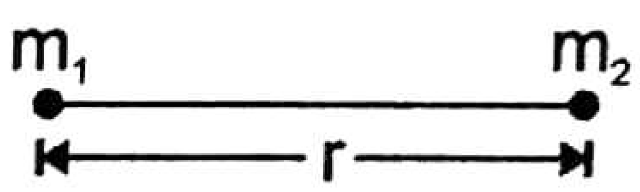
1. INTRODUCTION

The motion of celestial bodies such as the sun, the moon, the earth and the planets etc. has been a subject of fascination since time immemorial. Indian astronomers of the ancient times have done brilliant work in this field, the most notable among them being Arya Bhatt the first person to assert that all planets including the earth revolve round the sun.

A millennium later the Danish astronomer Tycobrahe (1546-1601) conducted a detailed study of planetary motion which was interpreted by his pupil Johnaase Kepler (1571-1630), ironically after the master himself had passed away. Kepler formulated his important findings in three laws of planetary motion. The basis of astronomy is gravitation.

2. UNIVERSAL LAW OF GRAVITATION : NEWTON'S LAW

According to this law "Each particle attracts every other particle. The force of attraction between them is directly proportional to the product of their masses and inversely proportional to square of the distance between them".



$F \propto \frac{m_1 m_2}{r^2}$ or $F = G\frac{m_1 m_2}{r^2}$

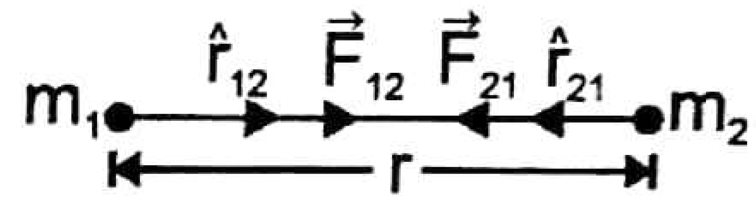
where $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$ is the universal gravitational constant.

Dimensional formula of G :

$$F = \frac{Fr^2}{m_1 m_2} = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1} L^3 T^{-2}]$$

Newton's Law of gravitation in vector form :

$$\vec{F}_{12} = -\frac{Gm_1 m_2}{r^2}\hat{r}_{12} \quad \& \quad \vec{F}_{21} = \frac{Gm_1 m_2}{r^2}\hat{r}_{21}$$



Where $\vec{F}_{12}$ is the force on mass m_1 exerted by mass m_2 and vice-versa.

Now $\hat{r}_{12} = -\hat{r}_{21}$, Thus $\vec{F}_{21} = \frac{-Gm_1 m_2}{r^2}\hat{r}_{12}$. Comparing above, we get $\vec{F}_{12} = -\vec{F}_{21}$

Important characteristics of gravitational force

(i) Gravitational force between two bodies form an action and reaction pair i.e. the forces are equal in magnitude but opposite in direction.

(ii) Gravitational force is a central force i.e. it acts along the line joining the centers of the two interacting bodies.

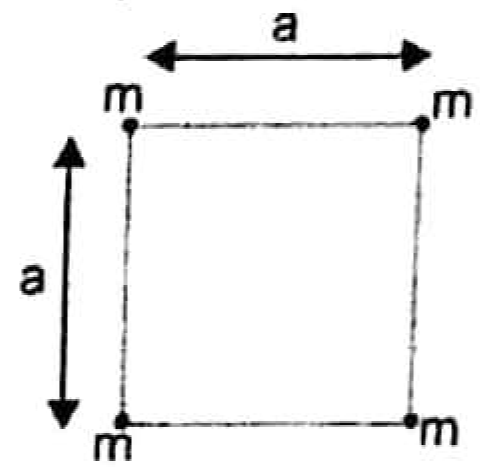
(iii) Gravitational force between two bodies is independent of the nature of the medium, in which they lie.

(iv) Gravitational force between two bodies does not depend upon the presence of other bodies.

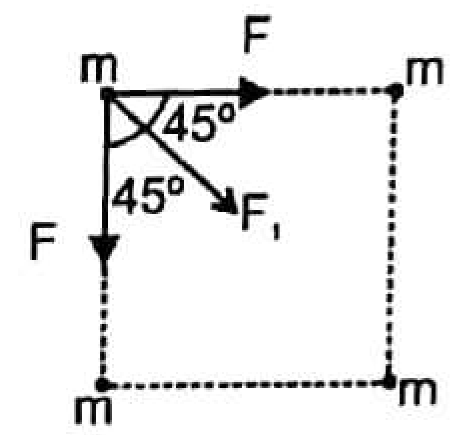
(v) Gravitational force is negligible in case of light bodies but becomes appreciable in case of massive bodies like stars and planets.

(vi) Gravitational force is long range-force i.e., gravitational force between two bodies is effective even if their separation is very large. For example, gravitational force between the sun and the earth is of the order of 10^{27} N although distance between them is 1.5×10^7 km

Example 3. Four point masses each of mass 'm' are placed on the corner of square of side 'a'. Calculat magnitude of gravitational force experienced by each particle.



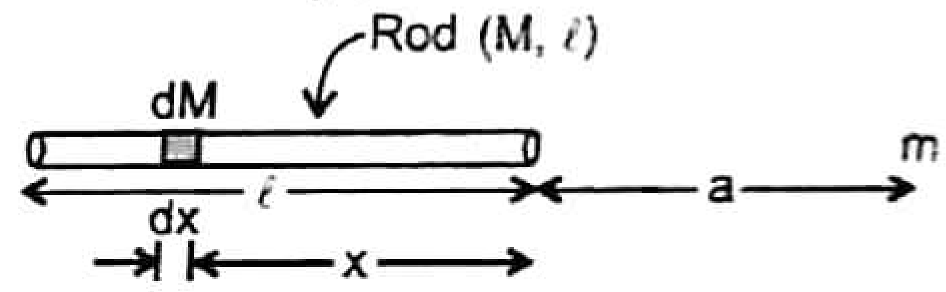
Solution :



F_r = resultant force on each particle = $2F\cos 45° + F_1$

$$= \frac{2G.m^2}{a^2}\cdot\frac{1}{\sqrt{2}} + \frac{Gm^2}{(\sqrt{2}a)^2} = \frac{G.m^2}{2a^2}(2\sqrt{2}+1)$$

Example 4. Find gravitational force exerted by point mass 'm' on uniform rod (mass 'M' and length 'ℓ')

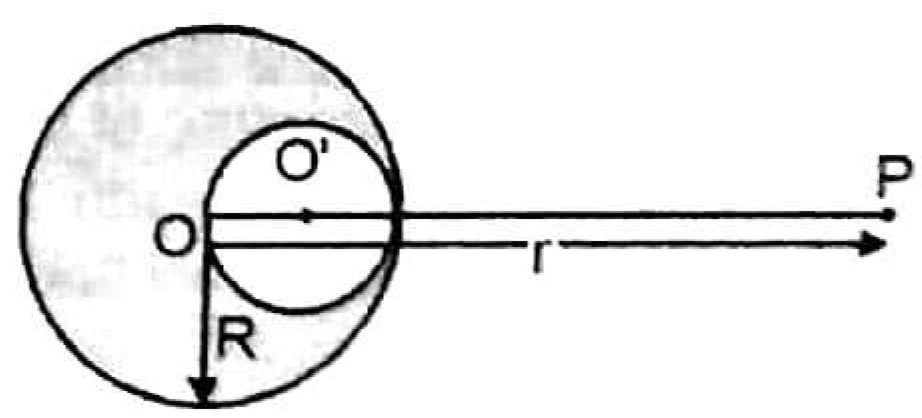


Solution : dF = force on element in horizontal direction = $\frac{G\cdot dM\cdot m}{(x+a)^2}$

where dM = $\frac{M}{\ell}$dx.

$$\therefore\ F = \int dF = \int_0^{\ell}\frac{G.Mmdx}{\ell(x+a)^2} = \frac{G.Mm}{\ell}\int_0^{\ell}\frac{dx}{(x+a)^2} = \frac{G.Mm}{\ell}\left[-\frac{1}{(\ell+a)}+\frac{1}{a}\right] = \frac{GMm}{(\ell+a)a}$$

Example 5. A solid sphere of lead has mass M and radius R. A spherical hollow is dug out from it (see figure). Its boundary passing through the centre and also touching the boundary of the solid sphere. Deduce the gravitational force on a mass m placed at P, which is distant r from O along the line of centres.



Solution : Let O be the centre of the sphere and O' that of the hollow (figure). For an external point the sphere behaves as if its entire mass is concentrated at its centre. Therefore, the gravitational force on a mass 'm' at P due to the original sphere (of mass M) is

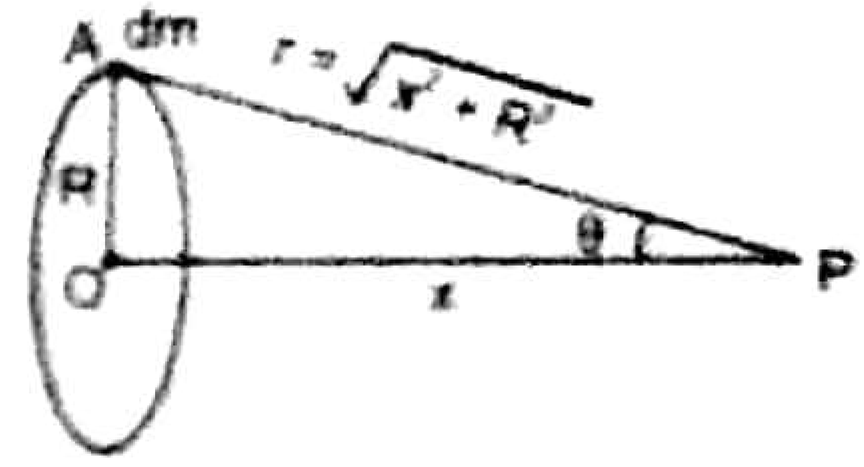
$F = G\frac{Mm}{r^2}$, along PO.

The diameter of the smaller sphere (which would be cut off) is R, so that its radius OO' is R/2. The force on m at P due to this sphere of mass M' (say) would be

$F' = G\frac{M'm}{(r-\frac{R}{2})^2}$ along PO'. $\quad [\because$ distance PO' $= r - \frac{R}{2}]$

Example 7. Calculate gravitational field intensity due to a uniform ring of mass M and radius R at a distance x on the axis from center of ring.

Solution : Consider any particle of mass dm. Gravitational field at point P due to dm



$$dE = \frac{Gdm}{r^2} \text{ along PA}$$

Component along PO is $dE \cos\theta = \dfrac{Gdm}{r^2}\cos\theta$

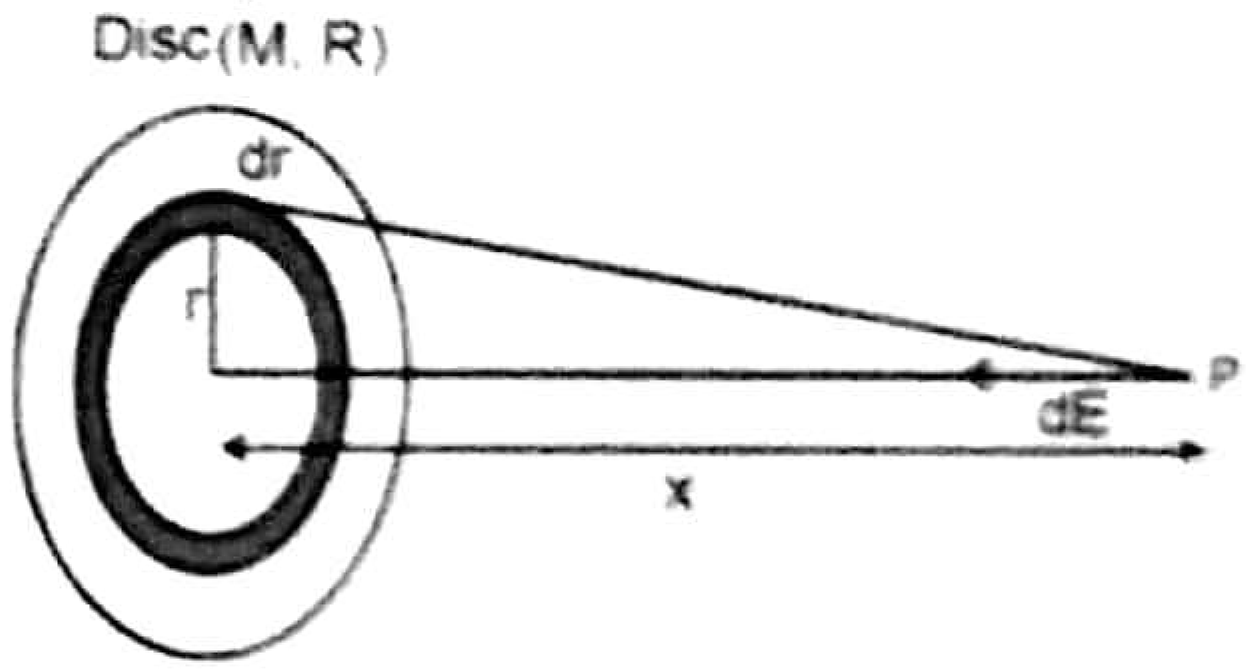
Net gravitational field at point P is

$$E = \int \frac{Gdm}{r^2}\cos\theta = \frac{G\cos\theta}{r^2}\int dm$$

$$= \frac{GMx}{\left(R^2 + x^2\right)^{3/2}} \text{ towards the center of ring}$$

Example 8. Calculate gravitational field intensity at a distance x on the axis from centre of a uniform disc of mass M and radius R.

Solution : Consider a elemental ring of radius r and thickness dr on surface of disc as shown in figure



Gravitational field due to elemental ring

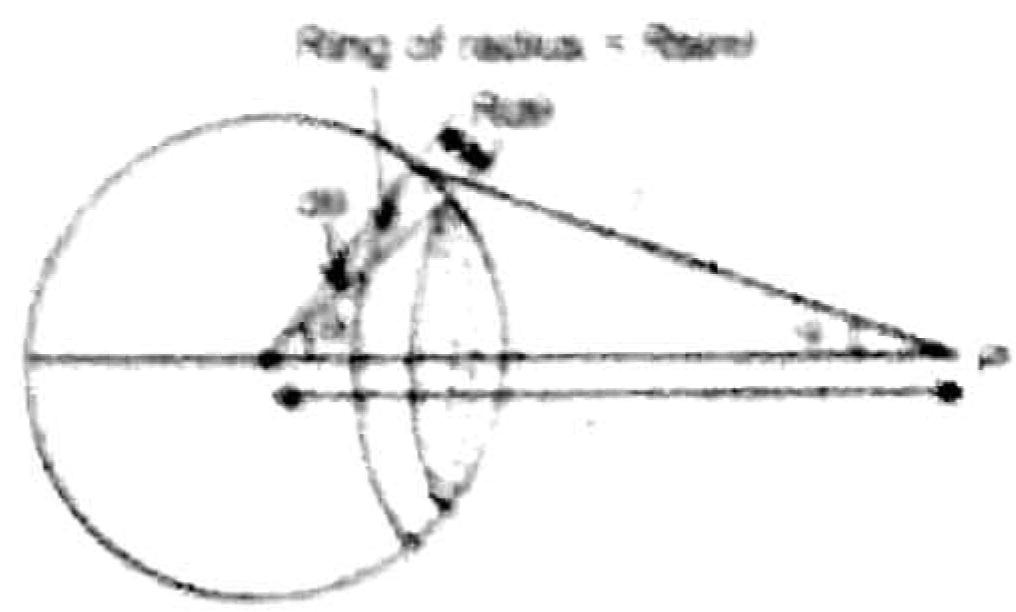
$$dE = \frac{GdMx}{(x^2 + r^2)^{3/2}} \quad \text{Here } dM = \frac{M}{\pi R^2} \cdot 2\pi r dr = \frac{2M}{R^2} r dr$$

$$\therefore \; dE = \frac{G.2Mxrdx}{R^2(x^2 + r^2)^{3/2}}$$

$$\therefore \; E = \int_0^R \left(\frac{2GMx}{R^2}\right)\frac{rdr}{(x^2 + r^2)^{3/2}} \qquad E = \frac{2GMx}{R^2}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}}\right]$$

Example 9. For a given uniform spherical shell of mass M and radius R, find gravitational field at a distance r from centre in following two cases. (a) $r \geq R$ (b) $r < R$

Solution :



$$dE = \frac{GdM}{\ell^2}\cdot\cos\alpha \qquad r \geq R$$

$$dM = \frac{M}{4\pi R^2} \times 2\pi R \sin\theta \, R d\theta$$

$$dM = \frac{M}{2}\sin\theta \, d\theta$$

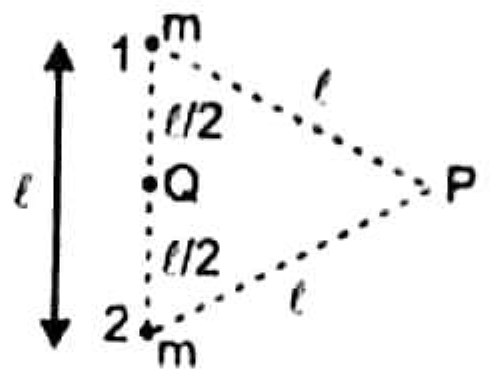
$$dE = \frac{GM \sin\theta \cos\alpha \, d\theta}{2\ell^2}$$

Now $\ell^2 = R^2 + r^2 - 2rR\cos\theta$ (1)

$R^2 = \ell^2 + r^2 - 2\ell r\cos\alpha$ (2)

$$\cos\alpha = \frac{\ell^2 + r^2 - R^2}{2\ell r}$$

Example 11.



