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Compendium of Important Concepts & Formulae

- Complete Syllabus
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- 3 Practice Test Papers

"SHORTENED VERSION of PHYSICS, CHEMISTRY & MATHS to review subjects at a glance."

Must Before Exam

JEE (MAIN+ADVANCED)

ENGLISH MEDIUM

Contents

COMPENDIUM OF FORMULAS
PHYSICS

UNIT & DIMENSION

- **Unit :** Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic standard called unit.

Fundamental Units :

S.No.	Physical Quantity	SI Unit	Symbol
1	Length	Metre	m
2	Mass	Kilogram	Kg
3	Time	Second	S
4	Electric Current	Ampere	A
5	Temperature	Kelvin	K
6	Luminous Intensity	Candela	Cd
7	Amount of Substance	Mole	mol

Supplementary Units :

S.No.	Physical Quantity	SI Unit	Symbol
1	Plane Angle	radian	r
2	Solid Angle	Steradian	Sr

Metric Prefixes :

S.No.	Prefix	Symbol	Value
1	Centi	c	10^{-2}
2	Mili	m	10^{-3}
3	Micro	μ	10^{-6}
4	Nano	n	10^{-9}
5	Pico	p	10^{-12}
6	Kilo	K	10^{3}
7	Mega	M	10^{6}

The dimensions of a physical quantity are the powers to which fundamental units are raised to represent the quantity.

Dimensional Analysis is used for :
(1) Conversion of one system of units into another.
(2) Checking the correctness of a given physical relation.
(3) Derivation of a formulae

Only like quantities can be added or subtracted from each other.

Angle θ is dimensionless.

Power of exponential is dimensionless.

RECTILINEAR MOTION

- **Average Velocity (in an interval) :** $\vec{v}_{av} = <\vec{v}> = \dfrac{\text{Total displacement}}{\text{Total time taken}} = \dfrac{\vec{r}_f - \vec{r}_i}{\Delta t}$

- **Average Speed (in an interval) :** Average Speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

- **Instantaneous Velocity (at an instant) :** $\vec{v}_{inst} = \lim_{\Delta t \to 0}\left(\dfrac{\Delta \vec{r}}{\Delta t}\right) = \dfrac{d\vec{r}}{dt}$

- **Average Acceleration (in an interval):** $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

- **Instantaneous Acceleration (at an instant):** $\vec{a} = \lim_{\Delta t \to 0}\left(\dfrac{\Delta \vec{v}}{\Delta t}\right) = \dfrac{d\vec{v}}{dt}$

- **Equations of Motion (for constant acceleration) :**

(a) $v = u + at$

(b) $s = ut + \dfrac{1}{2}at^2$; $s = vt - \dfrac{1}{2}at^2$; $x_f = x_i + ut + \dfrac{1}{2}at^2$

(c) $v^2 = u^2 + 2as$

(d) $s = \dfrac{(u+v)}{2}t$

(e) $s_n = u + \dfrac{a}{2}(2n-1)$

- **For freely falling bodies : (u = 0)** (taking upward direction as positive)

(a) $v = -gt$

(b) $s = -\dfrac{1}{2}gt^2$ $s = vt + \dfrac{1}{2}gt^2$ $h_f = h_i - \dfrac{1}{2}gt^2$

(c) $v^2 = -2gs$

(d) $s_n = -\dfrac{g}{2}(2n-1)$

- **Graphs in Uniformly Accelerated Motion along a straight line ($a \neq 0$) :**

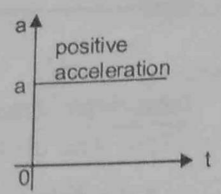
- x is a quadratic polynomial in terms of t. Hence x – t graph is a parabola.

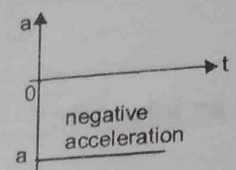
x-t graph

- v is a linear polynomial in terms of t. Hence v–t graph is a straight line of slope a.

v-t graph

- a–t graph is a horizontal line because a is constant.





a-t graph

- For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area under the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

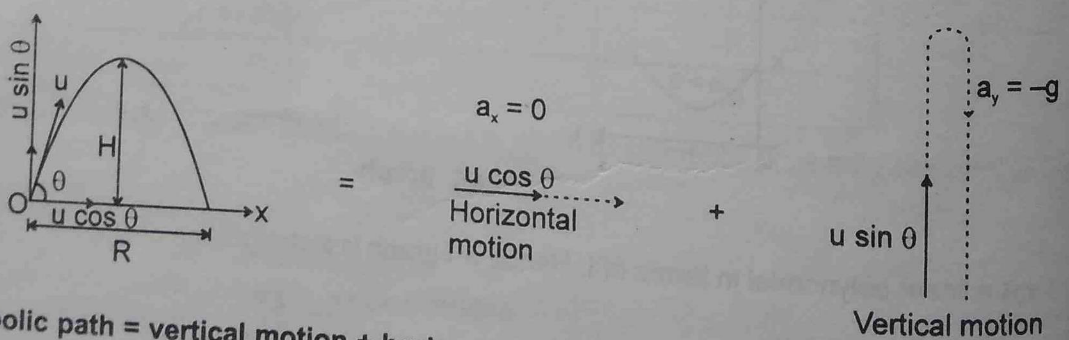
PROJECTILE MOTION

- **Projectile is any particle thrown at an angle other than 0° or 180° with net constant acceleration vector**

Or

Projectile is any particle which is thrown with $\vec{u}$, such that $\vec{a} \times \vec{u} \neq 0$, where $\vec{a}$ is net constant acceleration vector.

- **Projectile Motion :**
  - The motion of projectile is known as projectile motion.
  - It is an example of two dimensional motion with constant acceleration.
  - Projectile motion is considered as combination of two simultaneous motions in mutually perpendicular directions which are completely independent from each other i.e. horizontal motion and vertical motion.

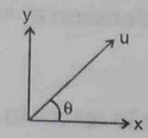


Parabolic path = vertical motion + horizontal motion.

Galileo's Statement :

Two perpendicular directions of motion are independent from each other. In other words any vector quantity directed along a direction remains unaffected by a vector perpendicular to it.

Parabolic path = vertical motion + horizontal motion. All the following formulae are for the shown positive x and y axes :



	Horizontal direction	Vertical direction
(a)	Initial velocity $u_x = u \cos\theta$	Initial velocity $u_y = u \sin\theta$
(b)	Acceleration $a_x = 0$	Acceleration $a_y = -g$
(c)	Velocity after time t, $v_x = u \cos\theta$	Velocity after time t, $v_y = u \sin\theta - gt$
(d)	$x = (u \cos\theta)\, t$;	$y = (u \sin\theta)\, t - \frac{1}{2} g (t^2)$

Time of flight : $T = \dfrac{2u\sin\theta}{g}$

Horizontal range : $R = \dfrac{u^2 \sin 2\theta}{g}$

Maximum height : $H = \dfrac{u^2 \sin^2\theta}{2g}$

Trajectory equation (equation of path) : $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

Other forms of trajectory equation :

- $y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2}$

- $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow \quad y = x\tan\theta\left[1 - \dfrac{gx}{2u^2\cos^2\theta\tan\theta}\right]$

$\Rightarrow \quad y = x\tan\theta\left[1 - \dfrac{gx}{2u^2\sin\theta\cos\theta}\right]$

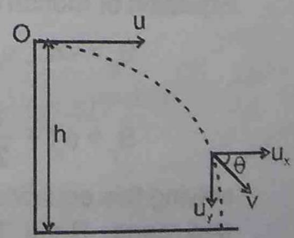
$\Rightarrow \quad y = x\tan\theta\left[1 - \dfrac{x}{R}\right]$

For maximum range : $\theta = 45^\circ$

Maximum range $= \dfrac{u^2}{g}$,

Range for θ and $90^\circ - \theta$ (complimentry angles) are same.

Projectile Thrown Parallel to the Horizontal from some height :



Consider a projectile thrown from point O at some height h from the ground with a velocity u. Now we shall study the characteristics of projectile motion by resolving the motion along horizontal and vertical directions.

Horizontal direction

(i) Initial velocity $u_x = u$

(ii) Acceleration $a_x = 0$

Vertical direction

Initial velocity $u_y = 0$

Acceleration $a_y = g$ (downward)

- **Time of flight :**

This is equal to the time taken by the projectile to return to ground. From equation of motion

$$t = \sqrt{\frac{2h}{g}}$$

- **Horizontal range :**

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

$$R = u\sqrt{\frac{2h}{g}}$$

- **Velocity at a general point P(x, y) :**

$V_x = u$

$V_y = gt$ (downwards)

$\therefore \quad v = \sqrt{u^2 + g^2t^2}$ and $\tan\theta = v_y/v_x$

- **Velocity with which the projectile hits the ground :**

$$V = \sqrt{u^2 + 2gh}$$

- **Trajectory equation :**

$$y = \frac{-1}{2}g \cdot \frac{x^2}{u^2}$$

This is trajectory equation of the particle projected horizontally from some height.

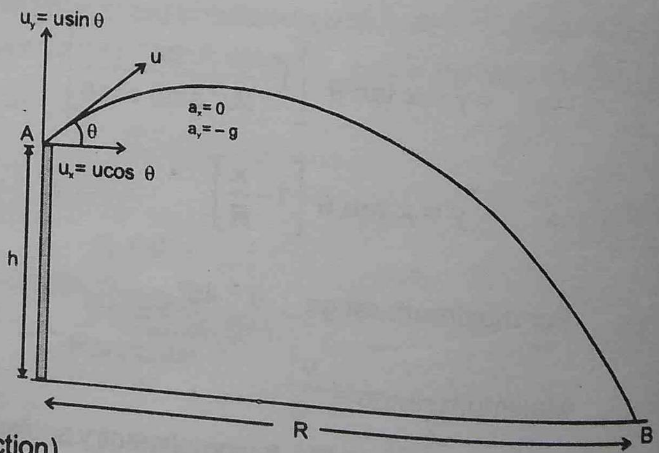
Projection from a tower :

Case (i) : Projection at an angle θ above horizontal

$u_x = u\cos\theta$;

$u_y = u\sin\theta$;

$a_y = -g$



Equation of motion between A & B (in Y direction)

$S_y = -h$, $u_y = u\sin\theta$, $a_y = -g$, $t = T$

$S_y = u_yt + \frac{1}{2}a_yt^2 \quad \Rightarrow \quad -h = u\sin\theta\, t - \frac{1}{2}gt^2$

solving this equation we will get time of flight, T.

