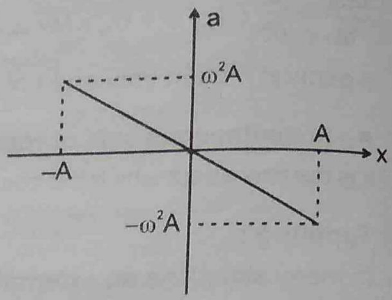
☞ **Acceleration :**

$a = -\omega^2 x \quad = -\omega^2 A \sin(\omega t + \phi)$

GRAPH OF ACCELERATION (A) VS DISPLACEMENT (x)

$a = -\omega^2 x$



6. **ENERGY OF SHM**

Kinetic Energy (KE)

$\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} k (A^2 - x^2)$ (as a function of x)

$= \frac{1}{2} m A^2\omega^2 \cos^2(\omega t + \theta) = \frac{1}{2} KA^2 \cos^2(\omega t + \theta)$ (as a function of t)

$KE_{max} = \frac{1}{2}kA^2 \quad ; \quad \langle KE \rangle_{0-T} = \frac{1}{4}kA^2 \quad ; \quad \langle KE \rangle_{0-A} = \frac{1}{3}kA^2$

Frequency of KE = 2 × (frequency of SHM)

Potential Energy (PE)

$\frac{1}{2} Kx^2$ (as a function of x) $= \frac{1}{2} kA^2 \sin^2(\omega t + \theta)$ (as a function of time)

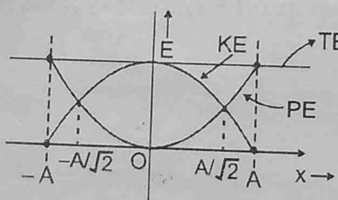
Total Mechanical Energy (TME)

Total mechanical energy = Kinetic energy + Potential energy

$$= \frac{1}{2} k (A^2 - x^2) + \frac{1}{2} Kx^2 = \frac{1}{2} KA^2$$

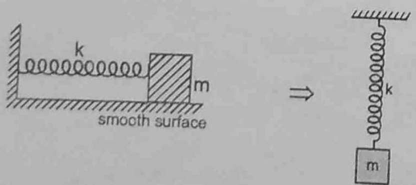
Hence total mechanical energy is constant in SHM.

Graphical Variation of energy of particle in SHM.



☞ **Spring-Mass System :**

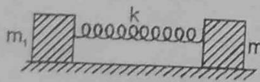
(1)



$\Rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$,

where y_0 is the stretch in the spring at equilibrium position.

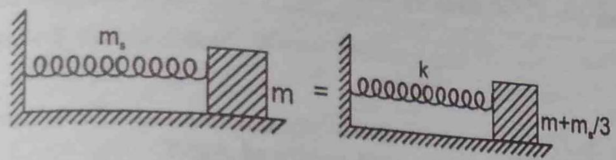
(2)



$T = 2\pi\sqrt{\frac{\mu}{K}}$, where $\mu = \frac{m_1 m_2}{(m_1 + m_2)}$

known as reduced mass

(3) If spring has mass m_s then



$$T = 2\pi\sqrt{\frac{m+\frac{m_s}{3}}{k}}$$

Combination of Springs :

Series Combination : $1/k_{eq} = 1/k_1 + 1/k_2$

Parallel combination : $k_{eq} = k_1 + k_2$

Note : Spring constant of spring is proportional to reciprocal of its natural length, i.e. $k \propto 1/\ell$

Simple Pendulum : $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{g_{eff.}}}$ (in accelerating Reference Frame);

where $g_{eff.} = |\vec{g} - \vec{a}|$ at mean position

Also $g_{eff} = \frac{\text{Net tension in string}}{\text{mass of bob}}$ at mean position

- General formula for time period of simple pendulum when ℓ is comparable to radius of Earth R.

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R}+\frac{1}{\ell}\right)}}$$ where, R = Radius of the earth

- Time period of simple pendulum of infinite length is maximum and is given by: $T = 2\pi\sqrt{\frac{R}{g}} = 84.6$ min

  (Where R is radius of earth)

- Time period of seconds pendulum is 2 sec and $\ell = 0.993$ m.

- Simple pendulum performs angular S.H.M. but due to small angular displacement, it is considered as linear S.H.M.

- If g remains constant & $\Delta\ell$ is change in length, then $\frac{\Delta T}{T}\times 100 = \frac{1}{2}\frac{\Delta\ell}{\ell}\times 100$

- If ℓ remain constant & Δg is change in acceleration then, $\frac{\Delta T}{T}\times 100 = -\frac{1}{2}\frac{\Delta g}{g}\times 100$

- If $\Delta\ell$ is change in length & Δg is change in acceleration due to gravity then,

$$\frac{\Delta T}{T}\times 100 = \left[\frac{1}{2}\frac{\Delta\ell}{\ell} - \frac{1}{2}\frac{\Delta g}{g}\right]\times 100$$

☞ Compound Pendulum / Physical pendulum :

Time period (T) : $T = 2\pi\sqrt{\dfrac{I}{mg\ell}}$

where, $I = I_{CM} + m\ell^2$; ℓ is distance between point of suspension and centre of mass.

☞ Torsional Pendulum :

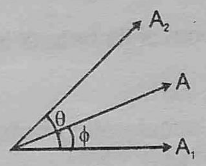
Time period (T) : $T = 2\pi\sqrt{\dfrac{I}{C}}$ where, C = Torsional constant

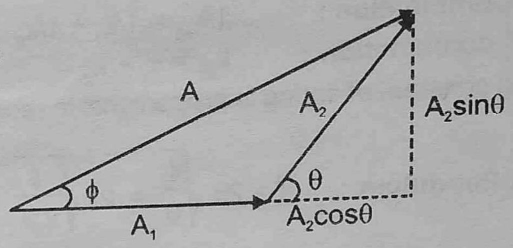
☞ Superposition of SHM's along the same direction :

$x_1 = A_1 \sin \omega t$ &

$x_2 = A_2 \sin (\omega t + \theta)$

If equation of resultant SHM is taken as $x = A \sin (\omega t + \phi)$





Phasor diagram

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

$$\& \quad \tan\phi = \frac{A_2 \sin\theta}{A_1 + A_2\cos\theta}$$

Superposition of SHM's in the perpendicular directions.

$X = A \sin\omega t$

$Y = B \sin\omega t$

If $\theta = 0, \pi$ then path will be straight line and motion will be S.H.M with amplitude $\sqrt{A^2 + B^2}$

If $\theta = \pi/2$ then path will be Ellipse and motion will not be S.H.M

☞ Forced oscillation and resonance :

If a force $F = F_0 \sin\omega t$ is applied to a body of mass m on which a restoring force – kx and a damping force –bv is acting then the motion is complicated for some time after this the body oscillates with frequency ω of the applied periodic force.

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\dfrac{b\omega}{m}\right)^2}}$$, where $\omega_0 = \sqrt{\dfrac{K}{m}}$, which it the natural angular freuqency.

If there is no damping force then

$$A = \frac{F_0/m}{(\omega^2 - \omega_0^2)}$$, where is the amplitude of resulting SHM.

On varying the ω of the applied force, this amplitude changes and becomes maximum when $\omega = \sqrt{\omega_0^2 - \left(\dfrac{b}{2m}\right)^2}$

$\omega \approx \omega_0$

This condition is called resonance and this frequency is called resonance frequency.

WAVES ON A STRING

General Equation of Wave Motion :

General differential equation of wave motion is $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ where v is wave speed and y is the value of disturbance at point x in space at time t. General solution of above equation is of the form:

$$y(x,t) = f\left(t \pm \frac{x}{v}\right)$$

where, y (x, t) should be finite everywhere.

$\Rightarrow$ $f\left(t + \frac{x}{v}\right)$ represents wave travelling in – ve x-axis.

$\Rightarrow$ $f\left(t - \frac{x}{v}\right)$ represents wave travelling in + ve x-axis.

If a travelling wave is sine or cosine function of $\left(t \pm \frac{x}{v}\right)$, the wave is said to be harmonic progressive wave, and its most general form is

$$y = A \sin(\omega t \pm kx + \phi)$$

Terms related to Wave Motion (for 1-D progressive sine wave) :

(a) **Amplitude (A) :** - Maximum displacement of vibrating particles from their mean position (every particle has same amplitude). Amplitude depends on source.

(b) **Time period (T) :** - Minimum time after which a wave repeats itself is called time period.
$T = 2\pi/\omega$ where ω is angular frequency.

(c) **Frequency (f) :** - Number of oscillations made by a particle in one second, frequency of oscillation of particle = frequency of wave $f = \frac{1}{T} = \frac{\omega}{2\pi}$. Frequency also depends on source and is independent of nature of medium in which wave propagates.

(d) **Wavelength (λ) :** Minimum distance between two particles oscillating in same phase or distance after which wave repeats itself.

$$\lambda = vT = \frac{2\pi v}{\omega} = \frac{2\pi}{k}$$

(e) **Wave number (or propagation constant) (k) :**

$$k = 2\pi/\lambda = \frac{\omega}{v} \text{ (rad m}^{-1}\text{)}$$

(f) **Phase of wave :** The argument of harmonic function $(\omega t \pm kx + \phi)$ is called phase of the wave.
Phase difference $(\Delta\phi)$: difference in phases of two particles at any time t.

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \qquad \text{Also. } \Delta\phi = \frac{2\pi}{T}.\Delta t$$

(g) **Wave speed (v) :** Speed with which disturbance propagates in a medium is called wave speed and it remains constant in a homogeneous medium. $v = \frac{dx}{dt}$

(h) **Velocity and acceleration of a particle :**

$$y = A \sin(\omega t - kx + \phi)$$

$$v_p = \frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx + \phi) = \omega \sqrt{(A^2 - y^2)}$$

$$a_p = \frac{\partial v_p}{\partial t} = -\omega^2 A \sin(\omega t - kx + \phi) = -\omega^2 y$$

☞ **Speed of transverse wave along a string/wire :**

Speed depends on

(a) Elasticity (measured by tension in string)

(b) Inertia (measured by mass per unit length)

$$v = \sqrt{\frac{T}{\mu}} \quad \text{where} \quad \begin{matrix} T = \text{Tension} \\ \mu = \text{mass per unit length} \end{matrix}$$

☞ **Power Transmitted along the string by a Sine Wave :**

When a travelling wave is established on a string, energy is transmitted along the direction of propagation of the wave, in form of potential energy and kinetic energy

Average Power $\langle P \rangle = 2\pi^2 f^2 A^2 \mu v$

Intensity $\quad I = \frac{\langle P \rangle}{s} = 2\pi^2 f^2 A^2 \rho v$

for a given medium (given μ & T) $\quad I \propto A^2 \quad \propto f^2$

Energy density : Energy in unit volume of a string due to wave propagation.

☞ **Superposition of Waves :**

When two or more than two waves pass through the same medium simultaneously then each particle of the medium is affected by both wave. This is called superposition of waves. The resultant displacement of each particle is obtained by vector sum of displacements produced by waves individually.

☞ **Reflection and Refraction of Waves :**

(a) If a wave enters a region where the wave velocity is smaller, then reflected wave gets inverted, if it enters a region where the wave velocity is larger, then reflected wave does not gets inverted. The transmitted wave is never inverted.

