

(ii) Now suppose, a hemispherical surface is placed in the direction through hemisphere.

$$\phi = \int \vec{E} \cdot d\vec{s} \cos \theta$$

where,  $\int d\vec{s} \cos \theta$  is

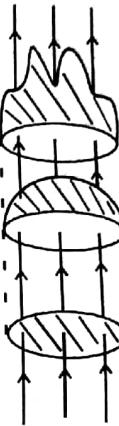
projection of the spherical surface Area on base.

$$\& \int d\vec{s} \cos \theta = \pi R^2$$

$\text{So, } \phi = E(\pi R^2)$  same Ans. as in previous case

So, we can conclude that

If the number of electric field lines passing through two surfaces are same, then flux passing through these surfaces will also be same, irrespective of the shape of surface



$$\phi_1 = \phi_2 = \phi_3 = E(\pi R^2)$$

✓ Case IV : Flux through a closed surface : Suppose there is a spherical surface and a charge  $q_i$  placed at centre.

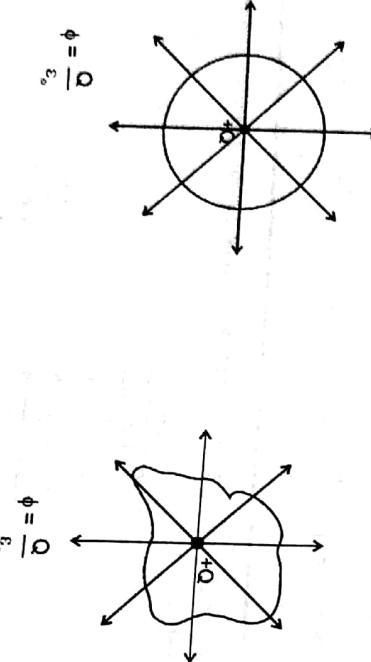
Flux through the spherical surface

$$\phi = \int \vec{E} \cdot d\vec{s} = \int E ds \quad (\text{as } \vec{E} \text{ is along } d\vec{s} \text{ (normal)})$$

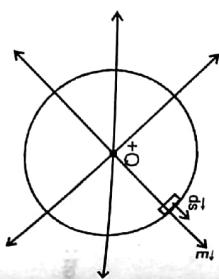
$$\therefore \phi = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \int ds \quad \text{where, } \int ds = 4\pi R^2$$

Now if the charge  $Q$  is enclosed by any other closed surface, still same no of lines of force will pass through the surface.

So, here also flux will be  $\phi = \frac{Q}{\epsilon_0}$ . That's what Gauss Theorem is.



$$\phi = \frac{Q}{\epsilon_0}$$



$$\phi_E = \oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \sum_{i=1}^n q_i$$

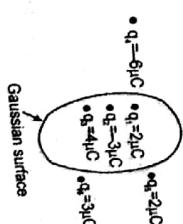
The circle on the sign of integration indicates that the integration is to be carried out over the closed surface.

#### Note :

- Flux through Gaussian surface is independent of its shape.
- Flux through Gaussian surface depends only on total charge present inside Gaussian surface.
- Flux through Gaussian surface is independent of position of charges inside Gaussian surface.
- Electric field intensity at the Gaussian surface is due to all the charges present inside as well as outside the Gaussian surface.
- In a closed surface incoming flux is taken negative, while outgoing flux is taken positive, because it is taken positive in outward direction.
- In a Gaussian surface,  $\phi = 0$  does not imply  $E = 0$  at every point implies  $\phi = 0$ .

### Solved Examples

Example 103. Find out flux through the given Gaussian surface.



$$\text{Solution : } \phi = \frac{Q_{in}}{\epsilon_0} = \frac{2q_1 - 3q_2 + 4q_3}{\epsilon_0} = \frac{3 \times 10^{-6}}{\epsilon_0} \text{ Nm}^2/\text{C}$$

## GAUSS'S LAW IN ELECTROSTATICS OR GAUSS'S THEOREM

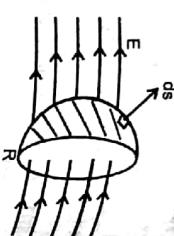
This law was stated by a mathematician Karl F Gauss. This law gives the relation between the electric field at a point on a closed surface and the net charge enclosed by that surface. This surface is called Gaussian surface. It is a closed hypothetical surface. Its validity is shown by experiments. It is used to determine the electric field due to some symmetric charge distributions.

### Statement and Details :

Gauss's law is stated as given below:

The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to  $\frac{1}{\epsilon_0}$  times the total charge enclosed within the surface. Here,  $\epsilon_0$  is the permittivity of free space.

If  $S$  is the Gaussian surface and  $\sum_{i=1}^n q_i$  is the total charge enclosed by the Gaussian surface, then according to Gauss's law,



Electrostatics

If a point charge  $q$  is placed at the centre of a cube, then find out flux through any one face of cube.

**Solution :** Flux through all 6 faces =  $\frac{q}{\epsilon_0}$ . Since, all the surfaces are symmetrical

$$\text{So, flux through one face} = \frac{1}{6} \frac{q}{\epsilon_0}$$

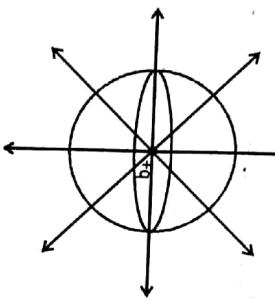
### 16.2 Flux through open surfaces using Gauss's Theorem :

#### Solved Examples

**Example 105.** A point charge  $+q$  is placed at the centre of curvature of a hemisphere. Find flux through hemispherical surface.



**Solution :** Lets put an upper half hemisphere. Now, flux passing through the entire sphere =  $\frac{q}{\epsilon_0}$



As the charge  $q$  is symmetrical to the upper half and lower half hemispheres, so half-half flux will emit from both the surfaces.

Flux emitting

$$\text{from lower half surface} = \frac{q}{2\epsilon_0}$$

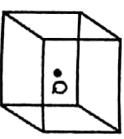
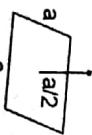
Flux emitting

$$\text{from upper half surface} = \frac{q}{2\epsilon_0}$$

**Example 106.** A charge  $Q$  is placed at a distance  $a/2$  above the centre of a horizontal, square surface of side  $a$  as shown in figure. Find the flux of the electric field through the square surface.

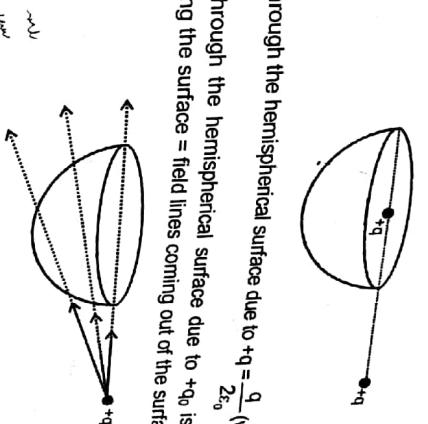
**Solution :**

We can consider imaginary faces of cube such that the charge lies at the centre of the cube. Due to symmetry, we can say that flux through the given area (which is one face of cube),  $\phi = \frac{Q}{6\epsilon_0}$



**Solution :**

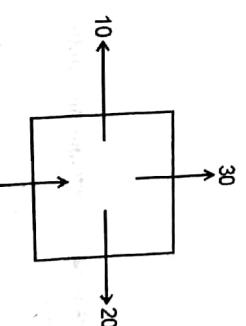
- Flux through the hemispherical surface due to  $+q$  =  $\frac{q}{2\epsilon_0}$  (we have seen in previous examples)
- Flux through the hemispherical surface due to  $+q_0$  is 0, because due to  $+q_0$ , field lines entering the surface = field lines coming out of the surface.



### 16.3 Finding $q_{in}$ from flux :

#### Solved Examples

**Example 108.**



Flux (in S.I. units) coming out and entering a closed surface is shown in the figure. Find charge enclosed by the closed surface.

Net flux through the closed surface =  $+20 + 30 + 10 - 15 = 45 \text{ N.m}^2/\text{C}$

From Gauss's theorem :  $\phi_{net} = \frac{q_{in}}{\epsilon_0}$

$$\text{or } 45 = \frac{q_{in}}{\epsilon_0} \quad \therefore \quad q_{in} = (45)\epsilon_0$$

### 16.4 Finding electric field from Gauss's Theorem :

From Gauss's theorem, we can say  $\int \vec{E} \cdot d\vec{s} = \phi_{net} = \frac{q_{in}}{\epsilon_0}$

#### 16.4.1 Finding E due to a spherical shell :

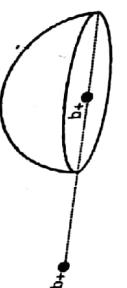
**Electric field outside the Sphere :**

Since, electric field due to a shell will be radially outwards. So lets choose a spherical Gaussian surface

Applying Gauss's theorem for this spherical Gaussian surface,

### 16.5 Electrostatic Potential :

#### Find flux through the hemispherical surface



$$\int \vec{E} \cdot \vec{ds} = \phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{q}{\epsilon_0}$$

$\downarrow$   
 $\int \vec{E} \cdot \vec{ds}$  (because value of  $E$  is constant at the surface)

$E \int ds$  (because total area of the spherical surface =  $4\pi r^2$ )

$$\Rightarrow E(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0} \quad \therefore E_{\text{out}} = \frac{q}{4\pi \epsilon_0 r^2}$$

**Electric field inside a spherical shell** : Lets choose a spherical Gaussian surface inside the shell Applying Gauss's theorem for this surface

$$\int \vec{E} \cdot \vec{ds} = \phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = 0$$



$$\int |\vec{E}| \cdot |\vec{ds}| \cos 0^\circ$$

$\downarrow$

$$E \int ds$$

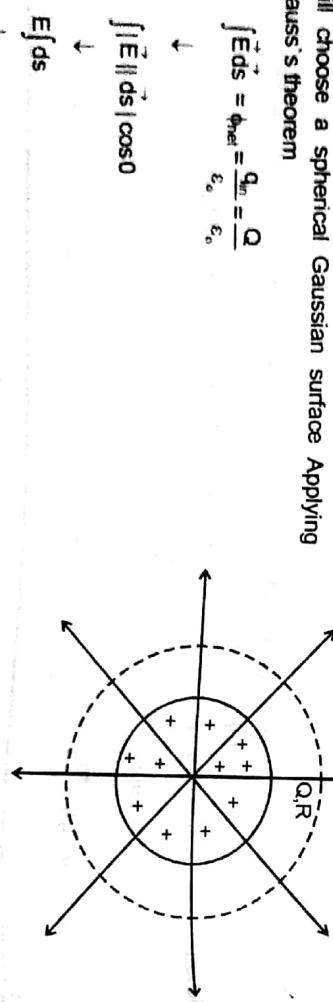
$\downarrow$

$$E(4\pi r^2) \Rightarrow E(4\pi r^2) = 0$$

**16.4.2 Electric field due to solid sphere (having uniformly distributed charge  $Q$  and radius  $R$ ) :**

**Electric field outside the sphere :**  
Direction of electric field is radially outwards, so we will choose a spherical Gaussian surface Applying Gauss's theorem

$$\int \vec{E} \cdot \vec{ds} = \phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

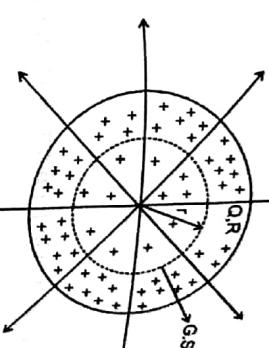
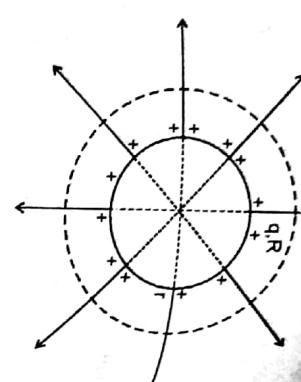
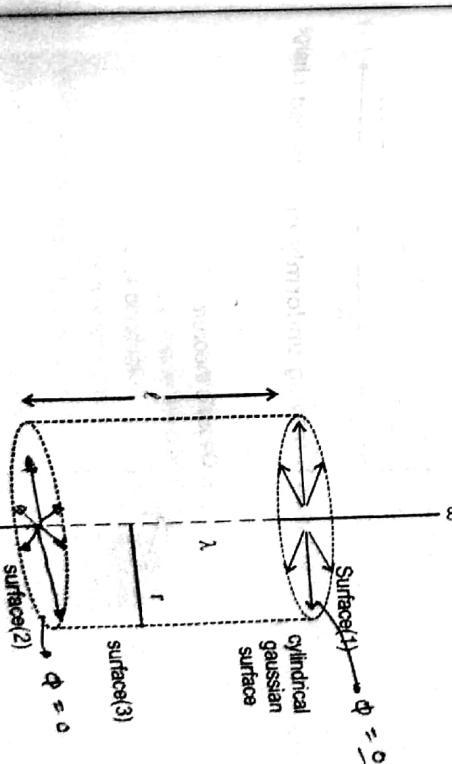


$$\int \vec{E} \cdot \vec{ds} = \phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\frac{4}{3}\pi r^3}{\epsilon_0} = \frac{qr^2}{\epsilon_0}$$

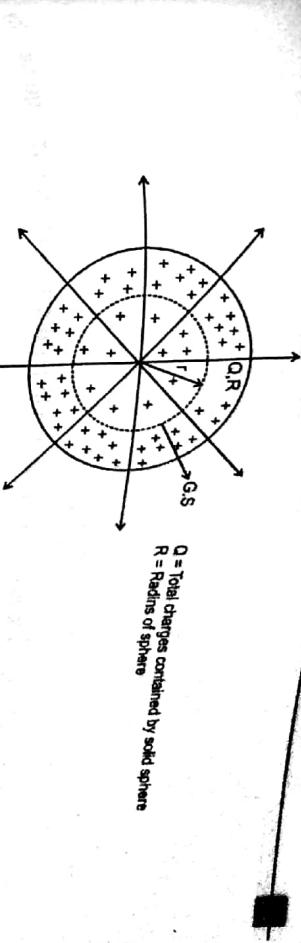
$$E(4\pi r^2) \Rightarrow E(4\pi r^2) = \frac{qr^2}{\epsilon_0 r^3}$$

$$E = \frac{qr}{4\pi \epsilon_0 R^3}$$

**16.4.3 Electric field due to infinite line charge (having uniformly distributed charged of charge density  $\lambda$ ) :**



$Q = \text{Total charges contained by solid sphere}$



For this choose a spherical Gaussian surface inside the solid sphere Applying Gauss's theorem for this surface

$$\int \vec{E} \cdot \vec{ds} = \phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\frac{4}{3}\pi r^3}{\epsilon_0} = \frac{qr^2}{\epsilon_0}$$

$\downarrow$

$$\int \vec{E} \cdot \vec{ds}$$

$\downarrow$

$$E(4\pi r^2) \Rightarrow E(4\pi r^2) = \frac{qr^2}{\epsilon_0 r^3}$$

$$E = \frac{kq}{r^2}$$

**Electric field due to infinitely wire is radial so we will choose cylindrical Gaussian surface as shown**

$$\text{or } E_{\text{out}} = \frac{Q}{4\pi \epsilon_0 r^2}$$

$$E(4\pi r^2) \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

Electric field due to infinitely wire is radial so we will choose cylindrical Gaussian surface as shown

Electric field due to infinitely wire is radial so we will choose cylindrical Gaussian surface as shown

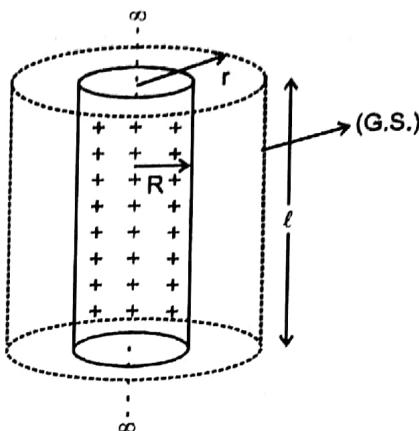
$$\phi_{\text{net}} \downarrow$$

$$\phi_1 = 0 \quad \phi_2 = 0 \quad \phi_3 \neq 0 = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$\phi_3 = \int \vec{E} \cdot d\vec{s} = \int E ds = E \int ds = E (2\pi r l)$$

$$\therefore E (2\pi r l) = \frac{\lambda l}{\epsilon_0} \quad \therefore E = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{2k\lambda}{r}$$

#### 16.4.4 Electric field due to infinitely long charged tube (having uniform surface charge density $\sigma$ and radius $R$ ) :



(i) **E outside the tube** : Lets choose a cylindrical Gaussian surface of length  $l$  :

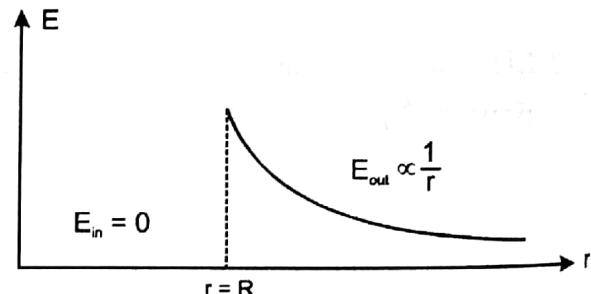
$$\therefore \phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma 2\pi R l}{\epsilon_0} \quad \Rightarrow \quad E_{\text{out}} \times 2\pi r l = \frac{\sigma 2\pi R l}{\epsilon_0} \quad \therefore \quad E = \frac{\sigma R}{r \epsilon_0}$$

(ii) **E inside the tube** :

Lets choose a cylindrical Gaussian surface inside the tube.

$$\phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = 0$$

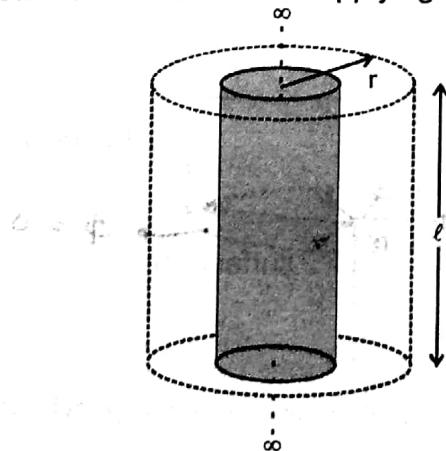
$$\text{So } E_{\text{in}} = 0$$



#### 16.4.5 E due to infinitely long solid cylinder of radius R (having uniformly distributed charge in volume (volume charge density $\rho$ )) :

(i) **E at outside point :-**

Lets choose a cylindrical Gaussian surface. Applying Gauss's theorem :



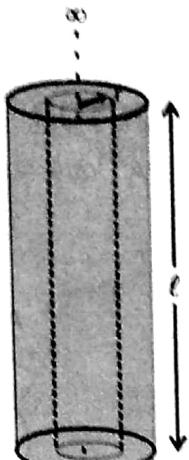
## Electrostatics

$$E \times 2\pi r l = \frac{q_{in}}{\epsilon_0} = \frac{\rho \times \pi R^2 l}{\epsilon_0}$$

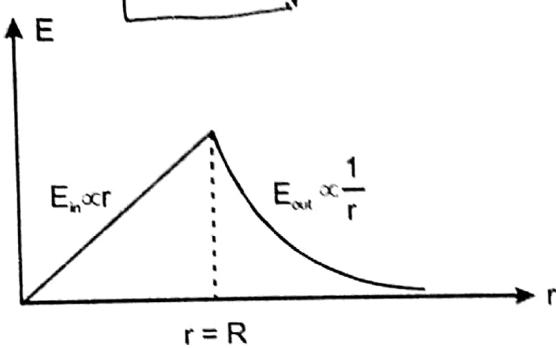
$$E_{out} = \frac{\rho R^2}{2r \epsilon_0} \rightarrow \text{outside}$$

(ii)  $E$  at inside point :

Lets choose a cylindrical Gaussian surface inside the solid cylinder. Applying Gauss's theorem

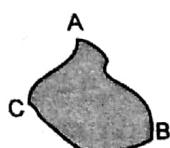


$$E \times 2\pi r l = \frac{q_{in}}{\epsilon_0} = \frac{\rho \times \pi r^2 l}{\epsilon_0} \Rightarrow E_{in} = \frac{\rho r}{2\epsilon_0}$$



## 17. CONDUCTOR AND IT'S PROPERTIES [FOR ELECTROSTATIC CONDITION]

- (i) Conductors are materials which contain large number of free electrons which can move freely inside the conductor.
- (ii) In electrostatics conductors are always equipotential surfaces.
- (iii) Charge always resides on outer surface of conductor.
- (iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.
- (v) Electric field is always perpendicular to conducting surface.
- (vi) Electric lines of force never enter into conductors.
- (vii) Electric field intensity near the conducting surface is given by formula  $\bar{E} = \frac{\sigma}{\epsilon_0} \hat{n}$



$$\bar{E}_A = \frac{\sigma_A}{\epsilon_0} \hat{n} ; \bar{E}_B = \frac{\sigma_B}{\epsilon_0} \hat{n} \text{ and } \bar{E}_C = \frac{\sigma_C}{\epsilon_0} \hat{n}$$

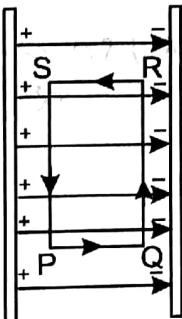
A positively charged body 'A' has been brought near a neutral brass sphere B mounted on a glass stand as shown in the figure. The potential of B will be:



? at which point ?

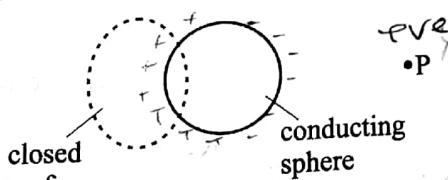
- (A) Zero      (B) Negative      (C) Positive      (D) Infinite

J-11. The amount of work done by electric field in joules in carrying a charge  $+q$  along the closed path PQRSTP between the oppositely charged metal plates is: (where, E is electric field between the plates)



- (A) zero      ✓  
 (B)  $q$   
 (C)  $qE(PQ + QR + SR + SP)$

J-12. Figure shows a closed surface which intersects a conducting sphere. If a positive charge is placed at the point P, the flux of the electric field through the closed surface:

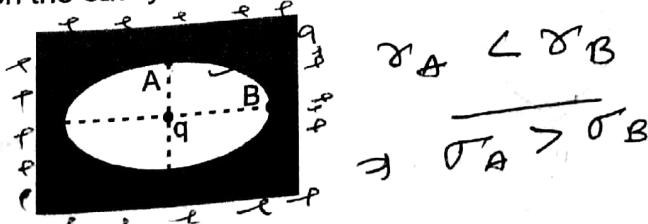


ELOF never enters conductors ✓  
 no E inside conductors ✓

- ✓ will become positive  
 (C) will become undefined

- (B) will remain zero  
 (D) will become negative

J-13. An ellipsoidal cavity is carved within a perfect conductor. A positive charge  $q$  is placed at the center of the cavity. The points A and B are on the cavity surface as shown in the figure. [JEE 1999 (Scr.), 3/100]



Then :

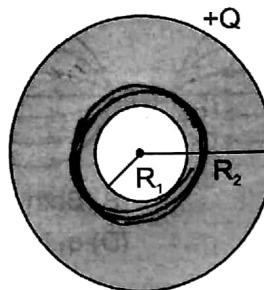
- (A) Electric field near A in the cavity = electric field near B in the cavity X  
 (B) Charge density at A = Charge density at B  
 (C) Potential at A = Potential at B → metal system is as we potential body  
 (D) Total electric field flux through the surface of the cavity is  $q/\epsilon_0$ .

