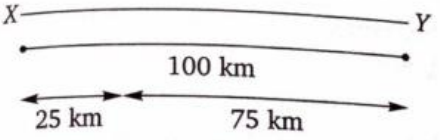
$= 375$ km

14. Consider the distance travelled by any one of them, then find the distance between X and M, where they meet.

$\therefore$ The distance travelled by P for the fourth meeting

$$= \frac{1}{4} \times 700 = 175 \text{ km}$$

Therefore P will be 75 km from Y. It means P will be 25 km away from X.



15. This is constant for any number of meeting and is equal to the ratio of their speeds. Hence, 1 : 3.

16. It is always twice the length of race course. Hence, between any two consecutive meeting they have to cover total 200 km distance to meet each other for the next meeting.

HINT
For the first meeting they will cover 100 km.
For the second meeting they will cover 300 km.
For the third meeting they will cover 500 km.
For the fourth meeting they will cover 700 km and so on.

17. To meet each other they will take equal time since they start their journey simultaneously.

18. To reach their destination the time taken by A and B is equal to the ratio of reciprocal to their speeds. Since when distance is constant time is inversely proportional to the respective speeds.

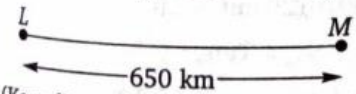
Hence, time taken by A and $B = \frac{1}{16} : \frac{1}{25} = 25 : 16$

19. $\frac{A\text{'s speed}}{B\text{'s speed}} = \sqrt{\frac{\text{Time taken by } B\ (t_B)}{\text{Time taken by } A\ (t_A)}}$

$$\frac{16}{25} = \sqrt{\frac{t_B}{t_A}}$$

$$\Rightarrow \quad \frac{t_B}{t_A} = \frac{256}{625} \Rightarrow \frac{t_A}{t_B} = \frac{625}{256}$$

20.



(Kamal) 40 km/h (Vimal) 30 km/h

In the first 3 hours Vimal covers 90 km.

So, the rest distance = 560 km

Now, Kamal and Vimal both travels together, towards each other.

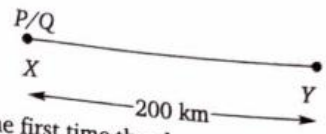
So, the time $= \frac{\text{Distance}}{\text{Speed}} = \frac{560}{70} = 8$ h

Thus, Vimal travels total = 3 + 8 = 11 h

Thus, the distance travelled by Vimal = 11 × 30 = 330 km

21. The distance covered by Kamal = 8 × 40 = 320 km

22.



To meet the first time they have to cover 200 × 2 = 400 km

Now, the time taken to meet each other $= \frac{400}{50} = 8$ h

23. Distance travelled by $P = 20 \times 8 = 160$ km

24. Distance travelled by $Q = 30 \times \frac{800}{50} = 480$ km

25. Distance travelled by $P = 20 \times \frac{1200}{50} = 480$ km

So, P will be 80 km away from X. Therefore ratio of distances

$$= \frac{80}{120} = \frac{2}{3}$$

26. The distance travelled by $P = \frac{1600}{50} \times 20 = 640$ km

Thus, P will be 160 km away from X.

[Since 640 = (200 × 2) + 240]

$\therefore$ Distance between D and X is 160 km.

27. This is possible only when the distance covered by P in n round be the multiple of 400.

28. X started at 12 : 00 noon

Y started at 3 pm

$$\text{Required time} = \frac{\text{Distance advanced in 3 h}}{\text{Relative speed}}$$

$$= \frac{40 \times 3}{20} = 6 \text{ h} \qquad (20 = 60 - 40)$$

Hence, Y will overtake X at 9 pm. $(3 + 6 = 9)$

29. $$\text{Required time} = \frac{\text{Distance advance} - \text{Required difference}}{\text{Relative speed}}$$

$$= \frac{120 - 30}{20} = \frac{90}{20} = 4.5 \text{ h}$$

Thus, at 7 : 30 pm X and Y will be 30 km apart.

30. $$\text{Required time} = \frac{\text{Distance advanced} + \text{Required difference}}{\text{Relative speed}}$$

$$= \frac{120 + 30}{20} = \frac{150}{20} = 7.5 \text{ h}$$

Thus at 10 : 30 pm X and Y will be 30 km apart.

31. Distance travelled by Y to overtake X

= Time taken × Speed of Y

$= 6 \times 60 = 360$ km

Thus, Y will overtake X at a distance of 360 km from P.

32. Required distance

$$= \left(\frac{\text{Distance advanced} + \text{Required difference}}{\text{Relative speed}}\right) \times (\text{Speed of } Y)$$

$$= \frac{120 + 360}{20} \times 60 = 1440 \text{ km}$$

33. Time (when X was 30 km ahead of Y) $= \frac{120 - 30}{20} = 4.5$ h

Time (when Y was 30 km ahead of X) $= \frac{120 + 30}{20} = 7.5$ h

Thus, required difference in time = 3 h

34. Speed = 36 km/h

$$= 36 \times \frac{5}{18} = 10 \text{ m/s}$$

35. Since in 60 minutes postman goes 36000 metre.

So in 1 minute postman goes $\frac{36000}{60} = 600$ metre

Thus, his required speed = 600 m/min

36. 1 mile = 1609.3 m = 1.6093 km

$\therefore$ 1 km $= \frac{1}{1.6093}$ mile = 0.6213882 mile

$\therefore$ Required speed = 36×0.6213882 mile/h

= 22.37 mile/h

37. Speed = 20 m/s $= 20 \times \frac{18}{5} = 72$ km/h

38. In one hour train goes 72 km. So in one minute train will go $\frac{72}{60}$.

$\therefore$ Speed $= \frac{6}{5} = 1.2$ km/min

39. The ratio of speeds of $A, B, C = 6 : 3 : 1$

$\therefore$ The ratio of time taken by $A, B, C = 1 : 2 : 6$

$\therefore$ Time taken by $A = 13$ min

40.

	A	:	B
Speed	2	:	3
Time	$3x$	:	$2x$

41. $\because$ $3x - 2x = 20 \Rightarrow x = 20$

$\therefore$ $3x = 60$ min = 1 h

42. Let the original speed be S_1 and time t_1 and distance be D

Now, $\frac{D/2}{2t_1} = S_2$

$\therefore$ $S_2 = \frac{D}{4t_1}$ and $S_1 = \frac{D}{t_1}$

$\therefore$ $\frac{S_1}{S_2} = \frac{D/t_1}{D/4t_1} = \frac{4}{1}$

43. You can go through options to check the required difference.

Alternatively: Required distance

$$= \frac{S_1 S_2}{(S_1 \sim S_2)} \times \text{Time difference}$$

$$= \frac{5 \times 8}{3} \times \frac{3}{2} = 20 \text{ km}$$

Alternatively: Take the LCM of distances then solve by unitary method.

$\therefore$ LCM of 5, 8 = 40

Now, consider 40 km as a distance, then there is a 3 hours difference in 40 km. So, 3/2 hours difference will be in 20 km.

Alternatively: Let x be the distance, then

$$\frac{x}{5} - \frac{x}{8} = \frac{3}{2}$$

$\Rightarrow$ $x = 20$ km

44.

	A	:	B
Speed	2	:	3
Time	3	:	2

$\because$ B takes 48 minutes so A will take 72 minutes

= 1 h 12 min

45. The ratio of distances = 160 : 140 = 8 : 7

$\therefore$ The ratio of speeds = 8 : 7

46. $\frac{x}{9} - \frac{x}{10} = \frac{20}{60}$

$x = 30$ km

47. Time taken by Abhinav = 4 h

Time taken by Praveen = 3.5 h

For your convenience take the product of times taken by both as a distance.

Then the distance = 14 km

Since, Abhinav covers half of the distance in 2 hours (*i.e.*, at 8 am)

Now, the rest half (*i.e.*, 7 km) will be covered by both Praveen and Abhinav.

Time taken by them $= \frac{7}{7.5} = 56$ min

Thus, they will cross each other at 8 : 56 am.

48. 9 km difference arises in the 99 km distance.

$\therefore$ 72 km difference will arise in the 792 km distance.

49. Apply the product constancy concept

Speed	Time	
$\frac{1}{5}\downarrow$	$\frac{1}{4}\uparrow$	= 15 min

Since, $\frac{x}{4} = 15$ min $\Rightarrow$ $x = 60$ min = 1 h

So, the original (or usual) time = 60 min = 1 h

50.

Speed	Time	
$\frac{1}{4}\downarrow$	$\frac{1}{3}\uparrow$	= 2 h

$\Rightarrow$ Usual time $= 2 \times 3 = 6$ h

51.

Speed	Time	
$\frac{1}{2}\uparrow$	$\frac{1}{3}\downarrow$	= 40 min

$\Rightarrow$ Usual time $= 3 \times 40 = 120$ min = 2 h

52.

Speed	Time	
$\frac{2}{20} = \frac{1}{10}\downarrow$	$\frac{1}{9}\uparrow$	= 10 min

$\Rightarrow$ Usual time $= 9 \times 10 = 90$ min $= \frac{3}{2}$ h

$\therefore$ Distance travelled = Speed $\times$ Time

$$= 20 \times \frac{3}{2} = 30 \text{ km}$$

53. Increase in speed = 1 km

Change in time = 25 min

Speed	Time	
$\frac{1}{5}\uparrow$	$\frac{1}{6}\downarrow$	= 25

$\Rightarrow$ Usual (normal) time $= 6 \times 25$ min $= \frac{5}{2}$ h

Distance = Normal speed × Normal time

$\therefore \quad = 5 \times \frac{5}{2} = 12.5$ km

54. Since, they are 1.5 km apart in each hour.
$\therefore$ 7.5 km apart they will be in 5 hours.
(Since, in the same direction speeds are subtracted)

55. In each hour they will be 720 km apart.
(Since in opposite direction speeds are added)

56. Let the distance = x km and usual rate = y km/h

$$\therefore \quad \frac{x}{y} - \frac{x}{(y+6)} = 4 \text{ h} \qquad \ldots(i)$$

$$\frac{x}{(y-4)} - \frac{x}{y} = 4 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{x}{y} - \frac{x}{(y+6)} = \frac{x}{(y-4)} - \frac{x}{y}$$

$$\Rightarrow \quad y = 24$$

Now putting the value of $y = 24$ in Eq (i), we get

$$x = 480$$

Alternatively:

$$\begin{matrix} S & & T \\ +6 & \times & -4 \end{matrix} \quad \Rightarrow \quad -4S + 6T = |-4 \times 6|$$

$$\begin{matrix} S & & T \\ -4 & \times & +4 \end{matrix} \quad \Rightarrow \quad 4S - 4T = |-4 \times 4|$$

$$\therefore \quad -4S + 6T = 24 \qquad \ldots(i)$$

$$4S - 4T = 16 \qquad \ldots(ii)$$

$\therefore$ Solving these two equations, we get

$$T = 20 \quad \text{and} \quad S = 24$$

$\therefore$ Distance = Speed × Time

$$= 24 \times 20 = 480 \text{ km}$$

57. Let the original speed be x km/h, then

$$\frac{600}{x} - \frac{600}{(x+20)} = 1$$

$$\Rightarrow \quad 600\left(\frac{x+20-x}{x(x+20)}\right) = 1$$

$$\Rightarrow \quad x^2 + 120x - 12000 = 0$$

$$\Rightarrow \quad (x-120)(x-100) = 0$$

$$\Rightarrow \quad x = 100 \quad \text{and} \quad x = 120$$

$\therefore$ Original speed = 100 km/h

Alternatively:

$$S_1 \times S_2 = \text{Distance} \times \frac{\text{Difference in speed}}{\text{Difference in time}}$$

$$= 600 \times \frac{20}{1} = 600 \times 20 = 12000$$

$$\Rightarrow \quad S_1 \times S_2 = 12000$$

$\therefore \quad S_1 = 100 \quad \text{and} \quad S_2 = 120$

(Factorise 12000 in such a way that the difference be 20) S_1 is the original speed and S_2 is the changed speed.

58. Very similar to question number 57.

$$180 \times \frac{5}{3} = 60 \times 5$$

$$S_1 \times S_2 = 300 = 15 \times 20$$

and $\quad S_1 = 15$ km/h

$\quad S_2 = 20$ km/h

Alternatively: $\quad S_1 \times (S_1 + 5) = 300$

$\Rightarrow \quad S_1 = 15$ km/h

59. Very similar to previous problems

$$t_1 \times t_2 = \text{Distance} \times \frac{\text{Difference in time}}{\text{Difference in speed}}$$

$$\Rightarrow \quad t_1 \times t_2 = 1680 \times \frac{6}{14}$$

$$\Rightarrow \quad t_1 \times t_2 = 120 \times 6 = 720 = 30 \times 24$$

$\quad t_1 = 30 \quad \text{and} \quad t_2 = 24$

[since $t_2 = (t_1 - 6) \Rightarrow t_1 \times (t_1 - 6) = 720 \Rightarrow t_1 = 30$]

Therefore, usual time = 30 h

60. Relative speecd $= \frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{60+40}{20} = 5 \text{ m/s}$$

$$\therefore \quad 5 \text{ m/s} = 5 \times \frac{18}{5} = 18 \text{ km/h}$$

Now, Relative speed = 18 km/h

= Speed of ambulance − Speed of school bus

18 = 30 − speed of school bus

Speed of school bus = 12 km/h

61.

Time	Speed
$\frac{1}{4}\downarrow$	$\frac{1}{3}\uparrow$

Since he has to increase his speed by $\frac{1}{3}$rd of the original speed.

So, the new speed = 15 + 5 = 20 km/h

Alternatively: $\quad 15 \times 4 = 3 \times x$

$\Rightarrow \quad x = 20$ km/h

62. Time (required) $= \frac{\text{Length of train}}{\text{Speed of train}} \quad \left(T = \frac{D}{S}\right)$

$$= \frac{350}{70} = 5 \text{ s}$$

63. Time taken to cross the bridge $= \frac{\text{Total length}}{\text{Speed of train}}$

$$= \frac{270+130}{40} = \frac{400}{40}$$

$$= 10 \text{ s}$$

64. Speed of train $= \frac{225}{9} = 25$ s

Now, Time taken to cross the tunnel $= \frac{225+450}{25} = 27$ s

65. Let the length of the tunnel be x m, then

$$\text{Time} = \frac{\text{Length of train + Length of tunnel}}{\text{Speed}}$$

$$60 = \frac{350 + x}{10} \qquad \left(\text{Speed} = 36 \times \frac{5}{18} = 10 \text{ m/s}\right)$$

$$\Rightarrow \quad x = 250 \text{ m}$$

66. $$\text{Speed of train} = \frac{\text{Length of train + Length of platform}}{\text{Time}}$$

$$= \frac{250 + 250}{25} = 20 \text{ m/s}$$

$$= 72 \text{ km/h}$$

67. Let the length of the Sabarmati express is x metre then,

$$\frac{x + 162}{18} = \frac{x + 120}{15} \text{ Speed of train}$$

$$\Rightarrow \quad x = 90 \text{ m}$$

68. $$\text{Time} = \frac{\text{Length of train}}{\text{Relative speed}}$$

$$= \frac{200}{19} = 10\frac{10}{19} \text{ s}$$

(Relative speed = 72 – 3.6 = 68.4 km/h = 19 m/s)

69. $$\text{Time} = \frac{\text{Length of train}}{\text{Relative speed}}$$

$$= \frac{350}{35} \times 6 = 60 \text{ s}$$

$$\left(\text{Relative speed} = 20 + 1 = 21 \text{ km/h} = 21 \times \frac{5}{18} = \frac{35}{6} \text{ m/s}\right)$$

70. Relative speed = 50 – 40 = 10 km/h $= \frac{50}{18}$ m/s

$$\therefore \quad \text{Time taken} = \frac{\text{Sum of length of the trains}}{\text{Relative speed}}$$

$$= \frac{200}{50} \times 18 = 72 \text{ s}$$

71. Relative speed = 50 + 40 = 90 km/h

$$= 90 \times \frac{5}{18} = 25 \text{ m/s}$$

$$\therefore \text{ Time taken} = \frac{\text{Sum of lengths of train}}{\text{Relative speed}} = \frac{200}{25} = 8 \text{ s}$$

72. Let the length of train be x m, then

$$\frac{x}{10} = \frac{120 + x}{18}$$

$$\Rightarrow \quad x = 150 \text{ m}$$

73. Let the speed of train be x km/h, then

$$\frac{175}{10} = (9 + x) \times \frac{5}{18}$$

$$\Rightarrow \quad x = 54 \text{ km/h}$$

74. Relative speed = Sum of speeds of two trains

$$= (60 + x)$$

$$\text{Time} = \frac{\text{Sum of length of two trains}}{\text{Relative speed}}$$

$$10 = \frac{250}{(60 + x) \times 5} \times 18$$

$$(60 + x) = 90$$

$$x = 30 \text{ km/h}$$

75. Let the length of the train be x metre, and let the speed of train be y km/h, then

$$x = (y + 3)\frac{5}{18} \times 36$$

and $$x = (y + 6) \times \frac{5}{18} \times 30$$

From Eq. (i) and (ii), we get

$$(y + 3) \times 36 = (y + 6) \times 30$$

$$y = 12 \text{ km/h}$$

$$\therefore \quad x = (y + 3) \times \frac{5}{18} \times 36$$

or $$x = 15 \times \frac{5}{18} \times 36$$

or $$x = 150 \text{ m}$$

76. Let the length of each train be x m, then

$S_1 = \frac{x}{4}$ and $S_2 = \frac{x}{5}$; S_1 and S_2 are speeds

Now, $$\text{required time} = \frac{2x}{\frac{x}{4} + \frac{x}{5}} = \frac{40}{9} \text{ s}$$

77.

