Primordial Black Holes: Tunnelling vs. No Boundary Proposal*

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Abstract

In the inflationary era, black holes came into existence together with the universe through the quantum process of pair creation. We calculate the pair creation rate from the no boundary proposal for the wave function of the universe. Our results are physically sensible and fit in with other descriptions of pair creation. The tunnelling proposal, on the other hand, predicts a catastrophic instability of de Sitter space to the nucleation of large black holes, and cannot be maintained.

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1 Introduction

1.1 Primordial Black Holes

We now have good observational evidence for black holes from stellar masses up to super-massive holes of 10^8 to 10^{10} solar masses and maybe even more. However, one can also speculate on the possible existence of black holes of much lower mass. These are the holes for which quantum effects can be important. Such holes could not form from the collapse of normal baryonic matter because degeneracy pressure will support white dwarfs or neutron stars below the Chandrasekhar limiting mass. One can express this limiting mass as $m_{\text{Planck}}(m_{\text{Planck}}/m_{\text{baryon}})^2$. Its value is about a solar mass, which might seem a coincidence, but there are good anthropic principle reasons why stars should be just on the verge of gravitational collapse.

This limiting mass applies only to the formation of black holes through the gravitational collapse of fermions. In the case of bosons the limiting mass is given by $m_{\text{Planck}}(m_{\text{Planck}}/m_{\text{boson}})$. To form a black hole by the gravitational collapse of bosons, they need to have a non-zero mass and either be stable or have a fairly long life. About the only candidate is the axion, which might have a mass of about 10^{-5}eV . In this case the limiting mass would be about the mass of the Earth, which is still quite high, and too large for quantum effects to be observable. To get black holes that are significantly smaller, one could not rely on gravitational collapse, but would have to shoot matter together with high energies. John Wheeler once calculated that if one made a hydrogen bomb with all the deuterium from the oceans, the centre would implode so violently that a little black hole would be formed. Perhaps fortunately, this experiment is unlikely to be performed. Thus the only place where tiny black holes might be formed is the early universe.

Previous discussions of black holes formed in the early universe have concentrated on black holes formed by matter coming together during the radiation era or first order phase transitions. Recent work on the critical behaviour of gravitational collapse has shown it is possible to form black holes in these situations. However, it is difficult because one has to arrange for matter to be fired together at high speed and accurately focused into a small region. Yet if too much matter is fired together it forms a closed universe on its own, with no connection with our universe. Such a separate universe would not be a black hole in our universe.

Black holes formed by collapse, or by hurling matter together, are not really primordial, in the sense that they do not form until a definite time after the beginning of the universe. On the other hand, the black holes we are going to consider form

by the quantum process of pair creation and are truly primordial, in that they can be considered to have existed since the beginning of the universe.

1.2 Inflation

It is generally assumed that the universe began with a period of exponential expansion called inflation. This era is characterised by the presence of an effective cosmological constant $\Lambda_{\rm eff}$ due to the vacuum energy of a scalar field ϕ . In chaotic inflation [1, 2] the effective cosmological constant typically starts out large and then decreases slowly until inflation ends when $\Lambda_{\rm eff} \approx 0$. Correspondingly, these models predict cosmic density perturbations which are proportional to the logarithm of the scale. On scales up to the current Hubble radius $H_{\rm now}^{-1}$, this agrees well with observations of near scale invariance. However, on much larger length scales of order $H_{\rm now}^{-1} \exp(10^5)$, perturbations are predicted to be on the order of one. Of course, this means that the perturbational treatment breaks down; but it indicates that black holes may be created.

Linde [3, 4] noted that in the early stages of inflation, when the strong density perturbations originate, the quantum fluctuations of the inflaton field are much larger than its classical decrease per Hubble time. He concluded that therefore there would always be regions of the inflationary universe where the field would grow, and so inflation would never end globally ("eternal inflation"). However, this approach only allows for fluctuations of the field. One should also consider fluctuations which change the topology of space-time. This topology change corresponds to the formation of a pair of black holes. The pair creation rate can be calculated using instanton methods, which are well suited to this non-perturbative problem.