(b) frequency of reflected and transmitted waves are same as that of incident wave

$$\omega_r = \omega_t = \omega_i$$

☞ **Standing/Stationary Waves :**

(a) When two waves of same frequency and amplitude travel in opposite directions at same speed, these waves superimpose to form standing waves.

(b) $\quad y_1 = A \sin(\omega t - kx + \theta_1)$

$\quad y_2 = A \sin(\omega t + kx + \theta_2)$

$$y_1 + y_2 = \left[2A \cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)\right] \sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right)$$

The quantity $2A \cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)$ represents resultant amplitude at x. At some position resultant amplitude is zero these are called **nodes**. At some positions resultant amplitude is 2A, these are called **antinodes.**

(c) Distance between successive nodes or antinodes = $\frac{\lambda}{2}$.

(d) Distance between successive nodes and antinodes = $\lambda/4$.

(e) All the particles in same segment (portion between two successive nodes) vibrate in same phase.

(f) The particles in two consecutive segments vibrate in opposite phase.

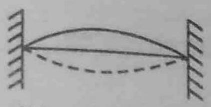
(g) Since nodes are permanently at rest so energy can not be transmitted across these.

Vibrations of strings (Standing wave) :

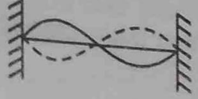
(a) Fixed at both ends :

1. Fixed ends will be nodes. So waves for which

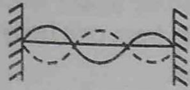
$L = \frac{\lambda}{2}$



$L = \frac{2\lambda}{2}$



$L = \frac{3\lambda}{2}$



are possible giving

$L = \frac{n\lambda}{2}$ or $\lambda = \frac{2L}{n}$ where n = 1, 2, 3, ...

as $v = \sqrt{\frac{T}{\mu}}$ $f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$, n = no. of loops

2. Higher frequencies are integral multiples of $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ called fundamental frequency and corresponding mode of vibration is fundamental mode.

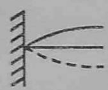
$f_2 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$ – 2nd harmonic (1st overtone)

$f_3 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$ – 3rd harmonic (2nd overtone)

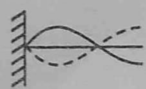
3. Any harmonic can be generated in a string fixed at both ends.

(b) String free at one end :

1. for fundamental mode $L = \frac{\lambda}{4}$ = or $\lambda = 4L$



First overtone $L = \frac{3\lambda}{4}$ Hence $\lambda = \frac{4L}{3} \Rightarrow$



so $f_1 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$ (First overtone)

Second overtone $f_2 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$ so $f_n = \frac{\left(n+\frac{1}{2}\right)}{2L}\sqrt{\frac{T}{\mu}} = \frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}}$

2. Only odd harmonics can be generated in a string free at one end and fixed at the other.
3. If one end is x = 0 then amplitude at any x is A sin kx where A is max amplitude.

Laws of transverse vibrations of a string - sonometer wire :

(a) Law of length $f \propto \frac{1}{L}$ so $\frac{f_1}{f_2} = \frac{L_2}{L_1}$; if T & μ are constant

(b) Law of tension $f \propto \sqrt{T}$ so $\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$; L & μ are constant

(c) Law of mass $f \propto \frac{1}{\sqrt{\mu}}$ so $\frac{f_1}{f_2} = \sqrt{\frac{\mu_2}{\mu_1}}$; T & L are constant.

HEAT AND THERMODYNAMICS

Heat Transfer :

Introduction :

The transfer of heat from one body to the other takes place through three routes.

(i) Conduction

(ii) Convection

(iii) Radiation

Conduction :

The process of transmission of heat energy in which heat is transferred from one particle of the medium to the other, but each particle of the medium stays at its own position is called conduction. Rate of flow of heat is given by

$$\boxed{\frac{dQ}{dt} = -KA\frac{dT}{dx}}$$

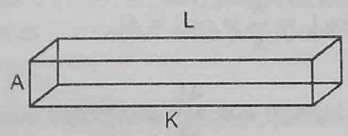
Steady state :

If the temperature of a cross-section at any position x in the above slab remains constant with time (remember, it does vary with position x), the slab is said to be in steady state.

Thermal Resistance to Conduction :

thermal resistance R is given as

$$R = \frac{L}{KA}$$



In terms of R, the amount of heat flowing though a slab in steady-state (in time t)

$$\frac{Q}{t} = \frac{(T_H - T_L)}{R} \quad \Rightarrow \quad i_T = \frac{T_H - T_L}{R}$$

Slabs in series and Parallel :

(1) **Slabs in series (in steady state)**

If two or more than two slabs are joined in series and are allowed to attain steady state, then equivalent thermal resistance is given by

$$R = R_1 + R_2 + R_3 + \ldots\ldots.$$

$$\frac{\ell_{eq}}{K_{eq}} = \frac{\ell_1}{K_1} + \frac{\ell_2}{K_2} + \ldots\ldots.$$

(2) **Slabs in parallel :**

If two or more than two rods are joined in parallel, the equivalent thermal resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots..$$

$$K_{eq}A_{eq} = K_1A_1 + K_2A_2 + \ldots..$$

Convection :

It is the process in which medium is required and heat transfer occurs by actual movement of medium. For example, heat transfer by air.

Radiation :

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

Prevost theory of exchange :

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature.

Perfectly black body :

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.

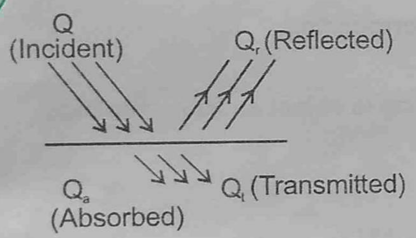
Absorption, reflection and emission of radiations :

$$Q = Q_r + Q_t + Q_a$$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

$$1 = r + t + a$$

where r = reflecting power , a = absorptive power



and t = transmission power.

(i) r = 0, t = 0, a = 1, perfect black body

(ii) r = 1, t = 0, a = 0, perfect reflector

(iii) r = 0, t = 1, a = 0, perfect transmitter

As all the radiations incident on a black body are absorbed, a = 1 for a black body.

Spectral Emissive power (E_λ) :

$$\frac{dE}{d\lambda} = E_\lambda \quad \Rightarrow \quad E = \int_0^\infty E_\lambda \, d\lambda$$

Emissivity:

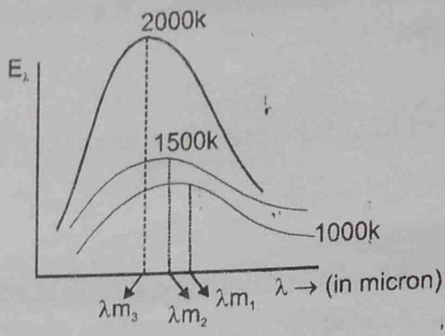
$$e = \frac{\text{Emissive power of a body at temperature } T}{\text{Emissive power of a black body at same temperature } T} = \frac{E}{E_0}.$$

Kirchoff's Law :

The ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all substances at the same temperature and is equal to the emissive power of a perfectly black body for the same wavelength and temperature.

$$\frac{E(\text{body})}{a(\text{body})} = E\,(\text{black body})$$

Nature of thermal radiations : (Wien's displacement law)



$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

This is called Wien's displacement law.
Here b = 0.282 cm-K, is the Wien's constant.

Stefan-Boltzmann's law :

$u = \sigma A T^4$ (for perfectly black body)
where σ is Stefan's constant $= 5.67 \times 10^{-8}\ W/m^2 k^4$

$u = e\sigma A T^4$ (for a body which is not a perfectly black body)

where e = emissivity (which is equal to absorptive power) which lies between 0 to 1

$\Delta u = u - u_0 = e\sigma A\,(T^4 - T_0^4)$

Newton's law of cooling :

For small temperature difference between a body and its surrounding,

$\frac{d\theta}{dt} \propto (\theta - \theta_0)$, where θ and θ_0 are temperature corresponding to object and surroundings.

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\theta_f = \theta_0 + (\theta_i - \theta_0)\, e^{-kt}$$

CALORIMETRY AND THERMAL EXPANSION

- Total heat given by hot objects = Total heat received by cold objects.
- Specific heat (s)

$$\Delta Q = ms\,\Delta\theta$$

when temperature changes but phase does not change.

- $$\Delta Q = mL$$

When phase changes but temperature does not change.

- Linear expansion

$$L = L_0\,(1 + \alpha\,\Delta\,\theta)$$

- Thermal stress

$$F/A = Y\,\alpha\,\Delta\,\theta$$

- Measured value = calibrated value $(1 + \alpha\,\Delta\,\theta)$

$$\alpha = \alpha_{object} - \alpha_{scale}$$

- Area expansion

$$A = A_0(1 + \beta\,\Delta\,\theta)$$

- Volume expansion

$$V = V_0\,(1 + \gamma\,\Delta\,\theta)$$

- Relation between α, β and γ for isotropic solid.

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$$

- Volume overflow

$$\Delta V = V_0\,(\gamma_L - \gamma_C)\,\Delta T$$

- Height of liquid in container

$$h = h_0\{1 + (\gamma_L - 2\alpha_s)\,\Delta T\}$$

- General formula for the conversion of temperature from one scale to another:

$$\frac{\text{Temp on one scale}(S_1) - \text{Lower fixed point }(S_1)}{\text{Upper fixed point }(S_1) - \text{Lower fixed point }(S_1)} = \frac{\text{Temp. on other scale}(S_2) - \text{Lower fixed point }(S_2)}{\text{Upper fixed point }(S_2) - \text{Lower fixed point }(S_2)}$$

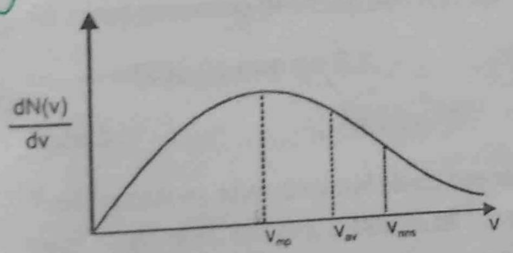
KINETIC THEORY OF GASES

- **Total translational K.E. of gas :**

$$= \frac{1}{2}M < V^2 > = \frac{3}{2}\,PV = \frac{3}{2}\,nRT$$

$$< V^2 > = \frac{3P}{\rho} \qquad V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_{mol}}} = \sqrt{\frac{3KT}{m}}$$

Maxwell's distribution law :



$$V_{rms} > V_{Av} > V_{mp}$$

- Important Points :

$$V_{rms} \propto \sqrt{T}, \quad \bar{V} = \sqrt{\frac{8KT}{\pi m}} = 1.59\sqrt{\frac{KT}{m}} \qquad V_{rms} = 1.73\sqrt{\frac{KT}{m}}$$

Most probable speed

$$V_{mp} = \sqrt{\frac{2KT}{m}} = 1.41\sqrt{\frac{KT}{m}} \qquad \therefore \quad V_{rms} > \bar{V} > V_{mp}$$

☞ **Degree of freedom :**

Mono atomic $f = 3$ (3+0) Diatomic $f = 5$ (T R 3+2) polyatomic $f = 6$ (3+3)

☞ **Maxwell's law of equipartition of energy :**

Total K.E. of the molecule = $\frac{1}{2}$ f KT

For an ideal gas :