	Pushpak		Bhopal
Time →	10	:	8
Speed →	8	:	10
⇒	4x	:	5x

Since, $5x - 4x = 10$

$$\Rightarrow \quad 5x = 50 \quad \text{and} \quad 4x = 40$$

$$\therefore \quad \text{Average speed} = \frac{2 \times 40 \times 50}{40 + 50} = 44\frac{4}{9} \text{ km/h}$$

78. $$\text{Required time} = \frac{\text{Distance advanced}}{\text{Relative speed}} = \frac{4 \times 40}{20} = 8 \text{ h}$$

Thus, the faster train will overtake at 8 pm.

79. Required distance = Time taken in overtaking × Faster's speed

$$= 8 \times 60 = 480 \text{ km from Meerut}$$

80. In 50 minute Rajdhani express can cover 50 km. So, the remaining distance = 650 km, which will be jointly covered by both trains

$$\therefore \quad \text{Time taken} = \frac{650}{(60 + 70)} = 5 \text{ h}$$

Distance from Lucknow = 5 × 70 = 350 km

81. $$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{560 + 360}{16} = 57\frac{1}{2} \text{ h}$$

82. $$\text{Average speed} = \frac{2 \times 80 \times 120}{200} = 96 \text{ km/h}$$

83. Distance = Average speed × Average time

or $$\text{Distance} = 96 \times \frac{25}{2}$$

or Distance = 1200 km

84. $$50 \times 30 = x \times 20$$

$$\Rightarrow \quad x = 75 \text{ km/h}$$

$$\frac{S_1}{S_2} = \sqrt{\frac{t_2}{t_1}}$$

88.

$$\frac{120}{S_2} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$$S_2 = 160 \text{ km/h}$$

$$S_1 + S_2 = \frac{240}{4} = 60$$

89.

$$S_1 - S_2 = \frac{240}{12} = 20$$

$$S_1 = 40 \text{ m/s} \quad \text{and} \quad S_2 = 20 \text{ m/s}$$

$$S_1 = 40 \times \frac{18}{5} = 144 \text{ km/h}$$

87. Time taken to cross the man $= \dfrac{\text{Length of the faster train}}{\text{Relative speed}}$

$$18 = \frac{x}{15 \times \frac{5}{18}} \quad \Rightarrow \quad x = 75 \text{ m}$$

88. Suppose the total distance be 300 km (LCM of 50 and 60) then in the first case it takes only 5 hours and in the second case it takes 6 hours.

Thus, in 6 hours trains halts for 1 hour.

Therefore in 1 hour train halts for 1/6 hour = 10 m

Alternatively: Difference in speeds = 10 km/h

Faster speed = 60 km/h

$\therefore$ Required time per hour $= \dfrac{10}{60} = \dfrac{1}{6}$ h = 10 min

89. If the speed of faster horse be f_S and that of slower horse be s_S, then

$$f_S + s_S = \frac{50}{1} = 50$$

and

$$\frac{50}{s_S} - \frac{50}{f_S} = \frac{5}{6}$$

Now, you can go through options.

The speed of slower horse is 20 km/h.

Since, $20 + 30 = 50$

and

$$\frac{50}{20} - \frac{50}{30} = \frac{5}{6}$$

90. Let he walked for x hours, then

$$5x + 25(10 - x) = 17 \times 10$$

$$\Rightarrow \quad x = 4$$

$$\therefore \quad 10 - x = 6 \text{ h}$$

Hence, distance travelled by auto $= 25 \times 6 = 150$ km.

91. Let the original speed be x km/h then,

$$\frac{36}{(x-6)} + \frac{36}{(x+6)} = 8$$

Now, you can go though options or solve it as follows

$$\frac{(x+6+x-6)}{(x^2-36)} = \frac{8}{36}$$

$$\Rightarrow \quad x = 12 \quad \text{and} \quad x = -3$$

Thus, the possible value of $x = 12$

$\therefore$ Time taken by faster speed = 2 h

92.

$$\text{Time} = \frac{\text{Distance advanced}}{\text{Relative speed}}$$

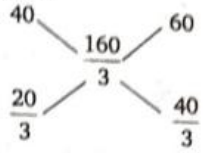
$$2 = \frac{2 \times x}{(30 - x)}$$

$$\Rightarrow \quad x = 15 \text{ km/h}$$

93. Prachi travels only for 3 hours, since half an hour she halts at Lucknow.

Now, the average speed (except the halt) $= \dfrac{160}{3}$ km/h

Therefore, by alligation



$$\Rightarrow \quad 1 : 2$$

Therefore, the ratio of time taken at 40 km/h and at 60 km/h is 1 : 2.

Thus, the distance between Lucknow and Kanpur

$= 2 \times 60 = 120$ km

94. Let the speed of the faster train be f_S and slower train be s_S, then

$$f_S + s_S = \frac{200 + 250}{18} = 25 \text{ m/s}$$

and

$$f_s - s_S = \frac{200 + 250}{60} = \frac{450}{60} = 7.5 \text{ m/s}$$

$$\therefore \quad f_S = 16.25 \text{ m/s}$$

$$= 16.25 \times \frac{18}{5} = 58.5 \text{ km/h}$$

95. Distance covered in 3 minutes $= 3 \times \dfrac{1000}{60} = 50$ m

Now he has to cover (500 + 50) metres in (30 − 3) minutes

$\therefore$ New speed $= \dfrac{550/1000}{27/60} = \dfrac{11}{9}$ km/h

96.

	Anil		Mukesh
Speed →	3	:	4
Time →	4	:	3

But

$$4x - 3x = \frac{1}{2} \text{ h}$$

$$\Rightarrow \quad 4x = 2 \text{ h} \quad \text{and} \quad 3x = 1.5 \text{ h}$$

Now, since Anil doubles the speed so time will be half of the actual time. Hence, new time will be 1 hour.

97. Average speed of Anil and Mukesh $= \dfrac{3x + 4x}{2} = 28$

$$\Rightarrow \quad x = 8$$

$\therefore$ Speed of Sameer $= 3 \times 8 = 24$ km/h

$\therefore$ Distance travelled $= 2 \times 24 = 48$ km

98. Let the speed of X and Y be the x km/h and y km/h respectively. Since they meet after 3 hours, so $x + y = 100$. Since, the faster train takes atleast $3 + 2 = 5$ hours to complete the 300 km journey. Hence, minimum possible speed for the slower train = 40 km/h at which speed it will take 7.5 h to complete the journey. $\left(7.5 = \dfrac{300}{40}\right)$

99. Let the time taken in first third part of the journey be x minutes, then the time required in second third part of the journey is $\frac{3x}{2}$ and in the last third part of the journey time required is $\frac{15x}{8}$.

Therefore, $x + \frac{3x}{2} + \frac{15x}{8} = 350$ min

$\Rightarrow$ $x = 80$ min

100.

Speed	Time	
$\frac{1}{5}\downarrow$	$\frac{1}{4}\uparrow$	$= 15$ min

Therefore usual time $= 4 \times 15 = 60$ min

Now,

Speed	Time	
$\frac{1}{4}\uparrow$	$\frac{1}{5}\downarrow$	$= 12$ min

(Since original time = 60 min)

Therefore he will be $15 + 12 = 27$ minutes early in comparison to the previous day.

101. Let the original speed be s km/h and scheduled time $= t$ hours

and total distance $= D$ km

then $s \times t = \frac{3}{4}D$ ...(i)

and $s \times (t + 3) = D$ ...(ii)

From Eq. (i) and (ii), we get

$$st = \frac{3}{4}[s(t+3)] \Rightarrow t = 9\text{ h}$$

and let $s = 1$ km/h, then $D = 12$ km

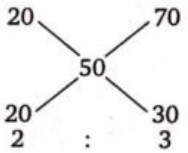
Again, since he doubles his speed after k hours then,

$$s_1t_1 + s_2t_2 = D$$

$$1 \times k + 2 \times (9 - k) = 12$$

$\Rightarrow$ $k = 6$ h

102. By alligation rule



Ratio of time = 2 : 3

$\therefore$ Ratio of distances $= 2 \times 20 : 3 \times 70 = 4 : 21$

Alternatively: $\frac{x}{20} + \frac{y}{70} = \frac{x+y}{50}$

$\Rightarrow$ $\frac{76x + 20y}{1400} = \frac{x+y}{50}$

$\Rightarrow$ $42x = 8y \Rightarrow \frac{x}{y} = \frac{4}{21}$

103. $\frac{\text{(Speed of wind)}}{\text{(Speed of car)}} = \frac{\text{(Time utilised)}}{\text{(Time saved)}}$

$$\frac{332}{x} = \frac{332}{28}$$

$\Rightarrow$ $x = 28$ m/s

104. Time $= \frac{\text{Total distance}}{\text{Relative speed}}$

$$\frac{4.5}{60} = \frac{450/1000}{x} \Rightarrow x = 6\text{ km/h}$$

Relative speed = Speed of car − Speed of man

$$6 = x - 6$$

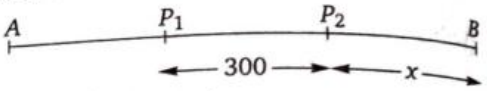
$\Rightarrow$ $x = 12$ km/h

105. $W + R \rightarrow 4$ h 20 min

$W + W \rightarrow 5$ h 20 min

$\therefore R + R \rightarrow 3$ h 20 min

106.



$P_1 \rightarrow$ Place of accident

$P_2 \rightarrow$ Imaginary place of accident

For the distance x

Speed	Time	
$\frac{1}{6}\downarrow$	$\frac{1}{5}\uparrow$	$= 1$ h

Thus, the usual time required for the distance x km is 5×1 hours.

For the distance $(x + 300)$

Speed	Time	
$\frac{1}{6}\downarrow$	$\frac{1}{5}\uparrow$	$= 2$ h

Thus, the usual time required for the distance $(x + 300)$ km $5 \times 2 = 10$ h.

It means he covers 300 km distance in 5 h

$\therefore$ Speed $= \frac{300}{5} = 60$ km/h (normal spe

107. Since, he can cover x km at 60 km/h in 5 h it means $x =$ km.

Therefore, total distance $= (120 + 300 + 300) = 720$ km

108. Circumference means one revolution.

Therefore, distance covered in 10 revolutions $= 300 \times 10 = 30$ m

i.e., 30 metre in 6 seconds.

$\therefore$ Speed of wheel $= \frac{30}{6}$ m/s $= 5$ m/s

$\therefore$ 5 m/s $= 5 \times \frac{18}{5} = 18$ km/h

109. Speed of man in still water $= \frac{S_D + S_U}{2}$

$$= \frac{12 + 8}{2} = 10\text{ km/h}$$

$S_D \rightarrow$ Speed in downstream (= Boat + River)

$S_U \rightarrow$ Speed in upstream (=Boat − River)

110. Speed of water current $= \frac{S_D - S_U}{2} = \frac{21 - 15}{2} = 3$ km/h

111. $S_D = 12$ km/h

$S_U = 5$ km/h

Speed of current $= \frac{12 - 5}{2} = 3.5$ km/h

112. $S_D \rightarrow 15$ km/h
$S_U \rightarrow 9$ km/h

$$\text{Speed of man} = \frac{15+9}{2} = 12\,\text{km/h}$$

113. Let the required distance be D km, then

$$\frac{D}{6} + \frac{D}{4} = \frac{5}{4}$$

$$D\left(\frac{10}{24}\right) = \frac{5}{4}$$

$$D = 3\,\text{km}$$

$\Rightarrow$

114. $\frac{16}{8} + \frac{16}{2} = 10\,\text{h}$

115. Let the downstream speed be D and upstream speed be U, then

$$\frac{48}{U} + \frac{72}{D} = 12$$

and

$$\frac{48}{D} + \frac{72}{U} = 13$$

$\therefore$ $48m + 72n = 12$ ...(i)

and $48n + 72m = 13$ ...(ii)

Solving Eq. (i) and (ii), we get

$$m + n = \frac{5}{24} \quad \text{and} \quad m - n = \frac{1}{24}$$

$\therefore$ $D = 12\,\text{km/h}$ and $U = 8\,\text{km/h}$

$\therefore$ Speed of current = 2 km/h

116.
$$D_S = \frac{9}{2} = 4.5\,\text{km/h}$$

$$U_S = \frac{9}{6} = 1.5\,\text{km/h}$$

$\therefore$ Speed of boat in still water $= \frac{4.5+1.5}{2} = 3\,\text{km/h}$

and Speed of river in still water $= \frac{4.5-1.5}{2} = 1.5\,\text{km/h}$

117. $\frac{D_T}{U_T} = \frac{1}{2} \Rightarrow \frac{D_S}{U_S} = \frac{2}{1} \Rightarrow \frac{B+S}{B-S} = \frac{2}{1} \Rightarrow \frac{B}{S} = \frac{3}{1}$

HINT D_T and U_T are the downstream and upstream times and D_S and U_S are the downstream and upstream speeds. Here we can use componendo and dividendo.

$\because$ $\frac{\text{Speed of boat}}{\text{Speed of stream}} = \frac{3}{1} = \frac{15}{x}$

$\therefore$ Speed of stream = 5 km/h

118. If t_1 and t_2 are the upstream and downstream times. Then time taken in still water is given by

$$\frac{2 \times t_1 \times t_2}{t_1 + t_2} = \frac{2 \times 12 \times 24}{36} = 16\,\text{h}$$

Alternatively: $D = (B+S) \times 12$

and $D = (B-S) \times 24$

where $(B+S)$ is downstream speed
$(B-S)$ is upstream speed

$\Rightarrow$ $\frac{B+S}{B-S} = \frac{2}{1}$

$\Rightarrow$ $\frac{B}{S} = \frac{3}{1}$ (By componendo and dividendo)

Now $D = 4S \times 12 = 48S$

$D = 48S = 16B$ (Distance = Time × Speed)

$\therefore$ Required time = 16 h

119. $\frac{D_S}{U_S} = \frac{B+S}{B-S} = \frac{3}{2}$

where $B \rightarrow$ Speed of boat in still water
$S \rightarrow$ Speed of current/stream

$\Rightarrow$ $\frac{2B}{2S} = \frac{5}{1}$ (By componendo and dividendo)

$\Rightarrow$ $\frac{B}{S} = \frac{5/2}{1/2} \Rightarrow B = \frac{5}{2}$

Average speed of downstream and upstream $= \frac{2 \times 3 \times 2}{3+2} = \frac{12}{5}$

$\therefore$ Required ratio $= \frac{5/2}{12/5} = \frac{25}{24}$

120. Let x be the upstream speed, then the downstream speed will be $(x+3)$.

$\therefore$ $\frac{3}{x} + \frac{3}{x+3} = 3$

$\Rightarrow$ $x^2 + x - 3 = 0$

$\Rightarrow$ $x = \frac{-1+\sqrt{13}}{2}$

$= \frac{-1+3.6}{2} = 1.3\,\text{km/h}$

$\therefore$ $(x+3) = 4.3\,\text{km/h}$

121. Upstream speed of first boat = 8 km/h

Upstream speed of second boat = 4 km/h

$\therefore$ Relative speed = 4 km/h (= 8 − 4)

$\therefore$ Required time $= \frac{10}{4} = 2.5\,\text{h}$

122. Downstream speed = $B + S$ $B \rightarrow$ Speed of boat

Upstream speed = $B - S$ $S \rightarrow$ Speed of stream

$\therefore$ $\frac{120}{B+S} + \frac{120}{B-S} = 15$

$\Rightarrow$ $\frac{B}{B^2 - S^2} = \frac{1}{16}$ ...(i)

Again $\frac{120}{B+2S} + \frac{120}{B-2S} = 24$

$\Rightarrow$ $\frac{B}{B^2 - 4S^2} = \frac{1}{10}$ ...(ii)

From Eq. (i) and (ii), we get

$B = (B^2 - S^2)16$

and $B = (B^2 - 4S^2)10$

$\Rightarrow$ $10B^2 - 40S^2 = 16B^2 - 16S^2$

$\Rightarrow$ $\frac{B^2}{S^2} = 9 \Rightarrow \frac{B}{S} = 3$

$B : S = 3 : 1$

i.e.,

Now, you can go through options or solve by equations. Since now you know the ratio of speeds of boat and stream. The correct choice is (b).

