Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. (i) 9 (ii) 20 (iii) 72 (iv) 326592

A-2. 90

A-3. (i) 40320 (ii) 37440 (iii) 2880 (iv) 1152 (v) 1152

A-4. (i) 4320 (ii) 720 (iii) 4320 (iv) 14400 (v) 14400 (vi) 2880 (vii) 6720 (viii) 720 (ix) 3600

A-5.	5 : 2	**A-6.**	200	**A-7.**	50
A-8.	$7.\,^7P_3$	**A-9.**	$^{10}P_4$	**A-10.**	738
A-11.	9999	**A-12.**	65	**A-13.**	1170
A-14.	91	**A-15.**	154	**A-16.**	886656
A-17.	229	**A-18.**	7350		

A-19. (i) 4^n (ii) 3^n (iii) $\frac{3^n-1}{2}+1$ (iv) 2^n (v) 2^{n-1} (vi) $^nC_1 . 3^{n-1}$

A-20. 63 **A-21.** 20 **A-22.** 62784

A-23. 280

Section (B) :

B-1. (i) 18 (ii) 23 (iii) 4

B-2. (i) 479 (ii) 256 (iii) 6 (iv) 473

B-3. 45 **B-4.** 36

Section (C) :

C-1.	12	**C-2.**	144	**C-3.**	48
C-4.	1	**C-5.**	55	**C-6.**	37

C-7. (i) $^{23}C_3$ (ii) $^{19}C_3$ (iii) $^{18}C_3 - 4.\,^9C_3$ (iv) $^{11}C_8$

C-8. 685

Section (D) :

D-1. $\frac{18!}{(3!)^4.(2!)^3\,4!\,3!}$ **D-2.** 360360 **D-3.** 70

D-4. (a) 25 (b) 150 (c) 270000

D-5. 90

D-7. (i) 8 (ii) 10

D-8. (i) 150, (ii) 6 (iii) 25 (iv) 2

D-9. (a) 44 (b) 109

D-10. 126

PART - II

Section (A) :

A-1.	(C)	**A-2.**	(D)	**A-3.**	(D)
A-4.	(B)	**A-5.**	(C)	**A-6.**	(A)
A-7.	(A)	**A-8.**	(D)	**A-9.**	(C)
A-10.	(C)	**A-11.**	(D)	**A-12.**	(A)
A-13.	(B)	**A-14.**	(C)	**A-15.**	(A)
A-16.	(A)	**A-17.**	(A)	**A-18.**	(B)

Section (B) :

B-1.	(D)	**B-2.**	(A)	**B-3.**	(A)
B-4.	(B)	**B-5.**	(A)		

Section (C) :

C-1.	(C)	**C-2.**	(C)	**C-3.**	(A)
C-4.	(A)	**C-5.**	(A)	**C-6.**	(A)
C-7.	(A)	**C-8.**	(C)		

Section (D) :

D.1	(B)	**D-2.**	(C)	**D-3.**	(D)
D-4.	(B)	**D-5.**	(D)		

PART - III

1. (A) → (r), (B) → (p), (C) → (q), (D) → (s)
2. (A) - (q) ; (B) - (p) ; (C) - (s) ; (D) - (r)

EXERCISE - 2

PART - I

1.	(A)	**2.**	(A)	**3.**	(C)
4.	(B)	**5.**	(D)	**6.**	(C)
7.	(A)	**8.**	(C)	**9.**	(C)
10.	(D)	**11.**	(C)	**12.**	(B)
13.	(A)	**14.**	(B)	**15.**	(A)
16.	(A)	**17.**	(A)	**18.**	(C)

PART - II

1.	15	**2.**	7	**3.**	0
4.	9	**5.**	10	**6.**	0
7.	48	**8.**	10	**9.**	2
10.	18	**11.**	42	**12.**	4
13.	21	**14.**	98	**15.**	12
16.	20	**17.**	10	**18.**	8
19.	1				

PART - III

1.	(CD)	**2.**	(AB)	**3.**	(ACD)
4.	(CD)	**5.**	(AB)	**6.**	(BC)
7.	(ABCD)	**8.**	(ABC)	**9.**	(ABCD)
10.	(AD)	**11.**	(BD)	**12.**	(BCD)
13.	(AB)	**14.**	(ABCD)	**15.**	(BCD)

PART - IV

1.	(A)	**2.**	(D)	**3.**	(B)
4.	(A)	**5.**	(D)	**6.**	(D)

EXERCISE - 3

PART - I

1. (C) **2.** (C)

3. (A) → (p), (B) → (s), (C) → (q), (D) → (q)

4.	(C)	**5.**	(D)	**6.**	(B)
7.	(A)	**8.**	(B)	**9.**	(7)
10.	(5)	**11.**	(C)	**12.**	5
13.	(A)				

PART - II

1.	(4)	**2.**	(3)	**3.**	(4)
4.	(4)	**5.**	(3)	**6.**	(1)
7.	(1)	**8.**	(4)	**9.**	(2)
10.	(2)	**11.**	(3)	**12.**	(1)

8. You are given 8 balls of different colour (black, white,...). The number of ways in which these balls can be arranged in a row so that the two balls of particular colour (say red & white) may never come together is:
(A) $8! - 2.7!$ (B) $6.7!$ (C) $2.6!.{}^7C_2$ (D) none

9. Consider the word 'MULTIPLE' then in how many other ways can the letters of the word 'MULTIPLE' be arranged ;
(A) without changing the order of the vowels equals 3359
(B) keeping the position of each vowel fixed equals 59
(C) without changing the relative order/position of vowels & consonants is 359
(D) using all the letters equals $4\cdot7! - 1$

10. The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if the letter C are separated from one another is:
(A) ${}^{13}C_3 \cdot \frac{12!}{5!\,3!\,2!}$ (B) $\frac{13!}{5!\,3!\,3!\,2!}$ (C) $\frac{14!}{3!\,3!\,2!}$ (D) $11.\frac{13!}{6!}$

11. The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \le n$ (where n is a positive integer) is
(A) ${}^{n+3}C_3$ (B) ${}^{n+4}C_4$ (C) ${}^{n+5}C_5$ (D) ${}^{n+4}C_n$

12. There are 10 seats in the first row of a theatre of which 4 are to be occupied. The number of ways of arranging 4 persons so that no two persons sit side by side is:
(A) ${}^7C_4$ (B) $4.{}^7P_3$ (C) ${}^7C_3.4!$ (D) 840

13. ${}^{50}C_{36}$ is divisible by
(A) 19 (B) 5^2 (C) 19^2 (D) 5^3

14. ${}^{2n}P_n$ is equal to
(A) $(n + 1)(n + 2) \ldots (2n)$ (B) $2^n [1 . 3 . 5 \ldots (2n - 1)]$
(C) $(2) . (6) . (10) \ldots (4n - 2)$ (D) $n! ({}^{2n}C_n)$

15. The number of ways in which 200 different things can be divided into groups of 100 pairs, is:
(A) $\frac{200!}{2^{100}}$ (B) $\left(\frac{101}{2}\right)\left(\frac{102}{2}\right)\left(\frac{103}{2}\right)\ldots\left(\frac{200}{2}\right)$
(C) $\frac{200!}{2^{100}\,(100)!}$ (D) (1. 3. 5...... 199)

PART - IV : COMPREHENSION

Comprehension # 1

There are 8 official and 4 non-official members, out of these 12 members a committee of 5 members is to be formed, then answer the following questions.

1. Number of committees consisting of at least two non-official members, are
(A) 456 (B) 546 (C) 654 (D) 466

2. Number of committees in which a particular official member is never included, are
(A) 264 (B) 642 (C) 266 (D) 462

Comprehenssion # 2

Let n be the number of ways in which the letters of the word "RESONANCE" can be arranged so that vowels appear at the even places and m be the number of ways in which "RESONANCE" can be arrange so that letters R, S, O, A, appear in the order same as in the word RESONANCE, then answer the following questions.

