12 GEOMETRY

This chapter tests nothing but your imagination. I think it is the drawing of MATHS. You need not to be an artist or painter, but at least you must be capable to visualize the geometrical figures. So before going for the tedious and complex geometrical problems it is prerequisite to be familiar with the fundamentals of the geometry. Basically it consists of a very large proportion of problems in CAT. Sometimes 8-10 problems related to this chapter in addition with mensuration are asked in CAT. Therefore it is advised that you must learn this chapter religiously for the sake of your score in CAT. Apart from CAT other entrance tests for MBA ask a plethora of questions as well.

Remember if you don't take seriously any theorem, axiom or concept you cannot perform well since the complexity of ideas, figures and concepts makes it a daunting task for you.

For the sake of convenience this chapter is divided into 5 parts.

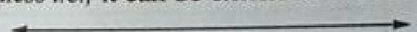
(i) Lines, angles and planes
(ii) Triangles
(iii) Quadrilaterals
(iv) Polygons
(v) Circles and Loci

"Well begun is half done"–Anonymous

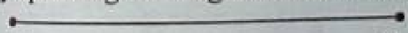
(1) LINE AND ANGLES

Point : The figure of which length, breadth and height cannot be measured is called a point. It is infinitesimal.

Line : A line is made up of a infinite number of points and it has only length *i.e.*, it does not has any thickness (or width). A line is endless *i.e.*, it can be extended in both directions.

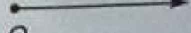


Line segment : A line segments has two end points, but generally speaking line segment is called a line.

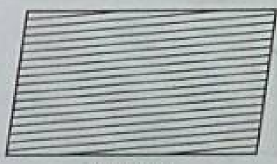


(line segment $\overrightarrow{AB}$)

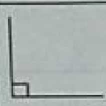
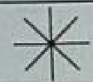
Ray : A ray extends indefinitely in one direction from any given point. This is exhibited by an arrow. The starting point is called as the initial point.



Plane : It is a flat surface, having length and breadth both, but no thickness. It is a two dimensional figure.



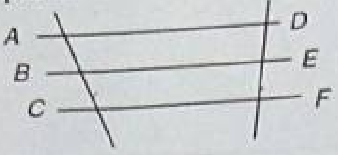
(Plane)

Type of Lines	Definition	Diagram
Parallel lines	Two lines, lying in a plane and having no common intersecting point are called parallel lines. The distance between two parallel lines is constant.	 **Symbol :** $\|\|$
Perpendicular lines	Two lines, which lie in a single plane and intersect eachother at right angle, are called perpendicular lines.	 **Symbol :** $\perp$
Concurrent lines	More than two straight lines intersecting at the same point.	

POINTS TO REMEMBER

1. A line contains infinitely many points.
2. The intersection of two different lines is a point.
3. Through a given point, there pass an infinite number of lines and these lines are called **concurrent lines.**
4. Only one line can pass through any two particular points.
5. When more than two points lie on a line, they are called as **collinear points** else they are called as **non-collinear** points.
6. Two lines can intersect maximum at one point. This point is called as **point of intersection** and these lines are called as **intersecting lines**.
7. There are an infinite number of planes which pass through a single (particular) point.
8. When more than three points lie in the same plane, they are called as **coplanar** else they are called as **non-coplanar.**
9. When more than one line lie in the same plane, then these lines are called as coplanar else they are called as non-coplanar.

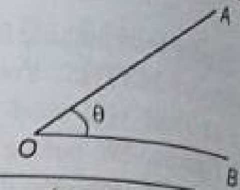
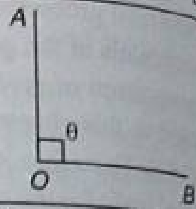
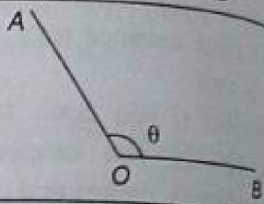
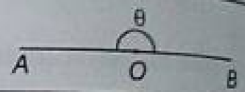
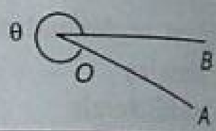
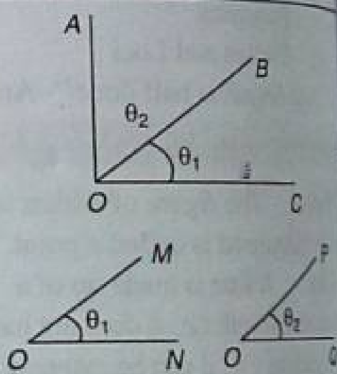
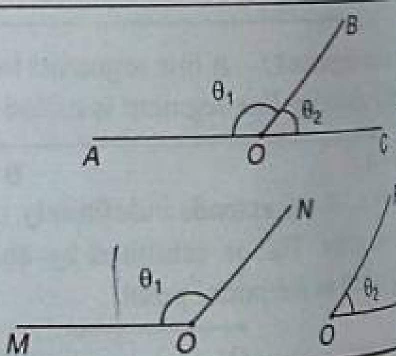
10. When two plane intersect each other, they form a line *i.e.*, intersecting region is a line.
11. Two different lines which are perpendicular to the same (a third line) line, necessarily parallel to each other, lying on the same plane.
12. When two or more parallel lines are intercepted by some other intercepting lines, then the ratio of corresponding intercepts are equal. An intercepting line is generally called as a transversal.

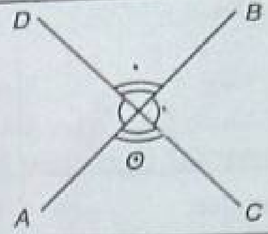
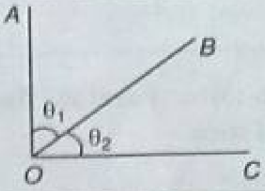
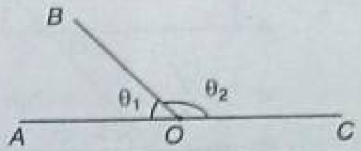
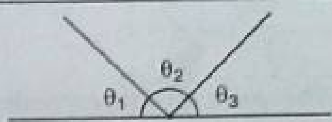
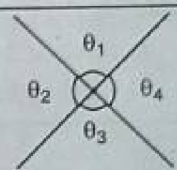
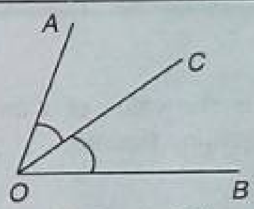
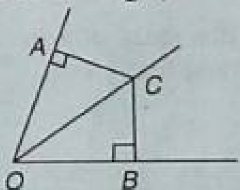
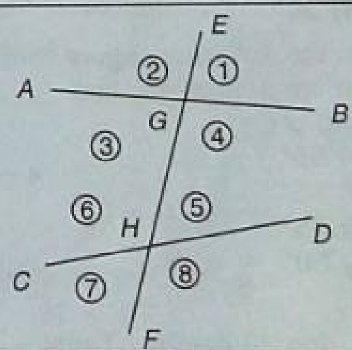


i.e., $\frac{AB}{BC} = \frac{DE}{EF}$

Angles : The amount of rotation about O, the vertex of the angle AOA', is called the magnitude of the angle.

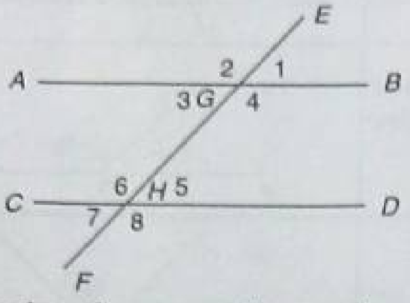
$m\angle AOA'$ denotes the measure of $\angle AOA'$. Angles are said to be congruent when their measure is same. (symbol : $\cong$)

Types of Angles	Property	Diagram
Acute	$0° < \theta < 90°$ ($\angle AOB$ is an acute angle)	
Right	$\theta = 90°$ ($\angle AOB$ is a right angle)	
Obtuse	$90° < \theta < 180°$ ($\angle AOB$ is an obtuse angle)	
Straight	$\theta = 180°$ ($\angle AOB$ is a straight angle)	
Reflex	$180° < \theta < 360°$ ($\angle AOB$ is a reflex angle)	
Complementary	$\theta_1 + \theta_2 = 90°$ Two angles whose sum is 90°, are complementary to each other	
Supplementary	$\theta_1 + \theta_2 = 180°$ Two angles whose sum is 180°, are supplementary to each other.	

Types of Angles	Property	Diagram
...ically opposite	$\angle DOA = \angle BOC$ and $\angle DOB = \angle AOC$	
...jacent angles	$\angle AOB$ and $\angle BOC$ are adjacent angles Adjacent angles must have a common side. (*e.g.*, OB)	
...ear pair	$\angle AOB$ and $\angle BOC$ are linear pair angles. One side must be common (*e.g.*, OB) and these two angles must be supplementary.	
...les on the one side of a ...e	$\theta_1 + \theta_2 + \theta_3 = 180°$	
...les round the point	$\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360°$	
...le bisector	OC is the angle bisector of $\angle AOB$. *i.e.*, $\angle AOC = \angle BOC = \frac{1}{2}(\angle AOB)$ When a line segment divides an angle equally into two parts, then it is said to be the angle bisector (OC)	 (Angle bisector is equidistant from the two sides of the angle) *i.e.*,  $AC = BC$
...rresponding angles	When two lines are intersected by a transversal, then they form four pairs of corresponding angles (a) $\angle AGE, \angle CHG \Rightarrow (\angle 2, \angle 6)$ (b) $\angle AGH, \angle CHF \Rightarrow (\angle 3, \angle 7)$ (c) $\angle EGB, \angle GHD \Rightarrow (\angle 1, \angle 5)$ (d) $\angle BGH, \angle DHF \Rightarrow (\angle 4, \angle 8)$	
...terior angles	These are following four angles (i) $\angle AGE \Rightarrow \angle 2$ (ii) $\angle CHF \Rightarrow \angle 7$ (iii) $\angle EGB \Rightarrow \angle 1$ (iv) $\angle DHF \Rightarrow \angle 8$	

Types of Angles	Property	Diagram
Interior angles	These are following four angles (i) $\angle AGH \Rightarrow \angle 3$ (ii) $\angle GHC \Rightarrow \angle 6$ (iii) $\angle BGH \Rightarrow \angle 4$ (iv) $\angle DHG \Rightarrow \angle 5$	
Alternate angles	These are two pairs of angles as following : (i) $\angle AGH, \angle GHD$ $(\angle 3, \angle 5)$ (ii) $\angle GHC, \angle BGH$ $(\angle 6, \angle 4)$	

When two parallel lines are intersected by a transversal then



1. The pairs of corresponding angles so formed are congruent.

i.e.,
$$\angle 2 = \angle 6$$
$$\angle 3 = \angle 7$$
$$\angle 1 = \angle 5$$
$$\angle 4 = \angle 8$$

NOTE If one pair of corresponding angles is congruent, pairs of corresponding angles are also congruent.

2. The pairs of alternate angles so formed are

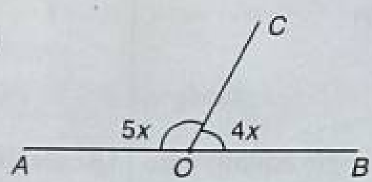
i.e., $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$

3. The pair of interior angles (*i.e.*, the interior the same side of a transversal) are supple

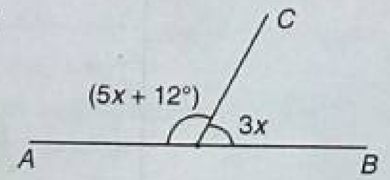
NOTE The converse of all the 3 rules is also true i.e. corresponding angles, or alternate angles or are equal (*i.e.*, congruent) then the two line are para intersected by a transversal.

INTRODUCTORY EXERCISE-1

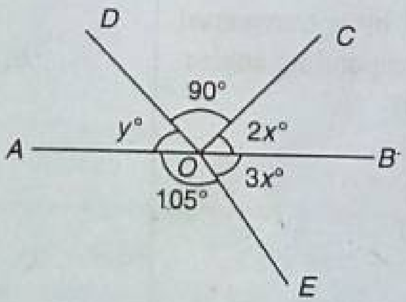
1. What is the value of x in the adjoining figure?
(a) 80
(b) 40
(c) 20
(d) 25



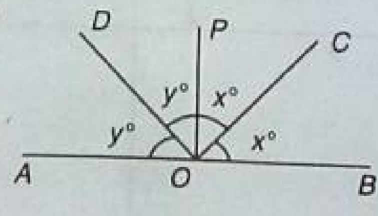
2. What is the value of x in the adjoining figure ?
(a) 18°
(b) 20°
(c) 21°
(d) 24°



3. In the following figure AB is a straight line. Find $(x+y)$:
(a) 55°
(b) 65°
(c) 75°
(d) 80°

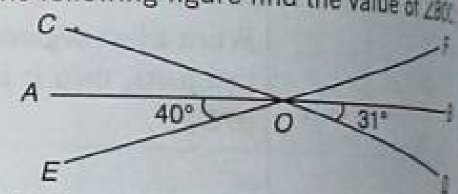


4. In the following figure $\angle BOP = 2x°$, $\angle AOP = 2y°$, OC and OD are angle bisectors of $\angle BOP$ and $\angle AOP$ respectively. Find the value of $\angle COD$:



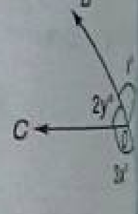
(a) 75°
(b) 90°
(c) 100°
(d) 120°

5. In the following figure find the value of $\angle BOC$



(a) 101°
(b) 149°
(c) 71°
(d) 140°

6. Find y, if $x° = 36°$, as per the given diagram :
(a) 36°
(b) 16°
(c) 12°
(d) 42°



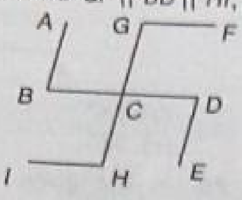
7. If $(2x + 17)°$, $(x + 4)°$ are complementary, find
(a) 63°
(b) 53°
(c) 35°
(d) 23°

8. If $(5y + 62)°$, $(22° + y)$ are supplementary, find
(a) 16°
(b) 32°
(c) 8°
(d) 1°

9. If two supplementary angles are in the ratio 13 : 5 find the greater angle :
(a) 130° (b) 65°
(c) 230° (d) 28°

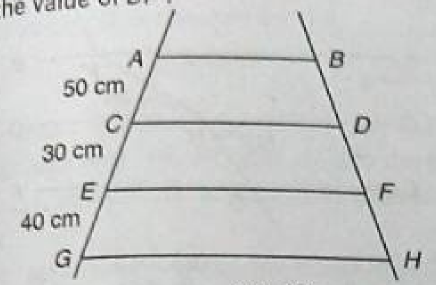
10. An angle is 30° more than one half of its complement. Find the angle in degrees :
(a) 60° (b) 50°
(c) 45° (d) 80°

11. In the given diagram AB|| GH || DE and GF || BD || HI, $\angle FGC = 80°$. Find the value of $\angle CHI$:



(a) 80°
(b) 120°
(c) 100°
(d) 160°

12. In the given figure AB || CD || EF || GH and BH = 100 cm. Find the value of DF :



(a) 26 cm (b) 40 cm
(c) 25 cm (d) 24 cm

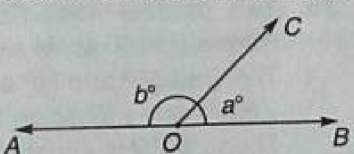
13. The complement of 65°50′ is :
(a) 24° 50′ (b) 24°10′
(c) 14° 50′ (d) 34° 10′

14. The supplement of 123° 45′ is :
(a) 56° 55′ (b) 56° 15′
(c) 55° 56′ (d) none of these

15. If two angles are complementary of each other, then each angle is :
(a) a right angle
(b) a supplementary angle
(c) an obtuse angle
(d) an acute angle

16. How many degrees are there in an angle which equals one-fifth of its supplement?
(a) 15° (b) 30°
(c) 75° (d) 150°

17. In the given figure, $\angle a$ is greater than one sixth of right angle, then :



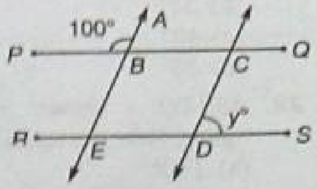
(a) $b > 165°$
(b) $b < 165°$
(c) $b \leq 165°$
(d) $b \geq 165°$

18. Let D be the mid–point of a straight line AB and let C be a point different from D such that AC = BC, then :
(a) $AC \perp AB$ (b) $\angle BDC = 90°$
(c) $\angle BDC$ is acute (d) $\angle BDC > 90°$

19. Answer on the basis of the following statements :
When two straight lines intersect, then :
1. adjacent angles are complementary
2. adjacent angles are supplementary
3. opposite angles are equal
4. opposite angles are supplementary
(a) 2 and 3 are correct (b) 1 and 4 are correct
(c) 1 and 3 are correct (d) 2 and 4 are correct

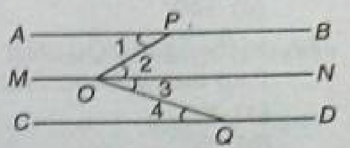
20. AB is a straight line and O is a point on AB, if a line OC is drawn not coinciding with OA or OB, then $\angle AOC$ and $\angle BOC$ are :
(a) equal (b) complementary
(c) supplementary (d) together equal to 100

21. In the adjoining figure AE || CD and BC || ED, then find Y :



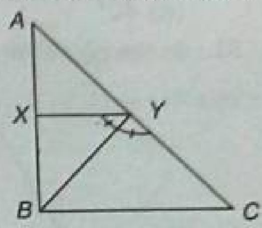
(a) 60°
(b) 80°
(c) 90°
(d) 75°

22. In the adjoining figure $\angle APO = 42°$ and $\angle CQO = 38°$. Find the value of $\angle POQ$:



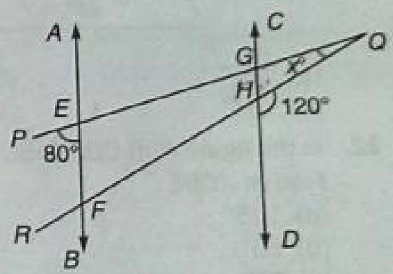
(a) 68°
(b) 72°
(c) 80°
(d) 126°

23. In a ΔABC, a line XY parallel to BC intersects AB at X and AC at Y :
If BY bisects $\angle XYC$, then $m\angle CBY : m\angle CYB$ is :



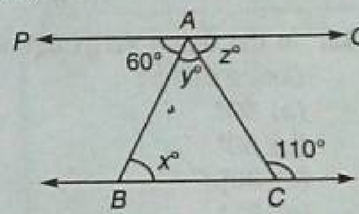
(a) 5 : 4
(b) 4 : 5
(c) 1 : 1
(d) 6 : 5

24. In the adjoining figure AB||CD and PQ, QR intersects AB and CD both at E, F and G, H respectively. Given that $m\angle PEB = 80°$, $m\angle QHD = 120°$ and $m\angle PQR = x°$, find the value of x :



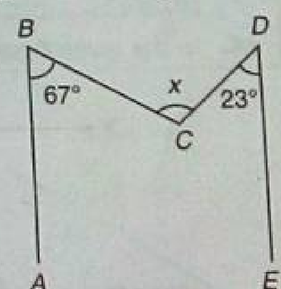
(a) 40° (b) 20°
(c) 100° (d) 30°

25. In the following figure, find the value of y :



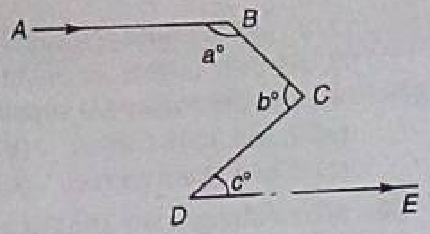
(a) 70°
(b) 60°
(c) 50°
(d) 80°

26. In the adjoining figure AB || DE, $\angle ABC = 67°$ and $\angle EDC = 23°$. Find $\angle BCD$:



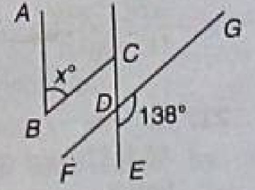
(a) 90°
(b) 44°
(c) 46°
(d) none of the above

27. In the given figure $AB \parallel DE$. Find $a° + b° - c°$:



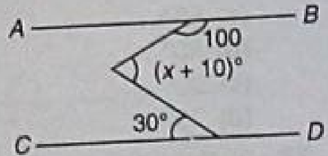
(a) 160°
(b) 120°
(c) 180°
(d) 210°

28. In the given figure $AB \parallel CE$ and $BC \parallel FG$. Find the value of $x°$:



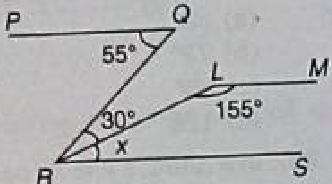
(a) 52°
(b) 32°
(c) 42°
(d) 36°

29. $AB \parallel CD$, shown in the figure. Find the value of x:



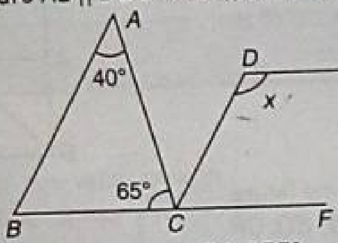
(a) 100°
(b) 90°
(c) 110°
(d) 140°

30. In the figure $PQ \parallel LM \parallel RS$. Find the value of $\angle LRS$:



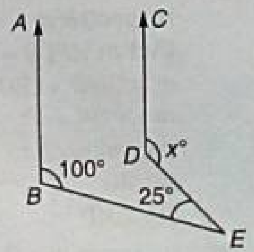
(a) 30°
(b) 25°
(c) 35°
(d) 40°

31. In the figure $AB \parallel DC$ and $DE \parallel BF$. Find the value of x:



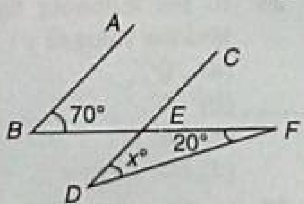
(a) 140°
(b) 155°
(c) 105°
(d) 115°

32. In the figure $AB \parallel CD$, $\angle ABE = 100$. Find $m\angle CDE$:



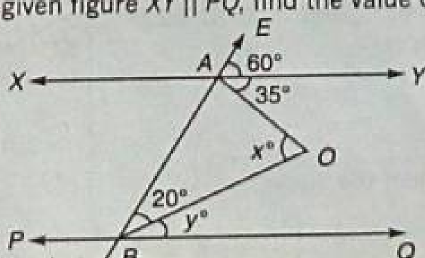
(a) 125°
(b) 55°
(c) 65°
(d) 75°

33. In the figure $AB \parallel CD$, find $x°$ (i.e., $\angle CDF$):



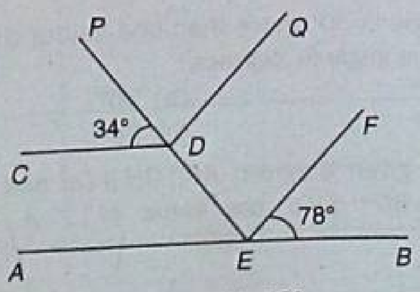
(a) 50°
(b) 90°
(c) 30°
(d) 70°

34. In the given figure $XY \parallel PQ$, find the value of x:



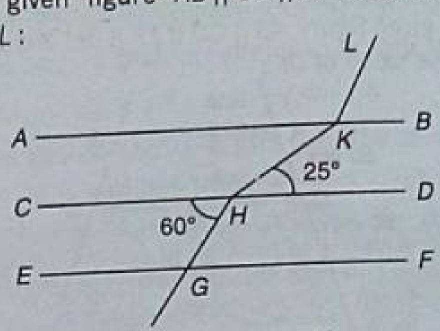
(a) 70°
(b) 40°
(c) 75°
(d) 15°

35. In the given figure $AB \parallel CD$ and $EF \parallel DQ$. Find the value of $\angle DEF$:



(a) 68°
(b) 78°
(c) 34°
(d) 39°

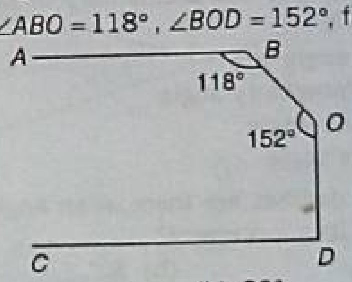
36. In the given figure $AB \parallel CD \parallel EF$ and $GH \parallel KL$. Find $m\angle HKL$:



(a) 85°
(b) 145°
(c) 120°
(d) 95°

37. Which one of the following statements is false?
(a) Two straight lines can intersect at only one point.
(b) Through a given point, only one straight line can be drawn.
(c) A line segment can be produced to any desired length.
(d) Through two given points, it is possible to draw one and only one straight line.