and range, $R = u_x \cdot T = u\cos\theta\, T$

Also, $v_y^2 = u_y^2 + 2a_yS_y$

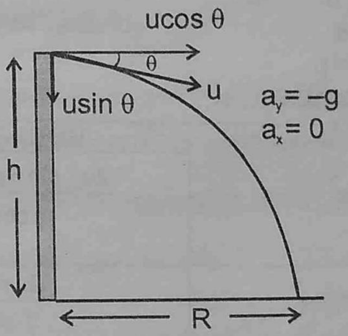
$= u^2 \sin^2\theta + 2gh$

$v_x = u\cos\theta$

$$v_B = \sqrt{v_y^2 + v_x^2}$$

$$v_B = \sqrt{u^2 + 2gh}$$

Case (ii) : Projection at an angle θ below horizontal



$u_x = u\cos\theta;$

$u_y = -u\sin\theta;$

$a_y = -g$

$S_y = u_yt + \frac{1}{2}a_yt^2$

$S_y = -h$, $u_y = -u\sin\theta$, $t = T$, $a_y = -g$

$\Rightarrow -h = -u\sin\theta\, T - \frac{1}{2}gT^2 \quad \Rightarrow \quad h = u\sin\theta + \frac{1}{2}gt^2$.

solving this equation we will get time of flight, T.

and range, $R = u_x T = u\cos\theta\, T$

$v_x = u\cos\theta$

$v_y^2 = u_y^2 + 2a_yS_y = u^2\sin^2\theta + 2(-g)(-h)$

$v_y^2 = u^2\sin^2\theta + 2gh$

$v_B = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

- Final velocity of projectile at which that hits the ground is independent of angle θ and equal to $\sqrt{u^2 + 2gh}$

☞ **Projection from a moving body**

A trolley moves horizontally with constant speed v. Consider a boy standing on the trolley who throws a ball with speed u at an angle of projection θ.. (w.r.t. the trolley)

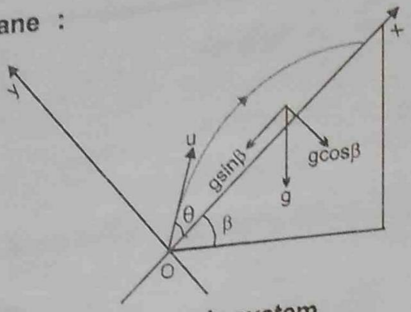
Case (i) : When ball is projected in the direction of motion of the trolley, horizontal component of ball's velocity = u cos θ + v. Initial vertical component of ball's velocity = u sin θ

Case (ii) : The ball is projected opposite to the direction of motion of the trolley
Horizontal component of ball's velocity = u cosθ – v
Initial vertical component of ball's velocity = u sinθ

Case (iii) : The ball projected upwards from a platform moving with speed v upwards.
Horizontal component of ball's velocity = u cos θ
Initial vertical component of ball's velocity = u sin θ + v

Case (iv) : The ball projected upwards from a platform moving with speed v downwards.
Horizontal component of ball's velocity = u cosθ
Initial vertical component of ball's velocity = u sinθ – v

Projection on an inclined plane :



axis system

$u_x = u \cos \theta$
$u_y = u \sin \theta$
$a_x = -g \sin \beta$
$a_y = -g \cos \beta$

	Up the Incline	Down the Incline
Range	$\frac{2u^2 \sin\theta \cos(\theta+\beta)}{g\cos^2\beta}$	$\frac{2u^2 \sin\theta \cos(\theta-\beta)}{g\cos^2\beta}$
Time of flight	$\frac{2u\sin\theta}{g\cos\beta}$	$\frac{2u\sin\theta}{g\cos\beta}$
Angle of projection with incline plane for maximum range	$\frac{\pi}{4}-\frac{\beta}{2}$	$\frac{\pi}{4}+\frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

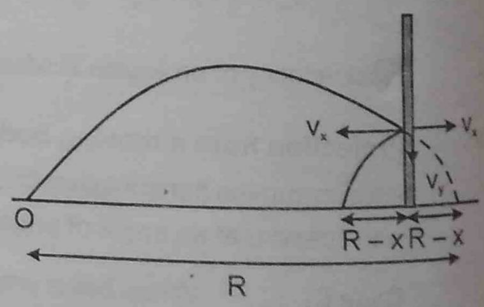
- Replace by $-\beta$ for down the inclined.

Elastic collision of a projectile with a smooth wall :

Suppose a projectile is projected with speed u at an angle θ from point O on the ground. Range of the projectile is R. If a wall is present in the path of the projectile at a distance x from the point O. The collision with the wall is elastic, path of the projectile changes after the collision as described below.

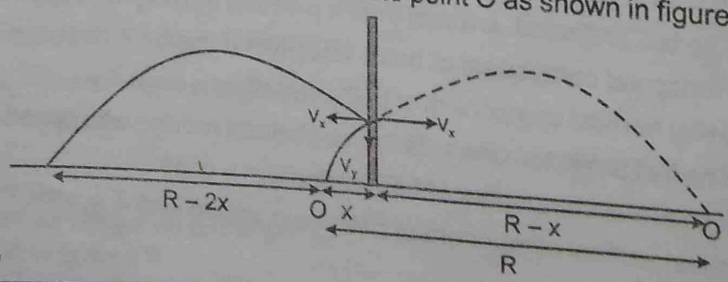
Case I : If $x \geq \frac{R}{2}$

Direction of x component of velocity is reversed but its magnitude remains the same and y component of velocity remains unchanged, therefore the remaining distance $(R-x)$ is covered in the backward direction and projectile falls a distance $(2x - R)$ ahead of the point O as shown in figure.



Case II : If $x < \frac{R}{2}$

Direction of x component of velocity is reversed but its magnitude remains the same and y component of velocity remains unchanged, therefore the remaining distance $(R-x)$ is covered in the backward direction and projectile falls a distance $(R-2x)$ behind the the point O as shown in figure.



RELATIVE MOTION

Relative Motion : Motion is a combined property of the object under study as well as the observer. There is no such thing like absolute motion or absolute rest. Motion is always defined with respect to an observer or reference frame.

Relative Motion in one Dimension :

Relative Positon : $\boxed{\vec{X}_{AB} = \vec{X}_A - \vec{X}_B}$

where $\vec{x}_A, \vec{x}_B \rightarrow$ position vector of A & B w.r.t. origin (ground).

Relative Velocity : Relative velocity of a particle A with respect to B is defined as the velocity with which A appears to move if B is considered to be at rest. In other words, it is the velocity with which A appears to move as seen by B considering itself to be at rest. If $\vec{v}_A$ is the velocity of A w.r.t. ground, $\vec{v}_B$ is velocity of B w.r.t. ground and $\vec{v}_{AB}$ is velocity of A w.r.t. B then we have, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

All velocities are relative & have no significance unless observer is specified. However, when we say "velocity of A", what we mean is , velocity of A w.r.t. ground which is assumed to be at rest.

Relative Acceleration : It is the rate at which relative velocity is changing.

$$\vec{a}_{AB} = \frac{d\vec{v}_{AB}}{dt} = \frac{d\vec{v}_A}{dt} - \frac{d\vec{v}_B}{dt} = \vec{a}_A - \vec{a}_B$$

Equations of Motion : (If relative acceleration is constant)

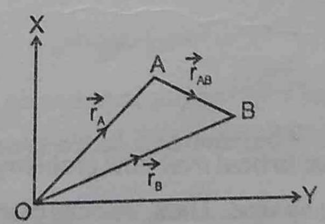
$$\vec{V}_{rel.} = \vec{u}_{rel.} + \vec{a}_{rel.}\, t$$

$$\vec{S}_{rel.} = \vec{u}_{rel.}\, t + \frac{1}{2}\vec{a}_{rel.}\, t^2$$

$$|\vec{V}_{rel.}|^2 = |\vec{u}_{rel.}|^2 + 2(\vec{a}_{rel.}).(\vec{S}_{rel.})$$

Velocity of Approach / Separation : It is the component of relative velocity of one particle w.r.t. another, along the line joining them. If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

Relative Motion in two Dimension :



$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$ where $\vec{r}_A, \vec{r}_B \rightarrow$ position vector of A & B w.r.t. origin (ground).

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ where $\vec{V}_A, \vec{V}_B \rightarrow$ velocity vector of A & B w.r.t. origin (ground).

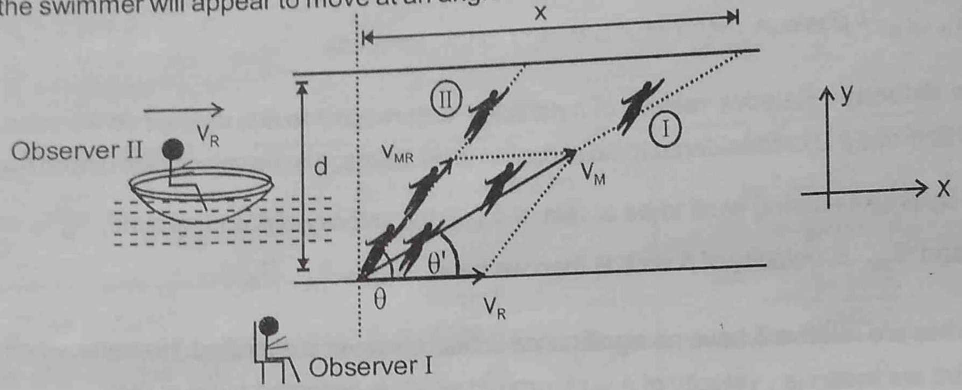
$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$ where $\vec{a}_A, \vec{a}_B \rightarrow$ acceleration vector of A & B w.r.t. origin (ground)

Relative motion in river flow :

Consider a man swimming in a river with a velocity of $\vec{v}_{MR}$ relative to river at an angle of θ with the river flow. The velocity of river is $\vec{V}_R$. Let there be two observers I and II, observer I is on ground and observer II is on a raft floating along with the river and hence moving with the same velocity as the river. Hence motion w.r.t. observer II is same as motion w.r.t. river. i.e. the man will appear to swim at an angle θ with the river flow for observer II.

For observer I the velocity of swimmer will be $\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$,

Hence the swimmer will appear to move at an angle θ' with the river flow.



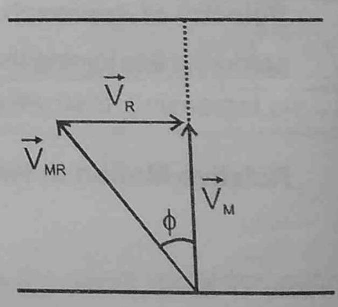
(I) : Motion of swimmer for observer I

(II) : Motion of swimmer for observer II

Drift : It is defined as the displacement of man in the direction of river flow. In the above figure drift is equal to x.

Crossing the river in shortest time : Time to cross the river will be minimum if man swims perpendicular to the river flow. $t_{min} = \dfrac{d}{V_{MR}}$.

Crossing the river in shortest path, Minimum Drift : The minimum possible drift is zero. In this case the man swims in the direction perpendicular to the river flow as seen from the ground. This path is known as shortest path. For minimum drift the man must swim at some angle ϕ with the perpendicular in backward direction, which is given by, $\sin\phi = \dfrac{V_R}{V_{MR}}$



Time to cross the river along the shortest path

$$t = \frac{d}{V_{MR}\cos\phi} = \frac{d}{\sqrt{V_{MR}^2 - V_R^2}}$$

Wind Airplane Problems : This is very similar to boat river flow problems. The only difference is that boat is replaced by aeroplane and river is replaced by wind. Thus, velocity of aeroplane with respect to wind

$$\vec{v}_{aw} = \vec{v}_a - \vec{v}_w$$

or $$\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$$

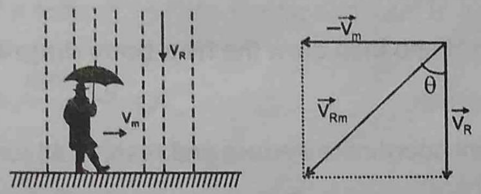
where, $\vec{v}_a$ = velocity of aeroplane w.r.t. ground and, $\vec{v}_w$ = velocity of wind.