## Electrostatics

- J-4. Three concentric conducting spherical shells carry charges as follows : +4Q on the inner shell, -2Q on the middle shell and -5Q on the outer shell. The charge on the inner surface of the outer shell is:  
 (A) 0      (B) 4Q      (C) -Q      (D) -2Q

- J-5. A charge  $q$  is uniformly distributed over a large plastic plate. The electric field at a point P close to the centre and just above the surface of the plate is 50 V/m. If the plastic plate is replaced by a copper plate of the same geometrical dimensions and carrying the same uniform charge  $q$ , the electric field at the point P will become:  
 (A) zero      (B) 25 V/m      (C) 50 V/m      (D) 100 V/m

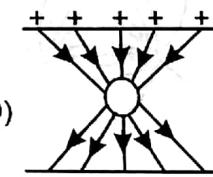
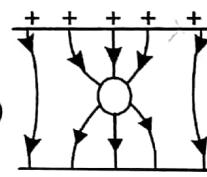
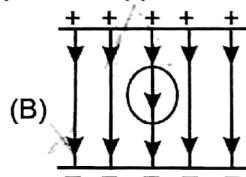
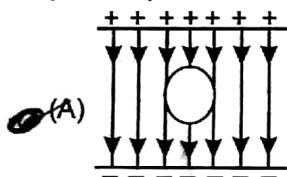
- J-6. Figure shows a thick metallic sphere. If it is given a charge +Q, then electric field will be present in the region



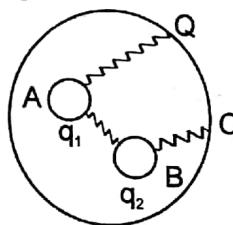
- (A)  $r < R_1$  only  
 (C)  $r \geq R_2$  only

- (B)  $r > R_1$  and  $R_1 < r < R_2$   
 (D)  $r \leq R_2$  only

- J-7. An uncharged sphere of metal is placed in a uniform electric field produced by two large conducting parallel plates having equal and opposite charges, then lines of force look like:



- J-8. Two small conductors A and B are given charges  $q_1$  and  $q_2$  respectively. Now they are placed inside a hollow metallic conductor (C) carrying a charge Q. If all the three conductors A, B and C are connected by conducting wires as shown, the charges on A, B and C will be respectively :



- (A)  $\frac{q_1 + q_2}{2}, \frac{q_1 + q_2}{2}, Q$   
 (C)  $\frac{q_1 + q_2 + Q}{2}, \frac{q_1 + q_2 + Q}{2}, 0$

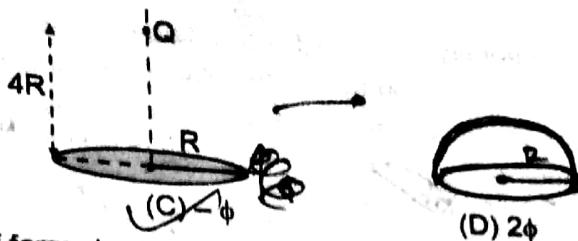
- (B)  $\frac{Q + q_1 + q_2}{3}, \frac{Q + q_1 + q_2}{3}, \frac{Q + q_1 + q_2}{3}$   
 (D) 0, 0,  $Q + q_1 + q_2$

- J-9. You are travelling in a car during a thunder storm. In order to protect yourself from lightning, would you prefer to :

- (A) Remain in the car  
 (C) Get out and be flat on the ground

- (B) Take shelter under a tree  
 (D) Touch the nearest electrical pole

A charged surface. The flux through the curved surface (taking direction of area vector along normal as positive), is -



(A) zero

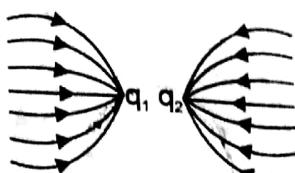
(B)  $\phi$

(C)  $< \phi$

(D)  $2\phi$

I-12.

The given figure gives electric lines of force due to two charges  $q_1$  and  $q_2$ . What are the signs of the two charges?



(A) Both are negative ✓

(C)  $q_1$  is positive but  $q_2$  is negative

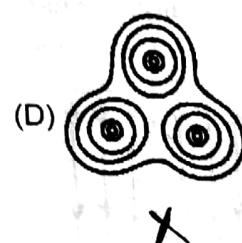
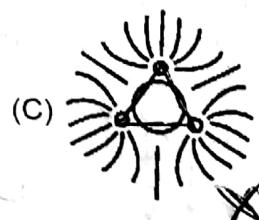
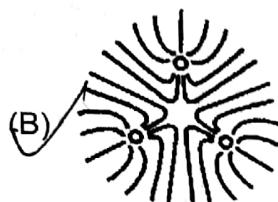
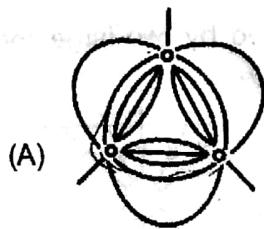
(B) Both are positive

(D)  $q_1$  is negative but  $q_2$  is positive

I-13.

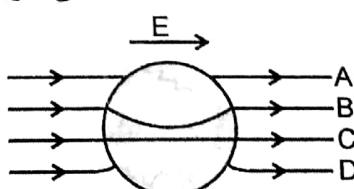
Three positive charges of equal value  $q$  are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in :

[JEE 2001(Scr.), 3/100]



### Section (J) : Conductor, it's properties & Electric Pressure

J-1. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in figure as : *EL OF never enter conductors* [JEE 1996, 2/100]



(A) A

(B) B

(C) C

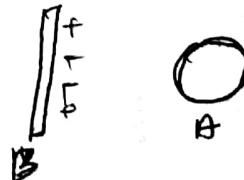
✓ (D) D

J-2.

A neutral spherical metallic object A is placed near a finite metal plate B carrying a positive charge. The electric force on the object will be :

- (A) away from the plate B  
(C) parallel to the plate B

- ✓ (B) towards the plate B  
(D) zero

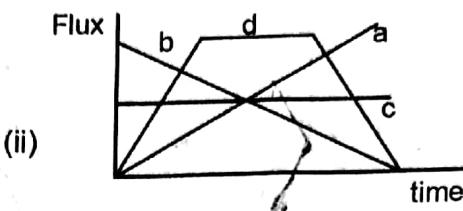
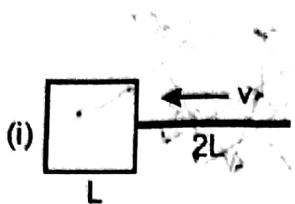


J-3.

A positive point charge  $q$  is brought near a neutral metal sphere.

- (A) The sphere becomes negatively charged.  
(B) The sphere becomes positively charged.  
✓ (C) The interior remains neutral and the surface gets non-uniform charge distribution.  
(D) The interior becomes positively charged and the surface becomes negatively charged.

- I-7. Figure (i) shows an imaginary cube of edge length L. A uniformly charged rod of length  $2L$  moves towards left at a small but constant speed  $v$ . At  $t = 0$ , the left end of the rod just touches the centre of the face of the cube opposite to it. Which of the graphs shown in fig.(ii) represents the flux of the electric field through the cube as the rod goes through it?



(A) a

(B) b

(C) c

(D) d

- I-8. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 20cm surrounding the total charge is 50 V-m. The flux over a concentric sphere of radius 40 cm will be:

(A) 50 V-m

(B) 75 V-m

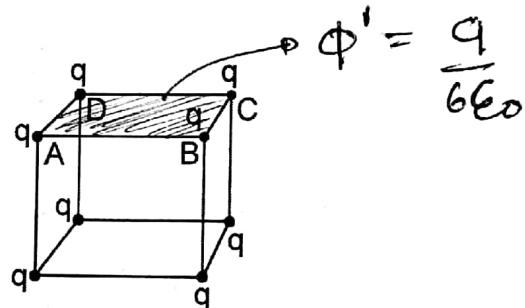
(C) 100 V-m

(D) 200 V-m

- I-9. Eight point charges (can be assumed as uniformly charged small spheres and their centres at the corner of the cube) having value  $q$  each are fixed at vertices of a cube. The electric flux through square surface ABCD of the cube is

$$q_{\text{in}} = \frac{q}{8} \times 8 = q$$

$$\Phi = \frac{q}{\epsilon_0} \rightarrow \text{Total}$$



$$(A) \frac{q}{24 \epsilon_0}$$

$$(B) \frac{q}{12 \epsilon_0}$$

$$(C) \frac{q}{6 \epsilon_0}$$

$$(D) \frac{q}{8 \epsilon_0}$$

- X** I-10. Figure shows two large cylindrical shells having uniform linear charge densities  $+\lambda$  and  $-\lambda$ . Radius of inner cylinder is 'a' and that of outer cylinder is 'b'. A charged particle of mass  $m$ , charge  $q$  revolves in a circle of radius  $r$ . Then, its speed 'v' is : (Neglect gravity and assume the radii of both the cylinders to be very small in comparison to their length.)

$$\lambda = \frac{Q}{h} \Rightarrow \sigma = \frac{Q}{2\pi r h} = \frac{\lambda}{2\pi r}$$

$$\Phi = \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

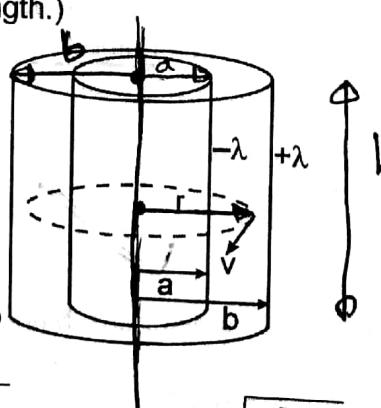
$$\Rightarrow E \cdot 2\pi r h = \frac{\lambda h}{\epsilon_0} \Rightarrow E = \frac{\lambda}{2\pi r \epsilon_0}$$

$$(A) \sqrt{\frac{\lambda q}{2\pi \epsilon_0 m}}$$

$$(B) \sqrt{\frac{2\lambda q}{\pi \epsilon_0 m}}$$

$$(C) \sqrt{\frac{\lambda q}{\pi \epsilon_0 m}}$$

$$(D) \sqrt{\frac{\lambda q}{4\pi \epsilon_0 m}}$$



$$F = qE = \frac{\lambda q}{2\pi r \epsilon_0}$$

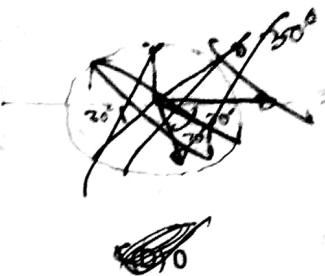
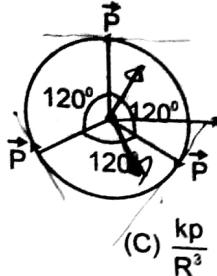
$$= \frac{m v^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{\lambda q}{2\pi m \epsilon_0}}$$

$$\sqrt{\frac{\lambda q}{4\pi \epsilon_0 m}}$$

- H-7. The force between two short electric dipoles separated by a distance  $r$  is directly proportional to :
- $r^2$
  - $r^4$
  - $r^{-2}$
  - $r^{-4}$

- H-8. Three dipoles each of dipole moment of magnitude  $p$  are placed tangentially on a circle of radius  $R$  in its plane positioned at equal angle from each other as shown in the figure. Then the magnitude of electric field intensity at the centre of the circle will be :



(A)  $\frac{4kp}{R^3}$

(B)  $\frac{2kp}{R^3}$

(C)  $\frac{kp}{R^3}$

### Section (I) : Electric lines of force, Flux calculation and Gauss's law

- I-1. A square of side 'a' is lying in xy plane such that two of its sides are lying on the axis. If an electric field  $\vec{E} = E_0 \hat{x}$  is applied on the square. The flux passing through the square is :

(A)  $E_0 a^3$

(B)  $\frac{E_0 a^3}{2}$

(C)  $\frac{E_0 a^3}{3}$

(D)  $\frac{E_0 a^2}{2}$

If electric field is uniform, then the electric lines of forces are:

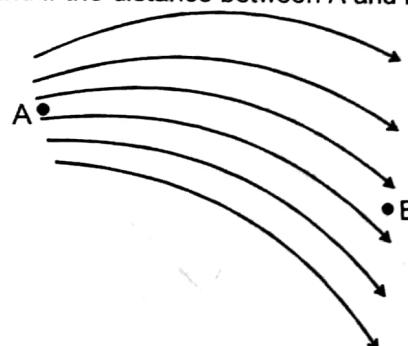
(A) Divergent

(B) Convergent

(C) Circular

(D) Parallel

- I-3. The figure shows the electric lines of force emerging from a charged body. If the electric fields at A and B are  $E_A$  and  $E_B$  respectively and if the distance between A and B is  $r$ , then



(A)  $E_A < E_B$

(B)  $E_A > E_B$

(C)  $E_A = \frac{E_B}{r}$

(D)  $E_A = \frac{E_B}{r^2}$

- I-4. Select the correct statement :

(A) The electric lines of force are always closed curves

(B) Electric lines of force are parallel to equipotential surface

(C) Electric lines of force are perpendicular to equipotential surface

(D) Electric line of force is always the path of a positively charged particle.

- I-5. If the electric flux entering and leaving a closed surface are respectively of magnitude  $\phi_1$  and  $\phi_2$ , then the electric charge inside the surface will be :

(A)  $\frac{\phi_2 - \phi_1}{\epsilon_0}$

(B)  $(\phi_1 - \phi_2)\epsilon_0$

(C)  $\epsilon_0(\phi_2 - \phi_1)$

$\phi_1 = q_1 / \epsilon_0$  entering  
 $\phi_2 = q_2 / \epsilon_0$  leaving

- I-6. An electric dipole is placed at the centre of a sphere. Mark the correct options.

(A) The electric field is zero at every point of the sphere.

(B) The flux of the electric field through the sphere is non-zero.

(C) The electric field is zero on a circle on the sphere.

(D) The electric field is not zero anywhere on the sphere.

**G-6.**

Let  $E$  be the electric field and  $V$ , the electric potential at a point.

- (A) If  $E \neq 0$ ,  $V$  cannot be zero  $\times$   
 (B) If  $E = 0$ ,  $V$  must be zero  $\times$   
 (C) If  $V = 0$ ,  $E$  must be zero  $\times$   
 (D) None of these

**G-6.**

The electric field in a region is directed outward and is proportional to the distance  $r$  from the origin.

Taking the electric potential at the origin to be zero, the electric potential at a distance  $r$ :

- (A) increases as one goes away from the origin. (B) is proportional to  $r^2$   
 (C) is proportional to  $r$  (D) is uniform in the region

**G-7.** A non-conducting ring of radius 0.5 m carries a total charge of  $1.11 \times 10^{-10}$  C distributed non-uniformly on its circumference producing an electric field  $\vec{E}$  every where in space. The value of the line integral

$$\int_{\ell=\infty}^{\ell=0} -\vec{E} \cdot d\vec{\ell} \quad (\ell = 0 \text{ being centre of the ring}) \text{ in volts is : (Approximately)}$$

[JEE 1997, 1]

(A) +2

(B) -1

(C) -2

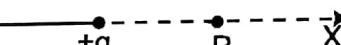
(D) zero

## Section (H) : Dipole

**H-1.** Due to an electric dipole shown in fig., the electric field intensity is parallel to dipole axis.



Equatorial



(A) at P only

(B) at Q only

(C) both at P and at Q (D) neither at P nor at Q

**H-2.** An electric dipole of dipole moment  $\vec{p}$  is placed at the origin along the  $x$ -axis. The angle made by electric field with  $x$ -axis at a point  $P$ , whose position vector makes an angle  $\theta$  with  $x$ -axis, is :

(where,  $\tan \alpha = \frac{1}{2} \tan \theta$ )

(A)  $\alpha$

(B)  $\theta$



(C)  $\theta + \alpha$

(D)  $\theta + 2\alpha$

**H-3.** An electric dipole consists of two opposite charges each of magnitude  $1.0 \mu\text{C}$ , separated by a distance of 2.0 cm. The dipole is placed in an external electric field of  $1.0 \times 10^5 \text{ N/C}$ . The maximum torque on the dipole is :

- (A)  $0.2 \times 10^{-3} \text{ N-m}$  (B)  $1.0 \times 10^{-3} \text{ N-m}$  (C)  $2.0 \times 10^{-3} \text{ N-m}$  (D)  $4.0 \times 10^{-3} \text{ N-m}$

**H-4.** A dipole of electric dipole moment  $P$  is placed in a uniform electric field of strength  $E$ . If  $\theta$  is the angle between positive directions of  $P$  and  $E$ , then the potential energy of the electric dipole is largest when  $\theta$  is :

- (A) zero (B)  $\pi/2$  (C)  $\pi$  (D)  $\pi/4$

**H-5.** Two opposite and equal charges of magnitude  $4 \times 10^{-8}$  coulomb each when placed  $2 \times 10^{-2}$  cm apart form a dipole. If this dipole is placed in an external electric field of  $4 \times 10^8 \text{ N/C}$ , the value of maximum torque and the work required in rotating it through  $180^\circ$  from its initial orientation which is along electric field will be : (Assume rotation of dipole about an axis passing through centre of the dipole):

- (A)  $64 \times 10^{-4} \text{ N-m}$  and  $44 \times 10^{-4} \text{ J}$  (B)  $32 \times 10^{-4} \text{ N-m}$  and  $32 \times 10^{-4} \text{ J}$   
 (C)  $64 \times 10^{-4} \text{ N-m}$  and  $32 \times 10^{-4} \text{ J}$  (D)  $32 \times 10^{-4} \text{ N-m}$  and  $64 \times 10^{-4} \text{ J}$

**H-6.** At a point on the axis (but not inside the dipole and not at infinity) of an electric dipole

(A) The electric field is zero

(B) The electric potential is zero

(C) Neither the electric field nor the electric potential is zero

(D) The electric field is directed perpendicular to the axis of the dipole

## Electrostatics

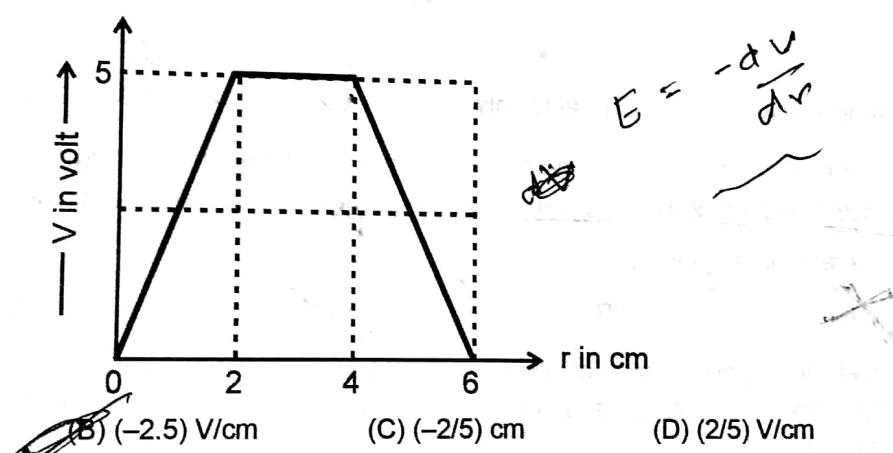
- E-2. You are given an arrangement of three point charges  $q$ ,  $2q$  and  $xq$  separated by equal finite distances so that electric potential energy of the system is zero. Then the value of  $x$  is :
- (A)  $-\frac{2}{3}$       (B)  $-\frac{1}{3}$       (C)  $\frac{2}{3}$       (D)  $\frac{3}{2}$

## Section (F) : Self Energy and energy density

- F-1. A uniformly charged sphere of radius 1 cm has potential of 8000 V at surface. The energy density near the surface of sphere will be:
- (A)  $64 \times 10^5 \text{ J/m}^3$       (B)  $8 \times 10^3 \text{ J/m}^3$       (C)  $32 \text{ J/m}^3$       (D)  $2.83 \text{ J/m}^3$
- F-2. If 'n' identical water drops (assumed spherical each) each charged to a potential energy  $U$  coalesce to form a single drop, the potential energy of the single drop is (Assume that drops are uniformly charged):
- (A)  $n^{1/3} U$       (B)  $n^{2/3} U$       (C)  $n^{4/3} U$       (D)  $n^{5/3} U$

## Section (G) : questions based on relation between $\vec{E}$ and $V$ :

- G-1. The variation of potential with distance  $r$  from a fixed point is shown in Figure. The electric field at  $r = 5 \text{ cm}$ , is :



- (A)  $(2.5) \text{ V/cm}$       (B)  $(-2.5) \text{ V/cm}$       (C)  $(-2/5) \text{ cm}$       (D)  $(2/5) \text{ V/cm}$

- G-2. In the above question, the electric force acting on a point charge of 2 C placed at the origin will be :

- (A) 2 N      (B) 500 N      (C) -5 N      (D) -500 N

- G-3. The electric potential  $V$  as a function of distance  $x$  (in metre) is given by

$$V = (5x^2 + 10x - 9) \text{ volt.} \quad E = -\frac{dV}{dx} = -10x - 10$$

The value of electric field at  $x = 1 \text{ m}$  would be :

- (A) 20 volt/m      (B) 6 volt/m      (C) 11 volt/m      (D) -23 volt/m

- G-4. A uniform electric field having a magnitude  $E_0$  and direction along positive  $x$ -axis exists. If the electric potential  $V$  is zero at  $x = 0$ , then its value at  $x = +x$  will be :

- (A)  $V_x = xE_0$       (B)  $V_x = -xE_0$       (C)  $V_x = x^2 E_0$       (D)  $V_x = -x^2 E_0$

$$V = - \int E_x dx$$

### Electrostatics

- C-10. A particle of charge  $Q$  and mass  $m$  travels through a potential difference  $V$  from rest. The momentum of the particle is :
- (A)  $\frac{mV}{Q}$       (B)  $2Q\sqrt{mV}$        (C)  $\sqrt{2mQV}$       (D)  $\sqrt{\frac{2QV}{m}}$

- C-11. If a uniformly charged spherical shell of radius 10 cm has a potential  $V$  at a point distant 5 cm from its centre, then the potential at a point distant 15 cm from the centre will be :
- (A)  $\frac{V}{3}$        (B)  $\frac{2V}{3}$       (C)  $\frac{3}{2}V$       (D)  $3V$

- C-12. A hollow uniformly charged sphere has radius  $r$ . If the potential difference between its surface and a point at distance  $3r$  from the centre is  $V$ , then the electric field intensity at a distance  $3r$  from the centre is :
- (A)  $V/6r$       (B)  $V/4r$       (C)  $V/3r$       ~~(D)~~ (D)  $V/2r$

- C-13. A hollow sphere of radius 5 cm is uniformly charged such that the potential on its surface is 10 volt. Then potential at centre of sphere will be :
- (A) Zero      (B) 10 volt      (C) Same as at a point 5 cm away from the surface      (D) Same as at a point 25 cm away from the centre

- C-14. A charge  $+q$  is fixed at each of the points  $x = x_0, x = 3x_0, x = 5x_0, \dots$  upto infinity on the  $x$ -axis and a charge  $-q$  is fixed at each of the points  $x = 2x_0, x = 4x_0, x = 6x_0, \dots$  upto infinity. Here  $x_0$  is a positive constant. Take the electric potential at a point due to a charge  $Q$  at a distance  $r$  from it to be

Then the potential at the origin due to the above system of charges is:

[JEE 1998 Screening, 2/200]

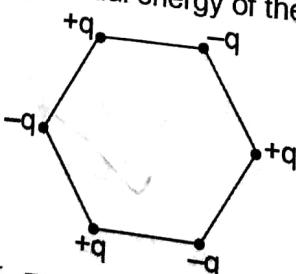
- (A) 0      (B)  $\frac{q}{8\pi\epsilon_0 x_0 \ln 2}$       (C)  $\infty$        (D)  $\frac{q \ln 2}{4\pi\epsilon_0 x_0}$

### Section (D) : Electric Potential Energy of a particle

- D-1. If a charge is shifted from a high potential region to low potential region, the electrical potential energy:
- (A) Increases      ~~(B)~~ Decreases       (C) May increase or decrease.      (D) Remains constant
- D-2. A particle of mass 2 g and charge  $1\mu C$  is held at rest on a frictionless horizontal surface at a distance of 1 m from a fixed charge of  $1\text{mC}$ . If the particle is released it will be repelled. The speed of the particle when it is at distance of 10 m from the fixed charge is:
- (A) 100 m/s       (B) 90 m/s      (C) 60 m/s      (D) 45 m/s

### Section (E) : Potential Energy of a system of Point Charges

- E-1. Six charges of magnitude  $+q$  and  $-q$  are fixed at the corners of a regular hexagon of edge length  $a$  as shown in the figure. The electrostatic potential energy of the system of charged particles is :

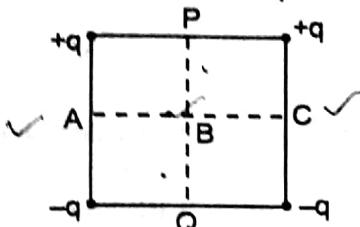


- (A)  $\frac{q^2}{\pi\epsilon_0 a} \left[ \frac{\sqrt{3}}{8} - \frac{15}{4} \right]$       (B)  $\frac{q^2}{\pi\epsilon_0 a} \left[ \frac{\sqrt{3}}{2} - \frac{9}{4} \right]$       (C)  $\frac{q^2}{\pi\epsilon_0 a} \left[ \frac{\sqrt{3}}{4} - \frac{15}{2} \right]$        (D)  $\frac{q^2}{\pi\epsilon_0 a} \left[ \frac{\sqrt{3}}{2} - \frac{15}{8} \right]$

### Section (C) : Electric Potential and Potential Difference

- C-1. At a certain distance from a point charge, the electric field is  $500 \text{ V/m}$  and the potential is  $3000 \text{ V}$ . What is the distance ?  
 (A)  $6 \text{ m}$       (B)  $12 \text{ m}$       (C)  $36 \text{ m}$       (D)  $144 \text{ m}$

- C-2. Figure represents a square carrying charges  $+q$ ,  $+q$ ,  $-q$ ,  $-q$  at its four corners as shown. Then the potential will be zero at points : (A, C, P and Q are mid points of sides)



- (A) A, B, C, P and Q      (B) A, B and C      (C) A, P, C and Q      (D) P, B and Q

- C-3. Two equal positive charges are kept at points A and B. The electric potential, while moving from A to B along straight line :  
 (A) continuously increases      (B) remains constant  
 (C) decreases then increases      (D) increases then decreases

- C-4. A semicircular ring of radius  $0.5 \text{ m}$  is uniformly charged with a total charge of  $1.5 \times 10^{-9} \text{ coul}$ . The electric potential at the centre of this ring is :  
 (A)  $27 \text{ V}$       (B)  $13.5 \text{ V}$       (C)  $54 \text{ V}$       (D)  $45.5 \text{ V}$