Find out potential at P and Q due to the two point mass system. Find out work done by external agent in bringing unit mass from P to Q. Also find work done by gravitational force.

Solution : (i) V_{P1} = potential at P due to mass 'm' at '1' = $-\dfrac{Gm}{\ell}$

$$V_{P2} = -\frac{Gm}{\ell}$$

$$\therefore \quad V_P = V_{P1} + V_{P2} = -\frac{2Gm}{\ell}$$

(ii) $V_{Q\,1} = -\dfrac{GM}{\ell/2} \quad \Rightarrow \quad V_{Q2} = -\dfrac{Gm}{\ell/2}$

$$\therefore \quad V_Q = V_{Q1} + V_{Q2} = -\frac{Gm}{\ell/2} - \frac{Gm}{\ell/2} = -\frac{4Gm}{\ell}$$

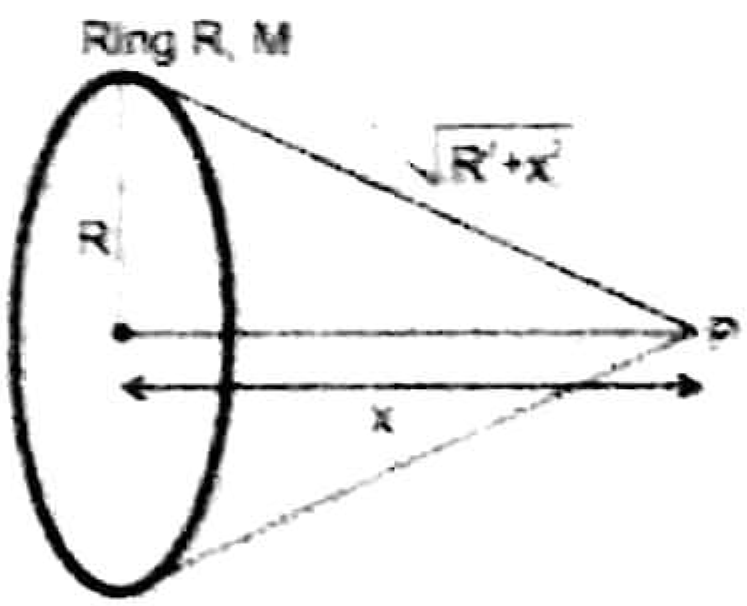
Force at point Q = 0

(iii) work done by external agent = $(V_Q - V_P) \times 1 = -\dfrac{2GM}{\ell}$

(iv) work done by gravitational force = $V_P - V_Q = \dfrac{2GM}{\ell}$

Example 12. Find potential at a point 'P' at a distance 'x' on the axis away from centre of a uniform ring of mass M and radius R.

Solution : Ring can be considered to be made of large number of point masses (m_1 , m_2 ....etc)



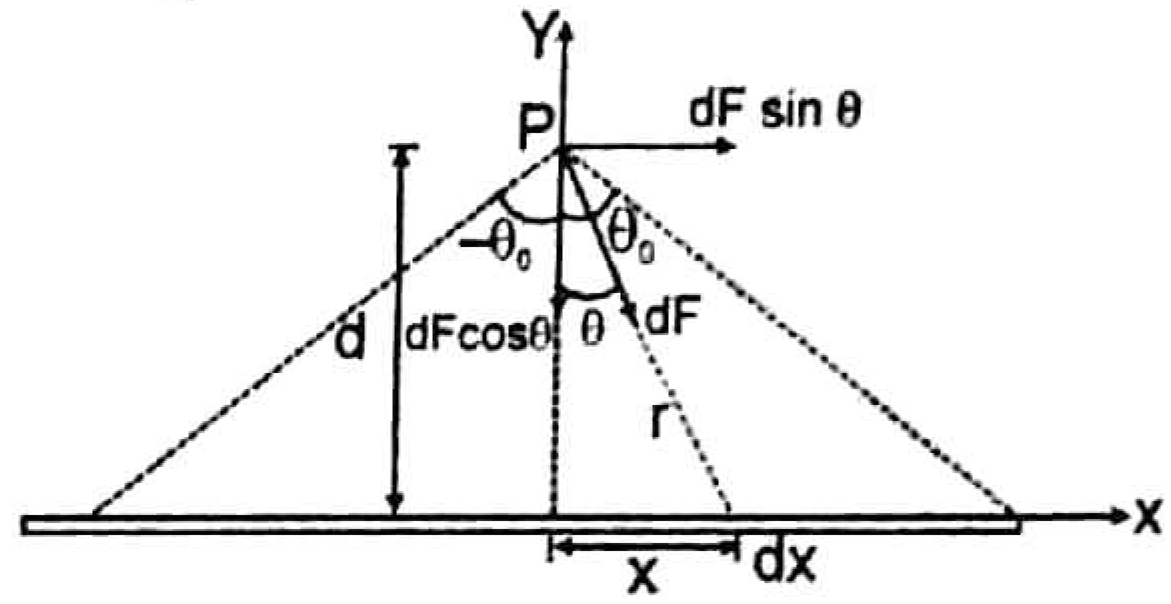
$$V_P = -\frac{Gm_1}{\sqrt{R^2+x^2}} - \frac{Gm_2}{\sqrt{R^2+x^2}} - \ldots\ldots$$

$$= -\frac{G}{\sqrt{R^2+x^2}}(m_1 + m_2\ldots\ldots) = -\frac{GM}{\sqrt{R^2+x^2}}$$

where $M = m_1 + m_2 + m_3 + \ldots\ldots\ldots\ldots$

Potential at centre of ring $= -\dfrac{G.M}{R}$

II. **A linear mass of finite length on its axis :**



(a) Potential :

$$\Rightarrow V = -\frac{GM}{L}\ell n\,(\sec\theta_0 + \tan\theta_0) = -\frac{GM}{L}\ell n\left\{\frac{L+\sqrt{L^2+d^2}}{d}\right\}$$

(b) Field intensity :

$$\Rightarrow E = -\frac{GM}{Ld}\sin\theta_0 = -\frac{GM}{d\sqrt{L^2+d^2}}$$

III. **An infinite uniform linear mass distribution of linear mass density λ, Here $\theta_0 = \frac{\pi}{2}$.**

And noting that $\lambda = \frac{M}{2L}$ in case of a finite rod

we get, for field intensity $E = \frac{2G\lambda}{d}$

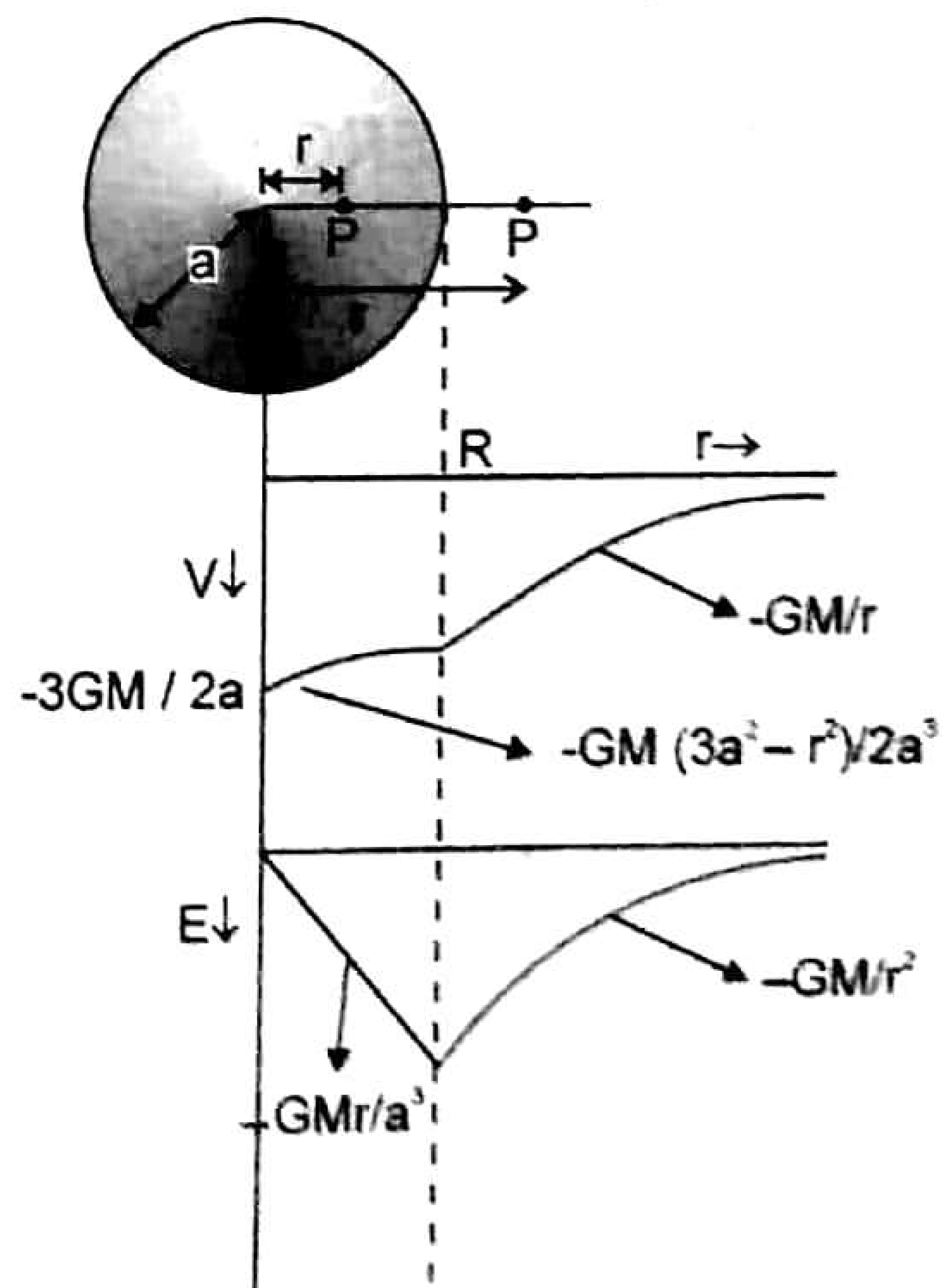
Potential for a mass-distribution extending to infinity is not defined. However even for such mass distributions potential-difference is defined. Here potential difference between points P_1 and P_2 respectively at distances d_1 and d_2 from the infinite rod, $V_{12} = 2G\lambda\ \ell n\frac{d_2}{d_1}$

IV. **Uniform Solid Sphere**

(a) **Point P inside the shell.** $r \le a$, then

$$V = -\frac{GM}{2a^3}(3a^2 - r^2)\ \&\ E = -\frac{GMr}{a^3},\ \text{and at the centre } V = -\frac{3GM}{2a} \text{ and } E = 0$$

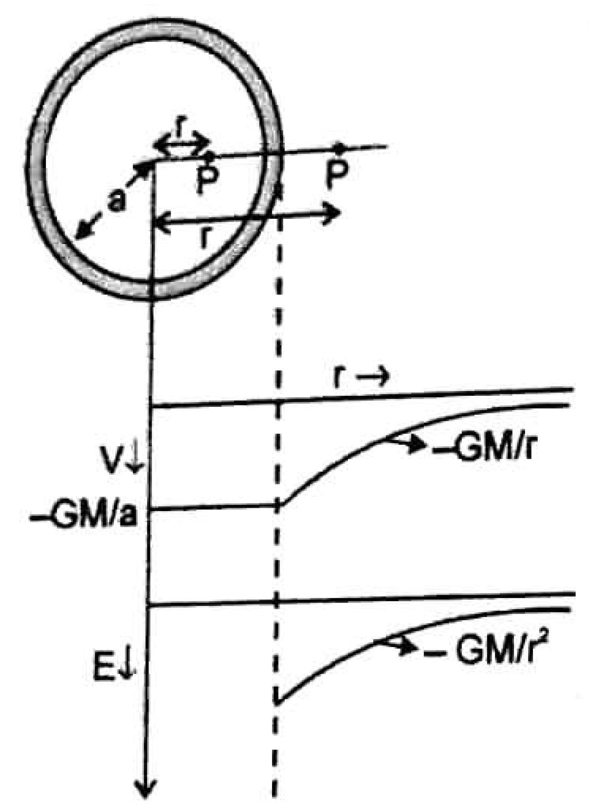
(b) **Point P outside the shell.** $r \ge a$, then $V = -\frac{GM}{r}$ & $E = -\frac{GM}{r^2}$



V. Uniform Thin Spherical Shell

(a) Point P Inside the shell. $r \le a$, then $V = \frac{-GM}{a}$ & $E = 0$

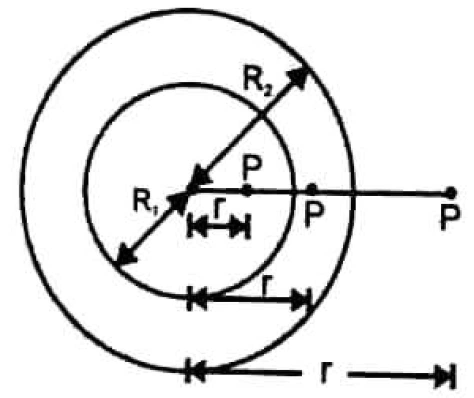
(b) Point P outside shell. $r \ge a$, then $V = \frac{-GM}{r}$ & $E = -\frac{GM}{r^2}$



VI. Uniform Thick Spherical Shell

(a) Point outside the shell $V = -G\frac{M}{r}$; $E = -G\frac{M}{r^2}$

(b) Point inside the Shell $V = -\frac{3}{2}GM\left(\frac{R_2 + R_1}{R_2^2 + R_1R_2 + R_1^2}\right)$

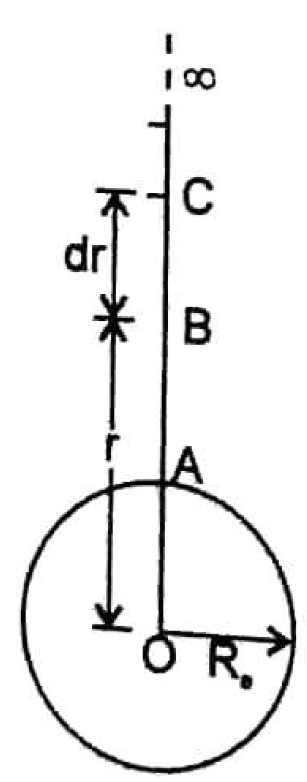


$E = 0$

(c) Point between the two surface $V = -\frac{GM}{2r}\left(\frac{3rR_2^2 - r^3 - 2R_1^3}{R_2^3 - R_1^3}\right)$; $E = -\frac{GM}{r^2}\frac{r^3 - R_1^3}{R_2^3 - R_1^3}$

7. GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy of two mass system is equal to the work done by an external agent in assembling them, while their initial separation was infinity. Consider a body of mass m placed at a distance r from another body of mass M. The gravitational force of attraction between them is given by, $F = \frac{GMm}{r^2}$.



Now, Let the body of mass m is displaced from point. C to B through a distance 'dr' towards the mass M, then work done by internal conservative force (gravitational) is given by,

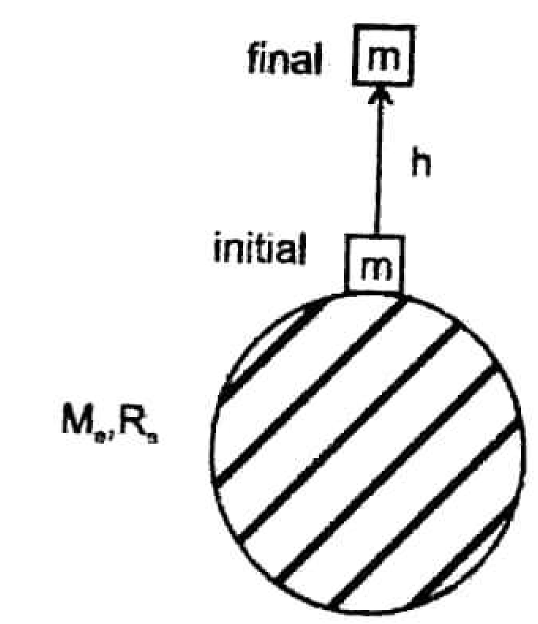
$$dW = F\,dr = \frac{GMm}{r^2}dr \quad \Rightarrow \quad \int dW = \int_{\infty}^{r} \frac{GMm}{r^2}dr$$

$\therefore$ Gravitational potential energy, $\boxed{U = -\frac{GMm}{r}}$

Increase in gravitational potential energy :

Suppose a block of mass m on the surface of the earth. We want to lift this block by 'h' height.

