

# Fundamentals of Deep Learning of Representations

Tel-Aviv University  
Deep Learning Master Class

UPCOMING MIT PRESS BOOK  
DRAFT CHAPTERS AVAILABLE  
ON MY WEB PAGE.

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November 7, 2014, Tel-Aviv



# Ultimate Goal

- **Understand the principles giving rise to intelligence**

# Focus

- **Learning:** mathematical and computational principles allowing one to learn from examples in order to acquire knowledge

# Breakthrough

- **Deep Learning:** machine learning algorithms inspired by brains, based on learning multiple levels of representation / abstraction.

# Impact

**Deep learning has revolutionized**

- **Speech recognition**
- **Object recognition**

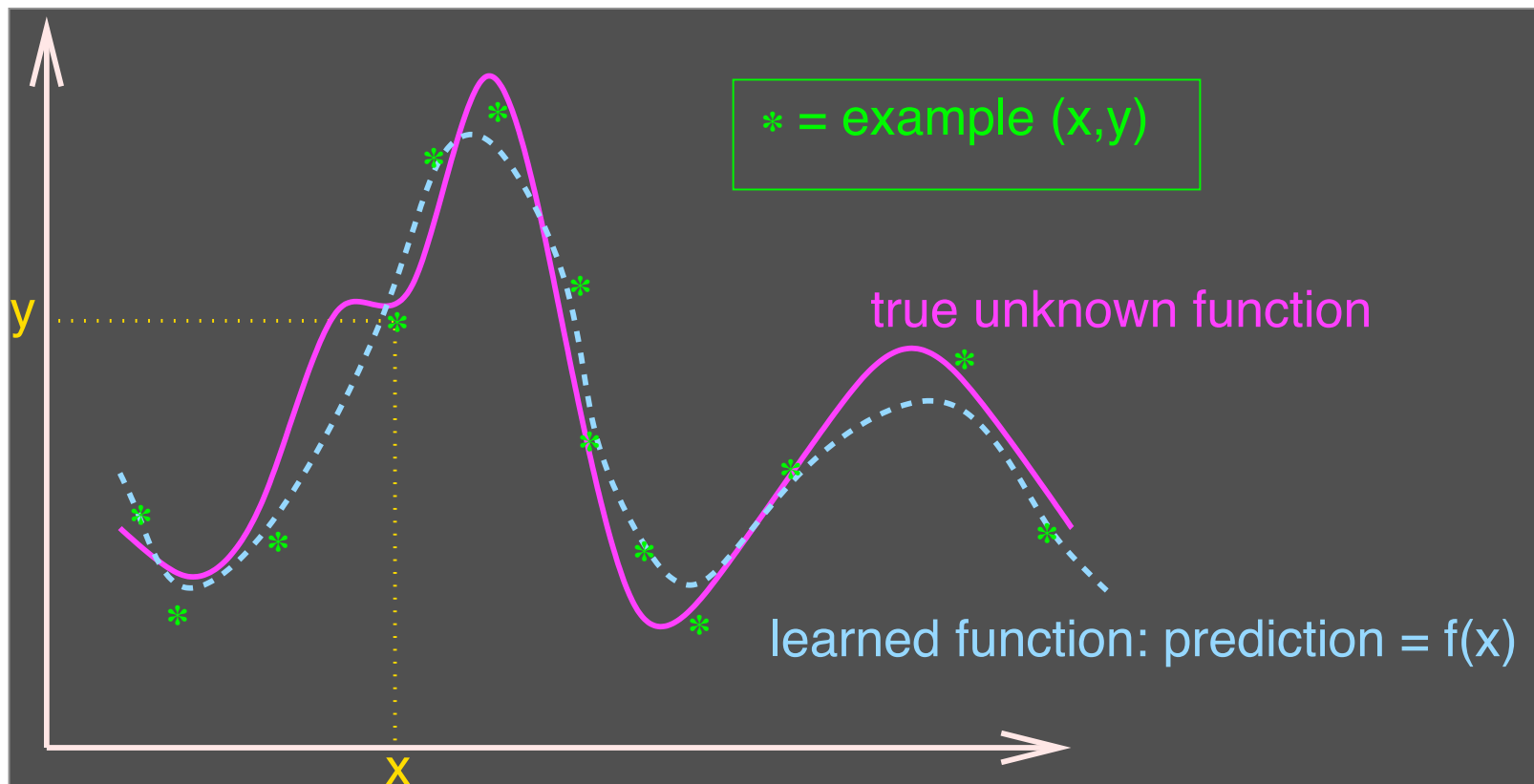
**More coming,** including other areas of computer vision, NLP, machine translation, dialogue, reinforcement learning...

# Technical Goals Hierarchy

## To reach AI:

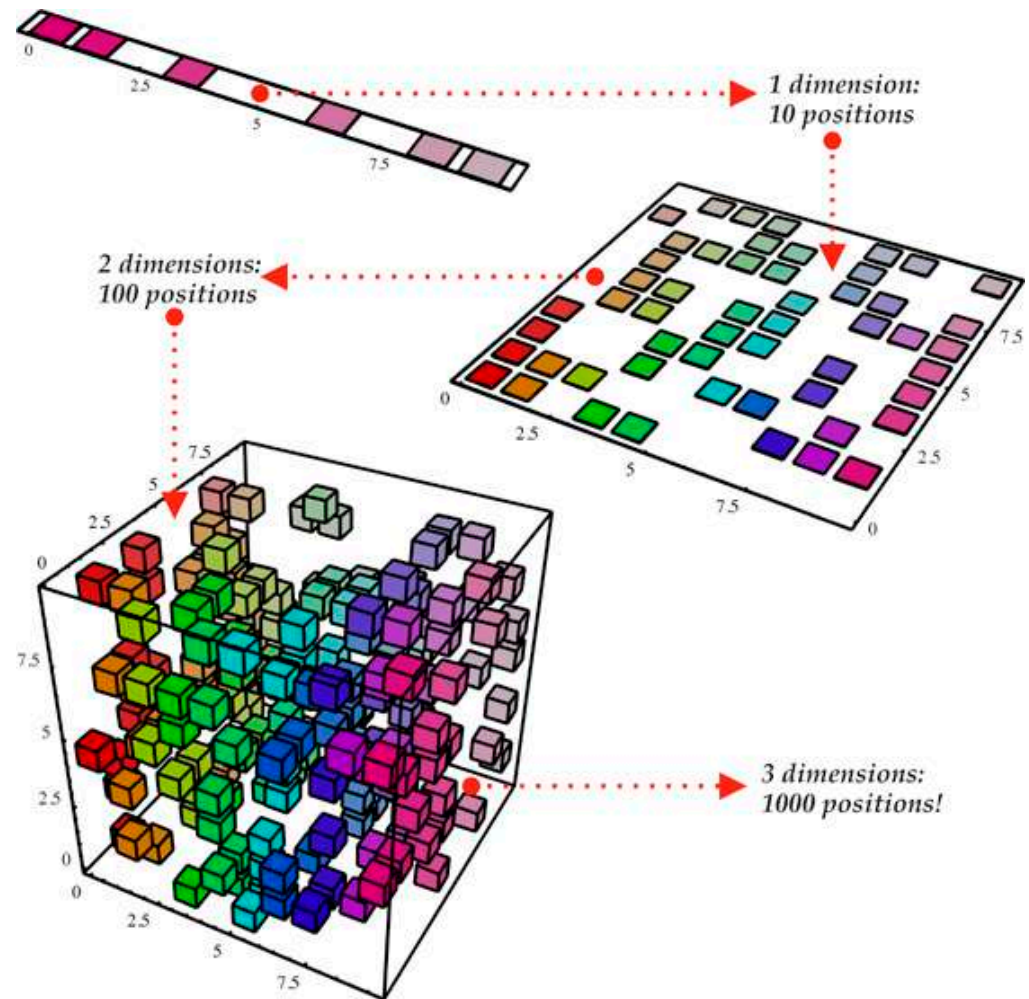
- Needs **knowledge**
- Needs **learning**  
(involves priors + *optimization/search* + *efficient computation*)
- Needs **generalization**  
(guessing where probability mass concentrates)
- Needs ways to fight the curse of dimensionality  
(exponentially many configurations of the variables to consider)
- Needs disentangling the underlying explanatory factors  
(making sense of the data)

# Easy Learning



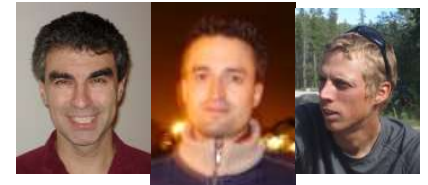
# ML 101. What We Are Fighting Against: The Curse of Dimensionality

To generalize locally,  
need representative  
examples for all  
relevant variations!



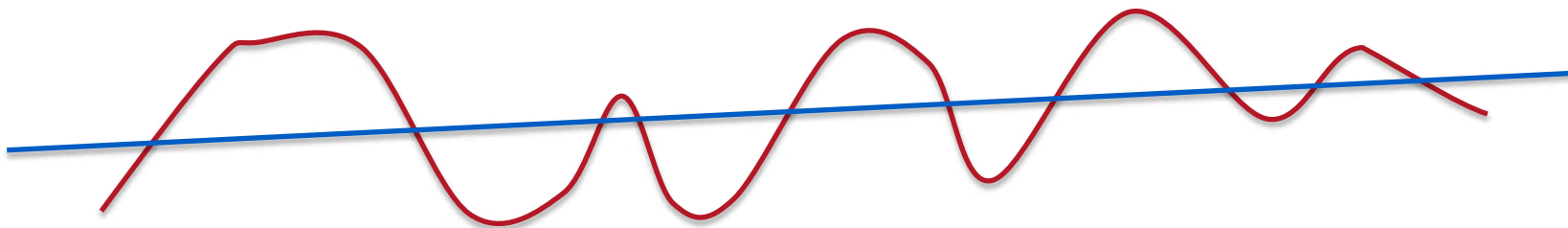


# Not Dimensionality so much as Number of Variations



(Bengio, Dellalleau & Le Roux 2007)

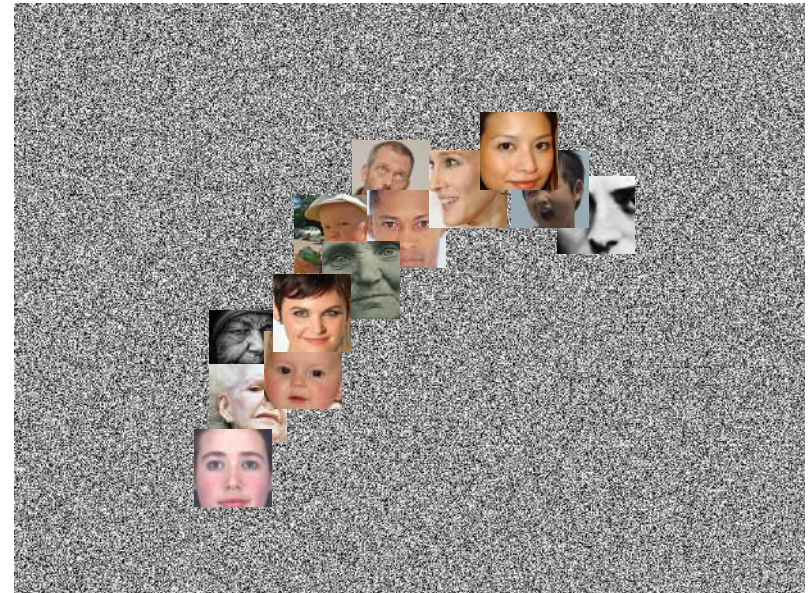
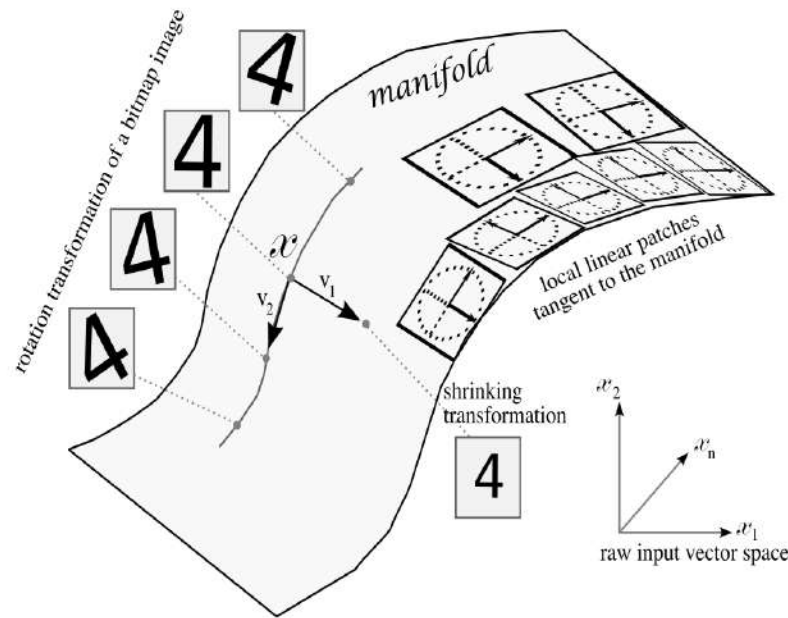
- **Theorem:** Gaussian kernel machines need at least  $k$  examples to learn a function that has  $2k$  zero-crossings along some line



- **Theorem:** For a Gaussian kernel machine to learn some maximally varying functions over  $d$  inputs requires  $O(2^d)$  examples

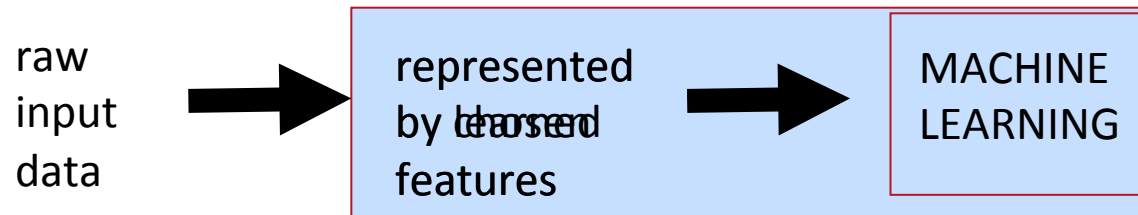
# For AI Tasks: Manifold structure

- examples **concentrate** near a lower dimensional “manifold”
- Evidence: most input configurations are unlikely