38. $AB \parallel CD$. $\angle ABO = 118°$, $\angle BOD = 152°$, find $\angle ODC$:



(a) 70°
(b) 80°
(c) 90°
(d) 34°

39. Two parallel lines AB and CD are intersected by a transversal EF at M and N respectively. The lines MP and NP are the bisectors of interior angles $\angle BMN$ and $\angle DNM$ on the same side of the transversal. Then $\angle MPN$ is equal to:
(a) 60°
(b) 90°
(c) 45°
(d) 120°

40. If the arms of one angle are respectively parallel to the arms of another angle, then the two angles are:
(a) either equal or supplementary
(b) neither equal not supplementary
(c) equal but not supplementary
(d) not equal but supplementary

41. AB and CD are two parallel lines. PQ cuts AB and CD at E and F respectively. EL is the bisector of $\angle BEF$. If $\angle LEB = 35°$ then $\angle CFQ$ will be :
(a) 70° (b) 55°
(c) 110° (d) 125°

42. A plane figure is bounded by straight lines only. If n is the number of these lines, then the least value of n is :
(a) 1 (b) 2
(c) 3 (d) 4

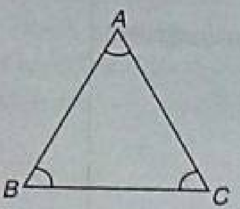
43. How many planes can pass through any three arbitrary points?
(a) 1 (b) 2
(c) 3 (d) 0

44. Out of the four arbitrary non-collinear points three points are taken at a time, then the number of planes that can be drawn through the three points is :
(a) 3 (b) 4
(c) 6 (d) 12

45. Maximum number of points of intersection of four lines on a plane is :
(a) 4 (b) 6
(c) 8 (d) 5

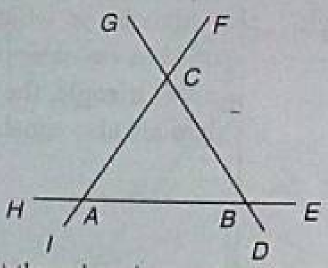
(2) TRIANGLES

Triangle : A three sided closed plane figure, which is formed by joining the three non-collinear points, is called as a triangle. It is denoted by the symbol Δ.



In the above Δ (triangle) ABC, A, B and C are three vertices, line segments AB, BC and AC are the three **sides** of the triangle. $\angle A$, $\angle B$ and $\angle C$ are the three **interior angles** of a triangle ABC.

In the adjoining figure $\angle FCB$, $\angle CBE$, $\angle ABD$, $\angle IAB$, $\angle HAC$, $\angle GCA$ are the **exterior angles** of the ΔABC.



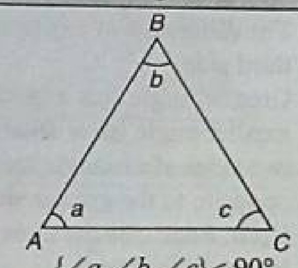
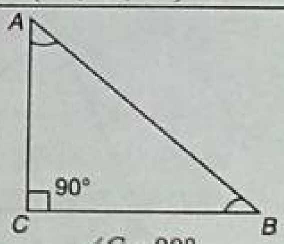
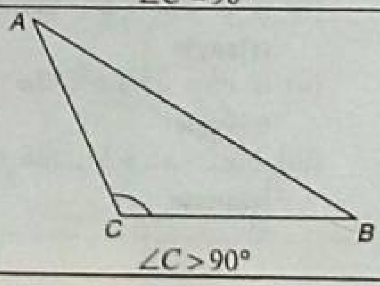
- Sum of the three interior angles of a triangle is always 180°.
- Exterior angle = Sum of two interior opposite angles
  e.g., $\angle CBE = \angle CAB + \angle BCA$

Perimeter of triangle is equal to sum of all the three sides i.e., $a + b + c$

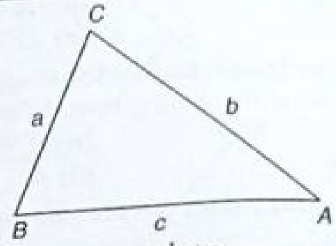
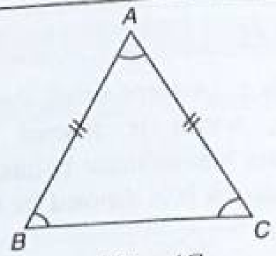
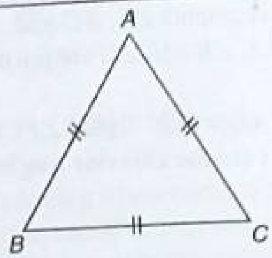
Semiperimeter of a triangle is half of the perimeter i.e., $s = \dfrac{a+b+c}{2}$, a, b, c are the length of three sides of a triangle.

Types of Triangles

(A) According to interior angles

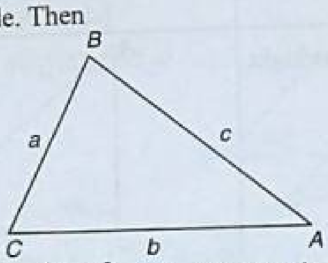
Types of Triangles	Property/Definition	Diagram
Acute angle triangle	Each of the angle of a triangle is less than 90° i.e., $a < 90°, b < 90°, c < 90°$	 $\{\angle a, \angle b, \angle c\} < 90°$
Right angled triangle	One of the angle is equal to 90°, then it is called as right angled triangle. Rest two angles are complementary to each other.	 $\angle C = 90°$
Obtuse angle triangle	One of the angle is obtuse (*i.e.*, greater than 90°), then it is called as obtuse angle triangle.	 $\angle C > 90°$

(B) According to the length of sides.

Types of triangles	Property/Definition	Diagram
Scalene triangle	A triangle in which none of the three sides is equal is called a scalene triangle (all the three angles are also different).	 $a \neq b \neq c$
Isosceles triangle	A triangles in which at least two sides are equal is called as isosceles triangle. In this triangle, the angles opposite to the congruent sides are also equal.	 $AB = AC$ $\angle B = \angle C$
Equilateral triangle	A triangle in which all the three sides are equal is called as, an equilateral triangle. In this triangle each angle is congruent and equal to 60°.	 $AB = BC = AC$ $\angle A = \angle B = \angle C = 60°$

FUNDAMENTAL PROPERTIES OF TRIANGLES

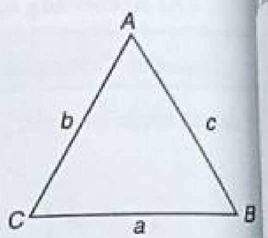
1. Sum of any two sides is always greater than the third side.
2. The difference of any two sides is always less than the third side.
3. Greater angle has a greater side opposite to it and smaller angle has a smaller side opposite to it *i.e.*, if two sides of a triangle are not congruent then the angle opposite to the greater side is greater.
4. Let a, b and c be the three sides of a ΔABC and c is the largest side. Then



   (i) if $c^2 < a^2 + b^2$, the triangle is **acute angle triangle**
   (ii) if $c^2 = a^2 + b^2$, the triangle is **right angled triangle**
   (iii) if $c^2 > a^2 + b^2$, the triangle is **obtuse angle triangle**

5. **Sine rule :** In a ΔABC, if a, b, c be the three sides opposite to the angles A, B, C respectively, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



6. **Cosine rule :** In a ΔABC, if a, b, c be the sides opposite to angle A, B and C respectively, then

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc},$$

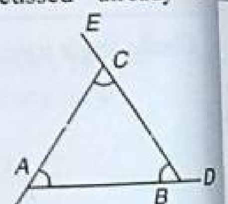
$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

(These rules have been discussed already in trigonometry.)

7. The sum of all the three interior angles is always 180°

   i.e., $\angle CAB + \angle ABC + \angle BCA = 180°$



8. The sum of three (ordered) exterior angles of a triangle is 360°

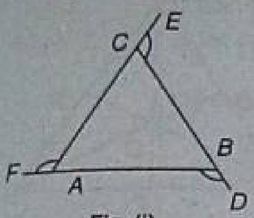


Fig. (i)

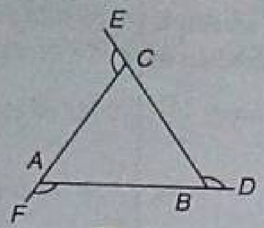
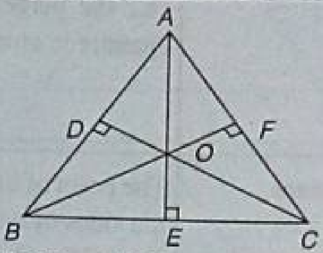
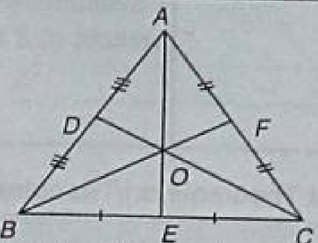
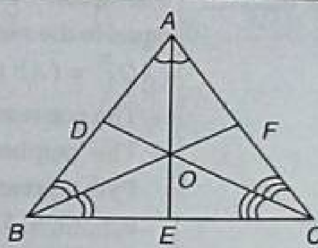
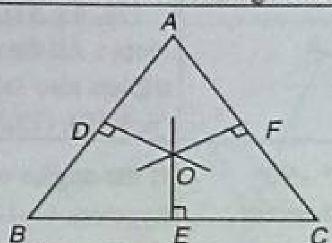
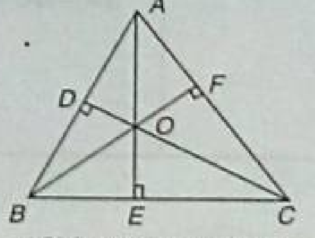


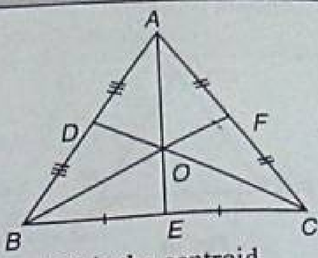
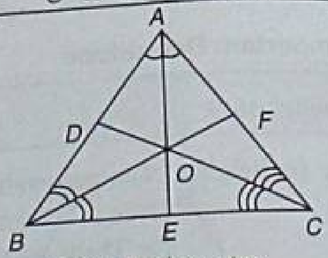
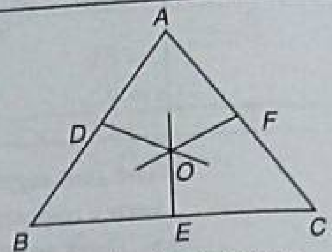
Fig. (ii)

In fig. (i) : $(\angle FAC + \angle ECB + \angle DBA) = 360°$
In fig. (ii) : $(\angle FAB + \angle DBC + \angle ECA) = 360°$

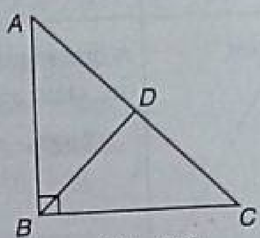
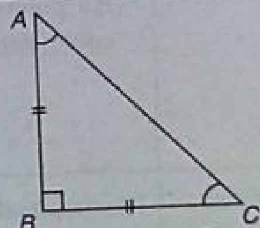
9. The sum of an interior angle and its adjacent exterior angle is 180°.
10. A triangle must has at least two acute angles.
11. In a triangle, the measure of an exterior angle equals the sum of the measures of the interior opposite angles.
12. The measure of an exterior angle of a triangle is greater than the measure of each of the opposite interior angles.

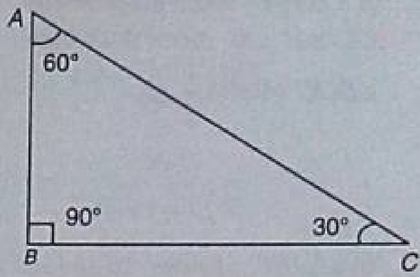
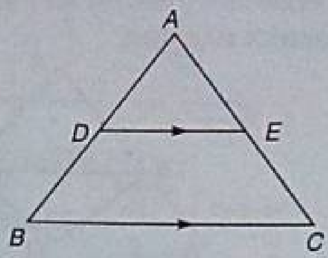
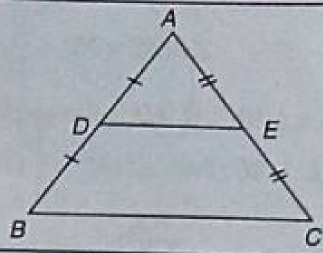
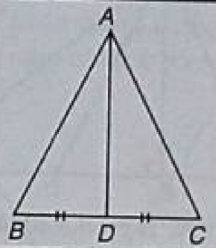
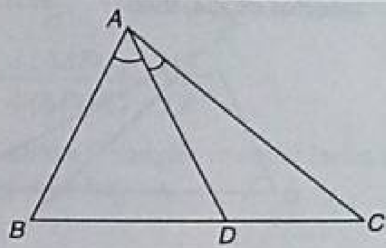
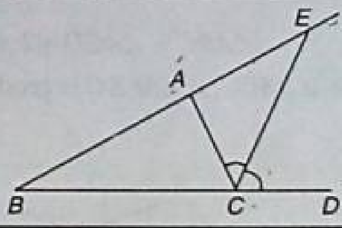
Some Important Definitions

Nomenclature	Property/Definition	Diagram
Altitude (or height)	The perpendicular drawn from the opposite vertex of a side in a triangle is called an altitude of the triangle. ➢ There are three altitudes in a triangle.	 AE, CD and BF are the altitudes
Median	The line segment joining the mid-point of a side to the vertex opposite to the side is called a median. ➢ There are three medians in a triangle. ➢ A median bisects the area of the triangle *i.e.*, $A(ABE) = A(AEC) = \frac{1}{2} A(\Delta ABC)$ etc.	 AE, CD and BF are the medians $(BE = CE,\ AD = BD,\ AF = CF)$
Angle bisector	A line segment which originates from a vertex and bisects the same angle is called an angle bisector. $\left(\angle BAE = \angle CAE = \frac{1}{2}\angle BAC\right)$ etc.	 AE, CD and BF are the angle bisectors.
Perpendicular bisector	A line segment which bisects a side perpendicularly (i.e, at right angle) is called a perpendicular bisector of a side of triangle. ➢ All points on the perpendicular bisector of a line are equidistant from the ends of the line.	 DO, EO and FO are the perpendicular bisectors.
Orthocentre	The point of intersection of the three altitudes of the triangle is called as the orthocentre. $\angle BOC = 180 - \angle A$ $\angle COA = 180 - \angle B$ $\angle AOB = 180 - \angle C$	 'O' is the orthocentre

Types of Triangles	Property/Definition	Diagram
Centroid	The point of intersection of the three medians of a triangle is called the centroid. A centroid divides each median in the ratio 2 : 1 (vertex : base) $\frac{AO}{OE}=\frac{CO}{OD}=\frac{BO}{OF}=\frac{2}{1}$	 'O' is the centroid.
Incentre	The point of intersection of the angle bisectors of a triangle is called the incentre. Incentre O is the always equidistant from all three sides *i.e.*, the perpendicular distance between the sides and incentre is always same for all the three sides.	 'O' is the incentre.
Circumcentre	The point of intersection of the perpendicular bisectors of the sides of a triangle is called the circumcentre. $OA=OB=OC=$ (circum radius) Circumcentre O is always equidistant from all the three vertices A, B and C.	 'O' is the circumcentre.

Important Theorems on Triangles

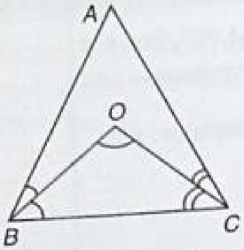
Theorem	Statement/Explanation	Diagram
Pythagoras theorem	The square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the other two sides. *i.e.*, $(AC)^2=(AB)^2+(BC)^2$ ➢ The converse of this theorem is also true. ➢ The numbers which satisfy this relation, are called Pythagorean triplets. *e.g.*, (3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (12, 35, 37), (16, 63, 65), (20, 21, 29), (28, 45, 53), (33, 56, 65) **Note :** All the multiples (or submultiples) of Pythagorean triplets also satisfy the relation. *e.g.*, (6, 8,10), (15,36, 39), (1.5, 2, 2.5) etc	 $\angle B=90^\circ$ $\overline{AC}\rightarrow$ Hypotenuse $AD=CD=BD$ (D is the mid-point of AC)
$45^\circ-45^\circ-90^\circ$ triangle theorem	If the angles of a triangle are 45°, 45° and 90°, then the hypotenuse (*i.e.*, longest side) is $\sqrt{2}$ times of any smaller side. ➢ Excluding hypotenuse rest two sides are equal. *i.e.*, $AB=BC$ and $AC=\sqrt{2}AB=\sqrt{2}BC$	 $\angle A=45^\circ$ $\angle B=90^\circ$ $\angle C=45^\circ$

Theorem	Statement/Explanation	Diagram
$30°-60°-90°$ triangle theorem	If the angles of a triangle are 30°, 60° and 90°,then the sides opposite to 30° angle is half of the hypotenuse and the side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse . *e.g.,* $AB=\frac{AC}{2}$ and $BC=\frac{\sqrt{3}}{2}AC$ $\therefore \quad AB:BC:AC=1:\sqrt{3}:2$	
Basic proportionality theorem (BPT) or Thales theorem	Any line parallel to one side of a triangle divides the other two sides proportionally. So if DE is drawn parallel to BC, it would divide sides AB and AC proportionally *i.e.,* $\frac{AD}{DB}=\frac{AE}{EC}$ or $\frac{AD}{AB}=\frac{AE}{AC}$ $\frac{AD}{DE}=\frac{AB}{BC}=\frac{AE}{DE}=\frac{AC}{BC}$	
Mid-point theorem	If the mid-points of two adjacent sides of a triangle are joined by a line segment, then this segment is parallel to the third side. *i.e.,* if $AD=BD$ and $AE=CE$ then $DE \parallel BC$	
Apollonius theorem	In a triangle, the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side. *i.e.,* $AB^2+AC^2=2(AD^2+BD^2)$	 $BD=CD$ AD is the median
Interior angle bisector theorem	In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. *i.e.,* $\frac{BD}{CD}=\frac{AB}{AC}$ and $BD\times AC-CD\times AB=AD^2$	
Exterior angle bisector theorem	In a triangle the angle bisector of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides *i.e.,* $\frac{BE}{AE}=\frac{BC}{AC}$	

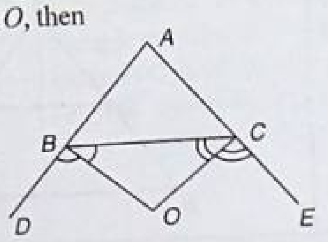
SOME USEFUL RESULTS

1. In a ΔABC, if the bisectors of $\angle B$ and $\angle C$ meet at O then

$$\angle BOC = 90^\circ + \frac{1}{2}\angle A$$

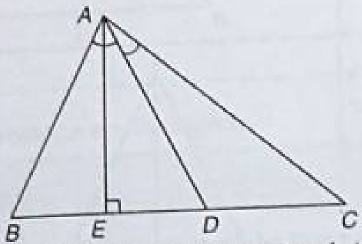


2. In a ΔABC, if sides AB and AC are produced to D and E respectively and the bisectors of $\angle DBC$ and $\angle ECB$ intersect at O, then

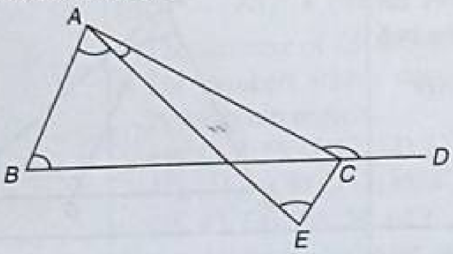


$$\angle BOC = 90^\circ - \frac{1}{2}\angle A$$

3. In a ΔABC, if AD is the angle bisector of $\angle BAC$ and $AE \perp BC$, then $\angle DAE = \frac{1}{2}(\angle ABC - \angle ACB)$

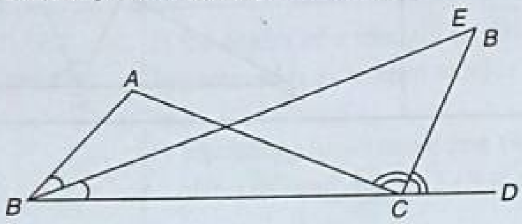


4. In a ΔABC, if BC is produced to D and AE is the angle bisector of $\angle A$, then



$$\angle ABC + \angle ACD = 2\angle AEC.$$

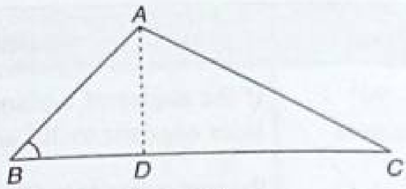
5. In a ΔABC, if side BC is produced to D and bisectors



of $\angle ABC$ and $\angle ACD$ meet at E, then

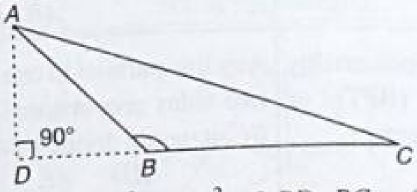
$$\angle BEC = \frac{1}{2}\angle BAC$$

6. In an acute angle ΔABC, AD is a perpendicular dropped on the opposite side of $\angle A$, then



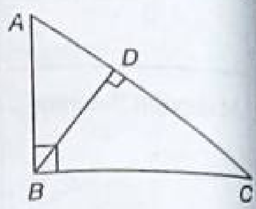
$$AC^2 = AB^2 + BC^2 - 2BD \cdot BC \quad (\angle B < 90^\circ)$$

7. In an obtuse angle ΔABC, AD is perpendicular dropped on BC. BC is produced to D to meet AD, then



$$AC^2 = AB^2 + BC^2 + 2\,BD \cdot BC \quad (\angle B > 90^\circ)$$

8. In a right angle ΔABC, $\angle B = 90^\circ$ and AC is hypotenuse. The perpendicular BD is dropped on hypotenuse AC from right angle vertex B, then



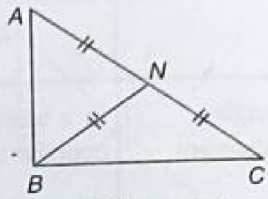
(i) $BD = \dfrac{AB \times BC}{AC}$ (ii) $AD = \dfrac{AB^2}{AC}$

(iii) $CD = \dfrac{BC^2}{AC}$ (iv) $\dfrac{1}{BD^2} = \dfrac{1}{AB^2} + \dfrac{1}{BC^2}$

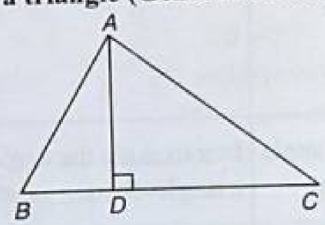
- In a right angled triangle, the median to the hypotenuse $= \frac{1}{2} \times$ hypotenuse

i.e., $BN = \dfrac{AC}{2}$

(as per the fig.)



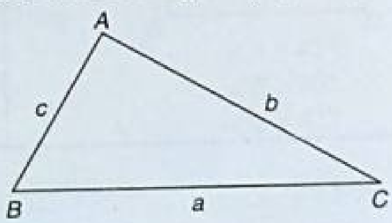
9. **Area of a triangle (General formula)**



$$A(\Delta) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$A(\Delta) = \frac{1}{2} \times BC \times AD \quad \text{(as per the figure.)}$$

10. **Area of scalene triangle** $= \sqrt{s(s-a)(s-b)(s-c)}$



Also, $A(\Delta) = r \times s = \dfrac{abc}{4R}$

where a, b and c are the sides of the triangle.

