PHYSICS 200 LABORATORY MANUAL

FOUNDATIONS OF PHYSICS II

Spring 2010

Contents

Introduction

Purpose

> By a careful and diligent study of natural laws I trust that we shall at least
> escape the dangers of vague and desultory modes of thought and acquire a habit
> of healthy and vigorous thinking which will enable us to recognise error in all
> the popular forms in which it appears and to seize and hold fast truth whether
> it be old or new... [But] I have no reason to believe that the human intellect is
> able to weave a system of physics out of its own resources without experimental
> labour. Whenever the attempt has been made it has resulted in an unnatural
> and self-contradictory mass of rubbush. *James Clerk Maxwell*

Physics and engineering rely on quantitative experiments. Experiments are designed sim-
plifications of nature: line drawings rather than color photographs. The hope is that by
stripping away the details, the essence of nature is revealed. (Of course, the critics of science
would argue that the essence of nature is lost in simplification: a dissected frog is no longer a
frog.) While the aim of experiment is appropriate simplification, the design of experiments
is anything but simple. Typically it involves days (weeks, months, ...) of "fiddling" before
the experiment finally "works". I wish this sort of creative problem-oriented process could
be taught in a scheduled lab period, but limited time and the many prerequisites make this
impossible. Look for more creative labs starting next year!

Thus this Lab Manual describes experiences ("labs") that are a caricature of experimental
physics. Our labs will typically emphasize thorough preparation, an underlying mathemati-
cal model of nature, good experimental technique, analysis of data (including the significance
of error) ... the basic prerequisites for doing science. But your creativity will be circum-
scribed. You will find here "instructions" which are not a part of real experiments (where
the methods and/or outcomes are not known in advance). In my real life as a physicist, I
have little use for "instructions", but I'm going to try and force you to follow them in this
course. (This year: follow what I say—not what I do.)

The goals of these labs are therefore limited. You will:

1. Perform experiments that illustrate the foundations of electricity and magnetism.

2. Become acquainted with some commonly used electronic lab equipment (meters,
   scopes, sources, etc).

3. Perform basic measurements and recognize the associated limitations (which, when
   expressed as a number, are called uncertainties or errors).

4. Practice the methods which allow you to determine how uncertainties in measured quantities propagate to produce uncertainties in calculated quantities.

5. Practice the process of verifying a mathematical model, including data collection, data display, and data analysis (particularly graphical data analysis with curve fitting).

6. Practice the process of keeping an adequate lab notebook.

7. Experience the process of "fiddling" with an experiment until it finally "works".

8. Develop an appreciation for the highs and lows of lab work. And I hope: learn to learn from the lows.

Lab Schedule

The lab schedule can be found in the course syllabus. You should be enrolled in a lab section for PHYS 200 and you should perform and complete the lab that day/time. Problems meeting the schedule should be addressed—well in advance—to the lab manager.

Materials

You should bring the following to each lab:

- Lab notebook. You will need three notebooks: While one is being graded, the others will be available to use in the following labs. The lab notebook should have quad-ruled paper (so that it can be used for graphs) and a sewn binding (for example, Ampad #26–251, available in the campus bookstores). The notebooks may be "used" (for example, those used in PHYS 191).

- Lab Manual (this one)

- The knowledge you gained from carefully reading the Lab Manual before you attended.

- A calculator, preferably scientific.

- A straightedge (for example, a 6" ruler).

- A pen (we prefer your lab book be written in ink, since you're not supposed to erase).

Before Lab:

Since you have a limited time to use equipment (other students will need it), it will be to your advantage if you come to the laboratory well prepared. Please read the description of the experiment carefully, and do any preliminary work *in your lab notebook* before you come to lab. Note carefully (perhaps by underlining) questions included in the lab description. Typically you will lose points if you fail to answer every question.

During Lab:

Note the condition of your lab station when you start so that you can return it to that state when you leave. Check the apparatus assigned to you. Be sure you know the function of each piece of equipment and that all the required pieces are present. If you have questions, ask your instructor. Usually you will want to make a sketch of the setup in your notebook. Prepare your experimental setup and decide on a procedure to follow in collecting data. Keep a running outline in your notebook of the procedure actually used. If the procedure used is identical to that in this Manual, you need only note "see Manual". Nevertheless, an outline of your procedure can be useful even if you aim to exactly follow the Manual. Prepare tables for recording data (leave room for calculated quantities). *Write your data in your notebook as you collect it!*

Check your data table and graph, and make sample calculations, if pertinent, to see if everything looks satisfactory before going on to something else. Most physical quantities will appear to vary continuously and thus yield a smooth curve. If your data looks questionable (e.g., a jagged, discontinuous "curve") you should take some more data near the points in question. Check with the instructor if you have any doubts.

Complete the analysis of data in your notebook and indicate your final results clearly. If you make repeated calculations of any quantity, you need only show one sample calculation. Often a spreadsheet will be used to make repeated calculations. In this case it is particularly important to report how each column was calculated. Tape computer-generated data tables, plots and least-squares fit reports into your notebook so that they can be examined easily. Answer all questions that were asked in the Lab Manual.

CAUTION: *for your protection and for the good of the equipment, please check with the instructor before turning on any electrical devices.*

Lab Notebook

Your lab notebook should represent a *detailed* record of what you have done in the laboratory. It should be complete enough so that you could look back on this notebook after a year or two and reconstruct your work.

Your notebook should include your preparation for lab, sketches and diagrams to explain the experiment, data collected, initial graphs (done as data is collected), comments on difficulties, sample calculations, data analysis, final graphs, results, and answers to questions asked in the Lab Manual. **NEVER** delete, erase, or tear out sections of your notebook that you want to change. Instead, indicate in the notebook what you want to change and why (such information can be valuable later on). Then lightly draw a line through the unwanted section and proceed with the new work.

DO NOT collect data or other information on other sheets of paper and then transfer to your notebook. Your notebook is to be a running record of what you have done, not a formal (all errors eliminated) report. There will be no formal lab reports in this course. When you have finished a particular lab, you turn in your notebook.

Ordinarily, your notebook should include the following items for each experiment.

NAMES. The title of the experiment, your name, your lab partner's name, and your lab station number.

DATES. The date the experiment was performed.

PURPOSE. A brief statement of the objective or purpose of the experiment.

THEORY. At least a listing of the relevant equations, and what the symbols represent. Often it is useful to number these equations so you can unambiguously refer to them.

   Note: These first four items can usually be completed before you come to lab.

PROCEDURE. This section should be an outline of what you did in lab. As an absolute minimum your procedure must clearly describe the data. For example, a column of numbers labeled "voltage" is not sufficient. You must identify how the voltage was measured, the scale settings on the voltmeter, etc. Your diagram of the apparatus (see below) is usually a critical part of this description, as it is usually easier to draw how the data were measured than describe it in words. Sometimes your procedure will be identical to that described in the Lab Manual, in which case the procedure may be abbreviated to something like: "following the procedure in the Lab Manual, we used apparatus Z to measure Y as we varied X". However there are usually details you can fill in about the procedure. Your procedure may have been different from that described in the Lab Manual. Or points that seem important to you may not have been included. And so on. This section is also a good place to describe any difficulties you encountered in getting the experiment set up and working.

DIAGRAMS. A sketch of the apparatus is almost always required. A simple block diagram can often describe the experiment better than a great deal of written explanation.

DATA. You should record in your notebook (or perhaps a spreadsheet) a concurrent record of your relevant observations (the _actual_ immediately observed data not a recopied version). You should record _all_ the numbers (including every digit displayed by meters) you encounter, including _units_ and _uncertainties._ If you find it difficult to be neat and organized while the experiment is in progress, you might try using the left-hand pages of your notebook for doodles, raw data, rough calculations, etc., and later transfer the important items to the right-hand pages. This section often includes computer-generated data tables, graphs and fit reports — just tape them into your lab book (one per page please).

You should examine your numbers as they are observed and recorded. Was there an unusual jump? Are intermediate data points required to check a suspicious change? The best way to do this is to graph your data as you acquire it or immediately afterward.

CALCULATIONS. Sample calculations should be included to show how results are obtained from the data, and how the uncertainties in the _results_ are related to the uncertainties in the _data_ (see Appendix C). For example, if you calculate the slope of a straight line, you should record your calculations in detail, something like:

$$\frac{v_2 - v_1}{t_2 - t_1} = \frac{(4.08 - .27) \text{ cm/wink}}{(15 - 1) \text{ wink}} = 0.27214 \text{ cm/wink}^2 \qquad (1)$$

The grader must be able to reproduce your calculated results based on what you have recorded in your notebook. The graders are told to totally disregard answers that appear without an obvious source. It is particularly important to show how each column in a spreadsheet hardcopy was calculated (quick & easy via 'self documenting' equations).

RESULTS/CONCLUSIONS. You should end each experiment with a conclusion that summarizes your results — what were your results, how successful was the experiment, and what did you learn from it.

This section should begin with a *carefully constructed* table that collects all of your important numerical results in one place. Numerical values should always include units, an appropriate number of significant digits and the experimental error.

You should also compare your results to the theoretical and/or accepted values. Does your experimental range of uncertainty overlap the accepted value? Based on your results, what does the experiment tell you?

DISCUSSION/CRITIQUE. As a service to us and future students we would appreciate it if you would also include a short critique of the lab in your notebook. Please comment on such things as the clarity of the Lab Manual, performance of equipment, relevance of experiment, timing of the experiment (compared to lecture) and if there is anything you particularly liked or disliked about the lab. This is a good place to blow off a little steam. Don't worry; you won't be penalized, and we use constructive criticisms to help improve these experiments.

QUICK REPORT. As you leave lab, each lab group should turn in a 3"×5" quick report card. You will be told in lab what information belongs on your card. These cards go directly to the instructor who will use them to identify problems.

Drop off your lab notebook in your lab instructor's box. Note: *The TA's cannot accept late labs. If for some reason you cannot complete a lab on time, please see the lab manager (Lynn Schultz) in PEngel 139 or call 363–2835. Late labs will only be accepted under exceptional circumstances. If an exception is valid, the lab may still be penalized depending on how responsibly you handled the situation (e.g., did you call BEFORE the lab started?).*

Grading

Each lab in your notebook will be graded separately as follows:

9–10 points:	A
8–8.9 points:	B
7–7.9 points:	C
6–6.9 points:	D
0–5.9 points:	Unsatisfactory

1. Field Superposition

Purpose

The universe is filled with sources of electric and magnetic fields. In this lab you will test the principle that the field produced by several sources simultaneously is just the vector sum of the fields produced by each source individually.

Introduction

From Coulomb's law we know the electric field produced by an isolated charged particle. In a universe of zillions of electrons and protons this result would be without value if we lacked a method of finding the field resulting from multiple sources. Electric fields (and also magnetic fields) combine in the simplest possible way: by vector addition[1].

In this experiment we find it convenient to work with magnetic fields rather than electric fields. We have a readily available source of natural magnetic field from the Earth; we can produce controlled magnetic fields using electromagnets; and we can easily measure the direction of the magnetic field using an ordinary compass. While you have not yet covered magnetic fields in lecture, all you will need to know about them for this lab is that the source of magnetic field is electric current and that the magnetic field produced by a current is proportional to that current. Permanent magnets result from orbiting, spinning electrons in iron; Electromagnets result from the easily measured current flowing through copper wires that make up the windings of the coil. (Similar electric currents flowing through the metallic core of the Earth power the Earth's magnetic field.) Just as an electric field is proportional to the charge that is its source, so the magnetic field of an electromagnet is proportional to its current.

Apparatus

- 1 power supply

- 1 digital multimeter

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[1]Electricity and magnetism is really just one example, as these two things are, as Einstein showed, really just different aspects of one thing: $F_{\mu\nu}$. Note that many other force fields (for example that in Einstein's theory of gravity called general relativity) do *not* satisfy this simple combination rule.
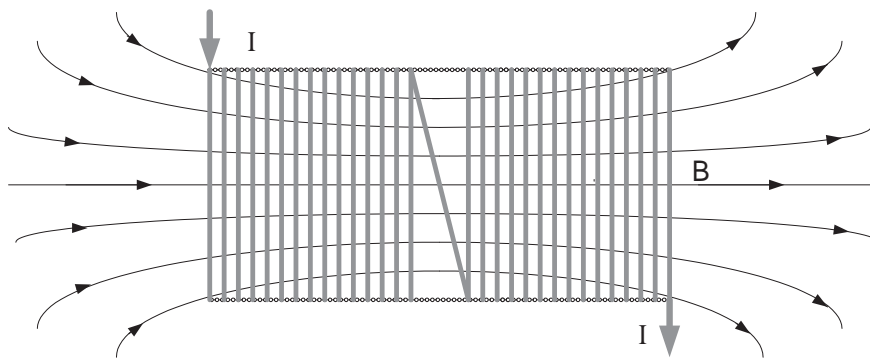
Figure 1.1: Magnetic field lines through a finite solenoid of coils carrying current I.

- 1 solenoid assembly

- 1 compass

- 1 set of leads

Theory

Solenoid

A solenoid is a length of wire wound around a cylinder. Mathematically it is easiest to think about a solenoid as a series of circular loops of wire each carrying the same current, but in fact the wire is a tightly spaced helix (spiral). Figure 1.1 shows the magnetic field lines of a solenoid carrying a current I. The result is perhaps a bit surprising: the magnetic field goes through the core of the cylinder as the current winds around the edge of the cylinder. In a month or so you'll learn how to calculate the magnetic field for such a solenoid, but for this lab it is enough to know that the magnetic field is everywhere proportional to the current. In particular, the magnetic field, B, at he center of our solenoid (where we will be doing our experiment) is given by:

$$B_s = aI. \tag{1.1}$$

where I is the electric current flowing around the solenoid (measured in ampere, denoted "A") and $a = 1.14 \times 10^{-3}$ T/A. (The unit of magnetic field is tesla, denoted "T".) The solenoid will be used as an adjustable source of magnetic field.

Earth's Magnetic Field

The magnetic field of the Earth varies with position and time. However, during the course of this lab, at your particular lab table, the Earth's field may be considered constant in magnitude and direction. (That is the currents producing the Earth's magnetic field will not change much during this experiment.) Generally speaking the Earth's field points north, however here in the northern hemisphere it also points down. The inclination of the field is quite large (over 70°) at our latitude.
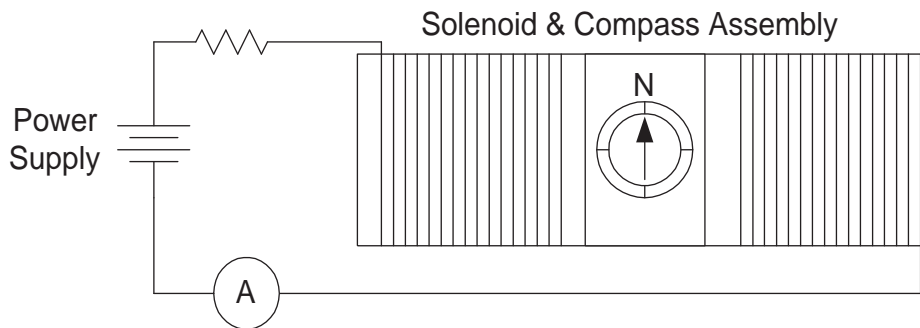
Figure 1.2: Solenoid circuit.

Compass

The needle of a compass points in the direction of the magnetic field it experiences. We say that a compass needle shows north, because that is (generally speaking) the direction of the Earth's magnetic field. The needle is a magnetic dipole made of a small permanent magnet. Just as an electric dipole feels a torque aligning it to the external electric field, so a magnetic dipole twists until it is aligned with the external magnetic field. We will use the compass to show us the direction of the magnetic field that results from the superposition of the Earth's field with that of the solenoid. While the compass is graduated in degrees, we will find it convenient to work in radians, and so the conversion factor $360° = 2\pi$ radian should be applied to all angles.

Setup

As shown in Figure 1.2, the compass is placed in the center of the solenoid, and the apparatus is oriented so that with no current flowing the compass needle points perpendicular to the coil-axis in the direction "N", i.e., $\theta = 0$. When a current is sent through the coil, the needle will be deflected to point in the direction of the resulting magnetic field. The current will be supplied by an adjustable power supply, and accurately measured with a digital multimeter. (For more information on the operation of the multimeter see Figure 4.2 on page 40.) If we denote the magnetic field of the Earth by B_e and the magnetic field of the solenoid by B_s, the resulting total magnetic field should be the vector sum of the two. As shown in Figure 1.3, we predict

$$\tan\theta = \frac{B_s}{B_e} = \frac{a}{B_e}\, I \tag{1.2}$$

Thus a graph of $\tan\theta$ vs. I should be a straight line.

Note: since the compass needle is only free to rotate in the horizontal plane, only the horizontal components of the magnetic field are detected. Thus B_e above is actually just the horizontal component of the Earth's magnetic field. (Of course, the solenoid has been oriented so that its field is fully horizontal.) At our latitude it turns out that the vertical component of the Earth's field is more than twice as large as the horizontal component.
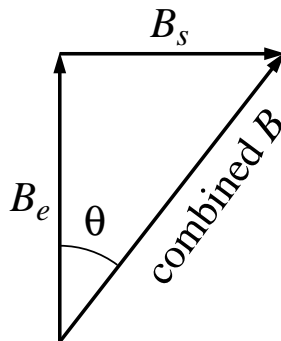
Figure 1.3: Adding two magnetic fields: B_e from the Earth and B_s from the solenoid. The angle of the resulting magnetic field is called θ.

Procedure

Position the solenoid/compass assembly such that the compass needle is aligned with north (N) on the compass housing. Without moving the assembly, connect the power supply in series with the assembly and multimeter as shown in Fig. 1.2. Set the multimeter FUNCTION to measure DC amps, denoted: ⎓A, on the 20 mA RANGE. After your instructor checks the circuit, take a series of readings from the compass and multimeter while increasing the current from power supply.

Record about ten well-spaced readings. (Readings should be spaced by about 2 mA or 5°, but it is a waste of time to try to make I or θ a round number. Don't include the $\theta = 0$, $I = 0$ starting condition as a measurement.)

Reverse the leads on the power supply and take a similar set of measurements. For this last set of measurements the current, the angle, and the tangent of the angle will be negative. When the measurements have been completed, turn off the power supply and check to see if the compass is still aligned to the north. If the solenoid/compass assembly has moved, the measurements should be redone.

Calculations will be easier if you put your measured values into a spreadsheet. The uncertainty in the current (δI) can be accurately determined from Table 4.1 on page 45, however for this lab an error of 0.5% should be accurate enough. The uncertainty in the compass reading ($\delta\theta$) must be estimated based on your ability to read the compass scale. Typically, for an analog scale, the uncertainty is estimated to be ± half of the smallest scale division. You may wish to assign a larger uncertainty if you believe the compass is unusually difficult to read. *Because computers generally assume angles are in radians, you will need to convert both θ and $\delta\theta$ to radians!*

Lab Report

1. From your uncertainty in θ, determine an uncertainty for each value of $y = \tan\theta$. According to calculus, this is:

$$\delta(\tan\theta) = \delta\theta/(\cos\theta)^2 = \delta\theta\left(1 + \tan^2\theta\right) = \delta\theta(1 + y^2) \tag{1.3}$$

(This formula can be entered into *WAPP$^+$* directly.) Alternatively you can estimate errors from the difference $|\tan(\theta + \delta\theta) - \tan\theta| \approx \delta(\tan\theta)$. Print your spreadsheet and include it in your notebook. (Remember to self-document your spreadsheet or show sample calculations.) Make sure you've clearly displayed how each column was calculated.

2. Use *WAPP$^+$* (Goggle "`wapp+`" to find it) to determine the line best approximating your $\tan\theta$ versus I data. (Enter the current and its uncertainty in amperes, not mA.) Tape the fit report and plot in your notebook.

3. Using the best-fit slope, calculate the Earth's magnetic field, B_e, at SJU. (Actually this is just the horizontal component of of the Earth's magnetic field.) A "ballpark" value for B_e is 1.5×10^{-5} T, but magnetic materials in the building will affect the result. Using the uncertainty in slope, calculate the resulting uncertainty in B_e.

4. Complete your lab report with a conclusion and lab critique.

2. Equipotentials and Electric Field Lines

A new concept appeared in physics, the most important invention since Newton's time: the field. It needed great scientific imagination to realize that it is not the charges nor the particles but the field in the space between the charges and the particles that is essential for the description of physical phenomena. The field concept proved successful when it led to the formulation of Maxwell's equations describing the structure of the electromagnetic field.

Einstein & Infield *The Evolution Of Physics* 1938

Purpose

To explore the concepts of electrostatic potential and electric field and to investigate the relationships between these quantities. To understand how equipotential curves are defined and how they can display the behavior of both the potential and field.

Introduction

Every electric charge in the universe exerts a force on every other electric charge in the universe. Alternatively we can introduce the intermediate concept of an electric field. We then say that the universe's charge distribution sets up an electric field $\vec{\mathbf{E}}$ at every point in space, and then that field produces the electric force on any charged particle that happens to be present. The electric field $\vec{\mathbf{E}}$ is defined in terms of things we can measure if we bring a "test charge" q to the spot where we want to measure $\vec{\mathbf{E}}$. The electric field is then defined in terms of the total force, $\vec{\mathbf{F}}$, experienced by that test charge q:

$$\vec{\mathbf{E}} = \vec{\mathbf{F}}/q \tag{2.1}$$

Note that the field exists independently of any charges that might be used to measure it. For example, fields may be present in a vacuum. At this point is is not obvious[1] that the

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[1] *Star Wars Episode IV*:
LUKE: You don't believe in the Force, do you?
HAN: Kid, I've flown from one side of this galaxy to the other. I've seen a lot of strange stuff, but I've

field concept really explains anything. The utility of the electric field concept lies in the fact that the electric field exists a bit independently of the charges that produce it. For example, there is a time delay between changes in the source charge distribution and the force on distant particles due to propagation delay as the field readjusts to changes in its source.

The electric force $\vec{F}$ does work $(\vec{F} \cdot \Delta\vec{r})$ on a charge, q, as it is moved from one point to another, say from A to B. This work results in a change in the electrical potential energy $\Delta U = U_B - U_A$ of the charge. In going from a high potential energy to a low potential energy, the force does positive work: $W_{AB} = -\Delta U$. Consider, for a moment, a line or surface along which the potential energy is constant. No work is done moving along such a line or surface, and therefore the force cannot have a component in this direction: Electric forces (and hence the electric fields) must be **perpendicular** to surfaces of constant potential energy. This concept is central to our experiment.

Example: Gravitational potential energy. Consider a level surface parallel to the surface of the Earth. The potential energy mgh does not change along such a surface. And of course, both the gravitational force $m\vec{g}$, and the gravitational field $\vec{g}$ are perpendicular to this surface, just as we would expect from the above reasoning. Electrical fields and potential energies are less intuitive because less familiar, but they work in much the same way.

