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Variable Correlation Digital Noise Source on FPGA — A Versatile Tool for Debugging Radio Telescope Backends

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Contemporary wideband radio telescope backends are generally developed on Field Programmable Gate Arrays (FPGA) or hybrid (FPGA+GPU) platforms. One of the challenges faced while developing such instruments is the functional verification of the signal processing backend at various stages of development. In the case of an interferometer or pulsar backend, the typical requirement is for one independent noise source per input, with provision for a common, correlated signal component across all the inputs, with controllable level of correlation. This paper describes the design of a FPGA-based variable correlation Digital Noise Source (DNS), and its applications to built-in testing and debugging of correlators and beamformers. This DNS uses the Central Limit Theorem-based approach for generation of Gaussian noise, and the architecture is optimized for resource requirements and ease of integration with existing signal processing blocks on FPGA.

Keywords: FPGA, noise source, variable correlation, radio telescope, correlator.

1. Introduction

A radio telescope array functions as an interferometer by cross-correlating the signals received from each antenna pair of the array. In a generic radio telescope receiver, the radio frequency signals from the astronomical source received by the antennas are first down-converted to Intermediate Frequency (IF) through heterodyning. These IF signals are further down-converted to base-band signals and are then digitized. The digital signals from different antennas are correlated pairwise for every pair of antennas in the digital backend system, for imaging the astronomical source. The amount of correlation measured between a pair of antennas varies for different astronomical sources and baseline lengths (Thompson *et al.*, 2001). A baseline is defined as a pair of spatially separated antennas. For observing compact radio sources with increased sensitivity, signals from all the array antennas are added using a beamformer.

Owing to high speed signal processing requirements, wide-band radio telescope digital backends are being implemented primarily using Application Specific Integrated Circuits (ASICs), Field Programmable Gate Arrays (FPGAs) or Graphics Processing Units (GPUs). These processing elements process the input digital data from the antennas to provide correlated or beamformed output. The signal at the input of these correlators (or beamformers) has a Gaussian distribution and possesses a flat spectrum similar to that of band-limited white noise. This signal is a summation of thermal noise from the receiver electronics and a relatively smaller amount of correlated noise due to the astronomical source.

The initial functional testing of these digital backends is conventionally carried out using analog noise sources (ANSs). Whereas this may be feasible for single dish radio telescopes, it becomes increasingly difficult for large arrays as ideally, one ANS per input is needed, each of which has to be completely uncorrelated (for simulating only receiver noise) or partially correlated with all the other

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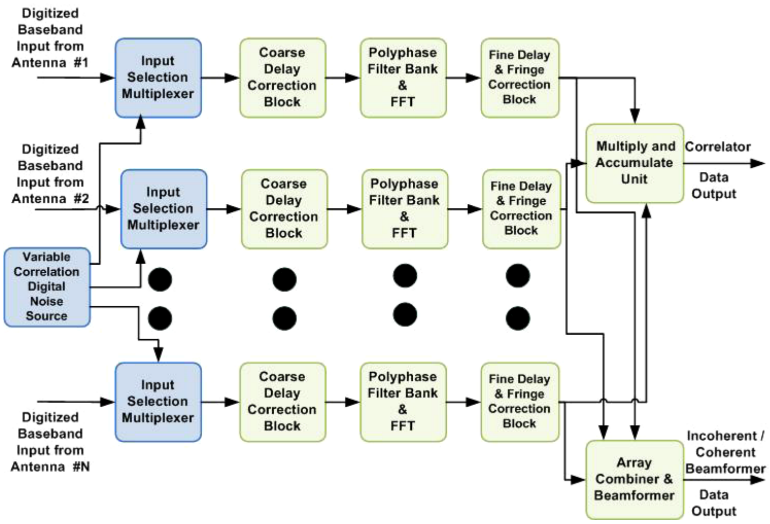

Fig. 1. DNS with other signal processing blocks in a typical FX correlator and beamformer.

noise sources (for simulating receiver noise plus sky signal from an astronomical source). A Digital Noise Source (DNS) for emulating the input signal, implemented on the same hardware (chip) along with the other signal processing blocks, offers an attractive option in such situations. Such a realization makes it easier to test the hardware and also benefits in-system testing during the normal operation of the backend. Figure 1 shows the basic block diagram of a typical FX correlator and beamformer along with the location of such a proposed DNS in the signal processing chain. The existing signal processing blocks are shown in green color, whereas the additional blocks are shown in blue color.

