

ELECTROSTATICS

1. INTRODUCTION

The branch of physics which deals with electric effect of static charge is called electrostatics.

2. ELECTRIC CHARGE

Charge of a material body or particle is the property (acquired or natural) due to which it produces and experiences electrical and magnetic effects. Some of naturally occurring charged particles are electrons, protons, α -particles etc.

Charge is a derived physical quantity & is measured in Coulomb in S.I. unit. In practice we use mC(10^{-3} C), μ C (10^{-6} C), nC(10^{-9} C) etc.

C.G.S. unit of charge = electrostatic unit = esu.

1 coulomb = 3×10^9 esu of charge

Dimensional formula of charge = [M⁰L⁰T¹I¹]

2.1 Properties of Charge

- (i) **Charge is a scalar quantity :** It adds algebraically and represents excess or deficiency of electrons.
- (ii) **Charge is of two types : (i) Positive charge and (ii) Negative charge** Charging a body implies transfer of charge (electrons) from one body to another. Positively charged body means loss of electrons i.e. deficiency of electrons. Negatively charged body means excess of electrons. This also shows that **mass of a negatively charged body > mass of a positively charged identical body**.
- (iii) **Charge is conserved :** In an isolated system, total charge (sum of positive and negative) remains constant whatever change takes place in that system.
- (iv) **Charge is quantized :** Charge on any body always exists in integral multiples of a fundamental unit of electric charge. This unit is equal to the magnitude of charge on electron ($1e = 1.6 \times 10^{-19}$ coulomb). So charge on anybody is $Q = \pm ne$, where n is an integer and e is the charge of the electron. **Millikan's oil drop experiment** proved the quantization of charge or atomicity of charge

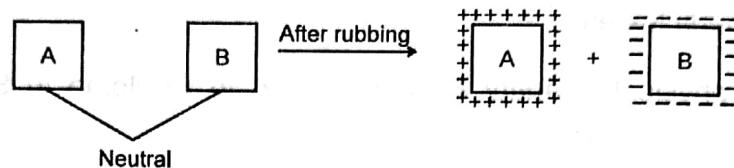
Note : Recently, the existence of particles of charge $\pm \frac{1}{3}e$ and $\pm \frac{2}{3}e$ has been postulated. These particles are called quarks but still this is not considered as the quantum of charge because these are unstable (They have very short span of life).

- (v) Like point charges repel each other while unlike point charges attract each other.
- (vi) Charge is always associated with mass, i.e., charge can not exist without mass though mass can exist without charge. The particle such as photon or neutrino which have no (rest) mass can never have a charge.
- (vii) **Charge is relativistically invariant:** This means that charge is independent of frame of reference i.e. charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge, the mass of a body depends on its speed and increases with increase in speed.
- (viii) A charge at rest produces only electric field around itself, a charge having uniform motion produces electric as well as magnetic field around itself while a charge having accelerated motion emits electromagnetic radiations.

2.2 Charging of a body

A body can be charged by means of (a) friction, (b) conduction, (c) induction, (d) thermionic ionization or thermionic emission (e) photoelectric effect and (f) field emission.

(a) Charging by Friction : When a neutral body is rubbed against other neutral body then some electrons are transferred from one body to other. The body which can hold electrons tightly, draws some electrons and the body which can not hold electrons tightly, loses some electrons. The body which draws electrons becomes negatively charged and the body which loses electrons becomes positively charged.



For example : Suppose a glass rod is rubbed with a silk cloth. As the silk can hold electrons more tightly and a glass rod can hold electrons less tightly (due to their chemical properties), some electrons will leave the glass rod and get transferred to the silk. So, in the glass rod there will be deficiency of electrons, therefore it will become positively charged. And in the silk, there will be some extra electrons, so it will become negatively charged

(b) Charging by conduction (flow): There are three types of materials in nature

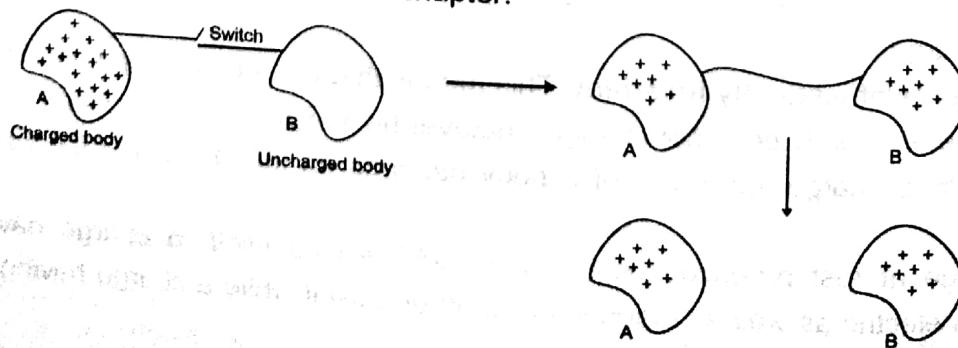
(i) Conductor : Conductors are the material in which the outer most electrons are very loosely bound, so they are free to move (flow). So in a conductor, there are large number of free electrons.

Ex. Metals like Cu, Ag, Fe, Al.....

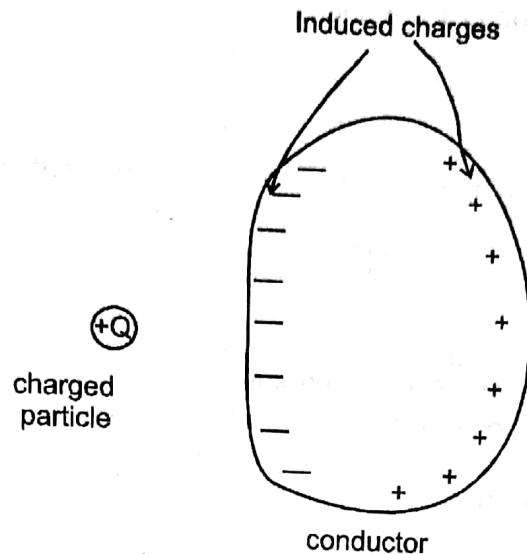
(ii) Insulator or Dielectric or Nonconductor : Non-conductors are the materials in which outer most electrons are very tightly bound, so that they cannot move (flow). Hence in a non-conductor there are no free electrons. Ex. plastic, rubber, wood etc.

(iii) Semiconductor : Semiconductors are the materials which have free electrons but very less in number.

Now lets see how the charging is done by conduction. In this method, we take a charged conductor 'A' and an uncharged conductor 'B'. When both are connected, some charge will flow from the charged body to the uncharged body. If both the conductors are identical & kept at large distance and connected to each other, then charge will be divided equally in both the conductors otherwise they will flow till their electric potential becomes same. Its detailed study will be done in last section of this chapter.



(c) **Charging by Induction** : To understand this, let's have introduction to induction.



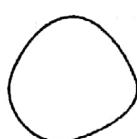
We have studied that there are lot of free electrons in the conductors. When a charged particle $+Q$ is brought near a neutral conductor, due to attraction of $+Q$ charge, many electrons ($-ve$ charges) come closer and accumulate on the closer surface.

On the other hand, a positive charge (deficiency of electrons) appears on the other surface. This phenomena is called induction and charges produced are called induced charges.

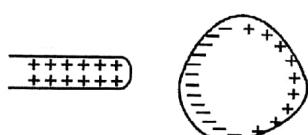
A body can be charged by induction in the following two ways :

Method I :

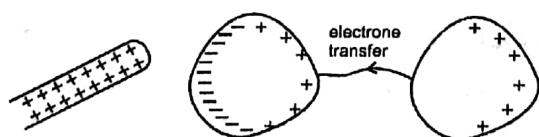
Step 1 : Take an isolated neutral conductor..



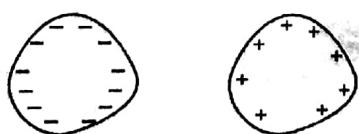
Step 2: Bring a charged rod near it. Due to the charged rod, charges will induce on the conductor.



Step 3 : Connect another neutral conductor with it. Due to attraction of the rod, some free electrons will move from the right conductor to the left conductor and due to deficiency of electrons positive charges will appear on right conductor and on the left conductor, there will be excess of electrons due to transfer from right conductor.



Step 4 : Now disconnect the connecting wire and remove the rod.

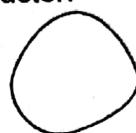


The first conductor will be negatively charged and the second conductor will be positively charged.

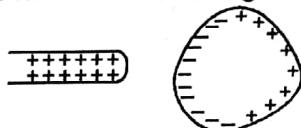
Electrostatics

Method II

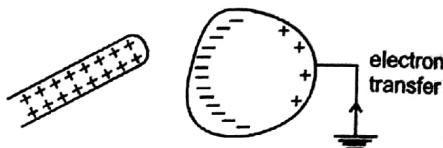
Step 1: Take an isolated neutral conductor.



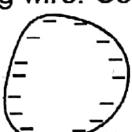
Step 2 : Bring a charged rod near it. Due to the charged rod, charges will induce on the conductor.



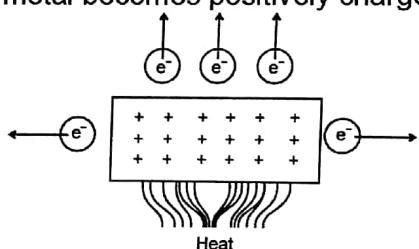
Step 3 : Connect the conductor to the earth (this process is called grounding or earthing). Due to attraction of the rod, some free electrons will move from earth to the conductor, so in the conductor there will be excess of electrons due to transfer from the earth, so net charge on conductor will be negative.



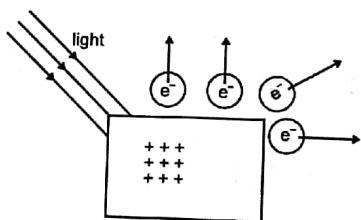
Step 4. Now disconnect the connecting wire. Conductor becomes negatively charged.



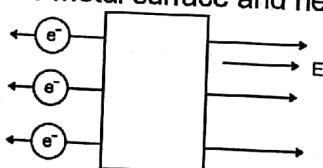
(d) **Thermionic emission** : When the metal is heated at a high temperature then some electrons of metals are ejected and the metal becomes positively charged.



(e) **Photoelectric effect** : When light of sufficiently high frequency is incident on metal surface then some electrons gain energy from light and come out of the metal surface and remaining metal becomes positively charged.



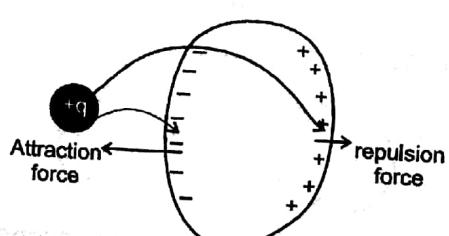
(f) **Field emission** : When electric field of large magnitude is applied near the metal surface then some electrons come out from the metal surface and hence the metal gets positively charged.



Solved Examples

Example 1.
Solution :

If a charged body is placed near a neutral conductor, will it attract the conductor or repel it?



If a charged body (+ve) is placed left side near a neutral conductor, (-ve) charge will induce at left surface and (+ve) charge will induce at right surface. Due to positively charged body -ve induced charge will feel attraction and the +ve induced charge will feel repulsion. But as the -ve induced charge is nearer, so the attractive force will be greater than the repulsive force. So the net force on the conductor due to positively charged body will be attractive. Similarly, we can prove for negatively charged body also.

From the above example we can conclude that, "A charged body can attract a neutral body." If there is attraction between two bodies then one of them may be neutral. But if there is repulsion between two bodies, both must be charged (similarly charged). So "repulsion is the sure test of electrification".

- Example 2.** A positively charged body 'A' attracts a body 'B' then charge on body 'B' may be:
(A) positive (B) negative (C) zero (D) can't say
Answer : B, C

- Example 3.** Five styrofoam balls A, B, C, D and E are used in an experiment. Several experiments are performed on the balls and the following observations are made :
(i) Ball A repels C and attracts B.
(ii) Ball D attracts B and has no effect on E.
(iii) A negatively charged rod attracts both A and E.

For your information , an electrically neutral styrofoam ball is very sensitive to charge induction and gets attracted considerably, if placed nearby a charged body. What are the charges, if any, on each ball ?

	A	B	C	D	E
(A)	+	-	+	0	+
(B)	+	-	+	+	0
(C)	+	-	+	0	0
(D)	-	+	-	-	0

- Answer :** C
Solution : From (i), as A repels C, so both A and C must be charged similarly. Either both are +ve or both are -ve. As A also attract B, so charge on B should be opposite of A or B may be uncharged conductor.

From (ii) as D has no effect on E, so both D and E should be uncharged and as B attracts uncharged D, so B must be charged and D must be an uncharged conductor.

From (iii), a -vely charged rod attracts the charged ball A, so A must be +ve and from exp. (i) C must also be +ve and B must be -ve.

- Example 4.** Charge conservation is always valid. Is it also true for mass?

Solution : No, mass conservation is not always. In some nuclear reactions, some mass is lost and it is converted into energy.

- Example 5.** What are the differences between charging by induction and charging by conduction ?

Solution : Major differences between two methods of charging are as follows :

- In induction, two bodies are close to each other but do not touch each other while in conduction they touch each other. (Or they are connected by a metallic wire)
- In induction, total charge of a body remains unchanged while in conduction it changes.
- In induction, induced charge is always opposite in nature to that of source charge while in conduction charge on two bodies finally is of same nature.

- If a glass rod is rubbed with silk, it acquires a positive charge because :

- | | |
|-------------------------------|------------------------------------|
| (A) protons are added to it | (B) protons are removed from it |
| (C) electrons are added to it | (D) electrons are removed from it. |

Answer : D



3. COULOMB'S LAW (INVERSE SQUARE LAW)

On the basis of experiments Coulomb established the following law known as Coulomb's law :

The magnitude of electrostatic force between two point charges is directly proportional to the product of charges and inversely proportional to the square of the distance between them.

$$\text{i.e. } F \propto q_1 q_2 \text{ and } F \propto \frac{1}{r^2} \Rightarrow F \propto \frac{q_1 q_2}{r^2} \Rightarrow F = \frac{K q_1 q_2}{r^2}$$

Important points regarding Coulomb's law :

(i) It is applicable only for point charges.

(ii) The constant of proportionality K in SI units in vacuum is expressed as $\frac{1}{4\pi\epsilon_0}$ and in any other

medium expressed as $\frac{1}{4\pi\epsilon}$. If charges are dipped in a medium then electrostatic force on one

charge is $\frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1 q_2}{r^2}$ where ϵ_0 and ϵ are called permittivity of vacuum and absolute permittivity

of the medium respectively. The ratio $\epsilon/\epsilon_0 = \epsilon_r$ is called relative permittivity of the medium, which is a dimensionless quantity.

(iii) The value of relative permittivity ϵ_r is constant for a medium and can have values between 1 to ∞ . For vacuum, by definition it is equal to 1. For air it is nearly equal to 1 and may be taken to be equal to 1 for calculations. For metals, the value of ϵ_r is ∞ and for water is 81. The material in which more charge can induce ϵ_r will be higher.

(iv) The value of $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$ & $\epsilon_0 = 8.855 \times 10^{-12} \text{ C}^2/\text{Nm}^2$.

Dimensional formula of ϵ is $[\text{M}^{-1} \text{ L}^{-3} \text{ T}^4 \text{ A}^2]$

(v) The force acting on one point charge due to the other point charge is always along the line joining these two charges. It is equal in magnitude and opposite in direction on two charges, irrespective of the medium in which they lie.

(vi) The force is conservative in nature i.e., work done by electrostatic force in moving a point charge along a closed loop of any shape is zero.

(vii) Since the force is a central force, in the absence of any other external force, angular momentum of one particle w.r.t. the other particle (in two particle system) is conserved.

(viii) In vector form formula can be given as below.

$$\vec{F} = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1 q_2}{|\vec{r}|^3} \vec{r} = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}; \quad (\text{q}_1 \text{ & q}_2 \text{ are to be substituted with sign.})$$

Here, \vec{r} is position vector of the test charge (on which force is to be calculated) with respect to the source charge (due to which force is to be calculated).

Solved Examples

Example 7. Find out the electrostatic force between two point charges placed in air (each of +1 C) if they are separated by 1m.

Solution : $F_e = \frac{k q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 1 \times 1}{1^2} = 9 \times 10^9 \text{ N}$



From the above result, we can say that 1 C charge is too large to realize. In nature, charge is usually of the order of μC

Example 8. A particle of mass m carrying charge q_1 is revolving around a fixed charge $-q_2$ in a circular path of radius r . Calculate the period of revolution and its speed also.

$$\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = m r \omega^2 = \frac{4\pi^2 m r}{T^2},$$

$$T^2 = \frac{(4\pi\epsilon_0)r^2(4\pi^2 m r)}{q_1 q_2} \quad \text{or} \quad T = 4\pi r \sqrt{\frac{\pi\epsilon_0 m r}{q_1 q_2}}$$

and also we can say that

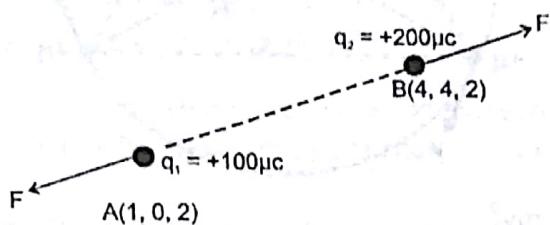
$$\frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{q_1 q_2}{4\pi\epsilon_0 m r}}$$

Example 9. A point charge $q_A = +100 \mu\text{c}$ is placed at point A (1, 0, 2) m and another point charge $q_B = +200 \mu\text{c}$ is placed at point B (4, 4, 2) m. Find :

- (i) Magnitude of electrostatic interaction force acting between them
- (ii) Find \vec{F}_A (force on A due to B) and \vec{F}_B (force on B due to A) in vector form

Solution :

(i)



$$\text{Value of } F : |\vec{F}| = \frac{k q_A q_B}{r^2} = \frac{(9 \times 10^9)(100 \times 10^{-6})(200 \times 10^{-6})}{\left(\sqrt{(4-1)^2 + (4-0)^2 + (2-2)^2}\right)^2} = 7.2 \text{ N}$$

$$\begin{aligned} \text{(ii) Force on B, } \vec{F}_B &= \frac{k q_A q_B}{|\vec{r}|^3} \vec{r} = \frac{(9 \times 10^9)(100 \times 10^{-6})(200 \times 10^{-6})}{\left(\sqrt{(4-1)^2 + (4-0)^2 + (2-2)^2}\right)^3} [(4-1)\hat{i} + (4-0)\hat{j} + (2-2)\hat{k}] \\ &= 7.2 \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j} \right) \text{ N} \end{aligned}$$

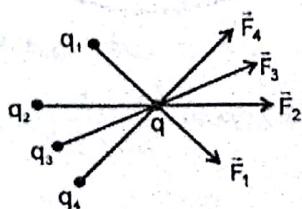
$$\text{Similarly } \vec{F}_A = 7.2 \text{ N} \left(-\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j} \right)$$

Action (\vec{F}_A) and Reaction (\vec{F}_B) are equal but in opposite direction.



4. PRINCIPLE OF SUPERPOSITION

The electrostatic force is a two body interaction i.e. electrical force

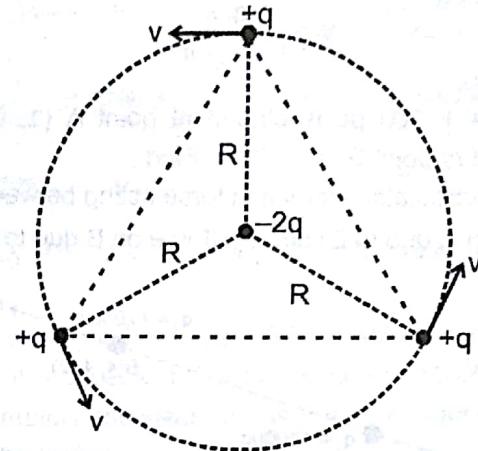


The electrostatic force is a two body interaction i.e. electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid i.e. force on charged particle due to number of point charges is the resultant of forces due to individual point charges. Therefore, force on a point test charge due to many charges is given by.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

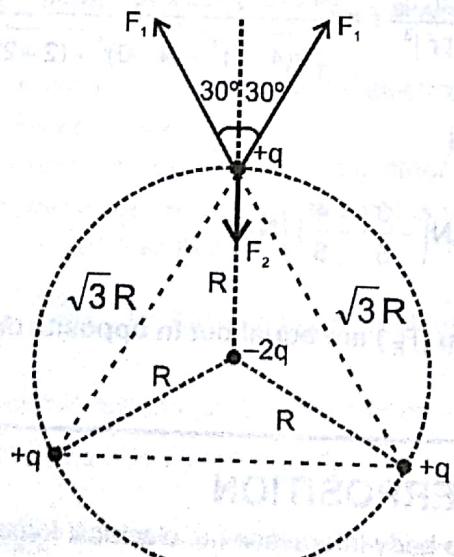
Solved Examples

- Example 10.** Three equal point charges of charge $+q$ each are moving along a circle of radius R and a point charge $-2q$ is also placed at the centre of circle (as shown in figure). If charges are revolving with constant and same speed in the circle then calculate speed of charges



Solution : $F_2 - 2F_1 \cos 30^\circ = \frac{mv^2}{R}$

$$\Rightarrow \frac{K(q)(2q)}{R^2} - \frac{2(Kq^2)}{(\sqrt{3}R)^2} \cos 30^\circ = \frac{mv^2}{R}$$



$$\Rightarrow v = \sqrt{\frac{kq^2}{Rm} \left[2 - \frac{1}{\sqrt{3}} \right]}$$

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Example 11.

Two equally charged identical small metallic spheres A and B repel each other with a force 2×10^{-5} N when placed in air (neglect gravitational attraction). Another identical uncharged sphere C is touched to B and then placed at the mid point of line joining A and B. What is the net electrostatic force on C?

Solution :

Let initially the charge on each sphere be q and separation between their centres be r . Then according to given problem :

$$F = \frac{1}{4\pi\epsilon_0} \frac{q \times q}{r^2} = 2 \times 10^{-5} \text{ N}$$

When sphere C touches B, the charge of B i.e. q will distribute equally on B and C as spheres are identical now charges on spheres;

$$q_B = q_C = (q/2)$$

So sphere C will experience a force

$$F_{CA} = \frac{1}{4\pi\epsilon_0} \frac{q(q/2)}{(r/2)^2} = 2F \text{ along } \overline{AB} \text{ due to charge on A.}$$

$$\text{and, } F_{CB} = \frac{1}{4\pi\epsilon_0} \frac{(q/2)(q/2)}{(r/2)^2} = F, \text{ along } \overline{BA} \text{ due to charge on B :}$$

So the net force F_C on C due to charges on A and B,

$$F_C = F_{CA} - F_{CB} = 2F - F = 2 \times 10^{-5} \text{ N along } \overline{AB}.$$

Example 12.

Five point charges, each of value q are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value $-q$ coulomb placed at the centre of the hexagon?