We can define a new quantity, the potential difference $\Delta V = V_B - V_A$ between points A and B, in the following way:

$$\Delta V = \frac{\Delta U}{q} = -\frac{W_{AB}}{q} \tag{2.2}$$

where W_{AB} is the work done by the electric force as the test charge q is moved from A to B. Although the potential energy difference $\Delta U = U_B - U_A$ does depend on q, the ratio $\Delta U/q$ (and therefore ΔV) is independent of q. That is, in a manner analogous to the electric field, potential difference does not depend on the charge q used in its definition, but only on the charge distribution producing it. Also, since $\vec{F}$ is a conservative force, W_{AB} (and hence the potential difference) does not depend on the path taken by q in moving from A to B.

Finally, note that only the potential *difference* has been defined. Specifying the potential itself at any point depends on assigning a value to the potential at some convenient (and arbitrary) reference point. Often we choose the potential to be zero "at infinity", i.e., far away from the source charges, but this choice is arbitrary. Moreover, if we measure only potential differences, the choice of "ground" (zero volts) doesn't matter.

Apparatus

- DC power supply

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never seen anything to make me believe there's one all-powerful force controlling everything. There's no mystical energy field that controls my destiny.

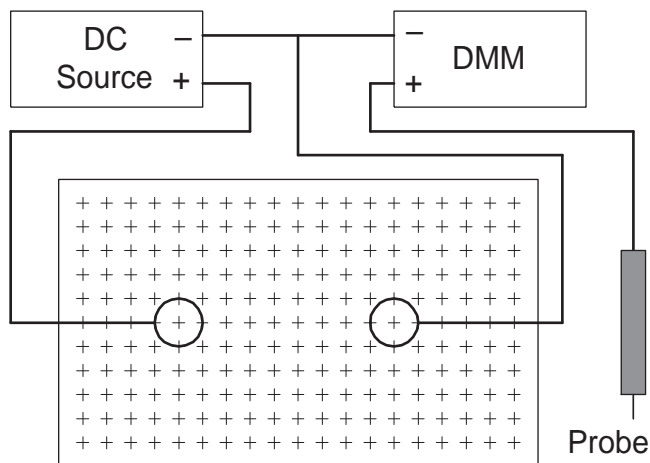Of course, we should replace "force" in the above with "field".

Figure 2.1: Electric field mapping circuit.

- digital multimeter (DMM)

- conductive paper with electrode configurations (2)

- card board

- graph paper

- push pins

You will be provided two electrode configurations drawn with a special type of conducting ink (graphite suspension) on a special carbon impregnated paper, as sketched in Fig. 2.1. The paper has a very high, but finite, resistance, which allows small currents to flow. Nevertheless, since there is a big difference between the resistance of the conducting ink and the paper, the potential drop within the ink-drawn electrodes is negligible (less than 1%) of that across the paper. The potential differences closely resemble those under strictly electrostatic (no current) conditions.

Theory

In this experiment you will use different configurations and orientations of electrodes which have been painted on very high resistance paper with conducting ink. When the electrodes are connected to a "battery" (actually a DC power supply: a battery eliminator) an electric field is set up that approximates the electrostatic conditions discussed in lecture. Under truly electrostatic conditions the charges would be fixed, and the battery could be disconnected and the electric field would continue undiminished. However, in this experiment the battery must continue to make up for the charge that leaks between the electrodes through the paper. These currents are required for the digital multimeter (DMM) to measure the potential difference. Nevertheless, the situation closely approximates electrostatic conditions and measurement of the potential differences will be similar to those obtained under strictly electrostatic conditions.
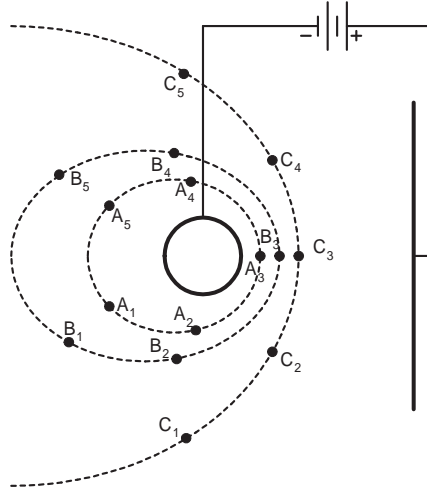
Figure 2.2: Some equipotential curves for the circle-line electrodes.

Consider Fig. 2.2 where the electrodes are a circle and a line. Suppose we fix one electrode of the digital multimeter (DMM) at some (arbitrarily chosen) point O and then, using the second electrode as a probe, find that set of points $A_1, A_2, A_3, \ldots$ such that the potential differences between any of these points and O are equal:

$$V_{A_1 O} = V_{A_2 O} = V_{A_3 O} = \ldots. \tag{2.3}$$

Hence the potentials at points $A_1, A_2, A_3, \ldots$ are equal. In fact, we can imagine a continuum of points tracing out a continuous curve, such that the potential at *all* the points on this curve is the same. Such a curve is called an *equipotential*. We label this equipotential V_A. We could proceed to find a second set of points, $B_1, B_2, B_3, \ldots$ such that the potential difference $V_{B_n O}$ is the same for $n = 1, 2, 3, \ldots$; i.e.,

$$V_{B_1 O} = V_{B_2 O} = V_{B_3 O} = \ldots. \tag{2.4}$$

This second set of points defines a second equipotential curve, which we label V_B. The actual values of the potentials assigned to these curves, V_A and V_B, depend on our choice of the reference potential V_O. However, the potential *difference* between any two points on these two equipotentials does not depend on our choice of V_O; that is,

$$(V_A - V_O) - (V_B - V_O) = V_A - V_B \tag{2.5}$$

and so the potential difference $V_A - V_B$ can be determined from the measured values V_{AO} and V_{BO}. A mapping of electrostatic equipotentials is analogous to a contour map of topographic elevations, since lines of equal elevation are gravitational equipotentials. The elevation contours are closed curves; that is, one could walk in such a way as to remain always at the same height above sea level and eventually return to the starting point. Likewise, electrostatic equipotentials are closed curves. The electric field at a given point is related to the spatial rate of change of the potential at that point, i.e., the *gradient*, in the potential. Continuing the analogy with the contour map, we find the electric field is

(a) Two equipotential curves and the minimum distance, Δx, between them.

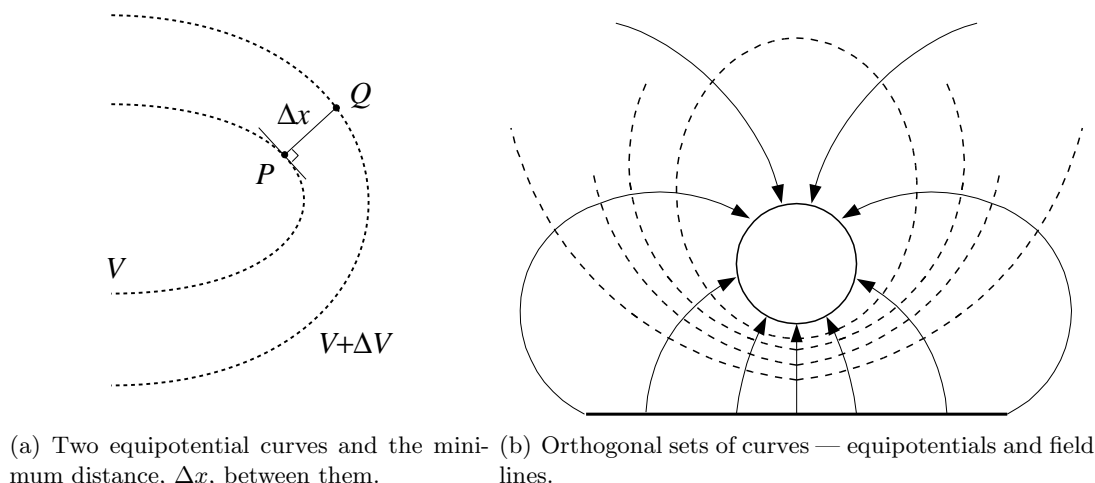(b) Orthogonal sets of curves — equipotentials and field lines.

Figure 2.3: Equipotential curves

analogous to the slope of the landscape, with the direction of the electric field corresponding to the direction of steepest slope at the point in question. Thus, as shown in Figure 2.3(a), with one equipotential at potential V and a second, nearby one, at potential $V + \Delta V$, the magnitude of the electric field at point P is approximated by

$$E \approx \left| -\frac{\Delta V}{\Delta x} \right|, \tag{2.6}$$

where Δx is the minimum distance between the two equipotentials. Using the minimum value of Δx guarantees that the 'derivative' $\left| -\frac{\Delta V}{\Delta x} \right|$ is largest at the point P. (The derivative in any direction is determined by the *gradient* of V: $dV = \vec{\boldsymbol{\nabla}} V \cdot d\vec{\mathbf{r}}$; the electric field is closely related to the gradient: $\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V$.) The direction of the electric field is in the direction of Δx, either from P to Q, or in the opposite direction, from Q to P, depending upon whether ΔV is negative or positive. The minus sign in Eq. (2.6) simply means that the electric field is in the direction of decreasing potential. One more property of the gradient is important to note: the gradient is perpendicular to the tangent to the equipotential at point P, as shown in Fig. 2.3(a). Thus, the electric field is perpendicular to the equipotential curves.

We can now envision two entire families of curves, as illustrated in Figure 2.3(b): the first, a set of equipotentials, and the second, a set of curves whose tangent at a point indicates the direction of the electric field at that point. The members of the latter family, the electric field lines, are also called "lines of force" because the electric field is in the direction in which a positive charge q would move if it were placed at that point. As mentioned earlier, the equipotentials form closed curves (no beginning, no end), while the field lines start on positive charge and end on negative charge. The two families of curves are orthogonal (perpendicular), because at any given point, say P, the tangent to the equipotential through P is perpendicular to the field line through P. The direction of a field line is in the direction of the electric field.

The arrows on field lines indicate the direction of the electric field.

Conventionally, equipotential are selected with their potentials in a sequence of uniform steps (i.e., constant ΔV). The spacing (Δx) between the equipotentials then immediately

gives the relative strength of the corresponding electric field at the point: small Δx corresponding to large E.

Conventionally, the spacing of the field lines is also designed to give a qualitative measure of the magnitude (strength) of the electric field; that is, the field is stronger where the field lines are more concentrated. (In 3 dimensional space we talk about the "density of field lines", meaning the number of lines per cross sectional area. In our plane figures, this corresponds to the separation between the field lines.) Once the equipotentials have been mapped, the field lines can be found by drawing the orthogonal family of curves.

Procedure

1. A pair of electrodes have been defined by a pattern painted with conducting silver ink on black high resistance paper. Record (copy or trace) the electrode patterns onto a piece of graph paper. Make copies for each lab partner.

2. Place one of the conductive sheets on the cork board. Connect each painted electrode to the power supply (battery eliminator) using a positive (+, red) or negative (−, black) wire and a pushpin. Do not break the conducting path of the painted electrode with a pushpin hole—instead the pushpin should be placed directly alongside the painted electrode. The pushpin will then press and hold the ring terminal of the wire to the painted electrode without scraping off the paint. You can hold the sheet in place with additional pins at corners. Connect the black (negative, common or "COM") terminal of the DMM to the negative terminal on the power supply. (Figure 4.2 on page 40 briefly describes the operation of the multimeter.) You will use the (red) probe connected to the other voltage terminal of the DMM to measure voltage difference.

3. Have your lab instructor check your set-up. The DMM should be set to read DC Volts (FUNCTION ⎓V), with a RANGE of 20 Volts. Set the power supply to about 10 Volts DC.

4. With the free probe of the DMM, touch the positive electrode and record the potential difference V_{PO} between the two points probed by the DMM. Press firmly to make good electrical contact; however, do not puncture or otherwise damage the paper (or the painted electrode). Touch the DMM probe to various points on the positive electrode. (This may not be possible if the electrode itself is very small, i.e., a "point charge".) Each electrode should be an equipotential; if the potential seems to vary check that you have firm connections to the electrode and contact your instructor if you cannot correct the problem.

   Touch the free probe of the DMM to the negative electrode and record the (nearly zero) reading V_{NO}. (Again: this electrode should also be an equipotential. If you cannot achieve an equipotential electrode by reseating the ring terminal on the painted electrode, contact your instructor.) In both readings, O refers to a fixed location at the negative terminal of the power supply. V_{NO} and V_{PO} are the potential differences between the negative (N) and positive (P) electrodes, and the reference O. Compute V_{PN}, the potential difference between the positive and negative electrodes.

5. In the vicinity of the negative electrode, find a point where the DMM reads 2.0 Volts as the free probe is touched to the paper. Remember, do not puncture or otherwise

damage the paper. Record this point as accurately as you can on the (white) graph paper.

Now move the free probe of the DMM to another nearby point where the reading is again 2.0 V and again record this point on your graph paper. Obtain a series of such points, all corresponding to a DMM reading of 2.0 V, by gradually moving the probe from point to point. Eventually the set of points will close on itself or, perhaps, go off the edge of the paper.

6. Repeat the previous step for some other DMM readings, say 3.0 V, 4.0 V, 5.0 V, 6.0 V, 7.0 V, and 8.0 V. You should obtain at least five sets of equipotential points.

7. Draw the equipotential curves on the graph paper by connecting the points at the given potential with a smooth curve. Each such curve is an equipotential and can be labeled by the appropriate DMM reading.

8. Repeat Steps 2 through 7 for the second electrode configuration.

Analysis

Show your equipotentials to the lab instructor. The instructor will assign a point — call it P — where you should find the electric field.

Carry out the following steps:

1. Draw the tangent to the equipotential through P. (P may have been selected so that is does not lie on an equipotential you have measured. Nevertheless you should be able to estimate the orientation of that unmeasured equipotential by comparison with the neighboring, measured equipotentials.)

2. Draw the perpendicular to this tangent through P.

3. Find the distance Δx, by measuring in cm using a ruler, between two adjacent equipotentials, which differ by ΔV. Find the magnitude and direction of the electric field $\vec{\mathbf{E}}$ at point P, i.e. $\vec{\mathbf{E}}_P$. Record the magnitude and indicate the direction by a vector placed at P on the graph paper. (Δx could also be defined as the shortest distance between the adjacent equipotentials measured along a line the goes through the point P.)

4. Sketch the field lines orthogonal to the equipotentials for this configuration of electrodes, and indicate their direction by small arrowheads. (See Figure 2.3(b).)

5. Repeat Step 4 for the second electrode configuration.

3. The Digital Oscilloscope

Purpose

To become familiar with two common lab instruments used with time varying electrical signals (AC). The function generator produces simple AC signals; the oscilloscope is used to measure AC signals.

Apparatus

- oscilloscope
- multimeter (DMM)
- function generator
- battery
- microphone
- BNC cables (2)
- "T" adapter
- Banana plug cable with BNC adapter

Introduction

Communication requires signals that change in time. Modern (high volume) electrical communication requires electrical signals (voltages, currents) that change rapidly. The digital multimeters we've used to measure voltage and current can not detect quickly changing signals (for example AM radio signals) and have significant limitations even at moderate frequencies (e.g., 20 kHz: the high-frequency end of audio signals).

This laboratory will introduce you to the oscilloscope, which will then be used to observe several different electrical signals and make quantitative measurements of their characteristics. Before coming to lab you should study the following description of the oscilloscope, and start to learn the uses of the controls. A drawing of the oscilloscope you will be using in the laboratory is included here to help you get started (see Figure 3.1 on page 28).

A typical oscilloscope has a lot of knobs and switches, and can be a bit intimidating at first acquaintance. It helps to keep in mind that for all the complications, the oscilloscope is nothing more than a very fast voltmeter that displays a graph of voltage as a function of time!

Types of oscilloscopes

Several methods are available to produce a visual display of voltage vs. time. For slowly varying signals a mechanical system such as a strip chart recorder is adequate. (Such recorders are often used for electrocardiogram (ECG) displays.) However, the inertia of a mechanical system makes rapidly changing signals difficult to measure. Even the comparatively low frequency (60 Hz — 60 cycles/sec) of our AC power lines cannot readily be measured with a mechanical system. Therefore, more sophisticated instruments must be employed for signals that change with time more quickly.

Analog Oscilloscope

Electrons, with the smallest mass (and hence inertia) of any charged particle, are ideally suited to replace the movable pen of a strip-chart recorder. This technology is at the heart of the *analog* oscilloscope. High speed electrons are produced at one end of a vacuum tube (the "cathode ray tube" or CRT); when they slam into the far end of the tube a bit of light is produced. These flashes of light are the ink that is used to draw the graph. The electron beam (the pen) is deflected by voltages applied to parallel plates near the source of the electron beam.

The CRT is by now an old technology, but it is by no means obsolete—many TVs and computer monitors continue to use this method of rapidly drawing pictures.

Digital Oscilloscope

Modern oscilloscopes are based on digital technology—they do not use the deflection of a moving electrons to measure voltages. The process is indirect, but even easier to understand. A very fast voltmeter simply repeatedly measures the voltage and a computer is used to display the results as a graph. Thus these oscilloscopes are essentially single-purpose computers with LCD displays. Lacking a big monitor and mouse, it can be a bit awkward moving through the menus used to control the display, but we hope this lab will be a first step in becoming proficient with this ubiquitous device.

These oscilloscopes measure voltage at the rate of up to one billion samples per second (that is, 10^9 samples/sec = GS/s) and display the results continuously in real time. Note that if the signal isn't repeating itself, a plot of a billion points per second is not going to be useful. Thus the aim of an oscilloscope is to obtain a "steady trace" that shows an apparently unchanging plot made by rapidly replacing nearly identical cycles. We can then observe "slow" changes in the signal. Alternatively a "snapshot" of the signal can be taken and displayed for as long as we like.

The large number of menu and action buttons on the front panel of the oscilloscope can be

confusing so, for the experiments we will be doing, only the necessary buttons and controls will be described. It might be helpful if you check off each step in the instructions when it has been completed. Accidentally skipping a step may cause confusion during later steps.

Lab Report

This report will follow a format that is different from the one used in your previous labs. You should keep a log and commentary on the steps given in the instructions. Include sketches of observed waveforms and answers to any questions asked in the instructions. There are a few calculations to be made but most the report will consist of your comments and observations.

Operating Basics

Think of an oscilloscope as a device that (usually) measures voltage as a function of time. Voltage is plotted on the vertical axis (y-axis), and time on the horizontal axis (x-axis).

The face of the oscilloscope is divided into functional areas. The display area on the left side of the instrument face is the computer's screen. The display generally shows a 8×10 grid used to plot voltage vs. time. (The approximately cm size units on the grid are known as DIVISIONs, so the y scale is typically given in VOLTS/DIV and the x scale is in SEC/DIV.) In addition to plots of the waveform(s) being measured, numerical waveform details, instrument control settings and menu options may be presented in the display.. The menu buttons near the top right hand side of the oscilloscope, cause different menu options to appear in the rhs of the display next to a column of buttons used for selection. These selection buttons are the action (i.e., selection) area. The control area brings together the most frequently used controls that change the time and voltage scales used in the graph. Voltage scales are selected by the VERTICAL controls and the time scale is selected by the HORIZONTAL control. Another set of controls (TRIGGER) allows flexibility in how the waveform is detected.

Procedure

A. Starting Out

Push in the power switch on the top of the oscilloscope and wait for the instrument to go though a self-check. In a few seconds the graphical display should appear along with a menu window. The menu window that appears will be whatever the previous user had set up before the scope was turned off. To make sure that all lab groups start with same settings, you will need to go though a few preliminary steps.

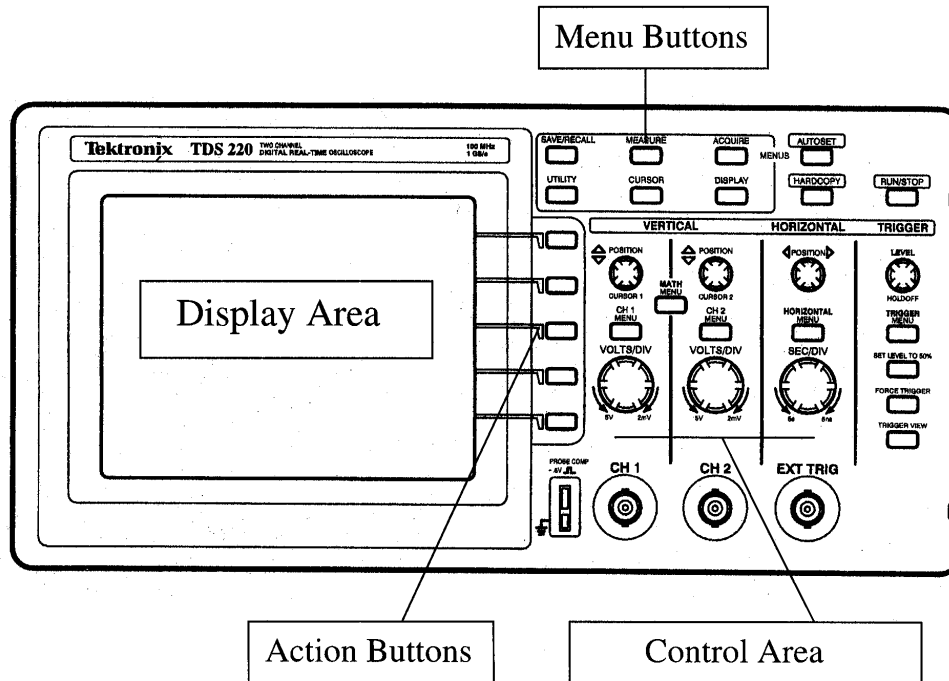1. Press the SAVE/RECALL button in the menu section.

Figure 3.1: Tektronix TDS 220 Digital Oscilloscope

2. When the SAVE/REC window appears in the action area, the top box in the menu should have **Setups** highlighted. If it doesn't, press the button to the right of the box until **Setups** is highlighted.

3. The third box will highlight a setup number. Make sure **1** is highlighted.

4. The bottom box and adjacent button is used to **Recall** the selected setup. Press the button and look at the bottom which should briefly display the message "**Setup 1 recalled**". The bottom of the display should then read:
   CH1 500mV   M 1.00ms   CH1 $\diagup$ 0.00V

   The first number refers to the vertical (y) scale: 0.5 VOLTS/DIV; the second refers to the horizontal (x) scale: .001 SEC/DIV.

5. You should never save a setup!

6. Press the MEASURE button in the menu section.

7. When the MEASURE menu window in the action area, you are ready to start the experiment.

Notice that the time axis will appear darker than the rest of the grid and, if you look closely, you will see some random dots appear and disappear near the time axis. The display is actively graphing voltage but, without a source being connected to the scope, only some random low-voltage "noise" is being observed.