Such a DNS can be realized using FPGA-based random number generators. FPGA-based random number generators for emulating input signals have been developed for several applications like channel emulation (Danger *et al.*, 2001; Ghazel *et al.*, 2001), Monte-Carlo simulations (Bachir & Brault, 2009), correlation radiometers (Pérez *et al.*, 2009) and built-in self test for integrated circuits (Agrawal *et al.*, 1993) amongst others.

This paper describes a multi-bit, hardware optimized, FPGA-based, variable correlation DNS along with its statistical and spectral properties. This variable correlation DNS is suitable for radio astronomy signal emulation and is amenable to integration with FPGA-based digital backend of a radio telescope. Furthermore, the results obtained from the testing of wideband correlators and enhancements in the design to achieve pulse emulation for testing pulsar backends, are also discussed. Since the design is hardware optimized, it can be used for any resource critical system that requires generation of Gaussian noise as a permanent feature along with the other design blocks.

2. Architecture of the DNS

Gaussian random number generators (GRNGs) can be broadly classified into two categories based on the accuracy of the Gaussian noise output — Exact and Approximate (Thomas *et al.*, 2007). Exact GRNGs need more hardware resources than approximate GRNGs. We have observed that the use of an approximate GRNG is generally sufficient for emulating statistics of the astronomical signal received by a radio telescope (detailed analysis in Sec. 3).

A Central Limit Theorem (CLT)-based approach was chosen for realizing GRNGs due to its low hardware resource requirements. The advantage of the CLT approach over Box–Muller method, a commonly used method for FPGA-based random number generation (Danger *et al.*, 2001; Ghazel *et al.*, 2001) is the saving of on-chip memory. CLT-based Gaussian random number generation requires addition of at least 12 uncorrelated random sequence generators having uniform distribution (Thomas *et al.*, 2007). Such uniformly distributed sequences can be generated using an irreducible polynomial, implemented in digital hardware using a Linear Feedback Shift Register (LFSR). The periodicity of such sequences (also called maximal length sequences) is $2^N - 1$ clock cycles, where N is

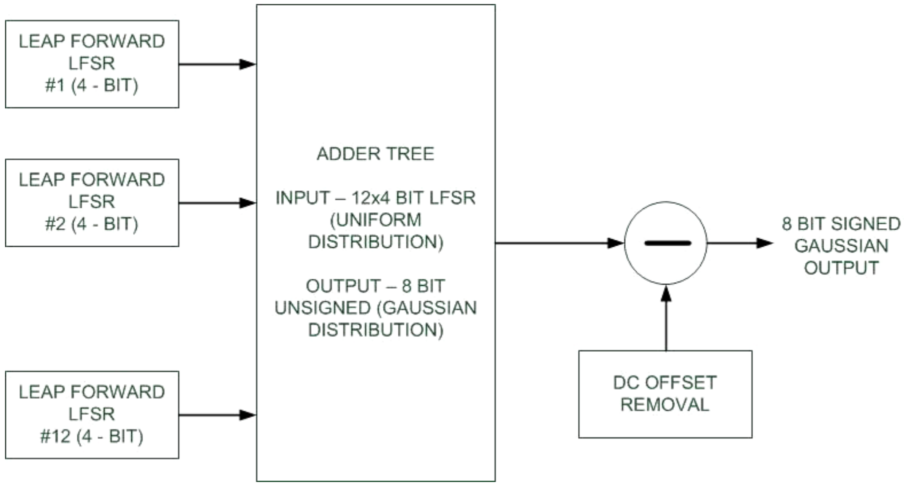

Fig. 2. Block diagram of DNS using CLT-based approach.

the order of the polynomial. In order to get a longer sequence, a higher order polynomial is required. The hardware utilization for CLT-based DNS increases with increase in the order of the polynomial and also with the increase in the number of output bits of the LFSR.