Work required in this process = increase in P.E. = $U_f - U_i = m(V_f - V_i)$



$$W_{ext} = \Delta U = (m)\left[\left(-\frac{GM_e}{R_e + h}\right) - \left(-\frac{GM_e}{R_e}\right)\right]$$

$$W_{ext} = \Delta U = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right)$$

$$= \frac{GM_e m}{R_e}\left(1 - \left(1 + \frac{h}{R_e}\right)^{-1}\right)$$

Binomial

(as h << R_e , we can apply ~~Bionomical~~ theorem)

$$W_{ext} = \Delta U = \frac{GM_e m}{R_e}\left(1 - \left(1 - \frac{h}{R_e}\right)\right) = (m)\left(\frac{GM_e}{R_e^2}\right)h$$

$W_{ext} = \Delta U = mgh$

* This formula is valid only when $h << R_e$

Solved Example

Example 17. A body of mass m is placed on the surface of earth. Find work required to lift this body by a height

(i) $h = \frac{R_e}{1000}$ (ii) $h = R_e$

Solution : (i) $h = \frac{R_e}{1000}$, as $h << R_e$, so

we can apply

$W_{ext} = U\uparrow = mgh$

$$W_{ext} = (m)\left(\frac{GM_e}{R_e^2}\right)\left(\frac{R_e}{1000}\right) = \frac{GM_e m}{1000R_e}$$

(ii) $h = R_e$, in this case h is not very less than R_e, so we cannot apply $\Delta U = mgh$

so we cannot apply $\Delta U = mgh$

$W_{ext} = U\uparrow = U_f - U_i = m(V_f - V_i)$

$$W_{ext} = m\left[\left(-\frac{GM_e}{R_e + R_e}\right) - \left(-\frac{GM_e}{R_e}\right)\right]$$

$$W_{ext} = -\frac{GM_e m}{2R_e}$$

Example 18. Calculate the velocity with which a body must be thrown vertically upward from the surface of the earth so that it may reach a height of 10 R, where R is the radius of the earth and is equal to 6.4×10^8 m. (Earth's mass $= 6 \times 10^{24}$ kg, Gravitational constant $G = 6.7 \times 10^{-11}$ N-m^2/kg^2)

Solution : The gravitational potential energy of a body of mass m on earth's surface is

$$U(R) = -\frac{GMm}{R}$$

where M is the mass of the earth (supposed to be concentrated at its centre) and R is the radius of the earth (distance of the particle from the centre of the earth). The gravitational energy of the same body at a height 10 R from earth's surface, i.e. at a distance 11R from earth's centre is

$$U(11R) = -\frac{GMm}{R}$$

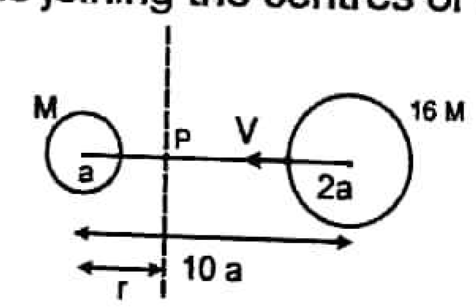
$\therefore$ change in potential energy $\quad U(11R) - U(R) = -\frac{GMm}{11\ R} - \left(-\frac{GMm}{R}\right) = \frac{10}{11}\frac{GMm}{R}$

This difference must come from the initial kinetic energy given to the body in sending it to that height. Now, suppose the body is thrown up with a vertical speed v, so that its initial kinetic energy is $\frac{1}{2}mv^2$. Then $\frac{1}{2}mv^2 = \frac{10}{11}\frac{GMm}{R}$ or $v = \sqrt{\left(\frac{20}{11}\frac{GM}{R}\right)}$.

Putting the given values : $v = \sqrt{\left(\frac{20 \times (6.7 \times 10^{-11} N - m^2/kg^2) \times (6 \times 10^{24} kg)}{11 \ (6.4 \times 10^6 m)}\right)} = 1.07 \times 10^4$ m/s.

Example 19. Distance between centres of two stars is 10 a. The masses of these stars are M and 16 M and their radii are a & 2a respectively. A body is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star?

Solution : Let P be the point on the line joining the centres of the two planets s.t. the net field at it is zero



Then, $\frac{GM}{r^2} - \frac{G.16M}{(10a-r)^2} = 0 \quad \Rightarrow \quad (10\ a - r)^2 = 16\ r^2$

$\Rightarrow \quad 10a - r = 4r \quad \Rightarrow \quad r = 2a$

Potential at point P, $\quad V_P = \frac{-GM}{r} - \frac{G.16M}{(10a-r)} = \frac{-GM}{2a} - \frac{2GM}{a} = \frac{-5GM}{2a}$.

Now if the particle projected from the larger planet has enough energy to cross this point, it will reach the smaller planet.

For this, the K.E. imparted to the body must be just enough to raise its total mechanical energy to a value which is equal to P.E. at point P.

i.e. $\frac{1}{2}mv^2 - \frac{G(16M)m}{2a} - \frac{GMm}{8a} = mV_P$

or, $\frac{v^2}{2} - \frac{8GM}{a} - \frac{GM}{8a} = \frac{-5GMm}{2a}$

or, $v^2 = \frac{45GM}{4a} \quad$ or, $\quad v_{min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$

8. GRAVITATIONAL SELF-ENERGY

The gravitational self-energy of a body (or a system of particles) is defined as the work done by an external agent in assembling the body (or system of particles) from infinitesimal elements (or particles) that are initially an infinite distance apart.

Gravitational self energy of a system of n particles

Potential energy of n particles at an average distance 'r' due to their mutual gravitational attraction is equal to the sum of the potential energy of all pairs of particle, i.e.,

$$U_s = -G \sum_{\substack{\text{all pairs} \\ j \neq i}} \frac{m_i m_j}{r_{ij}}$$

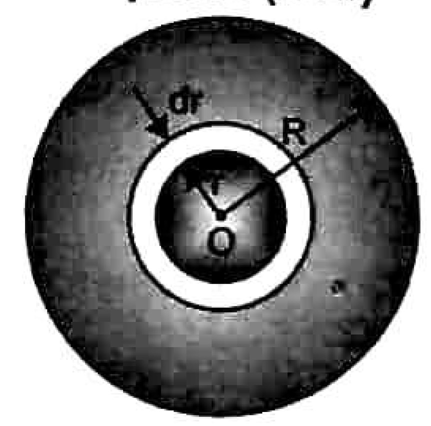
This expression can be written as $U_s = -\frac{1}{2}G\sum_{i=1}^{i=n}\sum_{\substack{j=1 \\ j\neq i}}^{j=n}\frac{m_i m_j}{r_{ij}}$

If consider a system of 'n' particles, each of same mass 'm' and separated from each other by the same average distance 'r', then self energy

or $\quad U_s = -\frac{1}{2}G\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n}\left(\frac{m^2}{r}\right)_{ij}$

Thus on the right hand side 'i' comes 'n' times while 'j' comes (n – 1) times. Thus $U_s = -\frac{1}{2}Gn(n-1)\frac{m^2}{r}$

Gravitational Self energy of a Uniform Sphere (star)



$$U_{sphere} = -G\frac{\left(\frac{4}{3}\pi r^3\rho\right)(4\pi r^2 dr\rho)}{r} \quad \text{where } \rho = \frac{M}{\left(\frac{4}{3}\right)\pi R^3} \quad = -\frac{1}{3}G(4\pi\rho)^2 r^4\, dr,$$

$$U_{star} = -\frac{1}{3}G(4\pi\rho)^2\int_0^R r^4 dr = -\frac{1}{3}G(4\pi\rho)^2\left[\frac{r^5}{5}\right]_0^R = -\frac{3}{5}G\left(\frac{4\pi}{3}R^3\rho\right)^2\frac{1}{R}.$$

$$\therefore \quad U_{star} = -\frac{3}{5}\frac{GM^2}{R}$$

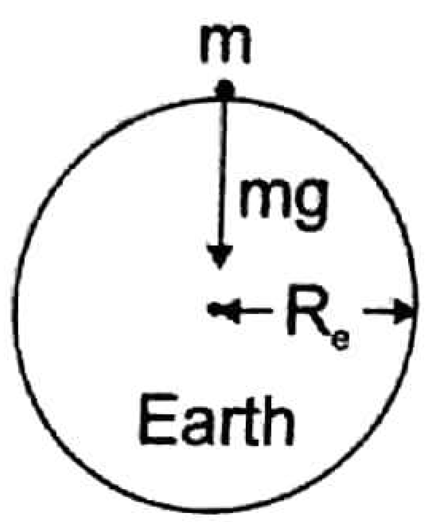
9. ACCELERATION DUE TO GRAVITY :

It is the acceleration, a freely falling body near the earth's surface acquires due to the earth's gravitational pull. The property by virtue of which a body experiences or exerts a gravitational pull on another body is called **gravitational mass m_G**, and the property by virtue of which a body opposes any change in its state of rest or uniform motion is called its **inertial mass m_I** thus if $\vec{E}$ is the gravitational field intensity due to the earth at a point P, and $\vec{g}$ is acceleration due to gravity at the same point, then $m_I\vec{g} = m_G\vec{E}$.

Now the value of inertial & gravitational mass happen to be exactly same to a great degree of accuracy for all bodies. Hence, $\vec{g} = \vec{E}$

The gravitational field intensity on the surface of earth is therefore numerically equal to the acceleration due to gravity (g), there. Thus we get,

$$g = \frac{GM_e}{R_e^2}$$



where, M_e = Mass of earth
R_e = Radius of earth

Note :
- Here the distribution of mass in the earth is taken to be spherical symmetrical so that its entire mass can be assumed to be concentrated at its center for the purpose of calculation of g.

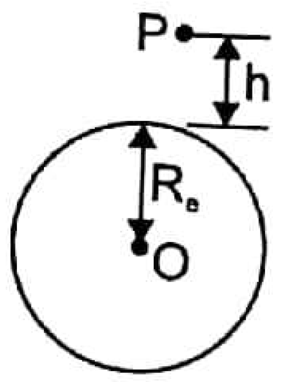
10. VARIATION OF ACCELERATION DUE TO GRAVITY

(a) Effect of Altitude

Acceleration due to gravity on the surface of the earth is given by, $g = \frac{GM_e}{R_e^2}$

Now, consider the body at a height 'h' above the surface of the earth, then the acceleration due to gravity at height 'h' given by

$$g_h = \frac{GM_e}{(R_e+h)^2} = g\left(1+\frac{h}{R_e}\right)^{-2} \simeq g\left(1-\frac{2h}{R_e}\right) \text{ when } h << R.$$



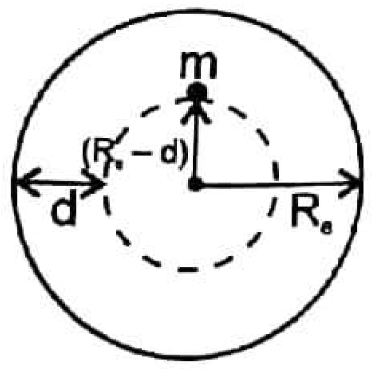
The decrease in the value of 'g' with height $h = g - g_h = \frac{2gh}{R_e}$. Then

percentage decrease in the value of 'g' $= \frac{g-g_h}{g} \times 100 = \frac{2h}{R_e} \times 100\%$

(b) Effect of depth

The gravitational pull on the surface is equal to its weight i.e. $mg = \frac{GM_e m}{R_e^2}$

$$\therefore \quad mg = \frac{G \times \frac{4}{3}\pi R_e^3 \rho m}{R_e^2} \quad \text{or} \quad g = \frac{4}{3}\pi G R_e \rho \quad ....(1)$$



When the body is taken to a depth d, the mass of the sphere of radius $(R_e - d)$ will only be effective for the gravitational pull and the outward shall will have no resultant effect on the mass. If the acceleration due to gravity on the surface of the solid sphere is g_d, then

$$g_d = \frac{4}{3}\pi G (R_e - d)\rho \quad ....(2)$$

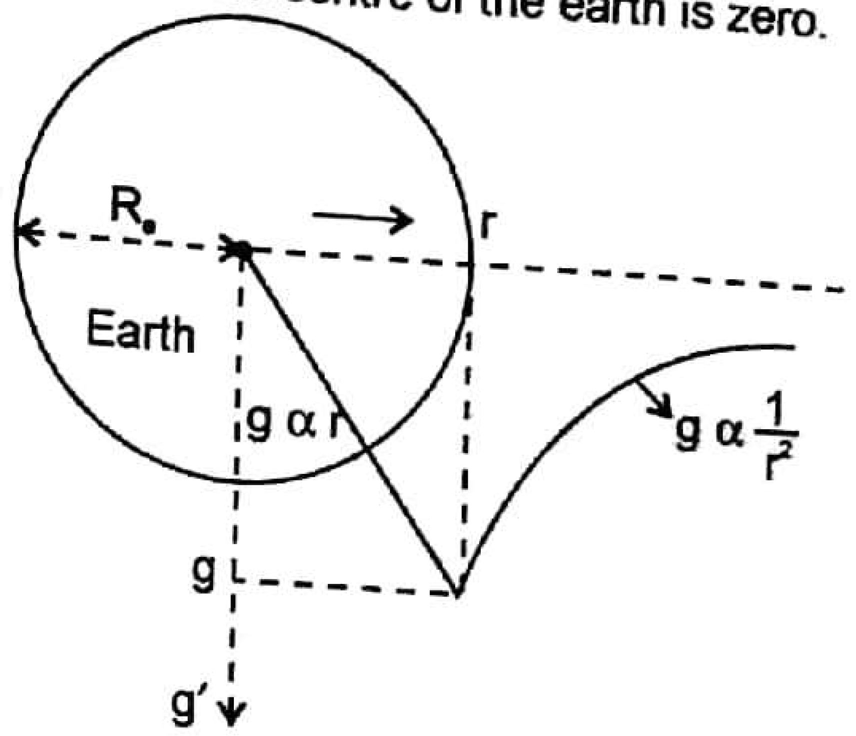
By dividing equation (2) by equation (1)

$$\Rightarrow \quad g_d = g\left(1-\frac{d}{R_e}\right)$$

IMPORTANT POINTS

(i) At the center of the earth, $d = R_e$, so $g_{centre} = g\left(1-\frac{R_e}{R_e}\right) = 0$.

Thus weight (mg) of the body at the centre of the earth is zero.

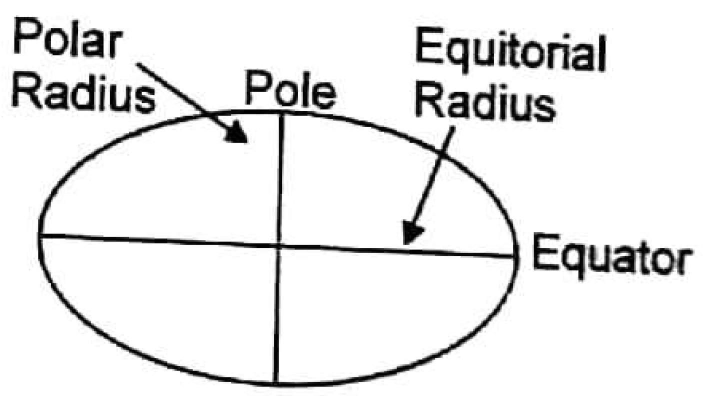


(ii) Percentage decrease in the value of 'g' with the depth $= \left(\frac{g-g_d}{g}\right) \times 100 = \frac{d}{R_e} \times 100$.