Internal energy $U = \frac{f}{2} nRT$

THERMODYNAMICS

☞ **Work done in isothermal process :** $W = [2.303\ nRT \log_{10} \frac{V_f}{V_i}]$

☞ **Internal energy in isothermal process :** $\Delta U = 0$

☞ **Work done in isochoric process :** $dW = 0$

☞ **Change in internal energy in isochoric process :** $\Delta U = n\frac{f}{2} R\Delta T = \Delta Q$ (heat given)

☞ **Isobaric process :**
Work done $\Delta W = nR(T_f - T_i)$
change in internal energy $\Delta U = nC_v \Delta T$
heat given $\Delta Q = \Delta U + W$

☞ **Molar heat capacity of an ideal gas in terms of R:**

$C_v = \frac{f}{2}R \quad C_p = \left(\frac{f}{2}+1\right)R$

(i) for mono atomic gas : $\frac{C_p}{C_v} = 1.67$

(ii) for diatomic gas : $\frac{C_p}{C_v} = 1.4$

(iii) for triatomic gas : $\frac{C_p}{C_v} = 1.33$

☞ **In general :** $\gamma = \frac{C_p}{C_v} = \left[1+\frac{2}{f}\right]$

Mayer's equation $\Rightarrow C_p - C_v = R$ for an ideal gas only

☞ **Adiabatic process :** Work done $W = \frac{nR(T_i - T_f)}{\gamma - 1}$

☞ **In cyclic process :** $\Delta U = 0$ $\Delta Q = W$

In a mixture of non-reacting gases :

$$\text{Mol. wt.} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$$

$$C_v = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2}, \quad C_p = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1 + n_2}$$

$$\gamma = \frac{C_{p(mix)}}{C_{v(mix)}} = \frac{n_1C_{p_1} + n_2C_{p_2} + ...}{n_1C_{v_1} + n_2C_{v_2} + ...}$$

MEASUREMENT ERROR

Least count :

Definition : The smallest quantity an instrument can measure.

Permissible error :

Error in measurement due to the limitation (least count) of the instrument, is called permissible error.
If we use mm scale then maximum permissible error $(\Delta\ell) = 1$ mm. Maximum uncertainty can be 1 mm.

Significant figures :

Significant figures are = Figures which are absolutely correct + The first uncertain figure

Common rules of counting significant figures :

Rule 1 :

All non-zero digits are significant

i.e. 123.56 has five S.F.

Rule 2 :

All zeros occurring between two non-zeros digits are significant

i.e. 1230.05 has six S.F.

Rule 3 :

Trailing zeroes after decimal place are significant (Shows the further accuracy)

i.e. 85 mm, 8.5 cm, 0.085 m all have same significant figure = 2.

<u>Number of S.F. is always conserved, change of units cannot change S.F.</u>

Rule 4:

In the number less than one, all zeros after decimal point and to the left of first non-zero digit are insignificant (arises only due to change of unit)

i.e. 0.000305 has three S.F.

$\Rightarrow$ 3.05×10^{-4} has three S.F.

Rule 5 :

The terminal or trailing zeros in a number without a decimal point are not significant. (Also arises only due to change of unit)

i.e. $154 \text{ m} = 15400 \text{ cm} = 15400 \text{ mm} = 154 \times 10^9 \text{ nm}$

all has only three S.F. all trailing zeros are insignificant

Rule 6 :

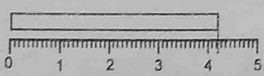
There are certain measurement, which are exact i.e.

Number of apples are = 12 (exactly) = 12.000000.......... ∞

This type of measurement is infinitely accurate so, it has ∞ S.F.

Errors in measurement :

When we use an mm scale : (a scale on which mm. marks are there)



We will measure length $\ell = 4.2$ cm., which is a more closer measurement. Here also if we observe closely, we'll find that the length is a bit more than 4.2, but we cannot say its length to be 4.21, or 4.22, or 4.20 as this scale can measure upto 0.1 cms (1 mm) only.

It can measure upto 0.1 cm accuracy

Its <u>least count</u> is 0.1 cm; maximum <u>uncertainty</u> in ℓ can be = 0.1cm; **<u>maximum possible error</u>** in ℓ can be = 0.1cm

Measurement of length = 4.2 cm. has two **<u>significant figures</u>** ; 4 and 2, in which 4 is absolutely correct , and 2 is reasonable correct (Doubtful) because uncertainty of 0.1 cm is there.

Max. Permissible Error in result due to error in each measurable quantity :

For getting maximum permissible error , sign should be adjusted, so that errors get added up to give maximum effect. let us understand this with an example.

Let Result f(x, y) contains two measurable quantity x and y

Let error in x is = $\pm \Delta x$ i.e. $x \in (x - \Delta x, x + \Delta x)$

error in y is = $\pm \Delta y$ i.e. $y \in (y - \Delta y, y + \Delta y)$

Case : (I) If $f(x, y) = x + y$

max possible error in f = $(\Delta f)_{max}$ = max of $(\pm \Delta x \pm \Delta y)$

$(\Delta f)_{max} = \Delta x + \Delta y$

Case : (II) If $f(x, y) = x - y$

max possible error in f = $(\Delta f)_{max}$ = max of $(\pm \Delta x \mp \Delta y)$

$\Rightarrow \quad (\Delta f)_{max} = \Delta x + \Delta y$

i.e. $f = 2x - 3y - z$

$(\Delta f)_{max} = 2\Delta x + 3\Delta y + \Delta z$

ELECTROSTATICS

Electric Charge :
Charge of a material body or particle is the property (acquired or natural) due to which it produces and experiences electrical and magnetic effects. Some of naturally occurring charged particles are electrons, protons, α-particles etc.

Units of Charge :
C.G.S. unit of charge = electrostatic unit = esu. (or stat coulomb)
1 coulomb = 3×10^9 esu of charge
Dimensional formula of charge = $[M^0L^0T^1I^1]$
[Where, I =Dimension of current]

Coulomb force between two point charges : $\vec{F} = \dfrac{1}{4\pi\varepsilon_0\varepsilon_r}\dfrac{q_1q_2}{|\vec{r}|^3}\vec{r} = \dfrac{1}{4\pi\varepsilon_0\varepsilon_r}\dfrac{q_1q_2}{|\vec{r}|^2}\hat{r}$,

where $\dfrac{1}{4\pi\varepsilon_0} = k = 9\times 10^9$ N-m²/C²

The electric field intensity at any point is the force experienced by unit positive charge, given by $\vec{E} = \dfrac{\vec{F}}{q_0}$ & $\vec{F} = q\vec{E}$, where $\vec{F}$ is force on q due to $\vec{E}$

Electric Potential :
If $(W_{\infty\to P})_{ext}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is :

$$V_p = \left.\frac{W_{\infty\to p})_{ext}}{q}\right]_{\Delta k=0}$$

Potential Difference :

$$V_A - V_B = \left.\frac{(W_{BA})_{ext}}{q}\right)_{\text{keeping KE constant or } K_i = K_f}$$

Potential difference between points A and B : $V_A - V_B = -\int_B^A \vec{E}.d\vec{r}$

or $\vec{E} = -\left[\hat{i}\dfrac{\partial}{\partial x}V + \hat{j}\dfrac{\partial}{\partial x}V + \hat{k}\dfrac{\partial}{\partial z}V\right] = -\left[\hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial x} + \hat{k}\dfrac{\partial}{\partial z}\right]V = -\nabla V = -\text{grad } V$

Formulae of $\vec{E}$ and V :

(i) For point charge, $\vec{E} = \dfrac{Kq}{|\vec{r}|^2}\cdot\hat{r} = \dfrac{Kq}{r^3}\vec{r}$ & $V = \dfrac{Kq}{r}$ [V is an scalar quantity]

(ii) For infinitely long line charge $\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \dfrac{2K\lambda\hat{r}}{r}$

V = not defined, $V_B - V_A = -2K\lambda \ln(r_B / r_A)$

(iii) For uniformly charged ring

$E_{axis} = \dfrac{KQx}{(R^2+x^2)^{3/2}}$, $E_{centre} = 0$

$V_{axis} = \dfrac{KQ}{\sqrt{R^2+x^2}}$, $V_{centre} = \dfrac{KQ}{R}$

where, x is the distance from centre along axis.

(iv) For thin uniformly charged disc (Surface charge density is σ) :

$$E_{axis} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2 + x^2}}\right] \quad \Rightarrow \quad V_{axis} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2 + x^2} - x\right]$$

(v) For infinite nonconducting thin sheet, $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$,

V = not defined, $V_B - V_A = -\frac{\sigma}{2\varepsilon_0}(r_B - r_A)$

(vi) For infinitely large charged conducting sheet, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

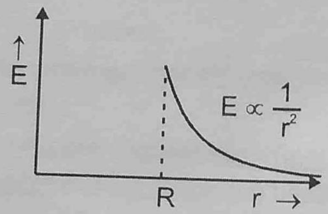
V = not defined, $V_B - V_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$

(vii) For uniformly charged hollow conducting/ nonconducting /solid conducting sphere :

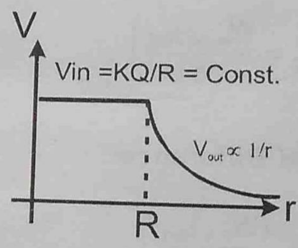
(a) For $r \geq R$, $\vec{E}_{out} = \frac{KQ}{|\vec{r}|^2}\hat{r}$, $V_{out} = \frac{KQ}{r}$

(b) For $r < R$, $\vec{E}_{in} = 0$, $V_{in} = \frac{KQ}{R}$

Graph of E v/s r :



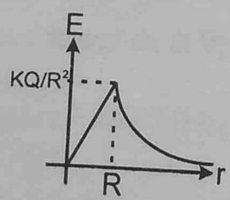
Graph of V v/s r :

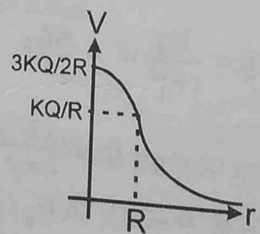


(viii) For uniformly charged solid nonconducting sphere (insulating material)

(a) For $r \geq R$, $\vec{E}_{out} = \frac{KQ}{|\vec{r}|^2}\hat{r}$, $V_{out} = \frac{KQ}{r}$

(b) For $r \leq R$, $\vec{E}_{in} = \frac{KQ\vec{r}}{R^3} = \frac{\rho\vec{r}}{3\varepsilon_0}$, $V_{in} = \frac{KQ}{2R^3}(3R^2 - r^2)$





☞ **Work done by external agent in taking charge from A to B is**
$(W_{ext})_{AB} = q(V_B - V_A)$ and $(W_{el})_{AB} = q(V_A - V_B)$. where $W_{el} \Rightarrow$ workdone by electric field

☞ **The electrostatic potential energy of a point charge**
U = qV

- U = PE of the system =

$$\frac{U_1 + U_2 + ...U_n}{2} = (U_{12} + U_{13} + ... + U_{1n}) + (U_{23} + U_{24} + ... + U_{2n}) + (U_{34} + U_{35} + ... + U_{3n}) ......$$

- Energy Density = $\frac{1}{2}\varepsilon E^2$; where ϵ = permittivity of given medium.