124. Upstream speed $= B - S$

Downstream speed $= B + S$

$$B - S = \frac{15}{5} = 3 \text{ km/h}$$

Again $B = 4S$

$\therefore$ $B - S = 3 = 3S$

$\Rightarrow$ $S = 1$ and $B = 4$ (km/h)

$\therefore$ $B + S = 5$ km/h

$\therefore$ Time during downstream $= \frac{15}{5} = 3$ h

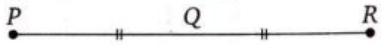
125. $\frac{24}{(5.5 - R)} - \frac{24}{(5.5 + R)} = 5$

$\Rightarrow$ $R = 2.5$ km/h; $R \rightarrow$ Speed of river/current

Again $(B_2 + R) = \frac{24}{4} = 6$

$\Rightarrow$ $(B_2 + 2.5) = 6 \Rightarrow B_2 = 3.5$ km/h

126.



$PQ = QR$

$P \rightarrow Q \rightarrow R$ (7 h)

It means $P \rightarrow Q$ (3.5 h)

Again $\{P \rightarrow Q$ and $Q \rightarrow P\}$ (8 h)

It means $Q \rightarrow P$ (4.5 h)

Therefore $R \rightarrow Q$ (4.5 h)

Thus, from R to P boat will take 9 hours.

HINT $P \rightarrow R$ (Downstream)
$R \rightarrow P$ (Upstream)

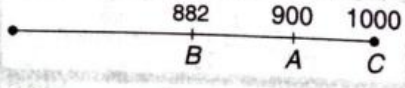
127.

$$\frac{40}{(B - S)} + \frac{55}{(B + S)} = 13$$

$$\frac{30}{(B - S)} + \frac{44}{(B + S)} = 10$$

HINT Go though options for quicker answer and prefer the value which can help in dividing 44 and 55.

Solutions for question number 128 and 129: *B is 2% slow than A and A is 10% slow than C. Therefore, in 1 km race.*

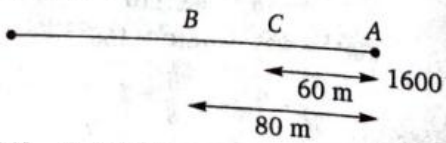


128. Since in 1000 m (1 km) race difference between A and B is 18 m.

So in 2 km race it will be 36 m.

129. In a 1 km race C can give B a start of 118 m. Therefore in a half km race C can give B a start of 59 m.

130. In a 1600 m race



$C \rightarrow 1540$, $B \rightarrow 1520$

The ratio of speeds of $C : B$ is 77 : 76. It means in 77 m race C beats B by 1 m. So, in 400 m race C will beat B by

$$400 \times \frac{1}{77} \text{ m} = 5\frac{15}{77} \text{ m}$$

131.

	Aman	Shakti
Time	190	200
Speed	$20x$	$19x$

$\because$ $20x = 1000$ m

$\Rightarrow$ $x = 50$ m

Again $20x - 19x = x = 50$ m

So, Aman can beat Shakti by 50 m.

132.

	A	:	B	
Speed	7	:	4	
Time	4	:	7	
Distance	4	:	7	(Since distance $\propto$ time)

Now, $7x - 4x = 300$

$3x = 300$

$\Rightarrow$ $x = 100$

$\therefore$ $7x = 700$

Thus, the winning post be 700 m away from the starting point.

133. Ratio of speeds of $A : B = 12 : 11$

and ratio of speeds of $B : C = 8 : 7$

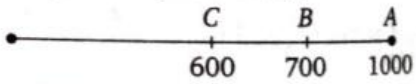
Therefore ratio of speeds of $A : B : C = 96 : 88 : 77$

So in 9600 m race A will beat C by 1900 m.

134. When Ameesha runs 1000 m, then Bipasha runs only 700 m [1000 – (100 + 200)]

when Ameesha runs 1000 m, then Celina runs only 600 m [1000 – (100 + 300)]

Therefore,



Now, since in 700 m race Bipasha beats Celina by 100 m, in 50 m race Bipasha will beat Celina by

$$50 \times \frac{100}{700} = 7.14 \text{ m}$$

Alternatively: Bipasha beats Celina by 14.28% of the distance then in 50 m race Bipasha will beat Celina by 7.14 m.

135.

		Distance	Time
Case I:	Ravi	1000	t_1
	Vinod	960	$t_1 + 19$
Case II:	Ravi	960	t_2
	Vinod	1000	$t_2 + 30$

Therefore, $\frac{1000}{t_1} = \frac{960}{t_2}$ = Speed of Ravi

$\Rightarrow$ $t_1 = \frac{25}{24} t_2$

Also $\frac{960}{t_1 + 19} = \frac{1000}{t_2 + 30}$ = Speed of Vinod

$\therefore$ $(t_2 + 30)\,24 = 25\left[\frac{25}{24} t_2 + 19\right]$

$\Rightarrow$ $49t_2 = 5880$

$t_2 = 120$ s

$\Rightarrow$ Required ratio $= \dfrac{960/t_2}{1000/t_2 + 30} = \dfrac{6}{5}$

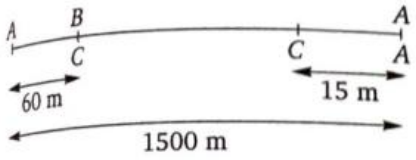
$\therefore$

137. Since, the speeds of Vinay and Versha are in the ratio of 5 : 3 i.e., when Vinay covers 5 rounds, then Versha covers 3 rounds, but first time Vinay and Versha meet when Vinay completes $2\frac{1}{2}$ round and Versha completes $1\frac{1}{2}$ round.

For Vinay to pass Versha 7th time, Vinay would have completed $7 \times 2\frac{1}{2}$ rounds. Since, each round is $1\frac{1}{2}$ km, the distance covered by Vinay is

$$7 \times 2\frac{1}{2} \times 1\frac{1}{2} = 7 \times \frac{5}{2} \times \frac{3}{2} = 26\frac{1}{4} \text{ km}$$

138.



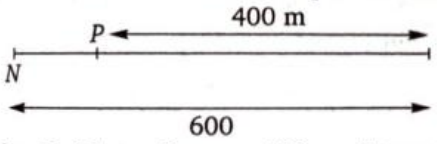
In the same time, when A covers 1500 m, B covers 1440 m and C covers 1425 m.

So, in 1440 m race B can give a start of 15 m.

$\therefore$ In 1500 m race B will give a start of

$$\frac{15}{1440} \times 1500 = 15\frac{5}{8} \text{ m}$$

139. In 600 m race Prabhat can have only start of 120 m. Now since he has more than 200 m start up so he will win the race.



Now, when Prabhat will cover 400 m distance, then in the same time Nishith will cover only 500 m. So, Prabhat will win by 100 m.

140. B is 25% slower than A and C is 20% slower than B.

Therefore in a game of 200 points B can have 150 points and C can have 120 points.

Thus, A can give C 80 points.

141. Since between 11 am and 1 pm and 11 pm and 1 am two hands of a clock coincide only once, each time.

142. Since between 2 am and 3 am (2 pm and 3 pm) and 8 am and 10 am (8 pm and 10 pm) two hands of a clock make 90° angle only 3 times in rest of the each hour two hands make 90° angle 2 times.

143. Since between 5 am and 7 am (5 pm and 7 pm) this happens only once. In rest each of the hours it happens one time.

144. Both (a) and (b) are correct.

Relative speed = Speed of minute-hand − Speed of hour-hand

$$= 6° - \frac{1}{2}° = 5\frac{1}{2}°$$

and $1 \text{ min} - \frac{1}{12} \text{ min} = \frac{11}{12} \text{ min}$

145. $\dfrac{5 \times 30}{11/2} = \dfrac{300}{11} = 27\dfrac{3}{11}$ min = 27 min 16 s

Therefore, required time = 5 : 27 : 16

146. The angle made by minute-hand = $5 \times 30 = 150°$

The angle made by hour-hand = $2 \times 30 + 25 \times \frac{1}{2} = 72.5°$

Hence, required angle = $150 - 72.5 = 77.5°$

147. $\dfrac{90° - 30°}{5.5} = \dfrac{60}{11} \times 2 = \dfrac{120}{11} = 10\dfrac{10}{11}$ minute

$= 10$ min 54 s

$\therefore$ Required time = 3 : 10 : 54

148. $\dfrac{210}{5.5} = \dfrac{210}{11} \times 2 = \dfrac{420}{11} = 38\dfrac{2}{11}$ min = 38 min 11 s

Therefore, required time = 7 : 38 : 11

149. The angle made by minute-hand = 90°

The angle made by hour-hand = $6 \times 30 + 15 \times \frac{1}{2} = 187.5°$

$\therefore$ Required difference = 97.5°

150. $\dfrac{90 - 35}{5.5} = \dfrac{55}{11} \times 2 = 10$ min

So, the required time = 3 : 10 : 00

Again $\dfrac{90 + 35}{5.5} = \dfrac{125}{11} \times 2 = \dfrac{250}{11} = 22\dfrac{8}{11}$ min

$= 22$ min 43 s

LEVEL 2

1.

	Cycle	Auto	Car	
Speed	x	$(5x - 20)$	$5x$	
Time		$(t + 1)$	t	
Distance	120	120	120	(in km)

$\therefore$ $\dfrac{120}{(5x - 20)} - \dfrac{120}{5x} = 1$

$\Rightarrow$ $x^2 - 4x - 96 = 0$

$\Rightarrow$ $x = 12$

$\therefore$ Average speed $= \dfrac{360}{(10 + 3 + 2)} = 24$ km/h

2. Time taken by cycle $= \dfrac{120}{12} = 10$ h

Time taken by auto $= \dfrac{120}{40} = 3$ h

Time taken by car $= \dfrac{120}{60} = 2$ h

Total time = 15 h

3. In last 5 hours she covers 240 km (120 + 120)

4. New time = 3 + 3 + 2 = 8 h

Hence, decrease in time = 7 h

$\therefore$ Percentage change $= \dfrac{7}{15} \times 100 = 46.66\%$

(15 – 8)

5. Time taken to meet Bipasha and Mallika $= \frac{1080}{(60+120)} = 6$ h

So, in 6 hours Bipasha covers 360 km and this 360 km distance Rani covers in $\frac{360}{90} = 4$ h.

Hence, Rani leaves Kolkata 2 hours later than Bipasha *i. e.*, at 8 am. Rani leaves Kolkata.

NOTE The distance 360 covered by Bipasha to meet Mallika can also be calculated by the ratio of their speeds.

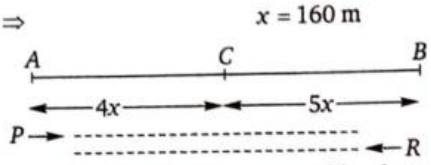
6. Note here the length of the train in which passenger is travelling is not considered since we are concerned with the passenger instead of train. So, the length of the bridge will be directly proportional to the time taken by the passenger respectively.

Therefore, $\frac{t_1}{t_2} = \frac{l_1}{l_2}$ $t \rightarrow$ Time, $l \rightarrow$ Length of bridge

$$\frac{7}{4} = \frac{280}{x}$$

$$\Rightarrow \quad x = 160 \text{ m}$$

7.



Note that the distances covered by them to meet at C are in the direct ratio of their speeds. Therefore

$$AC : BC = 4x : 5x$$

Now, for any particular person (say Pathik) the time required to cover different distances is directly proportional to the different distances. So, time taken by Pathik to cover AC and BC are the ratio of 4 : 5 (excluding staying or halt time at Chandni Chowk).

Thus time required to cover AC is 52 minutes only since he covers BC in 65 minutes.

But since he leaves Chandni Chowk for Bhavnagar at 9 : 27 am *i. e.*, 67 minutes later, when he left Andheri. It means he must have stayed at C for $(67 - 52) = 15$ minutes.

8. Let the length of the train be L metres and speeds of the train Arjun and Srikrishna be R, A and K respectively, then

$$\frac{L}{R-A} = 36 \qquad \ldots(i)$$

and $$\frac{L}{(R+K)} = 24 \qquad \ldots(ii)$$

From eq. (i) and (ii)

$$3(R-A) = 2(R+K)$$

$$\Rightarrow \quad R = 3A + 2K$$

In 30 minutes (*i. e.*, 1800 seconds), the train covers $1800R$ (distance) but the Arjun also covers $1800A$ (distance) in the same time.

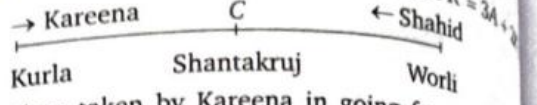
Therefore distance between Arjun and Srikrishna, when the train has just crossed Srikrishna

$$= 1800(R-A) - 24(A+K)$$

$$\therefore \quad \text{Time required} = \frac{1800(R-A) - 24(A+K)}{(A+K)}$$

$$= (3600 - 24) = 3576 \text{ s}$$

9.



(Since $R = 3A + 2K$)

Let the time taken by Kareena in going from K to S is x minutes and the time taken by Shahid in going from Worli to Shantakruj be y min.

Since, the new speed of Kareena is $\frac{2}{3}$, therefore time taken in returning $= \frac{3}{2}x$.

$$\therefore \quad x + \frac{3}{2}x = 120$$

$$\Rightarrow \quad x = 48 \text{ min}$$

but $$x = y$$

Again since the new speed of Shahid is $\frac{4}{3}$, therefore the time taken in returning $= \frac{3}{4}y$.

$$\therefore \quad \text{Total time} = y + \frac{3}{4}y$$

$$= 48 + 36 = 84 \text{ min}$$

10. Time taken to collide the two trains $= \frac{3}{2}$ h

So, in $\frac{3}{2}$ h bird travels $\frac{3}{2} \times 60 = 90$ km

11. Let there be l steps in the escalator and x be the speed (in steps/second) of escalator, then

$$\frac{l}{(5+x)} = 10 \quad \text{and} \quad \frac{l}{(5-x)} = 40$$

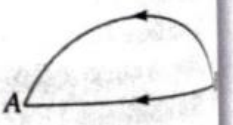
then $$\frac{5+x}{(5-x)} = \frac{40}{10} \quad \Rightarrow \quad x = 3$$

$\therefore$ Number of steps in the escalator $= l = 8 \times 10 = 80$

12. Let the radius be r, then difference in the distance

$$= (\pi r - 2r) = r(\pi - 2)$$

$$= r\left(\frac{22}{7} - 2\right) = 60 \times 3$$



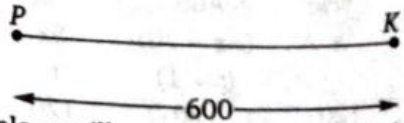
$$\Rightarrow \quad 2r = 315 \text{ m}$$

[$\pi r \rightarrow$ semiperimeter and $2r \rightarrow$ diameter]

13. Time taken by trains to collide $= \frac{560}{70} = 8$ h

In 8 h sparrow will cover $8 \times 80 = 640$ km

14.



In 18 h plane will cover $18 \times 120 = 2160$ km

Now, $2160 = (600 \times 2) + 600 + 360$

So, the plane will be 360 km away from Kargil it means it will be 240 km (600 – 360) away from Pukhwara.