The output of a single LFSR is a serial data stream. Thus, multiple LFSRs are required to achieve multi-bit output *e.g.* in order to generate an 8-bit output, each using 12 LFSRs, a total of 96 LFSRs is required. This would significantly increase the resource requirement. Instead, a leap forward LFSR was used to optimize hardware. A detailed design technique for generating a multi-bit output using leap forward LFSRs is described in Chu & Jones (1999) and Lauradoux (2007). Such a leap forward LFSR is required for each of the 12 uniform number generators. Outputs from each of these leap forward LFSRs are added (using a tree of adders) to converge to a Gaussian distributed output as shown in Fig. 2. At this stage, the hardware was optimized by generating a 4-bit output instead of 8-bit output from each of the leap forward LFSRs. The bit growth inside the adder tree leads to 8 bit output. In our design, it was observed that addition of 14 LFSRs, each providing 4-bit unsigned output, was required to converge to a Gaussian distribution with the desired statistical property of value of Kurtosis within the range of 2.8 to 3.2. It was observed through simulation that the sample Kurtosis (K) of the DNS output varies from $K = 2.99$ (in case of 14 unique polynomials with same seeds) to $K = 2.83$ (for a combination of two sets of polynomials with different seeds). However, as the use of 14 unique polynomials increases the overall design complexity, only two unique sets of polynomials with different seeds were used across 14 LFSRs, while achieving an acceptable value for the Kurtosis.

The output of the adder tree has a fixed DC offset (mean) and variance. The mean can be varied by adding or subtracting a fixed value and the variance can be changed by using a multiplier or divider (or a bit-shifter). In order to get 8-bit signed output, the mean value was subtracted from the generated output.

3. Statistical Characterization of the DNS

Characterization of the DNS was carried out by comparing its statistical properties with digitized samples from an ANS, and with the digitized time series captured from a radio telescope receiver chain. The data from the DNS, ANS and telescope time series were quantized to 8-bit precision. Statistical moments, Skewness and Kurtosis, were computed to check the Gaussian behavior of the DNS.

Skewness is the third statistical moment and is a measure of the asymmetry of the probability distribution. For a data set asymmetrically distributed about the central value, the skewness can be either positive or negative. A skewness of zero ($\pm$ the estimation error) indicates a symmetric distribution about the mean. As shown in Table 1, the sample skewness of the DNS is close to zero, as is the Skewness of the ANS as well as of the telescope data, which is a desirable feature.

Table 1. Comparison of major statistical parameters for signals recorded from different sources.

Parameter	Ideal value	DNS	ANS	Telescope data
Skewness	0	0.0091	−0.0249	−0.0312
Kurtosis	3	2.9962	3.1141	3.0183

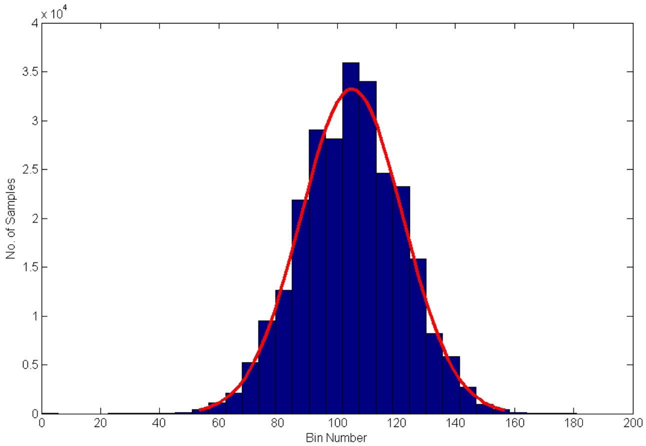

Fig. 3. Comparison between histogram of data captured from the DNS and best fit Gaussian density function.

A quantitative measure of Gaussian nature of a distribution is the fourth moment called Kurtosis. Kurtosis for Gaussian distribution is 3 (± the estimation error). A comparison of the values in Table 1 shows that the value of Kurtosis for the DNS is closer to the ideal, than that of the ANS and telescope data. In this case, the estimation error for Skewness is ±0.0117 and for Kurtosis is ±0.0095, considering 256 K samples of captured data. Figure 3 shows the best fit of a Gaussian density function to the histogram of the DNS. The slightly higher values for the ANS and telescope data, compared to the DNS, can be attributed to the presence of systematic errors in the setup, such as DC offset.