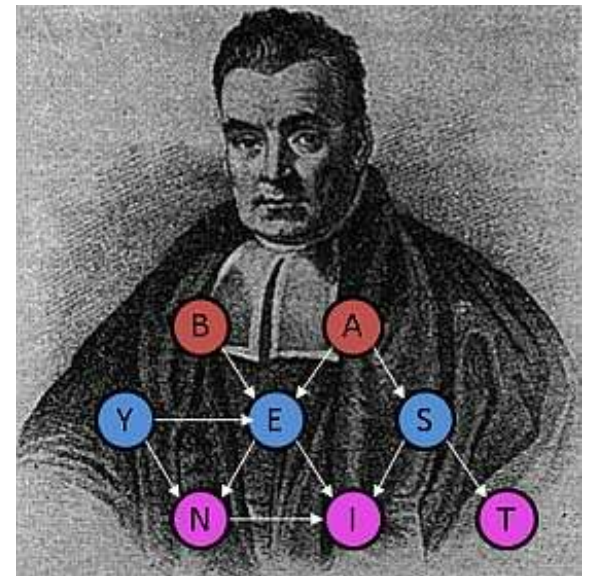


# Representation Learning

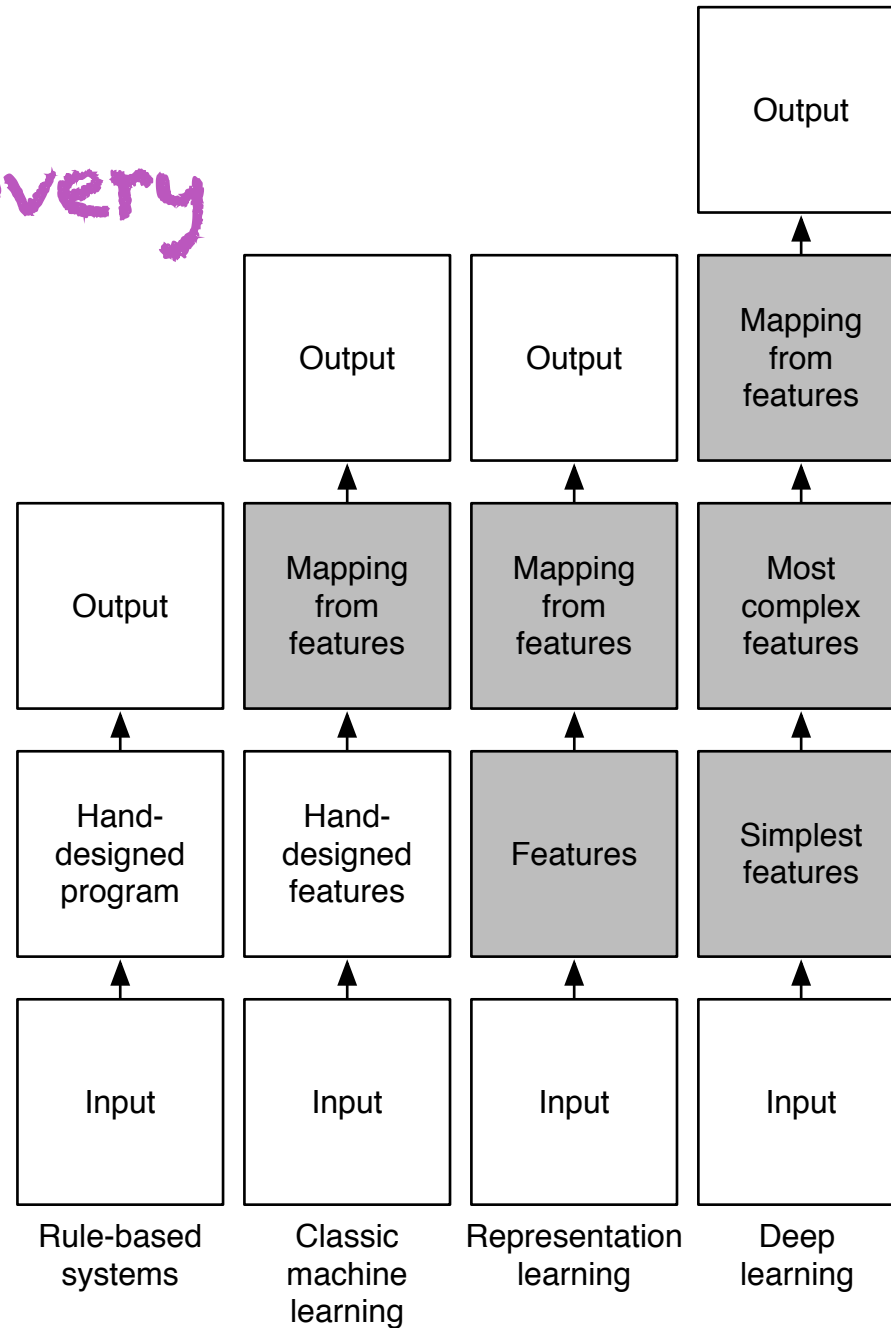
- Good **features** essential for successful ML: 90% of effort



- Handcrafting features vs learning them
- Good representation?
- **guesses**  
the features / factors / causes



# Automating Feature Discovery



# Learning multiple levels of representation

There is theoretical and empirical evidence in favor of multiple levels of representation

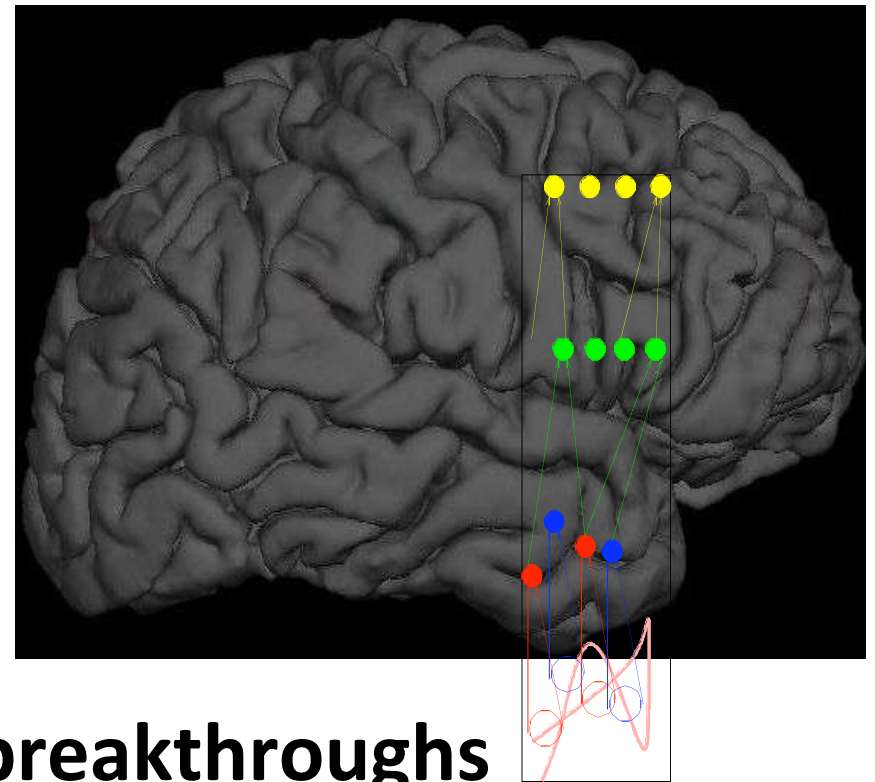
**Exponential gain for some families of functions**

Biologically inspired learning

Brain has a deep architecture

Cortex seems to have a generic learning algorithm

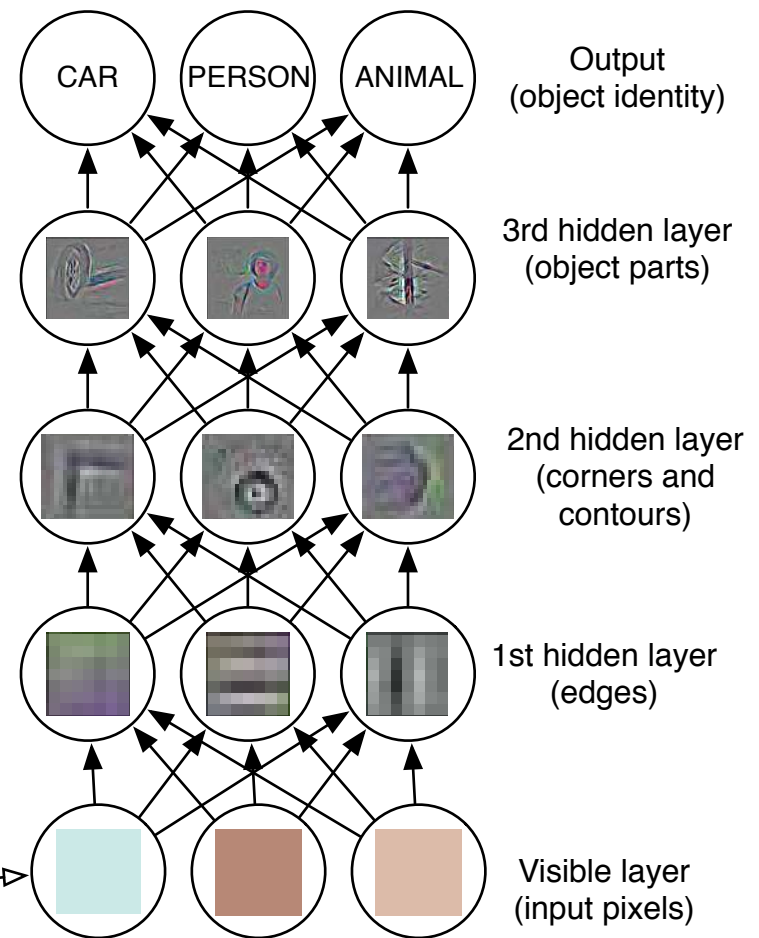
**Humans first learn simpler concepts and compose them**



**It works! Speech + vision breakthroughs**

# Composing Features on Features

Higher-level features  
are defined in terms of  
lower-level  
features





# Google Image Search:

Different object types represented in the same space

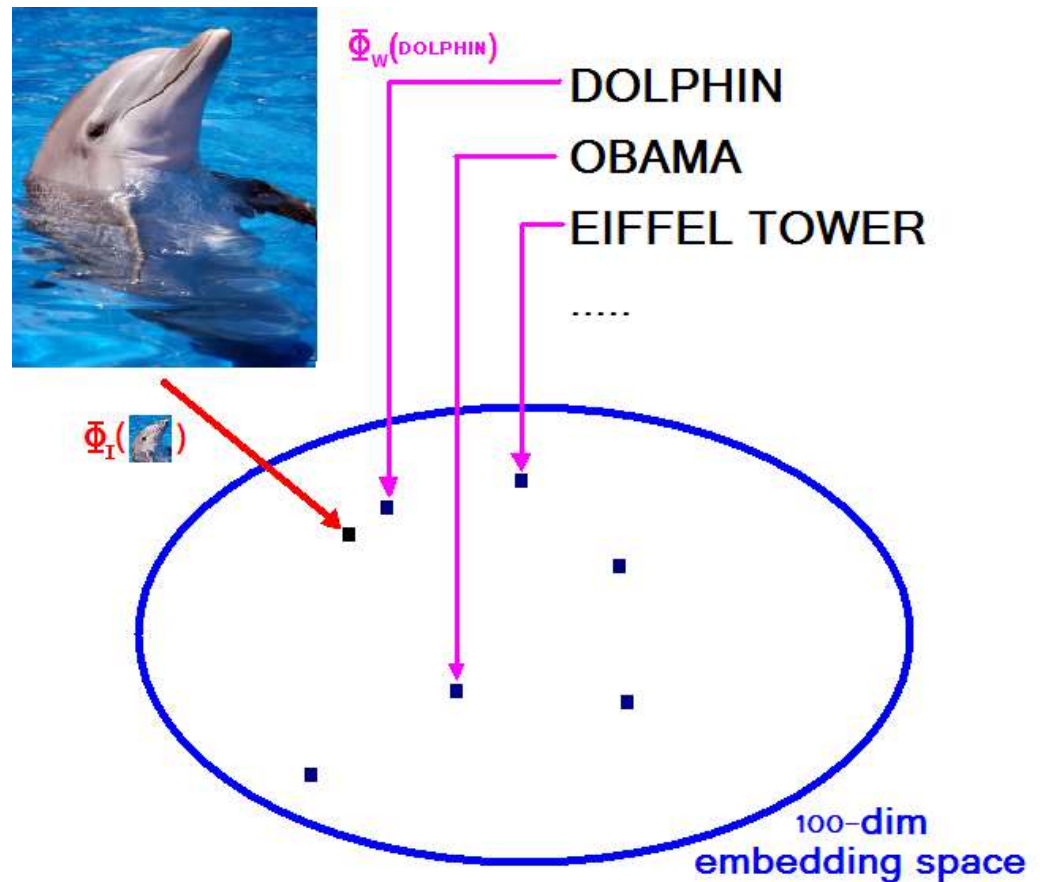


Google:

S. Bengio, J.  
Weston & N.  
Usunier



(IJCAI 2011,  
NIPS'2010,  
JMLR 2010,  
MLJ 2010)



*Learn  $\Phi_I(\cdot)$  and  $\Phi_W(\cdot)$  to optimize precision@k.*

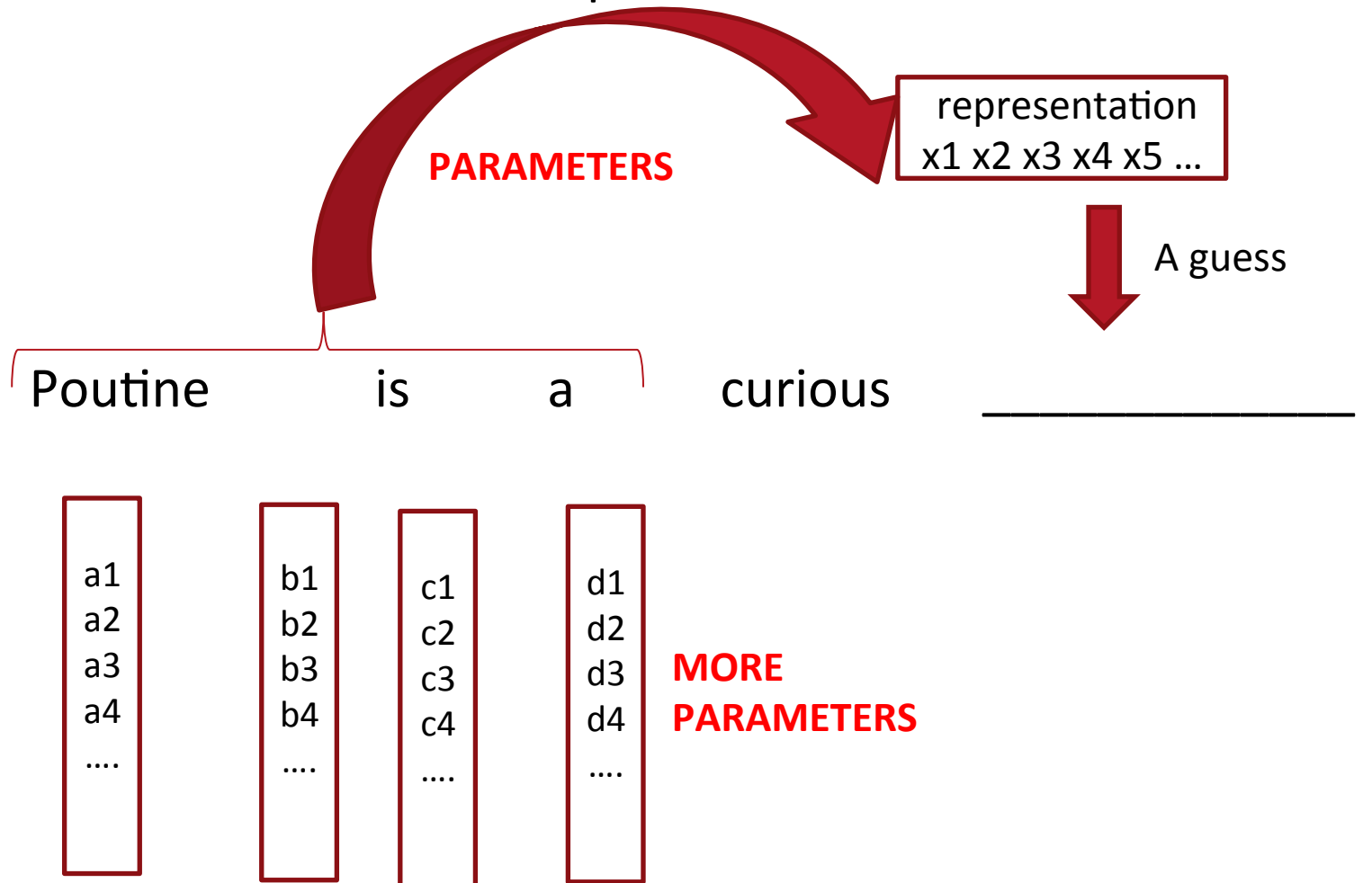
## Following up on (Bengio et al NIPS'2000) Neural word embeddings - visualization