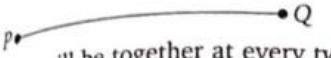
15.

	First hour	Second hour	Third hour
Initial speed	x	$3x$	$2x$
New speed	$3x$	$3x$	$3x$

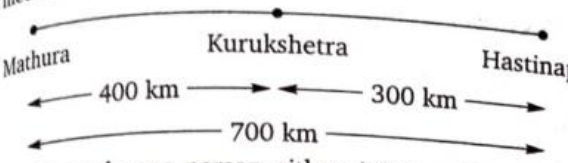
$\therefore$ Percentage increase in speed $= \frac{3x}{6x} \times 100 = 50\%$

Since speed is increased by (50%) $\frac{1}{2}$.

Therefore, time will reduce by (33.33%) $\frac{1}{3}$.

16. 

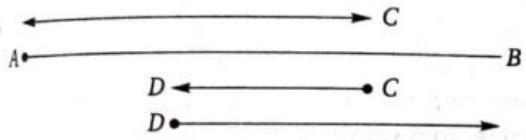
They will be together at every two hours. Therefore in 12 h they will be $(6 + 1) = 7$ times together at P and they will never meet altogether at Q.

17. 

Consider only one person either Arjun or Srikrishna since their speed is same and move together.

Now, the distance covered by Arjun and Abhimanyu is in the ratio of their speeds. So, Arjun will cover 500 km to meet Abhimanyu and thus Arjun has to return back 100 km for Kurukshetra. Therefore, Arjun will cover total 600 km distance.

18. Total time $= \frac{600}{25} = 24$ h

19. 

A is the starting point of journey.

B is the destination.

C → where Salman has got off.

D → where Priyanka picks up Akshay

Let $AD = l$ and $BC = k$ and $CD = x$

then $$\frac{CD + DB}{BC} = \frac{50}{10}$$

$$\frac{2x + k}{k} = \frac{5}{1}$$

$\Rightarrow$ $$\frac{x}{k} = \frac{2}{1}$$

Again $$\frac{AC + CD}{AD} = \frac{50}{10}$$

$$\frac{2x + l}{l} = \frac{5}{1}$$

$\Rightarrow$ $$\frac{x}{l} = \frac{2}{1}$$

$\Rightarrow$ $$x = 2k = 2l \quad \text{or} \quad k = l = \frac{x}{2}$$

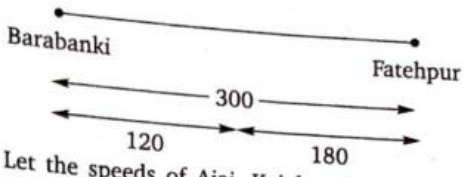
$\therefore$ $$k + x + l = 120$$

$\Rightarrow$ $k = 30$ km, $x = 60$ km and $l = 30$ km

Total distance travelled $= AC + CD + DB$

$= l + x + x + x + k = 240$ km

$\therefore$ Time (required) $= \frac{240}{50} = 4.8$ h

20. 

Let the speeds of Ajai, Kajol and Shahrukh be x, y and z respectively, then

$$\frac{y}{x} = \frac{180}{120} \Rightarrow x = \frac{2y}{3}$$

Note Kajol is faster since she covers 180 km while Ajai covers only 120 km in the same time.

Shahrukh meets Kajol 1.5 hours after Shahrukh himself starts and 2.5 hours after Kajol starts.

Hence, $$2.5y + 1.5z = 300$$

$\Rightarrow$ $$z = \frac{600 - 5y}{3}$$

Since $z \geq (y + 20) \Rightarrow \frac{600 - 5y}{3} \geq (y + 20)$

$\Rightarrow$ $$y \leq 67.5$$

or $$x \leq 45 \text{ km/h}$$

21. Let t be the time after Kajol starts, when she meets Ajai, then

$$t = \frac{300}{(x + y)}$$

This should be less than 2.5 or $(x + y) > 120$

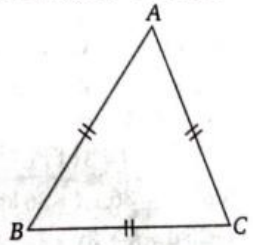
Since $$y = \frac{3x}{2} \Rightarrow y > 72$$

This $(y > 72)$ is greater than 67.5 km/h and hence Shahrukh will always overtake Ajai before he meets Kajol.

22. Speed of Raghupati $(R_P) = 60$ km/h

Speed of Raghav $(R_V) = 36$ km/h

Speed of Raja Ram $(RR) = 18$ km/h



$AB = AC = BC$

Time taken to cover AB by (RR) is 2 hours

$\therefore$ Time taken to cover AB by Raghav is 1 hour

$\therefore$ Time taken to cover AB by Raghupati = 36 min

$$\left(t_{RP} : t_{RV} : t_{RR} = \frac{1}{S_{RP}} : \frac{1}{S_{RV}} : \frac{1}{S_{RR}}\right)$$

$t \rightarrow$ Time, $S \rightarrow$ Speed

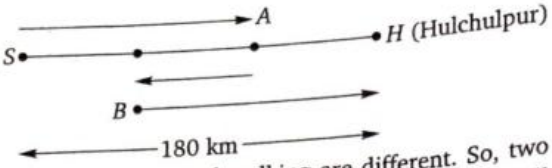
$AB = 2 \times 18 = 36$ km

23. Time $= \frac{3 \times 36}{60} = \frac{9}{5}$ h $= 1$ h 48 min

24. Distance from Bareilley $= \frac{60}{(60 + 18)} \times 36$

$= \frac{360}{13} = 27\frac{9}{13}$ km

25.



Since the speed of bike and walking are different. So, two people partially travelled by bike and rest by walking since all the three persons take equal time to reach the destination. It means initially Mohan will carry either Namit or Pranav to a point A, then this person reach to H by walking and Mohan return to B where he will pick up the third person and reach at H at the same time as the second person.

Let $SB = k$, $AB = x$ and $AH = l$

Now, $$\frac{SA + AB}{SB} = \frac{36}{6}$$

$$\frac{2x + k}{k} = \frac{6}{1}$$

$$\Rightarrow \quad \frac{x}{k} = \frac{5}{2}$$

and $$\frac{AB + BH}{AH} = \frac{36}{6}$$

$$\Rightarrow \quad \frac{2x + l}{l} = \frac{6}{1}$$

$$\Rightarrow \quad \frac{x}{l} = \frac{5}{2}$$

$$\therefore \quad x : k : l = 5 : 2 : 2$$

$$\Rightarrow \quad x + k + l = 180$$

$$\Rightarrow \quad x = 100, \quad k = 40 \quad \text{and} \quad l = 40 \text{ km}$$

Total distance travelled by bike $= SA + AB + BH$

$$= k + 3x + l = 380 \text{ km}$$

26.

$$\frac{2x + k}{k} = \frac{42}{6} = \frac{7}{1}$$

$$\Rightarrow \quad \frac{x}{k} = \frac{3}{1}$$

Similarly $$\frac{x}{l} = \frac{3}{1}$$

$$\therefore \quad x : k : l = 3 : 1 : 1$$

$$\therefore \quad x = 108, \quad k = 36, \quad l = 36 \text{ km}$$

Total distance travelled $= k + 3x + l = 396$ km

$$\therefore \quad \text{Required time} = \frac{396}{42} = 9\frac{3}{7} \text{ h}$$

27. Let the buses leave from both the stations at time intervals of T, then the distance between any two consecutive buses coming opposite to me = the distance betwen any two consecutive buses coming in the same direction as me $= VT$.
(where V is the velocity of the buses)
Let the speed of walking be W, then

$$\frac{VT}{V + W} = 20 \quad \text{and} \quad \frac{VT}{V - W} = 30$$

$$\therefore \quad \frac{V + W}{V - W} = \frac{30}{20} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{V}{W} = \frac{5}{1}$$

$$\therefore \quad \frac{VT}{V + W} = 20$$

$$\Rightarrow \quad \frac{5}{6} \times T = 20$$

$$\Rightarrow \quad T = 24 \text{ min}$$

28. Time taken by Abhinav = 36 h

Ideal time required by Abhinav $= \frac{600}{25} = 24$ h

It means Abhinav rests for $(36 - 24) = 12$ h

Now, the required time for Brijesh $= \frac{600}{30} = 20$ h

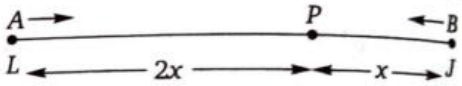
But Brijesh utilised those 12 hours in which Abhinav rest he needs only $(20 - 12) = 8$ hours extra.

Thus, the total time taken by Brijesh $= 36 + 8 = 44$ h

29. Downstream (Steamer) = 40 min
Downstream (Boat) = 60 min
Upstream (Steamer) = 60 min
Upstream (Boat) = 90 min

Required time $= 40 + 30 + 45 = 115$ min

30.



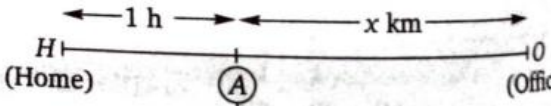
These two trains meet only at P and L i. e., there are only points.

31. For the first meeting they have to cover only $2x + x$ distance and for the further meeting for each next meet they have to cover $6x$ distance together.

Distance covered by A	$2x$	$2x$	$4x$	
Distance covered by B	x	$4x$	$2x$	
Point of meeting	P	L	P	
Total distance travelled	$3x$	$6x$	$6x$	

when A and B meet at P for the third time A goes $10x$ and goes $11x$.
Thus, the required ratio = 10 : 11

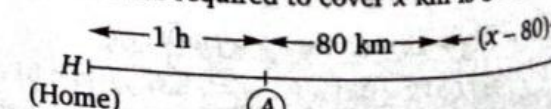
32.



1 hr

Speed	Time
$\frac{1}{6}\downarrow$	$\frac{1}{5}\uparrow = 36$ min

$\Rightarrow$ actual time required to cover x km is $5 \times 36 = 180$ min



1 hr

Speed	Time
$\frac{1}{6}\downarrow$	$\frac{1}{5}\uparrow = 20$

$\Rightarrow$ actual time required for $(x - 80)$ km $= 5 \times 20 =$
It means he can move $= x - (x - 80) = 80$ km in

$$(180 - 80) = 80 \text{ min}$$

It means his actual speed = 60 km/h

Thus, the total distance from his home to his office

$$= 60 \times 1 + 60 \times 3 = 240 \text{ km}$$

NOTE Since 1 hour he lost at the place of accident, so the actual delay due to reduced speed is always 1 hour less than it is found to be in both the cases.
It means due to reduced speed he becomes late only 36 minutes and 20 minutes in respective cases.

33. $$\frac{\text{Speed of wind (Sound)}}{\text{Relative speed of soldier and terrorist}} = \frac{\text{Time utilised}}{\text{Difference in time}}$$

$$\frac{1188}{x} = \frac{330}{5}$$

$$\Rightarrow \quad x = 18 \text{ km/h}$$

34. In case of increasing gap between two objects.

$$\frac{\text{Speed of sound}}{\text{Speed of tiger}} = \frac{\text{Time utilised}}{\text{Difference in time}}$$

$$\frac{1195.2}{x} = \frac{83}{7}$$

$$\Rightarrow \quad x = 100.8 \text{ km/h}$$

35. In 20 minutes the difference between man and his son

$$= 20 \times 20 = 400 \text{ m}$$

Distance travelled by dog when he goes towards son

$$= \frac{400}{40} \times 60$$

$$= 600 \text{ m and time required is 10 minutes}$$

In 10 minutes the remaining difference between man and son.

$$400 - (20 \times 10) = 200 \text{ m}$$

NOTE Relative speed of dog with child is 40 km/h and the same with man is 100 km/h.

Time taken by dog to meet the man $= \frac{200}{100} = 2 \text{ min}$

In 2 min the remaining distance between child and man

$$200 - (2 \times 20) = 160 \text{ m}$$

Now, the time taken by dog to meet the child again

$$= \frac{160}{40} = 4 \text{ min}$$

In 4 minutes he covers $4 \times 60 = 240$ m distance while going towards the son.
In 4 minute the remaining distance between man and child

$$= 160 - (4 \times 20) = 80 \text{ m}$$

Time required by dog to meet man once again

$$= \frac{80}{100} = 0.8 \text{ min}$$

In 0.8 min remaining distance between man and child

$$= 80 - (0.8 \times 20) = 64 \text{ m}$$

Now, time taken by dog to meet the child again

$$= \frac{64}{40} \times \frac{8}{5} \text{ min}$$

$\therefore$ Distance travelled by dog $= \frac{8}{5} \times 60 = 96$ m

Thus, we can observe that every next time dog just go 2/5th of the previous distance to meet the child in the direction of child.

So, we can calculate the total distance covered by dog in the direction of child with the help of GP formula.

Here, first term $(a) = 600$ and common ratio $(r) = \frac{2}{5}$

$\because$ Sum of the infinite GP $= \frac{a}{1-r}$

$$= \frac{600}{\left(1 - \frac{2}{5}\right)} = \frac{600}{3/5} = 1000 \text{ m}$$

36. Let Amarnath express takes x hours, then Gorakhnath express takes $(x - 2)$ hours.

$$\therefore \quad \frac{1}{x} + \frac{1}{(x-2)} = \frac{60}{80}$$

$$\Rightarrow \quad x = 4 \text{ h}$$

37. Distance travelled by them in first hour = 12 km

Distance travelled by them in second hour = 13 km

Distance travelled by them in third hour = 14 km and so on

Thus, in 9 hours they will cover exactly 144 km and in 9 h each will cover half-half the total distance.

$(8 \times 9 = 72$ and $4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 72)$

38. Speed of tiger = 40 m/min

Speed of deer = 20 m/min

Relative speed = 40 − 20 = 20 m/min

Difference in distances = $50 \times 8 = 400$ m

$\therefore$ Time taken in overtaking (or catching) $= \frac{400}{20} = 20$ min

$\therefore$ Distance travelled in 20 min = $20 \times 40 = 800$ m

39. The sum of their speeds $= \frac{615}{15} = 43$ km/h

Notice that they are actually exchanging their speeds. Only then they can arrive at the same time at their respective destinations. It means the difference in speeds is 3 km/h.

Thus, $\quad x + (x + 3) = 43$

$\Rightarrow \quad x = 20 \quad$ and $\quad x + 3 = 23$.

The concept is very similar to the case when after meeting each other they returned to their own places of departure. It can be solved through option also.

40. Let Pele covers x km in 1 hour. So Maradona takes (2 h − 40 min) = 1 h 20 min to cover x km. Let speed of Maradona and Pele be M and P respectively than

$$x = M \times \frac{4}{3} \quad \text{and} \quad x = P \times 1$$

$$\Rightarrow \quad \frac{M}{P} = \frac{3}{4}$$

Again $\quad \frac{300}{M} - \frac{300}{P} = 1$

$$\Rightarrow \quad \frac{300}{3k} - \frac{300}{4k} = 1$$

$$\Rightarrow \quad k = 25$$

$$\Rightarrow \quad M = 3k = 75 \text{ km/h}$$

and $\quad P = 4k = 100 \text{ km/h}$

(Through option it is very easy to solve.)

41. Initial speed of police = 10 m/s
Increased speed of police = 20 m/s
Speed of thief = 15 m/s
Initial difference between thief and police = 250 m
After 5 seconds difference between thief and police

$$= 250 - (5 \times 10) = 200 \text{ m}$$

After 10 seconds more the difference between thief and police
$= 200 + (5 \times 10) = 250$ m.
Now, the time required by police to catch the thief

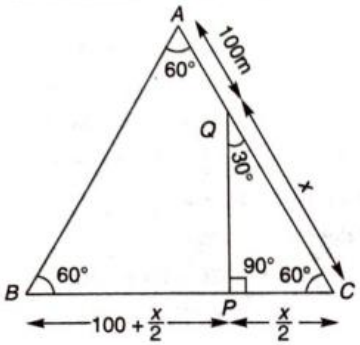
$$= \frac{250}{5} = 50 \text{ s}$$

Distance travelled $= 50 \times 20 = 1000$ m

Total time $= 50 + 15 = 65$ s

Total distance $= 1000 + (15 \times 10) = 1150$ m

42.



$$\frac{100 + \frac{x}{2}}{100} = \frac{100 + x}{(100 + x - 150)}$$

$$= \frac{\text{Speed of Bajrang}}{\text{Speed of Angad}}$$

$$\frac{200 + x}{200} = \frac{(100 + x)}{(x - 50)}$$

$$\Rightarrow \quad (200 + x)(x - 50) = 200(100 + x)$$

$$\Rightarrow \quad x^2 + 150x - 10000 = 20000 + 200x$$

$$\Rightarrow \quad x^2 - 50x - 30000 = 0$$

$$\Rightarrow \quad (x - 200)(x + 150) = 0$$

$$\Rightarrow \quad x = 200 \text{ km}$$

Therefore distance between Ayodhya and Banaras is 300 km since $AB = BC = AC$.
(With the help of trigonometry we can find the value of PC in terms of x i. e., $\cos 60° = \frac{PC}{QC} = \frac{1}{2}$. Hence $PC = \frac{x}{2}$)

43. Basically they will exchange their speeds just after half of the time required for the whole journey. It means after covering 210 km distance they will exchange their speeds.
Check it out graphically for more clarification.