Rain Problem : If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to observer will be :

$$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m$$

or $$v_{Rm} = \sqrt{v_R^2 + v_m^2}$$

and direction $\theta = \tan^{-1}\left(\frac{v_m}{v_R}\right)$ with the vertical as shown in figure.



Condition for uniformly moving particles to collide : If two particles are moving with uniform velocities and the relative velocity of one particle w.r.t. other particle is directed towards each other then they will collide.

If the initial position of two particles are $\vec{r}_1$ and $\vec{r}_2$ and their velocities are $\vec{v}_1$ and $\vec{v}_2$ then shortest distance between the particles, $d_{shortest} = \frac{|\vec{r}_{12} \times \vec{v}_{12}|}{|\vec{v}_{12}|}$ and time after which this situation will occur, $t = -\frac{\vec{r}_{12} \cdot \vec{v}_{12}}{|\vec{v}_{12}|^2}$

NEWTON'S LAWS OF MOTION

First Law of Motion :

" Every body preserves in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon."

A particle continues to be in its original state of rest or of motion until and unless an external agent called force is not applied on it. It happens due to its property of inertia.

Second Law of Motion :

"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

Third Law of Motion :

"To every action there is always opposed an equal and opposite reaction: to the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

From Third Law of Motion :

$\vec{F}_{AB} = -\vec{F}_{BA}$ $\Rightarrow$ $\vec{F}_{AB}$ = Force on A due to B

$\vec{F}_{BA}$ = Force on B due to A

☞ From second law of motion :

$$F_x = \frac{dP_x}{dt} = ma_x \quad \Rightarrow \quad F_y = \frac{dP_y}{dt} = ma_y \quad \Rightarrow \quad F_z = \frac{dP_z}{dt} = ma_z$$

☞ **Applications of Newton's Laws :**

When objects are in equilibrium

To solve problems involving objects in equilibrium:

Step 1: Make a sketch of the problem.

Step 2: Isolate a single object and then draw the **free-body diagram** for the object. Label all external forces acting on it.

Step 3: Choose a convenient coordinate system and resolve all forces into x and y components.

Step 4: Apply the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

Step 5: Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.

Eventually at step 5 you will have enough equations to solve for all unknown quantities.

☞ **Accelerating Objects :**

To solve problems involving objects that are in accelerated motion :

Step 1: Make a sketch of the problem.

Step 2: Isolate a single object and then draw the **free-body diagram** for that object. Label all external forces acting on it. Be sure to include all the forces acting on the chosen body, but be equally carefully not include any force exerted by the body on some other body. Some of the forces may be unknown; label them with algebraic symbols.

Step 3: Choose a convenient coordinate system, show location of coordinate axes explicitly in the free-body diagram, and then determine components of forces with reference to these axes and resolve all forces into x and y components.

Step 4: Apply the equations $\Sigma F_x = ma_x$ and $\Sigma F_y = m\,a_y$.

Step 5: Step 4 will give two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps. Eventually at step 5 you will have enough equations to solve for all unknown quantities.

☞ **Weighing Machine :**

A weighing machine does not measure the weight but measures the normal reaction force exerted by object on its upper surface.

Spring Force :

$$\vec{F} = -k\vec{x}$$

x is extension or compression from its natural length or deformation of the spring where K = spring constant.

Spring property :

$K \times \ell$ = constant

ℓ = Natural length of spring.

If spring is cut into two in the ratio m : n then spring constant is given by

$$\ell_1 = \frac{m\ell}{m+n}; \qquad k_1 = \frac{k(m+n)}{m} = K_m$$

$$\ell_2 = \frac{n.\ell}{m+n} \qquad k_2 = \frac{k(m+n)}{n} = k_n$$

$$k\ell = k_1\ell_1 = k_2\ell_2$$

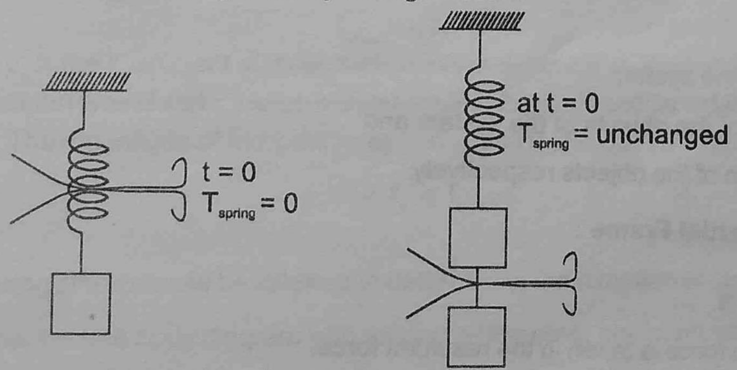
For series combination of springs

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + ...$$

For parallel combination of spring

$$k_{eq} = k_1 + k_2 + k_3 ....$$

Tension in a spring : If spring is cut then the tension in it immediately become zero but if two ends of a spring are not free i.e. connect with a support / block at its two ends then just after the phenomenon tension remains unchanged if any change occurs



Spring Balance :

It does not measure the weight. It measures the tension force exerted by the object at the hook.

String Constraint :

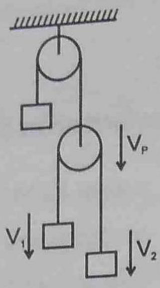
When two objects are connected through a string and if the string have the following properties :

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them. This relation is called string constraint.

Remember :

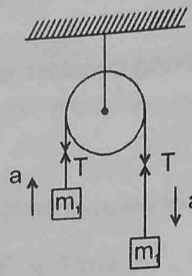


$$V_p = \frac{V_1 + V_2}{2}$$

$$a_p = \frac{a_1 + a_2}{2}$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$T = \frac{2m m_2 g}{m_1 + m_2}$$



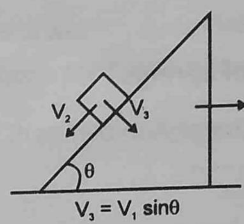
Wedge Constraint:

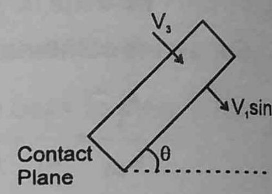
Conditions :

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.





In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

Newton's Law for a System :

$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots\ldots$$

$\vec{F}_{ext}$ = Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and

$\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects respectively.

Newton's Law for Non Inertial Frame :

$$\vec{F}_{Real} + \vec{F}_{Pseudo} = m\vec{a}$$

Net sum of real and pseudo force is taken in the resultant force.

$\vec{a}$ = Acceleration of the particle in the non inertial frame

$$\vec{F}_{Pseudo} = -m\ \vec{a}_{Frame}$$

Pseudo force is always directed opposite to the direction of the acceleration of the frame.

Pseudo force is an imaginary force and there is no action-reaction for it. So it has nothing to do with Newton's Third Law.

Reference Frame:

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

(a) Inertial reference frame: Frame of reference either stationary or moving with constant velocity.

(b) Non-inertial reference frame: A frame of reference moving with non-zero acceleration.

FRICTION

Part of the contact force that is tangential to the surface is called friction force. Microscopically friction force because of attraction between molecules of the two surfaces.

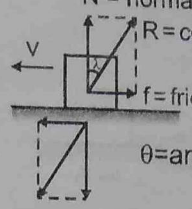
Friction force is of two type :

(A) Kinetic friction

(B) Static friction

$R \sin\theta = f$

$R \cos\theta = N$



Kinetic friction force :

Kinetic friction exists between two surfaces (in case of block), or two points (in case of sphere), or two line (in case of cylinder) when there is relative motion. It stops acting when relative motion ceases to exist.

Direction of Kinetic Friction.

It is opposite to the relative velocity of contact surfaces.

Note : Its direction is not opposite to the force applied it is opposite to the motion of he body considered which is in contact with the other surface.

Kinetic friction $f_k = \mu_k N$

The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.

Static friction :

When two surfaces in contact have relative velocity zero, but there is tendency of relative motion then the friction force acting will be static.

Direction of static friction

If there is tendency of sliding between the contact surfaces, it will act in such a direction to prevent sliding. Static friction is variable and self adjusting force. It can adjust its value upto a limit which is called limiting friction force (f_{smax}).

$$f_{smax} = \mu_s N$$

Here μ_s is the coefficient of static friction which depends on the nature of two contact surfaces.

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

$$0 \le f_s \le f_{smax}$$

Following steps should be followed in determining the direction of static friction force on an object.

(i) Draw the free body diagram with respect to the other object on which it is kept.

(ii) Include pseudo force also if contact surface is accelerating.

(iii) Decide the resultant force and the component parallel to the surface of this resultant force.

(iv) The direction of static friction is opposite to the above component of resultant force.

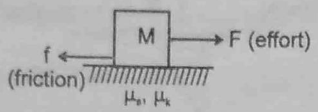
Here μ_s and μ_k are dimensionless quantities independent of shape and area of contact. It is a property of the two contact surfaces. In general $\mu_s > \mu_k$ for a given pair of surfaces. If it is not mentioned separately the $\mu_s = \mu_k$ can be taken.

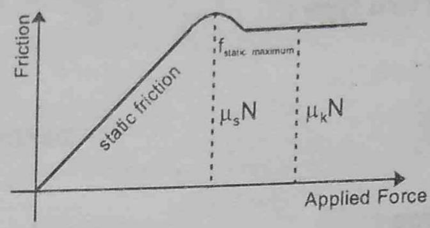
μ_s and μ_k can also be represented as angles. If θ_s and θ_k are angles of static friction and kinetic friction respectively, then

$\theta_s = \tan^{-1} \mu_s$

$\theta_k = \tan^{-1} \mu_k$

θ_s is also called angle of repose.





Rolling Friction :

When a body rolls on a surface the resistance offered by the surface is called as rolling friction.

Rolling friction is less than sliding friction

In sliding motion, elevation collide. This introduces friction. In rolling motion, elevations are crossed over. This avoids friction.

Rolling friction zero in ideal case :

In ideal rolling the contact with cylindrical surface of body and lower surface must be along a straight line. Elevations must be crossed over and no friction be present.

But no rolling is ideal. Due to deformation of moving cylindrical surface (wheel of a loaded truck), or deformation of lower surface (mud street), contact becomes over a flat surface. This introduces sliding, which causes friction.

