### Section (D) :

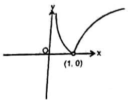
**D-1.** (i) $\left[\frac{1}{2}, 2\right) \cup (5, \infty)$ (ii) $[-1, (\sqrt{5}-1)/2)$

(iii) $x \in [3, \infty)$

(iv) $x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$

(v) $x = 2$

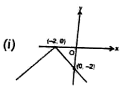
(vi)



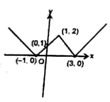

**D-2.** $x = \log_2 a$ , $a \in (0, 1]$

### Section (E) :

**E-1.** (i)




(ii)

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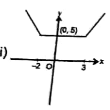


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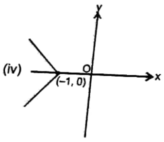
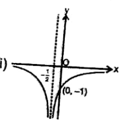
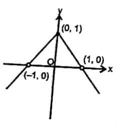


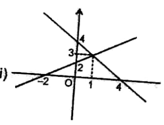

**E-2.** (i)



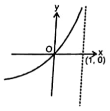

(ii)



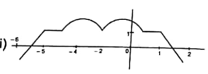

(iii)



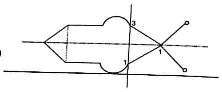

**E-3.**




**E-4.** (i)



(ii)




**E-5.** $\lambda \in (12, 16)$

### Section (F) :

**F-1.** $x = 0, 3/2$

**F-2** (i) $\{0\}$ (ii) $x \in [-1, 1)$

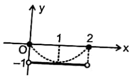
(iii) $x \in Z$ (iv) $\left[-\frac{1}{2}, \frac{1}{2}\right)$

**F-3.** (i) $x \in [-6, 1)$ (ii) $x \in [-5, 1)$

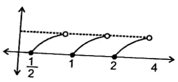
(iii) $x \in \phi$ (iv) $x \in [-1, 1)$

**F-4.** 5

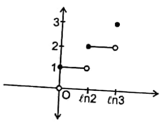
**F-5.** (i)




(ii)




(iii)




**F-6.** 3





## PART - II

### Section (A) :

**A-1.** (D) **A-2.** (B) **A-3.** (C)
**A-4.** (D) **A-5.** (C) **A-6.** (D)
**A-7.** (D) **A-8.** (D) **A-9.** (D)

### Section (B) :

**B-1.** (C) **B-2.** (B) **B-3.** (A)
**B-4.** (C) **B-5.** (A)

### Section (C) :

**C-1.** (D) **C-2.** (C) **C-3.** (B)
**C-4.** (B) **C-5.** (B)

### Section (D) :

**D-1.** (A) **D-2.** (C) **D-3.** (A)
**D-4.** (D)

### Section (E)

**E-1.** (B) **E-2.** (D) **E-3.** (A)
**E-4.** (B) **E-5.** (B)

### Section (F) :

**F-1.** (B) **F-2.** (D) **F-3.** (C)
**F-4.** (C) **F-5.** (C) **F-6.** (D)
**F-7.** (A) **F-8.** (D)

## PART - III

**1.** (A) → (r), (B) → (p), (C) → (q), (D) → (s)

**2.** (A → r,s), (B → r), (C → p), (D → p,q)

## EXERCISE - 2

## PART - I

**1.** (B) **2.** (B) **3.** (D)
**4.** (A) **5.** (A) **6.** (A)
**7.** (B) **8.** (D) **9.** (A)
**10.** (C) **11.** (D) **12.** (D)
**13.** (A) **14.** (D) **15.** (D)
**16.** (A)

## PART - II

**1.** 3 **2.** 81 **3.** x = 9
**4.** 15 **5.** 16 **6.** 1
**7.** 9 **8.** 3 **9.** 2
**10.** 1 **11.** 4 **12.** 0
**13.** 1 **14.** 0

## PART - III

**1.** (ABC) **2.** (ABC) **3.** (AB)
**4.** (BD) **5.** (AB) **6.** (D)
**7.** (ACD) **8.** (ACD) **9.** (ABC)
**10.** (ABCD) **11.** (ACD) **12.** (ABCD)
**13.** (AD) **14.** (BD) **15.** (AC)

## PART - IV

**1.** (A) **2.** (D) **3.** (B)
**4.** (C) **5.** (D)

## EXERCISE - 3

## PART - I

**2.** (A) **3.** (B) **4.** (D)
**5.** (C) **6.** $x = -1 - \sqrt{3}$ or $-4$
**7.** (A) **8.** (B)
**9.** $\{-1\} \cup [1, \infty)$
**10.** 4
**11.** (B) **12.** (D)

## PART - II

**1.** (3) **2.** (3) **3.** (2)

# Exercise-3

✍ Marked Questions may have for Revision Questions.

## PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Draw the graph of $y = |x|^{1/2}$ for $-1 \le x \le 1$.

2. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is :
(A) 4 (B) 1 (C) 3 (D) 2

3.✍ If p, q, r are any real numbers, then
(A) max (p, q) < max (p, q, r)
(B) min (p, q) = $\frac{1}{2}(p + q - |p - q|)$
(C) max (p, q) < min(p, q, r)
(D) None of these

4. Let $f(x) = |x - 1|$. Then
(A) $f(x^2) = (f(x))^2$ (B) $f(x + y) = f(x) + f(y)$ (C) $f(|x|) = |f(x)|$ (D) None of these

5. If x satisfies $|x - 1| + |x - 2| + |x - 3| \ge 6$, then
(A) $0 \le x \le 4$ (B) $x \le -2$ or $x \ge 4$ (C) $x \le 0$ or $x \ge 4$ (D) None of these

6. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$.

7. If p, q, r are positive and are in A.P., then roots of the quadratic equation $px^2 + qx + r = 0$ are real for
(A) $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$ (B) $\left|\frac{r}{p} - 7\right| < 4\sqrt{3}$ (C) all p and r (D) no p and r

8. The function $f(x) = |ax - b| + c|x|$ $\forall x \in (-\infty, \infty)$, where $a > 0, b > 0, c > 0$, assumes its minimum value only at one point if
(A) $a \ne b$ (B) $a \ne c$ (C) $b \ne c$ (D) $a = b = c$

9.✍ Find the set of all solutions of the equation $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$

10. The sum of all the real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is______.

11. If $\alpha$ & $\beta$ $(\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then
(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$ (C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

12. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min f(x) > max g(x), then the relation between b and c, is
(A) no relation (B) $0 < c < b/2$ (C) $|c| < \sqrt{2}|b|$ (D) $|c| > \sqrt{2}|b|$

## PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is :
(1) $(-\infty, \infty)$ (2) $(0, \infty)$ (3) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$

2. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where [x] denotes the greatest integer $\le x$) has no intgeral solution, then all possible values of a lie in the interval :
(1) $(-2, -1)$ (2) $(-\infty, -2) \cup (2, \infty)$ (3) $(-1, 0) \cup (0, 1)$ (4) $(1, 2)$

3. Let $\alpha$ and $\beta$ be the roots of equation $px^2 + qx + r = 0, p \ne 0$. If p, q ,r are in the A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is :
(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$ (3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$





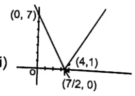
# Answers

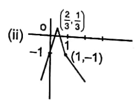
## EXERCISE - 1

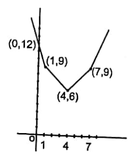
### PART - I

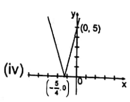
### Section (A)

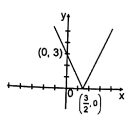
A-1. (i) $x^2 - 7x + 10$, $x > 5$ or $x \le 2$
$-(x^2 - 7x + 10)$, $2 < x \le 5$
(ii) $x^3 - x$, $x \in [-1, 0] \cup [1, \infty)$
$x - x^3$, $x \in (-\infty, -1) \cup (0, 1)$
(iii) $2^x - 2$, $x \ge 1$
$2 - 2^x$, $x < 1$
(iv) $x^2 - 6x + 10$, $x \in R$
(v) $x^2 - 2x + 1$, $x \ge 2$
$4x - x^2 - 3$, $1 \le x < 2$
$x^2 - 4x + 3$, $x < 1$
(vi) $x - 3$, $x \ge 3$
$3 - x$, $x < 3$
(vii) $2^{x-1} + x + 2 - 3^{x+1}$ $x \ge -1$
$2^{x-1} + x + 2 - 3^{-(x+1)}$ $-2 \le x < -1$
$2^{x-1} - x - 2 - 3^{-(x+1)}$ $x < -2$

A-2. (i)



(ii)



(iii)



(iv)



(v)



A-3. (i) x = 0, 8 (ii) x = –10, –6, 0, 4
(iii) x = 0, ± 4, 8

A-4. (i) ± 8 (ii) 0, 1
(iii) 0, 4 (iv) –2, 3

A-5. (i) 6 (ii) 0

A-6. (i) 2 (ii) Infinite
(iii) p < 4 no solution
p = 4 one solution
p > 4 Two solution

### Section (B) :

B-1. (i) $x \in (-\infty, 1] \cup [5, \infty)$
(ii) x = 5 or x = – 1
(iii) $x \in R - \{3\}$
(iv) $x \in [0, 6]$

B-2. (i) $x \in (-1, 0) \cup (0, 3)$
(ii) $x \in (-\infty, -4] \cup [-1, 1] \cup [4, \infty)$
(iii) $x \in (-5, -2) \cup (-1, \infty)$
(iv) $x \in \left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{1}{2}, \infty\right)$
(v) $x \in \left(-\frac{2}{3}, 4\right)$

B-3. (i) $x \in (-\infty, -1] \cup [0, \infty)$
(ii) $x \in (-\infty, 1] \cup [3, \infty)$
(iii) $x \in (-\infty, 0) \cup (1, \infty)$
(iv) $x \in (2, \infty)$
(v) (1, 5/3)
(vi) $(2, \infty)$

B-4. (i) $\{-1\} \cup [0, \infty)$ (ii) $[1, 2] \cup [3, 4]$
(iii) $x \in \left[-\frac{1}{2}, \infty\right)$

### Section (C) :

C-1. (i) $\log_{10}x + 2^{x-1} - 1$ $x \ge 1$
$-(\log_{10}x + 2^{x-1} - 1)$ $0 < x < 1$
(ii) $(\log_2 x)^2 - 3(\log_2 x) + 2$ $x \in (0, 2] \cup [4, \infty)$
$-((\log_2 x)^2 - 3(\log_2 x) + 2)$ $x \in (2, 4)$
(iii) $5^{x^2-4x+5} - 25$ $x \in (-\infty, 1] \cup [3, \infty)$
$25 - 5^{x^2-4x+5}$ $x \in (1, 3)$

C-2. x = 10/3, y = 20/3 & x = – 10, y = 20

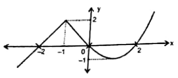
C-3. $x \in \left(0, \frac{1}{4}\right] \cup [2, \infty)$

5. If $a \neq 0$, then the inequation $|x - a| + |x + a| < b$

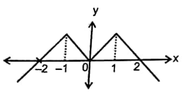
(A) has no solutions if $b \leq 2|a|$
(B) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b > 2|a|$
(C) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b < 2|a|$
(D) All above

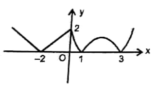
6. The equation $||x - a| - b| = c$ has four distinct real roots, then
(A) $a > b - c > 0$ (B) $c > b > 0$ (C) $a > c + b > 0$ (D) $b > c > 0$

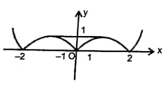
7. If graph of $y = f(x)$ is as shown in figure

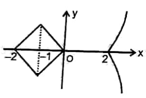



then which of the following options is/are correct ?

(A) Graph of $y = f(-|x|)$ is 


(B) Graph of $y = f(|x|)$ is 


(C) Graph of $y = |f(|x|)|$ is 


(D) Graph of $|y| = f(x)$ is 


8. Consider the equation $\left|x^2 - 4|x| + 3\right| = p$
(A) for $p = 2$ the equation has four solutions
(B) for $p = 2$ the equation has eight solutions
(C) there exists only one real value of $p$ for which the equation has odd number of solutions
(D) sum of roots of the equation is zero irrespective of value of $p$

9. Consider the equation $|\ell nx| + x = 2$, then
(A) The equation has two solutions
(B) Both solutions are positive
(C) One root exceeds one and other in less than one
(D) Both roots exceed one

10. The inequality $[2 - x] + 2[x - 1] \geq 0$ is satisfied by (where [.] denotes greatest integer function) :
(A) $x \in \{0\}$ (B) $x \in W$ (C) $x \in N$ (D) $x \in [1, \infty)$





11. Values of x satisfying $\left(\frac{1}{3}\right)^{\{x\}} > \frac{1}{\sqrt{3}}$ are (where {.} denotes the fractional part function)
(A) $\pi$ (B) $-1 + \frac{1}{\sqrt{2}}$ (C) $2 + \frac{1}{\sqrt[3]{9}}$ (D) $\frac{e}{2}$

12. Which of the following pairs of the inequations has same solution set. ([x] represents greatest integer function and {x} represents fractional part function)
(A) $[x] \leq 3$ and $[x] < 4$
(B) $[x] > 3$ and $[x] \geq 4$
(C) $[x] + [-x] \geq 0$ and $\{x\} + \{-x\} \leq 0$
(D) $\text{sgn}(x^2 + 1) > 0$ and $x^2 + 7x + 43 > 0$

13. Let $\alpha < \beta < \gamma$, be the real solutions of the equation $2e^{-\{x\}} = (x + 1)$, then. ({x} represent fractional of x).
(A) $\alpha\beta\gamma < 0$ (B) $\alpha\beta\gamma > 0$ (C) $\alpha + \beta + \gamma < 0$ (D) $\alpha + \beta + \gamma > 0$

14. Consider the equation $\text{sgn}(x^2 - 6x + p) = q$. Let 's' be the number of solutions of the equation, identify the correct assertions :
(A) $q = 0, s = 2 \Rightarrow p \leq 9$
(B) $q = 0, s = 2 \Rightarrow p < 9$
(C) $q = 0, s = 0 \Rightarrow p \geq 9$
(D) $q = 0, s = 0 \Rightarrow p > 9$

15. If P and Q are the sum and product respectively of all integral values of x satisfying the equation $|3[x] - 4x| = 4$, then (where [.] denotes represents greatest integer function)
(A) $P = 0$ (B) $P = 8$ (C) $Q = -16$ (D) $Q = -9$

## PART - IV : COMPREHENSION

**Comprehension # 1 (Q.1 to Q-3)**

Consider the equation $||x - 1| - |x + 2|| = p$. Let $p_1$ be the value of p for which the equation has exactly one solution. Also $p_2$ is the value of p for which the equation has infinite solution. Let $\alpha$ be the sum of all the integral values of p for which this equation has solution.

1. $p_1$ is equal to
(A) 0 (B) 1 (C) 2 (D) 3

2. $p_2$ is equal to
(A) 0 (B) 1 (C) 2 (D) 3

3. $\alpha$ is equal to
(A) 3 (B) 6 (C) 10 (D) 15

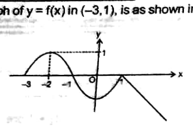
**Comprehension # 2 (Q.4 to Q-5)**

Number of solution of the equation f(x) = g(x) are same as number of point of intersection of the curves y = f(x) and y = g(x) hence answer the following questions.

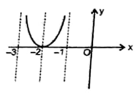
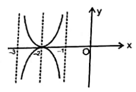
4. Number of the solution of the equation $2^x = |x - 1| + |x + 1|$ is $x = 2, 1$
(A) 0 (B) 1 (C) 2 (D) $\infty$

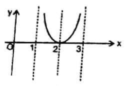
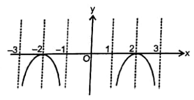
5. Number of the solution of the equation $x^2 = |x - 2| + |x + 2| - 1$ is
(A) 0 (B) 3 (C) 2 (D) 4

11. If graph of $y = f(x)$ in $(-3, 1)$, is as shown in the following figure



and $g(x) = \ell n(f(x))$, then the graph of $y = g(-|x|)$ is

(A) 
(B) 

(C) 
(D) 

12. If $x \geq 0$ and $y \geq 0$, then the area bounded by the graph of $[x] + [y] = 2$ is (where [ . ] denotes greatest integer function)
(A) 4 sq. unit. (B) 1 sq. unit (C) 2 sq. unit (D) 3 sq. unit

13. If the solution set of $[x] + \left[x + \frac{1}{2}\right] + \left[x - \frac{1}{3}\right] = 8$ is $[a, b)$, then $(a + b)$ equals to (where [ ] denotes greatest integer function)
(A) $\frac{19}{3}$ (B) $\frac{20}{3}$ (C) 6 (D) 7

14. Number of integral solutions of the inequation $x^2 - 10x + 25\text{sgn}(x^2 + 4x - 32) \leq 0$
(A) infinite (B) 6 (C) 7 (D) 8

15. If $[x + [2x]] < 3$, where [.]denotes the greatest integer function, then x is
(A) $[0, 1)$ (B) $\left(-\infty, \frac{3}{2}\right]$ (C) $(1, \infty)$ (D) $(-\infty, 1)$

16. The set of all values of x for which $\frac{\sqrt{-x^2 + 5x - 6}}{\sqrt{1 - 2\{x\}}} \geq 0$ is (where {.} denotes the fractional part function)
(A) $\left[2, \frac{5}{2}\right) \cup \{3\}$ (B) $(2, 3)$ (C) $\left(\frac{5}{2}, 3\right]$ (D) $\left[2, \frac{5}{2}\right) \cup \left(\frac{5}{2}, 3\right]$

## PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. If $|x + 2| + y = 5$ and $x - |y| = 1$ then find the value of $x + y$

2. Find the nonprime value of x satisfying the equation $|x - 1|^A = (x - 1)^7$, where $A = \log_3 x^2 - 2\log_x 9$.

3. Find the integral value of x satisfying the equation $\left|\log_{\sqrt{3}} x - 2\right| - \left|\log_3 x - 2\right| = 2$

4. Find the sum of all possible integral solutions of equation
$||x^2 - 6x + 5| - |2x^2 - 3x + 1|| = 3|x^2 - 3x + 2|$





5. The complete solution set of the inequality $(|x - 1| - 3)(|x + 2| - 5) < 0$ is $(a, b) \cup (c, d)$ then find the value of $|a| + |b| + |c| + |d|$

6. Find the absolute value of product of all the integers which do not belong to the solution set of the inequality $\left|\frac{3|x| - 2}{|x| - 1}\right| \geq 2$.

7. Let $f(x) = |x - 2|$ and $g(x) = |3 - x|$ and
A be the number of real solutions of the equation $f(x) = g(x)$
B be the minimum value of $h(x) = f(x) + g(x)$
C be the area of triangle formed by $f(x) = |x - 2|$, $g(x) = |3 - x|$ and x-axis and $\alpha < \gamma < \beta < \delta$ where $\alpha < \beta$ are the roots of $f(x) = 4$ and $\gamma < \delta$ are the roots of $g(x) = 4$, then find the value of sum of digits of $\frac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{ABC}$.