Solution :

Method-I : If there had been a sixth charge $+q$ at the remaining vertex of hexagon, force due to all the six charges on $-q$ at O would have been zero (as the forces due to individual charges will balance each other), i.e., $\bar{F}_R = 0$

Now if \bar{f} is the force due to sixth charge and \bar{F} due to remaining five charges.

$$\text{From } \bar{F} + \bar{f} = 0 \text{ i.e. } \bar{F} = -\bar{f}$$

$$\text{or, } |\bar{F}| = |\bar{f}| = \frac{1}{4\pi\epsilon_0} \frac{q \times q}{L^2} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{L^2}$$

$$\bar{F}_{\text{Net}} = \bar{F}_{OO} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{L^2} \text{ along } \overline{OD}$$

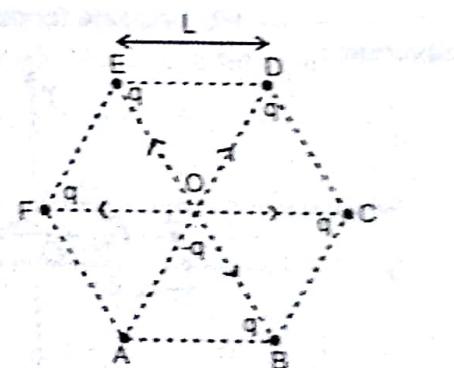
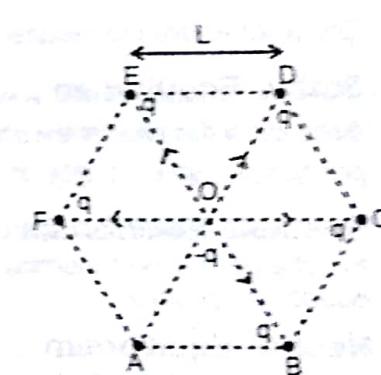
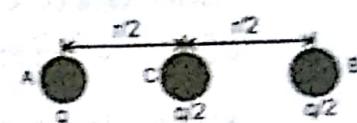
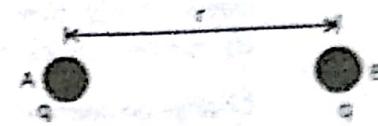
Method-II : In the diagram, we can see that force due to charge A and D are opposite to each other

$$\bar{F}_{OF} + \bar{F}_{OC} = \bar{0} \quad \dots(i)$$

$$\text{Similarly } \bar{F}_{OB} + \bar{F}_{OE} = \bar{0} \quad \dots(ii)$$

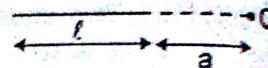
$$\text{So } \bar{F}_{OF} + \bar{F}_{OB} + \bar{F}_{OC} + \bar{F}_{OD} + \bar{F}_{OE} = \bar{F}_{\text{Net}}$$

$$\text{Using (i) and (ii) } \bar{F}_{\text{Net}} = \bar{F}_{OO} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{L^2} \text{ along } \overline{OD}.$$



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Example 13 A thin straight rod of length l carrying a uniformly distributed charge q is located in vacuum. Find the magnitude of the electric force on a point charge ' Q ' kept as shown in the figure.



Solution : As the charge on the rod is not point charge, therefore, first we have to find force on charge due to charge over a very small part on the length of the rod. This part, called element of length dy can be considered as point charge.

$$\text{Charge on element, } dq = \lambda dy = \frac{q}{l} dy$$

Electric force on ' Q ' due to element

$$= \frac{K \cdot dq \cdot Q}{y^2} = \frac{K \cdot Q \cdot q \cdot dy}{y^2 \cdot l}$$

All forces are along the same direction,

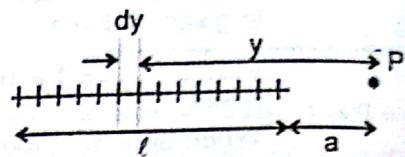
$\therefore F = \sum dF$. This sum can be calculated using integration,

$$\text{therefore, } F = \int_{y=a}^{a+l} \frac{KQq dy}{y^2 l} = \frac{KqQ}{l} \left[-\frac{1}{y} \right]_a^{a+l} = \frac{KQq}{l} \left[\frac{1}{a} - \frac{1}{a+l} \right] = \frac{KQq}{a(a+l)}$$

Note : (1) The total charge of the rod cannot be considered to be placed at the centre of the rod as we do in mechanics for mass in many problems.

Note : (2) If $a \gg l$ then, $F = \frac{KQq}{a^2}$

i.e. Behavior of the rod is just like a point charge.



5. ELECTROSTATIC EQUILIBRIUM

The point where the resultant force on a charged particle becomes zero is called equilibrium position.

5.1

Stable Equilibrium : A charge is initially in equilibrium position and is displaced by a small distance. If the charge tries to return back to the same equilibrium position then this equilibrium is called position of stable equilibrium.

5.2

Unstable Equilibrium : If charge is displaced by a small distance from its equilibrium position and the charge has no tendency to return to the same equilibrium position. Instead it goes away from the equilibrium position.

5.3

Neutral Equilibrium : If charge is displaced by a small distance and it is still in equilibrium condition then it is called neutral equilibrium.

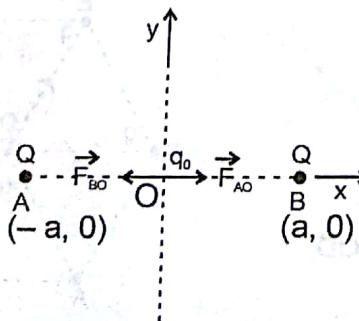
Solved Examples

Example 14. Two equal positive point charges ' Q ' are fixed at points $B(a, 0)$ and $A(-a, 0)$. Another test charge q_0 is also placed at $O(0, 0)$. Show that the equilibrium at ' O ' is

- Stable for displacement along X-axis.
- Unstable for displacement along Y-axis.

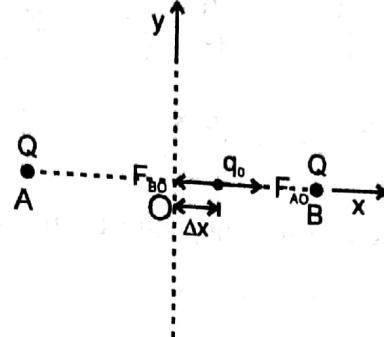
Solution :

-



$$\text{Initially } \vec{F}_{AO} + \vec{F}_{BO} = 0 \Rightarrow |\vec{F}_{AO}| = |\vec{F}_{BO}| = \frac{KQq_0}{a^2}$$

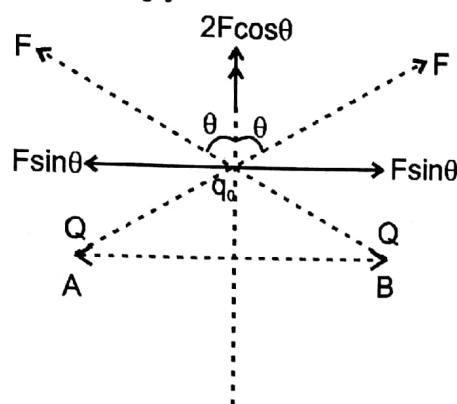
When charge is slightly shifted towards $+x$ axis by a small distance Δx , then.



$$|\vec{F}_{AO}| < |\vec{F}_{BO}|$$

Therefore, the particle will move towards origin (its original position). Hence, the equilibrium is stable.

(ii) When charge is shifted along y axis:



After resolving components, net force will be along y axis So, the particle will not return to its original position & it is unstable equilibrium. Finally, the charge will move to infinity.

Example 15. Two point charges of charge q_1 and q_2 (both of same sign) and each of mass m are placed such that gravitational attraction between them balances the electrostatic repulsion. Are they in stable equilibrium? If not then what is the nature of equilibrium?

Solution : In given example : $\frac{Kq_1q_2}{r^2} = \frac{Gm^2}{r^2}$

We can see that irrespective of distance between them charges will remain in equilibrium. If now distance is increased or decreased then there is no effect in their equilibrium. Therefore it is a neutral equilibrium.

Example 16. A particle of mass m and charge q is located midway between two fixed charged particles each having a charge q and a distance 2ℓ apart. Prove that the motion of the particle will be SHM if it is displaced slightly along the line connecting them and released. Also find its time period.

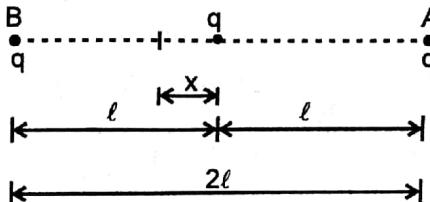
Solution : Let the charge q at the mid-point is displaced slightly to the left. The force on the displaced charge q due to charge q at A,

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(\ell+x)^2}$$

The force on the displaced charge q due to charge at B,

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(\ell-x)^2}$$

Net restoring force on the displaced charge q .



Electrostatics

$$F = F_2 - F_1 \text{ or } F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(\ell-x)^2} - \frac{1}{4\pi\epsilon_0} \frac{q^2}{(\ell+x)^2}$$

$$\text{or } F = \frac{q^2}{4\pi\epsilon_0} \left[\frac{1}{(\ell-x)^2} - \frac{1}{(\ell+x)^2} \right] = \frac{q^2}{4\pi\epsilon_0} \frac{4\ell x}{(\ell^2 - x^2)^2}$$

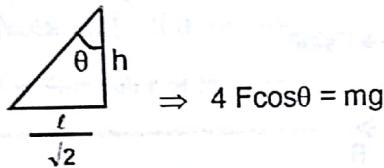
$$\text{Since } \ell \gg x, \therefore F = \frac{q^2 \ell x}{\pi\epsilon_0 \ell^4} \text{ or } F = \frac{q^2 x}{\pi\epsilon_0 \ell^3}$$

Hence we see that $F \propto x$ and it is opposite to the direction of displacement. Therefore, the motion is SHM.

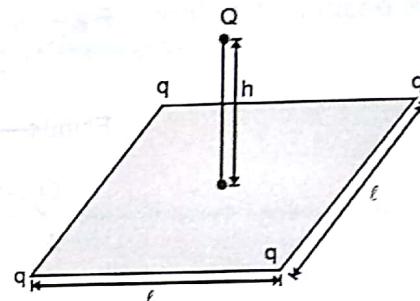
$$T = 2\pi \sqrt{\frac{m}{k}}, \text{ (here } k = \frac{q^2}{\pi\epsilon_0 \ell^3}) T = 2\pi \sqrt{\frac{m\pi\epsilon_0 \ell^3}{q^2}}$$

Example 17. Find out mass of the charge Q, so that it remains in equilibrium for the given configuration.

Solution :



$$\Rightarrow 4F \cos\theta = mg$$



$$\Rightarrow 4 \times \frac{KQq}{\left(\frac{\ell^2 + h^2}{2}\right)^{3/2}} h = mg \quad \therefore m = \frac{4KQqh}{g \left(\frac{\ell^2 + h^2}{2}\right)^{3/2}}$$

Example 18. Two identical charged spheres are suspended by strings of equal length. Each string makes an angle θ with the vertical. When suspended in a liquid of density $\sigma = 0.8 \text{ gm/cc}$, the angle remains the same. What is the dielectric constant of the liquid? (Density of the material of sphere is $\rho = 1.6 \text{ gm/cc}$.)

Solution : Initially as the forces acting on each ball are tension T , weight mg and electric force F , for its equilibrium along vertical

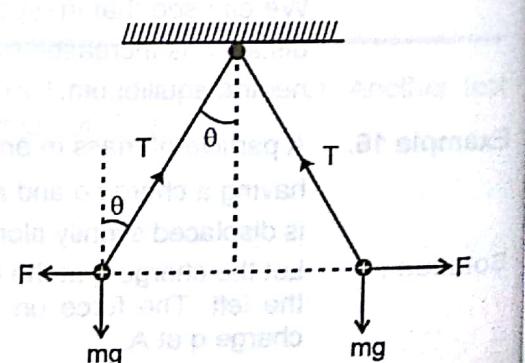
$$T \cos \theta = mg \quad \dots\dots(1)$$

and along horizontal

$$T \sin \theta = F \quad \dots\dots(2)$$

Dividing Eqn. (2) by (1), we have

$$\tan \theta = \frac{F}{mg} \quad \dots\dots(3)$$



When the balls are suspended in a liquid of density σ and dielectric constant K , the electric force will become $(1/K)$ times, i.e., $F' = (F/K)$ while weight

$$mg' = mg - F_B = mg - V\sigma g \quad [\text{as } F_B = V\sigma g, \text{ where } \sigma \text{ is density of material of sphere}]$$

$$\text{i.e., } mg' = mg \left[1 - \frac{\sigma}{\rho} \right] \quad [\text{as } V = \frac{m}{\rho}]$$

So, for equilibrium of ball,

$$\tan \theta = \frac{F'}{mg'} = \frac{F}{Kmg[1 - (\sigma/\rho)]} \quad \dots(4)$$

According to given information $\theta = 0$; so from equations (4) and (3), we have :

$$K = \frac{\rho}{(\rho - \sigma)} = \frac{(1.6)}{(1.6 - 0.8)} = 2 \quad \text{Ans.}$$

6. ELECTRIC FIELD

Electric field is the region around charged particle or charged body in which if another charge is placed, it experiences electrostatic force.

6.1 Electric field intensity \vec{E} : Electric field intensity at a point is equal to the electrostatic force experienced by a unit positive point charge both in magnitude and direction.

If a test charge q_0 is placed at a point in an electric field and experiences a force \vec{F} due to some charges (called source charges), the electric field intensity at that point due to source charges is given by $\vec{E} = \frac{\vec{F}}{q_0}$

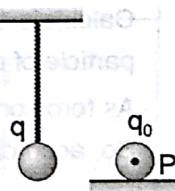
If the \vec{E} is to be determined practically then the test charge q_0 should be small otherwise it will affect the charge distribution on the source which is producing the electric field and hence modify the quantity which is measured.

Solved Examples

Example 19. A positively charged ball hangs from a long silk thread. We wish to measure E at a point P in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge q_0 at the point and measure F/q_0 . Will F/q_0 be less than, equal to, or greater than E at the point in question?

Solution : When we try to measure the electric field at point P , then after placing the test charge at P , it repels the source charge (suspended charge) and the measured

value of electric field $E_{\text{measured}} = \frac{F}{q_0}$ will be less than



the actual value E_{act} , that we wanted to measure.

6.2 Properties of electric field intensity \vec{E} :

- (i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.
- (ii) Direction of electric field due to positive charge is always away from it while due to negative charge, always towards it.
- (iii) Its S.I. unit is Newton/Coulomb.
- (iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

Electrostatics

(v) Electric force on a charge q placed in a region of electric field at a point where the electric field intensity is \vec{E} is given by $\vec{F} = q\vec{E}$.

Electric force on point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.

(vi) It obeys the superposition principle, that is, the field intensity at a point due to a system of charges is vector sum of the field intensities due to individual point charges.

$$\text{i.e. } \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

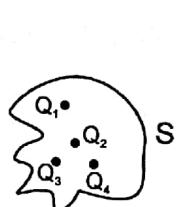
(vii) It is produced by source charges. The electric field will be a fixed value at a point unless we change the distribution of source charges.

Solved Examples

Example 20. Electrostatic force experienced by $-3\mu\text{C}$ charge placed at point 'P' due to a system 'S' of fixed point charges as shown in figure is $\vec{F} = (21\hat{i} + 9\hat{j})\mu\text{N}$.

(i) Find out electric field intensity at point P due to S.

(ii) If now, $2\mu\text{C}$ charge is placed and $-3\mu\text{C}$ is removed at point P then force experienced by will be.



Solution : (i) $\vec{F} = q\vec{E} \Rightarrow (21\hat{i} + 9\hat{j})\mu\text{N} = -3\mu\text{C}(\vec{E})$

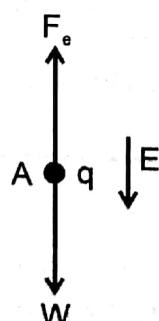
$$\Rightarrow \vec{E} = -7\hat{i} - 3\hat{j} \frac{\text{N}}{\text{C}}$$

(ii) Since the source charges are not disturbed the electric field intensity at 'P' will remain same.

$$\vec{F}_{2\mu\text{C}} = +2(\vec{E}) = 2(-7\hat{i} - 3\hat{j}) = (-14\hat{i} - 6\hat{j})\mu\text{N}$$

Example 21. Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge $-10\mu\text{C}$ and mass 10mg . (take $g = 10\text{ ms}^{-2}$)

Solution : As force on a charge q in an electric field \vec{E} is $\vec{F}_q = q\vec{E}$
So, according to given problem:

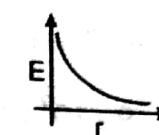
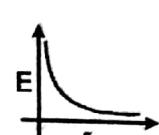
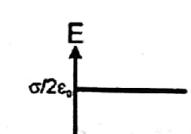
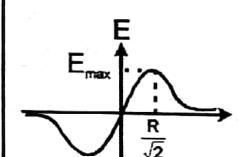
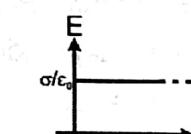
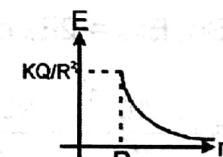
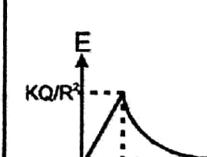


[W = weight of particle]

$$|\vec{F}_q| = |\vec{W}| \text{ i.e., } |q|\vec{E} = mg$$

$$\text{i.e., } E = \frac{mg}{|q|} = 10 \text{ N/C, in downward direction.}$$

List of formula for Electric Field Intensity due to various types of charge distribution :

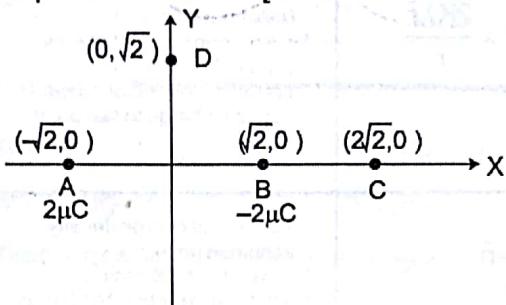
Name / Type	Formula	Note	Graph
Point charge	$\vec{E} = \frac{Kq}{ \vec{r} ^2} \cdot \hat{r}$	* q is source charge. * \vec{r} is vector drawn from source charge to the test point. outwards due to +charges & inwards due to -charges.	
Infinitely long line charge	$\frac{\lambda}{2\pi\epsilon_0 r} \hat{r} = \frac{2K\lambda \hat{r}}{r}$	* λ is linear charge density (assumed uniform) * r is perpendicular distance of point from line charge. * \hat{r} is radial unit vector drawn from the charge to test point.	
Infinite non-conducting thin sheet	$\frac{\sigma}{2\epsilon_0} \hat{n}$	* σ is surface charge density (assumed uniform) * \hat{n} is unit normal vector. * x = distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Uniformly charged ring	$E = \frac{KQx}{(R^2 + x^2)^{3/2}}$ $E_{\text{centre}} = 0$	* Q is total charge of the ring * x = distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Infinitely large charged conducting sheet	$\frac{\sigma}{\epsilon_0} \hat{n}$	* σ is the surface charge density (assumed uniform) * \hat{n} is the unit vector perpendicular to the surface.	
Uniformly charged hollow conducting/ nonconducting /solid conducting sphere	(i) for $r \geq R$ $\vec{E} = \frac{kQ}{ \vec{r} ^2} \hat{r}$ (ii) for $r < R$ $E = 0$	* R is radius of the sphere. * \vec{r} is vector drawn from centre of sphere to the point. * Sphere acts like a point charge placed at centre for points outside the sphere. * \vec{E} is always along radial direction. * Q is total charge ($= \sigma 4\pi R^2$). (σ = surface charge density)	
Uniformly charged solid nonconducting sphere (insulating material)	(i) for $r \geq R$ $\vec{E} = \frac{kQ}{ \vec{r} ^2} \hat{r}$ (ii) for $r \leq R$ $\vec{E} = \frac{kQ}{R^3} \vec{r}$	* \vec{r} is vector drawn from centre of sphere to the point * Sphere acts like a point charge placed at the centre for points outside the sphere * \vec{E} is always along radial dir ⁿ * Q is total charge ($= \rho \frac{4}{3}\pi R^3$). (ρ = volume charge density) * Inside the sphere $E \propto r$. * Outside the sphere $E \propto 1/r^2$.	

Example 22. Find out electric field intensity at point A (0, 1m, 2m) due to a point charge $-20\mu\text{C}$ situated at point B($\sqrt{2}$ m, 0, 1m).

Solution :

$$\begin{aligned} \mathbf{E} &= \frac{KQ}{|\vec{r}|^3} \vec{r} = \frac{KQ}{|\vec{r}|^2} \hat{r} \Rightarrow \vec{r} = \text{P.V. of A} - \text{P.V. of B} \quad (\text{P.V.} = \text{Position vector}) \\ &= (-\sqrt{2}\hat{i} + \hat{j} + \hat{k}) \quad |\vec{r}| = \sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2} = 2 \\ \mathbf{E} &= \frac{9 \times 10^9 \times (-20 \times 10^{-6})}{8} (-\sqrt{2}\hat{i} + \hat{j} + \hat{k}) = -22.5 \times 10^3 (-\sqrt{2}\hat{i} + \hat{j} + \hat{k}) \text{ N/C.} \end{aligned}$$

Example 23. Two point charges $2\mu\text{C}$ and $-2\mu\text{C}$ are placed at points A and B as shown in figure. Find out electric field intensity at points C and D. [All the distances are measured in meter].



Solution : Electric field at point C (E_A , E_B are magnitudes only and arrows represent directions).

Electric field due to positive charge is away from it while due to negative charge, it is towards the charge.
It is clear that $E_B > E_A$.

$\therefore E_{\text{Net}} = (E_B - E_A)$ towards negative X-axis

$$= \frac{K(2\mu\text{C})}{(\sqrt{2})^2} - \frac{K(2\mu\text{C})}{(3\sqrt{2})^2} \text{ towards negative X-axis} = 8000 (-\hat{i}) \text{ N/C}$$

Electric field at point D :

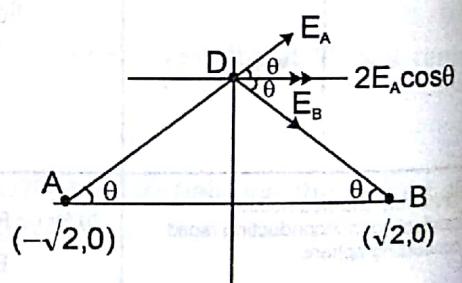
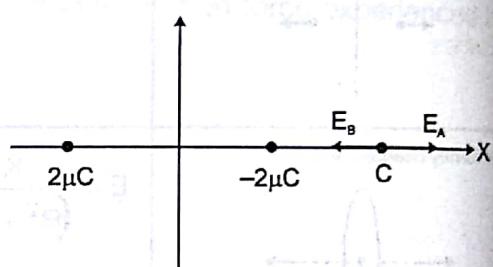
Since magnitude of charges are same and also $AD = BD$

So, $E_A = E_B$

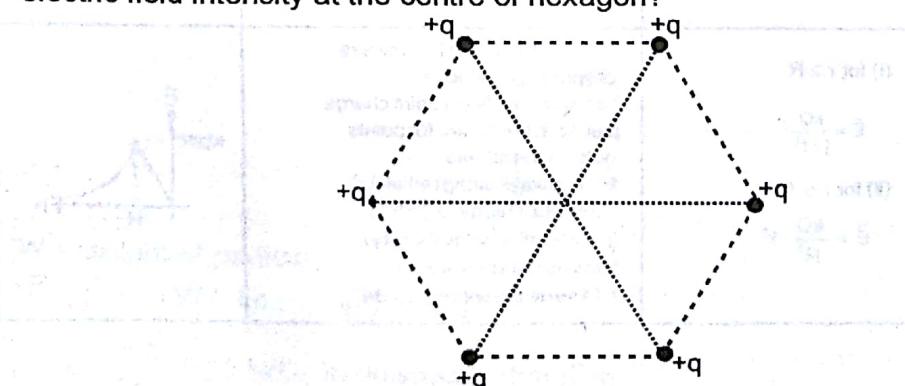
Vertical components of \vec{E}_A and \vec{E}_B cancel each other while horizontal components are in the same direction.

$$\text{So, } E_{\text{net}} = 2E_A \cos\theta = \frac{2K(2\mu\text{C})}{2^2} \cos 45^\circ$$

$$= \frac{K \times 10^{-6}}{\sqrt{2}} = \frac{9000}{\sqrt{2}} \hat{i} \text{ N/C.}$$

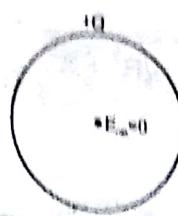


Example 24. Six equal point charges are placed at the corners of a regular hexagon of side 'a'. Calculate electric field intensity at the centre of hexagon?