3. The value of n is
(A) 360 (B) 720 (C) 240 (D) 840

4. The value of m is
(A) 3780 (B) 3870 (C) 3670 (D) 3760

Comprehension # 3

A mega pizza is to be sliced n times, and S_n denotes maximum possible number of pieces.

5. Relation between S_n & S_{n-1}
(A) $S_n = S_{n-1} + n + 3$ (B) $S_n = S_{n-1} + n + 2$ (C) $S_n = S_{n-1} + n + 2$ (D) $S_n = S_{n-1} + n$

6. If the mega pizza is to be distributed among 60 person, each one of them get atleast one piece then minimum number of ways of slicing the mega pizza is :
(A) 10 (B) 9 (C) 8 (D) 11

Exercise-3

Marked Questions may have for Revision Questions.
* Marked Questions may have more than one correct option.

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is **[IIT-JEE-2006, (3, –1), 184]**
(A) 252 (B) 254 (C) 225 (D) 224

2. The letters of the word **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an english dictionary. Then number of words that appear before the word **COCHIN** is **[IIT-JEE-2007,P-II, (3, – 1), 81]**
(A) 360 (B) 192 (C) 96 (D) 48

3. Consider all possible permutations of the letters of the word ENDEANOEL. Match the Statements/ Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS. **[IIT-JEE-2008,P-II, (6, 0), 81]**

Column I		Column II	
(A)	The number of permutations containing the word ENDEA is 5×4!	(p)	5! A
(B)	The number of permutations in which the letter E occurs in the first and the last positions is	(q)	2 × 5! C D
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(r)	7 × 5! X
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	(s)	21 × 5! B

4. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is 2 cases **[IIT-JEE-2009, Paper-I, (3, – 1), 240]**
(A) 55 (B) 66 (C) 77 (D) 88

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Number of five digits numbers divisible by 3 that can be formed using the digits 0, 1, 2, 3, 4, 7 and 8 if, each digit is to be used atmost one is N where 'N' is three digit number then sum of the digits of 'N'

2. The sides AB, BC & CA of a triangle ABC have 3, 4 & 5 interior points respectively on them. If the number of triangles that can be constructed using these interior points as vertices is k, then sum of digits in the number k equals. 205 → 2+0+5=7

3. Shubham has to make a telephone call to his friend Nisheeth, Unfortunately he does not remember the 7 digit phone number. But he remembers that the first three digits are 635 or 674, the number is odd and there is exactly one 9 in the number. The maximum number of trials that Shubham has to make to be successful is a four digit number of the form abcd then c equal. 2106

4. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways in which the division may be made is k, then sum of the digits in number of k equals. 630 → 6+3+0 = 9

5. Number of ways in which five vowels of English alphabets and ten decimal digits can be placed in a row such that between any two vowels odd number of digits are placed and both end places are occupied by vowels is 20(b!)(5!) then b equals.

6. The number of integers which lie between 1 and 10^6 and which have the sum of the digits equal to 12 is N where 'N' is a four digit number of the form abcd then (a – c) equals.

7. The number of ways in which 8 non-identical apples can be distributed among 3 boys such that every boy should get atleast 1 apple & atmost 4 apples is a four digit number of the form pqrs then p.q.r equals.

8. In a hockey series between team X and Y, they decide to play till a team wins '10' match. Then the number of ways in which team X wins is $\frac{{}^{20}C_m}{2}$ then m equals.

9. In a shooting competition a man can score 0, 2 or 4 points for each shot. Then the number of different ways in which he can score 14 points in 5 shots, is N then number of digits in 'N' equals 30 → 2 digit

10. Six persons A, B, C, D, E and F are to be seated at a circular table. The number of ways this can be done if A must have either B or C on his right and B must have either C or D on his right, is:

11. The number of permutations which can be formed out of the letters of the word "SERIES" taking three letters together, is: 42

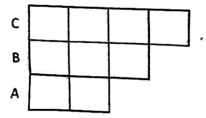
12. A box contains 6 balls which may be all of different colours or three each of two colours or two each of three different colours. The number of ways of selecting 3 balls from the box (if ball of same colour are identical), is N then sum of the digits in the number 'N' equals

13. Five friends F_1, F_2, F_3, F_4, F_5 book five seats C_1, C_2, C_3, C_4, C_5 respectively of movie KABIL independently (i.e. F_1 books C_1, F_2 books C_2 and so on). In how many different ways can they sit on these seats if no one wants to sit on his booked seat, more over F_1 and F_2 want to sit adjacent to each other.

14. The number of ways in which 5 X's can be placed in the squares of the figure so that no row remains empty is:

X – identical

Select only



98

15. Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4, is N then sum of the digit of the number 'N' equals 12

16. Six married couple are sitting in a room. Number of ways in which 4 people can be selected so that there is exactly one married couple among the four is N then number of divisors of N equals 20

240

17. Let P_n denotes the number of ways of selecting 3 people out of 'n' sitting in a row if no two of them are consecutive and Q_n is the corresponding figure when they are in a circle. If $P_n - Q_n = 6$, then 'n' is equal to:

18. The number of ways selecting 8 books from a library which has 10 books each of Mathematics, Physics, Chemistry and English, if books of the same subject are alike, is N then number of divisors of N.

19. The number of three digit numbers of the form xyz such that x < y and z ≤ y is N such that N is a three digit number of the form abc then (a + c – b) equals. 276 → 2+6−7 = 1

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. In an examination, a candidate is required to pass in all the four subjects he is studying. The number of ways in which he can fail is
(A) ${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$ (B) $4^4 - 1$
(C) $2^4 - 1$ (D) ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

2. The kindergarten teacher has 25 kids in her class. She takes 5 of them at a time, to zoological garden as often as she can, without taking the same 5 kids more than once. Then the number of visits, the teacher makes to the garden exceeds that of a kid by:
(A) ${}^{25}C_5 - {}^{24}C_4$ (B) ${}^{24}C_5$ (C) ${}^{25}C_5 - {}^{24}C_5$ (D) ${}^{24}C_4$

3. A student has to answer 10 out of 13 questions in an examination. The number of ways in which he can answer if he must answer atleast 3 of the first five questions is:
(A) 276 (B) 267
(C) ${}^{13}C_{10} - {}^5C_3$ (D) ${}^5C_3 . {}^8C_7 + {}^5C_4 . {}^8C_6 + {}^8C_5$

4. Number of ways in which 3 different numbers in A.P. can be selected from 1, 2, 3,...... n is:
(A) $\frac{(n-2)(n-4)}{4}$ if n is even (B) $\frac{n^2-4n+5}{2}$ if n is odd
(C) $\frac{(n-1)^2}{4}$ if n is odd (D) $\frac{n(n-2)}{4}$ if n is even

5. 2m white identical coins and 2n red identical coins are arranged in a straight line with (m + n) identical coins on each side of a central mark. The number of ways of arranging the identical coins, so that the arrangements are symmetrical with respect to the central mark.
(A) ${}^{m+n}C_m$ (B) ${}^{m+n}C_n$ (C) ${}^{m+n}C_{|m-n|}$ (D) ${}^{m+n}C_{|n-m|}$

6. The number of ways in which 10 students can be divided into three teams, one containing 4 and others 3 each, is
(A) $\frac{10!}{4!3!3!}$ (B) 2100 (C) ${}^{10}C_4 . {}^5C_3$ (D) $\frac{10!}{6!3!3!} . \frac{1}{2}$

7. If all the letters of the word 'AGAIN' are arranged in all possible ways & put in dictionary order, then
(A) The 50th word is NAAIG (B) The 49th word is NAAGI
(C) The 51st word is NAGAI (D) The 47th word is INAGA

2. Match the column

Column-I		Column-II
(A) There are 12 points in a plane of which 5 are collinear. The number of distinct convex quadrilaterals which can be formed with vertices at these points is:	(p)	185
(B) If 7 points out of 12 are in the same straight line, then the number of triangles formed is	(q)	420
(C) If AB and AC be two line segemets and there are 5, 4 points on AB and AC (other than A), then the number of quadrilateral, with vertices on these points equals	(r)	126
(D) The maximum number of points of intersection of 8 unequal circles and 4 straight lines	(s)	60