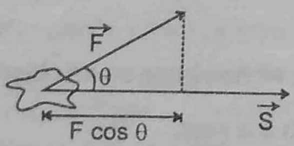
WORK, POWER AND ENERGY

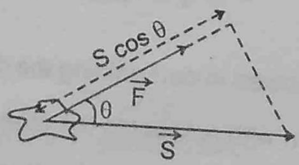
Work done by Constant Force :

$$W = \vec{F} \cdot \vec{S}$$

Since work is the dot product of two vectors therefore it is a scalar quantity.

$W = FS \cos\theta$ or $W = (F \cos\theta)S$





Work Done by Multiple Forces :

If several forces act on a particle, then we can replace $\vec{F}$ in equation $W = \vec{F} \cdot \vec{S}$ by the net force $\Sigma\vec{F}$ where

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

$$W = [\Sigma\vec{F}] \cdot \vec{S} \qquad ...(i)$$

$$W = \vec{F}_1 \cdot \vec{S} + \vec{F}_2 \cdot \vec{S} + \vec{F}_3 \cdot \vec{S} + ...$$

or $\quad W = W_1 + W_2 + W_3 + ...$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.

☞ **Dimensions of Work :**

[Work] = [Force] [Distance] = $[MLT^{-2}]\,[L]$ = $[ML^2T^{-2}]$

Work has one dimension in mass, two dimensions in length and '–2' dimensions in time,

☞ **Work in Terms of Rectangular Components :**

In terms of rectangular components, $\vec{F}$ and $\vec{S}$ may be written as :

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k}$

$\vec{F}.\vec{S} = F_xS_x + F_yS_y + F_zS_z$

☞ **Work Done by a Variable Force :**

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position. For a variable force work is calculated for infinitely small displacement and for this displacement force is assumed to be constant

$$\mathbf{dW = \vec{F}.d\vec{s}}$$

The total work done will be sum of infinitely small work

$$W_{A\to B} = \int_A^B \vec{F}.d\vec{s} = \int_A^B (\vec{F}\cos\theta)d\vec{s}$$

In terms of rectangular components,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$W_{A\to B} = \int_{x_A}^{x_B} F_X dx + \int_{x_A}^{x_B} F_Y dy + \int_{x_A}^{x_B} F_Z dz$$

☞ **Relation Between Momentum and Kinetic Energy :**

Important Points for K.E.

1. As mass m and v^2 ($\vec{v}.\vec{v}$) are always positive, kinetic energy is always positive scalar i.e, kinetic energy can never be negative.
2. The kinetic energy depends on the frame of reference,

$$K = \frac{p^2}{2m} \quad \text{and} \quad P = \sqrt{2\,m\,K} \quad ; \; P = \text{linear momentum}$$

☞ **Potential Energy :**

In case of conservative force

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r}$$

i.e., $U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = -W$, where W is work done by conservative forces

Whenever and wherever possible, we take the reference point at ∞ and assume potential energy to be zero there, i.e., If we take $r_1 = \infty$ and $U_1 = 0$ then

$$U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r} = -W$$

Types of Potential Energy :

(a) Elastic Potential Energy : $U = \frac{1}{2}ky^2$

where k is force constant and 'y' is the stretch or compression. Elastic potential energy is always positive.

(b) Electric Potential Energy : It is the energy associated with charged particles that interact via electric force. For two point charges q_1 and q_2 separated by a distance 'r',

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

As charge can be positive or negative, therefore electric potential energy can also be positive or negative.

(c) Gravitational Potential Energy : It is due to gravitational force. For two particles of masses m_1 and m_2 separated by a distance 'r', it is given by:

$$U = -G\frac{m_1m_2}{r}$$

which for a body of mass 'm' at height 'h' relative to surface of the earth reduces to U = mgh

Gravitational potential energy can be positive or negative.

Mechanical Energy :

Definition: Mechanical energy 'E' of an object or a system is defined as the sum of kinetic energy 'K' and potential energy 'U', i.e.,

$$E = K + U$$

Conservative Forces :

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.

$$F = -\frac{\partial U}{\partial r}$$

Conservative Force & Potential Energy :

We know that $F = -\frac{\partial U}{\partial r}$

Types of Equilibrium :

(a) **Stable equilibrium :** When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

Necessary conditions : $-\frac{dU}{dx} = 0$, and $\frac{d^2U}{dx^2} = +ve$ Potential energy is minimum.

(b) **Unstable Equilibrium :** When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition : $-\frac{dU}{dx} = 0$ potential energy is maximum i.e. $= \frac{d^2U}{dx^2} = -ve$

(c) **Neutral equilibrium :** In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.

Work-Energy Theorem :

According to work-energy theorem, the work done by all the forces on a particle is equal to the change in its kinetic energy.

$$W_C + W_{NC} + W_{PS} = \Delta K$$

Where, W_C is the work done by all the conservative forces.

W_{NC} is the work done by all non-conservative forces.

W_{PS} is the work done by all psuedo forces.

Modified Form of Work-Energy Theorem :

We know that conservative forces are associated with the concept of potential energy, that is

$$W_C = -\Delta U$$

So, Work-Energy theorem may be modified as

$$W_{NC} + W_{PS} = \Delta K + \Delta U \qquad W_{NC} + W_{PS} = \Delta E$$

Power :

Power is defined as the time rate of doing work.

The average power ($\overline{P}$ or p_{av}) delivered by an agent is given by

$$\overline{P} \text{ or } p_{av} = \frac{W}{t}$$

where W is the amount of work done in time t.

$$P = \frac{\vec{F}\cdot d\vec{S}}{dt} = \vec{F}\cdot\frac{d\vec{S}}{dt} = \vec{F}\,.\,\vec{v}$$

By definition of dot product,

$$P = Fv\cos\theta$$

where θ is the smaller angle between $\vec{F}$ and $\vec{v}$.

This P is called as instantaneous power if dt is very small.

CIRCULAR MOTION

Circular Motion:

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as the circular motion with respect to that fixed (or moving) point. That fixed point is called centre and the distance is called radius.

Kinematics of Circular Motion :

(A) Variables of Motion

(a) Angular Displacement 'θ'

Definition: Angle rotated by a position vector of the moving particle with some reference line is called angular displacement.

Important points:

1. It is dimensionless and has proper unit (SI unit) radian while other units are degree or revolution

$$2\pi \text{ rad} = 360° = 1 \text{ rev}$$

2. Inifinitly small angular displacement is a vector quantity but finite angular displacement is not because the addition of the small angular displacement is commutative while for large is not.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \qquad \text{but} \qquad \theta_1 + \theta_2 \neq \theta_2 + \theta_1$$

3.Direction of small angular displacement is decided by right hand thumb rule. When the fingures are directed along the motion of the point then thumb will reprents the direction of angular displacement.

4. Angular displacement can be different for different observers

(b) Angular Velocity ω :

(i) **Average Angular Velocity**

$$\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} \quad ; \quad \omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2

(ii) **Instantaneous Angular Velocity**

The rate at which the position vector of a particle w.r.t. the centre rotates, is called as instantaneous angular velocity with respect to the centre.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Important points :

1. It is an axial vector with dimensions $[T^{-1}]$ and SI unit rad/s.
2. For a rigid body as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about its own axis is $(2\pi/24)$ rad/hr.
3. If a body makes 'n' rotations in 't' seconds then angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{t}$$

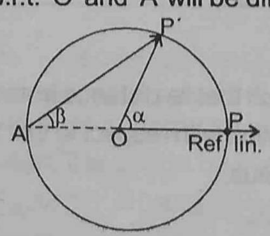
If T is the period and 'f' the frequency of uniform circular motion $\omega_{av} = \frac{2\pi \times 1}{T} = 2\pi f$

4. If $\theta = a - bt + ct^2$ then $\omega = \frac{d\theta}{dt} = -b + 2ct$

(c) Relative Angular Velocity :

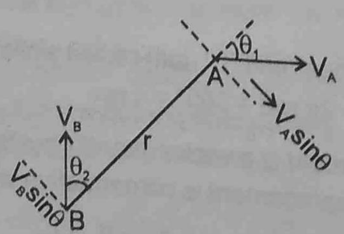
Angular velocity is defined with respect to the point from which the position vector of the moving particle is drawn

Here angular velocity of the particle w.r.t. 'O' and 'A' will be different



$$\omega_{PO} = \frac{d\alpha}{dt} \quad ; \quad \omega_{PO} = \frac{d\beta}{dt}$$

Definition: Relative angular velocity of a particle 'A' with respect to the other moving particle 'B' is the angular velocity of the position vector of 'A' with respect to 'B'. That means it is the rate at which position vector of 'A' with respect to 'B' rotates at that instant.



$$\omega_{AB} = \frac{(V_{AB})\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{Seperation between A and B}}$$

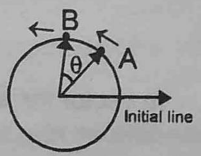
$(V_{AB})_\perp = V_A \sin\theta_1 + V_B \sin\theta_2$

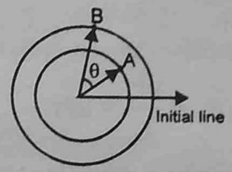
$r_{AB} = r$

Important points:

1. If two particles are moving on the same circle or different coplanar concentric circles in same direction with different uniform angular speed ω_A and ω_B respectively, the angular velocity of B relative to A for an observer at the center will be

$$\omega_{rel} = \omega_B - \omega_A = \frac{d\theta}{dt}$$

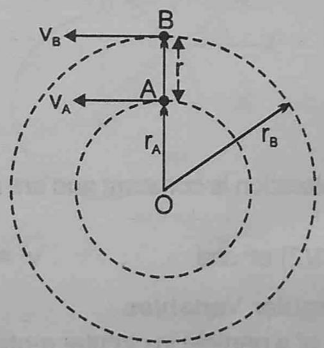




So the time taken by one to complete one revolution around O w.r.t. the other

$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1T_2}{T_1 - T_2}$$

2. If two particles are moving on two different concentric circles with different velocities then angular velocity of B relative to
A as observed by A will depend on their positions and velocities. consider the case when A and B are closet to each other moving in same direction as shown in figure. In this situation



$$v_{rel} = |\vec{v}_B - \vec{v}_A| = v_B - v_A$$

$$r_{rel} = |\vec{r}_B - \vec{r}_A| = r_B - r_A$$

so, $$\omega_{rel} = \frac{(v_{rel})_\perp}{r_{rel}} = \frac{v_B - v_A}{r_B - r_A}$$

$(v_{rel})_\perp$ = Relative velocity perpendicular to position vector

(d) Angular Acceleration α :

(i) **Average Angular Acceleration :** Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

(ii) **Instantaneous Angular Acceleration :** It is the limit of average angular acceleration as Δt approaches zero, i.e.,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$$

Impotant points:

1. It is also an axial vector with dimension $[T^{-2}]$ and unit rad/s^2.
2. If $\alpha = 0$, circular motion is said to be uniform.
3. As $\omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$, i.e., second derivative of angular displacement w.r.t. time gives angular acceleration.