Answer : Zero (By symmetry)

Similarly electric field due to a uniformly charged ring at the centre of ring :



- Note : (i) Net charge on a conductor remains only on the outer surface of a conductor. This property will be discussed in the article of the conductor. (article no.17)
(ii) On the surface of isolated spherical conductor charge is uniformly distributed.

6.3 Electric field due to a uniformly charged ring and arc.

Solved Examples

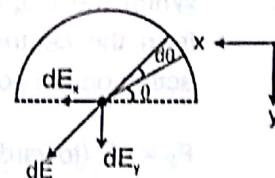
Example 25. Find out electric field intensity at the centre of a uniformly charged semicircular ring of radius R and linear charge density λ .

Solution : λ = linear charge density.

The arc is the collection of large no. of point charges. Consider a part of ring as an element of length $Rd\theta$ which subtends an angle $d\theta$ at centre of ring and it lies between 0 and $0 + d\theta$

$$d\vec{E} = dE_x \hat{i} + dE_y \hat{j}; E_x = \int dE_x = 0 \text{ (due to symmetry)}$$

$$\& E_y = \int dE_y = \int_0^{\pi} dE \sin\theta = \frac{K\lambda}{R} \int_0^{\pi} \sin\theta d\theta = \frac{2K\lambda}{R}$$



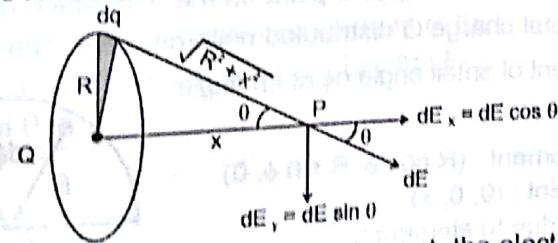
Example 26. Find out electric field intensity at the centre of uniformly charged quarter ring of radius R and linear charge density λ .

Solution : Refer to the previous question $d\vec{E} = dE_x \hat{i} + dE_y \hat{j}$ ∴ on solving $E_{net} = \frac{K\lambda}{R}(\hat{i} + \hat{j})$

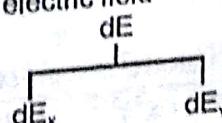


By use of symmetry and from the formula of electric field due to half ring.
Above answer can be justified.

(ii) Derivation of electric field intensity at a point on the axis at a distance x from centre of uniformly charged ring of radius R and total charge Q .



Consider an element of charge dq . Due to this element, the electric field at the point on axis, which is at a distance x from the centre of the ring is dE . There are two components of this electric field



Electrostatics

The y-component of electric field due to all the elements will be cancelled out to each other. So electric field intensity at the point will be only due to X-component of each element.

$$E_{\text{net}} = \int dE_x = \int dE \cos \theta = \int_0^Q \frac{K(dq)}{R^2 + x^2} \times \frac{x}{\sqrt{R^2 + x^2}} = \frac{kx}{(R^2 + x^2)^{3/2}} \int_0^Q dq$$

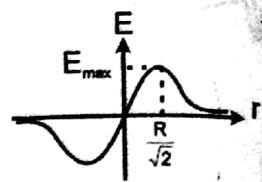
$$E_{\text{net}} = \frac{KQx}{[R^2 + x^2]^{3/2}}$$

Graph for variation of E with r.

E will be max when $\frac{dE}{dx} = 0$, that is at $x = \frac{R}{\sqrt{2}}$ and $E_{\text{max}} = \frac{2KQ}{3\sqrt{3} R^2}$

Case (i) : if $x \gg R$, $E = \frac{KQ}{x^2}$ Hence the ring will act like a point charge

Case (ii) : if $x \ll R$, $E = \frac{KQx}{R^3}$



Solved Example

Example 27. Positive charge Q is distributed uniformly over a circular ring of radius a. A point particle having a mass m and a negative charge $-q$, is placed on its axis at a distance y from the centre. Find the force on the particle. Assuming $y \ll a$, find the time period of oscillation of the particle if released from there. (Neglect gravity)

Solution : When the negative charge is shifted at a distance x from the centre of the ring along its axis then force acting on the point charge due to the ring :

$$F_E = qE \text{ (towards centre)} = q \left[\frac{KQy}{(a^2 + y^2)^{3/2}} \right]$$

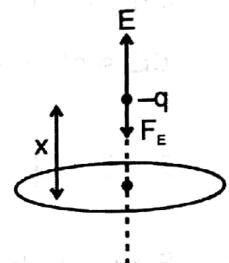
If $a \gg y$ then $a^2 + y^2 \approx a^2$

$$\therefore F_E = \frac{1}{4\pi\epsilon_0} \frac{Qqy}{a^3} \text{ (Towards centre)}$$

Since, restoring force $F_E \propto y$, therefore motion of charge the particle will be S.H.M.

Time period of SHM

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{\frac{Qq}{4\pi\epsilon_0 a^3}}} = \left[\frac{16\pi^3 \epsilon_0 m a^3}{Qq} \right]^{1/2}$$



Example 28.

Calculate electric field intensity at a point on the axis which is at distance x from the centre of a half ring, having total charge Q distributed uniformly on it. The radius of half ring is R.

Solution :

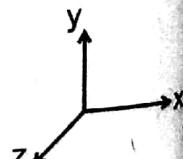
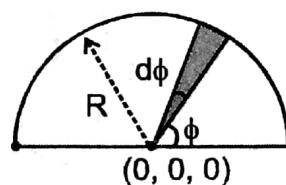
Consider an element of small angle $d\phi$ at an angle ϕ as shown.

Coordinates of element : $(R \cos \phi, R \sin \phi, 0)$

Coordinates of point : $(0, 0, x)$

Now electric field due to element :

$$dE = \frac{K(\lambda R d\phi) \cdot [-R \cos \phi \hat{i} - R \sin \phi \hat{j} + x \hat{k}]}{(R^2 \cos^2 \phi + R^2 \sin^2 \phi + x^2)^{3/2}} \Rightarrow E_x = \sum dE_x = - \int_0^\pi \frac{K\lambda R^2 \cos \phi d\phi}{(R^2 + x^2)^{3/2}} = 0$$



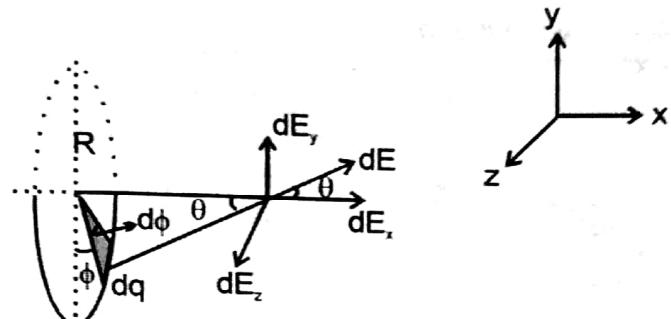
$$E_y = \sum dE_y = - \int_0^\pi \frac{K\lambda R^2 \sin \phi d\phi}{(R^2 + x^2)^{3/2}} = \frac{2K\lambda R^2}{(R^2 + x^2)^{3/2}} = \frac{2KQR}{\pi(R^2 + x^2)^{3/2}}$$

Electrostatics

$$E_z = \sum dE_z = \int_0^\pi \frac{K\lambda R x d\phi}{(R^2 + x^2)^{3/2}} = \frac{KQx}{(R^2 + x^2)^{3/2}}$$

$$E_{\text{net}} = \sqrt{E_x^2 + E_y^2 + E_z^2} = \frac{KQ}{(R^2 + x^2)^{3/2}} \cdot \sqrt{\frac{4R^2}{\pi^2} + x^2}$$

Alternate solution :



Consider an element of charge dq at an angle ϕ on circumference of half ring. Due to this element electric field at the point on axis, which is at a distance x from the centre of half ring is dE . This electric field can be resolved into three component.



$$E_z = \int_{-\pi/2}^{\pi/2} dE \sin \theta \sin \phi = 0$$

$$E_x = \int_{-\pi/2}^{\pi/2} dE \cos \theta = \frac{KQx}{[R^2 + x^2]^{3/2}} \quad \dots\dots(1)$$

$$E_y = \int_{-\pi/2}^{\pi/2} dE \sin \theta \cos \phi = \int \frac{Kdq}{R^2 + x^2} \sin \theta \cdot \cos \phi = \frac{2K\lambda R \sin \theta}{R^2 + x^2} \quad \dots\dots(2)$$

$$\therefore dq = \lambda R d\phi, \sin \theta = \frac{R}{\sqrt{R^2 + x^2}} \Rightarrow E_{\text{net}} = \sqrt{E_x^2 + E_y^2}$$

Example 29. Derive the expression of electric field intensity at a point 'P' which is situated at a distance x on the axis of uniformly charged disc of radius R and surface charge density σ . Also, derive results for

(i) $x \gg R$ (ii) $x \ll R$

The disc can be considered to be a collection of large number of concentric rings. Consider an element of the shape of rings of radius r and of width dr . Electric field due to this ring at P is

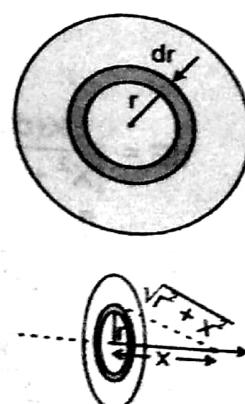
$$dE = \frac{K \cdot \sigma \cdot 2\pi r \cdot dr \cdot x}{(r^2 + x^2)^{3/2}}$$

$$\text{Put, } r^2 + x^2 = y^2$$

$$2rdr = 2ydy$$

$$\therefore dE = \frac{K \cdot \sigma \cdot 2\pi y \cdot dy \cdot x}{y^3} = 2K\sigma\pi x \frac{ydy}{y^3}$$

Electric field at P due to all rings is along the axis :



$$\begin{aligned}\therefore E &= \int dE \Rightarrow E = 2K\sigma\pi x \int_x^{\sqrt{R^2+x^2}} \frac{1}{y^2} dy = 2K\sigma\pi x \left[-\frac{1}{y} \right]_x^{\sqrt{R^2+x^2}} \\ &= 2K\sigma\pi x \left[+\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}} \right] = 2K\sigma\pi \left[1 - \frac{x}{\sqrt{R^2+x^2}} \right] \\ &= \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{R^2+x^2}} \right] \text{ along the axis}\end{aligned}$$

Cases : (i) If $x \gg R$

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + \frac{R^2}{x^2} + 1}} \right] = \frac{\sigma}{2\epsilon_0} \left[1 - \left(1 + \frac{R^2}{x^2} \right)^{-1/2} \right]$$

$$\frac{\sigma}{2\epsilon_0} = [1 - 1 + \frac{1}{2} \frac{R^2}{x^2} + \text{higher order terms}] = \frac{\sigma}{4\epsilon_0} \frac{R^2}{x^2} = \frac{\sigma\pi R^2}{4\pi\epsilon_0 x^2} = \frac{Q}{4\pi\epsilon_0 x^2}$$

i.e. behaviour of the disc is like a point charge.

(ii) If $x \ll R$

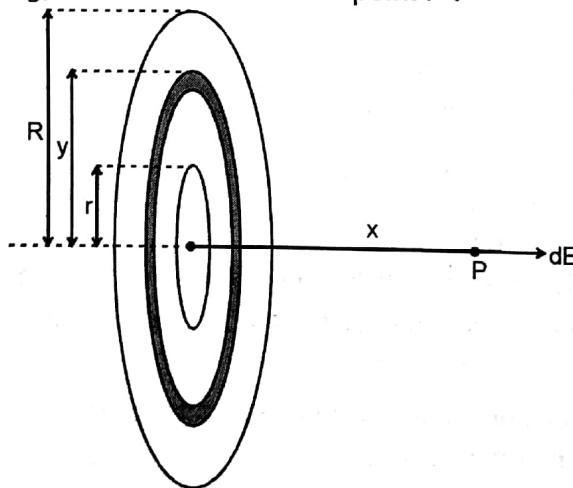
$$E = \frac{\sigma}{2\epsilon_0} [1 - 0] = \frac{\sigma}{2\epsilon_0}$$

i.e. behaviour of the disc is like infinite sheet.

Example 30. Calculate electric field at a point on axis, which at a distance x from centre of uniformly charged disc having surface charge density σ and R which also contains a concentric hole of radius r .

Solution :

Consider a ring of radius y ($r < y < R$) and width dy concentric with disc and in the plane of the disc. Due to this ring, the electric field at the point P :



$$dE = \frac{K(dq)x}{[x^2 + y^2]^{3/2}}$$

$$E_{\text{net}} = \int_r^R \frac{K x \cdot \sigma (2\pi y) dy}{[x^2 + y^2]^{3/2}} \quad [\because dq = \sigma 2\pi y dy]$$

$$E_{\text{net}} = \frac{2\pi\sigma}{2} \frac{kx}{x^2 + r^2} \int_{x^2 + r^2}^{x^2 + R^2} \frac{dt}{t^{3/2}}, \quad \text{put } x^2 + y^2 = t, 2y \cdot dy = dt$$

$$= \frac{\sigma x}{2\epsilon_0} \left[\frac{1}{\sqrt{x^2 + r^2}} - \frac{1}{\sqrt{x^2 + R^2}} \right] \text{ away from centre}$$

Alternate method

We can also use superposition principle to solve this problem.

- Assume a disc without hole of radius R having surface charge density $+\sigma$.
- Also assume a concentric disc of radius r in the same plane of first disc having charge density $-\sigma$. Now using derived formula in last example the net electric field at the centre is :

$$\vec{E}_{\text{net}} = \vec{E}_R + \vec{E}_r = \frac{\sigma}{2\epsilon_0} \times \left[\frac{1}{\sqrt{r^2 + x^2}} - \frac{1}{\sqrt{R^2 + x^2}} \right] \text{ away from centre.}$$

6.4

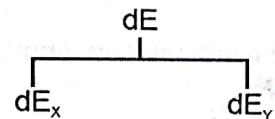
Electric field due to uniformly charged wire:

- Line charge of finite length :** Derivation of expression for intensity of electric field at a point due to line charge of finite size of uniform linear charge density λ . The perpendicular distance of the point from the line charge is r and lines joining ends of line charge distribution make angle θ_1 and θ_2 with the perpendicular line.

Consider a small element dx on line charge distribution at distance x from point A (see fig.). The charge of this element will be $dq = \lambda dx$. Due to this charge (dq), the intensity of electric field at the point P is dE .

$$\text{Then } dE = \frac{K(dq)}{r^2 + x^2} = \frac{K(\lambda dx)}{r^2 + x^2}$$

$$\text{There will be two components of this field : } E_x = \int dE_x = \int dE \cos \theta = \int \frac{K\lambda dx}{r^2 + x^2} \cos \theta$$



$$\text{Assuming, } x = r \tan \theta \Rightarrow dx = r \sec^2 \theta \cdot d\theta$$

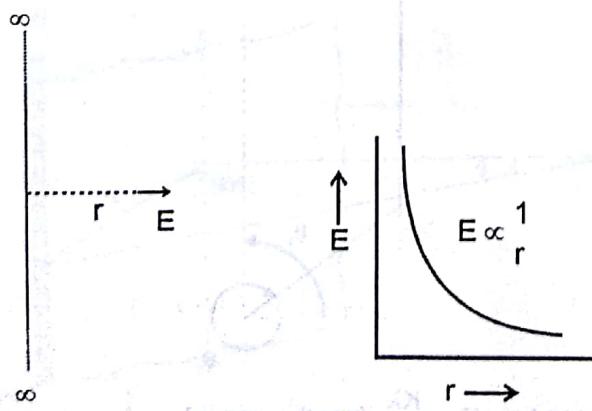
$$\text{so } E_x = \int_{-\theta_2}^{+\theta_1} \frac{K \lambda r \sec^2 \theta \cdot \cos \theta \cdot d\theta}{r^2 + r^2 \tan^2 \theta} = \frac{K \lambda}{r} \int_{-\theta_2}^{+\theta_1} \cos \theta \cdot d\theta = \frac{K \lambda}{r} [\sin \theta_1 + \sin \theta_2] \quad \dots\dots(1)$$

$$\text{Similarly y-component. } E_y = \frac{K \lambda}{r} \int_{-\theta_2}^{+\theta_1} \sin \theta \cdot d\theta = \frac{K \lambda}{r} [\cos \theta_2 - \cos \theta_1]$$

$$\text{Net electric field at the point: } E_{\text{net}} = \sqrt{E_x^2 + E_y^2}$$

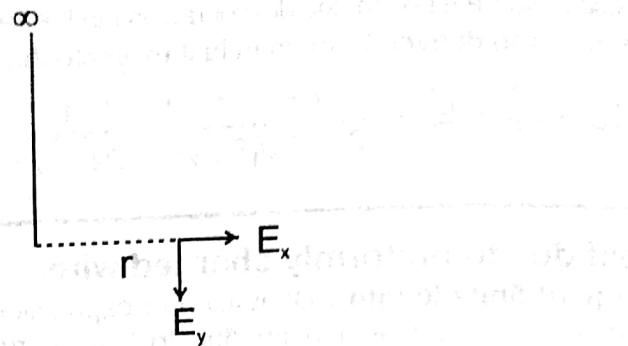
- We can derive a result for infinitely long line charge:** In above eq. (1) & (2), if we put $\theta_1 = \theta_2 = 90^\circ$, we can get required result.

$$E_{\text{net}} = E_x = \frac{2K\lambda}{r}$$



(iii) For Semi-infinite wire : $\theta_1 = 90^\circ$ and $\theta_2 = 0^\circ$, so,

$$E_x = \frac{K\lambda}{r}, E_y = \frac{K\lambda}{r}$$

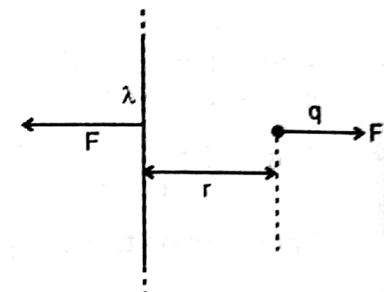


Solved Examples

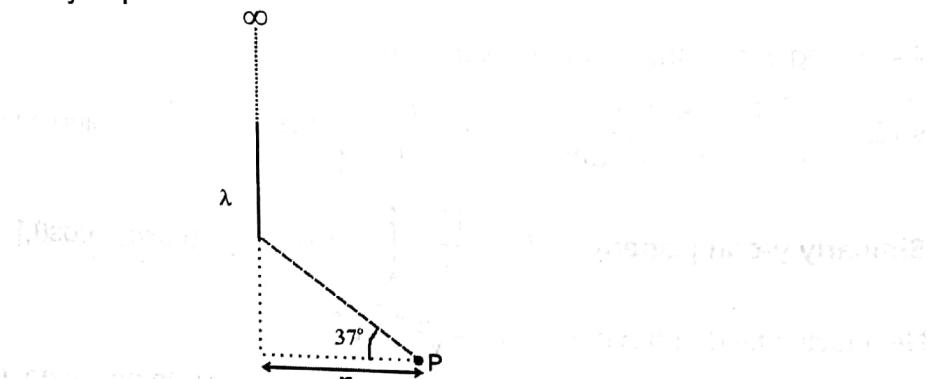
Example 31. A point charge q is placed at a distance r from a very long charged thread of uniform linear charge density λ . Find out total electric force experienced by the line charge due to the point charge. (Neglect gravity).

Solution : Force on charge q due to the thread, $F = \left(\frac{2K\lambda}{r}\right) \cdot q$

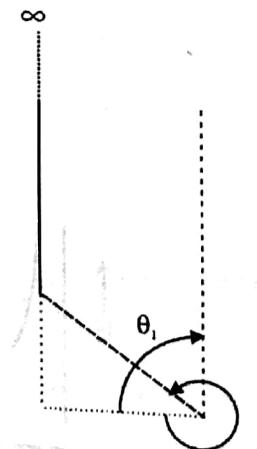
By Newton's III law, every action has equal and opposite reaction So, force on the thread $= \frac{2K\lambda}{r} \cdot q$ (away from point charge)



Example 32. Figure shows a long wire having uniform charge density λ as shown in figure. Calculate electric field intensity at point P.

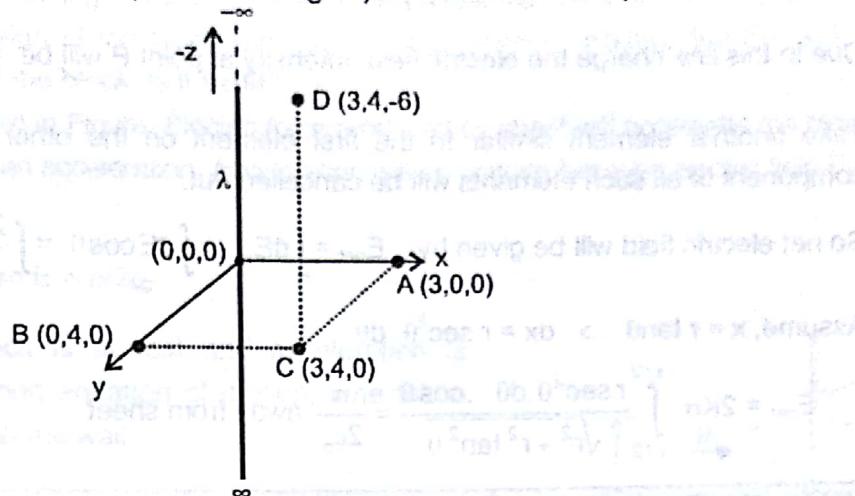


Solution : $\theta_1 = 90^\circ$ and $\theta_2 = 360^\circ - 37^\circ$ So



$$E_x = \frac{K\lambda}{r} [\sin\theta_1 + \sin\theta_2]; E_y = \frac{K\lambda}{r} [\cos\theta_2 - \cos\theta_1]$$

Example 33. Find electric field at point A, B, C, D due to infinitely long uniformly charged wire with linear charge density λ and kept along z-axis (as shown in figure). Assume that all the parameters are in S.I. units.



Solution : $E_A = \frac{2K\lambda}{3}(\hat{i}) \Rightarrow E_B = \frac{2K\lambda}{4}(\hat{j})$

$$E_C = \frac{2K\lambda}{5} \hat{OC} = \frac{2K\lambda}{5} \left(\frac{3\hat{i} + 4\hat{j}}{5} \right) \quad E_D = \frac{2K\lambda}{5} \left(\frac{3\hat{i} + 4\hat{j}}{5} \right) \Rightarrow E_D = E_C$$

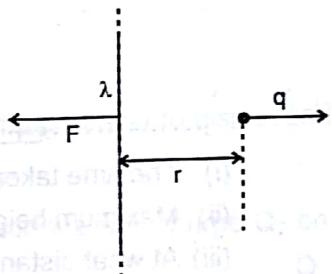
Example 34. A point charge q is placed at a distance r from a very long charged thread of uniform linear charge density λ . Find out total electric force experienced by the line charge due to the point charge. (Neglect gravity).