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. A train is going from London to Cambridge stops at 12 intermediate stations. 75 persons enter the train after London with 75 different tickets of the same class. Number of different sets of tickets they may be holding is:
(A) $^{78}C_3$ (B) $^{91}C_{75}$ (C) $^{84}C_{75}$ (D) $^{78}C_{74}$

2. A family consists of a grandfather, m sons and daughters and 2n grand children. They are to be seated in a row for dinner. The grand children wish to occupy the n seats at each end and the grandfather refuses to have a grand children on either side of him. In how many ways can the family be made to sit.
(A) $(2n)!\, m!\, (m-1)$ (B) $(2n)!\, m!\, m$ (C) $(2n)!\, (m-1)!\, (m-1)$ (D) $(2n-1)!\, m!\, (m-1)$

3. A bouquet from 11 different flowers is to be made so that it contains not less than three flowers. Then then number of different ways of selecting flowers to form the bouquet.
(A) 1972 (B) 1952 (C) 1981 (D) 1947

4. If $\alpha = x_1 x_2 x_3$ and $\beta = y_1 y_2 y_3$ be two three digit numbers, then the number of pairs of α and β that can be formed so that α can be subtracted from β without borrowing.
(A) $55 . (45)^2$ (B) $45 . (55)^2$ (C) $36 . (45)^2$ (D) 55^3

5. 'n' digits positive integers formed such that each digit is 1, 2, or 3. How many of these contain all three of the digits 1, 2 and 3 atleast once ?
(A) $3(n-1)$ (B) $3^n - 2.2^n + 3$ (C) $3^n - 3.2^n - 3$ (D) $3^n - 3.2^n + 3$

6. There are 'n' straight line in a plane, no two of which are parallel and no three pass through the same point. Their points of intersection are joined. Then the maximum number of fresh lines thus introduced is
(A) $\frac{1}{12} n(n-1)^2(n-3)$ (B) $\frac{1}{8} n(n-1)(n+2)(n-3)$
(C) $\frac{1}{8} n(n-1)(n-2)(n-3)$ (D) $\frac{1}{8} n(n+1)(n+2)(n-3)$

7. $X = \{1, 2, 3, 4, ... 2017\}$ and $A \subset X$; $B \subset X$; $A \cup B \subset X$ here $P \subset Q$ denotes that P is subset of Q($P \neq Q$). Then number of ways of selecting unordered pair of sets A and B such that $A \cup B \subset X$.
(A) $\frac{(4^{2017} - 3^{2017}) + (2^{2017} - 1)}{2}$ (B) $\frac{(4^{2017} - 3^{2017})}{2}$
(C) $\frac{4^{2017} - 3^{2017} + 2^{2017}}{2}$ (D) None of these

8. The number of ways in which 15 identical apples & 10 identical oranges can be distributed among three persons, each receiving none, one or more is:
(A) 5670 (B) 7200 (C) 8976 (D) 7296

9. The number of ways in which a mixed double tennis game can be arranged from amongst 9 married couple if no husband & wife plays in the same game is:
(A) 756 (B) 3024 (C) 1512 (D) 6048 $^4C_2 \times {}^7C_2 \times 2$

10. Two variants of a test paper are distributed among 12 students. Number of ways of seating of the students in two rows so that the students sitting side by side do not have identical papers & those sitting in the same column have the same paper is: 2 arrangement for test paper 12! for students
(A) $\frac{12!}{6!\,6!}$ (B) $\frac{(12)!}{2^5 . 6!}$ (C) $(6!)^2 . 2$ (D) $12! \times 2$

11. If n identical dice are rolled, then number of possible out comes are.
(A) 6^n (B) $\frac{6^n}{n!}$ (C) $^{(n+5)}C_5$ (D) None of these

12. Number of ways in which 2 Indians, 3 Americans, 3 Italians and 4 Frenchmen can be seated on a circle, if the people of the same nationality sit together, is:
(A) $2.(4!)^2(3!)^2$ (B) $2.(3!)^3.4!$ (C) $2.(3!)(4!)^3$ (D) $2.(3!)^2(4!)^3$

13. Number of ways in which a pack of 52 playing cards be distributed equally among four players so that each have the Ace, King, Queen and Jack of the same suit, is
(A) $\frac{36!\,.\,4!}{(9!)^4}$ (B) $\frac{36!}{(9!)^4}$ (C) $\frac{52!\,.\,4!}{(13!)^4}$ (D) $\frac{52!}{(13!)^4}$

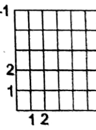
14. Seven person $P_1, P_2, ..., P_7$ initially seated at chairs $C_1, C_2 ...., C_7$ respectively. They all left there chairs symultaneously for hand wash. Now in how many ways they can again take seats such that no one sits on his own seat and P_1 sits on C_2 and P_2 sits on C_3 ?
(A) 52 (B) 53 (C) 54 (D) 55

15. Given six line segments of length $\boxed{2, 3, 4, 5, 6, 7}$ units, the number of triangles that can be formed by these segments is
(A) $^6C_3 - 7$ (B) $^6C_3 - 6$ (C) $^6C_3 - 5$ (D) $^6C_3 - 4$

16. There are m apples and n oranges to be placed in a line such that the two extreme fruits being both oranges. Let P denotes the number of arrangements if the fruits of the same species are different and Q the corresponding figure when the fruits of the same species are alike, then the ratio P/Q has the value equal to :
(A) $^nP_2 . {}^mP_m . (n-2)!$ (B) $^mP_2 . {}^nP_n . (n-2)!$ (C) $^nP_2 . {}^nP_n . (m-2)!$ (D) none

17. The number of intersection points of diagonals of 2009 sides regular polygon, which lie inside the polygon.
(A) $^{2009}C_4$ (B) $^{2009}C_2$ (C) $^{2008}C_4$ (D) $^{2008}C_2$

18. A rectangle with sides $2m - 1$ and $2n - 1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is



$^nP_r = \frac{n!}{(n-r)!}$ $^nC_r = \frac{n!}{r!(n-r)!}$ $^nC_n = 1$

(A) $(m + n - 1)^2$ (B) 4^{m+n-1} (C) $m^2 n^2$ (D) $m(m + 1)n(n + 1)$

Resonance® Educating for better tomorrow
Corporate Office: CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005
Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in
Toll Free : 1800 258 5555 | CIN: U80302RJ2007PLC024029

A-13. A box contains 2 white balls, 3 black balls & 4 red balls. In how many ways can three balls be drawn from the box if atleast one black ball is to be included in draw (the balls of the same colour are different).
(A) 60 (B) 64 (C) 56 (D) none

A-14. Eight cards bearing number 1, 2, 3, 4, 5, 6, 7, 8 are well shuffled. Then in how many cases the top 2 cards will form a pair of twin prime equals
(A) 720 (B) 1440 (C) 2880 (D) 2160

A-15. Number of natural number upto one lakh, which contains 1,2,3, exactly once and remaining digits any time is -
(A) 2940 (B) 2850 (C) 2775 (D) 2680

A-16. The sum of all the four digit numbers which can be formed using the digits 6,7,8,9 (repetition is allowed)
(A) 2133120 (B) 2133140 (C) 2133150 (D) 2133122

A-17. If the different permutations of the word *'EXAMINATION'* are listed as in a dictionary, then how many words (with or without meaning) are there in this list before the first word starting with M.
(A) 2268000 (B) 870200 (C) 807400 (D) 839440

A-18. Out of 16 players of a cricket team, 4 are bowlers and 2 are wicket keepers. A team of 11 players is to be chosen so as to contain at least 3 bowlers and at least 1 wicketkeeper. The number of ways in which the team be selected, is
(A) 2400 (B) 2472 (C) 2500 (D) 960

Section (B) : Problem based on distinct and identical objects and divisors

B-1. The number of proper divisors of $a^p b^q c^r d^s$ where a, b, c, d are primes & p, q, r, s ∈ N, is
(A) pqrs (B) (p + 1) (q + 1) (r + 1) (s + 1) – 4
(C) pqrs–2 (D) (p + 1) (q + 1) (r + 1) (s + 1) – 2