Next, connect a battery to the scope input—your instructor will show you how. Adjust the VOLTS/DIV control for CH 1 in the VERTICAL section of the control area so the increased
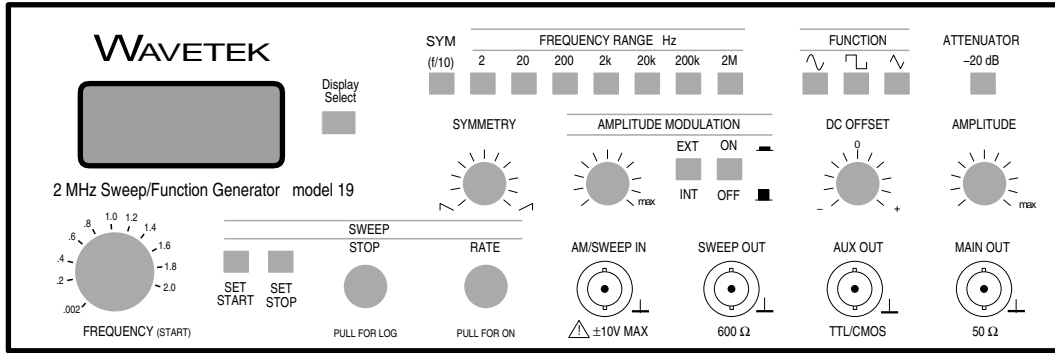
Figure 3.2: Wavetek Model 19 Function Generator

potential difference fits in the plot. For the selected scale (0.5 VOLTS/DIV) the y value should have increased by 3 DIV for a $1\frac{1}{2}$ V battery. Note that you can obtain an approximate measurement of the battery voltage by multiplying the y value (in DIVISIONS) by the scale factor (in VOLTS/DIV). It is important to understand that an oscilloscope is nothing more than a sophisticated voltmeter! Record your measured battery voltage.

B. Applying a Sine Wave with a Function Generator

A drawing of the Wavetek Model 19 function generator is given in Figure 3.2.

1. Connect a BNC[1] cable from the MAIN/OUT BNC of the function generator to the CH 1 input of the scope.

2. Before turning on the function generator, push in the 20K FREQUENCY RANGE button and the sine wave (leftmost: $\sim$) FUNCTION button. All other buttons should be off (out).

3. Adjust the FREQUENCY, SYMMETRY and DC OFFSET knobs to their mid position.

4. The AMPLITUDE knob should be set at approximately 25 percent.

5. Have a TA check your settings then turn the function generator on.

6. The scope display may appear chopped or distorted. Press the AUTOSET button near the upper right corner of the scope face. When you connect to a new voltage source the autoset feature may save some time in obtaining a good (steady, properly scaled) display of the waveform.

7. You should now see a recognizable sine wave on the display. Notice the information at the bottom of the display. The value of each scale division will be listed. Vertical (CH 1) units will be V or mV per DIV and horizontal (M) units will be ms or μs per DIV. A division (DIV) is about a cm on the screen; the screen is 8×10 divisions.

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[1]According to Wiki, this denotes "bayonet Neill-Concelman" connector. This coaxial cable connector is very commonly used when signals below 1 GHz are being transmitted.

It is important to remember that the function generator is the source of this signal, while the oscilloscope is just measuring and displaying the signal. An oscilloscope, when properly used, should have little or no effect on the signal it is displaying.

C. Scale Adjustments

1. In the CH 1 VERTICAL control area, try adjusting the VOLTS/DIV knob. Notice how the appearance of the waveform changes because the vertical scale units (listed at the bottom left of the display) have been changed. Please note that the actual signal (produced and controlled by the function generator) is not changing, rather you are only controlling the display of that signal on the scope.

2. Do the same with the HORIZONTAL SEC/DIV knob. Now bring the settings back to where you started by pressing the AUTOSET button.

D. Measure Menu

The top of the action area should be labeled MEASURE; if not push the MEASURE button in menu area. (Recall that you pushed the MEASURE menu button in part A-6.) The bottom four boxes in the action area should all read CH 1 and None. The top box should have Type highlighted.

1. Starting with the second box, press the button to the immediate right until it reads Freq.

2. For the third box select Period.

3. For the fourth, Pk-Pk (peak-to-peak voltage difference) and the fifth, Mean.

4. Look at the numbers displayed in each box. The frequency should be very close to the value displayed by the function generator. Try reading the graph scales directly and see if you agree with the period and peak-to-peak values displayed in the boxes. (Record your results both in DIVISIONS and converted to seconds and volts.) Sketch the displayed waveform. (Be sure to include scale factors!) Record the four values measured.

5. The mean should be close to zero since the sine wave voltage alternates between equal positive and negative peaks. The DC OFFSET knob on the function generator can be used to change the mean value — try it while observing the display.

6. Set the DC OFFSET knob as close to zero as possible before proceeding.

E. Triggering

In this exercise, the function generator will be adjusted to produce a sine wave of changing amplitude.

1. In the AMPLITUDE MODULATION section of the function generator, press the ON button and turn the knob fully clockwise to its maximum position.

   The display should appear unstable. What you are experiencing is persistence of vision. With a 100 μs SEC/DIV setting, it only takes one millisecond for a graphical display to be refreshed. Your vision cannot respond that quickly, so you are seeing multiple graphs. Of course, no change would be observed if the continuously plotted waveforms were identical.

2. To see a single graph, press the RUN/STOP button in the top-right menu area. If you press the button repeatedly, you will notice that the waveform changes in appearance. Previously persistence of vision caused an overlapping of these different waveforms and the display seemed unstable.

3. To improve the stability of the waveform display, the method by which the oscilloscope commences graphing (triggering) can be adjusted. To make this adjustment, in the TRIGGER section of the control area find and push the MENU button.

4. The TRIGGER window should now be displayed in the action area. Press the action button adjacent to the Mode box until Normal is highlighted.

5. Turn the LEVEL knob (in the TRIGGER control area) and notice the arrow (◄) vertically moving on the right side of the display. As you move this arrow up and down, you should notice a change in the stability of the display. If you move it too far the display will stop being refreshed just as it did when you pressed the RUN/STOP button.

6. You may find it helpful to increase the time per division (HORIZONTAL scale), in order to see more clearly how the amplitude of the sine wave is changing and the effect of changing the triggering level.

   By adjusting the trigger level you are changing the voltage which starts or triggers the graphing process. ("Starts" is perhaps the wrong word, as this trigger point is displayed by default in the center of the screen.) Choosing a fairly high trigger level can select a unique (the highest) peak to be repeatedly displayed resulting in a stable display. There are several other options for triggering which you may wish to investigate. The trigger window now in the action area offers triggering on a Rising or Falling voltage (Slope button). It also offers different triggering Modes. The Normal mode allows the oscilloscope to acquire a waveform only when it is triggered. The Auto mode keeps acquiring or graphing even without a trigger. The Single mode acquires a waveform then stops the display. The RUN/STOP button must be pressed to acquire another waveform. There are additional triggering options that will not be covered in this lab.

7. Turn off the AMPLITUDE MODULATION on the function generator.

F. Sine Wave Measurements

1. Using a "T" adapter connect the function generator to both the oscilloscope and the DMM. (The Digital Multi Meter is displayed in Fig. 4.2 on page 40.) Set the DMM FUNCTION to AC volts (∿ V) and RANGE to 20.
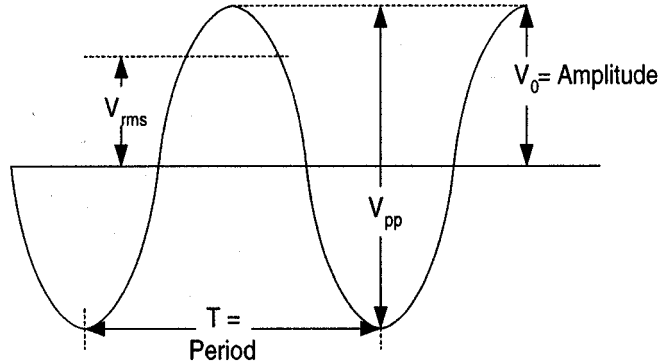
Figure 3.3: Relationships among V_0, V_{pp}, and V_{rms}

2. The AMPLITUDE of the function generator should be set anywhere from 25 to 50 percent. Use a frequency a bit less than 1 kHz. Make whatever adjustments are necessary to obtain a full screen display of the sine wave.

3. Press the CURSOR menu button and observe the cursor window in the action area.

4. Press the button to the right of the Type box until Voltage is highlighted. Notice that two dashed horizontal lines appear. These lines can be moved up and down with the two POSITION knobs in the VERTICAL section of the control area. The cursor window will display the voltage corresponding to the two lines along with the difference (Delta) between the two.

5. Adjust the two cursors to coincide with the positive and negative peaks of the sine wave. Record the peak-to-peak (Delta) voltage.

6. Read and record the voltage on the DMM. (The two values will not be the same.) Using Table 4.1 on page 45 determine the error in your DMM reading.

7. Now change the Type box to highlight Time. You will see two vertical dashed lines. The same two POSITION knobs, which were used to position the cursors before, can be used to position the new cursors.

8. Adjust the cursors to find the period (Delta) of one sine wave cycle.

G. Sine Wave Calculations

The voltage measured by the DMM is known as the rms (root-mean-square) voltage. For a sine wave, the relationship between peak-to-peak voltage and rms voltage is:

$$V_{\mathrm{rms}} = \frac{V_0}{\sqrt{2}} = \frac{V_{\mathrm{pp}}}{2\sqrt{2}}$$

Figure 3.3 illustrates relationships among the amplitude V_0, the peak-to-peak voltage V_{pp}, and the root-mean-square voltage V_{rms}. Be sure you understand these concepts before beginning the calculations outlined below.

1. Check your peak-to-peak (scope) and rms (DMM) measurements and see if they agree with the equation. The oscilloscope uncertainty can be determined from the Resolution and Accuracy table at the end of this chapter.

2. Calculate the sine wave frequency, f from the period, T using the formula:

$$f = \frac{1}{T}$$

   Compare the calculated frequency to that displayed by the function generator. The function generator uncertainty can be determined from the Resolution and Accuracy table.

3. We have used cursors here to measure quantities that could have been easily determined using the MEASURE menu. Typically the cursors are reserved for less standardized measurements.

4. Disconnect the DMM before proceeding.

H. Separating AC and DC Signals

1. Adjust the DC OFFSET knob on the function generator; the waveform should move upward on the display. (You may also need to make the trigger LEVEL more positive to obtain a stable waveform.) The function generator is now producing a signal of the form:

$$A \sin(\omega t) + B$$

   The constant voltage B is being combined with (added to) the sinusoidal signal of amplitude A. The oscilloscope has the ability to automatically ignore the constant voltage that is actually there (i.e., to separate and display just the AC part of a combined signal).

2. Press the CH 1 MENU button located in the VERTICAL section of the control area.

3. Find the Coupling box for CH 1 in the action area. Change the highlighted selection from DC to AC. Notice how the waveform is again centered vertically in the display. By selecting AC, the constant (DC) component of the waveform has been removed from the display—it is of course still present in the signal produced by the function generator. See that the DC OFFSET knob on the function generator is no longer effective in changing the display. (The mean value of the signal produced by the function generator is still changed by the DC OFFSET knob, but the scope is continuously removing that offset from the display.) The purpose of AC coupling is to allow you to focus on the changing component of the signal by removing the steady component.

I. Audio Signal

1. Disconnect the function generator from the scope and connect the microphone to the scope's CH 1 input. Select Coupling▶AC, if that is not already the case. Try to sing, hum or whistle a sustained tone into the microphone (press the AUTOSET button while trying to produce the tone).

The AUTOSET feature may not work if the signal is too weak. If this happens, a message will appear at the bottom of the display stating "unable to autoset". You will then need to manually adjust the vertical and horizontal scales along with the trigger to obtain a good display. Another problem that may occur when using AUTOSET is a change in the trigger coupling. AUTOSET may introduce a filter that rejects high frequencies, low frequencies or noise. To see if this has happened, hit to the TRIGGER MENU button and check the Coupling item in the action area. You may need to change it back to AC. As you become more familiar with the scope, you may decide to dispense with AUTOSET and always make manual selections.

2. Sketch and describe the waveform produced by your audio signal. (Scales please!)

3. Change the coupling selection back to DC before proceeding. (In general, use DC coupling unless the situation requires AC, i.e., a small AC signal on top of a large DC offset.)

J. Dual Channel Operation

This oscilloscope has the ability to measure and display two signals simultaneously. You've been using the CH 1 input; channel 2 (CH 2) has its inputs and controls just to the right of those for channel 1.

1. Disconnect the microphone. Connect the AUX BNC output of the function generator to channel 2 and the MAIN OUT BNC output to channel 1.

2. Set the AMPLITUDE of the function generator to approximately 25%. In the VERTICAL section of the control area, press the CH 2 MENU button. Notice the appearance of a second waveform.

3. Press AUTOSET. The two waveforms may be positioned vertically on the display by turning the two POSITION knobs. (Note that the POSITION knobs do not affect the signal produced by the function generator, just the display of that waveform.) Position the waveforms so they are both clearly visible. The vertical scales (VOLTS/DIV) for both waveforms may be changed independently but only one time scale can be selected. Try adjusting the two vertical scales.

4. Change the AMPLITUDE of the function generator and note how the two waveforms are affected differently.

5. From the TRIGGER MENU, either waveform can be used as a trigger by selecting the appropriate Source. Try both sources.

6. Close channel 2 by again hitting the CH 2 MENU button.

K. Complex Waveform

In this last exercise, you will investigate a signal that is a composite of several different waveforms. The electrical wiring and instruments in the lab room produce the signal.

1. Disconnect the BNC cable from channel 1 of the oscilloscope and replace it with a BNC/banana plug adapter connected to one of the longer banana plug leads. Make sure the adapter is firmly pushed onto the BNC jack for channel 1.

2. Use what you have learned about the oscilloscope settings to obtain a stable display. Determine at least two frequencies that are part of this complex waveform. (Use the the cursor tool.) Sketch the waveform and list any frequencies that you were able to measure. Recall: anytime you sketch a scope display it is critical to include the x and y scale factors. (They are displayed on the bottom of the screen.)

Analysis and Discussion

Be sure that all calculations, comparisons and answers to questions called for in the above sections are accurate and complete.

Critique of the Lab

Comment on the clarity of the Lab Manual, the performance of the equipment, the relevance of the experiment, and your pleasure or displeasure with the experiment. Please be honest. This critique will not affect your grade, and serves to help us improve the lab experience.

Resolution and Accuracy

Wavetek Model 19 Function Generator

Display Accuracy

Frequency: ± 1 digit on 2 kHz to 2 MHz ranges;
                 < or = 1.5% of full scale on 2 Hz to 200 Hz ranges

Amplitude: Typically 5% of range at 1 kHz.

DC offset: Typically 2% of reading.

Resolution: 0.05% maximum on all ranges

Tektronix TDS 200-Series Digital Oscilloscope

Vertical Measurement Accuracy in Average Acquisition Mode (> 16 waveforms)

DC measurement with   ± (4% × reading + 0.1 DIV + 1 mV)
vertical position at zero

DC measurement with   ± [3% × (reading + vertical position) + 1% of vertical
vertical position not at zero position + 0.2 DIV]. Add 2 mV for settings from
                             2 mV/DIV to 200 mV/DIV. Add 50 mV for settings
                             from > 200 mV/DIV to 5 V/DIV.

Delta volts measurement   ± (3% × reading + 0.05 DIV)

Horizontal Measurement Accuracy

Delta time measurement   ± (1 sample interval* + .01% × reading + 0.6 ns)
Single-shot sample mode

Delta time measurement   ± (1 sample interval* + .01% × reading + 0.4 ns)
> 16 averages

*Sample interval = (s/DIV)/250

4. Electrical Circuits

Purpose

To become familiar with the three most common electrical quantities: current, voltage, and resistance, and two of the most common lab instruments: power supply and digital multimeter. To practice drawing a schematic circuit and building a real circuit that matches.

Introduction

In this lab, you will investigate some of the electrical principles involved in simple DC (direct current) circuits. The electronic revolution, beginning with folks like Edison and Marconi and continuing today at Intel, is based on understanding these simple principles. These circuits, in their most basic form, continue today in things like flashlights and household wiring. But even the most up-to-date electronics (like a microprocessor with more than a billion interconnections in a chip the size of a dime) follow the circuit principles (Kirchhoff's Rules) explored in this lab. Of course in order to design and characterize circuits you will need to become familiar with two common electrical test instruments: the digital multimeter (DMM) and the power supply. You will do the following in this lab:

1. Become familiar with common resistors used in circuits and determine their resistance based on a color code.

2. Become familiar with the operation and use of a power supply.

3. Wire your own DC circuits based on your own schematic circuit diagrams.

4. Become familiar with the operation and use of multimeters by measuring currents, voltages, and resistances.

5. Note the importance of the polarity of meters in analyzing DC circuits.

6. Become aware of the accuracy limitations of meter readings.

7. Test the equations used to find the equivalent resistance of resistors in series and in parallel.

Apparatus

- 1 power supply

- 1 digital multimeter (DMM)

- 2 resistors

- 1 plug-in board with connecting wires
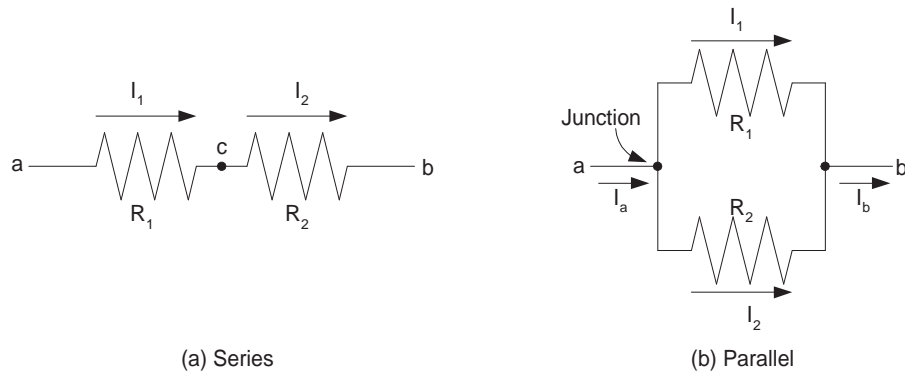
- 2 small bulbs

Series and Parallel Circuits

Figure 4.1: Resistor combinations.

Resistors

If two resistors of resistance R_1 and R_2 are connected *in series* as in Figure 4.1(a), the equivalent resistance, R_{eq}, between points a and b is given by,

$$R_{eq} = R_1 + R_2 \tag{4.1}$$

If two resistors of resistance R_1 and R_2 are connected *in parallel* as in Figure 4.1(b), the equivalent resistance, R_{eq}, between points a and b is given by,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{or} \qquad R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} \tag{4.2}$$

Note that for parallel circuits, the equivalent resistance is *smaller* than either resistor separately.

Currents in Series and Parallel Circuits

Current is the flow of electric charge (something like the flow of water). There can be no accumulation of electric charge in a circuit — what comes in must go out somewhere else in the circuit. Hence in Figure 4.1(a) the charge flowing from a into R_1 must flow out of R_1 into R_2 and through R_2 to b. As a result we can say that the current, I_1 through R_1 is equal to the current, I_2, through R_2. In a series circuit the current has no branching options, and hence must be the same throughout the circuit.

Now what about the current in Figure 4.1(b)? In this case, we say that the two resistors R_1 and R_2 are connected in parallel. The current from a, I_a, must split into two parts, with part of the current, I_1, going through R_1 and the balance, I_2, going through R_2. To avoid a buildup of charge at the junction where the current divides, the total current out of the junction, $I_1 + I_2$, must be equal to the total current into the junction, I_a. That is, $I_a = I_1 + I_2$. As we continue through to b, a similar argument applies to junction where the current recombines. The total current into the junction must equal the total current out, that is, $I_1 + I_2 = I_b$. Combining these two results leads us to conclude that

$$I_a = I_1 + I_2 = I_b \tag{4.3}$$

Voltages (Potential Differences) in Series and Parallel Circuits

Next we ask, what makes charge flow? Consider the analogy of water flowing in a system of pipes. Water flows because there is a difference of pressure between two points in the system. In electrical circuits, charge flows because there is a *potential difference* (a voltage drop) between the points.

Consider the circuit in Figure 4.1(a) again. There will be a current through the resistor R_1 if there is a potential difference between the two points a and c. We call the difference of potential between points a and c (the voltage across R_1) V_{ac}. Similarly the potential difference between the two points c and b (the voltage across R_2) is V_{cb}. Since the change of potential from a to c is V_{ac} and the change from c to b is V_{cb}, then the total change from a to b is

$$V_{ab} = V_{ac} + V_{cb} \tag{4.4}$$

In Figure 4.1(b), there must likewise be a well-defined potential difference between the points a and b. Consequently, in a parallel circuit, the voltage drop across the two resistors must be the same.

Moving water around a pipe loop requires a mechanical pump (a source of pressure difference). In an analogous fashion, moving electrons around a circuit loop requires a power source (a source of potential difference). Batteries and power supplies ("battery eliminators" that plug into a wall outlet) are common examples of such source of potential difference. Whatever the name, DC power supplies will have a positive (or "high potential"), terminal (often colored red) and a negative (low potential) terminal (often colored black). The potential difference, V, produced by our lab power supplies is adjustable whereas it is fixed
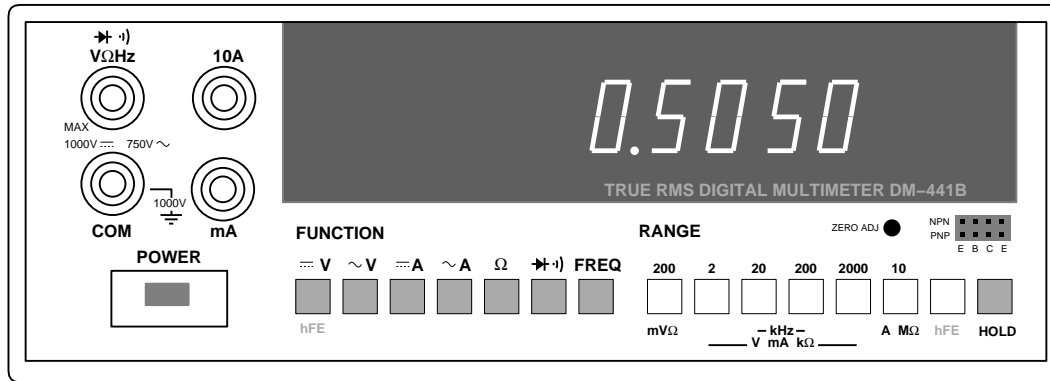
Figure 4.2: The DM-441B digital multimeter. When used as a volt/ohm meter, the leftmost inputs ("VΩHz" and "COM") are used. When used as an ammeter, the bottom inputs ("mA" and "COM") are used. "COM" refers to common or ground: the low terminal. The FUNCTION buttons determine the mode of the multimeter: V (voltmeter), A (ammeter), and Ω (ohmmeter). AC modes are denoted with ∿ and DC modes are denoted with ⎓. Note that accurate measurement requires you use the appropriate RANGE: the smallest possible without producing an overscale (a flashing display).

for a charged battery (e.g., a $1\frac{1}{2}$ V D cell, 9 V "transistor" battery, 12 V car battery, ...) The symbol used to represent a power supply is:


The positive electrode is the high potential terminal and the negative is the low potential terminal.