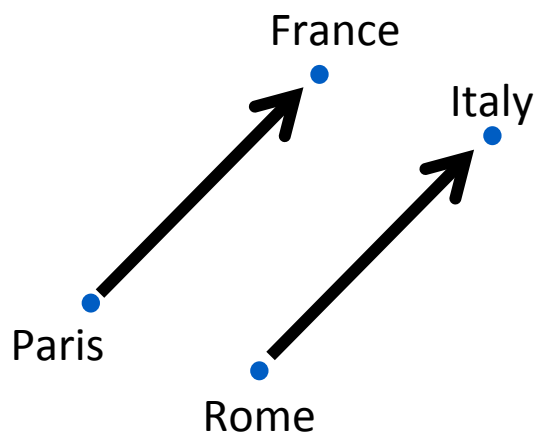
# Neural Language Models

- Meanings and their combination all 'learned' together. Minimal structure imposed.



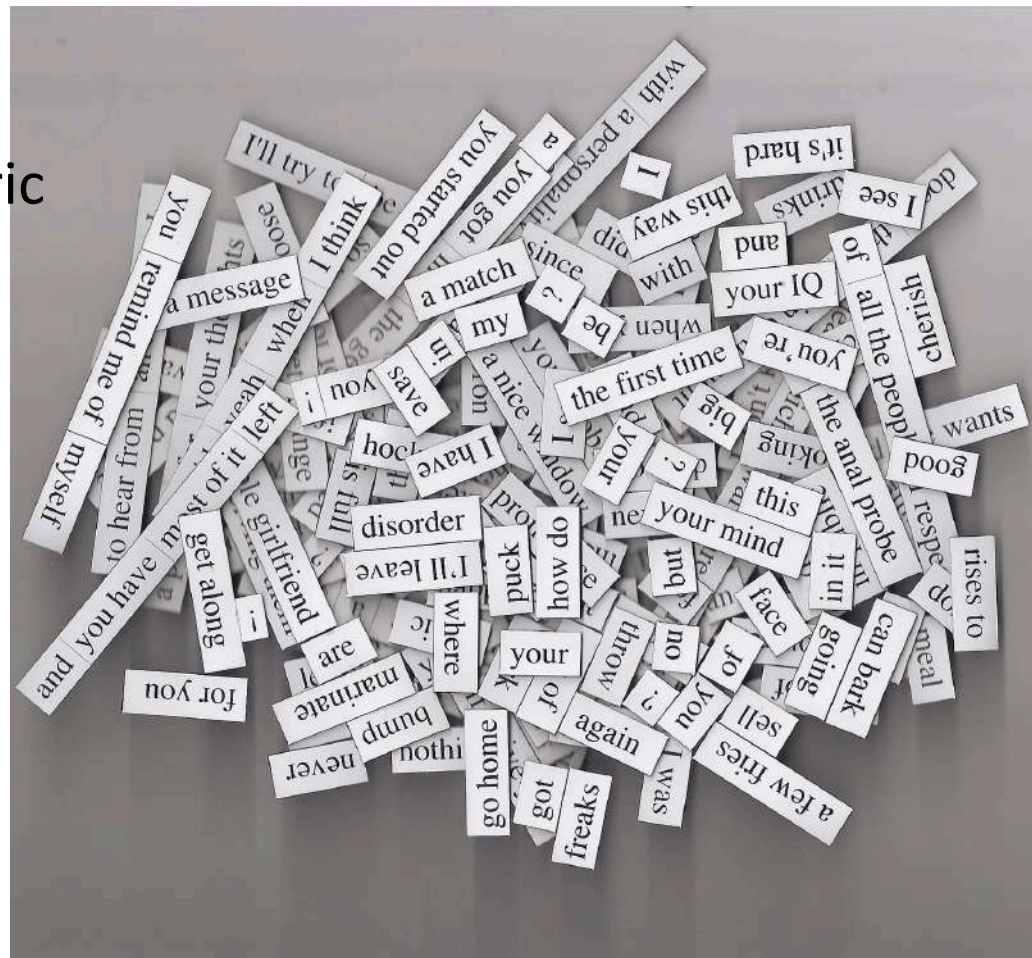
# Analogical Representations for Free (Mikolov et al, ICLR 2013)

- Semantic relations appear as linear relationships in the space of learned representations
- King – Queen  $\approx$  Man – Woman
- Paris – France + Italy  $\approx$  Rome



## The Next Challenge: Rich Semantic Representations for Word Sequences

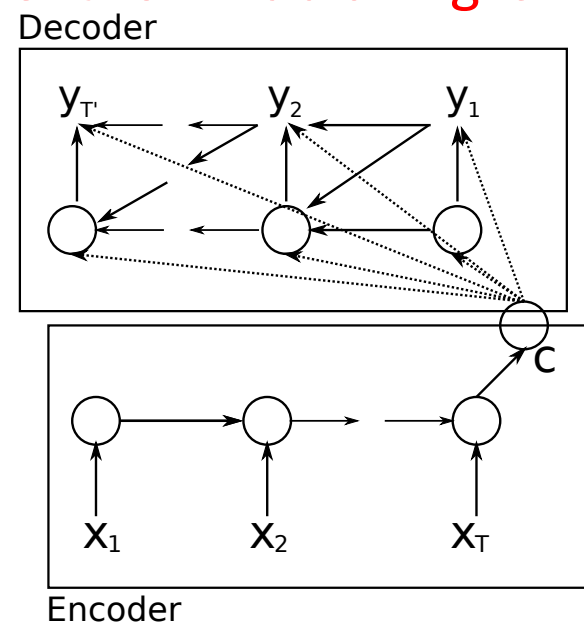
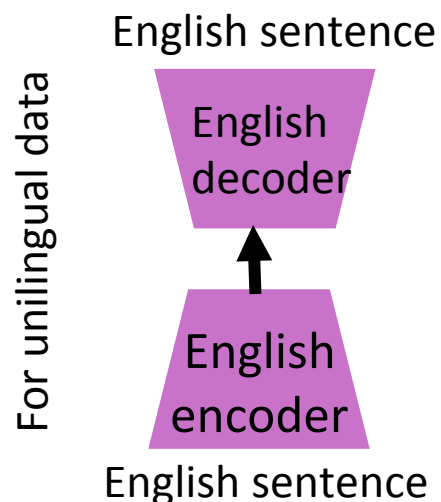
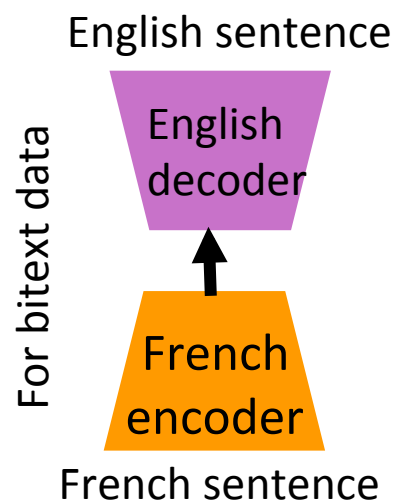
- Impressive progress in capturing word semantics  
Easier learning: non-parametric (table look-up)
- Optimization challenge for mapping sequences to rich & complete representations
- Good test case: machine translation



# Breakthroughs in Machine Translation

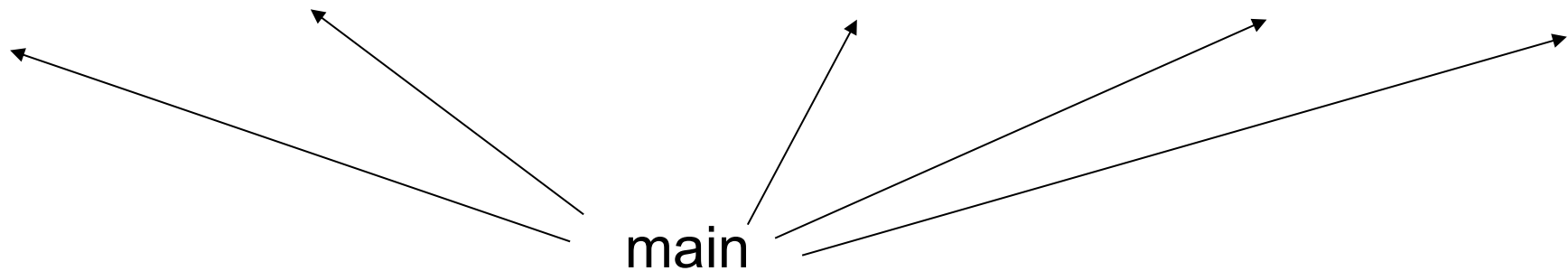
- (Cho et al, EMNLP 2014) Learning Phrase Representations using RNN Encoder–Decoder for Statistical Machine Translation
- (Sutskever et al, NIPS 2014) Sequence to sequence learning with neural networks, **3 BLEU points improvement for English-French**
- (Devlin et al, ACL 2014) Fast and Robust Neural Network Joint Models for Statistical Machine Translation

**Best paper award, 6 BLEU points improvement for Arabic-English**

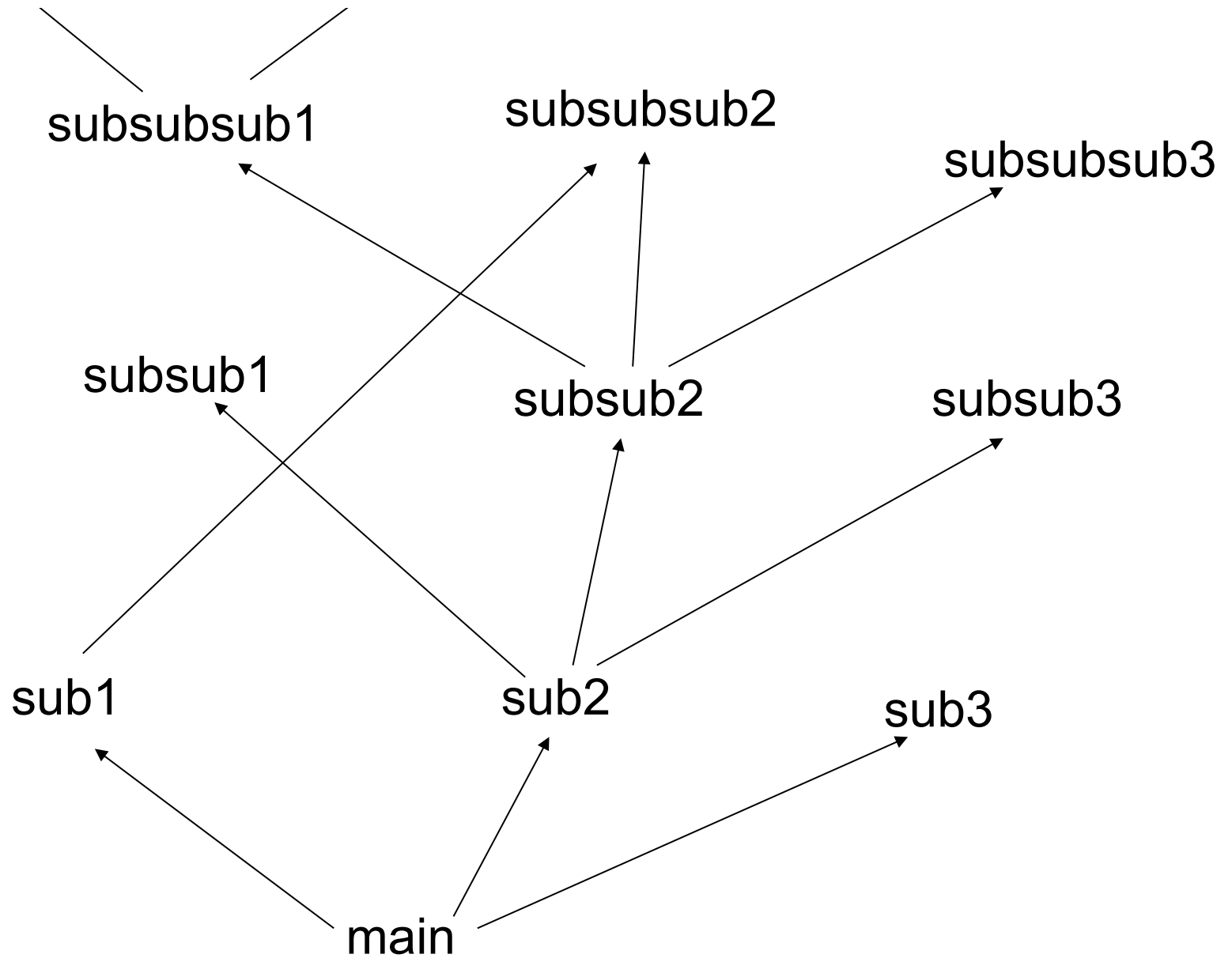


subroutine1 includes  
subsub1 code and  
subsub2 code and  
subsubsub1 code

subroutine2 includes  
subsub2 code and  
subsub3 code and  
subsubsub3 code and ...