8. Find the number of integers satisfying the inequality $\sqrt{\log_{1/2}^2 x + 4\log_2\sqrt{x}} < \sqrt{2}\,(4 - \log_{16} x^4)$.

9. If $\alpha$ and $\beta$ are the values of x for which $\{x\}, [x], x$ are in harmonic progression then find the value of $\frac{1}{|\alpha\beta|}$.
(where [x] and {x} denote integral and fractional part of x resp.)

10. Find the number of solutions of the equation : $2x + 3[x] - 4\{-x\} = 4$ (where [x] and {x} denote integral and fractional part of x resp.)

11. Find the reciprocal of the value of 'x' satisfying equation $|2x - 1| = 3[x] + 2\{x\}$. (where [ . ] and { . } denote greatest integer and fractional part function respectively) :

12. Find the number of solution(s) of the equation $x^2 - 4x + [x] + 3 = 0$ (where [x] denotes integral part of x)

13. If the product $\left[x - \frac{1}{2}\right]\left[x + \frac{1}{2}\right]$ is a prime number then $x \in [x_1, x_2) \cup [x_3, x_4)$ (where [.] represents greatest integer function). The value of $|x_1x_3 - x_2x_4|$ is

14. If [ . ] denotes the greatest integer less than or equal to x and $\lceil . \rceil$ denotes the least integer greater than or equal to x, then solution set of the inequality $[x]^2 + \lceil x \rceil^2 < 4$ is an interval $[\lambda, \mu]$ then $\lambda + \mu$ is equal to

## PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. If $f(x) = |x + 1| - 2|x - 1|$ then
(A) maximum value of f(x) is 2.
(B) there are two solutions of $f(x) = 1$.
(C) there is one solution of $f(x) = 2$.
(D) there are two solutions of $f(x) = 3$.

2. The solution set of inequality $|x| < \frac{a}{x}$, $a \in R$, is
(A) $\left(-\sqrt{-a}, 0\right)$ if $a < 0$ (B) $\left(0, \sqrt{a}\right)$ if $a > 0$ (C) $\phi$ if $a = 0$ (D) $(0, a)$ if $a > 0$

3. If a and b are the solutions of equation : $\log_5\left(\log_{64}|x| - \frac{1}{2} + 25^x\right) = 2x$, then
(A) $a + b = 0$ (B) $a^2 + b^2 = 128$ (C) $ab = 64$ (D) $a - b = 8$

4. Solution set of inequality $||x| - 2| \leq 3 - |x|$ consists of :
(A) exactly four integers
(B) exactly five integers
(C) Two prime natural number
(D) One prime natural number

F-4. The number of solution(s) of the equation $[x] + 2\{-x\} = 3x$, is/are (where [ ] represents the greatest integer function and $\{x\}$ denotes the fractional part of $x$):
(A) 1 (B) 2 (C) 3 (D) 0

F-5. Number of solutions of the equation $[2x] - 3\{2x\} = 1$ is (where [ . ] and { . } denote greatest integer and fractional part function respectively)
(A) 1 (B) 2 (C) 3 (D) 0

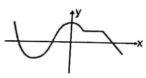
F-6. The complete solution set of the equation $\text{sgn}\left(\dfrac{x^2-5x+4}{\{x\}}\right) = -1$, where $\{\cdot\}$ is fractional part function, is $\{x\} \neq 0$
(A) (1, 4) (B) [1, 4] (C) $(-\infty, 1) \cup (4, \infty)$ (D) $(1, 2) \cup (2, 3) \cup (3, 4)$

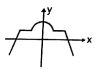
F-7. $\text{sgn}(x^3 - 4x^2 + 3x) = 1$, $x \in Z$ and $x \in [-5, 10]$, then number of possible values of $x$ is :
(A) 7 (B) 13 (C) 10 (D) 8

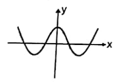
F-8. $f(x) = \begin{cases} 1 & x \in Q \\ -1 & x \in R - Q \end{cases}$. If $f(1) + f(2) + f(\pi) + f(p) = 0$, then p cannot be
(A) $-e$ (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 4

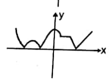
## PART - III : MATCH THE COLUMN

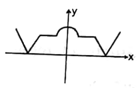
1. If $y = f(x)$ has following graph, then match the column.



(A) $y = |f(x)|$ (p)  B

(B) $y = f(|x|)$ (q)  C

(C) $y = f(-|x|)$ (r)  A

(D) $y = |f(|x|)|$ (s)  ✓ D

2. Match the following :

| Column-I | Column-II |
|---|---|
| (A) $(\log_2 x)^2 - \lvert\log_2 x\rvert - 6 = 0$, then $[x]$ can be ([x] represents greatest integer function) | (p) 2 C D |
| (B) Sum of all the solutions of the equation $2^{\lvert x\rvert} + 3^{\lvert x\rvert} + 4^{\lvert x\rvert} = 9$ is $1 - 1 = 0$ | (q) 3 D |
| (C) $p = \{x_1\} + \{x_2\} + \{x_3\}$, if p is a prime number, then p is ({x} represents fractional part function) | (r) 0 A B |
| (D) If $x^2 - \lvert x - 3\rvert - 3 = 0$, then $\lvert x\rvert$ can be | (s) 8 A |





# Exercise-2

**Marked Questions may have for Revision Questions.**

## PART - I : ONLY ONE OPTION CORRECT TYPE

1. Number of integral values of 'x' satisfying the equation $3^{|x+1|} - 2.3^x = 2.|3^x - 1| + 1$ are
(A) 1 (B) 2 (C) 3 (D) 4

2. $|x^2 + 6x + p| = x^2 + 6x + p \ \forall x \in R$ where p is a prime number then least possible value p is
(A) 7 (B) 11 (C) 5 (D) 13

3. If $(\log_{10}x)^2 - 4|\log_{10}x| + 3 = 0$, the product of roots of the equation is :
(A) 3 (B) $10^4$ (C) $10^8$ (D) 1

4. The equaiton $||x - 1| + a| = 4$ can have real solutions for x if a belongs to the interval
(A) $(-\infty, 4]$ (B) (0, 4) (C) $(4, \infty)$ (D) [4, 4]

5. The number of values of x satisfying the equation $|2x + 3| + |2x - 3| = 4x + 6$, is
(A) 1 (B) 2 (C) 3 (D) 4

6. Number of prime numbers satisfying the inequality $\log_3 \dfrac{|x^2-4x|+3}{x^2+|x-5|} \geq 0$ is equal to
(A) 1 (B) 2 (C) 3 (D) 4

7. Find the number of all the integral solutions of the inequality $\dfrac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \leq 0$
(A) 1 (B) 2 (C) 3 (D) 4

8. Find the complete solution set of the inequality $\dfrac{1-\sqrt{21-4x-x^2}}{x+1} \geq 0$
(A) $[2\sqrt{6}-2, 3]$ (B) $[-2-2\sqrt{6}, -1)$
(C) $[-2-2\sqrt{6}, -1] \cup [2\sqrt{6}-2, 3]$ (D) $[-2-2\sqrt{6}, -1) \cup [2\sqrt{6}-2, 3]$

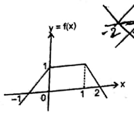
9. The solution set of the inequality $\dfrac{|x+2|-|x|}{\sqrt{4-x^3}} \geq 0$ is
(A) $[-1, \sqrt[3]{4})$ (B) $[1, \sqrt[3]{4})$ (C) $[-1, \sqrt[3]{2})$ (D) $[0, \sqrt[3]{4})$

10. If $f_1(x) = ||x| - 2|$ and $f_n(x) = |f_{n-1}(x) - 2|$ for all $n \geq 2$, $n \in N$, then number of solution of the equation $f_{2015}(x) = 2$ is
(A) 2015 (B) 2016 (C) 2017 (D) 2018

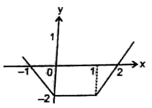
## Section (E) : Transformation of curves

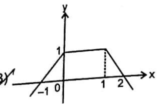
y = f(|x|)

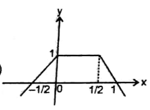
**E-1.** Given the graph of y = f(x), is

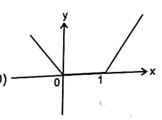


which of the following is graph of y = f(1 – x) ?

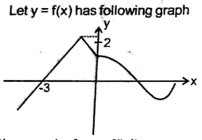
(A)


(B)


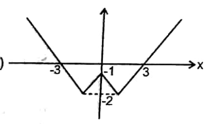
(C)


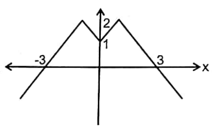
(D)


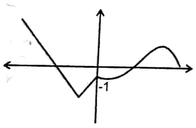
**E-2.** Let y = f(x) has following graph

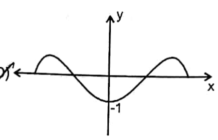


then graph of y = – f(|x|)

(A)


(B)


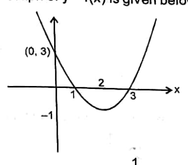
(C)


(D)


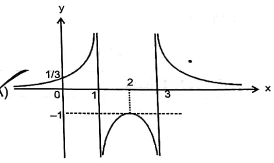


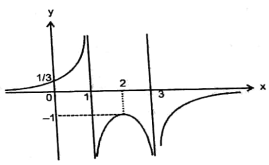


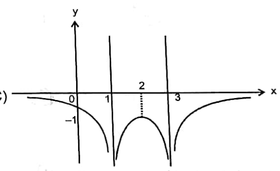
**E-3.** Graph of y = f(x) is given below :

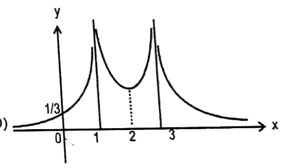


then graph of $y = \frac{1}{f(x)}$ is best represented by

(A)


(B)


(C)


(D)


**E-4.** Number of roots of equation $3^{|x|} - |2 - |x|| = 1$ is
(A) 0 (B) 2 (C) 4 (D) 7

**E-5.** Number of solutions of equation $x + 1 = x \cdot 2^x$ are
(A) 1 (B) 2 (C) 3 (D) 4

## Section (F) : Greatest Integer [.], Fractional part {.}, signum and Dirichlet's function

**F-1.** The value of $[e] - [-\pi]$ is, where [.] denotes greatest integer function
(A) 5 (B) 6 (C) 7 (D) 8

**F-2.** The number of solutions of the equation $2\{x\}^2 - 5\{x\} + 2 = 0$ is (where {.} denotes the fractional part function)
(A) no solution (B) 1 (C) 2 (D) infinite

**F-3.** Let $f(n) = \left[\frac{1}{2} + \frac{n}{100}\right]$, where [.] denotes the greatest integer function, then the value of $\sum_{n=1}^{151} f(n)$ is
(A) 101 (B) 102 (C) 104 (D) 103

$[-5, 1)$.

**F-3.** Find complete set of solution of following

(i) $-5 \le [x + 1] < 2$ $[-6, 1)$

(ii) $[x]^2 + 5[x] - 6 < 0$

(iii) $-1 < \{x\} < 0$ $\phi$

(iv) $-1 \le [x] \le 0$ $[-1, 1)$

**F-4.** Find the number of integers for which $\text{sgn}(x^2 - 2x - 8) = -1$ 5

$x^2 - 4x + 2x - 8 < 0 \Rightarrow (x+2)(x-4) < 0$

$\Rightarrow x \in (-2, 4) \to 5$

**F-5.** Draw the graph of

(i) $y = [x^2 - 2x]$, $0 \le x \le 2$

(ii) $y = \{\log_2 x\}$, $x \in [1/2, 4]$

(iii) $y = [e^x]$, $x \in (-\infty, \ell n3]$

**F-6.** $f(x) = \begin{cases} x & x \in Q \\ 1-x & x \in R - Q \end{cases}$. Find the value of $[f(1)] + |[(f(e)]|$ 3

## PART - II : ONLY ONE OPTION CORRECT TYPE

### Section (A) : Modulus Function & Equation

$t^2 - 3t + 2 = 0$

$\Rightarrow (t-2)(t-1) = 0$

$\Rightarrow t = 2$ or $1$

$\Rightarrow x = 2, -2, 1, -1$

**A-1.** The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is :
(A) 1 (B) 2 (C) 3 (D) 4

**A-2.** The minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$ is equal to
(A) 1 (B) 2 (C) 3 (D) 0

**A-3.** Solution of $|4x + 3| + |3x - 4| = 12$ is
(A) $x = -\frac{7}{3}, \frac{3}{7}$ (B) $x = -\frac{5}{2}, \frac{2}{5}$ (C) $x = -\frac{11}{7}, \frac{13}{7}$ (D) $x = -\frac{3}{7}, \frac{7}{5}$

**A-4.** If $||x - 1| - 5| = 2$, then number of distinct values of x is
(A) 1 (B) 2 (C) 3 (D) 4

**A-5.** If $|x^3 - 6x^2 + 11x - 6|$ is a prime number then number of possible integral values of x is
(A) 1 (B) 2 (C) 0 (D) 3

**A-6.** Solve for $x \in R$ $|x^2 - x - 6| = x + 2$.
(A) $x \in \{2, 4\}$ (B) $x \in \{-2, 4\}$ (C) $x \in \{-2, 2\}$ (D) $x \in \{-2, 2, 4\}$

**A-7.** If $||x - 3| - 4| = 1$ the sum of values of x is
(A) 3 (B) 6 (C) 9 (D) 12

**A-8.** Sum of the solutions of the equations $x^2 - 5|x| - 4 = 0$ is
(A) 8 (B) 2 (C) 10 (D) 0

**A-9.** Number of solutions of the equations $|2x^2 + x - 1| = |x^2 + 4x + 1|$
(A) 1 (B) 2 (C) 3 (D) 4

### Section (B) : Modulus Inequalities

**B-1.** $|x - 1| + |x + 2| \ge 3$, then complete solution set of this inequation is :
(A) $[1, \infty)$ (B) $(-\infty, -2]$ (C) R (D) $[-2, 1]$

**B-2.** The complete set of real 'x' satisfying $||x - 1| - 1| \le 1$ is :
(A) $[0, 2]$ (B) $[-1, 3]$ (C) $[-1, 1]$ (D) $[1, 3]$

**B-3.** If $|x^2 - 2x - 8| + |x^2 + x - 2| = 3|x + 2|$, then the set of all real values of x is
(A) $[1, 4] \cup \{-2\}$ (B) $[1, 4]$ (C) $[-2, 1] \cup [4, \infty)$ (D) $(-\infty, -2] \cup [1, 4]$





**B-4.** Solution set of the inequalities $|x^2 + x - 2| \le 0$ and $|x^2 - x + 2| \ge 0$ is
(A) $x \in [-2, 1]$ (B) $[-2, -1]$ (C) $\{-2, 1\}$ (D) $\{-2, -1, 1, 2\}$

**B-5.** The complete solution set of $|2x - 3| + |x + 5| \le |x - 8|$ is
(A) $\left[-5, \frac{3}{2}\right]$ (B) $(-\infty, -5]$ (C) $\left[\frac{3}{2}, \infty\right)$ (D) $(-\infty, -5] \cup \left[\frac{3}{2}, \infty\right)$

### Section (C) : Miscellaneous Modulus Equations & Inequations

**C-1.** The complete solution set of $2^x + 2^{|x|} \ge 2\sqrt{2}$ is
(A) $(-\infty, \infty)$ (B) $(-\infty, \log_2(\sqrt{2} - 1)]$
(C) $\left[\frac{1}{2}, \infty\right)$ (D) $(-\infty, \log_2(\sqrt{2}-1)] \cup \left[\frac{1}{2}, \infty\right)$

**C-2.** The complete solution set of $2|\log_3 x| + \log_3 x \ge 3$ is
(A) $\left[0, \frac{1}{3}\right) \cup [3, \infty)$ (B) $\left[\frac{1}{27}, 3\right]$ (C) $\left(0, \frac{1}{27}\right] \cup [3, \infty)$ (D) $\left(-\infty, \frac{1}{27}\right] \cup [3, \infty)$

**C-3.** If x, $|x + 1|$, $|x - 1|$ are three terms of an A.P., then number of possible values of x is – $-3/2, -1/2$
(A) 1 (B) 2 (C) 3 (D) 4

**C-4.** Number of real solution(s) of the equation $|x - 3|^{3x^2 - 10x + 3} = 1$ is : $1/3, 3, 4, 2$
(A) exactly four (B) exactly three (C) exactly two (D) exactly one

**C-5.** If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then value of $|x + y|$ is $(2x+y)(x-2y)$
(A) 2 (B) 4 (C) 6 (D) 2 or 4 or 6

### Section (D) : Irrational Inequations

**D-1.** Complete set of values of x satisfying the inequality $x - 3 < \sqrt{x^2 + 4x - 5}$ is
(A) $(-\infty, -5] \cup [1, \infty)$ (B) $(-5, 3]$ (C) $[3, 5)$ (D) $(-5, 3)$

$(2x+y)(x-2y) = 7$

**D-2.** Set of all real values of x satisfying the inequation $\sqrt{x^2 - 5x - 24} > x + 2$
(A) $(-\infty, 8]$ (B) $[8, \infty)$ (C) $(-\infty, -3]$ (D) $\left(-\infty, \frac{-28}{9}\right]$

$2x+y = 7$
$x-2y = 1$

**D-3.** Set of all real values of x satisfying the inequation $\sqrt{4 - x^2} \ge \frac{1}{x}$ is
(A) $[-2, 0) \cup [\sqrt{2-\sqrt{3}}, \sqrt{2+\sqrt{3}}]$ (B) $(-\infty, -2] \cup (\sqrt{2+\sqrt{3}}, \infty)$
(C) $(0, \sqrt{2-\sqrt{3}}] \cup [\sqrt{2+\sqrt{3}}, \infty)$ (D) $(-\infty, \sqrt{2-\sqrt{3}}] \cup [\sqrt{2+\sqrt{3}}, \infty)$

$2x+y = 1$
$x-2y = 7$

**D-4.** Set of values of x satisfying the inequality $\frac{\sqrt{x+7}}{x+1} > \sqrt{3 - x}$ is
(A) $(-2, -1) \cup (-1, 1) \cup (2, \infty)$ (B) $(-2, 1) \cup (2, 3]$
(C) $(-\infty, -2) \cup (2, 3]$ (D) $(-1, 1) \cup (2, 3]$

$2x+y = -7$
$x-2y = -1$

# Exercise-1

ఐ Marked Questions may have for Revision Questions.