The randomness property of the DNS was further evaluated by integrating successive blocks of digital noise data. The standard deviation of integrated data was compared with the expected $N^{-0.5}$ behavior for a random distribution. Figure 4 shows the normalized standard deviation for the DNS using 18- and 21-bit polynomials and compares it with that of a true random (normal) distribution. In this case, the digital noise samples, divided into blocks of 10 K samples each, were used to compute the standard deviation. The integration was carried

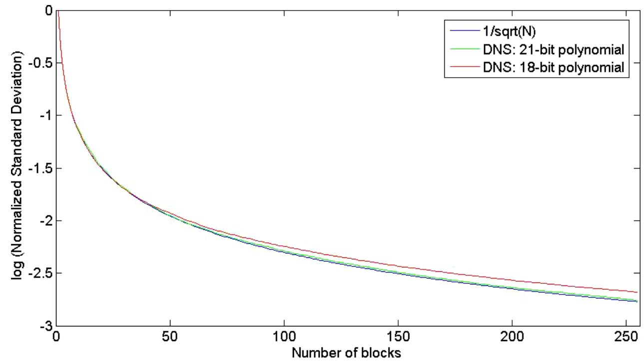

Fig. 4. Comparison of 18-bit and 21-bit polynomial-based DNS outputs upon accumulation.

out on sequences generated from 18- and 21-bit LFSR-based DNS for a duration longer than their respective periods. Since the DNS has a finite periodicity decided by the bit-width of the polynomial, integrating beyond the period leads to deviation from the integrated output of a true random distribution. Since the 21-bit polynomial has a longer period, it starts deviating much later in time than the 18-bit polynomial. This behavior also indicates that successive integration of digital noise data beyond the period of DNS leads to a compromise in the correlation properties of the DNS.

The above test shows that the integration time of the system under test will decide the required periodicity of the sequence generated from the DNS, which in-turn will depend on the bit-width of the LFSR.

4. Implementation and Testing of the DNS

The hardware implementation of the basic DNS was carried out using the Xilinx System Generator for realization on the Xilinx Virtex-5 FPGA. Two numbers of 8-bit wide DNS using polynomials of order 18 and 22, respectively, were implemented inside a FPGA correlator block to emulate two input signals whose auto-correlation and cross-correlation spectra were calculated by the correlator. Sample results are shown in Fig. 5. The first and second panels show autocorrelation spectra of noise sources #1 and #2, respectively, both of which show fairly flat white spectra. The low value of the normalized cross-correlation spectrum (third panel), and the random nature of the phase of the cross-correlation (fourth panel), confirm the uncorrelated nature of the two noise sources.

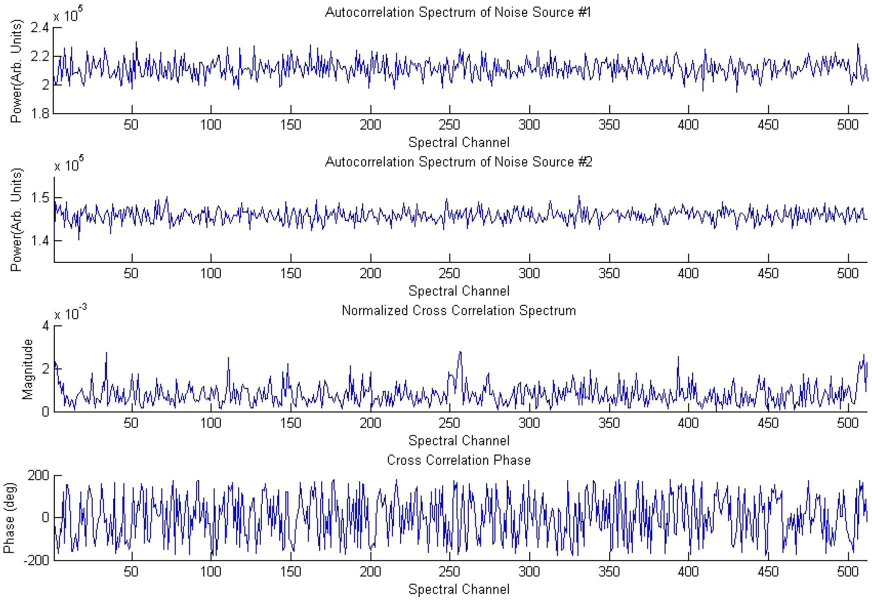

Fig. 5. Auto- and cross-correlation performance for the DNS.