B-2. N is a least natural number having 24 divisors. Then the number of ways N can be resolved into two factors is
(A) 12 (B) 24 (C) 6 (D) None of these

B-3. How many divisors of 21600 are divisible by 10 but not by 15?
(A) 10 (B) 30 (C) 40 (D) none

B-4. The number of ways in which the number 27720 can be split into two factors which are co-primes, is:
(A) 15 (B) 16 (C) 25 (D) 49

B-5. The number of words of 5 letters that can be made with the letters of the word **"PROPOSITION"**.
(A) 6890 (B) 7000 (C) 6800 (D) 6900

Section (C) : Problem based on circular arrangement and Multinomial theorem

C-1. The number of ways in which 8 different flowers can be strung to form a garland so that 4 particulars flowers are never separated, is:
(A) 4!. 4! (B) $\frac{8!}{4!}$ (C) 288 (D) none

C-2. The number of ways in which 6 red roses and 3 white roses (all roses different) can form a garland so that all the white roses come together, is
(A) 2170 (B) 2165 (C) 2160 (D) 2155

C-3. The number of ways in which 4 boys & 4 girls can stand in a circle so that each boy and each girl is one after the other, is:
(A) 3!. 4! (B) 4!. 4! (C) 8! (D) 7!

C-4. The number of ways in which 10 identical apples can be distributed among 6 children so that each child receives atleast one apple is :
(A) 126 (B) 252 (C) 378 (D) none of these

circular

C-5. The number of ways in which 5 beads, chosen from 8 different beads be threaded on to a ring, is:
(A) 672 (B) 1344 (C) 336 (D) none

C-6. Number of ways in which 3 persons throw a normal die to have a total score of 11, is
(A) 27 (B) 25 (C) 29 (D) 18

C-7. If chocolates of a particular brand are all identical then the number of ways in which we can choose 6 chocolates out of 8 different brands available in the market, is:.
(A) $^{13}C_6$ (B) $^{13}C_8$ (C) 8^6 (D) none

C-8. Number of positive integral solutions of $x_1 . x_2 . x_3 = 30$, is
(A) 25 (B) 26 (C) 27 (D) 28

Section (D) : Problem based on geometry / Dearrangement / exponent of prime/ Principal of exculusion/Grouping

D-1. Passengers are to travel by a double decked bus which can accommodate 13 in the upper deck and 7 in the lower deck. The number of ways that they can be divided if 5 refuse to sit in the upper deck and 8 refuse to sit in the lower deck, is
(A) 25 (B) 21 (C) 18 (D) 15

D-2. Number of ways in which 9 different toys be distributed among 4 children belonging to different age groups in such a way that distribution among the 3 elder children is even and the youngest one is to receive one toy more, is:
(A) $\frac{(5!)^2}{8}$ (B) $\frac{9!}{2}$ (C) $\frac{9!}{3!\,(2!)^3}$ (D) none

D-3. There are six letters $L_1, L_2, L_3, L_4, L_5, L_6$ and their corresponding six envelopes $E_1, E_2, E_3, E_4, E_5, E_6$. Letters having odd value can be put into odd value envelopes and even value letters can be put into even value envelopes, so that no letter go into the right envelopes, then number of arrangement equals.
(A) 6 (B) 9 (C) 44 (D) 4

D-4. How many ways can atleast 2 fruit be selected out of 5 Mangoes, 4 Apples, 3 Bananas and three different fruits.
(A) 959 (B) 953 (C) 960 (D) 954

D-5. The streets of a city are arranged like the lines of a chess board. There are m streets running North to South & 'n' streets running East to West. The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is:
(A) $\sqrt{m^2 + n^2}$ (B) $\sqrt{(m-1)^2 . (n-1)^2}$ (C) $\frac{(m+n)!}{m!\,.\,n!}$ (D) $\frac{(m+n-2)!}{(m-1)!\,.\,(n-1)!}$

PART - III : MATCH THE COLUMN

1. Match the column

Column – I		Column – II	
(A)	The total number of selections of fruits which can be made from, 3 bananas, 4 apples and 2 oranges is, it is given that fruits of one kind are identical	(p)	120 B
(B)	There are 10 true-false statements in a question paper. How many sequences of answers are possible in which exactly three are correct ? TTT FFFFFFF	(q)	286 C
(C)	The number of ways of selecting 10 balls from unlimited number of red, black, white and green balls is, it is given that balls of same colours are identical	(r)	59 A
(D)	The number of words which can be made from the letters of the word 'MATHEMATICS' so that consonants occur together ?	(s)	75600 D

C-4. In how many ways four '+' and five '–' sign can be arranged in a circles so that no two '+' sign are together.

C-5. Find number of negative integral solution of equation $x + y + z = -12$

C-6. In how many ways it is possible to divide six identical green, six identical blue and six identical red among two persons such that each gets equal number of item?

C-7. Find the number of solutions of $x + y + z + w = 20$ under the following conditions:
(i) x, y, z, w are whole number
(ii) x, y, z, w are natural number
(iii) x, y, z, w $\in$ {1, 2, 3, ..., 10}
(iv) x, y, z, w are odd natural number

C-8. Find total number of positive integral solutions of $15 < x_1 + x_2 + x_3 \le 20$.

Section (D) : Problem based on geometry / Dearrangement / exponent of prime/ Principal of exculusion/Grouping

D-1. In how many ways 18 different objects can be divided into 7groups such that four groups contains 3 objects each and three groups contains 2 objects each.

D-2. In how many ways fifteen different items may be given to A, B, C such that A gets 3, B gets 5 and remaining goes to C.

D-3. Find number of ways of distributing 8 different items equally among two children.

D-4. (a) In how many ways can five people be divided into three groups?
(b) In how many ways can five people be distributed in three different rooms if no room must be empty?
(c) In how many ways can five people be arranged in three different rooms if no room must be empty and each room has 5 seats in a single row.

D-5. Three ladies have brought one chlid each for admission to a school. The principal wants to interview the six persons one by one subject to the condition that no mother is interviewed before her chlid. Then find the number of ways in which interviews can be arranged

D-6. Prove that : $\dfrac{200!}{(10!)^{20}\,19!}$ is an integer no of ways in which 200 obj can be dist into 20 groups of 10 obj

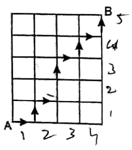
$\dfrac{200!}{(10!)^{20} \times 19!} \times 20$ ∴ integer $\dfrac{20(200)!}{(10!)^{20}\,20!}$

D-7. (i) Find exponent of 3 in 20 !
(ii) Find number of zeros at the end of 45!.

D-8. Five balls are to be placed in three boxes in how many diff. ways can be placed the balls so that no box remains empty if
(i) balls and boxes are diff,
(ii) balls identical and boxes diff.
(iii) balls diff. and boxes identical
(iv) balls as well as boxes are identical

D-9. A person writes letters to five friends and addresses on the corresponding envelopes. In how many ways can the letters be placed in the envelopes so that
(a) all letters are in the wrong envelopes?
(b) at least three of them are in the wrong envelopes?

D-10. A person is to walk from A to B. However, he is restricted to walk only to the right of A or upwards of A. but not necessarily in the order shown in the figure. Then find the number of paths from A to B.