## PART - I : SUBJECTIVE QUESTIONS

### Section (A) : Modulus Function & Equation

**A-1.** Write the following expression in appropriate intervals so that they are bereft of modulus sign

(i) $|x^2 - 7x + 10|$ (ii) $|x^3 - x|$ (iii) $|2^x - 2|$

(iv) $|x^2 - 6x + 10|$ (v) $|x - 1| + |x^2 - 3x + 2|$ (vi) $\sqrt{x^2 - 6x + 9}$

(vii) $2^{|x-1|} + |x + 2| - 3^{|x+1|}$

**A-2.** Draw the labled graph of following

(i) $y = |7 - 2x|$ (ii) $y = |x - 1| - |3x - 2|$

(iii) $y = |x - 1| + |x - 4| + |x - 7|$ (iv) $y = |4x + 5|$

(v) $y = |2x - 3|$

**A-3.** Solve the following equations

(i) $|x| + 2|x - 6| = 12$ (ii) $||x + 3| - 5| = 2$ (iii) $|||x - 2| - 2| - 2| = 2$

**A-4.** Solve the following equations :

(i) $x^2 - 7|x| - 8 = 0$ (ii) $|x^2 - x + 1| = |x^2 - x - 1|$

(iii) $|x^3 - 6x^2 + 11x - 6| = 6$ (iv) $|x^2 - 2x| + x = 6$

**A-5.** Find the sum of solutions of the following equations :

(i) $(x - 3)^2 + |x - 3| - 11 = 0$ (ii) $|x|^3 - 15x^2 - 8|x| - 11 = 0$

**A-6.** Find number of solutions of the following equations

(i) $|x - 1| + |x - 2| + |x - 3| = 9$ (ii) $|x - 1| + |x - 2| + |x - 3| + |x - 4| = 4$

(iii) $|x| + |x + 2| + |x - 2| = p,\ p \in R$

### Section (B) : Modulus Inequalities

**B-1.** Solve the following inequalities :

(i) $|x - 3| \ge 2$ (ii) $||x - 2| - 3| \le 0$

(iii) $||3x - 9| + 2| > 2$ (iv) $|2x - 3| - |x| \le 3$

**B-2.** Solve the following inequalities :

(i) $\left|1 + \frac{3}{x}\right| > 2$ (ii) $\left|\frac{3x}{x^2 - 4}\right| \le 1$ (iii) $\frac{|x+3| + x}{x+2} > 1$

(iv) $|x^2 + 3x| + x^2 - 2 \ge 0$ (v) $|x + 3| > |2x - 1|$

**B-3.ఐ** Solve the following inequalities

(i) $|x^3 - 1| \ge 1 - x$ (ii) $\left|x^2 - 4x + 4\right| \ge 1$ (iii) $\frac{|x+2| - x}{x} < 2$

(iv) $\frac{|x-2|}{x-2} > 0$ (v) $|x - 2| > |2x - 3|$

(vi) $|x + 2| + |x - 3| < |2x + 1|$

**B-4.** Solve the following equations

(i) $|x^3 + x^2 + x + 1| = |x^3 + 1| + |x^2 + x|$ (ii) $|x^2 - 4x + 3| + |x^2 - 6x + 8| = |2x - 5|$

(iii) $|x^2 + x + 2| - |x^2 - x + 1| = |2x + 1|$





### Section (C) : Miscellaneous Modulus Equations & Inequations

**C-1.** Write the following expression in appropriate intervals so that they are bereft of modulus sign

(i) $|\log_{10}x| + |2^{x-1} - 1|$ (ii) $|(\log_2 x)^2 - 3(\log_2 x) + 2|$ (iii) $|5^{x^2-4x+5} - 25|$

**C-2.ఐ** Solve the equations $\log_{100}|x + y| = 1/2$, $\log_{10}y - \log_{10}|x| = \log_{100}4$ for x and y. → + ve alwa

**C-3.ఐ** Solve the inequality
$(\log_2 x)^2 - |(\log_2 x) - 2| \ge 0$

### Section (D) : Irrational Inequations

**D-1.** Solve the following inequalities :

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$ (ii) $x - \sqrt{1 - |x|} < 0$ (iii) $\sqrt{x^2 - x - 6} < 2x - 3$

(iv) $\sqrt{x^2 - 6x + 8} \le \sqrt{x+1}$ (v) $\sqrt{x^2 - 7x + 10} + 9\log_4\left(\frac{x}{8}\right) \ge 2x + \sqrt{14x - 20 - 2x^2} - 13$

**D-2.** Solve the equation $\sqrt{a(2^x - 2) + 1} = 1 - 2^x$ for every value of the parameter a. $x = \log_2 a$, $a \in (0, 1]$

### Section (E) : Transformation of curves

**E-1.** Draw the graph of followings —

(i) $y = -|x + 2|$ (ii) $y = ||x - 1| - 2|$

(iii) $y = |x + 2| + |x - 3|$ (iv) $|y| + x = -1$

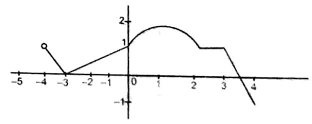
**E-2** Draw the graphs of the following curves :

(i) $y = -\frac{1}{|2x+1|}$ (ii) $\frac{y}{|x| - 1} = -1$

(iii) $|y - 3| = |x - 1|$ (iv) $y = \frac{|x^2 - 1|}{(x^2 - 1)}\ell nx$

**E-3.** Draw the graph of $y = \log_{1/2}(1 - x)$.

**E-4.** If $y = f(x)$ is shown in figure given below, then plots the graph for
(i) $y = f(|x + 2|)$ (ii) $|y - 2| = f(-3x)$.




**E-5.** Find the set of values of $\lambda$ for which the equation $|x^2 - 4|x| - 12| = \lambda$ has 6 distinct real roots. $\lambda \in (12, 16)$

### Section (F) : Greatest Integer [.], Fractional part {.}, signum and Dirichlet's function

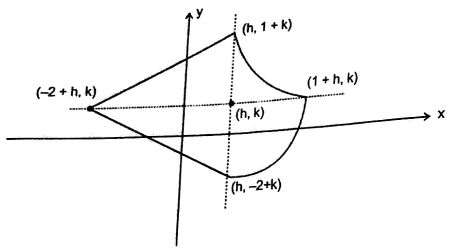
In this Section [ . ] and { . } denotes greatest integer and fractional part function respectively

**F-1.** Find the value(s) of x, if {x}, [x] & x are in A.P. :

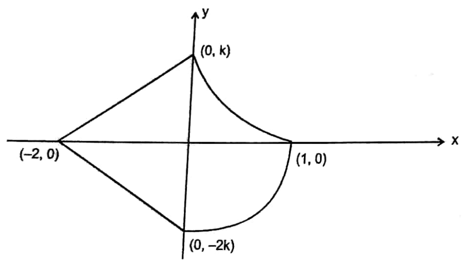
**F-2.** Solve the equation

(i) $4[x] = x + \{x\}$ (ii) $[x]^2 = -[x]$ (iii) $\{x\}^2 = -\{x\}$ (iv) $[2x] = [x]$

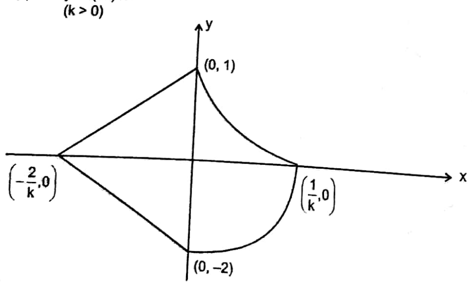
(a) $y - k = f(x - h)$ is




(b) $y = kf(x)$ is
$(k > 0)$



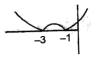

(c) $y = f(kx)$ is
$(k > 0)$



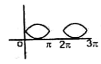




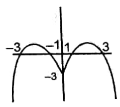

**Example # 4 :** $y = |x^2 + 4x + 3|$ **Solution :**



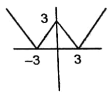

**Example # 5 :** $|y-1| = \sin x$ **Solution :**




**Example # 6 :** $y = -x^2 + 4|x| - 3$ **Solution :**



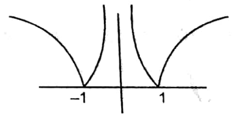

**Example # 7 :** $y = ||x| - 3|$ **Solution :**



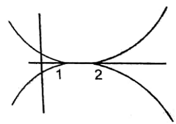

**Example # 8 :** $y = \sin\left(\frac{x}{3}\right)$ **Solution :** period is $6\pi$

**Example # 9 :** $y = |\sin x - 3|$ **Solution :** Graphical Transformation

**Example # 10 :** $y = |-\ell n|-x||$ **Solution :**




**Example # 11 :** $|y| = x^2 - 3x + 2$ **Solution :**

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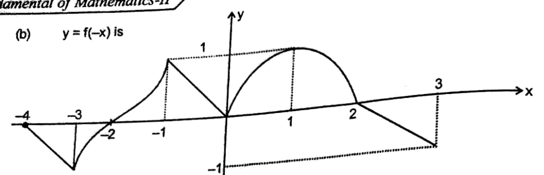
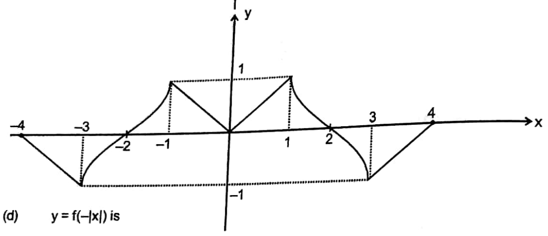
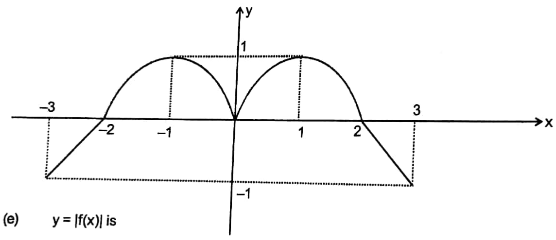
(b) y = f(−x) is



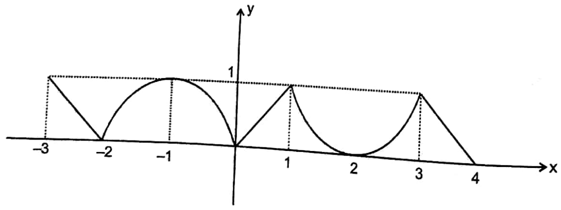

(c) y = f(|x|) is




(d) y = f(−|x|) is



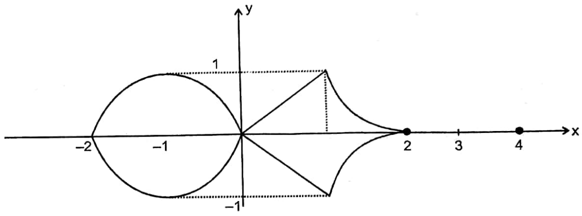

(e) y = |f(x)| is



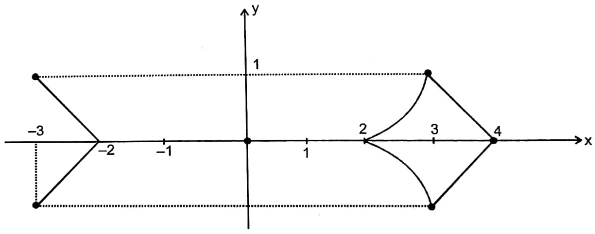





(f) |y| = f(x) is

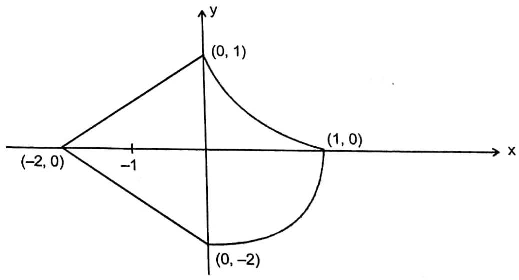



(g) |y| = − f(x)




**Graphical Trasformation :**

If graph of y = f(x) is

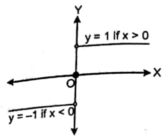



then graph of

## Signum function :

A function f (x) = sgn (x) is defined as follows :

$$f(x) = \text{sgn}(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$$




y = sgn x

It is also written as $\text{sgn}(x) = \begin{cases} \frac{|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$

**Note :** ☞ $\text{sgn}(f(x)) = \begin{cases} \frac{|f(x)|}{f(x)}; & f(x) \neq 0 \\ 0; & f(x) = 0 \end{cases}$

**Example # 2 :** Solve the equation [x] + {–x} = 2x, (where [.] and { . } represents greatest integer function and fractional part function respectively).

**Solution :** **case-I** $x \in I$

$x + 0 = 2x \Rightarrow x = 0$

**case-II** $x \notin I$

$[x] + 1 - \{x\} = 2x$

$[I + f] + 1 - \{I + f\} = 2(I + f)$

$I + 1 - f = 2I + 2f$

$\frac{1-I}{3} = f$ as $0 < f < 1$

$0 < \frac{1-I}{3} < 1$

$0 < 1 - I < 3$

$-1 < -I < 2$

$-2 < I < 1 \Rightarrow I = -1, 0$

$f = \frac{2}{3}, \frac{1}{3}$

Here $x = -\frac{1}{3}, \frac{1}{3}$ $\therefore$ Solutions are $x = 0, -\frac{1}{3}, \frac{1}{3}$

## Rational function :

A rational function is a function of the form, $y = f(x) = \frac{g(x)}{h(x)}$, where g (x) & h (x) are polynomial functions.

## Irrational function :

An irrational function is a function y = f(x) in which the operations of addition, substraction, multiplication, division and raising to a fractional power are used.

For example $y = \frac{x^3 + x^{1/3}}{2x + \sqrt{x}}$ is an irrational function

(a) The equation $\sqrt{f(x)} = g(x)$, is equivalent to the following system
$f(x) = g^2(x)$ & $g(x) \geq 0$

(b) The inequation $\sqrt{f(x)} < g(x)$, is equivalent to the following system
$f(x) < g^2(x)$ & $f(x) \geq 0$ & $g(x) \geq 0$

(c) The inequation $\sqrt{f(x)} > g(x)$, is equivalent to the following system
$g(x) \leq 0$ & $f(x) \geq 0$ or $g(x) \geq 0$ & $f(x) > g^2(x)$





**Example # 3 : Solve :** $x + 2 > 2\sqrt{1 - x^2}$

**Solution :** $4(1 - x^2) < (x + 2)^2$ and $x + 2 \geq 0$ & $1 - x^2 \geq 0$

$x \in \left(-\infty, \frac{-4}{5}\right) \cup (0, \infty)$ ...(1)

$x \in [-2, \infty)$ ...(2)

$x \in [-1, 1]$ ...(3)

$(1) \cap (2) \cap (3)$

$\left[-1, -\frac{4}{5}\right) \cup (0, 1]$

**Self Practice Problem :**

(1) $\sqrt{2x^2 + x - 6} < x$

(2) $\sqrt{5 - x} > x + 1$

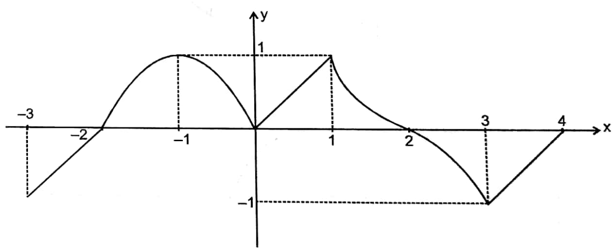
(3) $x + 3 + \sqrt{x^2 + 4x - 5} > 0$

(4) $\sqrt{x} - \sqrt{4 - x} \geq 1$

**Ans.** (1) $\left[\frac{3}{2}, 2\right)$ (2) $(-\infty, 1)$ (3) $(-\infty, -1] \cup [5, \infty)$ (4) $\left[\frac{4 + \sqrt{7}}{2}, 4\right]$

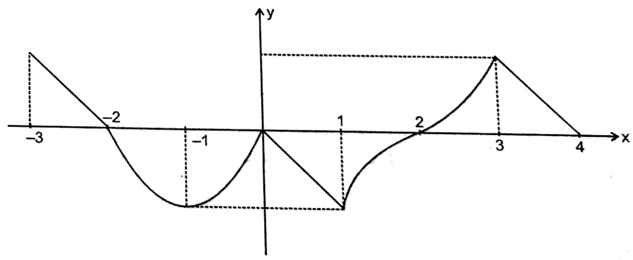
## Graphs Related to modulus :

If graph of y = f(x) is




then graph of

(a) y = – f(x) is



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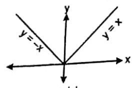
# Fundamentals of Mathematics-II

He is unworthy of the name of man who is ignorant of the fact that the diagonal of square is incommensurable with its side .......Plato

## Absolute value function / modulus function :

The symbol of modulus function is f (x) = |x| and is defined as: $y = |x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$.



y = |x|

**Properties of modulus :** For any a, b ∈ R

(i) $|a| \ge 0$ (ii) $|a| = |-a|$

(iii) $|a| \ge a$, $|a| \ge -a$ (iv) $|ab| = |a|\,|b|$

(v) $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$

(vi) $|a + b| \le |a| + |b|$ ; Equality holds when $ab \ge 0$

(vii) $|a - b| \ge ||a| - |b||$ ; Equality holds when $ab \ge 0$

**Example # 1 :** Solve the following linear equations

(i) $x|x| = 4$

(ii) $|x - 3| + 2|x + 1| = 4$

**Solution :** (i) $x|x| = 4$

If $x > 0$

$\therefore$ $x^2 = 4 \Rightarrow x = \pm 2$

$\therefore$ $x = 2$ $(\because x \ge 0)$

If $x < 0 \Rightarrow -x^2 = 4$

$\Rightarrow$ $x^2 = -4$ which is not possible

(ii) $|x - 3| + 2|x + 1| = 4$

<u>case I</u> : If $x \le -1$

$\therefore$ $-(x - 3) - 2(x + 1) = 4$

$\Rightarrow$ $-x + 3 - 2x - 2 = 4 \Rightarrow -3x + 1 = 4$

$\Rightarrow$ $-3x = 3 \Rightarrow x = -1$

<u>case II</u> : If $-1 < x \le 3$

$\therefore$ $-(x - 3) + 2(x + 1) = 4$

$\Rightarrow$ $-x + 3 + 2x + 2 = 4$

$\Rightarrow$ $x = -1$ which is not possible

<u>case III</u> : If $x > 3$

$x - 3 + 2(x + 1) = 4$

$3x - 1 = 4$

$\Rightarrow$ $x = 5/3$ which is not possible

$\therefore$ $x = -1$ **Ans.**





## Greatest integer function or step up function :

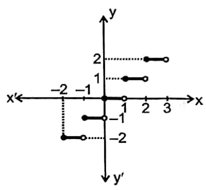
The function y = f (x) = [x] is called the greatest integer function where [x] equals to the greatest integer less than or equal to x. For example :

[3.2] = 3; [– 3.2] = – 4

for $-1 \le x < 0$ ; [x] = – 1 ; for $0 \le x < 1$ ; [x] = 0

for $1 \le x < 2$ ; [x] = 1 ; for $2 \le x < 3$ ; [x] = 2 and so on.