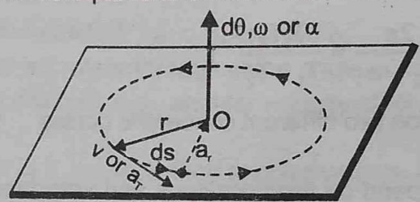
(e) Centripetal Acceleration :

Definition: Acceleration directed towards the center of the circle is called as centripetal acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r \qquad \text{In vector for } \vec{a}_r = \vec{\omega} \times \vec{v}$$

[B] Relations among Angular Variables :

These relations are also referred as equations of rotational motion and are –



$$\omega_f = \omega_i + \alpha t \qquad -(1)$$

$$\theta_f - \theta_i = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad -(2)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \qquad -(3)$$

These are valid only if angular acceleration is constant and are analogous to equations of translatory motion, i.e.,

$v = u + at$; $\quad s = ut + (1/2)\,at^2$ and $\quad v^2 = u^2 + 2as$

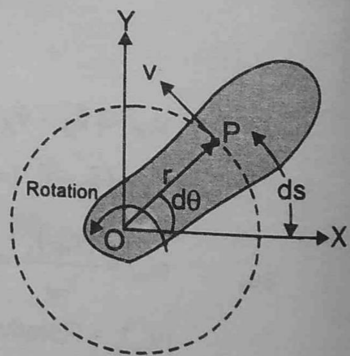
[C] Relations among Linear and Angular Variables :

This is also referred as kinematics of a particle in circular motion. For this consider a particle P moving on a circle of radius r.

$$ds = r\,d\theta \qquad .....(1)$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow v = r\omega \qquad .....(2)$$

$$\frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} \Rightarrow a_T = r\alpha \qquad .....(3)$$



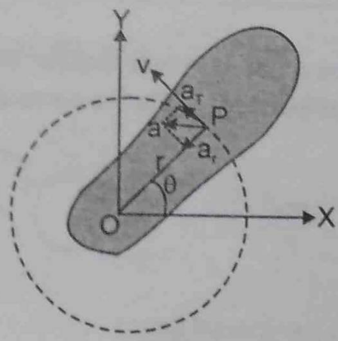
(A)

Here a_T is tangential acceleration directed along the tangent and is equal to the rate of change of speed. Note that net acceleration is rate of change of velocity.

In addition to tangential acceleration a_T in case of circular motion there is also a radial acceleration a_r given by

$$a_r = (v^2/r) = r\omega^2 \qquad ....(4)$$

In a circle as tangent and radius are always normal to each other so will a_T and a_r. This in turn implies that acceleration in case of circular motion.



(B)

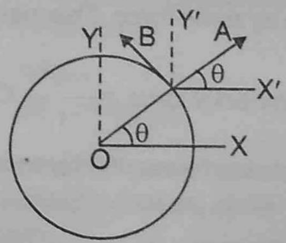
$$a = \sqrt{a_r^2 + a_T^2} \quad \text{....(5)}$$

Here it must be noted that a_T governs the magnitude of $\vec{v}$ while a_r its direction of motion so that if for r time of motion we have

$a_r = 0$ and $a_T = 0$; $a \rightarrow 0 \Rightarrow$ motion is uniform rectilinear.
$a_r = 0$ but $a_T \neq 0$; $a \rightarrow a_T \Rightarrow$ motion is accelerated rectilinear
$a_r \neq 0$ but $a_T = 0$; $a \rightarrow a_r \Rightarrow$ motion is uniform circular.
$a_r \neq 0$ and $a_T \neq 0$; $a \rightarrow \sqrt{a_r^2 + a_T^2}$ = motion is nonuniform circular.

Radial and Tangential unit vector :

$\hat{e}_t$ is along the tangent in the direction of increasing θ. We call $\hat{e}_r$ the radial unit vector and $\hat{e}_t$ the tangential unit vector.



or, $\quad \hat{e}_r = \hat{i}\cos\theta + \hat{j}\sin\theta$,

or, $\quad \hat{e}_t = \hat{i}\sin\theta + \hat{j}\cos\theta$

Centripetal Force :

The force which compels a body to describe circular motion is called centripetal force. It acts along the radius towards the centre of the circle.

Dynamics of circular motion :

In circular motion or motion along any curved path Newton's law is applied in two perpendicular directions one along the tangent and other perpendicular to it . i.e. towards centre.

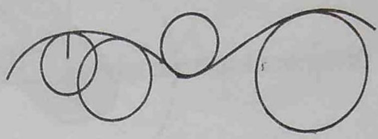
$$F_t = Ma_t = M\frac{dv}{dt} = M \cdot \alpha\, r$$

$$F_c = m\,\omega^2 r = \frac{mv^2}{r}$$

Radius of curvature :

Any curved path can be assume to be made of infinite circular arcs. Radius of curvature at a point is the radius of the circular arc at a particular point which fits the curve at that point.

$$F_c = \frac{mv^2}{R} \Rightarrow R = \frac{mv^2}{F_C} = \frac{mv^2}{F_\perp}$$



$F_\perp$ = Force perpendicular to mid point (centripetal force)
If the equation of trajectory of a particle is given we can find the radius of curvature of the instantaneous circle by using the formula ,

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}.$$

Centrifugal Force :

When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer B, who is sharing the motion along the circular path (i.e., the observer B also rotating with the body is released, it appears to B, as if it has been thrown off along the radius away from the centre by some force. This inertial force is called centrifugal force.)

Its magnitude is equal to that of the centripetal force. $= \frac{mv^2}{r}$. Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply N.L in that frame)

Centripetal Force and Centrifugal Force :

Concepts

This necessary resultant force towards the centre is called the centripetal force.

$$F = \frac{mv^2}{r} = m\omega^2 r$$

(i) A body moving with constant speed in a circle is not in equilibrium.

(ii) It should be remembered that in the absence of the centripetal force the body will move in a straight line with constant speed.

(iii) It is not a new kind of force which acts on bodies. In fact, any force which is directed towards the centre may provide the necessary centripetal force.

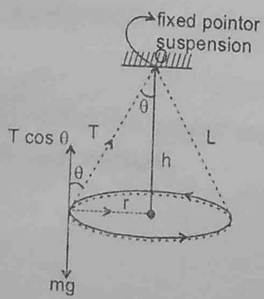
Conical pendulum :

$T\cos\theta = mg$

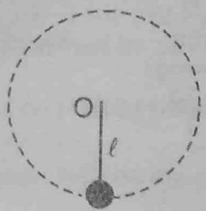
$T\sin\theta = m\omega^2 r$

$\therefore$ Time period =

$$2\pi\sqrt{\frac{L\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$$

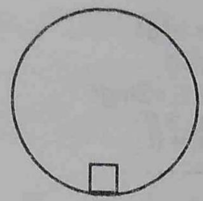


Minimum velocity for completing the loop :



(A)

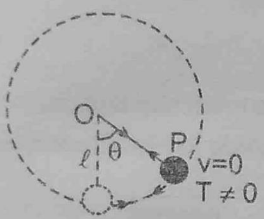
Mass attached to a string
condition for just looping the loop
tension at highest point = 0
Minimum velocity at lowest point = $\sqrt{5g\ell}$



(B)

Mass moving along a
smooth vertical circular loop.
condition for just looping the loop
normal at highest point = 0
Minimum velocity at lowest point = $\sqrt{5g\ell}$

Vertical circular Motion (for a mass attached to a string) :

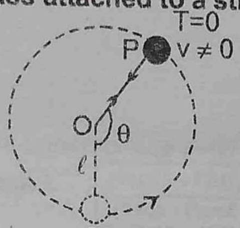


(A)

For Oscillation

$0 < v_L \leq \sqrt{2g\ell}$

$0 < \theta \leq 90°$

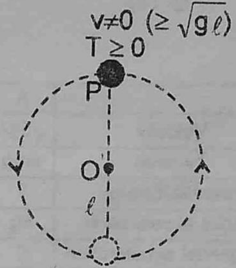


(B)

For Leaving the circular path after which motion converts into projectile motion.

$\sqrt{2g\ell} < v_L < \sqrt{5g\ell}$

$90° < \theta < 180°$



(C)

For Looping the loop

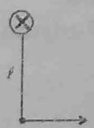
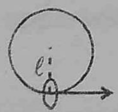
$v_L \geq \sqrt{5g\ell}$

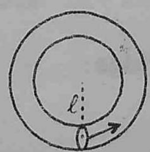
Note : These critical conditions are not true for the following cases :-

(1) A particle attached to a light rod rotated in vertical circle.

(2) A bead attached to a ring and rotated.

(3) A block rotated between smooth surfaces of a pipe.

(1)  (2) 

(3) 

$V_{min} = \sqrt{4g\ell}$ $V_{min} = \sqrt{4g\ell}$ $V_{min} = \sqrt{4g\ell}$

(for completing the circle)
(for completing the circle)
(for completing the circle)

If velocity at lowest point is just enough for looping the loop, value of various quantities.

(True for a point mass attached to a string or a mass moving on a smooth vertical circular track.)

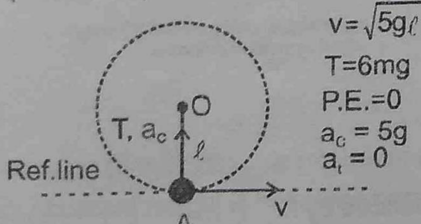


Fig. (1)

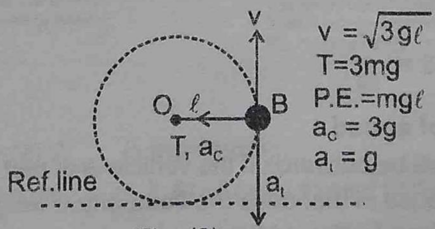


Fig. (2)

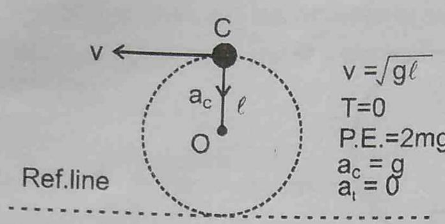


Fig. (3)

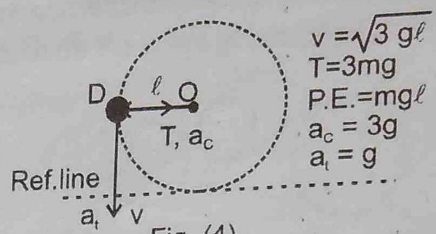


Fig. (4)

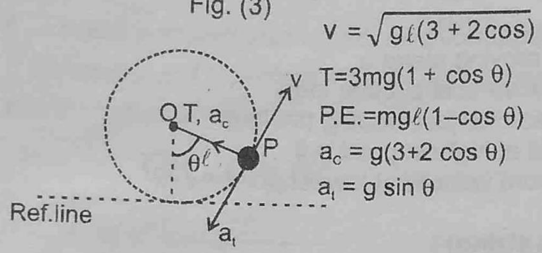


Fig. (5)