Multimeters

We will use multimeters that can be set in either a current-measuring mode or a voltage-measuring mode. In the current-measuring mode the multimeter is called an *ammeter*, represented by the symbol

and in the voltage-measuring mode, it is called a *voltmeter*, represented by the symbol
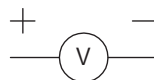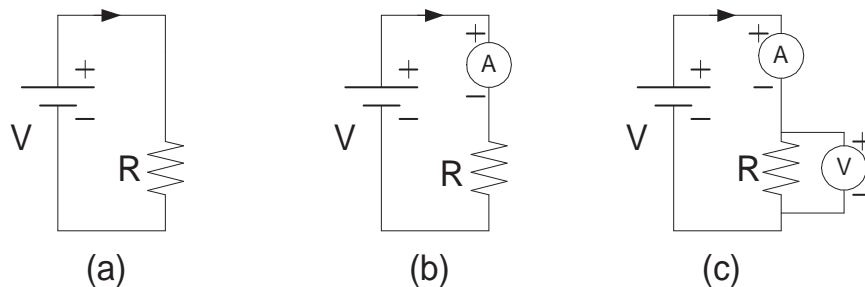
Figure 4.3: Simple circuit showing how to connect a multimeter as an ammeter and a voltmeter.

Please note: All meters come with specifications indicating their accuracy. See Table 4.1 at the end of this lab for DM-441B multimeter accuracies. Note that the uncertainty depends on the scale used, thus in addition to the result displayed by the DMM you must also <u>record the scale</u> used to properly calculate errors. Alternatively you can get in the habit of recording <u>every digit displayed by the DMM</u>, as that will also tell which scale was used and make error calculations easier (since the meaning of "1 dgt" will be clear). If you do not follow these instructions you will have to repeat your measurements!!!

Now we will put all this information together and build a circuit that will allow us to test Eqs. (4.3) and (4.4) experimentally.

Figure 4.3(a) is a simple circuit with a power supply and a resistor. Suppose that we want to measure the current through the resistor, and the potential difference (voltage drop) across it. We must determine how to place the meters in a circuit when we want to measure either current or voltage.

- Figure 4.3(b) shows where to place an ammeter to measure the current through the resistor. Note that to measure the current through the wire, we place the ammeter so that this current must pass (no choice) through the ammeter — hence the ammeter and the resistor are in series in the circuit. In order to measure the current in an existing circuit, a wire must be cut and an ammeter inserted (bridging the cut section of wire).

- Figure 4.3(c) shows where to place the multimeter in voltage measuring mode (voltmeter) to measure the potential difference (voltage) across the resistor. Note that we are measuring the difference of potential between two points — one on either side of the resistor. The voltmeter and resistor are parallel to each other in the circuit. In order to measure the voltage difference in an existing circuit, simply jump the two locations with the voltmeter.

In short: Ammeters substitute for a wire (in order to sample the full current). Inserting an ammeter into a circuit requires "cutting" a wire. On the other hand, using a voltmeters does not require disrupting the circuit; simply connect two locations and measure the potential difference.

Procedure

This lab involves two sets of (unrelated) measurements:

1. You will use resistor color codes (see Appendix) to determine the resistances of two resistors. Then, using the multimeter as an ohmmeter, you will measure the resistances of the two resistors separately and then in series and parallel circuits. The values obtained from the color codes and the multimeters should agree within experimental error.

2. You will use the multimeter in ammeter and voltmeter modes to measure currents and voltages in series and parallel circuits ("Kirchhoff's Rules").

Measurement of resistances using color codes and an ohmmeter

1. Use the color code listed in Table 4.2 below, to determine the resistance of your resistors, including an estimate of the uncertainty.

2. Use your multimeter to measure the same resistances. You must set the function to Ω and the range to a range that is larger than the resistance you want to measure. Ask the instructor for assistance. Measure and record the resistances of R_1 and R_2 separately. Leave room in your data table for the uncertainties.

3. Finally, use the multimeter to measure the equivalent resistances for series and parallel circuits.

   (a) Connect the two resistors in series (Fig. 4.1(a)) and measure their equivalent resistance in series. In your notebook draw a schematic circuit diagram showing how you connected the ohmmeter to the resistors.

   (b) Connect the two resistors in parallel (Fig. 4.1(b)) and measure their equivalent resistance in parallel. (Check that you are using the proper—smallest possible— scale.) In your notebook draw a schematic circuit diagram showing how you connected the ohmmeter to the resistors.

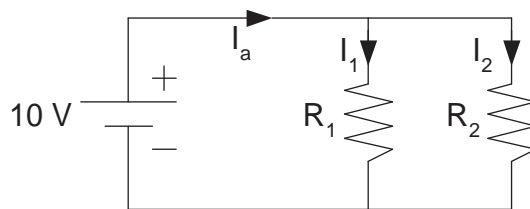## Measurement of currents and voltages in series and parallel circuits

Figure 4.4: Two resistors connected in a parallel circuit.

1. Prepare the circuit shown in Fig. 4.4. *DO NOT turn on the power supply until the circuit has been checked by the lab instructor.* Ask the lab instructor to explain the

operation of the power supply and learn how to adjust it to get the 10 V potential difference needed for these measurements.

2. Use Ohm's law to make rough calculations of the current you expect in the circuit. *When using the multimeter in current measuring mode you must set the range to a value higher than the current in the circuit to avoid damaging the meter.* Plan your procedure for measuring the currents, I_a, I_1, and I_2. You will need to rearrange the wires in your circuit for each measurement so that the ammeter is getting the desired current. *Draw a circuit diagram showing the placement of the ammeter in the circuit for each measurement.* Be sure to use the proper DMM inputs for current (mA), select the DC amps function (---A), and insert the meter with the correct "polarity", that is with the positive terminal connected to the high potential side of the circuit (positive of the meter facing the positive of the power supply). Check with the instructor BEFORE turning on any power. Measure and record the three currents.

3. Next, with the ammeter removed from the circuit, measure the voltage across R_1 and the voltage across R_2. (Use the proper DMM inputs for voltage, select the DC volts function (===V), and be sure to use the right polarity.) Draw the circuits used to measure the two voltages.
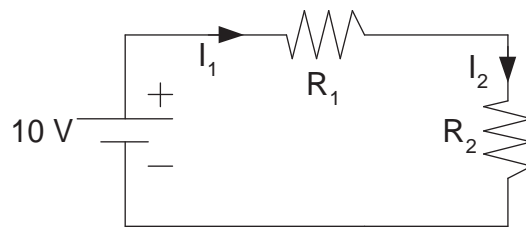

Figure 4.5: Two resistors connected in a series circuit.

4. Prepare the circuit shown in Fig. 4.5. *Have the circuit checked by the instructor before turning on any power.*

5. Measure and record the current through each resistor. Draw a circuit diagram showing the placement of the ammeter in the circuit for each measurement.

6. Measure and record the voltage across each resistor. Also measure the voltage of the power supply. Is this the same as the sum of the voltages across the resistors? Draw a circuit diagram showing the placement of the voltmeter in the circuit for each measurement.

DMM as a component checker

A DMM is often used to check the behavior of various electrical components. As a little project, we have put two small light bulbs in the lab. Using only a multimeter, find the faulty bulb. Explain carefully your procedure, and why you think each bulb is or is not good.

Analysis

Resistors

1. Compare the color code values for the resistances of R_1 and R_2 with your DMM measured values. Are they the same within the manufacturer's tolerance and the accuracy of the meter used? Make a table of the color code values and your own measurements.

2. Using the DMM measured values of R_1 and R_2, calculate the equivalent resistance for the resistors connected in series and in parallel. Compare these values with your DMM measured values. Do they agree within the accuracy of the meters? Use Appendix 3 as a guide to finding the uncertainty in the equivalent resistance of two resistors connected in parallel and Appendix B Eq. B.5 for resistors connected in series.

Currents and Voltages

1. For the parallel circuit, Fig. 4.4, does Eq. (4.3) hold within the accuracy of the meters? Explain.

2. Were the voltages across the parallel resistors the same?

3. For the series circuit, Fig. 4.5, does Eq. (4.4) hold within the accuracy of the meters? Explain.

4. Were the currents through series resistors the same?

5. What do your data tell you about the current drawn by two resistors when connected in series compared to when they are connected in parallel?

Appendix 1 — Multimeter Uncertainties

Most meters have accuracies specified by the manufacturer. Table 4.1 reproduces the specifications for the DM-441B multimeter provided by its manufacturer. Note that the accuracy depends on the function, and sometimes on the range. Your instructor will have the specifications for any other meters that we may use. In the table, the letters "dgt" stand for least significant digit — the right-most digit on the display; "4 dgt" means 4× whatever a 1 in the right-hand digit (and zeros everywhere else) would correspond to. (In the below table "Resolution" is the same thing as "1 dgt".) Note that the value of 1 dgt always changes with the range, so you need to record both the range and the value when you take data. Some of the expressions are a bit complex. It may help to use a spreadsheet to calculate them.

Table 4.1: DM-441B Multimeter Resolution and Accuracy

Function	Range	Resolution	Accuracy
Resistance	200Ω	0.01Ω	±(2% + 5 dgt)
	2 KΩ	0.1Ω	±(0.2% + 2 dgt)
	20 KΩ	1Ω	
	200 KΩ	10Ω	
	2000 KΩ	100Ω	±(0.5% + 2 dgt)
	20 MΩ	1 KΩ	
DC Voltage	200 mV	10μV	±(0.1% + 4 dgt)
	2 V	100μV	
	20 V	1 mV	
	200 V	10 mV	
	1000 V	100 mV	±(0.15% + 4 dgt)
DC Current	2 mA	0.1μA	±(0.5% + 1 dgt)
	20 mA	1μA	
	200 mA	10μA	
	2000 mA	100μA	
	10 A	1 mA	±(0.75% + 3 dgt)
AC Voltage	200 mV	10 μV	±(0.5% + 20 dgt)
(45 Hz – 1 kHz)	2 V	100 μV	
	20 V	1 mV	
	200 V	10 mV	
	750 V	100 mV	±(1% + 20 dgt)

Examples: A resistance reading of 71.49 Ω (on the 200 Ω scale) has an uncertainty of: $71.49 \times 2\% + 5$ dgt $= 1.43 + .05 = 1.48 \approx 1.5$ Ω. A DC voltage reading of 6.238 V (on the 20 V scale) has an uncertainty of: $6.238 \times 0.1\% + 4$ dgt $= .0062 + .004 = .0102 \approx .010$ V. An AC voltage reading of 6.238 V (on the 20 V scale) has an uncertainty of: $6.238 \times 0.5\% + 20$ dgt $= .0312 + .020 = .0412 \approx .041$ V.

Appendix 2 — Resistor Color Codes

Resistors have colored bands on them to specify their resistance. The colored bands around the body of a color-coded resistor represent its value in ohms. These colored bands are grouped toward one end of the resistor body. Starting with this end of the resistor, the first band represents the first digit of the resistance value; the second band represents the second digit; the third band represents the number by which the first two digits are multiplied. A fourth band of gold or silver represents a tolerance (or uncertainty) of $\pm 5\%$ or $\pm 10\%$ respectively. The absence of a fourth band indicates a tolerance of $\pm 20\%$. The physical size of a resistor is related to its power rating. Size increases progressively as the power rating is increased. The diameters of 1/2 watt, 1 watt, and 2 watt resistors are approximately 1/8", 1/4", and 5/16" respectively.

Color	1st digit	2nd digit	Multiplier	Tolerance
black	0	0	1	—
brown	1	1	10	—
red	2	2	100	—
orange	3	3	1000	—
yellow	4	4	10,000	—
green	5	5	100,000	—
blue	6	6	10^6	—
violet	7	7	10^7	—
gray	8	8	10^8	—
white	9	9	10^9	—
gold	—	—	0.1	5%
silver	—	—	0.01	10%

First digit
Second digit
Multiplier
Tolerance

Yellow = 4
Violet = 7
Orange = 1000
Silver = $\pm$ 10%

47,000 ohms
=
47 KΩ

Red = 2
Red = 2
Green = 100,000
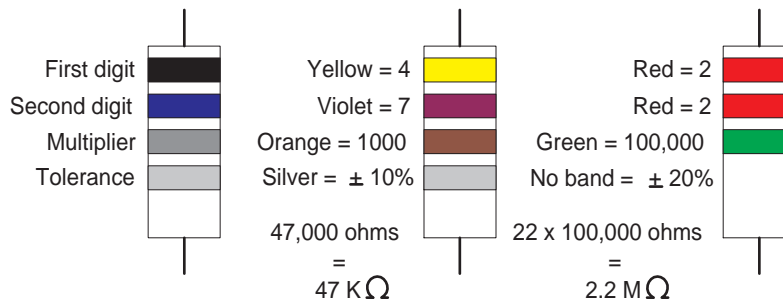No band = $\pm$ 20%

22 x 100,000 ohms
=
2.2 MΩ

Table 4.2: Resistor Color Code

Appendix 3 — Finding the uncertainty in the equivalent resistance of two resistors in parallel

1. We start with our measurements: $R_1 \pm \delta R_1$ and $R_2 \pm \delta R_2$.

2. The equivalent resistance is of course

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

3. We define a new variable $u \equiv 1/R$, in terms of which the above equation becomes

$$u_{eq} = u_1 + u_2.$$

4. We calculate the uncertainties in both u's using the techniques you learned in Physics 191:

$$\frac{\delta u}{u} = \frac{\delta R}{R} \text{ using the form } R = K/a.$$

In our case,

$$\delta u = u\frac{\delta R}{R}$$

$$\delta u = \frac{1}{R}\frac{\delta R}{R} \quad \text{or}$$

$$\delta u = \frac{\delta R}{R^2}$$

Use this result to find δu_1 and δu_2.

5. It is now easy to find $\delta u_{eq} = \sqrt{(\delta u_1)^2 + (\delta u_2)^2}$ (since the generic form is $R = a + b$)

6. Finally, to find δR_{eq} , we do the same process in reverse: Follow the reasoning above, and persuade yourself that

$$\delta R_{eq} = R_{eq}^2 \, \delta u_{eq}.$$

Use this procedure to find δR_{eq} from δR_1 and δR_2. A spreadsheet may help to make these calculations more organized and less tedious! It's even easier in *Mathematica*.

5. RC Circuits

Purpose

The purpose of this experiment is to investigate circuits with time-varying signals, which almost always involve capacitors.

Introduction

Ideally the current through a resistor responds instantaneously to changes in applied voltage, instant by instant following Ohm's Law: $I = V/R$. This is not true of capacitors: the resulting current flow through a capacitor depends on history and time, not just the applied voltage. This time lag between voltage and current is put to use in most every circuit which has changing voltages or currents ("AC"). For example, capacitors are used in 'filtering' AC signals (e.g., boosting the bass—low frequency—but not the treble), in fully electronic sound sources (synthesizer, audio oscillators), and in tuned circuits used to select TV or radio frequencies.

The rate at which a capacitor charges or discharges is governed by an exponential law. In this experiment, we will investigate the behavior of capacitors as they charge and discharge. Since we expect this behavior to involve exponential functions, the data analysis techniques described in the Appendices to the Physics 191 laboratory manual will be useful—you should review them as needed.

Theory

Figure 5.1 shows an idealized circuit for studying the charging and discharging of a capacitor.

Charging: We first consider the case when the switch is put in position 1 to charge the capacitor. Applying Kirchhoff's first rule to this circuit yields:
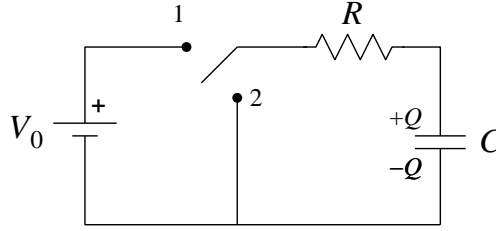
$$V_0 - IR - \frac{Q}{C} = 0, \tag{5.1}$$

Figure 5.1: Simple charging/discharging circuit for a capacitor.

where V_0 is the emf supplied by the power supply, Q the charge stored on the capacitor plates, and I the current in the circuit. Note that:

$$I = \frac{dQ}{dt},$$
(5.2)

and hence Eq. (5.1) can be written:

$$V_0 - R\frac{dQ}{dt} - \frac{1}{C}Q = 0.$$
(5.3)

This equation is a differential equation that can be solved for Q to give

$$Q = CV_0 \left[1 - \exp\left(-\frac{t}{RC}\right)\right]$$
(5.4)

where we have assumed that the charge on the capacitor at $t = 0$ was zero. We can use this equation to find both the voltage drop across the capacitor, V_C and the current in the circuit as functions of time. Consider first the voltage drop. If we divide Eq. (5.4) by the capacitance C and use the definition of capacitance $C = Q/V$, we obtain

$$V_C = Q/C = V_0 \left[1 - \exp\left(-\frac{t}{RC}\right)\right].$$
(5.5)

Now consider the current: If we take the derivative of Eq. (5.4) with respect to time, we obtain

$$I = \frac{dQ}{dt} = \frac{V_0}{R} \exp\left(-\frac{t}{RC}\right)$$
(5.6)

which allows us to find the voltage across the resistor using Ohm's law: $V_R = IR$.

Consider Eq. (5.4) first: It says that if we close the switch in position 1, the charge on the capacitor, Q, will increase with the time t. In the limit as $t \to \infty$, $\exp(-t/RC) \to 0$ and

$$Q \to Q_\infty = CV_0,$$
(5.7)

which is the final charge on the capacitor. Figure 5.2(a) shows Q plotted as a function of time — Q approaches Q_∞ asymptotically. Similarly, Eq. (5.5) shows that the voltage drop V_C across the capacitor approaches V_0 in the limit.

In all of these equations, the rate at which the capacitor charges is governed by the factor (t/RC), that is RC sets the scale for time variation. We find it useful to define the *time constant* $\tau \equiv RC$.
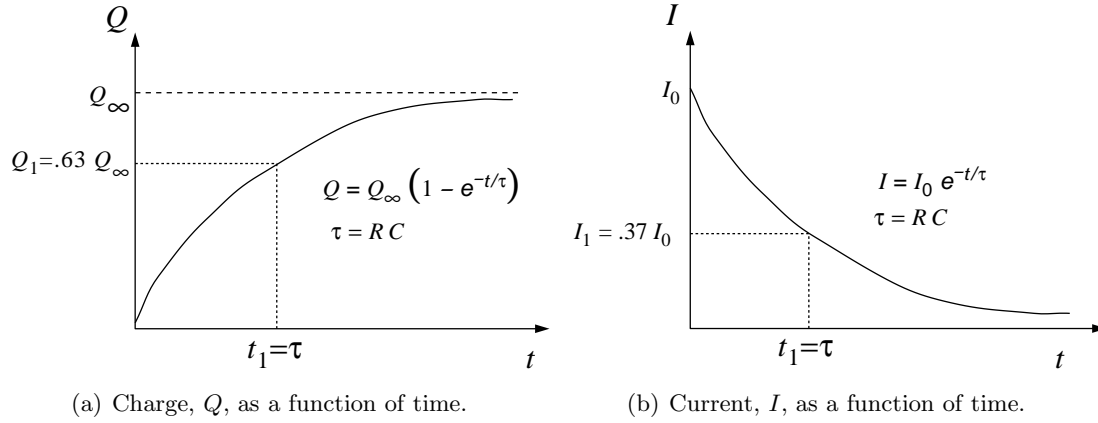
(a) Charge, Q, as a function of time.

(b) Current, I, as a function of time.

Figure 5.2: Charging capacitor: charge and current as a function of time.

Exercise: Show that RC has dimensions of time by working out the dimensions of both R and C in terms of the fundamental quantities mass, length, time, and electric charge. Note that we could have predicted this result in advance, since the argument of an exponential must be dimensionless.

To see the significance of the time constant, consider the following argument: If $t_1 = \tau = RC$ is put into Equation 5.4, we obtain:

$$Q_1 = CV_0 \left[1 - \exp\left(-1\right)\right] = CV_0 \left(1 - \frac{1}{e}\right), \qquad (5.8)$$

Thus,

$$Q_1 = Q_\infty \left(1 - \frac{1}{e}\right) \approx 0.63 \times Q_\infty \qquad (5.9)$$

This result says that at time $t_1 = RC$ the charge is within $1/e$ of its final value Q_∞.

Note that if $t = \tau = RC$ is used in Equation 5.6 we get:

$$I_1 = \frac{V_0}{R} \exp\left(-1\right) = \frac{1}{e}\left(\frac{V_0}{R}\right) = \frac{1}{e} I_0 \approx 0.37 \times I_0. \qquad (5.10)$$

Figure 5.2(b) shows I plotted as a function of t (Equation 5.6). I approaches zero asymptotically as $t \to \infty$. The current has its maximum value, I_0, initially (i.e., just after the switch is placed in position 1) and thereafter declines exponentially. Note that $I_0 = V_0/R$.

Discharging: Consider next the case when the capacitor is discharging. Suppose that the capacitor has a charge Q_0 on it and the switch is moved to position 2 (Figure 5.1), Kirchhoff's first rule yields:

$$IR - \frac{Q}{C} = 0. \qquad (5.11)$$

But $I = \frac{dQ}{dt}$, so Equation 5.11 becomes:

$$R\frac{dQ}{dt} - \frac{1}{C}Q = 0. \qquad (5.12)$$

The solution to this differential equation is:

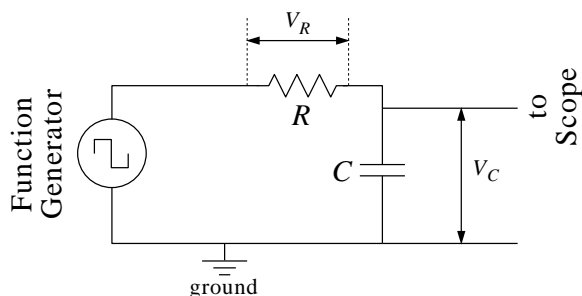$$Q = Q_0 \exp\left(-\frac{t}{RC}\right) \qquad (5.13)$$

Figure 5.3: Experimental setup to determine the time constant of an RC circuit.

where we have assumed the capacitor has charge Q_0 initially. If we again apply the definition of capacitance $C = Q/V$ to this result, we find that the voltage drop across the capacitor as a function of time is

$$V_C = V_0 e^{-t/\tau} \tag{5.14}$$

where we have substituted for the time constant $\tau = RC$. If we again consider the time $t_1 = \tau$, then:

$$V_1 = V_0 \ e^{-1} \approx 0.37 \times V_0. \tag{5.15}$$

Hence in a time equal to the time constant, the capacitor discharges to a voltage which is $1/e$ of its initial value. Note, also, that if we differentiate Equation 5.13 with respect to time we get the discharging current:

$$I = \frac{dQ}{dt} = -\frac{Q_0}{RC} \ e^{-t/\tau}. \tag{5.16}$$

The negative sign tells us that it has the opposite direction as the charging current, and since $C = Q_0/V_0$:

$$I = -\frac{V_0}{R} \ e^{-t/\tau} = -I_0 \ e^{-t/\tau}, \tag{5.17}$$

where $I_0 = V_0/R$ is just the initial current. Again, if desired, the voltage across the resistor could be found using Ohm's law: $V_R = IR$.