1.3 Pair Creation

Quantum pair creation is only possible on a background that provides a force which pulls the pair apart. In the case of a virtual electron-positron pair, for example, the particles can only become real if they appear in an external electric field. Otherwise they would just fall back together and annihilate. The same holds for black holes; examples in the literature include their pair creation on a cosmic string [5], where they are pulled apart by the string tension; or the pair creation of magnetically charged black holes on the background of Melvin's universe [6], where they are separated by a magnetic field. In our case, the black holes will be accelerated apart

by the inflationary expansion of the universe. While preventing classical gravitational collapse, this expansion provides a suitable background for the quantum pair creation of black holes.

After the end of inflation, during the radiation and matter dominated eras, the effective cosmological constant was nearly zero. Thus the only time when black hole pair creation was possible in our universe was during the inflationary era, when $\Lambda_{\rm eff}$ was large. Moreover, these black holes are unique since they can be so small that quantum effects on their evolution are important. Indeed, their evolution turns out to be quite interesting and non-trivial [7]. Here we will only describe the creation of black holes, summarising a more rigourous treatment [8]. We focus on the consequences for the choice of the prescription for the wave function of the universe.

In the standard semi-classical treatment of pair creation, one finds two instantons: one for the background, and one for the objects to be created on the background. From the instanton actions $I_{\rm bg}$ and $I_{\rm obj}$ one calculates the pair creation rate Γ :

$$\Gamma = \exp\left[-\left(I_{\text{obj}} - I_{\text{bg}}\right)\right],\tag{1.1}$$

where we neglect a prefactor. This prescription has been very successfully used by a number of authors recently [9, 10, 11, 12] for the pair creation of black holes on various backgrounds. It is motivated not only by analogies in quantum mechanics and quantum field theory [13, 14], but also by considerations of black hole entropy [15, 16, 17].

In this paper, however, we will obtain the pair creation rate through a somewhat more fundamental procedure. Since we have a cosmological background, we can apply the tools of quantum cosmology, and use the wave function of the universe to describe black hole pair creation. Two different prescriptions have been put forward for the calculation of this wave function: Vilenkin's tunnelling proposal [18], and the Hartle-Hawking no boundary proposal [19] (reviewed in Sec. 2). We will describe the creation of an inflationary universe by a de Sitter type gravitational instanton, which has the topology of a four-sphere, S^4 . In this picture, the universe starts out with the spatial size of one Hubble volume. After one Hubble time, its spatial volume will have increased by a factor of $e^3 \approx 20$. However, by the de Sitter no hair theorem, we can regard each of these 20 Hubble volumes as having been nucleated independently through gravitational instantons. With this interpretation, we are allowing for black hole pair creation, since some of the new Hubble volumes might have been created through a different type of instanton that has the topology $S^2 \times S^2$ and thus represents a pair of black holes in de Sitter space [20]. Using the

no boundary proposal, we assign probability measures to both instanton types. We then estimate the fraction of inflationary Hubble volumes containing a pair of black holes by the fraction Γ of the two probability measures. This is equivalent to saying that Γ is the pair creation rate of black holes on a de Sitter background.

In Sec. 3 we describe the relevant instantons and calculate the pair creation rate. The result is compared with that obtained from the tunnelling proposal in Sec. 4, where we demonstrate that the usual description of pair creation, Eq. (1.1), arises naturally from the no boundary proposal. We shall use units in which $m_P = \hbar = c = k = 1$.

2 The Wave Function of the Universe

The prescription for the wave function of the universe has long been one of the central, and arguably one of the most disputed issues in quantum cosmology. The two competing proposals differ in their choice of boundary conditions for the wave function.