## Graph of greatest integer function :




## Properties of greatest integer function :

(a) $x - 1 < [x] \le x$

(b) $[x \pm m] = [x] \pm m$ iff m is an integer.

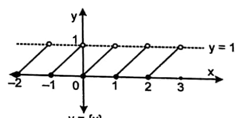
(c) $[x] + [y] \le [x + y] \le [x] + [y] + 1$

(d) $[x] + [-x] = \begin{cases} 0; & \text{if x is an integer} \\ -1 & \text{otherwise} \end{cases}$

**Note :** $[mx] \ne m[x]$

## Fractional part function:

It is defined as y = {x} = x – [x]. It is always non-negative and varies from [0, 1). The period of this function is 1 and graph of this function is as shown.



y = {x}

For example {2.1} = 2.1 – [2.1] = 2.1 – 2 = 0.1

{– 3.7} = – 3.7 – [– 3.7] = – 3.7 + 4 = 0.3

## Properties of fractional part function :

(a) $\{x \pm m\} = \{x\}$ iff m is an integer

(b) $\{x\} + \{-x\} = \begin{cases} 0 & , \text{ if x is an integer} \\ 1 & , \text{ otherwise} \end{cases}$

**Note:** $\{mx\} \ne m\,\{x\}$

**Section (B) :**

| | | | | | |
|---|---|---|---|---|---|
| B-1_. | (B) | B-2_. | (D) | B-3^. | (C) |
| B-4. | (D) | B-5. | (B) | B-6^. | (D) |
| B-7. | (D) | | | | |

**Section (C) :**

| | | | | | |
|---|---|---|---|---|---|
| C-1. | (A) | C-2. | (D) | C-3. | (B) |
| C-4. | (D) | C-5. | (D) | C-6. | (D) |
| C-7. | (D) | C-8. | (A) | C-9. | (A) |
| C-10. | (D) | C-11. | (D) | C-12_. | (C) |
| C-13_. | (B) | C-14. | (A) | C-15. | (D) |
| C-16. | (B) | C-17. | (C) | C-18. | (B) |

**Section (D) :**

| | | | | | |
|---|---|---|---|---|---|
| D-1. | (C) | D-2. | (A) | D-3. | (B) |
| D-4. | (A) | D-5. | (D) | D-6. | (D) |

**Section (E) :**

| | | | | | |
|---|---|---|---|---|---|
| E-1. | (B) | E-2. | (D) | E-3. | (B) |
| E-4. | (A) | | | | |

## PART - III

1. (A)$\to$r, (B)$\to$s, (C)$\to$p, (D)$\to$p, (E)$\to$q, (F)$\to$q
2. (A)$\to$r, (B)$\to$p, (C)$\to$s, (D)$\to$q
3. (A)$\to$r, (B)$\to$s, (C)$\to$p, (D)$\to$q

# EXERCISE - 2

## PART - I

| | | | | | |
|---|---|---|---|---|---|
| 1_. | (C) | 2. | (C) | 3. | (C) |
| 4. | (B) | 5. | (B) | 6. | (C) |
| 7. | (A) | 8. | (B) | 9. | (A) |
| 10. | (A) | 11. | (D) | 12. | (B) |
| 13. | (A) | | | | |

## PART - II

| | | | | | |
|---|---|---|---|---|---|
| 1. | 0 | 2. | 0 | 3. | 11 |
| 4. | 29 | 5. | 0 | 6. | 1 |
| 7. | 34 | 8. | 2 | 9. | 1 |
| 10. | 10 | 11. | 16 | 12. | x = 3 |
| 13. | 10 | 14. | 4 | 15. | 93 |
| 16. | 1 | 17. | 10 | 18. | 1 |
| 19. | 6 | 20. | (a) 12 (b) 47 | | |

## PART - III

| | | | | | |
|---|---|---|---|---|---|
| 1. | (ABC) | 2. | (BD) | 3. | (ABCD) |
| 4. | (ABC) | 5. | (ABCD) | 6. | (ABCD) |
| 7. | (AB) | 8. | (CD) | 9. | (ABCD) |
| 10. | (BC) | 11. | (ABD) | | |

## PART - IV

| | | | | | |
|---|---|---|---|---|---|
| 1. | (C) | 2. | (A) | 3. | (B) |
| 4. | (D) | 5. | (C) | 6. | (C) |
| 7. | (A) | 8. | (B) | | |

# EXERCISE - 3

## PART - I

1. (ABCD) 2. (C) 3. (A)
4. x = 8 5. x = 3 or – 3
6. (B)
7. (A) $\to$ (p), (r), (s) ;(B) $\to$ (q), (s);(C)$\to$(q), (s) ; (D) $\to$ (p), (r), (s)
8. (C)
9. 4.
10*. (A, B, C)

## PART - II

| | | | | | |
|---|---|---|---|---|---|
| 1. | (3) | 2. | (4) | 3. | (1) |
| 4. | (3) | 5. | (2) | 6. | (3) |
| 7. | (3) | 8. | (3) | 9. | (3) |
| 10. | (3) | 11. | (4) | 12. | (1) |
| 13. | (4) | | | | |

## PART - II : PREVIOUS YEARS PROBLEMS OF MAINS LEVEL

1. $\sqrt{5+\sqrt{5+\sqrt{5}}}$ + .......∞ is equal to [KCET-1996]

(1) 5 (2) $5+\sqrt{5}$ (3) $\frac{1+\sqrt{21}}{2}$ (4) $\frac{\sqrt{5}-1}{2}$

2. If $\log_p x = \alpha$ and $\log_q x = \beta$, then the value of $\log_{p/q} x$ is [KCET-1997]

(1) $\frac{\alpha-\beta}{\alpha\beta}$ (2) $\frac{\beta-\alpha}{\alpha\beta}$ (3) $\frac{\alpha\beta}{\alpha-\beta}$ (4) $\frac{\alpha\beta}{\beta-\alpha}$

3. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in G.P. Then x is equal to [KCET-1998]

(1) $\log_a(\log_b a)$ (2) $\log_a(\log_e a) + \log_a \log_b b$ (3) $-\log_a(\log_a b)$ (4) none of these

4. If $\log_x 256 = 8/5$, then x is equal to [KCET-2000]

(1) 64 (2) 16 (3) 32 (4) 8

5. If log 2, $\log(2^x - 1)$ and $\log(2^x + 3)$ are in A.P., then x is equal to [KCET-2000]

(1) 5/2 (2) $\log_2 5$ (3) $\log_2 3$ (4) $\log_3 2$

6. The rational number which is equal to the number $2.\overline{357}$ with recurring decimal is [DCE-2000]

(1) $\frac{2335}{1001}$ (2) $\frac{2370}{997}$ (3) $\frac{2355}{999}$ (4) none of these

7. The number $\log_2 7$ is [DCE-2000]

(1) an integer (2) a rational (3) an irrational (4) a prime number

8. The roots of the equation $\log_2(x^2 - 4x + 5) = (x - 2)$ are [KCET-2001]

(1) 4, 5 (2) 2, – 3 (3) 2, 3 (4) 3, 5

9. The number of 3.14150 rounded to 3 decimals is [DCE-2001]

(1) 3.14 (2) 3.141 (3) 3.142 (4) none of these

10. If x = 198 !, then value of the expression $\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+....+\frac{1}{\log_{198} x}$ equals [DCE-2005]

(1) –1 (2) 0 (3) 1 (4) 198

11. If $x^{2/3} - 7x^{1/3} + 10 = 0$ then, the value of x is [DCE-2006]

(1) {125} (2) {8} (3) $\phi$ (4) {125, 8}

12. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the intervel [DCE-2006]

(1) (2, ∞) (2) (1, 2) (3) (–2, –1) (4) none of these

13. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is [JEE(Main) 2016, (4, – 1), 120]

(1) – 4 (2) 6 (3) 5 (4) 3





# Answers

## EXERCISE - 1

### PART - I

**Section (A) :**

A-1 x = –1, y = –2, z = 1
A-2. 4 A-4. 84 A-5. 3
A-6. 47 A-7. 3 A-8. T
A-9. 10 A-10. 3x

**Section (B) :**

B-1. (i) $(-\infty,-2)\cup(-2,-1/2)\cup(1,\infty)$
(ii) (–17/25, –3/8)
(iii) $(-\infty,-1)\cup(5,\infty)$
(iv) $(-\infty,-20)\cup(23,\infty)$
(v) $\left(\frac{1}{2},3\right)$

B-2. (i) $[-\sqrt{2},-1)\cup(-1,\sqrt{2}]\cup[3,4)$
(ii) $(-\infty,-2]\cup(-1,4)$
(iii) $(-\infty,-5)\cup(1,2)\cup(6,\infty)$
(iv) $(-\infty,-7)\cup(-4,-2)$

B-3. (i) 4 (ii) 7 (iii) 9
(iv) 2 (v) 0

B-4. (i) 1 (ii) 7

**Section (C) :**

C-1. (i) 1 (ii) –72

C-2. (i) +ve (ii) –ve
(iii) +ve (iv) +ve
(v) +ve (vi) –ve
(vii) +ve (viii) –ve
(ix) –ve

C-3. (i) b – 2a (ii) a + 3b
(iii) $\frac{2b^2+3a^2}{ab}$ (iv) $\frac{4(2a+b)}{1-a+2b}$

C-4. (i) 2 (ii) 1
(iii) $7+\frac{1}{196}$ (iv) 0

C-5. (i) 1 (ii) 89

C-7. –1

C-9. (i) 3 (ii) ± 2
(iii) 3 (iv) 2
(v) 16 (vi) 8
(vii) {1/3} (viii) {– 4}
(ix) no root

C-10. (i) (2) (ii) 10 or $\frac{1}{100}$
(iii) $\{10^{-5}, 10^3\}$ (iv) 9
(v) $\log_2 6$

**Section (D) :**

D-1. (i) $\left[-\frac{1}{2},-\frac{1}{4}\right)\cup\left(\frac{3}{4},1\right]$
(ii) $(1,2)\cup(3,4)$ (iii) $\left(-\infty,\frac{1}{2}\right)$
(iv) $(-1,0)\cup(1,2)$ (v) $(-\infty,2)$
(vi) $\left(\frac{3}{4},1\right)\cup(1,3]$

D-2. 1

D-3. 2

D-4. (i) $(-\infty,-1)\cup(1,\infty)$
(ii) $x \in (-2,-1)\cup(-1,0)\cup(0,1)\cup(2,\infty)$

**Section (E) :**

E-1. (i) –3
(ii) 0
(iii) $a^3 + b^3 + c^3 - 3abc$
(iv) $-x^6 + 2x^3 - 1$

### PART - II

**Section (A) :**

A-1. (A) A-2. (A) A-3. (C)
A-4. (B) A-5. (B) A-6. (A)
A-7. (C) A-8. (B) A-9. (B)
A-10. (A)

**Comprehension # 2 (Q.4 to 5)**

Let $x = \sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$

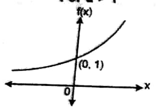
4. $x^3 + 3x$ is equal to
(A) 1 (B) 2 (C) 3 (D) 4

5. x is not equal to
(A) a rational number (B) an integer
(C) a composite number (D) a natural number

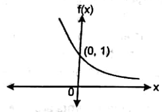
**Comprehension # 3 (Q.6 to 8)**

A function $f(x) = a^x (a > 0, a \neq 1, x \in R)$ is called an exponential function. Graph of exponential function can be as follows :

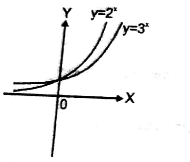
**Case - I**
**For a > 1**

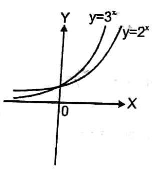


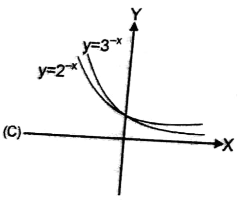

**Case - II**
**For 0 < a < 1**

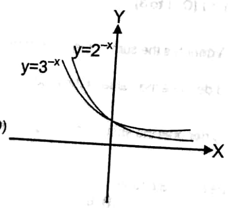



6*. Which of the following is correct :

(A)



(B)



(C)



(D)



7. Number of solutions of $3^x + x - 2 = 0$ is/are :
(A) 1 (B) 2 (C) 3 (D) 4

8.✍ The number of positive solutions of $\log_{1/2} x = 7^x$ is/are :
(A) 0 (B) 1 (C) 2 (D) 3





# Exercise-3

✍ Marked Questions may have for Revision Questions.
* Marked Questions may have more than one correct option.

## PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Indicate all correct alternatives, where base of the log is 2.
The equation $x^{(3/4)(\log x)^2 + \log x - (5/4)} = \sqrt{2}$ has : **[ JEE '89 ]**
(A) at least one real solution (B) exactly three real solutions
(C) exactly one irrational solution (D) complex roots

2. The number $\log_2 7$ is :
(A) an integer (B) a rational number (C) an irrational number (D) a prime number
**[ JEE '90 ]**

3. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has **[IIT-JEE-1997, Section-I, 2/100]**
(A) no solution (B) one solution (C) two solutions (D) more than two solutions

4.✍ Find all real numbers x which satisfy the equation
$2 \log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}\, x) = 1.$ **[REE - 1999, 6 ]**

5.✍ Solve the equation $\log_{3/4} \log_8 (x^2+7) + \log_{1/2} \log_{1/4} (x^2+7)^{-1} = -2.$ **[REE– 2000, 5]**

6. The number of solution(s) of $\log_4(x-1) = \log_2(x-3)$ is/are **[IIT-JEE-2002, Scr., (1, 0)/35]**
(A) 3 (B) 1 (C) 2 (D) 0

7. Let $f(x) = \dfrac{x^2-6x+5}{x^2-5x+6}$ **[IIT-JEE 2007, Paper-2, (6, 0), 81]**

| | Column – I | | Column – II |
|---|---|---|---|
| (A) | If $-1 < x < 1$, then f(x) satisfies | (p) | $0 < f(x) < 1$ |
| (B) | If $1 < x < 2$, then f(x) satisfies | (q) | $f(x) < 0$ |
| (C) | If $3 < x < 5$, then f(x) satisfies | (r) | $f(x) > 0$ |
| (D) | If $x > 5$, then f(x) satisfies | (s) | $f(x) < 1$ |

8.✍ Let $(x_0, y_0)$ be the solution of the following equations **[IIT-JEE 2011, Paper-1, (3, –1), 80]**
$(2x)^{\ell n 2} = (3y)^{\ell n 3}$
$3^{\ell n x} = 2^{\ell n y}$.
Then $x_0$ is
(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

9. The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots}}}\right)$ is **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

10*. If $3^x = 4^{x-1}$, then x = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**
(A) $\dfrac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\dfrac{2}{2-\log_2 3}$ (C) $\dfrac{1}{1-\log_4 3}$ (D) $\dfrac{2\log_2 3}{2\log_2 3 - 1}$

14. If $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$ then find the value of $64x^2$

15. Let a, b, c, d are positive integers such that $\log_a b = \frac{3}{2}$ and $\log_c d = \frac{5}{4}$. If $(a - c) = 9$, find the value of $(b - d)$.

16. If the product of all solutions of the equation $\frac{(2009)x}{2010} = (2009)^{\log_x (2010)}$ can be expressed in the lowest form as $\frac{m}{n}$ then the value of $(m - n)$ is

17. If the complete solution set of the inequality $(\log_{10} x)^2 \geq \log_{10} x + 2$ is $(0, a] \cup [b, \infty)$ then find the value of ab.

18. The complete solution set of the inequality $\frac{1}{\log_4 \frac{x+1}{x+2}} < \frac{1}{\log_4 (x+3)}$, is $(-a, \infty)$, then determine 'a'.

19. Find the number of integers which do not satisfy the inequality $\log_{1/2} (x + 5)^2 > \log_{1/2} (3x - 1)^2$.

20. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then find :
(a) the number of digits in $6^{15}$
(b) the number of zeros immediately after the decimal in $3^{-100}$

## PART- III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. If $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$, then $\frac{(a^k + b^k + c^k)^{\frac{1}{k}}}{(d^k + e^k + f^k)^{\frac{1}{k}}}$ is equal to : $(k \in N)$

(A) $\frac{a}{d}$ (B) $\frac{b}{e}$ (C) $\frac{c}{f}$ (D) None of these

2. Let $a > 2$, $a \in N$ be a constant. If there are just 18 positive integers satisfying the inequality $(x - a)(x - 2a)(x - a^2) < 0$ then which of the option(s) is/are correct?
(A) 'a' is composite
(B) 'a' is odd
(C) 'a' is greater than 8
(D) 'a' lies in the interval (3, 11)

3. Let $N = \frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is :
(A) a natural number (B) a prime number (C) a rational number (D) an integer

4. Values of x satisfying the equation $\log_5^2 x + \log_{5x}\left(\frac{5}{x}\right) = 1$ are

(A) 1 (B) 5 (C) $\frac{1}{25}$ (D) 3

5. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has :
(A) one irrational solution
(B) no prime solution
(C) two real solutions
(D) one integral solution





6. The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has
(A) exactly three real solution
(B) at least one real solution
(C) exactly one irrational solution
(D) complex roots.

7. The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is :
(A) {6, 3} (B) {3, 6} (C) {6, 12} (D) {12, 6}

8. Consider the quadratic equation, $(\log_{10} 8)x^2 - (\log_{10} 5)x = 2(\log_2 10)^{-1} - x$. Which of the following quantities are irrational.
(A) sum of the roots
(B) product of the roots
(C) sum of the coefficients
(D) discriminant

9. If $\log_a x = b$ for permissible values of a and x then identify the statement(s) which can be correct?
(A) If a and b are two irrational numbers then x can be rational.
(B) If a rational and b irrational then x can be rational.
(C) If a irrational and b rational then x can be rational.
(D) If a rational and b rational then x can be rational.

10. Which of the following statements are true
(A) $\log_2 3 < \log_{12} 10$
(B) $\log_6 5 < \log_7 8$
(C) $\log_3 26 < \log_2 9$
(D) $\log_{16} 15 > \log_{10} 11 > \log_7 6$

11. If $\frac{1}{2} \leq \log_{0.1} x \leq 2$, then
(A) maximum value of x is $\frac{1}{\sqrt{10}}$
(B) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$
(C) minimum value of x is $\frac{1}{10}$
(D) minimum value of x is $\frac{1}{100}$

## PART - IV : COMPREHENSION

**Comprehension # 1 (Q. 1 to 3)**

Let A denotes the sum of the roots of the equation $\frac{1}{5 - 4\log_4 x} + \frac{4}{1 + \log_4 x} = 3$.