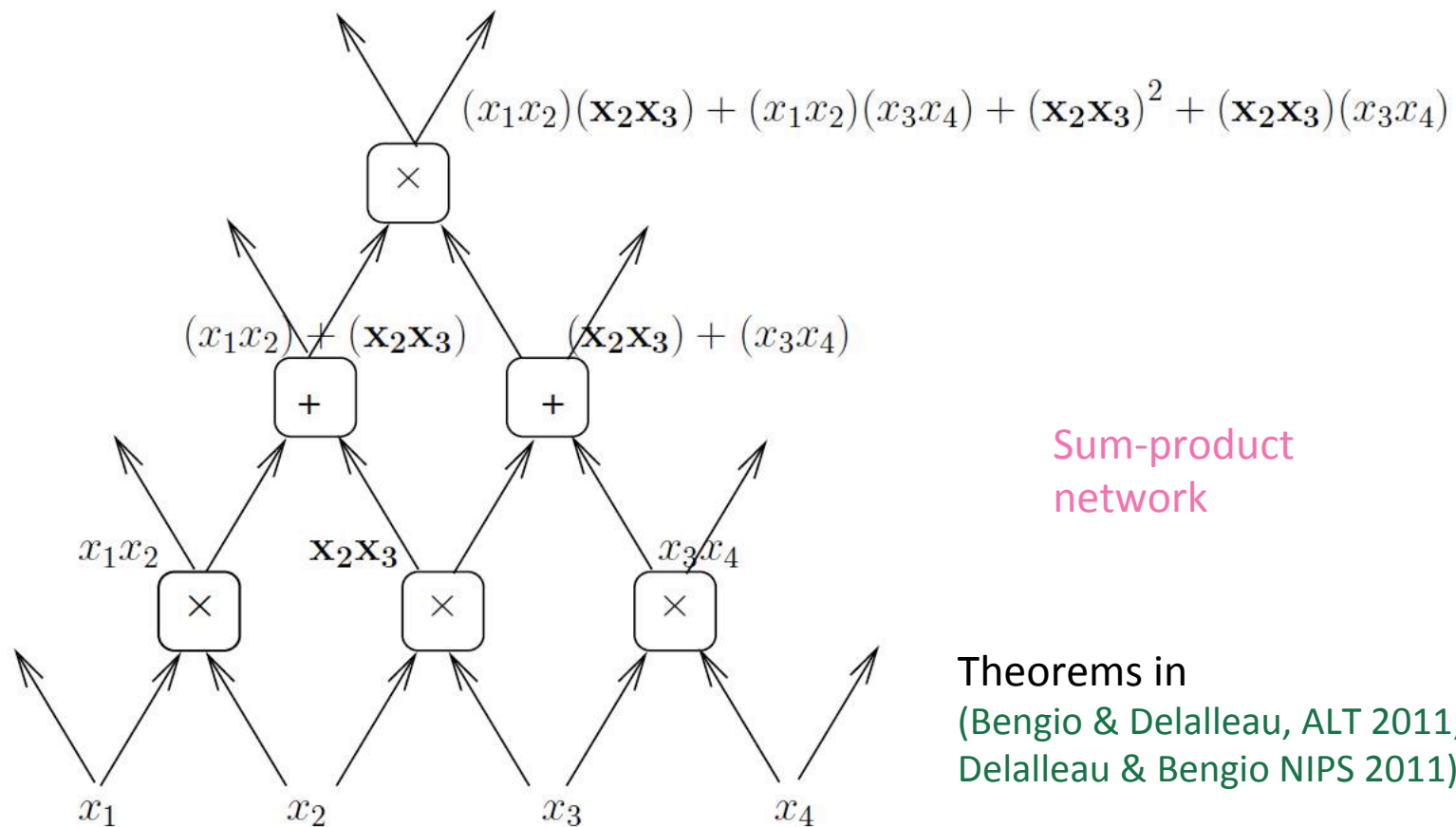
**“Shallow” computer program**



**“Deep” computer program**

# Sharing Components in a Deep Architecture

Polynomial expressed with shared components: advantage of depth may grow exponentially



Theorems in  
(Bengio & Delalleau, ALT 2011;  
Delalleau & Bengio NIPS 2011)

# Deep Architectures are More Expressive

Theoretical arguments:

2 layers of {  
Logic gates  
Formal neurons  
RBF units

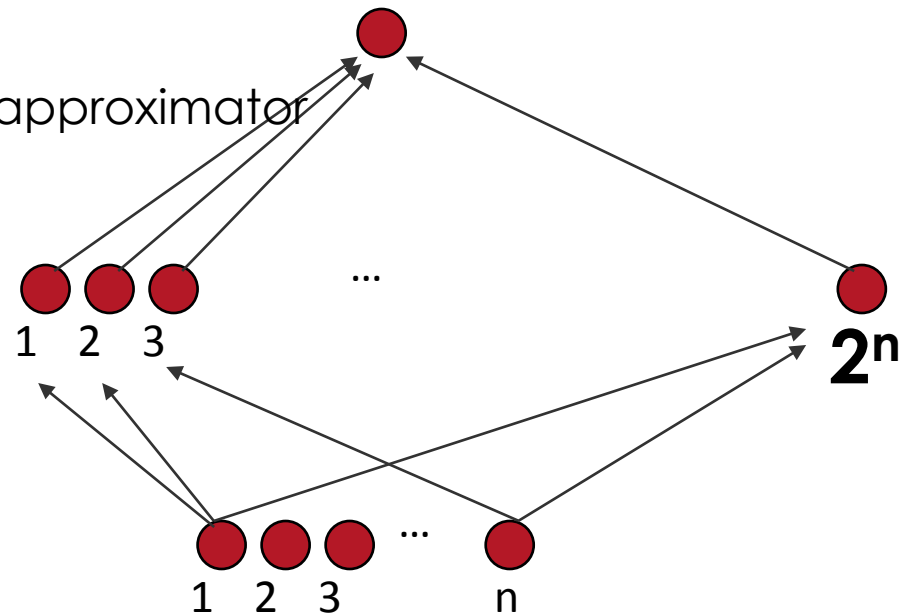
= universal approximator

RBMs & auto-encoders = universal approximator

## Theorems on advantage of depth:

(Hastad et al 86 & 91, Bengio et al 2007, Bengio & Delalleau 2011, Braverman 2011, Pascanu et al 2014)

Some functions compactly represented with  $k$  layers may require exponential size with 2 layers

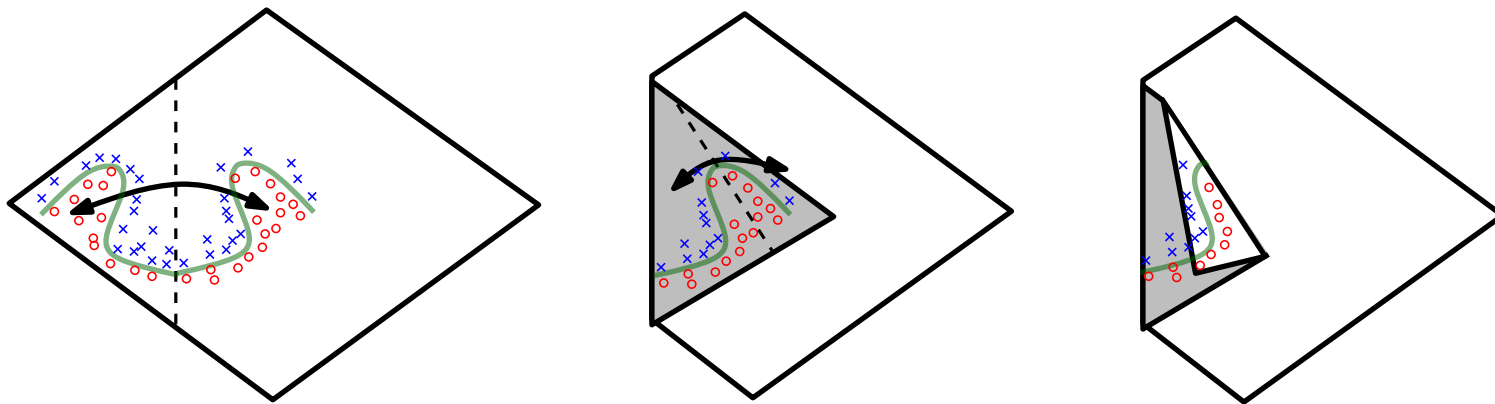




# New theoretical result: Expressiveness of deep nets with piecewise-linear activation fns

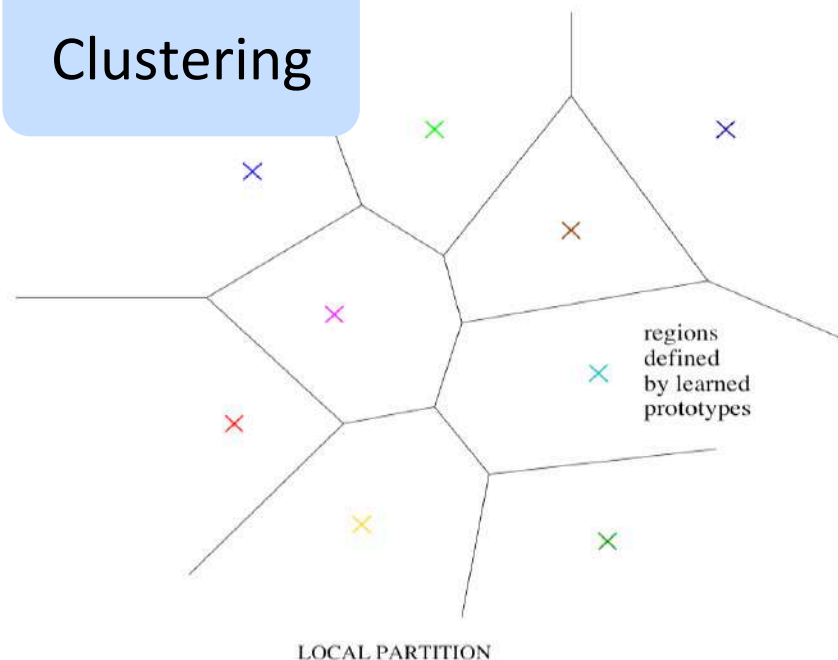
(Pascanu, Montufar, Cho & Bengio; ICLR 2014)

Deeper nets with rectifier/maxout units are exponentially more expressive than shallow ones (1 hidden layer) because they can split the input space in many more (not-independent) linear regions, with constraints, e.g., with abs units, each unit creates mirror responses, folding the input space:



# Non-distributed representations

## Clustering



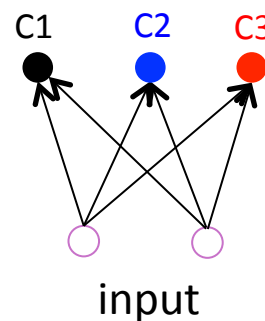
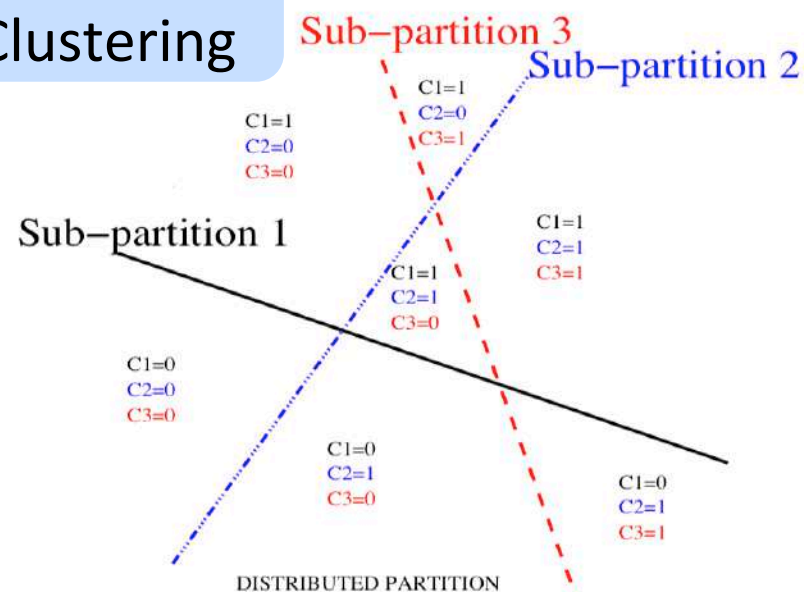
- Clustering, Nearest-Neighbors, RBF SVMs, local non-parametric density estimation & prediction, decision trees, etc.
- Parameters for each distinguishable region
- **# of distinguishable regions is linear in # of parameters**

→ No non-trivial generalization to regions without examples

# The need for distributed representations

- Factor models, PCA, RBMs, Neural Nets, Sparse Coding, Deep Learning, etc.
- Each parameter influences many regions, not just local neighbors
- **# of distinguishable regions grows almost exponentially with # of parameters**
- **GENERALIZE NON-LOCALLY TO NEVER-SEEN REGIONS**

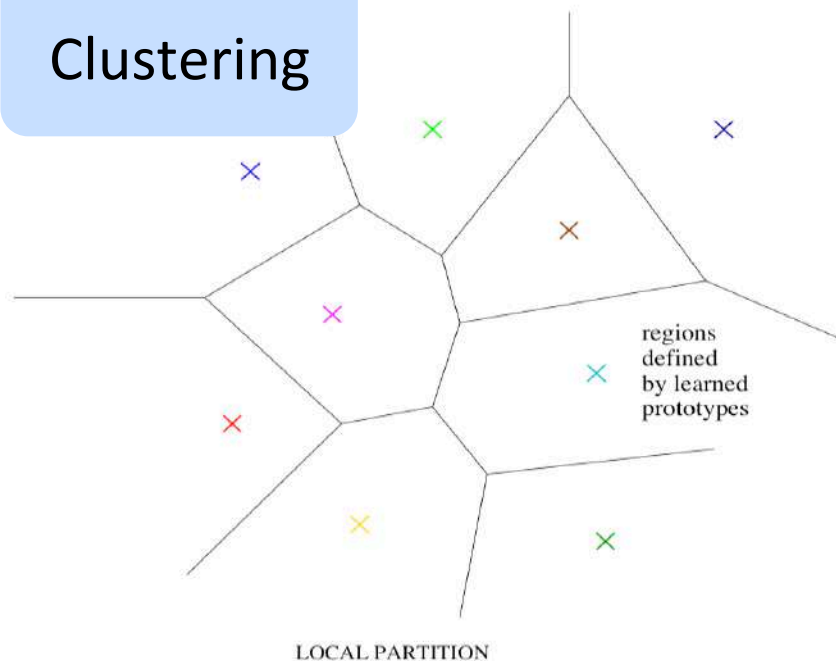
## Multi-Clustering



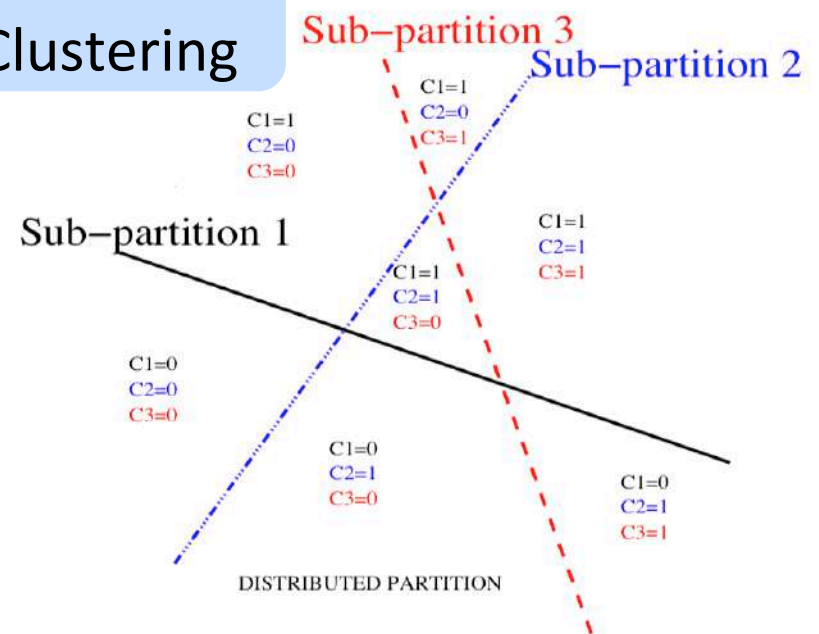
Non-mutually exclusive features/attributes create a combinatorially large set of distinguishable configurations