2.1 No Boundary Proposal

According to the no boundary proposal, the quantum state of the universe is defined by path integrals over Euclidean metrics $g_{\mu\nu}$ on compact manifolds M. From this it follows that the probability of finding a three-metric h_{ij} on a spacelike surface Σ is given by a path integral over all $g_{\mu\nu}$ on M that agree with h_{ij} on Σ . If the spacetime is simply connected (which we shall assume), the surface Σ will divide M into two parts, M_+ and M_- . One can then factorise the probability of finding h_{ij} into a product of two wave functions, Ψ_+ and Ψ_- . Ψ_+ (Ψ_-) is given by a path integral over all metrics $g_{\mu\nu}$ on the half-manifold M_+ (M_-) which agree with h_{ij} on the boundary Σ . In most situations Ψ_+ equals Ψ_- . We shall therefore drop the suffixes and refer to Ψ as the wave function of the universe. Under inclusion of matter fields, one arrives at the following prescription:

$$\Psi[h_{ij}, \Phi_{\Sigma}] = \int D(g_{\mu\nu}, \Phi) \exp\left[-I(g_{\mu\nu}, \Phi)\right], \qquad (2.1)$$

where (h_{ij}, Φ_{Σ}) are the 3-metric and matter fields on a spacelike boundary Σ and the path integral is taken over all compact Euclidean four geometries $g_{\mu\nu}$ that have Σ as their only boundary and matter field configurations Φ that are regular on them;

 $I(g_{\mu\nu},\Phi)$ is their action. The gravitational part of the action is given by

$$I_E = -\frac{1}{16\pi} \int_{M_+} d^4x \, g^{1/2} (R - 2\Lambda) - \frac{1}{8\pi} \int_{\Sigma} d^3x \, h^{1/2} K, \tag{2.2}$$

where R is the Ricci-scalar, Λ is the cosmological constant, and K is the trace of K_{ij} , the second fundamental form of the boundary Σ in the metric g.

We shall calculate the wave function semi-classically, using a saddle-point approximation to the path integral; and from the wave function we shall calculate the pair creation rate. The method can be outlined as follows. One is interested in two types of inflationary universes: one with a pair of black holes, and one without. They are characterised by spacelike sections of different topology. For each of these two universes, one has to find a classical Euclidean solution to the Einstein equations (an instanton), which can be analytically continued to match a boundary Σ of the appropriate topology. One then calculate the Euclidean actions I of the two types of saddle-point solutions. Semiclassically, it follows from Eq. (2.1) that the wave function is given by

$$\Psi = \exp\left(-I\right),\tag{2.3}$$

neglecting a prefactor. One can thus assign a probability measure to each type of universe:

$$P = |\Psi|^2 = \exp\left(-2I^{\text{Re}}\right),\tag{2.4}$$

where the superscript 'Re' denotes the real part. As explained in the introduction, the ratio of the two probability measures gives the rate of black hole pair creation on an inflationary background, Γ .

The probability measure P for the nucleation of a space-time should be proportional to the number of possible quantum states it contains, e^S . The entropy S of a space-time is given by the total of its horizon areas, divided by four; it follows that $S = -2I^{\text{Re}}$ in the cosmological case [17]. So Eq. (2.4) above does indeed reflect the number of internal states. If the black hole space-time has lower entropy than the background, one obtains $\Gamma < 1$. Then the pair creation will be suppressed, as it should be.

2.2 Tunnelling Proposal

The tunnelling proposal places different boundary conditions on the wave function at small geometries in the Euclidean region.

The action (2.2) is in general negative for a small boundary geometry h_{ij} . Thus $\Psi = e^{-I}$ is enhanced. The proponents of the tunnelling proposal feel, however, that the wave function ought to be suppressed in the Euclidean region because it is supposed to be forbidden. They are therefore forced to choose the

$$\Psi_{\rm TP} = \exp\left(+I\right) \tag{2.5}$$

solution of the Wheeler-DeWitt equation as the boundary condition at small h_{ij} . This has the obvious disadvantage that it does not reflect the entropy difference correctly. Transitions in the direction of lower entropy are enhanced, rather than suppressed. This will lead to absurd predictions in the context of pair creation.