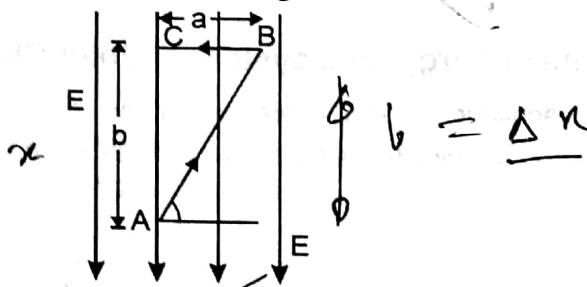
- C-5. When a charge of  $3 \text{ coul}$  is placed in a uniform electric field, it experiences a force of  $3000 \text{ newton}$ . The potential difference between two points separated by a distance of  $1 \text{ cm}$  along field within this field is :  
 (A)  $10 \text{ volt}$       (B)  $90 \text{ volt}$       (C)  $1000 \text{ volt}$       (D)  $3000 \text{ volt}$

- C-6. A  $5 \text{ coulomb}$  charge experiences a constant force of  $2000 \text{ N}$  when moved between two points separated by a distance of  $2 \text{ cm}$  in a uniform electric field. The potential difference between these two points is:  
 (A)  $8 \text{ V}$       (B)  $200 \text{ V}$       (C)  $800 \text{ V}$       (D)  $20,000 \text{ V}$

The kinetic energy which an electron acquires when accelerated (from rest) through a potential difference of  $1 \text{ volt}$  is called :

- (A)  $1 \text{ joule}$       (B)  $1 \text{ electron volt}$       (C)  $1 \text{ erg}$       (D)  $1 \text{ watt}$

- C-7. The potential difference between points A and B in the given uniform electric field is :



- (A)  $Ea$       (B)  $E\sqrt{(a^2 + b^2)}$       (C)  $Eb$       (D)  $(Eb/\sqrt{2})$

- C-8. An equipotential surface and an electric line of force :

- (A) never intersect each other      (B) intersect at  $45^\circ$   
 (C) intersect at  $60^\circ$       (D) intersect at  $90^\circ$

B-3. The maximum electric field intensity on the axis of a uniformly charged ring of charge  $q$  and radius  $R$  will be :

(A)  $\frac{1}{4\pi\epsilon_0} \frac{q}{3\sqrt{3}R^2}$

(B)  $\frac{1}{4\pi\epsilon_0} \frac{2q}{3R^2}$

(C)  $\frac{1}{4\pi\epsilon_0} \frac{2q}{3\sqrt{3}R^2}$

(D)  $\frac{1}{4\pi\epsilon_0} \frac{3q}{2\sqrt{3}R^2}$

B-4. A charged particle of charge  $q$  and mass  $m$  is released from rest in a uniform electric field  $E$ . Neglecting the effect of gravity, the kinetic energy of the charged particle after time 't' seconds is

(A)  $\frac{Eqm}{t}$

(B)  $\frac{E^2q^2t^2}{2m}$

(C)  $\frac{2E^2t^2}{mq}$

(D)  $\frac{Eq^2m}{2t^2}$

B-5. A flat circular fixed disc has a charge  $+Q$  uniformly distributed on the disc. A charge  $+q$  is thrown with kinetic energy  $K$ , towards the disc along its axis. The charge  $q$  :

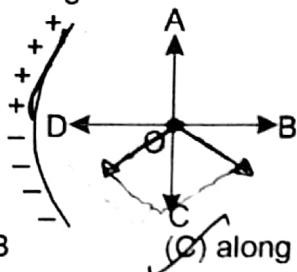
(A) may hit the disc at the centre

(B) may return back along its path after touching the disc

(C) may return back along its path without touching the disc

(D) any of the above three situations is possible depending on the magnitude of  $K$

B-6. The linear charge density on upper half of a segment of ring is  $\lambda$  and at lower half, it is  $-\lambda$ . The direction of electric field at centre O of ring is :



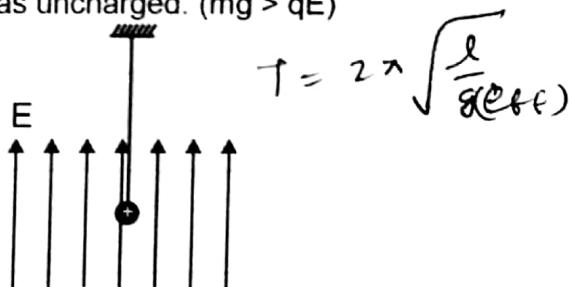
(A) along OA

(B) along OB

(D) along OD

B-7. A positively charged pendulum is oscillating in a uniform electric field as shown in Figure. Its time period of SHM as compared to that when it was uncharged. ( $mg > qE$ )

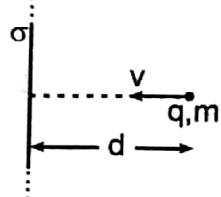
$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$



(A) Will increase  
(C) Will not change

(B) Will decrease  
(D) Will first increase then decrease

B-8. The particle of mass  $m$  and charge  $q$  will touch the infinitely large plate of uniform charge density  $\sigma$  if its velocity  $v$  is more than: {Given that  $\sigma q > 0$ }



(A) 0

(B)  $\sqrt{\frac{2\sigma q d}{m \epsilon_0}}$

(C)  $\sqrt{\frac{\sigma q d}{m \epsilon_0}}$

(D) none of these

B-9. There is a uniform electric field in X-direction. If the work done by external agent in moving a charge of  $0.2 \text{ C}$  through a distance of  $2 \text{ metre}$  slowly along the line making an angle of  $60^\circ$  with X-direction is  $4 \text{ joule}$ , then the magnitude of  $E$  is :

(A)  $\sqrt{3} \text{ N/C}$

(B)  $4 \text{ N/C}$

(C)  $5 \text{ N/C}$

(D)  $20 \text{ N/C}$

## PART - II : ONLY ONE OPTION CORRECT TYPE

### Section (A) : Properties of charge and Coulomb's Law

- A-1 A charged particle  $q_1$  is at position  $(2, -1, 3)$ . The electrostatic force on another charged particle  $q_2$  at  $(0, 0, 0)$  is :

(A)  $\frac{q_1 q_2}{56\pi \epsilon_0} (2\hat{i} - \hat{j} + 3\hat{k})$

(C)  $\frac{q_1 q_2}{56\pi \epsilon_0} (\hat{j} - 2\hat{i} - 3\hat{k})$

$$r = \sqrt{4 + 1 + 9} = \sqrt{14}$$

~~$$\frac{q_1 q_2}{56\sqrt{14}\pi \epsilon_0} (2\hat{i} - \hat{j} + 3\hat{k})$$~~

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

~~(D)  $\frac{q_1 q_2}{56\sqrt{14}\pi \epsilon_0} (\hat{j} - 2\hat{i} - 3\hat{k})$~~

- A-2 Three charges  $+4q$ ,  $Q$  and  $q$  are placed in a straight line of length  $\ell$  at points at distance  $0$ ,  $\ell/2$  and  $\ell$  respectively from one end of line. What should be the value of  $Q$  in order to make the net force on  $q$  to be zero?

(A)  $-q$

(B)  $-2q$

(C)  $-q/2$

(D)  $4q$

- A-3 Two similar very small conducting spheres having charges  $40 \mu C$  and  $-20 \mu C$  are some distance apart. Now they are touched and kept at the same distance. The ratio of the initial to the final force between them is :

(A)  $8 : 1$

(B)  $4 : 1$

(C)  $1 : 8$

(D)  $1 : 1$

- A-4 Two point charges placed at a distance  $r$  in air exert a force  $F$  on each other. The value of distance  $R$  at which they experience force  $4F$  when placed in a medium of dielectric constant  $K = 16$  is :

(A)  $r$

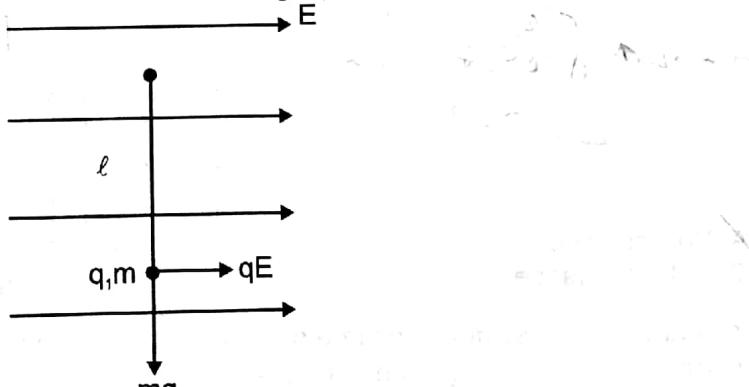
(B)  $r/4$

~~(C)  $r/8$~~

(D)  $2r$

### Section (B) : Electric Field

- B-1 A simple pendulum has a length  $\ell$  & mass of bob  $m$ . The bob is given a charge  $q$  coulomb. The pendulum is suspended in a uniform horizontal electric field of strength  $E$  as shown in figure, then calculate the time period of oscillation when the bob is slightly displaced from its mean position.



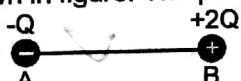
(A)  $2\pi \sqrt{\frac{\ell}{g}}$

(B)  $2\pi \sqrt{\left\{ \frac{\ell}{g + \frac{qE}{m}} \right\}}$

(C)  $2\pi \sqrt{\left\{ \frac{\ell}{g - \frac{qE}{m}} \right\}}$

~~(D)  $2\pi \sqrt{\frac{\ell}{g^2 + \left( \frac{qE}{m} \right)^2}}$~~

- B-2 Charges  $2Q$  and  $-Q$  are placed as shown in figure. The point at which electric field intensity is zero will be:



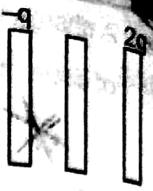
(A) Somewhere between  $-Q$  and  $2Q$

~~(B) Somewhere on the left of  $-Q$~~

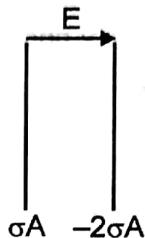
(C) Somewhere on the right of  $2Q$

(D) Somewhere on the perpendicular bisector of line joining  $-Q$  and  $2Q$

- ~~J-2.~~ Three identical metal plates with large equal surface areas are kept parallel to each other as shown in figure. The leftmost plate is given a charge  $-q$ , the rightmost a charge  $2q$  and the middle one remains neutral. Find the charge appearing on the outer surface of the leftmost plate.  $\frac{q}{2}$



- ~~J-3.~~ Two thin conducting plates (very large) parallel to each other carrying total charges  $\sigma A$  and  $-2\sigma A$  respectively (where  $A$  is the area of each plate), are placed in a uniform external electric field  $E$  as shown. Find the surface charge on each surface.



- ~~J-4.~~ Figure shows two conducting spheres separated by large distance and of radius 2cm and 3cm containing charges  $10\mu C$  and  $20\mu C$  respectively. When the spheres are connected by a conducting wire then find out following :

- (i) Ratio of the final charge.
- (ii) Final charge on each sphere.
- (iii) Ratio of final charge densities.  $\frac{1}{2}$
- (iv) Heat produced during the process.

$$\frac{Q_1'}{Q_2'} = \frac{R_1}{R_2} = \frac{2}{3}, Q_1' = \frac{2}{5} \cdot 30 = 12\mu C$$

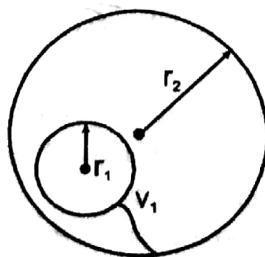
$$Q_2' = \frac{3}{5} \cdot 30 = 18\mu C$$

- ~~J-5.~~ Two concentric hollow conducting spheres of radius  $a$  and  $b$  ( $b > a$ ) contains charges  $Q_a$  and  $Q_b$  respectively. If they are connected by a conducting wire then find out following

- (i) Final charges on inner and outer spheres.  ~~$Q_a + Q_b \rightarrow$  outer, inner~~
- (ii) Heat produced during the process.

- ~~J-6.~~ There are two concentric metal shells of radii  $r_1$  and  $r_2$  ( $r_2 > r_1$ ). If initially, the outer shell has a charge  $Q$  and the inner shell is having zero charge and then inner shell is grounded. Find :
- (i) Charge on the inner surface of outer shell.
  - (ii) Final charges on each sphere.
  - (iii) Charge flown through wire in the ground.

- ~~J-7.~~ A metal sphere of radius  $r_1$  charged to a potential  $V_1$  is then placed in a thin-walled uncharged conducting spherical shell of radius  $r_2$ . Determine the potential acquired by the spherical shell after it has been connected for a short time to the sphere by a conductor.



**STATICS** \*

A charge 'q' is carried slowly from a point A ( $r, 135^\circ$ ) to a point B ( $r, 45^\circ$ ) following a path which is a quadrant of circle of radius 'r'. If the dipole moment is  $\vec{P}$ , then find out the work done by external agent.

$$V_2 = \frac{kP \cos 45^\circ}{r^2} = \frac{kP}{\sqrt{2}r^2}$$

$$W = q \Delta V = \frac{\sqrt{2}kP}{r^2}$$

$$V_1 = \frac{kP \cos \theta}{r^2} = \frac{-kP}{\sqrt{2}r^2}$$

$$F = \frac{1}{4\pi\epsilon_0}$$

H.4.2 Find out the magnitude of electric field intensity and electric potential due to a dipole of dipole moment  $\vec{P} = \hat{i} + \sqrt{3}\hat{j}$  kept at origin at following points.

- (i) (2, 0, 0)      (ii) (-1,  $\sqrt{3}$ , 0)

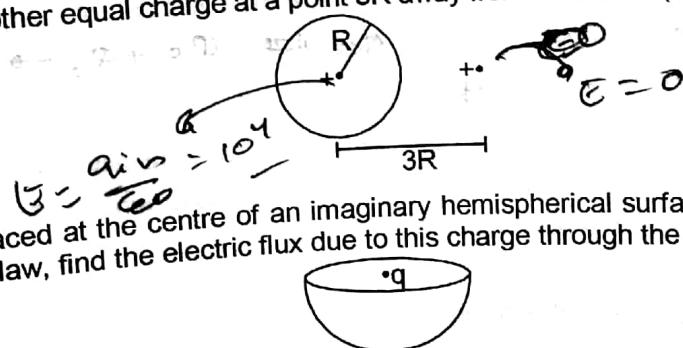
H.5. \* A molecule of a substance has permanent electric dipole moment equal to  $10^{-29}$  C-m. A mole of this substance is polarised (at low temperature) by applying a strong electrostatic field of magnitude ( $10^6 \text{Vm}^{-1}$ ). The direction of the field is suddenly changed by an angle of  $60^\circ$ . Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarisation to the sample.

### Section (I) : Electric lines of force, Flux calculation and Gauss's law

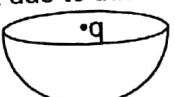
I.1 Find out the electric flux through an area  $10 \text{ m}^2$  lying in XY plane due to an electric field  $\vec{E} = 2\hat{i} - 10\hat{j} + 5\hat{k}$ .

I.2 In a uniform electric field E if we consider an imaginary cubical closed surface of side a, then find the net flux through the cube?

I.3 Find the flux of the electric field through a spherical surface of radius R due to a charge of  $8.85 \times 10^{-8} \text{ C}$  at the centre and another equal charge at a point  $3R$  away from the centre (Given :  $\epsilon_0 = 8.85 \times 10^{-12}$  units)



I.4 A charge q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the electric flux due to this charge through the given surface.

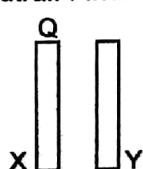


I.5 What do you predict by the given statement about the nature of charge (positive or negative) enclosed by the closed surface. "In a closed surface, lines which are leaving the surface are double than the lines which are entering it".

### Section (J) : Conductor, it's properties & Electric Pressure

J.1. Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure. The plate X is given a charge Q whereas the other is neutral. Find:

(a) The surface charge density at the inner surface of the plate X,  
(b) The electric field at a point to the left of the plates,  
(c) The electric field at a point in between the plates and  
(d) The electric field at a point to the right of the plates.



- F-2.** A spherical shell of radius  $R$  with a uniform charge  $q$  has point charge  $q_0$  at its centre. Find the work performed by the electric forces during the shell expansion slowly from radius  $R$  to  $2R$ . Also find out work done by external agent against electric forces.
- F-3.** Two identical non-conducting spherical shells having equal charge  $Q$ , which is uniformly distributed on it, are placed at a distance  $d$  apart. from where they are released. Find out kinetic energy of each sphere when they are at a large distance.
- F-4.** In a solid uniformly charged sphere of total charge  $Q$  and radius  $R$ , if energy stored outside the sphere is  $U_0$  joules then find out self energy of sphere in term of  $U_0$ ?

### Section (G) : Questions based on relation between $\vec{E}$ and $V$ :

**G-1.** If  $\vec{E} = 2y\hat{i} + 2x\hat{j}$ , then find  $V(x, y, z)$

**G-2.** If  $V = x^2y + y^2z$  then find  $\vec{E}(x, y, z)$

**G-3.** If  $V = 2r^2$  then find out (i)  $\vec{E}(1, 0, -2)$  (ii)  $\vec{E}(r = 2)$

**G-4.** An electric field  $\vec{E} = (10\hat{i} + 20\hat{j}) \text{ N/C}$  exists in the space. If the potential at the origin is taken to be zero, find the potential at  $(3m, 3m)$ .

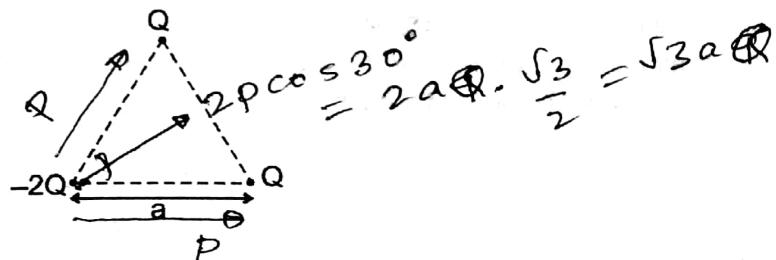
**G-5.** An electric field  $\vec{E} = Bx\hat{i}$  exists in space, where  $B = 20 \text{ V/m}^2$ . Taking the potential at  $(2m, 4m)$  to be zero, find the potential at the origin.

**G-6.** If  $E = 2r^2$ , then find  $V(r)$

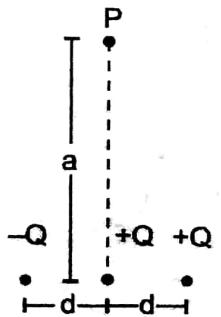
**G-7.** If  $\vec{E} = 2x^2\hat{i} - 3y^2\hat{j}$ , then find  $V(x, y, z)$

### Section (H) : Dipole

**H-1.** Three charges are arranged on the vertices of an equilateral triangle as shown in figure. Find the dipole moment of the combination.



**H-2.** Three point charges  $-Q$ ,  $Q$  and  $Q$  are placed on a straight line with distance  $d$  between charges as shown. Find the magnitude of the electric field at the point  $P$  in the configuration shown which is at a distance  $a$  from middle charge  $Q$  in the system provided that  $a \gg d$ . Take  $2Qd = p$ .



**Two identical charges,  $5\mu\text{C}$  each are fixed at a distance 8 cm and a charged particle of mass  $9 \times 10^{-6} \text{ kg}$  and charge  $-10 \mu\text{C}$  is placed at a distance 5 cm from each of them and is released. Find the speed of the particle when it is nearest to the two charges.**

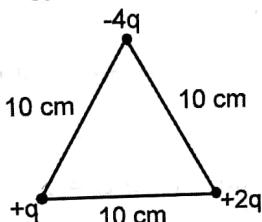
- E4.** A particle of mass  $m$ , charge  $q > 0$  and initial kinetic energy  $K$  is projected from infinity towards a heavy nucleus of charge  $Q$  assumed to have a fixed position.

- If the aim is perfect, how close to the centre of the nucleus is the particle when it comes instantaneously to rest?
- With a particular imperfect aim, the particle's closest approach to nucleus is twice the distance determined in (a). Determine speed of particle at the closest distance of approach.

### **Section (E) : Potential energy of a system of point charges**

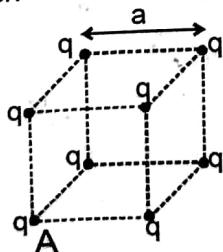
- E1.** Two positive point charges  $15 \mu\text{C}$  and  $10 \mu\text{C}$  are 30 cm apart. Calculate the work done in bringing them closer to each other by 15 cm.

- E2.** Three point charges are arranged at the three vertices of a triangle as shown in Figure. Given:  $q = 10^{-7} \text{ C}$ , calculate the electrostatic potential energy of the system.



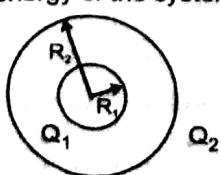
- E3.** Eight equal point charges each of charge ' $q$ ' and mass ' $m$ ' are placed at eight corners of a cube of side ' $a$ '.

- Find out potential energy of charge system
- Find out work done by external agent against electrostatic forces and by electrostatic forces to increase all sides of cube from  $a$  to  $2a$ .
- If all the charges are released at rest, then find out their speed when they are at the corners of cube of side  $2a$ .
- If keeping all other charges fixed, charge of corner 'A' is released then find out its speed when it is at infinite distance?
- If all charges are released simultaneously from rest then find out their speed when they are at a very large distance from each other.



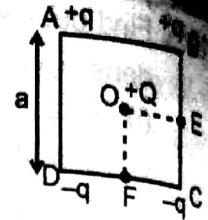
### **Section (F) : Self Energy and energy density**

- F-1.** Two concentric spherical shells of radius  $R_1$  and  $R_2$  ( $R_2 > R_1$ ) are having uniformly distributed charges  $Q_1$  and  $Q_2$  respectively. Find out total energy of the system.



## Electrostatics

- C-7. Four charges  $+q$ ,  $+q$ ,  $-q$ ,  $-q$  are fixed respectively at the corners of A, B, C and D of a square of side 'a' arranged in the given order. Calculate the electric potential of a square of side 'a' arranged in the given order. Calculate the electric potential and intensity at O (Center of square). If E and F are the midpoints of sides BC, CD respectively, what will be the work done by external agent in carrying a charge Q slowly from O to E and from O to F?

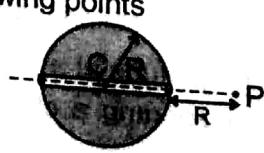


- C-8. A charge Q is distributed over two concentric hollow spheres of radius r and R ( $R > r$ ), such that the surface densities of charge are equal. Find the potential at the common centre.
- C-9. Two concentric hollow spheres of radii R and  $2R$  are charged. The inner sphere has a charge of  $1 \mu\text{C}$  and the outer sphere has a charge of  $2 \mu\text{C}$  of the same sign. The potential is  $9000 \text{ V}$  at a point P at a distance  $3R$  from the common centre O. What is the value of R?
- C-10. In front of a uniformly charged infinite non-conducting sheet of surface charge density  $\sigma$ , a point charge  $q_0$  is shifted slowly from a distance  $a$  to  $b$  ( $b > a$ ). If work done by external agent is  $W$ , then find out relation between the given parameters.
- C-11. An electric field of  $20 \text{ N/C}$  exists along the negative x-axis in space. Calculate the potential difference  $V_B - V_A$ , where the points A and B are given by :  
 (a)  $A = (0, 0)$ ;  $B = (0, 4\text{m})$       (b)  $A = (2\text{m}, 1\text{m})$ ;  $B = (4\text{m}, 3\text{m})$
- \* C-12. A uniform field of  $8 \text{ N/C}$  exists in space in positive x-direction.  
 (a) Taking the potential at the origin to be zero, write an expression for the potential at a general point  $(x, y, z)$ .  
 (b) At which points, the potential is  $160 \text{ V}$ ?  
 (c) If the potential at the origin is taken to be  $80\text{V}$ , what will be the expression for the potential at a general point?  
 (d) What will be the potential at the origin if the potential at infinity is taken to be zero ?
- C-13. A particle of charge  $+3 \times 10^{-9} \text{ C}$  is in a uniform field directed to the left. It is released from rest and moves a distance of  $5 \text{ cm}$ , after which its kinetic energy is found to be  $4.5 \times 10^{-5} \text{ J}$ .  
 (a) What work was done by the electrical force?  
 (b) What is the magnitude of the electrical field?  
 (c) What is the potential of the starting point with respect to the end point?

- C-14. In the previous problem, suppose that another force in addition to the electrical force acts on the particle so that when it is released from rest, it moves to the right. After it has moved  $5 \text{ cm}$ , the additional force has done  $9 \times 10^{-5} \text{ J}$  of work and the particle has  $4.5 \times 10^{-5} \text{ J}$  of kinetic energy.  
 (a) What work was done by the electrical force?  
 (b) What is the magnitude of the electric field?  
 (c) What is the potential of the starting point with respect to the end point?