# The need for distributed representations

## Clustering

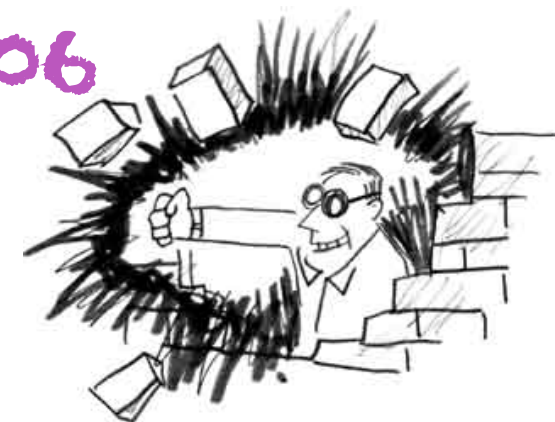


## Multi-Clustering



Learning a **set of features** that are not mutually exclusive can be **exponentially more statistically efficient** than having nearest-neighbor-like or clustering-like models

# Major Breakthrough in 2006



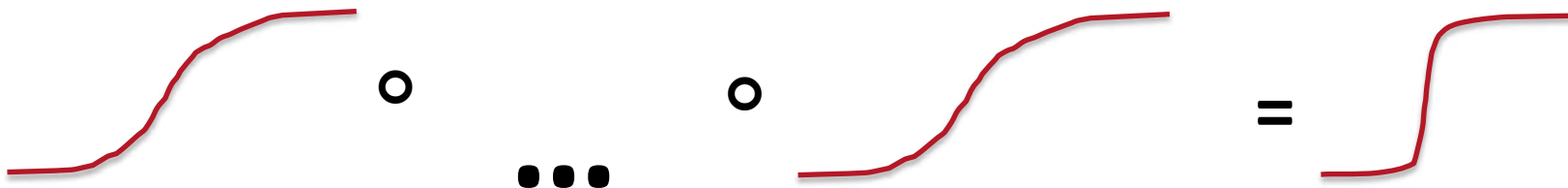
- Ability to train deep architectures by using layer-wise unsupervised learning, whereas previous purely supervised attempts had failed
- Unsupervised feature learners:
  - RBMs
  - Auto-encoder variants
  - Sparse coding variants



*(Bengio & LeCun 2007), Scaling Learning Algorithms towards AI*

## Issues with Back-Prop

- In very deep nets or recurrent nets with many steps, non-linearities compose and yield sharp non-linearity  
→ gradients vanish or explode
- Training deeper nets: harder optimization
- In the extreme of non-linearity: discrete functions, can't use back-prop
- Not biologically plausible?



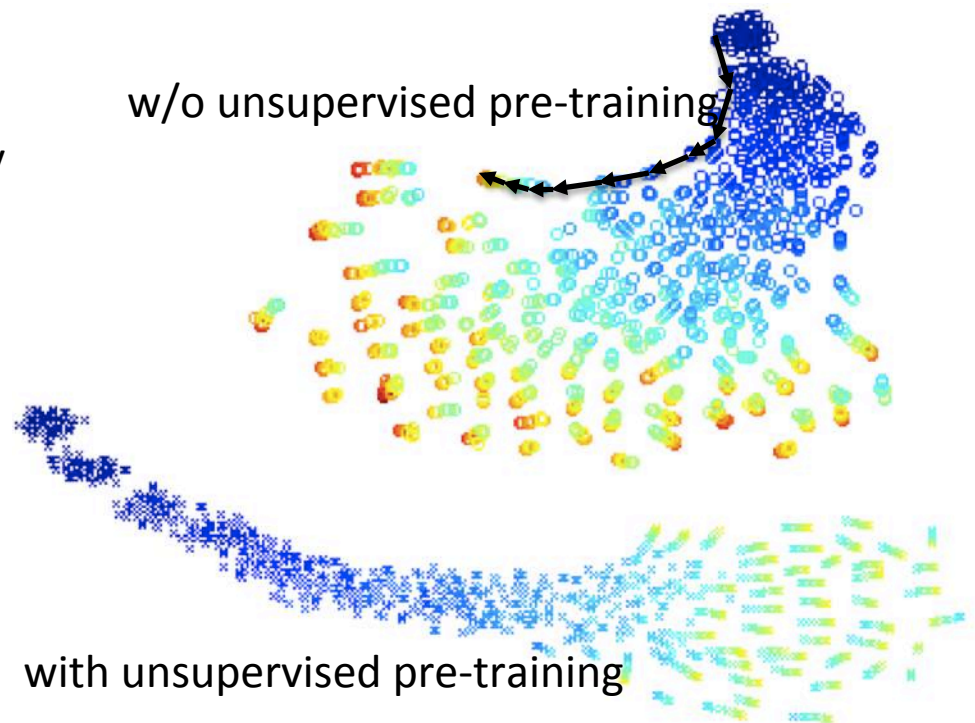
# Effect of Initial Conditions in Deep Nets

- (Erhan et al 2009, JMLR)
- Supervised deep net with vs w/o unsupervised pre-training → very different minima

Neural net trajectories in function space, visualized by t-SNE

No two training trajectories end up in the same place → huge number of effective local minima

w/o unsupervised pre-training



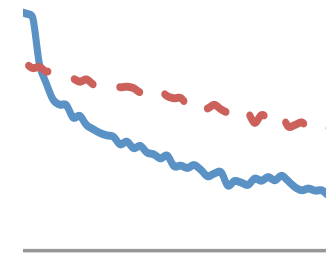
with unsupervised pre-training

# Order & Selection of Examples Matters

(Bengio, Louradour, Collobert & Weston, ICML'2009)



- Curriculum learning
  - (Bengio et al 2009, Krueger & Dayan 2009)
- **Start with easier examples**
- Faster convergence to a better local minimum in deep architectures



— curriculum  
- - no-curriculum



# Curriculum Learning

Guided learning helps training humans and animals



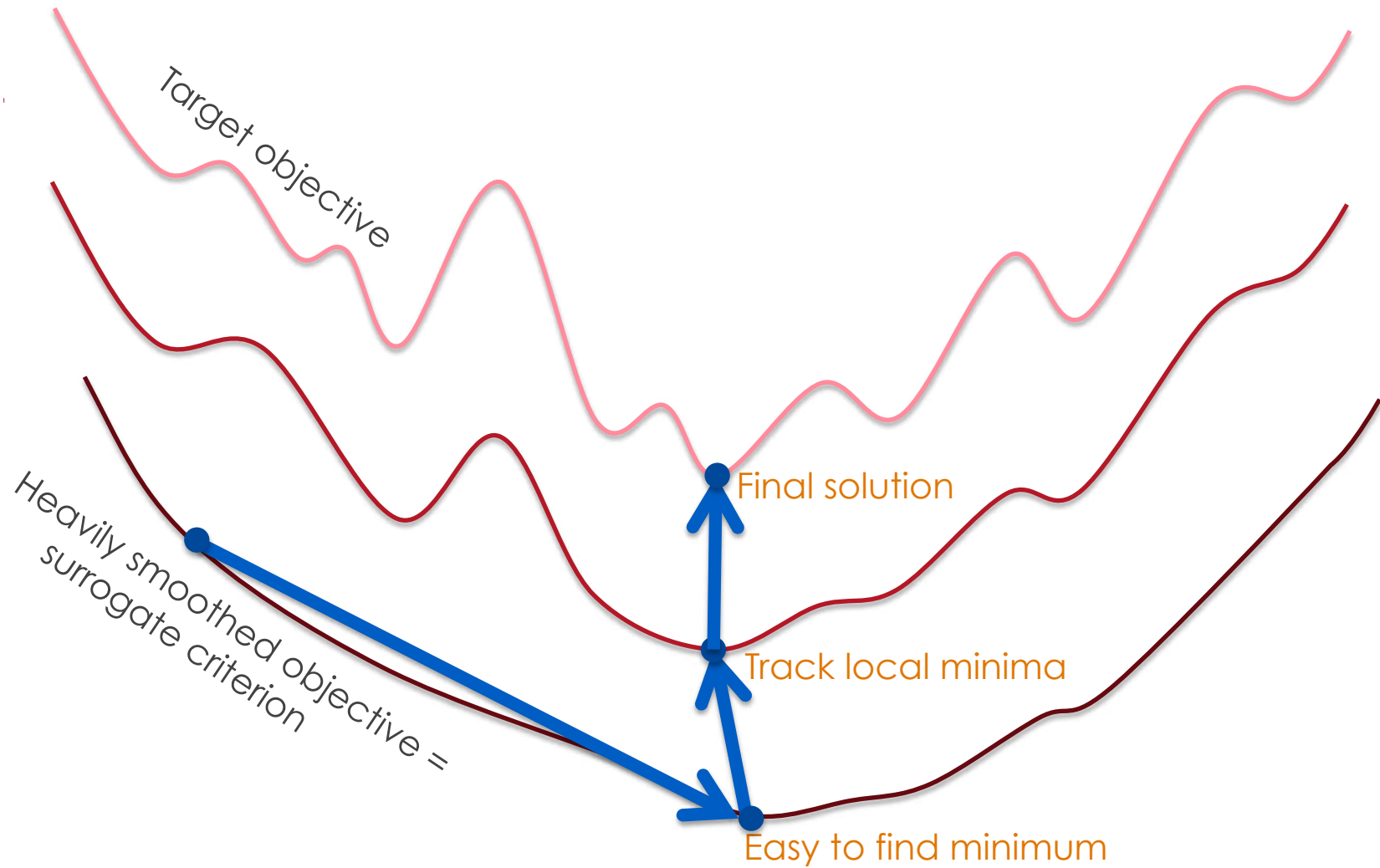
**Education**



**Shaping**

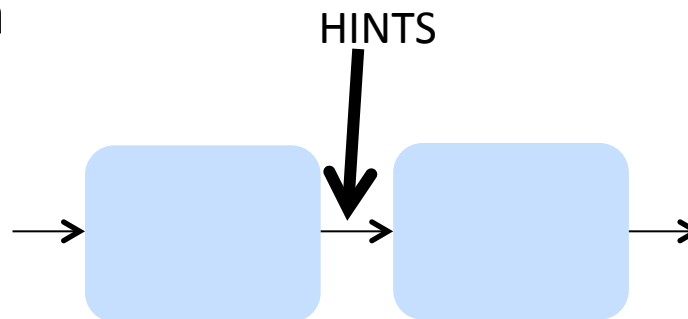
Start from simpler examples / easier tasks (Piaget 1952, Skinner 1958)

# Continuation Methods



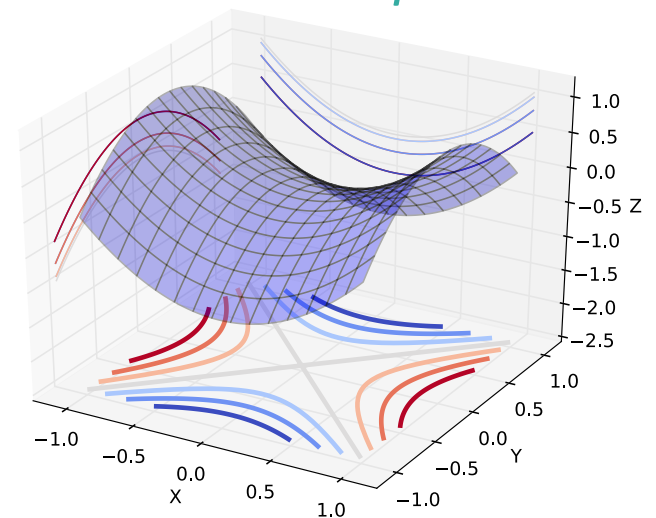
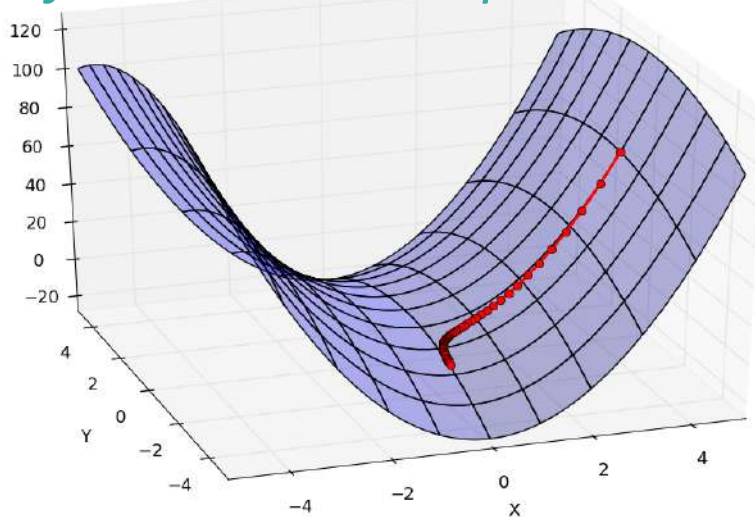
# Guided Training, Intermediate Concepts

- In (Gulcehre & Bengio ICLR'2013) we set up a task that seems almost impossible to learn by shallow nets, deep nets, SVMs, trees, boosting etc
- Breaking the problem in two sub-problems and pre-training each module separately, then fine-tuning, nails it
- *Need prior knowledge to decompose the task*
- **Guided pre-training** allows to find much better solutions, escape effective local minima