Solution : Force on charge q due to the thread, $F = \left(\frac{2K\lambda}{r} \right) \cdot q$

By Newton's III law, every action has equal and opposite

reaction, so force on the thread = $\frac{2K\lambda}{r} \cdot q$

(away from point charge)

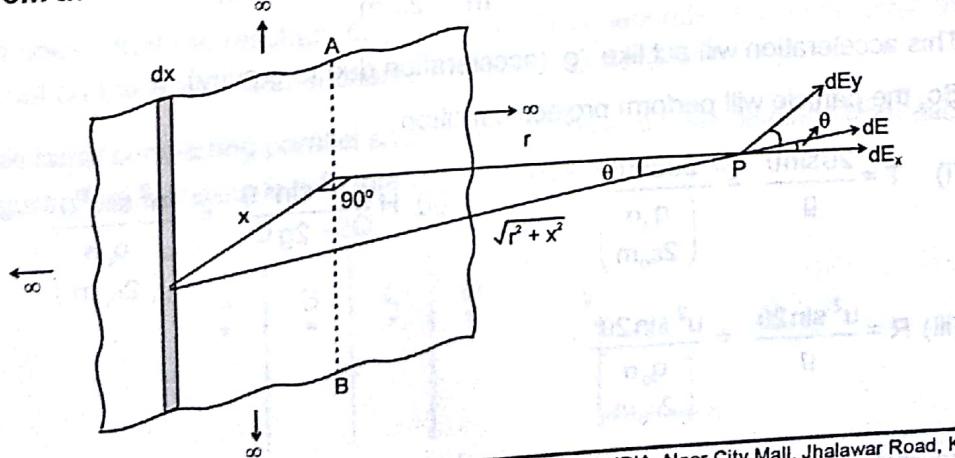


6.5 Electric field due to uniformly charged infinite sheet

$$E_{net} = \frac{\sigma}{2\epsilon_0}$$
 towards normal direction

ELECTRIC FIELD DUE TO AN INFINITELY LARGE, UNIFORMLY CHARGED SHEET

Derivation of expression for intensity of electric field at a point which is at a perpendicular distance r from the thin sheet of large size having uniform surface charge density σ .



Electrostatics

Assume a thin strip of width dx at distance x from line AB (see figure), which can be considered as a infinite line charge of charge density $\lambda = \sigma dx$

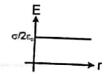
$$\text{Due to this line charge the electric field intensity at point P will be } dE = \frac{\sigma K(dx)}{\sqrt{r^2 + x^2}}$$

Take another element similar to the first element on the other side of AB. Due to symmetry, Y component of all such elements will be cancelled out.

$$\text{So net electric field will be given by: } E_{\text{net}} = \int dE_x = \int dE \cos 0 = \int \frac{2K(\sigma dx)}{\sqrt{r^2 + x^2}} \times \cos 0$$

$$\text{Assume, } x = r \tan \theta \Rightarrow dx = r \sec^2 \theta d\theta$$

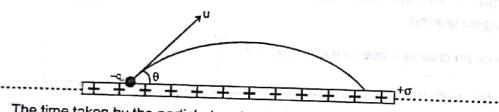
$$\therefore E_{\text{net}} = 2K\sigma \int_{-\pi/2}^{\pi/2} \frac{r \sec^2 \theta d\theta}{\sqrt{r^2 + r^2 \tan^2 \theta}} \cos 0 = \frac{\sigma}{2\epsilon_0} \text{ away from sheet}$$



- Note : (1) The direction of electric field is always perpendicular to the sheet.
(2) The magnitude of electric field is independent of distance from sheet.

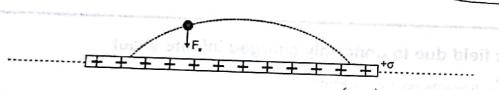
Solved Examples

Example 35. An infinitely large plate of surface charge density $+\sigma$ is lying in horizontal xy plane. A particle having charge $-q_0$ and mass m is projected from the plate with velocity u making an angle θ with sheet. Find :



- (i) The time taken by the particle to return on the plate.
(ii) Maximum height achieved by the particle.
(iii) At what distance will it strike the plate (Neglect gravitational force on the particle)

Solution :



$$\text{Electric force acting on the particle } F_e = q_0 E : F_e = (q_0) \left(\frac{\sigma}{2\epsilon_0} \right) \text{ downward}$$

$$\text{So, acceleration of the particle : } a = \frac{F_e}{m} = \frac{q_0 \sigma}{2\epsilon_0 m} = \text{uniform}$$

This acceleration will act like 'g' (acceleration due to gravity)

So, the particle will perform projectile motion.

$$(i) T = \frac{2u \sin \theta}{g} = \frac{2u \sin \theta}{\left(\frac{q_0 \sigma}{2\epsilon_0 m} \right)}$$

$$(ii) H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 \theta}{2 \left(\frac{q_0 \sigma}{2\epsilon_0 m} \right)}$$

$$(iii) R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 2\theta}{\left(\frac{q_0 \sigma}{2\epsilon_0 m} \right)}$$

Electrostatics

Example 36.

A block having mass m and charge Q is resting on a frictionless plane at a distance d from fixed large non-conducting infinite sheet of uniform charge density $-\sigma$ as shown in Figure. Assuming that collision of the block with the sheet is perfectly elastic, find the time period of oscillatory motion of the block. Is it SHM?

Solution :

The situation is shown in Figure. Electric force produced by sheet will accelerate the block towards the sheet producing an acceleration. Acceleration will be uniform because electric field E due to sheet is uniform.

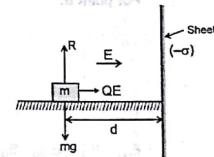
$$a = \frac{F}{m} = \frac{QE}{m}, \text{ where } E = \sigma/2\epsilon_0$$

As initially the block is at rest and acceleration is constant, from second equation of motion, time taken by the block to reach the wall

$$d = \frac{1}{2} at^2 \text{ i.e., } t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2md}{QE}} = \sqrt{\frac{4md\epsilon_0}{Q\sigma}}$$

As collision with the wall is perfectly elastic, the block will rebound with same speed and as now its motion is opposite to the acceleration, it will come to rest after traveling same distance d in same time t .

After stopping, it will again be accelerated towards the wall and so the block will execute oscillatory motion with 'span' d and time period.

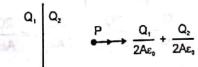


$$T = 2t = 2 \sqrt{\frac{2md}{QE}} = 2 \sqrt{\frac{4md\epsilon_0}{Q\sigma}}$$

However, as the restoring force $F = QE$ is constant and not proportional to displacement x , the motion is not simple harmonic.

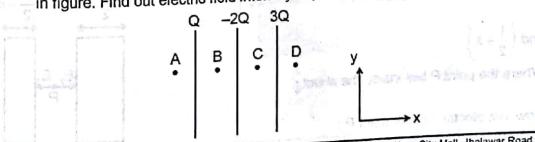
Example 37. If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\epsilon_0}$, where $A = Q_1 + Q_2$

$$\text{Solution : Electric field at point P : } \vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{Q_1}{2A\epsilon_0} \hat{n} + \frac{Q_2}{2A\epsilon_0} \hat{n} = \frac{Q_1 + Q_2}{2A\epsilon_0} \hat{n} = \frac{Q}{2A\epsilon_0}$$



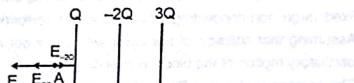
[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Example 38. Three large conducting parallel sheets are placed at a finite distance from each other as shown in figure. Find out electric field intensity at points A, B, C & D.



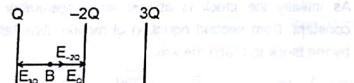
Electrostatics

Solution : For point A :



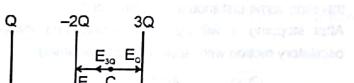
$$\vec{E}_{\text{net}} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = -\frac{Q}{2A\epsilon_0} \hat{i} - \frac{3Q}{2A\epsilon_0} \hat{i} + \frac{2Q}{2A\epsilon_0} \hat{i} = -\frac{Q}{A\epsilon_0} \hat{i}$$

For point B :



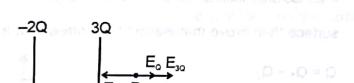
$$\vec{E}_{\text{net}} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = -\frac{3Q}{2A\epsilon_0} \hat{i} + \frac{2Q}{2A\epsilon_0} \hat{i} + \frac{Q}{2A\epsilon_0} \hat{i} = \vec{0}$$

For point C :



$$\vec{E}_{\text{net}} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = +\frac{Q}{2A\epsilon_0} \hat{i} - \frac{3Q}{2A\epsilon_0} \hat{i} - \frac{Q}{2A\epsilon_0} \hat{i} = -\frac{2Q}{A\epsilon_0} \hat{i}$$

For point D :



$$\vec{E}_{\text{net}} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = +\frac{Q}{2A\epsilon_0} \hat{i} + \frac{3Q}{2A\epsilon_0} \hat{i} - \frac{2Q}{2A\epsilon_0} \hat{i} = \frac{2Q}{A\epsilon_0} \hat{i}$$

Example 39. Determine and draw the graph of electric field due to infinitely large non-conducting sheet of thickness t and uniform volume charge density ρ as a function of distance x from its symmetry plane.

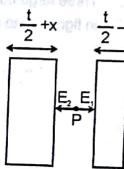
$$(a) x \leq \frac{t}{2} \quad (b) x \geq \frac{t}{2}$$

Solution : We can consider two sheets of thickness $(\frac{t}{2} - x)$

$$\text{and } \left(\frac{t}{2} + x\right)$$

Where the point P lies inside the sheet.

Now, net electric field at point P :



Electrostatics

$$E = E_1 - E_2 = \frac{Q_1}{2A\epsilon_0} - \frac{Q_2}{2A\epsilon_0} [Q_1 : \text{charge of left sheet}; Q_2 : \text{charge of right sheet}]$$

$$= \frac{Ap\left(\frac{t}{2} + x\right) - pA\left(\frac{t}{2} - x\right)}{2A\epsilon_0} = \frac{\rho x}{\epsilon_0}$$

For point which lies outside the sheet we can consider a complete sheet of thickness t

$$R = \frac{Q}{A\epsilon_0}, \quad \sigma = \frac{Q}{A} \Rightarrow \sigma = R\epsilon_0$$

$$E = \frac{\sigma t}{2\epsilon_0} = \frac{Qt}{2\epsilon_0}$$

Alternate : We can assume thick sheet to be made of large number of uniformly charged thin sheets. Consider an elementary thin sheet of width dx at a distance x from symmetry plane.

Charge in sheet = $pAdx$ (A : assumed area of sheet)

$$\text{Surface charge density, } \sigma = \frac{pAdx}{A}$$

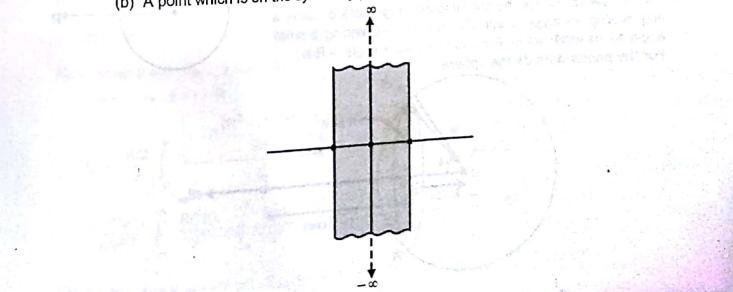
$$\text{so, electric field intensity due to elementary sheet : } dE = \frac{pdx}{2\epsilon_0}$$

$$(a) \text{ When } x < \frac{t}{2} \Rightarrow E_{\text{Net}} = \int_{-t/2}^x \frac{pdx}{2\epsilon_0} - \int_x^{t/2} \frac{pdx}{2\epsilon_0} = \frac{\rho x}{\epsilon_0}$$

$$(b) \text{ When } x > \frac{t}{2} \Rightarrow E_{\text{Net}} = \int_{-t/2}^{t/2} \frac{pdx}{2\epsilon_0} = \frac{\rho t}{2\epsilon_0}$$

Example 40. Thin infinite sheet of width w contains uniform charge distribution σ . Find out electric field intensity at following points :

- (a) A point which lies in the same plane at a distance d from one of its edges.
- (b) A point which is on the symmetry plane of sheet at a perpendicular distance d from it.



Solution:

- (a) Consider a thin strip of width dx . Linear charge density of strip : $\lambda = \sigma dx$

So, electric field due to this strip at point P

$$dE = \frac{2\lambda dx}{x}$$

$$E_{\text{net}} = \int_d^{\infty} \frac{\sigma dx}{2\pi\epsilon_0 x} = \frac{\sigma}{2\pi\epsilon_0} \ln\left(\frac{d+\infty}{d}\right)$$

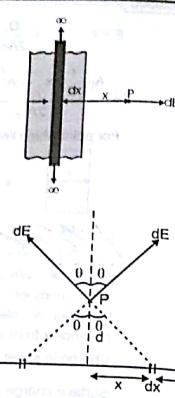
- (b) Consider a thin strip of width dx . Linear charge density of strip :

$$\lambda = \sigma dx$$

$$\therefore E_p = \int dE \cos \theta$$

$$\text{or } E_p = 2 \int_0^{w/2} \frac{\sigma dx}{2\pi\epsilon_0 \sqrt{d^2 + x^2}} \frac{d}{\sqrt{d^2 + x^2}}$$

$$= \frac{\sigma d}{\pi\epsilon_0} \int_0^{w/2} \frac{dx}{d^2 + x^2} = \frac{\sigma}{\pi\epsilon_0} \tan^{-1} \frac{w}{2d}$$



6.6 Electric field due to uniformly charged spherical shell

$E = \frac{KQ}{r^2}$ if $r \geq R$ \Rightarrow For the outside points & point on the surface, the uniformly charged spherical shell behaves as a point charge placed at the centre

$$E = 0 \quad r < R$$

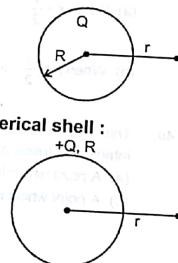
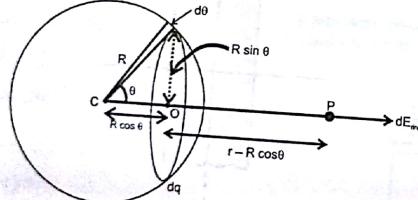
Electric field due to spherical shell outside it is always along the radial direction.

Finding electric field due to a uniformly charged spherical shell :

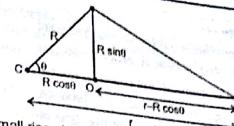
Suppose we have a spherical shell of radius R and charge $+Q$ uniformly distributed on its surface. We have to find electric field at a point P, which is at a distance 'r' from the centre of the sphere.

For this, we can divide the shell into thin rings. Let's consider a ring making an angle θ with the axis and subtending a small angle $d\theta$. Its width will be $Rd\theta$. (arc = radius \times angle = $Rd\theta$).

For the points outside the sphere :



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Electric field due to this small ring element :

$$dE = \frac{Kdqx}{[(\text{ring radius})^2 + x^2]^{3/2}}$$

$$\text{So, total electric field } E_{\text{net}} = \int \frac{Kdq x}{[(\text{ring radius})^2 + x^2]^{3/2}}$$

Here, radius of the ring element = $R \sin \theta$ & x = axial distance of point P from the ring = $r - R \cos \theta$

dq = charge of the small ring element. We can find dq by unitary method.

In $4\pi R^2$ Area, charge is Q

In unit Area, charge is $\frac{Q}{4\pi R^2}$.

In $(2\pi R \sin \theta) Rd\theta$ Area, charge = $\frac{Q}{4\pi R^2} \times (2\pi R \sin \theta) Rd\theta = dq$

Putting values of r and dq in equation ..(1)

$$\text{We get } E_{\text{out}} = \int_0^{\theta} \frac{K \left(\frac{Q}{4\pi R^2} \times 2\pi (R \sin \theta) Rd\theta \right) (r - R \cos \theta)}{[(R \sin \theta)^2 + (r - R \cos \theta)^2]^{3/2}} d\theta$$

(The first ring will make angle $\theta = 0$ and the last ring will make $\theta = 180^\circ$. So, limit will be from 0 to $0 = 180^\circ$)

Steps of integration : From above integral :

$$E_{\text{out}} = \frac{KQ}{2} \int_0^{\pi} \frac{(r - R \cos \theta) \sin \theta d\theta}{(R^2 + r^2 - 2Rr \cos \theta)^{3/2}}$$

$$\text{Now, let } z^2 = R^2 + r^2 - 2Rr \cos \theta \Rightarrow 2zdz = 0 + 0 - 2Rr(-\sin \theta) d\theta$$

$$\therefore zdz = Rr \sin \theta d\theta \quad \& \quad \cos \theta = \frac{R^2 + r^2 - z^2}{2Rr}$$

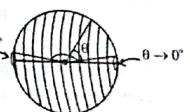
Now, when $\theta = 0 \rightarrow z = (r - R)$

$$0 = \pi \rightarrow z = r + R$$

$$\therefore E_{\text{out}} = \frac{KQ}{2} \int_{r-R}^{r+R} \frac{\left[r - R \frac{(R^2 + r^2 - z^2)}{2Rr} \right] zdz}{z^3} = \frac{KQ}{2} \int_{r-R}^{r+R} \frac{(2Rr^2 - R^3 - Rr^2 + Rz^2) dz}{2R^2 r^2 z^2}$$

$$= \frac{KQ}{4R^2 r^2} \left[\int_{r-R}^{r+R} \frac{Rr^2 dz}{z^2} - \int_{r-R}^{r+R} \frac{R^3 dz}{z^2} + \int_{r-R}^{r+R} \frac{Rdz}{z} \right] = \frac{KQ}{4R^2 r^2} \left[\frac{Rr^2}{z} + \frac{R^3}{z} + Rz \right]_{r-R}^{r+R}$$

On solving above, we will get : $E_{\text{out}} = \frac{KQ}{r^2}$ if $r > R$



For the points inside the sphere:

Now let's derive the electric field due to uniformly charged solid sphere at a point 'P' inside it. The sphere is divided into two parts, the rings on the left part of point 'P' will produce electric field towards right and the rings on right part will produce electric field towards left and $E_{net} = E_{right} - E_{left}$. For this, limit of integration is divided into two parts.

$$E_{net} = \left(\int_{\theta=0}^{\theta=\cos^{-1}(r/R)} \text{Electric field due to rings of right part} \right) - \left(\int_{\theta=\pi}^{\theta=\cos^{-1}(R/r)} \text{Electric field due to rings of left part} \right)$$

$$\text{As } z^2 = R^2 + r^2 - 2Rr \cos\theta$$

$$\text{When } \theta = \cos^{-1}\left(\frac{r}{R}\right) \Rightarrow z = \sqrt{R^2 - r^2}$$

$$\text{When } \theta = 0 \Rightarrow z = R - r$$

$$\text{When } \theta = \pi \Rightarrow z = R + r$$

From the result of previous case and just by changing limits we can write

$$E_{net} = \left[-\frac{Rr^2}{z} + \frac{R^3}{z} + Rz \right]_{R-r}^{R-\sqrt{R^2-r^2}} - \left[-\frac{Rr^2}{z} + \frac{R^3}{z} + Rz \right]_{\sqrt{R^2-r^2}}^{R+r}$$

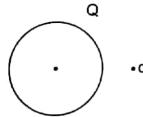
On solving this expression, we will get E and $E = 0$ if $r < R$.

Finding electric field due to shell by integration is very lengthy, so we will not use this method. The given hand-out was just for knowledge. The best method to find E due to shell is by Gauss theorem which we will study later.

Solved Examples

Example 41. Figure shows a uniformly charged sphere of radius R and total charge Q . A point charge q is situated outside the sphere at a distance r from centre of sphere. Find out the following :

- (i) Force acting on the point charge q due to the sphere.
- (ii) Force acting on the sphere due to the point charge.



Solution : (i) Electric field at the position of point charge $\vec{E} = \frac{KQ}{r^2} \hat{r}$

$$\text{So, } \vec{F} = \frac{KqQ}{r^2} \hat{r} \quad |\vec{F}| = \frac{KqQ}{r^2}$$

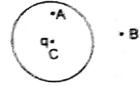
(ii) Since we know that every action has equal and opposite reaction so $\vec{F}_{sphere} = \frac{KqQ}{r^2} \hat{r}$

$$\vec{F}_{sphere} = \frac{KqQ}{r^2}$$

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Example 42.

Figure shows a uniformly charged thin sphere of total charge Q and radius R . A point charge q is also situated at the centre of the sphere. Find out the following :

- (i) Force on charge q
- (ii) Electric field intensity at A.
- (iii) Electric field intensity at B.
- (iv) Electric field at the centre of the uniformly charged hollow sphere = 0
- (v) Electric field at A



Solution :

$$\vec{E}_A = \vec{E}_{Sphere} + \vec{E}_q = 0 + \frac{Kq}{r^2} \hat{r}; r = CA$$

E due to sphere = 0, because point lies inside the charged hollow sphere.

$$(iii) \text{ Electric field } \vec{E}_B \text{ at point B} = \vec{E}_{Sphere} + \vec{E}_q = \frac{KQ}{r^2} \hat{r} + \frac{Kq}{r^2} \hat{r} = \frac{K(Q+q)}{r^2} \hat{r}; r = CB$$

Note : Here, we can also assume that the total charge of sphere is concentrated at the centre, for calculation of electric field at B.

Example 43. Two concentric uniformly charged spherical shells of radius R_1 and R_2 ($R_2 > R_1$) have total charges Q_1 and Q_2 respectively. Derive an expression of electric field as a function of r for following positions.



- (i) $r < R_1$
- (ii) $R_1 \leq r < R_2$
- (iii) $r \geq R_2$

Solution :

$$E_{net} = E_{inner} + E_{outer} = 0 + 0$$

- (i) For $r < R_1$, therefore, point lies inside both the spheres

$$\therefore E_{net} = E_{inner} + E_{outer}$$

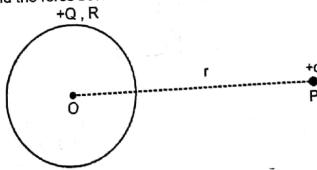
$$\frac{KQ_1}{r^2} \hat{r} + 0 = \frac{KQ_1}{r^2} \hat{r}$$

- (ii) For $R_1 \leq r < R_2$

point lies outside inner as well as outer sphere.

$$\text{Therefore, } E_{net} = E_{inner} + E_{outer} = \frac{KQ_1}{r^2} \hat{r} + \frac{KQ_2}{r^2} \hat{r} = \frac{K(Q_1 + Q_2)}{r^2} \hat{r}$$

Example 44. A spherical shell having charge $+Q$ (uniformly distributed) and a point charge $+q_0$ are placed as shown. Find the force between shell and the point charge ($r \gg R$).



(i) Force on the point charge $+q_0$ due to the shell $= q \vec{E}_{shell} = (q_0) \left(\frac{KQ}{r^2} \hat{r} \right) = \frac{KQq_0}{r^2} \hat{r}$ where \hat{r} is unit vector along OP.

From action-reaction principle, force on the shell due to the point charge will be equal and opposite.

$$\text{also be : } F_{shell} = \frac{KQq_0}{r^2} (-\hat{r})$$

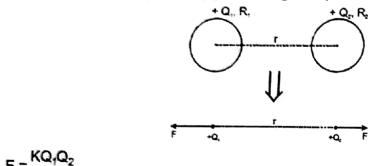
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Conclusion : To find the force on a hollow sphere due to outside charges, we can replace the sphere by a point charge kept at centre.

- Example 45.** Find force acting between two shells of radius R_1 and R_2 which have uniformly distributed charges Q_1 and Q_2 respectively and distance between their centers is r .

Solution : The shells can be replaced by point charges kept at centre, so force between them



$$F = \frac{KQ_1 Q_2}{r^2}$$

6.7 Electric field due to uniformly charged solid sphere:

Derive an expression for electric field due to solid sphere of radius R and total charge Q which is uniformly distributed in the volume, at a point which is at a distance r from centre for given two cases.