How can we measure the time constant in the laboratory? The theory indicates that, if we could measure Q, V, or I as a function of time, the time constant τ could be determined by fitting the theoretical equation (for $Q(t)$, $V(t)$, or $I(t)$) to the experimental data. Either the charging or discharging of the capacitor, or both, could be used. The circuit shown in Figure 5.3 will allow you to measure the voltage, V_C, across the capacitor; or if the capacitor and resistor are interchanged, you may measure on the oscilloscope the voltage across the resistor, V_R. (Why is it necessary to interchange these components, rather than just move the oscilloscope leads?) If you measure V_C, then Q may be obtained from $Q = V_C C$. If you measure V_R then I is obtained from $I = V_R/R$. In this lab you will measure V_C as a function of time, and then fit the resulting data to the expected exponential form Eq. 5.14.

To simulate the situation developed in the theory above the function generator should be set to the square wave mode. A square wave can be thought of as a d.c. voltage that is turned on and off at regular intervals. Thus, the square wave alternately charges the capacitor, and allows it to discharge. You will simultaneously observe both the square wave and V_C by using the both channels of the oscilloscope.

The oscilloscope will also be used to measure the voltage across the resistor. *If you are at all hazy on how to use the oscilloscope, you should review the write-up for the oscilloscope experiment you performed earlier this semester.*

Procedure

Qualitative Analysis

1. Set up the circuit. Initially set the resistance $R \sim 10$ kΩ, and capacitance $C \sim$ 0.1 μF. Connect the output from the function generator to the CH 1 scope input and V_C from your circuit to the CH 2 input. Have your lab instructor check your circuit.

2. Recall the oscilloscope's Setup 1. (See the digital oscilloscope lab, page 27, for details.) At this point the scope will only be displaying CH 1. Hit the CH 2 MENU button so the scope will display both channels.

3. Set the function generator to the 200 Hz FREQUENCY RANGE, an AMPLITUDE of about 50%, and a square wave (⎍) FUNCTION. Hit AUTOSET on the scope. Check to see that the scope is displaying a square wave and that the scope's display responds properly when you adjust the function generator's FREQUENCY and AMPLITUDE knobs. Note that you can use the VERTICAL POSITION knobs to locate the traces anywhere you want on the display. (For the below Quantitative Analysis, we will want them to exactly superimpose, but that may make viewing/sketching difficult.) Start with identical vertical scales (VOLTS/DIV the same for CH 1 and CH 2), but if CH 2 looks flat, blow up that scale so you can see its behavior—AUTOSCALE can help!

4. Observe the effects of changing the function generator frequency, f, as well as the values of R and C. (Keep the resistance in the range: 100 $\Omega < R < 100$ kΩ.)

5. Let $T = 1/f$ be the period of the square wave. Make careful sketches in your notebook of the wave forms you obtain for $T \gg RC$ and $T \ll RC$. (Adjust scope scale factors to maintain a meaningful plot of the signal: the signal should be at least 3 divisions high with at most two cycles on screen.) Be sure to give the values of T, R, C, and RC for each sketch and record each axis scale. Give a careful explanation of the shape of each waveform. (Note: $10 \gg 1$. If the disparity between T and RC is too large you may obscure what's going on.)

6. Rearrange your circuit to measure V_R. Note that since the scope measures voltage relative to ground, we must arrange the circuit so that one end of the resistor is at ground and the other end is attached to the scope probe. Thus the desired circuit will have resistor and the capacitor swapped compared to the circuit in Figure 5.3. The easy way to swap the R and C components is actually to just reverse the leads from the function generator... Can you see why this works?

   For the same conditions you used in part 5, make sketches of the wave forms observed.

Quantitative Analysis

Now, we will take data for V_C, the voltage across the capacitor, as a function of time. If these data are consistent with an exponential, we should be able to find the time constant

τ from the B value of the fit. (Of course you will want to compare the value of τ calculated from the fit to that suggested by theory: RC.)

First, be sure that you rearrange the circuit so that you are again measuring V_C, the voltage across the capacitor.

Find values of R and C that result in a nice discharging pattern on the oscilloscope. (TRIGGER Slope▶Falling may be helpful.) Use the HORIZONTAL POSITION knob to move the start of the discharge cycle to the middle of the first division of time, i.e., near the left hand side of the screen. (The end of the discharge cycle should be off the rhs of the screen.)
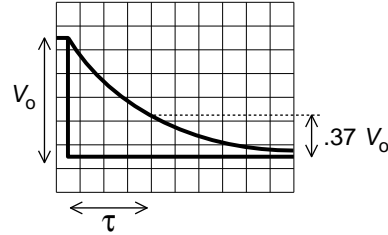
With CH 1 and CH 2 on the same scale and POSITIONed with the same zero value, the CH 2 trace should have discharged to essentially the CH 1 value by the right hand side of the display. You will need to fiddle a bit with the scope scales (both VOLTS/DIV and SEC/DIV) and POSITIONs (both HORIZONTAL and VERTICAL), the values of R and C, and the function generator frequency to achieve a proper display.

When you have achieved a proper display, hit the CURSOR button in the scope's menu area. In the action area select: Type▶Voltage and Source▶CH2. Line up the bottom cursor (using the POSITION knobs) with the final discharge voltage V_∞ (the bottom of the CH 1 square wave). Record the numerical value corresponding to V_∞. To measure V_C, move the other cursor so that it crosses the CH 2 curve at the desired time and record Delta as V_C.

Make a table reporting values of time and V_C at every full-division (vertical line) time division. In an adjacent column record the vertical scale used when the voltage was measured. Leave room for a column for the error in V_C (δV). (The formula for the uncertainty in voltage can be found in the manufacturer's specifications recorded on page 36.) As the voltage difference gets smaller you must change VERTICAL scales (maintaining the equality between the CH 1 scale and CH 2 scale). This will require vertically repositioning the graphs and the cursors to accommodate the changed scale. (Don't change the HORIZONTAL POSITION once you've started to take data!) Collect 8 or 9 data points (which often requires two or three scale changes).

Hints for Making Accurate V_C vs. Time Measurements

Proper instrument use always requires setting the proper scale (range), which is typically the most sensitive range without producing an out-of-range reading. For scope measurements you should change to a more sensitive range if what you're trying to measure is smaller than about 3 divisions. Usually such a change is easy: just turn the VOLTS/DIV knob, but for this experiment much more is required:

- The CH 1 vertical position must match the CH 2 vertical position, i.e., the zero-location of the two channels must match. In addition, the CH 1 and CH 2 vertical scales must match.

- Initially note the little markers: 1-▶ and 2-▶ on the left hand side of the screen: they mark the location of zero volts for each channel. You can assure zero-location equality

by moving these markers (using the POSITION knob in the VERTICAL section of the corresponding channel) so that they are superimposed. When you turn a POSITION knob the numerical position of the marker is displayed on the bottom of the screen. **Make sure these numerical values match for the two channels!**

- Once you enter CURSOR mode, the POSITION knobs do not move the plots. (Instead they move the cursors.) To exit CURSOR mode, and thus activate the POSITION knobs for vertical plot adjustment, you must hit either channel's MENU button.

- For the larger time data points, V_C approaches its asymptotic value (V_∞), and the CH 1 and CH 2 vertical scales must be changed to blow up the ever smaller difference between V_C and V_∞. At these more sensitive scales, the zero-point markers will be off the top of the screen, and so you must line them up just using the numerical value of the vertical position displayed on the bottom of the screen when the channel's POSITION knob is turned.

- There is a limit to selecting a properly sensitive scale: while the zero level may be off-screen it may not be more than 10 divisions high. You will typically reach this limit after 2–3 scale changes and then be forced to make relatively high-error measurements for the remaining points with Delta corresponding to a few divisions or even a fraction of a division.

- Record the value of the cursor marking V_∞. As you change vertical scales this numerical value should be the same! (But see following.)

- The V_∞ produced by the function generator may not be exactly constant, in which case you should measure its value at the same vertical grid line (time) as CH 2 V_C.

- Each time you record a voltage measurement, also record the scale in use for that measurement; **the scale is required to calculate the error!** (The scale is displayed at the bottom of the screen.)

- Use of a spreadsheet will make calculating δV easier.

- Be sure to accurately measure voltages at the grid time divisions as we will be neglecting the uncertainty in time.

Lab Report

Use $WAPP^+$ to fit an exponential function to your voltage vs. time data. Produce a normal plot and a semi-log plot. Attach these plots along with the fit report in your lab notebook. Comment on whether your data are indeed consistent with exponential behavior.

Calculate value of τ and its error using the computer fit B value and its error. (Since $\tau = -1/B$, what is the uncertainty in τ?) Calculate the expected value of τ (and its error) from the readings on the variable resistor and capacitor. (The readings on the variable resistors are good to $\pm 10\%$ and the readings on the variable capacitors are good to $\pm 20\%$. What is the uncertainty in the product RC?) Within the limits of error, are these τ equal? Make a nice table properly reporting both numerical values for τ (including error and units).

Critique of the Lab

Comment on the clarity of the Lab Manual, the performance of the equipment, the relevance of the experiment, etc. This critique will help us improve the lab experience.

6. Ohmic & Non-Ohmic Circuit Elements

Purpose

To extend your knowledge of DC circuits to nonlinear circuit elements.

Introduction

The purpose of this lab is to extend your knowledge of DC circuits to include circuit elements for which the relationship between current and voltage is not a simple linear function. Such circuit elements are called non-linear or non-ohmic devices. Non-ohmic components, particularly those involving semiconducting PN junctions, are critical to the operation of most any modern electronic device. (Before the invention of transistors in 1947, vacuum tubes were the non-linear components of choice.) In 1939 electrical engineers Bill Hewlett and Dave Packard made innovative use of the non-linear behavior of a light bulb in designing the Model 200A audio oscillator—the first product of the Hewlett-Packard Company.

In this experiment we will investigate three common circuit elements to investigate linear (ohmic) and two types of non-linear behavior. We will use digital multimeters (DMM) to record how the current, I, through a circuit element depends on the voltage drop, V, across it. We will go on to use techniques of graphical analysis that you learned in Physics 191 to investigate the functional relationship between I and V for a resistor, a diode, and a light bulb. You may find it useful to review the Appendices in the Physics 191 lab manual as you perform this analysis.

Apparatus

- 1 power supply

- 2 digital multimeters

- 1 circuit bread-board

- 1 resistor: 2.7 kΩ

- 1 diode (2N3904 transistor with collector and base wired together)

- 1 light bulb

Theory

Ohm's law states that the current flowing through a circuit element is proportional to the applied voltage:

$$V = IR, \qquad (R \text{ is a constant}) \tag{6.1}$$

The proportionality constant, R, is called the resistance of the circuit element and is independent of I and V. Circuit elements that obey this relationship are called linear or ohmic. Although there is no device that displays perfectly ohmic characteristics, many circuit elements are nearly ohmic over a wide range of currents and voltages.

In general, circuit elements display non-linear behavior. For such circuits, the current is not directly proportional to the applied voltage; $V/I \equiv R$ is not a constant. In such cases we convert Ohm's Law into a definition of a non-constant resistance:

$$R \equiv \frac{V}{I} \tag{6.2}$$

NOTE: Ohm's Law states that for many materials V/I is a constant—whereas for non-ohmic materials V/I changes along with the current and voltage. Equation (6.2) is not equivalent to Eq. (6.1), even though they look alike. Equation (6.1) indicates that for a linear circuit element a plot of I versus V yields a straight line. Equation (6.2) is a definition of a (potentially non-constant) R. For non-ohmic materials a plot of I versus V will show some sort of curvature. If the graph is a curve (i.e., if the slope changes) it is sometimes useful to define a *dynamic resistance*, r:

$$\frac{1}{r} \equiv \frac{dI}{dV} \tag{6.3}$$

Procedure

Examine the apparatus for this experiment. With the assistance of the lab instructor, become familiar with the operation of the multimeters and power supply. **Always have the lab instructor check your circuits before turning on the power supply!** Note that you will make graphs of current on the y-axis vs. voltage on the x-axis . If the graph is a straight line, the slope of this graph (a constant) will be the inverse of resistance, i.e., $1/R$. If the graph has curvature, the slope of the graph varies with V and is $1/r$.

Linear Circuit Elements

Build the circuit shown in Figure 6.1 using the circuit breadboard and banana plug connectors. You will measure current through the resistor as a function of voltage across the resistor for voltages up to 9 V. (Such measurements define what is called the IV characteristic of a circuit element.) Use the color code on the resistor (page 46) to find the resistance
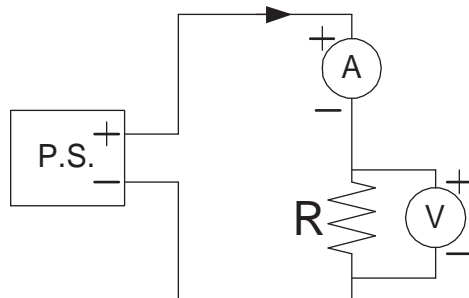
Figure 6.1: Linear circuit. Here P.S. stands for the power supply.

of the resistor, and then use Ohm's law, Eq. (6.1) to find the maximum current expected for this range of voltage. Set the initial range on each multimeter to accommodate the maximum current and voltage. Be sure to have your instructor check these range settings before you proceed. When the circuit is ready, make a table listing the current and voltage data and the errors associated with your measurements.

NOTE: The accuracies of the DM-441B multimeters are given on page 45. For this DMM (and most any meter) the accuracy depends on the range, so you must record the range used in addition to the value to calculate the error. Of course, you should always use any meter on its most sensitive scale that avoids an out-of-range reading. (On a DM-441B, an out-of-range measurement is displayed as a flashing display.) In analyzing your data it will be helpful if you record every digit (even trailing 0s) displayed on the DMM.

Measure the current as the voltage is varied from .5 V to 9 V; record at least a dozen well-spaced data points. (Why shouldn't you use 0 V as a point?) Plot the current I (on the y-axis) versus the voltage V (on the x-axis). The plotted data points should vary smoothly. If any data points look suspicious, recheck them while the circuit is still intact. When you are satisfied that the data accurately reflect the experimental conditions, turn down the voltage and shut off the power supply to prepare for the next part of the experiment.

Non-Linear Circuit Elements

Diode

Add a diode to your circuit, as shown in Fig. 6.2(a). The symbol

is used to represent a diode and the arrow indicates the direction of the current (positive charge). In this experiment, we are using a transistor with two terminals wired together as the diode—it turns out that its behavior is closer to that of a theoretically "ideal" diode than the inexpensive diodes used in power supplies.

You should take data from about 0.5 V (lower if your ammeter can handle very low currents) to about 0.8 volts, and measure using at least 10 voltages, approximately equally spaced, within this range. You will need to frequently adjust the range of the ammeter to

(a) Nonlinear circuit with a diode.   (b) Nonlinear circuit with a light bulb.
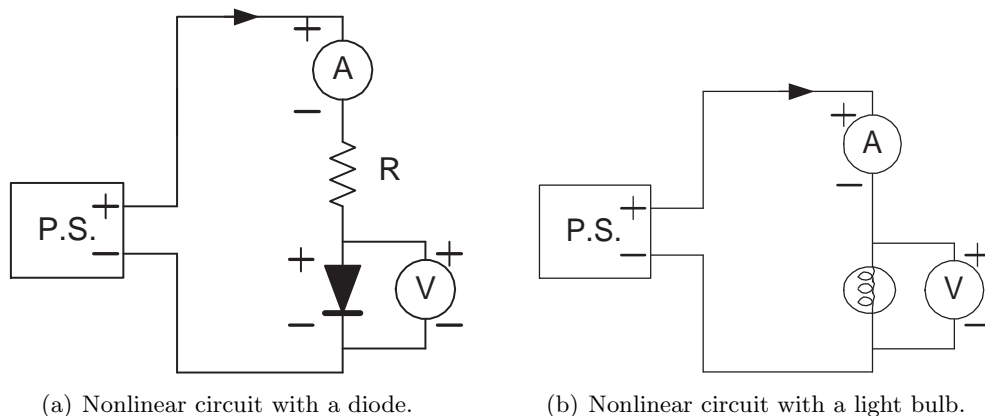
Figure 6.2: Nonlinear circuits.

accommodate this current, which should grow rapidly as the voltage is increased. **Have your instructor check the circuit before turning on the power supply — it can be tricky to be sure the polarity is correct.** As before, measure, record and plot the *IV* characteristics for the diode. Again: when the currents are small, use the lower ranges on the multimeter to reduce your experimental uncertainty—but remember that the uncertainties do depend on the ranges, so you must record the range used in addition to the DMM reading.

Next we will investigate the effects of changing the direction of the current flow through the diode. Reverse the direction of the diode in the circuit and observe the current as the voltage is increased to about 1 V. You need not record data points during this part of the experiment, but note your observations of the circuit's behavior in your lab notebook. As always, turn down the voltage when done and shut off the power supply.

Light Bulb

We will now investigate the *IV* characteristics of a light bulb using the circuit shown in Fig. 6.2(b); replace the diode with the bulb.

Do not exceed the voltage rating of the light bulb—if you aren't sure, stop at the point the bulb seems to be extremely bright. Collect *IV* data following a procedure similar to that used for the diode and resistor. Try to collect data using a wide range of voltages, with, say, $V_{min} \sim \frac{1}{10} V_{max}$ When you have completed taking data, turn down and shut off all equipment.

Analysis

Using $WAPP^+$ (Goggle "wapp+" to find it or use LINFIT) analyze the three sets of current vs. voltage data. There will be uncertainties in both voltage and current, so be sure you calculate error in both the x and y coordinates. (Show sample calculations or self-document your spreadsheet!) Attach in your notebook the fit reports and five plots (resistor: normal,

diode: normal and semilog, bulb: normal and log-log).

1. **Resistor:** Use the computer data analysis program to determine the resistance and its uncertainty. Comment on the quality of the least-squares fit. Does your result agree with the color code resistance value[1] to within the specified resistor tolerance?

2. **Diode:** Solid state physics predicts that an ideal diode will obey an exponential relationship:

$$I = I_0 \exp\left(\frac{eV}{kT}\right) \tag{6.4}$$

where $e = 1.602 \times 10^{-19}$ C is the absolute value of the charge of the electron, $k = 1.3805 \times 10^{-23}$ J/K is Boltzmann's constant, and T is the absolute temperature (in Kelvins). Fit your data to an exponential and use B to find the "theoretical" temperature (with an error). Measure the actual temperature in the lab, and see if the result of your least-squares fit is consistent with this result, within experimental uncertainty. Don't be too surprised if there is a mismatch: real diodes are different from ideal diodes!

3. **Bulb:** *Very* approximate theory suggests a power law relationship between I and V:

$$I = AV^{3/5} \tag{6.5}$$

Fit your data to both a power law and one of the other optional functional forms. Which one works best? (Why?) Does the better functional form actually produce a good fit? (Explain!) Compare the computer-fit power to crude theory's value of $\frac{3}{5}$.

In your lab report, qualitatively discuss the effects of varying the voltage across these devices.

Critique of the Lab

Comment on the clarity of the Lab Manual, the performance of the equipment, the relevance of the experiment, and your pleasure or displeasure with the experiment. Please be honest. This critique will not affect your grade, and serves to help us improve the lab experience.

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[1]see Table 4.2 page 46

7. e/m of the Electron

Purpose

In this experiment you will measure the charge-to-mass ratio e/m of the electron. Another important purpose of the experiment is to enhance your understanding of the use of electric and magnetic fields to control beams of charged particles. The production and control of a beam of charged particles (electrons or ions) is fundamental to the operation of particle accelerators (CERN and Fermilab), electron microscopes, mass spectrographs, televisions, microwave ovens, etc.

Introduction

In 1897 the British physicist J. J. Thomson reported on his investigations of "cathode rays" (an electrical current flowing through a vacuum) and concluded that the rays were really streams of negatively charged particles ("corpuscles"). This work is usually cited[1] as the discovery of the electron[2], the first substructure discovered in atoms[3]. While a great deal of electrical technology was created without an understanding of the fundamental nature of electricity (e.g., Edison: electric lighting, Bell: telephone), the discovery of the electron allowed the development of much more sophisticated devices like the triode vacuum tube[4] (for early radio) and the transistor[5] (the basis for all of modern electronics).