- Self Energy of a uniformly charged spherical shell = $U_{self} = \frac{KQ^2}{2R}$

- Self Energy of a uniformly charged solid non-conducting sphere = $U_{self} = \frac{3KQ^2}{5R}$

- **Electric Field Intensity Due to Dipole**

(i) On the axis, $\vec{E} = \frac{2K\vec{P}}{r^3}$ i.e. (Electric field is along dipole moment $\vec{P}$)

(ii) On the equatorial position : $\vec{E} = -\frac{K\vec{P}}{r^3}$ i.e.(Electric field is in a direction opposite to dipole moment $\vec{P}$)

(iii) Total electric field at general point O (r,θ) is

$$E_{res} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$$

- **Potential Energy of an Electric Dipole in External Uniform Electric Field :**

$$U = -\vec{P}.\vec{E}$$

- **Electric Dipole in Uniform External Electric Field :**

Torque, $\vec{\tau} = \vec{P} \times \vec{E}$; $\vec{F} = 0$

- **Electric Potential Due to Dipole at General Point (r, θ) :**

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} = \frac{\vec{P}.\vec{r}}{4\pi\epsilon_0 r^3}$$

Equipotential Surface :

Definition : If potential of a surface (imaginary or physically existing) is same throughout, then such surface is known as an equipotential surface.

Properties of equipotential surfaces :

(i) When a charge is shifted from one point to another point on an equipotential surface, then work done against electrostatic forces is zero.

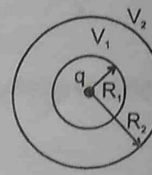
(ii) Electric field is always perpendicular to equipotential surfaces.

(iii) Two equipotential surfaces do not cross each other.

Examples of equipotential surfaces :

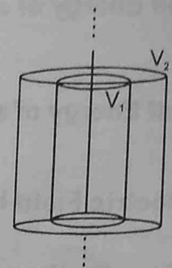
(i) **Point charge :**

Equipotential surfaces are concentric and spherical as shown in figure. In figure, we can see that sphere of radius R_1 has potential V_1 throughout its surface and similarly for other concentric sphere potential is same.



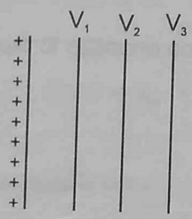
(ii) **Line charge :**

Equipotential surfaces have curved surfaces as that of coaxial cylinders of different radii.



(iii) **Uniformly charged large conducting / non conducting sheets:**

Equipotential surfaces are parallel planes.



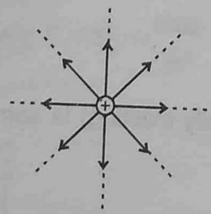
Note : In uniform electric field equipotential surfaces are always parallel planes.

Electric Lines of Force (ELOF) :

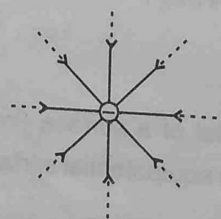
The line of force in an electric field is an imaginary line, the tangent to which at any point on it represents the direction of electric field at the given point.

Properties :

(i) Line of force originates out from a positive charge and terminates on a negative charge. If there is only one positive charge then lines start from positive charge and terminate at ∞. If there is only one negative charge then lines start from ∞ and terminate at negative charge.



ELOF of Isolated positive charge



ELOF of Isolated negative charge

(ii) The electric field intensity or electric intensity at a point is the number of lines of force streaming through per unit area normal to the direction of the intensity at that point. The intensity will be more where the density of lines is more.

(iii) Number of lines originating (terminating) from (on) a charge is directly proportional to the magnitude of the charge.

(iv) ELOF of resultant electric field can never intersect with each other.

(v) Electric lines of force produced by static charges do not form closed loop.

(vi) Electric lines of force end or start perpendicularly on the surface of a conductor.

(vii) Electric lines of force never enter into conductors.

Note: A charged particle need not follow an ELOF.

- The electric flux over the whole area is given by

$$\phi_E = \int_S \vec{E}.\vec{dS} = \int_S E_n dS$$

- Flux using Gauss's law,
  Flux through a closed surface

$$\phi_E = \oint \vec{E}\cdot\vec{dS} = \frac{q_{in}}{\varepsilon_0}.$$

Properties of Electric Flux

(i) Flux through Gaussian surface is independent of its shape.

(ii) Flux through Gaussian surface depends only on total charge present inside Gaussian surface.

(iii) Flux through Gaussian surface is independent of position of charges inside Gaussian surface.

(iv) Electric field intensity at the Gaussian surface is due to all the charges present inside as well as outside the Gaussian surface.

(v) In a closed surface incoming flux is taken negative, while outgoing flux is taken positive, because $\hat{n}$ is taken positive in outward direction.

(vi) In a Gaussian surface, $\phi = 0$ does not imply $E = 0$ at every point of the surface but $E = 0$ at every point implies $\phi = 0$.

Conductor and it's properties [For electrostatic condition]

(i) Conductors are materials which contain large number of free electrons which can move freely inside the conductor.

(ii) In electrostatics, conductors are always equipotential surfaces.

(iii) Charge always resides on outer surface of conductor.

(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.

(v) Electric field is always perpendicular to conducting surface.

(vi) Electric lines of force never enter into conductors.

(vii) Electric field intensity near the conducting surface is given by formula

$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$; [where, σ = charge density of conducting surface near that point]

(viii) When a conductor is grounded, it's potential becomes zero.

(ix) When an isolated conductor is grounded then its charge becomes zero.

(x) When two conductors are connected there will be charge flow till their potentials become equal.

(xi) **Electric pressure :** Electric pressure at the surface of a conductor is given by formula

$P = \frac{\sigma^2}{2\varepsilon_0}$; where, σ is the local surface charge density.

CURRENT ELECTRICITY

- $I_{av} = \frac{\Delta q}{\Delta t}$ and $i_{inst.} = \frac{dq}{dt}$ $\Rightarrow$ $q = \int i dt$ = area between current – time graph on time axis.
- Current $i = neAV_d$ n = no. of free electron per unit volume, A = cross–section area of conductor, V_d = drift velocity, e = charge on electron = 1.6×10^{-19} C
- Ohm's law V = IR
- $R = \frac{\rho \ell}{A}$ ρ = resistivity = $\frac{1}{\sigma}$, σ = conductivity
- Power $P = VI \Rightarrow P = I^2R = \frac{V^2}{R}$
- Energy = power × time (if power is constant.) otherwise energy, $E = \int P.dt$ where P is power.
- The rate at which the chemical energy of the cell in consumed = Ei
- The rate at which heat is generated inside the battery = i^2r
- Electric power output = $(\varepsilon - ir)\, i$
- Maximum power output when net internal resistance = net external resistance, R = r

  Maximum power output = $\frac{\varepsilon^2}{4r}$
- In series combination $R = R_1 + R_2 + R_3 + \ldots\ldots\ldots$
- In parallel combination $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots$
- Cell in series combination
- $E_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots\ldots + \varepsilon_n$ (write Emf's with polarity)

  $r_{eq} = r_1 + r_2 + r_3 + \ldots\ldots\ldots$
- Cells in parallel combination

$$E_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \ldots\ldots + \frac{\varepsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots\ldots + \frac{1}{r_n}}$$ (Use proper sign before the EMFs for polarity)

and $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots + \frac{1}{r_n}$$

- In ammeter shunt $(S) = \frac{I_G \times R_G}{I - I_G}$
- In voltmeter $V = I_G R_S + I_G R_G$

- Potential gradient in potentiometer : $x = \frac{\varepsilon}{R+r} \times \frac{R}{L}$

- $\frac{\theta_i + \theta_c}{2} = \theta_n$

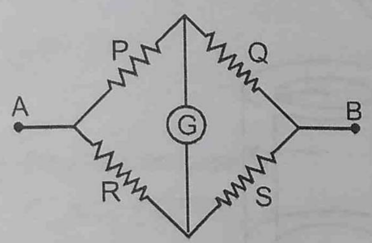
where, θ_i = inversion temperature

θ_c = Temperature of cold junction

θ_n = Neutral temperature

- In balanced wheat stone bridge

$\frac{P}{R} = \frac{Q}{S}$



CAPACITANCE

- (i) $q \propto V \quad \Rightarrow q = CV$
  q : Charge on positive plate of the capacitor
  C : Capacitance of capacitor.
  V : Potential difference between positive and negative plates.

(ii) Representation of capacitor : —| |— , —| (—

(iii) It is a scalar quantity having dimensions
$[C] = [M^{-1} L^{-2} T^4 A^2]$

(iv) S.I. Unit is Farad. (F).

(v) Energy stored in the capacitor : $U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} = \frac{QV}{2}$

(vi) Energy density = $\frac{1}{2} \in_0 \in_r E^2 = \frac{1}{2} \in_0 K E^2$

$K = \in_r$ = Relative permittivity of the medium (Dielectric Constant)

For vacuum, energy density = $\frac{1}{2} \in_0 E^2$

(vii) Types of Capacitors :

(a) Parallel plate capacitor

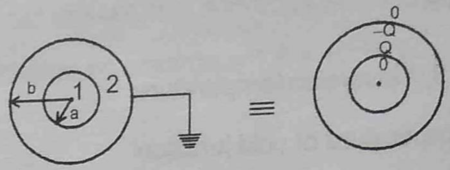
$C = \frac{\in_0 \in_r A}{d} = K \frac{\in_0 A}{d}$

A : Area of plates
d : distance between the plates(<< size of plate)

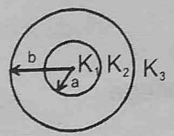
(b) Spherical Capacitor :

- Capacitance of an isolated spherical Conductor (hollow or solid)
  $C = 4\pi \epsilon_0 \epsilon_r R$
  R = Radius of the spherical conductor
- Capacitance of spherical capacitor

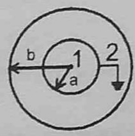
$$C = 4\pi\epsilon_0 \frac{ab}{(b-a)}$$



- $C = \dfrac{4\pi \epsilon_0 K_2 ab}{(b-a)}$

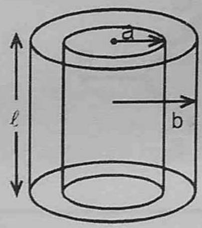


- $C = \dfrac{4\pi \epsilon_0 b^2}{(b-a)}$



(c) Cylindrical Capacitor : $\ell >> \{a,b\}$

Capacitance per unitlength = $\dfrac{2\pi \epsilon_0}{\ell n(b/a)}$ F/m



(viii) Capacitance of capacitor depends on
(a) Area of plates
(b) Distance between the plates
(c) Dielectric medium between the plates.

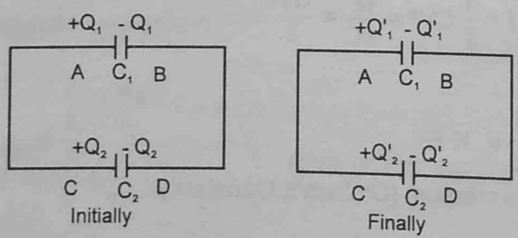
(ix) Electric field intensity between the plates of capacitor $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{V}{d}$

σ : Surface change density

(x) Force experienced by any plate of capacitor : $F = \dfrac{q^2}{2A\epsilon_0}$

☞ **Distribution of Charges on Connecting two Charged Capacitors:**

When two capacitors are C_1 and C_2 are connected as shown in figure



(a) Common potential : $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{\text{Total charge}}{\text{Total capacitance}}$

(b) $Q_1' = C_1V = \dfrac{C_1}{C_1 + C_2}(Q_1 + Q_2)$

$$Q_2' = C_2 V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

(c) Heat loss during redistribution :

$$\Delta H = U_i - U_f = \frac{1}{2}\,\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

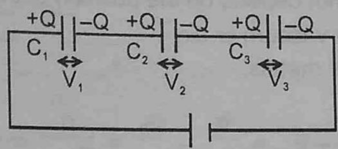
Note : (i) When plates of similar charges are connected with each other (+ with + and – with –) then put all values (Q_1, Q_2, V_1, V_2) with positive sign.

