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STRUCTURE OF ATOMIC NUCLEI AND NUCLEAR TRANSFORMATIONS

BY

G. GAMOW

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CONSTITUTION OF ATOMIC NUCLEI
AND RADIOACTIVITY

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PREFACE

DURING the five years which have passed since the first edition of this book the most astonishing developments have taken place both in the theoretical and in the experimental branches of nuclear physics The discovery of the neutron completely changed our views on the nature of the fundamental constituents of heavy nuclei: it made possible the general statistical model of Heisenberg in which heavy particles only are considered On the other hand, the discovery and investigation of induced negative and positive β-activities, the closer study of the continuous β-spectra, and a formulation of the theory of β-disintegration as developed by Fermi gave us a new insight into the relation of heavy and light elementary particles and the emission of electrons by radioactive nuclei Finally, widespread experimental research into different types of artificial nuclear transformation has brought this part of nuclear physics to a stage almost comparable with that of our present knowledge of molecular reactions, thus providing a large amount of new information concerning the interior of the nucleus and throwing some light on the origin of the chemical elements in the universe

All this made it necessary to rewrite the whole book completely, increasing its volume more than twice, though even this would still be too small an increase if all the available evidence concerning nuclear structure and transformations were to be discussed In fact, whereas in writing the first edition the author felt unhappy because there were not enough experimental facts and theoretical calculations, he now feels that there are perhaps too many.

Something must be said at the outset about the general development of nuclear theories In the case of the simplest nuclei such as the deuteron ($_1H^2$), and less rigorously for the third isotope of hydrogen ($_1H^3$) and the two isotopes of helium ($_2He^3$ and $_2He^4$), the appropriate wave equation for the two-, three-, or four-body problem may be solved by fairly straightforward methods. Such calculations, which are based on the assumption of strong attractive forces between elementary heavy particles, decreasing rapidly with increasing distance, lead to results which are in good agreement with experiment (evidence from mass defects, photoelectric disintegration of the deuteron, *scattering and recombination in collisions* between neutrons and protons, etc) We might even conclude that the theory of the simplest nuclei is not much more difficult than the theory of the simplest atoms

On the other hand, the theory of complex nuclei presents many more difficulties for mathematical solution We have to deal here with a large collection of particles interacting with one another through very strong forces this makes the problem much more complicated than that of a complex atom, in which the interaction between any two electrons may be considered as small compared with the forces due to the central body Briefly, the theory of complex nuclei is difficult mathematically precisely because everything is of the same order of magnitude and nothing can be neglected, in this respect it is analogous to the theory of liquids in which, as is well known, all attempts to follow a strictly rigorous treatment have practically been abandoned

Nevertheless there are many features of the spontaneous and induced transformations of complex nuclei which do not depend upon factors very closely connected with the nuclear interior They may be discussed with sufficient accuracy in terms of potential barriers corresponding to Coulomb forces, and with the help of the principle of the conservation of angular momentum (involving an additional potential barrier for centrifugal forces) Such considerations are more or less independent of any particular conceptions regarding internal structure Originally, whenever the attempt was made to take structural considerations into account, it was always assumed that each separate nuclear particle could be considered as moving in the average statistical field due to the rest of the nucleus Now the applicability of this treatment, which is very successful in the case of complex atoms (methods of Thomas-Fermi and of Hartree), is very questionable where complex nuclei are concerned As already indicated, the difference lies in the strong interaction and exchange of energy between the constituent parts of a heavy nucleus

In his recent critical investigation of this subject Bohr (*Nature*, **137** (1936), 344) especially stresses this point of view. According to him the division of a complex nucleus into 'the particle in question' and 'the rest of the nucleus'—represented by a certain distribution of potential energy—is not suited to the proper description of nuclear phenomena Instead, in the treatment of complex nuclei, we should take into account all particles, continuously interchanging the available energy amongst themselves. Thus the process of substitutional transformation, for example, should be considered not as the direct single collision of an incident particle entering the nucleus with one of the nuclear particles, resulting in the ejection of the latter, but rather as the capture of the incident particle, in such a way that all its kinetic energy is distributed amongst the other nuclear particles, followed by the splitting of the nucleus into two (or more) parts, in accordance with the available energy and the probability of such splitting (as defined by the corresponding potential barrier).

This point of view is of particular importance for the processes of collision between complex nuclei and neutrons, where, on account of the absence of Coulomb potential barriers, the probability of reaction is entirely determined by intranuclear conditions As we shall see, collisions between complex nuclei and neutrons are just those for which calculations, made on the basis of the simplified representation of the nucleus by a 'potential hole', are in disagreement with experimental evidence, failing, for example, to explain the absence of resonance phenomena in the scattering of slow neutrons in spite of the occurrence of such phenomena in respect of radiative capture (Chap XI) Bohr has, however, succeeded in showing, by means of general considerations, that this disagreement can be removed if we treat nuclear processes in the more complicated way indicated above

According to this new radical point of view most of the previous calculations concerning nuclear processes must be abandoned or considerably changed At present, however, rigorous mathematical methods for treatment of the mechanical system with very strong mutual interactions are lacking, the possibility being not excluded that such methods will never be available in a sufficiently simple form. In view of this fact it may be considered as still permissible to use to a certain extent the old methods of calculation, remembering, however, that the results obtained in this way may on certain points be very rough indeed As the manuscript of the present book was essentially finished before the critical investigations of Bohr appeared, only very few applications of the new treatment could be included in the text

In the present book no attempt has been made to go rigorously through all the mathematical calculations concerning different nuclear processes, the main aim being to consider questions of principle concerning nuclear structure and to understand the different nuclear processes from the point of view of the present quantum theory

Returning to the experimental results. they are given with sufficient completeness in the book to provide a good idea of nuclear properties and a general basis for theoretical considerations, but no attempt has been made to include all the material at present available For further details concerning experimental methods, as well as for particular facts, the reader is referred to *An Introduction to Nuclear Physics*, by N Feather (Camb Univ Press, 1936)

Finally, the author considers it his pleasant duty to express his gratitude to Dr Feather for his important help in the preparation of the present book, both in the matter of the revision of experimental data and also as regards orthography and conventional idiom Thanks are also due to Drs Blackett, Dee, and Feather, who supplied the cloud-chamber photographs of various nuclear transformations

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PLATES

VALUES OF PHYSICAL CONSTANTS

Velocity of light	$c = (2\cdot99796 \pm 0\cdot00004) \times 10^{10}$ cm. sec $^{-1}$
Quantum constant	$\hbar = \dfrac{h}{2\pi} = (1\cdot0420 \pm 0\cdot0013) \times 10^{-27}$ erg sec.
Newton's constant of gravitation	$K = \kappa c^2 = (6\cdot664 \pm 0\cdot0005) \times 10^{-8}$ cm ^{3} gm $^{-1}$ sec $^{-2}$
Elementary charge	$e = (4\cdot770 \pm 0\cdot005) \times 10^{-10}$ abs E S U
Mass of electron	$m_e = (9\cdot035 \pm 0\cdot010) \times 10^{-28}$ gm
Mass of proton	$M_p = (1\cdot6609 \pm 0\cdot0017) \times 10^{-24}$ gm
Mass of neutron	$M_n = (1\cdot6622 \pm 0\cdot0020) \times 10^{-24}$ gm
Atomic magneton	$\mu_e = \dfrac{e\hbar}{2mc} = 0\cdot917 \times 10^{-20}$ erg gauss^{-1}
Nuclear magneton	$\mu_{\text{nuc}} = \dfrac{e\hbar}{2Mc} = 0\cdot497 \times 10^{-23}$ erg gauss^{-1}
Reciprocal of fine-structure constant	$\dfrac{1}{\alpha} = \dfrac{\hbar c}{e^2} = 137\cdot29$
Classical radius of electron	$r_e = \dfrac{e^2}{mc^2} = 2\cdot8 \times 10^{-13}$ cm
Radius of normal hydrogen atom	$r_a = \dfrac{\hbar^2}{me^2} = 0\cdot53 \times 10^{-8}$ cm
Compton wave-length for electron	$\Lambda_e = \dfrac{2\pi\hbar}{mc} = 2\cdot4 \times 10^{-10}$ cm
Compton wave-length for proton	$\Lambda_p = \dfrac{2\pi\hbar}{Mc} = 1\cdot3 \times 10^{-13}$ cm

To transfer:	**multiply by:**
Electron Volts into *Ergs*	$1\cdot5911 \times 10^{-12}$
Ergs into *Electron Volts*	$0\cdot6285 \times 10^{+12}$
Mass Units† into *Ergs*	$1\cdot483 \times 10^{-3}$
Ergs into *Mass Units*	$0\cdot674 \times 10^{+3}$
Mass Units into *Electron Volts*	$0\cdot932 \times 10^{+9}$
Electron Volts into *Mass Units*	$1\cdot073 \times 10^{-9}$

Wave-length of light

$$= \frac{1\cdot9628 \times 10^{-16} \text{ cm.}}{\text{Energy of quanta (in ergs)}} = \frac{1\cdot2336 \times 10^{-4} \text{ cm}}{\text{Energy of quanta (in el. volts)}}.$$

† 'Mass unit' is defined as $\frac{1}{16}$ of the O^{16} mass.

PART I

STABLE NUCLEI

I

ELEMENTARY PARTICLES AND THE CONSTITUENT PARTS OF ATOMIC NUCLEI

1. Introduction

SINCE 1911, when Rutherford first put forward the nuclear model of the atom, research in atomic physics has progressed in two different directions

For the one part physicists have been occupied with the investigation of the arrangement and motion of the extranuclear electrons and the explanation of the main physical and chemical properties of the elements along these lines Through the efforts of Bohr and his school the quantum theory of atomic structure has been brilliantly developed Now, supplemented by quantum mechanics, as worked out by Heisenberg, Schrödinger, and Dirac, it is practically complete —in so far as its formal aspects are concerned. On the other hand, increasing attention has been paid to the problem of the nucleus It became clear at an early stage that the nucleus could not be regarded as an elementary particle, in its turn it had to be assigned a rather complicated structure The discovery of radioactive phenomena and their interpretation, due chiefly to Rutherford, in terms of the spontaneous disintegration of atomic nuclei showed that the nuclei of certain heavy elements, at least, are unstable. These change naturally into lighter nuclei with the emission of particles and the liberation of large amounts of energy Rutherford also succeeded, as early as 1919, in producing in a number of light elements, 'artificial' transformations of usually stable nuclei by bombarding them with fast particles. In this way a new and very productive epoch in physics began, and for the first time it became possible to obtain direct information concerning the internal structure of nuclei.

The first requisite for a theory of nuclear structure is a knowledge of the elementary constituent parts from which the nuclei of the different elements are built up. Originally, electrons and protons were regarded as the only possible fundamental units and, on this assumption, Gamow attempted to develop a satisfactory nuclear model. He assumed that whenever possible α-particles are formed within the nucleus (each α-particle containing four protons and two electrons), so that not more than three protons (0, 1, 2, or 3 according to the particular nucleus) are present in a free state. This is a natural assumption since in general the binding energy of an α-particle $(-{}_2He^4+4\,{}_1H^1+2e = 26{\cdot}8\times10^6$ e.v.) is large compared with the energy of binding of the α-units in the nucleus (for example, for carbon, the total binding energy for three α-particles would be $6{\cdot}7\times10^6$ e.v.). The presence of a certain number of unbound electrons (up to 28 for the heaviest nuclei) must also be admitted. On this picture, for example, the constitution of the aluminium nucleus ${}_{13}Al^{27}$ is $6\alpha+3p+2e$. Calculations of binding energies were made by considering as a first approximation the heavy nuclear constituents only and postulating, in addition to repulsive forces, forces of attraction between them decreasing very rapidly with increasing separation. The model then possesses many points of analogy with a small drop of liquid in which the distribution of density and potential is almost uniform, except that both quantities vary rapidly through the surface layer of the drop. On this model the nuclear radius is proportional to the cube root of the mass of the nucleus, and calculation shows that the total energy of the system decreases at first with increasing mass, passes through a minimum, and finally increases again when the effect of the repulsive Coulomb forces becomes all-important. In this calculation the electrons were omitted merely for sake of simplicity, but it must not be forgotten that serious difficulties, both theoretical and experimental, are encountered by any theory which assigns to them individual existence in the nucleus. For the fundamental relation

$$\Lambda = 2\pi\hbar/p, \tag{1}$$

between the de Broglie wave-length Λ of a particle and its momentum p, may be employed to restate the condition of applicability of the ordinary non-relativistic wave equation ($p \ll mc$), in the case of a particle of mass m moving in a region of geometrical dimensions l, in the form

$$l \gg 2\pi\hbar/mc. \tag{1'}$$

Now, when the protonic mass is substituted for m in the right-hand side of (1′) a value $1{\cdot}3\times10^{-13}$ cm. is obtained—roughly an order of magnitude smaller than the accepted values of nuclear radii—but when the electronic mass is employed these accepted values are exceeded by a factor of several hundreds. It must be concluded, therefore, that whilst a non-relativistic treatment is sufficient for protons or heavier particles, since these particles move inside the nucleus with velocities small compared with the velocity of light, nuclear electrons approach very near to that limiting velocity and so require a relativistic quantum theory for the description of their motions. Such a generalization of wave mechanics for the relativistic case was given by Dirac and has proved very useful in the description of many relativistic features of electron motion, notably those which are involved in the theories of the fine structure of spectral lines, the magnetic moment of the electron and the nature of the positron, but it is of no help in the present situation For, in Dirac's generalization, as in ordinary wave mechanics, electrons are considered as mathematical points in which the whole of the charge is concentrated. However, energy considerations show that this charge must be distributed throughout a finite region of which the minimum dimension is given by the so-called classical radius of the particle,

$$r_{\text{class}} = e^2/mc^2. \tag{2}$$

If the dimensions of the region in which the particle moves are not large compared with this, unknown effects, due to the interpenetration of the particles in one another's structure, will arise Thus the limit of applicability of any theory which neglects the charge distribution is given by the condition

$$l \gg e^2/mc^2 \tag{2'}$$

Substituting numerical values, the classical radii of the proton and the electron are obtained as $1{\cdot}5\times10^{-16}$ cm and $2{\cdot}8\times10^{-13}$ cm, respectively, and, of these, the classical electron radius is the same as the distance between nuclear particles For the description of electron behaviour within atomic nuclei, therefore, a more elaborate yet unknown theory is still required It is hardly likely that it can be obtained, on the basis of the general ideas of wave mechanics, by some more subtle relativistic generalization than that given by Dirac, since it has been shown that Dirac's equations represent the only possible relativistically invariant wave equations fulfilling the fundamental conditions of quantum theory It appears that future theory must give up the description of the electron as a particle in favour of some other description which may well differ in point of view from Dirac's present theory as radically as the modern wave mechanics differs from the old postulates of Bohr

The foregoing have been theoretical considerations; from the more directly experimental side it should be noted that in all processes in which nuclear electrons are intimately involved, extraordinary features are exhibited, some of which appear to contradict notions which are common to the classical and relativistic wave mechanics of a particle. Certain of these features will be treated more fully in later chapters.

Whilst, therefore, nuclei were regarded as constituted of heavy particles and electrons with strong mutual interaction between them, it had to be admitted that nothing could be said concerning the behaviour of the electrons involved It might reasonably have appeared impossible, on account of this restriction alone, to have put forward any satisfactory theory of nuclear processes whatever, but strangely enough, this was far from the case. For it proved possible to describe all processes in which electrons were not immediately concerned in terms of a non-relativistic theory which completely ignored any influence which they might have upon phenomena Successful descriptions were offered of such processes as the emission of heavy particles or radiation from nuclei in spontaneous or conditioned ('artificial') transformations Thus an absence of intrusion of nuclear electrons in heavy particle phenomena was set in evidence which naturally called for an explanation It was suggested that the 'disobedient' electrons might be tightly bound to heavy particles, to be set free only in the processes of β disintegration Actually, this came very near to what is currently accepted for the truth.

Another difficulty for the hypothesis that nuclei are constituted of α-particles, not more than three protons, and electrons, arises from a study of the experimental values of nuclear binding energies † These may be calculated, for any assumed nuclear structure, from the exact atomic masses of the different species In Table B‡ is given the complete list of the nuclei for which exact mass determinations have been made by Aston and by other investigators The binding energies calculated on the hypothesis that the maximum possible number of α-particles N_α is present in each nucleus are plotted against this N_α in Fig 1 It is immediately obvious that any curve which may be drawn through the points presents unexpected peculiarities in the region which belongs to the heavier elements. Here a general increase of internal nuclear energy with mass number is observed, but more detailed examination shows that the

† G. Gamow, *Proc. Roy. Soc* **126** (1930), 632. ‡ At the end of the book.

region in question may be resolved into a number of smaller domains in each of which the nuclear energy decreases as the mass number of the nucleus increases. The portion of the figure occupied by the heavy radioactive elements forms the only exception to this rule, showing increase of energy with increase of mass number. That the general trend of the second half of the curve is similar to that in the radioactive region is due, as is evident from the figure, to a number of sudden jumps in nuclear energy at definite points in the curve. These jumps must receive further consideration, they are the peculiarities which have already been referred to If they are disregarded,

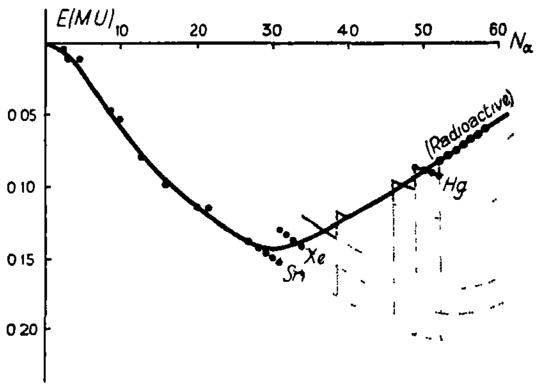


Fig 1

then throughout the whole of Fig 1, except for the radioactive elements, the internal energy of the nucleus decreases as the mass number increases, to extract a heavy particle from any of these nuclei then requires that work shall be done on the system All the species are stable species, therefore, for this type of change. Likewise the figure exhibits the instability of the radioactive nuclei against α-particle emission The discontinuities, on the other hand, do not correspond to known instances of instability in elements of mass number less than 207. To this extent they are anomalous and unexplained Rutgers† pointed out that they would not occur at all if the initial assumption of the maximum possible number of nuclear α-particles were suitably modified For, if two alternative constitutions be postulated for a given nucleus, identical in every respect, except that one of the α-particles in one structure is replaced by

† A. I. Rutgers, *Nature*, 5 March 1932.

four protons and two electrons in the other, then the binding energy calculated on the basis of the first assumption is obviously less than that corresponding to the second by the energy of binding of protons and electrons to form an α-particle Now this binding energy is known ($26{\cdot}8\times10^6$ e v) and corresponds fairly closely to the magnitude of the energy jumps in Fig 1 The explanation appeared reasonable, therefore, that the discontinuities under discussion are evidence for the failure of the maximum number of α-particles to form at certain stages of the nuclear synthesis No reason was suggested for this failure, however, and the explanation remained merely a formal one From the experimental side, whilst the bulk of evidence favoured the suggestion, the data were neither sufficiently precise nor complete to determine all the species in which a fresh α-particle might be regarded as failing to form, nor, in particular, to fix the lightest species in which the effect occurred. The descending energy curve characteristic of the lighter elements of itself tends to mask possible discontinuities of the type in question

At this stage the difficulties of a particular nuclear model have been explored and hypotheses noted which were suggested in an attempt to get rid of them A much more natural general explanation was forthcoming when the existence of another nuclear particle—the neutron—was proved. This explanation will now be considered.

2. The neutron-proton nuclear model

Early in 1932 it was shown by Chadwick that the very penetrating radiation emitted by beryllium and certain other light elements when bombarded by α-particles consists of electrically neutral material particles of mass about equal to the mass of a proton These particles have been called neutrons Since neutrons were obtained from a great many light elements it was natural to assume that they play an important part in nuclear structure. Introducing neutrons as nuclear constituents it became possible, as will now appear, to eliminate the two main difficulties which the earlier model encountered —and to achieve this a model constructed from neutrons and protons only was proposed On this model the number of protons, N_p, is evidently given directly by the atomic number Z, and the number of neutrons, N_n, by the difference between the mass number A and the atomic number. Taking, as previously, the aluminium nucleus $_{13}Al^{27}$ for example, its constitution is now $13p+14n$ The nucleus thus being formed from heavy particles only, it is evident that the ordinary wave mechanics may be applied to the description of all nuclear processes in which the emission (or absorption) of electrons does not take place. Conversely, when these processes do occur, they must be regarded as resulting from the transformation of a neutron into a proton (or vice versa) inside the nucleus ($n \rightarrow p+e$; $p \rightarrow n+\partial$) —admitting that for their detailed understanding a further development of theory is necessary

The difficulty concerning nuclear binding energies also disappears when the neutron-proton model is used, as was pointed out by Professor D. D Ivanenko † In fact, if α-particles be introduced into the new scheme as subsidiary nuclear units, as before, and the maximum possible number of such particles (constituted of two protons and two neutrons) be postulated, stable‡ nuclei must be regarded as constituted of α-particles, neutrons, and not more than one proton. For the aluminium nucleus ${}_{13}Al^{27}$, for example, the structural formula becomes $6\alpha+2n+1p$ In this case the number of α-particles is the same as was given by the original hypothesis (p 4) For heavier nuclei, on the other hand, the neutron-proton model does not admit of the formation of as many α-particles as the electron-proton model allows For bismuth, ${}_{83}Bi^{209}$, in particular, instead of $52\alpha+22e+1p$ the new model gives $41\alpha+44n+1p$. Here is the natural interpretation of the 'dissociation' of α-particles in certain heavy nuclei, which appeared to be indicated by the binding energy curve In Fig. 2, binding energies calculated on the new basis are plotted against the mass number and, for ease of comparison, the curve of Fig. 1 is also reproduced (dotted line). In the new plot no marked discontinuities appear and the energy curve descends monotonically until the region corresponding to the heavy radioelements is reached

In order to explain the stability of nuclei built up from neutrons and protons it is necessary, of course, to admit the existence of certain attractive forces between these constituent particles which overbalance, at small distances, the electrostatic repulsive forces between individual protons It was shown by Heisenberg|| that the relative numbers of neutrons and protons in existing nuclei (as evidenced in the relation between mass number and atomic number) provide data

† D. D. Ivanenko, *Nature*, **130** (1932), 892

‡ For these species $A-2Z \geqslant 0$

|| W. Heisenberg, *Zs f. Phys.* **77** (1932), 1, **78** (1932), 156, **80** (1933), 587.

from which the comparative values of these attractive forces (neutron-neutron, neutron-proton, and proton-proton interactions) may be deduced For, consider nuclei with the same total number of particles ($N_p+N_n = A =$ constant), but different atomic numbers. Then, since $N_n/N_p = (A-Z)/Z$, the ratio of the numbers of neutrons and protons will be different from one of these nuclei to the next. Let the mutual potential energies corresponding to the attractive forces for the three above-mentioned combinations of particles be $-I_{nn}(\bar{r})$, $-I_{np}(\bar{r})$, and $-I_{pp}(\bar{r})$, respectively,† where $\bar{r}$ is the average distance

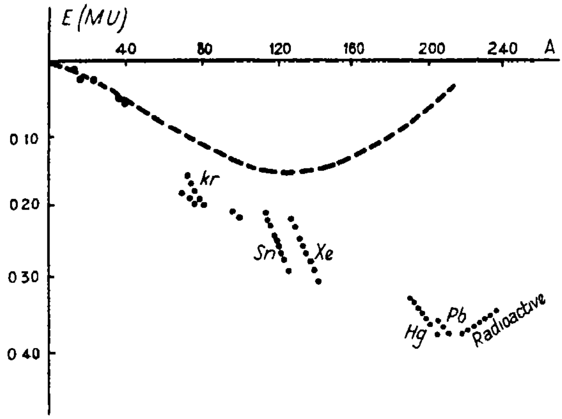


FIG. 2

between such pairs of particles in the nucleus The potential energy due to the Coulomb repulsion between two protons is then $+e^2/\bar{r}$. The relative number of neutrons and protons for the most stable nucleus of given mass number will be determined by the condition that, for A constant and Z varying, the nuclear binding energy is a maximum (internal energy of the nucleus a minimum) Now, considering each of the three interactions, n-n, n-p, p-p, separately, since the contribution to the total intra-nuclear energy from each is negative, it is obvious that, in isolation, each would tend to increase the number of corresponding pairs of particles in the nucleus Thus the n-n interaction would tend to increase the number of neutrons up to the limit $N_n = A$, N_p $(\equiv Z) = 0$. Similarly, on account of the p-p interaction acting alone, the number of protons would increase until $N_n = 0$, N_p $(\equiv Z) = A$; whilst, finally, for the

† The negative sign is used to emphasize the attractive character of the forces.

n-p attraction alone, the state of least energy would obviously be given by $N_n = N_p$, $Z = A/2$. The repulsive Coulomb forces between protons, on the other hand, being associated with positive potential energy, tend to diminish the number of protons in any nucleus. Since, for existing nuclei, the numbers of nuclear neutrons and protons are approximately equal,† it is evident that the main part of the nuclear binding energy cannot be ascribed to n-n interaction, otherwise neutrons would greatly predominate. Thus n-p forces play the predominating role in the nuclear structure, while the n-n and p-p forces are either vanishingly small or approximately equal to each other, thus showing no effect on the relative number of the two kinds of nuclear particles. In fact we shall see later that the forces between like particles amount to roughly a half of neutron-proton forces The relative increase in the number of neutrons in heavier elements may then be due either to the predominance of n-n forces over p-p forces, or to the Coulomb repulsion between protons (Coulomb repulsion predominating over p-p attraction at the mean distance $\bar{r}$) The latter alternative appears preferable since the existence of large Coulomb forces are required to explain the details of the spontaneous emission of positive particles (α-particles) by heavy nuclei The above conclusions are summarized, therefore, by the inequalities

$$I_{np}(\bar{r}) > I_{nn}(\bar{r}), I_{pp}(\bar{r}); \quad I_{nn}(\bar{r}) \cong I_{pp}(\bar{r}) \tag{3}$$

They are in good agreement also with more particular evidence, that supplied by the simplest existing nuclei Here the combination of one neutron and one proton represents a stable nucleus (${}_1H^2$, deuteron), whereas the forces between two neutrons or two protons are unable to keep these particles together and the systems ${}_0n^2$ and ${}_2He^2$ are, hitherto, unknown

3. The nature of the neutron-proton interaction

It has already been mentioned that it is necessary in describing β-disintegration to speak of the transformations of a neutron into a proton, or of a proton into a neutron, with the simultaneous emission of an electron or a positron, as the case may be. In this

† $Z = A/2$ for even mass numbers less than 40, and, although this relation then becomes less exact, even for the heaviest elements the discrepancy is not more than about 30 per cent.

connexion the question has frequently been raised whether the neutron should be considered as a complex particle, built up from a proton and an electron, or a proton regarded as formed from a neutron and a positron. If the first point of view be accepted, then the emission of a positron must be regarded as a complex process consisting in the formation of an electron pair ($e+\partial$) and the subsequent binding of the negative electron to a proton Accepting the second point of view an analogous explanation must be given for the emission of a negative electron. In either case the explanation is not very satisfactory, on this account.