# Saddle Points, not Local Minima

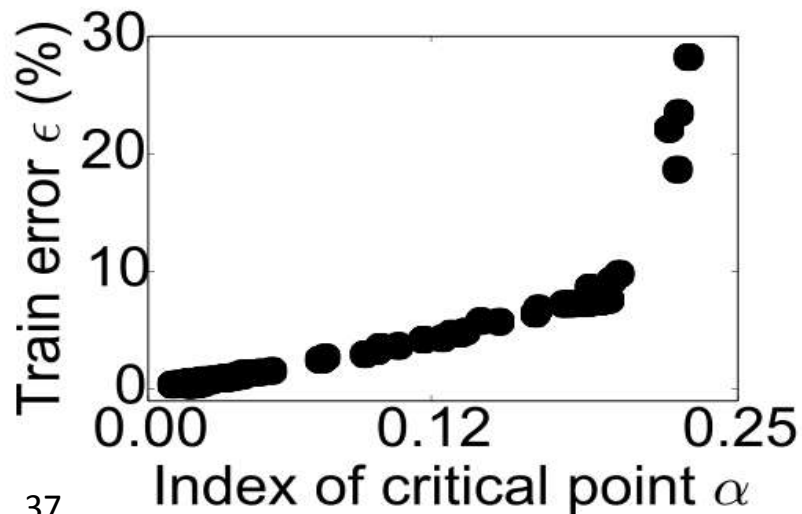
- Traditional thinking is that major obstacle for training deep nets is local minima
- Theoretical and empirical evidence suggest instead that saddle points are exponentially more prevalent critical points, and local minima tend to be of cost near that of global minimum
- (Pascanu, Dauphin, Ganguli, Bengio 2014): *On the saddle point problem for non-convex optimization.*



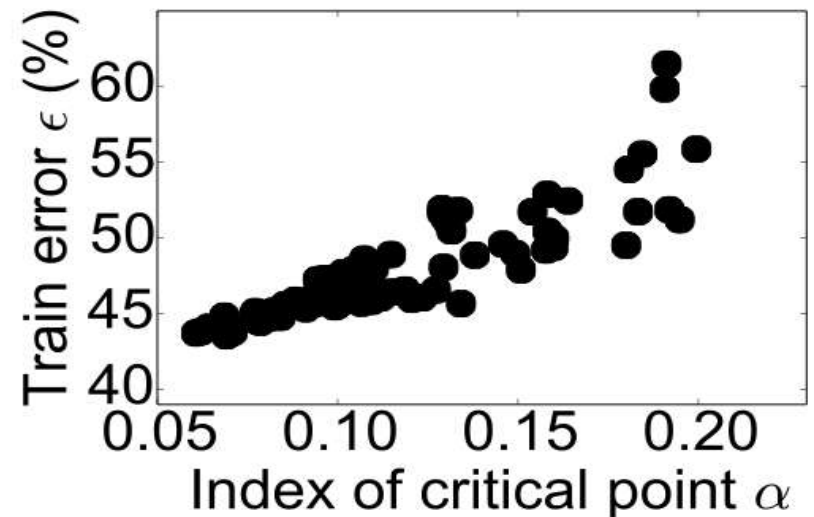
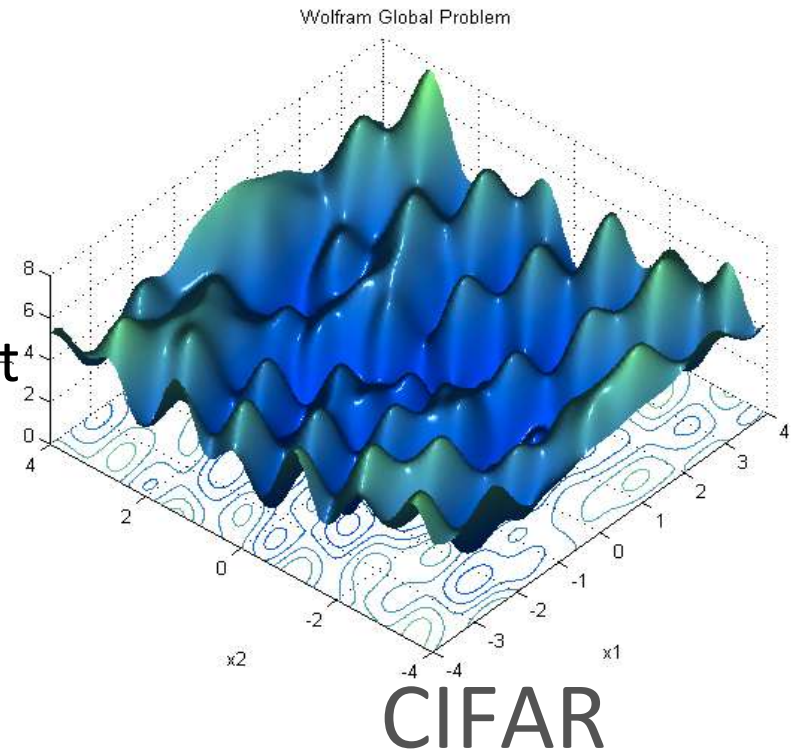
# Saddle Points

- Local minima dominate in low-D, but saddle points dominate in high-D
- Most local minima are close to the bottom (global minimum error)

MNIST

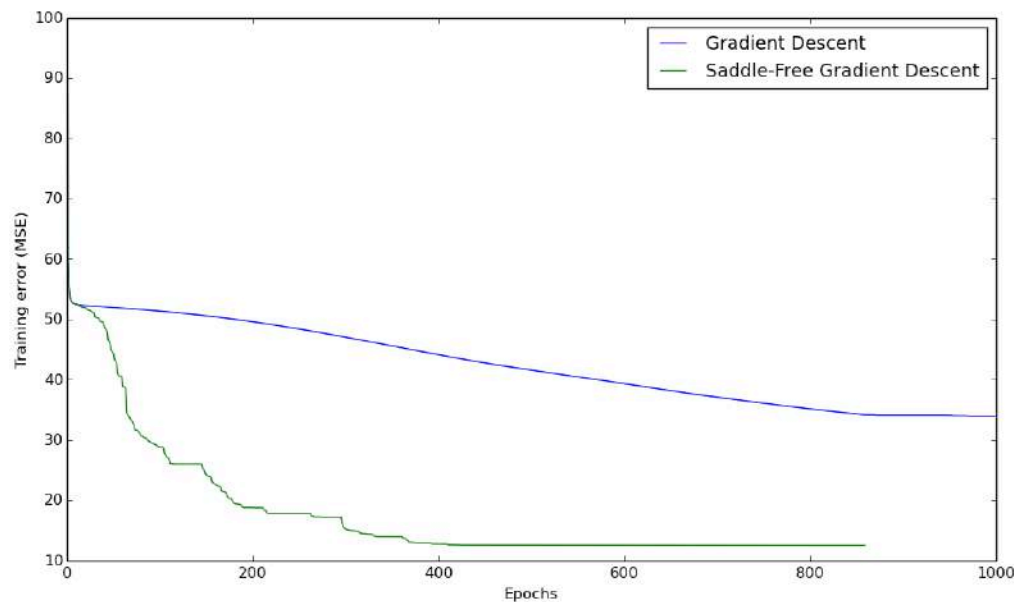


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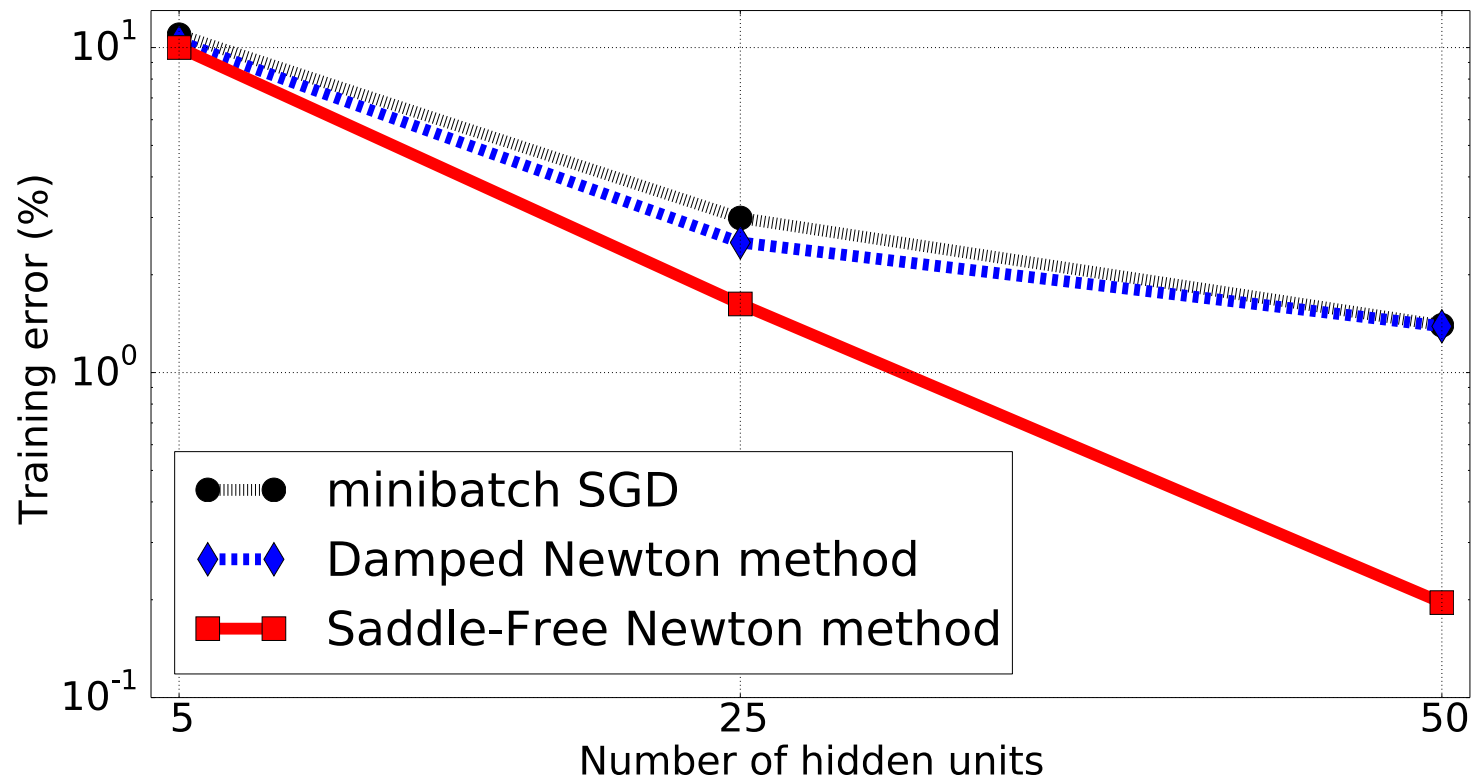
# It is possible to escape saddle points!

- NIPS'2014 paper, Dauphin et al.
- More work is ongoing to make it online
- Challenge: track the most negative eigenvector, which is easy in batch mode with power method, if we also track most positive, via  $v \leftarrow (H - \lambda I)v$



# Saddle-Free Optimization (Dauphin et al NIPS'2014)

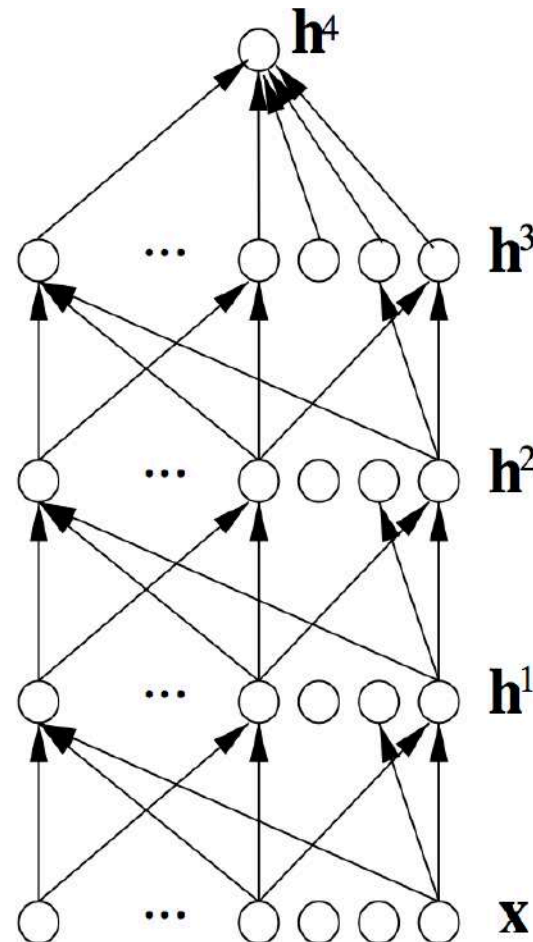
- Replace eigenvalues  $\lambda$  of Hessian by  $|\lambda|$





# Deep Supervised Neural Nets

- Now can train them even without unsupervised pre-training:  
**better initialization and non-linearities** (rectifiers, maxout), generalize well with large labeled sets and regularizers (dropout)
- **Unsupervised pre-training:**  
rare classes, transfer, smaller labeled sets, or as extra regularizer.