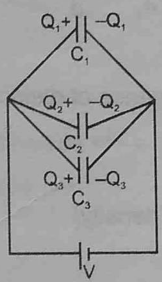
(ii) When plates of opposite polarity are connected with each other (+ with –) then take charge and potential of one of the plate to be negative.

Combination of capacitor :

(i) Series Combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\,;\; V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$





(ii) Parallel Combination :

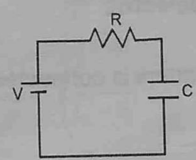
$$C_{eq} = C_1 + C_2 + C_3 + \ldots\ldots\ldots$$

$$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$$

Charging and Discharging of a capacitor :

(i) Charging of Capacitor (Capacitor initially uncharged):

$$q = q_0(1 - e^{-t/\tau})$$

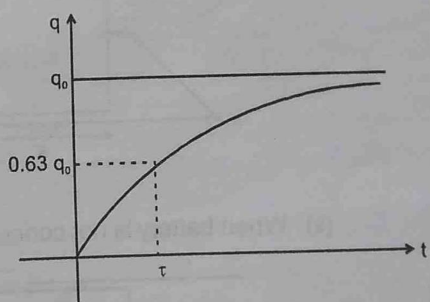


q_0 = Charge on the capacitor at steady

State $\Rightarrow$ $q_0 = CV$

τ : Time constant $= CR_{eq.}$

$$i = \frac{q_0}{\tau}\, e^{-t/\tau} = \frac{V}{R} e^{-t/\tau}$$



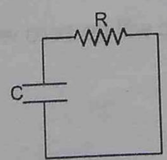
‣ 63% of maximum charge is deposited in one time constant.

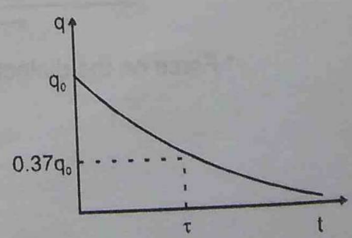
(ii) Discharging of Capacitor :

$$q = q_0\, e^{-t/\tau}$$

q_0 = Initial charge on the capacitor

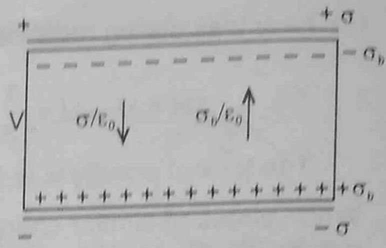
$$i = \frac{q_0}{\tau}\, e^{-t/\tau}$$





‣ 63% of discharging is complete in one time constant.

Capacitor with dielectric :



(i) Capacitance in the presence of dielectric :

$$C = \frac{K \epsilon_0 A}{d} = KC_0$$

C_0 = Capacitance in the absence of dielectric.

(ii) If thickness of dielectric slab is t, then its capacitance

$$C = \frac{\epsilon_0 A}{(d - t + t/k)},$$ where k is the dielectric constant of slab.

- It does not depend on the position of the slab.
- k = 1 for vaccum or air.
- $k = \infty$ for metals.

(iii) $E_{in} = E - E_{ind} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_b}{\epsilon_0} = \frac{\sigma}{K\epsilon_0} = \frac{V}{d}$

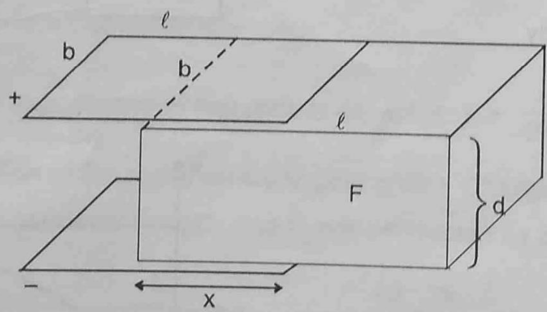
$E = \frac{\sigma}{\epsilon_0}$ Electric field in the absence of dielectric

E_{ind} : Induced electirc field

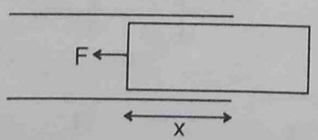
(iv) $\sigma_b = \sigma(1 - \frac{1}{K})$. (induced charge density)

Force on dielectric

(i) When battery is connected $F = \frac{\epsilon_0 b(K-1)V^2}{2d}$



(ii) When battery is not connected $F = \frac{Q^2}{2C^2}\frac{dC}{dx}$

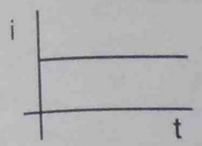


* Force on the dielectric will be zero when the dielectric is fully inside.

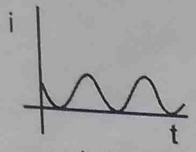
ALTERNATING CURRENT

AC and DC Current :

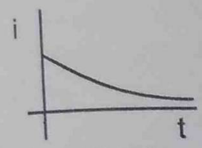
A current that changes its direction periodically is called alternating current (AC). If a current maintains its direction constant it is called direct current (DC).



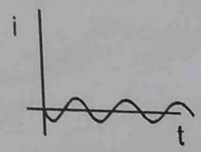
constant dc



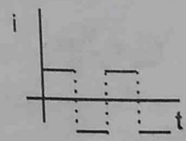
periodic dc



variable dc



ac



ac

Average Value :

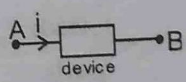
Average value of a function, from t_1 to t_2, is defined as $<f> = \dfrac{\int_{t_1}^{t_2} f.dt}{t_2 - t_1}$.

Root Mean Square Value:

Root Mean Square Value of a function, from t_1 to t_2, is defined as $f_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$.

Power Consumed or Supplied in an ac Circuit:

Instantaneous power P consumed by the device $= V\,i = (V_m \sin \omega t)\,(I_m \sin(\omega t + \phi))$



Average power consumed in a cycle $= \dfrac{\int_0^{\frac{2\pi}{\omega}} P dt}{\frac{2\pi}{\omega}} = \dfrac{1}{2} V_m I_m \cos \phi$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi = V_{rms} I_{rms} \cos \phi.$$

Here $\cos \phi$ is called **power factor**.

Some Definitions:

The factor **$\cos \phi$** is called **Power factor.**

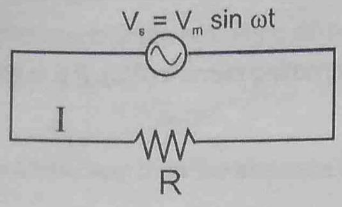
$I_m \sin \phi$ is called **wattless current.**

Impedance Z is defined as $Z = \dfrac{V_m}{I_m} = \dfrac{V_{rms}}{I_{rms}}$

ωL is called **inductive reactance** and is denoted by X_L.

$\dfrac{1}{\omega C}$ is called **capacitive reactance** and is denoted by X_C.

Purely Resistive Circuit:

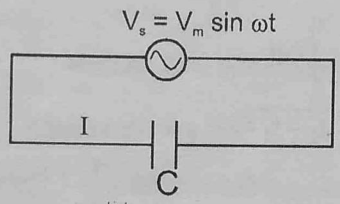


$$I = \frac{V_s}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R} \quad \Rightarrow \quad I_{rms} = \frac{V_{rms}}{R}$$

$$\langle P \rangle = V_{rms} I_{rms} \cos \phi = \frac{V_{rms}^2}{R}$$

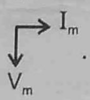
Purely Capacitive Circuit:

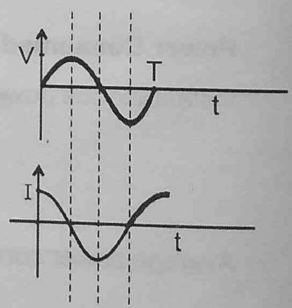


$$I = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{d(CV_m \sin \omega t)}{dt} = CV_m \omega \cos \omega t = \frac{V_m}{1/\omega C} \cos \omega t = \frac{V_m}{X_C} \cos \omega t = I_m \cos \omega t.$$

$X_C = \dfrac{1}{\omega C}$ and is called capacitive reactance.

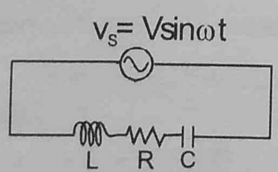
I_C leads V_C by $\pi/2$ Diagrammatically (phasor diagram) it is represented as

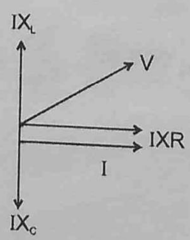


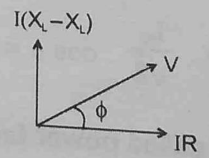


Since $\phi = 90°$, $\langle P \rangle = V_{rms} I_{rms} \cos \phi = 0$

RLC Series Circuit With An ac Source :







From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = IZ \qquad Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

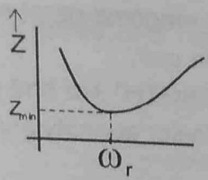
$$\tan \phi = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

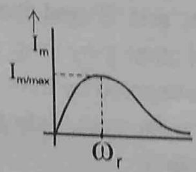
Resonance :

Amplitude of current (and therefore I_{rms} also) in an RLC series circuit is maximum for a given value of V_m and R , if the impedance of the circuit is minimum, which will be when X_L-X_C =0. This condition is called **resonance.**

So at resonance: $X_L - X_C = 0$.

or $\omega L = \frac{1}{\omega C}$ or $\omega = \frac{1}{\sqrt{LC}}$. Let us denote this ω as ω_r.





Quality factor : $Q = \frac{X_L}{R} = \frac{X_C}{R}$

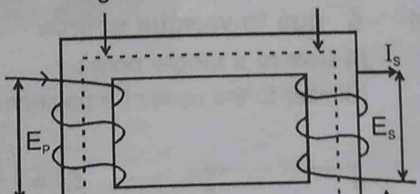
$$Q = \frac{\text{Resonance freq.}}{\text{Band width}} = \frac{\omega_R}{\Delta\omega} = \frac{f_R}{f_2 - f_1}$$

where f_1 & f_2 are half power frequencies.

Transformer ;

A transformer changes an alternating potential difference from one value to another of greater or smaller value using the principle of mutual induction . For an ideal tranformer $\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$, where denotations have their usual meanings.

E_s N and I are the emf, number of turns and current in the coils.



$N_s > N_P \Rightarrow$ $E_s > E_P \rightarrow$ step up transformer.

$N_s < N_P \Rightarrow$ $E_s < E_P \rightarrow$ step down transformer.