Contemporary digital backends for wide-band radio telescopes process data streams in parallel for real-time operation. For such cases, the basic implementation described above has been modified to provide four parallel streams for each input, which required four different realizations of the basic noise source.

It was observed that for such noise source realizations using parallel data streams, a heterogeneous combination of polynomials yields higher periodicity. For example, use of 18-, 19-, 20- and 21-order polynomial was made for the input channel with four parallel data streams. It was observed that for such a heterogeneous combination of polynomials, the total periodicity is the multiplication of the periods of the individual LFSRs.

Two such noise sources were integrated with a wide-band FPGA correlator having a bandwidth of 300 MHz, 1024 point FFT and 0.89s integration time. The order of polynomials used for the first source were 18, 19, 20 and 21 and for the second source were 22, 23, 24 and 25.

The resource utilization details for this DNS implementation are provided in Table 2. The resource utilization of the two sources adds up to about 3% of the Xilinx Virtex-5 SX95T -1 FF1136 FPGA. This approach does not require any on-chip memory. As the noise source is resource optimized, it can easily be integrated as a permanent, in-built test feature in operational correlators.

Table 2. Resource utilization of the variable correlation block.

Type of resource	Utilization	Utilization (%)
Single noise source		
No. of slice registers	654 out of 58,880	1
No. of slice LUTs	362 out of 58,880	0
Two uncorrelated noise sources		
No. of slice registers	1835 out of 58,880	3
No. of slice LUTs	882 out of 58,880	1

5. Applications of DNS

5.1. *Generation of noise outputs with programmable variable correlation*

As discussed earlier multiple noise sources with controlled amounts of correlation between them are required to emulate the astronomical signals received by an antenna array. Noise sources with programmable variable correlation can be implemented using different methods as described in Hartmann & Cho (2011). Here, we have generated outputs having desired amount of correlation by adding a variable component of noise from a common source, to the output of uncorrelated DNS, as shown in Fig. 6 for a 2 antenna simulation.

Noise sources #1, #2 and the common noise source are mutually uncorrelated. Varying the fraction of output from the common noise source added to the other two noise sources changes the cross-correlation between the final outputs #1 and

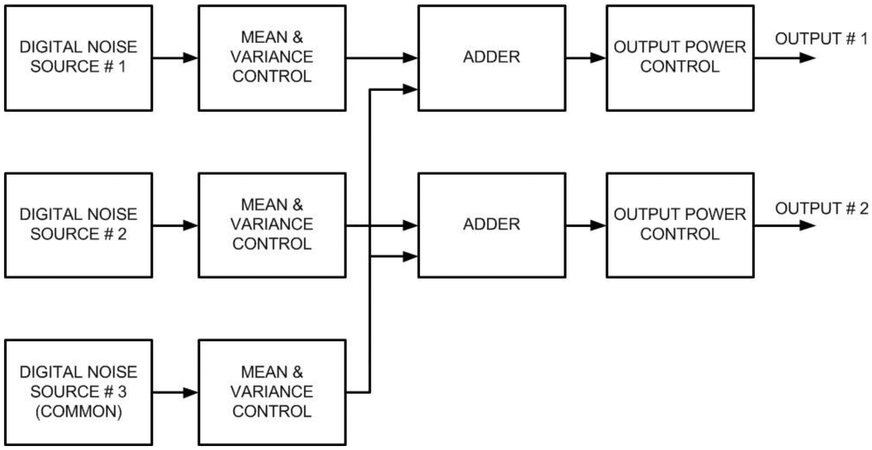

Fig. 6. Block diagram of variable correlation DNS.

#2. If $V_1 = V_2 = V$ are the variances of the two noise source outputs and if V_c is the variance of the common noise source, the normalized cross-correlation coefficient is $R = V_c/(V + V_c)$. The resolution of R is limited by the accuracy of the scaling circuit and the residual correlation amongst the noise source outputs, if any. The process of addition of the common noise to the output of the other two noise sources increases the variance. This has to be re-normalized to prevent saturation effects in the hardware, which is carried out by appropriate scaling of the power level at the output of this block.