## Section (D) : Electric potential energy of a point charge

- D-1. An  $\alpha$  particle is placed in an electric field at a point having electric potential  $5\text{V}$ . Find its potential energy?  
 D-2. Find the potential energy of a charge  $q_0$  placed at the centre of regular hexagon of side  $a$ , if charge  $q$  is placed at each vertex of regular hexagon?  
 D-3. A solid uniformly charged fixed non-conducting sphere of total charge  $Q$  and radius  $R$  contains a tunnel of negligible diameter. If a point charge ' $-q$ ' of mass ' $m$ ' is released at rest from point P as shown in figure then find out its velocity at following points



(i) At the surface of sphere

(ii) At the center

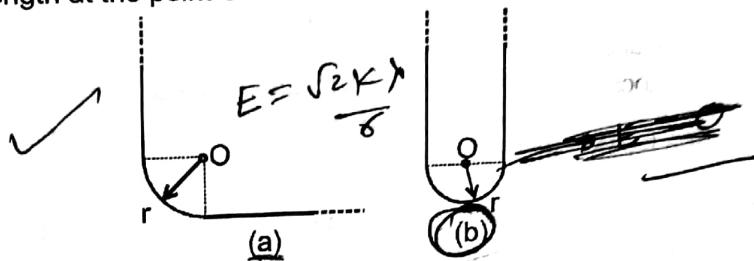
## Electrostatics

B10.1. Find out electric field intensity due to uniformly charged solid non-conducting sphere of volume charge density  $\rho$  and radius  $R$  at following points :

- At a distance  $r$  from surface of sphere (inside)
- At a distance  $r$  from the surface of sphere (outside)

B11. Repeat the question if sphere is a hollow non-conducting sphere of radius  $R$  and has uniform surface charge density  $\sigma$ .

B12. A thread carrying a uniform charge  $\lambda$  per unit length has the configuration shown in figure a and b. Assuming a curvature radius  $r$  to be considerably less than the length of the thread, find the magnitude of the electric field strength at the point O.



### Section (C) : Electric Potential and Potential Difference

C-1. A point charge  $20 \mu\text{C}$  is shifted from infinity to a point P in an electric field with zero acceleration. If the potential of that point is 1000 volt, then

- Find out work done by external agent against electric field?

- What is the work done by electric field?

- If the kinetic energy of charge particle is found to increase by  $10 \text{ mJ}$  when it is brought from infinity to point P, then what is the total work done by external agent?

- What is the work done by electric field in the part (iii)?

- If a point charge  $30 \mu\text{C}$  is released at rest at point P, then find out its kinetic energy at a large distance?

C-2. Two particles A and B having charges of  $4 \times 10^{-6} \text{ C}$  and  $-8 \times 10^{-6} \text{ C}$  respectively, are held fixed at a separation of 60 cm. Locate the point(s) on the line AB where the electric potential is zero.

C-3. Six equal point charges  $q_0$  each are placed at six corners of a regular hexagon of side 'a'. Find out work required to take a point charge 'q' slowly :

- From infinity to the centre of hexagon.

- From infinity to a point on the axis which is at a distance ' $\sqrt{3} a$ ' from the centre of hexagon.

- Does your answer to part (i) and (ii) depends on the path followed by the charge. N o

C-4. 20 J of work has to be done against an existing electric field to take a charge of  $0.05 \text{ C}$  from A to B. How much is the potential difference  $V_B - V_A$  ?

C-5. A charge of  $8 \text{ mC}$  is located at the origin. Calculate the work done by external agent in taking a small charge of  $-2 \times 10^{-9} \text{ C}$  from a point A(0, 0, 0.03 m) to a point B(0, 0.04 m, 0) via a point C(0, 0.06 m, 0.09 m).

C-6. A positive charge  $Q = 50 \mu\text{C}$  is located in the xy plane at a point having position vector  $\vec{r}_0 = (2\hat{i} + 3\hat{j}) \text{ m}$  where  $\hat{i}$  and  $\hat{j}$  are unit vectors in the positive directions of X and Y axis respectively. Find:

where  $\hat{i}$  and  $\hat{j}$  are unit vectors in the positive directions of X and Y axis respectively. Find:

- The electric intensity vector and its magnitude at a point having co-ordinates (8 m, -5 m).

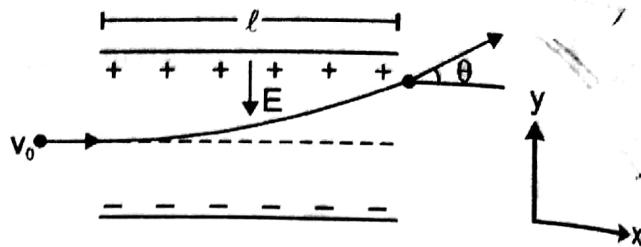
- Work done by external agent in transporting a charge  $q = 10 \mu\text{C}$  from (8 m, 6 m) to the point (4 m, 3 m).

## Electrostatics

### Section (B) : Electric Field

B.1. The electric force experienced by a charge of  $5 \times 10^{-8}$  C is  $25 \times 10^{-3}$  N. Find the magnitude of electric field at that position of the charge due to the source charges.

B.2. A uniform electric field  $E = 91 \times 10^{-6}$  V/m is created between two parallel, charged plates as shown in figure. An electron enters the field symmetrically between the plates with a speed  $v_0 = 4 \times 10^3$  m/s. The length of each plate is  $l = 1$  m. Find the angle of deviation of the path of the electron as it comes out of the field. (Mass of the electron is  $m = 9.1 \times 10^{-31}$  kg and its charge is  $e = -1.6 \times 10^{-19}$  C).



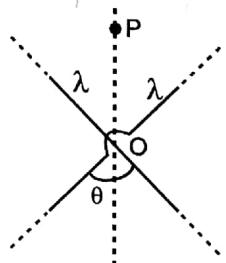
B.3. Two point particles A and B having charges of  $4 \times 10^{-6}$  C and  $-64 \times 10^{-6}$  C respectively are held at a separation of 90 cm. Locate the point(s) on the line AB or on its extension where the electric field is zero.

B.4. Three point charges  $q_0$  are placed at three corners of square of side a. Find out electric field intensity at the fourth corner.

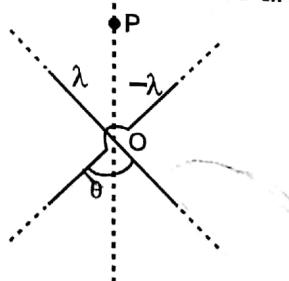
B.5. Two point charges  $3\mu\text{C}$  and  $2.5\mu\text{C}$  are placed at point A (1, 1, 2)m and B (0, 3, -1)m respectively. Find out electric field intensity at point C(3, 3, 3)m.

B.6. A hollow sphere of radius a carries a total charge Q distributed uniformly over its surface. A small area  $dA$  of the sphere is cut off. Find the electric field at the centre due to the remaining sphere.

B.7. (i) Two infinitely long line charges each of linear charge density  $\lambda$  are placed at an angle  $\theta$  as shown in figure. Find out electric field intensity at a point P, which is at a distance x from point O along angle bisector of line charges.



(ii) Repeat the above question if the line charge densities are  $\lambda$  and  $-\lambda$ . as shown in figure.

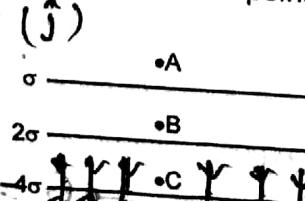


B.8. The bob of a simple pendulum has a mass of 60 g and a positive charge of  $6 \times 10^{-6}$  C. It makes 30 oscillations in 50 s above earth's surface. A vertical electric field pointing upward and of magnitude  $5 \times 10^4$  N/C is switched on. How much time will it now take to complete 60 oscillations? ( $g = 10 \text{ m/s}^2$ )

B.9. If three infinite charged sheets of uniform surface charge densities  $\sigma$ ,  $2\sigma$  and  $-4\sigma$  are placed as shown in figure, then find out electric field intensities at points A, B, C and D.

$$A: \frac{\sigma + 2\sigma - 4\sigma}{2\epsilon_0} = \frac{-\sigma}{2\epsilon_0} \quad (\uparrow)$$

$$B: \frac{2\sigma - 4\sigma - \sigma}{2\epsilon_0} = \frac{-3\sigma}{2\epsilon_0} \quad (\uparrow)$$



$$C: \frac{-4\sigma - 2\sigma - \sigma}{2\epsilon_0} = \frac{-7\sigma}{2\epsilon_0} \quad (\uparrow)$$

$$D: 4\sigma - \dots$$

**Exercise-1**

**Marked Questions can be used as Revision Questions.**

**PART - I : SUBJECTIVE QUESTIONS****Section (A) : Properties of charge and Coulomb's Law**

A-1 Two point charges  $q_1 = 2 \times 10^{-3}$  C and  $q_2 = -3 \times 10^{-6}$  C are separated by a distance  $x = 10\text{cm}$ . Find the magnitude and nature of the force between the two charges.

A-2 Two point charges  $q_1 = 20\mu\text{C}$  and  $q_2 = 25\mu\text{C}$  are placed at  $(-1, 1, 1)$  m and  $(3, 1, -2)$  m, with respect to a coordinate system. Find the magnitude and unit vector along electrostatic force on  $q_2$ ?

A-3 20 positively charged particles are kept fixed on the X-axis at points  $x = 1\text{ m}, 2\text{ m}, 3\text{ m}, \dots, 20\text{ m}$ . The first particle has a charge  $1.0 \times 10^{-6}$  C, the second  $8 \times 10^{-6}$  C, the third  $27 \times 10^{-6}$  C and so on. Find the magnitude of the electric force acting on a 1 C charge placed at the origin.  $1.89 \times 10^9$  N

- A-4 (i) Two charged particles having charge  $4.0 \times 10^{-6}$  C and mass  $24 \times 10^{-3}$  Kg each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.  
(ii) If suddenly string is cut then what is the acceleration of each particle?  
(iii) Are they having equal acceleration? *+ equal only in magnitudes but not in direction*

A-5 Two identical conducting spheres (of negligible radius), having charges of opposite sign, attract each other with a force of 0.108 N when separated by 0.5 meter. The spheres are connected by a conducting wire, which is then removed (when charge stops flowing), and thereafter repel each other with a force of 0.036 N keeping the distance same. What were the initial charges on the spheres?

A-6 Two small spheres, each of mass 0.1 gm and carrying same charge  $10^{-9}$  C are suspended by threads of equal length from the same point. If the distance between the centres of the sphere is 3 cm, then find out the angle made by the thread with the vertical. ( $g = 10 \text{ m/s}^2$ ) &  $\tan^{-1}\left(\frac{1}{100}\right) = 0.6^\circ$

A-7 The distance between two fixed positive charges  $4e$  and  $e$  is  $\ell$ . How should a third charge 'q' be arranged for it to be in equilibrium? Under what condition will equilibrium of the charge 'q' be stable (for displacement on the line joining  $4e$  and  $e$ ) or will it be unstable?

\* A-8 Three charges, each of value  $q$ , are placed at the corners of an equilateral triangle. A fourth charge  $Q$  is placed at the centre O of the triangle.

- (a) If  $Q = -q$ , will the charges at corners start to move towards centre or away from it.  
(b) For what value of  $Q$  at O will the charges remain stationary?

A-9 Two charged particles A and B, each having a charge  $Q$  are placed a distance  $d$  apart. Where should a third particle of charge  $q$  be placed on the perpendicular bisector of AB so that it experiences maximum force? Also find the magnitude of the maximum force.

## Electrostatics

**Solution :** Plates are conducting so net electric field inside these plates should be zero. So, electric field due to induced charges (on the surface of the plate) balance the outside electric field.

Here  $\vec{E}_i$  = induced electric field

For both plates,  $\vec{E}_i + \vec{E} = 0$

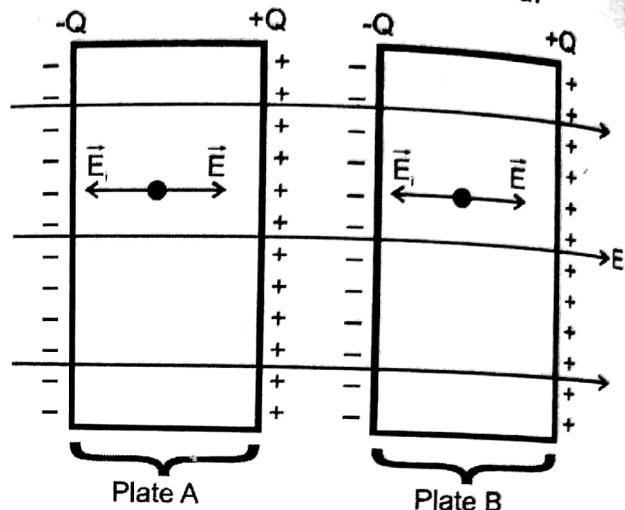
$$\Rightarrow \vec{E}_i = -\vec{E} \quad \dots\dots(1)$$

Let charge induced on surfaces are  $+Q$  and  $-Q$ , then

$$|\vec{E}_i| = \frac{Q}{A\epsilon_0}$$

By equation (1)

$$\frac{Q}{A\epsilon_0} = E \Rightarrow Q = AE\epsilon_0 \text{ Ans.}$$



### Problem 22.

A positive charge  $q$  is placed in front of a conducting solid cube at a distance  $d$  from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

**Solution :**

Here  $\vec{E}_i$  =  $\vec{E}$  electric field due to induced charges and

$E_q$  = electric field due to charge  $q$

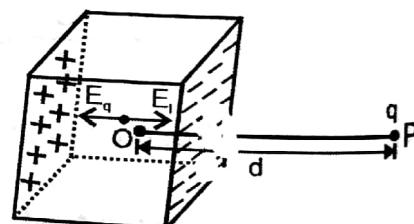
We know that net electric field in a conducting cavity is equal to zero.

i.e.  $\vec{E} = \vec{0}$  at the centre of the cube.

$$\Rightarrow \vec{E}_i + \vec{E}_q = \vec{0}$$

$$\vec{E}_i = -\vec{E}_q$$

$$\vec{E}_i = -\frac{kq}{d^2} \vec{PO} \text{ Ans.}$$



**Problem 16.** If  $E = 2r^2$  then find  $V(r)$

**Solution :** Given :  $E = 2r^2$

$$\text{we know that : } \int dv = - \int \vec{E} \cdot d\vec{r} = - \int 2r^2 dr \Rightarrow V(r) = \frac{-2r^3}{3} + c \quad \text{Ans.}$$

**Problem 17.** A charge  $Q$  is uniformly distributed over a rod of length  $\ell$ . Consider a hypothetical cube of edge  $\ell$  with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

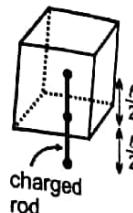
**Solution :** According to Gauss law : Flux depends upon charge inside the closed hypothetical surface. So, for minimum possible flux through the entire surface of the cube, charge inside it should be minimum.

$$\text{Linear charge density of rod} = \frac{Q}{\ell}$$

$$\text{and minimum length of rod inside the cube} = \frac{\ell}{2}$$

$$\text{So, charge inside the cube} = \frac{\ell}{2} \cdot \frac{Q}{\ell} = \frac{Q}{2}$$

$$\text{So, flux through the entire surface of the cube} = \frac{\Sigma q}{\epsilon_0} = \frac{Q}{2\epsilon_0}$$

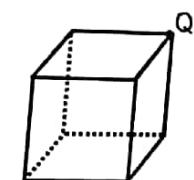


**Problem 18.** A charge  $Q$  is placed at a corner of a cube. Find the flux of the electric field through the six surfaces of the cube.

**Solution :** By Gauss's law,  $\phi = \frac{q_{in}}{\epsilon_0}$ . Here, since  $Q$  is kept at the corner, so only  $\frac{Q}{8}$  charge is inside the cube. (Since, complete charge can be enclosed by 8 such cubes)

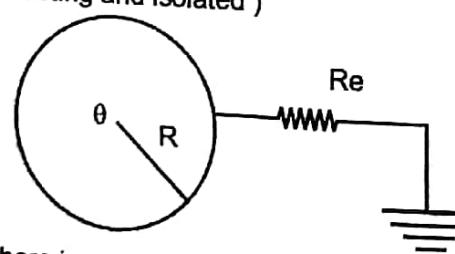
$$\therefore q_{in} = \frac{Q}{8}$$

$$\text{So, } \phi = \frac{q_{in}}{\epsilon_0} = \frac{Q}{8\epsilon_0} \quad \text{Ans.}$$



**Problem 19.** An isolated conducting sphere of charge  $Q$  and radius  $R$  is grounded by using a high resistance wire. What is the amount of heat loss ?

**Solution :** When sphere is grounded, its potential become zero which means all charge goes to earth (since sphere is conducting and isolated )



So, all energy in sphere is converted into heat

$$\text{So, total heat loss} = \frac{kQ^2}{2R}$$

### Electrostatics

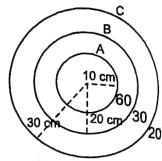
**Solution :** We know, that the electric field is always perpendicular to equipotential surface. So, making electric field lines perpendicular to the surface, we find that these lines are originating from the centre. So, the field is similar to that due to a point charge placed at the centre. So, comparing the given potentials with that due to point charge, we have,

$$V = \frac{kQ}{r} \Rightarrow kQ = V_A r_A = V_B r_B = V_C r_C = 6 \text{ V-m}$$

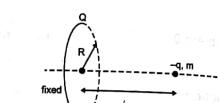
Hence, electric field at distance  $r$  can be given by

$$E = \frac{kQ}{r^2} = \frac{6}{r^2} \text{ V/m}$$

As the electric field lines are directed towards the decreasing potential. So, electric field is along radially outward direction.



**Problem 12.** A point charge of charge  $-q$  and mass  $m$  is released with negligible speed from a distance  $\sqrt{3}R$  on the axis of a fixed uniformly charged ring of charge  $Q$  and radius  $R$ . Find out its velocity when it reaches at the centre of the ring.



**Solution :** As potential due to uniformly charged ring at its axis (at  $x$  distance) is :

$$V = \frac{kQ}{\sqrt{R^2 + x^2}}$$

So, potential at point A due to ring

$$V_1 = \frac{kQ}{\sqrt{R^2 + 3R^2}} = \frac{kQ}{2R}$$

So potential energy of charge  $-q$  at point A

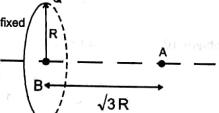
$$P.E._1 = \frac{-kQq}{2R} \text{ and potential at point B, } V_2 = \frac{kQ}{R}$$

So, potential energy of charge  $-q$  at point B :  $P.E._2 = \frac{-kQq}{R}$

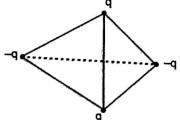
Now by energy conservation :  $P.E._1 + K.E._1 = P.E._2 + K.E._2$

$$\frac{-kQq}{2R} + 0 = \frac{-kQq}{R} + \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{kQq}{mR}$$

So velocity of charge  $-q$  at point B  $v = \sqrt{\frac{kQq}{mR}}$  Ans.



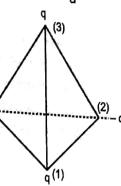
**Problem 13.** Four small point charges (each of equal magnitude  $q$ ) are placed at four corners of a regular tetrahedron of side  $a$ . Find out potential energy of charge system



### Electrostatics

**Solution :** Potential energy of system :  $U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$\therefore U = \frac{-kq^2}{a} + \frac{kq^2}{a} + \frac{-kq^2}{a} + \frac{-kq^2}{a} + \frac{kq^2}{a} + \frac{-kq^2}{a}$$



$$\text{Total potential energy of this charge system } U = \frac{-2kq^2}{a}$$

**Problem 14.** If  $V = x^2y + y^2z$  then find  $\vec{E}$  at  $(x, y, z)$

**Solution :** Given  $V = x^2y + y^2z$  and  $\vec{E} = -\frac{\partial V}{\partial r}$

$$\vec{E} = -\left[ \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right] \Rightarrow \vec{E} = -[2xy \hat{i} + (x^2+2yz) \hat{j} + y^2 \hat{k}]$$

**Problem 15.** Magnitude of electric field depends only on the  $x$ -coordinate as  $\vec{E} = \frac{20}{x^2} \hat{x}$  V/m. Find

(i) The potential difference between two points A(5m, 0) and B(10m, 0).

(ii) Potential at  $x = 5$  if  $V$  at  $\infty$  is 10 volt.

(iii) In part (i), does the potential difference between A and B depend on whether the potential at  $\infty$  is 10 volt or something else.

**Solution :** Given,  $\vec{E} = \frac{20}{x^2} \hat{x}$  V/m

We know that :  $\int dV = -\int \vec{E} \cdot d\vec{r}$

$$\int_{V_1}^{V_2} dV = - \int_{x_1}^{x_2} E_x dx = - \int_{x_1}^{x_2} \frac{20}{x^2} dx$$

$$\therefore \text{Potential difference, } \Delta V = \frac{20}{x} \Big|_{x_1}^{x_2} \Rightarrow V_2 - V_1 = \frac{20}{x_2} - \frac{20}{x_1}$$

(i) Potential difference between point A and B ( $\Delta V$  for A to B)

$$V_B - V_A = \frac{20}{10} - \frac{20}{5} = -2 \text{ volt}$$

(ii)  $\Delta V$  for  $x = \infty$  to  $x = 5$

$$V_5 - V_\infty = \frac{20}{5} - \frac{20}{\infty}$$

$$\therefore V_5 = 10 + 4 = 14 \text{ volt}$$

(iii) Potential difference between two points does not depend on reference value of potential. So, the potential difference between A and B does not depend on whether the potential at  $\infty$  is 10 volt or something else.

### Electrostatics

**Solution :** Net electric field =  $E_1 + E_2$

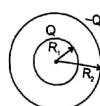
$E_1$  = field due to sphere of radius  $R_1$

$E_2$  = field due to sphere of radius  $R_2$

$$(i) E_1 = 0, E_2 = 0 \quad \therefore E_{\text{net}} = 0$$

$$(ii) E_1 = \frac{KQ}{r^2}, E_2 = 0 \quad \Rightarrow \vec{E} = \frac{Kq}{r^2} \hat{r}$$

$$(iii) \vec{E}_1 = \frac{Kq}{r^2} \hat{r}, \vec{E}_2 = \frac{Kq}{r^2} (-\hat{r}) \quad \Rightarrow \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = 0$$



### Problem 7.

A solid non conducting sphere of radius  $R$  and uniform volume charge density  $\rho$  has central origin. Find out electric field intensity in vector form at following positions :

- (i)  $(R, 0, 0)$       (ii)  $(0, R/2)$       (iii)  $(R, R, R)$

**Solution :** For uniformly charged non-conducting sphere, electric field inside the sphere :

$$\vec{E} = k \frac{Q\vec{r}}{R^3} = \frac{\rho\vec{r}}{3\epsilon_0} \quad (\text{for } r < R)$$

and electric field outside the sphere

$$\vec{E}_o = \frac{KQ}{r^2} \cdot \hat{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\rho \frac{4}{3}\pi R^3}{r^2} \hat{r} = \frac{\rho R^3}{3\epsilon_0 r^2} \cdot \hat{r} \quad (\text{for } r \geq R)$$

(i)  $(R, 0, 0)$  means it is at the surface  $\vec{r} = R\hat{r}$  and  $\hat{r} = \hat{i}$

$$\therefore \vec{E}_o = \frac{\rho R^3}{3\epsilon_0 R^2} (\hat{i}) = \frac{\rho R}{3\epsilon_0} \hat{i}$$

$$(\text{ii}) (0, 0, \frac{R}{2})$$

means point is inside the sphere

$$\vec{r} = \frac{R}{2} \hat{k} \quad \Rightarrow \quad \vec{E} = \frac{\rho R}{6\epsilon_0} \hat{k}$$

$$(\text{iii}) \text{ For position } (R, R, R)$$

$$\vec{r} = R(\hat{i} + \hat{j} + \hat{k}) \Rightarrow \vec{r} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}, r = R\sqrt{3}$$

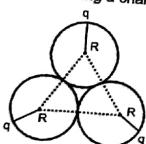
means point  $(R, R, R)$  is outside the sphere

$$\therefore \vec{E} = \frac{\rho R^3}{3\epsilon_0(3R^2)} \cdot \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} = \frac{\rho R}{9\sqrt{3}\epsilon_0} (\hat{i} + \hat{j} + \hat{k}) \quad \text{Ans.}$$

### Problem 8.

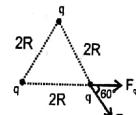
Three identical spheres each having a charge  $q$  (uniformly distributed) and radius  $R$ , are kept in such a way that each touches the other two. Find the magnitude of the electric force on any one sphere due to other two.

**Solution :** Given three identical spheres each having a charge  $q$  and radius  $R$  are kept as shown.



For any external point, sphere behaves like a point charge. So it becomes a triangle having point charges at its corners.

$$|\vec{F}_{qq}| = \frac{kq^2}{4R^2}$$



$$\text{So, net force } (F) = 2 \cdot \frac{kq^2}{4R^2} \cdot \cos \frac{60^\circ}{2} = 2 \cdot \frac{kq^2}{4R^2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} \cdot \frac{kq^2}{R^2} \quad \text{Ans.}$$

**Problem 9.** A uniform electric field of 20 N/C exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 40cm.

$$\vec{E} = -\frac{dv}{dr} \Rightarrow dv = -\vec{E} \cdot d\vec{r}$$

$$\text{for } \vec{E} = \text{constant} \Rightarrow \Delta V = -\vec{E} \cdot \vec{dr}$$

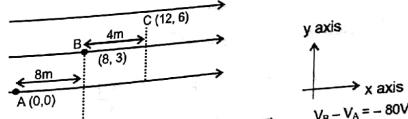
$$\Delta V = -20(-\hat{j}) \cdot (40 \times 10^{-2})\hat{j} = 8 \text{ volts.}$$

**Problem 10.** An electric field of 10 N/C exists along the x-axis in space. Calculate the potential difference  $V_B - V_A$ , where the points A and B are given by –

$$(a) A = (0, 0); B = (8m, 3m) \quad (b) A = (8m, 3m); B = (12m, 6m)$$

$$(c) A = (0, 0); B = (12m, 6m)$$

**Solution :** Electric field in x-axis means  $\vec{E} = 10\hat{i}$

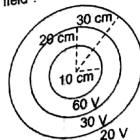


$$(a) |\Delta V_{AB}| = \vec{E} \cdot \vec{d} = 10\hat{i} \cdot 8\hat{i} = 80 \text{ V} \Rightarrow V_B - V_A = -80 \text{ V}$$

$$(b) |\Delta V_{BC}| = \vec{E} \cdot \vec{d} = 10\hat{i} \cdot 4\hat{i} = 40 \text{ volt} \Rightarrow V_C - V_B = -40 \text{ V}$$

$$(c) |\Delta V_{AC}| = \vec{E} \cdot \vec{d} = 10\hat{i} \cdot 12\hat{i} = 120 \text{ volt} \Rightarrow V_C - V_A = -120 \text{ V}$$

**Problem 11.** Some equi-potential surfaces are shown in figure. What can you say about the magnitude and direction of the electric field ?