B denotes the value of the product of m and n, if $2^m = 3$ and $3^n = 4$.

C denotes the sum of the integral roots of the equation $\log_{3x}\left(\frac{3}{x}\right) + (\log_3 x)^2 = 1$.

1. The value of A + B equals
(A) 10 (B) 6 (C) 8 (D) 4

2. The value of B + C equals
(A) 6 (B) 2 (C) 4 (D) 8

3. The value of A + C ÷ B equals
(A) 5 (B) 8 (C) 7 (D) 4

# Exercise-2

✍ Marked Questions may have for Revision Questions.
* Marked Questions may have more than one correct option.

## PART - I : ONLY ONE OPTION CORRECT TYPE

1.✍ If P(x) is polynomial of degree 4 with leading coefficient as three such that $P(1) = 2$, $P(2) = 8$, $P(3) = 18$, $P(4) = 32$, then the value of $P(5)$ is :
(A) 120 (B) 121 (C) 122 (D) 123

2. If $(a + b + c)^3 = a^3 + b^3 + c^3$ then $(a + b)(b + c)(c + a)$ is equal to :
(A) 3 (B) 1 (C) 0 (D) –1

3. If $\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} \le 4$, then the least and the highest values of $4x^2$ are:
(A) 36 & 81 (B) 9 & 81 (C) 0 & 81 (D) 9 & 36

4.✍ If $\frac{4\alpha}{\alpha^2+1} \ge 1$ and $\alpha + \frac{1}{\alpha}$ is an odd integer then number of possible values of $\alpha$ is
(A) 1 (B) 2 (C) 3 (D) 4

5. If $\log_a b = 2$; $\log_b c = 2$ and $\log_3 c = 3 + \log_3 a$ then $(a + b + c)$ equals
(A) 90 (B) 93 (C) 102 (D) 243

6. The sum of the solutions of the equation $9^x - 6 \cdot 3^x + 8 = 0$ is
(A) $\log_3 2$ (B) $\log_3 6$ (C) $\log_3 8$ (D) $\log_3 4$

7.✍ The expression: $\dfrac{\left(\dfrac{x^2+3x+2}{x+2}\right)+3x-\dfrac{x(x^3+1)}{(x+1)(x^2-x+1)}\log_2 8}{(x-1)(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 2)}$ reduces to
(A) $\frac{x+1}{x-1}$ (B) $\frac{x^2+3x+2}{(\log_2 5)x-1}$ (C) $\frac{3x}{x-1}$ (D) x

8.✍ If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$. The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals
(A) 489 (B) 469 (C) 464 (D) 400

9. Consider the statement : $x(\alpha - x) < y(\alpha - y)$ for all x, y with $0 < x < y < 1$. The statement is true
(A) if and only if $\alpha \ge 2$
(B) if and only if $\alpha > 2$
(C) if and only if $\alpha < -1$
(D) for no values of $\alpha$

10. The set of values of x satisfying simultaneously the inequalities $\dfrac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\frac{10}{7}(\log_2 5 - 1)\right)} \ge 0$ and $2^{x-3} - 31 > 0$ is :
(A) a unit set
(B) an empty set
(C) an infinite set
(D) a set consisting of exactly two elements.





11.✍ The solution set of the inequality $\frac{(3^x - 4^x)\cdot \ell n(x+2)}{x^2 - 3x - 4} \le 0$ is
(A) $(-\infty, 0] \cup (4, \infty)$ (B) $(-2, 0] \cup (4, \infty)$ (C) $(-1, 0] \cup (4, \infty)$ (D) $(-2, -1) \cup (-1, 0] \cup (4, \infty)$

12.✍ Number of integers for which $f(x) = \sqrt{\frac{1}{\log_{(3x-2)}(2x+3)} - \log_{(2x+3)}(x^2 - x + 1)}$ is defined is equal to-
(A) 1 (B) 2 (C) 3 (D) 4

13. If $\sqrt{\log_4\{\log_3\{\log_2(x^2 - 2x + a)\}\}}$ is defined $\forall x \in R$, then the set of values of 'a' is
(A) $[9, \infty)$ (B) $[10, \infty)$ (C) $[15, \infty)$ (D) $[2, \infty)$

## PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Find the sum of all the real solution(s) to the equation $(x + y)^2 = (x + 1)(y - 1)$.

2.✍ If $x^3 + y^3 + 1 = 3xy$, where $x \ne y$ determine the value of $x + y + 1$.

3. $\dfrac{\sqrt{31+\sqrt{31+\sqrt{31+....}}}}{\sqrt{1+\sqrt{1+\sqrt{1+.....}}}} = a - \sqrt{b}$ where $a, b \in N$, then find the value of $a + b$.

4. If $n, m \in N$ and $m = \frac{n^2 - n - 35}{n - 4}$, then find the value of m

5. Find the sum of all the real solutions of the inequality $\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \le 0$

6. Find the value of $(\log_3 12)(\log_3 72) - \log_3(192).\log_3 6$

7.✍ Let $x = (\log_{1/3} 5)(\log_{125} 343)(\log_{49} 729)$ and $y = 25^{3\log_{289} 11 \, \log_{28}\sqrt{17} \, \log_{1331} 784}$, then find the value of $x^2 + y^2$ is

8. If $c(a-b) = a(b-c)$ then find the value of $\frac{\log(a+c)+\log(a-2b+c)}{\log(a-c)}$
(Assume all terms are defined)

9.✍ If $\log_b a . \log_c a + \log_a b . \log_c b + \log_a c . \log_b c = 3$ (where a, b, c are different positive real numbers $\ne 1$), then find the value of abc.

10. If $4^A + 9^B = 10^C$, where $A = \log_{16} 4$, $B = \log_3 9$ & $C = \log_x 83$, then find x.

11. Find the natural number, x , which satisfies the equation $\log_{10}(x^2 - 12x + 36) = 2$

12. Find the value of x satisfying the equation $\log_{\frac{1}{2}}(x-1) + \log_{\frac{1}{2}}(x+1) - \log_{\frac{1}{\sqrt{2}}}(7-x) = 1$

13. Find the non negative square root of product of reciprocal of roots of the equation
$\log_{10}^2 x + \log_{10} x^2 = \log_{10}^2 2 - 1$

## Section (D) : Logarithmic inequalities

**D-1.** The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2-3x+2) \ge 2$ is

(A) $\left(\frac{1}{2}, 2\right)$ (B) $\left(1, \frac{5}{2}\right)$ (C) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (D) $(1, 2)$

**D-2.** If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(A) $(2, \infty)$ (B) $(1, 2)$ (C) $(-2, -1)$ (D) $\left(1, \frac{3}{2}\right)$

**D-3.** Solution set of the inequality $2 - \log_2(x^2 + 3x) \ge 0$ is :

(A) $[-4, 1]$ (B) $[-4, -3) \cup (0, 1]$

(C) $(-\infty, -3) \cup (1, \infty)$ (D) $(-\infty, -4) \cup [1, \infty)$

**D-4.** If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then 'x' lies in the interval

(A) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$ (B) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 2)$

(C) $(\sqrt{5}, 3\sqrt{5})$ (D) $\phi$

**D-5.** The set of all solutions of the inequality $(1/2)^{x^2-2x} < 1/4$ contains the set

(A) $(-\infty, 0)$ (B) $(-\infty, 1)$ (C) $(1, \infty)$ (D) $(3, \infty)$

**D-6.** The set of all the solutions of the inequality $\log_{1-x}(x-2) \ge -1$ is

(A) $(-\infty, 0)$ (B) $(2, \infty)$ (C) $(-\infty, 1)$ (D) $\phi$

## Section (E) : Determinants

**E-1.** The value of the determinant $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$ is equal to

(A) $-40$ (B) $40$ (C) $28$ (D) $52$

**E-2.** If $\begin{vmatrix} x^2-2x+3 & 7x+2 & x+4 \\ 2x+7 & x^2-x+2 & 3x \\ 3 & 2x-1 & x^2-4x+7 \end{vmatrix} = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$ the value of g is

(A) $2$ (B) $1$ (C) $-204$ (D) $-108$

**E-3.** The value of k for which determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix}$ vanishes, is

(A) $-3$ (B) $3$ (C) $-2$ (D) $2$

**E-4.** The absolute value of the determinant $\begin{vmatrix} -1 & 2 & 1 \\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix}$ is:

(A) $16\sqrt{2}$ (B) $8\sqrt{2}$ (C) $8$ (D) $2\sqrt{2}$





## PART - III : MATCH THE COLUMN

**1.** **Column-I**

(A) If $a = 3\left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to

(B) If $a = \sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}$, $b = \sqrt{11+6\sqrt{2}} - \sqrt{11-6\sqrt{2}}$, then the value of $\log_a b$ is equal to

(C) If $a = \sqrt{3+2\sqrt{2}}$, $b = \sqrt{3-2\sqrt{2}}$, then the value of $\log_a b$ is equal to

(D) If $a = \sqrt{7+\sqrt{7^2-1}}$, $b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to

(E) The number of zeroes at the end of the product of first 20 prime numbers, is

(F) The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is

**Column-II**

(p) $-1$

(q) $1$

(r) $2$

(s) $\frac{3}{2}$

(t) None

**2.** **Column-I**

(A) When the repeating decimal 0.363636............ is written as a rational fraction in the simplest form, the sum of the numerator and denominator is

(B) Given positive integer p, q and r with $p = 3^q \cdot 2^r$ and $100 < p < 1000$. The difference between maximum and minimum values of $(q + r)$, is

(C) If $\log_8 a + \log_8 b = (\log_8 a)(\log_8 b)$ and $\log_a b = 3$, then the value of 'a' is

(D) If $P = 3^{\sqrt{\log_3 2}} - 2^{\sqrt{\log_2 3}}$ then value of P is

**Column-II**

(p) 4

(q) 0

(r) 15

(s) 16

**3.** **Column-I**

(A) Anti logarithm of $(0.\overline{6})$ to the base 27 has the value equal to

(B) Characteristic of the logarithm of 2008 to the base 2 is

(C) The value of b satisfying the equation, $\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$ is

(D) Number of naughts after decimal before a significant figure comes in the number $\left(\frac{5}{6}\right)^{100}$, is

(Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$)

**Column-II**

(p) 5

(q) 7

(r) 9

(s) 10



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**B-3^.** The number of prime numbers satisfying the inequality $\frac{x^2-1}{2x+5} < 3$ is

(A) 1 (B) 2 (C) 3 (D) 4

**B-4.** The number of the integral solutions of $x^2 + 9 < (x+3)^2 < 8x + 25$ is :

(A) 1 (B) 3 (C) 4 (D) 5

**B-5.** The complete solution set of the inequality $\frac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} \geq 0$ is:

(A) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$
(B) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$
(C) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$
(D) $(-\infty, -5] \cup [1, 2] \cup [6, \infty)$

**B-6^.** Number of positive integral values of x satisfying the inequality

$$\frac{(x-4)^{2017}\,.\,(x+8)^{2016}\,(x+1)}{x^{2016}(x-2)^3\,.\,(x+3)^5\,.\,(x-6)\,(x+9)^{2018}} \leq 0 \text{ is}$$

(A) 0 (B) 1 (C) 2 (D) 3

**B-7.** Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$ is

(A) 0 (B) 1 (C) 2 (D) 3

## Section (C) : Logarithm

**C-1.** If $a^4 \cdot b^5 = 1$ then the value of $\log_a(a^5b^4)$ equals

(A) 9/5 (B) 4 (C) 5 (D) 8/5

**C-2.** $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c}$ has the value equal to

(A) abc (B) $\frac{1}{abc}$ (C) 0 (D) 1

**C-3.** $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to :

(A) 1/2 (B) 1 (C) 2 (D) 4

**C-4.** $(\log_2 10) . (\log_2 80) - (\log_2 5) . (\log_2 160)$ is equal to :

(A) $\log_2 5$ (B) $\log_2 20$ (C) $\log_2 10$ (D) $\log_2 16$

**C-5.** The ratio $\frac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to :

(A) $a^2 - a - 1$ (B) $a^2 + a - 1$ (C) $a^2 - a + 1$ (D) $a^2 + a + 1$

**C-6.** Let $x = 2^{\log 3}$ and $y = 3^{\log 2}$ where base of the logarithm is 10, then which one of the following holds good?

(A) $2x < y$ (B) $2y < x$ (C) $3x = 2y$ (D) $y = x$

**C-7.** If $\log_a(ab) = x$, then $\log_b(ab)$ is equal to

(A) $\frac{1}{x}$ (B) $\frac{x}{1+x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x-1}$





**C-8.** $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q(\log_r(\log_p x)) = 0$ then 'p' equals

(A) $r^{q/r}$ (B) rq (C) 1 (D) $r^{r/q}$

**C-9.** Which one of the following is the smallest?

(A) $\log_{10}\pi$ (B) $\sqrt{\log_{10}\pi^2}$ (C) $\left(\frac{1}{\log_{10}\pi}\right)^3$ (D) $\left(\frac{1}{\log_{10}\sqrt{\pi}}\right)$

**C-10.** $\log_{10}(\log_2 3) + \log_{10}(\log_3 4) + \log_{10}(\log_4 5) + \ldots\ldots + \log_{10}(\log_{1023} 1024)$ simplifies to

(A) a composite
(B) a prime number
(C) rational which is not an integer
(D) an integer

**C-11.** The sum of all the solutions to the equation $2\log_{10} x - \log_{10}(2x - 75) = 2$

(A) 30 (B) 350 (C) 75 (D) 200

**C-12.** If the solution of the equation $\log_x(125x) . \log_{25}^2 x = 1$ are $\alpha$ and $\beta$ $(\alpha < \beta)$. Then the value of $1/\alpha\beta$ is :

(A) 5 (B) 25 (C) 125 (D) 625

**C-13.** The positive integral solution of the equation $\log_x \sqrt{5} + \log_x 5x = \frac{9}{4} + \log_x^2 \sqrt{5}$ is :

(A) Composite number
(B) Prime number
(C) Even number
(D) Divisible by 3

**C-14.** The expression $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\ldots\ldots\sqrt[p]{p}}}}}_{\text{n radical sign}}$, where $p \geq 2$, $p \in N$ ; $n \in N$ when simplified is

(A) independent of p
(B) independent of p and of n
(C) dependent on both p and n
(D) positive

**C-15.** If $\log_x \log_{18}\left(\sqrt{2} + \sqrt{8}\right) = \frac{1}{3}$. Then the value of 1000 x is equal to

(A) 8 (B) 1/8 (C) 1/125 (D) 125

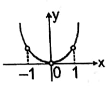
**C-16.** If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is

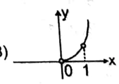
(A) zero (B) 1 (C) 2 (D) more than 2

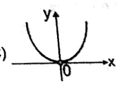
**C-17.** Number of real solutions of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is :

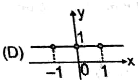
(A) zero (B) exactly 1 (C) exactly 2 (D) 4

**C-18.** The correct graph of $y = x^{\log_x x^2}$ is

(A) 


(B) 


(C) 


(D) 

**C-7.** Suppose n be an integer greater than 1. let $a_n = \frac{1}{\log_n 2002}$. Suppose $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then find the value of $(b-c)$

**C-8.** If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, show that $a^a \cdot b^b \cdot c^c = 1$.

**C-9.** Solve the following equations :

(i) $\log_x(4x-3) = 2$

(ii) $\log_2(\log_3(x^2-1)) = 0$

(iii) $4^{\log_2 x} - 2x - 3 = 0$

(iv) $\sqrt{5 - \log_2 x} = 3 - \log_2 x$

(v) $\log_4(\log_2 x) + \log_2(\log_4 x) = 2.$

(vi) $\log_4 \log_3 \log_2 x = 0$

(vii) $\log_3\left(\log_9 x + \frac{1}{2} + 9^x\right) = 2x.$

(viii) $2\log_4(4-x) = 4 - \log_2(-2-x).$

(ix) $x^{(\log_{\sqrt{x}} 2x)} = 4$

**C-10.** (i) Find the real value of 'x' satisfying the equation $x^{0.5\log_{\sqrt{x}}(x^2-x)} = 3^{\log_9 4}$.

(ii) Solve for x : $x^{\log_{10} x + 2} = 10^{\log_{10} x + 2}$

(iii) $x^{\frac{\log_{10} x + 5}{3}} = 10^{5 + \log_{10} x}$

(iv) Find the product of roots of the equation $(\log_3 x)^2 - 2(\log_3 x) - 5 = 0$

(v) Find product of roots of the equation $4^x - 7.2^x + 6 = 0$

## Section (D) : Logarithmic inequalities

**D-1.** Solve the following inequalities

(i) $\log_{\frac{5}{8}}\left(2x^2 - x - \frac{3}{8}\right) \ge 1$

(ii) $\log_{\frac{1}{2}}(x^2 - 5x + 6) > -1$

(iii) $\log_7 \frac{2x-6}{2x-1} > 0$

(iv) $\log_{1/4}(2-x) > \log_{1/4}\left(\frac{2}{x+1}\right)$

(v) $\log_{1/3}(2^{x+2} - 4^x) \ge -2$

(vi) $\log_x(4x-3) \ge 2$

**D-2.** Find the number of integers satisfying $\log_{1/5} \frac{4x+6}{x} \ge 0$

**D-3.** Find the number of positive integers not satisfying the inequality $\log_2(4^x - 2.2^x + 17) > 5$.

**D-4.** Solve the following inequalities :

(i) $\log_{(3x^2+1)} 2 < \frac{1}{2}$

(ii) $\log_{x^2}(2+x) < 1$

## Section (E) : Determinants

**E-1.** Evaluate the following determinants

(i) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{vmatrix}$ (ii) $\begin{vmatrix} 11 & 12 & 17 \\ 4 & 2 & 3 \\ 26 & 26 & 37 \end{vmatrix}$ (iii) $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ (iv) $\begin{vmatrix} x & x^2 & 1 \\ x^2 & 1 & x \\ 1 & x & x^2 \end{vmatrix}$





## PART - II : ONLY ONE OPTION CORRECT TYPE

## Section (A) : Number system, Polynomials & Divisional Algorithm

**A-1.** The number of real roots of the equation, $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ is :
(A) 0 (B) 1 (C) 2 (D) 3

**A-2.** Which of the following conditions imply that the real number x is rational?
(i) $x^{1/2}$ is rational (ii) $x^2$ and $x^5$ are rational (iii) $x^2$ and $x^4$ are rational
(A) (i) and (ii) only (B) (i) and (iii) only (C) (ii) and (iii) only (D) (i) (ii) and (iii)

**A-3.** If $x + \frac{1}{x} = 2$, then $x^2 + \frac{1}{x^2}$ is equal to
(A) 0 (B) 1 (C) 2 (D) 3

**A-4^.** Let $N = (2+1)(2^2+1)(2^4+1) \ldots\ldots (2^{32}+1) + 1$ and $N = 2^\lambda$ then the value of $\lambda$ is
(A) 63 (B) 64 (C) 65 (D) 66

**A-5.** If $(x+y)^2 = 2(x^2+y^2)$ and $(x-y+\lambda)^2 = 4$, $\lambda > 0$, then $\lambda$ is equal to :
(A) 1 (B) 2 (C) 3 (D) 4

**A-6.** If $\frac{a+3d}{a+9d} = \frac{a+d}{a+5d} = k$, then k is equal to (a, d > 0)
(A) 1/2 (B) 2 (C) 6 (D) 1/4

**A-7.** If $(x-a)$ is a factor of $x^3 - a^2x + x + 2$, then 'a' is equal to
(A) 0 (B) 2 (C) –2 (D) 1

**A-8.** The polynomials $P(x) = kx^3 + 3x^2 - 3$ and $Q(x) = 2x^3 - 5x + k$, when divided by $(x-4)$ leave the same remainder. The value of k is
(A) 2 (B) 1 (C) 0 (D) –1

**A-9.** Let f(x) be a polynomial function. If f(x) is divided by $x-1$, $x+1$ & $x+2$, then remainders are 5, 3 and 2 respectively. When f(x) is divided by $x^3 + 2x^2 - x - 2$, then remainder is :
(A) $x-4$ (B) $x+4$ (C) $x-2$ (D) $x+2$

**A-10.** If $2x^3 - 5x^2 + x + 2 = (x-2)(ax^2 - bx - 1)$, then a & b are respectively :
(A) 2, 1 (B) 2, – 1 (C) 1, 2 (D) – 1, 1/2

## Section (B) : Rational Inequalities

**B-1_.** The complete solution of $\frac{x^2-1}{x+3} \ge 0$ & $x^2 - 5x + 2 \le 0$ is :

(A) $x \in \left[\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\right]$ (B) $x \in \left[1, \frac{5+\sqrt{17}}{2}\right]$

(C) $x \in (-3, -1]$ (D) $x \in (-3, -1] \cup [1, \infty)$

**B-2_.** The solution of the inequality $2x - 1 \le x^2 + 3 \le x - 1$ is
(A) $x \in R$ (B) $[2-\sqrt{2}, 2+\sqrt{2}]$ (C) $[2-\sqrt{2}, 2]$ (D) $x \in \phi$

## Exercise-1

Marked Questions may have for Revision Questions.