(c) Effect of the surface of Earth

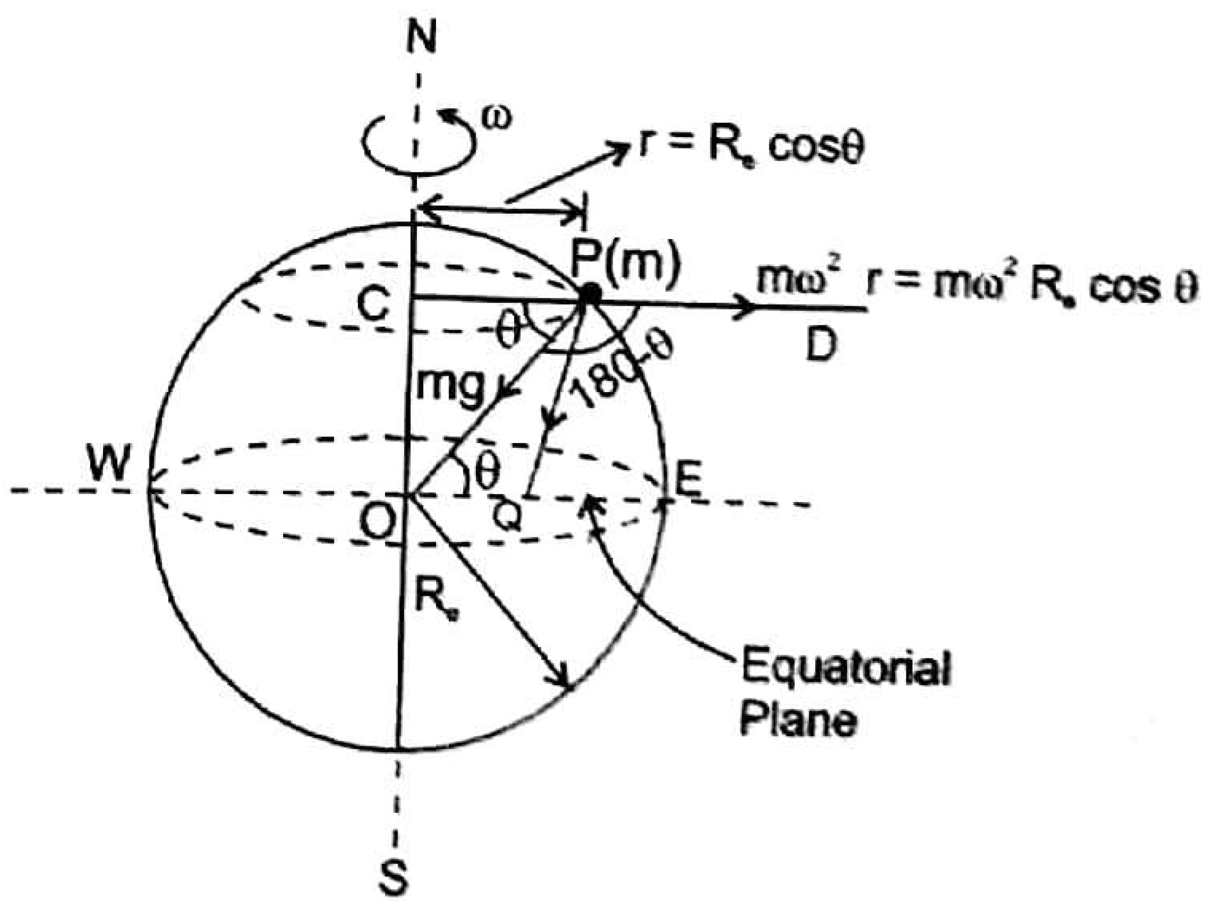
The equatorial radius is about 21 km longer than its polar radius.

We know, $g = \frac{GM_e}{R_e^2}$ Hence $g_{pole} > g_{equator}$. The weight of the body increase as the body taken from the equator to the pole.



(d) Effect of rotation of the Earth

The earth rotates around its axis with angular velocity ω. Consider a particle of mass m at latitude θ. The angular velocity of the particle is also ω.



According to parallelogram law of vector addition, the resultant force acting on mass m along PQ is

$F = [(mg)^2 + (m\omega^2 R_e \cos\theta)^2 + \{2mg \times m\omega^2 R_e \cos\theta\} \cos(180 - \theta)]^{1/2}$

$= [(mg)^2 + (m\omega^2 R_e \cos\theta)^2 - (2m^2 g\omega^2 R_e \cos\theta) \cos\theta]^{1/2}$

$$= mg\left[1 + \left(\frac{R_e\omega^2}{g}\right)^2 \cos^2\theta - 2\frac{R_e\omega^2}{g}\cos^2\theta\right]^{1/2}$$

At pole $\theta = 90° \Rightarrow g_{pole} = g$, At equator $\theta = 0 \Rightarrow g_{equator} = g\left[1 - \frac{R_e\omega^2}{g}\right]$.

Hence $g_{pole} > g_{equator}$

If the body is taken from pole to the equator, then $g' = g\left(1 - \frac{R_e\omega^2}{g}\right)$.

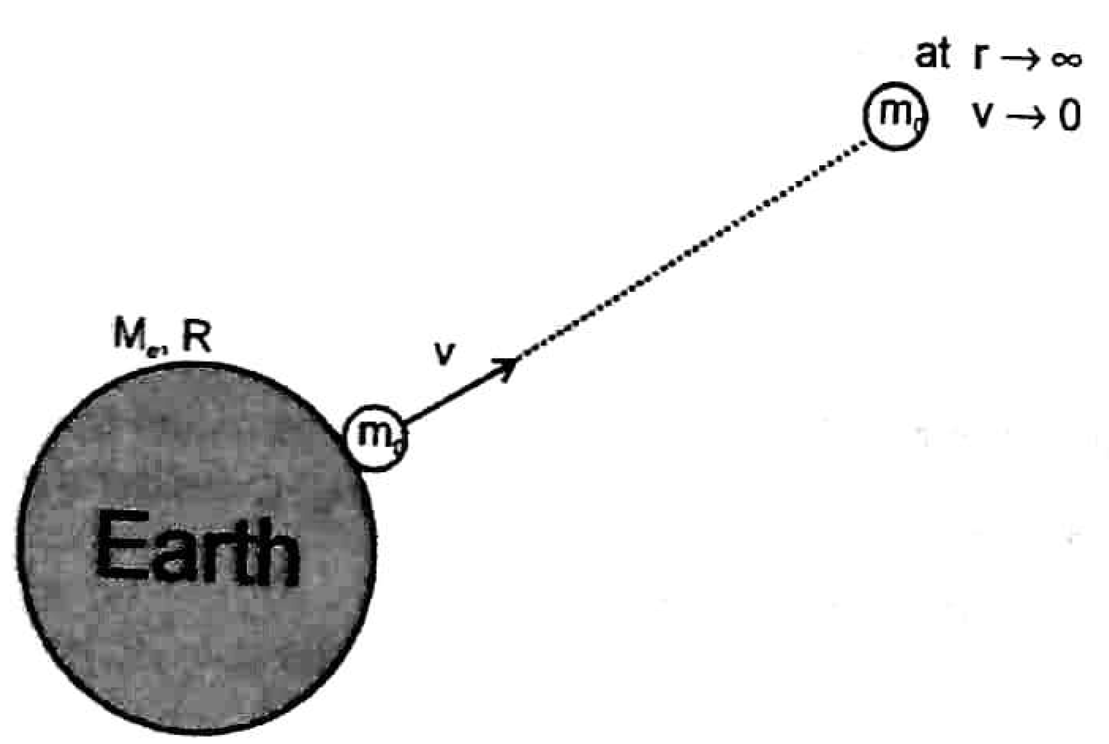
$$\text{Hence \% change in weight} = \frac{mg - mg\left(1 - \frac{R_e\omega^2}{g}\right)}{mg} \times 100 = \frac{m\ R_e\omega^2}{mg} \times 100 = \frac{R_e\omega^2}{g} \times 100$$

11. ESCAPE SPEED

The minimum speed required to send a body out of the gravity field of a planet (send it to $r \to \infty$)

11.1 Escape speed at earth's surface :

Suppose a particle of mass m is on earth's surface. We project it with a velocity V from the earth's surface, so that it just reaches $r \to \infty$ (at $r \to \infty$, its velocity become zero). Applying energy conservation between initial position (when the particle was at earth's surface) and find positions (when the particle just reaches to $r \to \infty$)



$K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv^2 + m_0\left(-\frac{GM_e}{R}\right) = 0 + m_0\left(-\frac{GM_e}{(r \to \infty)}\right) \Rightarrow v = \sqrt{\frac{2GM_0}{R}}$$

Escape speed from earth is surface $v_e = \sqrt{\frac{2GM_e}{R}}$

If we put the values of G, Me, R the we get $V_e = 11.2$ km/s.

11.2 Escape speed depends on :

(i) Mass (M_e) and size (R) of the planet

(ii) Position from where the particle is projected.

11.3 Escape speed does not depend on :

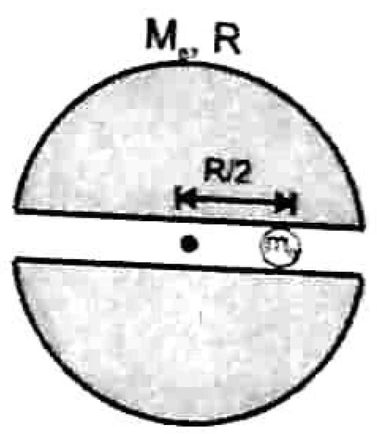
(i) Mass of the body which is projected (m_0)

(ii) Angle of projection.

If a body is thrown from Earth's surface with escape speed, it goes out of earth's gravitational field and never returns to the earth's surface. But it starts revolving around the sun.

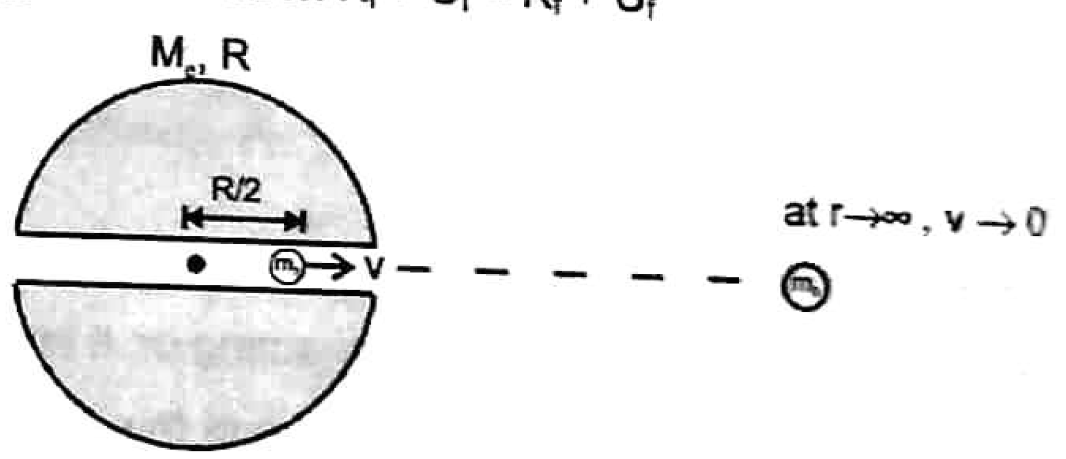
Solved Example

Example 20. A very small groove is made in the earth, and a particle of mass m_0 is placed at R/2 distance from the centre. Find the escape speed of the particle from that place.



Solution : Suppose we project the particle with speed v, so that it just reaches at ($r \to \infty$).

Applying energy conservation $K_i + U_i = K_f + U_f$



$$\frac{1}{2}m_0v^2 + m_0\left(-\frac{GM_e}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)\right) = 0 + 0$$

$v = \sqrt{\frac{11GM_e}{4R}} = V_e$ at that position.

Example 21. Find radius of such planet on which the man escapes through jumping. The capacity of jumping of person on earth is 1.5 m. Density of planet is same as that of earth.

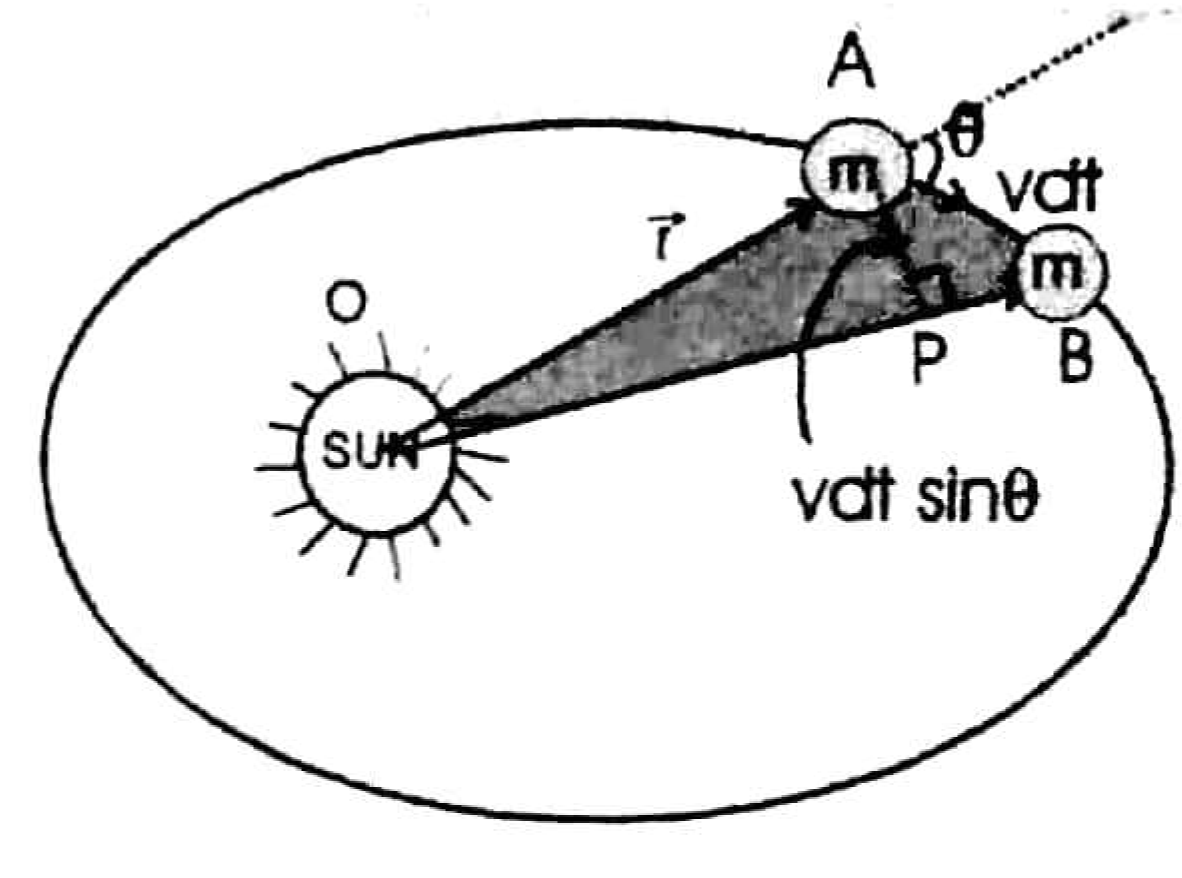
Solution : For a planet : $\frac{1}{2}mv^2 - \frac{GM_pm}{R_p} = 0 \quad \Rightarrow \frac{1}{2}mv^2 = \frac{GM_pm}{R_p}$

On earth $\to \frac{1}{2}mv^2 = m\left(\frac{GM_E}{R_E^2}\right)h$

$\therefore \quad \frac{GM_pm}{R_p} = \frac{GM_E m}{R_E^2}.h \Rightarrow \frac{M_p}{R_p} = \frac{M_Eh}{R_E^2}$

$\therefore$ Density (ρ) is same $\Rightarrow \frac{4/3\pi R_p^3\rho}{R_p} = \frac{4/3\pi R_E^2 h\rho}{R_E^2} \Rightarrow R_p = \sqrt{R_Eh}$

Suppose a planet is revolving around the sun and at any instant its velocity is v, and angle between radius vector ($\vec{r}$) and velocity ($\vec{v}$). In dt time, it moves by a distance vdt, during this dt time, area swept by the radius vector will be OAB which can be assumed to be a triangle



dA = 1/2 (Base) (Perpendicular height)

dA = 1/2 (r) (vdtsinθ)

so rate of area swept $\frac{dA}{dt} = \frac{1}{2} vr \sin\theta$

we can write $\frac{dA}{dt} = \frac{1}{2}\frac{mvr\sin\theta}{m}$

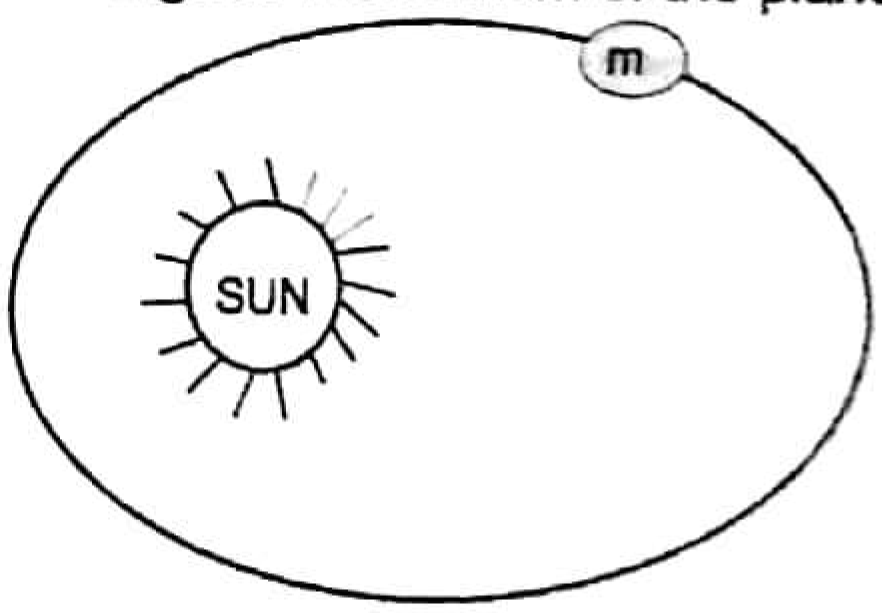
where mvr sinθ = angular momentum of the planet about the sun, which remains conserved (constant)

$\Rightarrow \frac{dA}{dt} = \frac{L_{planet/sun}}{2m}$ = constant

so **Rate of area swept by the radius vector is constant**

Solved Example

Example 22. Suppose a planet is revolving around the sun in an elliptical path given by $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$. Find time period of revolution. Angular momentum of the planet about the sun is L.