Energy Losses In Transformer are due to

1. Resistance of the windings.

2. Eddy Current.

3. Hysteresis.

4. Flux Leakage.

MAGNETIC EFFECT OF CURRENT AND MAGNETIC FORCE ON CHARGE OR CURRENT

☞ **Magnet :**

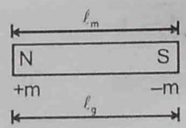
Two bodies even after being neutral (showing no electric interaction) may attract / repel strongly if they have a special property. This property is known as magnetism. The force with which they attract or repel is called magnetic force. Those bodies are called magnets.

☞ **Pole strength magnetic dipole and magnetic dipole moment :**

A magnet always has two poles 'N' and 'S' and like poles of two magnets repel each other and the unlike poles of two magnets attract each other they form action reaction pair.

These poles are quantitatively represented by their "POLE STRENGTH" +m and –m respectively (just like we have charges +q and –q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges –q and +q). It is called MAGNETIC DIPOLE and it has a MAGNETIC DIPOLE MOMENT. It is represented by $\vec{M}$. It is a vector quantity. It's direction is from –m to +m that means from 'S' to 'N')



$M = m.\ell_m$ here ℓ_m = magnetic length of the magnet. ℓ_m is slightly less than ℓ_g (it is geometrical length of the magnet = end to end distance)

☞ **Magnetic field and strength of magnetic field :**

Mathematically, $\vec{B} = \dfrac{\vec{F}}{m}$

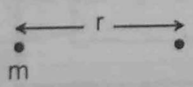
Here $\vec{F}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

S.I. unit of $\vec{B}$ is **Tesla** or **Weber/m²** (abbreviated as T and Wb/m²).

(a) **$\vec{B}$ due to various source**

(i) Due to a single pole :

(Similar to the case of a point charge in electrostatics)



$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}.$$

in vector form $\vec{B} = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{r^3}\vec{r}$

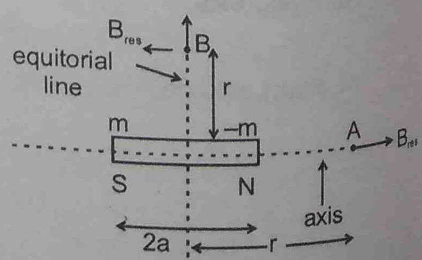
here m is with sign and $\vec{r}$ = position vector of the test point with respect to the source pole.

(ii) Due to a bar magnet :

(Same as the case of electric dipole in electrostatics) Independent case never found. Always 'N' and 'S' exist together as magnet.

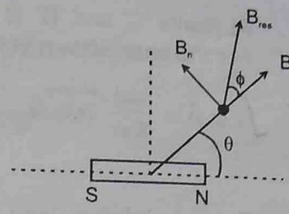
at A (on the axis) $= 2\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{\vec{M}}{r^3}$ for $a << r$

at B (on the equatorial) $= -\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{\vec{M}}{r^3}$ for $a << r$



At General point :

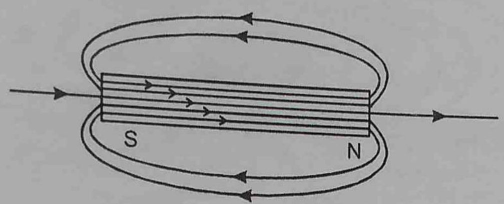
$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3} \Rightarrow B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$



$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$$

Magnetic lines of force of a bar magnet :

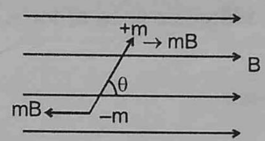


☞ **Magnet in an external uniform magnetic field :**

(same as case of electric dipole)

$F_{res} = 0$ (for any angle)

$\tau = MB\sin\theta$

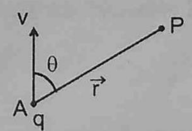


*here θ is angle between $\vec{B}$ and $\vec{M}$

in vector form $\vec{\tau} = \vec{M}\times\vec{B}$

☞ **Magnetic effects of current (and moving charge)**

$\vec{B}$ due to a point charge :



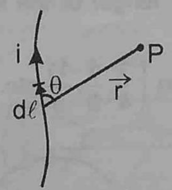
$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{qv\sin\theta}{r^2}$$; here θ = angle between $\vec{v}$ and $\vec{r}$

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q\vec{v}\times\vec{r}}{r^3};$$ with sign

$\vec{B}\perp\vec{v}$ and also $\vec{B}\perp\vec{r}$.

Direction of $\vec{B}$ will be found by using the rules of vector product.

Biot-savart's law (${\vec{B}}$ due to a wire)



$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{id\ell\sin\theta}{r^2}$$

$$\Rightarrow d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\vec{d\ell}\times\vec{r}}{r^3}$$

here $\vec{r}$ = position vector of the test point w.r.t. $\vec{d\ell}$

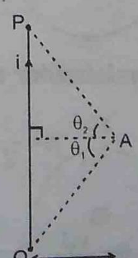
θ = angle between $\vec{d\ell}$ and $\vec{r}$. The resultant

$\vec{B}$ due to a straight wire :

Due to astraight wire 'PQ'

carrying a current 'i' the $\vec{B}$

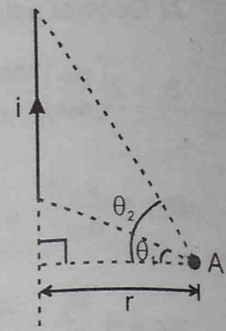
at A is given by the formula



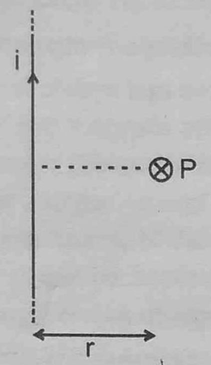
$$B = \frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

☞ At points 'C' and 'D' $\vec{B} = 0$ (think how).
For the case shown in figure

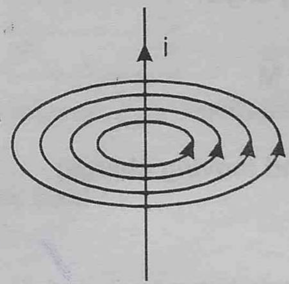
$$B \text{ at } A = \frac{\mu_0 i}{4\pi r}(\sin\theta_2 - \sin\theta_1) \otimes$$



☞ B due to infinitely long straight wire is $\mu_0 i/2\pi r$



☞ Magnetic lines of force by a current carrying straight wire are circular like shown in figure.

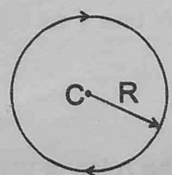


■ **$\vec{B}$ due to circular loop**
(a) At centre :

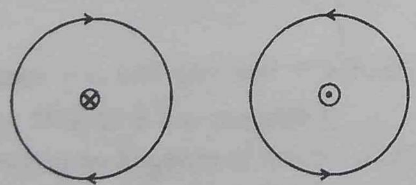
$$B = \frac{\mu_0 NI}{2R},$$

N = No. of turns in the loop.

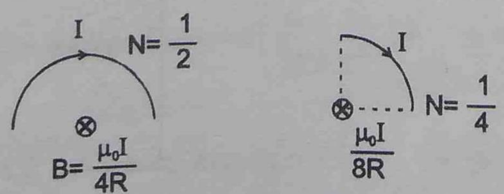
$$= \frac{\ell}{2\pi R}; \ \ell = \text{length of the loop.}$$



N can be fraction $\left(\frac{1}{4}, \frac{1}{3}, \frac{11}{3} \text{ etc.}\right)$ or integer.

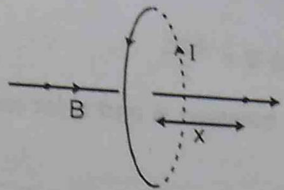


Semicircular and Quarter of a circle :



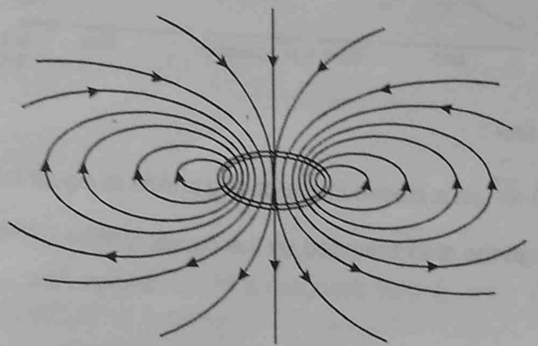
(b) **On the axis of the loop :**

$$B = \frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}}$$

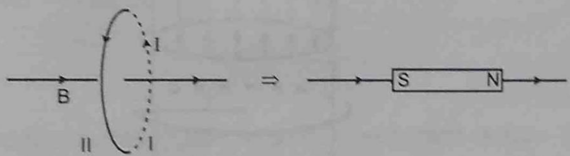


N = No. of turns (integer)

☞ magnetic lines of force due to current in the ring are like shown in figure.

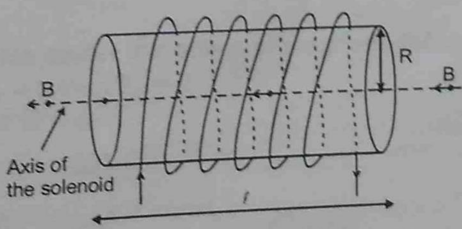


☞ The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.

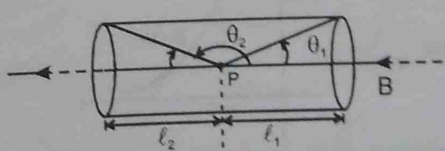


The side 'I' (the side from which the $\vec{B}$ emerges out) of the loop acts as 'NORTH POLE' and side II (the side in which the $\vec{B}$ enters) acts as the 'SOUTH POLE'.

Solenoid :



magnetic field at any genaral point P is given by



$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$

where n : number of turns per unit length.

For 'Ideal Solenoid' : (ℓ >> R or length is infinite)

magnetic field inside the solenoid at mide point on its axis is given by

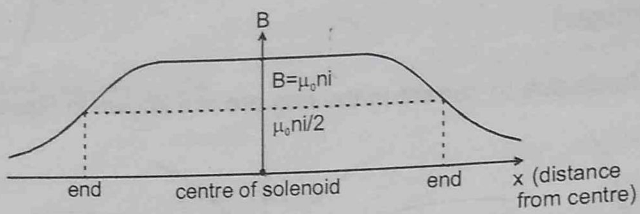
$B = \mu_0 ni$

magnetic field inside the solenoid can be consider same everywhere.