In the following two sections we shall discuss the saddle-point solutions needed to describe the pair creation of black holes on a cosmological background [8]. We shall use only the no boundary proposal to calculate the probability measures and the pair creation rate. The disastrous consequences of choosing the prescription (2.5), instead, will be discussed in Sec. 4.

3 Instantons

We shall assume spherical symmetry. Before we introduce a more realistic inflationary model, it is helpful to consider a simpler situation with a fixed positive cosmological constant Λ but no matter fields. We can then generalise quite easily to the case where an effective cosmological "constant" arises from a scalar field.

3.1 de Sitter Space

First we consider the case without black holes, a homogeneous isotropic universe. Since $\Lambda > 0$, its spacelike sections will simply be round three-spheres. The wave function is given by a path integral over all metrics on a four-manifold M_+ bounded by a round three-sphere Σ of radius a_{Σ} . The corresponding saddle-point solution is the de Sitter space-time. Its Euclidean metric is that of a round four-sphere of radius $\sqrt{3/\Lambda}$:

$$ds^{2} = d\tau^{2} + a(\tau)^{2} d\Omega_{3}^{2}, \tag{3.1}$$

where τ is Euclidean time, $d\Omega_3^2$ is the metric on the round three-sphere of unit radius, and

$$a(\tau) = \sqrt{\frac{3}{\Lambda}} \sin \sqrt{\frac{\Lambda}{3}} \tau. \tag{3.2}$$

We can regard Eq. (3.2) as a function on the complex τ -plane. On a line parallel to the imaginary τ -axis defined by $\tau^{\text{Re}} = \sqrt{\frac{3}{\Lambda}} \frac{\pi}{2}$, we have

$$a(\tau)|_{\tau^{\text{Re}} = \sqrt{\frac{3}{\Lambda}} \frac{\pi}{2}} = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}} \tau^{\text{Im}}.$$
 (3.3)

This describes a Lorentzian de Sitter hyperboloid, with $\tau^{\rm Im}$ serving as a Lorentzian time variable. One can thus construct a complex solution, which is the analytical continuation of the Euclidean four-sphere metric. It is obtained by choosing a contour in the complex τ -plane from 0 to $\tau^{\rm Re} = \sqrt{\frac{3}{\Lambda}} \frac{\pi}{2}$ (which describes half of the Euclidean four-sphere) and then parallel to the imaginary τ -axis (which describes half the Lorentzian hyperboloid). The geometry corresponding to this path is shown in (Fig. 1).

The Lorentzian part of the metric will contribute a purely imaginary term to the action. This will affect the phase of the wave function but not its amplitude. The real part of the action of this complex saddle-point metric will be the action of the Euclidean half-four-sphere:

$$I_{\text{de Sitter}}^{\text{Re}} = -\frac{3\pi}{2\Lambda}.$$
 (3.4)

Thus the magnitude of the wave function will still be $e^{3\pi/2\Lambda}$, corresponding to the probability measure

$$P_{\text{de Sitter}} = \exp\left(\frac{3\pi}{\Lambda}\right).$$
 (3.5)

3.2 Schwarzschild-de Sitter Space

We turn to the case of a universe containing a pair of black holes. Now the cross sections Σ have topology $S^2 \times S^1$. Generally, the radius of the S^2 varies along the S^1 . This corresponds to the fact that the radius of a black hole immersed in de Sitter space can have any value between zero and the radius of the cosmological horizon. The minimal two-sphere corresponds to the black hole horizon, the maximal two-sphere to the cosmological horizon. The saddle-point solution corresponding to this topology is the Schwarzschild-de Sitter universe. However, the Euclidean section of this spacetime typically has a conical singularity at one of its two horizons and thus does not represent a regular instanton [7, 20]. The only regular Euclidean solution is the degenerate case where the black hole has the maximum possible size. It is