It must be remembered, however, that the difficulties already discussed (§ 1), concerning the description of electron behaviour in a restricted region of space, still apply, and great care must be exercised in the formulation of any question regarding the complexity of neutron or proton in such models, which are themselves based upon old conceptions. Naturally, nothing can be stated with certainty, before a complete theory has been worked out, concerning the structure and mutual transformations of neutrons and protons, but it seems very likely already that the question 'which of these two particles is the simple one?' has no physical meaning Rather, neutrons and protons should be considered merely as two different states of the 'fundamental heavy particle', subject to transformations from one to the other with the emission or absorption of energy and the liberation of (negative or positive) charge

From such a point of view it appears reasonable, according to Heisenberg† and Majorana,‡ to consider the attractive force between neutron and proton as due to the exchange of electric charge between them in the same general way as in Heitler and London's theory of the interaction between hydrogen atom and hydrogen ion || The analogy must not be followed too closely, however, since the behaviour of electric charge in the present problem cannot be described by the same simple set of wave equations as in the case of interaction between atoms On the other hand, it gives immediately

† W Heisenberg, l c

‡ E Majorana, *Zs. f Phys* 82 (1933), 137

|| In this theory the forces between H and H^+ are explained as due to the exchange of an electron between the two hydrogen nuclei If the frequency of exchange is $\omega(r)$, for a distance r between the nuclei, the additional energy due to this exchange will be $h\omega$ As shown by Heitler and London, the variation of exchange energy with distance corresponds, in this particular case, to an attractive force very rapidly decreasing with increasing distance

certain results which are in good agreement with known facts concerning nuclei. The most important property of such exchange forces is that, unlike ordinary forces (the introduction of which into nuclear theory was attempted by Wigner†), they show the phenomenon of saturation, each particle being able to interact only with a given number of other particles This property of exchange forces is extremely important for the construction of a nuclear model, as otherwise it would be impossible to explain the well-established facts that nuclear radii vary approximately in proportion to the cube root of the mass number (the nuclear density being constant) and that the nuclear binding-energy is represented roughly by a linear function of the mass number. To illustrate this, let us consider the nucleus as constructed from a given number N of particles, interacting with ordinary forces If r_0 is the nuclear radius, the quantized velocity of each particle will be inversely proportional to r_0, and the total kinetic energy of the system will be proportional to N/r_0^2. If now a is the range of forces between nuclear particles (analogous to the 'radius of action' of molecules in liquids) each particle will interact with all other particles inside the volume a^3, so that the total potential energy of the system will be proportional to $-N^2I(a)a^3/r_0^3$, where $I(a)$ gives the average interaction potential up to the distance a between the particles The last expression changes at least as rapidly as $1/r_0^3$ (for $r_0 > a$) and it can easily be seen that no stable state (minimum of total energy) can be reached so long as $r_0 > a$. This leads us to the wrong conclusions that all nuclei have the same radius (equal to the range of the nuclear forces) and that the binding energy increases at least as fast as the square of the nuclear mass In order to obtain correct results it would be necessary to introduce a strong repulsion between nuclear particles at small distances ('rigid radii'), and this seems rather an inelegant assumption

The introduction of exchange forces showing the phenomenon of saturation removes the above difficulty, as now, whatever the number of particles inside the 'sphere of action', the particle in question can interact only with a given number of other particles The number of protons with which a given neutron can interact and vice versa (the number of possible valency-bindings between these two particles) depends on the special assumptions concerning the type of exchange which takes place between these two particles In his original

† E Wigner, *Phys. Rev.* 43 (1933), 252.

hypothesis Heisenberg assumed (by analogy with molecular interactions) that only the exchange of electric charge takes place between neutrons and protons (Fig. 8a) which leads to a single valency between these two particles. This hypothesis was modified by Majorana who supposed that both charge and spin of the particles are exchanged (Fig. 8b) in which case one obtains a double valency, each neutron being able to bind two protons and vice versa From the first hypothesis it would follow that a deuteron ($n \longleftrightarrow p$) would represent a saturated shell, whereas according to the second hypothesis the shell will be completed only for an α-particle (Fig 3). The experimental fact that α-particles and not deuterons possess an exceptionally high stability speaks definitely in favour of Majorana's type of exchange.

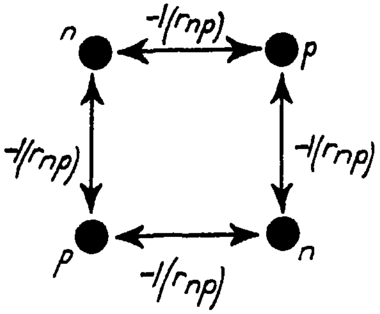


FIG. 3

4. The possible existence of negative protons†

After the discovery of the positive electron, which was predicted in Dirac's theory, the dissymmetry previously existing in the problem of the elementary particle has practically disappeared. It is true that many more negative electrons than positive electrons are observed in our common experience, but this may now be regarded as merely the local peculiarity of the part of the universe in which we are living. Dissymmetry still exists, however, in the matter of heavy particles, in order to get rid of which it would be necessary to postulate the existence of negative protons in addition to neutrons and positive protons These negative protons would be symmetrical to positive protons with respect to neutrons, their relation to ordinary protons would not, it will be shown, be analogous to that between positrons and electrons on Dirac's theory This conclusion may be reached as follows. Bohr has shown that Dirac's equations may be applied—with all the consequences which follow from them—to any particle, only if its radius is small compared with the length $2\pi\hbar/mc$, where m is the mass of the particle in question. This condition is easily fulfilled for an electron, as in this case $2\pi\hbar/mc = 2{\cdot}4\times10^{-10}$ cm.,

† G. Gamow, *Phys. Rev.* **45** (1934), 728, *Nature*, **135** (1935), 858

a value much larger than the electron radius. On the other hand, inserting the proton mass, $2\pi\hbar/Mc = 1\cdot3\times10^{-13}$ cm., and here the radius of the free proton is most probably of the same order of magnitude.† This means that a proton may not be considered as an elementary particle from the point of view of Dirac's theory—a conclusion in good agreement with the currently accepted assumption that a proton may split into a neutron and a positron, if subject to sufficiently strong fields. It implies, in addition, that certain other consequences of Dirac's theory do not necessarily apply: the magnetic moment of the proton need no longer be smaller than that of the electron in the ratio of the masses of these particles (actually, experiments by Stern and others show that it is not), and there is no longer any reason for the existence of 'Dirac's holes' for protons, with the connected phenomena of annihilation.

Thus the negative proton is to be regarded—if its existence be assumed—merely as an independent particle subject to intertransformation with a neutron involving the emission of an electron or a positron, as the case may be. The possible transformations between neutrons and protons (positive and negative) are then summarized by the following scheme (Fig. 4):

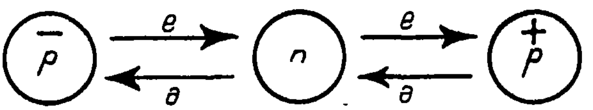


FIG. 4

As concerns the interaction between the newly introduced particle and other particles, general symmetry relations indicate that the forces between a negative proton and a neutron, and between two negative protons, must be identical with the corresponding forces for positive protons. Thus $I_{np-} = I_{np+}$ and $I_{p-p-} = I_{p+p+}$. Concerning the interaction between a positive and a negative proton, however, symmetry relations provide no information whatever, and a special hypothesis is required for the form of the potential energy I_{p+p-}.‡ Some information may be obtained from considerations of general stability similar to those of § 2. It has there been concluded that two different factors operate to determine the A/Z ratio for

† Compare Chapter IX.

‡ This is the potential energy over and above that corresponding to the Coulomb attraction between the two charges.

stable nuclear species: an attraction between neutrons and protons, tending to make the numbers of these two types of particle equal ($Z = A/2$), and the Coulomb repulsion between protons, tending to diminish the number of protons. Through the interplay of these factors stable nuclei may result which contain slightly more neutrons than protons—as stable nuclei actually do. If, however, negative protons are admitted as nuclear constituents the situation becomes more complicated and the state of minimum energy will depend, amongst other things, upon the unknown forces corresponding to the potential I_{p+p-}. Let it be assumed as a trial hypothesis that these forces are so small that they may be neglected. Then, carrying through the original argument and including the forces corresponding to I_{np-}, I_{p-p-}, as well as the Coulomb forces, the stable state becomes that in which $N_n = A/2$, $N_{p+} = N_{p-} = A/4$ Here the number of neutron-proton (p^+ or p^-) bindings is a maximum and the Coulomb energy a minimum ($Z \equiv N_{p+} - N_{p-} = 0$) In such a case, for each value of A the most stable state would correspond to $Z = 0$, and all nuclear species would be isotopes of the neutron Since this is contrary to known facts it is obvious that the initial hypotheses require revision If the hypothesis of a negative proton be not abandoned, therefore, the assumption that I_{p+p-} is negligible is evidently incorrect To assign to this potential energy a negative sign (attraction at small distances between negative and positive protons) merely aggravates the situation, the only possible assumption is that of a strong repulsion between these particles (positive potential energy) overcoming their Coulomb attraction when the distance is small Only in this case will the equilibrium number of negative protons in any nucleus remain within reasonable limits

An interesting consequence of the introduction of negative protons as nuclear units is the possibility of constructing isomeric nuclei, with both atomic number and mass number the same for each, differing one from the other in the matter of internal structure only. This would occur, for example, if a pair of neutrons were replaced by two protons, one positive and the other negative Such isomeric nuclei would differ, of course, as regards binding energy and spin, and, if they were unstable species, in their mode of disintegration also. The transformation of one isomer into the other (following the internal process $n+n \rightleftarrows p^+ + p^-$), with liberation, or absorption, of energy in the form of γ-radiation, should likewise be considered a possibility, although the probability of such a spontaneous (exothermic) double process may be shown (see Chap. VII) to be very small indeed. The corresponding half-value period might well be of the order of millions of years.

This suggestion of possible isomerism amongst nuclei appears at the present time to be definitely of use only in a limited number of cases. Nevertheless, experimental data are accumulating so rapidly to-day that it is well to bear in mind its potentialities in a wider field. Foremost amongst the well-attested cases is the β-β branching of UX_1, discovered many years ago by Hahn The accepted transformation scheme here is as follows

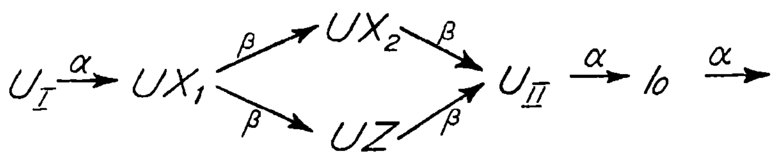


FIG. 5

According to this scheme the nucleus UZ is isomeric with UX_2, both having the same atomic number (91) and the same mass number (234) These two nuclei, however, have manifestly different properties the mean energy of the primary β-particles of UZ is very much less than that of the β-particles of UX_2, and the half-value periods are 6·7 hrs and 1 14 min , respectively It might be imagined, of course, that UX_2 is merely UZ in an excited state, but on any reasonable assumption it appears impossible to understand how such a state should persist for so long † If, on the other hand, these two nuclei be regarded as isomeric in the sense which has just been defined, no further difficulty arises, for the β-β branching at UX_1 may then be explained in terms of the two different possibilities for emission of a negative electron ($n \rightarrow p^{+}+e$ and $p^{-} \rightarrow n+e$)

A second possible example of nuclear isomerism may also be taken from amongst the heavy elements. If the report of Aston be finally confirmed it must be assumed that the species $_{82}Pb^{210}$ is present to the extent of 0·08 per cent in ordinary lead. This apparently stable

† To postulate a metastable state in this case is to encounter similar difficulties (cf. Chap IV), for, even if γ-radiation is prohibited, the excited nucleus will discharge its energy directly by interaction with one of the extranuclear electrons

species is then isomeric with the well-known β-active substance RaD It is possible that similar isomeric pairs will be found in which the unstable nucleus is known only through its production in conditioned transformations ('artificial radioactivity'). So far, however, no definite example has been accorded general acceptance.

II

NUCLEAR BINDING ENERGY AND STABILITY LIMITS

1. Structure of the simplest nuclei

OWING to the fact that we have not at present an exact expression for the interaction between neutrons and protons (function $I(r)$), the theory of even the simplest nuclei, such as the *deuteron* $_1H^2$ ($= n+p$), the nuclei $_1H^3$ ($=2n+1p$) and $_2He^3$ ($=1n+2p$), and the α-particle $_2He^4$ ($= 2n+2p$), cannot be worked out in such detail as is possible in the case of the hydrogen atom in atomic theory However, as was indicated by Wigner,† one can obtain very interesting results concerning the structure and binding energies of these nuclei by adopting rather general hypotheses as to the laws of interaction In fact, the values of binding energy obtained are somewhat insensitive to the exact details of the nuclear potential function employed, so long as the graphical representation is of the form of a narrow hole with more or less steep walls (rapid decrease of $I(r)$ with distance) Starting with the problem of the deuteron, Wigner took for the interaction energy between neutron and proton the expression

$$I(r) = I(0)\frac{4}{(1+e^{r/\rho})(1+e^{-r/\rho})}, \tag{1}$$

which fulfils the above-mentioned conditions and has the advantage that the corresponding wave equation possesses an analytic solution. Here $I(0)$ is the value of the potential energy for $r = 0$ and ρ corresponds to the half-breadth of the potential hole A graphical representation of such an interaction energy is given by the thin line in Fig 6 for the special values‡

$$\rho = 0{\cdot}050\times 10^{-12}\text{ cm}, \quad I(0) = 0\ 114\times Mc^2$$

(the reason for choosing these values will appear later) In Wigner's original treatment of the problem he assumes the forces between neutron and proton to be simply ordinary central forces As we have seen, however, this interaction must be considered, according to Majorana, as due to certain exchange forces originating in the exchange of charge and spin between the particles Thus the wave

† E Wigner, *Phys Rev* **43** (1933), 252

‡ Here and in the following pages the numerical values differ from those given in Wigner's original paper, they are calculated for the new value of the H^2 binding energy $= 0\ 0024$ M U, whereas Wigner accepted the old value 0·0016 M U

equation for a deuteron must be written in the form

$$\left[\frac{\hbar^2}{2M}(\nabla^2_{x_1}+\nabla^2_{x_2})+E\right]\Psi(x_1\sigma_1, x_2\sigma_2) = I(x_1-x_2)\Psi(x_2\sigma_1, x_1\sigma_2), \quad (2)$$

where x_1 and σ_1 are the space and spin coordinates of the neutron and x_2 and σ_2 the corresponding coordinates of the proton.†

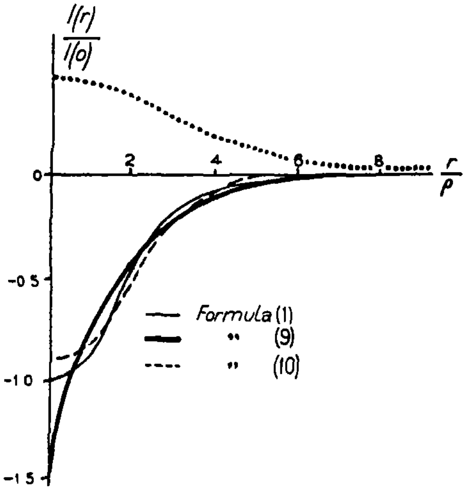


FIG 6

If, in (2), we make the substitution

$$\Psi(x_1\sigma_1, x_2\sigma_2) = \chi\left(\frac{x_1+x_2}{2}\right)\alpha(\sigma_1\sigma_2)u(r)P_{lm}(\theta,\phi), \quad (3)$$

(r,θ,ϕ) being spherical polar coordinates describing the relative positions of the particles and P_{lm} spherical harmonics of order l, we find that $u(r)$ must satisfy the equation

$$\frac{\hbar^2}{M}\left(\frac{1}{r}\frac{d^2}{dr^2}(ru)-\frac{l(l+1)}{r^2}u\right)+Eu = (-1)^l I(r)u. \quad (4)$$

It is obvious that the problem with Majorana's exchange forces is equivalent to the problem with ordinary central forces, except that for even values of l the forces are attractive and for odd values of l repulsive In so far as Wigner considers only the fundamental state of a deuteron, for which he adopts $l = 0$ (see, however, p. 61), the

† Assuming ordinary central forces between the particles (Wigner's original hypothesis) we should have, for the right-hand side of (2) $I(x_1-x_2)\Psi(x_1\sigma_1, x_2\sigma_2)$, whilst, with exchange of charge only (Heisenberg's hypothesis), for the same expression $I(x_1-x_2)\Psi(x_2\sigma_2, x_1\sigma_1)$.

problem here resolves itself into a simple case of a central force. Assuming $l = 0$, we obtain

$$\frac{\hbar^2}{M}\frac{1}{r}\frac{d^2}{dr^2}(ru)+Eu-I(r)u = 0. \tag{5}$$

The solutions of this equation for $I(r)$ given by (1) were investigated by Eckart. The lowest energy-level is

$$-\frac{E}{Mc^2} = \frac{5}{8}\left(\frac{\hbar}{Mc\rho}\right)^2+\frac{I(0)}{Mc^2}-\frac{3}{8}\left(\frac{\hbar}{Mc\rho}\right)^2\left(1+\frac{16I(0)\rho^2M}{\hbar^2}\right)^{\frac{1}{2}} \tag{6}$$

and the corresponding wave function is

$$u = \frac{\rho}{r}\frac{e^{r/\rho}-1}{e^{r/\rho}+1}\frac{1}{(1+e^{r/\rho})^{\nu}(1+e^{-r/\rho})^{\nu}}, \tag{7}$$

where

$$\nu = \left(-E\frac{M}{\hbar^2}\rho^2\right)^{\frac{1}{2}}. \tag{8}$$

The solution of the wave equation for the values of $I(0)$ and ρ as specified above is shown by the dotted line in Fig. 6 It will be noticed that the wave function spreads over a much wider region than the potential energy of interaction, from which it follows that the mean potential energy $\bar{I}$ has a much smaller value than $I(0)$. It is also possible to show that the mean kinetic energy $\bar{E}_{\rm kin}$ differs only very slightly from the absolute value of the mean potential energy; the value $|\bar{I}/\bar{E}_{\rm kin}|$ tending to unity for small values of ρ (i.e for a narrow potential hole)

Inserting the value for the binding energy of the deuteron ($2\ 25\times 10^6$ e v.) in the form $E' = E/Mc^2 = 0{\cdot}0024$, we obtain from equation (6) the relation between the depth and the breadth of the potential hole. This relation is represented graphically in Fig 7 by the light line From this we cannot obtain $I(0)$ and ρ, separately, however. to do so we must use the binding energy of some other nucleus, for example of the α-particle Comparing the binding energy of the deuteron ($0{\cdot}0024Mc^2$) with the corresponding values for the α-particle ($0{\cdot}0300Mc^2$) and heavier nuclei, we first notice the extraordinarily low value of deuteron binding energy with respect to that of other nuclei. This fact was explained by Wigner by making use of the above-mentioned result that for small values of ρ the potential and kinetic energies of the deuteron are almost compensated In fact we must remember that in the deuteron we have one interaction energy ($n \longleftrightarrow p$) and two kinetic energies (n and p), whereas in the α-particle there are four interactions $\begin{pmatrix} n & \longleftrightarrow & p \\ \updownarrow & & \updownarrow \\ p & \longleftrightarrow & n \end{pmatrix}$ and four kinetic energies ($2n$ and $2p$), and the same is generally true for heavier nuclei the number of interactions is approximately equal to the number of particles, on account of the double valency of neutrons and protons Now it is

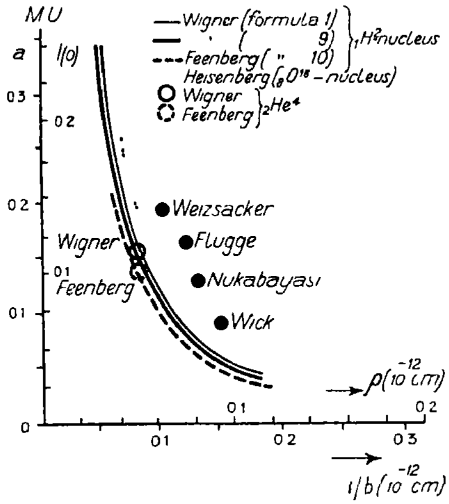


FIG 7

easy to understand that if, due to the small interaction-radius ρ, the potential energy of the deuteron is almost completely compensated by the kinetic energies of the two constituent particles, this cannot be the case for the α-particle and for heavier nuclei, in which the potential energy must play the prevailing role Wigner calculated that in order to obtain the correct ratio for the binding energies of α-particle and deuteron $\left(\frac{0\ 0300}{0{\cdot}0024} = 12\ 5\right)$ one must accept for ρ the value $0{\cdot}050 \times 10^{-12}$ cm This leads, according to previous considerations (formula (6)), to the value 0 114 M U for $I(0)$ Thus we have

$$I(0) = 169 \times 10^{-6}\,\text{erg} = 106 \times 10^{+6}\,\text{e.v}\ ,$$
$$\rho = 0{\cdot}050 \times 10^{-12}\,\text{cm}$$

In order to show that the results which are obtained do not depend upon the special choice of the interaction potential $I(0)$, Wigner carried out the calculations for the deuteron with a somewhat simpler expression for the interaction:†

$$I(r) = ae^{-br} \tag{9}$$

For this interaction, however, the wave equation cannot easily be solved analytically and numerical calculations of the eigenvalues have therefore been carried out by Wigner in this case ‡ To compare these calculations with those made on the basis of the interaction function (1), it is necessary to find the correlation between the coefficients a and $1/b$, on the one hand, and $I(0)$ and ρ, on the other, so that the two curves of potential energy against distance fit as well as possible

The best approximation can be obtained if we adopt $a = 1{\cdot}4I(0)$ and $1/b = 1{\cdot}6\rho$, as can be seen from Fig. 6, where the potential (9) with the above values of coefficients is plotted (heavy line) together with the potential (1) The heavy line in Fig 7 gives the plot of $a/1{\cdot}4$ and $(1/b)/1\ 6$ which, as can be seen from the figure, follows very closely the relation between $I(0)$ and ρ as obtained from the formula (6) The values of a and $1/b$ which fit the observed ratio of mass defects $\Delta\,{}_2He^4/\Delta\,{}_1H^2$ become now

$$a = 0{\cdot}160\,\mathrm{M\,U.} = 238\times10^{-6}\,\mathrm{erg} = 148\times10^{+6}\,\mathrm{e\ v}\ ;$$

$$1/b = 0{\cdot}08\times10^{-12}\,\mathrm{cm}$$

Finally, we should mention that, by choosing another law of interaction,

$$I(r) = \alpha e^{-\beta^2r^2}, \tag{10}$$

we may obtain approximate correspondence with the two previous expressions if $\alpha = 0\ 64a = 0{\cdot}9I(0)$ and $1/\beta = 1{\cdot}7(1/b) = 2{\cdot}7\rho$ (compare Fig 6) The values of α and $1/\beta$ calculated by Feenberg so as to give the correct binding energy of the deuteron are shown in the following table

α	0·130	0 065	0 041	0 029	0 022	M U.
$1/\beta$	0 11	0 16	0 22	0 27	0 33	$\times10^{-12}$ cm

† This interaction law, on account of its simplicity, is usually accepted nowadays in most of the calculations with nuclear models

‡ An analytic solution of the wave equation for the interaction law (9) has recently been given by Massey and Mohr (see ref. p 26)

The broken line in Fig 7 represents the relation between $a = \alpha/0{\cdot}62$ and $1/b = (1/\beta)/1{\cdot}7$ obtained in this way

The above results show that, to the degree of accuracy at present attainable, we can take any mathematically convenient expression for $I(r)$ provided it represents forces very rapidly decreasing with the distance between the particles.