## PART - I : SUBJECTIVE QUESTIONS

### Section (A) : Number system, Polynomials & Divisional Algorithm

**A-1.** If x, y, z ∈ Q such that $(2x-y)+(2x-y)\sqrt{3}=(x+z)+(x-y-1)\sqrt{5}$ then find x, y, z.

**A-2.** If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ and $\frac{2a^4b^2+3a^2c^2-5e^4f}{2b^6+3b^2d^2-5f^5}=\left(\frac{a}{b}\right)^n$ then find the value of n.

**A-3.** Show that $\frac{x^3+x^2+x+1}{x^3-x^2+x-1}=\frac{x^2+x+1}{x^2-x+1}$, is not possible for any $x \in R$

**A-4.** How many positive integer x are there such that 3x has 3 digits and 4x has four digits ?

**A-5.** If a, b, c are real and distinct numbers, then the value of $\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b).(b-c).(c-a)}$ is :

**A-6.** If a and b are positive integers such that $a^2-b^4=2009$, find a + b.

**A-7.** Find the number of positive integers x for which $f(x)=x^3-8x^2+20x-13$, is a prime number.

**A-8.** If both a and b are divisible by c and r is the remainder when a is divided by b then r is divisible by c.

**A-9.** If $f(x)=x^4-2x^3+3x^2-ax+b$ is a polynomial such that when it is divided by (x – 1) and (x + 1) the remainders are 5 and 19 respectively. If f(x) is divided by (x – 2), then find the remainder.

**A-10.** $f(x)=x^5+ax^3+bx$. The remainder when f(x) is divided by x + 1 is '–3', then find the remainder when it is divided by $x^2-1$

### Section (B) : Rational Inequalities

**B-1.** Solve the following Inequalities

(i) $\frac{x^2+4x+4}{2x^2-x-1}>0$ (ii) $\frac{7x-5}{8x+3}>4$ (iii) $\frac{x^4+x^2+1}{x^2-4x-5}>0$

(iv) $\frac{2x^2-3x-459}{x^2+1}>1$ (v) $\frac{x^2-5x+12}{x^2-4x+5}>3$

**B-2.** Solve the following Inequalities

(i) $\frac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)}\ge 0$ (ii) $\frac{(x+2)(x^2-2x+1)}{4+3x-x^2}\ge 0$

(iii) $\frac{x^4-3x^3+2x^2}{x^2-x-30}>0$ (iv) $\frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)}>1$





**B-3.** Find the number of integers satisfying the following Inequalities

(i) $x^4-5x^2+4\le 0$ (ii) $x^4-2x^2-63\le 0$

(iii) $x^2+6x-7\le 2$ (iv) $\frac{14x}{x+1}<\frac{9x-30}{x-4}$

(v) $\frac{x^2+2}{x^2-1}<-2$

**B-4.** Find the number of positive integers satisfying the following Inequalities

(i) $\frac{7}{(x-2)(x-3)}+\frac{9}{x-3}+1\le 0$ (ii) $\frac{x+7}{x-5}+\frac{3x+1}{2}\ge 0$ & x < 10

### Section (C) : Logarithm

**C-1.** Find the value of

(i) $\log_{10}5.\log_{10}20+(\log_{10}2)^2$ (ii) $5^{\log_{\sqrt{5}}2}+9^{\log_3 7}-8^{\log_2 5}$

**C-2.** Which of the following numbers are positive/negative

(i) $\log_{\sqrt{3}}\sqrt{2}$ +ve (ii) $\log_{1/7}(2)$ +ve (iii) $\log_{1/3}(1/5)$ +ve

(iv) $\log_3(4)$ +ve (v) $\log_7(2.11)$ +ve (vi) $\log_3(\sqrt{7}-2)$ +ve

(vii) $\log_4\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ +ve (viii) $\log_3\left(\frac{2\cdot\sqrt[3]{3}}{3}\right)$ –ve (ix) $\log_{10}(\log_{10}9)$ –ve

**C-3.** Let $\log_{10}2=a$ and $\log_{10}3=b$ then determine the following logarithms in terms of a and b.

(i) $\log_{10}\left(\sin^2\frac{\pi}{3}\right)$ (ii) $\log_{100}4+2\log_{100}27$

(iii) $\log_2 9+\log_3 8$ (iv) $\log_{\sqrt{45}}144$

**C-4.** Compute the following

(i) $\sqrt[3]{5^{\frac{1}{\log_7 5}}+\frac{1}{(-\log_{10}0.1)}}$ (ii) $\log_{0.75}\log_2\sqrt{\sqrt{\frac{1}{0.125}}}$

(iii) $\left(\frac{1}{49}\right)^{1+\log_7 2}+5^{-\log_{1/5}7}$ (iv) $7^{\log_3 5}+3^{\log_5 7}-5^{\log_3 7}-7^{\log_5 3}$

**C-5.** (i) Let n = 75600, then find the value of $\frac{4}{\log_2 n}+\frac{3}{\log_3 n}+\frac{2}{\log_5 n}+\frac{1}{\log_7 n}$

(ii) If $\log_2(\log_3(\log_4(x)))=0$ and $\log_3(\log_4(\log_2(y)))=0$ and $\log_4(\log_2(\log_3(z)))=0$ then find the sum of x, y and z is

**C-6.** Show that the number $\log_2 7$ is an irrational number.

**Note :**

If characteristic of a number (base of log is 10) is found to be n, then there would be (n + 1) digits in that number.

- Characteristic of log of $\frac{1}{10}$ = 0.1 with base 10 = – 1

Characteristic of log of $\frac{1}{100}$ = 0.01 with base 10 = – 2

Characteristic of log of $\frac{1}{1000}$ = 0.001 with base 10 = – 3

Characteristic of log of $\frac{3}{100}$ = 0.001 with base 10 = – 2

Characteristic of log of $\frac{3}{1000}$ = 0.006794 with base 10 = – 3

| Interval | Characteristic (base 10) | No. of zeros immediately after decimal | No. of integer resiprocal of which lies in interval. |
|---|---|---|---|
| [1/10, 1) | –1 | 0 | $9 = 9\times10^{1-1}$ |
| [1/100, 1/10) ≡ [0.01, 0.1) | –2 | 1 | $90 = 9\times10^{2-1}$ |
| $[1/10^3, 1/10^2)$ ≡ [0.0001, 0.01) | –3 | 2 | $900 = 9\times10^{3-1}$ |
| [0.0001, 0.001) | –4 | 3 | $9000 = 9\times10^{4-1}$ |
| | \| | | |
| | \| | | |
| | –n | (n – 1) | $= 9 \times 10^{n-1}$ |

- **Note :**

If characteristic of a number (base of log is 10) is found to be –n, then there would be (n –1) zeros immediately after decimal before first significant digit.

**Example # 26** Find the total number of digits in the number $18^{50}$.
(Given $\log_{10}2 = 0.3010$ ; $\log_{10}3 = 0.4771$) **Ans.** 63

**Solution.** $N = 18^{50}$

$\log_{10}N = 50 \log_{10}18 = 50 (0.3010 + 0.9542) = 50(1.2552) = 62.76$

Characterstic = $[\log_{10}N] = 62$

No. of digits = 62 + 1 = 63

**Self practice problem**

(21) Find the total number of zeros immediately after the decimal in $6^{-200}$.

**Ans.** (21) 155





**Expansion of a determinat :**

1. Expansion of two order determinant :

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

2. Expansion of $3^{rd}$ order determinant :

(i) With respect to first row :

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1(a_2b_3 - b_3a_3)$$

(ii) With respect to second column :

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + b_2 \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} - b_3 \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$= - b_1(a_2c_3 - a_3c_2) + b_2 (a_1c_3 - a_3c_1) - b_3(a_1c_2 - a_2c_1)$$

Similarly a determinant can be expanded with respect to any row or column.

**Example#27 :** Find the value of the determinant. $\begin{vmatrix} 2 & 3 & 4 \\ 6 & 5 & 7 \\ 1 & -3 & 2 \end{vmatrix}$

**Solution.** Expending the determinant w.r.t first row

$$\begin{vmatrix} 2 & 3 & 4 \\ 6 & 5 & 7 \\ 1 & -3 & 2 \end{vmatrix} = 2\begin{vmatrix} 5 & 7 \\ -3 & 2 \end{vmatrix} - 3 \begin{vmatrix} 6 & 7 \\ 1 & 2 \end{vmatrix} + 4 \begin{vmatrix} 6 & 5 \\ 1 & -3 \end{vmatrix}$$

= 2(10 + 21) – 3 (12 – 7) + 4 (–18 – 5)
= 62 – 15 – 92 = – 45

**Example # 23 :** Solve the inequality $\log_{1/3}(5x-1) > 0$.

**Solution.** by using the basic property of logarithm.

$$\begin{cases}5x-1<1\\5x-1>0\end{cases} \Rightarrow \begin{cases}5x<2\\5x>1\end{cases} \Rightarrow \begin{cases}x<\frac{2}{5}\\x>\frac{1}{5}\end{cases}$$

$\Rightarrow$ The solution of the inequality is given by $\left(\frac{1}{5},\frac{2}{5}\right)$ Ans.

**Example # 24 :** Solve the inequality $\log_{(2x+3)} x^2 < \log_{(2x+3)}(2x+3)$.

**Solution.** The given inequality is equivalent to the collection of the systems

$$\left[\begin{array}{ll}\begin{cases}0<2x+3<1\\x^2>2x+3\end{cases} & \text{(i)}\\ \begin{cases}2x+3>1\\0<x^2<2x+3\end{cases} & \text{(ii)}\end{array}\right.$$

Solving system (i) we obtain

$$\begin{cases}\frac{-3}{2}<x<-1\\(x-3)(x+1)>0\end{cases} \quad \text{(iii)}$$

System (iii) is equivalent to the collection of two systems

$$\left[\begin{array}{ll}\frac{-3}{2}<x<-1,\ x>3; & \text{(iv)}\\ \frac{-3}{2}<x<-1,\ x<-1 & \text{(v)}\end{array}\right.$$

system (iv) has no solution. The solution of system (v) is $x \in \left(\frac{-3}{2},-1\right)$,

solving system (ii) we obtain.

$$\begin{cases}x>-1\\(x-3)(x+1)<0\end{cases} \text{ or } \begin{cases}x>-1\\-1<x<3\end{cases} \Rightarrow x \in (-1, 3)$$

$$x \in \left(\frac{-3}{2},-1\right) \cup (-1, 3)$$

**Example # 25 :** Solve the inequation $\log_{\left(\frac{x^2-12x+30}{10}\right)}\left(\log_2 \frac{2x}{5}\right) > 0$.

**Solution.** This inequation is equivalent to the collection of following systems.

$$\begin{cases}\frac{x^2-12x+30}{10}>1,\\ \log_2\left(\frac{2x}{5}\right)>1,\end{cases} \quad \text{and} \quad \begin{cases}0<\frac{x^2-12x+30}{10}<1,\\ 0<\log_2\left(\frac{2x}{5}\right)<1\end{cases}$$

Solving the first system we have.

$$\begin{cases}x^2-12x+20>0\\ \frac{2x}{5}>2\end{cases} \Leftrightarrow \begin{cases}(x-10)(x-2)>0\\x>5\end{cases}$$





$$\Leftrightarrow \begin{cases}x<2 \text{ or } x>10\\x>5\end{cases}$$

Therefore the system has solution $x > 10$

Solving the second system we have.

$$\Rightarrow \begin{cases}0<x^2-12x+30<10\\1<\frac{2x}{5}<2\end{cases}$$

$$\Leftrightarrow \begin{cases}x^2-12x+30>0 \text{ and } x^2-12x+20<0\\ \frac{5}{2}<x<5\end{cases}$$

$$\Rightarrow \begin{cases}x<6-\sqrt{6} \text{ or } x>6+\sqrt{6} \text{ and } 2<x<10\\ \frac{5}{2}<x<5\end{cases}$$

$\therefore$ The system has solutions $\frac{5}{2} < x < 6-\sqrt{6}$ combining both systems, then solution of the original inequation is.

$x \in \left(\frac{5}{2}, 6-\sqrt{6}\right) \cup (10, \infty)$ **Ans.**

**Self practice problems :**

(20) Solve the following inequalities

(i) $\log_{3x+5}(9x^2+8x+8) > 2$
(ii) $\log_{0.2}(x^2-x-2) > \log_{0.2}(-x^2+2x+3)$
(iii) $\log_x(x^3-x^2-2x) < 3$

**Answers :** (20) (i) $\left(-\frac{4}{3},-\frac{17}{22}\right)$ (ii) $\left(2,\frac{5}{2}\right)$ (iii) $(2, \infty)$

## Characteristic & Mantissa

$[\log_a N]$ is called characteristic of log of N with base 'a'. It is always an integer.
$\{\log_a N\}$ is called mantissa of log of N with base 'a'. Mantissa $\in [0, 1)$

Characteristic of log of 1 with base 10 = 0
characteristic of log of 10 with base 10 = 1
characteristic of log of 100 with base 10 = 2
characteristic of log of 1000 with base 10 = 3
characteristic of log of 83.5609 with base 10 = 1
characteristic of log of 613.0965 with base 10 = 2

| Interval, | Cha.(Base 10) | number of digits in no | No. of integers in the interval |
|---|---|---|---|
| [1, 10) | 0 | 1 | $9 = 9 \times 10^0$ |
| [10, 100) | 1 | 2 | $90 = 9 \times 10^1$ |
| [100, 1000) | 2 | 3 | $900 = 9 \times 10^2$ |
| [100, 10000) | 3 | 4 | $9000 = 9 \times 10^3$ |
| | \| | | |
| | \| | | |
| | \| | | |
| | n | (n + 1) | $9 \times 10^n$ |

**NOTE :**

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_{1/a} a = -1$
- $\log_b a = \dfrac{1}{\log_a b}$
- $a^x = e^{x \ln a}$
- $a^{\log_c b} = b^{\log_c a}$

**Note :** (i) If the number and the base are on the same side of the unity, then the logarithm is positive.

(ii) If the number and the base are on the opposite sides of unity, then the logarithm is negative.