$s \rightarrow$ semiperimeter $= \dfrac{a+b+c}{2}$

$r \rightarrow$ inradius

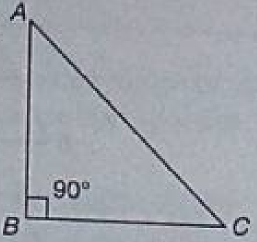
$R \rightarrow$ circumradius

11. **Area of right angled triangle**

$= \dfrac{1}{2} \times$ base $\times$ height

$= \dfrac{1}{2} \times BC \times AB$

(as per the figure)



12. **Area of an isosceles triangle**

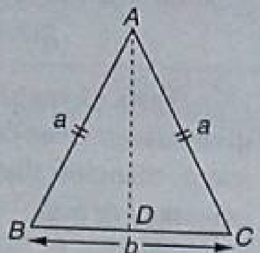
$= \dfrac{b}{4}\sqrt{4a^2 - b^2}$

$AB = AC$

and $\angle B = \angle C$

$\Delta ABD \cong \Delta ACD$

($AD \rightarrow$ Angle bisector, median, altitude and perpendicular bisector)

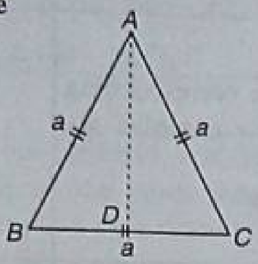


13. **Area of an equilateral triangle**

$= \dfrac{\sqrt{3}}{4}a^2$

$\left[A(\Delta) = \dfrac{1}{2} BC \times AD\right.$

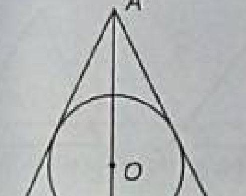
$\left. = \dfrac{1}{2} \times a \times \dfrac{\sqrt{3}}{2}a = \dfrac{\sqrt{3}}{4}a^2\right]$

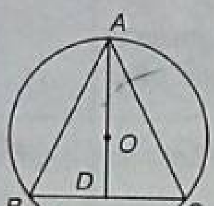


($a \rightarrow$ each side of the triangle)

$AD \rightarrow$ Altitude, median, angle bisector and perpendicular bisector also.

Inradius : $\dfrac{1}{3} \times$ height $= \dfrac{\text{side}}{2\sqrt{3}}$, $OD \rightarrow$ Inradius





Circumradius $= \dfrac{2}{3} \times$ height $= \dfrac{\text{side}}{\sqrt{3}}$

$OA \rightarrow$ Circumradius

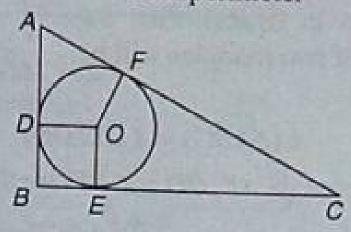
NOTE In equilateral triangle orthocentre centroid, incentre and circumcentre coincide at the same point.

- Circumradius $= 2 \times$ inradius
- For the given perimeter of a triangle, the area of equilateral triangle is maximum.
- For the given area of a triangle, the perimeter of equilateral triangle is minimum.

14. In a right angled triangle

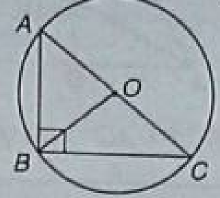
(i) Inradius $(r) = \dfrac{AB + BC - AC}{2}$

(ii) Inradius $(r) = \dfrac{\text{Area}}{\text{Semiperimeter}}$



$DO = EO = FO\ (r)$

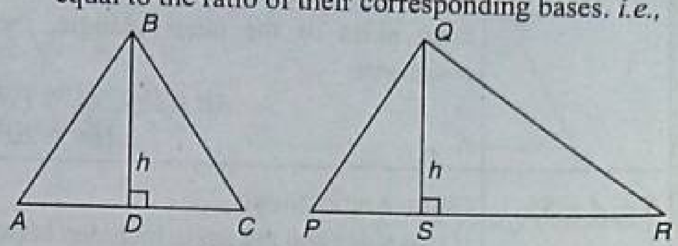
(iii) Circumradius $(R) = \dfrac{AC}{2} = \left(\dfrac{\text{hypotenuse}}{2}\right)$



$AO = CO = BO = (R)$

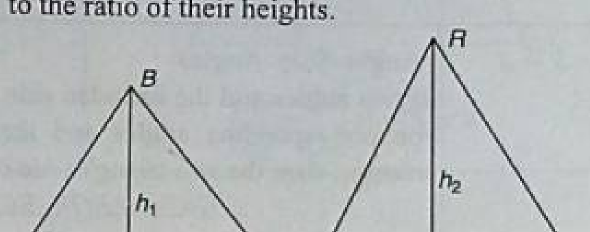
AC is the diameter.

15. The ratio of areas of two triangles of equal heights is equal to the ratio of their corresponding bases. *i.e.*,



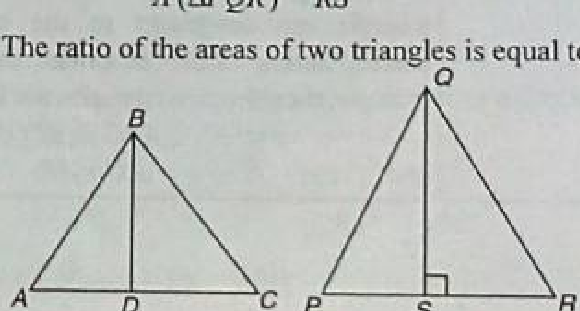
$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AC}{PR}$$

16. The ratio of areas of triangles of equal bases is equal to the ratio of their heights.



i.e., $$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BD}{RS}$$

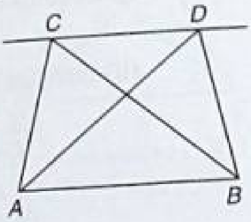
17. The ratio of the areas of two triangles is equal to the



ratio of the products of base and its corresponding height *i.e.*,

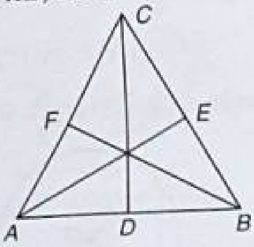
$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AC \times BD}{PR \times QS}$$

18. If the two triangles have the same base and lie between the same parallel lines (as shown in figure), then the area of two triangles will be equal.



i.e., $A(\Delta ABC) = A(\Delta ADB)$

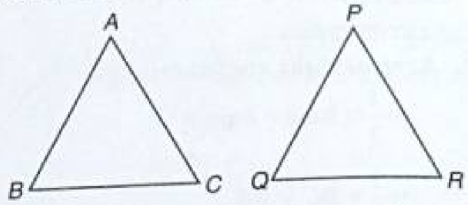
19. In a triangle AE, CD and BF are the medians then



$$3(AB^2 + BC^2 + AC^2) = 4(CD^2 + BF^2 + AE^2)$$

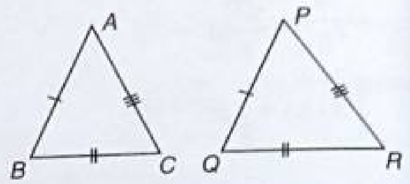
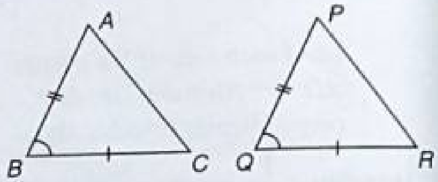
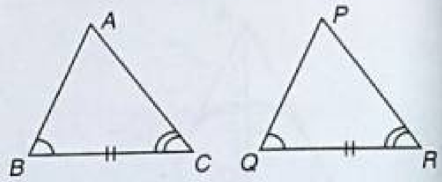
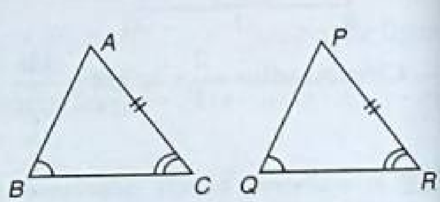
Congruency of triangles : Two triangles are said to be congruent if they are equal in all respects. *i.e.*,

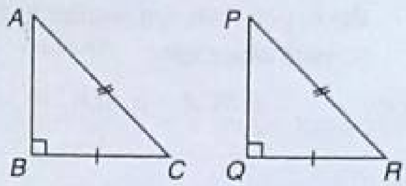
1. Each of the three sides of one triangle must be equal to the three respective sides of the other.
2. Each of the three angles of the one triangle must be equal to the three respective angles of the other.



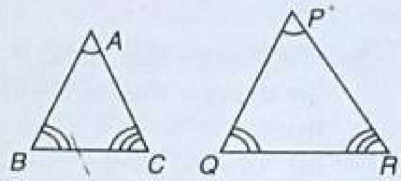
i.e., $\left.\begin{array}{l} AB = PQ \\ AC = PR \\ BC = QR \end{array}\right\}$ and $\left.\begin{array}{l} \angle A = \angle P \\ \angle B = \angle Q \\ \angle C = \angle R \end{array}\right\}$

Tests of congruency : With the help of the following given tests, we can deduce without having detailed information about triangles that whether the given two triangles are congruent or not.

Test	**Property**	**Diagram**
$S-S-S$	(Side–Side–Side) If the three sides of one triangle are equal to the corresponding three sides of the other triangle, then the two triangles are congruent. $AB \cong PQ, AC \cong PR, BC \cong QR$ $\therefore$ $\Delta ABC \cong \Delta PQR$	
$S-A-S$	(Side–Angle–Side) If two sides and the angle included between them are congruent to the corresponding sides and the angle included between them, of the other triangle then the two triangles are congruent. $AB \cong PQ, \angle ABC \cong \angle PQR, BC \cong QR$ $\therefore$ $\Delta ABC \cong \Delta PQR$	
$A-S-A$	(Angle–Side–Angle) If two angles and the included side of a triangle are congruent to the corresponding angles and the included side of the other triangle, then the two triangles are congruent. $\angle ABC \cong \angle PQR, BC \cong QR, \angle ACB \cong \angle PRQ$ $\therefore$ $\Delta ABC \cong \Delta PQR$	
$A-A-S$	(Angle–Angle–Side) If two angles and a side other than the included side of a one triangle are congruent to the corresponding angles and a corresponding side other than the included side of the other triangle, then the two triangles are congruent. $\angle ABC \cong \angle PQR, \angle ACB \cong \angle PRQ$ and $AC \cong PR$ (or $AB \cong PQ$)	

Test	Property	Diagram
R–H–S	(Right angle–Hypotenuse–Side) If the hypotenuse and one side of the right angled triangle are congruent to the hypotenuse and a corresponding side of the other right angled triangle, then the two given triangles are congruent. $AC \cong PR$, $\angle B = \angle Q$ and $BC \cong QR$ $\therefore \Delta ABC \cong \Delta PQR$	

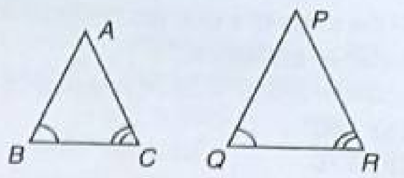
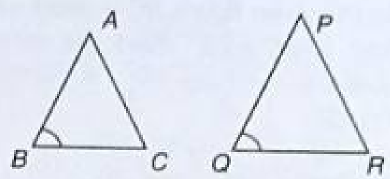
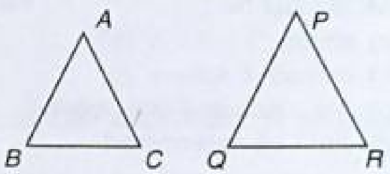
Similarity of triangles : Two triangles are said to be similar if the corresponding **angles** are congruent and their corresponding **sides** are in **proportion**. The symbol for similarity is '~'.



If $\Delta ABC \sim \Delta PQR$ then

$\angle ABC \cong \angle PQR$, $\angle BCA \cong \angle QRP$, $\angle BAC \cong \angle QPR$

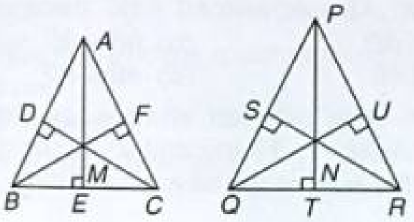
Tests for Similarity : Through the tests for similarity we can deduce the similarity of triangles with minimum required information.

Test	Property/Definition	Diagram
A – A	Angle-Angle If the two angles of one triangle are congruent to the corresponding two angles of the other triangle, then the two triangles are said to be similar $\angle ABC \cong \angle PQR$ $\angle ACB \cong \angle PRQ$ $\therefore$ $\Delta ABC \sim \Delta PQR$	
S-A-S	Side-Angle-Side If the two sides of one triangle are proportional to the corresponding two sides of the other triangle and the angle included by them are congruent, then the two triangles are similar. *i.e.*, $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle ABC = \angle PQR$ $\therefore$ $\Delta ABC \sim \Delta PQR$.	 $\frac{AB}{PQ} = \frac{BC}{QR} = K$ (K is any constant)
S-S-S	Side-Side-Side If the three sides of one triangle are proportional to the corresponding three sides of the other triangle, then the two triangles are similar. *i.e.*, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ $\therefore$ $\Delta ABC \sim \Delta PQR$	 $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = K$

NOTE When the corresponding sides are in proportion , then the corresponding angles are in proportion.

PROPERTIES OF SIMILAR TRIANGLES

If the two triangles are similar, then for the proportional/corresponding sides we have the following results.



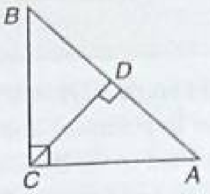
1. Ratio of sides = Ratio of heights (altitudes)
   = Ratio of medians
   = Ratio of angle bisectors
   = Ratio of inradii
   = Ratio of circumradii
2. Ratio of areas = Ratio of squares of corresponding sides.
   i.e., if $\Delta ABC \sim \Delta PQR$, then

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

NOTE Rule 1 can also apply with rule 2.

3. In a right angled triangle, the triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other too.

i.e., $\Delta BCA \sim \Delta BDC \sim \Delta CDA$.

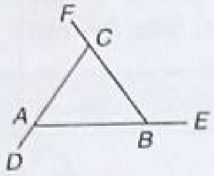


INTRODUCTORY EXERCISE-12.2

1. In a triangle ABC, if AB, BC and AC are the three sides of the triangle, then which of the statements is necessarily true?
   (a) $AB + BC < AC$ (b) $AB + BC > AC$
   (c) $AB + BC = AC$ (d) $AB^2 + BC^2 = AC^2$

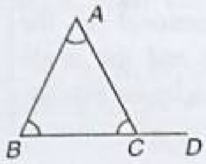
2. The sides of a triangle are 12 cm, 8 cm and 6 cm respectively, the triangle is :
   (a) acute (b) obtuse
   (c) right (d) can't be determined

3. If the sides of a triangle are produced then the sum of the exterior angles *i.e*, $\angle DAB + \angle EBC + \angle FCA$ is equal to :



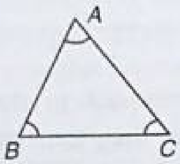
   (a) 180°
   (b) 270°
   (c) 360°
   (d) 240°

4. In the given figure BC is produced to D and $\angle BAC = 40°$ and $\angle ABC = 70°$. Find the value of $\angle ACD$:



   (a) 30°
   (b) 40°
   (c) 70°
   (d) 110°

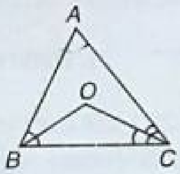
5. In a ΔABC, $\angle BAC > 90°$, then $\angle ABC$ and $\angle ACB$ must be :



   (a) acute
   (b) obtuse
   (c) one acute and one obtuse
   (d) can't be determined

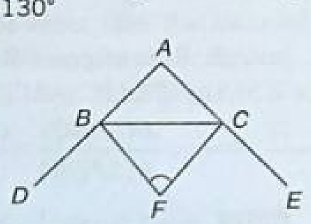
6. If the angles of a triangle are in the ratio 1 : 4 : 7, then the value of the largest angle is :
   (a) 135° (b) 84°
   (c) 105° (d) none of these

7. In the adjoining figure $\angle B = 70°$ and $\angle C = 30°$. BO and CO are the angle bisectors of $\angle ABC$ and $\angle ACB$. Find the value of $\angle BOC$:



   (a) 30° (b) 40°
   (c) 120° (d) 130°

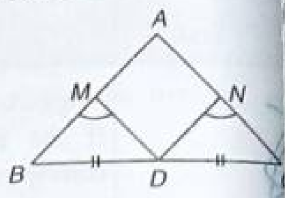
8. In the given diagram of ΔABC, $\angle B = 80°$, $\angle C = 30°$. BF and CF are the angle bisectors of $\angle CBD$ and $\angle BCE$ respectively. Find the value of $\angle BFC$:



   (a) 110° (b) 50°
   (c) 125° (d) 55°

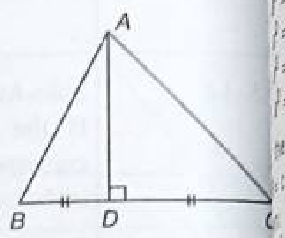
9. In an equilateral triangle, the incentre, circumcentre, orthocentre and centroid are :
   (a) concylic (b) coincident
   (c) collinear (d) none of these

10. In the adjoining figure D is the midpoint of BC of a ΔABC. DM and DN are the perpendiculars on AB and AC respectively and $DM = DN$, then the ΔABC is :



   (a) right angled (b) isosceles
   (c) equilateral (d) scalene

11. In the adjoining figure of ΔABC, AD is the perpendicular bisector of side BC. The triangle ABC is :

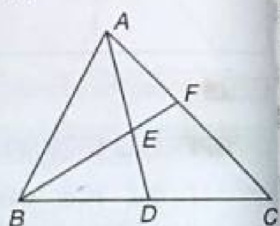


   (a) right angled
   (b) isosceles
   (c) scalene
   (d) equilateral

12. Triangle ABC is such that $AB = 9$ cm, $BC = 6$ cm, $AC = 7.5$ cm. Triangle DEF is similar to ΔABC. If $EF = 1$ cm then DE is :
    (a) 6 cm (b) 16 cm
    (c) 18 cm (d) 15 cm

13. In ΔABC, $AB = 5$ cm, $AC = 7$ cm. If AD is the angle bisector of $\angle A$. Then $BD : CD$ is :
    (a) 25 : 49 (b) 49 : 25
    (c) 6 : 1 (d) 5 : 7

14. In a ΔABC, D is the mid-point of BC and E is mid-point of AD, BF passes through E. What is the ratio of $AF : FC$?



    (a) 1 : 1
    (b) 1 : 2
    (c) 1 : 3
    (d) 2 : 3

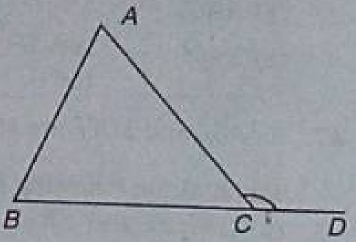
15. In a ΔABC, $AB = AC$ and $AD \perp BC$, then :
    (a) $AB < AD$ (b) $AB > AD$
    (c) $AB = AD$ (d) $AB \leq AD$

16. The difference between altitude and base of a right angled triangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle is :

(a) 24 cm (b) 31 cm
(c) 34 cm (d) can't be determined

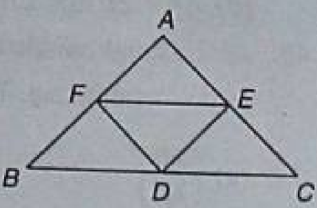
17. If AB, BC and AC be the three sides of a triangle ABC, then which one of the following is true?
(a) $AB - BC = AC$ (b) $(AB - BC) > AC$
(c) $(AB - BA) < AC$ (d) $AB^2 - BC^2 = AC^2$

18. In the triangle ABC, side BC is produced to D. $\angle ACD = 100°$ if $BC = AC$, then $\angle ABC$ is :



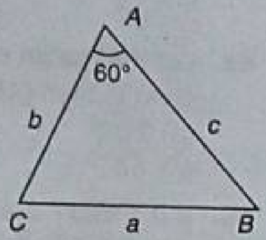
(a) $40°$
(b) $50°$
(c) $80°$
(d) can't be determined

19. In the adjoining figure D, E and F are the mid-points of the sides BC, AC and AB respectively. ΔDEF is congruent to triangle :



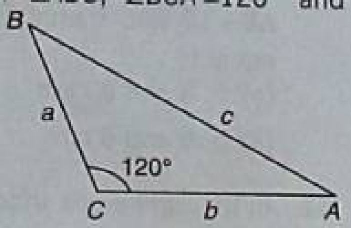
(a) ABC
(b) AEF
(c) CDE, BFD
(d) AFE, BFD and CDE

20. In the adjoining figure $\angle BAC = 60°$ and $BC = a$, $AC = b$ and $AB = c$, then :



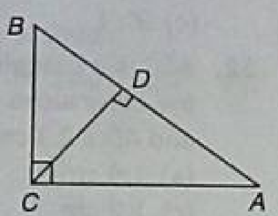
(a) $a^2 = b^2 + c^2$
(b) $a^2 = b^2 + c^2 - bc$
(c) $a^2 = b^2 + c^2 + bc$
(d) $a^2 = b^2 + 2bc$

21. In the adjoining figure of ΔABC, $\angle BCA = 120°$ and $AB = c$, $BC = a$, $AC = b$ then:



(a) $c^2 = a^2 + b^2 + ba$
(b) $c^2 = a^2 + b^2 - ba$
(c) $c^2 = a^2 + b^2 - 2ba$
(d) $c^2 = a^2 + b^2 + 2ab$

22. In a right angled ΔABC, $\angle C = 90°$ and CD is the perpendicular on the hypotenuse AB, $AB = c$, $BC = a$, $AC = b$ and $CD = p$, then :



(a) $\frac{p}{a} = \frac{p}{b}$
(b) $\frac{1}{p^2} + \frac{1}{b^2} = \frac{1}{a^2}$
(c) $p^2 = b^2 + c^2$
(d) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

23. If the medians of a triangle are equal, then the triangle is :
(a) right angled (b) isoscles
(c) equilateral (d) scalene

24. The incentre of a triangle is determined by the :
(a) medians
(b) angle bisector
(c) perpendicular bisectors
(d) altitudes

25. The circumcentre of a triangle is determined by the :
(a) altitudes
(b) median
(c) perpendicular bisector
(d) angle bisectors

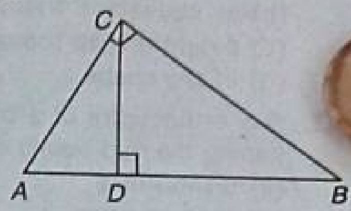
26. The point of intersection of the angle bisectors of a triangle is :
(a) orthocentre (b) centroid
(c) incentre (d) circumcentre

27. A triangle PQR is formed by joining the mid-points of the sides of a triangle ABC. 'O' is the circumcentre of ΔABC, then for ΔPQR, the point 'O' is :
(a) incentre (b) circumcentre
(c) orthocentre (d) centroid

28. If in a ΔABC, 'S' is the circumcentre then :
(a) S is equidistant from all the vertices of a triangle
(b) S is equidistant from all the sides of a triangle
(c) AS, BS and CS are the angular bisectors
(d) AS, BS and CS produced are the altitudes on the opposite sides.

