## Comment on 'Quantum Creation of an Open Universe', by Andrei Linde

S.W. Hawking\* and Neil Turok $^{\dagger}$  DAMTP, Silver St, Cambridge, CB3 9EW, U.K. (February 7, 2008)

## Abstract

We comment on Linde's claim that one should change the sign in the action for a Euclidean instanton in quantum cosmology, resulting in the formula  $P \sim e^{+S}$  for the probability of various classical universes. There are serious problems with doing so. If one reverses the sign of the action of both the instanton and the fluctuations, the latter are unsupressed and the calculation becomes meaningless. So for a sensible result one would have to reverse the sign of the action for the background, while leaving the sign for the perturbations fixed by the usual Wick rotation. The problem with this approach is that there is no invariant way to split a given four geometry into background plus perturbations. So the prescription would have to violate general coordinate invariance. There are other indications that a sign change is problematic. With the choice  $P \sim e^{+S}$  the nucleation of primordial black holes during inflation is unsuppressed, with a disastrous resulting cosmology. We regard these as compelling arguments for adhering to the usual sign given by the Wick rotation.

In a recent letter, we pointed out the existence of new finite action instanton solutions describing the birth of open inflationary universes according to the Hartle-Hawking no boundary proposal. Linde has written a response in which he claims that the Hartle-Hawking calculation of the probability for classical universes is wrong, and that the expression

$$P \sim e^{-S_E(i)} \tag{1}$$

for the probability P in terms of the Euclidean action for the instanton solution  $S_E(i)$  should be replaced by

$$P \sim e^{+S_E(i)}. (2)$$

<sup>\*</sup>email:S.W.Hawking@damtp.cam.ac.uk

<sup>†</sup>email:N.G.Turok@damtp.cam.ac.uk

Since the Euclidean action is very large and negative ( $S_E(i) \sim -10^8$  typically) for solutions of the type we describe, the difference between these two formulae is extremely significant. What hope for theory if we cannot resolve disagreements of this order!

Let us explain where these formulae come from. One starts from the full Lorentzian path integral for quantum gravity coupled to a scalar field,

$$\int [dg][d\phi]e^{iS[g,\phi]} \tag{3}$$

which in principle defines all correlation functions of physical observables. Unfortunately the integrand is highly oscillatory for large field values, and an additional prescription is needed to evaluate it. The prescription suggested by Hartle and Hawking was to perform the the analytic continuation to Euclidean time,  $t_E = it$ , and to continue the metric to a compact Euclidean metric. The sign of the Wick rotation that is involved is fixed by the requirement that non-gravitational physics be correctly reproduced, because the other sign would produce an action for non-gravitational field fluctuations that was unbounded below. So (3) becomes

$$\int [dg][d\phi]e^{-S_E[g,\phi]}. (4)$$

Having performed the Wick rotation, we now try to evaluate it. The only way we know how to do this is to use the saddle point method. That is we find a stationary point of the action, i.e. a solution to the classical Euclidean equations, and expand around it. We obtain

$$S_E \approx S_0 + S_2 + \dots \tag{5}$$

where  $S_0 = S_E(i)$  is the action of the classical solution (the instanton) and  $S_2$  is the action for the fluctuations. One computes the fluctuations by performing the Gaussian integral with the measure  $\exp(-S_2)$ . It is very important that  $S_2$  is positive so that the fluctuations about the background classical solution are suppressed. As is well known, the Euclidean action for gravity alone is not positive definite, so the positivity of  $S_2$  is not guaranteed, and has to be checked for the particular classical background in question. In the inflationary example  $S_2$  is known to be positive [3]. Physically this corresponds to the fact that the classical background is not gravitationally unstable.

Let us turn to Linde's paper. He would like to reverse the sign in the exponent, turning (2) into (1). This is because the Euclidean action for the instanton  $S_E(i) \sim -M_{Pl}^4/V(\phi_0)$  where  $M_{Pl}$  is the Planck mass and  $\phi_0$  the initial value of the scalar field. Values of the scalar field giving small values for the potential  $V(\phi_0)$  give a large negative action, and are thus favoured. Obviously, changing the sign of the action will instead mean that these are strongly disfavoured, and make large initial values of the scalar field more likely. Whilst this improves the prospects for obtaining large amounts of inflation, we do not believe the sign change is tenable. If one treats background and perturbations together, a change in the sign of  $S_0 = S_E(i)$  is accompanied by a change in the sign of  $S_2$ . But this is disastrous - the fluctuations are left unsuppressed and the description of the spacetime as a classical background with small fluctuations breaks down.

One could try to treat background and fluctuations separately, by performing Wick rotations of the opposite sign on them. However the problem is that there is no coordinate invariant way to separate the two. So any such prescription would have to violate general coordinate invariance.

Problems occur with changing the sign of the action in nonperturbative contexts too. For example, calculations by Bousso and one of us [4] have shown that if one adopted Linde's prescription the creation of universes with large numbers of black holes would have been favoured and their mass would have dominated the energy density, leaving the universe without a radiation dominated era.

Linde gives another, intuitive, argument against using the standard sign for the Euclidean action. He argues that the entropy S of de Sitter space is given by a quarter of its horizon area. This quantity is accurately approximated by  $S \approx -S_E(i)$ , the negative of the action for the Euclidean instanton. He then argues that "it seems natural to expect that the emergence of a complicated object of large entropy must be suppressed by  $\exp(-S)$ ". We find this hard to understand. The formula probability  $\propto \exp(+S)$  is the foundation of statistical physics. Likewise if one pictures the formation of the universe as the endpoint of some process, the rate is proportional to the phase space available in the final state, again given by  $\exp(+S)$ . His intuitive argument seems to us to support rather than contradict the sign we have adopted.

In summary, changing the sign of the Euclidean action is not something one can do without negative repercussions. If the sign happens to disfavour large amounts of inflation, we prefer to face up to that problem, as in [1]. Possible solutions include a) accepting that we live in a universe on the tail of the distribution, possibly for anthropic reasons or b) exploring open inflationary continuations of the type we proposed in the context of more fundamental theories of quantum gravity, such as supergravity or M-theory, to see whether large amounts of inflation are favoured for other reasons (one candidate such mechanism was mentioned in [1]).

## REFERENCES

- [1] S.W. Hawking and N. Turok, hep-th/9802030, Phys. Lett. **B** in press (1998).
- [2] A. Linde, gr-qc/9802038.
- [3] See e.g. S. Mukhanov, H. Feldman and R. Brandenberger, Phys. Rep. 215, 203 (1992).
- [4] R. Bousso and S.W. Hawking, Phys. Rev. **D54**, 6312 (1996).