44. The ratio of speeds
= The ratio of distances, when time is constant
∴ The ratio of distances covered by leopard to the tiger =
Again, ratio of rounds made by leopard to the tiger =
Hence, leopard makes 48 rounds, when tiger makes rounds.

45. Length of $DC = \frac{6000}{13}$ (for this, refer geometry section)

Total distance covered in the returning by Jai

$$= AD + CD$$

$$= \frac{2500}{13} + \frac{6000}{13} = \frac{8500}{13} \text{ km}$$

$$\text{Required time} = \frac{8500/13}{500/13} = 17 \text{ h}$$

Total distance covered by Jaya while returning

$$= BD + DC$$

$$= \frac{14400}{13} + \frac{6000}{13}$$

$$\therefore \quad \text{Required time} = \frac{20400/13}{1200/13} = 17$$

Hence, both will reach at the same time.
Alternatively: Since the ratio of speeds is same as the distances. So, they will take same time to reach the home.

46. The distance of route $ADC = \frac{8500}{13}$

and the distance of route $BNC = 1300$

and the time taken by Jai is $\frac{8500/13}{500/13} = 17$ h

and the time taken by Jaya is $\frac{1300}{1200/13} = \frac{169}{12}$ h $= 14\frac{1}{12}$ h

$= 14$ h 05 min

Hence, option (c) is correct.

47. Time saved in percentage $= \frac{175}{1020} \times 100 = 17.15\%$

48. Husband takes 17 hours and she takes 14 h 05 min +

= 17 h 05 min

So, she becomes late by 05 min than her husband.

49. $x^2 + (x + 100)^2 = (500)^2$ (Using Pythagorus theorem)

$$\Rightarrow \quad x = 300 \text{ km}$$

Now, let they change their speeds after t_1 hours and the rest time is t_2 then

$$30t_1 + 40t_2 = 800$$

$$40t_1 + 30t_2 = 900$$

Solving Eq. (i) and (ii), we get

$$t_1 = \frac{120}{7} \quad \text{and} \quad t_2 = \frac{50}{7}$$

50. Since it moves only one radian on every path and it has move 2π radian to reach directly eastward. Hence, it has run on more than 6 paths i. e., the last path is 7th one.
(∵ $n \times 1$ radian $\geq 2\pi$ radian)

$$\Rightarrow \quad n \geq 2\pi$$

$$n = 7, \quad \text{for interger values}$$

or

Hence, option (c) is correct.

51. Since it stops directly eastward of the shop so the total distance covered so for

$$= 7 + (1 + 2 + 3 + 4 + 5 + 6 + 2) = 30 \text{ km}$$

NOTE Total radial movement = 7 km
Again on the last path it will move only 2 km.

Actually it has to cover total 2π radian distance but on 6 paths it covers only 6 radian hence the remaining distance which will be covered on the 7th path *i. e.*, $2\pi - 6$

$$= 2 \times \frac{22}{7} - 6 = \frac{2}{7} \text{ radian}$$

But, the radius of the last path (*i. e.*, P_7) = 7 km

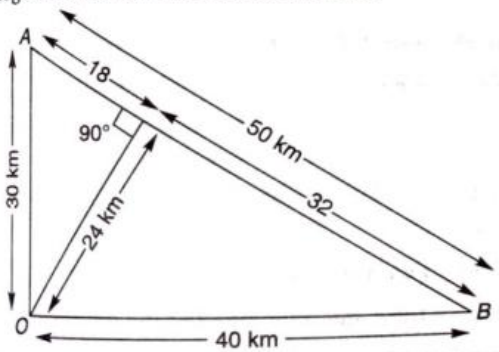
Hence, the distance covered in km $= \frac{2}{7} \times 7 = 2$ km

$$\left(\theta = \frac{\text{Arc}}{\text{Radius}}\right)$$

Thus, on the last path it moves only 2 km. Hence, (a) is the correct choice.

52. The ratio of distance covered on P_2 and $P_7 = \frac{2}{2} = \frac{1}{1}$

53. Since it is clear from the statement itself that ΔAOB is a right angle triangle and further OP must be perpendicular to AB then we can find that $AO = 30$ km and $BO = 40$ km by using Pythogorus theorem and its corrollaries.



HINT

$$OP^2 = OA^2 - AP^2$$

$$\therefore \quad OP^2 = 900 - x^2$$

and $$OB^2 = OP^2 + BP^2$$

$$\therefore \quad OB^2 = 900 - x^2 + 1024$$

and $$AB^2 = OA^2 + OB^2$$

$$\therefore \quad (x + 32)^2 = 900 + 900 - x^2 + 1024$$

$$\Rightarrow \quad x = 18 \text{ km}$$

Hence, $AP = 18$, $OA = 30$ and $OB = 40$ and $OP = 24$ km

Now, since jackal and cat reaches A at the same time, so the ratio of speeds = ratio of distances covered by them.

$$\therefore \quad \frac{\text{Speed of jackal}}{\text{Speed of cat}} = \frac{30}{18} = \frac{5}{3}$$

54. Again, since jackal and train both arrive at A at the same time and let the train was x km away from A, before entering into the tunnels, *i.e.*, when it makes a whistle then the ratio of distances covered by train and jackal.

$$= \frac{x}{30} = \frac{x + 50}{40}$$

$$\Rightarrow \quad x = 150 \text{ km}$$

Thus, the ratio of speeds of Jackal is to train is 1 : 5.

55. Since, when the train arrive at A, the jackal can move 30 km. So, at the time when train is at A the jackal will cover 6 km from P on PA in addition to 24 km at OP. Now, the rest distance at AP is 12 km this remaining distance will be covered by train and jackal according to their respective speeds.

So, distance covered by train $= 12 \times \frac{5}{6} = 10$ km

and distance covered by jackal $= 12 \times \frac{1}{6} = 2$ km

Hence, jackal will meet with train at M_1 which is 10 km away from A (inside AB).

NOTE It can be solved using options in lesser time.

56. It is obvious from the path of cat that if cat moves in the POA direction it will never meet with accident and now jackal follows the path OPB. Again when the train is at A then jackal will cover 30 km (*i. e.*, 24 (OP) + 6 km on PB).
So, the ratio of distances covered by jackal is to train = ratio of their respective speeds.
Now let the jackal and train meet each other at AB, $(6 + x)$ km away from P towards B, then

$$\frac{x}{24 + x} = \frac{1}{5}$$

$$\Rightarrow \quad 4x = 24 \quad \Rightarrow \quad x = 6$$

Hence, train meets with jackal at $(18 + 6 + 6) = 30$ km away from A.

Alternatively: $\frac{150 + 18 + 6 + x}{30 + x} = \frac{5}{1} \quad \Rightarrow \quad x = 6$

Hence, 18 + 6 + 6 = 30 km.
Thus, option (b) is correct.

57. The ratio of time taken by cat and jackal $= \frac{72/3}{96/5} = \frac{5}{4}$

Hence, option (c) is correct.

58. $$(6 - x) = (8 - 1.5x)$$

$$\Rightarrow \quad x = 4 \text{ cm}$$

So, it will take 4 hours to burn in such a way that they remain equal in length.

59. Total distance covered by them when they meet = $2W$

and $$\text{Total time} = \frac{2W}{b_1 + b_2}$$

$$\therefore \quad d_1 = \frac{2W}{(b_1 + b_2)} b_1 \quad \text{and} \quad d_2 = \frac{2W}{(b_1 + b_2)} b_2$$

60. Let the speed of boat be B and that of river be R. In 12 minutes the distance between boat and hat

$$= 12(B - R) + 12R = 12B$$

Now time taken by boat to reach to the hat

$$= \frac{12B}{(B + R) - R} = 12 \text{ min}$$

Total time = 24 min

In 24 minutes had flown off = 3 km

$\therefore \quad \frac{24}{60} \times R = 3$

$\Rightarrow \quad R = 7.5$ km/h

61. Akbar meets Birbal once $= \frac{500}{20 - 15} = 100$ s

Birbal meets Chanakya once $= \frac{500}{20 + 25} = 11\frac{1}{9}$ s

Akbar meets Chanakya once $= \frac{500}{15 + 25} = 12.5$ s

62. Time taken by them to meet $= \frac{600}{30 - 20} = 60$ s

Time taken to meet 5th time $= 5 \times 60 = 300$ s

Total duration of race $= \frac{3000}{30} = 100$ s

So, they will not meet 5th time in the race of 3000 metre.

63. Length of the track $= 2 \times \frac{22}{7} \times 175 = 1100$ m

Distance to be covered for the first meeting = 550 m

$$\text{Speed of Akkal} = \frac{1100}{100} = 11 \text{ m/s}$$

$$\text{Speed of Bakkal} = \frac{1100}{50} = 22 \text{ m/s}$$

Time taken from the start of the first meeting

$$= \frac{550}{(11 + 22)} = \frac{50}{3} \text{ s}$$

Time taken for Akkal and Bakkal to meet again at Love point = LCM of times taken by them to go around the track once.

= LCM of $\frac{1100}{11}$ and $\frac{1100}{22}$

= LCM of 100 and 50

= 100 s

So, the total required time $= \frac{50}{3} + 100 + 100$

$$= \frac{650}{3} = 216\frac{2}{3} \text{ s}$$

64. Since both rest for 6 seconds so when B is just about to start the journey A reaches there at the shallow end so they meet at they shallow end.

65. B runs around the track in 10 min.

i. e., Speed of B = 10 min per round

$\therefore$ A beats B by 1 round

Time taken by A to complete 4 rounds

= Time taken by B to complete 3 rounds

= 30 min

$\therefore$ A's speed $= \frac{30}{4}$ min per round

= 7.5 min per round

Hence, if the race is only of one round A's time over the course = 7 min 30 sec.

66. The ratio of speeds of $A, B, C = \frac{10}{49} : \frac{9}{50} : \frac{8}{51}$

Hence, A is the fastest.

67. Speed of this car $= \frac{400 + 200}{20} \times \frac{18}{5}$ km/h

= 108 km/h

68. The speeds of two persons is 108 km/h and 75 km/h. The first person covers 1080 km in 10 hours and thus he makes rounds. Thus, he will pass over another person 12 times any one of the direction.

69. Angle between two hands at 3 : 10 am

$= (90 + 5) - 60 = 35°$

So, the required angle = 70°, after 3 : 10 am

Total time required to make 70° angle when minute-hand ahead of hour-hand.

$$= \frac{90 + 70}{11/2} = \frac{320}{11} \text{ min}$$

So, at 3 h $\frac{320}{11}$ min the required angle will be formed.

Alternatively: Check through options.

70. **For the first watch:** When a watch creates the differe of 12 hours, it shows correct time.

So to create the difference of 12 h required time

$$= \frac{60 \times 12}{24} = 30 \text{ days}$$

For the second watch: To create the difference of 12 required time

$$= \frac{30 \times 12}{24} = 15 \text{ days}$$

So, after 30 days at the same time both watches show correct time.

HINT Take the LCM of 30 and 15.

71. To show the same time together the difference between watches must be 12 h.

Now, since they create 3 min differnce in 1 h

So they will create 12 h differnce in $\frac{1}{3} \times \frac{12 \times 60}{24}$

= 10 days later

72. To show the correct time again, watch must create difference. (Since in one round hour-hand covers 24 h.)

So, the required time $= \frac{4}{3} \times \frac{60 \times 24}{24} = 80$ day

73. $(n + 1)$ times in n days

74. Actually the watch gains (12 + 16)

= 28 min in $7 \times 24 \times 60$ min.

Thus, it gains 1 min in 360 minutes.

Therefore, it will gain (12 + 8) min in $\frac{20 \times 360}{60 \times 24} = 5$ da

Hence, (b) is the correct choice.

75. Actually they create a difference of 3 min per hour and the two watches are showing a difference of 66 minutes. Thus, they must have been corrected 22 hours earlier.
Now, the correct time can be found by comparing any one of the watch.
Since, second watch gains 1 min in 1 hour so it will must show 22 min extra than the correct time in 22 hours.
Hence, the correct time can be found by subtracting 22 min from 10 : 06.
Hence, (d) is the correct answer.

NOTE For quick answer go through options.

76. Incorrect watch covers 1452 min in 1440 min

So, it will cover 1 min in $\frac{1440}{1452}$ min

Therefore it will cover 4840 min in $\frac{1440}{1452} \times 4840$

$= 4800$ min

$= 80$ h

Therefore 80 h = 3 days and 8 h.

77. You must know that a correct watch coincide just after $65\frac{5}{11}$ min.

Therefore in every $65\frac{5}{11}$ hours the watch gains $\frac{2}{11}$.

Hence, in 24 hours it will gain $\frac{2}{11} \times \frac{11}{720} \times 24 \times 60 = 4$ min

78. In 72 hours my watch gains $(8 + 7) = 15$ min. To show the correct time watch must gain 8 minutes.
Since the watch gains 15 min in 72×60 min.
Therefore, the watch will gain 8 min in $\frac{72 \times 60 \times 8}{15}$ min

$= \frac{72 \times 60 \times 8}{15} = 38$ h 24 min

Hence, (a) is the correct choice.

80. To exchange the position both hands to cover 360° together.
In one minute, hour-hand moves $\frac{1°}{2}$ and in one minute, minute-hand moves 6°. Let the required time be t min, then

$$6t + \frac{1}{2}t = 360$$

$$\Rightarrow \quad t = \frac{360}{13} \times 2 = \frac{720}{13} = 55\frac{5}{13} \text{ min}$$

□□□

8 TIME AND WORK

This chapter is one of the easiest chapter for the students. Even an average student can perform better than in other chapters. There is basically one concept is involved in this chapter *i.e.,* concept of efficiency. So most of the problems are very similar in their basic characteristic. Almost every aptitude exam ask the problems from this chapter. On an average 2-3 problems from this chapter have been asked in past years in CAT.

As it is very clear to all of us that the work is directly related with time. As one can say if a particular person or machine works for more time then more work will be done and if it devotes less time then it yields less work *i.e.,* output of a machine or person is directly proportional to time, provided that he/she maintains his/her efficiency during the work.

CONCEPT OF EFFICIENCY

Suppose a person can complete a particular work in 2 days then we can say that each day he does half of the work or 50% work each day. Thus it is clear that his efficiency is 50% per day. Efficiency is generally considered with respect to the time. The time can be calculated either in days, hours, minutes or months etc. So if a person completes his work in 4 days, then his efficiency (per day) is 25%. Since each day he works 1/4th of the total work (*i.e.,* 25% of the total work).

I would like to mention that the calculation of percentage and conversion of ratios and fractions into percentage and vice versa is the prerequisite for this chapter.

Now, if a person can complete a work in n days then his one day's work $= \frac{1}{n}$

and this one day's work in terms of percentage is called his efficiency.

Also if a person can complete $\frac{1}{n}$ work in one day, then he can complete the whole work in n days.

Relation between work of 1 unit of time and percentage efficiency.

A person can complete his work in n days, then his one day's work $= \frac{1}{n}$, his percentage efficiency $= \frac{1}{n} \times 100$

No. of days/hours etc. required to complete the whole work	Work of 1 day/hour	Percentage efficiency
n	$1/n$	$100/n$
1	1/1	100%
2	1/2	50%
3	1/3	$33.33\% = 33\frac{1}{3}\%$
4	1/4	25%
5	1/5	20%
6	1/6	$16.66\% = 16\frac{2}{3}\%$
7	1/7	$14.28\% = 14\frac{2}{7}\%$
8	1/8	12.5%
9	1/9	$11.11\% = 11\frac{1}{9}\%$
10	1/10	10%

This table is very similar to the percentage fraction table given in the chapter of percentage. This table just manifests as a model for efficiency conversion.