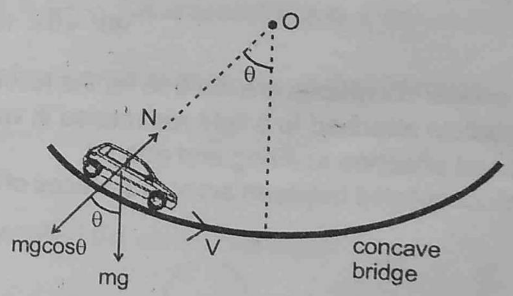
		A	B,D	C	P(general point)
1	Velocity	$\sqrt{5g\ell}$	$\sqrt{3g\ell}$	$\sqrt{g\ell}$	$\sqrt{g\ell(3+2\cos\theta)}$
2	Tension	6mg	3mg	0	$3mg(1+\cos\theta)$
3	Potential Energy	0	$mg\ell$	$2mg\ell$	$mg\ell(1-\cos\theta)$
4	Radial acceleration	5g	3g	g	$g(3+2\cos\theta)$
5	Tangential acceleration	0	g	0	$g\sin\theta$

Note :- From above table we can see ,
$T_{bottom} - T_{top} = T_C - T_A = 6$ mg ,
this difference in tension remain same even if

$$V > \sqrt{5g\ell}$$

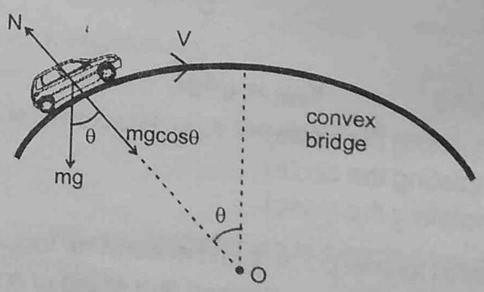
- **Normal reaction on a concave bridge :**

$$\Rightarrow \quad N = mg\cos\theta + \frac{mv^2}{r}$$



- **Normal reaction on a convex bridge:**

$$\Rightarrow \quad N = mg\cos\theta - \frac{mv^2}{r}$$



- **Skidding of vehicle on a level road :**

$$\Rightarrow \quad v_{safe} \le \sqrt{\mu g r}$$

- **Skidding of an object on a rotating platform :**

$$\Rightarrow \quad \omega_{max} = \sqrt{\mu g / r}$$

- **Bending of cyclist :**

$$\Rightarrow \quad \tan\theta = \frac{v^2}{rg}$$

- **Banking of a road :**

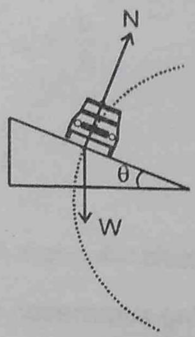
Friction f will be outwards if the vehicle is at rest
v = 0. Because in that case the component weight mg sinθ is balanced by f.
Banking of road without friction

$\Rightarrow \quad \tan\theta = \dfrac{v^2}{rg}$

(i) Friction f will be zero if $\quad v = \sqrt{rg\tan\theta}$

(ii) Friction f will be outwards if $\quad v < \sqrt{rg\tan\theta}$

(iii) Friction f will be inwards if $\quad v > \sqrt{rg\tan\theta}$



Maximum and minimum safe speed on a banked rough road

Where coefficient of static friction $\mu_s = \tan\lambda$

$$\therefore \quad v_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1 - \mu\tan\theta)}}$$

This corresponds to case (iii) where friction acts inwards.

$$\therefore \quad v_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}$$

This corresponds to case (ii) where friction acts outwards.

Centrifugal force (pseudo force) :

$\Rightarrow$ $f = m\omega^2 r$, acts outwards when the particle itself is taken as a frame.

Effect of earths rotation on apparent weight :

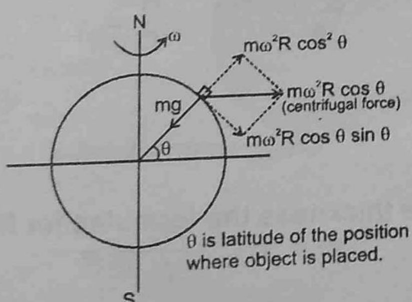


Figure (1) Earth's gravity & centrifugal force due rotation of Earth.

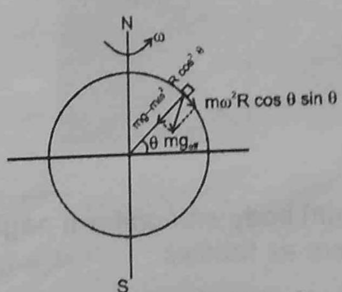


Figure (2) Resultant of Earth's gravity & centrifugal force is shown.

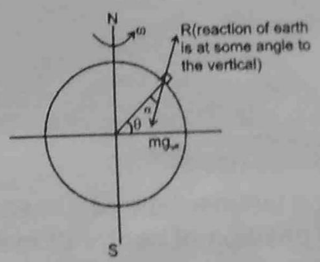


Figure (3) The reaction on an object kept at rest w.r.t. Earth is called a apparent weight $W_{app} = R = mg_{eff} = m(g - \omega^2 R\cos^2\theta)$

Note: At equator ($\theta = 0$) $W_{app.}$ is minimum and at pole ($\theta = \pi/2$) $W_{app.}$ is maximum. This apparent weight is not along normal but at some angle α w.r.t. it. At all point except poles and equator ($\alpha = 0$ at poles and equator)

CENTRE OF MASS

- **Mass Moment :** $\vec{M} = m\vec{r}$:

- **Centre of Mass of a System of 'N' Discrete Particles :**

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots\dots + m_n\vec{r}_n}{m_1 + m_2 + \dots\dots + m_n} \; ; \; \vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i} \qquad \vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

where $M\left(=\sum_{i=1}^{n} m_i\right)$ is the total mass of the system.

- **Centre of Mass of a Continuous Mass Distribution :**
For continuous mass distribution the centre of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation

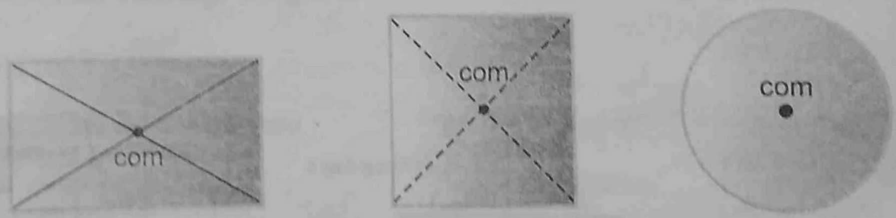
$$x_{cm} = \frac{\int x\,dm}{\int dm},\; y_{cm} = \frac{\int y\,dm}{\int dm},\; z_{cm} = \frac{\int z\,dm}{\int dm}$$

$\int dm = M$ (mass of the body)

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm.$$

Note: If an object has symmetric uniform mass distribution about x axis than y coordinate of COM is zero and vice-versa.

1. Centre of mass of a uniform rectangular, square or circular plate lies at its centre. Axis of symmetry plane of symmetry.



2. For a laminar type (2-dimensional) body with uniform negligible thickness the formulae for finding the position of centre of mass are as follows :

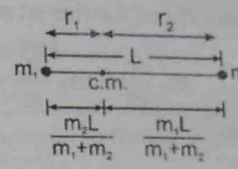
or $\quad \vec{r}_{COM} = \dfrac{A_1\vec{r}_1 + A_2\vec{r}_2 + \dots}{A_1 + A_2 + \dots}$ $\qquad$ Here, A stands for the area,

3. If some mass of area is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained from the following formulae:

$\vec{r}_{COM} = \dfrac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2}$ $\quad$ or $\quad \vec{r}_{COM} = \dfrac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$ $\quad$ (For laminar type)

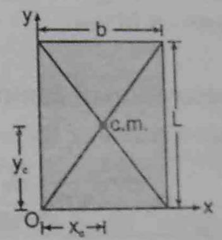
Centre of mass of some common systems :

- A system of two point masses $m_1 r_1 = m_2 r_2$
- The centre of mass lies closer to the heavier mass.



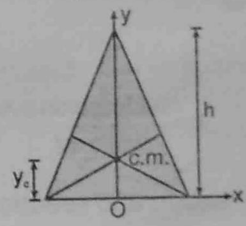
- Rectangular plate (By symmetry)

$$x_c = \frac{b}{2} \quad ; \quad y_c = \frac{L}{2}$$



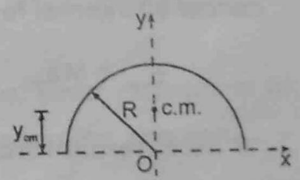
- A triangular plate (By qualitative argument)

at the centroid : $y_c = \frac{h}{3}$



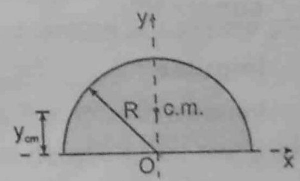
- A semi-circular ring

$$y_c = \frac{2R}{\pi} \qquad x_c = 0$$



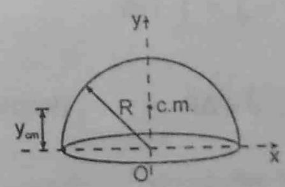
- A semi-circular disc

$$y_c = \frac{4R}{3\pi} \qquad x_c = 0$$



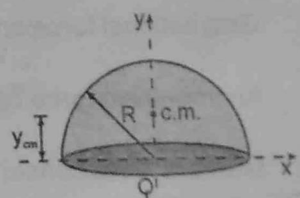
- A hemispherical shell

$$y_c = \frac{R}{2} \qquad x_c = 0$$



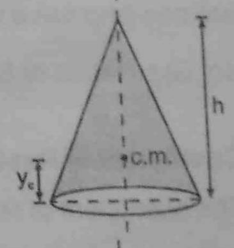
- A solid hemisphere

$$y_c = \frac{3R}{8} \qquad x_c = 0$$



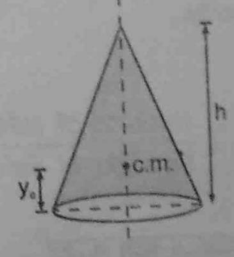
- A circular cone (solid)

$$y_c = \frac{h}{4}$$



- A circular cone (hollow)

$$y_c = \frac{h}{3}$$



Motion of Centre of Mass and Conservation of Momentum :

Velocity of centre of mass of system

$$\vec{v}_{cm} = \frac{m_1\frac{\overrightarrow{dr_1}}{dt} + m_2\frac{\overrightarrow{dr_2}}{dt} + m_3\frac{\overrightarrow{dr_3}}{dt} \ldots\ldots\ldots\ldots + m_n\frac{\overrightarrow{dr_n}}{dt}}{M} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \ldots\ldots\ldots + m_n\vec{v}_n}{M}$$

$$\vec{P}_{System} = M\vec{v}_{cm}$$

Acceleration of centre of mass of system :

$$\vec{a}_{cm} = \frac{m_1\frac{\overrightarrow{dv_1}}{dt} + m_2\frac{\overrightarrow{dv_2}}{dt} + m_3\frac{\overrightarrow{dv_3}}{dt} \ldots\ldots\ldots\ldots + m_n\frac{\overrightarrow{dv_n}}{dt}}{M} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 \ldots\ldots\ldots + m_n\vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{M} = \frac{\text{Net External Force} + \text{Net internal Force}}{M} = \frac{\text{Net External Force}}{M}$$

($\because$ action and reaction both of an internal force must be within the system. Vector summation will cancel all internal forces and hence net internal force on system is zero)

$$\vec{F}_{ext} = M\vec{a}_{cm}$$

"If no external force is acting on the system, net momentum of the system must remain constant".