$\dfrac{9!}{5!\,4!} = \dfrac{6 \cdot 7 \cdot 8 \cdot 9}{4 \cdot 3 \cdot 2} = 126$

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Fundamental principle of counting, problem based on selection of given object & arrangement of given object, rank of word

A-1. The number of signals that can be made with 3 flags each of different colour by hoisting 1 or 2 or 3 above the other, is: Permute (Arrange)
(A) 3 (B) 7 (C) 15 (D) 16

A-2. 8 chairs are numbered from 1 to 8. Two women & 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4, then the men select the chairs from among the remaining. The number of possible arrangements is:
(A) ${}^{6}C_3 \cdot {}^{4}C_4$ (B) $P_2 \cdot {}^{4}P_3$ (C) ${}^{4}C_3 \cdot {}^{4}P_3$ (D) ${}^{4}P_2 \cdot {}^{6}P_3$

A-3. Number of words that can be made with the letters of the word "GENIUS" if each word neither begins with G nor ends in S, is:
(A) 24 (B) 240 (C) 480 (D) 504

A-4. The number of words that can be formed by using the letters of the word 'MATHEMATICS' that start as well as end with T, is
(A) 80720 (B) 90720 (C) 20860 (D) 37528

A-5. The number of permutations that can be formed by arranging all the letters of the word '**NINETEEN**' in which no two E's occur together. is
(A) $\dfrac{8!}{3!\,3!}$ (B) $\dfrac{5!}{3! \times {}^{6}C_2}$ (C) $\dfrac{5!}{3!} \times {}^{6}C_3$ (D) $\dfrac{8!}{5!} \times {}^{6}C_3$.

A-6. 5 boys & 3 girls are sitting in a row of 8 seats. Number of ways in which they can be seated so that not all the girls sit side by side, is:
(A) 36000 (B) 9080 (C) 3960 (D) 11600

A-7. 10 different letters of an alphabet are given. Words with 5 letters are formed from these given letters, then the number of words which have atleast one letter repeated is:
(A) 69760 (B) 30240 (C) 99748 (D) none

A-8. In a conference 10 speakers are present. If S_1 wants to speak before S_2 & S_2 wants to speak after S_3, then the number of ways all the 10 speakers can give their speeches with the above restriction if the remaining seven speakers have no objection to speak at any number is :
(A) ${}^{10}C_3$ (B) ${}^{10}P_8$ (C) ${}^{10}P_3$ (D) $\dfrac{10!}{3}$

A-9. If all the letters of the word "QUEUE" are arranged in all possible manner as they are in a dictionary, then the rank of the word QUEUE is:
(A) 15th (B) 16th (C) 17th (D) 18th

A-10. The sum of all the numbers which can be formed by using the digits 1, 3, 5, 7 all at a time and which have no digit repeated, is
(A) 16 × 4! (B) 1111 × 3! (C) 16 × 1111 × 3! (D) 16 × 1111 × 4!.

A-11. How many nine digit numbers can be formed using the digits 2, 2, 3, 3, 5, 5, 8, 8, 8 so that the odd digits occupy even positions?
(A) 7560 (B) 180 (C) 16 (D) 60

A-12. There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. The number of ways in which they can be arranged in a row so that atleast one ball is separated from the balls of the same colour, is :
(A) 6 (7! – 4!) (B) 7 (6! – 4!) (C) 8! – 5! (D) none

Exercise-1

✎ Marked Questions may have for Revision Questions.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Fundamental principle of counting, problem based on selection of given object & arrangement of given object.

A-1. There are nine students (5 boys & 4 girls) in the class. In how many ways
- (i) One student (either girl or boy) can be selected to represent the class.
- (ii) A team of two students (one girl & one boy) can be selected.
- (iii) Two medals can be distributed. (no one get both)
- (iv) One prize for Maths, two prizes for Physics and three prizes for Chemistry can be distributed. (No student can get more than one prize in same subject & prizes are distinct)

A-2. There are 10 buses operating between places A and B. In how many ways a person can go from place A to place B and return to place A, if ~~he returns~~ in a different bus?

A-3. There are 4 boys and 4 girls. In how many ways they can sit in a row
- (i) there is no restriction.
- (ii) not all girls sit together.
- (iii) no two girls sit together.
- (iv) all boys sit together and all girls sit together
- (v) boys and girls sit alternatively.

A-4.✎ Find the number of words those can be formed by using all letters of the word 'DAUGHTER'. If
- (i) Vowels occurs in first and last place.
- (ii) Start with letter G and end with letters H.
- (iii) Letters G,H,T always occurs together.
- (iv) No two letters of G,H,T are consecutive
- (v) No vowel occurs together
- (vi) Vowels always occupy even place.
- (vii) Order of vowels remains same.
- (viii) Relative order of vowels and consonants remains same.
- (ix) Number of words are possible by selecting 2 vowels and 3 consonants.

A-5. Words are formed by arranging the letters of the word "STRANGE" in all possible manner. Let m be the number of words in which vowels do not come together and 'n' be the number of words in which vowels come together. Then find the ratio of m: n.(where m and n are coprime natural number)

A-6. In a question paper there are two parts part A and part B each consisting of 5 questions. In how many ways a student can answer 6 questions, by selecting atleast two from each part?

A-7. How many 3 digit even numbers can be formed using the digits 1, 2, 3, 4, 5 (repetition allowed)?

A-8. Find the number of 6 digit numbers that ends with 21 (eg. 537621), without repetition of digits.

A-9. The digits from 0 to 9 are written on slips of paper and placed in a box. Four of the slips are drawn at random and placed in the order. How many out comes are possible? ${}^{10}P_4$

A-10. Find the number of natural numbers from 1 to 1000 having none of their digits repeated. 9+81+648=738

A-11. A number lock has 4 dials, each dial has the digits 0, 1, 2, ..., 9. What is the maximum unsuccessful attempts to open the lock? No. of combinations = 10×10×10×10 = 10000

A-12.✎ In how many ways we can select a committee of 6 persons from 6 boys and 3 girls, if atleast two boys & atleast two girls must be there in the committee?

A-13.✎ In how many ways 11 players can be selected from 15 players, if only 6 of these players can bowl and the 11 players must include atleast 4 bowlers?

A-14. In how many ways a team of 5 can be chosen from 4 girls & 7 boys, if the team has atleast 3 girls.

A-15. A committee of 6 is to be chosen from 10 persons with the condition that if a particular person 'A' is chosen, then another particular person B must be chosen.

A-16. In how many ways we can select 5 cards from a deck of 52 cards, if each selection must include atleast one king.

A-17. How many four digit natural numbers not exceeding the number 4321 can be formed using the digits 1, 2, 3, 4, if repetition is allowed?

A-18. How many different permutations are possible using all the letters of the word MISSISSIPPI, if no two I's are together?

A-19.✎ If $A = \{1, 2, 3, 4 .....n\}$ and $B \subseteq A$; $C \subseteq A$, then the find number of ways of selecting
- (i) Sets B and C
- (ii) Order pair of B and C such that $B \cap C = \phi$
- (iii) Unordered pair of B and C such that $B \cap C = \phi$
- (iv) Ordered pair of B and C such that $B \cup C = A$ and $B \cap C = \phi$
- (v) Unordered pair of B and C such that $B \cup C = A$, $B \cap C = \phi$
- (vi) Ordered pair of B and C such that $B \cap C$ is singleton

A-20. For a set of six true or false statements, no student in a class has written all correct answers and no two students in the class have written the same sequence of answers. What is the maximum number of students in the class, for this to be possible.

A-21. How many arithmetic progressions with 10 terms are there, whose first term is in the set $\underbrace{\{1, 2, 3, 4\}}_{4}$ and whose common difference is in the set $\underbrace{\{3, 4, 5, 6, 7\}}_{5}$?

A-22. Find the number of all five digit numbers which have atleast one digit repeated.

A-23. There are 3 white, 4 blue and 1 red flowers. All of them are taken out one by one and arranged in a row in the order. How many different arrangements are possible (flowers of same colurs are similar)?

Section (B) : Problem based on distinct and identical objects and divisors

B-1. Let N = 24500, then find
- (i)✎ The number of ways by which N can be resolved into two factors.
- (ii) The number of ways by which 5N can be resolved into two factors.
- (iii) The number of ways by which N can be resolved into two coprime factors.

B-2. Find number of ways of selection of one or more letters from AAAABBCCCDEF
- (i) there is no restriction.
- (ii) the letters A & B are selected atleast once.
- (iii) only one letter is selected.
- (iv)✎ atleast two letters are selected

B-3. Find number of ways of selection of atleast one vowel and atleast one consonant from the word TRIPLE

B-4. Find number of divisors of 1980.
- (i) How many of them are multiple of 11 ? find their sum
- (ii) How many of them are divisible by 4 but not by 15.