The design of the above variable correlation control block was implemented on Xilinx System Generator and tested on Xilinx Virtex-5 SX95T FPGA along with the wide-band correlator designed using CASPER (https://casper.berkeley.edu/) library blocks. Table 3 shows the resource utilization of the variable correlation block.

5.2. *Testing beamformers using the DNS*

The beamformer mode of the digital backend of a radio telescope is used for observations of specific objects like pulsars. Pulsar signals have repetitive pulses with a specific period and duty cycle. The ON time and OFF time of the pulse depends on the properties of a particular pulsar. A noise source with outputs having variable correlation, as described in the previous section, can be used to generate pulsed noise for emulating a pulsar signal. The pulse ON time is emulated by addition of a common noise in the two uncorrelated noise outputs, whereas the OFF time is emulated by the absence of the common noise in the outputs. The ON-OFF switching of the common noise that is added to the pair of uncorrelated noise sources can be controlled by varying a combination of counters. A basic block diagram of pulse generation using such a DNS implementation is shown in Fig. 7.

The functioning of this design was tested with an incoherent beamformer (300 MHz, 1024-point FFT, 1.75 ms integration time) implemented on FPGA (Bist & Buch, 2012) developed at the Giant Metrewave Radio Telescope (GMRT), and the output is as shown in Fig. 8.

Table 3. Resource utilization of the variable correlation block.

Type of resource	Utilization	Utilization (%)
No. of slice registers	1918 out of 58,880	3
No. of slice LUTs	665 out of 58,880	1
No. of slice DSP48Es	28 out of 640	4

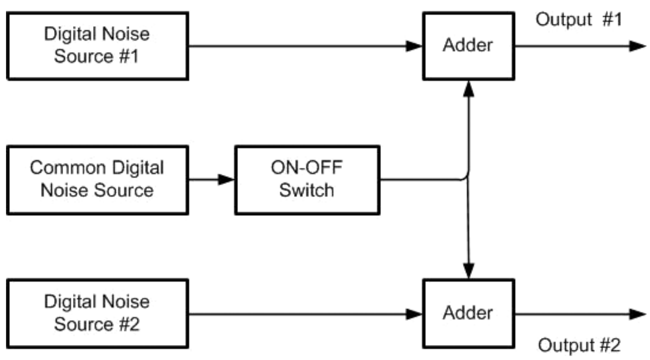

Fig. 7. Pulse generation using DNS.

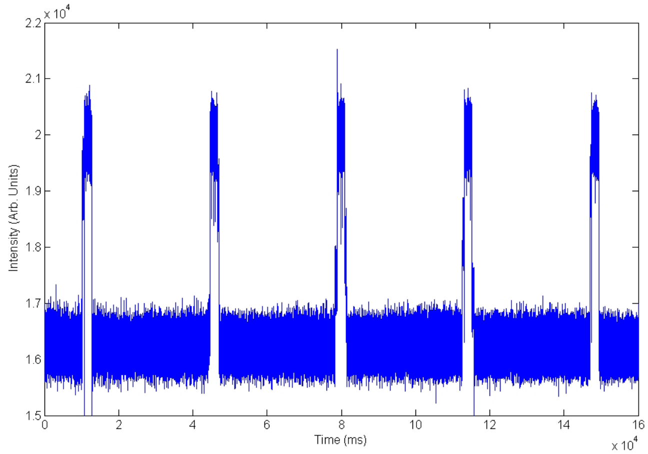

Fig. 8. Accumulated beam output value at 1.75 ms time resolution and 10% duty cycle.

6. Conclusion

This paper described the resource optimized design and architecture, FPGA implementation, and applications of a DNS for testing radio telescope backends. The method of generating variable correlation test signals using this noise source was also described. The ability to generate noise with variable correlation, extended to emulate pulsar signals with periodic ON-OFF switching, shows the utility of the source as a versatile tool for debugging radio telescope backends. The capability of such a DNS can be enhanced by generating noise of desired band-shape. Such a source can also be used for calibrating the entire signal processing chain of a digital backend.

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