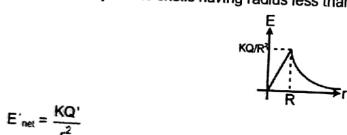
- (i) $r \geq R$ (ii) $r \leq R$

Assume an elementary concentric shell of charge dq . Due to this shell, the electric field at the point ($r > R$) will be:

$$dE = \frac{Kdq}{r^2} \quad [\text{from above result of hollow sphere}]$$

$$E_{\text{net}} = \int dE = \frac{KQ}{r^2}$$

For $r < R$, there will be no electric field due to shell of radius greater than r , so electric field at the point will be present only due to shells having radius less than r .

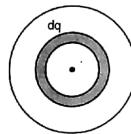


$$E_{\text{net}} = \frac{KQ'}{r^2}$$

$$\text{Here, } Q' = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = \frac{Qr^3}{R^3}$$

$$\therefore E_{\text{net}} = \frac{KQ'}{r^2} = \frac{KQr}{R^3}; \text{ away from the centre.}$$

Note : The electric field inside and outside the sphere is always in radial direction.



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Solved Examples

Example 46.

A solid non conducting sphere of radius R and uniform volume charge density ρ has its centre

- (i) $(R/2, 0, 0)$ (ii) $(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0)$ (iii) $(R, R, 0)$

Solution :

(i) At $(R/2, 0, 0)$: Distance of point from centre $= \sqrt{(R/2)^2 + 0^2 + 0^2} = R/2 < R$, so point lies inside the sphere, so $\vec{E} = \frac{K\vec{r}}{3\epsilon_0} = \frac{\rho}{3\epsilon_0} \left[\frac{R}{2} \hat{i} \right]$

(ii) At $(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0)$ distance of point from centre $= \sqrt{(R/\sqrt{2})^2 + (R/\sqrt{2})^2 + 0^2} = R = R$, so point lies at the surface of sphere, therefore

$$\vec{E} = \frac{KQ}{R^3} \vec{r} = \frac{K \frac{4}{3}\pi R^3 \rho}{R^3} \left[\frac{R}{\sqrt{2}} \hat{i} + \frac{R}{\sqrt{2}} \hat{j} \right] = \frac{\rho}{3\epsilon_0} \left[\frac{R}{\sqrt{2}} \hat{i} + \frac{R}{\sqrt{2}} \hat{j} \right]$$

(iii) The point is outside the sphere

$$\text{So, } \vec{E} = \frac{KQ}{r^3} \vec{r} = \frac{K \frac{4}{3}\pi R^3 \rho}{(2R)^3} \left[R \hat{i} + R \hat{j} \right] = \frac{\rho}{6\epsilon_0} \left[R \hat{i} + R \hat{j} \right]$$

Example 47.

A Uniformly charged solid non-conducting sphere of uniform volume charge density ρ and radius R is having a concentric spherical cavity of radius r . Find out electric field intensity at following points, as shown in the figure :

- (i) Point A (ii) Point B
(iii) Point C (iv) Centre of the sphere

Method-I :

- (i) For point A : We can consider the solid part of sphere to be made of large number of spherical shells which have uniformly distributed charge on its surface. Now, since point A lies inside all spherical shells so electric field intensity due to all shells will be zero.

$$\vec{E}_A = 0$$

- (ii) For point B : All the spherical shells for which point B lies inside will make electric field zero at point B. So electric field will be due to charge present from radius r to OB .

$$\text{So, } \vec{E}_B = \frac{K \frac{4}{3}\pi (OB^3 - r^3) \rho}{OB^3} \vec{OB} = \frac{\rho}{3\epsilon_0} \frac{[OB^3 - r^3]}{OB^3} \vec{OB}$$

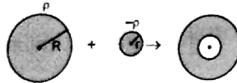
- (iii) For point C, similarly we can say that for all the shell points C lies outside the shell

$$\text{So, } \vec{E}_C = \frac{K \frac{4}{3}\pi (R^3 - r^3)}{|OC|^3} \vec{OC} = \frac{\rho}{3\epsilon_0} \frac{R^3 - r^3}{|OC|^3} \vec{OC}$$

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Method-II : We can consider that the spherical cavity is filled with charge density ρ and also, thereby making net charge density zero after combining. We can consider two concentric spheres: One of radius R and charge density ρ and other of radius r and charge density $-\rho$.

Applying superposition principle :



$$(i) \vec{E}_A = \vec{E}_p + \vec{E}_{-p} = \frac{\rho(\vec{OA})}{3\epsilon_0} + \frac{(-\rho(\vec{OA}))}{3\epsilon_0} = 0$$

$$(ii) \vec{E}_B = \vec{E}_p + \vec{E}_{-p} = \frac{\rho(\vec{OB})}{3\epsilon_0} + \frac{K\left[\frac{4}{3}\pi r^3(-\rho)\right]}{(OB)^3} \vec{OB}$$

$$= \left[\frac{\rho}{3\epsilon_0} - \frac{r^3\rho}{3\epsilon_0(OB)^3} \right] \vec{OB} = \frac{\rho}{3\epsilon_0} \left[1 - \frac{r^3}{OB^3} \right] \vec{OB}$$

$$(iii) \vec{E}_C = \vec{E}_p + \vec{E}_{-p} = \frac{K\left(\frac{4}{3}\pi R^3\rho\right)}{OC^3} \vec{OC} + \frac{K\left(\frac{4}{3}\pi r^3(-\rho)\right)}{OC^3} \vec{OC} = \frac{\rho}{3\epsilon_0(OC)^3} [R^3 - r^3] \vec{OC}$$

$$(iv) \vec{E}_O = \vec{E}_p + \vec{E}_{-p} = 0 + 0 = 0$$

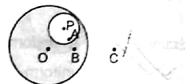
Example 48.

Solution :

In above question, if cavity is not concentric and centered at point P then repeat all the steps. Again assume ρ and $-\rho$ in the cavity, (similar to the previous example) :

$$(i) \vec{E}_A = \vec{E}_p + \vec{E}_{-p} = \frac{\rho(\vec{OA})}{3\epsilon_0} + \frac{(-\rho\vec{PA})}{3\epsilon_0}$$

$$\frac{\rho}{3\epsilon_0} [\vec{OA} - \vec{PA}] = \frac{\rho}{3\epsilon_0} [\vec{OP}]$$



Note : Here, we can see that the electric field intensity at point A is independent of position of point A inside the cavity. Also the electric field is along the line joining the centres of the sphere and the spherical cavity.

$$(ii) \vec{E}_B = \vec{E}_p + \vec{E}_{-p} = \frac{\rho(\vec{OB})}{3\epsilon_0} + \frac{K\left[\frac{4}{3}\pi r^3(-\rho)\right]}{(PB)^3} \vec{PB}$$

$$(iii) \vec{E}_C = \vec{E}_p + \vec{E}_{-p} = \frac{K\left(\frac{4}{3}\pi R^3\rho\right)}{(OC)^3} \vec{OC} + \frac{K\left(\frac{4}{3}\pi r^3(-\rho)\right)}{(PC)^3} \vec{PC}$$

$$(iv) \vec{E}_O = \vec{E}_p + \vec{E}_{-p} = 0 + \frac{K\left(\frac{4}{3}\pi r^3(-\rho)\right)}{(PO)^3} \vec{PO}$$

Example 49. A non-conducting solid sphere has volume charge density that varies as $\rho = \rho_0 r$, where ρ_0 is constant and r is distance from centre. Find out electric field intensities at following positions.

(i) $r < R$ (ii) $r \geq R$

Method I :

(i) For $r < R$:

The sphere can be considered to be made of large number of spherical shells. Each shell has uniform charge density on its surface. So the previous results of the spherical shell can

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be used. Consider a shell of radius x and thickness dx as an element. Charge on shell $dq = (4\pi x^2 dx) \rho_0 x$.

Electric field intensity at point P due to shell, $dE = \frac{Kdq}{x^2}$

Since all the shell will have electric field in same direction

$$\therefore E = \int_0^R dE = \int_0^r dE + \int_r^R dE$$

Due to shells which lie between region $r < x \leq R$, electric field at point P will be zero.

$$\therefore |\vec{E}| = \int_0^r \frac{Kdq}{x^2} + 0 = \int_0^r \frac{K \cdot 4\pi x^2 dx \rho_0 x}{r^2} = \frac{4\pi K \rho_0}{r^2} \left[\frac{x^4}{4} \right]_0^r = \frac{\rho_0 r^2}{4\epsilon_0}$$

$$(ii) \text{ For } r \geq R, \quad \vec{E} = \int_0^R dE = \int_0^R \frac{K \cdot 4\pi x^2 dx \rho_0 x}{r^2} = \frac{\rho_0 R^4}{4\epsilon_0 r^2} \hat{r}$$

Method II :

The sphere can be considered to be made of large number of spherical shells. Each shell has uniform charge density on its surface. So the previous results of the spherical shell can be used. we can say that all the shells for which point lies inside will make electric field zero at that point,

$$\text{So } \vec{E}_{(r < R)} = \frac{\int_0^r (4\pi x^2 dx) \rho_0 x}{r^2} = \frac{\rho_0 r^2}{4\epsilon_0} \hat{r}$$

(ii) Similarly, for $r \geq R$, all the shells will contribute in electric field, Therefore :

$$\vec{E}_{(r < R)} = \frac{\int_0^R (4\pi x^2 dx) \rho_0 x}{r^2} = \frac{\rho_0 R^4}{4\epsilon_0 r^2} \hat{r}$$

7. ELECTRIC POTENTIAL :

In electrostatic field, the electric potential (due to some source charges) at a point P is defined as the work done by external agent in taking a unit positive charge from a reference point (generally taken at infinity) to that point P without changing its kinetic energy.

7.1 Mathematical representation :

If $(W_{\infty \rightarrow P})_{\text{ext}}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is

$$V_p = \frac{W_{\infty \rightarrow P}}{q} \Big|_{q=0} = \frac{(-W_{\text{elc}})_{\infty \rightarrow P}}{q}$$

Note : (i) $(W_{\infty \rightarrow P})_{\text{ext}}$ can also be called as the work done by external agent against the electric force on a unit positive charge due to the source charge.

(ii) Write both W and q with proper sign.

7.2 Properties :

- (i) Potential is a scalar quantity, its value may be positive, negative or zero.
- (ii) S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{coulomb}}$ and its dimensional formula is $[\text{M}^1 \text{L}^2 \text{T}^{-3} \text{I}^{-1}]$.
- (iii) Electric potential at a point is also equal to the negative of the work done by the electric field taking the point charge from reference point (i.e. infinity) to that point.
- (iv) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinity. (taking $V_{\infty} = 0$).
- (v) Potential decreases in the direction of electric field.
- (vi) $V = V_1 + V_2 + V_3 + \dots$

7.3 Use of potential :

If we know the potential at some point (in terms of numerical value or in terms of formula) then we can find out the work done by electric force when charge moves from point 'P' to ∞ by the formula

$$W_{ep} |_{p \rightarrow \infty} = qV_p$$

Solved Example

Example 50 A charge $2\mu\text{C}$ is taken from infinity to a point in an electric field, without changing its velocity. work done against electrostatic forces is $-40\mu\text{J}$, then find the potential at that point.

Solution : $V = \frac{W_{ext}}{q} = \frac{-40\mu\text{J}}{2\mu\text{C}} = -20 \text{ V}$

Example 51 When charge $10 \mu\text{C}$ is shifted from infinity to a point in an electric field, it is found that work done by electrostatic forces is $-10 \mu\text{J}$. If the charge is doubled and taken again from infinity to the same point without accelerating it, then find the amount of work done by electric field against electric field.

Solution : $W_{ext}|_{\infty \rightarrow p} = -W_{el}|_{\infty \rightarrow p} = W_{el}|_{p \rightarrow \infty} = 10 \mu\text{J}$
because $\Delta KE = 0$

$$\therefore V_p = \frac{(W_{ext})_{\infty \rightarrow p}}{20\mu\text{C}} = \frac{10\mu\text{J}}{10\mu\text{C}} = 1 \text{ V}$$

So, if now the charge is doubled and taken from infinity then

$$1 = \frac{W_{ext}|_{\infty \rightarrow p}}{20\mu\text{C}} \Rightarrow \quad \text{or} \quad W_{ext}|_{\infty \rightarrow p} = 20 \mu\text{J}$$

$$\Rightarrow W_{el}|_{\infty \rightarrow p} = -20 \mu\text{J}$$

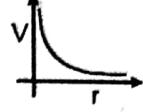
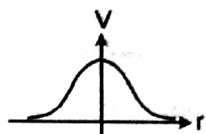
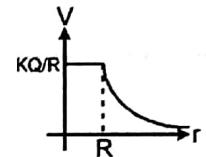
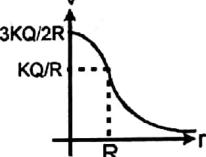
Example 52 A charge $3\mu\text{C}$ is released from rest from a point P where electric potential is 20 V then kinetic energy when it reaches infinity is :

Solution : $W_{el} = \Delta K = K_f - 0$

$$\therefore W_{el}|_{p \rightarrow \infty} = qV_p = 60 \mu\text{J} \quad \text{So, } K_f = 60 \mu\text{J}$$

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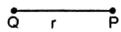
Electric Potential due to various charge distributions are given in table.

Name / Type	Formula	Note	Graph
Point charge	$\frac{kq}{r}$	* q is source charge. * r is the distance of the point from the point charge.	
Ring (uniform/nonuniform charge distribution)	at centre: $\frac{kQ}{R}$ at the axis: $\frac{kQ}{\sqrt{R^2 + x^2}}$	* Q is source charge. * x is the distance of the point on the axis from centre of ring	
Uniformly charged hollow conducting/nonconducting /solid conducting sphere	for $r \geq R$, $V = \frac{kQ}{r}$ for $r \leq R$, $V = \frac{kQ}{R}$	* R is radius of sphere * r is the distance from centre of sphere to the point * Q is total charge = $\sigma 4\pi R^2$.	
Uniformly charged solid nonconducting sphere .	For $r \geq R$, $V = \frac{kQ}{r}$ For $r \leq R$ $V = \frac{kQ(3R^2 - r^2)}{2R^3}$ $= \frac{\rho}{6\epsilon_0} (3R^2 - r^2)$	* R is radius of sphere * r is distance from centre to the point * $V_{\text{centre}} = \frac{3}{2} V_{\text{surface}}$. * Q is total charge = $\rho \frac{4}{3} \pi R^3$. * Inside the sphere potential varies parabolically * Outside the sphere potential varies hyperbolically.	
Infinite line charge	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula: $V_B - V_A = -2k\lambda \ln(r_B/r_A)$	
Infinite nonconducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{\epsilon_0} (r_B - r_A)$	
Infinite charged conducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{\epsilon_0} (r_B - r_A)$	

Electrostatics

7.4 Potential due to a point charge :

Derivation of expression for potential due to point charge Q , at a point which is at a distance r from the point charge.

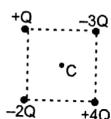


From definition of potential

$$V = \frac{W_{ext}(x \rightarrow p)}{q_0} = \frac{\int (q_0 E) \cdot dr}{q_0} = \int E \cdot dr \Rightarrow V = - \int \frac{KQ}{r^2} (-dr) \cos 180^\circ = \frac{KQ}{r}$$

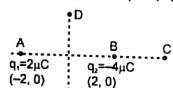
Solved Examples

Example 53. Four point charges are placed at the corners of a square of side ℓ . Calculate potential at the centre of square.



Solution : $V = 0$ at 'C'. [Use $V = \frac{Kq}{r}$]

Example 54. Two point charges $2\mu C$ and $-4\mu C$ are situated at points $(-2m, 0m)$ and $(2m, 0m)$ respectively. Find out potential at point C $(4m, 0m)$ and D $(0m, \sqrt{5} m)$.

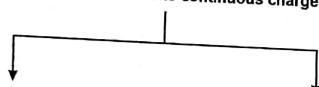


Solution : Potential at point C

$$V_C = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{6} + \frac{K(-4\mu C)}{2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{6} - \frac{9 \times 10^9 \times 4 \times 10^{-6}}{2} = -15000 V.$$

$$\text{Similarly, } V_D = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{\sqrt{(\sqrt{5})^2 + 2^2}} + \frac{K(-4\mu C)}{\sqrt{(\sqrt{5})^2 + 2^2}} = \frac{K(2\mu C)}{3} + \frac{K(-4\mu C)}{3} = -6000 V.$$

Finding potential due to continuous charges



If formula of E is tough, then we take

a small element and integrate

$$V = \int dv$$

If formula of E is easy then, we use

$$V = - \int_{r \rightarrow \infty}^{r=r} E \cdot dr$$

(i.e. for sphere, plate, infinite wire etc.)

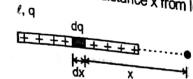
Electrostatics

Solved Examples

Example 55. A rod of length ℓ is uniformly charged with charge q . Calculate potential at point P.

Solution :

Take a small element of length dx , at a distance x from left end. Potential due to this small element



$$dV = \frac{K(dq)}{x} \quad \therefore \text{Total potential} \Rightarrow V = \int_{x=0}^{x=\ell} \frac{Kdq}{x}$$

$$\text{Where } dq = \frac{q}{\ell} dx \quad \Rightarrow \quad V = \int_{x=r}^{x=\ell+r} \frac{K \left(\frac{q}{\ell} dx \right)}{x} = \frac{Kq}{\ell} \log_e \left(\frac{\ell+r}{r} \right)$$

7.5 Potential due to a ring :

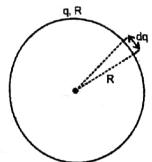
(i) Potential at the centre of uniformly charged ring :

Potential due to the small element dq

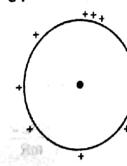
$$dV = \frac{Kdq}{R}$$

$$\therefore \text{Net potential : } V = \int \frac{Kdq}{R}$$

$$\therefore V = \frac{K}{R} \int dq = \frac{Kq}{R}$$

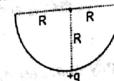


(ii) For non-uniformly charged ring potential at the center is



$$V = \frac{Kq_{\text{total}}}{R}$$

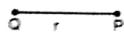
(iii) Potential due to half ring at center is :



$$V = \frac{Kq}{R}$$

7.4 Potential due to a point charge :

Derivation of expression for potential due to point charge Q , at a point which is at a distance r from the point charge.

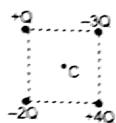


From definition of potential

$$V = \frac{W_{\text{ext}}(-q)}{q_0} = - \int \vec{E} \cdot d\vec{r} \Rightarrow V = - \int \frac{KQ}{r^2} (-dr) \cos 180^\circ = \frac{KQ}{r}$$

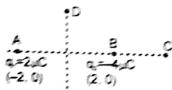
Solved Examples

Example 53. Four point charges are placed at the corners of a square of side ℓ . Calculate potential at the centre of square.



Solution : $V = 0$ at 'C' [Use $V = \frac{Kq}{r}$]

Example 54. Two point charges $2\mu\text{C}$ and $-4\mu\text{C}$ are situated at points $(-2\text{ m}, 0\text{ m})$ and $(2\text{ m}, 0\text{ m})$ respectively. Find out potential at point C $(4\text{ m}, 0\text{ m})$ and D $(0\text{ m}, \sqrt{5}\text{ m})$.



Solution : Potential at point C

$$V_C = V_{q_1} + V_{q_2} = \frac{K(2\mu\text{C})}{6} + \frac{K(-4\mu\text{C})}{2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{6} - \frac{9 \times 10^9 \times 4 \times 10^{-6}}{2} = -15000 \text{ V.}$$

$$\text{Similarly, } V_D = V_{q_1} + V_{q_2} = \frac{K(2\mu\text{C})}{\sqrt{(\sqrt{5})^2 + 2^2}} + \frac{K(-4\mu\text{C})}{\sqrt{(\sqrt{5})^2 + 2^2}} = \frac{K(2\mu\text{C})}{3} + \frac{K(-4\mu\text{C})}{3} = -6000 \text{ V.}$$

Finding potential due to continuous charges



If formula of E is tough, then we take

a small element and integrate

$$V = \int dv$$

If formula of E is easy then, we use

$$V = - \int_{r_1}^{r_2} \vec{E} \cdot d\vec{r}$$

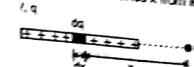
(i.e. for sphere, plate, infinite wire etc.)

Electrostatics

Solved Examples

Example 55. A rod of length ℓ is uniformly charged with charge q . Calculate potential at point P.

Solution : Take a small element of length dx , at a distance x from left end. Potential due to this small element



$$dV = \frac{K(dq)}{x}$$

$$\text{Total potential } \Rightarrow V = \int_{x=0}^{x=\ell} \frac{k dq}{x}$$

$$\text{Where } dq = \frac{q}{\ell} dx \Rightarrow V = \int_{x=r}^{x=r+\ell} \frac{K \left(\frac{q}{\ell} dx \right)}{x} = \frac{Kq}{\ell} \log_e \left(\frac{\ell+r}{r} \right)$$



7.5 Potential due to a ring :

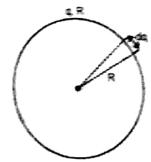
(i) Potential at the centre of uniformly charged ring :

Potential due to the small element dq

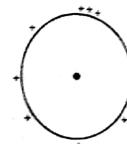
$$dV = \frac{Kdq}{R}$$

$$\therefore \text{Net potential : } V = \int \frac{Kdq}{R}$$

$$\therefore V = \frac{K}{R} \int dq = \frac{Kq}{R}$$

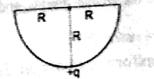


(ii) For non-uniformly charged ring potential at the center is



$$V = \frac{Kq_{\text{total}}}{R}$$

(iii) Potential due to half ring at center is :

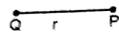


$$V = \frac{Kq}{R}$$

Electrostatics

7.4 Potential due to a point charge :

Derivation of expression for potential due to point charge Q , at a point which is at a distance r from the point charge.

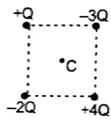


From definition of potential

$$V = \frac{W_{\text{ext}}(r \rightarrow \infty)}{q_0} = \frac{-\int (q_0 E) \cdot dr}{q_0} = \frac{-\int E \cdot dr}{q_0} \Rightarrow V = -\int_{\infty}^r \frac{KQ}{r^2} (-dr) \cos 180^\circ = \frac{KQ}{r}$$

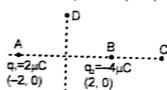
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Example 53. Four point charges are placed at the corners of a square of side ℓ . Calculate potential at the centre of square.



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$$V_C = V_{q_1} + V_{q_2} = \frac{K(2\mu\text{C})}{6} + \frac{K(-4\mu\text{C})}{2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{6} - \frac{9 \times 10^9 \times 4 \times 10^{-6}}{2} = -15000 \text{ V.}$$

$$\text{Similarly, } V_D = V_{q_1} + V_{q_2} = \frac{K(2\mu\text{C})}{\sqrt{(4)^2 + (\sqrt{5})^2}} + \frac{K(-4\mu\text{C})}{\sqrt{(4)^2 + (\sqrt{5})^2}} = \frac{K(2\mu\text{C})}{3} + \frac{K(-4\mu\text{C})}{3} = -6000 \text{ V.}$$

Finding potential due to continuous charges

If formula of E is tough, then we take a small element and integrate

$$V = \int dV$$

If formula of E is easy then, we use

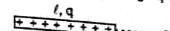
$$V = - \int_{r \rightarrow \infty}^r E \cdot dr$$

(i.e. for sphere, plate, infinite wire etc.)

Electrostatics

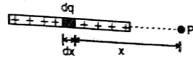
Solved Examples

Example 55. A rod of length ℓ is uniformly charged with charge q . Calculate potential at point P.



