# EVOLVING RELATIVISTIC FLUID SPACETIMES USING PSEUDOSPECTRAL METHODS AND FINITE DIFFERENCING

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We present a new code for solving the coupled Einstein-hydrodynamics equations to evolve relativistic, self-gravitating fluids. The Einstein field equations are solved on one grid using pseudospectral methods, while the fluids are evolved on another grid by finite differencing. We discuss implementation details, such as the communication between the grids and the treatment of stellar surfaces, and present code tests.

#### 1. Introduction

Numerical relativity has the potential to make indispensable contributions to our understanding of hydrodynamic compact object phenomena, such as neutron starneutron star (NSNS) binary merger, black hole-neutron star (BHNS) binary merger, and stellar core collapse. In such systems, both the spacetime metric and the fluid are dynamical, and they are strongly coupled.

The most common approach to numerically solving the coupled Einstein-hydrodynamics equations is by finite differencing. Finite difference (FD) techniques have been successfully used to simulate NSNS binaries, stellar collapse, and other interesting phenomena. FD algorithms usually converge to the exact solution as some power of the grid spacing. In addition, techniques have been developed which can evolve fluids with discontinuities stably and accurately. Unfortunately, FD codes usually require very large grids in order to obtain accurate results.

Einstein's equations can also be evolved using pseudospectral (PS) methods. For smooth functions, PS methods converge *exponentially* to the exact solution as the number of collocation points is increased. This allows PS methods to get accurate results with much smaller grids than those used by FD codes. A PS code for solving the Einstein equations has been developed by the Cornell-Caltech relativity group<sup>1,2</sup> and successfully used to carry out binary black hole inspiral simulations<sup>3</sup> which are both the most accurate and computationally cheapest of their kind.

There is a difficulty in evolving non-vacuum spactimes spectrally, however. Because of the possibility of stellar surfaces and shocks, the evolved variables are not always smooth at all derivatives. In these cases, spectral representations display Gibbs oscillations near the discontinuity which converge away only like a power of the number of collocation points, the order of convergence given by the order of the discontinuity. In some cases, the problem can be avoided by placing domain boundaries at discontinuities, but this is not practical for complicated shocks or strongly deformed stellar surfaces.

Another possibility would be a mixed approach: to evolve the metric fields, which are much smoother, using PS methods and evolve the hydrodynamic fields using

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shock-capturing FD methods. This would seem to utilize the strongest features of each method. This approach has been used successfully in a conformal gravity code to perform stellar collapse computations.<sup>4</sup> Here we extend this approach to full GR.

## 2. Numerical Algorithm

We integrate the hydrodynamic equations in conservative form. using piecewise parabolic reconstruction<sup>5</sup> together with a high-resolution central scheme.<sup>6</sup> We use uniform grids in three or two dimensions (the latter for axisymmetric systems). The vacuum outside the stars is handled by introducing a tenuous "atmosphere", together with a density floor and an internal energy ceiling, in these regions. The Einstein equations are evolved in the generalized harmonic system.<sup>2</sup> We find that filtering the metric variables is sufficient to stabilize the PS code in the presence of discontinuities. The interpolation from the spectral to the fluid grid would be very expensive if done directly, but we make the process much quicker using a technique introduced by Boyd.<sup>7</sup> Both codes use dual coordinate frames, which allow the grid to dynamically adjust to the motion of the system.<sup>3</sup>

Our finite difference code currently has no adaptive griding capability, but our two-grid approach gives us a few similar advantages. The finite difference grid need only cover the region containing the matter—a huge savings in many binary applications. The dual coordinate frame system allows the finite difference grid to move with the stars. Also, separate coordinate mappings can be applied to the two grids so that their resolutions can be controlled separately.

## 3. Tests

Our FD code has been successfully tested by evolving multi-dimensional shocks. In order to test the full FD plus PS code, we evolve equilibrium polytropes. We choose the domain decomposition of the PS grid to consist of a filled sphere (a "ball") centered on the star surrounded by several concentric spherical shells. Angular basis functions are spherical harmonics. For radial basis functions, we use Chebyshev polynomials on the shells and an appropriate set of functions<sup>8</sup> on the ball.

In Figure 1, we show results for four n=1 polytropes. Star A is a stable TOV star with central density  $\rho_{\rm c}/\rho_{\rm crit}=0.67$ , star B is an unstable TOV star with  $\rho_{\rm c}/\rho_{\rm crit}=1.33$ , star C has the same rest mass as star A but rotates uniformly with an angular velocity 80% of the mass shedding limit, and star C is a hypermassive rapidly differentially rotating star evolved by other groups. We choose the FD grid spacing so that 30 points cover a stellar radius. (We find that axisymmetric and 3D runs give similar results for all the models.) For stars A, B, and C, we choose our PS grids to have two shells, with the inner one containing the stellar surface. We use spherical harmonics up to L=7, and we use 7, 9, and 7 radial collocation points in the ball and the two shells, respectively. We find that the error in the PS grid is dominated by the shell containing the stellar surface, and it decreases quadratically with the grid spacing in this shell. We add a 1% pressure depletion to test stability.

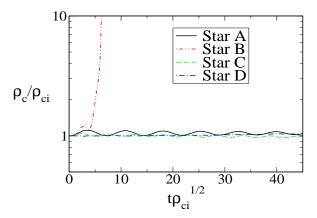


Fig. 1. Central density  $\rho_c$  as a function of time t for four equilibrium stars, where  $\rho_{ci} = \rho_c(t=0)$ .

For star D, we choose the ball with spherical harmonics up to L=14 and 18 radial points to cover the whole star.

We find that our code can accurately evolve equilibrium stars and distinguish stable from unstable configurations. We have also successfully evolved moving stars using the dual coordinate system to track the star's center of mass. These tests encourage us to think that our code might be able to produce accurate simulations of NSNS and BHNS binaries.

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