Theory

The Ealing e/m apparatus (Figure 7.1) will be used in this experiment. The apparatus consists of a vacuum tube that encloses an electron gun and an external pair of Helmholtz coils to produce a magnetic field. Inside the electron gun, electrons are boiled off the
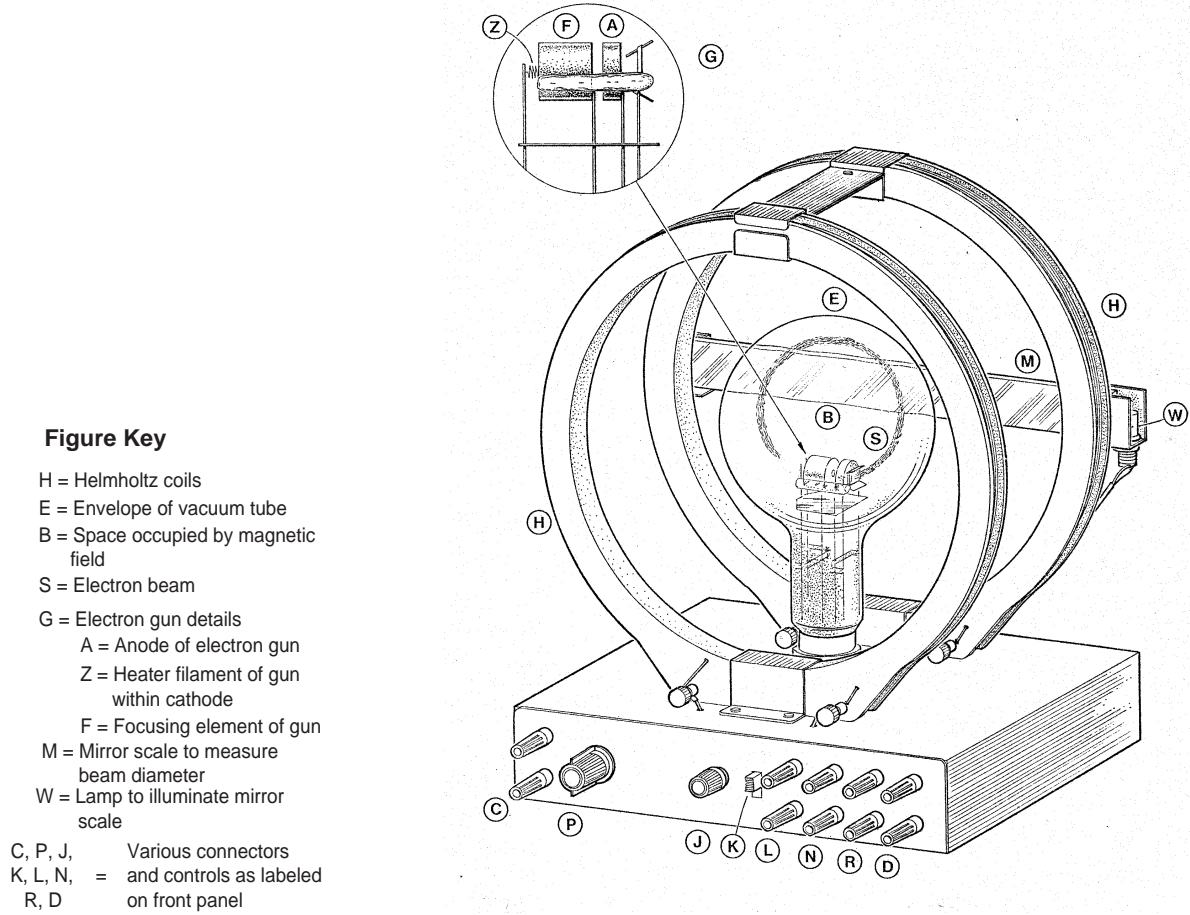
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[1]For example: http://www.aip.org/history/electron/

[2]The word "electron" was actually coined by G. Johnstone Stoney a bit earlier (1891) to denote the unit of charge found in electrolysis experiments. This idea of a chemical "atom of charge" goes back to the 1830s and Michael Faraday.

[3]To the Greek philosophers "atoms" were by definition indivisible, so the modern atom with parts (electrons and quarks) differs from the the original atom. While "splitting the atom" would be self-contradictory to Democritus, we retain the term "atom" for the chemical elements even as the search continues for true atoms: possible substructure inside electrons and quarks.

[4]Lee DeForest, 1907

[5]Shockley, Brattain and Bardeen, 1947

Figure Key

H = Helmholtz coils
E = Envelope of vacuum tube
B = Space occupied by magnetic
    field
S = Electron beam

G = Electron gun details
    A = Anode of electron gun
    Z = Heater filament of gun
      within cathode
    F = Focusing element of gun
M = Mirror scale to measure
    beam diameter
W = Lamp to illuminate mirror
    scale

C, P, J,   Various connectors
K, L, N, = and controls as labeled
 R, D   on front panel

Figure 7.1: Diagram of e/m apparatus.

heated cathode, collimated by the focusing element, and accelerated by the electric field produced by the large voltage difference between the cathode and the anode. Once outside the electron gun, the electric field is nearly zero, and the electrons respond to the magnetic field produced by the coils. In general, the force on an electron is governed by the Lorentz relation:

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right). \tag{7.1}$$

In this equation $\vec{\mathbf{F}}$ is the force vector on a particle of charge q, moving with velocity vector $\vec{\mathbf{v}}$ in an electric field, $\vec{\mathbf{E}}$, and a magnetic field, $\vec{\mathbf{B}}$. We denote the charge of an electron by $q = -e$.

Inside the electron gun, the electrons in the beam gain kinetic energy as they move to regions of lower potential energy (qV) due to conservation of energy. If the accelerating voltage is V the following relationship gives the kinetic energy of the electrons as they leave the gun:

$$eV = \frac{1}{2}mv^2. \tag{7.2}$$

If the electron beam is directed with velocity perpendicular to the magnetic field, the electrons will move in a circular path with the Lorentz force (perpendicular to both $\vec{\mathbf{B}}$ and $\vec{\mathbf{v}}$) providing the centripetal force. Under these circumstances, the magnitude of the Lorentz
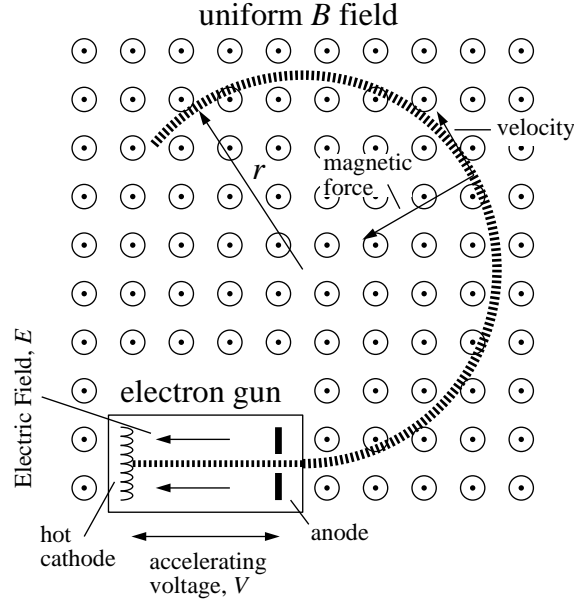
Figure 7.2: Schematic diagram of the e/m apparatus. Electrons from the hot cathode are accelerated by a potential difference V. They enter a uniform B field and move in a circle of radius r due to the magnetic force: $q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}$.

force, given by Equation (7.1), is:

$$F = evB \tag{7.3}$$

and the magnitude of the centripetal acceleration of the electron is:

$$a = \frac{v^2}{r}, \tag{7.4}$$

where r is the radius of the circular path. Applying Newton's second law, $F = ma$, we have

$$evB = m\,\frac{v^2}{r}, \tag{7.5}$$

where m is the mass of the electron. Though we may set up an expression for e/m immediately from Equation (7.5), we elect instead to solve for the velocity of the electron here in order to eliminate it from the final equation. Thus we have:

$$v = \frac{erB}{m}. \tag{7.6}$$

Now substitute this expression for v into Equation (7.2) and solve for e/m in terms of the laboratory parameters V, r, and B:

$$e/m = \frac{2V}{r^2 B^2}. \tag{7.7}$$

V and r are directly measurable and so we now consider how to find B.

As you may have already learned, the magnetic field B along the axis of a circular coil is given by

$$B = \frac{\mu_0 R^2 I}{2\left(R^2 + x^2\right)^{3/2}}, \tag{7.8}$$

where x is the distance along the axis of symmetry of a single current loop of radius R carrying a current I. Since there are N current loops in each coil and the loops are arranged so that their fields add constructively and contribute equally, we have a total field of

$$B = \frac{\mu_0 R^2 N I}{(R^2 + x^2)^{3/2}}. \tag{7.9}$$

The Helmholtz coil configuration calls for $x = R/2$ for each coil. If we substitute this result into Eq.(7.9), we obtain a total magnetic field between the coils of:

$$B = \frac{8}{\sqrt{125}} \frac{\mu_0 N I}{R}. \tag{7.10}$$

This value for B, derived for the central point on the axis between the Helmholtz coils, closely approximates[6] B at the position of the electron beam (which is typically a few cm from the center point).

Now substituting the permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ T $\cdot$ m/A; the number of turns in each coil, $N = 130$; and the radius of the coils, $R = 0.15$ m; into Equation (7.10) gives

$$B = \left(7.79 \times 10^{-4} \text{ T/A}\right) I. \tag{7.11}$$

Finally, we substitute the expression for B from Equation (7.11) into Equation (7.7) to arrive at an experimentally measurable value of e/m:

$$e/m = \frac{2V}{\left(6.07 \times 10^{-7}\right) I^2 r^2}. \tag{7.12}$$

Please derive the last two equations as part of your lab report—it would be a good idea to do so before coming to lab. Be sure you include units in your derivation, and in particular, show that the units of e/m are C/kg.

Procedure

PLEASE DO NOT TURN ON ANY POWER SUPPLIES WITHOUT THE LAB INSTRUCTOR'S APPROVAL!!!

Possible experimental approaches

In studying Equation (7.12) to determine an appropriate way to make measurements in the lab, we find several approaches are available to us. For example, Equation (7.12) could be written as:

$$V = \left[\frac{\left(6.07 \times 10^{-7}\right) \left(r^2\right) (e/m)}{2} \right] I^2. \tag{7.13}$$

---

[6]When typical values of r, x, and R from the Ealing apparatus are used in an exact formula for B off the axis and compared with (7.10), we are in error by less than 1%. Helmholtz coils are designed to produce a large region of nearly constant B.

Now if we keep r constant, a plot of V *vs.* I^2 would yield a slope, S, given by:

$$S = \frac{\left(6.07 \times 10^{-7}\right)\left(r^2\right)(e/m)}{2}. \tag{7.14}$$

Hence,

$$e/m = \frac{2S}{\left(6.07 \times 10^{-7}\right)r^2}. \tag{7.15}$$

Therefore, if we select a value of r and then record a sequence of values of I and V, one can plot V *vs.* I^2, determine the slope, and calculate e/m.

One might also hold the current constant, and measure V as a function of r. Feel free to explore such alternative approaches.

Error analysis

If you use the procedure outlined above, use the following scheme for error analysis:

- Error in the accelerating voltage V: If you are using the Pasco SF-9585A high voltage power supply, assume the uncertainty is the least count of the display (that is, 1 volt). (We don't have specifications for this power supply, but we have checked several of them with a multimeter, and this uncertainty looks reasonable to us. Feel free to check it yourself!)

- Error in I^2: First find the error in I using the table at the end of the Electrical Circuits experiment, page 45. Then use the techniques developed in the appendices of the Physics 191 lab manual to find the uncertainty in I^2.

- Error in radius r: Estimate this uncertainty from your measurements: How accurately can you measure r?

- Assuming the graph of V vs I^2 is a straight line, do a fit using errors in both x and y (that is, in both I^2 and V). The fit will give you the uncertainty in the slope S.

- Finally, calculate the uncertainty in e/m taking Eq. (7.15) as your starting point. Note that you will need to use both the uncertainty in the slope, and the uncertainty in r^2, which you should calculate from your estimate of the uncertainty in r using the techniques you learned in Physics 191.

Detailed experimental procedure

Connect a DC power supply through an ammeter to the Helmholtz coils (check polarity). This ammeter will measure I which should never exceed 2 A. *ONLY USE ENOUGH CURRENT TO BEND THE ELECTRON BEAM — TOO MUCH CURRENT COULD HARM THE COILS.*

Connect a 6 volt AC source to the electron gun heater (i.e., switch the Pasco SF-9585A AC power source to 6). Connect a high voltage DC source (up to 300 volts) to the electron gun electrodes (note polarity). The power supply voltmeter will measure V across the electrodes.

Measurement of the electron orbit radius r is difficult (because the beam is enclosed by glass and hence can not be placed adjacent to the scale) and mis-measurement is common. Parallax error will be avoided only if you carefully align the beam and its reflection in the mirrored scale. Measure the left and right sides of the orbit separately and be sure they lie on a diameter (i.e., the mirrored scale should go through the center of the circle). Also, for reasons that you should think about (and record in your notebook), use the outside edge of the electron beam when determining r.

If you select the approach discussed above you might select r equal to about 4.5 cm and vary V in about 10 volt steps from 150 to 250 volts. **CHECK WITH INSTRUCTOR BEFORE TURNING THE POWER ON!!** Be sure to record all pertinent information.

Lab Report

1. Tabulate experimental measurements and calculated quantities, particularly the uncertainties.

2. Use *WAPP+* (Goggle "`wapp+`" to find it or use LINFIT) to plot and fit your data. Include the plot and the fit report in your notebook.

3. Answer the following questions:

   (a) Why use the outside edge of the electron beam when determining r?

   (b) Describe, briefly, how electric and magnetic fields are used to control the particle beam in a circular particle accelerator.

4. Carefully report the results you found for e/m, including experimental error and units. Compare it to the accepted[7] value ($1.75882015 \pm 0.00000004 \times 10^{11}$ C/kg) and discuss possible reasons for any discrepancy.

5. Include any other conclusions, observations, discussion and a lab critique.

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[7]CODATA, 2006

8. Helmholtz Coils

Purpose

In this experiment we shall investigate the magnetic field, B, as a function of position, for three fairly simple current configurations. We will use the emf induced by a continuously changing B (Faraday's Law) to measure B.

Apparatus

- Helmholtz coils

- function generator

- oscilloscope

- search coil

- apparatus for mounting the search coil on the axis of the Helmholtz coils (bench clamps, heavy rods, and nylon cord)

Theory

One Current Loop

On the axis of a circular wire coil carrying a current I, the magnetic field B is given by

$$B = \frac{\mu_0 N I}{2} \frac{R^2}{(x^2 + R^2)^{3/2}}. \tag{8.1}$$

Here x is the distance along the axis, measured from the center of the coil; R is the radius of the coil; and N is the number of turns. The direction of B is along the axis of the coil. See your text and class notes for a full derivation and discussion of this result. Our purpose here will be to confirm the x dependence of Equation 8.1.

Helmholtz Coils

Two identical current loops on the same axis, and separated by a distance equal to the radius of either loop, constitute a set of Helmholtz coils[1] . (See Figure 7.1 on page 64 in the e/m experiment in this manual). Such a geometry produces a reasonably uniform magnetic field in the space between the coils:

$$B = \frac{8}{\sqrt{125}} \frac{\mu_0 N I}{R}. \tag{8.2}$$

The derivation of this result is long, but not terribly difficult, and may be found in most intermediate level texts on electricity and magnetism. In this experiment, we will see whether the field is in fact constant.

Helmholtz Coil with the direction of the current reversed in one coil

If we reverse the direction of the current in one coil, the fields of the two coils will both still be along the axis, but in opposite directions. Consequently, instead of adding in the region between the coils to produce a uniform field, they will cancel, and so produce a null field at the center of the coils. We will attempt to confirm this prediction, and in the process, will use an important feature of the oscilloscope.

Measurement of B

The Helmholtz coils are described in the e/m lab in this manual. The glass tube used for the e/m measurement will be removed. Measurement of magnetic fields is always a bit indirect. Unchanging magnetic fields can be measured with a Hall Probe, however these devices require calibration and analysis would be confused by the presence of the Earth's magnetic field. Consequently, in this experiment our field measuring system is designed to be sensitive only to changing magnetic fields. We will drive our coils with a sinusoidal current (using a function generator) so that a sinusoidally varying magnetic field will be produced:

$$B = B_0 \sin \omega t \tag{8.3}$$

Faraday's Law tells us that a time-varying magnetic field creates an electric field, and so induces a voltage in any conductor that happens to be present. We will, therefore, detect the magnetic field by the voltage induced in a small 'search' coil. By connecting the search coil to an oscilloscope, we will be able to measure the induced emf, and that emf will be proportional to the magnetic field. More precisely, Faraday's law states that
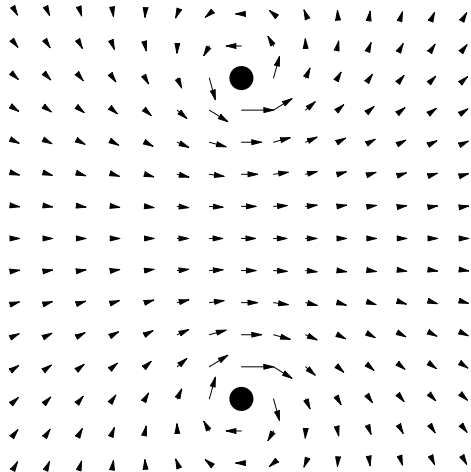
$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \tag{8.4}$$

where $\mathcal{E}$ is the emf induced in the search coil, and Φ_B is the magnetic flux through the search coil, defined by
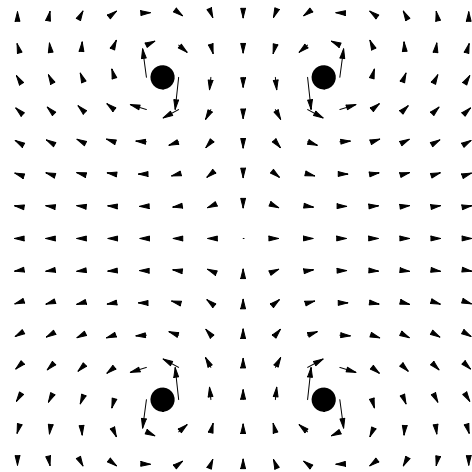
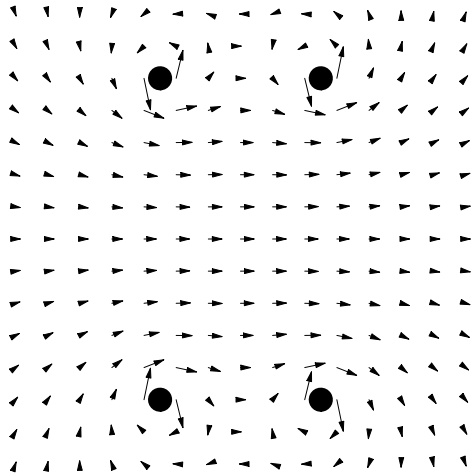$$\Phi_B = \int \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} \, dA \tag{8.5}$$

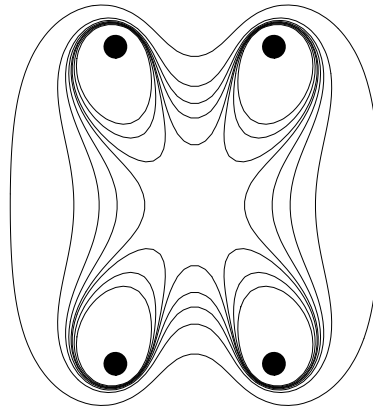[1]Named in honor of Hermann von Helmholtz (1821–94).

(a) Magnetic field near a single current loop.

(b) Magnetic field near a pair of opposing current loops. As in Helmholtz coils, the separation of the loops is equal to the loop radius.

(c) Magnetic field vectors near a pair of Helmholtz coils.

(d) A contour plot of the magnetic field *magnitude* near a pair of Helmholtz coils. Within the central 'octopus' B is within 1% of its value at the center of the pair of coils, B_0. In detail the contours are for: $.5B_0$, $.8B_0$, $.9B_0$, $.95B_0$, $.99B_0$, $1.01B_0$, $1.05B_0$, and $1.1B_0$.

Figure 8.1: The magnetic field in a plane bisecting current loops is displayed along with filled disks showing where the current loops intersect the plane

where the integral is over the effective area of the search coil. Both $\vec{B}$ and Φ_B vary sinusoidally with time, and hence the induced emf also varies sinusoidally with time:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\omega \, \cos \omega t \int \vec{B}_0 \cdot \hat{\mathbf{n}} \, dA \tag{8.6}$$

and expect to see a sine wave on the oscilloscope. The point to understand here is that the emf induced in the coil is proportional to B_0, the amplitude of the magnetic field. (The proportionality constant between the magnetic field and the voltage displayed on the scope is not difficult to calculate. However, we will not need it for this lab.)

Procedure

1. Mount the search coil so that it moves along the axis of the coils. Describe your procedure. How can you be sure that the search coil is precisely on-axis? **Be careful not to damage the coil or the cable connection to it**.

2. Power up the digital scope and recall the oscilloscope's Setup 1. (See the digital oscilloscope lab, page 27, for details.) We will want to MEASURE the Cyc RMS voltage on CH 1. (The measure menu is described in section D, page 30.) Before turning on the function generator, push in the 20K FREQUENCY RANGE button, the sine wave (leftmost: $\sim$) FUNCTION button, and turn the FREQUENCY and AMPLITUDE knobs to middle positions.

3. Single Coil: Connect the function generator to one of the Helmholtz coils (not both). Place the search coil at the center of the coil, and observe the signal from the search coil on the oscilloscope. Adjust the frequency of the function generator for maximum response, and record that frequency. You should define the point $x = 0$ to be at the point where the voltage is a maximum—approximately the center of the coil.

   Record the voltage of the search coil as a function of distance along the axis of the coil, using as wide a range of distances as possible. (Take at least 15 data points for values of x up to about 1 meter.) Of course, as x is increased the voltage will get smaller. Change the vertical scale (VOLTS/DIV) of the oscilloscope when the signal you're measuring is smaller than about 3 divisions. (Use of the proper scale allows us to ignore the constant in the complex error formula on page 36; Instead you can use a simple error estimate of 3% for RMS voltage.)

   Convert your independent variable to $z = x^2 + R^2$ where $R = .15 \pm .005$ m. Using Eq. C.12, the error in z is given by:

   $$\delta z = \sqrt{(2x\delta x)^2 + (2R\delta R)^2} \tag{8.7}$$

   Use a spreadsheet for this calculation! Fit your data to an equation of the form $y = Az^B$. Then see if your results are consistent with Eq. (8.1), which predicts $B = -1.5$.

   Be sure that your results are consistent with a power law, using the appropriate log-log graph. This step often helps correct problems in your procedure, if your data do not initially fall on a straight line.

4. Connect both Helmholtz coils and the function generator in series so that the current in the coils is going in the same direction. Measure the field as a function of position between the coils. Record data out to about $x = \pm 20$ cm. (Note that the point $x = 0$ is now defined to be the midpoint on the axis between the two coils.) Present your results graphically[2]. How nearly constant is the field between the coils?

5. Reverse the direction of the current through one of the Helmholtz coils — your instructor will show you how. See if the field has a sharp minimum at $x = 0$.

   While the Cyc RMS voltage reported by the scope will always be positive it is nevertheless possible to confirm that the direction of the field changes half-way between the Helmholtz coils. (That is the voltage should be recorded as negative on one side of the coil pair.) To observe this effect, TRIGGER the oscilloscope by connecting the function generator's AUX OUT to the EXT TRIG input of the oscilloscope. Using the scope's TRIGGER MENU set Source▶Ext for external triggering. *Be sure you understand how the external trigger works, and why you need to use it here*! Note that with the search coil at one side of the coil pair the central peak displayed on the scope will be positive, whereas on the other it will be negative. Record your observations, and explain how the scope's display shows that the direction of the field changes at $x = 0$ (i.e., that B changes sign there).

   Again, record the field (now with proper sign!) as a function of position between the coils. Record data out to about $x = \pm 20$ cm and present your results graphically.

Lab Report

Carefully summarize your results and answer all questions asked in the lab. Be sure to include a discussion of how well your single coil measurements agree with the theoretical predictions. As usual, attach your fit report and plots (single coil normal, single coil log-log, Helmholtz coil normal, and anti-Helmholtz coil normal) inside your notebook. Include any critique you can, commenting on the clarity of the Lab Manual, the performance of the equipment, the relevance of the experiment, etc.