Basically for faster and smarter calculation you have to have your percentage calculation very smart.

All the problems of this chapter can be solved through two methods :

1. Unitary method
2. Percentage efficiency

Unitary method is generally obsolete in respect to high level aptitude exam of CAT since it involves typical calculation of LCM each and every time. But when the problems are solved through percentage efficiency it becomes inevitable to save the time which in turn helps to do some more problems within the stipulated time. I admit that initially it might be difficult to solve for those students who are not so good and confident in percentage and fraction calculation, but a little bit of exercise Practice will yield an unexpected result in quicker calculations. Now I have some good examples to show you both the methods of solving the same problems.

EXAMPLE 1 *A can do a job in 12 days and B can do the same job in 6 days, in how many days working together they can complete the work?*

SOLUTION

$$A\text{'s 1 day's work} = \frac{1}{12}$$

$$B\text{'s 1 day's work} = \frac{1}{6}$$

$$\therefore (A+B)\text{'s 1 day's work} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$$

$$\therefore \text{Time taken by both to finish the whole work} = \frac{1}{1/4} = 4 \text{ days}$$

Alternatively :

$$\text{efficiency of } A = \frac{100}{12} = 8.33\%$$

and

$$\text{efficiency of } B = \frac{100}{6} = 16.66\%$$

Combined efficiency of A and B both $= 8.33 + 16.66 = 25\%$

$\therefore$ Time taken by both to finish the work (working together)

$$= \frac{100}{25} = 4 \text{ days}$$

As per my experience, I have found that in this chapter only selected (numerals) numbers are always used and thus there are almost 20 – 25 numbers are frequently used. So one can very easily remember (and calculate) the percentage efficiency but with different combinations to calculate the LCM becomes a very tedious job. Still you can choose your own method, which is comfortable to you.

EXAMPLE 2 *A can do a job in 10 days, B can do the same job in 12 days and C can do the same job in 15 days. In how many days they will finish the work together?*

SOLUTION

$$A\text{'s 1 day's work} = \frac{1}{10}$$

$$B\text{'s 1 day's work} = \frac{1}{12}$$

$$C\text{'s 1 day's work} = \frac{1}{15}$$

$$(A+B+C)\text{'s one day's work} = \frac{1}{10} + \frac{1}{12} + \frac{1}{15} = \frac{15}{60} = \frac{1}{4}$$

Since they can complete $\frac{1}{4}$ work in 1 day. So they will finish the whole work in $\frac{1}{1/4} = 4$ days.

Alternatively : A's efficiency = 10%

B's efficiency = 8.33%

C's efficiency = 6.66%

Combined efficiency of A, B and $C = 10 + 8.33 + 6.66 = 25\%$

Hence, they will take $\frac{100}{25} = 4$ days to finish the job working together

(Since in one day they complete 25% work)

RELATION BETWEEN EFFICIENCY AND TIME

Efficiency is inversely proportional to the time (*i.e.*, number of days, hours, minutes) etc.

For example if A is twice efficient as B, it means, A takes half the time to finish the same job as B requires working alone.

EXAMPLE 1 *A takes 16 days to finish a job alone, while B takes 8 days to finish the same job. What is the ratio of their efficiency and who is less efficient.*

SOLUTION Since A takes more time than B to finish the same job hence A is less efficient or

$$\text{efficiency of } A = \frac{100}{16} = 6.25\%$$

and

$$\text{efficiency of } B = \frac{100}{8} = 12.5\%$$

$$\text{ratio of efficiency of } A : B = \frac{1}{16} : \frac{1}{8} = 1 : 2$$

Hence, B is twice efficient as A.

EXAMPLE 2 *A is thrice efficient as B and A takes 20 days to do a work then in how many days B can finish the same job?*

SOLUTION Ratio of efficiency of $A : B = 3 : 1$

$\therefore$ Ratio of required days of $A : B = \frac{1}{3} : \frac{1}{1} = 1 : 3$

Now since A takes 20 days. So B will take 60 days to finish the job.

Ratio of number of days is equal to the ratio of the reciprocals of efficiency and *vice-versa*.

EXAMPLE 3 *P is thrice as efficient as Q and is therefore able to finish a piece of work in 60 days less than Q. Find the time in which P and Q can complete the work individually.*

SOLUTION

Efficiency of $P : Q = 3 : 1$

Required number of days of $P : Q = 1 : 3$

i.e., if P requires x days then Q requires $3x$ days

but $3x - x = 60$

$\Rightarrow$ $2x = 60$

$\Rightarrow$ $x = 30$ and $3x = 90$

Thus P can finish the work in 30 days and Q can finish the work in 90 days.

EXAMPLE 4 *A is twice as good a workman as B and is therefore able to finish a piece of work in 30 days less than B. In how many days they can complete the whole work; working together?*

SOLUTION Ratio of efficiency = 2 : 1 $(A : B)$

Ratio of required time = 1 : 2 $(A : B) \Rightarrow x : 2x$

but $2x - x = 30$

$\Rightarrow$ $x = 30$ and $2x = 60$

now efficiency of $A = 3.33\%$

and efficiency of $B = 1.66\%$

Combined efficiency of A and B together = 5%

$\therefore$ time required by A and B working together to finish the work

$$= \frac{100}{5} = 20 \text{ days.}$$

NOTE

$$\text{Efficiency} \propto \frac{1}{\text{number of time units}}$$

$\therefore$ Efficiency × time = constant work

Hence $$\text{Required time} = \frac{\text{work}}{\text{efficiency}}$$

Whole work is always considered as 1, in terms of fraction and 100%, in terms of percentage.

In general, number of day's or hours $= \frac{100}{\text{efficiency}}$

EXAMPLE 5 *A can do a works in x days while B can do the same work in y days then in how many days will they complete the work, working together?*

SOLUTION

$$A\text{'s one day's work} = \frac{1}{x}$$

$$B\text{'s one day's work} = \frac{1}{y}$$

So, both A and B completes $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$ work in one day.

Now, **by Unitary method**

$\frac{x+y}{xy}$ work can be completed in 1 day

$\therefore$ 1 (means complete) work will be finished in $\frac{1}{\frac{(x+y)}{xy}}$ days

$$= \frac{xy}{x+y} \text{ days.}$$

$\left(\because \text{Required time to complete the work} = \frac{1}{\text{one day's work}}\right)$

CONCEPT OF NEGATIVE WORK

In this case one person works but another destroyes it or cancells it. For example Sonu can write 20 pages per hour but his younger sister Rimjhim erases 10 pages per hour which Sonu writes.

It means finally Sonu writes 10 pages per hour since each hour his sister erases 10 pages out of 20 written pages. Take another example :

EXAMPLE 1 *A tub can be filled in 20 minutes but there is a leakage in it which can empty the full tub in 60 minutes. In how many minutes it can be filled?*

SOLUTION Filling efficiency = 5% $\left(\because 5 = \frac{100}{20}\right)$

emptying efficiency = 1.66% $\left(\because 1.66 = \frac{100}{60}\right)$

Net efficiency = 5 – 1.66 = 3.33%

$\therefore$ Required time to fill the tub $= \frac{100}{3.33} = 30$ minutes

Alternatively : In 1 minute tub is filling $= \frac{1}{20}$

In 1 minute tub is emptying $= \frac{1}{60}$

$\therefore$ In 1 minute, effective filling of tub $= \frac{1}{20} - \frac{1}{60}$

$$= \frac{2}{60} = \frac{1}{30}$$

Since $\frac{1}{30}$ part of tub is filled in 1 minute.

Therefore complete tub will be filled in $\frac{1}{1/30} = 30$ minutes

APPLICATION OF INVERSE PROPORTION (OR PRODUCT CONSTANCY)

As I have already discussed thoroughly the concept of inverse proportion and product constancy in ratio-proportion and it has been widely used in profit loss chapter also.

Since the efficiency or rate of work done in one unit of time (mentioned) is inversely proportional to the time *i.e.*, if the rate of work done is greater then the time required will be less and if the rate of work done is less then the time required for the same amount of work will be more.

This product constancy method is limited to the constant work, if the amount of work gets changed, then it does not work, then we have to take help from the unitary method.

When more than one man/machine work on a particular work/project, the rate of work is calculated as the strength of workers working in a particular time. So the amount of work is defined in terms of man-days or man-hours or man-days-hours.

EXAMPLE 1 *If 20 persons can do a piece of work in 7 days then calculate the number of persons required to complete the work in 28 days*

SOLUTION Number of persons × days = work

$$20 \times 7 = 140 \text{ man-days}$$

Now, $$x \times 28 = 140 \text{ man-days}$$

$$\Rightarrow \quad x = 5$$

Therefore in second case the required number of person is 5

Second Method : Since work is constant, therefore

$$M_1 \times D_1 = M_2 \times D_2 = \text{work done}$$

$$20 \times 7 = M_2 \times 28$$

$$\Rightarrow \quad M_2 = 5$$

Third Method : Men × days = work, which is constant

$$\downarrow \frac{3}{4}\begin{pmatrix} 20 & 7 \\ 5 & 28 \end{pmatrix} 3\uparrow$$

Since number of days is increased by 3 times (*i.e.*, 300%). So the number of men will be decreased by $\frac{3}{4}$ times (*i.e.*, 75%)

(remember percentage change graphic)

EXAMPLE 2 *If 25 men can do a piece of work in 36 days working 10 hours a day, then how many men are required to complete the work working 6 hours a day in 20 days?*

SOLUTION

$$M_1 \times D_1 \times H_1 = M_2 \times D_2 \times H_2$$

$$25 \times 36 \times 10 = M_2 \times 20 \times 6$$

$$\Rightarrow \quad M_2 = 75 \text{ persons}$$

Alternatively :

$$\begin{matrix} & \text{Men} & \times & \text{Time} & \\ 2\uparrow & \begin{pmatrix} 25 & & 360 \\ 75 & & 120 \end{pmatrix} & & \frac{2}{3}\downarrow \end{matrix}$$

[Since product (work) is constant]

By percentage change graphic, when time is decreased by $\frac{2}{3}$ (*i.e.*, 66.66%), number of men is increased by 2 times (*i.e.*, 200%)

EXAMPLE 3 *A contractor employed 30 men to complete the project in 100 days. But later on he realised that just after 25 days only 20% of the work had been completed.*

(a) How many extra days, than the scheduled time are required?

(b) To complete the work on the scheduled time how many men he has to increase?

(c) If the amount of work is also increased by 20% of the actual work, then how many extra days are required (in comparison with scheduled time) but the number of men remained constant.

(d) How many men should be increased so that the work will be completed in 25 days less than the scheduled time.

SOLUTION (a) Men × days = work done

$$30 \times 25 = 750 = 20\% \text{ of the actual work}$$

Now, the work to be done is 4 times than the work done but the number of days is only 3 times. So he is required 4 times the number of days, thus he has to work for extra 25 days.

(b)

$$M_1 D_1 = M_2 D_2$$

$$4\,(30 \times 25) = M_2 \times 75$$

$$\Rightarrow \quad M_2 = 40$$

(Since, the work to be done is 4 times of the work done. Hence he requires 4 times man-days.)

Alternatively : Since the new product (*i.e.*, work) is 4 times of the original product (*i.e.*, work). But the new product is being multiplied by 3. Thus to make it 4 times we have to multiply it by 4/3. Thus without changing number of days we get the new value of number of men which is 40 (being multiplied by 4/3)

Therefore he has to increase 10 more men.

(c) New work = 3750 man-days

and the available number of men = 30

$$\therefore \quad \text{number of days required} = \frac{3750}{30} = 125$$

So, he has to work for extra 50 days where

$$50 = (125 - 75)$$

(d) Work = 3000 unit (man days)

number of available days = 50 (75 – 25)

$$\therefore \quad \text{number of men required} = \frac{3000}{50} = 60$$

Thus he has to increase (60 – 30) = 30 more men.

EXAMPLE 4 *16 workers working 6 hours a day can build a wall of length 150 metres, breadth 20 m and height 12 m in 25 days. In how many days 12 workers, working 8 hours a day can build a wall of length 800 m, breadth 15 m and height 6 m.*

SOLUTION Here work is the volume of the wall. So the work force should be increased/decreased in the ratio of volume of the work. Therefore

$$\frac{L_1 B_1 H_1}{L_2 B_2 H_2} = \frac{M_1 D_1 T_1}{M_2 D_2 T_2}$$

where L, B, H are length, breadth and height of the wall respectively and M, D, T are men, days and time in hours per day, respectively. 1 indicates the first case, while 2 indicates second case. So the ratio of work force remains constant as the ratio of volume and work.

$$\therefore \quad \frac{150 \times 20 \times 12}{800 \times 15 \times 6} = \frac{16 \times 6 \times 25}{12 \times 8 \times D_2}$$

$$\Rightarrow \quad D_2 = 50$$

hence the required number of days = 50

Please notice that the volume of work becomes twice (in the second case) so the work force will also be twice to the previous work force.

RELATION BETWEEN EFFICIENCIES

In this case the efficiencies of different persons are different but when they work in a group, so the efficiency of the group is required to know the time taken. For example 3 men can do a work in 4 days while 12 boys can do the same work in 3 days. It means we need $3 \times 4 = 12$ man-days *i.e.*, 12 men can finish the job in 1 day.

Similarly we need $12 \times 3 = 36$ boys days *i.e.*, 36 boys can finish the same job in 1 day.

Here we can see that to finish the work in only 1 day 12 men are needed while 36 boys are needed. Thus we can conclude that work of 12 men is equal to the work of 36 boys. Therefore efficiency of 12 men is equal to 36 boys *i.e.*, we can say the efficiency of 1 man is equal to 3 boys. Thus a man is thrice efficient as a boy or we can say that a man is two times more efficient than a boy.

EXAMPLE 1 *6 boys and 8 women finish a job in 6 days and 14 boys and 10 women finish the same job in 4 days. In how many days working together 1 boy and 1 woman can finish the work?*

SOLUTION In this kind of questions we find the work force required to complete the work in 1 day (or given unit of time) then we equate the work force to find the relationship between the efficiencies (or work rate) between the different workers.

Therefore $6B + 8W = 6$ days

$\Rightarrow$ $6(6B + 8W) = 1$ day (inversely proportional)

$\Rightarrow$ $36B + 48W = 1$ (by unitary method)

Again $14B + 10W = 4$ days

$\Rightarrow$ $56B + 40W = 1$

So, here it is clear that either we employ $36B$ and $48W$ to finish the work in 1 day or $56B$ and $40W$ to finish the same job in 1 day. Thus, we can say

$\Rightarrow$ $36B + 48W = 56B + 40W$

$\Rightarrow$ $20B = 8W$

$\Rightarrow$ $W = 2.5B$

Thus a woman is 2.5 times as efficient as a boy.

Now, since $36B + 48W = 1$

$\Rightarrow$ $36B + 48 \times (2.5B) = 1$

$\Rightarrow$ $156B = 1$

i.e., to finish the job in 1 day 156 boys are required or the amount of work is 156 boys-days.

Again $1W + 1B = 2.5B + 1B = 3.5B$

Now, since 156 boys can finish the job in 1 day

So 1 boy can finish the job in 1×156 days

$\therefore$ 3.5 boys can finish the job in $\frac{1 \times 156}{3.5} = 44\frac{4}{7}$ days

NOTE There is a great difference between 'and' & 'or'
For example 4 men and 8 women can do a piece of work in 10 days : means it is unknown that who is faster or slower (*i.e.*, we don't know the relation between efficiencies of a man and a woman)
Again, 4 men or 8 women can do a piece of work in 10 days : means 4 men are equal to 8 women. Hence a man is twice efficient as a woman.

EXAMPLE 2 *6 men and 8 women can do a job in 10 days. In how many days can 5 men and 9 women do the same job?*

SOLUTION Since we don't know the relation between work rate (or efficiency) between a man and that of a woman so we can't find the required number of days.

EXAMPLE 3 *6 men and 8 women can do a job in 10 days. In how many days can 3 men and 4 women finish the same job working together?*

SOLUTION Notice here we don't know the relation of efficiencies but we can solve the problem due to clear relation between the work force.

Since $6M + 8W = 10$ days

$2(3M + 4W) = 10$ days

$\Rightarrow$ $(3M + 4W) = 20$ days

Since, the work force has become half of the original force so number of days must be double.