Solution : Rate of area swept $\frac{dA}{dt} = \frac{L}{2m}$ = constant

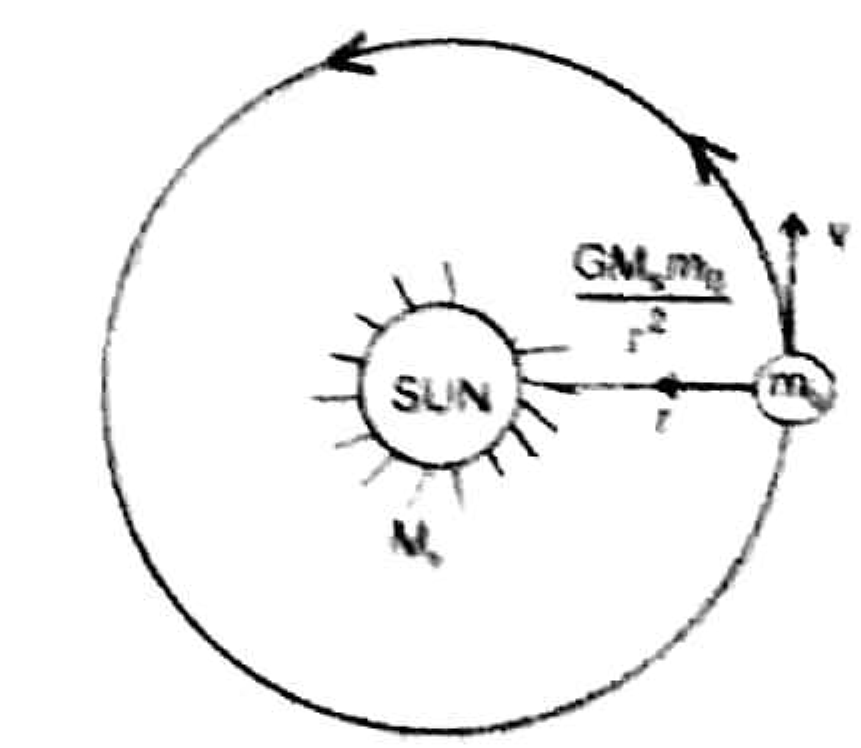
$$\Rightarrow dA = \frac{L}{2m}dt \ ; \quad \int_{A=0}^{A=\pi ab} dA = \int_{t=0}^{t=T} \frac{L}{2m}dt \Rightarrow \pi ab = \frac{L}{2m}T \Rightarrow T = \frac{2\pi mab}{L}$$

12.3 Kepler's law of time period :

Suppose a planet is revolving around the sun in circular orbit

then $\frac{m_0 v^2}{r} = \frac{GM_s m_0}{r^2}$

$$v = \sqrt{\frac{GM_s}{r}}$$



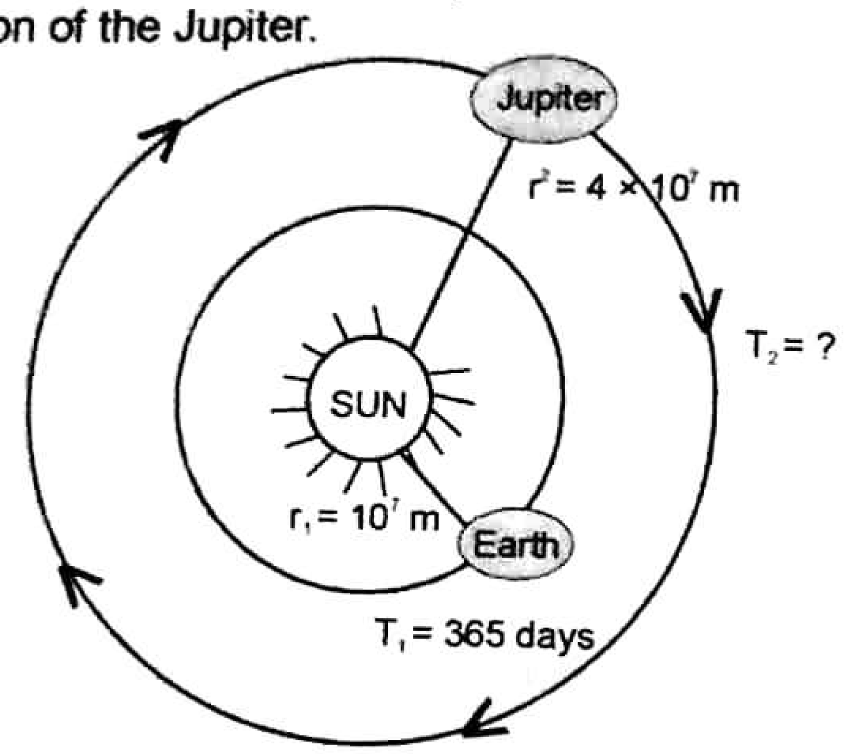
Time period of revolution is $T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM_s}}$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3 \Rightarrow T^2 \propto r^3$$

For all the planet of a sun, $T^2 \propto r^3$

Solved Example

Example 23. The Earth and Jupiter are two planets of the sun. The orbital radius of the earth is 10^7 m and that of Jupiter is 4×10^7 m. If the time period of revolution of earth is T = 365 days, find time period of revolution of the Jupiter.



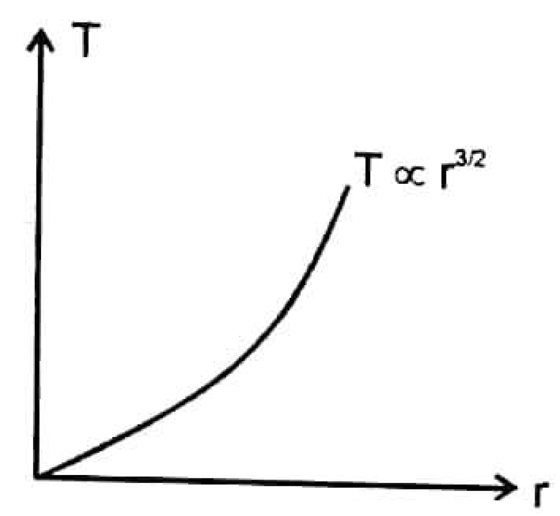
Solution : For both the planets

$T^2 \propto r^3$

$$\left(\frac{T_{jupiter}}{T_{earth}}\right)^2 = \left(\frac{T_{jupiter}}{r_{earth}}\right)^3 \Rightarrow \left(\frac{T_{jupiter}}{365\text{days}}\right)^2 = \left(\frac{4 \times 10^7}{10^7}\right)^3$$

$T_{Jupiter} = 8 \times 365$ days

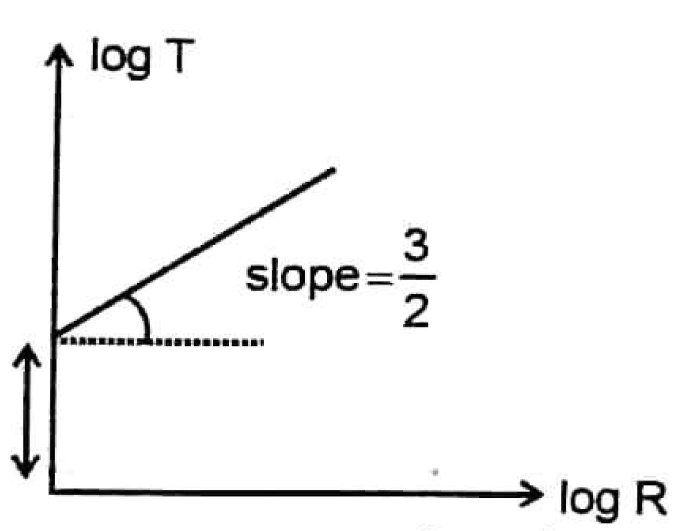
Graph of T vs r :



Graph of log T v/s log R :

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)R^3 \Rightarrow 2\log T = \log\left(\frac{4\pi^2}{GM_s}\right) + 3\log R$$

$$\log T = \frac{1}{2}\log\left(\frac{4\pi^2}{GM_s}\right) + \frac{3}{2}\log R$$



$$C = \frac{1}{2}\log\left(\frac{4\pi^2}{GM_s}\right)$$

* If planets are moving in elliptical orbit, then $T^2 \propto a^3$ where a = semi major axis of the elliptical path.

Example 24. A satellite is launched into a circular orbit 1600 km above the surface of the earth. Find the period of revolution if the radius of the earth is R = 6400 km and the acceleration due to gravity is 9.8 m/sec^2. At what height from the ground should it be launched so that it may appear stationary over a point on the earth's equator?

Solution : The orbiting period of a satellite at a height h from earth's surface is $T = \dfrac{2\pi r^{3/2}}{gR^2}$ where r = R + h

then, $T = \dfrac{2\pi(R+h)}{R}\sqrt{\left(\dfrac{R+h}{g}\right)}$

Here, R = 6400 km, h = 1600 km = R/4.

Then $T = \dfrac{2\pi\left(R+\frac{R}{4}\right)}{R}\sqrt{\left(\dfrac{R+\frac{R}{4}}{g}\right)} = 2\pi(5/4)^{3/2}\sqrt{\dfrac{R}{g}}$

Putting the given values : $T = 2 \times 3.14 \times \sqrt{\left(\dfrac{6.4 \times 10^6 m}{9.8\ m/s^2}\right)}\ (1.25)^{3/2}$ = 7092 sec = 1.97 hours

Now, a satellite will appear stationary in the sky over a point on the earth's equator if its period of revolution round the earth is equal to the period of revolution of the earth round its own axis which is 24 hours. Let us find the height h of such a satellite above the earth's surface in terms of the earth's radius. Let it be nR. then

$$T = \frac{2\pi(R+nR)}{R}\sqrt{\left(\frac{R+nR}{g}\right)} = 2\pi\sqrt{\left(\frac{R}{g}\right)}\ (1+n)^{3/2} = 2 \times 3.14\sqrt{\left(\frac{6.4 \times 10^6 \text{meter/sec}}{9.8\text{meter/sec}^2}\right)}(1+n)^{3/2}$$

$= (5075 \text{ sec})\ (1+n)^{3/2} = (1.41\text{hours})\ (1+n)^{3/2}$

For T = 24 hours, we have

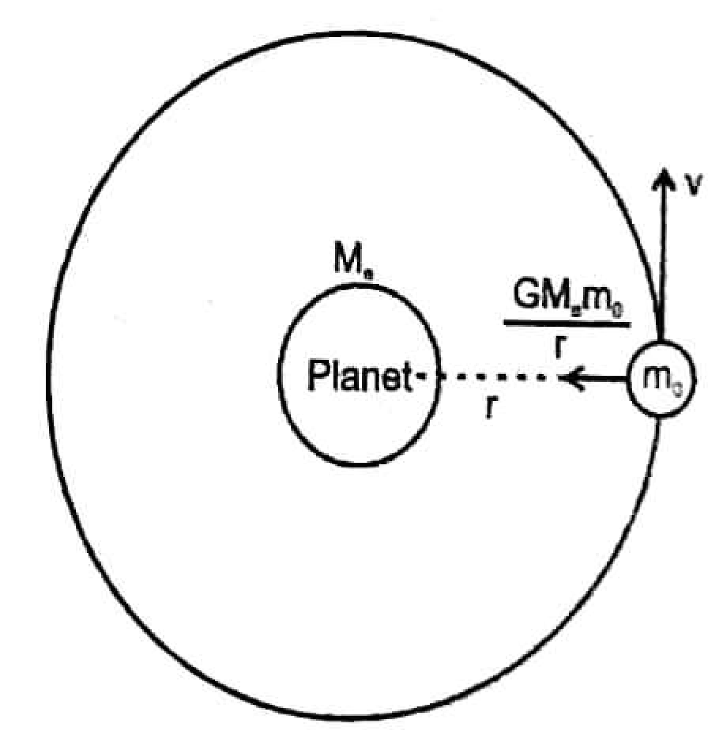
$(24 \text{ hours}) = (1.41) \text{ hours}\ (1+n)^{3/2}$

or $(1+n)^{3/2} = \dfrac{24}{1.41} = 17$

or $1 + n = (17)^{2/3} = 6.61$ or $n = 5.61$

The height of the geo-stationary satellite above the earth's surface is nR = 5.61 × 6400 km = 3.59×10^4 km.

13. CIRCULAR MOTION OF A SATELLITE AROUND A PLANET



Suppose at satellite of mass m_0 is at a distance r from a planet. If the satellite does not revolve, then due to the gravitational attraction, it may collide to the planet.

To avoid the collision, the satellite revolve around the planet, for circular motion of satellite.

$\Rightarrow \dfrac{GM_e m_0}{r^2} = \dfrac{m_0 v^2}{r}$ ....(1)

$\Rightarrow v = \sqrt{\dfrac{GM_e}{r}}$ this velocity is called orbital velocity (v_0)

$v_0 = \sqrt{\dfrac{GM_e}{r}}$

13.1 Total energy of the satellite moving in circular orbit :

(i) $KE = \frac{1}{2}m_0v^2$ and from equation (1)

$$\frac{m_0v^2}{r} = \frac{GM_em_0}{r^2} \Rightarrow m_0v^2 = \frac{GM_em_0}{r} \Rightarrow KE = \frac{1}{2}m_0v^2 = \frac{GM_em_0}{2r}$$

(ii) Potential energy $U = -\frac{GM_em_0}{r}$

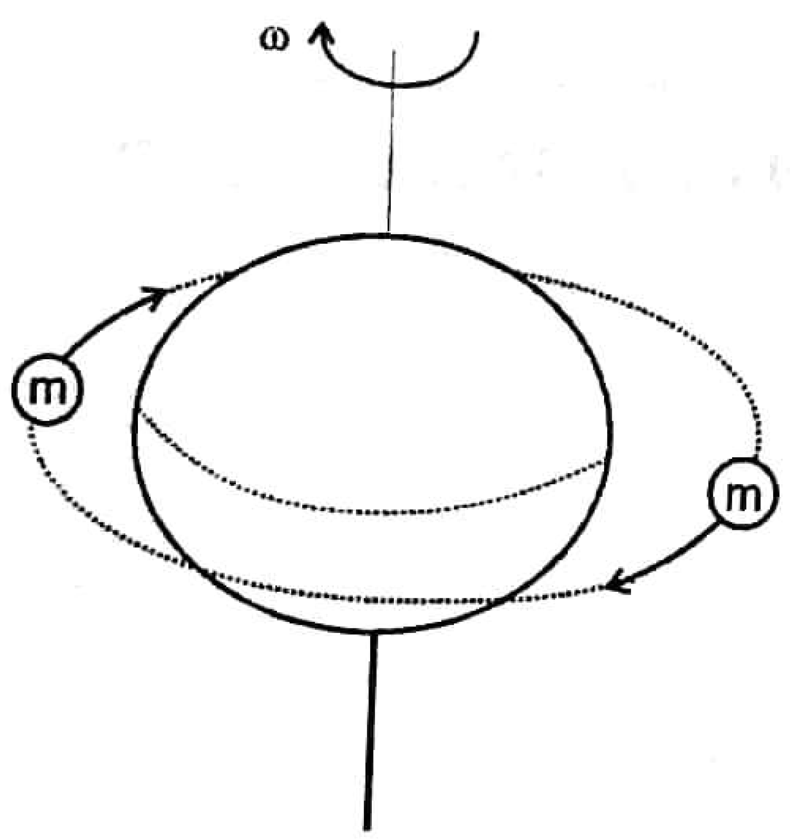
$$\text{Total energy} = KE + PE = \left(\frac{GM_em_0}{2r}\right) + \left(\frac{-GM_em_0}{r}\right)$$

$$TE = -\frac{GM_em_0}{2r}$$

Total energy is –ve. It shows that the satellite is still bounded with the planet.