The structure of the other simple nuclei,

$$_1\mathrm{H}^3\,(=2n+1p),\quad {}_2\mathrm{He}^3\,(=1n+2p),\quad \text{and}\quad {}_2\mathrm{He}^4\,(=2n+2p),$$

has also been discussed by Wigner and later in more detail by Feenberg.† Accepting the Majorana type of exchange force (exchange of charge and spin), we must write, for such a many-particle problem, the equation

$$\frac{\hbar^2}{2M}\nabla^2\Psi + \sum I(r_{n_k p_l})P_{n_k p_l}\Psi = 0, \tag{11}$$

where Ψ is a function of the coordinates of all particles in question and $P_{n_k p_l}$ the operator corresponding to the exchange of space coordinates between the n_kth neutron and the p_lth proton. For $I(r)$ Feenberg has employed the expression (10) As a first approximation we may represent the wave function Ψ as the product of different wave functions involving the relative distances between particles.

Thus, for the three-body problems ($_1\mathrm{H}^3$ and $_2\mathrm{He}^3$) we may write

$$\Psi = \Psi(r_{13})\Psi(r_{23})\Psi(r_{12}), \tag{12}$$

where the indices 1 and 2 refer to the two similar particles (neutrons in $_1\mathrm{H}^3$ and protons in $_2\mathrm{He}^3$) and the index 3 refers to the remaining particle (proton in $_1\mathrm{H}^3$ and neutron in $_2\mathrm{He}^3$) In the four-body problem of an α-particle we have

$$\Psi = \Psi(r_{13})\Psi(r_{14})\Psi(r_{23})\Psi(r_{24})\Psi(r_{12})\Psi(r_{43}), \tag{13}$$

where 1 and 2 refer to neutrons and 3 and 4 to protons The total energy of the system is given by

$$E = \int\ .\int \Psi H \Psi^* \, d\omega, \tag{14}$$

where H is the Hamiltonian operator for the problem (the operator in equation (11)) For the wave functions Ψ Feenberg adopts the expressions

$$\Psi_1 = Ne^{-\frac{1}{2}\nu r^2} \quad \text{and} \quad \Psi_2 = Ne^{-\frac{1}{2}\mu r^2}. \tag{15}$$

† E. Feenberg, *Phys. Rev* **47** (1935), 850

Here Ψ_1 denotes a wave function which depends on the relative distance between two dissimilar particles (such as $\Psi(r_{13})$; $\Psi(r_{14})$ etc.) and Ψ_2 a function having reference to similar particles (such as $\Psi(r_{12})$, $\Psi(r_{34})$, etc). After rather complicated integrations the expression for the eigenvalues (14) becomes

$$E^0({}_1\mathrm{H}^3) = 6\{(2+n)/(5+n)\}\beta^2\sigma - \\ -16\alpha\{(1+2n)/(5+6n+n^2)\}^{3/2}\{\sigma/(\sigma+1)\}^{3/2}, \quad (16)$$

$$E^0({}_2\mathrm{He}^3) = 6\{(2+n)/(5+n)\}\beta^2\sigma - \\ -16\alpha\{(1+2n)/(5+6n+n^2)\}^{3/2}\{\sigma/(\sigma+1)\}^{3/2}+\tfrac{1}{4}(\nu+2\mu)^{1/2}, \quad (17)$$

with
$$n = \frac{\mu}{\nu}, \qquad \sigma = \frac{\nu(5+n)}{4\beta^2}; \quad (18)$$

and

$$E^0({}_2\mathrm{He}^4) = 6\{(2+n)/(3+n)\}\beta^2\sigma - \\ -64\times 2^{1/2}\alpha\{(n+1)^{3/2}/(n+3)^3\}\{\sigma/(\sigma+1)\}^{3/2}+\tfrac{1}{4}(2\nu+2\mu)^{1/2}, \quad (19)$$

with
$$n = \frac{\mu}{\nu}, \qquad \sigma = \frac{\nu(3+n)}{2\beta^2}. \quad (20)$$

In (17) and (19) the last terms represent the result of Coulomb interaction between protons, which is, of course, absent in the case of ${}_1\mathrm{H}^3$ ($= 2n+1p$)

Accepting for n the values 1·5 and 1·7 in three- and four-body problems, respectively (these values have been found by trial to give the best results), Feenberg obtained the following tabulated results for the binding energies of these light nuclei, for different values of $1/\beta$.

TABLE I

$1/\beta\times 10^{13}$ *cm*	αM *U*	${}_1H^3$ $-E_0 M$ *U.*	${}_2He^3$ $-E_0 M$ *U*	${}_2He^4$ $-E_0 M$ *U*
0 200	0 046	0 0024	0 0018	0 016
0 127	0 098	−0 0011(!)	−0 0018(!)	0 018
0 104	0 140	−0 0038(!)	−0 0047(!)	0 020

These are to be compared with the experimental values 0·0091, 0·0076, and 0·030. We see at once that the results are not very satisfactory, the binding energy of the ${}_2\mathrm{He}^4$-nucleus for

$$1/\beta = 0{\cdot}1\times 10^{-12}\ \mathrm{cm}.$$

being still only two-thirds of empirical value, whereas the corresponding binding energies of the nuclei ${}_1\mathrm{H}^3$ and ${}_2\mathrm{He}^3$ are negative. In order to improve the results Feenberg has proposed certain modifications of his calculations based on the comparison of equations (16), (17), and (19) with the equation for an 'equivalent' two-body problem (This method is based on considerations of analogy and plausibility and for the details we refer the reader to the original paper.) He finds in this way that the correct value of mass defect of an α-particle may be obtained for $1/\beta = 0{\cdot}13\times10^{-12}$ cm. and $\alpha = 0{\cdot}095$ † With these values of the coefficients the binding energies of ${}_1\mathrm{H}^3$ and ${}_2\mathrm{He}^3$ become 0·0060 and 0·0052, in better agreement with experiment

Some further calculations of the binding energies of the nuclei ${}_1\mathrm{H}^3$ and ${}_2\mathrm{He}^3$ have been carried out by Massey and Mohr‡ who particularly investigated the validity of the variational method and showed that the results obtained by this method are very sensitive to the expression adopted for the zero-approximation. Instead of Feenberg's distributions (15) they have adopted a somewhat different distribution,

$$\Psi = \frac{1}{r}(e^{-Ar}-e^{-Br}), \tag{21}$$

which gives better results when this method is applied to the deuteron They finally obtained by the direct variational method for the binding energies of ${}_1\mathrm{H}^3$ and ${}_2\mathrm{He}^3$ the values 0 0055 and 0 0060 respectively

2. Statistical calculation of nuclear binding energy

In the last chapter (p 4) it has been mentioned that a nucleus constructed from a large number of similar particles, interacting with forces decreasing rapidly with increasing distance, may be treated in many respects in the same way as a small drop of liquid. This idea was used by Heisenberg,‖ who applied to the collection of neutrons and protons forming a nucleus the quantum-statistical method of Thomas and Fermi in the form in which it was developed by Dirac.

Let us consider a nucleus built up from N_n neutrons and N_p protons To represent the space coordinates of the particles we shall employ vectors $\mathbf{w}$, for the spin coordinates variables σ, and we shall distinguish throughout between coordinates referring to neutrons and protons, respectively, by using different subscripts, roman numerals

† This corresponds to $a = 0\ 153$, $1/b = 0\ 077\times10^{-12}$ cm.

‡ H Massey and C Mohr, *Proc Roy Soc.* **152** (1935), 693.

‖ W. Heisenberg, *Reports of the Solvay Congress*, 1933.

for neutrons and arabic numerals for protons. In this way the complete wave function of the system may be written in the form

$$\Phi = \begin{vmatrix} \phi_{\mathrm{I}}(\mathbf{w}_{\mathrm{I}}, \sigma_{\mathrm{I}}) & \cdot\;\cdot & \phi_{N_n}(\mathbf{w}_{\mathrm{I}}, \sigma_{\mathrm{I}}) \\ \phi_{\mathrm{I}}(\mathbf{w}_{\mathrm{II}}, \sigma_{\mathrm{II}}) & \cdot\;\cdot & \phi_{N_n}(\mathbf{w}_{\mathrm{II}}, \sigma_{\mathrm{II}}) \\ \cdots & & \cdots \\ \phi_{\mathrm{I}}(\mathbf{w}_{N_n}, \sigma_{N_n}) & \cdot & \phi_{N_n}(\mathbf{w}_{N_n}, \sigma_{N_n}) \end{vmatrix} . \begin{vmatrix} \phi_1(\mathbf{w}_1, \sigma_1) & \cdot & \phi_{N_p}(\mathbf{w}_1, \sigma_1) \\ \phi_1(\mathbf{w}_2, \sigma_2) & \cdot & \phi_{N_p}(\mathbf{w}_2, \sigma_2) \\ \cdots & & \cdots \\ \phi_1(\mathbf{w}_{N_p}, \sigma_{N_p}) & \cdot\;\cdot & \phi_{N_p}(\mathbf{w}_{N_p}, \sigma_{N_p}) \end{vmatrix}, \tag{22}$$

where the different ϕ's are the wave functions for the separate particles Moreover, since the spin-interaction between the constituent parts of the nucleus is small compared with the other forces involved, we may replace $\phi(\mathbf{w}, \sigma)$ by the product $\chi(\mathbf{w}).\zeta(\sigma)$ and define the densities of neutron and proton distributions inside the nucleus by the expressions

$$\rho_n(\mathbf{w}) = \sum_{K=1}^{K=N_n} \chi_K^* \chi_K, \qquad \rho_p(\mathbf{w}) = \sum_{k=1}^{k=N_p} \chi_k^* \chi_k, \tag{23}$$

respectively Similarly, we may introduce the notion of average densities, $\bar{\rho}_n = N_n/V$ and $\bar{\rho}_p = N_p/V$, V ($\equiv \frac{4}{3}\pi r_0^3$) being the nuclear volume Then, according to the method of Thomas and Fermi, the kinetic energy of the system is given by the equation

$$E_{\mathrm{kin}} = \frac{\hbar^2}{M} \frac{16\pi^3}{5} \left(\frac{3}{8\pi}\right)^{\frac{5}{3}} \int (\rho_n^{\frac{5}{3}} + \rho_p^{\frac{5}{3}})\, d\mathbf{w}, \tag{24}$$

where M is the mass of the particles in question (neutron or proton mass) and $d\mathbf{w}$ the element of space in the neighbourhood of the point $\mathbf{w}$. It remains now to derive an expression for the potential energy corresponding to the exchange forces between neutrons and protons Two possibilities are open, either we may consider the exchange of electric charge only (Heisenberg's original hypothesis) or the exchange of both charge and spin between the particles (first suggested by Majorana and later accepted by Heisenberg)

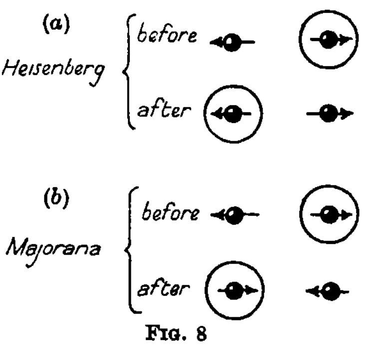


FIG. 8

These two possibilities are represented schematically in Fig 8 (*a*) and (*b*) As will appear, the former hypothesis necessitates a single-valency bond between neutron and proton, the latter hypothesis a double bond. The experimental fact that the α-particle rather than the deuteron possesses an extraordinarily large binding energy suggests that the latter hypothesis is correct, the system of two neutrons and two protons forming a saturated shell. This being the case, the mathematical expression for the potential energy corresponding to the exchange forces, for a pair of particles, takes on the form

$$-I(r_{K,l})P_{K,l}, \tag{25}$$

where $I(r_{K,l})$ is a function which decreases rapidly with increasing distance between the particles (K, l),† and $P_{K,l}$ is the operator which corresponds to the interchange of space coordinates $\mathbf{w}_K$ and $\mathbf{w}_l$.‡ The negative sign in (25) is included, as before (§ 1), to emphasize the attractive nature of the forces for moderate distances This attraction, it will be remembered, approaches zero when the distance becomes very small The total potential energy corresponding to exchange forces in the nuclear system is now

$$U_{\text{exch}} = -\int \Phi^* \sum_{K,k} I(r_{K,k})P_{K,k}\Phi\, d\mathbf{w}$$
$$= -\sum_{\sigma\sigma'}\iint \sum_{K=1}^{K=N_n} \phi_K^*(\mathbf{w},\sigma)\phi_K(\mathbf{w}',\sigma)I(|\mathbf{w}-\mathbf{w}'|)\times$$
$$\times \sum_{k=1}^{k=N_p} \phi_k^*(\mathbf{w}',\sigma')\phi_k(\mathbf{w},\sigma')\, d\mathbf{w}d\mathbf{w}', \tag{26}$$

or, separating space and spin coordinates, and carrying out the summation with respect to the latter,

$$U_{\text{exch}} = -\iint \sum_{K=1}^{K=N_n} \chi_K^*(\mathbf{w})\chi_K(\mathbf{w}')I(|\mathbf{w}-\mathbf{w}'|)\times$$
$$\times \sum_{k=1}^{k=N_p} \chi_k^*(\mathbf{w}')\chi_k(\mathbf{w})\, d\mathbf{w}d\mathbf{w}'. \tag{27}$$

In the Thomas-Fermi method densities are given by expressions

$$\rho_n(\mathbf{w}) = \frac{1}{4\pi^3\hbar^3}\int\limits_0^{P_n(\mathbf{w})} d\mathbf{p}, \qquad \rho_p(\mathbf{w}) = \frac{1}{4\pi^3\hbar^3}\int\limits_0^{P_p(\mathbf{w})} d\mathbf{p}, \tag{28}$$

respectively, in which P_n and P_p are determined by the potential energy in the neighbourhood of the point $\mathbf{w}$. In a similar way we

† Here $r_{K,l} = |\mathbf{w}_K - \mathbf{w}_l|$

‡ On Heisenberg's original hypothesis (exchange of charge only) we should write for the potential energy

$$-I(r_{K,l})P'_{K,l}, \tag{25'}$$

where the operator $P'_{K,l}$ corresponds now to the interchange of space and spin coordinates may write, according to Dirac,

$$\sum_{K=1}^{K=N_n} \chi_K^*(\mathbf{w})\chi_K(\mathbf{w}') = \rho_n(\mathbf{w},\mathbf{w}') = \frac{1}{4\pi^3\hbar^3} \int\limits_0^{P_n\left(\frac{\mathbf{w}+\mathbf{w}'}{2}\right)} \exp\left\{-\frac{i}{\hbar}\mathbf{p}(\mathbf{w}-\mathbf{w}')\right\} d\mathbf{p},$$

$$\sum_{k=1}^{k=N_p} \chi_k^*(\mathbf{w})\chi_k(\mathbf{w}') = \rho_p(\mathbf{w},\mathbf{w}') = \frac{1}{4\pi^3\hbar^3} \int\limits_0^{P_p\left(\frac{\mathbf{w}+\mathbf{w}'}{2}\right)} \exp\left\{-\frac{i}{\hbar}\mathbf{p}(\mathbf{w}-\mathbf{w}')\right\} d\mathbf{p}, \tag{29}$$

and the expression (27) finally becomes

$$U_{\text{exch}} = -\frac{1}{16\pi^6\hbar^6} \iint d\mathbf{w}d\mathbf{w}' \int\limits_0^{P_n} d\mathbf{p} \int\limits_0^{P_p} d\mathbf{p}' \times$$

$$\times \exp\left\{-\frac{i}{\hbar}(\mathbf{p}-\mathbf{p}')(\mathbf{w}-\mathbf{w}')\right\} I(|\mathbf{w}-\mathbf{w}'|). \tag{30}$$

A very rough approximation to the energy expression (30) may be obtained if use is made of the fact that the above-defined functions $\rho_n(\mathbf{w},\mathbf{w}')$, $\rho_p(\mathbf{w},\mathbf{w}')$ decrease very rapidly with increasing distance $|\mathbf{w}-\mathbf{w}'|$, especially if it be assumed that this decrease is more rapid than that of the potential energy function $I(|\mathbf{w}-\mathbf{w}'|)$. For then we may write

$$U_{\text{exch}} \sim \begin{cases} -2\int d\mathbf{w}\, \rho_p(\mathbf{w}) I(0) = -2N_p I(0), & \text{for } N_p < N_n \\ -2\int d\mathbf{w}\, \rho_n(\mathbf{w}) I(0) = -2N_n I(0), & \text{for } N_n < N_p. \end{cases} \tag{31}$$

With the final result in this form it is easy to see that the accepted exchange interaction (25) gives rise to a double-valency bond between neutrons and protons †

† Carrying through an analogous calculation employing (25′) for the exchange interaction for a single pair of particles (i e assuming Heisenberg's original hypothesis concerning the nature of the exchange forces) we should have

$$U_{\text{exch}} = -\sum_{\sigma\sigma'} \iint \sum_{K=1}^{K=N_n} \phi_K^*(\mathbf{w},\sigma)\phi_K(\mathbf{w}',\sigma') I(|\mathbf{w}-\mathbf{w}'|) \sum_{k=1}^{k=N_p} \phi_k^*(\mathbf{w}',\sigma')\phi_k(\mathbf{w},\sigma)\, d\mathbf{w}d\mathbf{w}'$$

$$= \tfrac{1}{2} \iint \sum_{K=1}^{K=N_n} \chi_K^*(\mathbf{w})\chi_K(\mathbf{w}') I(|\mathbf{w}-\mathbf{w}'|) \sum_{k=1}^{k=N_p} \chi_k^*(\mathbf{w}')\chi_k(\mathbf{w})\, d\mathbf{w}d\mathbf{w}'$$

$$\sim \begin{cases} -N_p I(0), & \text{for } N_p < N_n \\ -N_n I(0), & \text{for } N_n < N_p, \end{cases}$$

showing that exchange forces of this type give rise to the single-valency bond between neutron and proton.

A second and better approximation for (30) may be written as follows:

$$U_{\text{exch}} \sim -\frac{1}{16\pi^6\hbar^6}\int d\mathbf{w}\int d\sigma \int\limits_0^{P_n(\mathbf{w})} d\mathbf{p} \int\limits_0^{P_p(\mathbf{w})} d\mathbf{p}' \exp\Bigl\{-\frac{i}{\hbar}(\mathbf{p}-\mathbf{p}')\sigma\Bigr\} I(|\sigma|)$$

$$= -\int d\mathbf{w}\, f\{\rho_n(\mathbf{w}), \rho_p(\mathbf{w})\}. \tag{32}$$

Here f is a function, symmetrical in the variables ρ_n, ρ_p, which vanishes for $\rho_n = \rho_p = 0$ and, for large values of these densities, tends to the limit $2\rho_p I(0)$ or $2\rho_n I(0)$ according as ρ_n or ρ_p is the greater

In the process of deriving an expression for the total energy of the composite nucleus we have now obtained expressions for the kinetic energy of the constituent particles and for the potential energy corresponding to the exchange forces The potential energy corresponding to Coulomb repulsive forces between protons may evidently be written

$$U_{\text{coul}} = +\frac{1}{2}\iint \frac{e^2}{|\mathbf{w}-\mathbf{w}'|}\rho_p(\mathbf{w})\rho_p(\mathbf{w}')\, d\mathbf{w}d\mathbf{w}', \tag{33}$$

and this, in the case of a uniform distribution of positive charge, reduces to

$$U_{\text{coul}} = +\tfrac{3}{5}e^2N_p^2\Bigl(\frac{3V}{4\pi}\Bigr)^{-\frac{1}{3}}, \tag{34}$$

where V is the nuclear volume. Using (24), (32), and (33) the expression for the total energy of the nucleus becomes

$$E = E_{\text{kin}}+U_{\text{exch}}+U_{\text{coul}}$$

$$= \int \Bigl[\frac{\hbar^2}{M}\frac{16\pi^3}{5}\Bigl(\frac{3}{8\pi}\Bigr)^{\frac{5}{3}}\{\rho_n^{\frac{5}{3}}(\mathbf{w})+\rho_p^{\frac{5}{3}}(\mathbf{w})\}-f\{\rho_n(\mathbf{w}), \rho_p(\mathbf{w})\}\Bigr]\, d\mathbf{w} +$$

$$+\frac{1}{2}\iint \frac{e^2}{|\mathbf{w}-\mathbf{w}'|}\rho_p(\mathbf{w})\rho_p(\mathbf{w}')\, d\mathbf{w}d\mathbf{w}' \tag{35}$$

Applying to this expression for E—regarded as a function of $\rho_n(\mathbf{w})$, $\rho_p(\mathbf{w})$—the calculus of variations, with the conditions

$$\int \rho_n(\mathbf{w})\, d\mathbf{w} = N_n, \qquad \int \rho_p(\mathbf{w})\, d\mathbf{w} = N_p,$$

we may now obtain $\rho_n(\mathbf{w})$ and $\rho_p(\mathbf{w})$. Omitting the last member in (35), or neglecting the effect of Coulomb repulsion, a very simple result is obtained It is

$$\rho_n(\mathbf{w}) = N_n/V = \text{const}\,, \quad \rho_p(\mathbf{w}) = N_p/V = \text{const.}, \tag{36}$$

implying that in this case the distributions of neutrons and protons throughout the nucleus are both uniform. The effect of repulsive forces between protons is obviously to make the density of protons greater near the surface of the nucleus than at the centre, but, due to the smallness of these forces compared with exchange forces, this effect may be of importance only for very heavy nuclei. To a fair approximation, therefore, we may accept (36) and (34), and so obtain for E

$$E = \frac{\hbar^2}{M}\frac{16\pi^3}{5}\left(\frac{8}{3\pi}\right)^{\frac{5}{3}}(N_n^{\frac{5}{3}}+N_p^{\frac{5}{3}})V^{-\frac{2}{3}}-f\left(\frac{N_n}{V},\frac{N_p}{V}\right)V+\tfrac{3}{5}e^2N_p^2\left(\frac{3V}{4\pi}\right)^{-\frac{1}{3}}. \tag{37}$$

The condition for minimum total energy (maximum energy of binding) will then define the stable state of the system and may be used, in particular, to calculate V. We have

$$-\frac{\hbar^2}{M}\frac{32\pi^3}{15}\left(\frac{8}{3\pi}\right)^{\frac{5}{3}}(N_n^{\frac{5}{3}}+N_p^{\frac{5}{3}})V^{-\frac{5}{3}}-f\left(\frac{N_n}{V},\frac{N_p}{V}\right)+\frac{N_n}{V}\frac{\partial f}{\partial\rho_n}+$$
$$+\frac{N_p}{V}\frac{\partial f}{\partial\rho_p}-\frac{e^2}{5}\left(\frac{3}{4\pi}\right)^{-\frac{1}{3}}\left(\frac{N_p}{V}\right)^2V^{\frac{2}{3}} = 0. \tag{38}$$

Here, except in the last term, derived from Coulomb forces, the volume V enters only in the combinations N_n/V, N_p/V, and this means that, when Coulomb forces may be neglected, the nuclear volume is proportional to the total number of particles, provided N_n/N_p remains within fairly narrow limits Thus the nuclear density is roughly constant for nuclei of different mass numbers. The effect of the Coulomb forces, which have hitherto been neglected, is obviously to decrease slightly the mean nuclear density in the heavier nuclei; for these nuclei the radius increases with mass number somewhat more rapidly than the cube root Such deviations, however, may be neglected to a first approximation, and altogether, except for the heaviest nuclei.