Thus required number of days = 20

EXAMPLE 4 *A can complete a work in 12 days, B in 15 days. Find the time taken by them :*

(a) when A and B worked together.

(b) when A and B worked alternatively started by A.

(c) when A and B worked alternatively started by B.

(d) if A started two days later, in comparison to B.

(e) if B started two days later, in comparison to A.

(f) if A leaves two days before the actual completion of the work.

(g) if B leaves two days before the actual completion of the work.

(h) if A leaves two days before the scheduled completion of the work.

(i) if B leaves two days before the scheduled completion of the work.

(j) if B does negative work with his same work rate.

SOLUTION (a) A's efficiency = 8.33%

B's efficiency = 6.66%

Combined efficiency = 15%

So, the required time $= \frac{100}{15} = 6\frac{2}{3}$ days.

(b) In every two days A and B work 15%. So in 12 days they will complete 90% work. Now on the 13th day, A will finish 8.33% of the remaining (*i.e.*, 10% work) and the rest 1.66% will be finished by B on 14th day by taking time $= \frac{1.66}{6.66} = \frac{1}{4}$ day.

Thus, total required time $= 12 + 1 + \frac{1}{4} = 13\frac{1}{4}$ days.

See the chart below :

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Work	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	1.66
	15		15		15		15		15		15			
	$15 \times 6 = 90\%$												10%	

(c) Here the difference is in the last day/days only.

See the chart below :

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Work	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	3.33
	15		15		15		15		15		15		10	
	$15 \times 6 = 90\%$												10%	

So, after the completion of 90% work in 12 days, on the 13th day 6.66% work will be finished by B and then 3.33% (the last part of the work) will be finished by A by taking time $= \frac{3.33}{8.33} = \frac{2}{5}$ day.

Thus, the required time $= 12 + 1 + \frac{2}{5} = 13\frac{2}{5}$ days.

(d) It means B worked for 2 days more than A has worked for. Now, since B completes $2 \times 6.66 = 13.33\%$ work in two days. So the remaining work will be done by A and B together.

Therefore the time taken by A and B, working together

$$= \frac{86.66}{15} \frac{\text{(remaining work)}}{\text{(combined efficiency)}} = 5\frac{7}{9} \text{ days}$$

Thus the total required time to finish the whole work

$$= 2 + 5\frac{7}{9} = 7\frac{7}{9} \text{ days.}$$

(e) It means A alone worked for 2 days which is equal to

$$8.33 \times 2 = 16.66\%$$

So, the remaining (83.33%) work will be done by A and B together.

$\therefore$ Time taken together $= \frac{83.33}{15} = 5\frac{5}{9}$ days

Thus, the total required time $= 2 + 5\frac{5}{9} = 7\frac{5}{9}$ days.

(f) 'A leaves two days before the actual completion of the work' means in the last two days B has worked alone which is $6.66 \times 2 = 13.33\%$

Therefore the remaining work *i.e.*, 86.66% work was only done by A and B together in $\frac{86.66}{15} = 5\frac{7}{9}$ days

Thus, the required time $= 2 + 5\frac{7}{9} = 7\frac{7}{9}$ days.

NOTE This case is exactly same as part (d) of the same example. The difference is only in order of days. In part (d) B worked alone initially for 2 days and in part (f) B worked alone in the last 2 days.

(g) Applying the same concept as above we can calculate the required number of days which is same as in case (e)

Work	$A + B$	A
	$15 \times 5\frac{2}{5}$	8.33×2
	83.33%	16.66%

(h) 'A leaves two days before the scheduled completion of the work' means A works with B only for $4\frac{2}{3}$ days since the scheduled time is $6\frac{2}{3}$ [see the case (a)]

Scheduled time is the time in which work could be finished by working together without any change in efficiency or work force.

Now since both A and B have worked together for $4\frac{2}{3}$ days it means the last 2 days work (scheduled) which is equal to 30% (since if A and B working together can finish 30% work in 2 days) will be done by B only.

$\therefore$ required time to complete 30% work by B alone

$$= \frac{30}{6.66} = 4.5 \text{ days}$$

Thus the total required time to finish the whole work

$$= 4\frac{2}{3} + 4\frac{1}{2} = 9\frac{1}{6} \text{ days.}$$

(i) Very similar to the case (h) A and B worked only for $4\frac{2}{3}$ days (2 days less than the scheduled time $6\frac{2}{3}$ days)

(It is obvious that in $4\frac{2}{3}$ days A and B, working together completes 70% work, but we can directly calculate that they can complete 30% if they work together, in 2 days)

Thus 30% work is done by A alone

$\therefore$ time taken $= \frac{30}{8.33} = 3\frac{3}{5}$ days

Thus, the total time $= 4\frac{2}{3} + 3\frac{3}{5} = 8\frac{4}{15}$ days.

(j) Combined efficiency of A and $B = (8.33) + (-6.66)$

$= 1.66\%$

$\therefore$ Time required by A and B working together

$$= \frac{100}{1.66} = 60 \text{ days}$$

(Negative work means, B works against A)

EXAMPLE 5 *A can complete a work in 10 days, B in 12 days and C in 15 days. All of them began the work together, but A had to leave the work after 2 days of the start and B 3 days before the completion of the work. How long did the work last?*

SOLUTION See the diagram and then interpret the language of the question.

Initial 2 days		Last 3 days
$A + B + C$	$B + C$	C
$25 \times 2 = 50\%$	$15\% \times 2$ $[100 - (50 + 20)]$	$6.66 \times 3 = 20\%$

Since initially for 2 days all of them A, B and C work together so they complete the 50% work

and for the last 3 days only C works which is equal to 20% work.

Thus, the remaining work $= 30\%[100 - (50 + 20)]$

This 30% work was done by B and C in 2 days $= \left(\frac{30}{15}\right)$.

NOTE

Efficiency of $A = 10\%$

Efficiency of $B = 8.33\%$

Efficiency of $C = 6.66\%$

So, the total number of required days $= 2 + 2 + 3 = 7$ days.

EXERCISE

LEVEL 1

1. A can finish a piece of work in 12 days while B can do it in 15 days. If both work at it together, what time will they take to do the work?
(a) 6 days (b) 8 days
(c) $6\frac{2}{3}$ days (d) $10\frac{1}{5}$ days

2. A can do a piece of work in 10 days, B can do it in 12 days and C can do it in 15 days. In how many days they can complete the work, working together?
(a) 6 (b) 4
(c) 5 (d) 10

3. A can do a piece of work in 5 days, B can do it in 10 days. With the help of C, they finish the work in 2 days. In how many days C alone can do the whole work?
(a) 3 (b) 4
(c) 5 (d) 6

4. A can do a piece of work in 8 days, B can do it in 16 days, while C can do it in 80 days. In how many days they can complete the whole work, working together?
(a) 5 (b) 6
(c) $8\frac{2}{3}$ (d) $20\frac{2}{5}$

5. A can do a piece of work in 10 days, B can do it in 15 days working together they can finish the work in :
(a) 9 (b) 8
(c) 10 (d) 6

6. A can do a piece of work in 24 days, while B can do it in 30 days. With the help of C they can finish the whole work in 12 days. How much time is required for C to complete the work, alone?
(a) 100 days (b) 120 days
(c) 125 days (d) 72 days

7. A can do a piece of work in 10 days. B can do it in 24 days. If C also works with them then it takes only 6 days to complete the whole work. In how many days C alone can complete the whole work?
(a) 25 (b) 40
(c) 50 (d) 75

8. A can do a piece of work in 14 days while B can do it in 21 days. In how many days, working together they will complete the whole work?
(a) 10.5 (b) 8
(c) 8.4 (d) 9

9. A can do a piece of work in 24 days. If B is 60% more efficient than A, then the number of days required by B to do the same piece of work is :
(a) 10 (b) 15
(c) 12 (d) 9.6

10. A is twice as good a workman as B and together they finish a piece of work in 14 days. In how many days can A alone finish the work?
(a) 18 (b) 20
(c) 21 (d) 24

11. A is twice as good a workman as B and together they finish a piece of work in 7 days. In how many days can A alone finish the work?
(a) 10 (b) 10.5
(c) 12 (d) 11

12. A is thrice as good a workman as B and takes 10 days less to do a piece of work than B takes. The number of days taken by B to finish the work is :
(a) 9 (b) 18
(c) 15 (d) 12

13. A is twice as good a workman as B and therefore A takes 6 days less than B to finish the work individually. If A and B working together complete the work in 4 days, then how many days are required by B to complete the work alone?
(a) 12 (b) 18
(c) 8 (d) 6

14. A is thrice efficient as B and C is twice as efficient as B. What is the ratio of number of days taken by A, B and C, when they work individually?
(a) 2 : 6 : 3 (b) 2 : 3 : 6
(c) 1 : 2 : 3 (d) 3 : 1 : 2

15. Ajit is 3 times as efficient as Bablu, then the ratio of number of days required by each to work alone, completely?
(a) 3 : 1 (b) 1 : 3
(c) 6 : 3 (d) 3 : 6

16. X is 4 times more efficient as Y, then the number of days taken by X and Y are in the ratio :
(a) 8 : 1 (b) 1 : 5
(c) 6 : 8 (d) 5 : 1

17. If A takes 4/5 days as B takes and working together they require $\frac{20}{3}$ days to complete the whole work. What is the efficiency of B ?
(a) $6\frac{2}{3}\%$ (b) 16%
(c) 5.55% (d) $8\frac{1}{3}\%$

18. A is thrice as efficient as B. Working together they complete the work in 3 days. If B takes 8 days more than A, what is the number of days taken by A to finish the whole work, alone?
(a) 4 (b) 2
(c) 12 (d) 16

19. A and B can do a piece of work in 8 days, B and C can do the same work in 12 days and A and C complete it in 8 days. In how many days A, B and C can complete the whole work, working together?
(a) 4 (b) 6
(c) 12 (d) 9

20. A and B can do a piece of work in 12 days, B and C in 15 days C and A in 20 days. In how many days can C alone do it?
(a) 60 (b) 50
(c) 25 (d) 24

21. Ganga, Jamuna and Saraswati can do a piece of work, working together, in 1 day. Ganga is thrice efficient as Jamuna and Jamuna takes the twice the number of days as Saraswati takes to do it alone. What is the difference between the number of days taken by Ganga and Saraswati?
(a) 1 (b) 2
(c) 3 (d) 4

22. A can finish a work in 12 days and B can do it in 15 days. After A had worked for 3 days, B also joined A to finish the remaining work. In how many days, the remaining work will be finished?
(a) 3 (b) 4
(c) 5 (d) 6

23. Raja can do a piece of work in 14 days, while Rani can do the same work in 21 days. They started the work together but 3 days before the completion of the work, Raja left the work. The total number of days to complete the work is :
(a) 7 (b) 8.5
(c) 5 (d) $10\frac{1}{5}$

24. A and B can complete a task in 30 days when working together after A and B have been working together for 11 days, B is called away and A, all by himself completes the task in the next 28 days. Had A been working alone, the number of days taken by him to complete the task would have been :
(a) $33\frac{3}{19}$ (b) $19\frac{6}{25}$
(c) $44\frac{4}{19}$ (d) none of these

25. Sonu can do a piece of work in 20 days. He started the work and left after some days, when 25% work was done. After it Abhijeet joined and completed it working for 10 days. In how many days Sonu and Abhijeet can do the complete work, working together?
(a) 6 (b) 8
(c) 10 (d) 12

26. Efficiency of Asha is 25% more than Usha and Usha takes 25 days to complete a piece of work. Asha started a work alone and then usha joined her 5 days before actual completion of the work. For how many days Asha worked alone?
(a) 9 (b) 11
(c) 10 (d) 15

27. Krishna can do a work in 10 days while Mohan can do the same work in 20 days. They started work together. After 3 days Krishna left the work and Mohan completed it. For how many days Mohan worked alone more than the number of days required when both worked together?
(a) $4\frac{1}{3}$ (b) $3\frac{1}{4}$
(c) $2\frac{3}{5}$ (d) $3\frac{2}{3}$

28. Kareena can do a piece of work in 9 days and Karishma can do the same work in 18 days. They started the work. After 3 days Shahid joined them, who can complete alone the same whole work in 3 days. What is the total number of days in which they had completed the work?
(a) 12 (b) 8
(c) 4 (d) 6

29. Kavita, Babita and Samita started a work. 5 days later Samita left the work and Babita left the work after working 8 days. In how many more days Kavita would have completed the rest work if they take 20, 60 and 30 days individually to finish a work?
(a) 4 (b) 5
(c) 6 (d) 8

30. The ratio of efficiency of A is to C is 5 : 3. The ratio of number of days taken by B is to C is 2 : 3. A takes 6 days less than C, when A and C completes the work individually. B and C started the work and left after 2 days. The number of days taken by A to finish the remaining work is :
(a) 4.5 (b) 5
(c) 6 (d) $9\frac{1}{3}$

31. Anand can do a piece of work in 45 days, but Bahuguna can do the same work in 5 days less, than Anand, when working alone. Anand and Bahuguna both started the work together but Bahuguna left after some days and Anand finished the remaining work in 56 days with half of his efficiency but he did the work with Bahuguna with his complete efficiency. For how many days they had worked together?
(a) 6 (b) 8
(c) 9 (d) 12

32. Chandni and Divakar can do a piece of work in 9 days and 12 days respectively. If they work for a day alternatively, Chandni beginning, in how many days, the work will be completed?
(a) $10\frac{1}{4}$ (b) $9\frac{1}{5}$
(c) 11.11 (d) 10

33. Fatima and Zahira can do a piece of work in 12 days and 15 days respectively. If they work for alternate day and Fatima starts the work first, then in how many days the work will be completed?
(a) $12\frac{1}{5}$ (b) $13\frac{1}{4}$
(c) $13\frac{1}{5}$ (d) 15

34. In the previous question (number 33) if Zahira starts first then in how many days, the work will be completed?
(a) $14\frac{1}{5}$ (b) 14
(c) $13\frac{1}{5}$ (d) $13\frac{2}{5}$

35. The number of days required by A, B and C to work individually is 6, 12 and 8 respectively. They started a work doing it alternatively. If A has started then followed by B and so on, how many days are needed to complete the whole work?
(a) 8 (b) 7.5
(c) 8.5 (d) $9\frac{1}{2}$

36. In the previous question if the order of working days be as $B, C, A, B, C, A \ldots$ (starting with B and followed by C and A respectively), then in how many days the work will be completed?

(a) $7\frac{3}{4}$ (b) 8

(c) $8\frac{1}{4}$ (d) 9

37. A takes 6 days less than B to do a certain job and 2 days more than C. A and B together can do the work in the same time as C. In how many days B alone can do the complete work?

(a) 10 (b) 14

(c) 12 (d) 16

38. A and B undertook a work for Rs. 350. A got Rs. 150 more than that of B, when they worked together. B takes 9 days more than A, when they work individually. In how many days A and B working together can do the whole work :

(a) 5 (b) $4\frac{2}{7}$

(c) $4\frac{5}{7}$ (d) $5\frac{4}{7}$

39. Alen and Border can do a work individually in 21 and 42 days respectively. In how many days they can complete the work, working alternatively?

(a) 14 (b) 28

(c) 42 (d) 35

40. C takes twice the number of days to do a piece of work than A takes. A and B together can do it in 6 days while B and C can do it in 10 days. In how many days A alone can do the work?

(a) 60 (b) 30

(c) 6 (d) 7.5

41. When A, B and C are deployed for a task, A and B together do 70% of the work and B and C together do 50% of the work. Who is most efficient?

(a) A (b) B

(c) C (d) can't be determined

42. Colonel, Major and General started a work together for Rs. 816. Colonel and Major did $\frac{8}{17}$ of the total work, while Major and General together did $\frac{12}{17}$ of the whole work. What is the amount of the least efficient person?

(a) Rs. 256 (b) Rs. 144

(c) Rs. 85 (d) can't be determined

43. Sharma is 20% less efficient than Kelkar. If Kelkar can do a piece of work in 24 days. The number of days required by Sharma to complete the same work alone?

(a) 20 (b) 30

(c) 28.8 (d) can't be determined

44. 30 persons can do a piece of work in 24 days. How many more people are required to complete the work in 20 days?

(a) 4 (b) 5

(c) 6 (d) none of these

45. 12 women can do a piece of work in 20 days. If the 4 women deny to work, then how many more days are required?