14. GEO - STATIONARY SATELLITE :

We know that the earth rotates about its axis with angular velocity ω_{earth} and time period T_{earth} = 24 hours. Suppose a satellite is set in an orbit which is in the plane of the equator, whose ω is equal to ω_{earth}, (or its T is equal to T_{earth} = 24 hours) and direction is also same as that of earth. Then as seen from earth, it will appear to be stationery. This type of satellite is called geo- stationary satellite. For a geo-stationary satellite,



$w_{satellite} = w_{earth}$

$\Rightarrow T_{satellite} = T_{earth} = 24$ hr.

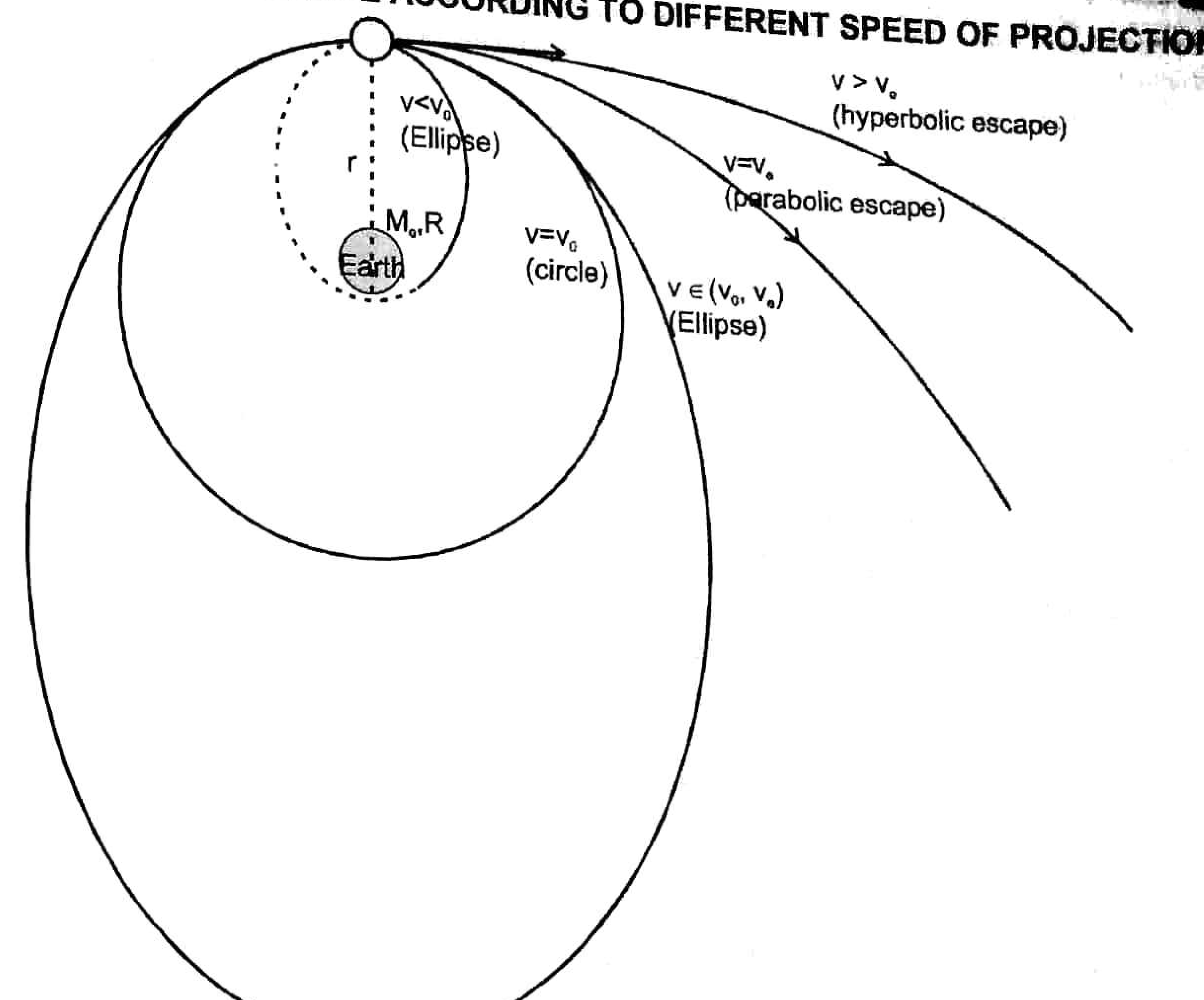
So time period of a geo-stationery satellite must be 24 hours. To achieve T = 24 hour, the orbital radius geo-stationary satellite :

$$T^2 = \left(\frac{4\pi^2}{GM_e}\right) r^3$$

Putting the values, we get orbital radius of geo stationary satellite $r = 6.6\ R_e$ (here Re = radius of the earth)

height from the surface $h = 5.6\ R_e$.

15. PATH OF A SATELLITE ACCORDING TO DIFFERENT SPEED OF PROJECTION



Suppose a satellite is at a distance r from the centre of the earth. If we give different velocities (v) to the satellite, its path will be different

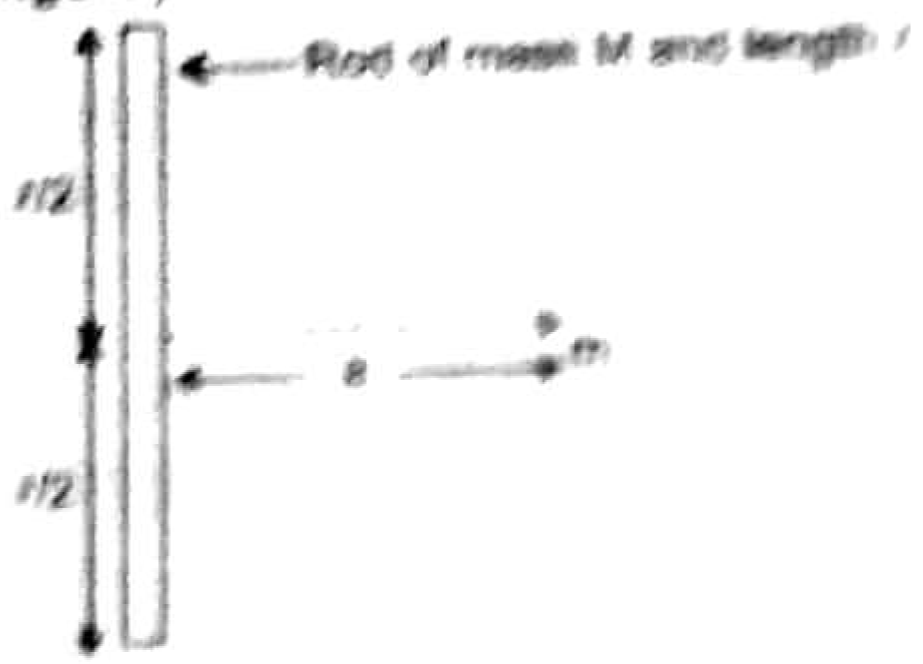
(i) If $v < v_0 \left(\text{or } v < \sqrt{\frac{GM_e}{r}}\right)$ then the satellite will move is an elliptical path and strike the earth's surface. But if size of earth were small, the satellite would complete the elliptical orbit, and the centre of the earth will be at is farther focus.

(ii) If $v = v_0 \left(\text{or } v = \sqrt{\frac{GM_e}{r}}\right)$, then the satellite will revolve in a circular orbit.

(iii) If $v_0 > v > v_0 \left(\text{or } \sqrt{\frac{2GM_e}{r}} > v > \sqrt{\frac{GM_e}{r}}\right)$, then the satellite will revolve in a elliptical orbital, and the centre of the earth will be at its nearer focus.

(iv) If $v = v_e \left(\text{or } v = \sqrt{\frac{2GM_e}{r}}\right)$, then the satellite will just escape with parabolic path.

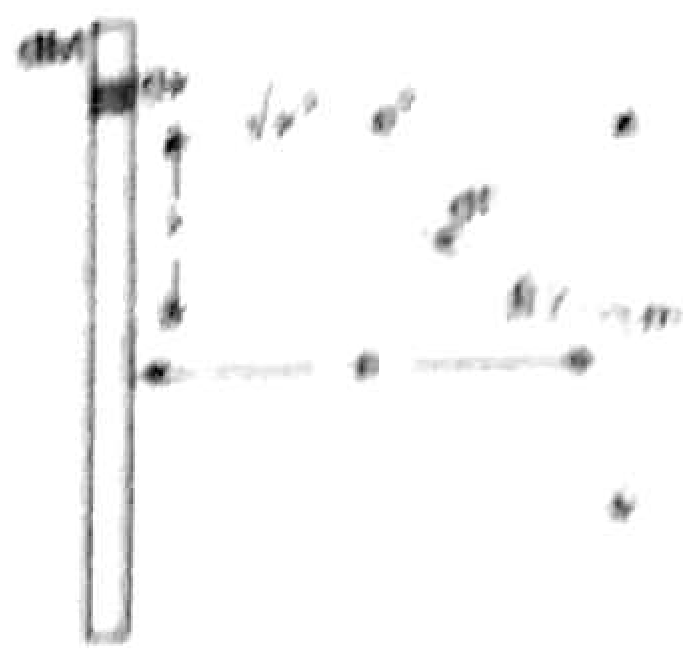
Problem 1. Calculate the force exerted by point mass m on rod of mass M and length l (Placed as shown in figure)



Solution : Direction of force is changing at every element. We have to make components of force and then integrate

Net vertical force = 0

$$dF = \text{force on element} = \frac{G\,dM\,m}{(x^2+a^2)}$$



$$dF_h = dF\cos\theta = \text{force on element in horizontal direction} = \frac{G\,dM\,m}{(x^2+a^2)}\cos\theta$$

$$F_h = \int \frac{GMm\cos\theta\,dx}{l(x^2+a^2)} = \frac{GMm}{l}\int_{-l/2}^{l/2}\frac{\cos\theta\,dx}{(x^2+a^2)} = \frac{GMm}{la^2}\int \frac{\cos\theta\,dx}{\sec^2\theta}$$

where $x = a\tan\theta$ then $dx = a\sec^2\theta\,d\theta$

$$= \frac{GMm}{la}\left[\sin\theta\right]_{-l/2}^{l/2} \qquad \tan\theta = \frac{x}{a} \text{ then } \sin\theta = \frac{x}{\sqrt{x^2+a^2}}$$

$$= \frac{GMm}{la}\left[\frac{x}{\sqrt{x^2+a^2}}\right]_{-l/2}^{l/2} = \frac{GMml}{la\sqrt{\frac{l^2}{4}+a^2}} = \frac{GMm}{a\sqrt{\frac{l^2}{4}+a^2}}$$

Problem 2. Three identical bodies of mass M are located at the vertices of an equilateral triangle with side L. At what speed must they move if they all revolve under the influence of one another's gravity in a circular orbit circumscribing the triangle while still preserving the equilateral triangle?

Solution : Let A, B and C be the three masses and O the centre of the circumscribing circle. The radius of this circle is

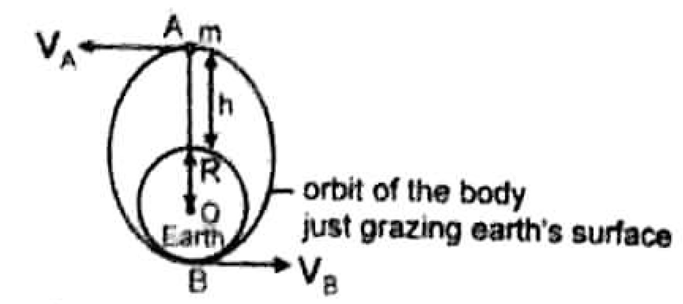
$$R = \frac{L}{2}\sec 30^\circ = \frac{L}{2}\times\frac{2}{\sqrt{3}} = \frac{L}{\sqrt{3}}$$

Let v be the speed of each mass M along the circle. Let us consider the motion of the mass at A. The force of gravitational attraction on it due to the masses at B and C are

$$\frac{GM^2}{L^2} \text{ along AB and } \frac{GM^2}{L^2} \text{ along AC}$$

The resultant force is therefore

Problem 4. For a particle projected in a transverse direction from a height h above Earth's surface, find the minimum initial velocity so that it just grazes the surface of earth path of this particle would be an ellipse with center of earth as the farther focus, point of projection as the apogee and a diametrically opposite point on earth's surface as perigee.



Solution : Suppose velocity of projection at point A is v_A & at point B, the velocity of the particle is v_B.

then applying Newton's 2nd law at point A & B, we get, $\frac{mv_A^2}{r_A} = \frac{GM_e m}{(R+h)^2}$ & $\frac{mv_B^2}{r_B} = \frac{GM_e m}{R^2}$

Where r_A & r_B are radius of curvature of the orbit at points A & B of the ellipse, but $r_A = r_B = r$(say).

Now applying conservation of energy at points A & B

$$\frac{-GM_e m}{R+h} + \frac{1}{2}mv_A^2 = \frac{-GM_e m}{R} + \frac{1}{2}mv_B^2$$

$$\Rightarrow GM_e m\left(\frac{1}{R} - \frac{1}{(R+h)}\right) = \frac{1}{2}(mv_B^2 - mv_A^2) = \left(\frac{1}{2}\rho GM_e m\left(\frac{1}{R^2} - \frac{1}{(R+h)^2}\right)\right)$$

$$\text{or, } r = \frac{2R(R+h)}{2R+h} = \frac{2Rr}{R+r} \qquad \therefore \quad V_A^2 = \frac{rGM_e}{(R+h)^2} = 2GM_e\frac{R}{r(r+R)}$$

where r = distance of point of projection from earth's centre = R + h.

Problem 5. A rocket starts vertically upward with speed v_0. Shown that its speed v at height h is given by $v_0^2 - v^2 = \frac{2hg}{1+\frac{h}{R}}$, where R is the radius of the earth and g is acceleration due to gravity at earth's surface. Hence deduce an expression for maximum height reached by a rocket fired with speed 0.9 times the escape velocity.

Solution : The gravitational potential energy of a mass m on earth's surface and that a height h is given by

$$U(R) = -\frac{GMm}{R} \text{ and } U(R+h) = -\frac{GMm}{R+h}$$

$$\therefore \quad U(R+h) - U(R) = -GMm\left(\frac{1}{R+h} - \frac{1}{R}\right) = \frac{GMmh}{(R+h)R} = \frac{mhg}{1+\frac{h}{R}} \quad [\because GM = gR^2]$$

This increase in potential energy occurs at the cost of kinetic energy which correspondingly decreases. If v is the velocity of the rocket at height h, then the decrease in kinetic energy is $\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2$.

$$\text{Thus, } \frac{1}{2}mv_0^2 - \frac{1}{2}mv^2 = \frac{mhg}{1+\frac{h}{R}}, \text{ or } \quad v_0^2 - v^2 = \frac{2gh}{1+\frac{h}{R}}$$

Let h_{max} be the maximum height reached by the rocket, at which its velocity has been reduced to zero. Thus, substituting v = 0 and h = h_{max} in the last expression, we have

$$v_0^2 = \frac{2gh_{max}}{1+\frac{h_{max}}{R}} \text{ or } = v_0^2\left(1+\frac{h_{max}}{R}\right)2\,gh_{max} \text{ or } v_0^2 = h_{max}\left(2g - \frac{v_0^2}{R}\right) \text{ or } h_{max} = \frac{v_0^2}{2g - \frac{v_0^2}{R}}$$

Now, it is given that $v_0 = 0.9 \times$ escape velocity $= 0.9 \times \sqrt{(2gR)}$

$$\therefore h_{max} = \frac{(09\times0.9)2gR}{2g - \frac{(09\times0.9)2gR}{R}} = \frac{1.62gR}{2g-1.62R} = \frac{1.62R}{0.38} = 4.26\text{ R}$$

Exercise-1

✍ Marked Questions can be used as Revision Questions.

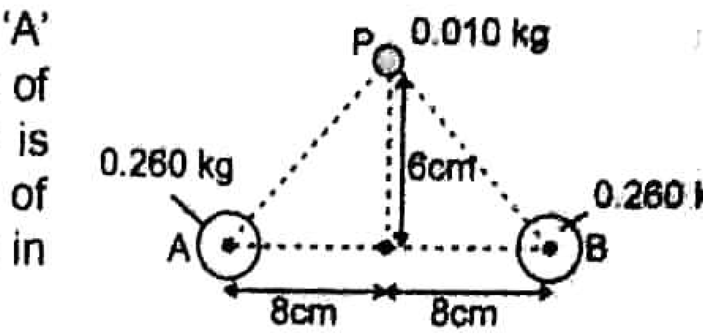
PART - I : SUBJECTIVE QUESTIONS

Section (A) : Universal law of gravitation

A-1. The typical adult human brain has a mass of about 1.4 kg. What force does a full moon exert on such a brain when it is directly above with its centre 378000 km away ? (Mass of the moon = 7.34×10^{22} kg)

A-2. Two uniform solid spheres of same material and same radius 'r' are touching each other. If the density is 'ρ' then find out gravitational force between them.