So far we have made no definite assumption concerning the form of the potential energy function $I(r)$. it is necessary to do this now, if the total energy formula (37) is to be given explicit form. In his calculations Heisenberg adopts the expression for $I(r)$ previously used by Wigner in the deuteron theory,

$$I(r) = ae^{-br}. \tag{39}$$

Here a and b are unknown constants characteristic of the special case of neutron-proton exchange. Adopting (39), we may then write for the corresponding f function, as defined by (32),

$$f(\rho_n,\rho_p) = \frac{ab^3}{6\pi^3}\Big[4L_n L_p+\{1+3(L_n^2+L_p^2)\}\log\frac{1+(L_n-L_p)^2}{1+(L_n+L_p)^2}+$$
$$+4(L_n^3+L_p^3)\tan^{-1}(L_n+L_p)-4(L_n^3-L_p^3)\tan^{-1}(L_n-L_p)\Big], \quad (40)$$

where
$$L_n = \frac{1}{b}(3\pi^2\rho_n)^{\frac{1}{3}}= \frac{1}{b}\left(3\pi^2\frac{N_n}{V}\right)^{\frac{1}{3}} \quad\text{and}\quad L_p = \frac{1}{b}\left(3\pi^2\frac{N_p}{V}\right)^{\frac{1}{3}}. \quad (41)$$

With this value for f, and the value of V obtained from (38), we are finally in a position to obtain, from (37), the total energy of our nucleus This energy will be expressed in terms of the numbers of neutrons and protons in the nucleus (N_n and N_p) and the constants a and $1/b$ of the exchange energy expression. Adopting for the coefficients the values $a = 0{\cdot}0273$ M U and $1/b = 0\ 8\times 10^{-12}$ cm. so as to fit the mass defects of A and Kr (not shown in Fig 6 as being far removed from the other values), Heisenberg obtains the following expression for the binding energy,†

$$E = 0{\cdot}00347N_p-0{\cdot}0364N_n+0{\cdot}01211\frac{N_n^2}{N_p}+$$
$$+N_p^{\frac{2}{3}}\left(3\ 19-0{\cdot}715\frac{N_n}{N_p}\right).10^{-4}+0{\cdot}049, \quad (42)$$

where the last constant term does not follow from previous considerations but is introduced in order to take account of the failure of the statistical method for lighter nuclei This expression gives a satisfactory fit with the experimental curve of mass defects, on the other hand, it gives too large values for nuclear radii (due to the comparatively large value assumed for $1/b$) Heisenberg's numerical values for a and $1/b$ have been improved by Wick,‡ who based his calculations on the empirical values of the binding energy and nuclear radius for $_8O^{16}$ Wick obtains

$$1/b = 0{\cdot}15\times 10^{-12}\text{ cm} \ ; \quad a = 0{\cdot}094\text{ M U}$$

in better agreement with the values obtained from the lightest nuclei.

A somewhat different formula has been obtained by Nukabayasi,|| following Heisenberg's general line of calculations He adopted for $1/b$ the value $0{\cdot}13\times 10^{-12}$ cm. (obtained by Rabi from neutron-scattering in hydrogen) and calculated, from the known mass defects

† W Heisenberg, l c. ‡ G C Wick, *Nuovo Cimento*, **11** (1934).
|| K. Nukabayasi, *Zs. f. Phys.* **97** (1935), 211.

of O^{16}, Kr^{86}, and Tl^{205}, the following values of a: 0·121, 0·132, 0·133 M.U Taking, as an average, $a = 0{\cdot}129$ M U., with this value of $1/b$, Nukabayasi obtained the following energy expression, to be compared with Heisenberg's,

$$E = -0{\cdot}01026(N_n+N_p)+0{\cdot}000659N_p^2(N_n+N_p)^{-\frac{1}{3}} - \\ -0{\cdot}00000452N_p^4(N_n+N_p)^{-\frac{5}{3}} \quad \text{(in M U.)}, \qquad (43)$$

and for the nuclear radii,

$$r_0 = 0{\cdot}1395(N_n+N_p)^{-\frac{1}{3}}[1+0{\cdot}0137N_p^2(N_n+N_p)^{-\frac{4}{3}}]\times 10^{-12}\text{ cm}\,, \quad (44)$$

by taking account of the deviations from constant density due to Coulomb forces. Graphical representation of the results calculated from (43) and (44) is provided by Fig 9† and Fig 10,‡ together with

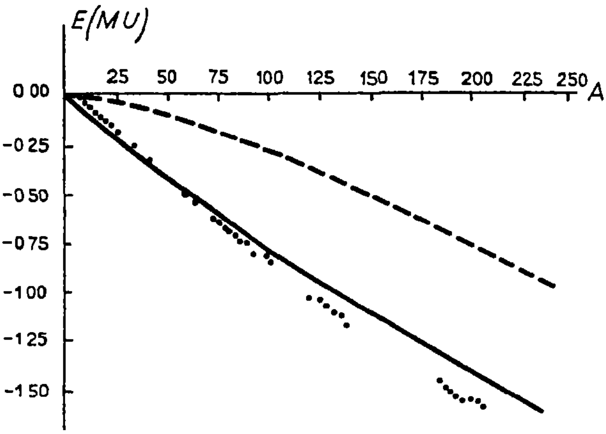


Fig. 9

the experimental values for mass defects and nuclear radii We see that the agreement between experimental and theoretical values can be considered as a satisfactory one

Comparing the values of a and $1/b$ as obtained by different authors from the statistical theory of heavier nuclei with those obtained by Wigner for the deuteron and α-particle, we find a rather large disagreement, which can be easily seen in Fig 7 where the results of

† The broken line in this figure represents the part of nuclear energy due to Coulomb forces

‡ The radii of light nuclei in this diagram are calculated from the data on artificial transformations and those for heavy nuclei from α-decay constants

different calculations are plotted. Clearly, this disagreement must be due to the simplifications adopted in our nuclear model and also, perhaps, to the failure of the Thomas-Fermi statistical method.

As the first important step towards precision for our nuclear model we must take into consideration the conditions existing near the surface of the nucleus and responsible for an effect which may conveniently be referred to as a surface-tension effect. Apart from the

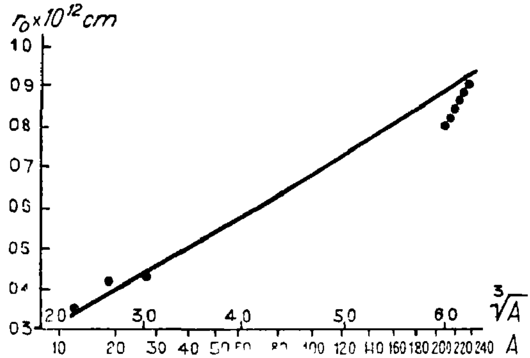


FIG. 10

ordinary surface tension originating in the fact that the radius of interaction of nuclear particles ($\sim 0{\cdot}1 \times 10^{-12}$ cm.) is not vanishingly small in comparison with nuclear radii (from $\sim 0{\cdot}3 \times 10^{-12}$ cm. to $\sim 0{\cdot}9 \times 10^{-12}$ cm.), we must also take into account that, representing the motion of nuclear particles by wave functions with not very high quantum numbers, it is impossible to obtain a uniform density distribution inside the nucleus with an abrupt jump to zero-value at the boundary. The kinetic energy of a particle the motion of which is represented by the wave function Ψ is determined by

$$E_{\text{kin}} = \frac{\hbar^2}{2M} (\operatorname{grad} \Psi)^2 \tag{45}$$

This expression leads to an infinite value if the density anywhere abruptly drops to zero. Conversely, from (45) we conclude that the decrease of density from the intranuclear value to zero must take place over a distance of the order of magnitude

$$d \sim \frac{\hbar}{\sqrt{(2M\bar{E}_{\text{kin}})}}, \tag{45'}$$

where $\bar{E}_{\text{kin}}$ is the average kinetic energy of the nuclear particle. This gradual decrease of density towards the boundary gives rise to an additional surface tension of pure quantum-mechanical character; we may call this tension 'kinetic surface tension' to distinguish it from the ordinary 'potential surface tension' mentioned above. It is thus easy to see that the effect of surface tension will be more important for lighter nuclei, for which the thickness of the surface layer may be of the same order of magnitude as the nuclear radius. This effect will show itself in a diminishing of nuclear binding energies and an increase in nuclear radii for these nuclei. The effect of surface tension on the general expression for binding energy has been investigated by Weizsacker,† who has been able to show that the kinetic tension, in general, plays a more important role than the potential tension (due to the small value of $1/b$) and is first to be taken into account This can be done in the statistical method by representing the wave functions in any small element of volume, not by harmonic waves, but by waves with linearly varying amplitudes It leads to an additional term in the kinetic-energy expression which has the form

$$E'_{\mathrm{kin}} = \frac{\hbar^2}{32\pi^2 M}\frac{(\mathrm{grad}\,\rho)^2}{\rho}. \tag{46}$$

The table below gives the numerical results of Weizsacker's calculations showing the most probable values for a and $1/b$,‡ the nuclear radius r_0, the thickness of the surface layer d, and the internal energies per particle as originating in volume, surface, and Coulomb forces.

TABLE II

$a = 0\ 198\,\mathrm{M\,U}, \quad 1/b = 0{\cdot}103\times 10^{-12}\,\mathrm{cm}$

Nucleus	$r_0\times 10^{12}$ *cm*	$d\times 10^{12}$ *cm*	$E_v M U$	$E_s M U$	$E_c M U$	$E = E_v+E_s+E_c$	E_{exp} $(M\ U.)$
$_8\mathrm{O}^{16}$	0 41	0 18	−0 159	+0 064	+0 013	−0 082	−0 083
$_{42}\mathrm{Mo}^{100}$			−0 159	+0 035	+0 031	−0 093	−0 088
$_{80}\mathrm{Hg}^{200}$	0 94		−0 159	+0 027	+0 045	−0 086	−0 082

In spite of the generally good agreement with the experimental values of binding energies and nuclear radii, the calculations of Weizsäcker lead again to values of a and $1/b$ different from those obtained by Wigner (see Fig. 7). The same disagreement has also been obtained by Flugge,‖ who applied Weizsacker's method of calculation to the lighter nuclei ($Z \leqslant 14$), for which the thickness of

† C F v Weizsäcker, *Zs f Phys* **96** (1935), 431.

‡ Estimated so as to give the correct binding energy of oxygen and the best approximation to the binding energies of heavier nuclei.

‖ S Flugge, *Zs f Phys* **96** (1935) 459.

the surface layer is not less than the nuclear radius and the density decreases steadily from the centre towards the boundary He obtains $a = 0{\cdot}167$ M U , $1/b = 0{\cdot}12\times 10^{-12}$ cm.

An attempt to reach better agreement with Wigner's values was then made by Heisenberg,† who, considering the previous disagreement as due to failure of the Thomas-Fermi statistical method, in his new calculations for light nuclei used the more elaborate method of self-consistent functions (Hartree's method) in somewhat simplified form As the zero-approximation Heisenberg chose a model of a light nucleus consisting of neutrons and protons harmonically oscillating between the limits given by the boundary of the nucleus. The normalized eigenfunctions for such harmonic oscillators (oscillating along the x-axis), for different quantum numbers, can be written in the form

$$\left.\begin{aligned}
\psi_0 &= \sqrt[4]{\left(\frac{\gamma}{\pi}\right)}e^{-\frac{1}{2}\gamma x^2}, \\
\psi_1 &= \sqrt[4]{\left(\frac{\gamma}{\pi}\right)}\frac{2\gamma^{\frac{1}{2}}x}{\sqrt{2}}e^{-\frac{1}{2}\gamma x^2}, \\
\psi_2 &= \sqrt[4]{\left(\frac{\gamma}{\pi}\right)}\frac{4\gamma x^2-2}{\sqrt{(2^2 . 2!)}}e^{-\frac{1}{2}\gamma x^2}, \\
\psi_3 &= \sqrt[4]{\left(\frac{\gamma}{\pi}\right)}\frac{8\gamma^{\frac{3}{2}}x^3-12\gamma^{\frac{1}{2}}x}{\sqrt{(2^3\,3!)}}e^{-\frac{1}{2}\gamma x^2}, \\
&\dots
\end{aligned}\right\}\tag{47}$$

where
$$\gamma = \frac{2\pi M}{h}\nu \quad (\nu = \text{frequency}).\tag{48}$$

For such a model the Pauli principle as applied to neutrons and protons will lead to the assumption of saturated shells for $_2He^4$, $_8O^{16}$, $_{20}Ca^{40}$, with the following quantum numbers (n_x, n_y, n_z) for levels occupied by two neutrons and two protons each

$_2He^4$			$_8O^{16}$			$_{20}Ca^{40}$		
n_x	n_y	n_z	n_x	n_y	n_z	n_x	n_y	n_z
0	0	0	0	0	0	0	0	0
			1	0	0	1	0	0
			0	1	0	0	1	0
			0	0	1	0	0	1
						2	0	0
						0	2	0
						0	0	2
						1	1	0
						1	0	1
						0	1	1

† W Heisenberg, *Zs f Phys* **96** (1935), 473.

For the interaction potential between neutrons and protons Heisenberg employs the expression

$$I(r) = \alpha e^{-\beta^2 r^2} \tag{49}$$

previously used by Feenberg, as being mathematically more convenient of manipulation Estimating the binding energy for this model and comparing it with the experimental values (for He, O, and Ca) he obtains numerical relations between α and $1/\beta$ which are shown in the following tables

${}_2He^4$

α	0 128	0 110	0 097	0 086	M.U.
$1/\beta$	0 132	0 144	0 157	0 170	$\times 10^{-12}$ cm

${}_8O^{16}$

α	0 163	0 130	0 106	0 090	M U
$1/\beta$	0 118	0 137	0 166	0 176	$\times 10^{-12}$ cm

${}_{20}Ca^{40}$

α	0 212	0 153	0 117	0 094	M U
$1/\beta$	0 103	0 128	0 154	0 180	$\times 10^{-12}$ cm

This relation (for ${}_8O^{16}$) is also plotted in Fig 7 (translated into a and $1/b$) and it can be seen that the curve which represents it is much closer to the curves calculated exactly from the deuteron model.

Since the calculations of this section are essentially statistical in nature, being valid only for nuclei composed of a large number of particles, they cannot distinguish between nuclei with even and odd numbers of neutrons or protons, or be expected to reproduce the periodicity in binding energy† which is characteristic of the formation of new saturated α-units, as further particles are added to the original nucleus In actual fact, however, there is found a slight but systematic difference in binding energy between nuclei having an even number of protons and those with an odd number (even and odd atomic number), and this difference is very important for the question of nuclear stability which is discussed in the next section It arises from the fact that when the number of protons is even all these are used for the formation of α-shells, whereas when that number is odd one proton is always left uncombined and, in consequence, somewhat less strongly bound in the nucleus The total binding energy is thus somewhat greater for nuclei with even atomic number

† Calculated in terms of neutrons and protons only,

than for the neighbouring nuclei for which the atomic number is odd We might introduce two different expressions in place of (16), or (22), to be used according as N_p is odd or even, and, as a first approximation, might assume that the corresponding energies, for $N_p+N_n = \text{const}$, differed merely by a constant quantity, whatever the value of N_p Since β-transformation necessarily results in a change from N_p odd to N_p even, or vice versa, it is obvious that this difference in binding energy is of great importance when β-stability limits are under discussion

3. Nuclear stability limits

Having obtained an expression for the total energy of the nucleus as a function of the numbers of neutrons and protons contained in it (or of the mass and atomic numbers, A ($\equiv N_n+N_p$) and Z ($\equiv N_p$)), we are now in a position to consider the general stability laws for nuclei so constituted † Let

$$E = E(N_n, N_p) \tag{50}$$

be the exact expression for the nuclear energy, given approximately by (37). The process of spontaneous disintegration of the nucleus into two or more heavy parts is energetically possible if the sum of the internal energies‡ of all resulting nuclei is less than the internal energy of the original nucleus For then surplus energy will be available to be distributed as kinetic energy between the particles liberated in the transformation The condition for possible disintegration may thus be written

$$\sum_i E(N_n^i, N_p^i) < E(N_n, N_p), \tag{51}$$

where

$$\sum_i N_p^i = N_p, \qquad \sum_i N_n^i = N_n \tag{52}$$

In particular, the conditions for the possibility of the emission of a neutron or a proton become

$$E(N_n-1, N_p)+0 < E(N_n, N_p) \tag{53}$$

and

$$E(N_n, N_p-1)+0 < E(N_n, N_p), \tag{54}$$

respectively, whilst for the emission of an α-particle one must have

$$E(N_n-2, N_p-2)+E(2, 2) < E(N_n, N_p) \tag{55}$$

† Cf Chap I, § 4

‡ For the quantity E we shall henceforward use the term 'internal energy'. The binding 'energy' of a nucleus (a positive quantity for a stable nucleus) is given by $-E$.

Disintegration processes which involve the emission of electrons, positive or negative, may also be considered in an analogous manner if this emission be regarded as the result of an internal nuclear transformation of a proton into a neutron, or vice versa (p. 9). For such transformations, however, (52) is invalid, since the numbers of neutrons and protons are not separately conserved In spite of this fact, and although certain difficulties still remain in our interpretation of the process of β-disintegration, we may therefore write

$$E(N_n-1, N_p+1) < E(N_n, N_p)+(M_n-M_p-m)c^2 \tag{56}$$

and

$$E(N_n+1, N_p-1) < E(N_n, N_p)+(M_p-M_n-m)c^2 \tag{56'}$$

as the conditions for the spontaneous emission of an electron or a positron from the nucleus (N_n, N_p). The quantities

$$\Delta_n = (M_n-M_p-m)c^2 \text{ and } \Delta_p = (M_p-M_n-m)c^2,$$

which might be called the internal energies of neutron and proton, respectively (M_n = neutron mass, M_p = proton mass, m = electron (positron) mass), are introduced to represent the energy which becomes available through the transformation of a neutron into a proton, or vice versa We may notice here that Δ_n and Δ_p evidently fulfil the condition

$$\Delta_n+\Delta_p = -2mc^2 \sim -10^6 \text{ e v} \tag{57}$$

Applying these considerations to a single neutron or a single proton we see from (56), (56′) that the reactions $n \rightarrow p+e$, $p \rightarrow n+\text{ə}$ become possible if $\Delta_n > 0$ or $\Delta_p > 0$, respectively Conversely, if free protons and free neutrons are both stable, the internal energies of both particles must be negative, and (57) leads to the results $|\Delta_p| \leqslant 2mc^2$, $|\Delta_n| \leqslant 2mc^2$ In this case the mass difference between neutron and proton, $M_n-M_p \equiv \Delta_n/c^2+m \equiv -\Delta_p/c^2-m$, may be positive or negative, but in absolute value it must be less than the mass of the electron These considerations, however, will be changed if we consider the relative stability of a neutron and a hydrogen atom In fact, if the mass of the neutron is not exactly equal to that of the hydrogen atom a transformation in one direction or the other is always possible with the liberation of energy (in the form of a γ-ray quantum) and we must conclude that, in general, either neutron or hydrogen atom is unstable (i e. one of the transformations, $n' \rightarrow [\mathrm{H}'+e]+E$ or $[\mathrm{H}'+e] \rightarrow n'+E$, is exothermic). On the other hand, if we suppose that in the process of transformation a neutrino of finite mass m_ν is emitted, we again obtain a stable region of breadth $2m_\nu c^2$.

Consider, now, the general shape of the internal energy surface $E(N_n, N_p)$, which, for graphical convenience, we first transform to new coordinates N_n+N_p and N_n/N_p and write as $E(N_n+N_p, N_n/N_p)$. We have already seen that for a given mass number ($N_n+N_p =$ const.) the binding energy is a maximum (internal energy a minimum) for a definite value of N_n/N_p, this value being one, exactly, for certain

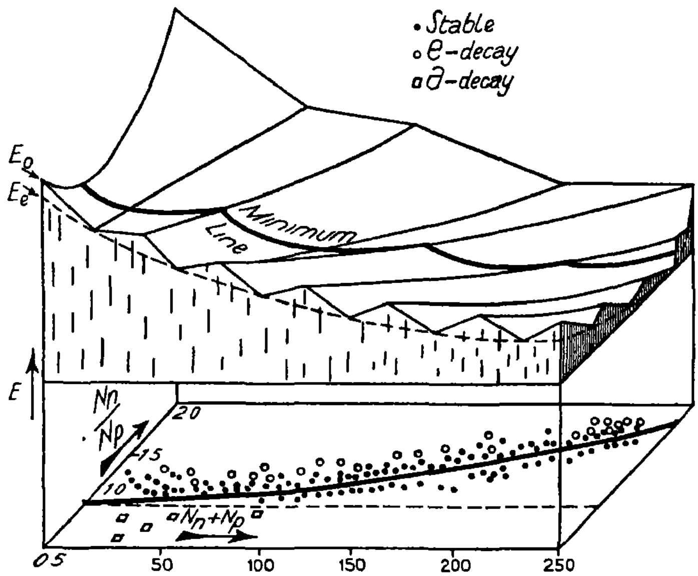


Fig 11

light nuclei, and somewhat greater than one for heavier nuclei We have also seen that with increasing mass number the internal energy decreases less and less rapidly, owing to the increase of potential energy corresponding to the electrostatic repulsion between protons. Fig 11 exhibits the general form of the internal energy surface as a function of N_n/N_p and N_n+N_p The periodic irregularities ('trenches') on the surface illustrate the fact, already referred to, that the internal energy of a nucleus of even mass number is usually less than might be expected from a consideration of the energies of neighbouring nuclei of odd mass number. A heavy line has been drawn on the surface this is the locus of the minima of all sections of the surface by planes parallel to the $E, N_n/N_p$ plane Such sections are all-important in any consideration of β-transformation, whilst the locus itself is evidently the border-line between regions on the surface corresponding to the possibility of electron and positron disintegration, respectively. The condition (55) which determines the possibility of spontaneous α-disintegration requires that the point $\{E(N_n, N_p), N_n+N_p, N_n/N_p\}$ be not below the point

$$\{E(N_n-2, N_p-2), N_n+N_p-4, (N_n-2)/(N_p-2)\}$$

on the diagram by a distance greater than that corresponding to the binding energy of two neutrons and two protons in an α-particle

$$(27{\cdot}9\times 10^6 \text{ e v } = 44\ 6\times 10^{-6} \text{ erg}),$$

measured on the same scale Due to the general flattening of the energy-surface for heavier nuclei this condition will be fulfilled for sufficiently high values of N_n+N_p Consequently on our schematic energy surface a second α-limiting line may be drawn to mark the boundary of the region characterized by the possibility of α-disintegration, this region evidently lies to the right of the line The figure also shows the projection of the β-limiting line on the $N_n+N_p, N_n/N_p$ plane The points plotted in this plane represent nuclear species at present known, stable nuclei being represented by full circles (●) and electron and positron active nuclei† by open circles (○) and open squares (□), respectively It will be evident on first inspection that the experimental evidence is in accord with the general shape of the internal energy surface Having reached this conclusion we may inquire in greater detail into certain matters of interest.

Hitherto we have referred simply to the 'internal energy surface' and in so doing have somewhat obscured the important fact that only integral values of N_n, N_p have any significance as belonging to real nuclei Thus only a limited number of points on the internal energy surface represent actual values of nuclear energy The dotted line, separating those portions of the surface characterized by electron and positron activity, respectively, passes between these points rather than through any large number of them We are thus led to extend our investigation and inquire how broad a region around this line may be characterized by stability of nuclei against β-transformation,

† The artificial production and the properties of unstable light nuclei are treated in greater detail in Chap X.

either by electron or positron emission. For the points corresponding to stable nuclei must obviously lie in such a region To answer this question we consider in detail the energy curve for isobaric nuclei which is obtained, as already mentioned, as the section of the internal energy surface by the appropriate plane parallel to E, N_n/N_p, the plane $N_n+N_p = A$ This section is shown schematically in Fig. 12, which is drawn for $A = 10$. Again, rather than the section of a continuous surface (giving a smooth curve), we consider a number of points which may be regarded as lying on two similar curves running approximately at a constant vertical separation These two curves refer to nuclei with odd and even atomic numbers, respectively, and their separate occurrence gives rise to a very interesting effect Although it may be impossible for a given nucleus to disintegrate with the emission of a single electron (or positron) it appears that sometimes such a nucleus may be unstable with respect to the emission of two electrons (or positrons) simultaneously Thus the transformation $a \rightarrow b$ is impossible, but the transformation $a \rightarrow c$, energetically permitted This second change involves the emission of two positrons We shall see later, however (Chap VII), that the process of simultaneous emission of two nuclear electrons (or positrons) possesses extremely small probability (10^{-17} year^{-1} for $E_1+E_2 = 10\times 10^6$ e v.), so that it would easily escape detection in most cases For all practical purposes the stable nucleus represented by c and the 'metastable' nucleus represented by a would be equally devoid of any detectable radioactivity It may be that a considerable fraction of the nuclear species discovered by Aston is constituted of metastable nuclei As far as present knowledge goes there occur 48 isobaric pairs and 3 isobaric triplets among the 'stable' nuclei with mass-numbers 1 to 209 In most of these cases one of the pair is very probably metastable in the sense here described a suggestion which is supported

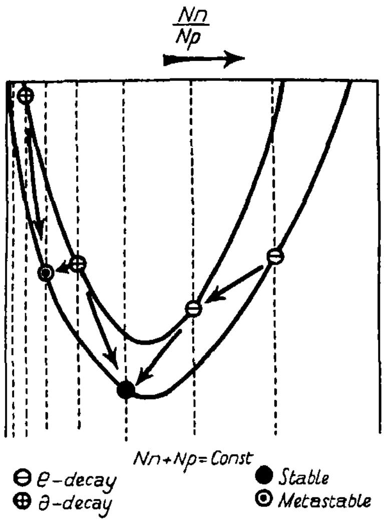


FIG. 12

by the fact that the difference in atomic number between the members of an isobaric pair is, in all cases except two or three, equal to two units—a difference consistent with the possibility of the transformation of one nucleus into the other only by double electronic emission In the case of isobars differing only by one unit in atomic number we must conclude (according to what was said on p 40) either that one of these nuclei is really unstable with a very long lifetime on account of the small energy of the emitted β-particle or that both nuclei are stable, this stability being ensured by the necessity of the emission of a neutrino with finite mass Considerations in this direction may lead to the setting of a lower limit to the mass of the hypothetical neutrino.