Impulse :

Impulse of a force F action on a body is defined as :-

$$\vec{J} = \int_{t_i}^{t_f} F dt$$

$\vec{J} = \Delta\vec{P}$ **(impulse - momentum theorem)**

Important points :

- Gravitational force and spring force are always non-Impulsive.
- An impulsive force can only be balanced by another impulsive force.

Note: Momentum is conserved for an instantaneous process (short duration process) if the external forces are non-Impulsive in nature.

eg. Gravitation or Spring force

Coefficient of Restitution (e) :

The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r\,dt}{\int F_d\,dt} = s\frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

Note : e is independent of shape and mass of object but depends on the material. The coefficient of restitution is constant for two particular objects.

(a) $e = 1$
- Impulse of Reformation = Impulse of Deformation
- Velocity of separation = Velocity of approach
- Kinetic energy of particles after collision may be equal to that of before collision.
- Collision is elastic.

(b) $e = 0$
- Impulse of Reformation = 0
- Velocity of separation = 0
- Kinetic energy of particles after collision is not equal to that of before collision.
- Collision is perfectly inelastic .

(c) $0 < e < 1$
- Impulse of Reformation < Impulse of Deformation
- Velocity of separation < Velocity of approach
- Kinetic energy of particles after collision is not equal to that of before collision.
- Collision is Inelastic.

Important Point :

In case of elastic collision, if rough surface is present then

$k_f < k_i$ (because friction is impulsive)

☞ No component of impulse act along common tangent direction. Hence, linear momentum or linear velocity of individual particles (if mass is constant) remain unchanged along this direction.

☞ Net impulse on both the particles is zero during collision. Hence, net momentum of both the particles remain conserved before and after collision in any direction.

☞ Definition of coefficient of restitution can be applied along common normal direction, i.e., along common normal direction we can apply

Relative speed of separation = e (relative speed of approach)

☞ **Variable Mass System :**

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity $\vec{v}_{rel}$ (w.r.t. the system),

then the force exerted by this mass on the system has magnitude $\mu|\vec{v}_{rel}|$.

☞ **Thrust Force ($\vec{F}_t$) :**

$$\vec{F}_t = \vec{v}_{rel}\left(\frac{dm}{dt}\right)$$

☞ **Rocket propulsion :**

If gravity is ignored and initial velocity of the rocket u = 0;

$$v = v_r \ln\left(\frac{m_0}{m}\right).$$

AIEEE TIP :

Collision

In an elastic, one-dimensional collision between a very heavy and a very light particle, to find the velocity of the light paticle after collision, reverse the velocity of the light particle before collision, and add to it twice the velocity of the heavy particle (which remains unchanged).

Impulse :

In a force-time graph, the impulse is equal to the area under the curve.

Thus, mu = area = $\frac{1}{2}$ FT.

RIGID BODY DYNAMICS

Rigid Body :

A rigid body is defined as a system of particles in which distance between each pair of particles remains constant (with respect to time).

- **W.r.t. any point of the rigid body the angular velocity of all other points of the that rigid body is same.**

I. Pure Translational Motion :

A body is said to be in pure translational motion, if the displacement of each particle of the system is same during any time interval. During such a motion, all the particles have same displacement $(\vec{s})$, velocity $(\vec{v})$ and acceleration $(\vec{a})$ at an instant.

II. Pure Rotational Motion :

Every point of the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval. Such a motion is called pure rotation.

III. Combined Translational and Rotational Motion :

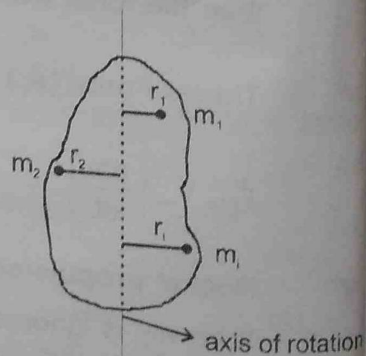
A body is said to be in combined translational and rotational motion if all points in the body rotate about an axis of rotation and the axis of rotation moves with respect to the ground. Any general motion of a rigid body can be viewed as a combined translational and rotational motion.

Moment of Inertia (I) about an axis :

(i) Moment of inetia of a system of n particles about an axis is defined as :

$I = m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots\ldots\ldots\ldots\ldots + m_n r_n^2$

i.e. $$I = \sum_{i=1}^{n} m_i r_i^2$$



where, r_i = It is perpendicular distance of mass m_i from axis of rotation

SI units of Moment of inertia is Kgm².

Moment of inertia is a scalar positive quantity.

(ii) For a continuous system :

$$I = \int r^2 (dm)$$

where dm = mass of a small element

r = perpendicular distance of the mass element dm from the axis

- **Moment of Inertia depends on :**

(i) density of the material of body
(ii) shape & size of body
(iii) axis of rotation

In totality we can say that it depends upon distribution of mass relative to axis of rotation.

- Moment of inertia does not change if the mass :
  (i) is shifted parallel to the axis of the rotation because r_i does not change.
  (ii) is rotated about axis of rotation in a circular path because r_i does not change.

(iii) **Moment of inertia of a large object can be calculated by integrating M.I.of an element of the object:**

$$I = \int dI_{element}$$ where dI = moment of inertia of a small element

- **Element chosen should be such that :** either perpendicular distance of axis from each point of the element is same or the moment of inertia of the element about the axis of rotation is known.

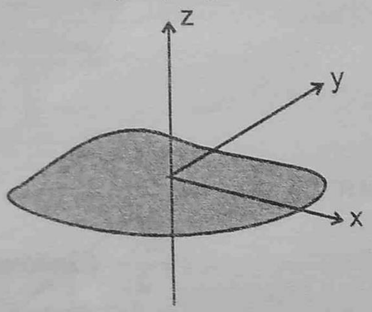
Two Important Theorems on Moment of Inertia :

(i) **Perpendicular Axis Theorem [Only applicable to plane laminar bodies (i.e. for 2-dimensional objects only)].**

If axis 1 & 2 are in the plane of the body and perpendicular to each other.

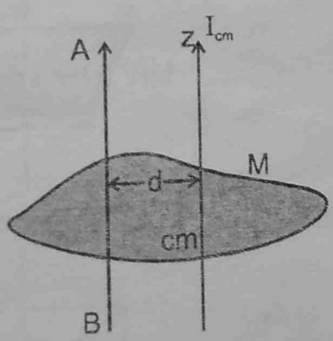
Axis 3 in perpendicular to plane of 1 & 2 .

Then, $I_3 = I_1 + I_2$



(ii) **Parallel Axis Theorem (Applicable to planer as well as 3 dimensional objects):**

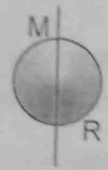
$$I_{AB} = I_{cm} + Md^2$$



☞ List of some useful formula :

Object — **Moment of Inertia**

Solid Sphere

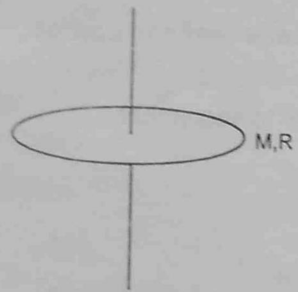


$\frac{2}{5}MR^2$ (Uniform)

Hollow Sphere

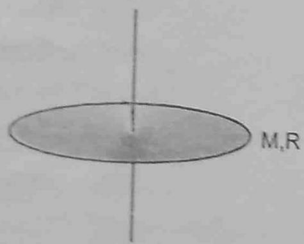
$\frac{2}{3}MR^2$ (Uniform)

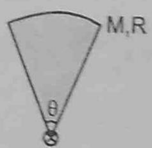
Ring.



MR^2 (Uniform or Non Uniform)

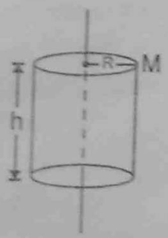
Disc

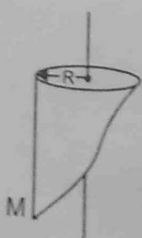




$\frac{MR^2}{2}$ (Uniform)

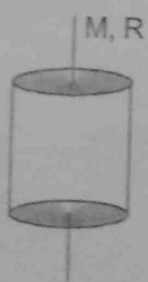
Hollow cylinder





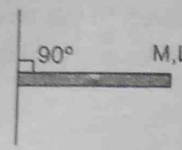
MR^2 (Uniform or Non Uniform)

Solid cylinder

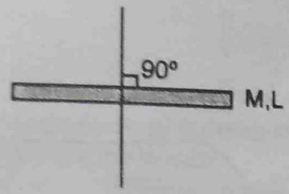


$\frac{MR^2}{2}$ (Uniform)

Thin Rod

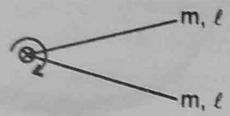


$$\frac{ML^2}{3} \text{ (Uniform)}$$



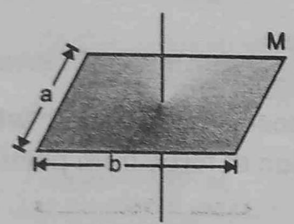
$$\frac{ML^2}{12} \text{ (Uniform)}$$

Two Thin Rods



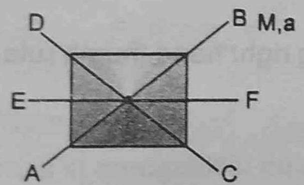
$$\frac{2m\ell^2}{3} \text{ (Uniform)}$$

Rectangular Plate



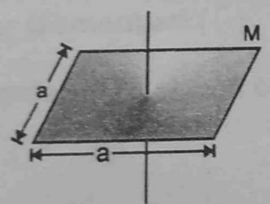
$$I = \frac{M(a^2 + b^2)}{12} \text{ (Uniform)}$$

Square Plate



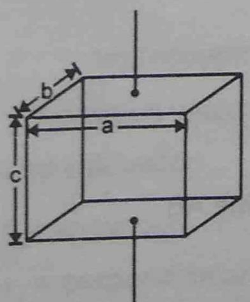
$$I_{AB} = I_{CD} = I_{EF} = \frac{Ma^2}{12} \text{ (Uniform)}$$

Square Plate



$$\frac{Ma^2}{6} \text{ (Uniform)}$$

Cuboid



$$\frac{M(a^2 + b^2)}{12} \text{ (Uniform)}$$

Radius of Gyration :

As a measure of the way in which the mass of rigid body is distributed with respect to the axis of rotation, we define a new parameter, the radius of gyration (K). It is related to the moment of inertia and total mass of the body

$$I = MK^2$$

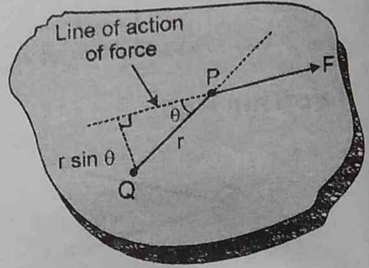
where I = Moment of Inertia of a body

M = Mass of a body

K = Radius of gyration

$$K = \sqrt{\frac{I}{M}}$$

Torque :



Torque represents the capability of a force to produce change in the rotational motion of the body.