A-3. Two uniform spheres, each of mass 0.260 kg are fixed at points 'A' and 'B' as shown in the figure. Find the magnitude and direction of the initial acceleration of a sphere with mass 0.010 kg if it is released from rest at point 'P' and acted only by forces of gravitational attraction of sphere at 'A' and 'B'(give your answer in terms of G).



Section (B) : Gravitational field and potential

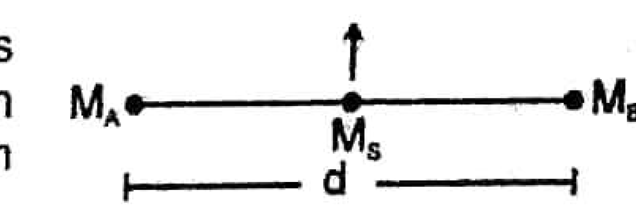
B-1. The gravitational potential in a region is given by V = (20x + 40y)J/kg. Find out the gravitational field (in newton/kg) at a point having co-ordinates (2, 4). Also find out the magnitude of the gravitational force on a particle of 0.250 kg placed at the point (2, 4).

B-2. Radius of the earth is 6.4×10^6 m and the mean density is 5.5×10^3 kg/m^3. Find out the gravitational potential at the earth's surface.

Section (C) : Gravitational Potential Energy and Self Energy

C-1. A body which is initially at rest at a height R above the surface of the earth of radius R, falls freely towards the earth. Find out its velocity on reaching the surface of earth. (Take g = acceleration due to gravity on the surface of the Earth).

C-2.✍ Two planets A and B are fixed at a distance d from each other as shown in the figure. If the mass of A is M_A and that of B is M_B, then find out the minimum velocity of a satellite of mass M_S projected from the mid point of two planets to infinity.



Section (D) : Kepler's law for Satellites, Orbital speed and Escape speed

D-1. A satellite is established in a circular orbit of radius r and another in a circular orbit of radius 1.01 r. How much nearly percentage the time period of second-satellite will be larger than the first satellite.

D-2.✍ Two identical stars of mass M, orbit around their centre of mass. Each orbit is circular and has radius R, so that the two stars are always on opposite sides on a diameter.
(a) Find the gravitational force of one star on the other.
(b) Find the orbital speed of each star and the period of the orbit.
(c) Find their common angular speed.
(d) Find the minimum energy that would be required to separate the two stars to infinity.
(e) If a meteorite passes through this centre of mass perpendicular to the orbital plane of the stars. What value must its speed exceed at that point if it escapes to infinity from the star system.

D-3. Two earth satellites A and B each of equal mass are to be launched into circular orbits about earth's centre. Satellite 'A' is to orbit at an altiude of 6400 km and B at 19200 km. The radius of the earth is 6400 km. Determine-
(a) the ratio of the potential energy
(b) the ratio of kinetic energy
(c) which one has the greater total energy

D-4. The Saturn is about six times farther from the Sun than The Mars. Which planet has :
(a) the greater period of revolution ? (b) the greater orbital speed and
(c) the greater angular speed ?

Section (E) : The Earth and Other Planets Gravity

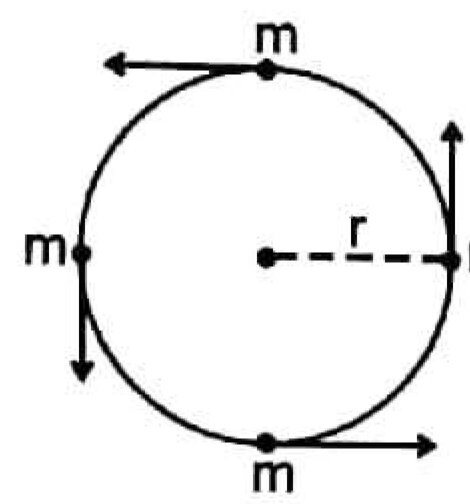
E-1. The acceleration due to gravity at a height $(1/20)^{th}$ the radius of the earth above earth's surface is $9\ m/s^2$. Find out its approximate value at a point at an equal distance below the surface of the earth.

E-2. If a pendulum has a period of exactly 1.00 sec. at the equator, what would be its period at the south pole ? Assume the earth to be spherical and rotational effect of the Earth is to be taken.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Universal law of gravitation

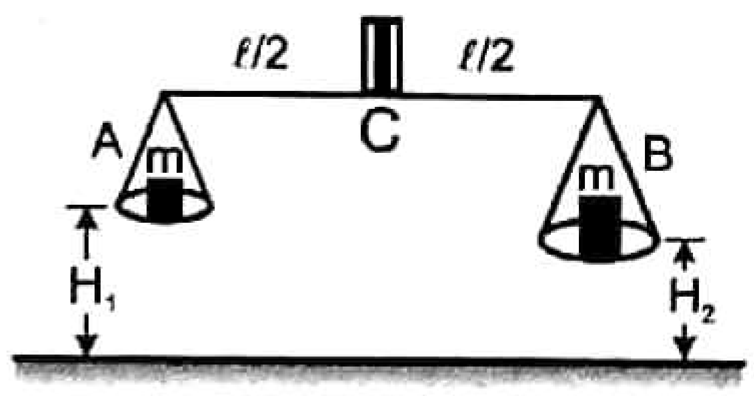
A-1. Four similar particles of mass m are orbiting in a circle of radius r in the same direction and same speed because of their mutual gravitational attractive force as shown in the figure. Speed of a particle is given by



(A) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ (B) $\sqrt[3]{\frac{Gm}{r}}$

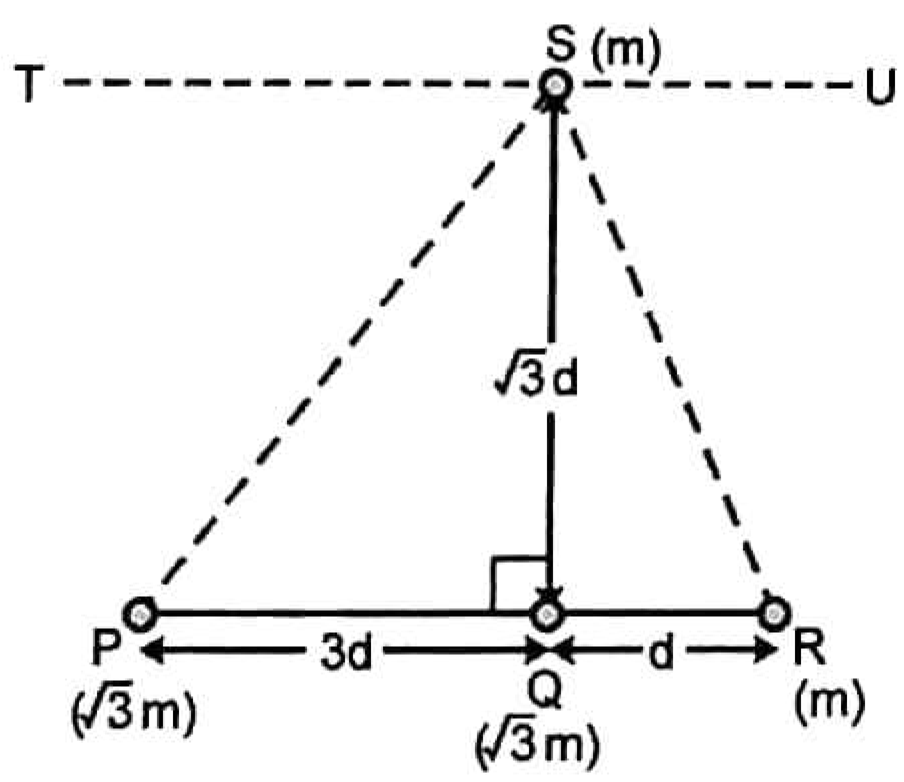
(C) $\sqrt{\frac{Gm}{r}\left(1+2\sqrt{2}\right)}$ (D) zero

A-2. Two blocks of masses m each are hung from a balance as shown in the figure. The scale pan A is at height H_1 whereas scale pan B is at height H_2. Net torque of weights acting on the system about point 'C', will be (length of the rod is ℓ and H_1 & H_2 are << R) ($H_1 > H_2$)



(A) $mg\left(\frac{1-2H_1}{R}\right)\ell$ (B) $\frac{mg}{R}(H_1-H_2)\ell$ (C) $\frac{2mg}{R}(H_1+H_2)\ell$ (D) $2mg\frac{H_2H_1}{H_1+H_2}\ell$

A-3. Three particles P, Q and R are placed as per given figure. Masses of P, Q and R are $\sqrt{3}$ m, $\sqrt{3}$ m and m respectively. The gravitational force on a fourth particle 'S' of mass m is equal to



(A) $\frac{\sqrt{3}GM^2}{2d^2}$ in ST direction only

(B) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}Gm^2}{2d^2}$ in SU direction

(C) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction only

(D) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}Gm^2}{2d^2}$ in ST direction

A-4. Three identical stars of mass M are located at the vertices of an equilateral triangle with side L. The speed at which they will move if they all revolve under the influence of one another's gravitational force in a circular orbit circumscribing the triangle while still preserving the equilateral triangle :

(A) $\sqrt{\frac{2GM}{L}}$ (B) $\sqrt{\frac{GM}{L}}$ (C) $2\sqrt{\frac{GM}{L}}$ (D) not possible at all

C-2. Three equal masses each of mass 'm' are placed at the three-corners of an equilateral triangle of side 'a'.

(a) If a fourth particle of equal mass is placed at the centre of triangle, then net force acting on it, is equal to :

(A) $\frac{Gm^2}{a^2}$ (B) $\frac{4Gm^2}{3a^2}$ (C) $\frac{3Gm^2}{a^2}$ (D) zero

(b) In above problem, if fourth particle is at the mid-point of a side, then net force acting on it, is equal to:

(A) $\frac{Gm^2}{a^2}$ (B) $\frac{4Gm^2}{3a^2}$ (C) $\frac{3Gm^2}{a^2}$ (D) zero

(c) If above given three particles system of equilateral triangle side a is to be changed to side of 2a, then work done on the system is equal to :

(A) $\frac{3Gm^2}{a}$ (B) $\frac{3Gm^2}{2a}$ (C) $\frac{4Gm^2}{3a}$ (D) $\frac{Gm^2}{a}$

(d) In the above given three particle system, if two particles are kept fixed and third particle is released. Then speed of the particle when it reaches to the mid-point of the side connecting other two masses:

(A) $\sqrt{\frac{2Gm}{a}}$ (B) $2\sqrt{\frac{Gm}{a}}$ (C) $\sqrt{\frac{Gm}{a}}$ (D) $\sqrt{\frac{Gm}{2a}}$

Section : (D) Kepler's law for Satellites, Orbital Velocity and Escape Velocity

D-1. Periodic-time of satellite revolving around the earth is - (ρ is density of earth)

(A) Proportional to $\frac{1}{\rho}$ (B) Proportional to $\frac{1}{\sqrt{\rho}}$

(C) Proportional ρ (D) does not depend on ρ.

D-2. An artificial satellite of the earth releases a package. If air resistance is neglected the point where the package will hit (with respect to the position at the time of release) will be

(A) ahead (B) exactly below

(C) behind (D) it will never reach the earth

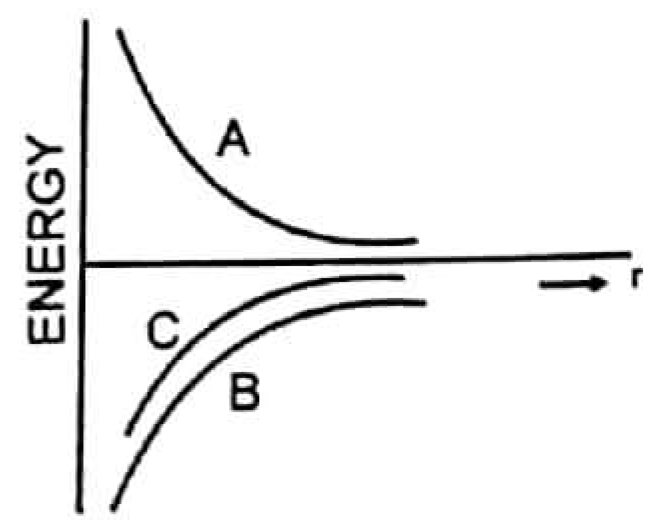
D-3. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C

(A) A shows the kinetic energy, B the total energy and C the potential energy of the system

(B) C shows the total energy, B the kinetic energy and A the potential energy of the system

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system

(D) A and B are the kinetic and potential energies respectively and C is the total energy of the system



D-4. A planet of mass m revolves around the sun of mass M in an elliptical orbit. The minimum and maximum distance of the planet from the sun are r_1 & r_2 respectively. If the minimum velocity of the planet is $\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$ then it's maximum velocity will be :

(A) $\sqrt{\frac{2GMr_2}{(r_1+r_2)r_1}}$ (B) $g\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$ (C) $\sqrt{\frac{2Gmr_2}{(r_1+r_2)r_1}}$ (D) $\sqrt{\frac{2GM}{r_1+r_2}}$

D-5. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be :

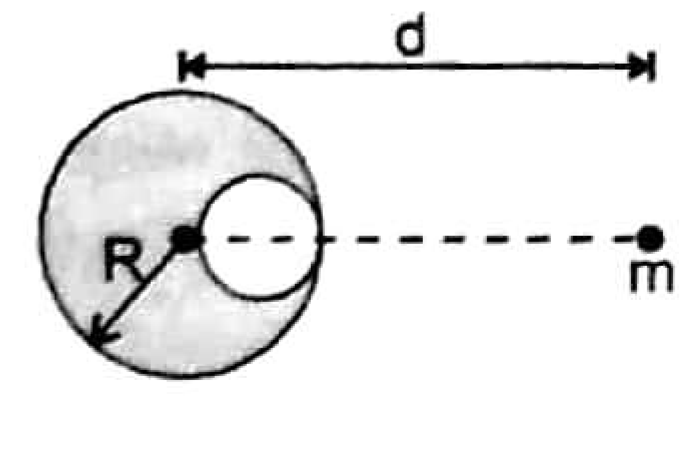
(A) $11\sqrt{2}$ km/s (B) 22 km/s (C) 11 km/s (D) $11/\sqrt{2}$ m/s

Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. A spherical hollow cavity is made in a lead sphere of radius R, such that its surface touches the outside surface of the lead sphere and passes through its centre. The mass of the sphere before hollowing was M. With what gravitational force will the hollowed-out lead sphere attract a small sphere of mass 'm', which lies at a distance d from the centre of the lead sphere on the straight line connecting the centres of the spheres and that of the hollow, if d = 2R :



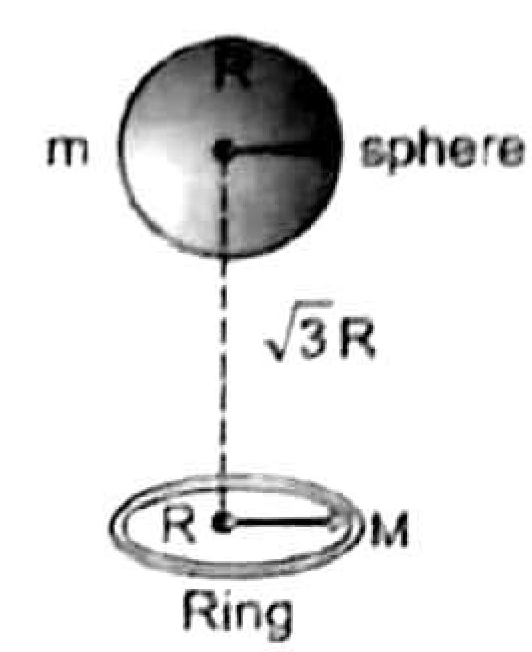
(A) $\frac{7GMm}{18R^2}$ (B) $\frac{7GMm}{36R^2}$ (C) $\frac{7GMm}{9R^2}$ (D) $\frac{7GMm}{72R^2}$

2. A straight rod of length ℓ extends from $x = \alpha$ to $x = \ell + \alpha$ as shown in the figure. If the mass per unit length is $(a + bx^2)$. The gravitational force it exerts on a point mass m placed at x = 0 is given by

(A) $Gm\left[a\left(\frac{1}{\alpha}-\frac{1}{\alpha+\ell}\right)+b\ell\right]$ (B) $\frac{Gm(a+bx^2)}{\ell^2}$

(C) $Gm\left[\alpha\left(\frac{1}{a}-\frac{1}{a+\ell}\right)+b\ell\right]$ (D) $Gm\left[a\left(\frac{1}{\alpha+\ell}-\frac{1}{\alpha}\right)+b\ell\right]$