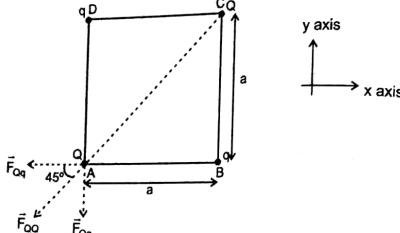
### Electrostatics

**Problem 3.** Two charges  $Q$  each, are placed at two opposite corners of a square. A charge  $q$  is placed at each of the other two corners.

- (a) If the resultant force on  $Q$  is zero, how are  $Q$  and  $q$  related?
- (b) Could  $q$  be chosen to make the resultant force on each charge zero?

**Solution :**

- (a) Let on a square ABCD, charges are placed as shown



Now, forces on charge  $Q$  (at point A) due to other charge are  $\vec{F}_{\infty}$ ,  $\vec{F}_{qq}$  and  $\vec{F}_{qQ}$  respectively as shown in figure.

$$F_{\text{net}} \text{ on } Q = \vec{F}_{\infty} + \vec{F}_{qq} + \vec{F}_{qQ} \quad (\text{at point A})$$

But  $F_{\text{net}} = 0$

So,  $\Sigma F_x = 0$

$$\Sigma F_x = -F_{\infty} \cos 45^\circ - F_{qQ}$$

$$\Rightarrow \frac{KQ^2}{(\sqrt{2}a)^2} \cdot \frac{1}{\sqrt{2}} + \frac{KQq}{a^2} = 0 \Rightarrow q = -\frac{Q}{2\sqrt{2}} \quad \text{Ans.}$$

- (b) For resultant force on each charge to be zero :

From previous data, force on charge  $Q$  is zero when  $q = -\frac{Q}{2\sqrt{2}}$ . If for this value of charge  $q$ , force on  $q$  is zero, then and only then the value of  $q$  exists for which the resultant force on each charge is zero.

#### Force on $q$ :

Forces on charge  $q$  (at point D) due to other three charges are  $\vec{F}_{qq}$ ,  $\vec{F}_{qQ}$  and  $\vec{F}_{\infty}$  respectively as shown in figure.

Net force on charge  $q$  :

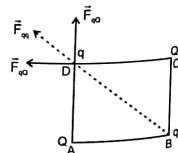
$$\vec{F}_{\text{net}} = \vec{F}_{qq} + \vec{F}_{qQ} + \vec{F}_{\infty} \quad \text{But } \vec{F}_{\text{net}} = 0$$

So,  $\Sigma F_x = 0$

$$\Sigma F_x = -\frac{Kq^2}{(\sqrt{2}a)^2} \cdot \frac{1}{\sqrt{2}} - \frac{KQq}{a^2} \Rightarrow q = 2\sqrt{2} - Q$$

But from previous condition,  $q = -\frac{Q}{2\sqrt{2}}$

So, no value of  $q$  makes the resultant force on each charge zero.



### Electrostatics

**problem 4.**

An infinitely large non-conducting sheet of thickness  $t$  and uniform volume charge density  $\rho$  is given in which left half of the sheet contains charge density  $\rho$  and right half contains charge density  $2\rho$ . Find the electric field at the symmetry plane of this sheet.

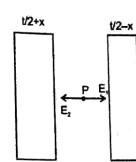
We can consider two sheets of thickness  $\left(\frac{t}{2}-x\right)$  and  $\left(\frac{t}{2}+x\right)$ .

When a point lies inside the sheet,

$$\text{Net electric field at point } P : E = E_1 - E_2 = \frac{Q_1}{2A\varepsilon_0} - \frac{Q_2}{2A\varepsilon_0}$$

$$[Q_1 : \text{charge on left sheet}; Q_2 = \text{charge of right sheet}] \\ = \frac{A\rho\left(\frac{t}{2}+x\right) - 2\rho A\left(\frac{t}{2}-x\right)}{2A\varepsilon_0} = \frac{\rho\left[3x - \frac{t}{2}\right]}{2\varepsilon_0}$$

$$\text{At the symmetry plane, } x = 0 \quad \text{So, } E = -\frac{\rho t}{4\varepsilon_0} \quad \text{Ans.}$$



**Problem 5.**

Figure shows a uniformly charged thin non-conducting sphere of total charge  $Q$  and radius  $R$ . If point charge  $q$  is situated at point 'A' which is at a distance  $r < R$  from the centre of the sphere, then find out following:

- (i) Force acting on charge  $q$ .
- (ii) Electric field at centre of sphere.
- (iii) Electric field at point B.

**Solution :**

- (i) Electric field inside a hollow sphere = 0

$\therefore$  Force on charge  $q$ .

$$F = qE = q \times 0 = 0$$

- (ii) Net electric field at centre of sphere

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2$$

$E_1$  = field due to sphere = 0

$E_2$  = field due to this charge =  $\frac{Kq}{r^2}$

$$\therefore E_{\text{net}} = \frac{Kq}{r^2}$$

(iii) Electric field at B due to charge on sphere,  $\vec{E}_1 = \frac{KQ}{r_1^2} \vec{r}_1$  and due to charge  $q$  at A,  $\vec{E}_2 = \frac{Kq}{r_2^2} \vec{r}_2$

$$\text{So, } \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = \frac{KQ}{r_1^2} \vec{r}_1 + \frac{Kq}{r_2^2} \vec{r}_2 \text{ where } r_1 = CB \text{ and } r_2 = AB$$

**Problem 6.**

Figure shows two concentric spheres of radii  $R_1$  and  $R_2$  ( $R_2 > R_1$ ) which contain uniformly distributed charges  $Q$  and  $-Q$  respectively. Find out electric field intensities at the following positions :



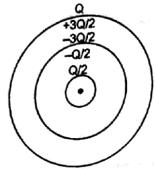
(iii)  $r \geq R_2$

(i)  $r < R_1$

(ii)  $R_1 \leq r < R_2$

### Electrostatics

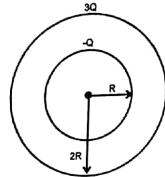
$$\therefore \text{Charge on innermost shell} = \frac{Q}{2} \text{ & Charge on outermost shell} = \frac{5Q}{2}$$



Charge on middle shell =  $-2Q$

Final charge distribution is as shown in figure.

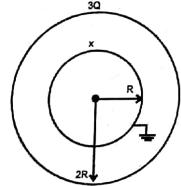
**Example 120.** Two conducting hollow spherical shells of radii  $R$  and  $2R$  carry charges  $-Q$  and  $3Q$  respectively. How much charge will flow into the earth if inner shell is grounded?



**Solution :** When inner shell is grounded to the Earth then the potential of inner shell will become zero because potential of the Earth is taken to be zero.

$$\frac{kx}{R} + \frac{k3Q}{2R} = 0$$

or  $x = \frac{-3Q}{2}$ , (the charge that has appeared on inner shell after grounding)



$$\Rightarrow \frac{-3Q}{2} - (-Q) = \frac{-Q}{2} \quad [\text{hence, charge flown into the Earth} = \frac{Q}{2}]$$

**Example 121.** An isolated conducting sphere of charge  $Q$  and radius  $R$  is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time, what is the amount of heat loss?

**Solution :** When two conducting spheres of equal radii are connected, charge is equally distributed on them (Result VI). So, we can say that heat loss of system :

$$\Delta H = U_i - U_f = \left( \frac{Q^2}{8\pi\epsilon_0 R} - 0 \right) - \left( \frac{Q^2/4}{8\pi\epsilon_0 R} + \frac{Q^2/4}{8\pi\epsilon_0 R} \right) = \frac{Q^2}{16\pi\epsilon_0 R}$$

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### Electrostatics

### Solved Miscellaneous Problems

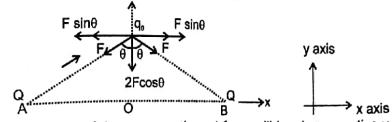
#### Problem 1.

Two equal positive point charges 'Q' each are fixed at points B( $a, 0$ ) and A( $-a, 0$ ). Another negative point charge  $q_0$  is also placed at O( $0, 0$ ) then prove that the equilibrium at 'O' is

- (i) Stable for displacement in Y-direction.
- (ii) Unstable for displacement in X-direction.

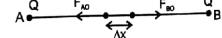
- (iii) When charge is shifted along y-axis:

Let x-y direction as :-



After resolving into components, net force will be along negative y-axis so the particle will return to its original position. So, it is stable equilibrium

- (iv) When negative charge  $q_0$  is shifted along x-axis.



$$\text{Initially, } \vec{F}_{AO} + \vec{F}_{BO} = \vec{0} \Rightarrow |\vec{F}_{AO}| = |\vec{F}_{BO}| = \frac{KQq_0}{a^2}$$

When charge  $q_0$  is slightly shifted towards +x axis by small distance  $\Delta x$  then  $|\vec{F}_{BO}| > |\vec{F}_{AO}|$

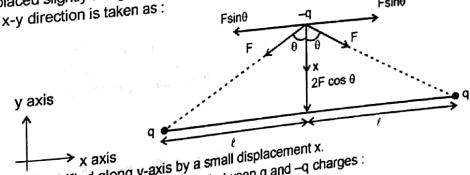
Also, these forces are attractive forces (due to negative charge)  
Therefore, the particle will move towards positive x-axis and will not return to its original position So, it is unstable equilibrium for negative charge.

#### Problem 2.

A particle of mass  $m$  and charge  $-q$  is located midway between two fixed charged particles each having a charge  $q$  and a distance  $2r$  apart. Prove that the motion of the particle will be SHM if it is displaced slightly along perpendicular bisector and released. Also find its time period.

#### Solution :

Let x-y direction is taken as :



Particle is shifted along y-axis by a small displacement  $x$ .

After resolving component of forces between  $q$  and  $-q$  charges:  
By figure,  $F_{net}$  in x-axis = 0 [  $F_{net}$  = net force on  $-q$  charge]

Net force on  $-q$  charge in y direction =  $-2F \cos \theta = -2 \cdot \frac{kqq}{(x^2 + r^2)^{1/2}} \cdot \frac{x}{(x^2 + r^2)^{1/2}}$

$$|\vec{F}| = \frac{2Kq^2x}{(x^2 + r^2)^{3/2}} \Rightarrow ma = \frac{2Kq^2x}{r^3} \quad (\text{for } x \ll r) \quad (a = \text{acceleration of } -q \text{ charge})$$

$$\Rightarrow a = \frac{2Kq^2}{mr^3} \cdot x \quad (\text{downwards})$$

This is equation of S.H.M. ( $a = -\omega^2 x$ )

So, time period of this charge  $(-q)$  :

$$T = 2\pi \sqrt{\frac{m}{2Kq^2}}$$

Ans.

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### Electrostatics

Note : Electric field at 'A' due to  $-q$  of  $S_1$  and  $+q$  of  $S_2$  is zero individually because they are uniformly distributed

$$\text{At point B : } V_B = \frac{Kq}{OB} + \frac{K(-q)}{OB} + \frac{Kq}{R_2} = \frac{Kq}{R_2}, E_B = 0$$

$$\text{At point C : } V_C = \frac{Kq}{OC}, \vec{E}_C = \frac{Kq}{OC^3} \vec{OC}$$

(ii) Force on point charge Q :

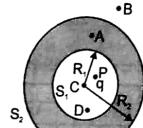
(Note : Here, force on 'Q' will be only due to 'q' of  $S_2$  (see result (iii))

$$\vec{F}_Q = \frac{KqQ}{r^2} \vec{r} \quad (r = \text{distance of 'Q' from centre 'O'})$$

Force on point charge q :

$$\vec{F}_q = 0 \quad (\text{using result (iii) & charge on } S_1 \text{ uniform})$$

**Example 117.** An uncharged conductor of inner radius  $R_1$  and outer radius  $R_2$  contains a point charge  $q$  placed at point P (not at the centre) as shown in figure. Find out the following :



(i)  $V_C$

(v)  $E_B$

(ii)  $V_A$

(vi) Force on charge Q, if it is placed at B.

(iii)  $V_B$

(iv)  $E_A$

**Solution :** (i)  $V_C = \frac{Kq}{CP} + \frac{K(-q)}{R_1} + \frac{Kq}{R_2}$

Note :  $-q$  on  $S_1$  is non-uniformly distributed. Still it produces potential  $\frac{K(-q)}{R_1}$  at 'C' because 'C' is at distance  $R_1$  from each point of ' $S_1$ '.

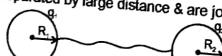
(ii)  $V_A = \frac{Kq}{R_2}$

(iv)  $E_A = 0$  (point is inside metallic conductor)

(v)  $E_B = \frac{Kq}{CB^2}$

(vi)  $F_Q = \frac{KQq}{CB^2}$

(vi) **Sharing of charges :** Two conducting hollow spherical shells of radii  $R_1$  and  $R_2$  having charges  $Q_1$  and  $Q_2$  respectively and separated by large distance & are joined by a conducting wire



Let final charges on spheres are  $q_1$  and  $q_2$  respectively. Potential on both spherical shell becomes equal after joining. Therefore,

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2} \quad \dots \text{(i)}$$

$$\text{and, } q_1 + q_2 = Q_1 + Q_2 \quad \dots \text{(ii)}$$

$$\text{from (i) and (ii) : } q_1 = \frac{(Q_1 + Q_2)R_1}{R_1 + R_2} \quad \dots \text{(iii)}$$

$$q_2 = \frac{(Q_1 + Q_2)R_2}{R_1 + R_2}$$

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### Electrostatics

$$\text{ratio of charges : } \frac{q_1}{q_2} = \frac{R_1}{R_2} \Rightarrow \frac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \frac{R_1}{R_2}$$

$$\therefore \text{ratio of surface charge densities : } \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

$$\text{Ratio of final charges : } \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\text{Ratio of final surface charge densities : } \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

### Solved Example

**Example 118.** The two conducting spherical shells are joined by a conducting wire which is cut after some time when charge stops flowing. Find out the charge on each sphere after that.



**Solution :**

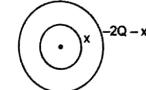
After cutting the wire, the potential of both the shells is equal

$$\text{Thus, potential of inner shell, } V_{in} = \frac{Kx + K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$$

$$\text{and potential of outer shell, } V_{out} = \frac{Kx + K(-2Q-x)}{2R} = \frac{-KQ}{R}$$

$$\text{As, } V_{out} = V_{in} \Rightarrow \frac{-KQ}{R} = \frac{K(x-2Q)}{2R} \Rightarrow -2Q = x - 2Q \Rightarrow x = 0$$

So, charge on inner spherical shell = 0 and outer spherical shell =  $-2Q$ .



**Example 119.** Find charge on each spherical shell after joining the inner most shell and outer most shell by a conducting wire. Also find charges on each surface.

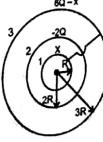


**Solution :**

Let the charge on the innermost sphere be  $x$ .

Finally potential of shell 3 = Potential of shell 3

$$\therefore \frac{Kx}{R} + \frac{K(-2Q)}{2R} + \frac{K(6Q-x)}{3R} = \frac{Kx}{3R} + \frac{K(-2Q)}{3R} + \frac{K(6Q-x)}{3R}$$



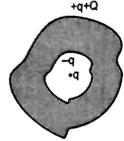
$$3x - 3Q + 6Q - x = 4Q; 2x = Q; x = \frac{Q}{2}$$

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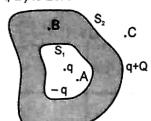
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### Electrostatics

(ii) If a charge  $q$  is kept inside the cavity of a conductor and conductor is given a charge  $Q$ , then  $-q$  charge will be induced on inner surface and total charge on the outer surface will be  $q + Q$ . (it can be proved using Gauss theorem)



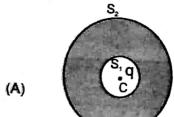
(iii) Resultant field, due to  $q$  (which is inside the cavity) and induced charge on  $S_1$ , at any point outside  $S_1$  (like B, C) is zero. Resultant field due to  $q + Q$  on  $S_2$  and any other charge outside  $S_2$ , at any point inside of surface  $S_2$  (like A, B) is zero.



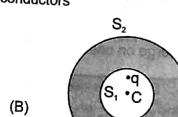
(iv) Resultant field in a charge free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also, this result is true. No charge will be induced on the inner most surface of the conductor.



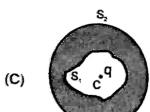
(v) Charge distribution for different types of cavities in conductors



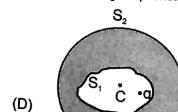
charge is at the common centre ( $S_1, S_2 \rightarrow$  spherical)



charge is not at the common centre ( $S_1, S_2 \rightarrow$  spherical)

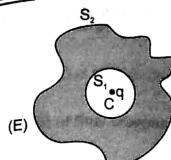


charge is at the centre of  $S_2$  ( $S_2 \rightarrow$  spherical)

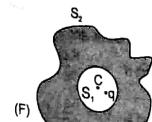


charge is not at the centre of  $S_2$  ( $S_2 \rightarrow$  spherical)

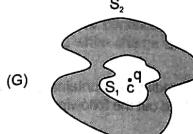
### Electrostatics



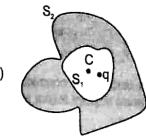
charge is at the centre of  $S_1$  (Spherical)



charge not at the centre of  $S_1$  (Spherical)



charge is at the geometrical centre



charge is not at the geometrical centre

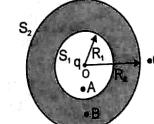
Using the result that  $\bar{E}_{\text{res}}$  in the conducting material should be zero and using result (ii) we can show that

Case	A	B	C	D	E	F	G	H
$S_1$	Uniform	Nonuniform	Nonuniform	Nonuniform	Uniform	Nonuniform	Nonuniform	Nonuniform
$S_2$	Uniform	Uniform	Uniform	Uniform	Nonuniform	Nonuniform	Nonuniform	Nonuniform

Note : In all cases, charge on inner surface  $S_1 = -q$  and on outer surface  $S_2 = q$ . The distribution of charge on ' $S_1$ ' will not change even if some charges are kept outside the conductor (i.e. outside the surface  $S_2$ ). But the charge distribution on ' $S_2$ ' may change if some charges(s) is/are kept outside the conductor.

### Solved Examples

Example 116. An uncharged conductor of inner radius  $R_1$  and outer radius  $R_2$  contains a point charge  $q$  at the centre as shown in figure



- Find  $\bar{E}$  and  $V$  at points A, B and C
- If a point charge  $Q$  is kept outside the sphere at a distance ' $r$ ' ( $>R_2$ ) from centre, then find out resultant force on charge  $Q$  and charge  $q$ .

Solution :

At point A :



$$V_A = \frac{Kq}{OA} + \frac{Kq}{R_2} + \frac{K(-q)}{R_1}, \bar{E}_A = \frac{Kq}{OA^3} \hat{OA}$$

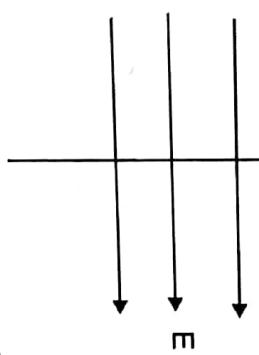
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**Example 115** An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces.

Q



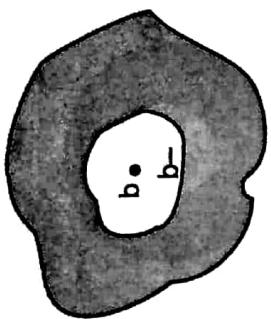
**Solution :** Let there is x charge on left side of plate and  $Q - x$  charge on right side of plate

$$\therefore E_p = 0$$

$$\begin{aligned} \therefore \frac{x}{2A\epsilon_0} + E &= \frac{Q-x}{2A\epsilon_0} & \text{or} & \frac{x}{A\epsilon_0} = \frac{Q}{2A\epsilon_0} - E \\ \therefore x &= \frac{Q}{2} - EA\epsilon_0 & \text{and} & Q - x = \frac{Q}{2} + EA\epsilon_0 \end{aligned}$$

So, charge on one side is  $\frac{Q}{2} - EA\epsilon_0$  and other side  $\frac{Q}{2} + EA\epsilon_0$

**Note :** Solve this question for  $Q = 0$  without using the above answer and match that answer with the answer that you will get by putting  $Q = 0$  in the above answer.

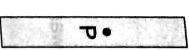


**17.1 Some other important results for a closed conductor:**

- (i) If a charge  $q$  is kept in the cavity then  $-q$  will be induced on the inner surface and  $+q$  will be induced on the outer surface of the conductor (it can be proved using Gauss theorem)

### Elastostatics

Two large parallel conducting sheets (placed at finite distance) are given charges  $Q$  and  $2Q$  respectively. Find out charges appearing on all the surfaces.



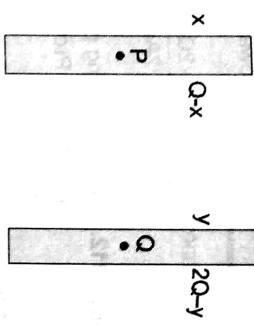
**Solution :**

Let there is  $x$  amount of charge on left side of first plate. So, on its right side charge will be  $Q-x$ . Similarly, for second plate there is  $y$  charge on left side and  $2Q-y$  charge is on right side.

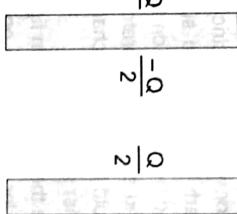
$$E_p = 0 \text{ (By property of conductor)}$$

$$\Rightarrow \frac{x}{2A\epsilon_0} - \left\{ \frac{Q-x}{2A\epsilon_0} + \frac{y}{2A\epsilon_0} + \frac{2Q-y}{2A\epsilon_0} \right\} = 0$$

We can also say that charge on left side of  $P$  = charge on right side of  $P$



$$\frac{+3Q}{2} \quad \frac{-Q}{2} \quad \frac{Q}{2} \quad \frac{+3Q}{2}$$



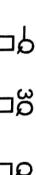
$$x = Q - x + 2Q - y \Rightarrow x = \frac{3Q}{2}, Q - x = \frac{-Q}{2}$$

$$\text{Similarly, for point } Q : x + Q - x + y = 2Q - y$$

$$\Rightarrow y = Q/2, 2Q - y = 3Q/2$$

So, final charge distribution of plates is

**Example 114.** Figure shows three large metallic plates with charges  $-Q$ ,  $3Q$  and  $Q$  respectively. Determine the final charges on all the surfaces.



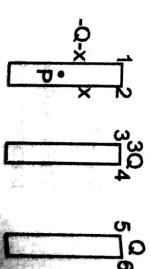
**Solution :**

We assume that charge on surface 2 is  $x$ . Following conservation of charge, we see that surfaces 1 has charge  $(-Q-x)$ . The electric field inside the metal plate is zero. So, field at  $P$  is zero.

Resultant field at  $P$

$$E_p = 0 \Rightarrow \frac{-Q-x}{2A\epsilon_0} = \frac{x+3Q+Q}{2A\epsilon_0} \text{ or } -Q-x = x+4Q$$

$$\therefore x = \frac{-5Q}{2}$$



**solution :** (i)  $E_A = E_Q + E_{-2Q} + E_{3Q}$ . Here  $E_Q$  means electric field due to 'Q'.

$$E_A = \frac{(Q - 2Q + 3Q)}{2A\epsilon_0} = \frac{2Q}{2A\epsilon_0} = \frac{Q}{A\epsilon_0}, \text{ towards left}$$

$$(ii) E_B = \frac{Q - (-2Q + 3Q)}{2A\epsilon_0} = 0$$

$$(iii) E_C = \frac{(Q - 2Q) - (3Q)}{2A\epsilon_0} = \frac{-4Q}{2A\epsilon_0} = \frac{-2Q}{A\epsilon_0}, \text{ towards right}$$

$$\Rightarrow \frac{2Q}{A\epsilon_0} \text{ towards left}$$

$$(iv) E_D = \frac{(Q - 2Q + 3Q)}{2A\epsilon_0} = \frac{2Q}{2A\epsilon_0} = \frac{Q}{A\epsilon_0}, \text{ towards right}$$

**Example 112.** Two conducting plates A and B are placed parallel to each other. A is given a charge  $Q_1$  and B a charge  $Q_2$ . Prove that the charges on the inner facing surfaces are of equal magnitude and opposite sign.