In the above discussion the conditions for the spontaneous emission of neutrons, protons, α-particles, and electrons from nuclei have been discussed in terms of the finite differences between internal energy values for real nuclei (N_n, N_p integral). For purposes of calculation we shall now substitute differential coefficients for differences, as we may plausibly do when we remember that the approximate numerical expressions (42), (43) for internal energy, from which we shall start, are applicable only when the number of constituent particles in the nucleus is fairly large. Then, instead of the expressions (53) to (57) we have

for neutron disintegration, $$\frac{\partial E}{\partial N_n} > 0 \tag{58}$$

for proton disintegration, $$\frac{\partial E}{\partial N_p} > 0 \tag{59}$$

for α-disintegration, $$2\left(\frac{\partial E}{\partial N_n}+\frac{\partial E}{\partial N_p}\right) > \Delta\alpha \tag{60}$$

for electron disintegration, $$\frac{\partial E}{\partial N_n}-\frac{\partial E}{\partial N_p} > -\Delta_n \tag{61}$$

for positron disintegration, $$\frac{\partial E}{\partial N_n}-\frac{\partial E}{\partial N_p} < +\Delta_p \tag{62}$$

The last two conditions show us that the β-stable nuclei are located between two close limits, and the central line of this region may be defined by the equation

$$\frac{\partial E}{\partial N_n}-\frac{\partial E}{\partial N_p} = 0. \tag{63}$$

Using his numerical formula (42) for the binding energy Heisenberg has given a graphical representation of the stability limits defined by the above formulae In Fig. 13 the α-decay limit and the central line of β-stability are plotted in the $(N_n+N_p,\ N_n/N_p)$ coordinate system and may be compared with the points representing known stable and unstable nuclei. It will be seen that the calculated curves give a good representation of the general positions of α- and β-limits although they are shifted relative to the real limits If we plot in this diagram the limits calculated from Nukabayasi's

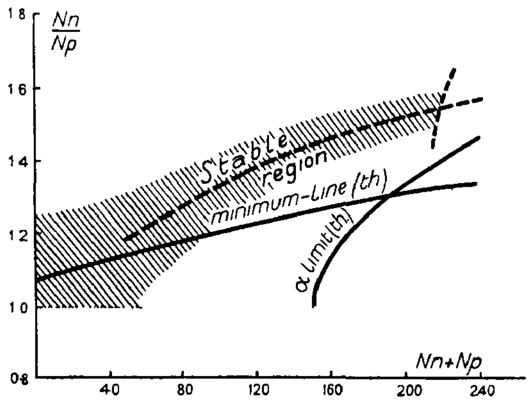


FIG 13

formula (43) we again obtain results which are good qualitatively but bad quantitatively This shows that calculations based on the above statistical method, though sufficiently accurate to represent the general change of binding energies, are too rough to represent in detail the delicate energy balance of nuclear transformations

Our general picture may be somewhat improved by the following considerations First of all the α-stability limit shown in Fig 13 represents the division between nuclei which are absolutely stable against α-disintegration and those possessing the energetical possibility of disintegration with arbitrarily small release of energy Such nuclei, however, would be characterized by extremely long lifetimes (see Chap V), being indistinguishable experimentally from stable nuclei If we assume that α-disintegration cannot be established with certainty if the corresponding half-value period is greater than, for example, 100 times the effective half-value period for Samarium we must conclude that the empirical α-stability limit will correspond to a positive energy balance, being shifted with respect to the true stability limit in the direction of increasing mass-number. A similar consideration may be applied to the limits of β-stability with the result that the band representing the stable region will become slightly broader. Assuming, for example, that a β-active species with half-value period longer than 100 times that of potassium will empirically be considered as stable, we must conclude that the empirical β-stability limits will be shifted upwards for e-emission and downwards for ∂-emission Since this effect is much smaller than the broadening corresponding to metastability (p 42), however, it is of no great importance in actual fact. Finally, calculating the location of the limits for the spontaneous emission of neutrons and protons ((58) and (59)), we find that they are located far out from the regions of α- and β-stability (the neutron limit being represented by a line located far above the diagram of Fig 13 and the proton limit far below), so that we can hardly hope to observe elements showing such spontaneous emission

4. Nuclear shells and periodic properties

We have seen, in the previous sections, that statistical treatment of nuclear structure gives us a very satisfactory description of the general laws governing the system of different nuclei. The next step in the survey must be a more detailed investigation of the conditions obtaining in each given nucleus, and, in particular, a detailed study of the distribution of nuclear neutrons and protons between different quantum-levels in the nucleus Here we may expect the formation of saturated shells, with the appearance of certain periodicities in the run of nuclear radii, binding energies, etc., with the mass-number In light nuclei, as we have already mentioned, the potential distribution has the shape of a smooth hole and the motion of nuclear neutrons and protons may be approximated to by replacing these particles by harmonic oscillators, as was done by Heisenberg in calculations referred to above † As we have seen, in this case we should expect to have saturated shells in $_2He^4$, $_8O^{16}$, and $_{20}Ca^{40}$, and we might look for certain irregularities of the binding-energy curve in these points However, a consideration of this portion of the mass-defect curve shows that we cannot as yet form

† loc cit. 36

any definite conclusions as to the correctness of this hypothesis. For heavier nuclei it seems most reasonable to accept the distribution of potential represented by a 'rectangular' hole ($U = U_0$ for $r < r_0$, $U = 0$ for $r > r_0$). In this case the solutions of the wave equation may be expressed in terms of Bessel functions, and so may be evaluated from the ordinary tables With reference to spherical polar coordinates (r, θ, ϕ) the solutions may be written

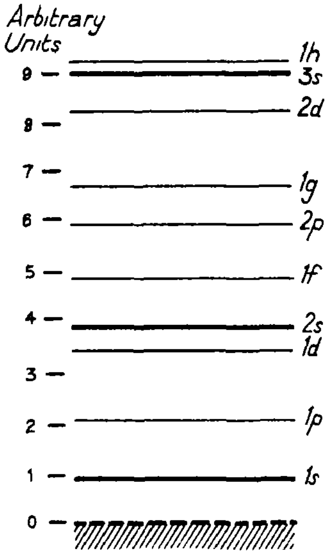


FIG 14

$$\psi = \frac{1}{r^{\frac{1}{2}}} J_{l+\frac{1}{2}}\left(\sqrt{\left\{\frac{2M}{\hbar}(U-E)\right\}}\,r\right) P_l(\theta, \phi), \tag{64}$$

P_l denoting, as before, a spherical harmonic of order l The condition $\psi(r_0) = 0$, r_0 being the nuclear radius, then fixes a set of discrete E values which determine the different quantum states of the nucleus. These are represented on an arbitrary scale in Fig 14 Using the ordinary notation (total and azimuthal quantum numbers) the sequence of these energy levels is $1_0, 2_1, 3_2, 2_0, 4_3, 3_1, 5_4, 4_2, 6_5, 3_0$, †

An attempt has been made by Elsasser‡ to develop a nuclear model by the processes of filling up these levels with neutrons and protons, without regard to the possible formation of α-shells as units of structure This attempt was based on the well-known result that the maximum number of particles (neutrons or protons) which may be accommodated, in a level of azimuthal quantum number l, is $2(2l+1)$ If these considerations are applicable also to lighter nuclei the numbers of particles of each kind in the first three completed levels are 2, 6, and 10, respectively On this assumption the nuclei $_2He^4$, $_8O^{16}$, and $_{18}A^{36}$ represent systems with saturated shells It is important at this stage to inquire whether there is any further evidence on this point are we justified in assuming the existence of

† This order depends upon the form of the potential function chosen For actual nuclei it is to be expected, therefore, that a different sequence may hold, especially for higher quantum numbers

‡ W M. Elsasser, *J de Phys.* **4** (1933), 549

α-particles as individual particles within the nuclear structure? May we reasonably modify Heisenberg's neutron-proton model, already discussed, and consider the nucleus as built up from α-particles, neutrons, and protons, or—expressing the question in its equivalent mathematical form—is it possible to represent the wave function describing a complex nucleus ($A = 4n$), as composed of the wave function for the centres of mass of α-particles, together with the wave functions representing the internal structure of each α-particle?

To answer this question it is necessary to investigate the stability of the α-particle when exposed to the action of the forces existing in the nucleus. This problem was considered by Ehrenfest and Oppenheimer, who showed that the existence of individual α-particles in the nucleus is dependent upon the individual α-particle possessing an energy of binding which is large compared with the energy corresponding to the forces of interaction between α-particles According to this criterion it is to be expected that α-particles in fact exist as such in complex nuclei, since, even in the case of the nucleus ${}_8O^{16}$, the total energy of binding, regarded as holding four α-particles together, is only $12{\cdot}7\times 10^6$ e v, which is to be compared with $27{\cdot}9\times 10^6$ e v for the energy of binding of neutrons and protons in an α-particle However, the above given criterion is based on the assumption that the elementary units from which nuclei are built interact with each other by means of central forces of some kind, it will not hold if these forces are of the type of valency forces, showing the phenomena of saturation In so far as the forces between neutrons and protons most probably belong to this class, it is very important to work out the criterion for this special case also. This has been done by Elsasser †

Elsasser shows that with forces exhibiting saturation phenomena the criterion for the individual existence of α-particles is that the radius of the α-particle shall be small compared with the average distance between these particles in the nucleus ‡ Now the radius of the α-particle is about $0\ 3\times 10^{-12}$ cm (see Chap IX) and the average distance between the α-particles in a nucleus of the order of $0\ 3\times 10^{-12}$ cm. We must thus conclude that, according to this criterion, α-

† W. M. Elsasser, *J de Phys* 5 (1934), 71.

‡ This criterion automatically goes over into the first if the number of valencies between constituent particles becomes very large.

particles do not exist as individual constituent parts of complex nuclei.

Yet these considerations do not prevent us from retaining the notion of α-shells (each formed of a saturated group of two neutrons and two protons) in the same way as we speak of the electronic shells in the outer atom For we have already noted the periodicity in nuclear binding energies which is most simply explained in this way —and this general point of view is important in explaining the main facts concerning β-stability If we adopt this viewpoint then, we should employ the above discussed quantum levels only for the neutrons which are not constituents of saturated α-shells. Then the formation of complete neutronic shells will not take place until the heavier nuclei are reached ($A > 2Z$), and the change in A necessary for the completion of a new shell will be considerably greater than according to the former system

These considerations make it an interesting study to examine the systematics of the existing stable nuclei with a view to finding such anomalies as would indicate the formation of new saturated units of nuclear structure This question was first investigated by Beck† from the point of view of the original proton-electron model of the nucleus ‡ Beck plotted the number of nuclear electrons against the mass-number and was able to point out certain discontinuities in the distribution of points corresponding to the known nuclei More recent investigations in this direction are due to Landé, who, on the basis of the neutron-proton model, investigated periodicity in nuclear structure || The investigation was carried further by Gamow,†† who brought it into connexion with the general stability rules, and in this way was able to explain certain anomalies in the energies of α-disintegration amongst the radioactive elements The latter argument may be developed as follows

In § 3 of this chapter the general shape of the nuclear energy surface (independent variables N_n+N_p and N_n/N_p) has already been discussed, and the locus of the minima for cross-sections of the surface by planes at right angles to the axis of (N_n+N_p) has been accepted as the central line of the region corresponding to nuclei stable as against

† G Beck, *Zs f Phys* **47** (1928), 407, **50** (1928), 548; **61** (1930), 615

‡ From rather a different standpoint a somewhat similar investigation had previously been made by Harkins

|| A Landé, *Phys Rev* **43** (1933), 620

†† G. Gamow, *Zs f Phys.* **89** (1934), 592

β-disintegration. We may map out the position of this minimum line† empirically, within quite narrow limits, by a consideration of the occurrence of isobaric pairs, as follows. The minimum line in question must obviously pass between each pair of points in the energy diagram which corresponds to two stable nuclei possessing the same value of (N_n+N_p) When the plane $N_n+N_p = A$ contains only one point the minimum is less exactly fixed, but even here it may be considered to lie between points neighbouring the existing

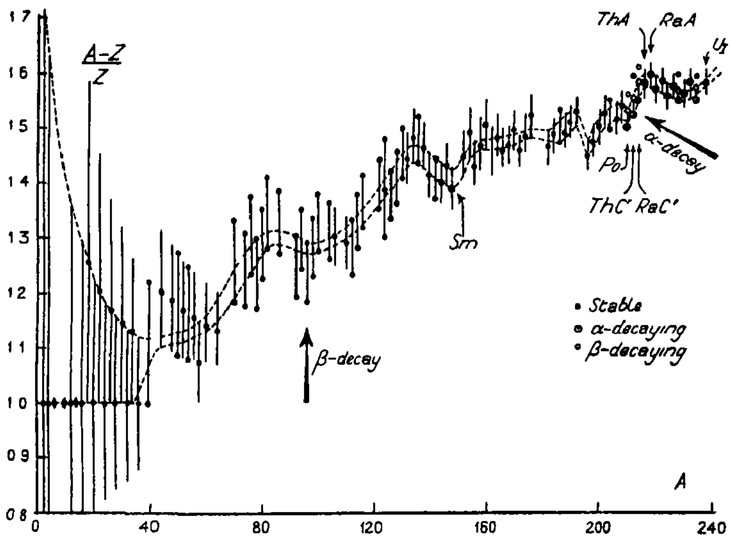


FIG. 15

point on either side and corresponding to those (unstable) nuclei which would transform into the known nucleus by electron or positron emission The vertical lines in Figs 15 and 16‡ join pairs of points on an N_n+N_p, N_n/N_p diagram which correspond either to stable‖ isobaric pairs or to pairs of unknown and hypothetically unstable nuclei which may be regarded as forming isobaric triplets with known nuclei of the appropriate A values These vertical lines constitute the 'gates' directing the course of the minimum line It is evident from their disposition that it is impossible to draw a curve which intersects them all and at the same time shows N_n/N_p increasing

† More exactly, of its projection upon the N_n+N_p, N_n/N_p plane

‡ Drawn separately for nuclei with even and odd masses

‖ We are concerned at present with β-stability, only, radioactive nuclei emitting α-particles must therefore here be considered 'stable'.

monotonically with N_n+N_p. The minimum line which has been drawn in the figure shows clearly the existence both of ascending and of descending portions in its length. This periodicity we assume to arise in a fundamental periodicity characteristic of the structure of the heavy nuclei. New periods appear to begin for mass-numbers 90–100, 140–50 (around Samarium), and 220–30 (the region of the radioactive elements) † Others are suggested in less certain places. We may notice here that the plots made separately for nuclei of even and odd mass-number show much the same peculiarities

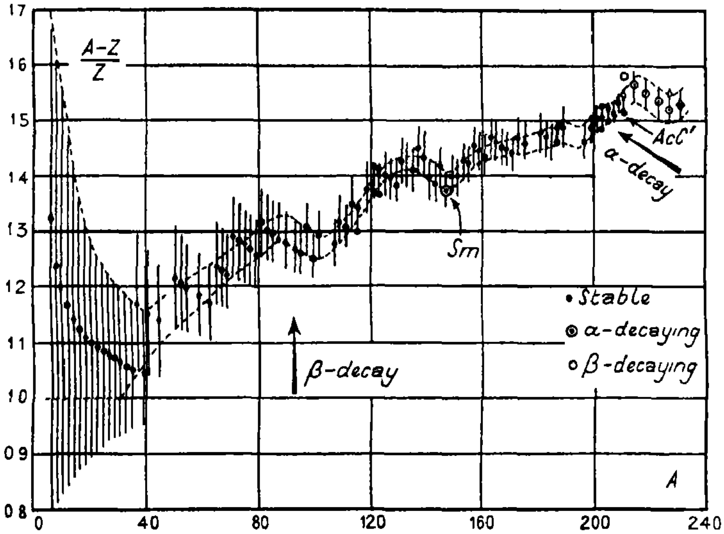


Fig 16

It is easy to see that the fluctuating form of the minimum curve will be reflected in the energy-balance shown in certain nuclear reactions Let us consider in some detail the region of radioactive elements The process of α-disintegration results in a decrease in N_n+N_p and an increase in N_n/N_p (already greater than unity). This effect is shown by the arrow in the figure It will be seen that the minimum curve in places is parallel to this arrow and in other places almost at right angles to it The amount of energy released in α-disintegration will therefore depend markedly upon the position with respect to the minimum line of the point representing the initial nucleus in the figure If we consider as normal those trans-

† A sharp minimum near $A = 200$ may not be real, but due to our ignorance about the isotopic constitution of Pt (G Racah, *Zs f Phys* **93** (1935), 704)

formations in which the nuclear reference point moves roughly parallel to the minimum line, we may expect anomalous disintegrations in which the energy-release is either greater than normal, or, sometimes, less than normal In the former case the initial point has been below the line, in the latter above it On this basis the strange fact that the C′ bodies in all three series show the largest energy of α-disintegration, although they are only just outside the α-stability limit, may be easily understood In Figs 15 and 16 the points representing ThC′, RaC′, and AcC′ are located in just such positions that they move, following α-transformations, down towards the bottom of the 'valley' as defined by the minimum line On the other hand, the points representing the lead isotopes are most probably situated so near the bottom of the valley already as to be perfectly stable as against α-disintegration The point 'RaF' (Po), although it falls near the same ascending portion of the minimum line as does 'RaC′', is so near the beginning of that portion as to be moved, following α-transformation, parallel to the adjacent (descending) portion of the line Polonium, as is well known, emits α-particles of much smaller energy than does RaC′ In like manner the points representing the α-active elements of the three main sequences, Io to RaA, etc , are sufficiently close to a descending portion of the minimum curve for the disintegration energies in these sequences to follow a perfectly regular order In order to explain the low energies of disintegration characteristic of thorium, uranium, and protoactinium we must suppose that the corresponding points are located near the bottom of the energy 'valley', otherwise the fact that these elements are rather far removed from the α-stability limit would speak for a high disintegration energy

Samarium, again, forms an interesting illustration of the use of the energy surface in determining stability and instability of nuclei. It will be seen from Figs 15 and 16 that the Samarium isotopes are represented by points in such a region of the diagram that a considerable energy anomaly is to be expected which may even make spontaneous α-disintegration possible for a certain nucleus or nuclei, although all neighbouring nuclei are perfectly stable in this respect. This is a striking result in view of the established α-activity of one at least of the isotopes of Samarium. Thus we see that even the simple study of the relative numbers of neutrons and protons in different nuclei allows us to reach rather interesting conclusions concerning nuclear structure and at the same time permits us to explain certain anomalies of behaviour which must almost certainly be connected with the completion of saturated shells in the nucleus at various stages in its synthesis. One may hope that further investigations along these lines will add considerably to our understanding of more detailed problems of structure Much has already been done, with rather overlapping results, by Bartlett, Gapon, Ivanenko, Elsasser, Guggenheim, and others; it is not referred to in detail here because the author was never able in studying these articles to remember the beginning when he was reading the end

Periodicity in nuclear structure must obviously be reflected in the curve of binding energy against mass-number, in much the same way as extranuclear periodicity is revealed by the energy curves of Bohr and Coster, but unfortunately Aston's results, here, are still unequal to the demands which would be placed upon them by such an application Actually, an attempt was made by Landé† to get certain information concerning the periodic properties of nuclear structure from the empirical mass-defect curve However, due to the above-mentioned uncertainty in experimental values, no definite conclusions can be drawn from the curves plotted by him

† A Landé, l c.

III

SPINS AND MAGNETIC MOMENTS OF NUCLEI

1. The determination of the mechanical and magnetic moments of nuclei

THE suggestion that atomic nuclei must be regarded as possessing both mechanical and magnetic spin moments was made first by Pauli in an attempt to explain the phenomenon of 'hyperfine structure' of spectral lines The formal explanation in terms of the splitting of electronic energy-levels was thus grounded in the physical result that magnetic interaction between such a nucleus and the extra-nuclear electrons necessarily gives rise to different possible orientations of orbital angular momentum with respect to nuclear spin, and endows these separate atomic configurations with slightly different energies From this point of view the very small separations found experimentally in the normal hyperfine structure patterns were to be explained by the smallness of nuclear magnetic moments compared with the Bohr magneton ($\mu_e = e\hbar/2mc$)—the magnetic spin moment of the electron Nowadays it is general to express nuclear magnetic moments in terms of nuclear magnetons ($\mu_{\text{nuc}} = e\hbar/2Mc$), here M is the mass of the elementary heavy particle. However, as we shall see later, μ_{nuc} is not equal to the magnetic moment of the proton, which is about three times larger The mechanical moment (angular momentum, or—more loosely—'spin') of a nucleus, as of any atomic system, is expressed in terms of the usual quantum unit $\hbar = h/2\pi$

In favourable cases the nuclear spin may be directly determined from the number of components in the hyperfine structure pattern For, let j quantum units be the total angular momentum of the atomic electron configuration (compounded of orbital momentum and electron spin) and i be the angular momentum of the nucleus in the same units Then the resultant angular momentum f will have different values corresponding to the different possible relative orientations of the vectors j and i Applying the rules of addition of quantum vectors we see that f may take the values

$$\left.\begin{array}{llll} f_0 = j+i, & f_1 = j+i-1, & \ldots, & f_{2i} = j-i \quad (j \geqslant i) \\ f_0 = i+j, & f_1 = i+j-1, & \ldots, & f_{2j} = i-j \quad (i \geqslant j) \end{array}\right\}, \tag{1}$$

so that, on account of the nuclear spin, each simple energy-level will split into $2i+1$ or $2j+1$ sub-levels according as $i \leqslant j$ or $i \geqslant j$. The relative separations of these sub-levels may also be calculated if it be assumed that the additional energy due to the magnetic interaction of the nuclear and electronic angular momenta is proportional to the scalar product of nuclear spin and orbital momentum, i.e. proportional to $\cos(i,j)$ Simple calculations show that the relative energy-differences between neighbouring hyperfine structure levels are given by the ratios

$$\left.\begin{array}{ll} f_0 \; f_1 \cdot f_2 \quad \cdot f_{2i-1} & (j \geqslant i) \\ f_0 \; f_1 \; f_2 \cdot \quad \cdot f_{2j-1} & (i \geqslant j) \end{array}\right\}, \tag{2}$$

f_r being defined by the equations (1) This is the so-called interval rule first proposed by Landé †

A slightly different method of determining nuclear spin depends upon the utilization of the Paschen-Back effect which takes place when very intense magnetic fields are employed Then the ordinary Zeeman splitting of spectral lines is so modified that each component of the normal pattern is resolved into $2i+1$ equally spaced components, the observation of which gives i directly, without any prior knowledge of the angular momenta of atomic electrons The Paschen-Back effect occurs when the applied field is able to overcome the magnetic interaction between nucleus and electron shells, so that i and j are independently orientated in the field In practice, however, it has been found difficult to realize this condition in more than a few cases

The determination of nuclear magnetic moment, in addition to spin, should also be possible from a further analysis of hyperfine structure data For absolute values of the separations of sub-levels in the atomic-energy diagram must obviously depend upon the value of the magnetic moment of the nucleus, or, more exactly, upon the projection of this moment along the spin axis of the nucleus. Hitherto the chief difficulties in such a determination have been theoretical rather than experimental General perturbation theory shows that the additional energy of an extranuclear electron due to the magnetic

† As an example of the application of these results we give here the analysis of the hyperfine structure of Pr, for which the energy-level ${}^5K_7$, corresponding to $j = 7$, is shown to be split into six components with relative energy-differences 19 0, 16 7, 14 4, 12 7, 10 4 As the number of sub-levels is less than $2j+1$ ($= 15$) we conclude that in this case $i < j$, and the relation $2i+1 = 6$ gives directly $i = \frac{5}{2}$ For these values of i and j the relative separations predicted by (2) become

19 17 15 13 11,

in good agreement with the observed values.

moment of the nucleus is given by

$$\Delta E = \tfrac{8}{3}\pi\mu_e\mu_{\text{nuc}}g(i)[f(f+1)-i(i+1)-\tfrac{3}{4}]\Psi^2(0), \tag{3}$$

for an S-term ($l = 0$), and

$$\Delta E = \frac{l(l+1)}{j(j+1)}\mu_e\mu_{\text{nuc}}g(i)[f(f+1)-i(i+1)-j(j+1)]\int\frac{\Psi\overline{\Psi}}{\sqrt{3}}\,d\omega, \tag{3'}$$

for higher terms ($l > 0$) Here l, j, i, and f are the quantum numbers specifying, respectively, the orbital and total angular momenta of the electron, the mechanical spin moment of the nucleus, and the resultant of this moment with the total angular momentum of the electron For different sub-levels f takes different values according to (1) Here, also, $i.g(i)$ nuclear magnetons is the effective magnetic moment of the nucleus, and Ψ the wave function for the spectral term in question The difficulties which have been mentioned concern the evaluation of Ψ For atoms with more than one electron the appropriate wave functions cannot be written down in an analytical form, they must be calculated either by the statistical method of Thomas and Fermi or by the more exact method of self-consistent wave solutions, due to Hartree In either case, since the formulae (3), (3′) are very sensitive to small changes in Ψ, the final values for $g(i)$, obtained by applying these formulae to the experimental data, may be rather inaccurate

Nuclear spin, as distinct from nuclear magnetic moment, may sometimes be deduced from an examination of the band spectra of certain molecules For a homonuclear diatomic molecule two possibilities exist the wave-functions may be symmetric or antisymmetric in two nuclear spins Since, however, the two possibilities may each be realized in several ways, in general different statistical weights must be assigned to the two molecular types. Simple considerations show that the ratio of the statistical weights is given by the relation

$$\frac{W_{\text{sym}}}{W_{\text{anti}}} = \frac{i+1}{i}, \tag{4}$$

where i represents the nuclear spin, as before Consequently, since transitions involving change of molecular type do not normally occur, corresponding electronic bands in the spectra of such elementary substances must also show this general ratio of intensities In actual fact, corresponding bands overlap to the extent of forming a single-band system of which consecutive lines belong, alternately, first to one type of molecule and then to the other Hence neighbouring lines in the (rotational) structure of the band must be governed by the intensity relation (4), and it becomes possible to determine i from the measured ratio of line intensity. Since, however, this measurement cannot be carried out with very high accuracy, the method can only be applied satisfactorily when the nuclear spin is small. Otherwise equation (4) shows that the difference in intensity of neighbouring lines is not very considerable. From a further analysis of the band spectrum it is also possible to determine whether or not the wave solution corresponding to parallel nuclear spins is also symmetrical in the space coordinates of the nuclei, that is, whether the quantum statistics of Bose-Einstein or those of Fermi-Dirac apply.