If material of the solid cylinder has relative permeability 'μ_r' then $B = \mu_0\mu_r ni$

At the ends $B = \frac{\mu_0 ni}{2}$

(v) **Graph between B and x for ideal solenoid :**

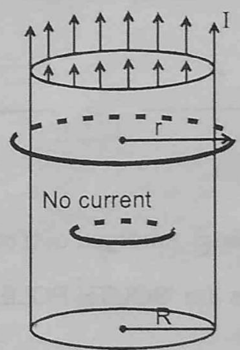


AMPERE's circuital law :

The line integral $\oint \vec{B}\cdot\vec{d\ell}$ on a closed curve of any shape is equal to μ_0 (permeability of free space) tim the net current I through the area bounded by the curve.

$$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I$$

Hollow current carrying infinitely long cylinder :
(I is uniformly distributed on the whole circumference)

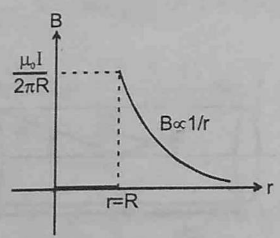


(i) for $r \geq R$

$$B = \frac{\mu_0 I}{2\pi r}$$

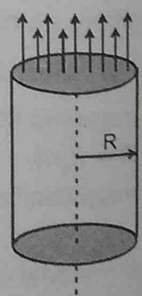
(ii) $r < R$, $B_{in} = 0$

Graph :



Solid infinite current carrying cylinder:

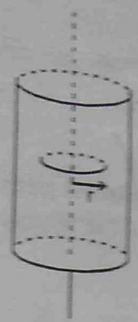
Assume current is uniformly distributed on the whole cross section area



current density $J = \frac{I}{\pi R^2}$

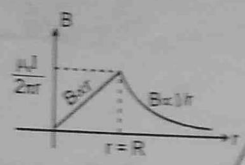
Case (I) : $r \le R$

$$\vec{B} = \frac{\mu_0 \vec{J} \times \vec{r}}{2}$$



Case (II) : $r \ge R$

$$\vec{B} = \frac{\mu_0 R^2}{2r^2}(\vec{J} \times \vec{r})$$



Magnetic force on moving charge :

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula :

$\vec{F} = q(\vec{v} \times \vec{B})$ Put q with sign.

$\vec{v}$: Instantaneous velocity

$\vec{B}$: Magnetic field at that point.

Note :

$\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$

$\because \vec{F} \perp \vec{v}$ $\therefore$ power due to magnetic force on a charged particle is zero. (use the formula of power $P = \vec{F} \cdot \vec{v}$ for its proof).

Since the $\vec{F} \perp \vec{B}$ so work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.

If $\vec{V} \parallel \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

Motion of charged particles under the effect of magnetic force

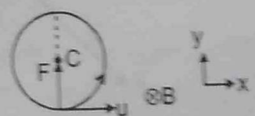
* Particle released if v = 0 then $F_M = 0$

  $\therefore$ particle will remain at rest

  $\vec{V} \parallel \vec{B}$ here $\theta = 0$ or $\theta = 180°$

  $\therefore F_m = 0$ $\quad \therefore \vec{a} = 0$ $\quad \therefore \vec{V}$ = const.

$\therefore$ particle will move in a straight line with constant velocity



* Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B}$ = uniform

$\therefore$ F_m = quB = constant

now $\quad quB = \dfrac{mu^2}{R} \Rightarrow R = \dfrac{mu}{qB}$ = constant.

The particle moves in a curved path whose radius of curvature is same every where, such curve in a plane is only a circle.

$\therefore$ $\quad$ path of the particle is circular.

$$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$$

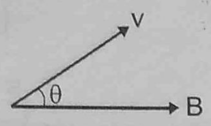
here $\quad$ p = linear momentum ;

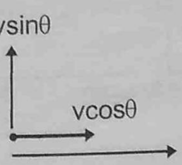
k = kinetic energy

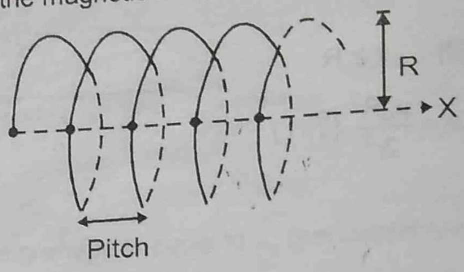
now $v = \omega R \Rightarrow \omega = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f$

Time period $T = 2\pi m/qB$
frequency $f = qB/2\pi m$

Helical path :
If the velocity of the charge is not perpendicular to the magnetic field, the resultant path is helix.







Radius $q(v\sin\theta)B = \dfrac{m(v\sin\theta)^2}{R} \Rightarrow R = \dfrac{mv\sin\theta}{qB}$

$\omega = \dfrac{v\sin\theta}{R} = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f.$

and pitch = $(v\cos\theta)T$

Charged Particle in $\vec{E}$ & $\vec{B}$

When a charged particle moves with velocity $\vec{V}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$, then. Net force experienced by it is given by following equation.

$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$

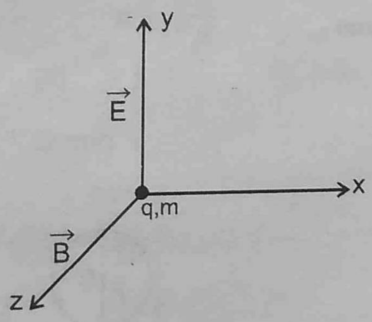
Combined force is known as Lorentz force.

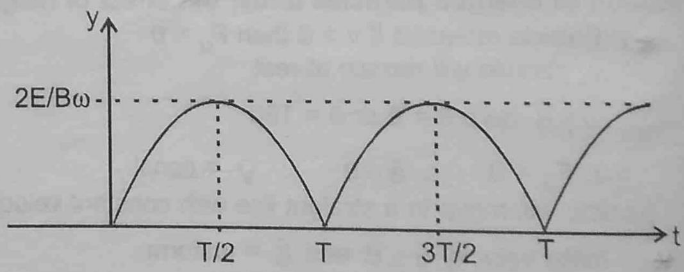
Case I : $\vec{E} \,||\, \vec{B} \,||\, \vec{v}$ E B v

In above situation particle passes undeviated but its velocity will change due to electric field. Magnetic force on it = 0.

Case - II : $\vec{E} = E\hat{j}$ and $\vec{B} = B\hat{k}$

and charge q is released at origin





then its path will be cyclotron

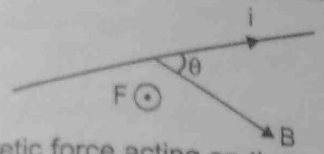
its velocity in y direction varies as $v_y = \dfrac{E}{B}.\sin\omega t$.

y coordinate at any time t is $y = \dfrac{E}{B}.\omega\left[1 - \cos\omega t\right]$

and x cooridnate can be given as $x = \dfrac{E}{B}\left(t - \dfrac{1}{\omega}.\sin\omega t\right)$

Magnetic force on a current carrying wire :

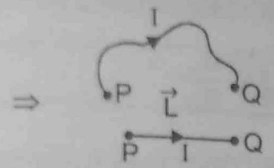
Suppose a conducting wire, carrying a current i, is placed in a magnetic field.



Magnetic force acting on the wire

$$\overrightarrow{F_{res}} = i\vec{L} \times \vec{B}$$

Here $\vec{L} = \int d\vec{\ell}$ = vector length of the wire = vector connecting the end points of the wire.



Note :

If a current loop of any shape is placed in a uniform $\vec{B}$ then $\left(\overrightarrow{F_{res}}\right)_{magnetic}$ on it = 0 $(\because \vec{L} = 0)$.

Magnetic moment of a current carrying coil :

M = NiA

N is the number of turns

i is the current in the coil

A is the area of the coil.

Torque on a current loop :

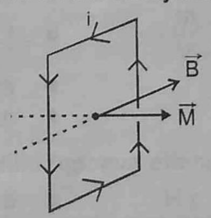
When a current-carrying coil is placed in a uniform magnetic field the net force on it is always zero.

Torque acting on a current carrying coil is

$\tau = MB \sin\theta$

In vector form

$\vec{\tau} = \vec{M} \times \vec{B}$

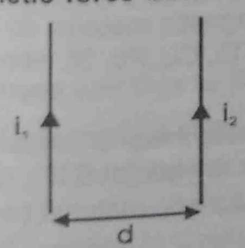


where $\vec{M}$ is the magnetic moment of current carrying coil.

$\vec{B}$ is the magnetic field.

θ is the angle between $\vec{M}$ & $\vec{B}$

Magnetic force between two parallel current carrying straight wires



$$F = \frac{\mu_0 i_1 i_2}{2\pi d}$$

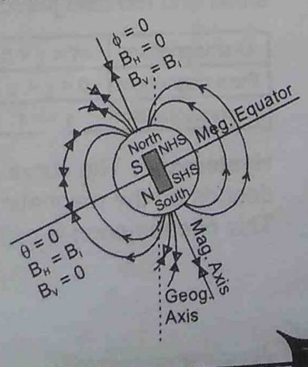
Where F is the force on per unit length of each wire.

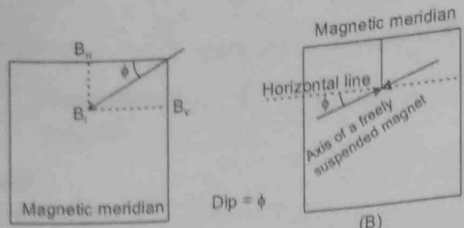
If i_1 and i_2 are in same direction then force is attracting and if in opposite direction then force is repulsive.

Terrestrial Magnetism (Earth's Magnetism) :

(a) **Variation or Declination θ :** At a given place the angle between the geographical meridian and the magnetic meridian is called declination, i.e.,

(b) **Inclination or Angle of Dip ϕ :** It is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian at the given place.





(A) Dip = ϕ (B)

(c) **Horizontal Component of Earth's Magnetic Field B_H :** At a given place it is defined as the component of earth's magnetic field along the horizontal in the magnetic meridian. It is represented by B_H and is measured with the help of a **vibration** or **deflection magnetometer**.

If at a place magnetic field of earth is B_I and angle of dip ϕ, then in accordance with figure (a).

$B_H = B_I \cos\phi$ and $B_v = B_I \sin\phi$ so that, $\tan\phi = \dfrac{B_V}{B_H}$

and $B_I = \sqrt{B_H^2 + B_V^2}$

Magnetic properties of matter :

Magnetic intensity (H) : it is a quantity related to currents in coils and conductors

$H = \dfrac{B}{\mu_0}$

- it is a vector quantity
- its dimension is $L^{-1}A$
- its SI unit is Am^{-1}

Magnetisation (M): It is equal to the magnetic moment per unit volume.

$M = \dfrac{m}{V}$

- It is a vector quantity
- its dimension is $L^{-1}A$
- its SI unit is Am^{-1}

Magnetic susceptibility (χ) : It is a measure how a magnetic material responds to an external field.

$M = \chi H$

- it is dimensionless quantity

Also, $\mu_r = 1 + \chi$

where, μ_r is called relative permeability and it is a dimensionless quality.

Also, $\mu_r = \mu_0 (1 + \chi)$

where, μ_0 is absolute permeability of free space.

Diamagnetism : The individual atoms (or ions or molecules) of a diamagnetic material do not posses a permanent dipole moment of their own. (some diamagnetic materials are Bi, Cu, Pb, Si, nitrogen (at STP), H_2O, NaCl)

Paramagnetism: The individual atoms (or ions or molecules) of a diamagnetic material posses a permanent dipole moment of their own.(some paramagnetic materials are Al, Na, Ca, oxygen(at STP), $CuCl_2$).

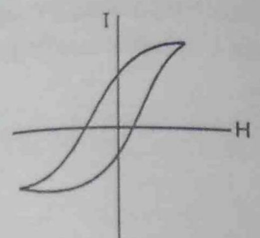
Ferromagnetism: The individual atoms (or ions or molecules) of a diamagnetic material posses a dipole moment of their own. (some ferromagnetic materials are Fe, Co, Ni, Ga)

In terms of susceptibility χ, a material is diamagnetic if χ is negative. Paramagnetic if χ is positive and small and ferromagnetic if χ is large and positive.