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[2]For plotting without function fitting, follow the <u>link</u> near the top of the $WAPP^+$ page.

9. AC Circuits

Introduction

Circuits with time dependent signals ("AC circuits") are more interesting and useful than circuits where the voltages and currents are constant. It should be no surprise that going from $V(t) = $ constant to $V(t) = $ anything opens up lots of possible behaviors, but even if we limit the voltages and currents to sinusoidal behavior, we have three (rather than just one) parameters to consider— amplitude, A, frequency f, and phase ϕ:

$$A \sin(2\pi f t + \phi) \tag{9.1}$$

It will save space in writing equations if we define the *angular frequency*, $\omega = 2\pi f$.

The three basic AC circuit elements are the capacitor (C), the inductor (L) and the resistor (R). In order to understand the behavior of different AC circuit element combinations, we will look at one simple yet important circuit, the series LRC circuit, and examine how the amplitude and phase angle vary with frequency.

Theory

First consider the AC behavior of resistors, capacitors, and inductors individually. For the purposes of discussion, let the source of emf that we connect these elements across be described by

$$V(t) = V_0 \sin(\omega t + \phi), \tag{9.2}$$

Resistor: If a resistor is connected across the AC voltage source described by Eq. (9.2), then the resulting current can be determined from the relationship $I = V/R$. Therefore:

$$I(t) = \frac{V(t)}{R} = \frac{V_0}{R} \sin(\omega t + \phi) = I_0 \sin(\omega t + \phi). \tag{9.3}$$

Note that the current has the same frequency f and phase ϕ as the applied emf and that the relationship between the peak voltage V_0 and peak current I_0 is that given by Ohm's law: $V_0 = I_0 R$. We usually measure AC quantities in root-mean-square (rms) terms, but converting both peak values to rms leaves the Ohm's law relationship intact, so $V_{rms} = I_{rms} R$.

Capacitor: A capacitor is a device whose charge is directly proportional to the applied voltage:

$$Q = CV \tag{9.4}$$

Recalling that the current is the derivative of the charge with respect to time we can write:

$$I = \frac{dQ}{dt} = C\,\frac{dV}{dt}. \tag{9.5}$$

If the capacitor is connected across the source of emf described by Eq. (9.2), then the resulting current is

$$I(t) = C\,\frac{d}{dt}\left(V_0 \sin(\omega t + \phi)\right) = \omega C V_0 \cos(\omega t + \phi) = I_0 \cos(\omega t + \phi). \tag{9.6}$$

This can be rearranged as:

$$I(t) = \omega C V_0 \sin\left(\omega t + \phi + \frac{\pi}{2}\right). \tag{9.7}$$

Note that the phase angle for the current in this case is $(\phi + \pi/2)$—the current leads the applied voltage by $\pi/2$ or $90°$ (a.k.a. the voltage lags the current by $90°$: ICE)—and the amplitude is $I_0 = \omega C V_0$

We can rearrange the sinusoidal amplitudes this equation to look like the Ohm's law:

$$V_0 = I_0\,\frac{1}{\omega C} = I_0\,X_C \tag{9.8}$$

where quantity $1/\omega C$ has units of resistance (ohms) and is called the *capacitive reactance*, X_C.

Inductor: An inductor is a device that can store/release magnetic energy. The magnetic energy is a result of the device's magnetic field which is in turn proportional to the current flowing through the device. In an AC circuit, this energy is alternately stored and released as the current through the inductor causes the magnetic field to increase and decrease. The induced emf created by the inductor's changing magnetic field is $\mathcal{E} = -L\,dI/dt$. Therefore, when an inductor is connected to the source of emf described by Eq. (9.2) the loop law becomes:

$$V_0 \sin(\omega t + \phi) = L\,\frac{dI}{dt}. \tag{9.9}$$

Integrating both sides with respect to t gives a relationship involving the current I and the applied emf:

$$I(t) = -\frac{V_0}{\omega L}\cos(\omega t + \phi) = \frac{V_0}{\omega L}\sin\left(\omega t + \phi - \frac{\pi}{2}\right). \tag{9.10}$$

Now the phase angle $(\phi - \pi/2)$ indicates that the current lags behind the applied voltage by $\pi/2$, a quarter cycle. (Or equivalently the voltage leads the current by $90°$: ELI.) The
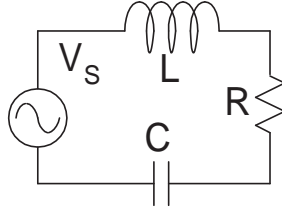
Figure 9.1: LRC series circuit with an AC source.

quantity ωL has units of resistance (ohms) and is called the *inductive reactance*, $X_L = \omega L$. As before we find an Ohm's Law like relationship between the peak (or rms) quantities:

$$V_0 = I_0\,\omega L = I_0\,X_L \tag{9.11}$$

LRC Circuit: When these three elements are combined together in one circuit, as in Fig. 9.1, the analysis becomes a bit more complicated. To make the algebra as simple as possible we assume[1]

$$I(t) = I_0 \cos(\omega t) \tag{9.12}$$

and seek the (phase shifted) source voltage as a function of time, $V(t)$, which is consistent with this choice:

$$V(t) = V_0 \cos(\omega t + \varphi) \tag{9.13}$$

To find the phase angle φ and the current amplitude I_0, we start with the equation derived from the loop law:

$$V_0 \cos(\omega t + \varphi) - \Delta V_R - \Delta V_C - \Delta V_L = 0. \tag{9.14}$$

Here ΔV_R is just the voltage drop across the resistor,

$$\Delta V_R = IR = I_0 R \cos(\omega t), \tag{9.15}$$

ΔV_C is the voltage drop across the capacitor,

$$\Delta V_C = \frac{Q}{C} = \frac{\int I\,dt}{C} = \frac{I_0}{\omega C}\,\sin(\omega t) = I_0 X_C \sin(\omega t), \tag{9.16}$$

and ΔV_L is the voltage drop across the inductor,

$$\Delta V_L = L\,\frac{dI}{dt} = -\omega L I_0 \sin(\omega t). \tag{9.17}$$

Thus, the loop law gives the following:

---

[1]There is no loss of generality in this choice; it amounts to choosing the instant $t = 0$ to be when the current peaks.

$$V_0 \cos(\omega t + \varphi) = RI_0 \cos(\omega t) - \omega L I_0 \sin(\omega t) + \frac{1}{\omega C} I_0 \sin(\omega t)$$

$$V_0 \cos(\varphi) \cos(\omega t) - V_0 \sin(\varphi) \sin(\omega t) = RI_0 \cos(\omega t) - \left(\omega L - \frac{1}{\omega C}\right) I_0 \sin(\omega t)$$

$$= RI_0 \cos(\omega t) - (X_L - X_C) I_0 \sin(\omega t)$$

We can conclude:

$$V_0 \cos \varphi = RI_0$$
$$V_0 \sin \varphi = (X_L - X_C) I_0 \qquad (9.18)$$

Dividing yields the phase angle, φ:

$$\tan \varphi = \frac{\sin \varphi}{\cos \varphi} = \frac{X_L - X_L}{C}. \qquad (9.19)$$

Adding the squares yields:

$$V_0^2 = V_0^2 \left(\cos^2 \varphi + \sin^2 \varphi\right) = \left(R^2 + (X_L - X_C)^2\right) I_0^2 \qquad (9.20)$$

Solving now for I_0 we find that:

$$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}. \qquad (9.21)$$

Now we have the expressions for the frequency-dependent amplitude I_0 and phase angle φ of the voltage (relative to the current) in the circuit, in terms of the constant parameters V_0, L, C, and R. Note that the frequency f dependence is hidden within the reactances:

$$X_C = \frac{1}{\omega C}, \qquad X_L = \omega L, \qquad (9.22)$$

where $\omega = 2\pi f$.

To simplify a bit more and to help in studying the amplitude and phase functions, define the **impedance** of the circuit as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}, \qquad (9.23)$$

At Resonance

At some frequency it will happen that $X_C = X_L$. Clearly, the impedance [Eq. (9.23)] is then minimized, becoming simply R, and the phase angle [Eq. (9.19)] goes to zero. Since $2\pi f = \omega$, this frequency satisfies

$$\frac{1}{2\pi f C} = 2\pi f L, \tag{9.24}$$

so the resonance frequency is found to be:

$$f_{\text{res}} = \frac{1}{2\pi\sqrt{LC}}. \tag{9.25}$$

This equation can be rearranged to yield:

$$C = \frac{1}{L(2\pi)^2}\, f_{\text{res}}^{-2} \tag{9.26}$$

which is a better match for your fitting equation.

At resonance, Z reaches its minimum value, R, so the current amplitude reaches its maximum value, V_0/R. And the phase relationships of the voltages around the circuit at resonance are summarized as:

$$
\begin{aligned}
V_S &= V_0\cos\omega t \\
V_R &= I_0 R\cos\omega t \\
V_C &= \frac{1}{\omega C} I_0\sin\omega t = \frac{1}{\omega C} I_0\cos(\omega t - 90°) \\
V_L &= -\omega L I_0\sin\omega t = \omega L I_0\cos(\omega t + 90°)
\end{aligned}
\tag{9.27}
$$

In this form it is clear that the voltage drop across the resistor is *in phase* with the applied voltage, the voltage drop across the capacitor *lags* the applied voltage by a quarter cycle, and the voltage drop across the inductor *leads* the applied voltage by a quarter cycle.

Near Resonance

The voltage across the resistor is given by Ohm's law: $V_R = I_0 R$, where the current I_0 is given by Eq. 9.21. Using the expression for reactance Eq. 9.22, we find:

$$V_R = \frac{V_0 R}{\sqrt{R^2 + \left(\frac{1}{2\pi f C} - 2\pi f L\right)^2}} \tag{9.28}$$

This equation defines a bell-shaped function displayed in Fig. 9.2.

Procedure

Oscilloscope connections

We want to examine the amplitudes and phase relationships of voltages across the different components in an *LRC* circuit. This cannot be done using a DMM. (The DMM does
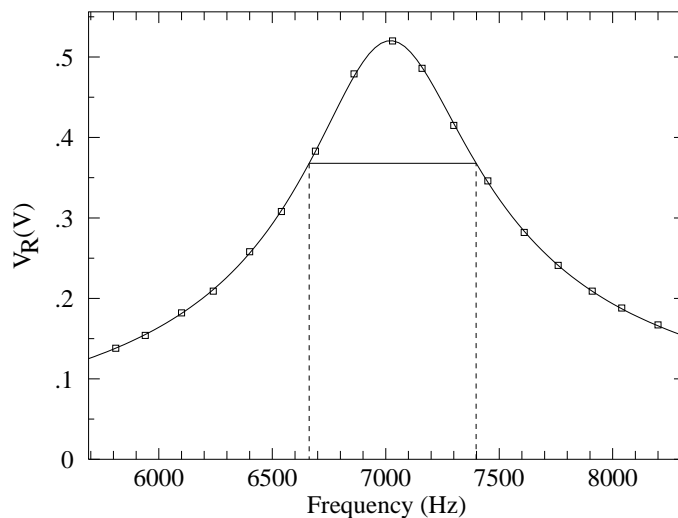
Figure 9.2: The graph of V_R vs. f is bell-shaped with a maximum voltage, V_{maxR} (here about 0.52 V) at the resonance frequency f_{res} (here about 7000 Hz). The full width of the resonance curve is defined by range of frequencies for which $V_R > V_{\mathrm{maxR}}/\sqrt{2}$. (Here the full width, denoted by a horizontal line, is measured at $V_R = 0.52/\sqrt{2} \approx .37$ V: $\Delta f = 7400 - 6660 = 740$ Hz.)
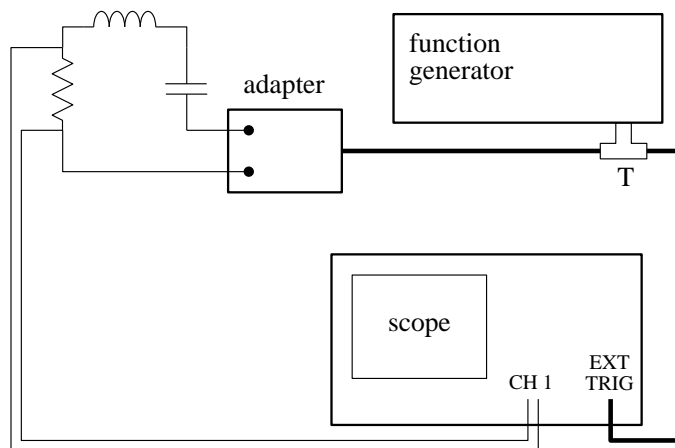

Figure 9.3: Oscilloscope connections to LRC series circuit: the circuit is indirectly driven by the function generator and hence the circuit's only connect to ground is through the scope.

not report phase information—just rms voltage—and in addition it cannot measure at the "high" frequencies used in this lab.)

The oscilloscope is a high speed voltmeter and it can display waveforms and hence phase relationships. However, a scope's black lead is internally connected to ground and hence can't be connected to any other voltage without creating a "short circuit". This problem can be avoiding if the circuit being measured has no connection to ground. Then the scope's black lead can be connected any place in the circuit—forcing that place to be ground—but creating no ground conflict. Thus your circuit must be driven indirectly through the function generator adapter box.

In displaying successively the V_R, V_L, and V_C waveforms on the scope, you must make sure the scope always displays the spot $t = 0$ in the same location. (Of course, you should also maintain the same horizontal scale.) Thus (as in the Helmholtz Coils experiment), you use the *external trigger* feature of the scope. Use a T adapter to send the function generator's output both to the circuit and to the EXT TRIG input of the scope. Using the TRIGGER MENU, select Source▶Ext . When the input signal is increasing and goes through a value of zero, the oscilloscope is told to start drawing the trace of the measured signals. This triggering point should be in the center of the display and is marked with a little triangle at the top of the display. (Turning the HORIZONTAL POSITION knob allows you to move the location of the trigger point on the display.)

If the trace of the measured signal starts at zero and is increasing (i.e., looks like a sine wave), then that signal is in phase with the triggering signal. On the other hand if the trace of the input signal starts at its maximum value and then decreases (i.e., looks like a cosine wave), then you know that the input signal *leads* the triggering signal by 90°. Also if the input signal starts at its minimum value and then increases (i.e., looks like an inverted cosine wave), then you know that the input signal *lags* the triggering signal by 90°. External triggering is a very powerful tool and is used frequently in all kinds of applications. If you are having trouble, don't be afraid to ask the lab instructor for help!

1. After turning the scope on, press the SAVE/RECALL menu button. (See page 27.) Make sure the number 5 is displayed in the Setup box. Press the button next to the Recall box. The message "Setup 5 recalled" should temporarily appear in the lower left section of the display and bottom of the display should then read:
   CH1 20.0mV   M 100μs   Ext $\diagup$ 0.00V

2. Press the TRIGGER MENU button. Notice that the slope is set to Rising and the source is set to Ext. When you use the AUTOSET feature, the trigger source is reset to CH1; you must then by hand reselect Source▶Ext.

   If you become lost in a series of menus, Setup 5 can always be recalled from the SAVE/RECALL menu.

Note that each channel (i.e., input) on the oscilloscope actually has two connections. One, called the pin, is the center conductor of the coaxial cable and is the red (positive) input. The other, called shell or black, is a shield that is wrapped around the central conductor to reduce stray electrical signals (noise). This surrounding shell is grounded—actually connected to the ground in the wall outlet—shielding the center signal wire.

In this lab we have chosen to evade the problem of the scope's ground by disconnecting the *LRC* circuit from any conflicting ground using a transformer. When this is not an option, voltage differences can be displayed by connecting CH1 and CH2 inputs to the circuit. The scope can then calculate directly the voltage difference between those two probes using the MATH functions.

Measurements near resonance: V_R versus f

1. Select $C = .01$ μF and $R = 100$ Ω. Set the function generator RANGE to 20 kHz, the frequency knob to about .8 (8 kHz) and the AMPLITUDE knob to about 50%. Connect the scope across the resistor. The trace should show a sine wave, if you cannot achieve a nice display, hit the AUTOSET button on the scope (then reselect trigger Source▶Ext!). As you turn the frequency knob, the amplitude and phase of the trace should vary. At the resonant frequency the amplitude is maximized (and the phase shift should be nearly zero).

2. Press the MEASURE menu button and make sure the Source/Type is CH1/Cyc RMS. Now you can easily read the rms voltage across the resistor (V_R). Record the resonant frequency and the corresponding maximum rms voltage V_{maxR}.

3. Now examine the amplitude around resonance by varying the function generator frequency, f, and reading the resulting V_R from the scope. As you move away from resonance on the low frequency side, record f and V_R for eight points. Select your data points to get a good representation of relationship. (For example, about half of your points should have V_R above $V_{maxR}/2$.) Repeat for frequencies above resonance. The result should be about 16 data points showing a bell-shaped response. (As reported on page 36, the basic frequency meter accuracy is ± 1 digit. You can use a scope accuracy of 3% if you maintain a proper scale.)

4. Plot[2] your V_R vs. f data, and sketch in a smooth curve to illustrate what happens near resonance. The results should look a bit like Fig. 9.2.

Measurements at resonance: $f_{\rm res}$ vs. C

1. Select $C = .22$ μF and a function generator RANGE of 2 kHz. Find the resonant frequency $f_{\rm res}$ by finding the frequency that maximizes V_R. Estimate the uncertainty in $f_{\rm res}$, by determining the frequency range over with V_R does not change significantly. The uncertainty in the substitution box C is 10%.

2. Now switch to the next smaller C value, and find the resulting $f_{\rm res}$. Continue doing this, and make a table of your f versus C results with uncertainties. (It will help to use consistent units of μF and kHz or F and Hz.) At some point you will have to increase the function generator RANGE. Continue to $C = 1000$ pF.

3. Analyze your $(f_{\rm res}, C)$ data using *WAPP*$^+$ (LINFIT cannot do this fit.). There are two common rules for determining which variable belongs on the x-axis:

   - Put the controlled variable (here C) on the x-axis and the measured value (here f) on the y-axis.

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[2]For plotting without function fitting, follow the <u>link</u> near the top of the *WAPP*$^+$ page.

- Put the small error quantity on the x-axis and the large error quantity on the y-axis. The 10% error in C is in this case almost certainly larger than your error estimate for f.

Following this <u>last</u> rule use $C = Y$ and $f = X$ and fit the data to the power law function $Y = AX^B$. In this case Eq. (9.26) requires $B = -2$, so you must hold B fixed with that value. You will use the A parameter from the fit to find the inductance L.

Measurements of phase relationships at resonance

1. Return to $C = .01 \ \mu$F and adjust the frequency for resonance. Reduce the function generator amplitude to approximately 20%. (This frequency and amplitude should be maintained for the following measurements.) Make sure the scope is set for External triggering. Use the oscilloscope to measure the rms voltage across the resistor. Record the rms voltage and make a careful sketch of the waveform. Be sure to record time and voltage scale factors! (See page 28 if you've forgotten where the scale factors are displayed. It will be necessary to change the vertical (voltage) scale; do not change the time scale.) Pay careful attention to the trace around the trigger point (marked with a triangle at the top of the display).

2. Without changing the order of the oscilloscope leads, move the leads around the circuit to measure the voltages across the capacitor and inductor. For each record the voltage, sketch the waveforms and record time and voltage scale factors. If these signals are too large to measure on any scale, reduce the function generator amplitude, and remeasure V_R as per #1 above.

3. Finally, measure the voltage, V_S, across the source (i.e., across the pins of the adapter)..
   Record the voltage and sketch the waveform.

Lab Report

1. V_R vs. f Plot:

   (a) The power dissipated by the resistor is $P = V_R I_R = V_R^2/R$. At resonance this reaches a maximum value $P_{max} = V_{maxR}^2/R$. There are two frequencies at which the power is half the maximum value; that is, where $V_R = V_{maxR}/\sqrt{2}$. Calculate $V_{maxR}/\sqrt{2}$ and, using your sketched curve, find these two frequencies and their difference Δf, which is an indication of the width of the resonance.

   (b) Resonant circuits are usually specified by the dimensionless "Q-factor", where

   $$Q \equiv \frac{f_{\text{res}}}{\Delta f} \tag{9.29}$$

   Large Q corresponds to a "sharper" (narrower) resonance curve. Calculate Q for your circuit.

2. C vs. f_{res} fit: Calculate L (don't forget units and error) from the fit value for A.

3. From your sketches of the waveforms of V_R, V_C, V_L and V_S, determine the phase differences between the voltages. Do they agree with the theory, as summarized in Eqs. (9.27)?

4. Include any critique you can, commenting on the clarity of the Lab Manual, the performance of the equipment, the relevance of the experiment, etc.

Appendix A — Least Squares Fits

References

Our treatment of the method of least squares in this appendix is necessarily brief and incomplete. At some point in your career, you are likely to need to learn more than can be provided here about least-squares fits. The following books provide increasingly sophisticated discussions of the least-squares method:

- John R. Taylor, *An Introduction to Error Analysis*, second edition, (University Science Books, 1997).

  The Physics Department requires this book for all second year and higher laboratory courses, and we recommend it for anyone planning to major in physics or engineering.

- Philip R. Bevington & D. Keith Robinson, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, 2002)

- William H. Press et al, *Numerical Recipes: The Art of Scientific Computing*, third edition, (Cambridge University Press, 2007) especially chapter 15.
  `http://www.nrbook.com/b/bookfpdf/f15-2.pdf`

Introduction

The foundation of "curve fitting" is determining the function that provides the "best possible" approximation to a set of experimental observations $\{(x_i, y_i)\} \quad i = 1, 2, \ldots, N$. While your eye is an excellent judge of "best possible" we seek a standardized (and automated) method of determining the "best" (and particularly the uncertainty that should be attached to the resulting parameters—e.g., slope and intercept of an approximating line).

Method of Least Squares

The method of least squares is the most commonly used technique[1] for fitting data to a function. Suppose, for example, that we have a set of experimental data that lie roughly

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[1] `http://www.physics.csbsju.edu/stats/fitting_lines.html` discusses some other, less common, approaches.

along a straight line. We want to find the "best" values of slope B and intercept A for the equation

$$y = A + Bx \tag{A.1}$$

that will describe the data. The problem is that the data don't usually fall exactly on any one line. This is hardly surprising as the data generally have experimental uncertainties. The method of least squares provides a commonly used mathematical criterion for finding the "best" fit. Suppose, for example, that our data points are (x_i, y_i) for $i = 1, 2, \ldots N$.