29. If AD, BE, CF are the altitudes of ΔABC whose orthocentre is H, then C is the orthocentre of :
(a) ΔABH (b) ΔBDH
(c) ΔABD (d) ΔBEA

30. In a right angled ΔABC, $\angle C = 90°$ and CD is the perpendicular on hypotenuse AB if $BC = 15$ cm and $AC = 20$ cm then CD is equal to :



(a) 18 cm (b) 12 cm
(c) 17.5 cm (d) can't be determined

31. In an equilateral ΔABC, if a, b and c denote the lengths of perpendiculars from A, B and C respectively on the opposite sides, then :
(a) $a > b > c$ (b) $a > b < c$
(c) $a = b = c$ (d) $a = c \neq b$

32. What is the ratio of side and height of an equilateral triangle?
(a) $2 : 1$ (b) $1 : 1$
(c) $2 : \sqrt{3}$ (d) $\sqrt{3} : 2$

33. The triangle is formed by joining the mid-points of the sides AB, BC and CA of ΔABC and the area of ΔPQR is 6 cm^2, then the area of ΔABC is :
(a) 36 cm^2 (b) 12 cm^2
(c) 18 cm^2 (d) 24 cm^2

34. One side other than the hypotenuse of right angle isosceles triangle is 6 cm. The length of the perpendicular on the hypotenuse from the opposite vertex is :
(a) 6 cm (b) $6\sqrt{2}$ cm
(c) 4 cm (d) $3\sqrt{2}$ cm

35. Any two of the four triangles formed by joining the mid-points of the sides of a given triangle are :
(a) congruent

(b) equal in area but not congruent
(c) unequal in area and not congruent
(d) none of the above

36. The internal bisectors of $\angle B$ and $\angle C$ of ΔABC meet at O. If $\angle A = 80°$ then $\angle BOC$ is :
(a) 50° (b) 160°
(c) 100° (d) 130°

37. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :
(a) centroid (b) incentre
(c) circumcenre (d) orthocentre

38. Incentre of a triangle lies in the interior of :
(a) an isosceles triangle only
(b) a right angled triangle only
(c) any equilateral triangle only
(d) any triangle

39. In a triangle PQR, $PQ = 20$ cm and $PR = 6$ cm, the side QR is :
(a) equal to 14 cm (b) less than 14 cm
(c) greater than 14 cm (d) none of these

40. The four triangles formed by joining the pairs of mid-points of the sides of a given triangle are congruent if the given triangle is :
(a) an isosceles triangle
(b) an equilateral triangle
(c) a right angled triangle
(d) of any shape

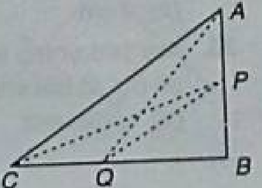
41. O is orthocentre of a triangle PQR, which is formed by joining the mid-points of the sides of a ΔABC, O is :
(a) orthocentre (b) incentre
(c) circumcentre (d) centroid

42. In a ΔABC, a line PQ parallel to BC cuts AB at P and AC at Q. If BQ bisects $\angle PQC$, then which one of the following relation is always true :
(a) $BC = CQ$ (b) $BC = BQ$
(c) $BC \neq CQ$ (d) $BC \neq BQ$

43. Which of the following is true, in the given figure, where AD is the altitude to the hypotenuse of a right angled ΔABC?
(i) ΔCAD and ΔABD are similar
(ii) ΔCDA and ΔADB are congruent.
(iii) ΔADB and ΔCAB are similar
Select the correct answer using the codes given below :
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

44. If D is such a point on the side, BC of ΔABC that $\frac{AB}{AC} = \frac{BD}{CD}$, then AD must be a/an :
(a) altitude of ΔABC
(b) median of ΔABC
(c) angle bisector of ΔABC
(d) perpendicular bisector of ΔABC

45. In right angled ΔABC, $\angle B = 90°$, if P and Q are points on the sides AB and BC respectively, then :



(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$
(b) $AQ^2 + CP^2 = AC^2 + PQ^2$
(c) $(AQ^2 + CP^2) = \frac{1}{2}(AC^2 + PQ^2)$
(d) $(AQ + CP) = \frac{1}{2}(AC + PQ)$

46. If ABC is a right angled triangle at B and M, N are the mid-points of AB and BC, then $4(AN^2 + CM^2)$ is equal to :
(a) $4AC^2$ (b) $6AC^2$
(c) $5AC^2$ (d) $\frac{5}{4}AC^2$

47. If ΔABC and ΔDEF are so related that $\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}$, then which of the following is true ?
(a) $\angle A = \angle F$ and $\angle B = \angle D$
(b) $\angle C = \angle F$ and $\angle A = \angle D$
(c) $\angle B = \angle F$ and $\angle C = \angle D$
(d) $\angle A = \angle E$ and $\angle B = \angle D$

48. ABC is a right angle triangle at A and AD is perpendicular to the hypotenuse. Then $\frac{BD}{CD}$ is equal to :
(a) $\left(\frac{AB}{AC}\right)^2$ (b) $\left(\frac{AB}{AD}\right)^2$
(c) $\frac{AB}{AC}$ (d) $\frac{AB}{AD}$

49. Let ABC be an equilateral triangle. Let $BE \perp CA$ meeting CA at E, then $(AB^2 + BC^2 + CA^2)$ is equal to :
(a) $2BE^2$ (b) $3BE^2$
(c) $4BE^2$ (d) $6BE^2$

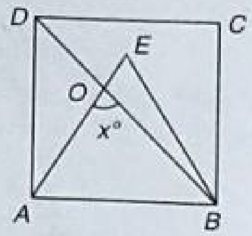
50. If D, E and F are respectively the mid-points of sides BC, CA and AB of a ΔABC. If $EF = 3$ cm, $FD = 4$ cm and $AB = 10$ cm, then DE, BC and CA respectively will be equal to :
(a) 6, 8 and 20 cm (b) 4, 6 and 8 cm
(c) 5, 6 and 8 cm (d) $\frac{10}{3}$, 9 and 12 cm

51. In the right angle triangle $\angle C = 90°$. AE and BD are two medians of a triangle ABC meeting at F. The ratio of the area of ΔABF and the quadrilateral $FDCE$ is :
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 2 : 3

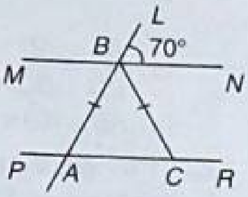
52. ABC is a triangle and DE is drawn parallel to BC cutting the other sides at D and E. If $AB = 3.6$ cm, $AC = 2.4$ cm and $AD = 2.1$ cm, then AE is equal to :
(a) 1.4 cm (b) 1.8 cm
(c) 1.2 cm (d) 1.05 m

53. Consider the following statements :
(1) If three sides of a triangle are equal to three sides of another triangle, then the triangles are congruent.
(2) If three angles of a triangle are equal to three angles of another triangle respectively, then the two triangles are congruent. Of these statements :
(a) 1 is correct and 2 is false
(b) both 1and 2 are false
(c) both 1 and 2 are correct
(d) 1 is false and 2 is correct

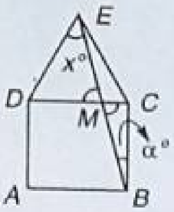
54. In the figure ΔABE is an equilateral triangle in a square $ABCD$. Find the value of angle x in degrees :
(a) 60°
(b) 45°
(c) 75°
(d) 90°



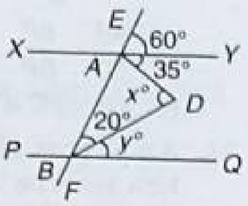
55. In the given diagram $MN \parallel PR$ and $m\angle LBN = 70^\circ$, $AB = BC$. Find $m\angle ABC$:
(a) 40°
(b) 30°
(c) 35°
(d) 55°



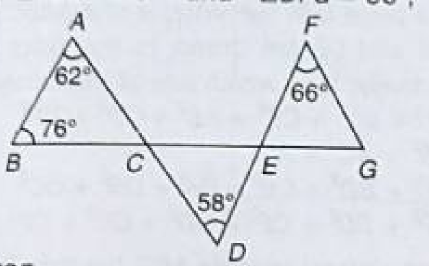
56. In the given diagram, equilateral triangle EDC surmounts square $ABCD$. Find the $m\angle BED$ represented by x. Where $m\angle EBC = \alpha^\circ$:
(a) 45°
(b) 60°
(c) 30°
(d) none of the above



57. In the given diagram $XY \parallel PQ$. Find $m\angle x^\circ$ and $m\angle y^\circ$:
(a) 75° and 40°
(b) $45^\circ, 60^\circ$
(c) $75^\circ, 45^\circ$
(d) 60° and 45°

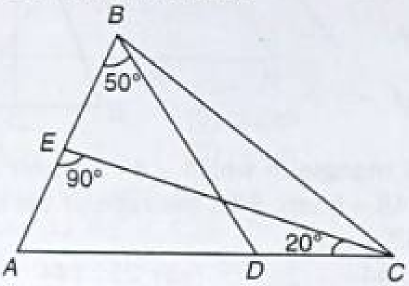


58. In the adjoining figure $m\angle CAB = 62^\circ$, $m\angle CBA = 76^\circ$
$m\angle ADE = 58^\circ$ and $\angle DFG = 66^\circ$,



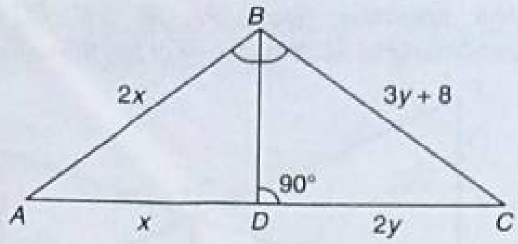
Find $m\angle FGE$:
(a) 44° (b) 34°
(c) 36° (d) none of these

59. In the given figure $CE \perp AB$, $m\angle ACE = 20^\circ$ and $m\angle ABD = 50^\circ$. Find $m\angle BDA$:



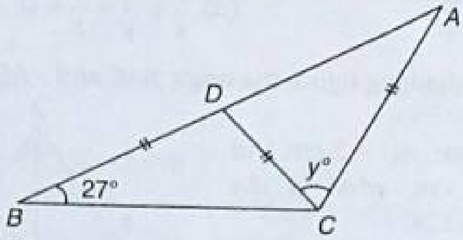
(a) 50° (b) 60°
(c) 70° (d) 80°

60. In the ΔABC, BD bisects $\angle B$, and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the value of x and y :



(a) 6, 12 (b) 10, 12
(c) 16, 8 (d) 8, 15

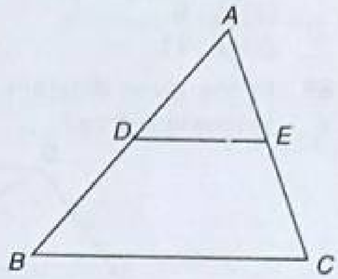
61. In the following figure $ADBC$. $BD = CD = AC$, $m\angle ABC = 27^\circ$, $m\angle ACD = y$. Find the value of y :



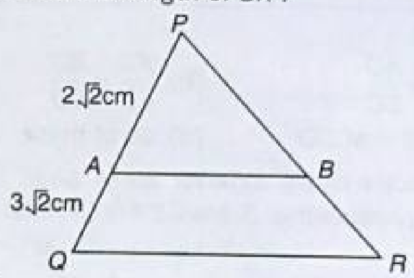
(a) 27° (b) 54°
(c) 72° (d) 58°

62. ABC is an isosceles triangle with $AB = AC$. Side BA is produced to D such that $AB = AD$. Find $m\angle BCD$:
(a) 60° (b) 90°
(c) 120° (d) can't be determined

63. In ΔABC, $AC = 5$ cm. Calculate the length of AE where $DE \parallel BC$. Given that $AD = 3$ cm and $BD = 7$ cm :
(a) 2 cm
(b) 1 cm
(c) 1.5 cm
(d) 2.5 cm

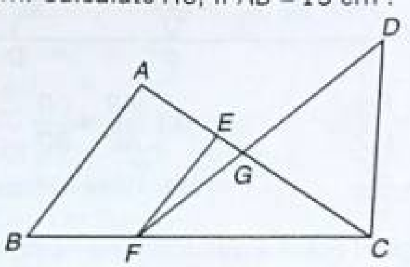


64. In ΔPQR, $AP = 2\sqrt{2}$ cm, $AQ = 3\sqrt{2}$ cm and $PR = 10$ cm, $AB \parallel QR$. Find the length of BR :



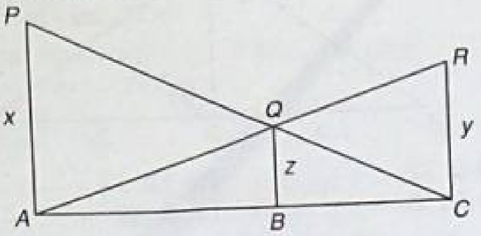
(a) $6\sqrt{2}$ cm (b) 6 cm
(c) $5\sqrt{2}$ cm (d) none of these

65. In the adjoining figure (not drawn to scale) AB, EF and CD are parallel lines. Given that $EG = 5$ cm, $GC = 10$ cm and $DC = 18$ cm. Calculate AC, if $AB = 15$ cm :



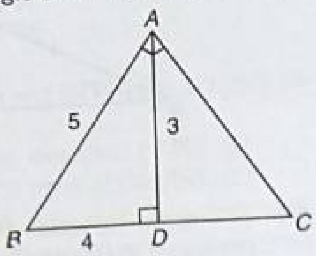
(a) 21 cm (b) 25 cm
(c) 18 cm (d) 28 cm

66. In the adjoining figure PA, QB and RC are each perpendicular to AC. Which one of the following is true :



(a) $x + y = z$ (b) $xy = 2z$
(c) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (d) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

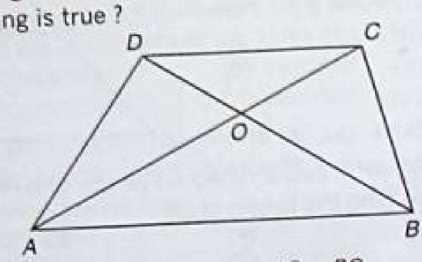
67. In the adjoining figure the angle BAC and $\angle ADB$ are right angles.
$BA = 5$ cm, $AD = 3$ cm and $BD = 4$ cm, what is the length of DC ?



(a) 2.5
(b) 3
(c) 2.25
(d) 2

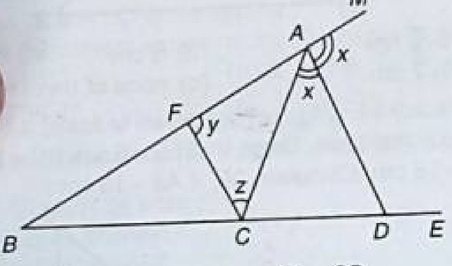
68. The areas of the similar triangles are in the ratio of 25 : 36. What is the ratio of their respective heights?
(a) 5 : 6 (b) 6 : 5
(c) 1 : 11 (d) 2 : 3

69. In the given diagram $AB \parallel CD$, then which one of the following is true ?



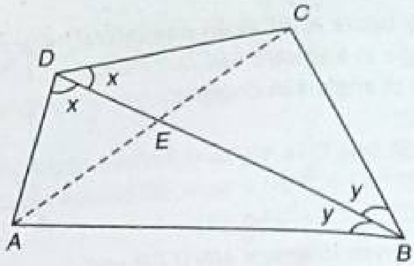
(a) $\frac{AB}{CD} = \frac{AO}{OC}$ (b) $\frac{AB}{CD} = \frac{BO}{OD}$
(c) $\Delta AOB \sim \Delta COD$ (d) all of these

70. The bisector of the exterior $\angle A$ of ΔABC intersects the side BC produced to D. Here CF is parallel to AD.



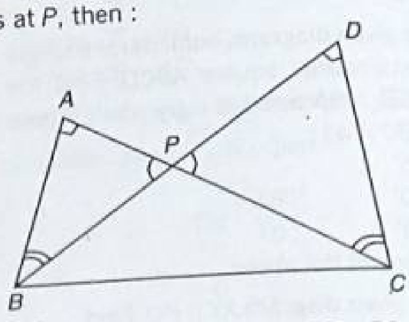
(a) $\frac{AB}{AC} = \frac{BD}{CD}$ (b) $\frac{AB}{AC} = \frac{CD}{BD}$
(c) $\frac{AB}{AC} = \frac{BC}{CD}$ (d) none of these

71. The diagonal BD of a quadrilateral $ABCD$ bisects $\angle B$ and $\angle D$, then :



(a) $\frac{AB}{CD} = \frac{AD}{BC}$ (b) $\frac{AB}{BC} = \frac{AD}{CD}$
(c) $AB = AD \times BC$ (d) none of these

72. Two right triangles ABC and DBC are drawn on the same hypotenuse BC on the same side of BC. If AC and DB intersects at P, then :

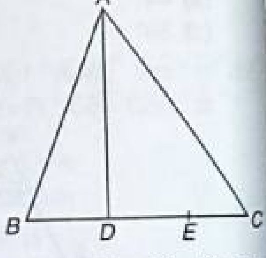


(a) $\frac{AP}{PC} = \frac{BP}{DP}$ (b) $AP \times DP = PC \times BP$
(c) $AP \times PC = BP \times DP$ (d) $AP \times BP = PC \times PD$

73. A man goes 150 m due east and then 200 m due north. How far is he from the starting point?
(a) 200 m (b) 350 m
(c) 250 m (d) 175 m

74. From a point O in the interior of a ΔABC perpendiculars OD, OE and OF are drawn to the sides BC, CA and AB respectively, then which one of the following is true ?
(a) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$
(b) $AB^2 + BC^2 = AC^2$
(c) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2$
(d) $AF^2 + BD^2 + CE^2 = OD^2 + OE^2 + OF^2$

75. In an equilateral triangle ABC, the side BC is trisected at D. Find the value of AD^2 :



(a) $\frac{9}{7}AB^2$
(b) $\frac{7}{9}AB^2$
(c) $\frac{3}{4}AB^2$
(d) $\frac{4}{5}AB^2$

76. ABC is a triangle in which $\angle A = 90°$. $AN \perp BC$, $AC = 12$ cm and $AB = 5$ cm. Find the ratio of the areas of ΔANC and ΔANB :
(a) 125 : 44 (b) 25 : 144
(c) 144 : 25 (d) 12 : 5

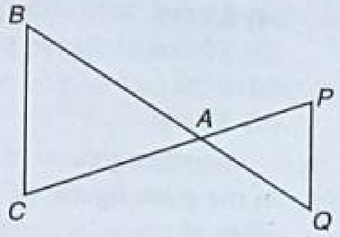
77. A vertical stick 15 cm long casts it shadow 10 cm long on the ground. At the same time a flag pole casts a shadow 60 cm long. Find the height of the flag pole :

(a) 40 cm (b) 45 cm
(c) 90 cm (d) none of these

78. Vertical angles of two isoceles triangles are equal. Then corresponding altitudes are in the ratio 4 : 9. Find the ratio of their areas :

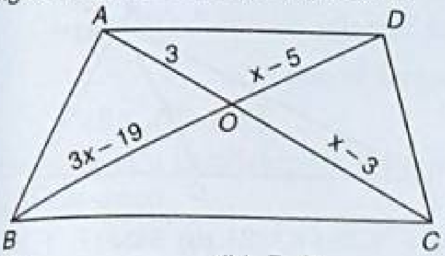
(a) 16 : 49 (b) 16 : 81
(c) 16 : 65 (d) none of these

79. In the figure $\Delta ACB \sim \Delta APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $AP = 2.8$ cm, find CA :



(a) 8 cm
(b) 6.5 cm
(c) 5.6 cm
(d) none of these

80. In the figure $BC \parallel AD$. Find the value of x :

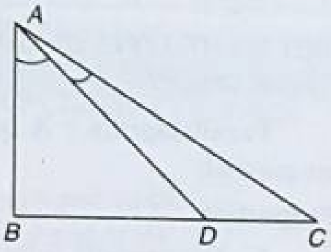


(a) 9, 10 (b) 7, 8
(c) 10, 12 (d) 8, 9

81. In an equilateral triangle of side $2a$, calculate the length of its altitude :

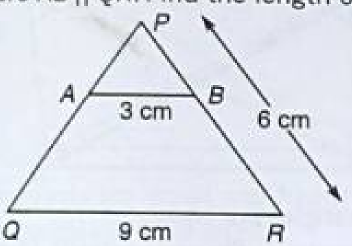
(a) $2a\sqrt{3}$ (b) $a\sqrt{3}$
(c) $a\dfrac{\sqrt{3}}{2}$ (d) none of these

82. In figure AD is the bisector of $\angle BAC$. If $BD = 2$ cm, $CD = 3$ cm and $AB = 5$ cm. Find AC :



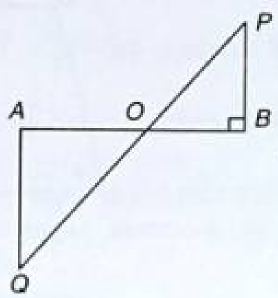
(a) 6 cm
(b) 7.5 cm
(c) 10 cm
(d) 15 cm

83. In the figure $AB \parallel QR$. Find the length of PB :



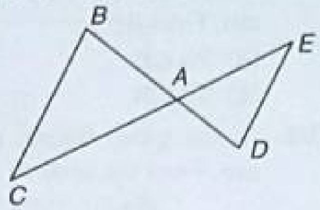
(a) 2 cm (b) 3 cm
(c) 2.5 cm (d) 4 cm

84. In the figure QA and PB are perpendicular to AB. If $AO = 10$ cm, $BO = 6$ cm and $PB = 9$ cm. Find AQ :



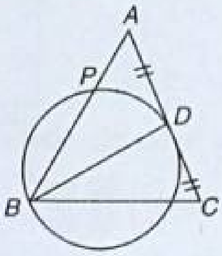
(a) 8 cm
(b) 9 cm
(c) 15 cm
(d) 12 cm

85. In the given figure $AB = 12$ cm, $AC = 15$ cm and $AD = 6$ cm. $BC \parallel DE$, find the length of AE :



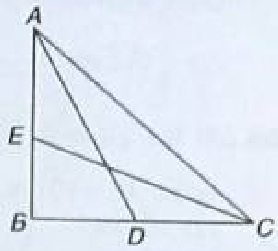
(a) 6 cm
(b) 7.5 cm
(c) 9 cm
(d) 10 cm

86. In the figure, ABC is a triangle in which $AB = AC$. A circle through B touches AC at D and intersects AB at P. If D is the mid-point of AC, Find the value of AB :



(a) $2AP$
(b) $3AP$
(c) $4AP$
(d) none of the above

87. In figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \dfrac{3\sqrt{5}}{2}$ cm, find the length of CE :



(a) $2\sqrt{5}$ cm
(b) 2.5 cm
(c) 5 cm
(d) $4\sqrt{2}$ cm

88. In a ΔABC, $AB = 10$ cm, $BC = 12$ cm and $AC = 14$ cm. Find the length of median AD. If G is the centroid , find length of GA :

(a) $\dfrac{5}{3}\sqrt{7}, \dfrac{5}{9}\sqrt{7}$ (b) $5\sqrt{7}, 4\sqrt{7}$
(c) $\dfrac{10}{\sqrt{3}}, \dfrac{8}{3}\sqrt{7}$ (d) $4\sqrt{7}, \dfrac{8}{3}\sqrt{7}$

89. ΔABC is a right angled at A and AD is the altitude to BC. If $AB = 7$ cm and $AC = 24$ cm. Find the ratio of AD is to AM if M is the mid–point of BC :

(a) 25 : 41 (b) 32 : 41
(c) $\dfrac{336}{625}$ (d) $\dfrac{625}{336}$

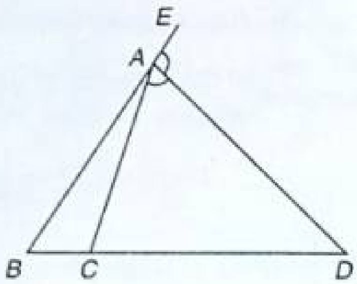
90. Area of $\Delta ABC = 30\text{ cm}^2$. D and E are the mid-points of BC and AB respectively. Find $A(\Delta BDE)$:

(a) 10 cm (b) 7.5 cm
(c) 15 cm (d) none of these

91. The three sides of a triangles are given which one of the following is not a right angle :

(a) 20, 21, 29 (b) 16, 63, 65
(c) 56, 90, 106 (d) 36, 35, 74

92. In the figure AD is the external bisector of $\angle EAC$, intersects BC produced to D. If $AB = 12$ cm, $AC = 8$ cm and $BC = 4$ cm, find CD :

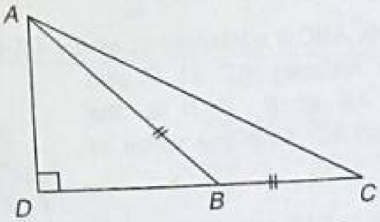


(a) 10 cm
(b) 6 cm
(c) 8 cm
(d) 9 cm

93. In ΔABC, $AB^2 + AC^2 = 2500$ cm^2 and median $AD = 25$ cm, Find BC :

(a) 25 cm (b) 40 cm
(c) 50 cm (d) 48 cm

94. In the given figure, $AB = BC$ and $\angle BAC = 15°$, $AB = 10$ cm. Find the area of ΔABC :

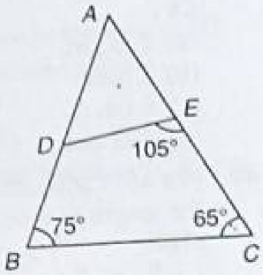


(a) 50 cm^2 (b) 40 cm^2
(c) 25 cm^2 (d) 32 cm^2

95. In ABC, G is the centroid, $AB = 15$ cm, $BC = 18$ cm and $AC = 25$ cm. Find GD, where D is the mid-point of BC :

(a) $\frac{1}{3}\sqrt{86}$ cm (b) $\frac{2}{3}\sqrt{86}$ cm
(c) $\frac{8}{3}\sqrt{15}$ cm (d) none of these

96. In the given figure , if $\frac{DE}{BC} = \frac{2}{3}$ and if $AE = 10$ cm. Find AB :

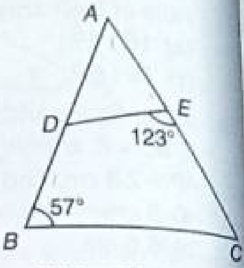


(a) 16 cm
(b) 12 cm
(c) 15 cm
(d) 18 cm

97. Find the maximum area that can be enclosed in a triangle of perimeter 24 cm :

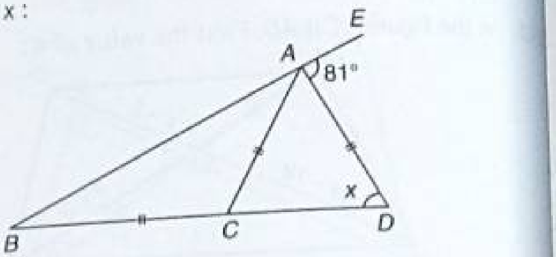
(a) 32 cm^2 (b) $16\sqrt{3}$ cm^2
(c) $16\sqrt{2}$ cm^2 (d) 27 cm^2

98. In the figure $AD = 12$ cm, $AB = 20$ cm and $AE = 10$ cm. Find EC :



(a) 14 cm
(b) 10 cm
(c) 8 cm
(d) 15 cm

99. In the given figure, $BC = AC = AD$, $\angle EAD = 81°$. Find the value of x :



(a) 45° (b) 54°
(c) 63° (d) 36°

100. What is the ratio of inradius to the circumradius of a right angled triangle?

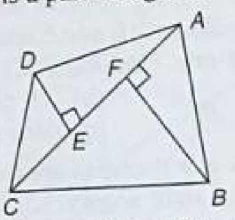
(a) 1 : 2 (b) $1 : \sqrt{2}$
(c) 2 : 5 (d) can't be determined

(3) QUADRILATERALS

A four sided closed figure is called a quadrilateral. It is denoted by symbol '▭'.