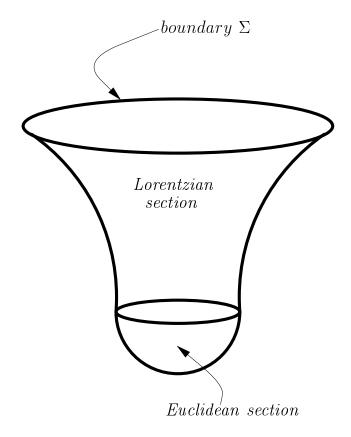


Figure 1: The creation of a de Sitter universe. The lower region is half of a Euclidean four—sphere, embedded in five—dimensional Euclidean flat space. The upper region is a Lorentzian four—hyperboloid, embedded in five—dimensional Minkowski space.

also known as the Nariai solution and given by the topological product of two round two-spheres:

$$ds^{2} = d\tau^{2} + a(\tau)^{2} dx^{2} + b(\tau)^{2} d\Omega_{2}^{2}, \tag{3.6}$$

where x is identified with period 2π , $d\Omega_2^2 = d\theta^2 + \sin^2\theta \ d\varphi^2$, and

$$a(\tau) = \sqrt{\frac{1}{\Lambda}} \sin \sqrt{\Lambda} \tau, \ b(\tau) = \sqrt{\frac{1}{\Lambda}} = \text{const.}$$
 (3.7)

In this case the radius b of the S^2 is constant in the S^1 direction. The black hole and the cosmological horizon have equal radius and no conical singularities are present. There will be no saddle-point solution unless we specify $b_{\Sigma} = 1/\sqrt{\Lambda}$. Then

the only variable we are free to choose on Σ is the radius a_{Σ} of the one-sphere. In the Lorentzian section, the one-sphere expands rapidly,

$$a(\tau)|_{\tau^{\text{Re}} = \sqrt{\frac{1}{\Lambda}} \frac{\pi}{2}} = \sqrt{\frac{1}{\Lambda}} \cosh \sqrt{\Lambda} \tau^{\text{Im}},$$
 (3.8)

while the two-sphere (and, therefore, the black hole radius) remains constant. Again we can construct a complex saddle-point, which can be regarded as half a Euclidean $S^2 \times S^2$ joined to half of the Lorentzian solution. The real part of the action will be the action of the half of a Euclidean $S^2 \times S^2$:

$$I_{\rm SdS}^{\rm Re} = -\frac{\pi}{\Lambda}.\tag{3.9}$$

The corresponding probability measure is

$$P_{\rm SdS} = \exp\left(\frac{2\pi}{\Lambda}\right). \tag{3.10}$$

We divide this by the probability measure (3.5) for a universe without black holes to obtain the pair creation rate of black holes in de Sitter space:

$$\Gamma = \frac{P_{\text{SdS}}}{P_{\text{de Sitter}}} = \exp\left(-\frac{\pi}{\Lambda}\right).$$
 (3.11)

Thus the probability for pair creation is very low, unless Λ is close to the Planck value, $\Lambda = 1$.

3.3 Effective Cosmological Constant

Of course the real universe does not have a large cosmological constant. However, in inflationary cosmology it is assumed that the universe starts out with a very large effective cosmological constant, which arises from the potential V of a scalar field ϕ . The exact form of the potential is not critical. So for simplicity we chose V to be the potential of a field with mass m, but the results would be similar for a $\lambda \phi^4$ potential. To account for the observed fluctuations in the microwave background [21], m has to be on the order of 10^{-5} to 10^{-6} [22]. The wave function Ψ will now depend on the three-metric h_{ij} and the value of ϕ on Σ . For $\phi > 1$ the potential acts like an effective cosmological constant