Section (C) : Problem based on circular arrangement and Multinomial theorem

C-1. In how many ways 5 persons can sit at a round table, if two of the persons do not sit together?

C-2. In how many ways four men and three women may sit around a round table if all the women are together?

C-3. Seven persons including A, B, C are seated on a circular table. How many arrangements are possible if B is always between A and C ?

Negative binomial expansion :

$(1 - x)^{-n} = 1 + {}^{n}C_1 x + {}^{n+1}C_2\, x^2 + {}^{n+2}C_3 x^3 + \ldots\ldots$ to ∞ , if $-1 < x < 1$.

Coefficient of x^r in this expansion $= {}^{n+r-1}C_r$ $(n \in N)$

Result : Number of ways in which it is possible to make a selection from $m + n + p = N$ things, where p are alike of one kind, m alike of second kind & n alike of third kind, taken r at a time is given by coefficient of x^r in the expansion of

$(1 + x + x^2 + \ldots\ldots + x^p)\ (1 + x + x^2 + \ldots\ldots + x^m)\ (1 + x + x^2 + \ldots\ldots + x^n)$.

For example the number of ways in which a selection of four letters can be made from the letters of the word **PROPORTION** is given by coefficient of x^4 in

$(1 + x + x^2 + x^3)\ (1 + x + x^2)\ (1 + x + x^2)\ (1 + x)\ (1 + x)\ (1 + x)$.

Method of fictious partition :

Number of ways in which n identical things may be distributed among p persons if each person may receive none, one or more things is ${}^{n+p-1}C_n$.

Example # 20 : Find the number of solutions of the equation $x + y + z = 6$, where $x, y, z \in W$.

Solution : Number of solutions = coefficient of x^6 in $(1 + x + x^2 + \ldots\ldots x^6)^3$
$= $ coefficient of x^6 in $(1 - x^7)^3\ (1 - x)^{-3}$
$= $ coefficient of x^6 in $(1 - x)^{-3}$
$= {}^{3+6-1}C_6 = {}^{8}C_2 = 28$.

Example # 21 : In a bakery four types of biscuits are available. In how many ways a person can buy 10 biscuits if he decide to take atleast one biscuit of each variety?

Solution : Let the person select x biscuits from first variety, y from the second, z from the third and w from the fourth variety. Then the number of ways = number of solutions of the equation
$x + y + z + w = 10$.
where $x = 1, 2, \ldots\ldots\ldots, 7$
$y = 1, 2, \ldots\ldots\ldots, 7$
$z = 1, 2, \ldots\ldots\ldots, 7$
$w = 1, 2, \ldots\ldots\ldots, 7$
So, number of ways = coefficient of x^{10} in $(x + x^2 + \ldots\ldots + x^7)^4$
$= $ coefficient of x^6 in $(1 + x + \ldots\ldots + x^6)^4$
$= $ coefficient of x^6 in $(1 - x^7)^4\ (1 - x)^{-4}$
$= $ coefficient x^6 in $(1 - x)^{-4}$
$= {}^{4+6-1}C_6 = {}^{9}C_3 = 84$

Self Practice Problems:

(23) Three distinguishable dice are rolled. In how many ways we can get a total 15?

(24) In how many ways we can give 5 apples, 4 mangoes and 3 oranges (fruits of same species are similar) to three persons if each may receive none, one or more?

Ans. (23) 10 (24) 3150

Formation of Groups :

Number of ways in which $(m + n + p)$ different things can be divided into three different groups containing m, n & p things respectively is $\dfrac{(m+n+p)!}{m!\,n!\,p!}$,

If $m = n = p$ and the groups have identical qualitative characteristic then the number of groups $= \dfrac{(3n)!}{n!\ n!\ n!\ 3!}$.

Note : If 3n different things are to be distributed equally among three people then the number of ways $= \dfrac{(3n)!}{(n!)^3}$.

Example # 22 : 12 different toys are to be distributed to three children equally. In how many ways this can be done ?

Solution : The problem is to divide 12 different things into three different groups.

Number of ways $= \dfrac{12!}{4!\,4!\,4!} = 34650$.

Example # 23 : In how many ways 10 persons can be divided into 5 pairs?

Solution : We have each group having 2 persons and the qualitative characteristic are same (Since there is no purpose mentioned or names for each pair).

Thus the number of ways $= \dfrac{10!}{(2!)^5\ 5!} = 945$.

Self Practice Problems :

(25) 9 persons enter a lift from ground floor of a building which stops in 10 floors (excluding ground floor), if it is known that persons will leave the lift in groups of 2, 3, & 4 in different floors. In how many ways this can happen?

(26) In how many ways one can make four equal heaps using a pack of 52 playing cards?

(27) In how many ways 11 different books can be parcelled into four packets so that three of the packets contain 3 books each and one of 2 books, if all packets have the same destination?

Ans. (25) 907200 (26) $\dfrac{52!}{(13!)^4\ 4!}$ (27) $\dfrac{11!}{(3!)^4\ 2}$

Derrangements :

Number of ways in which 'n' letters can be put in 'n' corresponding envelopes such that no letter goes to correct envelope is $n!\left(1 - \dfrac{1}{1!} + \dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \ldots\ldots\ldots + (-1)^n \dfrac{1}{n!}\right)$

Example # 24 : In how many ways we can put 5 writings into 5 corresponding envelopes so that no writing go to the corresponding envelope?

Solution : The problem is the number of dearragements of 5 digits.

This is equal to $5!\left(\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \dfrac{1}{5!}\right) = 44$.

Example # 25 : Four slip of papers with the numbers 1, 2, 3, 4 written on them are put in a box. They are drawn one by one (without replacement) at random. In how many ways it can happen that the ordinal number of atleast one slip coincide with its own number?

Solution : Total number of ways $= 4! = 24$.
The number of ways in which ordinal number of any slip does not coincide with its own number is the number of dearrangements of 4 objects $= 4!\left(\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!}\right) = 9$

Thus the required number of ways. $= 24 - 9 = 15$

Self Practice Problems:

(28) In a match the column question, Column I contain 10 questions and Column II contain 10 answers written in some arbitrary order. In how many ways a student can answer this question so that exactly 6 of his matchings are correct ?

(29) In how many ways we can put 5 letters into 5 corresponding envelopes so that atleast one letter go to wrong envelope ?

Ans. (28) 1890 (29) 119

Example # 13 : In how many ways the letters of the word "ARRANGE" can be arranged without altering the relative positions of vowels & consonants?

Solution : The consonants in their positions can be arranged in $\frac{4!}{2!}$ = 12 ways.

The vowels in their positions can be arranged in $\frac{3!}{2!}$ = 3 ways

$\therefore$ Total number of arrangements = 12 × 3 = 36

Self Practice Problems :

(14) How many words can be formed using the letters of the word ASSESSMENT if each word begin with A and end with T?

(15) If all the letters of the word ARRANGE are arranged in all possible ways, in how many of words we will have the A's not together and also the R's not together?

(16) How many arrangements can be made by taking four letters of the word MISSISSIPPI?

Ans. (14) 840 (15) 660 (16) 176.

Selection of one or more objects

(a) Number of ways in which atleast one object may be selected out of 'n' distinct objects, is

$$^nC_1 + {^nC_2} + {^nC_3} + \ldots\ldots\ldots\ldots + {^nC_n} = 2^n - 1$$

(b) Number of ways in which atleast one object may be selected out of 'p' alike objects of one type, 'q' alike objects of second type and 'r' alike objects of third type, is

$$(p + 1)(q + 1)(r + 1) - 1$$

(c) Number of ways in which atleast one object may be selected from 'n' objects where 'p' alike of one type, 'q' alike of second type and 'r' alike of third type and rest $n - (p + q + r)$ are different, is $(p + 1)(q + 1)(r + 1)\, 2^{n-(p+q+r)} - 1$

Example # 14 : There are 12 different books in a shelf. In how many ways we can select atleast one of them?