Properties

1. Sum of four interior angles is 360°.
2. The figure formed by joining the mid-points of a quadrilateral is a parallelogram.



3. The sum of opposite sides of a quadrilateral circumscribed about a circle, is always equal.
4. Area of quadrilateral $= \frac{1}{2} \times$ one of the diagonals $\times$ sum of the perpendiculars drawn to the diagonals from the opposite vertices.

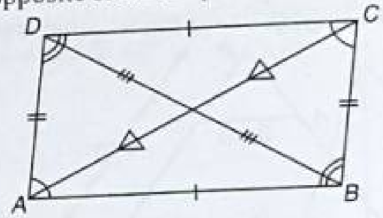
i.e., $A(\square ABCD) = \frac{1}{2} \times AC \times (DE + BF)$

DIFFERENT TYPES OF QUADRILATERALS ARE GIVEN BELOW

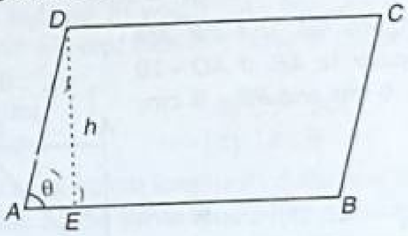
Parallelogram : A quadrilateral whose opposite side are parallel.

Properties

1. The opposite sides are parallel and equal.



2. Opposite angles are equal.
3. Sum of any two adjacent angles is 180°.
4. Diagonals bisect each other.

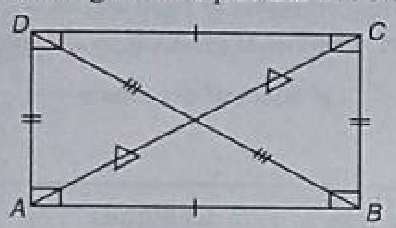


5. Diagonals need not be equal in length.
6. Diagonals need not bisect at right angle.
7. Diagonals need not bisect angles at the vertices.
8. Each diagonal divides a parallelogram into two congruent triangles.
9. Lines joining the mid-points of the adjacent sides of a **quadrilateral** form a **parallelogram.**
10. Lines joining the mid-points of of the adjacent sides of a **parallelogram** is a **parallelogram.**
11. The parallelogram that is inscribed in a circle is a rectangle.
12. The parallelogram that is circumscribed about a circle is a rhombus.
13. (a) Area of a parallelogram = base × height
    (b) Area of parallelogram
    = product of any two adjacent sides × sine of the included angles.
    $= AB \times AD \times \sin\theta$
14. Perimeter of a parallelogram = 2 (sum of any two adjacent sides)
15. $(AC)^2 + (BD)^2 = (AB)^2 + (BC)^2 + (CD)^2 + (AD)^2$
    $= 2(AB^2 + BC^2)$
16. Parallelogram that lie on the same base and between the same parallel lines are equal in area.
17. Area of a triangle is half of the area of a parallelogram which lie on the same base and between the same parallel lines.
18. A parallelogram is a rectangle if its diagonals are equal.

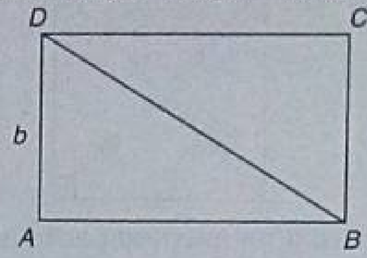
Rectangle : A parallelogram in which all the four angles at vertices are right (*i.e.*, 90°), is called a rectangle.

Properties

1. Opposite sides are parallel and equal.
2. Opposite angles are equal and of 90°.



3. Diagonals are equal and bisect each other, but not necessarily at right angles.
4. When a rectangle is inscribed in a circle, the diameter of the circle is equal to the diagonal of the rectangle.
5. For the given perimeter of rectangles, a square has maximum area.
6. The figure formed by joining the mid-points of the adjacent sides of a rectangle is a rhombus.
7. The quadrilateral formed by joining the mid-points of intersection of the angle bisectors of a parallelogram is a rectangle.
8. Every rectangle is a parallelogram.
9. Area of a rectangle = length × breadth $(= l \times b)$

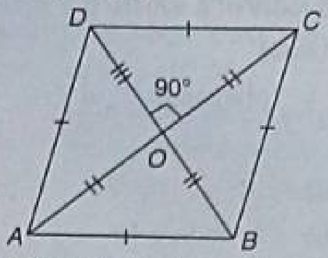


10. Diagonals of a rectangle $= \sqrt{l^2 + b^2}$
11. Perimeter of a rectangle $= 2(l + b)$
    $l \rightarrow$ length and $b \rightarrow$ breadth

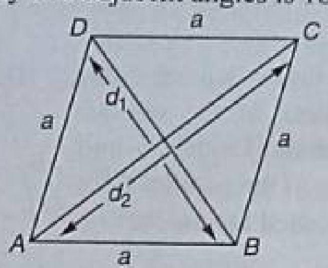
Rhombus : A parallelogram in which all sides are equal, is called a rhombus.

Properties

1. Opposite sides are parallel and equal.
2. Opposite angles are equal.
3. Diagonals bisect each other at right angle, but they are not necessarily equal.



4. Diagonals bisect the vertex angles.
5. Sum of any two adjacent angles is 180°.

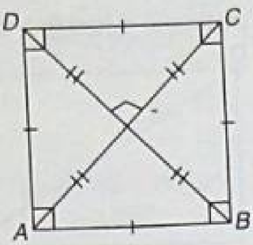


6. Figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle.
7. A parallelogram is a rhombus if its diagonals are perpendicular to each other.
8. (a) Area of a rhombus $= \frac{1}{2} \times$ product of diagonals
    $= \frac{1}{2} \times d_1 \times d_2$
    (b) Area of a rhombus = Product of adjacent sides × sine of the included angle.

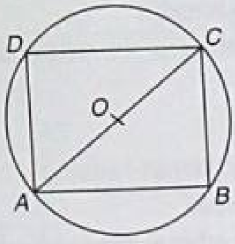
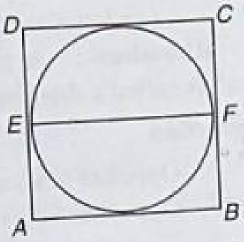
Square : A rectangle whose all sides are equal or a rhombus whose all angles are equal is called a square. Thus each rhombus is a parallelogram, a rectangle and a rhombus.

Properties

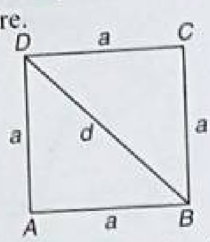
1. All side are equal and parallel.
2. All angles are right angles.
3. Diagonals are equal and bisect each other at right angle.



4. Diagonal of an inscribed square is equal to diameter of the inscribing circle.
5. Side of a circumscribed square is equal to the diameter of the inscribed circle.

6. The figure formed by joining the mid-points of the adjacent sides of a square is a square.



7. Area = (side)^2

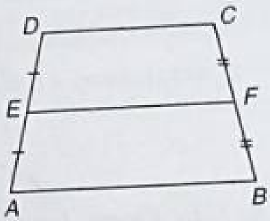
$$= a^2 = \frac{(\text{diagonal})^2}{2} = \frac{d^2}{2}$$

8. Diagonal = side $\sqrt{2} = a\sqrt{2}$
9. Perimeter = 4 × side = $4a$

Trapezium : A quadrilateral whose only one pair of sides is parallel and other two sides are not parallel.

Properties

1. The line joining the mid-points of the oblique (non-parallel) sides is half the sum of the parallel sides and is called the median.



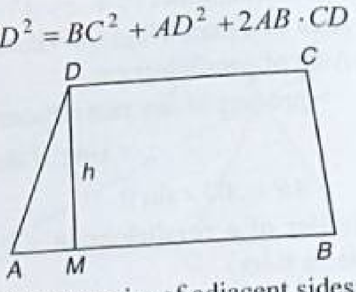
(*i.e.*, Median $= \frac{1}{2} \times$ sum of parallel sides

$$= \frac{1}{2} \times (AB + DC) = EF)$$

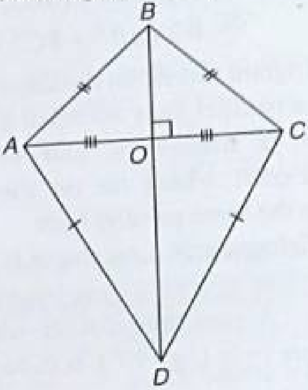
2. If the non-parallel sides are equal then the diagonals will also be equal to each other.
3. Diagonals intersect each other proportionally in the ratio of lengths of parallel sides.
4. By joining the mid-points of adjacent sides of a trapezium four similar triangles are obtained.
5. If a trapezium is inscribed in a circle, then it is an isosceles trapezium with equal oblique sides.
6. Area of trapezium $= \frac{1}{2} \times$ (sum of parallel sides × height)

$$= \frac{1}{2} \times (AB + CD) \times h$$

7. $AC^2 + BD^2 = BC^2 + AD^2 + 2AB \cdot CD$



Kite : In a kite two pairs of adjacent sides are equal.



Properties

1. $AB = BC$ and $AD = CD$
2. Diagonals intersect at right angles.
3. Shorter diagonal is bisected by the longer diagonal.
4. Area $= \frac{1}{2} \times$ product of diagonals

S. No.	Property	Parallelogram	Rectangle	Rhombus	Square
1.	Opposite sides are equal	✔	✔	✔	✔
2.	All sides are equal	✗	✗	✔	✔
3.	Opposite sides are parallel	✔	✔	✔	✔
4.	Opposite angles are equal	✔	✔	✔	✔
5.	All angles are equal and right angle	✗	✔	✗	✔
6.	Diagonals bisect each other	✔	✔	✔	✔
7.	Diagonals bisect each other at right angles	✗	✗	✔	✔
8.	Diagonals bisect vertex angles	✗	✗	✔	✔
9.	Diagonals are equal	✗	✔	✗	✔
10.	Diagonals form four triangles of equal area	✔	✔	✔	✔
11.	Diagonals form four congruent triangles	✗	✗	✔	✔

INTRODUCTORY EXERCISE-12.3

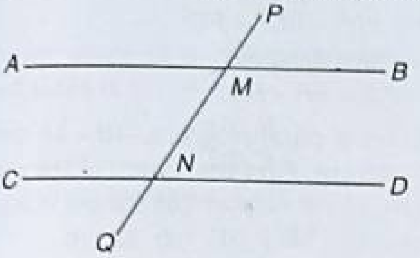
1. The measures of the angles of a quadrilateral taken in order are proportional to $1:2:3:4$, then the quadrilateral is :
(a) parallelogram (b) trapezium
(c) rectangle (d) rhombus

2. Find the measure of largest angle of a quadrilateral if the measures of its interior angles are in the ratio of $3:4:5:6$:
(a) $60°$ (b) $120°$
(c) $90°$ (d) can't be determined

3. $ABCD$ is a parallelogram, P and Q are the points on the diagonal AC such that $AP = QC$, then quadrilateral $BPDQ$ is a :
(a) trapezium (b) parallelogram
(c) square (d) none of these

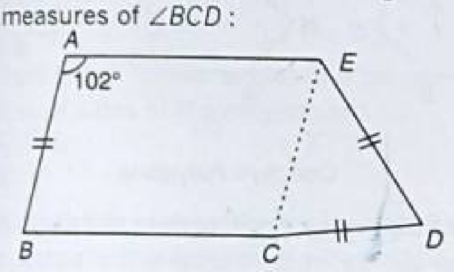
4. In a parallelogram $ABCD$, bisectors of consecutive angles A and B intersect at P. Find the measure of $\angle APB$:
(a) $90°$ (b) $60°$
(c) $120°$ (d) data insufficient

5. AB and CD are two parallel lines and a transversal PQ intersects AB and CD at M and N respectively. The bisector of the interior angles form a quadrilateral :



(a) rectangle (b) square
(c) parallelogram (d) none of these

6. In the given figure $AE = BC$ and $AE \parallel BC$ and the three sides AB, CD and ED are equal in length. If $m\angle A = 102°$, find measures of $\angle BCD$:



(a) $138°$ (b) $162°$
(c) $88°$ (d) none of these

7. $ABCD$ is a square, A is joined to a point P on BC and D is joined to a point Q on AB. If $AP = DQ$ and AP intersects DQ at R then $\angle DRP$ is :
(a) $60°$ (b) $120°$
(c) $90°$ (d) can't be determined

8. A point X inside a rectangle $PQRS$ is joined to the vertices then, which of the following is true :
(a) $A(\Delta PSX) = A(\Delta RXQ)$
(b) $A(\Delta PSX) + A(\Delta PXQ) = A(\Delta RSX) + A(\Delta RQX)$
(c) $A(\Delta PXS) + A(\Delta RXQ) = A(\Delta SRX) + A(\Delta PXQ)$
(d) none of the above

9. $\square ABCD$ is a parallelogram. $m\angle DAB = 30°$, $BC = 20$ cm and $AB = 20$ cm. Find the area of parallelogram :
(a) 150 cm^2 (b) 200 cm^2
(c) 400 cm^2 (d) 260 cm^2

10. The length of a side of a rhombus is 10 m and one of its diagonal is 12 m. The length of the other diagonal is :
(a) 15 m (b) 18 m
(c) 16 m (d) can't be determined

11. $ABCD$ is a parallelogram and BD is a diagonal. $\angle BAD = 65°$ and $\angle DBC = 45°$, then $m\angle BDC$ is :
(a) $65°$ (b) $70°$
(c) $20°$ (d) none of these

12. If $ABCD$ is a parallelogram in which P and Q are the centroids of ΔABD and ΔBCD, then, PQ equals :
(a) AQ (b) AP
(c) BP (d) DQ

13. Two parallelograms stand on equal bases and between the same parallels. The ratio of their areas is :
(a) $1:1$ (b) $\sqrt{2}:1$
(c) $1:3$ (d) $1:2$

14. If a rectangle and a parallelogram are equal in area and have the same base and are situated on the same side, then the ratio of perimeter of rectangle and that of parallelogram is k, then k is :
(a) $k > 1$ (b) $k < 1$
(c) $k = 1$ (d) can't be determined

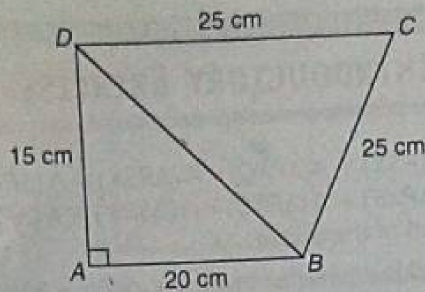
15. If area of a parallelogram with sides l and b is A and that of a rectangle with sides l and b is B, then :
(a) $A < B$ (b) $A = B$
(c) $A > B$ (d) none of these

16. $ABCD$ is a parallelogram and M is the mid-point of BC. AB and DM are produced to meet at N, then :
(a) $AN = \sqrt{3}AB$ (b) $AN = 2AB$
(c) $AN^2 = \frac{3}{2}AB^2$ (d) $AN^2 = 2AB^2$

17. In a rectangle $ABCD$, P, Q are the mid-points of BC and AD respectively and R is any point on PQ, then ΔARB equals :
(a) $\frac{1}{2}(\square ABCD)$ (b) $\frac{1}{3}(\square ABCD)$
(c) $\frac{1}{4}(\square ABCD)$ (d) none of these

18. Diagonals of a parallelogram are 8 m and 6 m respectively. If one of side is 5m, then the area of parallelogram is :
(a) 18 m^2 (b) 30 m^2
(c) 24 m^2 (d) 48 m^2

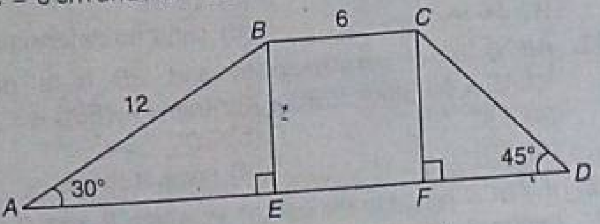
19. In the given figure $AD = 15$ cm, $AB = 20$ cm and $BC = CD = 25$ cm. Find the area of $\square ABCD$:



(a) $\frac{25}{4}(24+25\sqrt{3})\text{ cm}^2$ (b) $24(25+24\sqrt{3})\text{ cm}^2$

(c) $\frac{25}{2}(24+25\sqrt{3})\text{ cm}^2$ (d) none of these

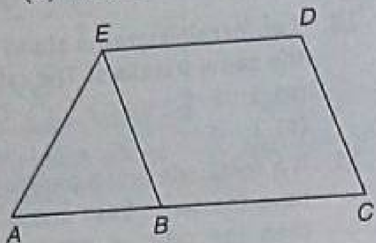
20. In the trapezium $ABCD$, $\angle BAE = 30°$, $\angle CDF = 45°$. $BC = 6$ cm and $AB = 12$ cm. Find the area of trapezium :



(a) $18(3+\sqrt{3})\text{ cm}^2$ (b) $36\sqrt{3}\text{ cm}^2$

(c) $12(3+2\sqrt{3})\text{ cm}^2$ (d) none of these

21. Area of quadrilateral $ACDE$ is 36 cm^2, B is the mid-point of AC. Find the area of ΔABE if $AC \parallel DE$ and $BE \parallel DC$:



(a) 10 cm^2

(b) 9 cm^2

(c) 12 cm^2

(d) can't be determined

22. A square and a rhombus have the same base and the rhombus is inclined at 30°. What is the ratio of area of the square to the area of the rhombus :

(a) $\sqrt{2}:1$ (b) $2:1$

(c) $1:1$ (d) $2:\sqrt{3}$

23. Find the area of a quadrilateral with sides 17, 25, 30 and 28 cm and one of its diagonal is 26 cm :

(a) 450 cm^2 (b) 360 cm^2

(c) 540 cm^2 (d) 720 cm^2

24. $PQRS$ is a parallelogram in which $\angle P = 70°$, $\angle Q = 90°$ and $\angle R = 100°$. How many points in the plane of the quadrilateral are there such that a point is equidistant from its vertices ?

(a) 0 (b) 1

(c) 2 (d) 3

25. $ABCD$ is a parallelogram. The diagonals AC and BD intersect at a point O. If E, F, G and H are the mid-points of AO, DO, CO and BO respectively, then the ratio of $(EF + FG + GH + HE)$ to $(AD + DC + CB + BA)$ is :

(a) $1:1$ (b) $1:2$

(c) $1:3$ (d) $1:4$

26. If $ABCD$ is a rhombus, then :

(a) $AC^2 + BD^2 = 4AB^2$ (b) $AC^2 + BD^2 = AB^2$

(c) $AC^2 + BD^2 = 2AB^2$ (d) $2(AC^2 + BD^2) = 3AB^2$

27. If P is a point within a rectangle $ABCD$, then :

(a) $AP^2 + PC^2 = BP^2 + PD^2$

(b) $AP^2 + BP^2 = PC^2 + PD^2$

(c) $AP + PC = BP + PD$

(d) $AP \times PC = BP \times PD$

28. $\square ABCD$ is a parallelogram, $AB = 14$ cm, $BC = 18$ cm and $AC = 16$ cm. Find the length of the other diagonal :

(a) 24 cm (b) 28 cm

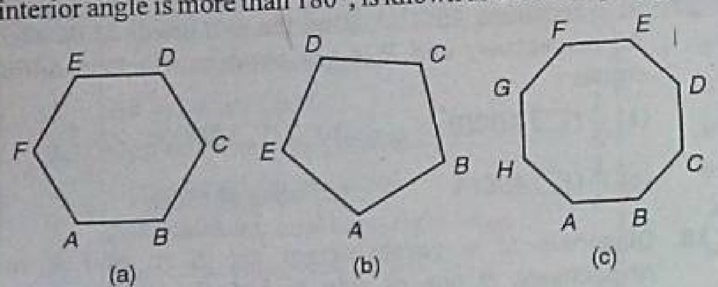
(c) 36 cm (d) 32 cm

(4) POLYGONS

Polygon : It is a closed plane figure bounded by three or more than three straight lines.

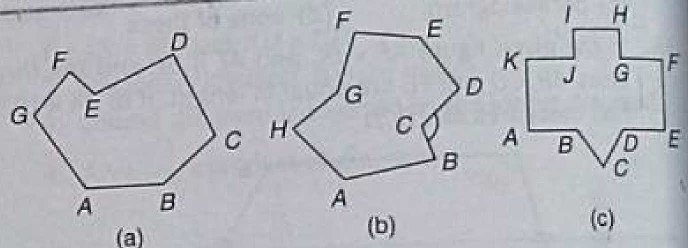
e.g., triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon and decagon.