Solution :

Take a small element of length dx , at a distance x from left end. Potential due to this small element



$$dV = \frac{K(dq)}{x}$$

$$\therefore \text{Total potential} \Rightarrow V = \int_{x=0}^{x=\ell} \frac{Kdq}{x}$$

$$\text{Where } dq = \frac{q}{\ell} dx \Rightarrow V = \int_{x=r}^{x=\ell} \frac{K \left(\frac{q}{\ell} dx \right)}{x} = \frac{Kq}{\ell} \log_e \left(\frac{\ell+r}{r} \right)$$

7.5 Potential due to a ring :

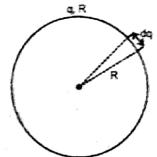
(i) Potential at the centre of uniformly charged ring :

Potential due to the small element dq

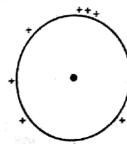
$$dV = \frac{Kdq}{R}$$

$$\therefore \text{Net potential} : V = \int \frac{Kdq}{R}$$

$$\therefore V = \frac{K}{R} \int dq = \frac{Kq}{R}$$

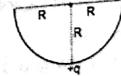


(ii) For non-uniformly charged ring potential at the center is



$$V = \frac{Kq_{\text{total}}}{R}$$

(iii) Potential due to half ring at center is :



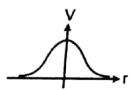
$$V = \frac{Kq}{R}$$

Electrostatics

(iv) Potential at the axis of a ring : Calculation of potential at a point on the axis which is a distance x from centre of uniformly charged (total charge Q) ring of radius R . Consider an element of charge dq on the ring. Potential at point P due to charge dq will be

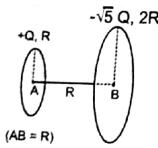
$$dV = \frac{K(dq)}{\sqrt{R^2 + x^2}}$$

∴ Net potential at point P due to all such elements will be : $V = \int dV = \frac{KQ}{\sqrt{R^2 + x^2}}$



Solved Examples

Example 56. Figure shows two rings having charges Q and $-\sqrt{5}Q$. Find Potential difference between A and B ($V_A - V_B$).



Solution : $V_A = \frac{KQ}{R} + \frac{K(-\sqrt{5}Q)}{\sqrt{(2R)^2 + (R)^2}} : V_B = \frac{K(-\sqrt{5}Q)}{2R} + \frac{K(Q)}{\sqrt{(R)^2 + (R)^2}}$

From above, we can easily find $V_A - V_B$.

Example 57. A point charge q_0 is placed at the centre of uniformly charged ring of total charge Q and radius R . If the point charge is slightly displaced with negligible force along the axis of the ring then find its speed when it reaches a large distance.

Solution :

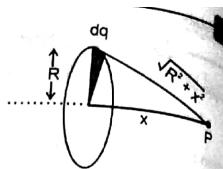
$$\begin{aligned} W_{el} &= \Delta K = \frac{1}{2}mv^2 - 0 \Rightarrow \text{Now } W_{el, \text{center}} = q_0 V_c = q_0 \frac{KQ}{R}. \\ \frac{Kq_0 Q}{R} &= \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2Kq_0 Q}{mR}} \end{aligned}$$

7.6 Potential due to uniformly charged disc :

$V = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + x^2} - x)$, where σ is the charge density and x is the distance of the point on the axis from the center of the disc & R is the radius of disc.

Finding potential due to a uniformly charged disc:

A disc of radius ' R' has surface charge density (charge/area) = σ . We have to find potential at its axis, at point 'P' which is at a distance x from the centre.



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For this, we can divide the disc into thin rings and let's consider a thin ring of radius r and thickness dr . Suppose charge on the small ring element $= dq$. Potential due to this ring at point 'P' is:

$$dV = \frac{Kdq}{\sqrt{r^2 + x^2}}$$

$$\text{So, net potential : } V_{\text{net}} = \int \frac{Kdq}{\sqrt{r^2 + x^2}}$$

$$\text{Here, } \sigma = \text{charge/area} = \frac{dq}{d(\text{area})}$$

$$\text{So, } dq = \sigma \times d(\text{area}) = \sigma (2\pi r dr)$$

(Here, $d(\text{area}) = \text{area of the small ring element} = (\text{length of ring}) \times (\text{width of the ring}) = (2\pi r) \cdot (dr)$)

$$\text{So, } V_{\text{net}} = \int_{r=0}^{r=R} \frac{K\sigma(2\pi r dr)}{\sqrt{r^2 + x^2}}$$

To integrate it, let $r^2 + x^2 = y^2$

$$2r dr = 2y dy. \text{ Substituting we will get :}$$

$$V_{\text{net}} = \int_{r=0}^{r=R} \frac{1}{4\pi\epsilon_0} \frac{\sigma(2\pi) y dy}{y} \Rightarrow V_{\text{net}} = \frac{\sigma}{2\epsilon_0} [y]_{r=0}^{r=R}$$

$$V_{\text{net}} = \frac{\sigma}{2\epsilon_0} \left(\sqrt{R^2 + x^2} \right)_{r=0}^{r=R} \Rightarrow V_{\text{net}} = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + x^2} - x)$$

If a test charge q_0 is placed at point P, then potential energy of this charge q_0 due to the disc is $U = q_0 V$

$$\Rightarrow U = q_0 \left[\frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + x^2} - x) \right]$$

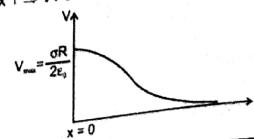
$$\text{Graph of } V \text{ vs } x : V = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + x^2} - x)$$

at $x = 0$, $V = \frac{\sigma R}{2\epsilon_0}$ to check whether V will increase with x or decrease, let's multiply and divide by conjugate.

$$V = \frac{\sigma}{2\epsilon_0} \left(\sqrt{R^2 + x^2} - x \right) \times \frac{(\sqrt{R^2 + x^2} + x)}{(\sqrt{R^2 + x^2} + x)}$$

$$\Rightarrow V = \frac{\sigma R^2}{2\epsilon_0} \left(\frac{1}{(\sqrt{R^2 + x^2} + x)} \right)$$

Now, we can say that as $x \uparrow \Rightarrow V \downarrow$ so curve will be



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Electrostatics

7.7 Potential Due To Uniformly Charged Spherical shell :

Derivation of expression for potential due to uniformly charged hollow sphere of radius R and total charge Q, at a point which is at a distance r from centre for the following situation

(i) $r > R$ (ii) $r < R$

Assume a ring of width $Rd\theta$ at angle θ from X axis (as shown in figure). Potential due to the ring at point P will be

$$dV = \frac{K(dq)}{\sqrt{(r - R\cos\theta)^2 + (R\sin\theta)^2}}$$

Where $dq = 2\pi R \sin\theta (Rd\theta)$ where $Q = 4\pi R^2 \sigma$

$$\text{then, net potential } V = \frac{KQ}{2} \int_0^\pi \frac{\sin\theta d\theta}{\sqrt{(r - R\cos\theta)^2 + (R\sin\theta)^2}}$$

$$\text{Solving this eq. we find } V = \frac{KQ}{r} \quad (\text{for } r > R)$$

$$\& \quad V = \frac{KQ}{R} \quad (\text{for } r < R)$$

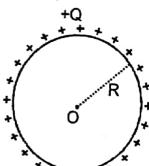
Alternate Method : As the formula of E is easy, we use $V = - \int_{r \rightarrow \infty}^{r=r} \vec{E} \cdot d\vec{r}$

$$(i) \text{ At outside point } (r \geq R) : V_{\text{out}} = - \int_{r \rightarrow \infty}^r \left(\frac{KQ}{r^2} \right) dr \Rightarrow V_{\text{out}} = \frac{KQ}{r} = \frac{KQ}{(\text{Distance from centre})}$$

For outside point, the hollow sphere acts like a point charge.

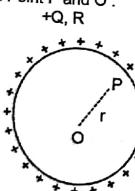
(ii) **Potential at the centre of the sphere ($r = 0$)** : As all the charges are at a distance R from the centre,

$$\text{So, } V_{\text{centre}} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphere})}$$



(iii) **Potential at inside point ($r < R$)** : Suppose we want to find potential at point P, inside the sphere.

∴ Potential difference between Point P and O :



$$V_P - V_O = - \int_0^P \vec{E}_{\text{in}} \cdot d\vec{r} \quad \text{Where, } E_{\text{in}} = 0$$

$$\text{So } V_P - V_O = 0 \Rightarrow V_P = V_O = \frac{KQ}{R} \Rightarrow V_{\text{in}} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphere})}$$

Electrostatics

7.8 Potential Due To Uniformly Charged Solid Sphere :

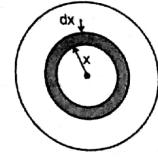
Derivation of expression for potential due to uniformly charged solid sphere of radius R and total charge Q (distributed in volume), at a point which is at a distance r from centre for the following situations.

(i) $r \geq R$ (ii) $r \leq R$

Consider an elementary shell of radius x and width dx

$$(i) \text{ For } r \geq R : V = \int_0^R \frac{K \cdot 4\pi x^2 dx p}{r} = \frac{KQ}{r}$$

$$(ii) \text{ For } r \leq R : V = \int_0^R \frac{K \cdot 4\pi x^2 dx p}{r} + \int_R^R \frac{K \cdot 4\pi x^2 dx p}{x} \\ = \frac{KQ}{2R^3} (3R^2 - r^2) \Rightarrow p = \frac{Q}{4 \cdot \pi R^3}$$



From definition of potential

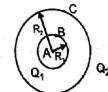
$$(i) \text{ For } r \geq R : V = - \int_{\infty}^r \frac{KQ}{r^2} \vec{r} \cdot dr = \frac{KQ}{r}$$

$$(ii) \text{ For } r \leq R : V = \int_{\infty}^R \frac{KQ}{r^2} dr - \int_R^r \frac{KQ}{R^3} dr$$

$$V = \frac{KQ}{R} - \frac{KQ}{2R^3} [r^2 - R^2] = \frac{KQ}{2R^3} [2R^2 - r^2 + R^2] = \frac{KQ}{2R^3} (3R^2 - r^2)$$

Solved Examples

Example 58. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out potential



- (i) at point A
- (ii) at surface of smaller shell (i.e. at point B)
- (iii) at surface of larger shell (i.e. at point C)
- (iv) at $r \leq R_1$
- (v) at $R_1 \leq r \leq R_2$
- (vi) at $r \geq R_2$

Solution :

- Using the results of hollow sphere as given in the table 7.4.
- (i) $V_A = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$
 - (ii) $V_B = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$
 - (iii) $V_C = \frac{KQ_1}{R_2} + \frac{KQ_2}{R_2}$
 - (iv) for $r \leq R_1$, $V = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$
 - (v) for $R_1 \leq r \leq R_2$, $V = \frac{KQ_1}{r} + \frac{KQ_2}{R_2}$
 - (vi) for $r \geq R_2$, $V = \frac{KQ_1}{r} + \frac{KQ_2}{r}$

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Example 59. Two hollow concentric non-conducting spheres of radius a and b ($a > b$) contain charges Q_a and Q_b , respectively. Prove that potential difference between the two spheres is independent of charge on outer sphere. If outer sphere is given an extra charge, is there any change in potential difference?

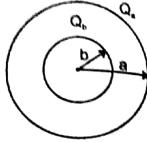
Solution :

$$V_{\text{inner sphere}} = \frac{KQ_a}{b} + \frac{KQ_b}{a}$$

$$V_{\text{outer sphere}} = \frac{KQ_a}{a} + \frac{KQ_b}{a}$$

$$V_{\text{inner sphere}} - V_{\text{outer sphere}} = \frac{KQ_b}{b} - \frac{KQ_b}{a}$$

$$\therefore \Delta V = KQ_b \left[\frac{1}{b} - \frac{1}{a} \right]$$



Which is independent of charge on outer sphere. If outer sphere is given any extra charge, then there will be no change in potential difference.

8. POTENTIAL DIFFERENCE

The potential difference between two points A and B is work done by external agent against electric field in taking a unit positive charge from A to B with no change in kinetic energy between initial and final points i.e. $\Delta K = 0$ or $K_i = K_f$

(a) Mathematical representation :

If $(W_{A \rightarrow B})_{\text{ext}}$ = Work done by external agent against electric field in taking the unit charge from A to B

$$\text{Then, } V_B - V_A = \frac{(W_{A \rightarrow B})_{\text{ext}}}{q} \Big|_{\Delta K=0} = \frac{-(W_{A \rightarrow B})_{\text{electric}}}{q} = \frac{U_B - U_A}{q} = \frac{- \int_A^B \vec{F}_e \cdot d\vec{r}}{q} = \frac{\vec{E} \cdot \vec{dr}}{q}$$

Note : Take W and q both with sign

(b) Properties :

- (i) The difference of potential between two points is called potential difference. It is also called voltage.
- (ii) Potential difference is a scalar quantity. Its S.I. unit is also volt.
- (iii) If V_A and V_B be the potential of two points A and B, then work done by an external agent in taking the charge q from A to B is $(W_{\text{ext}})_{AB} = q(V_B - V_A)$ or $(W_e)_{AB} = q(V_A - V_B)$.
- (iv) Potential difference between two points is independent of reference point.

8.1 Potential difference in a uniform electric field :

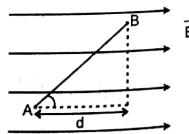
$$V_B - V_A = -\vec{E} \cdot \vec{AB}$$

$$\Rightarrow V_B - V_A = -|E| |AB| \cos \theta$$

$$= -|E| d$$

where d = effective distance between A and B along electric field.

or we can also say that $E = \frac{\Delta V}{\Delta d}$



Electrostatics

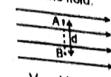
Special Cases :

Case-1. Line AB is parallel to electric field.



$$\therefore V_A - V_B = Ed$$

Case-2. Line AB is perpendicular to electric field.



$$\therefore V_A - V_B = 0 \Rightarrow V_A = V_B$$

Note : In the direction of electric field potential always decreases.

Solved Examples

Example 60. $1\mu\text{C}$ charge is shifted from A to B and it is found that work done by an external force is $40\mu\text{J}$ in doing so against electrostatic forces, then find potential difference $V_A - V_B$

$$(W_{AB})_{\text{ext}} = q(V_B - V_A) \Rightarrow 40\mu\text{J} = 1\mu\text{C}(V_B - V_A) \Rightarrow V_A - V_B = -40\text{ V}$$

Example 61. A uniform electric field is present in the positive x-direction. If the intensity of the field is 5N/C then find the potential difference $(V_B - V_A)$ between two points A (0m, 2m) and B (5m, 3m)

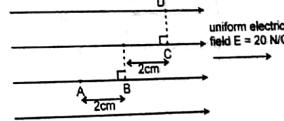
$$V_B - V_A = -\vec{E} \cdot \vec{AB} = -(5\hat{i})(5\hat{i} + \hat{j}) = -25\text{V}$$

$$\text{The electric field intensity in uniform electric field, } E = \frac{\Delta V}{\Delta d}$$

Where ΔV = potential difference between two points.

Δd = effective distance between the two points.
(projection of the displacement along the direction of electric field.)

Example 62. Find out following



$$(i) V_A - V_B \quad (ii) V_B - V_C \quad (iii) V_C - V_A \quad (iv) V_D - V_C$$

$$(v) V_A - V_D$$

(vi) Arrange the order of potential for points A, B, C and D.

$$(i) |\Delta V_{AB}| = Ed = 20 \times 2 \times 10^{-2} = 0.4 \text{ so, } V_A - V_B = 0.4\text{ V}$$

because In the direction of electric field potential always decreases.

$$(ii) |\Delta V_{BC}| = Ed = 20 \times 2 \times 10^{-2} = 0.4 \text{ so, } V_B - V_C = 0.4\text{ V}$$

$$(iii) |\Delta V_{CA}| = Ed = 20 \times 4 \times 10^{-2} = 0.8 \text{ so, } V_C - V_A = -0.8\text{ V}$$

because In the direction of electric field potential always decreases.

$$(iv) |\Delta V_{DC}| = Ed = 20 \times 0 = 0 \text{ so, } V_D - V_C = 0$$

because the effective distance between D and C is zero.

$$(v) |\Delta V_{AD}| = Ed = 20 \times 4 \times 10^{-2} = 0.8 \text{ so, } V_A - V_D = 0.8\text{ V}$$

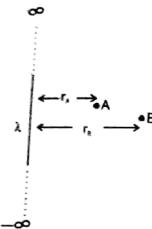
because In the direction of electric field potential always decreases.

$$(vi) The order of potential is : V_A > V_B > V_C = V_D$$

Electrostatics

8.2 Potential difference due to infinitely long wire :

Derivation of expression for potential difference between two points, which have perpendicular distances r_A and r_B from infinitely long line charge of uniform linear charge density λ :

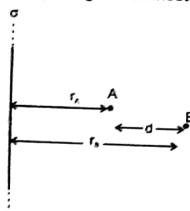


$$\text{From definition of potential difference : } V_{AB} = V_B - V_A = - \int_{r_A}^{r_B} \vec{E} \cdot d\vec{r} = - \int_{r_A}^{r_B} \frac{2K\lambda}{r} \hat{r} \cdot d\vec{r}$$

$$\therefore V_{AB} = -2K\lambda \ln \left(\frac{r_B}{r_A} \right)$$

8.3 Potential difference due to infinitely long thin sheet:

Derivation of expression for potential difference between two points, having separation d in the direction perpendicular to a very large uniformly charged thin sheet of uniform surface charge density σ .



Let the points A and B have perpendicular distance r_A and r_B respectively then from definition of potential difference.

$$V_{AB} = V_B - V_A = - \int_{r_A}^{r_B} \vec{E} \cdot d\vec{r} = - \int_{r_A}^{r_B} \frac{\sigma}{2\epsilon_0} \hat{r} \cdot d\vec{r} \Rightarrow V_{AB} = - \frac{\sigma}{2\epsilon_0} (r_B - r_A) = - \frac{\sigma d}{2\epsilon_0}$$

EQUIPOTENTIAL SURFACE :

Definition : If potential of a surface (imaginary or physically existing) is same throughout, then such surface is known as an equipotential surface.

Properties of equipotential surfaces :

- (i) When a charge is shifted from one point to another point on an equipotential surface, then work done against electrostatic forces is zero.
- (ii) Electric field is always perpendicular to equipotential surfaces.
- (iii) Two equipotential surfaces do not cross each other.

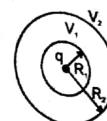
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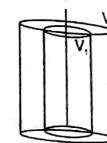
Electrostatics

9.3 Examples of equipotential surfaces :

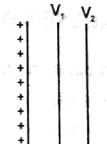
- (i) **Point charge :** Equipotential surfaces are concentric and spherical as shown in figure. In figure, we can see that sphere of radius R , has potential V , throughout its surface and similarly for other concentric sphere potential is same.



- (ii) **Line charge :** Equipotential surfaces have curved surfaces as that of coaxial cylinders of different radii.



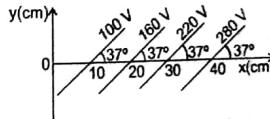
- (iii) **Uniformly charged large conducting / non conducting sheets :**
Equipotential surfaces are parallel planes.



Note : In uniform electric field equi-potential surfaces are always parallel planes.

Solved Examples

Example 63. Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field ?



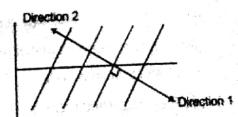
Solution : Here, we can say that the electric will be perpendicular to equipotential surfaces.

$$\text{Also, } |\vec{E}| = \frac{\Delta V}{\Delta d}$$

Where, ΔV = potential difference between two equipotential surfaces.

Δd = perpendicular distance between two equipotential surfaces.

$$\text{So } |\vec{E}| = \frac{60}{(10 \sin 37^\circ) \times 10^{-2}} = 1000 \text{ V/m}$$



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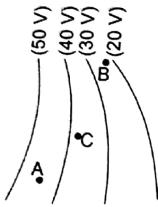
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Now there are two perpendicular directions: either direction 1 or direction 2 as shown in figure, since we know that in the direction of electric field, electric potential decreases, so the correct direction is direction 2.

Hence $E = 1000 \text{ V/m}$, making an angle 127° with the x-axis

Example 64. Figure shows some equipotential surfaces produced by some charges. At which point, the value of electric field is greatest?

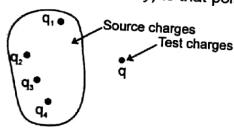


Solution : E is larger where equipotential surfaces are closer. ELOF are \perp to equipotential surfaces. In the figure, we can see that for point B, they are closer so E at point B is maximum

10. ELECTROSTATIC POTENTIAL ENERGY

10.1 Electrostatic potential energy of a point charge due to many charges :

The electrostatic potential energy of a point charge at a point in electric field is the work done in taking the charge from reference point (generally at infinity) to that point without change in kinetic energy.



Its Mathematical formula is $U = W_{\text{ext} \rightarrow \text{Pext}}|_{\Delta K=0} = qV = -W_{P \rightarrow \infty \text{ele}}$

Here, q is the charge whose potential energy is being calculated and V is the potential at its position due to the source charges.

Note : Always put q and V with sign.

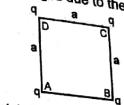
10.2 Properties :

- Electric potential energy is a scalar quantity but may be positive, negative or zero.
- Its unit is same as unit of work or energy i.e. joule (in S.I. system). Some times energy is also given in electron-volts. Where, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
- Electric potential energy depends on reference point. (Generally Potential Energy at $r = \infty$ is taken zero)

Electrostatics

Solved Examples

Example 65. The four identical charges (q each) are placed at the corners of a square of side a . Find the potential energy of one of the charges due to the remaining charges.



Solution : The electric potential of point A due to the charges placed at B, C and D is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{a} + \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{2}a} + \frac{1}{4\pi\epsilon_0} \frac{q}{a} = \frac{1}{4\pi\epsilon_0} \left(2 + \frac{1}{\sqrt{2}}\right) \frac{q}{a}$$

\therefore Potential energy of the charge at A is $qV = \frac{1}{4\pi\epsilon_0} \left(2 + \frac{1}{\sqrt{2}}\right) \frac{q^2}{a}$.

Example 66. A particle of mass 40 mg and carrying a charge $5 \times 10^{-6} \text{ C}$ is moving directly towards a fixed positive point charge of magnitude 10^{-3} C . When it is at a distance of 10 cm from the fixed point charge it has speed of 50 cm/s . At what distance from the fixed point charge will the particle come momentarily to rest? Is the acceleration constant during the motion?

Solution : If the particle comes to rest momentarily at a distance r from the fixed charge, then from conservation of energy, we have ?

$$\frac{1}{2}mu^2 + \frac{1}{4\pi\epsilon_0} \frac{Qq}{a} = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

Substituting the given data, we get: $\frac{1}{2} \times 40 \times 10^{-6} \times \frac{1}{2} \times \frac{1}{2} = 9 \times 10^9 \times 5 \times 10^{-6} \times 10^{-3} \left[\frac{1}{r} - 10 \right]$

$$\text{or, } \frac{1}{r} - 10 = \frac{5 \times 10^{-6}}{9 \times 5 \times 10^{-6}} = \frac{100}{9} \Rightarrow \frac{1}{r} = \frac{190}{9} \Rightarrow r = \frac{9}{190} \text{ m}$$

$$\text{or, i.e., } r = 4.7 \times 10^{-2} \text{ m. As here, } F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \quad \text{So, acc.} = \frac{F}{m} \propto \frac{1}{r^2}$$

i.e., Acceleration is not constant during the motion.

Example 67. A proton moves from a large distance with a speed u m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons in terms of mass of proton m and its charge e .