(a) 6 (b) 10

(c) 15 (d) none of these

46. 35 boys can do a piece of work in 15 days. The work was completed in 25 days. How many boys did not turn up for the job?

(a) 14 (b) 20

(c) 6 (d) 7

47. 24 men can complete a job in 40 days. The number of men required to complete the job in 32 days is :

(a) 30 (b) 40

(c) 25 (d) 50

48. 16 men finished one-third work in 6 days. The number of additional men are required to complete the job in next 6 days :

(a) 10 (b) 8

(c) 16 (d) 32

49. If 10 persons can do a job in 20 days, then 20 person with twice the efficiency can do the same job in :

(a) 5 days (b) 40 days

(c) 10 days (d) 20 days

50. A certain job was assigned to a group of men to do in 20 days. But 12 men did not turn up for the job and the remaining men did the job in 32 days. The original number of men in the group was :

(a) 32 (b) 36

(c) 42 (d) 40

51. 30 workers can finish a work in 20 days. After how many days should 9 workers leave the job so that the work is completed in total 26 days :

(a) 12 (b) 10

(c) 6 (d) none of these

52. 25 men can complete a job in 30 days. After how many days should the strength of work force be increased by 50 men so that the work will be completed in $\frac{2}{3}$rd of the actual time :

(a) 15 (b) 10

(c) 18 (d) 5

53. A group of workers can complete a job in 120 days. If there were 4 more such workers then the work could be finished in 12 days less. What was the actual strength of workers?

(a) 30 workers (b) 40 workers

(c) 42 workers (d) 36 workers

54. Mr. Modi can copy 40 pages in 10 minutes, Mr Xerox and Mr. Modi both working together can copy 250 in 25 minutes. In how many minutes Mr. Xerox can copy 36 pages?

(a) 5 minutes (b) 6 minutes

(c) 3 minutes (d) 12 minutes

55. 20 persons completed $\frac{1}{3}$rd of the work in 12 days. How many more person are required to finish the rest work in next 12 days?

(a) 20

(c) 18 (b) 12

(d) 40

56. A contractor undertook a work to complete in 60 days. But just after 20 days he observed that only $\frac{1}{5}$th of the project work had been completed. To complete the work in time (i.e.

in rest days) minimum how many workers he had to increase, if there were initially 75 workers were deployed for the task?
(a) 25 (b) 50
(c) 75 (d) can't be determined

57. 6 men or 10 women can reap a field in 15 days, then the number of days that 12 men and 5 women will take to reap the same field is :
(a) 5 (b) 6
(c) 8 (d) 12

58. If 2 men or 3 women or 4 boys can do a piece of work in 52 days, then the same piece of work will be done by 1 man, 1 woman and 1 boy in :
(a) 48 days (b) 36 days
(c) 45 days (d) none of these

59. 2 men or 5 women or 7 boys can finish a work in 469 days, then the number of days taken by 7 men, 5 women and 2 boys to finish the work is :
(a) 134 (b) 106
(c) 100 (d) 98

60. 6 children and 2 men complete a certain piece of work in 6 days. Each child takes twice the time taken by a man to finish the work. In how many days will 5 men finish the same work?
(a) 6 (b) 8
(c) 9 (d) 15

61. 2 men and 3 women finish 25% of the work in 4 days, while 6 men and 14 women can finish the whole work in 5 days. In how many days will 20 women finish it?
(a) 20 (b) 25
(c) 24 (d) 88

Directions : For question number 62 and 63:
A can do a work in 15 days and B can do it in 18 days. With the help of C, all of them complete the work in 6 days.

62. How long will it take C to finish the work alone?
(a) 30 days (b) 22 days
(c) $\frac{45}{2}$ days (d) 25 days

63. A, B and C received total Rs. 27,000 for the whole work. What is the share of C, if the money is distributed in the ratio of amount of work done, individually?
(a) 2700 (b) 7200
(c) 14400 (d) 6300

Directions : Solve the following questions individually.

64. 314 weavers weaves 6594 shawls in $\frac{1}{6}$ hours. What is the number of shawls weaved per hour by an average weaver?
(a) 42 (b) 21
(c) 102 (d) 126

65. Three men and two women can do a piece of work in 4 days, while two men and three women can do the same work in 5 days. Rs 44 is given to a woman for her contribution towards work, per day. What is the amount received by a man per day?
(a) Rs. 88 (b) Rs. 144
(c) Rs. 154 (d) can't be determined

66. 30 girls can finish a work in 40 days. After how many days should 10 girls leave the work so that it may be finished in 46 days, if all the 30 girls started the work?
(a) 18 days (b) 28 days
(c) 22 days (d) 30 days

67. If 8 women collect 200 kg of tea leaves in 10 hours. How many more (in kg) of tea leaves will 12 women collect in 8 hours?
(a) 24 kg (b) 40 kg
(c) 50 kg (d) 100 kg

68. 450 man-days of work can be completed by a certain number of men in some days. If the number of people (men) are increased by 27, then the number of day required to complete the same work is decreased by 15. The number of days are required to complete the three times work (than the previous/actual work) by 27 men?
(a) 50 (b) 60
(c) 54 (d) 45

69. 4 boys and 5 girls can do a piece of work in 10 days. 6 boys and 6 girls can do the same work in 7 days. In how many days can 2 boys and 7 girls complete the same work, working together?
(a) 15 days (b) 14 days
(c) 21 days (d) 18 days

70. If 20 engineers and 20 workers can together construct a 20 km road in 20 days. 40 engineers and 40 workers together construct 40 km road in how many days?
(a) 10 (b) 20
(c) 40 (d) can't be determined

71. $(x-2)$ men can do a piece of work in x days and $(x+7)$ men can do 75% of the same work in $(x-10)$ days. Then in how many days can $(x+10)$ men finish the work?
(a) 27 days (b) 12 days
(c) 25 days (d) 18 days

72. A man, a woman and a girl worked for a contractor for the same period. A man is twice efficient as a woman and a woman is thrice efficient as a girl Rs. 10000 were given to all of them. What is the sum of money received by a woman and a girl together?
(a) Rs. 5500 (b) Rs. 4500
(c) Rs. 4000 (d) Rs. 6000

73. 33 men can do a job in 30 days. If 44 men started the job together and after every day of the work, one person leaves. What is the minimum number of days required to complete the whole work?
(a) 21 (b) 42
(c) 45 (d) none of these

74. Abhishek can do a piece of work in 40 days. He alone worked at it for 8 days and then Bacchhan completed alone the rest work in 24 days. In how many days they will complete the whole work, working together?
(a) $17\frac{1}{7}$ days (b) $18\frac{1}{7}$ days
(c) $9\frac{1}{6}$ days (d) 14 days

75. C is twice efficient as A. B takes thrice as many days as C. A takes 12 days to finish the work alone. If they work in pairs (*i.e.*, AB, BC, CA) starting with AB on the first day then BC on the second day and AC on the third day and so on, then how many days are required to finish the work?

(a) $6\frac{1}{5}$ days (b) 4.5 days

(c) $5\frac{1}{9}$ days (d) 8 days

76. Ahluwalia and Bimal together take 6 days to finish the work. Bimal and Jalan together take 10 days to finish the work. What is the difference between number of days taken by Ahluwalia and Jalan when they worked alone to complete the whole work?

(a) 12 days (b) 16 days

(c) 15 days (d) can't be determined

77. B is twice efficient as A and A can do a piece of work in 15 days. A started the work and after a few days B joined him. They completed the work in 11 days, from the starting. For how many days they worked together?

(a) 1 day (b) 2 days

(c) 6 days (d) 5 days

78. A, B and C can complete a piece of work in 15, 30 and 40 days respectively. They started the work together and A left 2 days before the completion of the work and B left 4 days before the completion of the work. In how many days was the work completed?

(a) $7\frac{3}{10}$ (b) $10\frac{2}{15}$

(c) $10\frac{7}{30}$ (d) none of these

79. There was a leakage in the container of the refined oil. If 11 kg oil is leaked out per day then it would have lasted for 50 days, if the leakage was 15 kg per day, then it would have lasted for only 45 days. For how many days would the oil have lasted, if there was no leakage and it was completely used for eating purpose?

(a) 80 days (b) 72 days

(c) 100 days (d) 120 days

80. A contractor undertook to complete the work in 40 days and he deployed 20 men for his work. 8 days before the scheduled time he realised that $\frac{1}{3}$rd of the work was still to be done. How many more men were required to complete the work in stipulated time?

(a) 16 (b) 15

(c) 20 (d) 25

81. 7 Indian and 4 Chinese finish a job in 5 days. 7 Japanese and 3 Chinese finish the same job in 7 days. Given that the efficiency of each person of a particular nationality is same but different from others. One Indian, one Chinese and one Japanese will complete the work in :

(a) $18\frac{3}{13}$ days (b) $20\frac{5}{12}$ days

(c) $21\frac{6}{14}$ days (d) $20\frac{7}{12}$ days

82. A, B and C are three book binders. A takes 8 minutes, B takes 12 minutes and C takes 16 minutes to bind a book. If they work each day for 12 hours, then on an average, how many books each one bind per day?

(a) 65 (b) 52

(c) 48 (d) 70

83. A piece of work can be completed by 10 men and 6 women in 18 days. Men works 9 hours per day while women works 7.5 hours per day. Per hour efficiency of a woman is $\frac{2}{3}$rd of a man's efficiency. In how many days 10 men and 9 women complete the work?

(a) 16 days (b) 20 days

(c) 30 days (d) 25 days

84. B and C are equally efficient, but the efficiency of A is half of each B and C. A and B started a work and 3 days later C joined them. If A alone can do the work in 14 days, then in how many more days the work will be completed?

(a) 1 (b) 2

(c) 3 (d) 4.5

85. A can do a piece of work in 10 days, B in 15 days. They work together for 5 days, the rest of the work is finished by C in two more days. If they get Rs. 3000 as wages for the whole work, what are the daily wages of A, B and C respectively (in Rs.):

(a) 200,250,300 (b) 300,200,250

(c) 200,300,400 (d) none of these

86. A can do a piece of work in 2 hours, B can do thrice the work in 8 hours and 'C' can do the same work as A in 8 hours. If all of them work together, how long it would take them to complete the work :

(a) 1 hour (b) 2 hours

(c) 3 hours (d) 4 hours

87. A is twice efficient as B and together they do the same work in as much time as C and D together. If C and D can complete the work in 20 and 30 days respectively, working alone, then in how many days A can complete the work individually :

(a) 12 days (b) 18 days

(c) 24 days (d) 30 days

88. 4 men and 2 boys can finish a piece of work in 5 days. 3 women and 4 boys can finish the same work in 5 days. Also 2 men and 3 women can finish the same work in 5 days. In how many days 1 man, 1 woman and one boy can finish the work at their double efficiency?

(a) $4\frac{8}{13}$ (b) $4\frac{7}{13}$

(c) $3\frac{7}{13}$ (d) none of these

89. If m men can do a work in r days, then the number of days taken by $(m + n)$ men to do it is :

(a) $\frac{m+n}{mn}$ (b) $\frac{m+n}{mr}$

(c) $\frac{mr}{(m+n)}$ (d) $\frac{(m+n)r}{mn}$

90. Pipe A can fill a tank in 36 minutes and pipe B can fill it in 45 minutes. If both the pipes are opened to fill an empty tank, in how many minutes will it be full?

(a) 15 (b) 24
(c) 20 (d) 25

91. Tap A can fill the empty tank in 12 hours, but due to a leak in the bottom it is filled in 15 hours. If the tank is full and then tap A is closed then in how many hours the leak can empty it?
(a) 45 hours (b) 48 hours
(c) 52 hours (d) 60 hours

92. Pipe A and B can fill a cistern in 10 hours and 15 hours respectively. When a third pipe C which works as an outlet pipe is also open then the cistern can be filled in 18 hours. The outlet pipe can empty a full cistern in :
(a) 12 hours (b) 8 hours
(c) 9 hours (d) 14 hours

93. A cistern has a leak which would empty it in 6 hours. A tap is turned on which fills the cistern @ 10 liters per hour and then it is emptied in 15 hours. What is the capacity of the cistern?
(a) 100 litres (b) 166.66 litres
(c) 60.66 litres (d) none of these

94. Tap a fills a tank in 10 hours and B can fill it in 15 hours. Both are opened simultaneously. Sometimes later tap B was closed, then it takes total 8 hours to fill up the whole tank. After how many hours B was closed?
(a) 2 (b) 3
(c) 4 (d) 5

95. Tap A can fill a tank in 20 hours, B in 25 hours but tap C can empty a full tank in 30 hours. Starting with A, followed by B and C each tap opens alternatively for one hour period till the tank gets filled up completely. In how many hour the tank will be filled up completely?
(a) $51\frac{11}{15}$ (b) $52\frac{2}{3}$
(c) $24\frac{4}{11}$ (d) none of these

96. If one pipe A can fill a tank in 20 minutes, then 5 pipes, each of 20% efficiency of A, can fill the tank in :
(a) 80 min (b) 100 min
(c) 20 min (d) 25 min

97. Pipe A basically used as inlet pipe and pipe B is used as outlet pipe. Pipes A and B both are opened simultaneously, all the time. When pipe A fills the tank and B empty the tank, it will take double the time than when both the pipe fill the tank. When pipe B is used for filling the tank, its efficiency remains constant. What is the ratio of efficiency of pipe A and pipe B respectively?
(a) 3 : 1 (b) 5 : 2
(c) 1 : 3 (d) 3 : 2

98. Pipe A can fill the tank in 4 hours, while pipe B can fill it in 6 hours working separately. Pipe C can empty whole the tank in 4 hours. He opened the pipe A and B simultaneously to fill the empty tank. He wanted to adjust his alarm so that he could open the pipe C when it was half-filled, but he mistakenely adjusted his alarm at a time when his tank would be 3/4th filled. What is the time difference between both the cases, to fill the tank fully :
(a) 48 min. (b) 54 min.
(c) 30 min. (d) none of these

99. Two pipes A and B can fill a cistern in 15 hours and 10 hours respectively. A tap C can empty the full cistern in 30 hours. All the three taps were open for 2 hours, when it was remembered that the emptying tap had been left open. It was then closed. How many hours more would it take for the cistern to be filled?
(a) 30 min. (b) 1.2 hours
(c) 24 min. (d) 35 min.

100. Pipe A can fill an empty tank in 30 hours while B can fill it in 45 hours. Pipe A and B are opened and closed alternatively *i.e.*, first pipe A is opened, then B, again A and then B and so on for 1 hour each time without any time lapse. In how many hours the tank will be filled when it was empty, initially?
(a) 36 (b) 54
(c) 48 (d) 60

LEVEL 2

1. A, B and C three weavers have to supply an order of 100 shawls. A can weave a shawl in 2 hours, B in 3 hours and C in 4 hours respectively. It is known that even being a joint contract each one weaves his own shawl completely *i.e.*, no other weaver help to the rest weavers. In how many hours they will complete the order irrespective of day or night?
(a) 93 hours (b) 100 hours
(c) $92\frac{4}{13}$ hours (d) 94 hours

2. Arun and Satyam can complete a work individually in 12 working days and 15 working days respectively with their full efficiencies. Arun does work only on Monday, Wednesday and Friday while Satyam does the work on Tuesday, Thursday and Saturday. Sunday is always off. But Arun and Satyam both works with half of their efficiencies on Friday and Saturday respectively. If Arun started the work on Ist January which falls on Monday followed by Satyam on the next day and so on (*i.e.*, they work collectively in alternate days), then on which day work will be completed?
(a) Tuesday (b) Wednesday
(c) Thursday (d) Friday

3. Kaushalya can do a work in 20 days, while Kaikeyi can do the same work in 25 days. They started the work jointly. Few days later Sumitra also joined them and thus all of them completed the whole work in 10 days. All of them were paid total Rs. 700. What is the share of Sumitra?
(a) Rs. 130 (b) Rs. 185
(c) Rs. 70 (d) can't be determined

4. A and B can complete the work individually in 24 days and 30 days respectively, working 10 hours a day. Work is to be done in two shift. Morning shift lasts for 6 hours and evening shift lasts for 4 hours. On the first day A works in the morning shift while B works in the evening shift. Next day A works in the evening shift while B works in the morning shift and so on. It means they work alternatively with respect to their shifts. Thus they work on this pattern till the work is completed. On which day the work got completed?