Convex Polygon: A polygon in which none of its interior angle is more than 180°, is known as a convex polygon.



Convex Polygons

Concave Polygon: A polygon in which atleast one interior angle is more than 180°, then it is said to be concave.



Concave Polygons

Regular Polygon : A polygon in which all the sides are equal and also all the interior angles are equal, is called a regular polygon.

Formulae : For regular polygons

1. Sum of all interior angles $= (n-2) \times 180$
$= (2n-4) \times 90°$

2. Each interior angle $= 180 -$ exterior angle

3. Each exterior angle $= \left(\dfrac{360}{\text{Number of sides}}\right)$ (in degrees)

4. Sum of all exterior angle = 360° (always constant)

5. Number of sides in a polygon $= \dfrac{360}{\text{Exterior angle}}$

$$= 2(x+1) \qquad \left(x = \frac{\text{Interior angle}}{\text{Exterior angle}}\right)$$

6. Number of diagonals $= {}^nC_2 - n = \dfrac{n(n-1)}{2} - n$

$$= \frac{n^2 - 3n}{2} = \frac{n(n-3)}{2}$$

where $n \rightarrow$ number of sides of a polygon.

7. Star : Sum of angles of a n point star $= (n-4)\pi$

8. Area of a polygon $= \dfrac{na^2}{4} \times \cot\left(\dfrac{180}{n}\right)$;

where $n \rightarrow$ number of sides
$a \rightarrow$ length of sides

No. of sides	Polygon	Sum of all the angles	Each interior angle	Each exterior angle	No. of diagonals
3	Triangle	180°	60°	120°	0
4	Quadrilateral	360°	90°	90°	2
5	Pentagon	540°	108°	72°	5
6	Hexagon	720°	120°	60°	9
7	Heptagon	900°	$\left(128\frac{4}{7}\right)^\circ$	$\left(51\frac{3}{7}\right)^\circ$	14
8	Octagon	1080°	135°	45°	20
9	Nonagon	1260°	140°	40°	27
10	Decagon	1440°	144°	36°	35

INTRODUCTORY EXERCISE-12.4

1. Each interior angle of a regular polygon is 140°. The number of sides is :
(a) 10 (b) 8
(c) 6 (d) 9

2. Each angle of a regular hexagon is :
(a) 60° (b) 120°
(c) 90° (d) none of these

3. If one of the interior angles of a regular polygon is equal to 5/6 times of one of the interior angles of a regular pentagon, then the number of sides of the polygon is :
(a) 3 (b) 4
(c) 6 (d) 8

4. The sum of the interior angles of a polygon is 1260°. The number of sides of the polygon is :
(a) 6 (b) 7
(c) 8 (d) 9

5. If each interior angle of a regular polygon is 3 times its exterior angle, the number of sides of the polygon is :
(a) 4 (b) 5
(c) 6 (d) 8

6. Difference between the interior and exterior angles of regular polygon is 60°. The number of sides in the polygon is :
(a) 5 (b) 6
(c) 8 (d) 9

7. A polygon has 54 diagonals. The number of sides in the polygon is :
(a) 7 (b) 9
(c) 12 (d) none of these

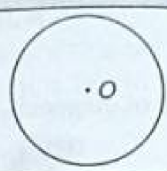
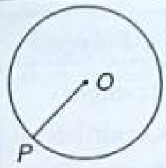
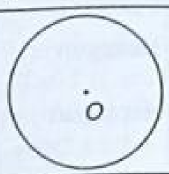
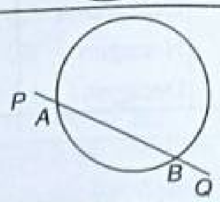
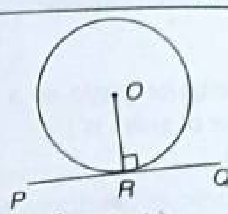
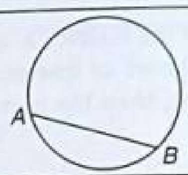
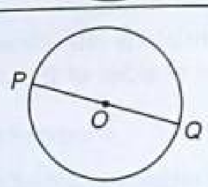
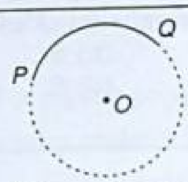
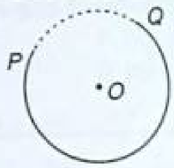
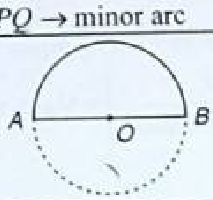
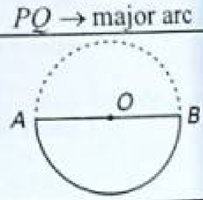
8. The ratio between the number of sides of two regular polygon 1 : 2 and the ratio between their interior angle is 3 : 4. The number of sides of these polygons are respectively :
(a) 3, 6 (b) 4, 8
(c) 6, 9 (d) 5, 10

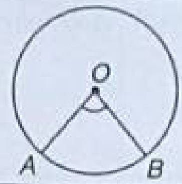
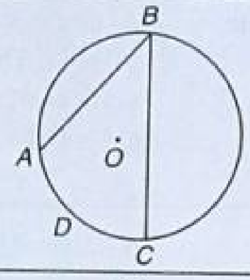
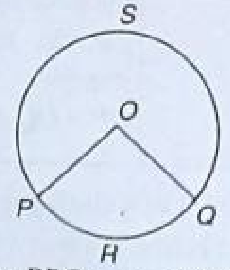
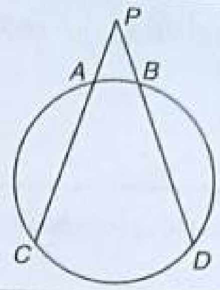
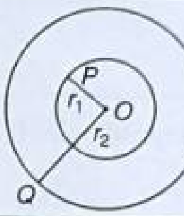
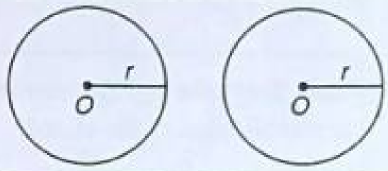
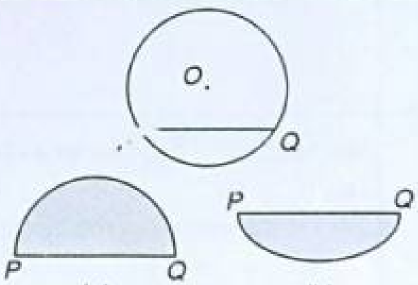
9. The sum of all the interior angles of a regular polygon is four times the sum of its exterior angles. The polygon is :
(a) hexagon (b) triangle
(c) decagon (d) nonagon

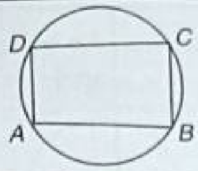
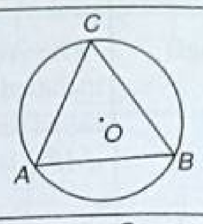
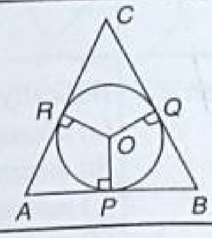
10. The ratio of the measure of an angle of a regular nonagon to the measure of its exterior angle is :
(a) 3 : 5 (b) 5 : 2
(c) 7 : 2 (d) 4 : 5

(5) CIRCLE

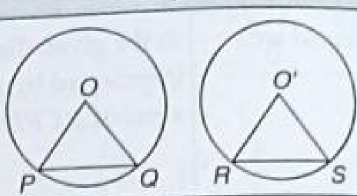
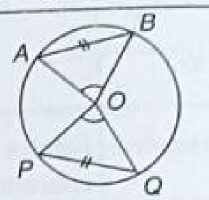
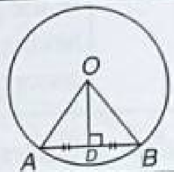
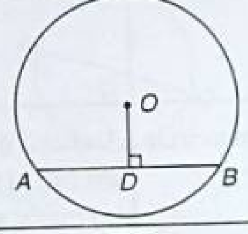
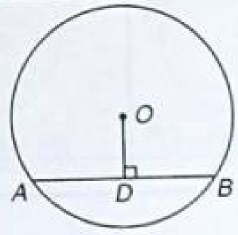
Circle : A circle is a set of points on a plane which lie at a fixed distance from a fixed point.

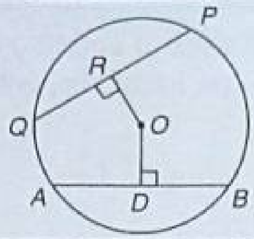
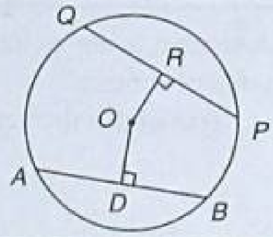
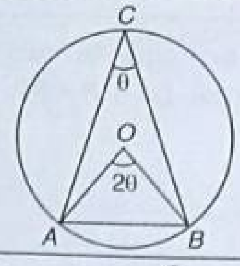
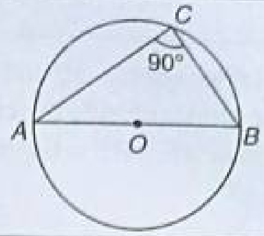
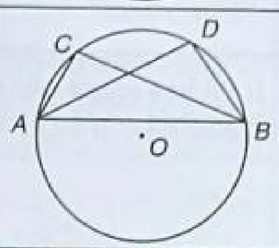
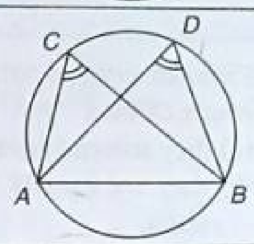
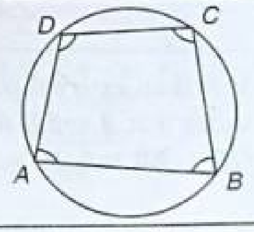
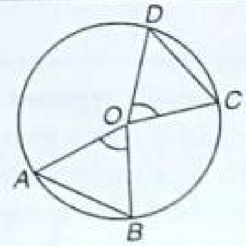
Nomenclature	Definition	Diagram
Centre	The fixed point is called the centre. In the given diagram 'O' is the centre of the circle.	
Radius	The fixed distance is called a radius. In the given diagram OP is the radius of the circle. (Point P lies on the circumference)	
Circumference	The circumference of a circle is the distance around a circle, which is equal to $2\pi r$. ($r \rightarrow$ radius of the circle)	
Secant	A line segment which intersects the circle in two distinct points, is called as secant. In the given diagram secant PQ intersects circle at two points at A and B.	
Tangent	A line segment which has one common point with the circumference of a circle *i.e.*, it touches only at only one point is called as tangent of circle. The common point is called as point of contact. In the given diagram PQ is a tangent which touches the circle at a point R.	 (R is the point of contact) **Note:** Radius is always perpendicular to tangent.
Chord	A line segment whose end points lie on the circle. In the given diagram AB is a chord.	
Diameter	A chord which passes through the centre of the circle is called the diameter of the circle. The length of the diameter is twice the length of the radius. In the given diagram PQ is the diameter of the circle. ($O \rightarrow$ is the centre of the circle)	
Arc	Any two points on the circle divides the circle into two parts the smaller part is called as minor arc and the larger part is called as major arc. It is denoted as '$\frown$'. In the given diagram $\overset{\frown}{PQ}$ is arc.	 $PQ \rightarrow$ minor arc  $PQ \rightarrow$ major arc
Semicircle	A diameter of the circle divides the circle into two equal parts. Each part is called as semicircle.	 

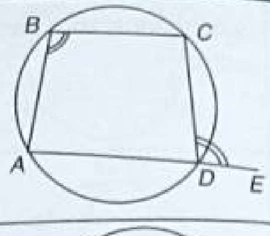
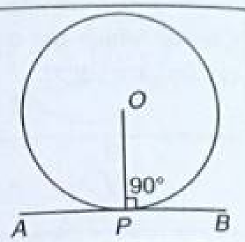
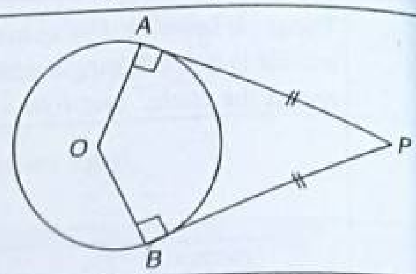
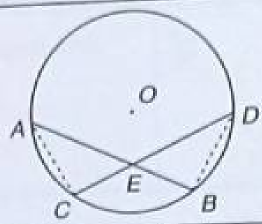
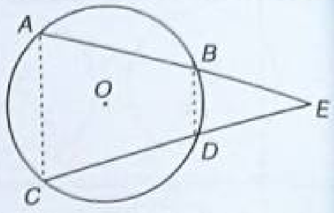
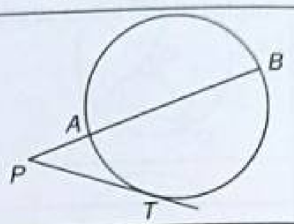
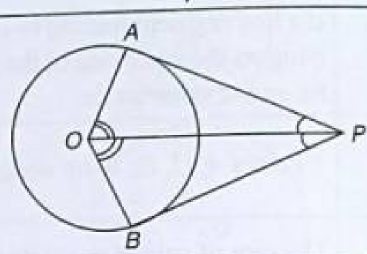
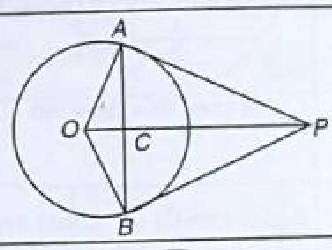
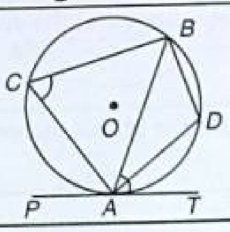
Nomenclature	Definition	Diagram
Central angle	An angle formed at the centre of the circle, is called the central angle. In the given diagram $\angle AOB$ is the central angle.	
Inscribed angle	When two chords have one common end point, then the angle included between these two chords at the common point is called the inscribed angle. $\angle ABC$ is the inscribed angle by the arc $\widehat{ADC}$.	
Measure of an arc	Basically it is the central angle formed by an arc. *e.g.*, (a) measure of a circle $= 360°$ (b) measure of a semicircle $= 180°$ (c) measure of a minor arc $= \angle POQ$ (d) measure of a major arc $= 360 - \angle POQ$	 m (arc PRQ) $= m\angle POQ$ m (arc PSQ) $= 360° - m$ (arc PRQ)
Intercepted arc	In the given diagram $\widehat{AB}$ and $\widehat{CD}$ are the two intercepted arcs, intercepted by $\angle CPD$. The end points of the arc must touch the arms of $\angle CPD$ *i.e.*, CP and DP.	
Concentric circles	Circles having the same centre at a plane are called the concentric circles. In the given diagram there are two circles with radii r_1 and r_2 having the common (or same) centre. These are called as concentric circles.	
Congruent circles	Circles with equal radii are called as congruent circles.	
Segment of a circle	A chord divides a circle into two regions. These two regions are called the segments of a circle. (a) major segment (b) minor segment	 (a) (b)

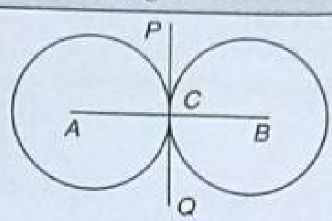
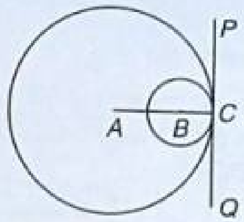
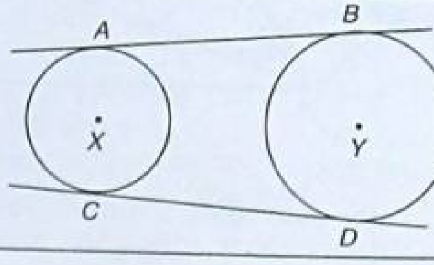
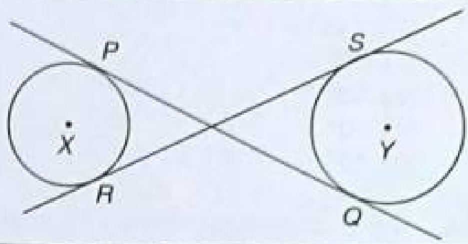
Nomenclature	Definition	Diagram
Cyclic quadrilateral	A quadrilateral whose all the four vertices lie on the circle.	
Circum-circle	A circle which passes through all the three vertices of a triangle. Thus the circumcentre is always equidistant from the vertices of the triangle. $OA = OB = OC$ (circumradius)	
Incircle	A circle which touches all the three sides of a triangle *i.e.*, all the three sides of a triangle are tangents to the circle is called an incircle. Incircle is always equidistant from the sides of a triangle. $OP = OQ = OR$ (inradius of the circle)	

- Two arcs of a circle (or of congruent circles) are congruent if their degree measures are equal.
- There is one and only one circle passes through three non-collinear points.

S. No.	Theorem	Diagram
1.	In a circle (or in congruent circles) equal chords are made by equal arcs. $\{OP = OQ\} = \{O'R = O'S\}$ and $\widehat{PQ} = \widehat{RS}$ $\therefore$ $PQ = RS$	
2.	Equal arcs (or chords) subtend equal angles at the centre $\widehat{PQ} = \widehat{AB}$ (or $PQ = AB$) $\therefore$ $\angle POQ = \angle AOB$	
3.	The perpendicular from the centre of a circle to a chord bisects the chord *i.e.*, if $OD \perp AB$ $\therefore$ $AB = 2AD = 2BD$	
4.	The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. $\because$ $AD = DB$ $\therefore$ $OD \perp AB$	
5.	Perpendicular bisector of a chord passes through the centre. *i.e.*, if $OD \perp AB$ and $AD = DB$ $\therefore$ O is the centre of the circle.	

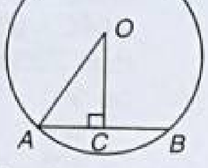
S.No.	Theorem	Diagram
6.	Equal chords of a circle (or of congruent circles) are equidistant from the centre. $\because$ $AB = PQ$ $\therefore$ $OD = OR$	
7.	Chords which are equidistant from the centre in a circle (or in congruent, circles) are equal. $\because$ $OD = OR$ $\therefore$ $AB = PQ$	
8.	The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. $m\angle AOB = 2m\angle ACB$.	
9.	Angle in a semicircle is a right angle.	
10.	Angles in the same segment of a circle are equal *i.e.*, $\angle ACB = \angle ADB$	
11.	If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle. $\angle ACB = \angle ADB$ $\therefore$ Points A, C, D, B are concyclic *i.e.*, lie on the circle.	
12.	The sum of pair of opposite angles of a cyclic quadrilateral is 180°. $\angle DAB + \angle BCD = 180°$ and $\angle ABC + \angle CDA = 180°$ (Inverse of this theorem is also true)	
13.	Equal chords (or equal arcs) of a circle (or congruent circles) subtended equal angles at the centre. $AB = CD$ (or $\overset{\frown}{AB} = \overset{\frown}{CD}$) $\therefore$ $\angle AOB = \angle COD$ (Inverse of this theorem is also true)	

S. No.	Theorem	Diagram
14.	If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. $m\angle CDE = m\angle ABC$	
15.	A tangent at any point of a circle is perpendicular to the radius through the point of contact. (Inverse of this theorem is also true)	
16.	The lengths of two tangents drawn from an external point to a circle are equal. *i.e.*, $AP = BP$	
17.	If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E), then $AE \times BE = CE \times DE$	 
18.	If PB be a secant which intersects the circle at A and B and PT be a tangent at T then $PA \cdot PB = (PT)^2$	
19.	From an external point from which the tangents are drawn to the circle with centre O, then (a) they subtend equal angles at the centre. (b) they are equally inclined to the line segment joining the centre of that point. $\angle AOP = \angle BOP$ and $\angle APO = \angle BPO$	
20.	If P is an external point from which the tangents to the circle with centre O touch it at A and B then OP is the perpendicular bisector of AB. $OP \perp AB$ and $AC = BC$	
21.	**Alternate segment theorem :** If from the point of contact of a tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram. $\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$	

S. No.	Theorem	Diagram
22.	The point of contact of two tangents lies on the straight line joining the two centres. (a) When two circles touch externally then the distance between their centres is equal to sum of their radii, *i.e.*, $AB = AC + BC$ (b) When two circles touch internally the distance between their centres is equal to the difference between their radii *i.e.*, $AB = AC - BC$	
23.	For the two circles with centre X and Y and radii r_1 and r_2. AB and CD are two Direct Common Tangents (DCT), then the length of DCT $= \sqrt{(\text{distance between centres})^2 - (r_1 - r_2)^2}$	
24.	For the two circles with centre X and Y and radii r_1 and r_2. PQ and RS are two transverse common tangent, then length of TCT $= \sqrt{(\text{distance between centres})^2 - (r_1 + r_2)^2}$	

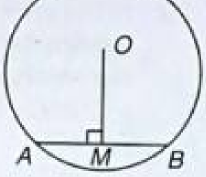
Introductory Exercise-12.5

1. In the given figure, O is the centre of the circle. Radius of the circle is 17 cm. If $OC = 8$ cm, then the length of the chord AB is :



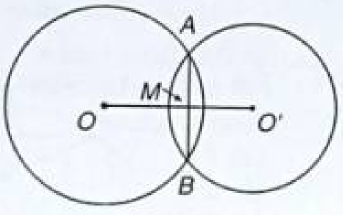
(a) 35 cm
(b) 30 cm
(c) 15 cm
(d) 18 cm

2. In the given figure $OM \perp AB$, radius of the circle is 5 cm and length of the chord $AB = 8$ cm. Find the measure of OM :



(a) 3 cm
(b) 2.5 cm
(c) 2 cm
(d) 6 cm

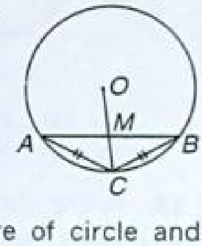
3. In the given figure, two circles with their respective centres intersect each other at A and B and AB intersects OO' at M, then $m\ \angle OMA$ is :



(a) 60°
(b) 80°
(c) 90°
(d) can't be determined

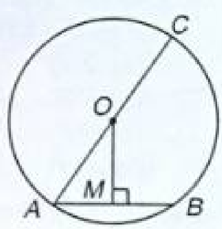
4. In the above question (no. 3) what is the ratio of $AM : BM$?
(a) 5 : 6 (b) 3 : 2
(c) 1 ; 1 (d) can't be determined

5. In the given figure the two chords AC and BC are equal. The radius OC intersect AB at M, then $AM : BM$ is :



(a) 1 : 1
(b) $\sqrt{2} : 3$
(c) $3 : \sqrt{2}$
(d) none of the above

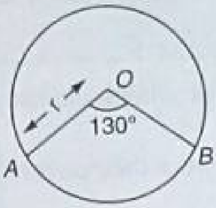
6. In the adjoining figure, O is the centre of circle and diameter $AC = 26$ cm. If chord $AB = 10$ cm, then the distance between chord AB and centre O of the circle is :



(a) 24 cm
(b) 16 cm
(c) 12 cm
(d) none of the above

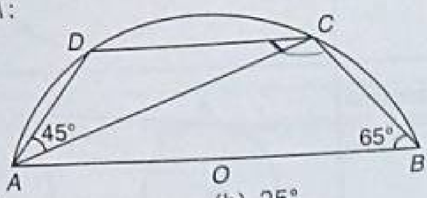
7. In the adjoining circle $C(0, r)$ the degree measure of minor arc $AB = 130°$. Find the degree measure of major arc :

(a) 230°
(b) 260°
(c) 310°
(d) none of the above



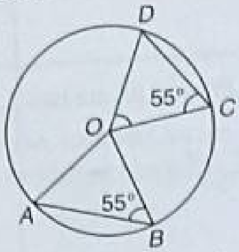
8. If the diagonals of a cyclic quadrilateral are equal, then the quadrilateral is :
(a) rhombus (b) square
(c) rectangle (d) none of these