Since nuclear spin is important in any consideration of the interaction between identical particles it should be possible to draw conclusions regarding its magnitude from a complete analysis of the data referring to the scattering, when two such particles are involved (see Chapter IX).

The direct determination of the magnetic moment associated with an atomic system was first successfully carried out by Stern and Gerlach, who measured the deviation of a beam of neutral atoms in passing through an inhomogeneous magnetic field. In this way they deduced the resultant moments for the atoms in question. Stern and his collaborators† have since applied the same method to the determination of the magnetic moments of proton and deuteron. Much more refined measurements were, of course, necessary here, since the nuclear moments are roughly a thousand times smaller than the atomic moments. A slightly different method has been worked out by Rabi‡ and used for a number of elements. Inhomogeneous magnetic fields are again employed, but whereas, for proton and deuteron moment Stern employed molecular hydrogen, in Rabi's method atomic beams are always used. The data obtained by different methods are collected in Table C.||

2. Spin and magnetic moment in relation to the nuclear model

In so far as nuclei are regarded as built up from neutrons and protons it is most important, for present considerations, to know the

† O. Stern and R. Frisch, *Zs. f. Phys.* **85** (1933), 4; O. Stern and I. Estermann, ibid. **86** (1933), 132.

‡ J. Rabi, J. Kellogg, and J. Zacharias, *Phys. Rev.* **46** (1934), 157.

|| At the end of the book.

spin moments characteristic of these elementary particles. The mechanical moment of the proton was first determined from observations of the alternation of intensity in the band spectrum of the hydrogen molecule. The value found, $\frac{1}{2}$ quantum unit, is that to be expected for an elementary rotator. The neutron spin, on the other hand, has not so far been directly determined, but, from the fact that the deuteron (consisting of one proton and one neutron) has a mechanical moment of one unit, it must be assumed that that of the neutron is half-integral, and the simplest assumption is to assign to the neutron, also, the quantum number $i = \frac{1}{2}$ This assumption, however, that the nuclear spins of both neutron and proton are half-integral (and, in particular, each equal to $\frac{1}{2}$ quantum unit), is in open contradiction with the principle of the conservation of angular momentum in the process of β-disintegration, if original notions be retained. Thus, considering either of the elementary reactions $n \rightarrow p+e$, $p \rightarrow n+\partial$, the total angular momentum of the resultants will be compounded from the half-integral momentum of the heavy particle (p or n), the half-integral momentum of the electron (e or ∂), and the integral orbital momentum of relative motion of the two particles. The latter quantity, as is well known, is necessarily integral in terms of the quantum unit The total angular momentum must, therefore, also be integral in terms of this unit The angular momentum of the original particle is, however, half-integral, so that an amount of angular momentum represented by a half-integer has been lost (or gained) in the process This difficulty is quite general and applies to all cases in which there is spontaneous emission of an electron from a nucleus For it is obvious from Table C that, for all elements which have been investigated, the nuclear spin is integral or half-integral according as the nuclear mass-number is even or odd † Now the process of β-disintegration does not produce any change in the mass-number of the nucleus, the above experimental result, therefore, signifies that the change in nuclear spin between initial and final nuclei must be an integral number of quantum units. The mechanical moment of the emitted electron (or positron) and the relative orbital motion in the final system together account for a half-integral component. As before, there is a failure to balance initial and final values —and a discrepancy half-integral in amount.

† This result in itself is clearly favourable to the hypothesis that all nuclei are constituted of protons and neutrons, each particle with half-integral spin

This non-conservation of angular momentum in the process of β-disintegration, together with the non-conservation of energy, which will be discussed in detail in Chapter VII, represents the main puzzle in the problem of the emission of electrons from nuclei. As will be evident later, two different hypotheses have been proposed in an effort to explain the facts. According to Bohr, the principle of conservation of energy, established for those processes which can be satisfactorily described by classical or wave mechanics, is no longer valid for transformation phenomena occurring between elementary particles and must be discarded in any future theory of these phenomena Another point of view is expressed by Pauli, who bases his explanation on the existence of particles of a new kind, neutrinos, particles devoid of charge and possessing very small mass. He supposes that these particles are emitted along with the electrons in β-disintegration in such circumstances as to carry away the 'missing' energy and the balance of the angular momentum For the latter purpose it is, of course, necessary to endow the neutrino with a half-integral mechanical moment (most simply, $\frac{1}{2}$ quantum unit). This whole question will be treated fully in the chapter dealing with β-disintegration, for the present the above brief mention must suffice

Before the magnetic moment of the proton was determined experimentally it was general to assume that its value would prove to be 1/1840 of that of the electron † This expectation was based on the assumption that the proton could be described, as the electron was satisfactorily described, in terms of Dirac's theory of the elementary charged particle According to this theory the magnetic moment of an elementary particle is inversely proportional to its mass However, the experiments of Stern, Estermann, and Frisch‡ and of Rabi, Kellogg, and Zacharias‡ demonstrated without doubt that the magnetic moment of the proton was actually nearly three times greater than was expected || This was very surprising until it was pointed out by Bohr that the proton did not satisfy all the conditions necessary for an elementary particle in Dirac's theory. Bohr showed that for the purposes of the theory a particle may be considered as elementary only when its real radius is small compared with the critical length $l = \hbar/mc$, where m is the mass of the particle in question. This condition is always fulfilled if the radius of the

† 1/1840 is the inverse ratio of the masses

‡ See references on p 56.

|| Most recent value. $\mu_p = 2\ 9\mu_{\text{nuc}}$.

particle is given on the classical assumption of pure electromagnetic mass, for then, to an order of magnitude,

$$r = e^2/mc^2 \quad \text{and} \quad r/l = e^2/\hbar c = 7{\cdot}3 \times 10^{-3},$$

which is certainly small compared with unity. Actually, the idea of pure electromagnetic mass is not generally held at the present time,† but in the case of the electron, at least, it is almost impossible that it should give a value of the radius several hundred times larger, for in such a case it would be as large as the volume occupied by the K orbits in heavy atoms Here no disturbance due to the finite size of the electron has ever been observed. The situation is rather different in the case of the proton The critical length now is already an order of magnitude smaller than the nuclear radius, being $2{\cdot}1 \times 10^{-14}$ cm, moreover, in the case of the proton, there is good reason to suppose that the real radius is very much greater than the classical radius ($1{\cdot}5 \times 10^{-16}$ cm) It has already been shown that the stability of the nucleus can only be secured on the basis of a structure formed of constituent particles interacting only at short distances (effective radius). Some idea of the effective radius of the proton may obviously be obtained on this supposition In the uranium nucleus there are 238 particles (neutrons and protons) within a sphere of radius $\sim 0{\cdot}9 \times 10^{-12}$ cm, we must therefore assign the range of action $\sim 0{\cdot}15 \times 10^{-12}$ cm to the individual particles. This is greater than the above defined critical length for the case of the proton A closely similar value is given by the results of experiments concerning the scattering of neutrons in hydrogen (see Chap IX) At this distance, these experiments show, strong attraction between the two particles begins. In just one respect, however, each of these estimates leaves something to be desired, since the sum of the radii of neutron and proton is obtained directly, rather than the radius of either separately The recent experiments on scattering of protons in hydrogen (see Chap IX) definitely show that the radius of the proton is of the order of magnitude 10^{-13} cm In that case Bohr's criterion for the applicability of Dirac's theory definitely returns a negative answer in respect of the proton, r/l is not less than unity and the proton cannot be regarded as an elementary particle for the purposes of the theory

One is naturally led to inquire what equations should be used for

† It is self-evidently false for the neutron.

the relativistic quantum description of the motion of the proton, if Dirac's equations fail in this case. For one remembers that Dirac's equations constitute the only mathematically possible wave equations for a particle which are consistent with the principle of relativity (p. 5) It seems that the most plausible way of avoiding this paradoxical situation is by the assumption that the conditions never actually arise in which a relativistic treatment of the motion of the proton is required. In order that such a quantum-mechanical treatment is necessary to describe the motion of a material particle, the particle in question must be situated in an extremely intense field of force, and it is not unlikely that under these conditions the transformation of a proton into a neutron and a positron (and the corresponding disintegration of the neutron) takes place so frequently that there is no longer any physical sense in referring to the motion of a single particle Inside atomic nuclei the velocities of protons and neutrons are still small compared with the velocity of light (Chap II) and in most cases it is sufficient to apply the ordinary Schrödinger's wave equation to the problem of their motion On the other hand, it seems probable that the phenomena of β-emission are in some way to be ascribed to the fact that the velocity of the proton (or neutron) is not vanishingly small compared with the velocity of light, in this case they will need for their explanation relativistic quantum equations in terms of which the behaviour (motion and transformations) of heavy elementary particles may be described

The magnetic moment of the neutron, like the corresponding angular momentum, has not been measured directly, and it is obvious that its determination will present serious difficulties from the experimental point of view. However, an attempt has been made by Schuler to form an estimate of its magnitude from a consideration of the magnetic moments of proton and deuteron Stern and Estermann and Rabi, Kellogg, and Zacharias have shown that the magnetic moment of the deuteron is only about one nuclear magneton † If neutron and proton in the composite nucleus $_1H^2$ have zero orbital momentum, then obviously the magnetic moment of the neutron must equal the difference between the magnetic moments of deuteron and proton, that is about -2 nuclear magnetons ‡ This is the con-

† Most recent value $\mu_d = 0\ 85\mu_{\text{nuc}}$

‡ The negative sign indicates that the axes of spin and magnetic moment are oppositely directed.

clusion arrived at by Schüler. On the same assumption the nuclear spin of the deuteron, 1 unit, is regarded as the resultant of the equal and parallel spins of neutron and proton, and the fundamental state of the nucleus is an S state (orbital angular momentum zero). This is in line with the general theorem in wave mechanics that the fundamental state of any system of two particles, acting upon one another with central forces, is always an S state, but, on the other hand, the validity of this general result can be seriously questioned in so far as the deuteron is concerned For it must not be forgotten that the radii of the interacting particles, in this case, are of the same order as their distance apart, so that the particles may even penetrate into one another's structure † If the internal structure and the laws of interaction of proton and neutron cannot satisfactorily be described in terms of the equations of wave mechanics, the applicability of the above-mentioned theorem is immediately suspect Moreover, if this theorem is not valid for the deuteron, then for any nucleus constituted of protons and neutrons the fundamental state may be characterized by an azimuthal quantum number different from zero ($j \neq 0$) For the deuteron two other possible assumptions would ascribe the resultant angular momentum to the combinations $-\frac{1}{2}+\frac{1}{2}+1$ (P state) or $-\frac{1}{2}-\frac{1}{2}+2$ (D state) In these two cases a magnetic moment would be associated with the orbital momentum (1 unit or 2 units, as the case may be) and until its contribution was known no certain conclusion could be drawn from the data in question Up to the present no trustworthy method of calculating the magnetic moment associated with such orbital momentum has been developed, the ordinary simple relation between these two quantities being certainly invalid for the nucleus In fact we may apply the relation (magnetic moment)/(mechanical moment) $= e/2m$‡ only when the radius of the orbit is large compared with the radius of the particle, a condition which is evidently not fulfilled for the proton in the nucleus. For the isolated proton, rotating about its own axis, (magnetic moment)/(mechanical moment) $\sim 6e/2m$, the proton is then said to be characterized by a gyromagnetic ratio of about 6 For a proton in the nucleus it is to be expected that the appropriate value for this ratio will lie somewhere between 6

† Roughly speaking one may say that the radius of the first S orbit may be smaller than the sum of the radii of the particles in question, so that this S state will be excluded from purely 'geometrical' reasons

‡ e and m are, respectively, the charge and mass of the particle in question

and 1. The precise value must depend, amongst other things, on the relative sizes of orbit and particle, in each case As an extreme instance we may consider the two protons in a hydrogen molecule, where the separation is very large indeed compared with the radius of the proton For this system the gyromagnetic ratio is 1, as might be expected †

In the absence of a complete theory of the elementary heavy particle it is obvious that deductions from observed values of nuclear magnetic moments must be made with the greatest caution. Thus, on the basis of a D state for the deuteron and a gyromagnetic ratio of about 2 for the proton, it is possible to account for the magnetic moment of ${}_1H^2$ whilst assigning zero magnetic moment to the neutron

3. Isotope displacement and nuclear radii

In addition to magnetic hyperfine structure there is another effect of the nucleus on the energy-levels of atomic electrons which is of some importance This is the so-called isotope displacement of spectral lines which was first observed by Schuler for several lines in the spectrum of thallium‡ and has since been shown to be a phenomenon of fairly frequent occurrence with the heavier elements. It consists in a rather small displacement separating the lines due to the different isotopes of the element under investigation Obviously the various components of this isotopic hyperfine structure have intensities proportional to the relative abundances of the corresponding nuclear species in the mixed element They may or may not exhibit magnetic hyperfine structure also, and when, as is often the case, different components show different degrees of structure, a very complicated pattern may result

As a characteristic example of this phenomenon we may consider the structure of the mercury green line, $\lambda = 5{,}461$ Å U , which is due to the transition $6\,{}^3P_2 \rightarrow 7\,{}^3S_1$ ‖ This structure is shown in Fig 17 with intensities of the various components indicated by the lengths of the lines According to the analysis of Schuler, the components f, g, k, and l must be ascribed to the isotopes of even

† For the system neutron-proton at great separation the gyromagnetic ratio would be expected to be 0 5, since only one of the particles is charged.

‡ H Schuler and J. Keyston, *Zs f Phys* **70** (1931), 1.

‖ H. Schüler, J. Keyston, and E Jones, ibid. **72** (1931), 423, **74** (1932), 631.

mass-number, $_{80}Hg^{198}$, $_{80}Hg^{200}$, $_{80}Hg^{202}$, and $_{80}Hg^{204}$, which presumably possess no spin ($i = 0$) since they give rise to single lines, whilst $_{80}Hg^{199}$ is responsible for the three components c, k, and p and $_{80}Hg^{201}$ for the eight components a, b, d, e, f, m, n, and o. The nuclear spins of these species with odd mass-number have been shown to be given by $i = \frac{1}{2}$ and $i = \frac{3}{2}$, respectively. Fig. 17 shows the components due to the even-numbered isotopes together with

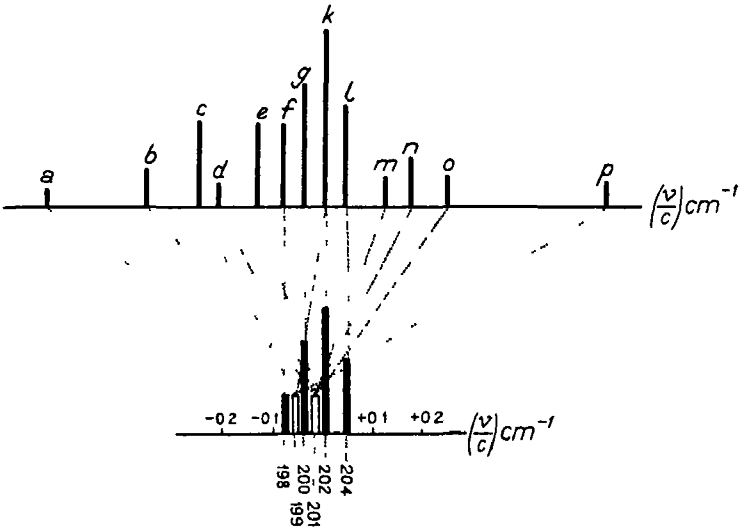


FIG. 17

the 'centres of gravity' of those due to the odd-numbered species. The *total* intensities in the corresponding hyperfine structure patterns are represented in this figure We see that the relative intensities due to the different components arrived at in this way, 12 : 11 · 27 · 10 · 32 · 8 (per cent of the total intensity), are in good agreement with the relative abundances of the isotopes as determined by Aston, namely,

$$_{80}Hg^{198} \quad _{80}Hg^{199} \quad _{80}Hg^{200} \quad _{80}Hg^{201} \quad _{80}Hg^{202} \,.\, _{80}Hg^{204}$$
$$= 9{\cdot}89 \quad 16\ 45 \,.\, 23\ 77 : 13{\cdot}67 \quad 29{\cdot}27 \,.\, 6{\cdot}85.$$

Fig. 17 also shows that the isotope displacement is a monotonic function of the mass-number concerned, though the components due to the odd-numbered isotopes do not fall quite symmetrically between those due to the isotopes of even mass-number. The average wave-number separation for successive even-numbered isotopic components ($\Delta M = 2$), in this particular case, is 28×10^{-3} cm.$^{-1}$

Turning now to the theoretical aspect of the problem it soon becomes evident that we must look for an explanation of isotope displacement in terms of deviations from the simple Coulomb law of force in the immediate neighbourhood of the nucleus An explanation in terms of nuclear motion alone (compare the original calculations concerning the spectra of hydrogen and singly ionized helium) was shown to be quite inadequate to cover either the magnitude of the effect or the rather particular conditions in which it is observed A complete discussion of the phenomenon, however, is very complicated and belongs more to atomic theory than to the theory of the nucleus we shall content ourselves here with the general treatment for the case of an S electron, where conditions are simplest The first calculations in this direction are due to Bartlett,† but by reason of the very rough approximations employed—neglect of relativity corrections and the screening effects of other atomic electrons—they could not be expected to lead to an accurate result Somewhat more detailed calculations have since been made by Racah‡ on the basis of Dirac's equation for the motion of the electron and taking count of screening, independently, calculations of a similar nature have been carried out by Breit and Rosenthal For the field of the nucleus, effective for the atomic electron, Racah accepts the Coulomb potential $U = Ze/r$ for values of $r > r_0$, and, within the critical radius r_0, the constant potential $U = Ze/r_0$ He works out the solutions of Dirac's equations for such a field on the assumption that they are asymptotically equivalent to the Schrödinger eigenfunctions for large values of r Then it appears that the electron density in the centre is given by the expression (for an S orbit)

$$|\psi(0)|^2 = \frac{2(1+\rho)\psi_0^2(0)}{[\Gamma(2\rho+1)]^2}\left(\frac{2Zr_0}{a}\right)^{2\rho-2}, \tag{5}$$

where $\rho = (1-\gamma^2)^{\frac{1}{2}}, \quad \gamma = Z\alpha = Ze^2/\hbar c \sim Z/137,$

$a = \hbar^2/me^2$ (Bohr's radius for hydrogen) and $\psi_0(0)$ is the value given by the unrelativistic solution at the origin, which can be worked out numerically by the statistical method of Thomas-Fermi. For the

† J. H Bartlett, *Nature*, 128 (1931), 408.

‡ G Racah, *Nature*, 14 May 1932

change in wave number due to the finite radius r_0 this result gives

$$\Delta\left(\frac{\nu}{c}\right) = \frac{4(1+\rho)\psi_0^2(0)}{[\Gamma(2\rho+1)]^2\hbar c}\left(\frac{2Z}{a}\right)^{2\rho-2}\int\limits_0^{r_0}\left(\frac{Ze^2}{r}-\frac{Ze^2}{r_0}\right)r^{2\rho}\,dr$$

$$= \frac{4(1+\rho)\psi_0^2(0)}{[\Gamma(2\rho+1)]^2(2\rho+1)\hbar c}\left(\frac{2Z}{a}\right)^{2\rho-2}\frac{Ze^2r_0^{2\rho}}{2\rho}. \tag{6}$$

If, therefore, the nuclear radii, in the case of two isotopes, differ by an amount δr_0, the relative change in wave number between the corresponding S levels considered becomes

$$\delta\Delta\left(\frac{\nu}{c}\right) = \frac{4(1+\rho)\psi_0^2(0)}{[\Gamma(2\rho+1)]^2(2\rho+1)\hbar c}\left(\frac{2Z}{a}\right)^{2\rho-2} Ze^2r_0^{2\rho-1}\,\delta r_0 \tag{7}$$

Assuming for the electron density $\psi_0^2(0)$ in the centre of a mercury atom the value 3×10^{26} cm.$^{-3}$, obtained by numerical computation by the Thomas-Fermi method, and using the experimental value $\delta\Delta\left(\frac{\nu}{c}\right) = 28\times 10^{-3}$ cm^{-1} for the relative isotope displacement in the case of mercury, Racah's formula shows that the fractional change in nuclear radius for two 'neighbouring' mercury isotopes is given by $\delta r_0/r_0 \sim 10^{-4}$. This change appears to be too small, on the basis of the hypothesis of constant nuclear density, for then

$$\delta r_0/r_0 = \delta M/3M \sim 10^{-2}.$$

However, Breit† has claimed that the value for $\psi_0^2(0)$ used by Racah is too high, as the result of cumulative errors in numerical calculations. Using the simple Landé formula

$$\psi_0^2(0) = l(l+1)\left(\overline{\frac{1}{r^3}}\right) = \frac{R\alpha^2 Z_i Z_0^2}{\mu_0^2(2l+1)n_0^2}, \tag{8}$$

Breit obtains for the density at the origin smaller values, leading to values of $\delta r_0/r_0$ in good agreement with the hypothesis of constant nuclear density

For a discussion of more complicated cases of isotopic shift (for electronic orbits other than S) readers interested in these questions are referred to the original papers on this subject

In conclusion we must mention an important observation of Schuler and Schmidt,‡ who, investigating the hyperfine structure of certain rare earths, found deviations from the interval-rule of Landé (p 54).

† G Breit and J Rosenthal, *Phys Rev* **41** (1932), 459

‡ H Schuler and T Schmidt, *Zs f Phys* **94** (1935), 457, **95** (1935), 265.

Fig. 18 represents the structure of the line 6,865 ($a^8S^0_{7/2}-z^{10}P_{9/2}$) of Europium as measured by these authors The distribution to be expected from the interval-rule (2) is plotted in the lower part of the diagram and we can clearly see that there are systematic deviations between the two sets of components. Schüler found from measurements on this and other spectral lines that the deviations in question are rather closely proportional to the square of the cosine of the angle between i and j, so that the complete interaction between the atomic electrons and the nucleus is determined by the factor

$$\cos(i,j)+\alpha\cos^2(i,j), \tag{9}$$

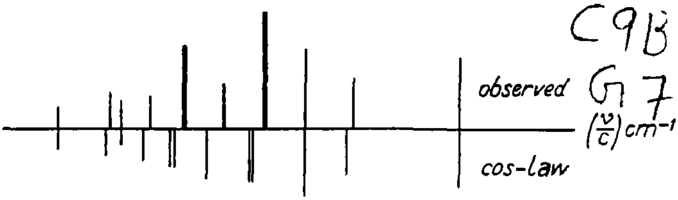


FIG 18

where α is a small quantity different for different nuclei Schuler attempts to account for the observed deviations from the cosine rule for a given nucleus by postulating a certain degree of asymmetry for the nucleus In fact, if we assume that a nucleus deviates from spherical symmetry, we must expect, except for S orbits, that the overlapping of the wave function of the atomic electron with the space occupied by the nuclear particles will be different for different orientations of orbital momentum and nuclear spin (symmetry-axis of the nucleus) This will lead to small corrections to the perturbation energies for different components of the hyperfine structure and one can even conclude that these corrections will be, to the first approximation, proportional to $\cos^2$ of the angle between the angular momentum and nuclear spin. In this way Schuler and Schmidt were able to show that the nucleus $_{80}Hg^{201}$ is characterized by a quadrupole moment $q = 0\,5\times10^{-24}$ absolute units and that this nucleus, being asymmetrical, is somewhat elongated in the direction of the spin axis. We thus see that the detailed investigations of hyperfine structure will give us important information not only about the spins but also about the symmetry properties of different nuclei

IV

ELECTROMAGNETIC RADIATION OF NUCLEI

1. γ-ray spectra

THE atomic nucleus, being a quantized system, is in general capable of existence in any one of a number of states of different energy. Being raised to a higher state it may be characterized by definite probabilities of transition to the various states which have smaller energy than the state in question when such a transition occurs it will usually be with the emission of the excess energy in the form of a quantum of electromagnetic radiation. This is the formal description of those nuclear processes which are involved in the emission, in spontaneous and artificial transformations of nuclei, of a quantum radiation to which the name γ-radiation is generally given.