$$\Lambda_{\text{eff}}(\phi) = 8\pi V(\phi). \tag{3.12}$$

There will again be complex saddle-points which can be regarded as a Euclidean solution joined to a Lorentzian solution. Due to the time dependence of $\Lambda_{\rm eff}$, however, one cannot find a path in the τ -plane along which the Euclidean and Lorentzian metrics will be exactly real [8]. Apart from this subtlety, the saddle point solutions are similar to those for a fixed cosmological constant, with the time-dependent $\Lambda_{\rm eff}$ replacing Λ . The radius of the pair created black holes will now be given by $1/\sqrt{\Lambda_{\rm eff}}$. As before, the magnitude of the wave function comes from the real part of the action, which is determined by the Euclidean part of the metric. This real part will be

$$I_{S^3}^{\text{Re}} = -\frac{3\pi}{2\Lambda_{\text{eff}}(\phi_0)} \tag{3.13}$$

in the case without black holes, and

$$I_{S^2 \times S^1}^{\text{Re}} = -\frac{\pi}{\Lambda_{\text{eff}}(\phi_0)} \tag{3.14}$$

in the case with a black hole pair. Here ϕ_0 is the value of ϕ in the initial Euclidean region. Thus the pair creation rate is given by

$$\Gamma = \frac{P_{S^2 \times S^1}}{P_{S^3}} = \exp\left[-\frac{\pi}{\Lambda_{\text{eff}}(\phi_0)}\right]. \tag{3.15}$$

4 Tunnelling vs. No Boundary Proposal

In the previous sections we have used the no boundary proposal to calculate the pair creation rate of black holes during inflation. Let us interpret the result, Eq. (3.15). Since $0 < \Lambda_{\rm eff} \le 1$, we get $\Gamma < 1$, and so black hole pair creation is suppressed. In the early stages of inflation, when $\Lambda_{\rm eff} \approx 1$, the suppression is week, and black holes will be plentifully produced . However, those black holes will be very small, with a mass on the order of the Planck mass. Larger black holes, corresponding to lower values of $\Lambda_{\rm eff}$ at later stages of inflation, are exponentially suppressed. A detailed analysis of their evolution [7] shows that the small black holes typically evaporate immediately, while sufficiently large ones grow with the horizon and survive long after inflation ends.

We now understand how the standard prescription for pair creation, Eq. (1.1), arises from the no boundary proposal: By Eq. (2.4),

$$\Gamma = \frac{P_{S^2 \times S^1}}{P_{S^3}} = \exp\left[-\left(2I_{S^2 \times S^1}^{\text{Re}} - 2I_{S^3}^{\text{Re}}\right)\right],\tag{4.1}$$

where I^{Re} denotes the real part of the Euclidean action of a complex saddle-point solution. But we have seen that this real part is equal to half of the action of the complete Euclidean solution. Thus $I_{\text{obj}} = 2I_{S^2 \times S^1}^{\text{Re}}$ and $I_{\text{bg}} = 2I_{S^3}^{\text{Re}}$, and we recover Eq. (1.1).

Let us return to the tunnelling proposal and see what results it would have produced. $\Psi_{\rm TP}$ is given by e^{+I} rather than e^{-I} . This choice of sign is inconsistent with Eq. (1.1), as it leads to the inverse result for the pair creation rate: $\Gamma_{\rm TP} = 1/\Gamma$. In our case, we would get $\Gamma_{\rm TP} = \exp(+\pi/\Lambda_{\rm eff})$. Thus black hole pair creation would be enhanced, rather than suppressed. This means that de Sitter space would decay: it would be catastrophically unstable to the formation of black holes. Since the radius of the black holes is given by $1/\sqrt{\Lambda_{\rm eff}}$, the black holes would be more likely the larger they were. Clearly, the tunnelling proposal cannot be maintained. On the other hand, Eq. (3.15), which was obtained from the no boundary proposal, is physically very reasonable. It allows topological fluctuations near the Planckian regime, but suppresses the formation of large black holes at low energies.

We summarise. The cosmological pair production of black holes provides an ideal theoretical laboratory in which to examine the question of the boundary conditions for the wave function of the universe. The results could not be more decisive. The no boundary proposal leads to physically sensible results, while the tunnelling proposal predicts a disastrous enhancement of black hole production.

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