For any point, the deviation

$$y_i - (A + Bx_i) \tag{A.2}$$

is the difference between the experimental value y_i and the value calculated from the (as yet unknown) equation for a straight line at the corresponding location x_i. Usually this deviation will have approximately the same magnitude as your experimental uncertainty or "error bar." The method of least squares finds the "best" fit by finding the values of A and B that minimize these deviations not just for a single data point but for all the data. More specifically, it does so by finding the minimum in the quantity called the reduced chi-squared statistic:

$$\bar{\chi}^2 \approx \frac{1}{N} \sum_{i=1}^{N} \frac{[y_i - f(x_i)]^2}{\sigma_i^2}. \tag{A.3}$$

For our special case in which the function is a straight line, this equation becomes

$$\bar{\chi}^2 = \frac{1}{N-2} \sum_{i=1}^{N} \frac{[y_i - (A + Bx_i)]^2}{\sigma_i^2} \tag{A.4}$$

where N is the number of data points, $f(x)$ is the fitting function, and the σ's are the standard deviations—or more intuitively, the uncertainties (or "errors")—associated with each y value. In the second expression, $N - 2$ is the number of "degrees of freedom" for N data points fit to a line.

Once a fit has been found (i.e., we have found the values of A and B that minimize reduced chi-square), we need some way of determining how good the fit is, that is: are the data well described by this line or is the relationship more complex (for example, quadratic). One good visual check is to compare the fitted solution to the data graphically — that is, plot the function and your data on the same graph, and see how well the function describes your data. Another easy way to check the fit is to see if the deviations are significantly larger than the corresponding uncertainty or show a pattern, for example, being consistently positive in some ranges of x and negative in others. These deviations are listed on the computer-generated fit report.

IMPORTANT: It turns out that our reduced chi-squared statistic provides another, more abstract criterion for goodness of fit. Statistical theory tells us that a good fit corresponds to a reduced chi-squared value of about one.

It is not hard to understand this result intuitively. Notice that each term in the chi-square sum, (deviation/error)^2, should be nearly one and we have N such terms, which in the end

d (cm)	T (°C)
1	16 ± 4
2	18
3	35
4	44
5	58
6	62
7	64
8	70

Table A.3: Sample data of temperature as a function of distance along a metal rod.

are divided by $N - 2$. That is reduced chi-square represents something like the average deviation divided by the expected deviation squared. Take a look at the reduced chi-square equation, and be sure you understand this point.

More or less arbitrarily, the fitting program assumes that any value of reduced chi-square ($\bar{\chi}^2$) between 0.25 and 4 represents a reasonably good fit. Otherwise, the program warns you that your fit may not be valid.

There are two common reasons why the value of the reduced chi-squared statistic might not be close to one. First, you may have picked an inappropriate fitting function: If your data describe a curve and you try to fit them to a straight line, for example, you will certainly get a bad fit! Look at a graph of your data along with the fitting function and see how well the two agree.

Second, you may have overestimated or underestimated your experimental uncertainties. In this case, your fit may be all right, but your uncertainties are nevertheless leading to a value of reduced chi-squared significantly larger or smaller than one. In this case, **DO NOT** try to correct the problem by guessing at the errors until you get a reasonable reduced chi-squared value!! Instead, see if you can figure out why your estimates of the experimental error are off. If you can't, talk to your instructor or simply say in your lab notebook that your estimates of experimental error are apparently wrong, and you are not sure why. This approach is not only honest, it is much more in the spirit of scientific inquiry! And as a practical matter, whoever grades your lab is more likely to be favorably impressed by a statement that you don't understand something than by an obvious attempt to "fudge" your error estimates!

Using Errors in Both x and y

Most treatments of the method of least squares, and many computer programs that use it, assume that the only experimental uncertainties are in y — in other words, that any uncertainty in x can be neglected. This assumption is reasonably accurate in a surprising number of cases. Nevertheless, in many experiments there will be significant uncertainties in both the x and y values and our fitting program is designed to include both of them.
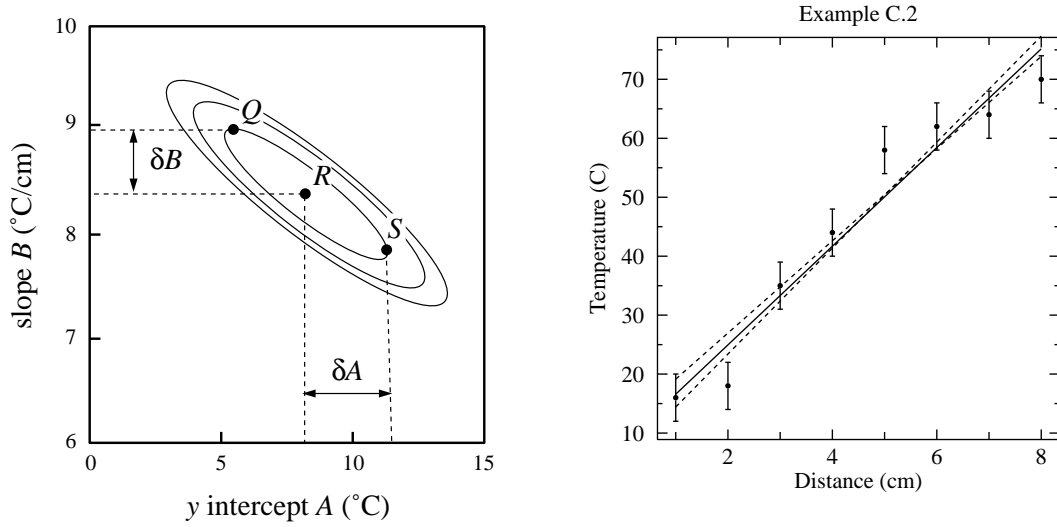
(a) Constant curves (ellipses) for $\bar{\chi}^2$ are displayed as a function of the test line's slope B and y intercept A. The minimum value of $\bar{\chi}^2$ occurs at the point R. The constant curves are for $\bar{\chi}^2$ $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$ above that minimum.

(b) The dashed lines show the extreme cases corresponding to points Q (maximizing the slope) and S (maximizing the y intercept). The best curve, corresponding to the $\bar{\chi}^2$ minimum R, is displayed as a solid line.

Figure A.1: Minimizing $\bar{\chi}^2$ for the dataset in Table A.3.

An Example, with Details

Consider the example data reproduced here (p. 87, Table A.3) from the 191 *Lab Manual.*

If we plug this data into our definition of reduced chi-square $\bar{\chi}^2$ (Eq. A.4) we find:

$$\bar{\chi}^2 = \frac{19945 - 734A + 8A^2 - 4006B + 72AB + 204B^2}{96} \tag{A.5}$$

(Each term in the sum is a quadratic form in A and B; sum those 8 quadratic forms and divide by $N - 2 = 6$ yields the above. It is a mess to confirm this by hand; I used *Mathematica.*) So reduced chi-square $\bar{\chi}^2$ is a simple quadratic function of A and B. We can find the minimum of this function by finding the point where the derivatives are zero:

$$\frac{\partial \bar{\chi}^2}{\partial A} = \frac{-734 + 16A + 72B}{96} = 0 \tag{A.6}$$

$$\frac{\partial \bar{\chi}^2}{\partial B} = \frac{-4006 + 72A + 408B}{96} = 0 \tag{A.7}$$

$$\tag{A.8}$$

The result is $(A, B) = (115/14, 703/84) \approx (8.21, 8.37)$.

Reduced chi-square $\bar{\chi}^2$ is a function of two variables much as the height of a mountain range is a function of the x, y location on the surface of the Earth. We can display such height information in the form of a topographical map, where we connect all the points

that are at the same altitude. In a similar way we can display the function $\bar{\chi}^2(A, B)$, by showing the collection of points where $\bar{\chi}^2(A, B)$ equals some constant. (See Figure A.1(a).) Since $\bar{\chi}^2(A, B)$ is a simple quadratic form the resulting constant curves are ellipses, and the 'topography' is quite simple with a single valley, oriented diagonally, with minimum at the point R. The uncertainly in the best fit parameters is determined by topography around that minimum: how much can a parameter vary before the resulting line is a detectably worst fit. The definition of 'detectably worst' is where the chi-square sum has increased by one unit above the minimum. The extreme points Q and S are displayed in the above figure along with the corresponding lines.

In more complicated situations, the chi-square topography can become much more complex, with several local valleys (which makes finding the global minimum more difficult). In the simplest cases (like that discussed above) the derivatives of chi-square are easily solved linear equations with a unique solution (hence just one valley). These cases are known as linear least squares.

You may be surprised at the relatively small range of variation implied by the parameter errors and displayed in Fig. A.1(b). Recall that the parameters are estimates of average behavior revealed by the assumed unbiased deviations. So much as the standard deviation of the mean is smaller than the standard deviation (and in fact can get arbitrarily small with sufficiently large data sets), so too the parameter uncertainties become increasingly small with large data sets. As a result the systematic errors must dominate the random errors for sufficiently large data sets, in which case the computer reported errors are not relevant.

Introduction to *WAPP*$^+$

There are many computer programs that can do least-squares fits. In this laboratory we will use the web-based program *WAPP*$^+$ located at:
`http://www.physics.csbsju.edu/stats/WAPP2.html`.
I suggest you add it to your favorites[2].

On the **opening page** of *WAPP*$^+$ you must report the format of your data, particularly for the uncertainty in your data:

- No y error

  Select this if your y data has no error (unlikely) or if you have no estimate for that error (rare) or if you have been told that the error is "negligible".

- Enter a formula for y error

  It is not uncommon to have a simple formula for your errors. In the simplest possible case your error may be a constant. Often you will know the error as a fraction (or percentage) of the y value. Very occasionally you may know a fairly complex formula for the error. (For example, in Physics 200, one error is calculated from: `(y^2+1)*pi/90`.) If you have such a formula, you may supply it to the program to

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[2]a Google for "wapp+" will provide a link or you can hit the statistics link on the Physics homepage: `www.physics.csbsju.edu`

calculate the errors for you. Alternatively you could use a spreadsheet or calculator to calculate those errors and just enter them along with your data.

- Enter y error for each data point

  Sometimes errors are estimated as part of the experiment by repeating the experiment and looking for deviations. In this case the errors are likely to be different for each data point and there will be no formula to apply.

Exactly analogous boxes must be checked to indicate the nature of x errors.

There are two ways of entering your data into the computer: copy & paste from a spreadsheet (the usual approach because using a spreadsheet is often required for other parts of the experiment) or entering each number into its own box on the web page. I would always opt for the former ("bulk" or copy & paste), but if you really want to you may use the latter ("pointwise data entry"). The following paragraphs assume $\boxed{\text{Do Bulk}}$ was selected.)

On the **second page** of $WAPP^+$ you must enter your data+errors and select which function applies to your data.

If you selected to enter errors with a formula, boxes to enter those formulae appear on the top of the page. For example, in Lab 1 you are told that the error in x is 0.5% and the error in y is given by the formula: $\delta\theta(1 + y^2)$, where $\delta\theta$ might be estimated as .035 rad, so you would enter:

y-error= $\boxed{\texttt{.035*(1+y\^{}2)}}$
x-error= $\boxed{\texttt{.005*x}}$

If you selected individualized errors, you will enter those errors along with the corresponding data. Two options are provided: you may do a "Block Copy & Paste" of all the data and errors (and perhaps columns to be ignored) or you may use the "Column Copy & Paste" area to separately paste each column of the requested data. (In either case the column headings should be a reminder of what you reported you were going to provide. E.g., if you are presented with a "X Errors" option or a "XE" column, you told the program on the previous page that you would supply individualized x-errors.)

I usually use the "Block Copy & Paste" pseudo spreadsheet. There $WAPP^+$ presents a big area to enter numbers with multiple-choice column headers:

column1	**column2**	**column3**	**column4**	**column5**	**column6**
None	None	None	None	None	None

Often I arrange my spreadsheet so that $x, \delta x, y, \delta y$ are in adjacent columns in which case I would select:

column1	**column2**	**column3**	**column4**	**column5**	**column6**
X	X Errors	Y	Y Errors	None	None

Sometimes you will have a column of irrelevant data between required data: no problem, just tell $WAPP^+$ to $\boxed{\text{Ignore}}$ the irrelevant column.

Note: The apparent grid of cells and columns in the "Block Copy & Paste" web-form,

in fact, have no meaning: it's just an image to remind you that this area will be filled by pasting data from a spreadsheet. The pasted data does not have to line up in columns (the white space between the numbers is what denotes column boundaries). You do need to assure that the number of numbers in each row matches the number of columns not selected as None. In addition, pasted data must be numbers not text (for example: column labels). Finally an extra Enter at the end of the last data item may avoid IE transfer problems.

If you decide to use "Column Copy & Paste" simply select the appropriate single column of values in your spreadsheet and paste it into the appropriate *WAPP*+ column. Typically several separate copy&paste moves will be needed to fill all the columns. Of course, each column must contain the data in the same order. (Note: you can also enter some data via "Block Copy & Paste" with the remaining data supplied via "Column Copy & Paste".)

Near the bottom of this page you select which function should fit the data.

When you Submit Data , the **third page** of *WAPP*+ arrives: this is your "fit report". Almost always you will want to select the top region of this page, print it out, and tape it into your notebook. Here is page generated by the data from Table A.3.

An analysis of data submitted by computer: linphys1.physics.csbsju.edu on 1-APR-2010 at 13:35 indicates that a function of the form:
—**Linear**— $y = A + Bx$
can fit the 8 data points with a reduced chi-squared of 1.7

```
              FIT
PARAMETER     VALUE         ERROR
   A =        8.214          3.1
   B =        8.369          0.62


NO x-errors


                  ACTUAL      ERROR IN        CALCULATED    DEVIATION
 POINT    X         Y            Y                Y         FROM FIT
   1     1.00      16.0     +-  4.0             16.6         -0.583
   2     2.00      18.0     +-  4.0             25.0         -6.95
   3     3.00      35.0     +-  4.0             33.3          1.68
   4     4.00      44.0     +-  4.0             41.7          2.31
   5     5.00      58.0     +-  4.0             50.1          7.94
   6     6.00      62.0     +-  4.0             58.4          3.57
   7     7.00      64.0     +-  4.0             66.8         -2.80
   8     8.00      70.0     +-  4.0             75.2         -5.17
```

Data Reference: 626A

It is perhaps worth reminding you that computers are usually ignorant of sigfigs and units: I would report these results as: $A = 8 \pm 3$ °C and $B = 8.4 \pm .6$ °C/cm. Alternatively $A = 8.2 \pm 3.1$ °C and $B = 8.37 \pm .62$ °C/cm is also OK; other possibilities are incorrect and will result in lost points.

The **Data Reference: 626A** at the bottom is useful and should be retained. If some problem (e.g., a computer crash) should occur, it is likely that this reference will allow you to get a

copy of your data from the web.

The bottom part of this page allows you to make various types of plots of your results. If you click on $\boxed{\text{Make Plot}}$ a fourth page pops up with links to the actual plots. Most commonly you will click on the PDF File, Adobe Acrobat will launch and your plot will be displayed, and if desired printed.

How to enter formulas: The usual syntax applies: $+ - $ * $/$ for add, subtract, multiply, and divide respectively; ˆ or ** for powers. Please note:

$$\mathtt{A/B*C} \;\; = \;\; \frac{AC}{B} \tag{A.9}$$

$$\mathtt{A/(B*C)} \;\; = \;\; \frac{A}{BC} \tag{A.10}$$

Mathematical functions for formulas:

ABS	ACOS	ASIN	ATAN
COS	COSH	ERF	ERFC
EXP	INT	LOG	LOG10
NINT	RAN	SCALE	SIGN
SIN	SINH	SQRT	TAN
TANH	GAMMA	K	NORM
INORM	PI		

Appendix B — Meter Uncertainty Specifications

DM-441B Digital Multimeter

Table B.4 reproduces the specifications for the DM-441B multimeter provided by its man-
ufacturer. Note that the accuracy depends on the function, and sometimes on the range.
In the table, the letters "dgt" stand for least significant digit — the right-most digit on the
display; "4 dgt" means 4× whatever a 1 in the right-hand digit (and zeros everywhere else)
would correspond to. (In the below table "Resolution" is the same thing as "1 dgt".) Note
that the value of 1 dgt always changes with the range, so you need to record both the range
and the value when you take data.

Table B.4: DM-441B Multimeter Resolution and Accuracy

Function	Range	Resolution	Accuracy
Resistance	200Ω	0.01Ω	±(2% + 5 dgt)
	2 kΩ	0.1Ω	±(0.2% + 2 dgt)
	20 kΩ	1Ω	
	200 kΩ	10Ω	
	2000 kΩ	100Ω	±(0.5% + 2 dgt)
	20 MΩ	1 kΩ	
DC Voltage	200 mV	10μV	±(0.1% + 4 dgt)
	2 V	100μV	
	20 V	1 mV	
	200 V	10 mV	
	1000 V	100 mV	±(0.15% + 4 dgt)
DC Current	2 mA	0.1μA	±(0.5% + 1 dgt)
	20 mA	1μA	
	200 mA	10μA	
	2000 mA	100μA	
	10 A	1 mA	±(0.75% + 3 dgt)
AC Voltage	200 mV	10 μV	±(0.5% + 20 dgt)
(45 Hz – 1 kHz)	2 V	100 μV	
	20 V	1 mV	
	200 V	10 mV	
	750 V	100 mV	±(1% + 20 dgt)

Examples: A resistance reading of 71.49 Ω (on the 200 Ω scale) has an uncertainty of:
$71.49 \times 2\% + 5$ dgt $= 1.43 + .05 = 1.48 \approx 1.5$ Ω. A DC voltage reading of 6.238 V (on the 20 V
scale) has an uncertainty of: $6.238 \times 0.1\% + 4$ dgt $= .0062 + .004 = .0102 \approx .010$ V. An AC
voltage reading of 6.238 V (on the 20 V scale) has an uncertainty of: $6.238 \times 0.5\% + 20$ dgt $=$
$.0312 + .020 = .0412 \approx .041$ V.

Wavetek Model 19 Function Generator

Display Accuracy

Frequency:	$\pm$ 1 digit on 2 kHz to 2 MHz ranges; < or = 1.5% of full scale on 2 Hz to 200 Hz ranges
Amplitude:	Typically 5% of range at 1 kHz.
DC offset:	Typically 2% of reading.
Resolution:	0.05% maximum on all ranges

Tektronix TDS 200-Series Digital Oscilloscope

Vertical Measurement Accuracy in Average Acquisition Mode (> 16 waveforms)

DC measurement with vertical position at zero	$\pm$ (4% $\times$ reading + 0.1 DIV + 1 mV)
DC measurement with vertical position not at zero	$\pm$ [3% $\times$ (reading + vertical position) + 1% of vertical position + 0.2 DIV]. Add 2 mV for settings from 2 mV/DIV to 200 mV/DIV. Add 50 mV for settings from > 200 mV/DIV to 5 V/DIV.
Delta volts measurement	$\pm$ (3% $\times$ reading + 0.05 DIV)

Horizontal Measurement Accuracy

Delta time measurement Single-shot sample mode	$\pm$ (1 sample interval* + .01% $\times$ reading + 0.6 ns)
Delta time measurement > 16 averages	$\pm$ (1 sample interval* + .01% $\times$ reading + 0.4 ns)

*Sample interval = (s/DIV)/250

Appendix C — Uncertainty Formulae

Appendix C—Uncertainty Formulae

In the below equations $a, b, c, \ldots$ have uncertainties $\delta a, \delta b, \delta c, \ldots$ whereas $K, k_1, k_2, \ldots$ are "constants" (like π or 2) with zero error or quantities with so small error that they can be treated as error-free (like the mass of a proton: $m_p = (1.67262158 \pm .00000013) \times 10^{-27}$ kg). This table reports the error in a calculated quantity R (which is assumed to be positive). Note that a quantity like δa is called an *absolute* error; whereas the quantity $\delta a / a$ is called the *relative* error (or, when multiplied by 100, the *percent* error). The odd Pythagorean-theorem-like addition (e.g., $\delta R = \sqrt{\delta a^2 + \delta b^2}$) is called "addition in quadrature". Thus the formula for the error in $R = K\, ab/cd$ could be stated as "the percent error in R is the sum of the percent errors in a, b, c and d added in quadrature".

Equation	Uncertainty	
$R = a + K$	$\delta R = \delta a$	(C.1)
$R = K\, a$	$\dfrac{\delta R}{R} = \dfrac{\delta a}{|a|} \quad \text{or} \quad \delta R = |K|\, \delta a$	(C.2)
$R = \dfrac{K}{a}$	$\dfrac{\delta R}{R} = \dfrac{\delta a}{|a|} \quad \text{or} \quad \delta R = \dfrac{|K|\, \delta a}{a^2}$	(C.3)
$R = K\, a^{k_1}$	$\dfrac{\delta R}{R} = \dfrac{|k_1|\, \delta a}{|a|} \quad \text{or} \quad \delta R = |k_1 K a^{k_1 - 1}|\, \delta a$	(C.4)
$R = a \pm b$	$\delta R = \sqrt{\delta a^2 + \delta b^2}$	(C.5)
$R = k_1 a + k_2 b$	$\delta R = \sqrt{(k_1 \delta a)^2 + (k_2 \delta b)^2}$	(C.6)
$R = K\, ab$	$\dfrac{\delta R}{R} = \sqrt{\left(\dfrac{\delta a}{a}\right)^2 + \left(\dfrac{\delta b}{b}\right)^2}$	(C.7)
$R = K\, \dfrac{a}{b}$	$\dfrac{\delta R}{R} = \sqrt{\left(\dfrac{\delta a}{a}\right)^2 + \left(\dfrac{\delta b}{b}\right)^2}$	(C.8)
$R = f(a)$	$\delta R = |f'(a)|\, \delta a$	(C.9)
$R = K\, \dfrac{ab}{cd}$	$\dfrac{\delta R}{R} = \sqrt{\left(\dfrac{\delta a}{a}\right)^2 + \left(\dfrac{\delta b}{b}\right)^2 + \left(\dfrac{\delta c}{c}\right)^2 + \left(\dfrac{\delta d}{d}\right)^2}$	(C.10)
$R = K\, \dfrac{a^{k_1} b^{k_2}}{c^{k_3} d^{k_4}}$	$\dfrac{\delta R}{R} = \sqrt{\left(\dfrac{k_1 \delta a}{a}\right)^2 + \left(\dfrac{k_2 \delta b}{b}\right)^2 + \left(\dfrac{k_3 \delta c}{c}\right)^2 + \left(\dfrac{k_4 \delta d}{d}\right)^2}$	(C.11)
$R = f(a, b, c, d)$	$\delta R = \sqrt{\left(\dfrac{\partial f}{\partial a}\, \delta a\right)^2 + \left(\dfrac{\partial f}{\partial b}\, \delta b\right)^2 + \left(\dfrac{\partial f}{\partial c}\, \delta c\right)^2 + \left(\dfrac{\partial f}{\partial d}\, \delta d\right)^2}$	(C.12)