9. In the given figure, AB is diameter of the circle. C and D lie on the semicircle. $\angle ABC = 65°$ and $\angle CAD = 45°$. Find $m\ \angle DCA$:



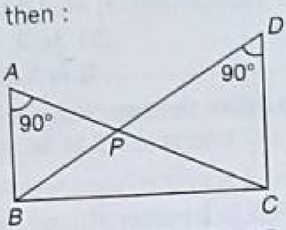
(a) 45° (b) 25°
(c) 20° (d) none of these

10. In the given figure, chords AB and CD are equal. If $\angle OBA = 55°$, then $m\ \angle COD$ is :
(a) 65°
(b) 55°
(c) 70°
(d) 50°



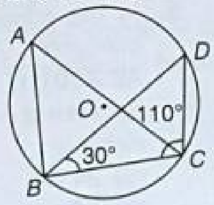
11. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is a :
(a) rectangle (b) square
(c) parallelogram (d) cyclic quadrilateral

12. ΔABC and ΔDBC have a common base and drawn towards one sides. $\angle BAC = \angle BDC = 90°$. If AC and DB intersect at P, then :

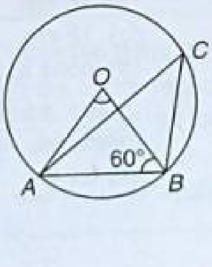


(a) $AP \times PC = BP \times PD$ (b) $AP \times BP = PC \times PD$
(c) $AP \times PD = PC \times BP$ (d) none of these

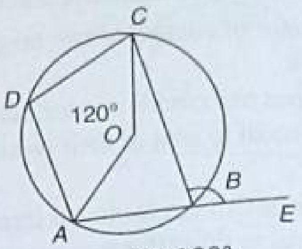
13. In the given figure, $\angle BAC$ and $\angle BDC$ are the angles of same segments. $\angle DBC = 30°$ and $\angle BCD = 110°$. Find $m\ \angle BAC$ is :
(a) 35°
(b) 40°
(c) 55°
(d) 60°



14. In the given figure, O is the centre of the circle. $\angle ABO = 60°$. Find the value of $\angle ACB$:
(a) 40°
(b) 60°
(c) 50°
(d) 30°

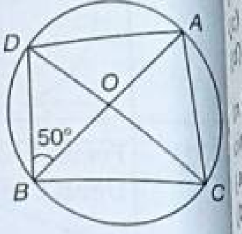


15. In the given figure, $\angle AOC = 120°$. Find $m\ \angle CBE$, where O is the centre :

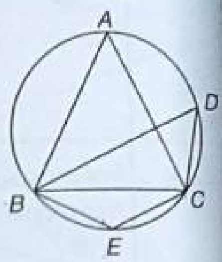


(a) 60° (b) 100°
(c) 120° (d) 150°

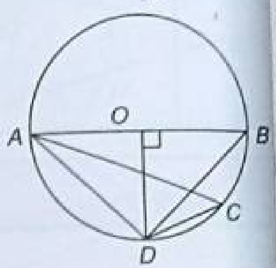
16. In the adjoining figure, O is the centre of the circle and $\angle OBD = 50°$. Find the $m\ \angle BAD$:
(a) 60°
(b) 40°
(c) 80°
(d) 45°



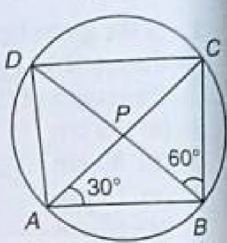
17. In the given figure, ΔABC is an equilateral triangle. Find $m\ \angle BEC$:
(a) 120°
(b) 60°
(c) 80°
(d) none of the above



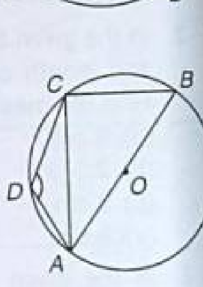
18. In the given figure, AB is the diameter of the circle. Find the value of $\angle ACD$:
(a) 30°
(b) 60°
(c) 45°
(d) 25°



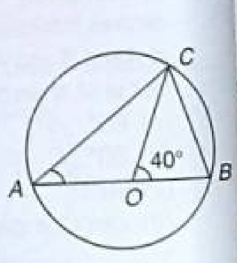
19. In the given figure, $ABCD$ is a cyclic quadrilateral and diagonals bisect each other at P. If $\angle DBC = 60°$ and $\angle BAC = 30°$, then $\angle BCD$ is :
(a) 90°
(b) 60°
(c) 80°
(d) none of the above



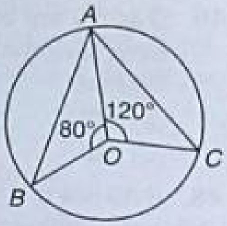
20. In the given figure, $ABCD$ is a cyclic quadrilateral and AB is the diameter. $\angle ADC = 140°$, then find $m\ \angle BAC$:
(a) 45°
(b) 40°
(c) 50°
(d) none of the above



21. In the given figure, $\angle COB = 40°$, AB is the diameter of the circle. Find $m\ \angle CAB$:
(a) 40°
(b) 20°
(c) 30°
(d) none of the above

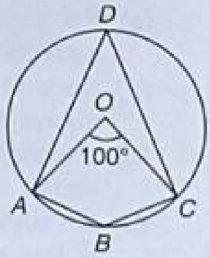


22. In the given figure, O is the centre of circle. $\angle AOB = 80°$ and $\angle AOC = 120°$. Find $m\angle BAC$:



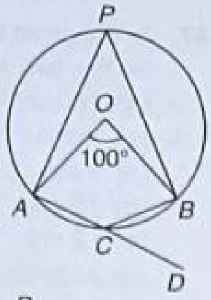
(a) 120°
(b) 80°
(c) 100°
(d) none of the above

23. In the given figure, O is the centre of the circle and $\angle AOC = 100°$. Find the ratio of $m\angle ADC : m\angle ABC$:



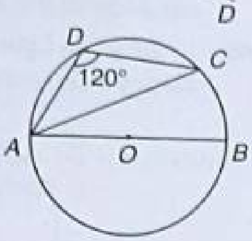
(a) 5 : 6
(b) 1 : 2
(c) 5 : 13
(d) none of the above

24. In the given figure, O is the centre of circle, $\angle AOB = 100°$. Find $m\angle BCD$:



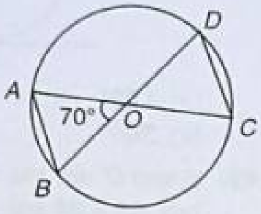
(a) 80°
(b) 60°
(c) 50°
(d) 40°

25. In the given figure, AB is the diameter of the circle. $\angle ADC = 120°$. Find $m\angle CAB$:



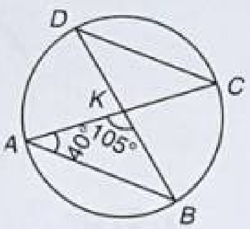
(a) 20°
(b) 30°
(c) 40°
(d) can't be determined

26. In the given figure, O is the centre of the circle. $\angle AOB = 70°$, find $m\angle OCD$:



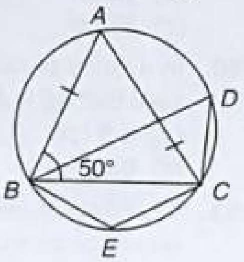
(a) 70°
(b) 55°
(c) 65°
(d) 110°

27. In the given figure, $\angle CAB = 40°$ and $\angle AKB = 105°$. Find $\angle KCD$:



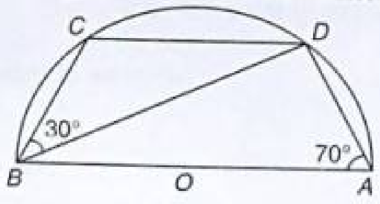
(a) 65°
(b) 35°
(c) 40°
(d) 72°

28. In the given figure, ABC is an isosceles triangle in which $AB = AC$ and $m\angle ABC = 50°$, $m\angle BDC$:



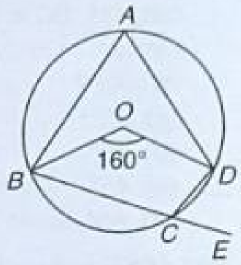
(a) 80°
(b) 60°
(c) 65°
(d) 100°

29. In the given figure, AB is the diameter. $m\angle BAD = 70°$



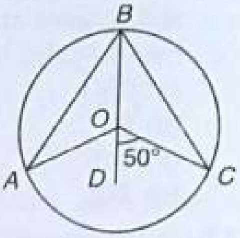
and $m\angle DBC = 30°$. Find $m\angle BDC$:
(a) 25° (b) 30°
(c) 40° (d) 60°

30. Find the value of $\angle DCE$:



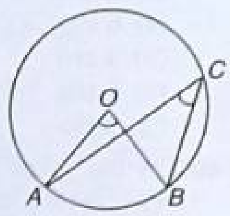
(a) 100°
(b) 80°
(c) 90°
(d) 75°

31. 'O' is the centre of the circle, line segment BOD is the angle bisector of $\angle AOC$, $m\angle COD = 50°$. Find $m\angle ABC$:



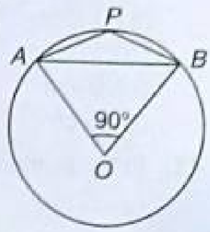
(a) 25°
(b) 50°
(c) 100°
(d) 120°

32. In the given figure, O is the centre of the circle and $\angle ACB = 25°$. Find $\angle AOB$:



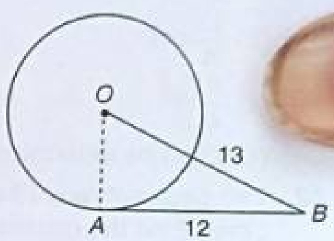
(a) 25°
(b) 50°
(c) 75°
(d) 60°

33. In the given figure, O is the centre of the circle. $\angle AOB = 90°$. Find $m\angle APB$:



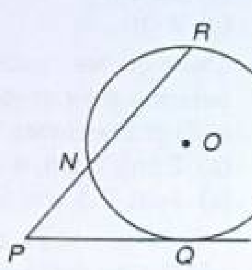
(a) 130°
(b) 150°
(c) 135°
(d) can't be determined

34. In the given figure, O is the centre of the circle. AB is tangent. $AB = 12$ cm and $OB = 13$ cm. Find OA:



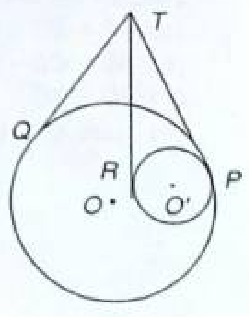
(a) 6.5 cm
(b) 6 cm
(c) 5 cm
(d) none of these

35. In the given figure, PQ is the tangent of the circle. Line segment PR intersects the circle at N and R. $PQ = 15$ cm, $PR = 25$ cm, find PN:



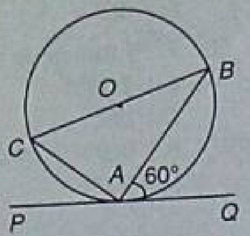
(a) 15 cm
(b) 10 cm
(c) 9 cm
(d) 6 cm

36. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find $TQ : TR$:



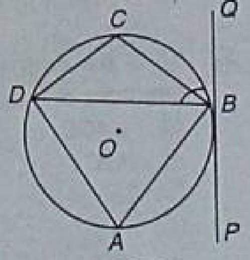
(a) 8 : 7 (b) 7 : 8
(c) 5 : 4 (d) 1 : 1

37. In the given figure, PAQ is the tangent. BC is the diameter of the circle. $m\angle BAQ = 60°$, find $m\angle ABC$:



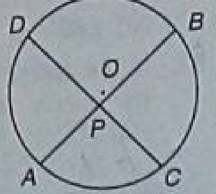
(a) 25°
(b) 30°
(c) 45°
(d) 60°

38. $ABCD$ is a cyclic quadrilateral PQ is a tangent at B. If $\angle DBQ = 65°$, then $\angle BCD$ is :



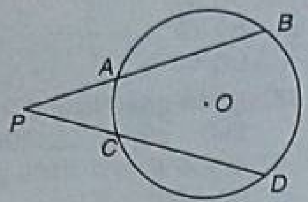
(a) 35°
(b) 85°
(c) 115°
(d) 90°

39. In the given figure, $AP = 2$ cm, $BP = 6$ cm and $CP = 3$ cm. Find DP :



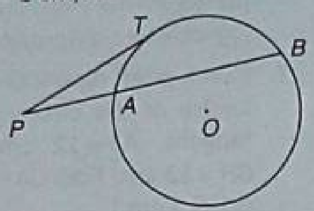
(a) 6 cm
(b) 4 cm
(c) 2 cm
(d) 3 cm

40. In the given figure, $AP = 3$ cm, $BA = 5$ cm and $CP = 2$ cm. Find CD :



(a) 12 cm
(b) 10 cm
(c) 9 cm
(d) 6 cm

41. In the given figure, tangent $PT = 5$ cm, $PA = 4$ cm, find AB:

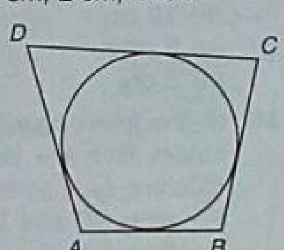


(a) $\frac{7}{4}$ cm
(b) $\frac{11}{4}$ cm
(c) $\frac{9}{4}$ cm
(d) can't be determined

42. Two circles of radii 13 cm and 5 cm touch internally each other. Find the distance between their centres :
(a) 18 cm (b) 12 cm
(c) 9 cm (d) 8 cm

43. Three circles touch each other externally. The distance between their centre is 5 cm, 6 cm and 7 cm. Find the radii of the circles :
(a) 2 cm, 3 cm, 4 cm (b) 3 cm, 4 cm, 1 cm
(c) 1cm, 2.5 cm, 3.5 cm (d) 1 cm, 2 cm, 4 cm

44. A circle touches a quadrilateral $ABCD$. Find the true statement :

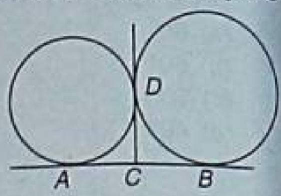


(a) $AB + BC = CD + AD$
(b) $AB + CD = BC + AD$
(c) $BD = AC$
(d) none of the above

45. O and O' are the centres of two circles which touch each other externally at P. AB is a common tangent. Find $\angle APO$:
(a) 90° (b) 120°
(c) 60° (d) data insufficient

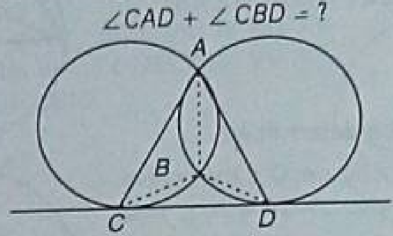
46. If AB is a chord of a circle, P and Q are two points on the circle different from A and B, then :
(a) the angle subtended by AB at P and Q are either equal or supplementary.
(b) the sum of the angles subtended by AB at P and Q is always equal two right angles.
(c) the angles subtended at P and Q by AB are always equal.
(d) the sum of the angles subtended at P and Q is equal to four right angles.

47. In the given figure, AB and CD are two common tangents to the two touching circle. If $CD = 6$ cm, then AB is equal to :



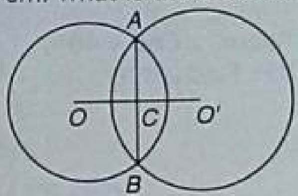
(a) 9 cm
(b) 15 cm
(c) 12 cm
(d) none of the above

48. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then :
$\angle CAD + \angle CBD = ?$



(a) 120° (b) 90°
(c) 360° (d) 180°

49. O and O' are the centres of circle of radii 20 cm and 37 cm. $AB = 24$ cm. What is the distance OO' ?



(a) 51 cm (b) 45 cm
(c) 35 cm (d) 48 cm

50. In a circle of radius 5 cm, AB and AC are the two chords such that $AB = AC = 6$ cm. Find the length of the chord BC :
(a) 4.8 cm (b) 10.8 cm
(c) 9.6 cm (d) none of these

51. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is :
(a) 23 cm (b) 30 cm
(c) 15 cm (d) none of these

52. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :
(a) $\sqrt{3}:2$ (b) $\sqrt{3}:1$
(c) $\sqrt{5}:1$ (d) none of these

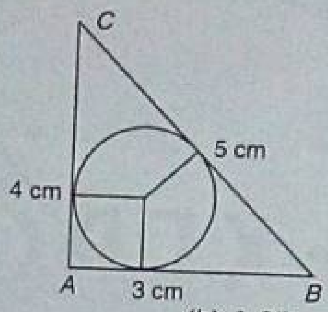
53. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the other circle which is outside the inner circle, is of length :
(a) $2\sqrt{2}$ cm (b) $3\sqrt{2}$ cm
(c) $2\sqrt{3}$ cm (d) $4\sqrt{2}$ cm

54. Through any given set of four points P, Q, R, S it is possible to draw :
(a) atmost one circle
(b) exactly one circle
(c) exactly two circles
(d) exactly three circles

55. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is :
(a) 4 cm (b) 6 cm
(c) 8 cm (d) 10 cm

56. The number of common tangents that can be drawn to two given circles is at the most :
(a) 1 (b) 2
(c) 3 (d) 4

57. ABC is a right angled triangle $AB = 3$ cm, $BC = 5$ cm and $AC = 4$ cm, then the inradius of the circle is :



(a) 1 cm (b) 1.25 cm
(c) 1.5 cm (d) none of these

58. A circle has two parallel chords of lengths 6 cm and 8 cm. If the chords are 1 cm apart and the centre is on the same side of the chords, then a diameter of the circle is of length :
(a) 5 cm (b) 6 cm
(c) 8 cm (d) 10 cm

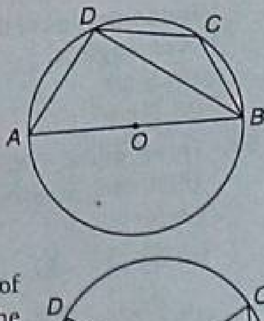
59. Three equal circles of unit radius touch each other. Then, the area of the circle circumscribing the three circles is :
(a) $6\pi(2+\sqrt{3})^2$ (b) $\frac{\pi}{6}(2+\sqrt{3})^2$
(c) $\frac{\pi}{3}(2+\sqrt{3})^2$ (d) $3\pi(2+\sqrt{3})^2$

60. The radius of a circle is 20 cm. The radii (in cm) of three concentric circles drawn in such a manner that the whole area is divided into four equal parts, are :
(a) $20\sqrt{2}, 20\sqrt{3}, 20$ (b) $\frac{10\sqrt{3}}{3}, \frac{10\sqrt{2}}{3}, \frac{10}{3}$
(c) $10\sqrt{3}, 10\sqrt{2}, 10$ (d) 17, 14, 9

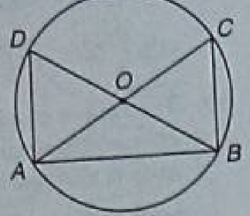
EXERCISE

LEVEL 1

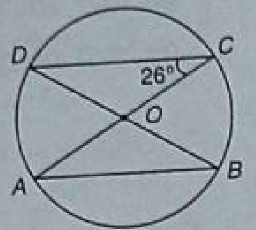
1. In the adjoining figure AB is a diameter of the circle and $\angle BCD = 130°$. What is the value of $\angle ABD$?
(a) 30°
(b) 50°
(c) 40°
(d) none of the above



2. In the given figure O is the centre of the circle and $\angle BAC = 25°$, then the value of $\angle ADB$ is :
(a) 40°
(b) 55°
(c) 50°
(d) 65°

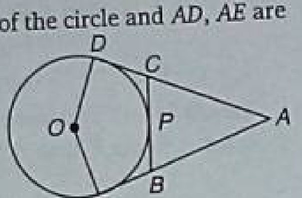


3. In the given figure O is the centre of the circle and $\angle OCD = 26°$, find $\angle AOD$:
(a) 52°
(b) 154°
(c) 128°
(d) data insufficient

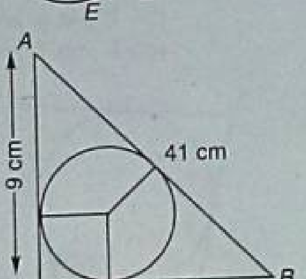


4. Three sides of a triangle ABC are a, b, c. $a = 4700$ cm, $b = 4935$ cm and $c = 6815$ cm. The internal bisector of $\angle A$ meets BC at P, and the bisector passes through incentre 'O'. What is ratio of $PO : OA$?
(a) $3 : 2$
(b) $2 : 3$
(c) $2 : 5$
(d) can't be determined

5. In the given circle O is the centre of the circle and AD, AE are the two tangents. BC is also a tangent, then :
(a) $AC + AB = BC$
(b) $3AE = AB + BC + AC$
(c) $AB + BC + AC = 4AE$
(d) $2AE = AB + BC + AC$

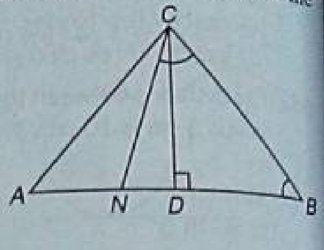


6. What is the inradius of the incircle shown in the figure?
(a) 9 cm
(b) 4 cm
(c) can't be determined
(d) none of the above



7. One of the diagonal of a parallelogram is 18 cm, whose adjacent sides are 16 cm and 20 cm respectively. What is the length of other diagonal?
(a) $2\sqrt{247}$ cm
(b) 13 cm
(c) 15.2 cm
(d) 28.5 cm

8. In the adjoining figure CD is perpendicular on AB. CN is the angle bisector of $\angle ACB$. $\angle ACN = 50°$, then the angle NCD is :
(a) $\frac{1}{3}\angle ACB$
(b) $\angle CAB - \angle CBA$
(c) $\frac{1}{2}(\angle CBA \sim \angle BAC)$
(d) can't be determined

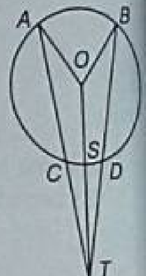


9. In a ΔPQR, points M and N are on the sides PQ and PR respectively such that $PM = 0.6.PQ$ and and $NR = 0.4\ PR$. What percentage of the area of the triangle PQR does that of triangle PMN form?
(a) 60%
(b) 50%
(c) 36%
(d) 55%

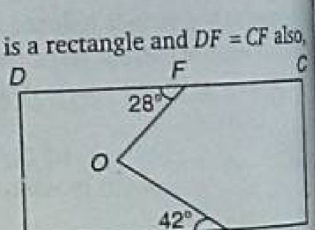
10. In a trapezium $ABCD$, the diagonals AC and BD intersect each other at O such that $OB : OD = 3 : 1$ then the ratio of areas of $\Delta AOB : \Delta COD$ is :
(a) $3 : 1$
(b) $1 : 4$
(c) $9 : 1$
(d) can't be determined

11. Three cities Fatehpur, Barabanki and Lucknow are very famous. Fatehpur is 42 km, away from Barakanki and the distance between Fatehpur and Lucknow is 66 km. Which of the following cannot be the distance between Barabanki and Lucknow?
(a) 28 km
(b) 98 km
(c) 32 km
(d) 23 km

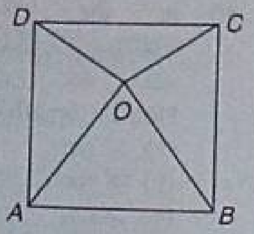
12. In the adjoining figure 'O' is the the centre of circle AC and BD are the two chords of circle which meets at T outside the circle. OT bisects CD, $OA = OB = 8$ cm and $OT = 17$ cm. What is the ratio of distance of AC and BD from the centre of the circle?
(a) $15 : 17$
(b) $8 : 15$
(c) $8 : 9$
(d) none of the above



13. In the adjoining figure $ABCD$ is a rectangle and $DF = CF$ also, $AE = 3BE$. what is the value of $\angle EOF$, if $\angle DFO = 28°$ and $\angle AEO = 42°$?
(a) 14°
(b) 42°
(c) 70°
(d) 90°

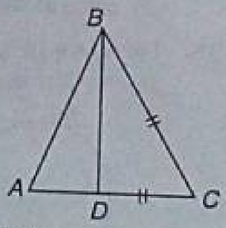


14. $ABCD$ is a square and AOB is an equilateral triangle. What is the value of $\angle DOC$?