Solution : As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach, both will move with same velocity. So, if v is the common velocity of each particle at closest approach, then by 'conservation of momentum' of the two proton system.

$$mu = mv + mv \text{ i.e., } v = \frac{1}{2}u$$

$$\text{And by conservation of energy, } \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

$$\Rightarrow \frac{1}{2}mu^2 - m\left(\frac{u}{2}\right)^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad [\text{as } v = \frac{u}{2}]$$

$$\Rightarrow \frac{1}{4}mu^2 = \frac{e^2}{4\pi\epsilon_0 r} \Rightarrow r = \frac{e^2}{\pi m \epsilon_0 u^2}$$

Electrostatics

11. ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

(This concept is useful when more than one charges move.)

It is the work done by an external agent against the internal electric field required to make a system of charges in a particular configuration from infinite separation without change in their kinetic energies.

11.1 Types of system of charges :

- (i) Point charge system (ii) Continuous charge system.

11.2 Derivation for a system of point charges:

(i) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position. The final work required. PE of the system is algebraic sum of all the works.

Let W_1 = Work done in bringing first charge.

W_2 = Work done in bringing second charge against force due to 1st charge.

W_3 = Work done in bringing third charge against force due to 1st and 2nd charge.

$$PE = W_1 + W_2 + W_3 + \dots \quad (\text{This will contain } \frac{n(n-1)}{2} = {}^nC_2 \text{ terms})$$

(ii) Method of calculation (to be used in problems) :

U = sum of the interaction energies of the charges.

$$= (U_{12} + U_{13} + \dots + U_{1n}) + (U_{23} + U_{24} + \dots + U_{2n}) + (U_{34} + U_{35} + \dots + U_{3n}) \dots$$

(iii) Method of calculation useful for symmetrical point charge systems.

Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges.

$$= (U_{12} + U_{13} + \dots + U_{1n})$$

U_2 = PE of second charge due to all other charges.

$$= (U_{21} + U_{23} + \dots + U_{2n}) \text{ then } U = PE \text{ of the system} = \frac{U_1 + U_2 + \dots + U_n}{2}$$

Solved Examples

Example 68. Find out potential energy of the two point charge system having charges q_1 and q_2 separated by distance r .

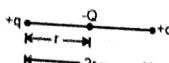
Solution : Let both the charges be placed at a very large separation initially.

Let, W_1 = work done in bringing charge q_1 in absence of q_2 = $q_1(V_r - V_i) = 0$

W_2 = work done in bringing charge q_2 in presence of q_1 = $q_2(V_r - V_i) = q_2(Kq_1/r - 0)$

$$\therefore PE = W_1 + W_2 = 0 + Kq_1q_2/r = Kq_1q_2/r$$

Example 69. Figure shows an arrangement of three point charges. The total potential energy of the arrangement is zero. Calculate the ratio $\frac{q}{Q}$.

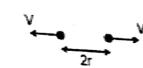


$$\begin{aligned} \text{Solution : } U_{sys} &= \frac{1}{4\pi\epsilon_0} \left[\frac{-qQ}{r} + \frac{(+q)(+q)}{2r} + \frac{Q(-q)}{r} \right] = 0 \\ -Q + \frac{q}{2} - Q = 0 &\quad \text{or} \quad 2Q = \frac{q}{2} \quad \text{or} \quad \frac{q}{Q} = \frac{4}{1} \end{aligned}$$

Electrostatics

Example 70. Two point charges, each of mass m and charge q are released when they are at a distance r from each other. What is the speed of each charged particle when they are at a distance $2r$? According to momentum conservation, both the charge particles will move with same speed. Now applying energy conservation:

$$0 + 0 + \frac{Kq^2}{r} = 2 \frac{1}{2} mv^2 + \frac{Kq^2}{2r} \Rightarrow v = \sqrt{\frac{Kq^2}{2mr}}$$



Example 71.

Two charged particles each having equal charges 2×10^{-5} C are brought from infinity to within a separation of 10 cm. Calculate the increase in potential energy during the process and the work required for this purpose.

$$\Delta U = U_f - U_i = U_f - 0 = U_f$$

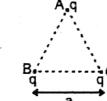
We have to simply calculate the electrostatic potential energy of the given system of charges

$$\therefore \Delta U = U_f = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-5} \times 2 \times 10^{-5} \times 100}{10} J = 36 J$$

\therefore Work required = 36 J = equal to change in potential energy of system

Example 72.

Three equal charges q each are placed at the corners of an equilateral triangle of side a .



(i) Find out potential energy of charge system.

(ii) Calculate work required to decrease the side of triangle to $a/2$.

Solution :

(iii) If the charges are released from the shown position and each of them has same mass m then find the speed of each particle when they lie on triangle of side $2a$.

(i) Method I (Derivation)

Assume all the charges are at infinity initially.

Work done in putting charge q at corner A

$$\Rightarrow W_1 = q(v_f - v_i) = q(0 - 0)$$

Since potential at A is zero in absence of charges, work done in putting q at corner B in presence of charge at A :

$$\Rightarrow W_2 = \left(\frac{Kq}{a} - 0 \right) q = \frac{Kq^2}{a}$$

Similarly work done in putting charge q at corner C in presence of charge at A and B :

$$\Rightarrow W_3 = q(v_f - v_i) = q \left[\left(\frac{Kq}{a} + \frac{Kq}{a} \right) - 0 \right] = \frac{2Kq^2}{a}$$

$$\text{So, net potential energy } PE = W_1 + W_2 + W_3 = 0 + \frac{Kq^2}{a} + \frac{2Kq^2}{a} = \frac{3Kq^2}{a}$$

(ii) Method II (using direct formula) :

$$U = U_{12} + U_{13} + U_{23} = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a}$$

$$U = U_f - U_i = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a} \text{ Joules}$$

(iii) Work required to decrease the sides $W = U_f - U_i = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a}$

$$U_f - U_i = K_f - K_i \Rightarrow \frac{3Kq^2}{a} - \frac{3Kq^2}{2a} = 3(\frac{1}{2}mv^2) - 0 \Rightarrow v = \sqrt{\frac{Kq^2}{am}}$$

Electrostatics

Example 73. Four identical point charges q each are placed at four corners of a square of side a . Find potential energy of the charge system



Solution : Method 1 (using direct formula) : $U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$= \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} = \left[\frac{4Kq^2}{a} + \frac{2Kq^2}{a\sqrt{2}} \right] = \frac{2Kq^2}{a} \left[2 + \frac{1}{\sqrt{2}} \right]$$

Method 2 [Using, $U = \frac{1}{2} (U_1 + U_2 + \dots)$] :

U_1 = total P.E. of charge at corner 1 due to all other charges.

U_2 = total P.E. of charge at corner 2 due to all other charges.

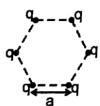
U_3 = total P.E. of charge at corner 3 due to all other charges.

U_4 = total P.E. of charge at corner 4 due to all other charges.

Since, due to symmetry, $U_1 = U_2 = U_3 = U_4$

$$U_{\text{Net}} = \frac{U_1 + U_2 + U_3 + U_4}{2} = 2U_1 = 2 \left[\frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a} \right] = \frac{2Kq^2}{a} \left[2 + \frac{1}{\sqrt{2}} \right]$$

Example 74. Six equal point charges q each are placed at six corners of a hexagon of side a . Find potential energy of charge system



Solution : $U_{\text{Net}} = \frac{U_1 + U_2 + U_3 + U_4 + U_5 + U_6}{2}$

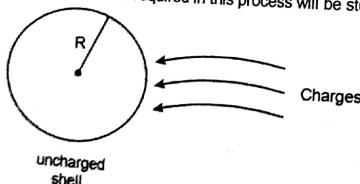
Due to symmetry $U_1 = U_2 = U_3 = U_4 = U_5 = U_6$ So $U_{\text{Net}} = 3U_1 = \frac{3Kq^2}{a} \left[2 + \frac{2}{\sqrt{3}} + \frac{1}{2} \right]$

11.3 Derivation of electric potential energy for continuous charge system :

This energy is also known as self energy.

(i) Finding P.E. (Self Energy) of a uniformly Charged spherical shell :

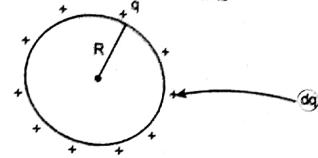
For this, let's use method 1 : Take an uncharged shell. Now bring charges one by one from infinity to the surface of the shell. The work required in this process will be stored as potential Energy.



uncharged shell

Electrostatics

Suppose, we have given charge q to the sphere and now we are giving extra charge dq to it. Work required to bring dq charge from infinity to the shell is



$$dW = (dq)(V_f - V_i) \Rightarrow dW = (dq) \left(\frac{Kq}{R} - 0 \right) = \frac{Kq}{R} dq$$

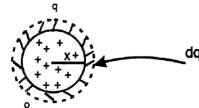
$$\Rightarrow \text{Total work required to give charge } Q \text{ is } W = \int_{q=0}^{Q=q} \frac{Kq}{R} dq = \frac{KQ^2}{2R}$$

This work will be stored in the form of P.E. (self energy)

$$\text{So, P.E. of a charged spherical shell : } U = \frac{KQ^2}{2R}$$

(ii) Self energy of uniformly charged solid sphere :

In this case we have to assemble a solid charged sphere. So we bring the charges one-by-one from infinity to the sphere so that the size of the sphere increases.



Suppose we have given charge q to the sphere, and its radius becomes 'x'. Now we are giving extra charge dq to it, which will increase its radius by 'dx'

\therefore Work required to bring charge dq from infinity to the sphere

$$= dq(V_f - V_i) = (dq) \left(\frac{Kq}{x} - 0 \right) = \frac{Kqdq}{x}$$

\therefore Total work required to give charge Q to the sphere

$$W = \int \frac{Kqdq}{x}, \text{ where } q = \rho \left(\frac{4}{3} \pi x^3 \right)$$

$$\& dq = \rho (4 \pi x^2 dx) \Rightarrow W = \int_{x=0}^{x=R} \frac{\rho \left(\frac{4}{3} \pi x^3 \right) \rho (4 \pi x^2 dx)}{x}$$

$$\text{Solving, well get : } W = \frac{3 KQ^2}{5 R} = U_{\text{self}} \text{ (for a solid sphere)}$$

Solved Examples

Example 75. A spherical shell of radius R with uniform charge q is expanded to a radius $2R$. Find the work performed by the electric forces and external agent against electric forces in this process.

Solution : $W_{\text{ext}} = U_f - U_i = \frac{q^2}{16\pi\epsilon_0 R} - \frac{q^2}{8\pi\epsilon_0 R} = -\frac{q^2}{16\pi\epsilon_0 R}$

$$W_{\text{elec}} = U_i - U_f = \frac{q^2}{8\pi\epsilon_0 R} - \frac{q^2}{16\pi\epsilon_0 R} = \frac{q^2}{16\pi\epsilon_0 R}$$

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Example 73. Four identical point charges q each are placed at four corners of a square of side a . Find potential energy of the charge system

$$q_1, \dots, q_4$$

Solution :

$$\text{Method 1 (using direct formula)}: U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$= \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} = \frac{2Kq^2}{a} \left[2 + \frac{1}{\sqrt{2}} \right]$$

Method 2 [Using, $U = \frac{1}{2}(U_1 + U_2 + \dots)$]:

U_1 = total P.E. of charge at corner 1 due to all other charges.

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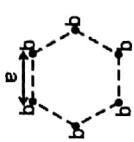
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U_4 = total P.E. of charge at corner 4 due to all other charges.

Since, due to symmetry, $U_1 = U_2 = U_3 = U_4$

$$U_{\text{net}} = \frac{U_1 + U_2 + U_3 + U_4}{2} = 2U_1 = 2 \left[\frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} \right] = \frac{2Kq^2}{a} \left[2 + \frac{1}{\sqrt{2}} \right]$$

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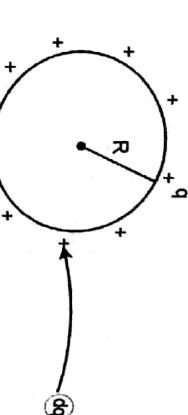


Solution :

$$U_{\text{net}} = \frac{U_1 + U_2 + U_3 + U_4 + U_5 + U_6}{2}$$

$$\text{Due to symmetry } U_1 = U_2 = U_3 = U_4 = U_5 = U_6 \quad \text{So } U_{\text{net}} = 3U_1 = \frac{3Kq^2}{a} \left[2 + \frac{2}{\sqrt{3}} + \frac{1}{2} \right]$$

Suppose, we have given charge q to the sphere and now we are giving extra charge dq from infinity to the shell. Work required to bring charge dq from infinity to the sphere so that the size of the sphere increases.



$$dW = (dq)(V_f - V_i) \quad \Rightarrow \quad dW = (dq)\left(\frac{Kq}{R} - 0\right) = \frac{Kq}{R}dq$$

$$\Rightarrow \text{Total work required to give charge } Q \text{ is } W = \int_{q_0}^{q_0+Q} \frac{Kq}{R} dq = \frac{KQ^2}{2R}$$

$$\text{So, P.E. of a charged spherical shell: } U = \frac{KQ^2}{2R}$$

(ii) Self energy of uniformly charged solid sphere:

In this case we have to assemble a solid charged sphere. So we bring the charges one-by-one from infinity to the sphere so that the size of the sphere increases.



Suppose we have given charge q to the sphere, and its radius becomes ' x '. Now we are giving extra charge dq to it, which will increase its radius by ' dx '

\therefore Work required to bring charge dq from infinity to the sphere

$$= dq(V_f - V_i) = (dq)\left(\frac{Kq}{x} - 0\right) = \frac{Kqdq}{x}$$

\therefore Total work required to give charge Q to the sphere

$$W = \int_x^R \frac{Kqdq}{x}, \text{ where } q = \rho \left(\frac{4}{3}\pi x^3 \right)$$

$$\& dq = \rho (4\pi x^2 dx) \quad \Rightarrow \quad W = \int_{x_0}^{x_R} K \frac{\rho \left(\frac{4}{3}\pi x^3 \right) (4\pi x^2 dx)}{x}$$

$$\text{Solving, we get: } W = \frac{3}{5} \frac{KQ^2}{R} = U_{\text{self}} \text{ (for a solid sphere)}$$

Example 75. A spherical shell of radius R with uniform charge q is expanded to a radius $2R$. Find the work performed by the electric forces and external agent against electric forces in this process.

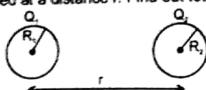
Solution :

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$$W_{\text{elec}} = U_i - U_f = \frac{q^2}{8\pi\epsilon_0 R} - \frac{q^2}{16\pi\epsilon_0 R}$$

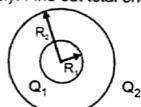
Electrostatics

Example 76. Two non-conducting hollow uniformly charged spheres of radii R_1 and R_2 with charge Q_1 and Q_2 respectively are placed at a distance r . Find out total energy of the system.



$$\text{Solution : } U_{\text{total}} = U_{\text{self}} + U_{\text{interaction}} = \frac{Q_1^2}{8\pi\epsilon_0 R_1} + \frac{Q_2^2}{8\pi\epsilon_0 R_2} + \frac{Q_1 Q_2}{4\pi\epsilon_0 r}$$

Example 77. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out total energy of the system.



$$\text{Solution : } U_{\text{total}} = U_{\text{self}1} + U_{\text{self}2} + U_{\text{interaction}} = \frac{Q_1^2}{8\pi\epsilon_0 R_1} + \frac{Q_2^2}{8\pi\epsilon_0 R_2} + \frac{Q_1 Q_2}{4\pi\epsilon_0 R_2}$$

11.4 Energy density :

Def : Energy density is defined as energy stored in unit volume in any electric field. Its mathematical formula is given as following :

$$\text{Energy density} = \frac{1}{2}\epsilon_0 E^2$$

where E = electric field intensity at that point

ϵ_0 = $\epsilon_0 F$, electric permittivity of medium

Solved Examples

Example 78. Find out energy stored in an imaginary cubical volume of side a in front of a infinitely large non-conducting sheet of uniform charge density σ .

Solution : Energy stored : $U = \int \frac{1}{2}\epsilon_0 E^2 dV$; where dV is small volume

$$\therefore U = \frac{1}{2}\epsilon_0 E^2 \int dV \quad \because E \text{ is constant}$$

$$\therefore U = \frac{1}{2}\epsilon_0 \frac{\sigma^2}{4\epsilon_0^2} \cdot a^3 = \frac{\sigma^2 a^3}{8\epsilon_0}$$

Example 79. Find out energy stored in the electric field of uniformly charged thin spherical shell of total charge Q and radius R .

Solution : We know that electric field inside the shell is zero, so the energy is stored only outside the shell, which can be calculated by using energy density formula.

$$U_{\text{self}} = \int_{x=R}^{x+\infty} \frac{1}{2}\epsilon_0 E^2 dV$$

Consider an elementary shell of thickness dx and radius x ($x > R$).

Volume of the shell = $(4\pi x^2 dx) = dV$

Electrostatics

$$\begin{aligned} U &= \int_R^\infty \frac{1}{2}\epsilon_0 \left[\frac{KQ}{x^2} \right]^2 \cdot 4\pi x^2 dx = \frac{1}{2}\epsilon_0 K^2 Q^2 4\pi \int_R^\infty \frac{1}{x^2} dx \\ &= \frac{4\pi\epsilon_0}{2} \frac{Q^2}{(4\pi\epsilon_0)^2} \left(\frac{1}{R} \right) = \frac{Q^2}{8\pi\epsilon_0 R} = \frac{KQ^2}{2R}. \end{aligned}$$

Example 80. Find out energy stored inside a solid non-conducting sphere of total charge Q and radius R . [Assume charge is uniformly distributed in its volume.]

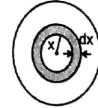
Solution : We can consider solid sphere to be made of large number of concentric spherical shells. Also electric field intensity at the location of any particular shell is constant.

$$\therefore U_{\text{inside}} = \int_0^R \frac{1}{2}\epsilon_0 E^2 dV$$

Consider an elementary shell of thickness dx and radius x .

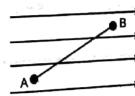
Volume of the shell = $(4\pi x^2 dx)$

$$\begin{aligned} U_{\text{inside}} &= \int_0^R \frac{1}{2}\epsilon_0 \left[\frac{KQx}{R^3} \right]^2 \cdot 4\pi x^2 dx = \frac{1}{2}\epsilon_0 \frac{K^2 Q^2 4\pi}{R^6} \int_0^R x^4 dx \\ &= \frac{4\pi\epsilon_0}{2R^6} \frac{Q^2}{(4\pi\epsilon_0)^2} \cdot \frac{R^5}{5} = \frac{Q^2}{40\pi\epsilon_0 R} = \frac{KQ^2}{10R}. \end{aligned}$$



12. RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL :

12.1 For uniform electric field :



(i) Potential difference between two points A and B

$$V_B - V_A = -\vec{E} \cdot \vec{AB}$$

12.2 Non uniform electric field

$$(i) E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} = -\left[\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right] V = -\nabla V = -\text{grad } V$$

Where, $\frac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

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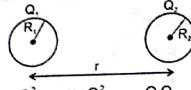


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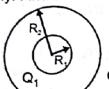
Electrostatics

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$$U_{\text{self}} = \int_{R \rightarrow \infty} \frac{1}{2} \epsilon_0 E^2 dV$$

Consider an elementary shell of thickness dx and radius x ($x > R$).
Volume of the shell = $(4\pi x^2 dx) = dV$

Electrostatics

Example 80. Find out energy stored inside a solid non-conducting sphere of total charge Q and radius R . [Assume charge is uniformly distributed in its volume.]

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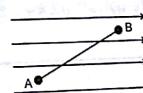
$$\therefore U_{\text{inside}} = \int_0^R \frac{1}{2} \epsilon_0 E^2 dV$$

Consider an elementary shell of thickness dx and radius x .
Volume of the shell = $(4\pi x^2 dx)$

$$U_{\text{inside}} = \int_0^R \frac{1}{2} \epsilon_0 \left[\frac{KQx}{R^3} \right]^2 \cdot 4\pi x^2 dx = \frac{1}{2} \epsilon_0 \frac{K^2 Q^2 4\pi}{R^5} \int_0^R x^4 dx \\ = \frac{4\pi \epsilon_0}{2R^5} \frac{Q^2}{(4\pi \epsilon_0)^2} \cdot \frac{R^5}{5} = \frac{Q^2}{40\pi \epsilon_0 R} = \frac{KQ^2}{10R}$$

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$\frac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

Electrostatics

12.3 If electric potential and electric field depends only on one coordinate, say r :

(i) $\vec{E} = -\frac{\partial V}{\partial r} \hat{r}$ where, \hat{r} is a unit vector along increasing r .

(ii) $\int dV = -\int \vec{E} \cdot d\vec{r} \Rightarrow V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{r}$

$d\vec{r}$ is along the increasing direction of r .

(iii) The potential of a point $V = - \int_{\infty}^r \vec{E} \cdot d\vec{r}$

Solved Examples

Example 81. A uniform electric field is along x -axis. The potential difference $V_A - V_B = 10$ V is between two points A (2m, 3m) and B (4m, 8m). Find the electric field intensity.

Solution : $E = \frac{\Delta V}{\Delta d} = \frac{10}{2} = 5$ V/m. (It is along +ve x -axis)

Example 82. $V = x^2 + y$, Find \vec{E} .

Solution : $\frac{\partial V}{\partial x} = 2x, \frac{\partial V}{\partial y} = 1$ and $\frac{\partial V}{\partial z} = 0$

$$\vec{E} = -\left(\hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z}\right) = -(2x\hat{i} + \hat{j})$$

∴ Electric field is non-uniform.

Example 83. For given $\vec{E} = 2x\hat{i} + 3y\hat{j}$, find the potential at (x, y) if V at origin is 5 volts.

Solution : $\int dV = -\int \vec{E} \cdot d\vec{r} = -\int_0^x E_x dx - \int_0^y E_y dy \Rightarrow V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2} \Rightarrow V = -\frac{2x^2}{2} - \frac{3y^2}{2} + 5.$

13. ELECTRIC DIPOLE

13.1 Electric Dipole

If two point charges, equal in magnitude 'q' and opposite in sign separated by a distance 'a' such that the distance of field point $r >> a$, the system is called a dipole. The electric dipole moment is defined as a vector quantity having magnitude $p = (q \times a)$ and direction from negative charge to positive charge.

Note: [In chemistry, the direction of dipole moment is assumed to be from positive to negative charge.] The C.G.S unit of electric dipole moment is debye which is defined as the dipole moment of two equal and opposite point charges each having charge 10^{-10} Franklin and separation of 1 Å, i.e.,

$$1 \text{ debye (D)} = 10^{-10} \times 10^{-8} = 10^{-18} \text{ Fr} \times \text{cm}$$

or $1 \text{ D} = 10^{-18} \times \frac{C}{3 \times 10^8} \times 10^{-2} \text{ m} = 3.3 \times 10^{-30} \text{ C} \times \text{m}$.

S.I. Unit is coulomb × metre = C.m

Electrostatics

Solved Examples

Example 84. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A : (0, 0, -0.15 m) and B : (0, 0, +0.15 m) respectively. What is the net charge and electric dipole moment of the system?

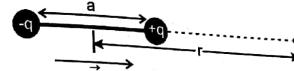
Net charge = $2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

Electric dipole moment, $P = (\text{Magnitude of charge}) \times (\text{Separation between charges}) = 2.5 \times 10^{-7} [0.15 + 0.15] \text{ C m} = 7.5 \times 10^{-8} \text{ C m}$

The direction of dipole moment is from B to A.