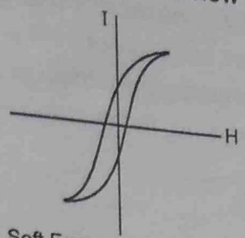
Diamagnetism	$-1 < \chi < 0$	$0 \le \mu_r < 1$	$\mu < \mu_0$
Paramagnetism	$0 < \chi < \epsilon$	$1 < \mu_r < 1 + \epsilon$	$\mu > \mu_0$
Ferromagnetism	$\chi >> 1$	$\mu_r >> 1$	$\mu >> \mu_0$

Hysteresis : The curve between B and H in ferromagnetic materials is complex. It is often not linear and depnds on the magnetic history of the sample.
This phenomenon is called hysteresis.

Magnetic hysteresis loops between the intensity of magnetization(I) and H for hard ferromagnetic ma and soft ferromagnetic materials are shown below :



Hard Ferromagnetic materials



Soft Ferromagnetic materials

where $I = \frac{B}{\mu_0} - H$

- Area of hysteresis loop is proportional to the thermal energy developed per unit of the volume of the material.

ELECTROMAGNETIC INDUCTION

- **Magnetic flux** is mathematically defined as

$$\phi = \int \vec{B}.d\vec{s}$$

- **Faraday's laws of electromagnetic induction**

When magnetic flux passing through a loop changes with time or magnetic lines of force are cut by a conducting wire then an emf is produced in the loop or in that wire.

The magnitude of induced emf is equal to the rate of change of flux w.r.t. time in case of loop. In case of a wire it is equal to the rate at which magnetic lines of force are cut by a wire.

$$\epsilon = -\frac{d\phi}{dt}$$

Negative sign implies that emf induce in such a way opposition of change in flux.

- **Lenz's Law (is based on conservation of energy principle)**

According to this law, emf will be induced in such a way that it will oppose the cause which has produced it.

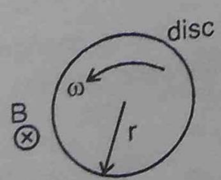
If a wire AB is moving with constant velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ then emf induced in any path joining A and B is same which is equal to $(\overrightarrow{AB}).(\vec{V}\times\vec{B})$ where L is the distance between A and B.

- **Induced emf Due to Rotation of Ring**

Emf induced in a conducting rod of length ℓ rotating with angular speed ω about its one end, in a uniform magnetic field B perpendicular to plane of rotation is 1/2 B ω ℓ^2.

- **EMF Induced in a rotating disc :**

Emf between the centre and the edge of disc of radius r rotating in a magnetic field B = $\frac{B\omega r^2}{2}$



Self induction

Self induction is induction of emf in a coil due to its own current change. Total flux $N\phi$ passing through a coil due to its own current is proportional to the current and is given as $N\phi = Li$ where L is called coefficient of self induction or inductance. The inductance L is purely a geometrical property.

If current in the coil changes by ΔI in a time interval Δt, the average emf induced in the coil is given as

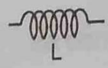
$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}.$$

The instantaneous emf is given as $\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$

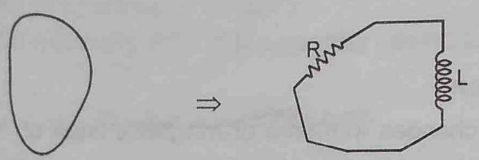
Self inductance of solenoid = $\mu_0 n^2 \pi r^2 \ell$.

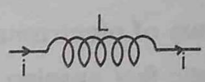
Inductor :

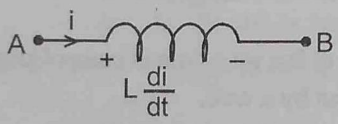
It is represent by



electrical equivalence of loop





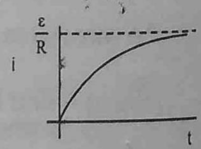


$$V_A - L\frac{di}{dt} = V_B$$

Energy stored in an inductor $= \frac{1}{2}LI^2$

Growth Of Current in Series R–L Circuit :

If a circuit consists of a cell, an inductor L and a resistor R and a switch S ,connected in series and the switch is closed at t = 0, the current in the circuit i will increase as $i = \frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})$



1. Final current in the circuit = $\frac{\varepsilon}{R}$, which is independent of L.
2. After one time constant , current in the circuit =63% of the final current.

Properties of R-L Circuit

1. At t = 0 inductor behaves like open circuited.
2. At t = ∞ (after very long time) inductor behaves like as short circuited wire.
3. Current in inductor is not abrubthyl changes.

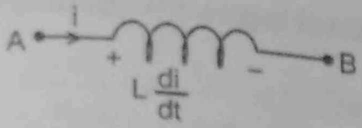
Decay of current in the circuit containing resistor and inductor:

Let the initial current in a circuit containing inductor and resistor be I_0. Current at a time t is given as $i = I_0 e^{\frac{-Rt}{L}}$

Mutual inductance : is induction of EMF in a coil (secondary) due to change in current in another coil (primary). If current in primary coil is i, total flux in secondary is proportional to i, i.e. $N\phi$ (in secondary) $\propto$ i.

or $N\phi$ (in secondary) = M i.

Equivalent self inductance :



$$L = \frac{V_A - V_B}{di/dt} \qquad ..(1)$$

Series combination :

$L = L_1 + L_2$ (neglecting mutual inductance)

$L = L_1 + L_2 + 2M$ (if coils are mutually coupled and they have winding in same direction)

$L = L_1 + L_2 - 2M$ (if coils are mutually coupled and they have winding in opposite direction)

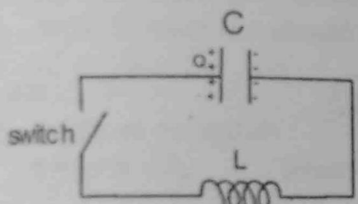
Parallel Combination :

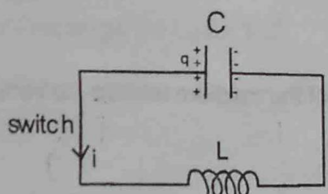
$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ (neglecting mutual inductance)

For two coils which are mutually coupled it has been found that $M \le \sqrt{L_1L_2}$ or $M = k\sqrt{L_1L_2}$ where k is called coupling constant and its value is less than or equal to 1.

L.C. Oscillations

Let a capacitor be charged to Q and then connected in series with an inductor with the help of a switch as shown in the figure.





Let the switch be closed at t=0.Let at a time t=t, the charge on the capacitor be q and the current in the circuit be i where $i = -\frac{dq}{dt}$. Writing Kirchoff's equation,we get $\frac{q}{C} = L\frac{di}{dt} = -L\frac{d^2q}{dt^2}$ or $\frac{d^2q}{dt^2} + \frac{q}{LC} = 0$.

Comparing with the standard differential equation it can be easily seen that $\omega^2 = 1/LC$ and therefore time period $T = 2\pi\sqrt{LC}$.

GEOMETRICAL OPTICS

Plane mirror:

(1) $\angle i = \angle r$

(2) $\delta = \pi - 2i$ or $\pi + 2i$.

(3) $x_{im} = -x_{om}$, $y_{im} = y_{om}$ and $z_{im} = z_{om}$
(x-axis is the normal to plane mirror and origin is at the mirror)

(4) $v_{(im)x} = -v_{(om)x}$; $v_{(im)y} = v_{(om)y}$; $v_{(im)z} = v_{(om)z}$.
(x-axis is the normal to plane mirror and origin is at the mirror)

Spherical Mirror:

(1) $\frac{1}{v} + \frac{1}{u} = \frac{2}{R} = \frac{1}{f}$ [For paraxial rays]

(2) Lateral magnification (or transverse magnification) $m = \frac{h_i}{h_o} = -\frac{v}{u}$

(3) In case of successive reflection
$m_{net} = m_1 \times m_2 \times m_3$ ....

(4) $\frac{dv}{du} = -\frac{v^2}{u^2}$.(du implies small change in position of object and dv implies corresponding small change in position of image.)

(5) $\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$,where $\frac{dv}{dt}$ is the velocity of image along principal axis (w.r.t. pole) and $\frac{du}{dt}$ is the velocity of object along principal axis (w.r.t. pole).

(6) Optical power of a mirror (in dioptre) $= -\frac{1}{f}$ (f in metre).

(7) Longitudinal magnification $= \frac{v_2 - v_1}{u_2 - u_1}$ (it will always be inverted)

(8) Newton's Formula: $XY = f^2$

(where x is distance of object from focus, y is distance of image from focus)

Refraction :

(1) Refractive index of the medium relative to vacuum $= n = \frac{c}{v} = \sqrt{\mu_r \in_r}$

(2) Snell's law

$$\frac{\text{Sin } i}{\text{Sin } r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

(n_1, λ_1, v_1 → refractive index, wavelength, velocity in the medium of incidence)

(3) Frequency of light does not change during refraction

(4) $\delta = |i - r|$

(5) ${}_1n_2 = \frac{1}{{}_2n_1}$

Lateral shift of light rays due to refraction through a parallel slab :

$$d = \frac{t\sin(i-r)}{\cos r}$$

Apparent Depth and shift of Submerged Object :

(1) at near normal incidence (small angle of incidence i) apparent depth (d′) is given by:

$$d' = \frac{d}{n_{relative}}$$ (where d and d' are distance of object and image from surface)

(2) Apparent shift $= d\left(1 - \frac{1}{n_{rel}}\right)$

where $n_{rel} = \frac{n_{incidence}}{n_{refraction}}$

Refraction through a Composite Slab

(1) Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1\,rel}} + \frac{t_2}{n_{2\,rel}} + \frac{t_3}{n_{3\,rel}} + \ldots\ldots + \frac{t_n}{n_{n\,rel}}$$

(i.e., $n_{1rel} = n_1/n_0$, $n_{2\,rel} = n_2/n_0$ etc, n_0 is refractive index of medium in which observer is present)

(2) Apparent shift

$$= t_1\left[1-\frac{1}{n_{1rel}}\right] + t_2\left[1-\frac{1}{n_{2rel}}\right] + \ldots\ldots + t_n\left[1-\frac{1}{n_{n\,rel}}\right]$$

Critical Angle For Total Internal Reflection (T. I. R.) :

$$C = \sin^{-1}\frac{n_r}{n_d}$$

(1) Light is incident on the interface from denser medium.

(2) Angle of incidence should be greater than the critical angle (i > C) for T.I.R.

Prism :

(1) **Angle of deviation :**

$\delta = i + e - A.$ (always valid)

(2) For minimum deviation,

$$i = e \text{ and } r_1 = r_2 = \frac{A}{2}$$

(3) $n_{rel} = \dfrac{\sin\left[\frac{A+\delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}$ (medium on both side should be same)

(4) For a thin prism ($A \le 10°$) and for small value of i, all values of $\delta = (n-1)A$

Dispersion. :

(I) $n(\lambda) = a + \dfrac{b}{\lambda^2}$ (Cauchy's formula) (where a and b are constants)

(1) For prism of small 'A' and with small 'i' :

$\theta = (n_v - n_r)A$

(2) Deviation of beam(also called mean deviation) $\delta = \delta_y = (n_y - 1)A$

(3) Dispersive power (property of material)

$$\omega = \frac{\delta_v - \delta_r}{\delta_y} = \frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y}$$

$n_y = \dfrac{n_v + n_r}{2}$ if value of n_y is not given in the problem