- **Torque about a point :**

Torque of force $\vec{F}$ about a point $\vec{\tau} = \vec{r} \times \vec{F}$

Where $\vec{F}$ = force applied

P = point of application of force

Q = Point about which we want to calculate the torque.

$\vec{r}$ = position vector of the point of application of force w.r.t. the point about which we want to determine the torque.

$|\vec{\tau}| = r F \sin\theta \qquad = r_{\perp} F \qquad = rF_{\perp}$

Where θ = angle between the direction of force and the position vector of P wrt. Q.

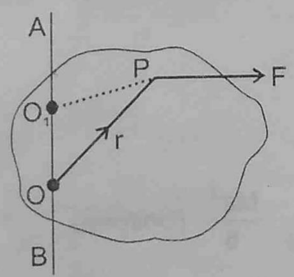
$r_{\perp} = r \sin\theta$ = perpendicular distance of line of action of force from point Q ,it is also called force arm.

$F_{\perp} = F \sin\theta$ = component of $\vec{F}$ perpendicular to $\vec{r}$

SI unit of torque is Nm

Torque is a vector quantity and its direction is determined using right hand thumb rule and it is always perpendicular to the plane of rotation of the body.

- **Torque about an axis :**



The torque of a force $\vec{F}$ about an axis AB is defined as the component of torque of $\vec{F}$ about any point O on the axis AB, along the axis AB.

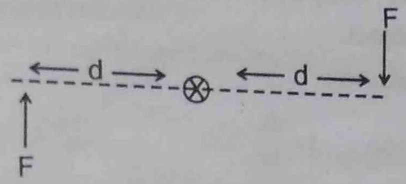
In the given figure torque of $\vec{F}$ about O is $\vec{\tau}_0 = \vec{r} \times \vec{F}$

The torque of $\vec{F}$ about AB, τ_{AB} is component of $\vec{\tau}_0$ along line AB.

There are four cases of torque of a force about an axis.:

- **Force Couple :**

  A pair of forces each of same magnitude and acting in opposite direction but having lines of action at some distance is called a force couple. Torque due to couple = Magnitude of one force × distance between their lines of action.



Magnitude of torque = $\tau = F\,(2d)$

- A couple does not exert a net force on an object even though it exerts a torque.
- Net torque due to a force couple is same about any point.

- **Point of Application of Force :**

Point of Application of force is the point at which, if net force is assumed to be acting, then it will produce same translational as well as rotational effect, as was produced earlier.

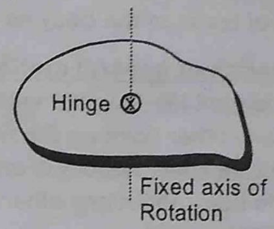
- **Rotation about a fixed axis :**

$\vec{\tau}_{ext}\big)_{Hinge} = I_{Hinge}\ \vec{\alpha}$

Rotational Kinetic Energy = $\frac{1}{2}.I.\omega^2$

$\vec{P} = M\vec{v}_{CM}$

$\vec{F}_{external} = M\vec{a}_{CM}$



Net external force acting on the body has two component tangential and centripetal.

$\Rightarrow \quad F_C = ma_c = m\frac{v^2}{r_{CM}} = m\omega^2 r_{CM} \qquad \Rightarrow \quad F_t = ma_t = m\alpha\, r_{CM}$

- **Equilibrium :**

A system is in mechanical equilibrium if it is in translational as well as rotational equilibrium.

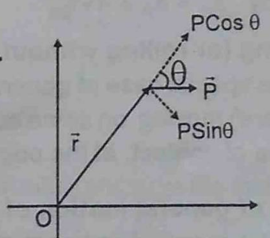
For this : $\overrightarrow{F_{net}} = 0 \qquad \overrightarrow{\tau_{net}} = 0$ (about every point)

- **Angular Momentum ($\vec{L}$)**

- **Angular momentum of a particle about a point.**

$\vec{L} = \vec{r} \times \vec{P} \qquad \Rightarrow \qquad L = rp\sin\theta$

or $|\vec{L}| = r_\perp \times P \qquad$ or $\qquad |\vec{L}| = P_\perp \times r$



Where $\vec{P}$ = linear momentum of particle

$\vec{r}$ = position vector of particle with respect to point O about which angular momentum is to be calculated .

θ = angle between vectors $\vec{r}$ & $\vec{P}$

$r_\perp$ = perpendicular distance of line of motion of particle from point O.

$P_\perp$ = component of linear momentum perpendicular to $\vec{r}$.

SI unit of angular momentum is kgm^2/sec.

- **Angular momentum of a rigid body rotating about fixed axis :**

$L_H = I_H \omega$

L_H = Angular momentum of object about axis of rotation.

I_H = Moment of Inertia of rigid , body about axis of rotation.

ω = angular velocity of the object.

- **Conservation of Angular Momentum :**

Newton's 2nd law in rotation : $\vec{\tau} = \frac{d\vec{L}}{dt}$

where $\vec{\tau}$ and $\vec{L}$ are about the same axis.

☞ Angular momentum of a particle or a system remains constant if $\tau_{ext} = 0$ about the axis of rotation.

Impulse of Torque : $\int \tau dt = \Delta J$

$\Delta J \rightarrow$ Change in angular momentum.

☞ **Combined Translational and Rotational motion of a rigid body**

The general motion of a rigid body can be thought of as a sum of two independent motions. A translation of some point of the body plus a rotation about this point . A most convenient choice of the point is the centre of mass of the body as it greatly simplifies the calculations.

<u>**Kinematics of general motion of a rigid body :**</u> For a rigid body as earlier stated value of angular displacement (θ) , angular velocity (ω), angular acceleration (α) is same for all points on the rigid body about any other point on the rigid body.

Hence if we know velocity of any one point (say A) on the rigid body and angular velocity of any point on the rigid body about any other point on the rigid body (say ω), velocity of each point on the rigid body can be calculated.

since distance AB is fixed

$\vec{V}_{BA} \perp \overrightarrow{AB}$

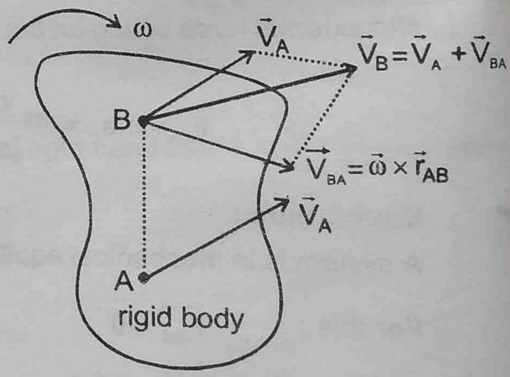
we know that $\omega = \frac{V_{BA\perp}}{r_{BA}}$

$V_{BA\perp} = V_{BA} = \omega r_{BA}$

in vector form $\vec{V}_{BA} = \vec{\omega} \times \vec{r}_{BA}$

Now from relative velocity : $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$\vec{V}_B = \vec{V}_A + \vec{V}_{BA} \Rightarrow \boldsymbol{\vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}_{BA}}$



similarly $\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{BA}$ [for any rigid system]

- **Pure Rolling (or rolling without sliding) :**

Pure rolling is a special case of general rotation of a rigid body with circular cross section (e.g. wheel, disc, ring, sphere) moving on some surface. Here, there is no relative motion between the rolling body and the surface of contact, at the point of contact

- **Dynamics of general motion of a rigid body :**

This motion can be viewed as translation of centre of mass and rotation about an axis passing through centre of mass

(i) $\vec{\tau}_{cm} = I_{cm} \vec{\alpha}$ (ii) $\vec{F}_{ext} = M\vec{a}_{cm}$

(iii) $\vec{P}_{system} = M\vec{v}_{cm}$ (vi) Total K.E.= $\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$ (v) $\vec{L}_{CM} = I_{CM}\vec{\omega}$

(vi) Angular momentum about point A = $\vec{L}$ about C.M. + $\vec{L}$ of C.M. about A

$\vec{L}_A = I_{cm}\vec{\omega} + \vec{r}_{cm} \times M\vec{v}_{cm}$

- $\frac{dL_A}{dt} = \frac{d}{dt}\left(I_{cm}\vec{\omega} + \vec{r}_{cm} \times M\vec{v}_{cm}\right) \neq I_A \frac{d\vec{\omega}}{dt}$. Notice that torque equation can be applied to a rigid body in a general motion only and only about an axis through centre of mass.

  - **Instantaneous axis of rotation :**
  It is the axis about which the combined translational and rotational motion appears as pure rotational motion.

- **Toppling :**
  In many situations an external force is applied to a body to cause it to slide along a surface. In certain cases, the body may tip over before sliding ensues. This is known as toppling.

SIMPLE HARMONIC MOTION

- **S.H.M.** If the restoring force/ torque acting on the body in oscillatory motion is directly proportional to the displacement of body/particle and is always directed towards equilibrium position then the motion is called Simple Harmonic Motion (SHM).

$$F = -kx$$

or $$m\frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad \text{[differential equation of SHM]}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \text{where } \omega = \sqrt{\frac{k}{m}}$$

It's solution is $x = A\sin(\omega t + \phi)$

General equation of S.H.M. is $x = A\sin(\omega t + \phi)$; where $(\omega t + \phi)$ is phase of the motion and ϕ is initial phase of the motion.

- **Angular Frequency (ω) :** $\omega = \frac{2\pi}{T} = 2\pi f$

- **Frequency (f) :** Number of oscillations completed in unit time interval is called frequency of oscillations, $f = \frac{1}{T} = \frac{\omega}{2\pi}$, its units is sec^{-1} or Hz.

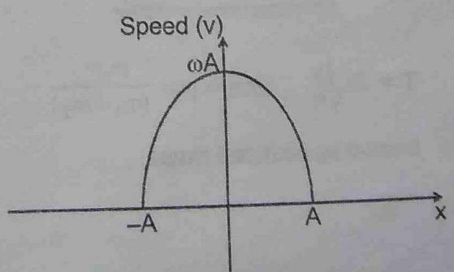
- **Time period (T) :** $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

- **Speed :** $v = \omega\sqrt{A^2 - x^2} \quad = \omega A\cos(\omega t + \phi)$, where ϕ = initial phase of the motion.

GRAPH OF SPEED (v) VS DISPLACEMENT (x):

$$v = \omega\sqrt{A^2 - x^2} \qquad v^2 = \omega^2\left(A^2 - x^2\right)$$

$$v^2 + \omega^2x^2 = \omega^2A^2 \qquad \frac{v^2}{\omega^2A^2} + \frac{x^2}{A^2} = 1$$



GRAPH WOULD BE AN ELLIPSE