(a) $120°$
(b) $150°$
(c) $125°$
(d) can't be determined

15. In the triangle ABC, $BC = CD$ and $(\angle ABC - \angle BAC) = 30°$. The measure of $\angle ABD$ is :

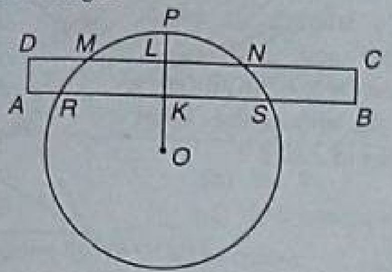


(a) $30°$
(b) $45°$
(c) $15°$
(d) can't be determined

16. In a trapezium $ABCD$, AB is a parallel to CD. BD is perpendicular to AD. AC is perpendicular to BC. If $AD = BC = 15$ cm and $AB = 25$ cm , then the area of the trapezium is :

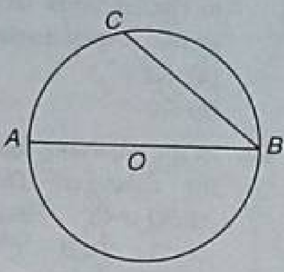
(a) 192 cm^2 (b) 232 cm^2
(c) 172 cm^2 (d) none of these

17. In the adjoining figure O is the centre of the circle. The radius OP bisects a rectangle $ABCD$, at right angle. $DM = NC = 2$ cm and $AR = SB = 1$ cm and $KS = 4$ cm and $OP = 5$ cm. What is the area of the rectangle?



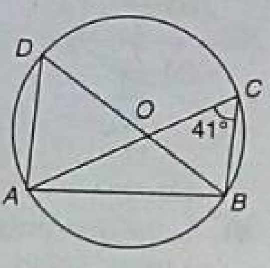
(a) 8 cm^2 (b) 10 cm^2
(c) 12 cm^2 (d) None of these

18. In the given figure of circle AB is the diameter with length 20 cm and BC is 16 cm, then find the length of CO when CO is perpendicular on AB :



(a) 9.6 cm
(b) 8.4 cm
(c) 10 cm
(d) data insufficient

19. In the adjoining figure BD is the diameter of the circle and $\angle BCA = 41°$. Find $\angle ABD$:



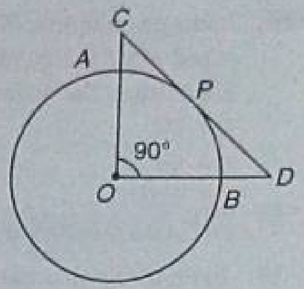
(a) $41°$
(b) $49°$
(c) $22.5°$
(d) $20.5°$

20. Each interior angle of a regular polygon exceeds its exterior angle by $132°$. How many sides does the polygon have ?

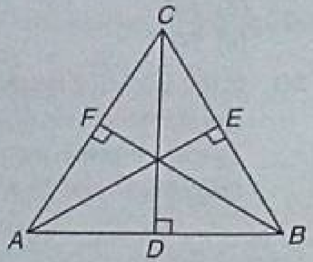
(a) 9 (b) 15
(c) 12 (d) none of these

21. In a circle O is the centre and $\angle COD$ is right angle. $AC_1 = BD$ and CD is the tangent at P. What is the value of $AC + CP$, if the radius of the circle is 1 metre?

(a) 105 cm
(b) 141.4 cm
(c) 138.6 cm
(d) can't be determined

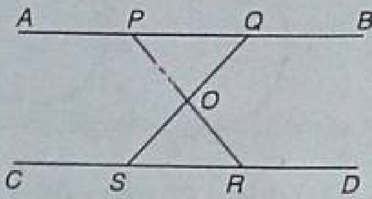


22. In the given triangle ABC, CD, BF and AE are the altitudes. If the ratio of $CD : AE : BF = 2 : 3 : 4$, then the ratio of $AB : BC : CA$ is :



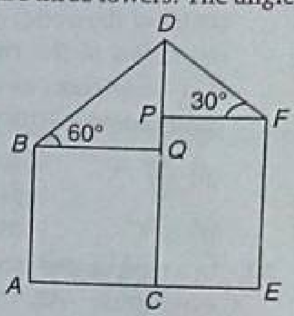
(a) $4 : 3 : 2$
(b) $2 : 3 : 4$
(c) $4 : 9 : 16$
(d) $6 : 4 : 3$

23. In the adjoining figure $AB \| CD$ and $PQ = SR$, then :



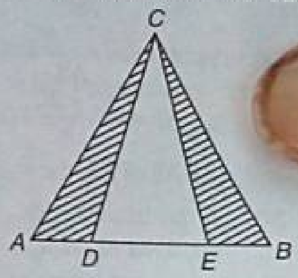
(a) $PQ = PS$
(b) $SR = RP$
(c) $PS = QR$
(d) $AP = RD$

24. In the given figure AB, CD and EF are three towers. The angle of elevation of the top of the tower CD from the top of the tower AB is $60°$ and that from EF is $30°$. $BD = 2\sqrt{3}$ m, $CD : EF = 5 : 4$ and $DF = 4$ m. What is the height of the tower AB ?



(a) 6 m
(b) 12 m
(c) 7 m
(d) none of the above

25. In the equilateral triangle ABC, $AD = DE = BE$, D and E lies on the AB. If each side of the triangle (*i.e.*, AB, BC and AC) be 6 cm, then the area of the shaded region is :

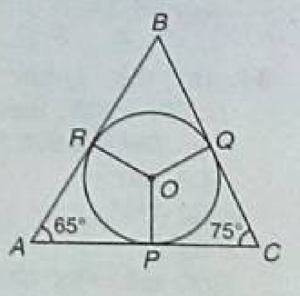


(a) 9 cm^2
(b) $6\sqrt{3} \text{ cm}^2$
(c) $5\sqrt{3} \text{ cm}^2$
(d) none of the above

26. In the above question (number 25) what is the perimeter of triangle CDE ?

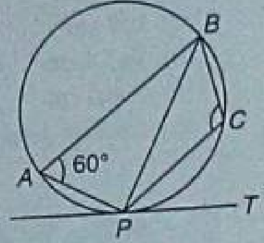
(a) $(3 + 2\sqrt{5})$ cm (b) $2(1 + 2\sqrt{7})$ cm
(c) $2(1 + 3\sqrt{7})$ cm (d) none of these

27. In a triangle ABC, O is the centre of incircle PQR, $\angle BAC = 65°$, $\angle BCA = 75°$, find $\angle ROQ$:



(a) $80°$
(b) $120°$
(c) $140°$
(d) can't be determined

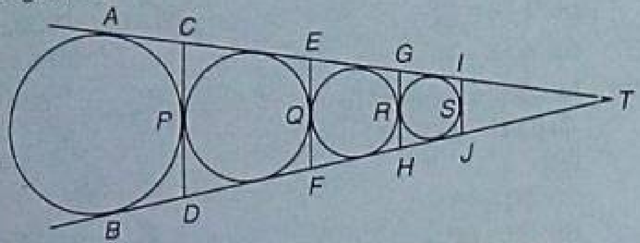
28. In the given figure, PT is a tangent at P and $ABCP$ is a quadrilateral. $\angle BAP$ is $60°$, then the value of $\angle PCB$ is :



(a) $60°$
(b) $90°$
(c) $120°$
(d) data insufficient

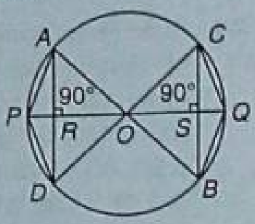
29. In the above question (number 28), what is the value of $\angle TPC$?
(a) $30°$ (b) $60°$
(c) $90°$ (d) can't be determined

30. In the adjoining figure AT and BT are the to two tangents at A and B respectively CD is also a tangent at P.
There are some more circles touching each other and the tangents AT and BT also. Which one of the following is true?



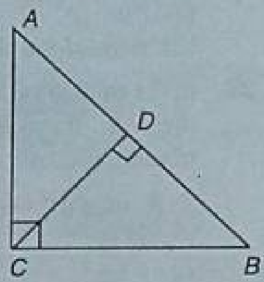
(a) $PC + CT = PD + DT$ (b) $RG + GT = RH + HT$
(c) $PC + QE = CE$ (d) all of these

31. In the adjoining figure O is the centre of the circle. $\angle AOD = 120°$. If the radius of the circle be 'r', then find the sum of the areas of quadrilaterals $AODP$ and $OBQC$:



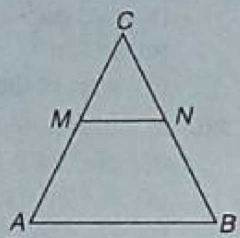
(a) $\frac{\sqrt{3}}{2}r^2$ (b) $3\sqrt{3}r^2$
(c) $\sqrt{3}r^2$ (d) none of these

32. In a right angled triangle ABC, CD is the perpendicular on the hypotenuse AB. Which of the following is correct?



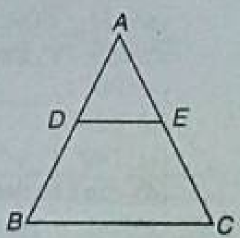
(a) $CD = \frac{AC \times BC}{AB}$
(b) $AD = \frac{AC \times AC}{AB}$
(c) $BD = \frac{BC \times BC}{AB}$
(d) All of the the above

33. In the triangle ABC, MN is parallel to AB. Area of trapezium $ABNM$ is twice the area of triangle CMN. What is ratio of $CM : AM$?



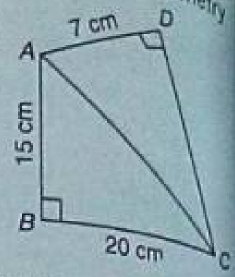
(a) $\frac{1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}-1}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) none of these

34. In the given figure $AD = AE$, $\angle BDE = 100°$, then what is the value of $\angle DBC + \angle BCE$?



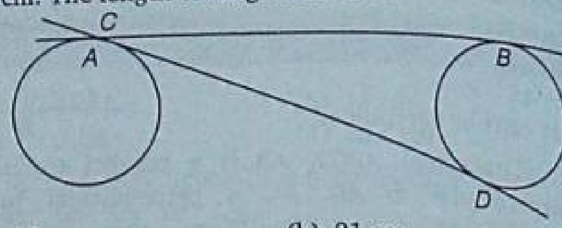
(a) $200°$
(b) $160°$
(c) $80°$
(d) can't be determined

35. In the given quadrilateral $ABCD$, $AB = 15$ cm, $BC = 20$ cm and $AD = 7$ cm, $\angle ABC = \angle ADC = 90°$. Find the length of side CD :



(a) 12 cm
(b) 18 cm
(c) 24 cm
(c) none of the above

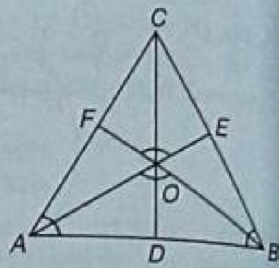
36. There are two circles each with radius 5 cm. Tangent AB is 26 cm. The length of tangent CD is :



(a) 15 cm (b) 21 cm
(c) 24 cm (d) can't be determined

37. One of diagonal of a parallelogram is 10 cm and an angle of the parallelogram is $\pi/4$. If its height be 8 cm then find the area of the parallelogram :
(a) 112 cm^2 (b) 88 cm^2
(c) 92 cm^2 (d) 104 cm^2

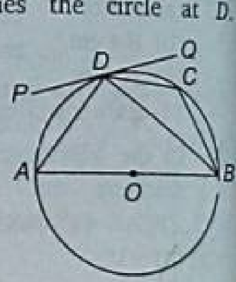
38. ABC is a triangle in which $\angle CAB = 80°$ and $\angle ABC = 50°$, AE, BF and CD are the altitudes and O is the orthocentre. What is the value of $\angle AOB$?



(a) $65°$
(b) $70°$
(c) $50°$
(d) $130°$

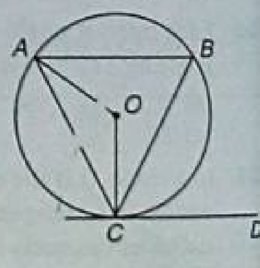
39. ABC is a triangle in which 35 times the smallest angle is equal to the 26 times largest angle. What is the measure of the second largest angle?
(a) $63°$ (b) $58°$
(c) $70°$ (d) $42°$

40. In the adjoining figure O is the centre of the circle and AB is the diameter. Tangent PQ touches the circle at D. $\angle BDQ = 48°$. Find the ratio of $\angle DBA : \angle DCB$:



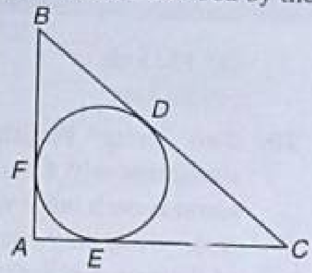
(a) $\frac{22}{7}$
(b) $\frac{7}{22}$
(c) $\frac{7}{12}$
(d) can't be determined

41. In the given diagram O is the centre of the circle and CD is a tangent. $\angle CAB$ and $\angle ACD$ are supplementary to each other $\angle OAC = 30°$. Find the value of $\angle OCB$:



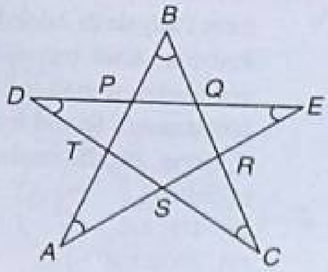
(a) $30°$
(b) $20°$
(c) $60°$
(d) none of the above

42. In the given diagram an incircle DEF is circumscribed by the right angled triangle in which $AF = 6$ cm and $EC = 15$ cm. Find the difference between CD and BD:



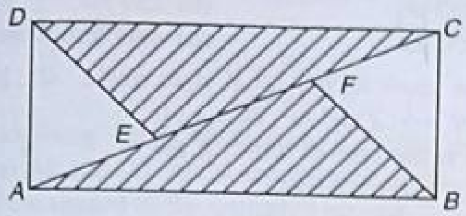
(a) 1 cm
(b) 3 cm
(c) 4 cm
(d) can't be determined

43. In the adjoining figure, a star is shown. What is the sum of the angles A, B, C, D and E?



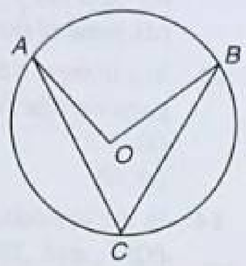
(a) 120°
(b) 180°
(c) 240°
(d) can't be determined

44. $ABCD$ is a rectangle of dimensions 6 cm × 8 cm. DE and BF are the perpendiculars drawn on the diagonal of the rectangle. What is the ratio of the shaded to that of unshaded region?



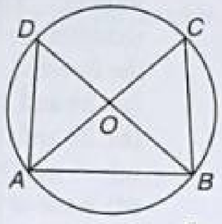
(a) 7 : 3
(b) 16 : 9
(c) $4 : 3\sqrt{2}$
(d) data insufficient

45. In the adjoining figure 'O' is the centre of circle. $\angle CAO = 25°$ and $\angle CBO = 35°$. What is the value of $\angle AOB$?



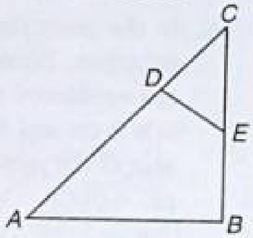
(a) 55°
(b) 110°
(c) 120°
(d) data insufficient

46. In the given diagram $ABCD$ is a cyclic quadrilateral $\angle OCB = 50°$ and $\angle BOC = 110°$. Find the value of $\angle DAO$:



(a) 20°
(b) 30°
(c) 50°
(d) can't be determined

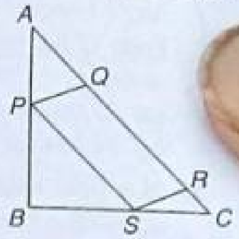
47. ABC and CDE are right angled triangle. $\angle ABC = \angle CDE = 90°$, D lies on AC and E lies on BC. $AB = 24$ cm, $BC = 60$ cm. If $DE = 10$ cm, then CD is :



(a) 28 cm
(b) 35 cm
(c) 25 cm
(d) can't be determined

48. In the above question (number 48) what is the ratio of $CE : BE$?
(a) $\sqrt{29} : (12 - \sqrt{29})$
(b) $12 : \sqrt{29}$
(c) $7 : \sqrt{21}$
(d) none of these

49. In the given diagram ΔABC is an isosceles right angled triangle, in which a rectangle is inscribed in such a way that the length of the rectangle in twice of breadth. Q and R lie on the hypotenuse and P, S lie on the two different smaller sides of the triangle. What is the ratio of the areas of the rectangle and that of triangle :

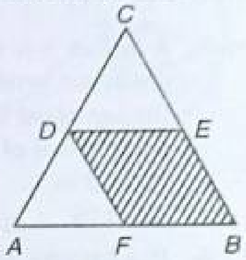


(a) $\sqrt{2} : 1$
(b) $1 : \sqrt{2}$
(c) 1 : 2
(d) $\sqrt{3} : 2$

50. An n sided polygon has 'n' diagonals, then the value of n is :
(a) 4
(b) 6
(c) 7
(d) 5

51. How many distinct equilateral triangles can be formed in a regular nonagon having at two of their vertices as the vertices of nonagon?
(a) 72
(b) 36
(c) 66
(d) none of these

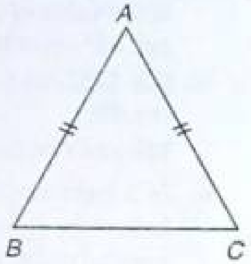
52. ABC is a triangle in which D, E and F are the mid-points of the sides AC, BC and AB respectively. What is the ratio of the area of the shaded to the unshaded region in the triangle?



(a) 1 : 1
(b) 3 : 4
(c) 4 : 5
(d) none of the above

53. Three circles each of units radius intersects each other at P, Q and R. P, Q and R are the centres of the three circles. What is the sum of length of the arcs PQ, QR and PR ?
(a) $\sqrt{3}\pi$
(b) $2\sqrt{2}\pi$
(c) π
(d) none of these

54. ABC is an isosceles triangle in which $AB = AC$ and $(\angle A) = 2(\angle B)$. $AB = 4$ cm. What is the ratio of inradius to the circumradius?



(a) 1 : 2
(b) $(\sqrt{2} - 1) : 1$
(b) $1 : (2\sqrt{2} - 1)$
(c) none of the above

Answers

INTRODUCTORY EXERCISE-12.1

1. (c)	2. (c)	3. (b)	4. (b)	5. (b)	6. (a)	7. (d)	8. (a)	9. (a)	10. (b)
11. (a)	12. (c)	13. (b)	14. (b)	15. (d)	16. (b)	17. (b)	18. (b)	19. (a)	20. (c)
21. (b)	22. (c)	23. (c)	24. (b)	25. (c)	26. (a)	27. (c)	28. (c)	29. (a)	30. (b)
31. (c)	32. (a)	33. (a)	34. (c)	35. (a)	36. (b)	37. (b)	38. (c)	39. (b)	40. (a)
41. (c)	42. (c)	43. (a)	44. (b)	45. (b)					

INTRODUCTORY EXERCISE-12.2

1. (b)	2. (b)	3. (c)	4. (d)	5. (a)	6. (c)	7. (d)	8. (d)	9. (b)	10. (b)
11. (b)	12. (c)	13. (d)	14. (b)	15. (b)	16. (b)	17. (c)	18. (b)	19. (d)	20. (b)
21. (a)	22. (d)	23. (c)	24. (b)	25. (c)	26. (c)	27. (c)	28. (a)	29. (a)	30. (b)
31. (c)	32. (c)	33. (d)	34. (d)	35. (a)	36. (d)	37. (b)	38. (d)	39. (c)	40. (d)
41. (c)	42. (a)	43. (c)	44. (c)	45. (b)	46. (c)	47. (a)	48. (a)	49. (c)	50. (c)
51. (a)	52. (a)	53. (a)	54. (c)	55. (a)	56. (a)	57. (a)	58. (b)	59. (b)	60. (c)
61. (c)	62. (b)	63. (c)	64. (b)	65. (b)	66. (c)	67. (c)	68. (a)	69. (d)	70. (a)
71. (b)	72. (c)	73. (c)	74. (a)	75. (b)	76. (c)	77. (c)	78. (b)	79. (c)	80. (d)
81. (b)	82. (b)	83. (a)	84. (c)	85. (b)	86. (c)	87. (a)	88. (d)	89. (c)	90. (b)
91. (d)	92. (c)	93. (c)	94. (c)	95. (b)	96. (c)	97. (b)	98. (a)	99. (b)	100. (d)

INTRODUCTORY EXERCISE-12.3

1. (b)	2. (b)	3. (b)	4. (a)	5. (a)	6. (b)	7. (c)	8. (c)	9. (b)	10. (c)
11. (b)	12. (b)	13. (a)	14. (b)	15. (a)	16. (b)	17. (c)	18. (c)	19. (a)	20. (a)
21. (c)	22. (b)	23. (c)	24. (a)	25. (b)	26. (a)	27. (a)	28. (b)		

INTRODUCTORY EXERCISE-12.4

1. (d)	2. (b)	3. (b)	4. (d)	5. (d)	6. (b)	7. (c)	8. (d)	9. (c)	10. (c)

INTRODUCTORY EXERCISE-12.5

1. (b)	2. (a)	3. (c)	4. (c)	5. (a)	6. (c)	7. (a)	8. (c)	9. (c)	10. (c)
11. (d)	12. (a)	13. (b)	14. (d)	15. (c)	16. (b)	17. (a)	18. (c)	19. (a)	20. (c)
21. (b)	22. (b)	23. (c)	24. (c)	25. (b)	26. (b)	27. (b)	28. (a)	29. (c)	30. (b)
31. (b)	32. (b)	33. (c)	34. (c)	35. (c)	36. (d)	37. (b)	38. (c)	39. (b)	40. (b)
41. (c)	42. (d)	43. (a)	44. (b)	45. (a)	46. (a)	47. (c)	48. (d)	49. (a)	50. (c)
51. (b)	52. (b)	53. (d)	54. (a)	55. (c)	56. (b)	57. (a)	58. (d)	59. (c)	60. (c)

LEVEL-1

1. (c)	2. (d)	3. (a)	4. (c)	5. (d)	6. (b)	7. (a)	8. (c)	9. (c)	10. (c)
11. (d)	12. (d)	13. (c)	14. (b)	15. (c)	16. (a)	17. (b)	18. (a)	19. (b)	20. (b)
21. (b)	22. (d)	23. (c)	24. (c)	25. (b)	26. (b)	27. (c)	28. (c)	29. (d)	30. (d)
31. (c)	32. (d)	33. (c)	34. (b)	35. (c)	36. (c)	37. (a)	38. (d)	39. (b)	40. (b)
41. (a)	42. (a)	43. (b)	44. (b)	45. (c)	46. (a)	47. (c)	48. (a)	49. (c)	50. (d)
51. (c)	52. (a)	53. (c)	54. (b)						

LEVEL-2

1. (b)	2. (d)	3. (d)	4. (c)	5. (b)	6. (c)	7. (d)	8. (b)	9. (b)	10. (b)
11. (a)	12. (d)	13. (d)	14. (b)	15. (b)	16. (a)	17. (a)	18. (c)	19. (d)	20. (b)
21. (c)	22. (c)	23. (b)	24. (a)	25. (d)	26. (c)	27. (a)	28. (d)	29. (a)	30. (a)
31. (a)	32. (a)	33. (a)	34. (a)	35. (b)	36. (b)	37. (c)	38. (b)	39. (a)	40. (b)
41. (b)	42. (a)	43. (a)	44. (b)	45. (c)	46. (c)	47. (c)	48. (a)	49. (b)	50. (c)
51. (c)	52. (a)	53. (b)	54. (a)	55. (b)					