In so far as the energy-differences between nuclear levels are in general much larger than the corresponding differences for the extra-nuclear system, the wave-lengths of γ-rays are much shorter than the wave-lengths of atomic radiations and the ordinary methods of spectroscopy using ruled gratings or crystals are hardly suited to them When these methods have been used—as originally by Rutherford and Andrade—they have been successful only for the less energetic components of the radiation The usual method of γ-ray spectroscopy, worked out independently by Ellis and Meitner, is based on the phenomenon of 'internal conversion' of γ-rays This phenomenon can be regarded, to some extent, as a special case of the photoelectric effect, in that the γ-ray quantum is absorbed in the same atom from the nucleus of which it has just been emitted As in the external photoelectric effect an electron is thereby ejected from one of the atomic levels That this description of the phenomenon is imperfect need not concern us here, but we shall see later that there is also another possibility—the direct mechanical transmission of energy from the excited nucleus to one of the atomic electrons Because the internal photoelectric effect may happen for any electronic shell in the atom there will arise, for every line in the γ-ray spectrum of the nucleus, a number of secondary electronic groups having energies $h\nu-E_K$, $h\nu-E_{L_\mathrm{I}}$, $h\nu-E_{L_\mathrm{II}}$, $h\nu-E_{L_\mathrm{III}}$, $h\nu-E_{M_\mathrm{I}}$,..., where E_K, E_{L_I}, E_{L_II}, E_{L_III}, E_{M_I},.. are the binding energies corresponding to the different electron levels and ν is the frequency of the γ-ray line Now it has been shown that the γ-radiation associated with any particle-disintegration of a radioactive nucleus is in fact characteristic of the excited product of the disintegration, the binding energies above used must thus be understood to refer to this nucleus and not to its immediate parent The method of Ellis and Meitner, therefore, consists in determining the energies of the secondary electron groups† by magnetic analysis and applying the above relations to deduce the energies and so the frequencies in terms of which the primary γ-ray spectra are to be specified As an example of the numerical procedure, the table below contains the results of the work of Meitner on the natural β-ray spectrum of actinium X and its analysis in terms of the γ-rays which are emitted from actinon left excited by the previous α-disintegration, AcX $\xrightarrow{\alpha}$ An Whilst possessing many advantages, the method of internal conversion is less satisfactory when intensities of γ-ray lines are in question, rather than the corresponding energies The intensities of the natural β-ray lines may be measured, it is true, but since the coefficient of internal conversion varies with the frequency and the type of radiation involved (see § 3), these do not give a simple

TABLE III

Analysis of secondary β-ray spectrum emitted by AcX ($Z = 88$)

Energy of sec β-group $\times 10^6$ *erg*	*Intensity*	*Level of origin for* $Z = 86$	*Energy of the level* $\times 10^6$ *erg*	*Energy of the γ-ray* $\times 10^6$ *erg*	*Deviations per cent*
0 0723	80	K	0 1554	0 2277	0 6
0 2003	50	L_I	0 0286	0 2289	
0 2204	25	M_I	0 0071	0 2275	
0 0889	100	K	0 1554	0 2443	0 6
0 2159	60	L_I	0 0286	0 2445	
0 2358	15	M_I	0 0071	0 2429	
0 0950	40	K	0 1554	0 2504	1 0
0 2204	25	L_I	0 0286	0 2490	
0 2407	15	M_I	0 0071	0 2478	
0 1658	40	K	0 1554	0 3212	2 0
0 2860	15	L_I	0 0286	0 3146	
0 2717	100	K	0 1554	0 4271	0 2
0 3990	30	L_I	0 0286	0 4276	

† These may easily be distinguished from the distribution of primary β-particles, which have all energies between wide limits (see Chap VII)

measure of the γ-ray intensities which are required. In many cases, however, they constitute the full extent of our information, and in Table D (at the end of the book), in which all the available data concerning the γ-ray spectra of the radioactive substances are collected, the numbers in the last column represent the intensities of the secondary electron groups originating in the K-shell. To this extent they provide only a very rough indication of the relative intensities of the γ-ray lines concerned.

A method of investigation of γ-ray spectra which does permit of more trustworthy estimates of intensity being made is due to Skobelzyn. It depends upon the observation of the Compton electrons produced in the gas of a Wilson expansion chamber and the determination of their energies by means of an applied magnetic field. The source of γ-rays is placed at some distance from the chamber. The method has its disadvantages, however: energies are not measured with such high accuracy as before, and only the stronger lines in the spectrum are easily detected. Both methods require fairly strong γ-ray sources for their success, they are applicable, therefore, only to the cases of spontaneous disintegration of radioactive bodies. For the considerably less intense γ-rays emitted in the process of artificial nuclear transformation the old method of analysis of the absorption curves is usually applied. According to Klein and Nishina the absorption coefficient for high-energy radiation is given by the expression

$$\sigma = 4{\cdot}96\times10^{-25}.n\left[\frac{1+\alpha}{\alpha^2}\left\{\frac{2(1+\alpha)}{1+2\alpha}-\frac{1}{\alpha}\log(1+2\alpha)\right\}+ \right.$$
$$\left. +\frac{1}{2\alpha}\log(1+2\alpha)-\frac{1+3\alpha}{(1+2\alpha)^2}\right], \qquad (1)$$

where $$\alpha = h\nu/mc^2 \qquad (1')$$

and n is the total number of atomic electrons per c.c. of the absorbing substance. However, recent investigations have shown that this formula, which takes count only of absorption due to Compton scattering, should be supplemented by an additional term representing the effect of the formation of pairs of positive and negative electrons by interaction of the γ-rays with the nuclei. This term, which becomes especially important for high quantum energies, was discussed by Oppenheimer and Plessett and will be considered in more detail in the Appendix at the end of the book.

The resultant total absorption coefficient is plotted in Fig 70 (in the Appendix) as a function of $h\nu$, in the case where lead is the absorbing substance We see that to a given absorption coefficient may correspond two different values of the quantum energy—a fact which sometimes may lead to much confusion With this reservation, however, Fig. 70 of the Appendix may be used whenever it is possible to analyse a crude absorption curve into pure exponential components In this way a fairly good idea may frequently be obtained regarding the spectral composition of the γ-radiation emitted in the process of artificial transmutation, although the information so obtainable is necessarily much less exact than that which may be reached by the methods above described when applied to the naturally radioactive substances

2. Selection principles and the intensities of γ-rays

The general selection rules which have become well established for radiative transitions occurring in the outer atom are applicable to the nucleus also, they may be formulated in terms of the change of nuclear angular momentum (spin) in the process of γ-emission. If the radiating nuclear particle can be considered as moving in a spherically symmetrical field of forces due to the rest of the nucleus (which, as we shall see later, affords some approximation to conditions in real nuclei), the selection principles for γ-ray emission may be formulated in the following way.

1. If the initial and final states involved in the transition both possess angular momentum (spin) equal to zero,† that is if the transition is of the type

$$i = 0 \rightarrow i = 0, \tag{2}$$

no γ-ray emission is possible, the transition in question may take place only through the intervention of an extranuclear particle, resulting in the *mechanical discharge* of the energy of nuclear excitation This process is analogous to that occurring in 'collisions of the second kind' between atoms.

2 If the initial and final states are such that the difference in spin quantum number is unity,

$$\Delta i = \pm 1, \tag{3}$$

† In the case of radiation by a complex system such as an atomic nucleus it is more convenient to speak of the change in total angular momentum, rather than the change in angular momentum of any constituent particle.

radiative transitions are permitted In this case the field of the emitted radiation is the ordinary dipole field—the most common type of radiation field when atomic and molecular radiations are in question.

3 If the change in spin quantum number is two units, or is zero (except as covered by rule 1),

$$\Delta i = \pm 2 \text{ or } \Delta i = 0 \quad (i \neq 0), \tag{4}$$

the emitted radiation is a quadrupole radiation possessing in general a much smaller transition probability than a similar dipole radiation. In optical spectra a few weak lines arise in quadrupole radiative transitions

4 Transitions corresponding to larger spin differences,

$$|\Delta i| \geqslant 3, \tag{5}$$

possess still smaller probabilities, the radiation fields by which the emission is described in such cases are characterized by higher symmetry (octopole, etc) Analogous transitions have never been found to occur in the outer atom Somewhat different selection rules apply to such cases as cannot be discussed in terms of the 'splitting' of the nucleus into 'radiating particle' and 'the rest of the nucleus'. Then we must consider the γ-radiation as due to transitions between quantum states of the whole nucleus, and in such a case (1) the transition $i = 0 \rightarrow i = 0$ still remains absolutely prohibited, (2) dipole transitions are possible for $\Delta i = 0$ or ± 1, (3) quadrupole transitions for $\Delta i = 0, \pm 1, \pm 2$, (4) octopole transitions for $\Delta i = 0, \pm 1, \pm 2, \pm 3$, etc

In all cases we should also apply the Laporte rule concerning transitions between 'even' and 'odd' terms† of the nuclear quantum-level system Just as in the case of atomic radiation this rule can be formulated as follows

(1) For electric dipole radiation only the transitions *even* $\rightleftarrows$ *odd* are permitted

(2) For electric quadrupole only *even* $\rightleftarrows$ *even* or *odd* $\rightleftarrows$ *odd*

(3) For electric octopole again *even* $\rightleftarrows$ *odd* and so on, alternately, for electric radiations of higher degree of symmetry.

(4) On the other hand, for all kinds of magnetic radiations (which

† The terms for which the wave functions can be transformed into each other by mirror-reflection.

might be important in the nucleus) only *even* $\rightleftarrows$ *even* or *odd* $\rightleftarrows$ *odd* transitions are permitted.

For ordinary electric dipole radiation, arising in a transition from the kth to the lth state of the radiating system, the transition probability, as is well known, is given by the formula

$$\kappa_{k,l}^{d} = \frac{4}{3}\frac{Z^2e^2}{\hbar c^3}\omega_{k,l}^3|r_{k,l}|^2, \tag{6}$$

where $\omega_{k,l}$ is the 'frequency'† of the γ-ray which is emitted and $Zer_{k,l}$ is the matrix element corresponding to the dipole moment of the system. From the general wave-mechanical relation

$$\sum_{l=0}^{l=k-1}\frac{2M\omega_{k,l}}{\hbar}|r_{k,l}|^2 = 1, \tag{7}$$

it follows that
$$|r_{k,l}|^2 \leqslant \frac{\hbar}{2M\omega_{k,l}}, \tag{8}$$

and it is clear that the upper limit of the dipole-radiation probability is given by the inequality

$$\kappa_{k,l}^{d} \leqslant \frac{2}{3}\frac{Z^2e^2}{M\hbar^2c^3}(\hbar\omega_{k,l})^2 \tag{9}$$

Since the probability of quadrupole radiation is smaller than that of dipole radiation by the factor $(r_0/\Lambda)^2$,

$$\kappa_{k,l}^{q} \leqslant \left(\frac{r_0}{\Lambda}\right)^2\frac{2}{3}\frac{Z^2e^2}{M\hbar^2c^3}(\hbar\omega_{k,l})^2 \tag{10}$$

Here r_0 is the nuclear radius and Λ the wave-length of the emitted radiation As above stated, in similar circumstances the probabilities of radiative transitions of higher order are much smaller even than is given by (10)

As an example of the application of the above results we may calculate the probabilities of emission of a quantum of radiation of energy, $h\nu$, one million electron volts by an oscillating proton (or α-particle, since Z^2/M is very nearly the same for these two particles) We have

$$\kappa^d \leqslant 2.10^{15}\ \text{sec}^{-1};\quad \kappa^q \leqslant 2.10^{11}\ \text{sec.}^{-1},\ \text{respectively}$$

At this stage it is necessary to introduce an important correction, first worked out by Perrin,‡ which takes into account the motion of the rest of the nucleus We suppose that the radiating system consists of two 'particles' (the excited particle and the rest of the

† $\omega = 2\pi\nu$, thus $\hbar\omega = h\nu$

‡ F Perrin, *C R* **195** (1932), 775

nucleus) of masses M_1 and M_2 and charges $Z_1 e$ and $Z_2 e$ Then, as is well known, we may reduce the problem to that of the motion of a single particle of mass $M_1 M_2/(M_1+M_2)$ In this case the electric moment of the system relative to the centre of mass is given by

$$\frac{M_2}{M_1+M_2} Z_1 er - \frac{M_1}{M_1+M_2} Z_2 er = \frac{M_1 M_2}{M_1+M_2}\left(\frac{Z_1}{M_1}-\frac{Z_2}{M_2}\right)er, \tag{11}$$

and the probability of dipole-radiation emission becomes

$$\kappa_{k,l}'^{d} \leqslant \frac{2}{3}\frac{e^2}{\hbar^2 c^3}(\hbar\omega_{k,l})^2 \frac{M_1 M_2}{M_1+M_2}\left(\frac{Z_1}{M_1}-\frac{Z_2}{M_2}\right)^2. \tag{12}$$

We may use the expressions (9) and (12) to obtain the ratio κ'/κ for the corresponding emission probabilities relative to different possibilities of dipole transitions in an excited RaC′ nucleus We have, for example,

for an excited α-particle, $\kappa'/\kappa = 0\ 047$,

for an excited neutron, $\kappa'/\kappa = 0\ 18$

It will be noted that in each case the correction which is introduced results in a remarkable lowering of the transition probability finally obtained, that for emission by an α-particle transition being roughly twenty times smaller than previously calculated—smaller, in fact, than the probability of a similar neutron transition (in which the dipole moment is wholly associated with the rest of the nucleus). If the probability of dipole-radiation transition is to be still further reduced in any case this reduction must come from some special symmetry of nuclear structure affecting the value of the matrix element corresponding to the dipole moment of the system

3. Internal conversion of γ-rays

It has already been stated that the phenomenon of internal conversion of γ-rays consists in the emission of an extranuclear electron in place of a nuclear γ-ray quantum—and it has been pointed out that this phenomenon may for some purposes be considered simply as an internal photoelectric effect, taking place in the radiating atom. That this point of view can no longer be accepted as completely satisfactory is due to the general wave-mechanical result that the wave functions representing the motions of atomic electrons have finite amplitude in the region of the nucleus The pure photo effect is therefore complicated by direct interaction between the excited nucleus and the electrons, and the possibility of definitely non-radiative transitions arises in consequence These transitions are analogous to collisions of the second kind investigated theoretically by Klein and Rosseland Since in practice there is no means of separating the effects of these two processes of absorption, it is more reasonable to consider the phenomenon of internal conversion as a single process of transmission of the energy of nuclear excitation to the atomic electrons through the action of extra- and intranuclear fields of force Only in the special case ($i = 0 \rightarrow i = 0$) is this action confined to one only of these fields, in this case the periodic components of the extranuclear electromagnetic fields vanish absolutely, γ-radiation is completely forbidden, and internal conversion is due entirely to the interaction of atomic electrons and the excited nucleus in the region 'within the nucleus'

In the general case the total nuclear transition probability may be written as the sum of two terms, for dipole radiation, in the above notation, as $\kappa(\omega)+\mu(\omega)$, where $\mu(\omega)$ denotes the probability of the ejection of an atomic electron simultaneously with the nuclear energy-change In so far as the secondary electron may originate in any one of the extranuclear levels of the atom, $\mu(\omega)$ may be written in the form $\mu(\omega) = \mu_K(\omega)+\mu_{L_I}(\omega)+\ldots$, where successive terms on the right-hand side refer to successive levels in order of decreasing energy of binding The ratio of the number of emitted electrons to the total number of nuclear transitions involving the energy-quantum $\hbar\omega$ is defined as the coefficient of internal conversion for nuclear radiation of the corresponding frequency We have

$$\alpha(\omega) = \mu(\omega)/\{\kappa(\omega)+\mu(\omega)\} \text{ or } \mu(\omega)/\kappa(\omega) = \alpha(\omega)/\{1-\alpha(\omega)\} \qquad (13)$$

Again, $\alpha(\omega)$ is the sum of a number of terms, $\alpha_K(\omega)$, $\alpha_{L_I}(\omega)$, $\ldots$, corresponding to the various μ's, these terms may be regarded as giving the internal-conversion coefficients relative to specified electron levels in the atom In any case the value of the internal-conversion coefficient depends upon the atomic electron level involved and upon the symmetry characteristics of the nuclear transition in question

Applying standard methods of wave-mechanical perturbation theory the probability $\mu(\omega)$ may be calculated from the equation

$$\mu(\omega) = \frac{2\pi}{\hbar} \sum \left| \int \psi_f^*[eA_0+e\rho_3\,\sigma\mathbf{A}]\psi_0\, d\tau \right|^2, \qquad (14)$$

where ψ_0 and ψ_f are the wave functions describing the orbital and free motion of the electron, A_0 and $\mathbf{A}$ the scalar and vector potentials of the radiative field of the nucleus, and ρ_3 and σ are the matrices in Dirac's theory. For the purpose in hand the relativistic form of the equation for the electron must be used Calculations on the basis of (14) were carried out by Hulme† on the assumption that the electromagnetic field of the nucleus is that of a dipole situated at its

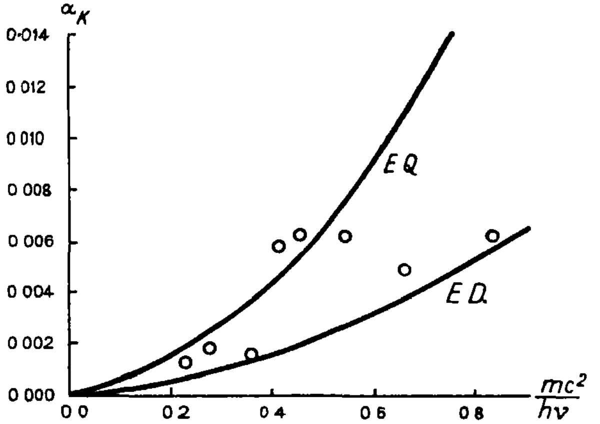


Fig 19

centre This field may be represented by the potentials

$$\left.\begin{aligned} A_0 &= B\frac{i}{r}\exp\left\{2\pi i\left(\frac{r}{\Lambda}-\nu t\right)\right\}\cos\theta\left(1+\frac{1}{qr}\right)+\text{complex conjugate}, \\ A_z &= B\frac{i}{r}\exp\left\{2\pi i\left(\frac{r}{\Lambda}-\nu t\right)\right\}, \\ A_x &= A_y = 0, \end{aligned}\right\} \quad (15)$$

with
$$q = \frac{2\pi\nu}{c} = \frac{\omega}{c}$$

Having obtained $\mu(\omega)$ as above described, $\kappa(\omega)$ is given by dividing the rate of radiation of energy by the field by $\hbar\omega$, the quantum energy of the radiation As is well known, the rate of radiation of energy by the field specified by (15) is $\frac{4}{3}\frac{B^2\omega^2}{c}$, thus $\kappa(\omega) = \frac{4}{3}\frac{B^2\omega}{\hbar c}$. From $\mu(\omega)$ and $\kappa(\omega)$ the internal-conversion coefficient $\alpha(\omega)$ is derived by equation (13) The values obtained by Hulme in this way for the internal-conversion coefficients relative to K-shell absorption in the RaC′ atom ($Z = 84$) are plotted as a function of $mc^2/\hbar\omega$ in Fig 19

† H R Hulme, *Proc Roy Soc* **138** (1932), 643 Previous calculations by Swirles and Casimir were based on much less exact approximations, for that reason they will not be further discussed at this stage

(curve *ED*) The experimental values, also given in the following table, are those of Ellis and Aston† for the eight most intense γ-rays

TABLE IV

Internal conversion of γ-rays of RaCC′ and RaBC

(*RaCC′*)

$h\nu \times 10^6$ e v	α_K	*Number of γ-quanta per disintegration*
0 612	0 0061	0 658
0 773	0 0048	0 065
0 941	0 0061	0 067
1 130	0 0062	0 206
1 248	0 0057	0 063
1 390	0 0014	0 064
1 426	> 0 1 (= 1?)	(= 0?)
1 778	0 0016	0 258
2 219	0 0013	0 074

(*RaBC*)

$h\nu \times 10^6$ e v	α_K	*Number of γ-quanta per disintegration*
0 243	0 364	0 115
0 297	0 186	0 258
0 354	0 117	0 450

emitted by this nucleus Five of these points fit with fair accuracy on the theoretical curve for dipole radiation The three points which do not fit represent considerably larger internal-conversion coefficients as we shall see later they must be explained on the basis of nuclear quadrupole radiation Internal-conversion coefficients relative to the electronic L levels were also evaluated by Hulme, with the following result In the limiting case, $\hbar\omega \to \infty$, for RaC′,

$$\alpha_K \quad \alpha_{L_{\mathrm{I}}} \quad \alpha_{L_{\mathrm{II}}} \quad \alpha_{L_{\mathrm{III}}}$$
$$= 1 \quad 0\,149 \quad 0\,0013 \quad 0{\cdot}0066$$

This is in satisfactory agreement with the values experimentally obtained

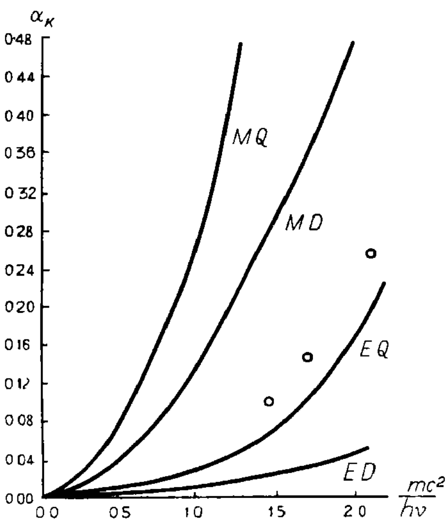


FIG 20

Fig 20 gives the internal conversion coefficients for somewhat softer γ-rays They have been calculated for $Z = 83$ in order to be compared with the values of

† E D Ellis and G H Aston, *Proc Roy Soc.* **129** (1930), 180

Ellis and Aston for the γ-rays emitted after the β-particle change RaB $\rightarrow$ RaC The experimental values from the above table are to be considered in relation to curve *ED*. It will be seen that experimental values are all considerably greater than those given by this curve, which may be taken to prove that the nuclear transitions in question cannot be described by a dipole field.

In order to account for the large values found for the internal-conversion coefficient in such cases, Taylor and Mott† carried out the calculations for quadrupole radiation, in terms of the potentials

$$A_0 = -C\frac{1}{r}\exp\left\{2\pi i\left(\frac{r}{\Lambda}-\nu t\right)\right\}\left\{2P_2(\cos\theta)\left[1+\frac{3i}{qr}-\frac{3}{q^2r^2}\right]+1\right\}+ \\ + \text{complex conjugate},$$

$$A_z = -3C\frac{1}{r}\exp\left\{2\pi i\left(\frac{r}{\Lambda}-\nu t\right)\right\}\cos\theta\left(1+\frac{i}{qr}\right)+\text{complex conjugate},$$

$$A_x = A_y = 0. \tag{16}$$

In this case $\kappa(\omega) = \dfrac{12}{5}\dfrac{C^2\omega}{\hbar c}$ γ-quanta per sec The curves *EQ* of Figs 19 and 20 give the results of these calculations It is evident that these curves go some way towards explaining the experimental results, although the agreement for the soft γ-rays of RaB–C is still far from exact This may be due in part to the lower accuracy of the calculations for γ-rays of small quantum energy and to the neglect of the screening effect of the atomic electrons It may be noticed here that apart from the electric dipole and quadrupole radiation discussed above there is also the possibility of *magnetic* dipole and quadrupole radiation due to oscillations of the magnetic moments of the system. Taylor‡ has shown that if some of the nuclear γ-rays have a magnetic origin the coefficients of internal conversion must be considerably larger than is the case for ordinary electric radiation The theoretical values of these coefficients, in the case of magnetic radiation, are shown in Fig. 20 by curves *MD* and *MQ* We must notice, however, that since the magnetic radiation possesses, in general, very small probability, it has to be taken into account only if electric dipole and quadrupole transitions are not permitted.