**Example#18:** Find the value of the followings :

(i) $\log_2 72 + \log_2\left(\dfrac{32}{81}\right) + \log_2\left(\dfrac{9}{64}\right)$ **Ans.** 2

(ii) $7^{\frac{1}{\log_{25} 49}}$ **Ans.** 5

**Solution.** (i) $\log_2 72 + \ldots\ldots\ldots$

$$= \log_2\left\{2^3.3^2.\frac{2^5}{3^4}.\frac{3^2}{2^6}\right\} = \log_2 4 = 2$$

(ii) $7^{\frac{1}{\log_{25} 49}} = 7^{\log_{49} 25} = 7^{\frac{2}{2}\log_7 5} = 5^{\log_7 7} = 5$

### Self practice problem :

(13) Find the value of the followings :

(i) $\log_{49} 343$ (ii) $4\log_{27} 243$

(iii) $\log_{(1/100)} 1000$ (iv) $\log_{(7-4\sqrt{3})}(7+4\sqrt{3})$

(v) $\log_{125} 625$

(14) $\log_8 9.\log_9 10 \ldots\ldots\ldots \log_{63} 64$

(15) Find the value of $\log \cot 1° + \log \cot 2° + \log \cot 3° + \ldots. + \log \cot 89°$

**Ans.** (13) (i) 3/2 (ii) 20/3 (iii) – 3/2 (iv) –1 (v) 4/3

(14) 2 (15) 0

## Logarithmic Equation :

The equality $\log_a x = \log_a y$ is possible if and only if $x = y$ i.e.

$$\log_a x = \log_a y \Leftrightarrow x = y$$

Always check validity of given equation, $(x > 0, y > 0, a > 0, a \neq 1)$

**Example#19 :** $\log_x(4x - 3) = 2$

**Solution.** Domain : $x > 0, 4x - 3 > 0, x \neq 1$

Hence $4x - 3 = x^2 \Rightarrow x^2 - 4x + 3 = 0$ **Ans.** $x = 3$

$x = 3$ or $x = 1$ (rejected as not in domain)

**Exmaple#20 :** $\log_2(\log_3\{\log_5(x^2+4)\}) = 0$

**Solution.** $\log_3(\log_5(x^2+4)) = 2^0 = 1$

$\Rightarrow \log_5(x^2+4) = 3^1 = 3$ **Ans.** $x = \pm 11$

$\Rightarrow (x^2+4) = 5^3 = 125 \Rightarrow x^2 = 121 \Rightarrow x = \pm 11$





**Example#21 :** $\log_2(x^2) + \log_2(x + 2) = 4$

**Solution.** $\log_2(x^2(x+2)) = 4 \Rightarrow x^3 + 2x^2 - 16 = 0 \Rightarrow (x-2)\underbrace{(x^2+4x+8)}_{D<0} = 0$ **Ans.** $x = 2$

$x = 2$

**Self practice problem**

(16) $3^{3\log_3 x} = 27$ (17) $(\log_{10} x)^2 - (\log_{10} x) - 6 = 0$

(18) $3(\log_7 x + \log_x 7) = 10$ (19) $(x+2)^{\log_2(x+2)} = 8(x+2)^2$

**Ans.** (16) $x = 3$ (17) $x = 10^3, \dfrac{1}{10^2}$ (18) $x = 343, \sqrt[3]{7}$ (19) $x = 6$ or $-3/2$

## Logarithmic Inequality :

Let 'a' is a real number such that

(i) If $a > 1$, then $\log_a x > \log_a y \Rightarrow x > y$

(ii) If $a > 1$, then $\log_a x < \alpha \Rightarrow 0 < x < a^\alpha$

(iii) If $a > 1$, then $\log_a x > \alpha \Rightarrow x > a^\alpha$

(iv) If $0 < a < 1$, then $\log_a x > \log_a y \Rightarrow 0 < x < y$

(v) If $0 < a < 1$, then $\log_a x < \alpha \Rightarrow x > a^\alpha$

**Form - I :** $f(x) > 0, g(x) > 0, g(x) \neq 1$

| | Form | | Collection of system |
|---|---|---|---|
| (a) | $\log_{g(x)} f(x) \geq 0$ | $\Leftrightarrow$ | $\begin{cases} f(x) \geq 1, & g(x) > 1 \\ 0 < f(x) \leq 1, & 0 < g(x) < 1 \end{cases}$ |
| (b) | $\log_{g(x)} f(x) \leq 0$ | $\Leftrightarrow$ | $\begin{cases} f(x) \geq 1, & 0 < g(x) < 1 \\ 0 < f(x) \leq 1, & g(x) > 1 \end{cases}$ |
| (c) | $\log_{g(x)} f(x) \geq a$ | $\Leftrightarrow$ | $\begin{cases} f(x) \geq (g(x))^a, & g(x) > 1 \\ 0 < f(x) \leq (g(x))^a, & 0 < g(x) < 1 \end{cases}$ |
| (d) | $\log_{g(x)} f(x) \leq a$ | $\Leftrightarrow$ | $\begin{cases} 0 < f(x) \leq (g(x))^a, & g(x) > 1 \\ f(x) \geq (g(x))^a, & 0 < g(x) < 1 \end{cases}$ |

**From - II : When the inequality of the form**

| | Form | | Collection of system |
|---|---|---|---|
| (a) | $\log_{\phi(x)} f(x) \geq \log_{\phi(x)} g(x)$ | $\Leftrightarrow$ | $\begin{cases} f(x) \geq g(x), \phi(x) > 1, \\ 0 < f(x) \leq g(x); 0 < \phi(x) < 1 \end{cases}$ |
| (b) | $\log_{\phi(x)} f(x) \leq \log_{\phi(x)} g(x)$ | $\Leftrightarrow$ | $\begin{cases} 0 < f(x) \leq g(x), \phi(x) > 1, \\ f(x) \geq g(x) > 0, 0 < \phi(x) < 1 \end{cases}$ |

**Example # 22:** Solve the logarithmic inequality $\log_{1/5}(2x^2 + 7x + 7) \geq 0$.

**Solution.** Since $\log_{1/5} 1 = 0$, the given inequality can be written as.

$\log_{1/5}(2x^2 + 7x + 7) \geq \log_{1/5} 1$

when the domain of the function is taken into account the inequality is equivalent to the system of ineqaulities.

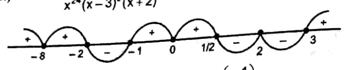
$$\begin{cases} 2x^2 + 7x + 7 > 0 \\ 2x^2 + 7x + 7 \leq 1 \end{cases}$$

Solving the ineqaualities by using method of

intervals $x \in \left[-2, \dfrac{-3}{2}\right]$

**Example# 17 :** Solve the inequality if $f(x) = \dfrac{(x-2)^{10}(x+1)^3\left(x-\frac{1}{2}\right)^5(x+8)^2}{x^{24}(x-3)^3(x+2)^5}$ is > 0 or < 0.

**Solution.** Let $f(x) = \dfrac{(x-2)^{10}(x+1)^3\left(x-\frac{1}{2}\right)^5(x+8)^2}{x^{24}(x-3)^3(x+2)^5}$ the poles and zeros are $0, 3, -2, -1, \frac{1}{2}, -8, 2$




If $f(x) > 0$, then $x \in (-\infty, -8) \cup (-8, -2) \cup (-1, 0) \cup \left(0, \frac{1}{2}\right) \cup (3, \infty)$

and if $f(x) < 0$, then $x \in (-2, -1) \cup \left(\frac{1}{2}, 2\right) \cup (2, 3)$ **Ans.**

## Various types of functions :

**(i) Polynomial Function :**

If a function f is defined by $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ where n is a non **negative integer** and $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers and $a_0 \neq 0$, then f is called a polynomial function of degree n.

**Note :** There are two polynomial functions, satisfying the relation; $f(x).f(1/x) = f(x) + f(1/x)$, which are $f(x) = 1 \pm x^n$

**(ii) Constant function :**

A function $f : A \to B$ is said to be a constant function, if every element of A has the same f image in B. Thus $f : A \to B$; $f(x) = c$, $\forall x \in A$, $c \in B$ is a constant function.

**(iii) Identity function :**

The function $f : A \to A$ defined by $f(x) = x\ \forall x \in A$ is called the identity function on A and is denoted by $I_A$. It can be observed that identity function is a bijection.

**(iv) Algebraic Function :**

y is an algebraic function of x, if it is a function that satisfies an algebraic equation of the form, $P_0(x)y^n + P_1(x)y^{n-1} + \ldots\ldots + P_{n-1}(x)y + P_n(x) = 0$ where n is a positive integer and $P_0(x)$, $P_1(x)\ldots\ldots$ are polynomials in x. e.g. $y = |x|$ is an algebraic function, since it satisfies the equation $y^2 - x^2 = 0$.

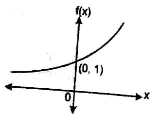
**Note :** All polynomial functions are algebraic but not the converse.

A function that is not algebraic is called **Transcendental Function.**

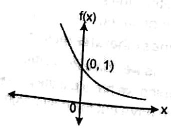
## Exponential Function

A function $f(x) = a^x = e^{x \ln a}$ $(a > 0, a \neq 1, x \in R)$ is called an exponential function. Graph of exponential function can be as follows :

**Case - I**
**For a > 1**




**Case - II**
**For 0 < a < 1**










## Logarithm of A Number :

The logarithm of the number N to the base 'a' is the exponent indicating the power to which the base 'a' must be raised to obtain the number N. This number is designated as $\log_a N$. Hence:

$$\log_a N = x \Leftrightarrow a^x = N,\ a > 0,\ a \neq 1\ \&\ N > 0$$

If a = 10, then we write log b rather than $\log_{10} b$.
If a = e, we write $\ell n\, b$ rather than $\log_e b$. Here 'e' is called as Napier's base & has numerical value equal to 2.7182.

**Remember**

$\log_{10} 2 \approx 0.3010$ ; $\log_{10} 3 \approx 0.4771$
$\ell n\, 2 \approx 0.693$ ; $\ell n\, 10 \approx 2.303$

## Domain of Definition :

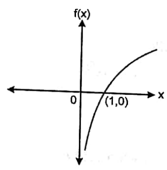
The existence and uniqueness of the number $\log_a N$ can be determined with the help of set of conditions, $a > 0\ \&\ a \neq 1\ \&\ N > 0$.

The base of the logarithm 'a' must not equal unity otherwise numbers not equal to unity will not have a logarithm and any number will be the logarithm of unity.

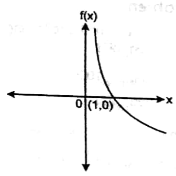
## Graph of Logarithmic function :

$f(x) = \log_a x$ is called logarithmic function where $a > 0$ and $a \neq 1$ and $x > 0$. Its graph can be as follows:

**Case-I**
**For a > 1**




**Case-II**
**For 0 < a < 1**




## Fundamental Logarithmic Identity :

$$a^{\log_a N} = N,\ a > 0,\ a \neq 1\ \&\ N > 0$$

## The Principal Properties of Logarithm:

Let M & N are arbitrary positive numbers, $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ and $\alpha, \beta$ are any real numbers, then :

(i) $\log_a(M.N) = \log_a M + \log_a N$ ; in general $\log_a(x_1 x_2 \ldots\ldots x_n) = \log_a x_1 + \log_a x_2 + \ldots\ldots + \log_a x_n$

(ii) $\log_a(M/N) = \log_a M - \log_a N$

(iii) $\log_a M^\alpha = \alpha.\log_a M$

(iv) $\log_{a^\beta} M = \dfrac{1}{\beta}\log_a M$

(v) $\log_b M = \dfrac{\log_a M}{\log_a b}$ (base changing theorem)

**Example # 14 :** If a, b, c, d, e are in continued proportion, then prove that
$(ab + bc + cd + de)^2 = (a^2 + b^2 + c^2 + d^2)\ (b^2 + c^2 + d^2 + e^2)$

**Solution.** If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e}$, then we have $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{\sqrt{(a^2+b^2+c^2+d^2)}}{\sqrt{(b^2+c^2+d^2+e^2)}} = k$ (say)

i.e.
$a = bk \quad \therefore \quad ab = b^2k$
$b = ck \quad \therefore \quad bc = c^2k$
$c = dk \quad \therefore \quad cd = d^2k$
$d = ek \quad \therefore \quad de = e^2k$

Again $(a^2 + b^2 + c^2 + d^2) = k^2 (b^2 + c^2 + d^2 + e^2)$ ...........(i)

Now L.H.S.
$= (ab + bc + cd + de)^2$
$= (kb^2 + kc^2 + kd^2 + ke^2)^2$
$= k^2 (b^2 + c^2 + d^2 + e^2)^2$
$= k^2 (b^2 + c^2 + d^2 + e^2)(b^2 + c^2 + d^2 + e^2)$
$= (a^2 + b^2 + c^2 + d^2)(b^2 + c^2 + d^2 + e^2)$ (Note) (use (i))

Hence $(ab + bc + cd + de)^2 = (a^2 + b^2 + c^2 + d^2)(b^2 + c^2 + d^2 + e^2)$

**Self practice problem**

(11) If $(a^2 + b^2 + c^2)(x^2 + y^2 + z^2) = (ax + by + cz)^2$, show that $x : a = y : b = z : c$.

(12) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then find the value of $\frac{4a^4b^2 + 9a^2c^2 - \sqrt{7}e^4f}{4b^6 + 9b^2d^2 - \sqrt{7}f^5}$ in terms of c and d.

**Answer :** (12) $\frac{c^4}{d^4}$

## Cross multiplication :

If two equations containing three unknowns are
$a_1x + b_1y + c_1z = 0$ ............(i)
$a_2x + b_2y + c_2z = 0$ ............(ii)
Then by the rule of cross multiplication

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1} \qquad ............(iii)$$

In order to write down the denominators of x, y and z in (iii) apply the following rule, "write down the coefficients of x, y and z in order beginning with the coefficients of y and repeat them as in the diagram"

$$\begin{array}{cccc} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{array}$$

Multiply the coefficients across in the way indicated by the arrows; remembering that informing the products any one obtained by descending is positive and any one obtained by ascending is negative.

**Example # 15 :** Find the ratios of $x : y : z$ from the equations $7x = 4y + 8z$, $3z = 12x + 11y$.

**Solution.** By transposition we have $7x - 4y - 8z = 0$, $12x + 11y - 3z = 0$,
Write down the coefficients, thus

$$\begin{array}{cccc} -4 & -8 & 7 & -4 \\ 11 & -3 & 12 & 11, \end{array}$$

hence we obtain the products
$(-4) \times (-3) - 11 \times (-8)$, $(-8) \times 12 - (-3) \times 7$, $7 \times 11 - 12 \times (-4)$,
or 100, $-75$, 125

$\therefore \quad \frac{x}{100} = \frac{y}{-75} = \frac{z}{125}$, that is, $\frac{x}{4} = \frac{y}{-3} = \frac{z}{5}$.





**Example # 16 :** Eliminate x, y, z from the equations
$a_1x + b_1y + c_1z = 0$ ..........(1)
$a_2x + b_2y + c_2z = 0$ ..........(2)
$a_3x + b_3y + c_3z = 0$ ..........(3)

From (2) and (3), by cross multiplication, $\frac{x}{b_2c_3 - b_3c_2} = \frac{y}{c_2a_3 - c_3a_2} = \frac{z}{a_2b_3 - a_3b_2}$;
denoting each of these ratios by k, by multiplying up, substituting in(1), and dividing through out by k, we obtain
$\mathbf{a_1(b_2c_3 - b_3c_2) + b_1(c_2a_3 - c_3a_2) + c_1(a_2b_3 - a_3b_2) = 0}$
This relation is called the **eliminant** of the given equations.

## Intervals :

Intervals are basically subsets of R and are commonly used in solving inequalities or in finding domains. If there are two numbers $a, b \in R$ such that $a < b$, we can define four types of intervals as follows :

| Name | Representation | Discription |
|---|---|---|
| Open Interval | (a, b) | $\{x : a < x < b\}$ i.e. end points are not included. |
| Close Interval | [a, b] | $\{x : a \le x \le b\}$ i.e. end points are also included. This is possible only when both a and b are finite. |
| Open - Closed Interval | (a, b] | $\{x : a < x \le b\}$ i.e. a is excluded and b is included. |
| Close - Open Interval | [a, b) | $\{x : a \le x < b\}$ i.e. a is included and b is excluded. |

**Note :** (1) The infinite intervals are defined as follows :
(i) $(a, \infty) = \{x : x > a\}$ (ii) $[a, \infty) = \{x : x \ge a\}$
(iii) $(-\infty, b) = \{x : x < b\}$ (iv) $(\infty, b] = \{x : x \le b\}$
(v) $(-\infty, \infty) = \{x : x \in R\}$
(2) $x \in \{1, 2\}$ denotes some particular values of x, i.e. $x = 1, 2$
(3) If there is no value of x, then we say $x \in \phi$ (null set)

## General Method to solve Inequalities :

**(Method of intervals (Wavy curve method)**

Let $$g(x) = \left(\frac{(x-b_1)^{k_1}(x-b_2)^{k_2} --- (x-b_n)^{k_n}}{(x-a_1)^{r_1}(x-a_2)^{r_2} --- (x-a_n)^{r_n}}\right) \qquad ... (i)$$

Where $k_1, k_2 .....k_n$ and $r_1, r_2 ........r_n \in N$ and $b_1, b_2.....b_n$ and $a_1, a_2 ......a_n$ are real numbers. Then to solve the inequality following steps are taken.

**Steps : -**

Points where numerator becomes zero are called zeros or roots of the function and where denominator becomes zero are called poles of the function.

(i) First we find the zeros and poles of the function.

(ii) Then we mark all the zeros and poles on the real line and put a vertical bar there dividing the real line in many intervals.

(iii) Determine sign of the function in any of the interval and then alternates the sign in the neghbouring interval if the poles or zeros dividing the two interval has appeared odd number of times otherwise retain the sign.

(iv) Thus we consider all the intervals. The solution of the $g(x) > 0$ is the union of the intervals in which we have put the plus sign and the solution of $g(x) < 0$ is the union of all intervals in which we have put the minus sign.

**Self practice problems :**

(8) Find the value of

(i) $\left(\frac{1}{3}\right)^{-10}.27^{-3}+\left(\frac{1}{5}\right)^{-4}.(25)^{-2}+\left(64^{-\frac{1}{9}}\right)^{-3}$ (ii) $\frac{(5\sqrt{3}+\sqrt{50})(5-\sqrt{24})}{\sqrt{75}-5\sqrt{2}}$

**Answer :** (8) (i) 8 (ii) 1

**Ratio :**

1. If A and B be two quantities of the same kind, then their ratio is A : B; which may be denoted by the fraction $\frac{A}{B}$ (This may be an integer or fraction)
2. A ratio may represented in a number of ways e.g. $\frac{a}{b}=\frac{ma}{mb}=\frac{na}{nb}=$..... where m, n,..... are non-zero numbers.
3. To compare two or more ratio, reduced them to common denominator.
4. Ratio between two ratios may be represented as the ratio of two integers e.g. $\frac{a}{b}:\frac{c}{d}:\frac{a/b}{c/d}=\frac{ad}{bc}$ or ad : bc.
5. Ratios are compounded by multiplying them together i.e. $\frac{a}{b}\cdot\frac{c}{d}\cdot\frac{e}{f}\ldots=\frac{ace}{bdf}\ldots$
6. If a : b is any ratio then its duplicate ratio is $a^2 : b^2$ ; triplicate ratio is $a^3 : b^3$ ..... etc.
7. If a : b is any ratio, then its sub-duplicate ratio is $a^{1/2} : b^{1/2}$ ; sub-triplicate ratio is $a^{1/3} : b^{1/3}$ etc.

**Example # 11 :** What term must be added to each term of the ratio 5 : 37 to make it equal to 1 : 3?

**Solution.** Let x be added to each term of the ratio 5 : 37.

Then $\frac{x+5}{x+37}=\frac{1}{3}$ $\Rightarrow$ $3x + 15 = x + 37$ i.e. $x = 11$

**Example # 12 :** If x : y = 3 : 4, then find the ratio of 7x – 4y : 3x + y

**Solution.** $\frac{x}{y}=\frac{3}{4}$ $\therefore$ $4x = 3y$ or $x=\frac{3}{4}y$

Now $\frac{7x-4y}{3x+y}=\frac{7.\frac{3}{4}y-4y}{3.\frac{3}{4}y+y}$ (putting the value of x)

$=\frac{\frac{21}{4}y-4y}{\frac{9}{4}y+y}=\frac{5y}{13y}=\frac{5}{13}$ i.e. 5 : 13 **Ans.**

**Self practice problem**

(9) If $\frac{a}{b}=\frac{2}{3}$ and $\frac{b}{c}=\frac{4}{5}$, then find value of $\frac{a+b}{b+c}$.

(10) If sum of two numbers is C and their quotient is $\frac{p}{q}$, then find number.