3. A uniform ring of mass M is lying at a distance $\sqrt{3}$ R from the centre of a uniform sphere of mass m just below the sphere as shown in the figure where R is the radius of the ring as well as that of the sphere. Then gravitational force exerted by the ring on the sphere is :



(A) $\frac{GMm}{8R^2}$ (B) $\frac{GMm}{3R^2}$

(C) $\sqrt{3}\frac{GMm}{R^2}$ (D) $\sqrt{3}\frac{GMm}{8R^2}$

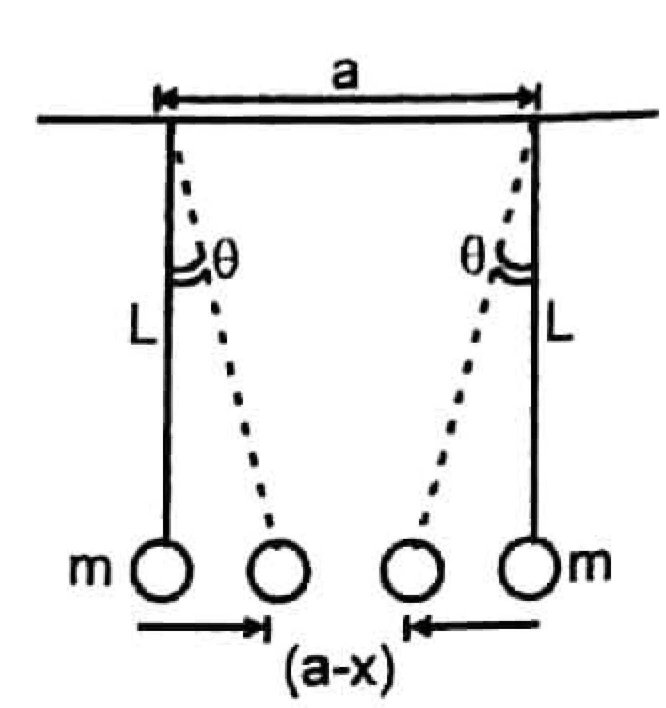
4. The gravitational potential of two homogeneous spherical shells A and B (separated by large distance) of same surface mass density at their respective centres are in the ratio 3 : 4. If the two shells coalesce into single one such that surface mass density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to :

(A) 3 : 2 (B) 4 : 3 (C) 5 : 3 (D) 3 : 5

5. A projectile is fired from the surface of earth of radius R with a speed kv_e in radially outward direction (where v_e is the escape velocity and k < 1). Neglecting air resistance, the maximum hight from centre of earth is

(A) $\frac{R}{k^2+1}$ (B) k^2R (C) $\frac{R}{1-k^2}$ (D) kR

6. Two small balls of mass m each are suspended side by side by two equal threads of length L as shown in the figure. If the distance between the upper ends of the threads be a, the angle θ that the threads will make with the vertical due to attraction between the balls is



(A) $\tan^{-1}\frac{(a-x)g}{mG}$ (B) $\tan^{-1}\frac{mG}{(a-x)^2g}$

(C) $\tan^{-1}\frac{(a-x)^2g}{mG}$ (D) $\tan^{-1}\frac{(a^2-x^2)g}{mG}$

SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A projectile is fired vertically up from the bottom of a crater (big hole) on the moon. The depth of the crater is R/100, where R is the radius of the moon. If the initial velocity of the projectile is the same as the escape velocity from the moon surface. The maximum aproximate height attained by the projectile above the lunar (moon) surface is xR. Find value of x.

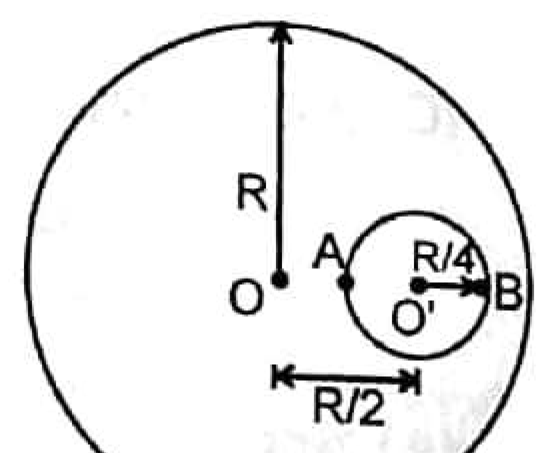
2. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period becomes (in hrs).

3. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth $\frac{NmgR}{2}$ is. Find the value of N :

4. The gravitational field in a region is given by $\vec{E} = (3\hat{i} - 4\hat{j})$ N/kg. Find out the work done (in joule) in displacing a particle of mass 1 kg by 1 m along the line 4y = 3x + 9.

5. In a solid sphere of radius 'R' and density 'ρ' there is a spherical cavity of radius R/4 as shown in figure. A particle of mass 'm' is released from rest from point 'B' (inside the cavity). Find out Velocity (in mm/sec.) of the particle at the instant when it strikes the cavity

$(R = 3m,\ \rho = \frac{10}{\pi} \times 10^3\ kg/m^3,\ G = \frac{20}{3} \times 10^{-11}\ Nm^2kg^{-2})$



6. A ring of radius R = 8m is made of a highly dense-material. Mass of the ring is $m_R = 2.7 \times 10^9$ kg distributed uniformly over its circumference. A particle of mass (dense) $m_p = 3 \times 10^8$ kg is placed on the axis of the ring at a distance x_0 = 6m from the centre. Neglect all other forces except gravitational interaction. Determine speed (in cm/sec.) of the particle at the instant when it passes through centre of ring. :

7. Our sun, with mass 2×10^{30} kg revolves on the edge of our milky way galaxy, which can be assumed to be spherical, having radius 10^{20}m. Also assume that many stars, identical to our sun are uniformly distributed in the spherical milky way galaxy. If the time period of the sun is 10^{15} second and number of stars in the galaxy are nearly $3 \times 10^{(a)}$, find value of 'a' (take $\pi^2 = 10$, G $\frac{20}{3} \times 10^{-11}$ in MKS)

8. Assume earth to be a sphere of uniform mass density. The energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles is x × 10^{31} J. Find the value of x. given the product of mass of earth and radius of earth to be 2.5×10^{31} kg-m and g = 10 m/s

9. The two stars in a certain binary star system move in circular orbits. The first star, α moves in an orbit of radius 1.00×10^9 km. The other star, β moves in an orbit of radius 5.00×10^8 km. What is the ratio of masses of star β to the star α ?

10. If the radius of earth is R and height of a satellite above earth's surface is R then find the minimum co-latitude (in degree) which can directly receive a signal from satellite.

9. A double star is a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to
(A) $r^{3/2}$ (B) r (C) $m^{1/2}$ (D) $m^{-1/2}$

10. An orbiting satellite will escape if :
(A) its speed is increased by $(\sqrt{2}-1)100\%$
(B) its speed in the orbit is made $\sqrt{1.5}$ times of its initial value
(C) its KE is doubled
(D) it stops moving in the orbit

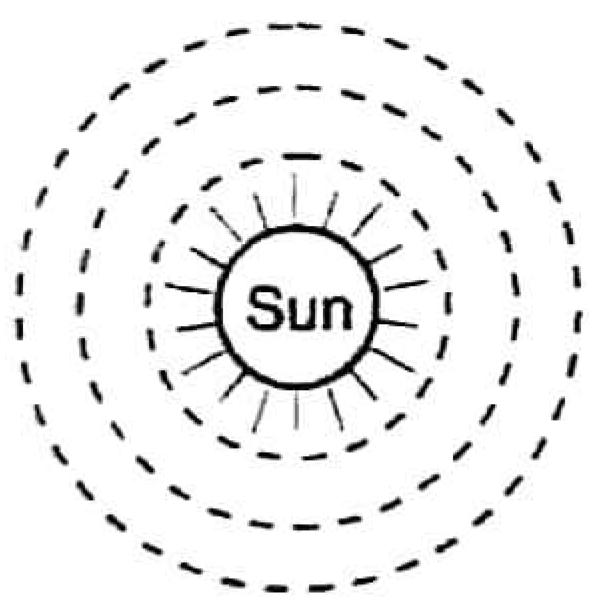
11. In case of an orbiting satellite if the radius of orbit is decreased :
(A) its Kinetic Energy decreases (B) its Potential Energy decreases
(C) its Mechanical Energy decreases (D) its speed decreases

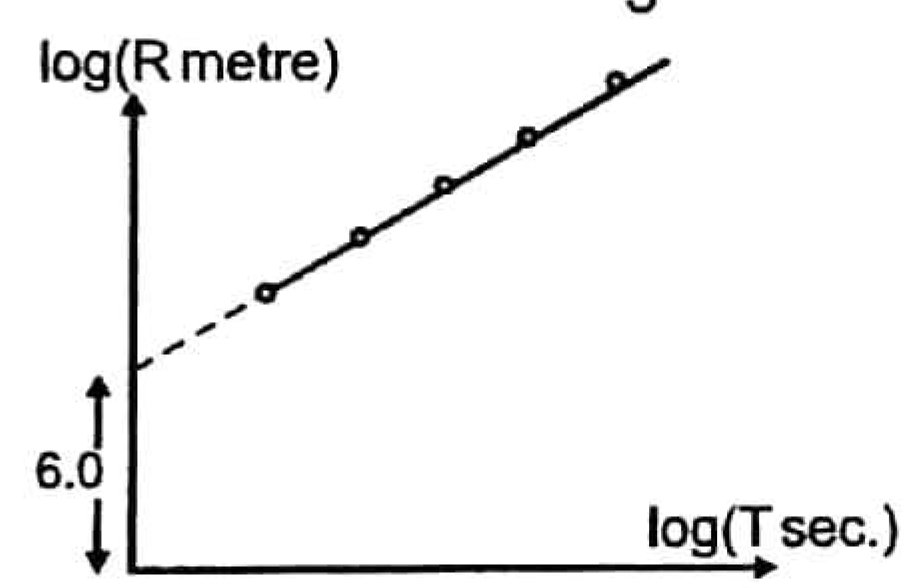
12. In case of earth :
(A) gravitational field is zero, both at centre and infinity
(B) gravitational potential is zero, both at centre and infinity
(C) gravitational potential is same, both at centre and infinity but not zero
(D) gravitational potential is minimum at the centre

PART - IV : COMPREHENSION

Comprehension - 1

Many planets are revolving around the fixed sun, in circular orbits of different radius (R) and different time period (T). To estimate the mass of the sun, the orbital radius (R) and time period (T) of planets were noted. Then $\log_{10} T$ v/s $\log_{10} R$ curve was plotted.
The curve was found to be approximately straight line (as shown in figure) having y intercept = 6.0
(Neglect the gravitational interaction among the planets [Take $G = \frac{20}{3} \times 10^{-11}$ in MKS, $\pi^2 = 10$]

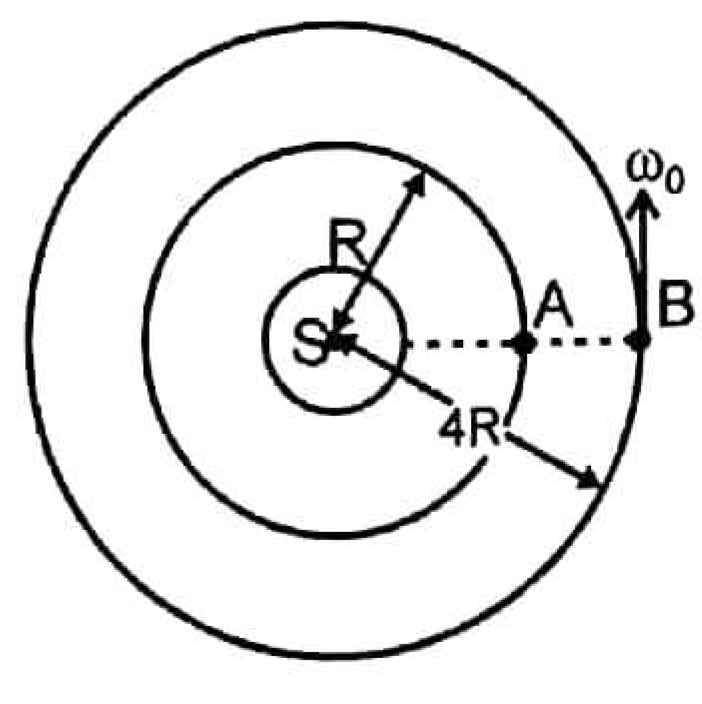




1. The slope of the line should be :
(A) 1 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{19}{4}$

2. Estimate the mass of the sun :
(A) 6×10^{29} kg (B) 5×10^{20} kg (C) 8×10^{25} kg (D) 3×10^{35} kg

3. Two planets A and B, having orbital radius R and 4R are initially at the closest position and rotating in the same direction. If angular velocity of planet B is ω_0, then after how much time will both the planets be again in the closest position ? (Neglect the interaction between planets).



(A) $\frac{2\pi}{7\omega_0}$ (B) $\frac{2\pi}{9\omega_0}$
(C) $\frac{2\pi}{\omega_0}$ (D) $\frac{2\pi}{5\omega_0}$

EXERCISE–1

PART - I

Section (A)

A-1. 4.8×10^{-5} N **A-2.** $\frac{4}{9}\pi^2\rho^2 G r^4$

A-3. 31.2 G m/sec^2 = 2.1×10^{-9} m/s^2, towards mid point

Section (B)

B-1. $-20\hat{i}-40\hat{j}$, $|\vec{F}| = 5\sqrt{5}$ N, $\vec{F} = -5\hat{i}-10\hat{j}$

B-2. 6.3×10^7 J/Kg

Section (C)

C-1. $\sqrt{gR}$ **C-2.** $2\sqrt{\frac{G(M_A+M_B)}{d}}$

Section (D)

D-1. 1.5%

D-2. (a) $F = \frac{GM^2}{4R^2}$ (b) $\sqrt{\frac{GM}{4R}}$; $T = 4\pi\sqrt{\frac{R^3}{GM}}$

(c) $\sqrt{\frac{GM}{4R^3}}$ (d) $\frac{GM^2}{4R}$ (e) $\sqrt{\frac{4GM}{R}}$

D-3. (a) $\frac{U_A}{U_B} = \frac{25600}{12800} = 2$ (b) $\frac{K_A}{K_B} = \frac{m_A}{m_B}\frac{r_B}{r_A} = 2$

(c) B is having more energy.

D-4. (a) The Saturn (b) The Mars (c) The Mars

Section (E)

E-1. $\frac{19}{2}$ m/s^2 = 9.5 m/s^2

E-2. $T = 1 - \frac{1}{2}\frac{4\pi^2}{(86400)^2} \times 6400 \times \frac{10^3}{9.8} = 0.998$ s

PART - II

Section (A)

A-1 (A) **A-2.** (B) **A-3.** (C)
A-4. (B)

Section (B)

B-1. (B) **B-2.** (D) **B-3.** (D)
B-4. (D) **B-5.** (C) **B-6.** (D)

Section (C)

C-1. (C) **C-2.** **(a)** (D), **(b)** (B), **(c)** (B), **(d)** (B)

Section : (D)

D-1. (B) **D-2.** (D) **D-3.** (D)
D-4. (A) **D-5.** (C)

Section (E)

E-1. (A)

PART - III

1.

I	II	III
A	p	w
B	r	u
C	q	v
D	p	t

2. (A) – p,r (B) – p,r (C) – q,r (D) – p,r

EXERCISE–2

PART - I

1.	(B)	2.	(A)	3.	(D)
4.	(C)	5.	(C)	6.	(B)
7.	(C)	8.	(D)	9.	(A)
10.	(D)	11.	(B)	12.	(A)
13.	(C)	14.	(C)	15.	(D)

PART - II

1.	99	2.	40	3.	1
4.	0	5.	2mm/sec.	6.	9
7.	11	8.	15	9.	2
10.	30°				

PART - III

1.	(ABCD)	2.	(ACD)	3.	(ABC)
4.	(BC)	5.	(BD)	6.	(BC)
7.	(ACD)	8.	(ABCD)	9.	(AD)
10.	(AC)	11.	(BC)	12.	(AD)

PART - IV

1.	(C)	2.	(A)	3.	(A)
4.	(B)	5.	(C)	6.	(A)
7.	(D)	8.	(B)	9.	(B)

EXERCISE–3

PART - I

1. (D) **2.** (A)→(p);(B)→(q, r);(C)→(p);(D)→(q, r)

3.	(C)	4.	(A)	5.	(A)
6.	6	7.	3	8.	(B)
9.	(BD)	10.	(BD)	11.	(B)
12.	2	13.	7	14.	(BC)
15.	(C)				

PART - II

1.	(1)	2.	(2)	3.	(4)
4.	(4)	5.	(1)	6.	(4)
7.	(1)	8.	(4)	9.	(2)
10.	(3)	11.	(1)		