# Why Unsupervised Learning?

- Recent progress mostly in supervised DL
- $\exists$  real challenges for unsupervised DL
- Potential benefits:
  - Exploit tons of unlabeled data
  - Answer new questions about the variables observed
  - Regularizer – transfer learning – domain adaptation
  - Easier optimization (local training signal)
  - Structured outputs

# Invariance and Disentangling

- Invariant features
- Which invariances?
- Alternative: learning to disentangle factors
- Good disentangling →  
avoid the curse of dimensionality



# Emergence of Disentangling

- (Goodfellow et al. 2009): sparse auto-encoders trained on images
  - some higher-level features more invariant to geometric factors of variation
- (Glorot et al. 2011): sparse rectified denoising auto-encoders trained on bags of words for sentiment analysis
  - different features specialize on different aspects (domain, sentiment)

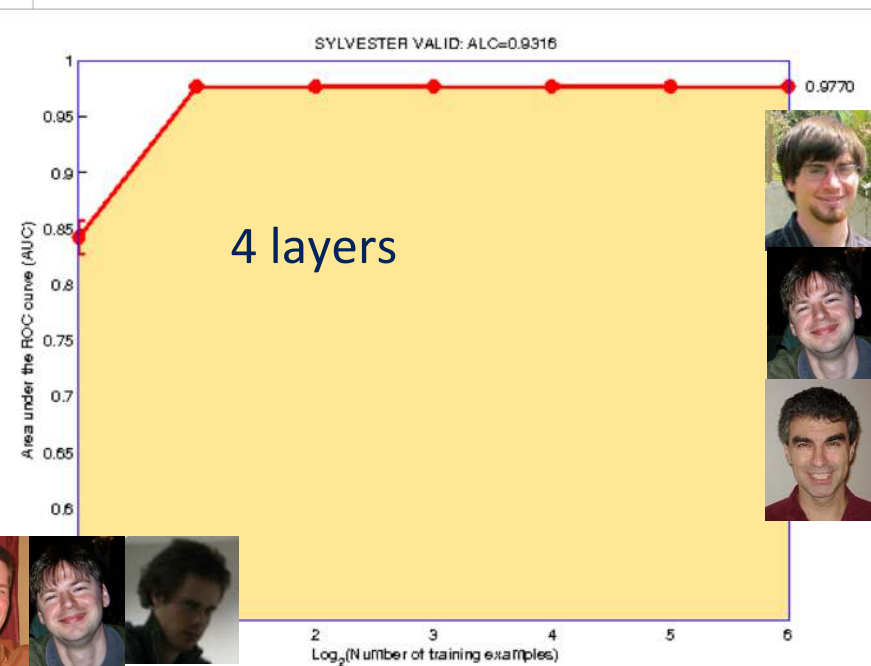
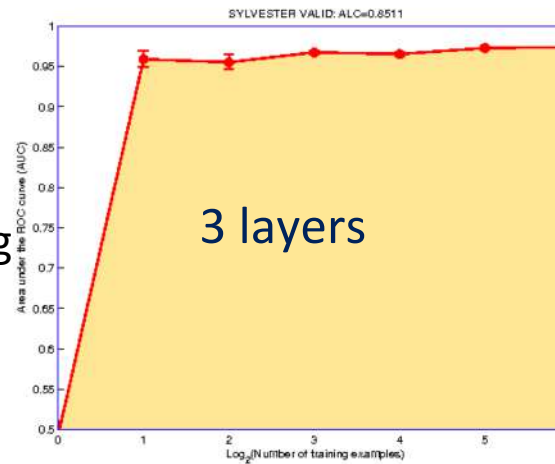
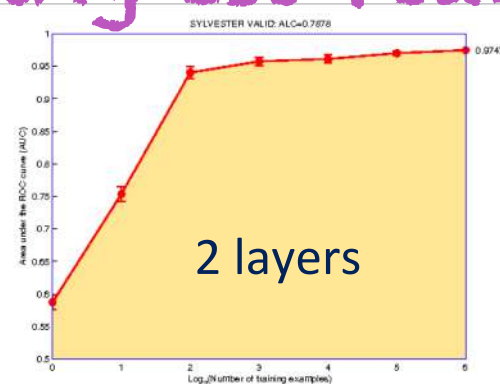
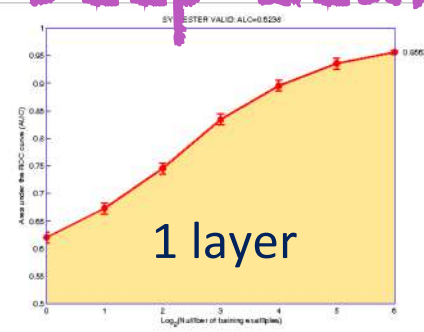
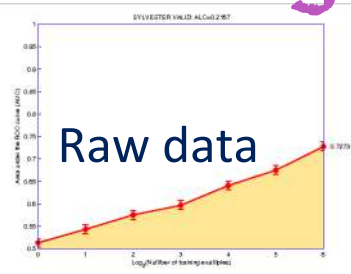


# WHY?

# How do humans generalize from very few examples?

- They **transfer** knowledge from previous learning:
  - Representations
  - Explanatory factors
- Previous learning from: unlabeled data
  - + labels for other tasks
- **Prior: shared underlying explanatory factors, in particular between  $P(x)$  and  $P(Y|x)$**

# Unsupervised and Transfer Learning Challenge + Transfer Learning Challenge: Deep Learning 1st Place

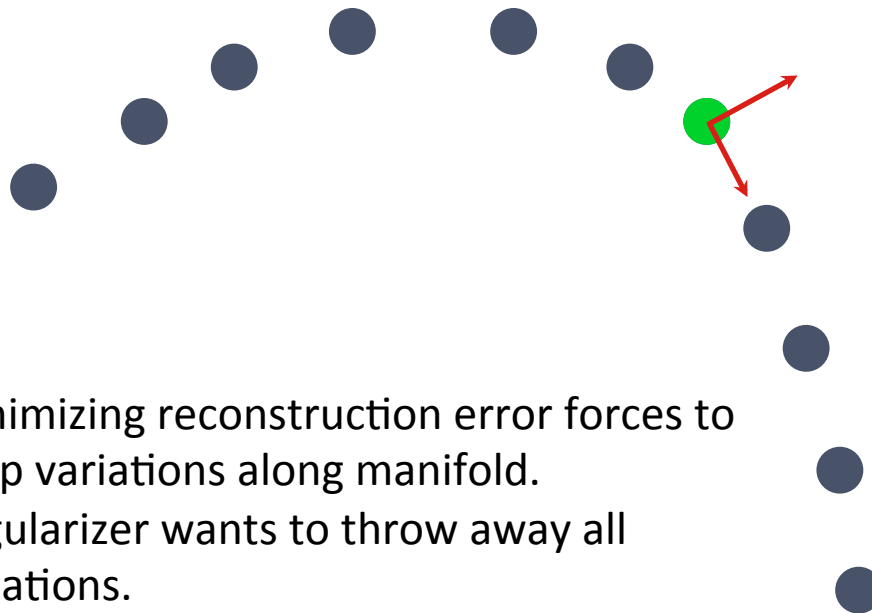


NIPS'2011  
Transfer  
Learning  
Challenge  
Paper:  
ICML'2012

ICML'2011  
workshop on  
Unsup. &  
Transfer Learning



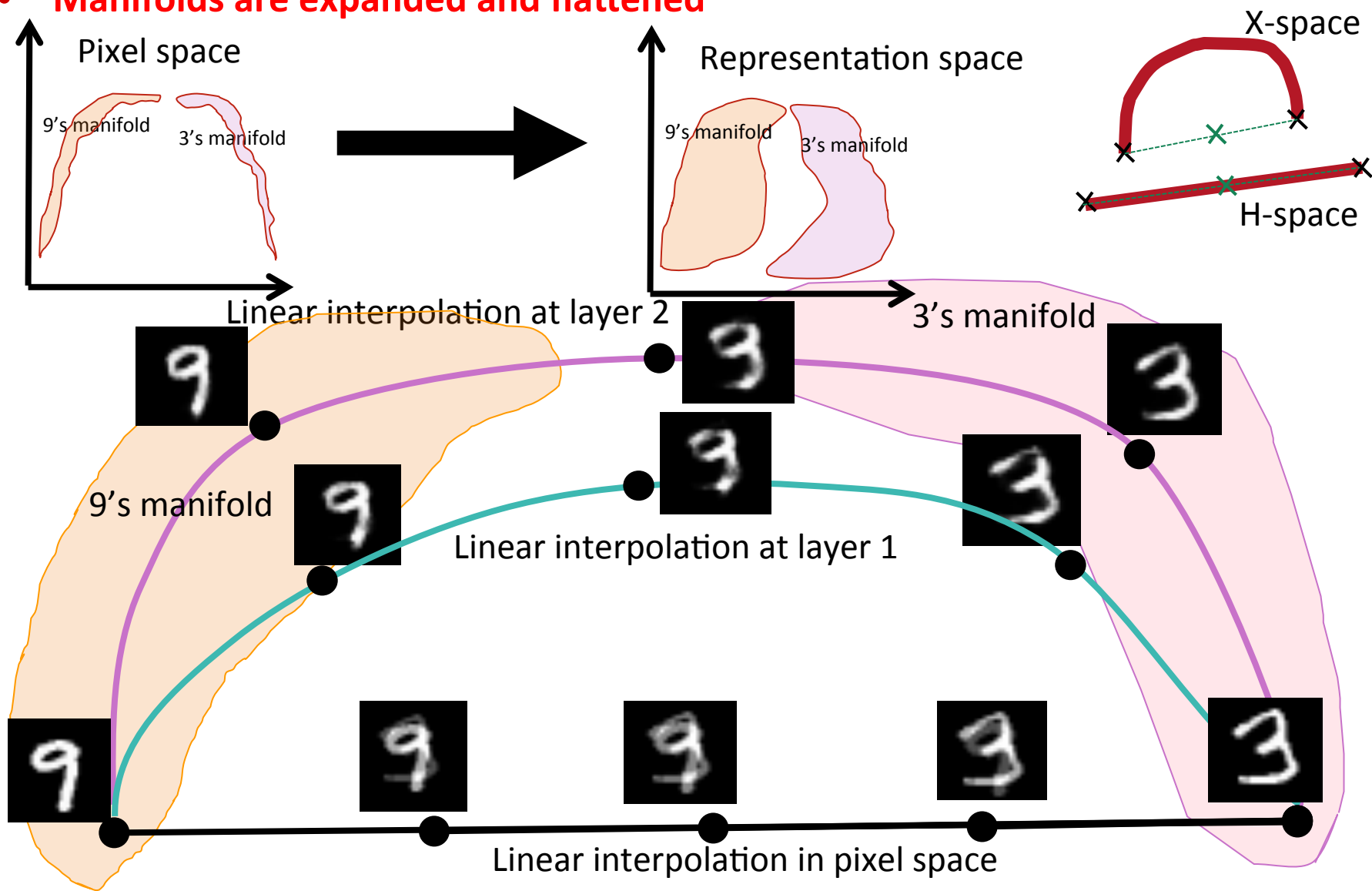
# Auto-Encoders Learn Salient Variations, Like a non-linear PCA



- Minimizing reconstruction error forces to keep variations along manifold.
- Regularizer wants to throw away all variations.
- With both: keep ONLY sensitivity to variations ON the manifold.

# Space-Filling in Representation-Space

- Deeper representations → abstractions → disentangling
- Manifolds are expanded and flattened



# Why Unsupervised Representation Learning? Because of Causality.

- If Ys of interest are among the causal factors of X, then

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

is tied to  $P(X)$  and  $P(X|Y)$ , and  $P(X)$  is defined in terms of  $P(X|Y)$ , i.e.

- The best possible model of X (unsupervised learning) MUST involve Y as a latent factor, implicitly or explicitly.
- Representation learning SEEKS the latent variables H that explain the variations of X, making it likely to also uncover Y.
- We need 3 pieces:
  - latent variable model  $P(H)$ ,
  - generative decoder  $P(X|H)$ , and
  - approximate inference encoder  $Q(H|X)$ .



# Challenges with Graphical Models with Latent Variables

- Latent variables help to avoid the curse of dimensionality
- But they come with intractabilities due to sums over an exponentially large number of terms (marginalization):
  - Exact inference ( $P(h|x)$ ) is typically intractable
  - With undirected models, the normalization constant and its gradient are intractable

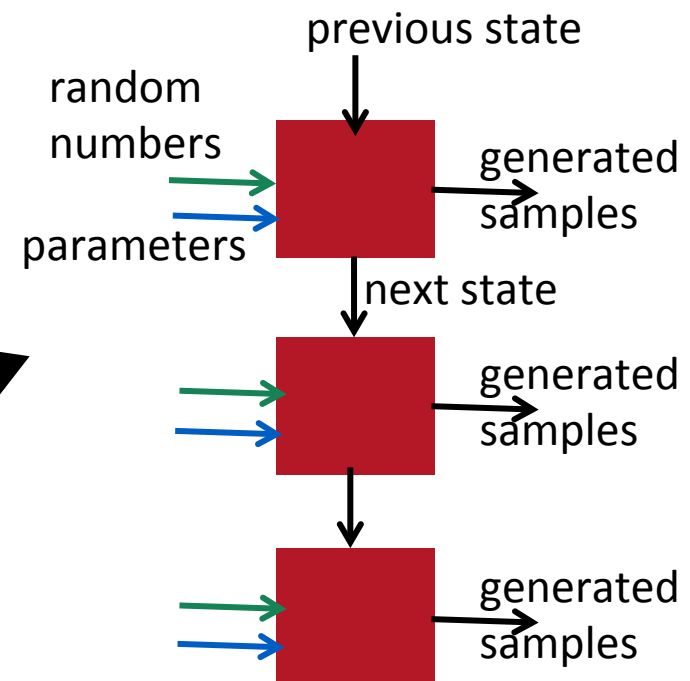
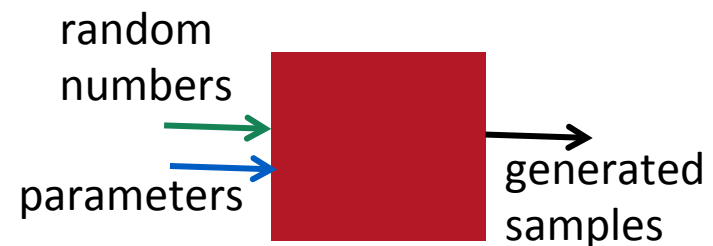
# Issues with Boltzmann Machines

- Sampling from the MCMC of the model is required in the inner loop of training
- As the model gets sharper, mixing between well-separated modes stalls

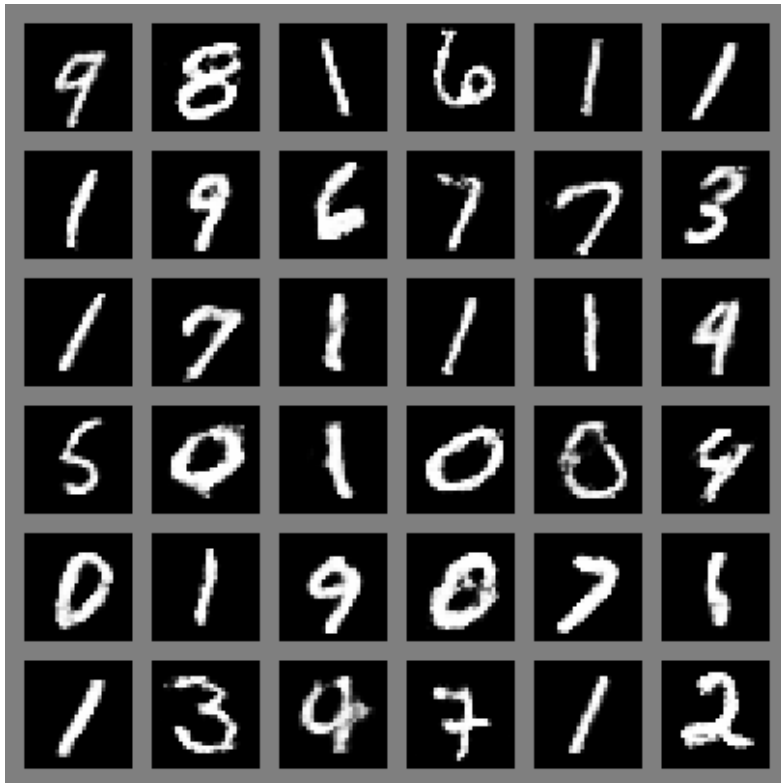


# Bypassing Normalization Constants with Generative Black Boxes

- **Instead of parametrizing  $p(x)$ , parametrize a machine which generates samples**
- (Goodfellow et al, NIPS 2014, Generative adversarial nets) for the case of ancestral sampling in a deep generative net. Variational auto-encoders are closely related.
- (Bengio et al, ICML 2014, Generative Stochastic Networks), learning the transition operator of a Markov chain that generates the data.