**Solution :**

Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.

The distribution should be like the one shown in figure. To find the value of  $q$ , consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is  $A$ . Using the equation,  $E = \sigma / (2\epsilon_0)$ , the electric field at P

$$\text{Due to the charge } Q_1 - q = \frac{Q_1 - q}{2A\epsilon_0} \text{ (downward)}$$

$$\text{Due to the charge } +q = \frac{q}{2A\epsilon_0} \text{ (upward),}$$

$$\text{Due to the charge } -q = \frac{q}{2A\epsilon_0} \text{ (downward),}$$

$$\text{and due to the charge } Q_2 + q = \frac{Q_2 + q}{2A\epsilon_0} \text{ (upward).}$$

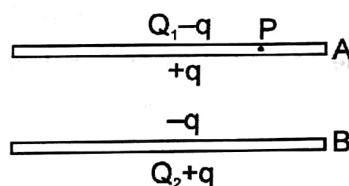
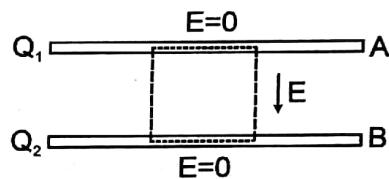
The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$E_p = \frac{Q_1 - q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{Q_2 + q}{2A\epsilon_0}$$

As the point P is inside the conductor, this field should be zero. Hence,

$$Q_1 - q - q + q - Q_2 - q = 0 \quad \text{or,} \quad q = \frac{Q_1 - Q_2}{2}$$

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.



## Electrostatics

$$2E_r = \frac{\sigma}{\epsilon_0} \Rightarrow E_r = \frac{\sigma}{2\epsilon_0}$$

Now, we can easily find the pressure from eqn.(1)

$$P = (E_r)(\sigma) = \frac{\sigma}{2\epsilon_0} (\sigma) = \frac{\sigma^2}{2\epsilon_0}$$

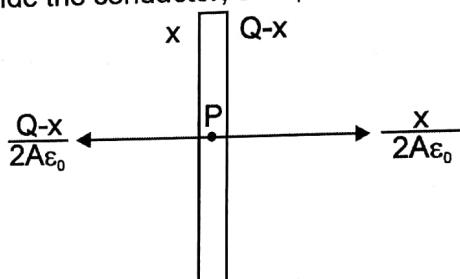
So, electrostatic pressure at the surface of the conductor  $P = \frac{\sigma^2}{2\epsilon_0}$

where,  $\sigma$  = local surface charge density.

## Solved Examples

**Example 109.** Prove that if an isolated (isolated means no charges are near the sheet) large conducting sheet is given a charge then the charge distributes equally on its two surfaces.

**Solution :** Let there is  $x$  charge on left side of sheet and  $Q-x$  charge on right side of sheet.  
Since, point P lies inside the conductor, so  $E_P = 0$



$$\text{or } \frac{x}{2A\epsilon_0} - \frac{Q-x}{2A\epsilon_0} = 0 \Rightarrow \frac{2x}{2A\epsilon_0} = \frac{Q}{2A\epsilon_0}$$

$$\Rightarrow x = \frac{Q}{2} \quad \& \quad Q-x = \frac{Q}{2}$$

So, charge is equally distributed on both sides.

**Example 110.** If an isolated infinite sheet contains charge  $Q_1$  on its one surface and charge  $Q_2$  on its other surface, then prove that electric field intensity at a point in front of sheet will be  $\frac{Q}{2A\epsilon_0}$ , where

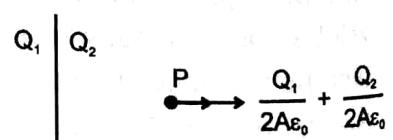
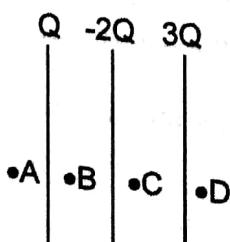
$$Q = Q_1 + Q_2$$

**Solution :** Electric field at point P :

$$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{Q_1}{2A\epsilon_0} \hat{n} + \frac{Q_2}{2A\epsilon_0} \hat{n} = \frac{Q_1 + Q_2}{2A\epsilon_0} \hat{n} = \frac{Q}{2A\epsilon_0} \hat{n}$$

[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

**Example 111.** Three large conducting sheets placed parallel to each other at finite distance contain charges  $Q$ ,  $-2Q$  and  $3Q$  respectively. Find electric field at points A, B, C and D



## ELECTROSTATIC PRESSURE AT THE SURFACE OF THE CONDUCTOR

Suppose a conductor is given some charge. Due to repulsion, all the charges will reach the surface of the conductor. But the charges will still repel each other. So an outward force will be felt by each charge due to others. Due to this force, there will be some pressure at the surface, which is called electrostatic pressure.



To find the electrostatic pressure, let's take a small surface element having Area 'ds'.

Force on this element due to the remaining charges :

$$dF = \begin{pmatrix} \text{electric field at} \\ \text{that place due to} \\ \text{remaining charges} \end{pmatrix} \begin{pmatrix} \text{charge of} \\ \text{the small} \\ \text{element} \end{pmatrix}$$

Let electric field at that point due to the remaining charges =  $E_r$

and charge of the small element =  $dq = \sigma ds$

$$\Rightarrow dF = (E_r) (dq) = (E_r) (\sigma ds)$$

So, pressure on this small element

$$P = \frac{dF}{ds} = \frac{(E_r)(\sigma ds)}{ds} \Rightarrow P = (E_r)(\sigma) \quad \dots(1)$$

Now to find pressure, we have to find  $E_r$  (electric field at that position due to the remaining charges)

Suppose,

Electric field due to the small element near the surface =  $E_s$

Electric field due to the remaining part near the surface =  $E_r$

At a point just outside the surface, electric field due to the small element ( $E_s$ ) will be normally outwards, and electric field due to the remaining part ( $E_r$ ) will also be normally outwards.

So Net electric field just outside the surface =  $E_s + E_r$  and we have proved

$$\text{that electric field just outside the conductor surface} = \frac{\sigma}{\epsilon_0} \quad \dots(2)$$

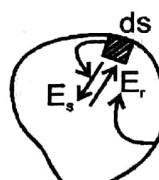
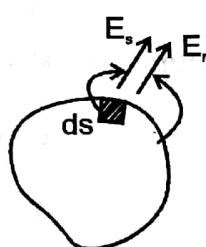
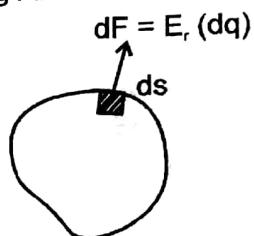
$$\Rightarrow E_s + E_r = \frac{\sigma}{\epsilon_0} \quad \dots(2)$$

Now, let's see the electric field just inside the metal surface. Here, electric field due to the remaining charges ( $E_r$ ) will be in the same direction (normally outward), but the electric field due to the small element will be in opposite direction (normally inward)

So net electric field just inside the metal surface =  $E_r - E_s$  and we know that electric field inside a conductor = 0

$$\text{So, } E_r - E_s = 0 \Rightarrow E_r = E_s \quad \dots(3)$$

from eqn. (2) and eqn. (3), we can say that :



## Electrostatics

(viii) When a conductor is grounded its potential becomes zero.



(ix) When an isolated conductor is grounded then its charge becomes zero.

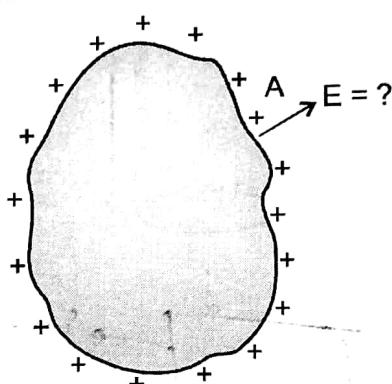
(x) When two conductors are connected there will be charge flow till their potentials become equal.

(xi) Electric pressure : Electric pressure at the surface of a conductor is given by formula

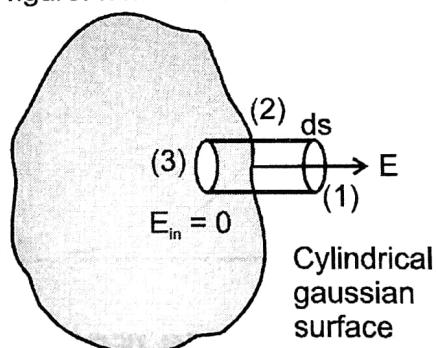
$$P = \frac{\sigma^2}{2\epsilon_0}, \text{ where } \sigma \text{ is the local surface charge density.}$$

## FINDING FIELD DUE TO A CONDUCTOR

Suppose we have a conductor and at any 'A', local surface charge density =  $\sigma$ . We have to find electric field just outside the conductor surface.



For this, let's consider a small cylindrical Gaussian surface, which is partly inside and partly outside the conductor surface, as shown in figure. It has a small cross section area  $ds$  and negligible height.



Applying Gauss's theorem for this surface :

$$\phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma ds}{\epsilon_0}$$

flux through surface (1)      flux through surface (2)      flux through surface (3)  
 $\phi_1 = Eds$        $\phi_2 = 0$        $\phi_3 = 0$   
 (because  $E$  is normal to the surface of conductor)      ( $E$  is normal to curved Gaussian surface)      (as  $E$  inside the conductor = 0)

$$\text{So, } Eds = \frac{\sigma ds}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0}$$

Electric field just outside the surface of conductor :

$$E = \frac{\sigma}{\epsilon_0} \text{ (direction will be normal to the surface)}$$

In vector form:  $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$  (Here,  $\hat{n}$  = unit vector normal to the conductor surface)

We are going to prove that the electric field strength is zero at the so-called incentre, the centre of the triangle's inscribed circle (which has radius  $r$  in the figure).

Let us consider a small length of rod at position  $P$  on one of the sides of the triangle; let it subtend an angle  $\Delta\varphi$  at the incentre (see figure). Its distance from the incentre is  $r/\cos\varphi$ . Its small length  $\Delta x$  can be found by noting that  $P$  is at a distance  $x = r \tan\varphi$  along the rod from the fixed point  $Q$  and so  $\Delta x = (r \Delta\varphi) / (\cos^2\varphi)$ .

Consequently the charge it carries is  $\lambda\Delta q = \frac{\lambda r \Delta\varphi}{\cos^2\varphi}$

where  $\lambda$  is the linear charge density on the rods. The magnitude of the elementary contribution of this

$$\text{small piece to the electric field at the incentre is } \Delta E = \frac{1}{4\pi\epsilon_0} \frac{\Delta q \cos^2\varphi}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda r \Delta\varphi}{r^2}$$

It can be seen from this result that the same electric field (in both magnitude and direction) would be produced by an arc of the inscribed circle that subtends  $\Delta\varphi$  at the circle's centre and carries the same linear charge density  $\lambda$  as the rod.

Summing up the contributions of the small arc pieces corresponding to all three sides of the triangle, we will, because of the circular symmetry, obtain zero net field. It follows that the electric field strength produced by the charged sides of the triangle is also zero at the incentre.

According to Newton's third law, the insulating plate acts on the point charge with a force of the same magnitude (but opposite direction) as the point charge does on the plate. We calculate the magnitude of this latter force. Divide the plate (notionally) into small pieces, and denote the area of the  $i^{\text{th}}$  piece by  $\Delta A_i$ . Because of the uniform charge distribution, the charge on this small piece is  $\Delta Q_i = \frac{Q}{d^2} \Delta A_i$ ,

and so the electric force acting on it is  $F_i = E_i Q_i$ , where  $E_i$  is the magnitude of the electric field produced by the point charge  $q$  at the position of the small piece.

The force acting on the insulating plate, as a whole, can be calculated as the vector sum of the forces acting on the individual pieces of the plate. Because of the axial symmetry, the net force is perpendicular to the plate, and so it is sufficient to sum the perpendicular components of the forces :

$$F = \sum_i F_i \cos\theta_i = \sum_i E_i \frac{Q}{d^2} \Delta A_i \cos\theta_i = \frac{Q}{d^2} \sum_i E_i \Delta A_i \cos\theta_i$$

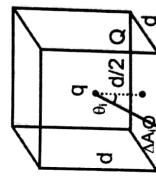
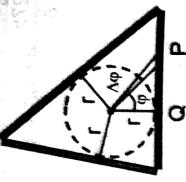
where  $\theta_i$  is the angle between the normal to the plate and the line that connects the point charge to the  $i^{\text{th}}$  piece of it.

The sum in the given expression is nothing other than the electric flux through the square sheet produced by the point charge  $q$  :  $\psi = \sum_i E_i \Delta A_i \cos\theta_i$

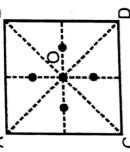
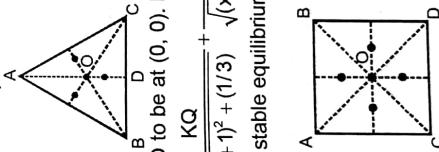
and can be evaluated as follows. Let us imagine that a cube of edge  $d$  is constructed symmetrically around the point charge (see figure). Then, the distance of the point charge from each side of the cube is just  $d/2$ . According to Gauss's law, the total electric flux passing through the six sides of the cube is  $q/\epsilon_0$  and so the flux through a single side is one-sixth of this :  $\psi = \frac{q}{6\epsilon_0}$

Using this and our previous observations, we calculate the magnitude of the force acting on the point charge due to the presence of the charged insulating plate as  $F = \frac{Qq}{6\epsilon_0 d^2}$

$$38. \quad F = \frac{(2\sqrt{2}-1)q^2}{32\pi\epsilon_0 \ell^2} \quad 39. \quad \sigma = -\frac{q\ell}{2\pi(\ell^2+r^2)^{3/2}}, \quad q_{\text{ind}} = -q \quad 40. \quad (\text{a}) \sigma = \frac{\lambda}{2\pi\ell}; \quad (\text{b}) \sigma(r) = \frac{\lambda}{2\pi\sqrt{\ell^2+r^2}}$$



19.  $\sqrt{\frac{275}{8}} = 5.86 \text{ m/s}$
22.  $6a\epsilon_0 x$
24.  $\phi = 4\pi R a, Q = 4\pi R a \epsilon_0$
26. Locus is a circle (equation depends on choice of coordinate system)
28. (i)  $E = \frac{\rho_0 r}{3 \epsilon_0} \left(1 - \frac{3r}{4R}\right)$  for  $r < R, E = \frac{\rho_0 R^3}{12 \epsilon_0 r^2}$  for  $r > R$  (ii)  $E_{\max} = \frac{1}{9} \frac{\rho_0 R}{\epsilon_0}$  for  $r_m = \frac{2}{3} R$ .
29. (a)  $\bar{F} = \frac{2KQq \left( \frac{a}{\sqrt{3}} - \delta \right)}{\left( a^2 + \left( \frac{a}{\sqrt{3}} - \delta \right)^2 \right)^{3/2}} - \frac{KQq}{\left( \frac{2a}{\sqrt{3}} + \delta \right)^2}$  Here  $K = 1/4\pi\epsilon_0$  and direction is upward (towards A)
- (b) Using binomial approximation,  $\bar{F} = KQq \frac{9\sqrt{3}}{16} \frac{\delta}{a^3}$  (upward) which is linear in  $\delta$ . Hence charge will oscillate simple harmonically about O when released.
- (c)  $\bar{F}_D = \frac{KQq}{3a^2}$  (downward)
- (d) For small  $\delta$  force on the test charge is upwards while for large  $\delta$  (eg. at D) force is downwards. So there is a neutral point between O and D. By symmetry there will be neutral points on other medians also. In figure x. Below all possible (4) neutral points are shown by •.
- (e) Let the distance along OP be  $x$  and O to be at (0, 0). Electric potential of a test charge along OP can be written as  $V(x) = \frac{KQ}{\sqrt{x^2 + (4/3)}} + \frac{KQ}{\sqrt{(x+1)^2 + (1/3)}} + \frac{KQ}{\sqrt{(x-1)^2 + (1/3)}} \approx KQ \sqrt{\frac{3}{4} \left( 3 + \frac{9}{16} x^2 \right)}$
- We can see that  $V(x) \propto x^2$ , hence it is a stable equilibrium.
- (f) Equilibrium points are indicated by •.
- (g)  $N + 1$
30.  $q_1 \sin^2 \frac{\alpha}{2} = q_2 \sin^2 \frac{\beta}{2}$
32. (a) 20 N right hand side (b) 35 N right hand side (c) 35 N left hand side
33.  $\frac{8Q}{-11}$
34.  $\frac{kq^2}{2b} - \frac{kq^2}{2a}$
35.  $q' = 4\sqrt{2}q$
31.  $\sigma_1 = \sigma_4, \sigma_2 = \epsilon_0 E, \sigma_3 = -\epsilon_0 E$



39. A point charge  $q$  is located between two mutually perpendicular conducting half-planes. Its distance from each half-plane is equal to  $\ell$ . Find the modulus of the vector of the force acting on the charge.

40. A point charge  $q$  is located at a distance  $\ell$  from an infinite conducting plane. Determine the surface density of charges induced on the plane as a function of separation  $r$  from the base of the perpendicular drawn to the plane from the charge.

A very long straight thread is oriented at right angles to an infinite conducting plane; its end is separated from the plane by a distance  $\ell$ . The thread carries a uniform charge of linear density  $\lambda$ . Suppose the point O is the trace of the thread on the plane. Find the surface density of the induced charge on the plane

- (a) At the point O
- (b) As a function of a distance  $r$  from the point O.

## HLP Answers

1.  $q = 4\ell \sqrt{4\pi \epsilon_0 m \sin\left(\frac{\alpha}{2}\right)} \sin \frac{\alpha}{2}$

2.  $T = 2\pi \sqrt{\frac{2m}{3\lambda\sigma}}$

3. (i)  $\frac{\sqrt{3}kq_0^2}{a^2}$ , away from the charges along perpendicular bisector of line joining remaining two charges.

(ii)  $\frac{\sqrt{3}kq_0^2}{a^2}$ ; towards the charges along perpendicular bisector of line joining remaining two charges.

(iii)  $\frac{16\sqrt{3}kq_0^2}{a^2}$ ; away from the charges along angle bisector.

4.  $V = \frac{5\rho R^2}{12\epsilon_0}$       5.  $\frac{\sigma}{8\epsilon_0}, \frac{\sigma}{8\epsilon_0|d|}$       6.  $r = \sqrt{\frac{2\pi\epsilon_0 mv^2}{\lambda q}} - 1$

7.  $3.36 \times 10^{-8} \text{ C}, 8.15 \times 10^{-3} \text{ N}$

8.  $-25 \times 10^{-9} \text{ C}$

9.  $0.505 \times 10^{-12} \text{ C/m}^2$ , No 10.  $u = \sqrt{58.1} = 7.6 \text{ m/s}$

11.  $F = \frac{q\lambda}{4\pi\epsilon_0 R}$

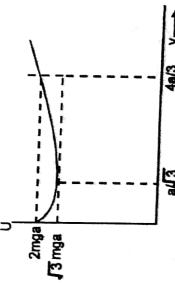
12. (a)  $\frac{\lambda}{4\epsilon_0 R}$       (b)  $E = \frac{\lambda_0 R^2}{4\epsilon_0 (X^2 + R^2)^{3/2}}$ . For  $x \gg R$  this strength  $E \approx \frac{\rho}{4\pi\epsilon_0 X^3}$ , where  $\rho = \pi R^2 \lambda_0$ .

13.  $|\phi| = \frac{q}{\epsilon_0} \left( 1 - \frac{1}{\sqrt{1 + (R/\ell)^2}} \right)$ . The sign of  $\phi$  depends on how the direction of the normal to the circle is chosen.

14.  $q = 2\pi\alpha R^2, E = \frac{1}{2} \frac{\alpha}{\epsilon_0}$

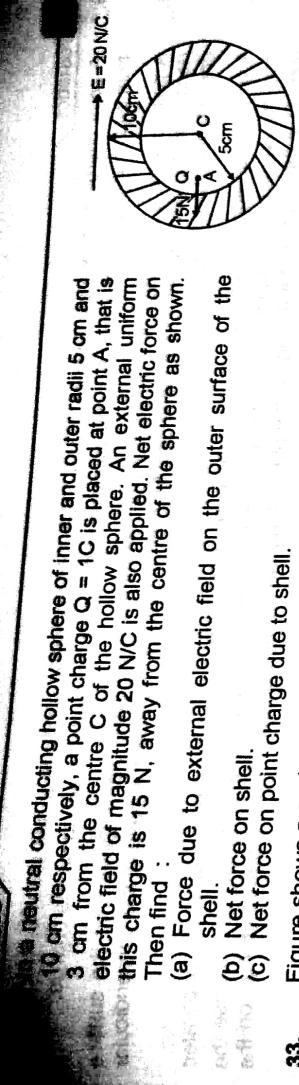
15.  $V = \frac{\sigma R}{2\epsilon_0}, E = \frac{\sigma}{4\epsilon_0}$

16. (i)  $H = 4a/3$       (ii)  $U(y) = 2mg + \left[ \sqrt{y^2 + a^2} - y \right] mg$ ; at equilibrium  $\frac{du}{dy} = 0 \Rightarrow y = \frac{a}{\sqrt{3}}$



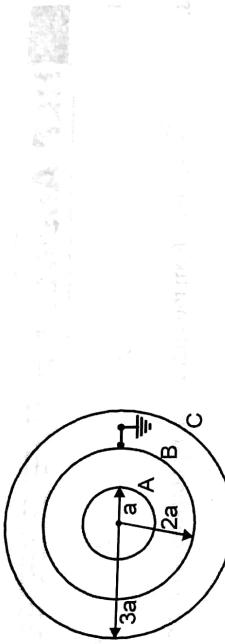
17.  $V = \left( \frac{9\epsilon_0 L}{2m} \right)^{1/3}$

18.  $V_0 = 3 \text{ m/s}; \text{K.E. at origin } (27 - 10\sqrt{6}) \times 10^{-4} \text{ J} = 2.5 \times 10^{-4} \text{ J}$



**33.** A neutral conducting hollow sphere of inner and outer radii 5 cm and 3 cm respectively, a point charge  $Q = 1\text{C}$  is placed at point A, that is 3 cm from the centre C of the hollow sphere. An external uniform electric field of magnitude 20 N/C is also applied. Net electric force on this charge is 15 N, away from the centre of the sphere as shown.  
 Then find :  
 (a) Force due to external electric field on the outer surface of the shell.  
 (b) Net force on shell.  
 (c) Net force on point charge due to shell.

- 33.** Figure shows a system of three concentric metal shells A, B and C with radii  $a$ ,  $2a$  and  $3a$  respectively. Shell B is earthed and shell C is given a charge  $Q$ . Now if shell C is connected to shell A, then find the final charge on the shell B

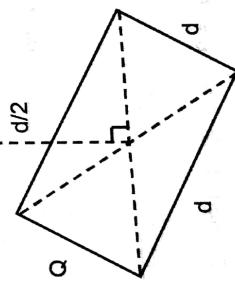


- 34.** A point charge  $q$  is brought from infinity (slowly so that heat developed in the shell is negligible) and is placed at the centre of a conducting neutral spherical shell of inner radius  $a$  and outer radius  $b$ , then find work done by external agent :

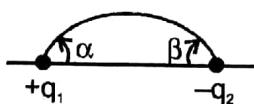
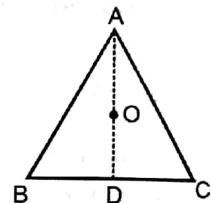


- 35.** Consider two solid dielectric spheres of radius  $a$ , separated by a distance  $R(R \gg a)$ . One of the spheres has a charge  $q$ , and the other is neutral (see figure.) We double the distance between two sphere. How much charge should reside on the first sphere now so that the force between the spheres remains the same ?

- 36.** A triangle is made from thin insulating rods of different lengths, and the rods are uniformly charged, i.e. the linear charge density on each rod is uniform and the same for all three rods. Find a particular point in the plane of the triangle at which the electric field strength is zero.  
**37.** A square of side  $d$ , made from a thin insulating plate, is uniformly charged and carries a total charge of  $Q$ . A point charge  $q$  is placed on the symmetrical normal axis of the square at a distance  $d/2$  from the plate. How large is the force acting on the point charge?