Solution : We may select 1 book, 2 books,........, 12 books.

$\therefore$ The number of ways = $^{12}C_1 + {^{12}C_2} + \ldots\ldots + {^{12}C_{12}} = 2^{12} - 1 = 4095$

Example # 15 : There are 11 fruits in a basket of which 6 are apples, 3 mangoes and 2 bananas (fruits of same species are identical). How many ways are there to select atleast one fruit?

Solution : Let x be the number of apples being selected

y be the number of mangoes being selected and

z be the number of bananas being selected.

Then x = 0, 1, 2, 3, 4, 5,6

y = 0, 1, 2, 3

z = 0, 1, 2

Total number of triplets (x, y, z) is 7 × 4 × 3 = 84

Exclude (0, 0, 0)

$\therefore$ Number of combinations = 84 – 1 = 83.

Self Practice Problems

(17) In a shelf there are 6 physics, 4 chemistry and 3 mathematics books. How many combinations are there if (i) books of same subject are different? (ii) books of same subject are identical?

(18) From 5 apples, 4 mangoes & 3 bananas, in how many ways we can select atleast two fruits of each variety if (i) fruits of same species are identical? (ii) fruits of same species are different?

Ans. (17) (i) 8191 (ii) 139 (18) (i) 24 (ii) $2^{12} - 4$

Results : Let $N = p^a\, q^b\, r^c \ldots\ldots$ where p, q, r...... are distinct primes & a, b, c..... are natural numbers then:

(a) The total numbers of divisors of N including 1 & N is = (a + 1) (b + 1) (c + 1)........

(b) The sum of these divisors is =

$(p^0 + p^1 + p^2 + \ldots + p^a)(q^0 + q^1 + q^2 + \ldots + q^b)(r^0 + r^1 + r^2 + \ldots + r^c)\ldots\ldots$

(c) Number of ways in which N can be resolved as a product of two factors is

$$= \begin{cases} \frac{1}{2}(a+1)(b+1)(c+1)\ldots & \text{if N is not a perfect square} \\ \frac{1}{2}[(a+1)(b+1)(c+1)\ldots+1] & \text{if N is a perfect square} \end{cases}$$

(d) Number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

Example # 16 : Find the number of divisors of 1350. Also find the sum of all divisors.

Solution : $1350 = 2 \times 3^3 \times 5^2$

$\therefore$ Number of divisors = (1+ 1) (3 + 1) (2 + 1) = 24

sum of divisors = $(1 + 2)(1 + 3 + 3^2 + 3^3)(1 + 5 + 5^2) = 3720$.

Example # 17 : In how many ways 8100 can be resolved into product of two factors?

Solution : $8100 = 2^2 \times 3^4 \times 5^2$

Number of ways = $\frac{1}{2}[(2 + 1)(4 + 1)(2 + 1) + 1] = 23$

Self Practice Problems :

(19) How many divisors of 9000 are even but not divisible by 4? Also find the sum of all such divisors.

(20) In how many ways the number 8100 can be written as product of two coprime factors?

Ans. (19) 12, 4056 (20) 4

Circular Permutation :

The number of circular permutations of n different things taken all at a time is $(n - 1)!$.

If clockwise & anti–clockwise circular permutations are considered to be same, then it is $\frac{(n-1)!}{2}$.

Note: Number of circular permutations of n things when p are alike and the rest are different, taken all at a time, distinguishing clockwise and anticlockwise arrangement is $\frac{(n-1)!}{p!}$.

Example # 18 : In how many ways can we arrange 6 different flowers in a circle? In how many ways we can form a garland using these flowers?

Solution : The number of circular arrangements of 6 different flowers = (6 – 1)! = 120

When we form a garland, clockwise and anticlockwise arrangements are similar. Therefore, the number of ways of forming garland = $\frac{1}{2}(6 - 1)! = 60$.

Example # 19 : In how many ways 6 persons can sit at a round table, if two of them prefer to sit together?

Solution : Let $P_1, P_2, P_3, P_4, P_5, P_6$ be the persons, where P_1, P_2 want to sit together.

Regard these person as 5 objects. They can be arranged in a circle in (5 – 1)! = 24 ways. Now P_1, P_2 can be arranged in 2! ways. Thus the total number of ways = 24 × 2 = 48.

Self Practice Problems :

(21) In how many ways letters of the word 'MONDAY' can be written around a circle, if vowels are to be separated in any arrangement ?

(22) In how many ways we can form a garland using 3 different red flowers, 5 different yellow flowers and 4 different blue flowers, if flowers of same colour must be together?

Ans. (21) 72 (22) 17280

Example # 5 : How many 3 digit numbers can be formed by using the digits 0, 1, 2, 3, 4, 5. In how many of these we have atleast one digit repeated?

Solution : We have to fill three places using 6 objects (repetition allowed), 0 cannot be at 100[th] place.

The number of numbers = 180. $\overset{5}{\square}\overset{6}{\square}\overset{6}{\square}$

Number of numbers in which no digit is repeated = 100 $\overset{5}{\square}\overset{5}{\square}\overset{4}{\square}$

$\therefore$ Number of numbers in which atleast one digit is repeated = 180 – 100 = 80

Example # 6 : How many functions can be defined from a set A containing 5 elements to a set B having 3 elements? How many of these are surjective functions?

Solution : Image of each element of A can be taken in 3 ways.

$\therefore$ Number of functions from A to B = $3^5 = 243$.

Number of into functions from A to B = $2^5 + 2^5 + 2^5 - 3 = 93$.

$\therefore$ Number of onto functions = 150.

Self Practice Problems :

(7) How many functions can be defined from a set A containing 4 elements to a set B containing 5 elements? How many of these are injective functions?

(8) In how many ways 5 persons can enter into a auditorium having 4 entries?

Ans. (7) 625, 120 (8) 1024.

Combination :

If ${}^nC_r$ denotes the number of combinations (selections) of n different things taken r at a time, then

$${}^nC_r = \frac{n!}{r!\,(n-r)!} = \frac{{}^nP_r}{r!} \text{ where } r \le n \text{ ; } n \in N \text{ and } r \in W.$$

NOTE : (i) ${}^nC_r = {}^nC_{n-r}$

(ii) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(iii) ${}^nC_r = 0$ if $r \notin \{0, 1, 2, 3........, n\}$

Example # 7 : There are fifteen players for a cricket match.

(i) In how many ways the 11 players can be selected?

(ii) In how many ways the 11 players can be selected including a particular player?

(iii) In how many ways the 11 players can be selected excluding two particular players?

Solution : (i) 11 players are to be selected from 15

Number of ways = ${}^{15}C_{11} = 1365$.

(ii) Since one player is already included, we have to select 10 from the remaining 14

Number of ways = ${}^{14}C_{10} = 1001$.

(iii) Since two players are to be excluded, we have to select 11 from the remaining 13.

Number of ways = ${}^{13}C_{11} = 78$.

Example # 8 : If ${}^{49}C_{3r-2} = {}^{49}C_{2r+1}$, find 'r'.

Solution : ${}^nC_r = {}^nC_s$ if either r = s or r + s = n.

Thus $3r - 2 = 2r + 1 \Rightarrow r = 3$

or $3r - 2 + 2r + 1 = 49 \Rightarrow 5r - 1 = 49 \Rightarrow r = 10$

$\therefore$ r = 3, 10

Example # 9 : A regular polygon has 20 sides. How many triangles can be drawn by using the vertices, but not using the sides?

Solution : The first vertex can be selected in 20 ways. The remaining two are to be selected from 17 vertices so that they are not consecutive. This can be done in ${}^{17}C_2 - 16$ ways.

$\therefore$ The total number of ways = $20 \times ({}^{17}C_2 - 16)$

But in this method, each selection is repeated thrice.

$\therefore$ Number of triangles = $\dfrac{20 \times ({}^{17}C_2 - 16)}{3} = 800$.

Example # 10 : 15 persons are sitting in a row. In how many ways we can select three of them if adjacent persons are not selected ?

Solution : Let $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}$ be the persons sitting in this order.