# Adversarial Nets

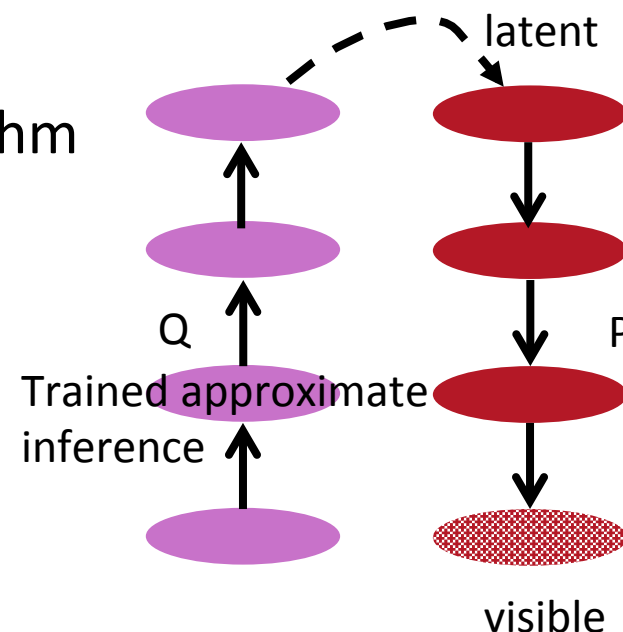


Each movie = linear interpolation  
between 2 random samples in  
representation-space



# Ancestral Sampling with Learned Approximate Inference

- Helmholtz machine & Wake-Sleep algorithm
  - (Dayan, Hinton, Neal, Zemel 1995)
- Variational Auto-Encoders
  - (Kingma & Welling 2013, ICLR 2014)
  - (Gregor et al ICML 2014)
  - (Rezende et al ICML 2014)
  - (Mnih & Gregor ICML 2014)
- Reweighted Wake-Sleep (Bornschein & Bengio 2014)
- Target Propagation (Bengio 2014)
- Deep Directed Generative Auto-Encoders (Ozair & Bengio 2014)
- NICE (Dinh et al 2014)



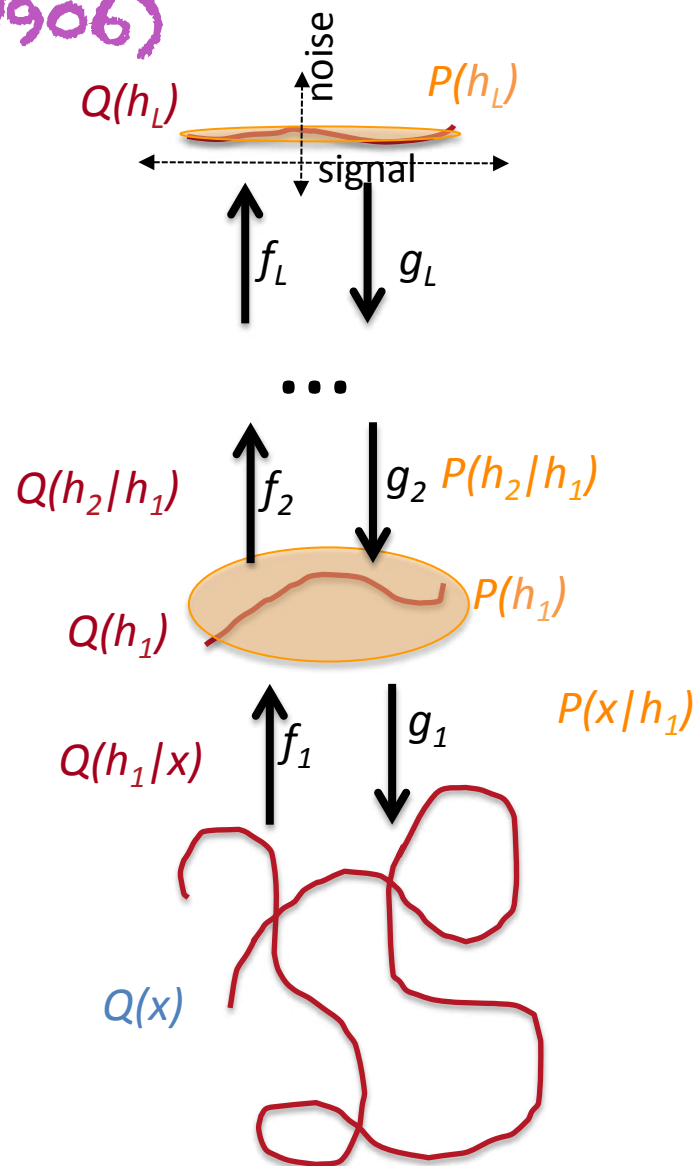
# Extracting Structure By Gradual Disentangling and Manifold Unfolding

(Bengio 2014, arXiv 1407.7906)

Each level transforms the data into a representation in which it is easier to model, unfolding it more, contracting the noise dimensions and mapping the signal dimensions to a factorized (uniform-like) distribution.

$$\min KL(Q(x, h) || P(x, h))$$

for each intermediate level  $h$



# NICE:

## Nonlinear Independent Component Estimation

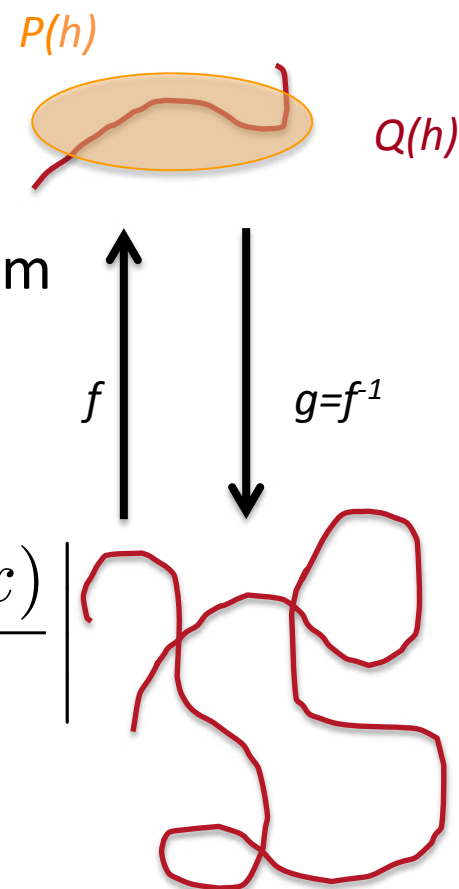
(Dinh, Krueger & Bengio 2014, arxiv 1410.8516)

- Perfect auto-encoder  $g=f^{-1}$
- No need for reconstruction error
- Deterministic encoder, no need for entropy term
- But need to correct for density scaling
- **Exact tractable likelihood**

$$\log p_X(x) = \log p_H(f(x)) + \log \left| \det \frac{\partial f(x)}{\partial x} \right|$$

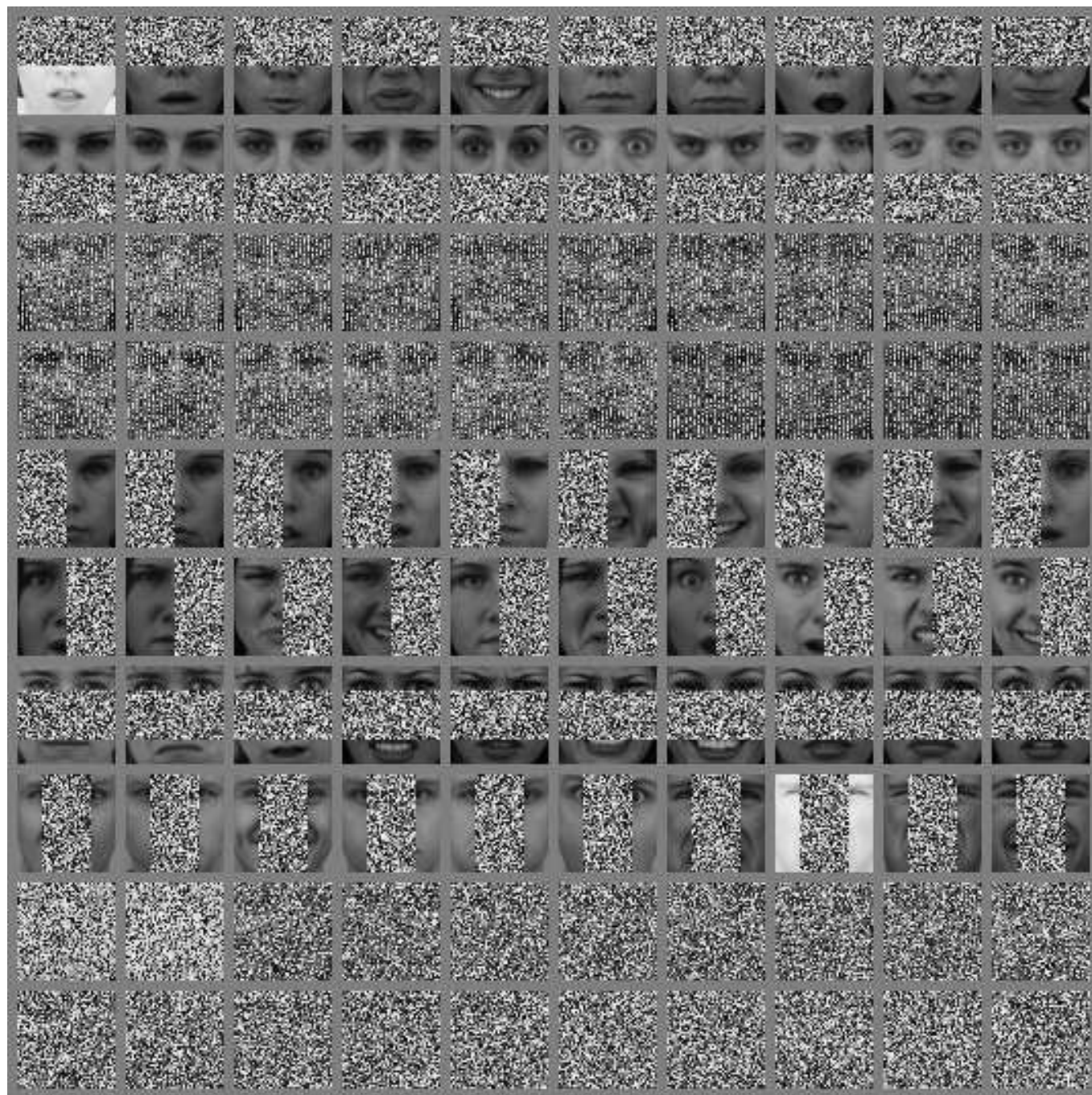
factorized prior

$$P_H(h) = \prod_i P_{H_i}(h_i)$$





NICE  
Inpainting  
Movies  
(not  
conv.)





# Unfolding AND Disentangling

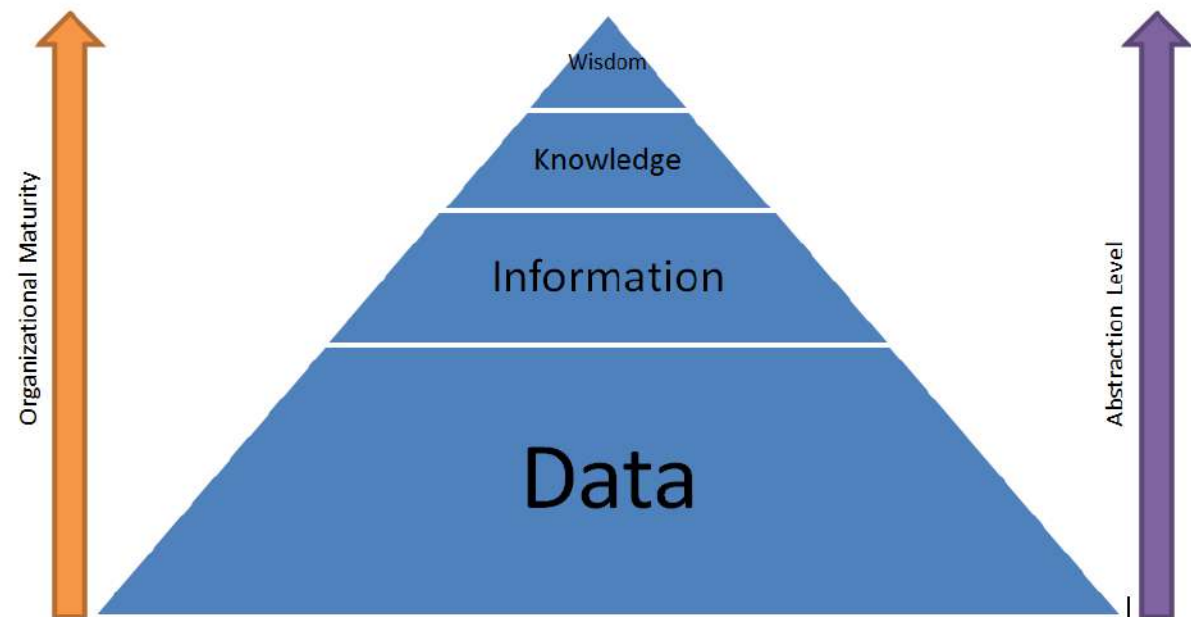
- The previous criteria may allow us to unfold and flatten the data manifold
- What about disentangling the underlying factors of variation?
- Is it enough to assume they are marginally independent?
- They are not conditionally independent...
- There may be intrinsic ambiguities what makes the disentangling job impossible → need more prior knowledge.

# Broad Priors as Hints to Disentangle the Factors of Variation

- *Multiple factors*: distributed representations
- Multiple levels of abstraction: *depth*
- *Semi-supervised* learning: Y is one of the factors explaining X
- *Multi-task* learning: different tasks share some factors
- *Manifold* hypothesis: probability mass concentration
- Natural *clustering*: class = manifold, well-separated manifolds
- Temporal and spatial *coherence*
- *Sparsity*: most factors irrelevant for particular X
- *Simplicity* of factor dependencies (in the right representation)

# Learning Multiple Levels of Abstraction

- The big payoff of deep learning is to allow learning higher levels of abstraction
- Higher-level abstractions disentangle the factors of variation, which allows much easier generalization and transfer



# Conclusions

- Deep Learning has become a crucial machine learning tool:
  - Int. Conf. on Learning Representation 2013 & 2014 a huge success!  
Conference & workshop tracks, open to new ideas ☺
- Industrial applications (Google, IBM, Microsoft, Baidu, Facebook, Samsung, Yahoo, Intel, Apple, Nuance, BBN, ...)
- Potential for more breakthroughs and approaching the “understanding” part of AI by
  - Scaling computation
  - Numerical optimization (better training much deeper nets, RNNs)
  - Bypass intractable marginalizations and exploit broad priors and layer-wise training signals to learn more disentangled abstractions for unsupervised & structured output learning

LISA team: **Merci! Questions?**