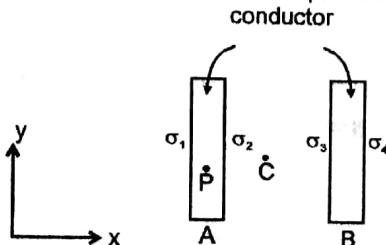


- 25.** A positive charge is distributed in a spherical region with charge density  $\rho = \rho_0 r$  for  $r \leq R$  (where  $\rho_0$  is a positive constant and  $r$  is the distance from centre). Find out electric potential and electric field at following locations.  
 (a) At a distance  $r$  from centre inside the sphere. (b) At a distance  $r$  from centre outside the sphere.
- 26.** Two point charges  $q$  and  $-2q$  are placed at a distance 6m apart on a horizontal plane ( $x-y$  plane). Find the locus of the zero potential points in the  $x-y$  plane.
- 27.** Two metallic balls of radii  $R_1$  and  $R_2$  are kept in vacuum at a large distance compared to their radii. Find the ratio of the charges on the two balls for which electrostatic energy of the system is minimum. What is the potential difference between the two balls for this ratio? Total charge of the balls is constant. Neglect the interaction energy. (charge distribution on each ball is uniform)
- 28.** A ball of radius  $R$  carries a positive charge whose volume density depends only on the separation  $r$  from the ball's centre as  $\rho = \rho_0 (1 - r/R)$ , where  $\rho_0$  is a constant. Assuming the permittivities of the ball and the environment to be equal to unity, find :  
 (i) The magnitude of the electric field strength as a function of the distance  $r$  both inside and outside the ball;  
 (ii) The maximum intensity  $E_{\max}$  and the corresponding distance  $r_m$ .
- 29.** Consider an equilateral triangle ABC of side  $2a$  in the plane of the paper as shown. The centroid of the triangle is O. Equal charges ( $Q$ ) are fixed at the vertices A, B and C. In what follows consider all motion and situations to be confined the plane of the paper.  
 (a) A test charge ( $q$ ), of same sign as  $Q$ , is placed on the median AD at a point at a distance  $\delta$  below O. Obtain the force ( $\vec{F}$ ) felt by the test charge.  
 (b) Assuming  $\delta \ll a$  discuss the motion of the test charge when it is released.  
 (c) Obtain the force ( $\vec{F}_D$ ) on this test charge if it is placed at the point D as shown in the figure.  
 (d) In the figure below mark the approximate locations of the equilibrium point(s) for this system. Justify your answer.  
 (e) Is the equilibrium at O stable or unstable if we displace the test charge in the direction of OP? The line PQ is parallel to the base BC. Justify your answer.  
 (f) Consider a rectangle ABCD. Equal charges are fixed at the vertices A, B, C and D. O is the centroid. In the figure below mark the approximate locations of all the neutral points of the system for a test charge with same sign as the charges on the vertices. Dotted lines are drawn for the reference.  
 (g) How many neutral points are possible for a system in which  $N$  charges are placed at the  $N$  vertices of a regular  $N$  sided polygon?
- 30.** An electrostatic field line leaves at angle  $\alpha$  from point charge  $q_1$  and connects with point charge  $-q_2$  at angle  $\beta$  (see figure).



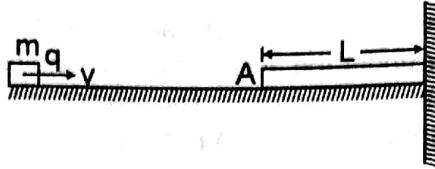
Then find the relationship between  $\alpha$  and  $\beta$ :

- 31.** Figure shows two infinitely large conducting plates A and B. If electric field at C due to charge densities  $\sigma_1, \sigma_2, \sigma_3$  and  $\sigma_4$  is  $E \hat{i}$ , find  $\sigma_2$  and  $\sigma_3$  in terms of  $E$ . State whether this much information is sufficient to find  $\sigma_1$  and  $\sigma_4$  in terms of  $E$ . Derive a relation between  $\sigma_1$  and  $\sigma_4$ .

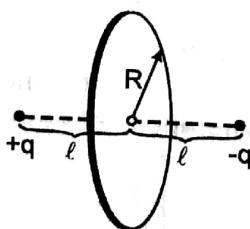


15. Find the electric field potential and strength at the centre of a hemisphere of radius R charged uniformly with the surface density  $\sigma$ .
16. A non-conducting disc of radius a and uniform positive surface charge density  $\sigma$  is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has  $\frac{q}{m} = \frac{4\epsilon_0 g}{\sigma}$ . [JEE 1999 (Mains), 5+5 /100]
- Find the value of H if the particle just reaches the disc.
  - Sketch the potential energy of the particle as a function of its height and find its equilibrium position.
17. The potential difference between two large parallel plates is varied as  $V = at$ ; a is a positive constant and t is time. An electron starts from rest at  $t = 0$  from the plate which is at lower potential. If the distance between the plates is L, mass of electron m and charge on electron -e then find the velocity of the electron when it reaches the other plate.
18. Four point charges  $+8\mu C$ ,  $-1\mu C$ ,  $-1\mu C$  and  $+8\mu C$ , are fixed at the points,  $-\sqrt{\frac{27}{2}} m$ ,  $-\sqrt{\frac{3}{2}} m$ ,  $+\sqrt{\frac{3}{2}} m$  and  $+\sqrt{\frac{27}{2}} m$  respectively on the y-axis. A particle of mass  $6 \times 10^{-4} \text{ kg}$  and of charge  $+0.1 \mu C$  moves along the -x direction. Its speed at  $x = +\infty$  is  $v_0$ . Find the least value of  $v_0$  for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given :  $1/(4\pi\epsilon_0) = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$  [JEE 2000 (Mains), 10/ 100]
19. A small ball of mass  $2 \times 10^{-3} \text{ kg}$  having a charge of  $1 \mu C$  is suspended by a string of length  $0.8 \text{ m}$ . Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution. ( $g = 10 \text{ m/s}^2$ ) [JEE 2001 (Mains), 5/100 ; REE – 1996]
20. Three point charges Q,  $2Q$  and  $8Q$  are to be placed on a  $10 \text{ cm}$  long straight line. Find the position where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge Q due to the other two charges? [JEE 1987]
21. A particle of charge  $-q$  and mass m moves in a circular orbit about a fixed charge  $+Q$ . Show that the " $r^3 \propto T^2$ " law, is satisfied, where r is the radius of orbit and T is time period.
22. The field potential in a certain region of space depends only on the x coordinate as  $\phi = -ax^3 + b$ , where a and b are constants. Find the distribution of the space charge  $p(x)$ .
23. A conducting sphere  $S_1$  of radius r is attached to an insulating handle. Another conducting sphere  $S_2$  of radius R is mounted on an insulating stand.  $S_2$  is initially uncharged.  $S_1$  is given a charge Q, brought into contact with  $S_2$  and removed.  $S_1$  is then recharged such that the charge on it is again Q & it is again brought into contact with  $S_2$  & removed. This procedure is repeated n times
  - Find the electrostatic energy of  $S_2$  after n such contacts with  $S_1$ . [JEE 1998 (Mains), 7+1/200]
  - What is the limiting value of this energy as  $n \rightarrow \infty$  ?
24. The electric field strength depends only on the x and y coordinates according to the law  $\vec{E} = a(x\hat{i} + y\hat{j})/(x^2 + y^2)$ , where a is a constant,  $\hat{i}$  and  $\hat{j}$  are the unit vectors of the x and y axes. Find the flux of the vector  $\vec{E}$  through a sphere of radius R with its centre at the origin of coordinates. Using the above result, also calculate total charge enclosed by the sphere.

6. Figure shows a rod of length  $L$  which is uniformly charged with linear charge density  $\lambda$  kept on a horizontal surface. Right end of rod is in contact with a vertical fixed wall. A block of mass  $m$  and charge  $q$  is projected with a velocity  $v$  from a point very far from rod in the line of rod. Find the distance of closest approach between the block & the left end A of the rod.

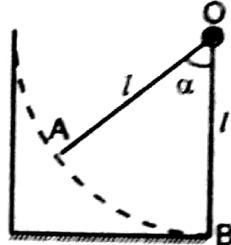


7. A charged ball of mass  $5.88 \times 10^{-4}$  kg is suspended from two silk strings of equal lengths so that the strings are inclined at  $90^\circ$  with each other. Another ball carrying a charge which is equal in magnitude but opposite in sign of the first one is placed vertically below the first one at a distance of  $4.2 \times 10^{-2}$  m. Due to this, the tension in the strings is doubled. Determine the charge on the ball and the tension in the strings after electrostatic interaction. ( $g = 9.8 \text{ m/s}^2$ )
8. A charge of  $16 \times 10^{-9}$  C is fixed at the origin of coordinates. A second charge of unknown magnitude is at  $x = 3\text{m}$ ,  $y = 0$  and a third charge of  $12 \times 10^{-9}$  C is at  $x = 6\text{m}$ ,  $y = 0$ . What is the value of the unknown charge if the resultant field at  $x = 8\text{m}$ ,  $y = 0$  is  $20.25 \text{ N/C}$  directed towards positive x-axis?
9. Two large conducting plates are placed parallel to each other with a separation of  $2.00 \text{ cm}$  between them. An electron starting from rest near one of the plates reaches the other plate in  $2.00 \text{ microseconds}$ . Find the surface charge density on the inner surfaces. Can you find out the charge density on outer surface?
10. A ball of mass  $10^{-2}$  kg & having charge  $+3 \times 10^{-6}$  C is tied at the end of a  $1 \text{ m}$  long thread. The other end of the string is fixed and a charge of  $-3 \times 10^{-6}$  C is placed at this end. The ball can move in a circular orbit of radius  $1 \text{ m}$  in a vertical plane. Initially the ball is at the bottom. Find the minimum initial horizontal velocity of the ball so that it will be able to complete the full circle. [ $g = 10 \text{ m/s}^2$ .] [REE 1996, 5]
11. A system consists of a thin charged wire ring of radius  $R$  and a very long uniformly charged thread oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to  $q$ . The charge of the thread (per unit length) is equal to. Find the interaction force between the ring and the thread.
12. A thin non-conducting ring of radius  $R$  has a linear charge density  $\lambda_0 = \lambda_0 \cos \phi$ , where  $\lambda_0$  is a constant,  $\phi$  is the azimuthal angle. Find the magnitude of the electric field strength  
 (a) At the centre of the ring.  
 (b) On the axis of the ring as a function of the distance  $x$  from its centre. Investigate the obtained function at  $x \gg R$ .
13. Two point charges  $q$  and  $-q$  are separated by the distance  $2l$  (Figure). Find the flux of the electric field strength vector across a circle of radius  $R$ .

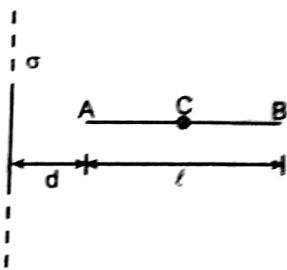


14. A system consists of a ball of radius  $R$  carrying a spherically symmetric charge and the surrounding space filled with a charge of volume density  $\rho = \alpha/r$ , where  $\alpha$  is a constant,  $r$  is the distance from the centre of the ball. Find the ball's charge for which the magnitude of the electric field strength vector is independent of  $r$  outside the ball. How high is this strength? The permittivities of the ball and surrounding space are assumed to be equal to unity.

1. An electrometer consists of a fixed vertical metal bar OB at the top of which is attached a thin rod OA which gets deflected from the bar under the action of an electric charge (fig.). The rod can rotate in vertical plane about fixed horizontal axis passing through O. The reading is taken on a quadrant graduated in degrees. The length of the rod is  $\ell$  and its mass is m. What will be the charge when the rod of such an electrometer is deflected through an angle  $\alpha$  in equilibrium. Find the answer using the following two assumptions:  
The charge on the electrometer is equally distributed between the bar & the rod and the charges are concentrated at point A on the rod & at point B on the bar.

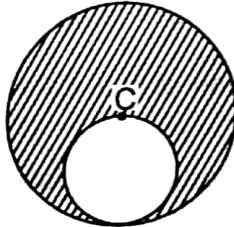


2. A uniform rod AB of mass m and length  $\ell$  is hinged at its mid point C. The left half (AC) of the rod has linear charge density  $-\lambda$  and the right half (CB) has  $+\lambda$ , where  $\lambda$  is constant. A large non conducting sheet of uniform surface charge density  $\sigma$  is also present near the rod. Initially, the rod is kept perpendicular to the sheet. The end A of the rod is initially at a distance d. Now the rod is rotated by a small angle in the plane of the paper and released. Prove that the rod will perform SHM and find its time period.

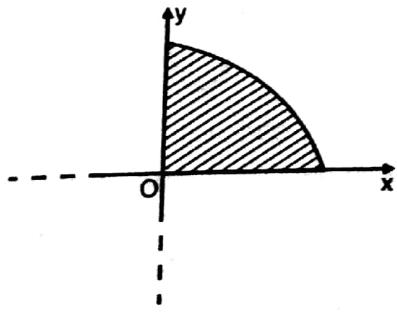


3. (i) Three equal charges  $q_0$  each are placed at three corners of an equilateral triangle of side 'a'. Find out force acting on one of the charge due to other two charges?  
(ii) In the above question, if one of the charge is replaced by negative charge then find out force acting on it due to other charges ?  
(iii) Repeat the part (i) if magnitude of each charge is doubled and side of triangle is reduced to half.

4. A solid sphere of radius 'R' is uniformly charged with charge density  $\rho$  in its volume. A spherical cavity of radius  $\frac{R}{2}$  is made in the sphere as shown in the figure. Find the electric potential at the centre of the sphere.



5. A uniform surface charge of density  $\sigma$  is given to a quarter of a disc extending upto infinity in the first quadrant of x-y plane. The centre of the disc is at the origin O. Find the z-component of the electric field at the point  $(0, 0, z)$  and the potential difference between the points  $(0, 0, d)$  &  $(0, 0, 2d)$ .



What is the charge on shell A ?

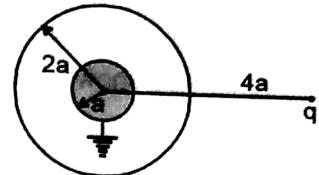
- (A)  $5.31 \times 10^{-6}$  C      (B)  $-5.31 \times 10^{-6}$  C      (C)  $-3.54 \times 10^{-6}$  C      (D)  $-1.77 \times 10^{-6}$  C

6. In which range of the values of  $r$  is the electric field zero ?

- (A) 0 to  $r_A$       (B)  $r_A$  to  $r_B$   
(C) For  $r > r_B$       (D) For no range of  $r$ , electric field is zero.

### Comprehension 3

A solid conducting sphere of radius 'a' is surrounded by a thin uncharged concentric conducting shell of radius  $2a$ . A point charge  $q$  is placed at a distance  $4a$  from common centre of conducting sphere and shell. The inner sphere is then grounded.



The charge on solid sphere is :

- (A)  $-\frac{q}{2}$       (B)  $-\frac{q}{4}$       (C)  $-\frac{q}{8}$       (D)  $-\frac{q}{16}$

Pick up the correct statement.

- (A) Charge on surface of inner sphere is non-uniformly distributed.  
(B) Charge on inner surface of outer shell is non-uniformly distributed.  
(C) Charge on outer surface of outer shell is non-uniformly distributed.  
(D) All the above statements are false.

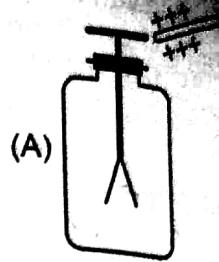
9. The potential of outer shell is.

- (A)  $\frac{q}{32\pi\epsilon_0 a}$       (B)  $\frac{q}{16\pi\epsilon_0 a}$       (C)  $\frac{q}{8\pi\epsilon_0 a}$       (D)  $\frac{q}{4\pi\epsilon_0 a}$

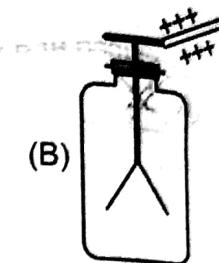
## Exercise-3

If we perform these steps one by one.

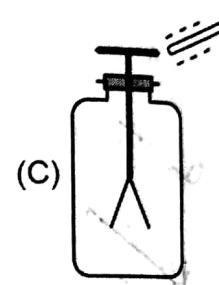
- (i) A positively charged rod is brought closer to initially uncharged knob



- (ii) Then the positively charged rod is touched to the knob



- (iii) Now the +vely charged rod is removed, and a negatively charged.



rod of same magnitude is brought closer at same distance

In which case, the leaves will converge (come closer), as compared to the previous state ?

- (A) (i) (B) (i) and (iii)  
(C) only (iii) (D) In all cases, the leaves will diverge

3.

In an electroscope, both leaves are hinged at the top point O. Each leaf has mass m, length  $\ell$  and gets charge q. Assuming the charge to be concentrated at ends A and B only, the small angle of deviation ( $\theta$ ) between the leaves in static equilibrium, is equal to :

$$(A) \left( \frac{4kq^2}{\ell^2 mg} \right)^{1/3} \quad (B) \left( \frac{kq^2}{\ell^2 mg} \right)^{1/3} \quad (C) \left( \frac{2kq^2}{\ell^2 mg} \right)^{1/2} \quad (D) \left( \frac{64kq^2}{\ell^2 mg} \right)^{1/3}$$

### Comprehension 2

A charged particle is suspended at the centre of two thin concentric spherical charged shells, made of non conducting material. Figure A shows cross section of the arrangement. Figure B gives the net flux  $\phi$  through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere.

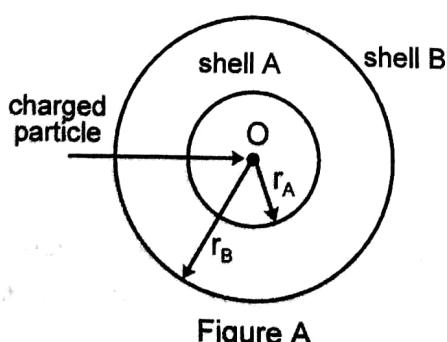


Figure A

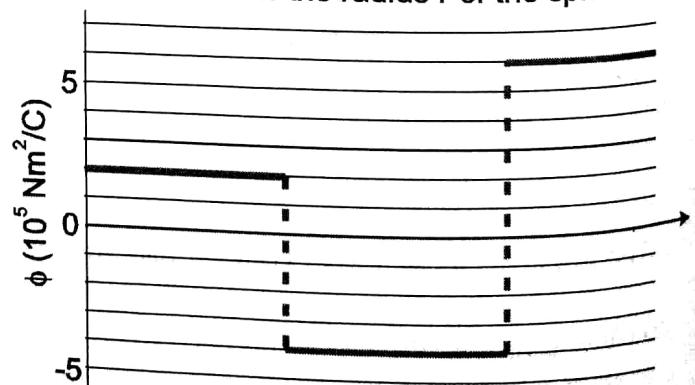


Figure B

4. What is the charge on the central particle ?

- (A)  $0.2 \mu C$  (B)  $2 \mu C$  (C)  $1.77 \mu C$  (D)  $3.4 \mu C$

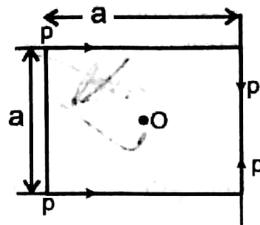
16. A and B are two conducting concentric spherical shells. A is given a charge  $Q$  while B is uncharged. If now B is earthed as shown in figure. Then :
- The charge appearing on inner surface of B is  $-Q$  ✓
  - The field inside and outside A is zero.
  - The field between A and B is not zero. ✓
  - The charge appearing on outer surface of B is zero. ✓
17. A large nonconducting sheet M is given a uniform charge density. Two uncharged small metal spheres A and B are placed near the sheet as shown in figure.
- ~~(A) M attracts A~~ ✓ ~~(B) M attracts B~~ ✓ ~~(C) M attracts B~~ ✓ ~~(D) B attracts A~~ ✓
- 
- A point charge ' $q$ ' is within an electrically neutral conducting shell whose outer surface is a sphere of radius  $R$ . The centre of outer surface is at O. Consider a point P outside the conductor as shown in the figure. The magnitude of electric field at P
- due to charge induced on inner surface of the conductor is zero ✗
  - due to charge induced on inner surface of the conductor is  $\frac{kq}{(r')^2}$
  - due to charge induced on outer surface of the conductor is  $\frac{kq}{r^2}$
  - due to charge induced on surface of the conductor is  $\frac{kq}{r^2}$  ✗
- 

#### PART - IV : COMPREHENSION

~~11.~~ An electric dipole is kept in the electric field produced by a point charge.

- (A) dipole will experience a force.  
(B) dipole will experience a torque.  
(C) it is possible to find a path (not closed) in the field on which work required to move the dipole is zero.  
(D) dipole can be in stable equilibrium.

~~12.~~ Four short dipoles each of dipole moment 'p' are placed at the vertices of a square of side  $a$ . The direction of the dipole moments are shown in the figure.



~~(A)~~ Electric field at O is  $\frac{\sqrt{2} p}{2\pi \epsilon_0 a^3}$

~~(B)~~ Electric field at O is  $\frac{\sqrt{2} p}{\pi \epsilon_0 a^3}$

~~(C)~~ Electrostatic potential at O is zero

~~(D)~~ Net dipole moment is  $2p$

~~13.~~ HCV

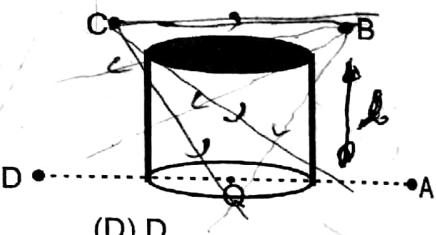
Figure shows a charge  $Q$  placed at the centre of open face of a cylinder as shown in figure. A second charge  $q$  is placed at one of the positions A, B, C and D, out of which positions A and D are lying on a straight line parallel to open face of cylinder. In which position(s) of this second charge, the flux of the electric field through the cylinder remains unchanged?

(A) A

(B) B

(C) C

(D) D

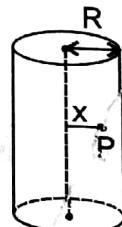


~~14.~~

A long cylindrical volume (of radius  $R$ ) contains a uniformly distributed charge of density  $\rho$ . Consider a point P inside the cylindrical volume at a distance  $x$  from its axis as shown in the figure. Here  $x$  can be more than or less than  $R$ . Electric field at point P is :

$$E = \frac{\rho \pi R^2 x}{2\pi R^2 \epsilon_0} = \frac{\rho x}{2\epsilon_0}$$

$$E = \frac{\rho \cdot \pi x^2}{2\pi R^2 \epsilon_0} = \frac{\rho x}{2\epsilon_0}$$



~~(A)~~  $\frac{\rho x}{2\epsilon_0}$  if  $x < R$

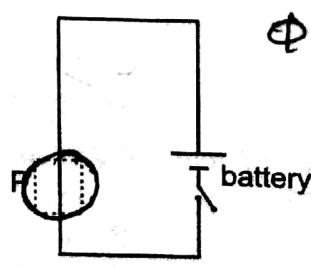
~~(B)~~  $\frac{\rho x}{\epsilon_0}$  if  $x < R$

~~(C)~~  $\frac{\rho R^2}{4\epsilon_0 x}$  if  $x > R$

~~(D)~~  $\frac{\rho R^2}{2\epsilon_0 x}$  if  $x > R$

~~15.~~

An imaginary closed surface P is constructed around a neutral conducting wire connected to a battery and a switch as shown in figure. As the switch is closed, the free electrons in the wire start moving along the wire. In any time interval, the number of electrons entering the closed surface P is equal to the number of electrons leaving it. On closing the switch, the flux of the electric field through the closed surface:



$$\Phi = \frac{q_{in}}{\epsilon_0}$$

~~(A)~~ remains unchanged ~~(B)~~ remains zero

~~(C)~~ is increased

An oil drop has a charge  $-9.6 \times 10^{-19}$  C and mass  $1.6 \times 10^{-15}$  gm. When allowed to fall, due to air resistance force it attains a constant velocity. Then if a uniform electric field is to be applied vertically to make the oil drop ascend up with the same constant speed, which of the following are correct. ( $g = 10 \text{ ms}^{-2}$ ) (Assume that the magnitude of resistance force is same in both the cases)

- (A) The electric field is directed upward      (B) The electric field is directed downward ✓  
 (C) The intensity of electric field is  $\frac{1}{3} \times 10^2 \text{ N C}^{-1}$       (D) The intensity of electric field is  $\frac{1}{6} \times 10^5 \text{ N C}^{-1}$

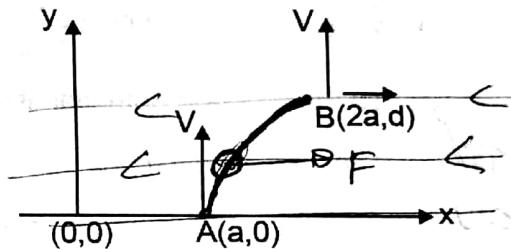
A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre.

- (A) Increases as r increases, for  $r \leq R$       (B) decreases as r increases, for  $0 < r < \infty$ .  
 (C) decreases as r increases, for  $R < r < \infty$ .      (D) is discontinuous at  $r = R$

$$\frac{kq}{r^2}$$

A uniform electric field of strength E exists in a region. An electron (charge  $-e$ , mass m) enters a point A with velocity  $V\hat{j}$ . It moves through the electric field & exits at point B. Then :

- (A)  $E = -\frac{2amv^2}{ed^2}\hat{i}$ .  
 (B) Rate of work done by the electric field at B is  $\frac{4ma^2v^3}{d^3}$ .  
 (C) Rate of work by the electric field at A is zero.  
 (D) Velocity at B is  $\frac{2av}{d}\hat{i} + v\hat{j}$ .



Which of the following quantities depends on the choice of zero potential or zero potential energy ?

- (A) Potential at a particular point  
 (B) Change in potential energy of a two-charge system  
 (C) Potential energy of a two-charge system  
 (D) Potential difference between two points

$$\frac{kq_1 q_2}{r}$$

The electric field intensity at a point in space is equal in magnitude to :

- (A) Magnitude of the potential gradient there  
 (B) The electric charge there  
 (C) The magnitude of the electric force, a unit charge would experience there  
 (D) The force, an electron would experience there

The electric field produced by a positively charged particle, placed in an xy-plane is  $7.2(4\hat{i} + 3\hat{j}) \text{ N/C}$  at the point (3 cm, 3cm) and  $100\hat{i} \text{ N/C}$  at the point (2 cm, 0).