If three are selected (non consecutive) then 12 are left out.

Let P,P,P,P,P,P,P,P,P,P,P,P be the left out & q, q, q be the selected. The number of ways in which these 3 q's can be placed into the 13 positions between the P's (including extremes) is the number ways of required selection.

Thus number of ways = ${}^{13}C_3 = 286$.

Example # 11 : In how many ways we can select 4 letters from the letters of the word MISSISSIPPI?

Solution : M

IIII

SSSS

PP

Number of ways of selecting 4 alike letters = ${}^2C_1 = 2$.

Number of ways of selecting 3 alike and 1 different letters = ${}^2C_1 \times {}^3C_1 = 6$

Number of ways of selecting 2 alike and 2 alike letters = ${}^3C_2 = 3$

Number of ways of selecting 2 alike & 2 different = ${}^3C_1 \times {}^3C_2 = 9$

Number of ways of selecting 4 different = ${}^4C_4 = 1$

Total number of ways = 2 + 6 + 3 + 9 + 1 = 21

Self Practice Problems :

(9) In how many ways 7 persons can be selected from among 5 Indian, 4 British & 2 Chinese, if atleast two are to be selected from each country ?

(10) Find a number of different seven digit numbers that can be written using only three digits 1, 2 & 3 under the condition that the digit 2 occurs exactly twice in each number ?

(11) In how many ways 6 boys & 6 girls can sit at a round table so that girls & boys sit alternate?

(12) In how many ways 4 persons can occupy 10 chairs in a row, if no two sit on adjacent chairs?

(13) In how many ways we can select 3 letters of the word PROPORTION ?

Ans. (9) 100 (10) 672 (11) 86400 (12) 840 (13) 36

Arrangement of n things, those are not all different :

The number of permutations of 'n' things, taken all at a time, when 'p' of them are same & of one type, q of them are same & of second type, 'r' of them are same & of a third type & the remaining n – (p + q + r) things are all different, is $\dfrac{n!}{p!\,q!\,r!}$.

Example # 12 : In how many ways we can arrange 3 red flowers, 4 yellow flowers and 5 white flowers in a row? In how many ways this is possible if the white flowers are to be separated in any arrangement? (Flowers of same colour are identical).

Solution : Total we have 12 flowers 3 red, 4 yellow and 5 white.

Number of arrangements = $\dfrac{12!}{3!\,4!\,5!} = 27720$.

For the second part, first arrange 3 red & 4 yellow

This can be done in $\dfrac{7!}{3!\,4!} = 35$ ways

Now select 5 places from among 8 places (including extremes) & put the white flowers there.

This can be done in ${}^8C_5 = 56$.

$\therefore$ The number of ways for the 2[nd] part = 35 × 56 = 1960.

PERMUTATION AND COMBINATION

There can never be surprises in logic...Wittgenstein, Ludwig

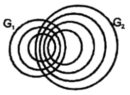
The most fundamental application of mathematics is counting. There are many natural methods used for counting

This chapter is dealing with various known techniques those are much faster than the usual counting methods.

We mainly focus, our methods, on counting the number of arrangements (Permutations) and the number of selections (combinations), even although we may use these techniques for counting in some other situations also .

Let us start with a simple problem

A group G_1 of 3 circles C_1, C_2, C_3 having different centers are situated in such a way that C_2 lie entirely inside C_1 ; C_3 lie entirely inside C_2. Another group G_2 of 4 circles C_1', C_2', C_3', C_4' are also situated in a similar fashion. The two groups of circles are in such a way that each member of G_1 intersect with every member of G_2, as shown in the following figure



(i) How many centres the circles altogether has ?
(ii) How many common chords are obtained ?

The answer to the first part is "3 + 4 = 7" and answer to the second part is "3 × 4 = 12". The method in which we calculated first part of the problem is called as "addition rule" and the method we used to calculate its second part is called as the "multiplication rule". These rules altogether are the most important tools in counting, popularly known as "the fundamental counting principle".

Fundamental counting principle :

Suppose that an operation O_1 can be done in m different ways and another operation O_2 can be done in n different ways.

(i) **Addition rule :** The number of ways in which we can do exactly one of the operations O_1, O_2 is m + n

(ii) **Multiplication rule :** The number of ways in which we can do both the operations O_1, O_2 is mn.

Note : The addition rule is true only when O_1 & O_2 are mutually exclusive and multiplication rule is true only when O_1 & O_2 are independent (The reader will understand the concepts of mutual exclusiveness and independence, in the due course)

Example # 1 : There are 8 buses running from Kota to Jaipur and 10 buses running from Jaipur to Delhi. In how many ways a person can travel from Kota to Delhi via Jaipur by bus?

Solution : Let E_1 be the event of travelling from Kota to Jaipur & E_2 be the event of travelling from Jaipur to Delhi by the person.

E_1 can happen in 8 ways and E_2 can happen in 10 ways.

Since both the events E_1 and E_2 are to be happened in order, simultaneously, the number of ways = 8 × 10 = 80.

Example # 2 : How many numbers between 10 and 10,000 can be formed by using the digits 1, 2, 3, 4, 5 if
(i) No digit is repeated in any number. (ii) Digits can be repeated.

Solution : (i) Number of two digit numbers = 5 × 4 = 20
Number of three digit numbers = 5 × 4 × 3 = 60
Number of four digit numbers = 5 × 4 × 3 × 2 = 120
Total = 200

(ii) Number of two digit numbers = 5 × 5 = 25
Number of three digit numbers = 5 × 5 × 5 = 125
Number of four digit numbers = 5 × 5 × 5 × 5 = 625
Total = 775

Self Practice Problems :

(1) How many 4 digit numbers are there, without repetition of digits, if each number is divisible by 5 ?

(2) Using 6 different flags, how many different signals can be made by using atleast three flags, arranging one above the other?

Ans. (1) 952 (2) 1920

Arrangements :

If ^nP_r denotes the number of permutations (arrangements) of n different things, taking r at a time, then

$$^nP_r = n\,(n-1)\,(n-2)\,.....\,(n-r+1) = \frac{n!}{(n-r)!}$$

NOTE : (i) Factorials of negative integers are not defined.
(ii) 0 ! = 1 ! = 1
(iii) ^nP_n = n ! = n. (n – 1) !
(iv) (2n) ! = 2^n. n ! [1. 3. 5. 7... (2n – 1)]

Example # 3 : How many three digit can be formed using the digits 1, 2, 3, 4, 5, without repetition of digits? How many of these are even?

Solution : Three places are to be filled with 5 different objects.
∴ Number of ways = $^5P_3$ = 5 × 4 × 3 = 60
For the 2nd part, unit digit can be filled in two ways & the remaining two digits can be filled in $^4P_2$ ways.
∴ Number of even numbers = 2 × $^4P_2$ = 24.

Example # 4 : If all the letters of the word 'QUEST' are arranged in all possible ways and put in dictionary order, then find the rank of the given word.

Solution : Number of words beginning with E = $^4P_4$ = 24
Number of words beginning with QE = $^3P_3$ = 6
Number of words beginning with QS = 6
Number of words beginning withQT = 6.
Next word is 'QUEST'
∴ its rank is 24 + 6 + 6 + 6 + 1 = 43.

E → 4! = 24
Q → E → 3! = 6
S S → 3! = 6
T T → 3! = 6
U U → E → 1st
S
T
43rd rank

Self Practice Problems :

(3) Find the sum of all four digit numbers (without repetition of digits) formed using the digits 1, 2, 3, 4, 5.

(4) Find 'n', if $^{n-1}P_3 : {}^nP_4$ = 1 : 9.

(5) Six horses take part in a race. In how many ways can these horses come in the first, second and third place, if a particular horse is among the three winners (Assume No Ties)?

(6) Find the sum of all three digit numbers those can be formed by using the digits. 0, 1, 2, 3, 4.

Ans. (3) 399960 (4) 9 (5) 60 (6) 27200

Result : Let there be 'n' types of objects, with each type containing atleast r objects. Then the number of ways of arranging r objects in a row is n^r.