13.2 Electric Field Intensity Due to Dipole :

(i) At the axial point :-



$$\vec{E} = \frac{Kq}{\left(r - \frac{a}{2}\right)^2} - \frac{Kq}{\left(r + \frac{a}{2}\right)^2} \text{ (along the } \vec{P}) = \frac{Kq(2ra)}{\left(r^2 - \frac{a^2}{4}\right)^2} \hat{P}$$

$$\text{If } r \gg a \text{ then, } \vec{E} = \frac{Kq2ra}{r^4} \hat{P} = \frac{2K\vec{P}}{r^3},$$

As the direction of electric field at axial position is along the dipole moment (\vec{P})

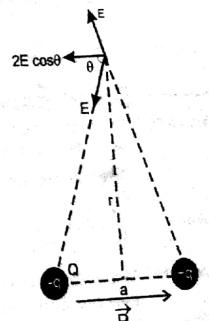
$$\text{So, } \vec{E}_{\text{axial}} = \frac{2K\vec{P}}{r^3}$$

(ii) Electric field at perpendicular Bisector (Equatorial Position)

$$E_{\text{net}} = 2E \cos \theta \text{ (along } -\hat{P})$$

$$\begin{aligned} \vec{E}_{\text{net}} &= 2 \left(\frac{Kq}{\left(\sqrt{r^2 + \left(\frac{a}{2}\right)^2}\right)^2} \right) \frac{a}{2} \frac{1}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} (-\hat{P}) \\ &= \frac{Kqa}{\left(r^2 + \left(\frac{a}{2}\right)^2\right)^{3/2}} (-\hat{P}) \end{aligned}$$

$$\text{If } r \gg a \text{ then } \vec{E}_{\text{net}} = \frac{K\vec{P}}{r^3} (-\hat{P})$$



As the direction of \vec{E} at equatorial position is opposite of \vec{P} so we can write in vector form:

$$\vec{E}_{\text{eqt}} = -\frac{K\vec{P}}{r^3}$$

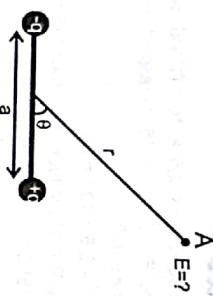
Electrostatics

(iii) Electric field at general point (r, θ):

Two charges $\pm 10\mu C$ are placed 5×10^{-3} m apart as shown in figure. At a point Q which is 0.15 m away from O, on the equatorial line.

solution :

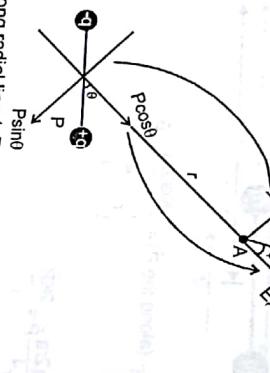
In the given problem, $r \gg a$.
Two charges $\pm 10\mu C$ are placed 5×10^{-3} m apart as shown in figure. Determine the electric field at a point Q which is 0.15 m away from O, on the equatorial line.



For this, let's resolve the dipole moment \vec{P} into components.

$$E_r = \frac{K(P\sin\theta)}{r^2}$$

$$E_\theta = \frac{K(P\cos\theta)}{r^2}$$



One component is along radial line (= $P\cos\theta$) and other component is \perp to the radial line (= $P\sin\theta$)

From the given figure $E_{\text{net}} = \sqrt{E_r^2 + E_\theta^2}$

$$\tan\phi = \frac{E_\theta}{E_r} = \frac{\frac{K(P\sin\theta)}{r^2}}{\frac{K(P\cos\theta)}{r^2}} = \frac{\tan\theta}{2}$$

Solved Examples

The electric field due to a short dipole at a distance r , on the axial line, from its mid-point is the same as that of electric field at a distance r' on the equatorial line, from its mid-point.

Determine the ratio $\frac{r}{r'}$.

Solution :

$$\frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{p}{r'^3} \quad \text{or} \quad \frac{2}{r^3} = \frac{1}{r'^3} \quad \text{or} \quad \frac{r^3}{r'^3} = 2 \quad \text{or}, \quad \frac{r}{r'} = 2^{1/3}$$

We cannot use formula of short dipole here because distance of the point is comparable to the distance between the two point charges.

$$E_{\text{res}} = E_r + E_\theta = \frac{K(5\mu C)}{(2cm)^2} - \frac{K(5\mu C)}{(6cm)^2} = \frac{144}{(144 \times 10^{-6}) NC^{-1}} = 10^8 NC^{-1}$$

Two charges, each of $5\mu C$ but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity of a point that is at a distance 4 cm from the mid point on the axial line of the dipole.

Example 8.6.

Two charges, each of $5\mu C$ but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity of a point that is at a distance 4 cm from the mid point on the axial line of the dipole.

Solution :

$$\frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{p}{r'^3} \quad \text{or} \quad \frac{2}{r^3} = \frac{1}{r'^3} \quad \text{or} \quad \frac{r^3}{r'^3} = 2 \quad \text{or}, \quad \frac{r}{r'} = 2^{1/3}$$

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Determine the ratio $\frac{r}{r'}$.

Solution :

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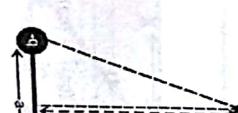
Solution :

$$\frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{p}{r'^3} \quad \text{or} \quad \frac{2}{r^3} = \frac{1}{r'^3} \quad \text{or} \quad \frac{r^3}{r'^3} = 2 \quad \text{or}, \quad \frac{r}{r'} = 2^{1/3}$$

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$$V = \frac{Kq}{r^2 + \left(\frac{a}{2}\right)^2} + \frac{K(-q)}{r^2 + \left(\frac{a}{2}\right)^2} = 0$$



(iii) Potential at general point (r, θ) : Let's resolve the dipole moment \vec{P} into components : $P\cos\theta$ along radial line and $P\sin\theta \perp$ to the radial line

For the $P\cos\theta$ component, the point A is an axial point,

$$\text{So, potential at A due to } P\cos\theta = \frac{K(P\cos\theta)}{r^2}$$

And for $P\sin\theta$ component, the point A is an equatorial point.

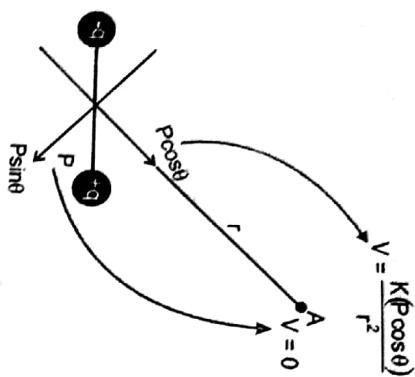
So potential at A due to $P\sin\theta = 0$

So potential at A due to $P\sin\theta = 0$

So potential at A due to $P\sin\theta = 0$

13.4 Dipole in uniform electric field

(i) Dipole is placed along electric field:



In this case, $F_{net} = 0$, $\tau_{net} = 0$, so it is an equilibrium state. And it is a stable equilibrium position.

(ii) If the dipole is placed at angle theta from E:

$$V_{net} = \frac{K(P \cos \theta)}{r^2}$$

$$\therefore V = \frac{K \left(\vec{P} \cdot \vec{r} \right)}{r^3}$$

$$\therefore V = \frac{K \left(P \cos \theta \right)}{r^2}$$

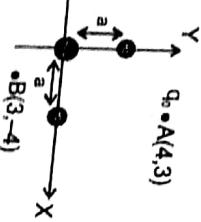
Solved Examples

Example 88.

- (i) Find potential at point A and B due to the small charge - system fixed near origin (Distance between the charges is negligible).

- (ii) Find work done to bring a test charge q_0 from point A to point B, slowly. All parameters are in S.I. units.

$$q_0 \bullet A(4,3)$$



Solution :

(i) Dipole moment of the system is $\vec{P} = (qa)\hat{i} + (qa)\hat{j}$

Potential at point A due to the dipole

$$V_A = K \left(\frac{\vec{P} \cdot \vec{r}}{r^3} \right) = K \left[(qa) \hat{i} + (qa) \hat{j} \right] \cdot \left(4\hat{i} + 3\hat{j} \right) = \frac{K(qa)}{125} \quad (7)$$

$$\Rightarrow V_B = K \left[(qa) \hat{i} + (qa) \hat{j} \right] \cdot \left(3\hat{i} - 4\hat{j} \right) = \frac{-K(qa)}{125} \quad (5)$$

$$\therefore W_{A \rightarrow B} = U_B - U_A = q_0 (V_B - V_A) = q_0 \left[\frac{-K(qa)}{125} - \frac{K(qa)}{125} - \frac{(K(qa))}{125} \right]$$

$$= W_{A \rightarrow B} = \frac{-Kq_0 q_A}{125} \quad (8)$$

- Example 89. A dipole is formed by two point charges $-q$ and $+q$, each of mass m , and both the point charges are connected by a rod of length l and mass m . This dipole is placed in uniform electric field E . If the dipole is disturbed by a small angle θ from stable equilibrium position, prove that its motion will be almost SHM. Also find its time period. If the dipole is disturbed by θ angle, $\tau_{net} = -PE \sin \theta$ (Here -ve sign indicates that direction of torque is opposite to θ)

Solved Examples

Example 89.

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If the dipole is disturbed by θ angle, $\tau_{net} = -PE \sin \theta$ (Here -ve sign indicates that direction of torque is opposite to θ)

Solution :

If θ is very small, $\sin \theta \approx \theta$
 $\therefore \tau_{net} = -(PE)\theta$
 $\tau_{net} \propto (-\theta)$ so motion will be almost SHM & $C = PE$ (where, $P = ql$)

$$\therefore T = 2\pi \sqrt{\frac{l}{C}}$$

$$\therefore T = 2\pi \sqrt{\frac{m(l + m(\frac{l}{2})^2)}{12PE}} = 2\pi \sqrt{\frac{m(l + m^2/4)}{12qE}} = 2\pi \sqrt{\frac{7ml^2}{12qE}}$$

$$T = \pi \sqrt{\frac{7ml^2}{3qE}}$$

Electrostatics

(iii) Potential energy of a dipole placed in uniform electric field :

$$U_B - U_A = - \int_A^B \vec{F} \cdot d\vec{r} \quad (\text{for translational motion})$$

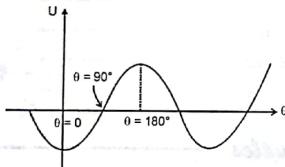
$$\text{Here, } U_B - U_A = - \int_{-\pi/2}^{\pi/2} (-PE \sin \theta) d\theta \quad (\text{for rotational motion})$$

In the case of dipole, at $\theta = 90^\circ$, P.E. is assumed to be zero.

$$U_B - U_{90^\circ} = - \int_{-\pi/2}^{90^\circ} (-PE \sin \theta) d\theta \quad (\text{As the direction of torque is opposite of } \theta)$$

$$U_B - 0 = - PE \cos 0$$

$0 = 90^\circ$ is chosen as reference, so that the lower limit comes out to be zero.



$$U_B = - \vec{P} \cdot \vec{E}$$

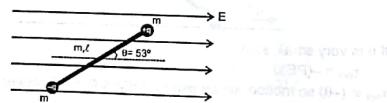
From the potential energy curve, we can conclude :

- (i) At $\theta = 0$, there is minimum of P.E. so it is a stable equilibrium position.
- (ii) At $\theta = 180^\circ$, there is maxima of P.E. so it is a position of unstable equilibrium.

Solved Examples

Example 90.

Two point masses of mass m and equal and opposite charge of magnitude q are attached on the corners of a non-conducting uniform rod of mass m and the system is released from rest in uniform electric field E as shown in figure from $\theta = 53^\circ$.



- (i) Find angular acceleration of the rod just after releasing
- (ii) What will be angular velocity of the rod when it passes through stable equilibrium.
- (iii) Find work required to rotate the system by 180° .

Solution :

$$(i) \tau_{\text{net}} = PE \sin 53^\circ = I \alpha$$

$$\therefore \alpha = \frac{(qE) \cdot \left(\frac{4}{5}\right)}{\frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2} = \frac{48qE}{35ml}$$

Electrostatics

(ii) From energy conservation : $K_i + U_i = K_f + U_f$

$$\therefore 0 + (-PE \cos 53^\circ) = \frac{1}{2} I\omega^2 + (-PE \cos 0^\circ)$$

$$\text{where } I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2 = \frac{7ml^2}{12} \therefore \frac{1}{2} I\omega^2 = PE(1 - \frac{3}{5}) = \frac{2}{5} PE$$

$$\therefore \frac{1}{2} \times \frac{7ml^2}{12} \times \omega^2 = \frac{2}{5} qE \quad \text{or} \quad \omega = \sqrt{\frac{48qE}{35ml}}$$

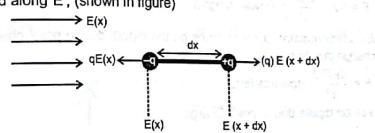
$$(iii) \because W_{\text{ext}} = U_f - U_i$$

$$\therefore W_{\text{ext}} = (-PE \cos(180^\circ + 53^\circ)) - (-PE \cos 53^\circ)$$

$$\text{or } W_{\text{ext}} = (qE)\left(\frac{3}{5}\right) + (qE)\left(\frac{3}{5}\right) \Rightarrow W_{\text{ext}} = \left(\frac{6}{5}\right) qE$$

13.5 Dipole in non-uniform electric field :

If the dipole is placed along \vec{E} , (shown in figure)

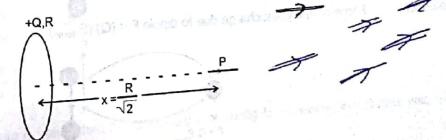


Then, Net force on the dipole : $F_{\text{net}} = qE(x+dx) - qE(x)$

$$F_{\text{net}} = q \frac{E(x+dx) - E(x)}{dx} (dx) ; \text{ here } (q(dx)) = P \therefore F_{\text{net}} = P \left(\frac{dE}{dx} \right) dx$$

Solved Examples

Example 91. A short dipole is placed on the axis of a uniformly charged ring (total charge $-Q$, radius R) at a distance $\frac{R}{\sqrt{2}}$ from centre of ring as shown in figure. Find the Force on the dipole due to the ring



Solution : $\therefore F = P \left(\frac{dE}{dx} \right)$

$$\therefore F = P \frac{d}{dx} \left(\frac{KQx}{(R^2 + x^2)^{3/2}} \right) ; \text{ (at } x = \frac{R}{\sqrt{2}} \text{)}$$

Solving we get, $F = 0$

13.6 Force between a dipole and a point charge :

Solved Examples

Example 92. A short dipole of dipole moment P is placed near a point charge Q as shown in figure. Find force on the dipole due to the point charge.

Solution :



$$\text{Force on the point charge due to the dipole } F = (Q) E_{\text{dipole}}$$

$$F = (Q) \left(\frac{2Kp}{r^3} \right) \text{ (towards right)}$$

From action reaction concept, force on the dipole due to point charge will be equal to the force on charge due to dipole

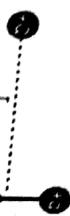
$$F = \frac{2KpQ}{r^3} \text{ (towards left)}$$

is force on dipole due to point Charge

13.7 Force between two dipoles :

Solved Examples

Example 93. A short dipole of dipole moment P is placed near a point charge Q as shown in figure. Find force on the dipole due to the point charge.



Solution :

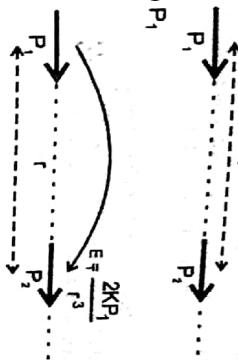
$$\text{Force on the point charge due to dipole } F = (Q) E_{\text{dipole}}$$

$$F = (Q) \left(\frac{kp}{r^3} \right) (\downarrow)$$

So force on the dipole due to the point charge will also be $F = \left(\frac{kpQ}{r^3} \right) (\uparrow)$ (but in opposite direction) as shown

Find force on short dipole P_2 due to short dipole P_1 if they are placed at a distance r as shown in figure.

Force on P_2 due to P_1



$$F_2 = (P_2) \left(\frac{dE_1}{dr} \right)$$

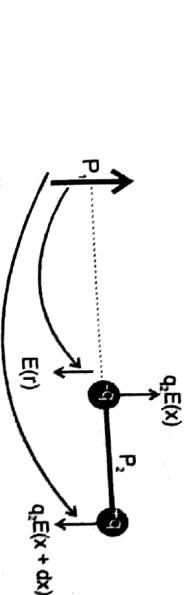
$$\therefore F_2 = (P_2) \left(\frac{d}{dr} \left(\frac{2Kp_1}{r^3} \right) \right) \text{ or } F_2 = -\frac{6Kp_1 p_2}{r^4}$$

Here - sign indicates that this force will be attractive (opposite to r)

Example 95. Find force on short dipole P_2 due to short dipole P_1 , if they are placed a distance r apart as shown in figure.



Solution :



$$F_{\text{net}} = q_2 E(x + dx) - q_2 E(x)$$

$$F_{\text{net}} = q_2 \left(\frac{E(x + dx) - E(x)}{dx} \right) dx$$

$$\text{Or } F_{\text{net}} = (P_2) \left(\frac{dE}{dx} \right)$$

(Usually this formula is valid when the dipole is placed along E . However, in this case also, we are getting the same formula)

$$\therefore F_{\text{net}} = (P_2) \left(\frac{d}{dr} \left(\frac{2Kp_1}{r^3} \right) \right)$$

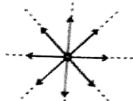
$\Rightarrow F_{\text{net}} = \frac{3Kp_1 p_2}{r^4}$ (in magnitude) & (direction upwards)

14. ELECTRIC LINES OF FORCE (ELOF)

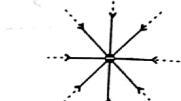
The line of force in an electric field is an imaginary line, the tangent to which at any point on it represents the direction of electric field at the given point.

14.1 Properties :

- (i) Line of force originates out from a positive charge and terminates on a negative charge. If there is only one positive charge, then lines starts from positive charge and terminates at ∞ . If there is only one negative charge, then lines starts from ∞ and terminates at negative charge.



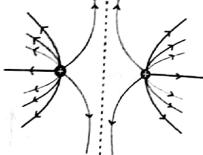
ELOF of isolated positive charge



ELOF of isolated negative charge



ELOF due to positive and negative charges



ELOF due to two positive charges

- (ii) Two lines of force never intersect each other because there cannot be two directions of \vec{E} at a single point

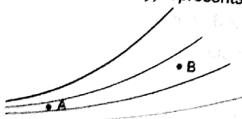


impossible

- (iii) Electric lines of force produced by static charges do not form closed loops. If lines of force make a closed loop, then work done to move a $+q$ charge along the loop will be non-zero. So it will not be conservative field. So these type of lines of force are not possible in electrostatics.



- (iv) The number of lines per unit area (line density) represents the magnitude of electric field.



$E_A > E_B$

If lines are dense $\Rightarrow E$ will be more

If lines are rare $\Rightarrow E$ will be less and if $E = 0$, no line of force will be found there

- (v) Number of lines originating (terminating) at a charge is proportional to the magnitude of charge

Electrostatics

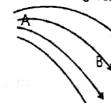
Solved Examples

- Example 96. If number of electric lines of force from charge q are 10, then find out number of electric lines of force from $2q$ charge.
 Solution : No. of ELOF \propto charge
 $10 \propto q \Rightarrow 20 \propto 2q$
 So, number of ELOF will be 20.

- (vi) Electric lines of force end or start perpendicularly on the surface of a conductor.
 (vii) Electric lines of force never enter into conductors.

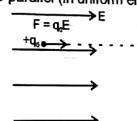
Solved Examples

- Example 97. Some electric lines of force are shown in figure. For points A and B



- (A) $E_A > E_B$ (B) $E_B > E_A$ (C) $V_A > V_B$ (D) $V_B > V_A$

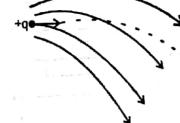
- Solution : Lines are more dense at A, so $E_A > E_B$ in the direction of electric field, potential decreases so $V_A > V_B$
 Example 98. If a charge is released in electric field, will it follow lines of force?
 Solution : Case I : If lines of force are parallel (in uniform electric field) :



In this type of field, if a charge is released, force on it will be $q_0 E$ and its direction will be along.

So the charge will move in a straight line, along the lines of force.

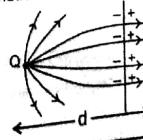
Case II : If lines of force are curved (in non-uniform electric field) :



The charge will not follow lines of force

- Example 99. A charge $+Q$ is fixed at a distance d in front of an infinite metal plate. Draw the lines of force indicating the directions clearly.

- Solution : There will be induced charge on two surfaces of conducting plate, so ELOF will start from $+Q$ charge and terminate at conductor and then will again start from other surface of conductor.



15. ELECTRIC FLUX

Consider some surface in an electric field \vec{E} . Let us select a small area element $d\vec{S}$ on this surface. The electric flux of the field over the area element is given by $d\phi_t = \vec{E} \cdot d\vec{S}$

Direction of $d\vec{S}$ is normal to the surface. It is along \hat{n}

$$\text{or } d\phi_t = EdS \cos 0^\circ$$

$$\text{or } d\phi_t = (E \cos \theta) dS$$

$$\text{or } d\phi_t = E_n dS$$

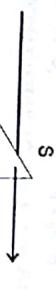
where E_n is the component of electric field in the direction of $d\vec{S}$.

$$\text{The electric flux over the whole area is given by } \phi_t = \int_S \vec{E} \cdot d\vec{S} = \int_S E_n dS$$

If the electric field is uniform over that area then $\phi_t = \vec{E} \cdot \vec{S}$

Special Cases :

- Case I :** If the electric field is normal to the surface, then angle of electric field \vec{E} with normal will be zero



$$\text{So } \phi = ES \cos 0^\circ \quad \text{or} \quad \phi = ES$$

- Case II :** If electric field is parallel to the surface (grazing), then angle made by \vec{E} with normal = 90°



$$\text{So } \phi = ES \cos 90^\circ = 0$$

Physical Meaning :

The electric flux through a surface represents the total number of electric lines of force crossing the surface. It is a property of electric field

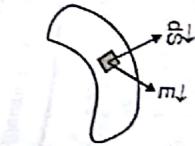
- (i) The SI unit of electric flux is $\text{Nm}^2 \text{C}^{-1}$ (Gauss) or J m C^{-1}
- (ii) Electric flux is a scalar quantity. (It can be positive, negative or zero)

Electrostatics Solved Examples

Example 100. The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$ with $E_0 = 2.0 \times 10^3 \text{ N/C}$. Find the flux of this field through a rectangular surface of area 0.2 m^2 parallel to the Y-Z plane.

$$\text{Solution : } \phi_t = \vec{E} \cdot \vec{S} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j} \right) \cdot (0.2\hat{i}) = 240 \frac{\text{N}}{\text{m}^2} \frac{\text{C}}{\text{C}}$$

Example 101. A point charge Q is placed at the corner of a square of side a , then find the flux through the square.



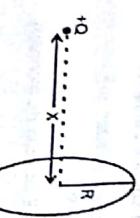
Solution :

The electric field due to Q at any point of the square will be along the plane of square and the electric field lines are perpendicular to square; so $\phi = 0$. In other words we can say that no lines is crossing the square so, flux = 0.

Example 102. Find the electric flux due to a point charge 'Q' through the circular region of radius R if the charge is placed on the axis of ring at a distance x .



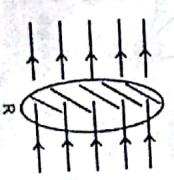
Solution : We can divide the circular region into small rings.



Lets take a ring of radius r and width dr . Flux through this small element $d\phi_t = E \cdot ds \cos 0^\circ$

$$\therefore \phi_{t,r} = \int E ds \cos 0^\circ = \int_{r=0}^{r=R} \frac{KQ}{(x^2 + r^2)} (2\pi r dr) \left(\frac{x}{\sqrt{x^2 + r^2}} \right) = \frac{Q}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

Case III : Curved surface in uniform electric field. Suppose a circular surface of radius R is placed in a uniform electric field as shown.



Flux passing through the surface $\phi = E (\pi R^2)$