- (A) The x-coordinate of the charged particle is  $-2\text{cm}$ .  
 (B) The charged particle is placed on the x-axis.  
 (C) The charge of the particle is  $10 \times 10^{-12} \text{ C}$ .  
 (D) The electric potential at the origin due to the charge is  $9V$ .

At distance of 5cm and 10cm outwards from the surface of a uniformly charged solid sphere, the potentials are 100V and 75V respectively. Then :

- (A) Potential at its surface is 150V.      (B) The charge on the sphere is  $(5/3) \times 10^{-9} \text{ C}$ .  
 (C) The electric field on the surface is  $1500 \text{ V/m}$ .      (D) The electric potential at its centre is 225V.

10. The electric potential decreases uniformly from 180 V to 20 V as one moves on the X-axis from  $x = -2 \text{ cm}$  to  $x = +2 \text{ cm}$ . The electric field at the origin :

- (A) must be equal to  $40 \text{ V/cm}$ .  
 (B) may be equal to  $40 \text{ V/cm}$ .  
 (C) may be greater than  $40 \text{ V/cm}$ .  
 (D) may be less than  $40 \text{ V/cm}$ .

13. Small identical balls with equal charges of magnitude 'q' each are fixed at the vertices of a regular 2014-gon (a polygon of 2014 sides) with side 'a' =  $4\mu\text{m}$ . At a certain instant, one of the balls is released and a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by  $K = 9 \times 10^9 \text{ J}$  at a sufficiently large distance from the polygon. Determine the charge q in mC.
14. The electric potential varies in space according to the relation:  $V = 3x + 4y$ . A particle of mass 10 Kg starts from rest from point (2, 3.2) under the influence of this field. Find the speed in m/s of the particle when it crosses the x-axis. The charge on the particle is +1C. Assume V and (x, y) are in S.I. units.
15. The electric field in a region is given by  $\vec{E} = E_0 x \hat{i}$ . The charge contained inside a cubical volume bounded by the surface  $x = 0, x = 2\text{m}, y = 0, y = 2\text{m}, z = 0$  and  $z = 2\text{m}$  is  $n\epsilon_0 E_0$ . Find the value of n.
16. The volume charge density as a function of distance X from one face inside a unit cube is varying as shown in the figure. Find the total flux (in S.I. units) through the cube (If  $\rho_0 = 8.85 \times 10^{-12} \text{ C/m}^3$ ):
- 
- density
- $12\rho_0$
- $1/4 \quad 3/4 \quad 1$  (in m)
17. A very long uniformly charged thread oriented along the axis of a circle of radius  $R = 1\text{m}$ , rests on its centre with one of the ends. The charge per unit length on the thread is  $\lambda = 16\epsilon_0$ . Find the flux of the vector E through the circle area in ( $\text{Vm}$ ).
18. The electric field in a region is radially outward with magnitude  $E = 2r$ . The charge contained in a sphere of radius  $a = 2\text{m}$  centred at the origin is  $4x\pi\epsilon_0$ . Find the value of x.
19. Two isolated metallic solid spheres of radii R and  $2R$  are charged such that both of these have same charge density  $12\mu\text{C/m}^2$ . The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere in  $\mu\text{C/m}^2$ .
20. A metallic sphere of radius R is cut in two parts along a plane whose minimum distance from the sphere's centre is  $h = \frac{R}{2}$  and the sphere is uniformly charged by a total electric charge Q. The minimum force necessary to hold the two parts of the sphere together is  $\frac{3kQ^2}{pR^2}$ . Then find the value of p?

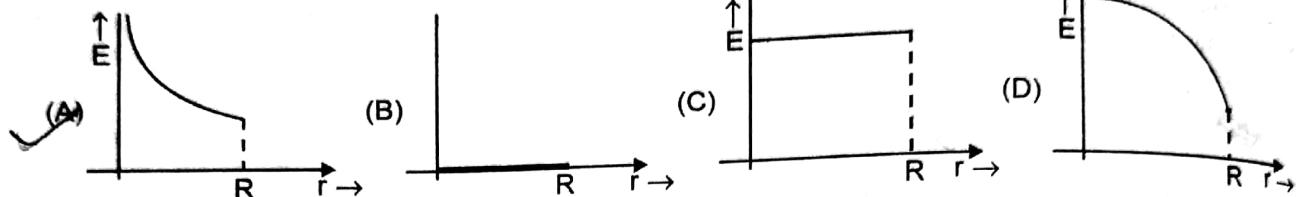
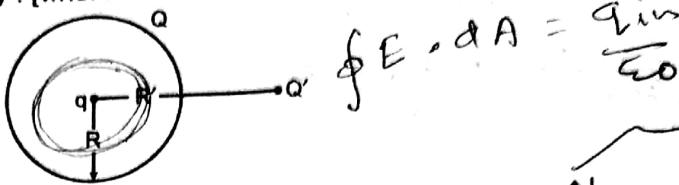
### PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Select the correct alternative :
- (A) The charge gained by the uncharged body from a charged body due to conduction is equal to half of the total charge initially present.  $\rightarrow$  depends on size & shape of body
- (B) The magnitude of charge increases with the increase in velocity of charge
- (C) Charge cannot exist without matter although matter can exist without net charge
- (D) Between two non-magnetic substances repulsion is the true test of electrification (electrification means body has net charge)
2. Two equal negative charges  $-q$  each are fixed at the points  $(0, a)$  and  $(0, -a)$  on the y-axis. A positive charge Q is released from rest at the point  $(2a, 0)$  on the x-axis. The charge Q will :
- (A) Execute simple harmonic motion about the origin
- (B) At origin velocity of particle is maximum. because  $F_{\text{net}} = 0$
- (C) Move to infinity
- (D) Execute oscillatory but not simple harmonic motion

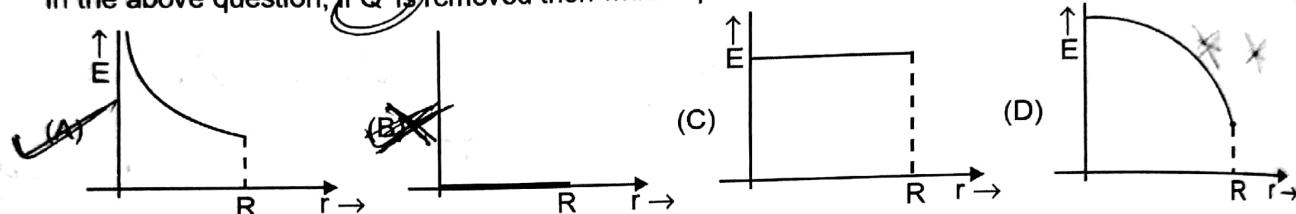
## Electrostatics

6. An infinitely long string uniformly charged with a linear charge density  $\lambda_1$  and a segment of length  $\ell$  uniformly charged with linear charge density  $\lambda_2$  lie in a plane at right angles to each other and separated by a distance  $r_0$  as shown in figure. The force with which these two interact is  $\frac{\lambda_1 \lambda_2}{4\pi\epsilon_0} \ln(x)$ . If  $\ell = r_0$ , then find the value of  $x$ .
- 
7. A cavity of radius  $r$  is present inside a solid dielectric sphere of radius  $R$ , having a volume charge density of  $\rho$ . The distance between the centres of the sphere and the cavity is  $a$ . An electron  $e$  is kept inside the cavity at an angle  $\theta = 45^\circ$  as shown. The electron (mass  $m$  and charge  $-e$ ) touches the sphere again after time  $\left(\frac{P\sqrt{2}mr\epsilon_0}{eap}\right)^{1/2}$ ? Find the value of  $P$ . Neglect gravity.
- 
8. A solid conductor sphere having a charge  $Q$  is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be  $30V$ . If the shell is now given a charge  $-3Q$ , the new potential difference between the same two surfaces is  $x V$ . Find the value of  $x$ :
9. A hollow sphere having uniform charge density  $\rho$  (charge per unit volume) is shown in figure. If  $b = 2a$  and potential difference between A and B is  $\frac{\rho a^2}{n\epsilon_0}$ . Then find the value of  $n$ :
- 
10. Two identical particles of mass  $m$  carry a charge  $Q$  each. Initially, one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first particle from a large distance, with a speed  $V$ . The closest distance of approach is  $\frac{xQ^2}{4\pi\epsilon_0 mv^2}$ . Find the value of  $x$ .
11. A particle having charge  $+q$  is fixed at a point O and a second particle of mass  $m$  and having charge  $-q_0$  moves with constant speed in a circle of radius  $r$  about the charge  $+q$ . The energy required to be supplied to the moving charge to increase radius of the path to  $2r$  is  $\frac{qq_0}{n\pi\epsilon_0 r}$ . Find the value of  $n$ .
12. A positive charge  $+Q$  is fixed at a point A. Another positively charged particle of mass  $m$  and charge  $+q$  is projected from a point B with velocity  $u$  as shown in the figure. The point B is at large distance from A and at distance ' $d$ ' from the line AC. The initial velocity is parallel to the line AC. The point C is at very large distance from A. Find the minimum distance (in meter) of  $+q$  from  $+Q$  during the motion. Take  $Qq = 4\pi\epsilon_0 mu^2 d$  and  $d = (\sqrt{2} - 1)$  meter.
-

18. A charge 'q' is placed at the centre of a conducting spherical shell of radius  $R$ , which is given a charge  $Q$ . An external charge  $Q'$  is also present at distance  $R'$  ( $R' > R$ ) from 'q'. Then the resultant field will be best represented for region  $r < R$  by : [where  $r$  is the distance of the point from q]



19. In the above question, if  $Q'$  is removed then which option is correct :



## PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Two small ~~equally charged identical~~ conducting balls are suspended from long threads from the same point. The charges and masses of the balls are such that they are in equilibrium. The distance between them is  $a = (108)^{\frac{1}{3}}$  cm (the length of the threads  $L \gg a$ ). One of the ball is discharged. After sometime both balls comes to rest in equilibrium. What will be the distance  $b$  (in cm) between the balls when equilibrium is restored?

2. Two identical spheres of same mass and specific gravity (which is the ratio of density of a substance and density of water) 2.4 have different charges of  $Q$  and  $-3Q$ . They are suspended from two strings of same length  $\ell$  fixed to points at the same horizontal level, but distant  $\ell$  from each other. When the entire set up is transferred inside a liquid of specific gravity 0.8, it is observed that the inclination of each string in equilibrium remains unchanged. The dielectric constant of the liquid is K. Find the value of  $4K$ .

3. Two small balls of masses  $3m$  and  $2m$  and each having charges  $Q$  are connected by a string passing over a fixed pulley. Calculate the acceleration of the balls (in  $m/sec^2$ ) if the whole assembly is located in a uniform electric field  $E = \frac{mg}{2Q}$  acting vertically downwards. Neglect any interaction between the balls.

$$\text{Take } g = 10 \text{ m/s}^2 \quad 8/5 = 2m/s^2$$

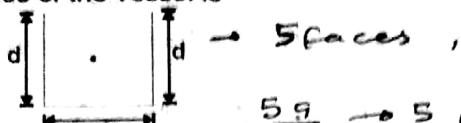
4. Two like charged, infinitely long parallel wires with the same linear charge density of  $3 \times 10^{-8} \text{ C/cm}$  are 2 cm apart. The work done against electrostatic force per unit length to be done in bringing them closer by 1 cm is  $\frac{x}{100} \text{ J/m}$ : Find the integer closest to x.

- The electric field at a point A on the perpendicular bisector of a uniformly charged wire of length  $\ell = 3\text{m}$  and total charge  $q = 5 \text{ nC}$  is  $x \text{ V/m}$ . The distance of A from the centre of the wire is  $b = 2\text{m}$ . Find the value of x.

10. A charge  $q$  is placed at the centre of the cubical vessel (with one face open) as shown in figure. The flux of the electric field through the surface of the vessel is

$$\Phi = \frac{q}{\epsilon_0} \rightarrow \text{enough}$$

~~6 faces~~



~~5 faces~~

~~$\frac{1}{6\epsilon_0}$~~

~~(A) zero~~

(B)  $q/\epsilon_0$

(C)  $\frac{q}{4\epsilon_0}$

$$\frac{5q}{6\epsilon_0} \rightarrow 5 \text{ faces}$$

~~(D)  $5q/6\epsilon_0$~~

11. The electric field above a uniformly charged nonconducting sheet is  $E$ . If the nonconducting sheet is now replaced by a conducting sheet, with the charge same as before, the new electric field at the same point is :

~~(A)  $2E$~~

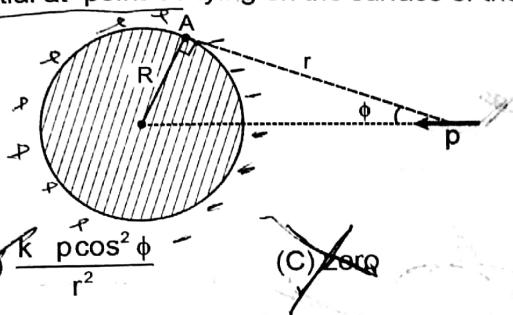
~~(B)  $E$~~

(C)  $\frac{E}{2}$

(D) None of these

12. A dipole having dipole moment  $p$  is placed in front of a solid uncharged ~~conducting~~ sphere as shown in the diagram. The net potential at point A lying on the surface of the sphere is :

~~\*~~



$V_A = V_{\text{dipole}}$

~~+  
V<sub>induced</sub>~~

~~(A)  $\frac{kpcos\phi}{r^2}$~~

~~(B)  $\frac{kpcos^2\phi}{r^2}$~~

~~(C) zero~~

(D)  $\frac{2kpcos^2\phi}{r^2}$

13. The net charge given to an isolated conducting solid sphere:

~~(A) must be distributed uniformly on the surface~~ (B) may be distributed uniformly on the surface  
~~(C) must be distributed uniformly in the volume~~ (D) may be distributed uniformly in the volume.

14. The net charge given to a solid insulating sphere: ~~is not isolated necessarily~~

(A) must be distributed uniformly in its volume  
(B) may be distributed uniformly in its volume  
(C) must be distributed uniformly on its surface

~~(D) the distribution will depend upon whether other charges are present or not.~~

15. A charge  $Q$  is kept at the centre of a conducting sphere of inner radius  $R_1$  and outer radius  $R_2$ . A point charge  $q$  is kept at a distance  $r (> R_2)$  from the centre. If  $q$  experiences an electrostatic force 10 N then assuming that no other charges are present, electrostatic force experienced by  $Q$  will be :

~~(A) -10 N~~ (B) 0 (C) 20 N (D) none of these

16. Two uniformly charged non-conducting hemispherical shells each having uniform charge density  $\sigma$  and radius  $R$  form a complete sphere (not stuck together) and surround a concentric spherical conducting shell of radius  $R/2$ . If hemispherical parts are in equilibrium then minimum surface charge density of inner conducting shell is:

(A)  $-2\sigma$  (B)  $-\sigma/2$  (C)  $-\sigma$  (D)  $2\sigma$

17. A solid ~~metallic~~ sphere has a charge  $+3Q$ . Concentric with this sphere is a ~~conducting~~ spherical shell having charge  $-Q$ . The radius of the sphere is  $a$  and that of the spherical shell is  $b (> a)$ . What is the electric field at a distance  $r (a < r < b)$  from the centre?

(A)  $\frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

(B)  $\frac{1}{4\pi\epsilon_0} \frac{3Q}{r}$

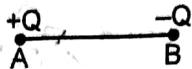
~~(C)  $\frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$~~

(D)  $\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

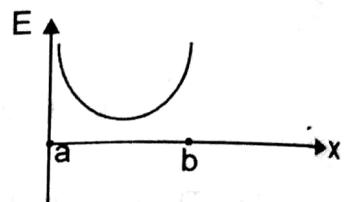
## Electrostatics

3. Five balls, numbered 1 to 5, are suspended using separate threads. Pairs (1, 2), (2, 4), (4, 1) show electrostatic attraction, while pairs (2, 3) and (4, 5) show repulsion. Therefore ball 1 :  
 (A) Must be positively charged      (B) Must be negatively charged  
 (C) May be neutral      (D) Must be made of metal

4. Two point charges of same magnitude and opposite sign are fixed at points A and B. A third small point charge is to be balanced at point P by the electrostatic force due to these two charges. The point P :  
 (A) lies on the perpendicular bisector of line AB      (B) is at the mid point of line AB  
 (C) lies to the left of A      (D) none of these.



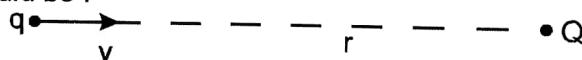
5. Two point charges a & b, whose magnitudes are same are positioned at a certain distance from each other with a at origin. Graph is drawn between electric field strength at points between a & b and distance x from a. E is taken positive if it is along the line joining from a to b. From the graph, it can be decided that



- (A) a is positive, b is negative      (B) a and b both are positive  
 (C) a and b both are negative      (D) a is negative, b is positive

6. A solid sphere of radius R has a volume charge density  $\rho = \rho_0 r^2$  (Where  $\rho_0$  is a constant and r is the distance from centre). At a distance x from its centre (for  $x < R$ ), the electric field is directly proportional to :  
 (A)  $1/x^2$       (B)  $1/x$       (C)  $x^3$       (D)  $x^2$

7. A charged particle 'q' is shot from a large distance with speed v towards a fixed charged particle Q. It approaches Q upto a closest distance r and then returns. If q were given a speed '2v', the closest distance of approach would be :



- (A) r      (B)  $2r$       (C)  $\frac{r}{2}$       (D)  $\frac{r}{4}$

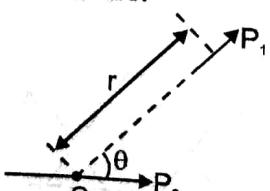
- \* 8. For an infinite line of charge having charge density  $\lambda$  lying along x-axis, the work required in moving charge q from C to A along arc CA is :

$$\begin{aligned} \Delta V &= \int_{\alpha}^{2a} E \cdot dr \\ &= \frac{2k\lambda}{\pi\epsilon_0} \cdot \int_{\alpha}^{2a} \frac{1}{r} \cdot dr = 2k\lambda \ln 2 \end{aligned}$$

*sloopy*

(A)  $\frac{q\lambda}{\pi\epsilon_0} \log_e \sqrt{2}$       (B)  $\frac{q\lambda}{4\pi\epsilon_0} \log_e \sqrt{2}$       (C)  $\frac{q\lambda}{4\pi\epsilon_0} \log_e 2$       (D)  $\frac{q\lambda}{2\pi\epsilon_0} \log_e \frac{1}{2}$

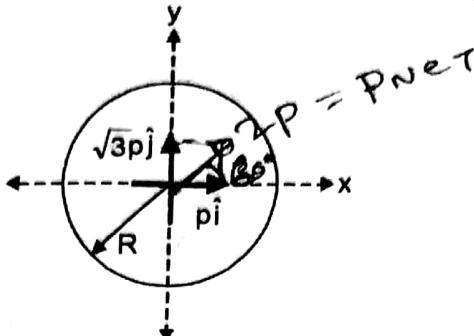
9. Two short electric dipoles are placed as shown ( $r$  is the distance between their centres). The energy of electric interaction between these dipoles will be:



(C is centre of dipole of moment  $P_2$ )

- (A)  $\frac{2k P_1 P_2 \cos\theta}{r^3}$       (B)  $\frac{-2k P_1 P_2 \cos\theta}{r^3}$       (C)  $\frac{-2k P_1 P_2 \sin\theta}{r^3}$       (D)  $\frac{-4k P_1 P_2 \cos\theta}{r^3}$

Column I gives a situation in which two dipoles of dipole moment  $\vec{p}_1$  and  $\sqrt{3}\vec{p}_1$  are placed at origin. A circle of radius R with centre at origin is drawn as shown in figure. Column II gives coordinates of certain positions on the circle. Match the statements in Column I with the statements in Column II.



$$\cos 60^\circ = \frac{1}{2}, \sin 60^\circ = \frac{\sqrt{3}}{2}$$

#### Column-I

(A) The coordinate(s) of point on circle where potential is maximum

#### Column-II

(p)  $\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$  ~~A C~~

(B) The coordinate(s) of point on circle where potential is zero

(q)  $\left(-\frac{R}{2}, -\frac{\sqrt{3}R}{2}\right)$  ~~C~~

(C) The coordinate(s) of point on circle where magnitude of electric field

(r)  $\left(-\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$  ~~B D~~

intensity is  $\frac{1}{4\pi\epsilon_0} \frac{4p}{R^3}$ .  $\theta = 0^\circ, 180^\circ$

(s)  $\left(\frac{\sqrt{3}R}{2}, -\frac{R}{2}\right)$  ~~B D~~

(D) The coordinate(s) of point on circle where magnitude of electric field

intensity is  $\frac{1}{4\pi\epsilon_0} \frac{2p}{R^3}$

## Exercise-2

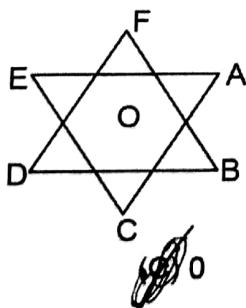
~~Marked Questions~~ can be used as Revision Questions.

### PART - I : ONLY ONE OPTION CORRECT TYPE

1. A total charge of  $20 \mu\text{C}$  is divided into two parts and placed at some distance apart. If the charges experience maximum coulombian repulsion, the charges should be :

- (A)  $5\mu\text{C}, 15\mu\text{C}$       (B)  $10\mu\text{C}, 10\mu\text{C}$       (C)  $12\mu\text{C}, 8\mu\text{C}$       (D)  $\frac{40}{3}\mu\text{C}, \frac{20}{3}\mu\text{C}$

The magnitude of electric force on  $2\mu\text{c}$  charge placed at the centre O of two equilateral triangles each of side  $10\text{ cm}$ , as shown in figure is P. If charge A, B, C, D, E & F are  $2\mu\text{c}$ ,  $2\mu\text{c}$ ,  $2\mu\text{c}$ ,  $-2\mu\text{c}$ ,  $-2\mu\text{c}$ ,  $-2\mu\text{c}$  respectively, then P is :



- (A)  $21.6\text{ N}$       (B)  $64.8\text{ N}$

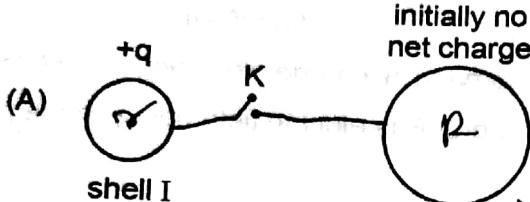
(D)  $43.2\text{ N}$

### PART - III : MATCH THE COLUMN

Column I gives certain situations involving two thin conducting shells connected by a conducting wire via a key K. In all situations, one sphere has net charge  $+q$  and other sphere has no net charge. After the key K is pressed, column II gives some resulting effects. Match the figures in Column I with the statements in Column II.

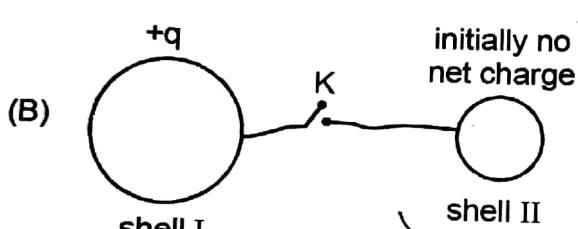
**Column-I**

**Column-II**



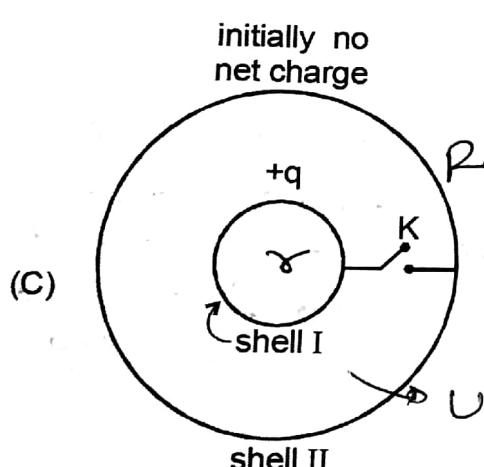
(p) Charge flows through connecting wire.

A B C



(q) Potential energy of system of spheres decreases.

A B C



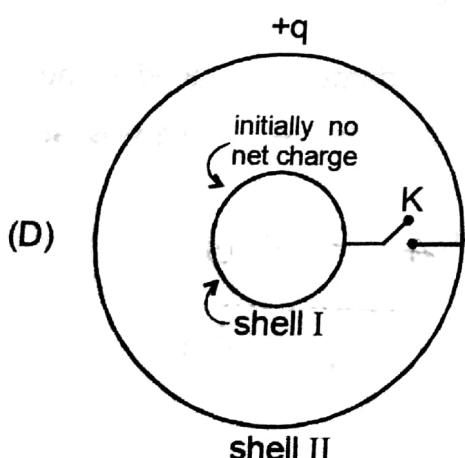
(r) No heat is produced.

D

$$U_i = \frac{kq^2}{2r}, \quad U_f = \frac{kq^2}{2R}$$

$$\Delta U = \frac{kq^2}{2r} \left( \frac{1}{R} - \frac{1}{r} \right) < 0$$

Heat produced



(s) The shell I has no charge after equilibrium is reached.

C D

If  $\Delta U \rightarrow -ve$ , then heat is produced