**Answers.** (9) $\frac{20}{27}$ (10) $\frac{pc}{p+q}$, $\frac{qc}{p+q}$





**Proportion :**

When two ratios are equal, then the four quantities compositing them are said to be proportional. If $\frac{a}{b}=\frac{c}{d}$, then it is written as a : b = c : d or a : b :: c : d

1. 'a' and 'd' are known as extremes and 'b and c' are known as means.
2. An important property of proportion : Product of extremes = product of means.
3. If a : b = c : d, then
   b : a = d : c (Invertando) i.e. $\frac{a}{b}=\frac{c}{d}$ $\Rightarrow$ $\frac{b}{a}=\frac{d}{c}$
4. If a : b = c : d, then
   a : c = b : d (Alternando) i.e. $\frac{a}{b}=\frac{c}{d}$ $\Rightarrow$ $\frac{a}{c}=\frac{b}{d}$
5. If a : b = c : d, then
   $\frac{a+b}{b}=\frac{c+d}{d}$ (Componendo) i.e. $\frac{a}{b}=\frac{c}{d}$ $\Rightarrow$ $\frac{a}{b}+1=\frac{c}{d}+1$
6. If a : b = c : d, then
   $\frac{a-b}{b}=\frac{c-d}{d}$ (Dividendo) i.e. $\frac{a}{b}=\frac{c}{d}$ $\Rightarrow$ $\frac{a}{b}-1=\frac{c}{d}-1$
7. If a : b = c : d, then
   $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (Componendo and dividendo)

i.e. $\frac{a}{b}=\frac{c}{d}$ $\Rightarrow$ $\frac{a}{b}+1=\frac{c}{d}+1$ $\Rightarrow$ $\frac{a+b}{b}=\frac{c+d}{d}$ .......(1)

$\frac{a}{b}-1=\frac{c}{d}-1$ $\Rightarrow$ $\frac{a-b}{b}=\frac{c-d}{d}$ .......(2)

Dividing equation (1) & (2) we obtain $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

**Example # 13 :** If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, then show that $\frac{x^3+a^3}{x^2+a^2}+\frac{y^3+b^3}{y^2+b^2}+\frac{z^3+c^3}{z^2+c^2}=\frac{(x+y+z)^3+(a+b+c)^3}{(x+y+z)^2+(a+b+c)^2}$

**Solution.** $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$ (constant)

$\therefore$ x = ak; y = bk; z = ck

Substituting these values of x, y, z in the given expression

$\frac{x^3+a^3}{x^2+a^2}+\frac{y^3+b^3}{y^2+b^2}+\frac{z^3+c^3}{z^2+c^2}=\frac{(x+y+z)^3+(a+b+c)^3}{(x+y+z)^2+(a+b+c)^2}$

we obtain

L.H.S. $=\frac{a^3k^3+a^3}{a^2k^2+a^2}+\frac{b^3k^3+b^3}{b^2k^2+b^2}+\frac{c^3k^3+c^3}{c^2k^2+c^2}=\frac{a^3(k^3+1)}{a^2(k^2+1)}+\frac{b^3(k^3+1)}{b^2(k^2+1)}+\frac{c^3(k^3+1)}{c^2(k^2+1)}$

$=\frac{a(k^3+1)}{k^2+1}+\frac{b(k^3+1)}{k^2+1}+\frac{c(k^3+1)}{k^2+1}=\frac{(k^3+1)}{(k^2+1)}\cdot(a+b+c)$

Now R.H.S $=\frac{(ak+bk+ck)^3+(a+b+c)^3}{(ak+bk+ck)^2+(a+b+c)^2}=\frac{k^3(a+b+c)^3+(a+b+c)^3}{k^2(a+b+c)^2+(a+b+c)^2}$

$=\frac{(k^3+1)(a+b+c)^3}{(k^2+1)(a+b+c)^2}=\frac{(k^3+1)}{(k^2+1)}\cdot(a+b+c)$

We see that L.H.S. = R.H.S.

**Some Important Identities:**

(1) $(a+b)^2 = a^2 + 2ab + b^2 \qquad = (a-b)^2 + 4ab$

(2) $(a-b)^2 = a^2 - 2ab + b^2 \qquad = (a+b)^2 - 4ab$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(5) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(6) $a^3 + b^3 = (a+b)^3 - 3ab(a+b) = (a+b)(a^2+b^2-ab)$

(7) $a^3 - b^3 = (a-b)^3 + 3ab(a-b) = (a-b)(a^2+b^2+ab)$

(8) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(9) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$

(10) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$= \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

(11) $a^4 - b^4 = (a+b)(a-b)(a^2+b^2)$

(12) $a^4 + a^2 + 1 = (a^2+1)^2 - a^2 = (1+a+a^2)(1-a+a^2)$

**Example # 5 :** If $\left(a+\frac{1}{a}\right)^2 = 3$, then $a^3 + \frac{1}{a^3}$ equals :

(A) $6\sqrt{3}$ (B) $3\sqrt{3}$ (C) 0 (D) $7\sqrt{7}$ (E) $6\sqrt{3}$

**Solution.** $a + \frac{1}{a} = \pm\sqrt{3}$

$$a^3 + \frac{1}{a^3} = \left(a+\frac{1}{a}\right)^3 - 3\left(a+\frac{1}{a}\right) = \pm 3\sqrt{3} \mp 3\sqrt{3} = 0.$$

**Example # 6 :** Show that the expression, $(x^2-yz)^3 + (y^2-zx)^3 + (z^2-xy)^3 - 3(x^2-yz).(y^2-zx).(z^2-xy)$ is a perfect square and find its square root.

**Solution.** $(x^2-yz)^3 + (y^2-zx)^3 + (z^2-xy)^3 - 3(x^2-yz)(y^2-zx)(z^2-xy)$

$= a^3 + b^3 + c^3 - 3abc$ where $a = x^2 - yz$, $b = y^2 - zx$, $c = z^2 - xy$

$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= \frac{1}{2}(a+b+c)((a-b)^2 + (b-c)^2 + (c-a)^2)$

$= \frac{1}{2}(x^2+y^2+z^2-xy-yz-zx)[(x^2-yz-y^2+zx)^2 + (y^2-zx-z^2+xy)^2 + (z^2-xy-x^2+yz)^2]$

$= \frac{1}{2}(x^2+y^2+z^2-xy-yz-zx)[\{x^2-y^2+z(x-y)\}^2 + \{y^2-z^2+x(y-z)\}^2 + \{z^2-x^2+y(z-x)\}^2]$

$= \frac{1}{2}(x^2+y^2+z^2-xy-yz-zx)(x+y+z)^2[(x-y)^2 + (y-z)^2 + (z-x)^2]$

$= (x+y+z)^2(x^2+y^2+z^2-xy-yz-zx)^2 = (x^3+y^3+z^3-3xyz)^2$ (which is a perfect square)

its square roots are

$\pm(x^3+y^3+z^3-3xyz)$





**Self practice problems :**

(5) If x, y, z are all different real numbers, then prove that

$$\frac{1}{(x-y)^2} + \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} = \left(\frac{1}{x-y} + \frac{1}{y-z} + \frac{1}{z-x}\right)^2.$$

(6) Factorise the expression, $(x+y+z)^3 - x^3 - y^3 - z^3$ into linear factors.

(7) Factorize

(i) $1 + x^4 + x^8$ (ii) $x^4 + 4$

**Answers :** (6) $3(x+y)(y+z)(z+x)$

(7) (i) $(x^4 - x^2 + 1)(x^2 + x + 1)(x^2 - x + 1)$ (ii) $(x^2 - 2x + 2)(x^2 + 2x + 2)$

## Definition of Indices :

If 'a' is any non zero real or imaginary number and 'm' is the positive integer, then $a^m = a.\ a.\ a.\ ...a$ (m times). Here a is called the base and m is called the index, power or exponent.

**Law of Indices :**

(1) $a^0 = 1$, $(a \neq 0)$

(2) $a^{-m} = \frac{1}{a^m}$, $(a \neq 0)$

(3) $a^{m+n} = a^m . a^n$, where m and n are rational numbers

(4) $a^{m-n} = \frac{a^m}{a^n}$, where m and n are rational numbers, $a \neq 0$

(5) $(a^m)^n = a^{mn}$

(6) $a^{p/q} = \sqrt[q]{a^p}$

**Example # 7 :** Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$; the result is :

(A) $a^{16}$ (B) $a^{12}$ (C) $a^8$ (D) $a^4$ (E) $a^2$

**Solution.** $a^{9(1/6)(1/3)4} . a^{9(1/3)(1/6)4} = a^2 . a^2 = a^4$.

**Example # 8 :** Simplify $a\left(\frac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1} + b\left(\frac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}$

**Solution.** The given expression is equal to

$$a\left(\frac{2b\sqrt{a}}{\sqrt{a}+\sqrt{b}}\right) + b\left(\frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right) = 2ab\left(\frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}} + \frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right) = 2ab$$

**Example # 9 :** Evaluate $\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{7+\sqrt{48}}}}$

**Solution.** $\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{7+\sqrt{48}}}} = \sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{4+3+2\sqrt{12}}}}$

$= \sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{4}+\sqrt{3}}}$

$= \sqrt{3+\sqrt{3}+\sqrt{4+2\sqrt{3}}} = \sqrt{3+\sqrt{3}+\sqrt{3}+1} = \sqrt{4+2\sqrt{3}} = \sqrt{3} + 1$

**Example # 10 :** Find rational numbers a and b, such that $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$

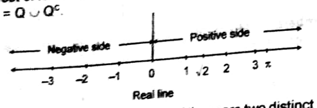
**Solution.** $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}} = a + b\sqrt{5}$

$\frac{61+24\sqrt{5}}{-29} = a + b\sqrt{5} \Rightarrow a = -\frac{61}{29}, b = -\frac{24}{29}$ **Ans.**

(xi) **Irrational numbers :** The numbers which can not be expressed in p/q form where $p, q \in I$ and $q \neq 0$ i.e. the numbers which are not rational are called irrational numbers and their set is denoted by $Q^c$ (i.e. complementary set of Q) e.g. $\sqrt{2}$, $1 + \sqrt{3}$ etc. Irrational numbers can not be expressed as recurring decimals.

**Note :** $e \approx 2.71$ is called Napier's constant and $\pi \approx 3.14$ are irrational numbers.

(xii) **Real numbers :** Numbers which can be expressed on number line are called real numbers. The complete set of rational and irrational numbers is the set of real numbers and is denoted by R. Thus $R = Q \cup Q^c$.



Real line

All real numbers follow the order property i.e. if there are two distinct real numbers a and b then either $a < b$ or $a > b$.

**Note :**
(a) Integers are rational numbers, but converse need not be true.
(b) Negative of an irrational number is an irrational number.
(c) Sum of a rational number and an irrational number is always an irrational number
e.g. $2 + \sqrt{3}$
(d) The product of a non zero rational number & an irrational number will always be an irrational number
(e) If $a \in Q$ and $b \notin Q$, then ab = rational number, only if a = 0.
(f) Sum, difference, product and quotient of two irrational numbers need not be a irrational number or we can say, result may be a rational number also.

(xiii) **Complex number :** A number of the form a + ib is called a complex number, where $a, b \in R$ and $i = \sqrt{-1}$. Complex number is usually denoted by Z and the set of complex number is represented by C. Thus $C = \{a + ib : a, b \in R \text{ and } i = \sqrt{-1}\}$

**Note :** It may be noted that $N \subset W \subset I \subset Q \subset R \subset C$.

**Divisibility test :**

| S.No. | Divisibility of | Test |
|---|---|---|
| 1 | 2 | The digit at the unit place of the number is divisible by 2. |
| 2 | 3 | The sum of digits of the number is divisible by 3. |
| 3 | 4 | The last two digits of the number together are divisible by 4. |
| 4 | 5 | The digit of the number at the unit place is either 0 or 5. |
| 5 | 6 | The digit at the unit place of the number is divisible by 2 & the sum of all digits of the number is divisible by 3. |
| 6 | 8 | The last 3 digits of the number all together are divisible by 8. |
| 7 | 9 | The sum of all it's digits is divisible by 9. |
| 8 | 10 | The digit at unit place is 0. |
| 9 | 11 | The difference between the sum of the digits at even places and the sum of digits at odd places is 0 or multiple of 11. e.g.1298, 1221, 123321, 12344321, 1234554321, 123456654321 |





**Remainder theorem :** Let p(x) be any polynomial of degree greater than or equal to one and 'a' be any real number. If p(x) is divided by (x – a), then the remainder is equal to p(a).

**Factor theorem :** Let p(x) be a polynomial of degree greater than or equal to 1 and 'a' be a real number such that p(a) = 0, then (x – a) is a factor of p(x). Conversely, if (x – a) is a factor of p(x), then p(a) = 0.

**Example # 1 :** Show that (x + 3) is a factor of the polynomial $x^3 + 3x^2 + 4x + 12$.

**Solution.** Let $p(x) = x^3 + 3x^2 + 4x + 12$ be the given polynomial. By factor theorem, (x – a) is a factor of a polynomial p(x) iff p(a) = 0. Therefore, in order to prove that x + 3 is a factor of p(x), it is sufficient to show that p(–3) = 0. Now, $p(x) = x^3 + 3x^2 + 4x + 12$
$\Rightarrow$ $p(-3) = -27 + 27 - 12 + 12 = 0$
Hence, (x + 3) is a factor of $p(x) = x^3 + 3x^2 + 4x + 12$.

**Example # 2 :** Without actual division prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

**Solution.** Let $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$ and $g(x) = x^2 - 3x + 2$ be the given polynomials. Then
$g(x) = x^2 - 3x + 2 = x^2 - 2x - x + 2 = x(x - 2) - 1(x - 2)$
$= (x - 1)(x - 2)$
In order to prove that f(x) is exactly divisible by g(x), it is sufficient to prove that x – 1 and x – 2 are factors of f(x). For this it is sufficient to prove that f(1) = 0 and f(2) = 0.
Now, $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$ $\Rightarrow$ $f(1) = 2 \times 1^4 - 6 \times 1^3 + 3 \times 1^2 + 3 \times 1 - 2$
$f(1) = 0$
and, $f(2) = 2 \times 2^4 - 6 \times 2^3 + 3 \times 2^2 + 3 \times 2 - 2$
$f(2) = 0$
$\Rightarrow$ Hence (x – 1) and (x – 2) are factors of f(x).
$\Rightarrow$ g(x) = (x – 1) (x – 2) is a factors of f(x).
Hence f(x) is exactly divisible by g(x).

**Example # 3 :** The polynomials $P(x) = kx^3 + 3x^2 - 3$ and $Q(x) = 2x^3 - 5x + k$, when divided by (x – 4) leave the same remainder. The value of k is
(A) 2 (B) 1 (C) 0 (D) –1

**Solution.** $P(4) = 64k + 48 - 3 = 64k + 45$
$Q(4) = 128 - 20 + k = k + 108$
given P(4) = Q(4)
$\therefore$ $64k + 45 = k + 108$ $\Rightarrow$ $63k = 63 \Rightarrow k = 1$

**Example # 4 :** If a two-digit number is divided by the number having same digits written in reverse order, we get 4 as quotient and 3 as remainder and if the number is divided by the sum of the digits then 8 as a quotient and 7 as a remainder is obtained. Find the number.

**Solution.** Let 10x+y be the required number.
$\therefore$ $10x+y = 4(10y+x) + 3$................. (i)
and $10x+y = 8(x+y) + 7$,......................(ii)
on solving (i) and (ii)
we get x=7, y=1
$\therefore$ the number is equal to 71

**Self practice problems :**

(1) Determine the remainder when the polynomial $P(x) = x^4 - 3x^2 + 2x + 1$ is divided by x – 1

(2) Find the value of a, if x – a is a factor of $x^3 - a^2x + x + 2$.

(3) Using factor theorem, show that a – b, b – c and c – a are the factors of $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

(4) A polynomial in x of the third degree which will vanish when x = 1 & x = –2 and will have the values 4 & 28 when x = –1 and x = 2 respectively is ______.

**Answers :** (1) 1 (2) a = – 2 (4) $f(x) = 3x^3 + 4x^2 - 5x - 2$

# Fundamentals of Mathematics-I

*He is unworthy of the name of man who is ignorant of the fact that the diagonal of square is incommensurable with its side .......Plato*

## Number system :

**(i)** **Natural numbers :** The counting numbers 1, 2, 3, 4, .... are called Natural Numbers. The set of natural numbers is denoted by N. Thus N = {1, 2, 3, 4, ....}.

**(ii)** **Whole numbers :** Natural numbers including zero are called whole numbers. The set of whole numbers is denoted by W. Thus W = {0, 1, 2, .........}

**(iii)** **Integers :** The numbers ... – 3, – 2, – 1, 0, 1, 2, 3 .... are called integers and the set is denoted by I or Z. Thus I (or Z) = {.. – 3, – 2, – 1, 0, 1, 2, 3...}

**Note :** (a) Positive integers $I^+$ = {1, 2, 3 ....} = N
(b) Negative integers $I^-$ = {....., –3, –2, –1}.
(c) Non-negative integers (whole numbers) = {0, 1, 2, ......}.
(d) Non-positive integers = {......, –3, –2, –1, 0}.

**(iv)** **Even Integers :** Integers which are divisible by 2 are called even integers.
e.g. 0, ± 2, ± 4,.......

**(v)** **Odd Integers :** Integers which are not divisible by 2 are called odd integers.
e.g. ± 1, ± 3, ± 5, ± 7......

**(vi)** **Prime numbers :** Natural numbers which are divisible by 1 and itself only are called prime numbers.
e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ........

**(vii)** **Composite number :** Let 'a' be a natural number, 'a' is said to be composite if, it has atleast three distinct factors.
e.g. 4, 6, 8, 9, 10, 12, 14, 15 .........

**Note :** (i) 1 is neither a prime number nor a composite number.
(ii) Numbers which are not prime are composite numbers (except 1).
(iii) '4' is the smallest composite number.
(iv) '2' is the only even prime number.

**(viii)** **Co-prime numbers :** Two natural numbers (not necessarily prime) are called coprime, if there H.C.F (Highest common factor) is one.
e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) (15, 16) etc.
These numbers are also called as **relatively prime** numbers.

**Note :** (a) Two prime number(s) are always co-prime but converse need not be true.
(b) Consecutive natural numbers are always co-prime numbers.

**(ix)** **Twin prime numbers :** If the difference between two prime numbers is two, then the numbers are called twin prime numbers.
e.g. {3, 5}, {5, 7}, {11, 13}, {17, 19}, {29, 31}

**Note :** Number between twin prime numbers is divisible by 6 (except (3, 5)).

**(x)** **Rational numbers :** All the numbers that can be represented in the form p/q, where p and q are integers and q ≠ 0, are called rational numbers and their set is denoted by Q. Thus $Q = \{\frac{p}{q} : p, q \in I \text{ and } q \neq 0\}$. It may be noted that every integer is a rational number since it can be written as p/1. It may be noted that all recurring decimals are rational numbers.

**Note :** Maximum number of different decimal digits in $\frac{p}{q}$ is equal to q, i.e. $\frac{11}{9}$ will have maximum of 9 different decimal digits.