The occurrence of dipole and quadrupole transitions of comparable intensity points to the conclusion that the probability of nuclear

† H M Taylor and N F. Mott, *Proc Roy Soc.* **138** (1932), 665

‡ H. M. Taylor, ibid **146** (1934), 178.

dipole radiation is relatively much smaller than might at first be imagined. This is in line with other considerations advanced at the end of the last section We shall see later (Chapter VI) that the average lifetime of an excited nucleus, estimated from the relative numbers of γ-rays and long-range α-particles in the corresponding groups, is more nearly represented by the quadrupole formula (10) than by the dipole formula (9)

The calculations of Hulme and of Taylor and Mott are based on the hypothesis of dipole and quadrupole moments localized in extremely minute regions in the centre of the nucleus By considering the effect of the finite size of the nucleus and the possibility that the expressions for the various potentials, (15) and (16), may require modification for values of r less than the nuclear radius, Fowler† was able to show, however, that the value of the integral (14) changes but slightly if these deviations take place inside the region $r \sim 10^{-12}$ cm Only in the case of the prohibited radiative transition ($i = 0 \rightarrow i = 0$) do such considerations become all-important In this case the direct interaction of an atomic electron with the excited nucleus represents the one method of the release of energy The probability of this process may be evaluated as to order of magnitude from the perturbation formula

$$\mu' \sim \frac{2\pi}{\hbar}[\tfrac{4}{3}\pi r_0^3 \overline{V}\psi_0(0)\psi_f(0)]^2, \tag{17}$$

where $\overline{V}$ is the average interaction potential between electron and nucleus for values of the separation of the order of r_0 Using the relativistic expressions for the wave functions, we obtain, for a transition energy of 10^6 e v communicated to a K-electron in an atom of RaC′, $\mu' \sim 10^{11}$ sec $^{-1}$ If the interaction is greater than has been supposed much larger values of μ' become possible These values are of the same order as those calculated above for quadrupole radiative transitions of the nucleus, they indicate that, whenever the mean life of an excited nucleus is of the order of 10^{-12} to 10^{-11} sec , internal conversion by direct interaction is an important mode of transference of energy At present the only well-established case of non-radiative nuclear transition is that associated with the quantum energy $1\,414 \times 10^6$ e v in RaC–C′ Secondary electron groups corresponding to this quantum are particularly intense, but no evidence

† R H Fowler, *Proc Roy Soc* **129** (1930), 1, see also M Delbruck and G Gamow, *Zs. f. Phys.* **72** (1931), 492.

of the corresponding γ-ray has been found in emission Before the above simple explanation of the results was advanced it was necessary to assume that for this particular radiation an internal-conversion coefficient of unity was applicable—one formed the strange picture of the K-electrons absorbing a large fraction of the outgoing quanta and of the remaining electrons being able completely to absorb the rest Secondary electron groups were observed corresponding to absorption in the K, L, and M levels, respectively Assuming the correctness of the present explanation, that we are dealing with a non-radiative nuclear transition, very important conclusions may be drawn We conclude that the two nuclear levels involved are both S levels, in other words that these two states of excitation possess zero spin. Moreover, as will appear later, the observed correlation between γ-ray energies and the energies of the long-range α-particles from RaC′ indicates that the transition in question takes place between one of the excited states of this nucleus and the ground state We conclude, therefore, that in its ground state the nucleus RaC′ has no spin $i = 0$

4. Systems of nuclear energy-levels for γ-rays

Many attempts have been made to construct systems of nuclear energy-levels for γ-rays merely by inspection of the known energies of the radiations with a view to discovering combining terms, in the way which proved successful for the optical levels of the atoms Owing, however, to the relatively small accuracy of the γ-ray measurements this direct method was not very successful one could never be certain that a particular numerical agreement was not entirely fortuitous The fact that several level-systems might be suggested for a given set of γ-rays showed that chance agreement was a common feature of them all.†

In theory, a knowledge of the intensities of the various γ-rays provides a possible test for any system of levels which may be proposed For the correct system must fulfil the condition that the total intensity of the transitions from a given level is never smaller than the intensity of the transitions to this level (the difference of these intensities represents the degree of initial excitation of the level in question), and the further condition that the total intensity of

† The author once constructed thirteen possible schemes for the γ-rays of RaC–C′, all of which have since been shown to be wrong

transitions across any line in the diagram between two levels is never greater than unity. In practice, however, our knowledge of γ-ray intensities is far from exact.

Further evidence may sometimes be obtained with the help of the selection rules, already discussed, especially if the character (dipole or quadrupole) of the transition is known from an interpretation of internal-conversion coefficients However, the only certain information comes at present from measurements of the energies of long-range α-particles and α-particles of the fine-structure groups (Chap. VI) Energy-differences between the main group of particles and the various sub-groups give directly energy-differences between the ground level and various excited levels of the nucleus in any such case These levels having thus been fixed, other levels may be added to the scheme in order to accommodate all the known γ-ray transitions This method will be considered further, together with the problem of nuclear excitation by α-emission, in Chapter VI, whilst the process of nuclear excitation by β-emission and the γ-radiation connected therewith will be discussed in Chapter VIII

PART II

SPONTANEOUS NUCLEAR TRANSFORMATIONS

V

SPONTANEOUS α-DISINTEGRATION

1. General features of α-disintegration

We have already seen that there are two different possibilities for the spontaneous disintegration of atomic nuclei: the emission of an α-particle and the emission of a negative or positive β-particle (e or ∂). If we take some element located outside the limit of α-stability on the stability diagram, the emission of α-particles will raise the value of N_n/N_p (because $(N_n-2)/(N_p-2) > N_n/N_p$ if $N_n/N_p > 1$) until the emission of a negative electron becomes possible. In this way we may understand the existence of sequences of radioactive disintegrations starting with some heavy element and, through a process of successive α- and β-emissions, leading to a stable product, on the boundary of α-stability. In α-disintegration the charge-number decreases by two and the mass-number by four units, in β-disintegration the charge-number increases by one unit and the mass-number does not change. Thus it is clear that we may expect the existence of four different series of radioactive disintegrations, or four different radioactive families, corresponding respectively to mass-numbers of the type $4n$, $4n+1$, $4n+2$, and $4n+3$. Three of these families have been actually found in nature: they are known as the Thorium family ($4n$), the Uranium family ($4n+2$), and the Actinium family ($4n+3$). The remaining family of the type ($4n+1$) has not been found in nature evidently for the same reason as determines the very rare occurrence of elements of this type in general (see Chap. XII), however, some elements belonging to this family have been produced artificially by bombarding other radioactive elements by neutrons (see Chaps. X and XI). The stable end-products of the radioactive families have all the same charge-number and are isotopes of lead (${}_{82}Pb^{208}$, [${}_{82}Pb^{205}$], ${}_{82}Pb^{206}$, ${}_{82}Pb^{207}$), which shows that the line limiting the α-stable region (compare Figs. 13, 15, 16), runs very close to the points representing these isotopes on the side of larger mass-numbers (N_n+N_p). In Fig. 21 schemes for the known radioactive families are shown, vertical arrows indicating α-emission and horizontal arrows β-emission.

We notice that the continuous series of α-transformations is sometimes interrupted by pairs of successive β-transformations, as is to be expected from what has been said above. The branching of the radioactive series observed in the C products is of special interest and is due to the fact that the points representing these nuclei are located in the part of the energy-surface (Fig 11) where both kinds of transformations are energetically possible We should also pay

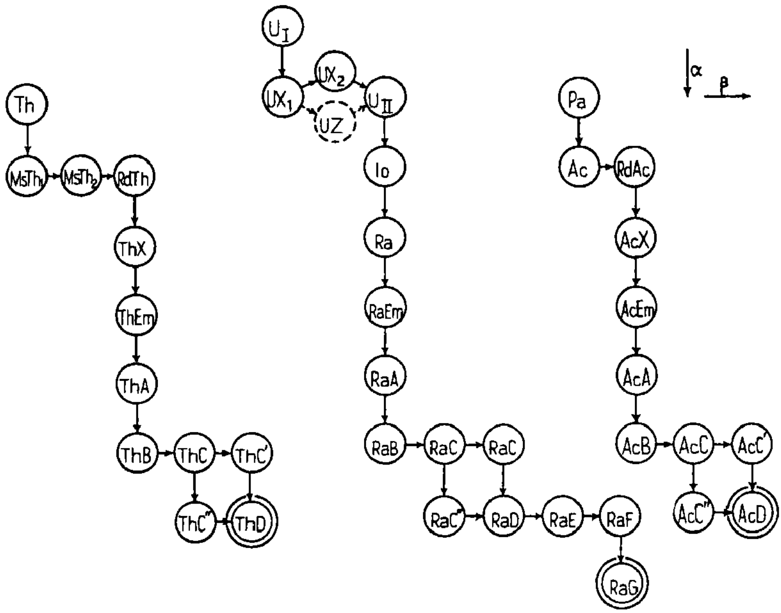


FIG 21

attention to an unusual branching, at the beginning of the U family, consisting of alternative β-disintegrations, a possible explanation of this effect, based on the hypothesis of negative protons, has been mentioned on p 17

One of the most important characteristics of a decaying nucleus is its *decay constant* (usually denoted by λ), giving the probability of disintegration per unit time If N_0 and N are the numbers of nuclei, which at times t_0 and t have not yet disintegrated, the number of disintegrations in the time-interval $(t, t+dt)$ is given by

$$-dN = \lambda N\,dt. \tag{1}$$

Integrating, we have $$N = N_0\,e^{-\lambda(t-t_0)}, \tag{1'}$$

which gives the exponential decrease of the amount of the radioactive body in question. The average life τ of this body can be defined as the period of time after which $\frac{1}{e} = 0{\cdot}368$ of the original activity remains, evidently it satisfies the relation $\tau = 1/\lambda$. The decay constant has a quite definite value for each radioactive body and varies within very wide limits from 10^{-18} sec^{-1} to 10^{+5} sec^{-1}, the average life τ varying from thousands of millions of years to a very small fraction of a second When multiple branching occurs, the observed decay constant is made up of the constants of individual disintegrations If λ_i ($i = 1, 2,$) represent the probabilities of different types of disintegration, we have for the total number of disintegrating atoms

$$-dN = \sum \lambda_i N\,dt = N \sum \lambda_i\,dt, \tag{2}$$

which shows that the effective decay constant will be simply the sum of the individual constants The numbers of atoms disintegrating in the different ways are evidently

$$N_K = \frac{\lambda_K}{\sum \lambda_i} N\,dt. \tag{2'}$$

If we measure the total decay constant λ and the percentages of disintegrations following the different modes we can easily calculate the individual decay constants from the formulae

$$\lambda_K = \lambda \frac{N_K}{\sum N_i}. \tag{2''}$$

The decay constants for the various spontaneous α- and β-disintegrations are given in Tables E and F (at the end of the book)

Another important characteristic of radioactive decay is the energy of disintegration, composed of the kinetic energy of the emitted particle (or particles), the recoil energy of the remaining nucleus, and the energies of any γ-rays which may be emitted in the process The energy of the emitted particles can usually be estimated most directly by measuring the deviation of a beam of these particles in a known magnetic field (method of magnetic spectra), but very often the absorption method is employed In the case of heavier particles, such as α-particles or protons, the number of particles penetrating a layer of absorbing material decreases abruptly after the thickness of the material reaches a certain value, depending on the energy of the particles in question, this limiting thickness is known as the range of the particles It is generally given in 'standard air' (15° C and 760 mm) In the case of β-particles the coefficient of absorption (as defined for electromagnetic radiation) is usually given.

The results of magnetic measurements show that the α-particles emitted by a given radioelement are to a high degree homogeneous in respect of energy, although in certain cases several well-defined groups are present, corresponding to different modes of disintegration, as will be discussed in the next chapter Information concerning

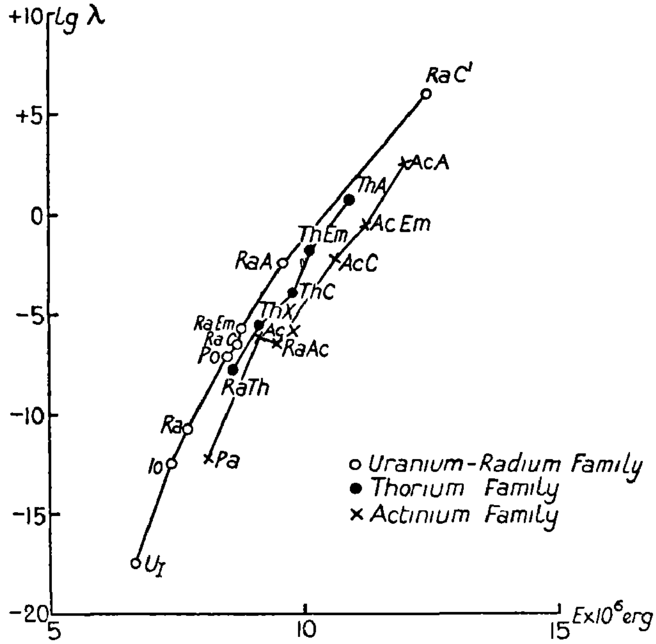


FIG 22

the α-particles emitted from all α-active bodies is given in Table E, which contains the energies and the corresponding ranges of all groups of particles—whether these are single groups or the component groups of complex spectra Considering the energies of different α-disintegrations in conjunction with the corresponding decay constants, we perceive a very interesting regularity decay constants increase very rapidly with increasing energy of disintegration Plotting $\log \lambda$ against the energy of disintegration, E, Geiger and Nuttall found points distributed on almost straight lines, different for different radioactive families but parallel to each other The plot representing the Geiger-Nuttall rule, based on the most recent measurements, is shown in Fig 22 As we can see from the figure, there are certain deviations from perfect regularity, for example the point representing AcX does not lie on the curve Furthermore, the points in the Geiger-Nuttall diagram do not actually lie on straight lines at all, but are much better represented by curves slightly concave to the axis of energy.

2. The theory of α-disintegration

In order to understand the extremely long periods of life of α-active bodies, and the connexion between period and energy of disintegration, we must investigate more closely the process of radioactive ejection of particles

Very important information concerning the process of α-decay was provided by the experiments of Rutherford and Chadwick on the scattering of fast α-particles by uranium It is known that experiments of this kind with light elements have made it possible to detect deviations from the Coulomb law of force at small distances, thus providing information regarding the additional forces operative within the nucleus. The experiments with uranium showed no such deviations whatsoever, although the α-particles of thorium C′ were employed, and these are capable of approaching to within about 3×10^{-12} cm of the centre of the nucleus We can only conclude that the attractive forces do not become appreciable except at distances much smaller still—and here we have not at our disposal sufficiently energetic α-particles to make further test of this hypothesis possible This negative, and at first sight not very important, result gives rise to a difficulty, however, which has been made the starting-point of a complete theory of α-disintegration, proposed by Gamow† and, independently, by Gurney and Condon ‡

The variation of the potential energy of an α-particle in the neighbourhood of a uranium nucleus is given diagrammatically in Fig 23 As the distance decreases the potential energy increases at first, to reach a maximum at some value of the distance certainly smaller than 3×10^{-12} cm , as the above-mentioned experiments show The maximum potential energy, on the same evidence, is certainly greater than 14×10^{-6} erg ($8{\cdot}8\times10^{6}$ e v), the kinetic energy of the α-particles from thorium C′ As the distance is still further decreased the potential energy decreases also and the intranuclear attractive forces come into play At the centre of the nucleus the potential

† G. Gamow, *Zs f Phys* **51** (1928), 204, G Gamow and F Houtermans, ibid **52** (1928), 496

‡ R. W. Gurney and E U. Condon, *Nature*, **122** (1928), 439, *Phys Rev.* **33** (1929), 127.

energy is negative but finite (we have already seen that the potential inside the nucleus may be considered as approximately constant).

The difficulty, which the experiments of Rutherford and Chadwick involve, concerns the fact that the uranium nucleus is itself radioactive, emitting a comparatively slow α-particle of energy $6{\cdot}6\times10^{-6}$ erg, only about half that of the α-particle from thorium C′ It appears difficult to imagine how such an α-particle is able to escape from the nucleus, since it must traverse on the way a 'potential barrier' of energy summit certainly twice as high as the total energy of the α-particle itself. it appears, rather, that it must remain for ever inside

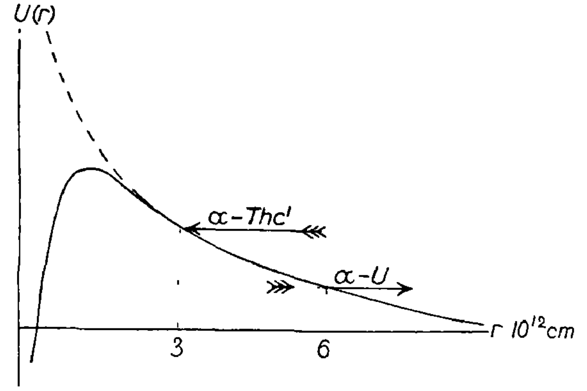


FIG 23

the nucleus, since it has not sufficient energy to surmount the barrier. This is the point of view of classical physics; the difficulty which it entails disappears completely when the wave mechanics is applied to this problem The apparently paradoxical behaviour of the nuclear particle in this case has a full analogy in certain phenomena attending the reflection of light It is well known that if a beam of light falls on the boundary between two media at an angle of incidence greater than the critical angle, then, according to geometrical optics, the phenomenon of total reflection will occur, all the light will be reflected at the surface and no disturbance will enter the second medium. According to the wave theory of light, however, the process of total reflection is much more complicated On this theory the disturbance in the second medium is not everywhere zero, but within the space of a few wave-lengths decreases exponentially to become entirely negligible at greater distances This 'forbidden' penetration of the disturbance cannot be described in terms of rays of light at all, the lines representing the directions of energy-flow being curved and returning to the surface again. If, now, the second medium be confined to a thin sheet, of thickness less than the range of penetration of the disturbance already considered, and if it is backed on the far side by a further portion of the first medium, a small fraction of the disturbance which has penetrated the sheet will emerge from the far side. This transmission of energy is obviously in contradiction to the predictions of purely geometrical optics· it is, however, established by experiment The change from classical mechanics to the

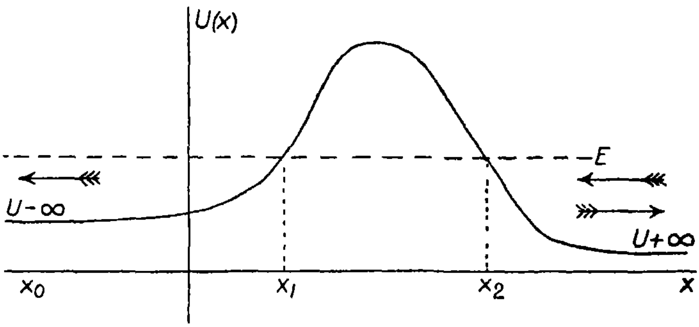


Fig 24

wave mechanics introduces an exactly similar possibility for the transmission of a particle through a potential barrier which would otherwise be insurmountable We describe the motion of the nuclear particle in the present case by a quasi-stationary de Broglie wave which gradually decreases in amplitude inside the nucleus on account of the leakage transmission through the finite potential barrier. Obviously, this description predicts the possibility of observing the particle, eventually, outside the nucleus, the small transparency of potential barriers in general provides the formal explanation of the extremely long lifetimes exhibited by some α-ray bodies

We proceed now to develop general formulae for the coefficient of transparency of potential barriers, considering, first of all, the one-dimensional case Let a particle of total energy E fall (from the right) on a potential barrier represented by an arbitrary potential energy function $U(x)$, as shown in Fig 24 We suppose that in a certain region ($x_1 < x < x_2$) the value of the function $U(x)$ is greater than E, and that for large distances from this region, on either side, constant values, $U_{-\infty}$ and $U_{+\infty}$, respectively, are reached. We write the

Schrödinger wave equation in the form

$$\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{2mi}{\hbar}\frac{\partial\psi(x,t)}{\partial t}-\frac{2m}{\hbar^2}U(x)\psi(x,t) = 0, \tag{3}$$

where m is the mass of the particle and $\hbar\,(= h/2\pi)$ is the quantum constant. Since the potential energy $U(x)$ is assumed to be independent of the time, the solution may be written in the form

$$\psi(x,t) = \Psi(x)e^{-(i/\hbar)Et}, \tag{4}$$

in which Ψ satisfies the equation

$$\frac{d^2\Psi}{dx^2}+\frac{2m}{\hbar^2}[E-U(x)]\Psi(x) = 0. \tag{5}$$

Writing
$$\Psi(x) = e^{s(x)}, \tag{6}$$
equation (5) reduces to

$$\frac{d^2s}{dx^2}+\left(\frac{ds}{dx}\right)^2+\frac{2m}{\hbar^2}[E-U(x)] = 0 \tag{7}$$

This equation can be solved by the method of successive approximations, so long as the first term is small compared with the second. As we shall see later, this condition, namely

$$\left|\frac{d^2s/dx^2}{(ds/dx)^2}\right| \equiv \left|\frac{d}{dx}\left(\frac{1}{ds/dx}\right)\right| \ll 1, \tag{8}$$

usually holds good for the potential barriers which occur in the theory of radioactive disintegration We proceed, therefore, by the method of approximations, and obtain the first approximation from

$$\left(\frac{ds_1}{dx}\right)^2+\frac{2m}{\hbar^2}[E-U(x)] = 0. \tag{9}$$

Integrating, we have

$$s_1 = \pm\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_0}^{x}[U(x)-E]^{\frac{1}{2}}\,dx+C_1. \tag{10}$$

For the second approximation we substitute from (10) in the first term of (7), writing s_1 for s We have

$$\left(\frac{ds_2}{dx}\right)^2 = \frac{2m}{\hbar^2}[U(x)-E]\mp\frac{\sqrt{(2m)}}{\hbar}\frac{d}{dx}[U(x)-E]^{\frac{1}{2}}, \tag{11}$$

or
$$\frac{ds_2}{dx} = \pm\frac{\sqrt{(2m)}}{\hbar}[U(x)-E]^{\frac{1}{2}}-\frac{1}{2}\frac{(d/dx)[U(x)-E]^{\frac{1}{2}}}{[U(x)-E]^{\frac{1}{2}}}. \tag{12}$$

Integrating this expression, we obtain

$$s_2 = \pm\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_0}^{x}[U(x)-E]^{\frac{1}{2}}\,dx-\tfrac{1}{2}\log[U(x)-E]^{\frac{1}{2}}+C_2. \qquad (13)$$

We may proceed in exactly the same way to the third approximation, but already it is of no great importance. Thus we have, finally, with sufficient accuracy

$$\Psi(x) = e^{s_1(x)} = C_2[U(x)-E]^{-\frac{1}{4}}\exp\left(\pm\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_0}^{x}[U(x)-E]^{\frac{1}{2}}\,dx\right). \quad (14)$$

This solution is valid only in those regions for which the condition (8) is satisfied Using the first approximation for s, we write this condition in the form

$$\frac{\hbar}{\sqrt{(2m)}}\left|\frac{d}{dx}\left(\frac{1}{[U(x)-E]^{\frac{1}{2}}}\right)\right| \ll 1. \qquad (15)$$

An estimate of the order of magnitude of the quantity occurring in (15) may be made in terms of its approximate equivalent,

$$\frac{\hbar}{\sqrt{(2m)}}\frac{1}{2}\frac{\Delta U(x)}{\Delta x[U(x)-E]^{\frac{3}{2}}}. \qquad (16)$$

Remembering that, in nuclear potential barriers, we have to do with energies of the order of 10^{-5} erg, and with changes of potential energy of roughly this amount over distances of the order of 10^{-12} cm , we substitute in (16) $E \sim U(x) \sim \Delta U(x) \sim 10^{-5}$ erg, $\Delta x \sim 10^{-12}$ cm , and obtain (for $m \sim 6\times 10^{-24}$ gm) the value $\sim \frac{1}{20}$. Near the points x_1, x_2 (Fig 24) the left-hand side of (15) becomes infinite, but otherwise our numerical calculation shows that in practical cases the condition (8) is usually satisfied In order to carry out the integration in the regions around x_1, x_2 we need to know the analytical form of $U(x)$, and, having substituted this in (7), to obtain the exact solution of the wave equation If, however, these regions (in which $[U(x)-E]^{-\frac{1}{4}}$ varies too rapidly with x) are small enough, we can make the solutions for the neighbouring regions on the two sides satisfy the boundary conditions of continuity and thus deduce the complete solution by exclusively approximate methods

Let us return, for the moment, to the more physical aspect of the problem in hand We have postulated a beam of particles moving from right to left and falling on the right-hand side of the potential barrier of Fig 24 Our mathematical solution must represent these particles as well as those reflected, and so moving from left to right, to the right of the barrier, and those transmitted (moving from right to left, to the left of the barrier) In particular, at large distances to the left of the barrier (where $U = U_{-\infty}$) the solution must represent a harmonic wave travelling from right to left Thus in the general solution (14) we put $U = U_{-\infty}$ and choose the negative sign of the exponent to secure the correct direction of motion (compare with (4)). We obtain in this way

$$\Psi_{x \ll x_1} = C(E-U_{-\infty})^{-\frac{1}{4}}\exp\Big(-i\frac{\sqrt{(2m)}}{\hbar}(E-U_{-\infty})^{\frac{1}{2}}(x-x_0)\Big), \tag{17}$$

representing a wave travelling from right to left, in a region where $E > U_{-\infty}$ †

In the region $x_1 < x < x_2$ the difference $[U(x)-E]$ becomes positive and both solutions, apart from the arbitrary constants, are real, one of them increasing and the other decreasing rapidly as x increases. To make the solutions continuous at $x = x_1$ we must take inside the barrier the general form with complex coefficients.

$$\Psi_{x_1<x<x_2} = C[E-U(x)]^{-\frac{1}{4}}\Big\{b_+\exp\Big(\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_1}^{x}[U(x)-E]^{\frac{1}{2}}\,dx\Big)+ \\ +b_-\exp\Big(-\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_1}^{x}[U(x)-E]^{\frac{1}{2}}\,dx\Big)\Big\}. \tag{18}$$

It is easy to see that the boundary conditions at $x = x_1$ (continuity of Ψ and $d\Psi/dx$) require

$$\left.\begin{aligned} b_+ &= \tfrac{1}{2}(1-i)\exp\Big(-i\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_0}^{x_1}[E-U(x)]^{\frac{1}{2}}\,dx\Big), \\ b_- &= \tfrac{1}{2}(1+i)\exp\Big(-i\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_0}^{x_1}[E-U(x)]^{\frac{1}{2}}\,dx\Big) \end{aligned}\right\} \tag{19}$$

At the other end of the barrier, where $x = x_2$, the second term in (18) is very small compared with the first In that case the solution reduces to

$$\Psi_{x=x_2} = \tfrac{1}{2}C(1-i)[E-U(x_2)]^{-\frac{1}{4}}\exp\Big(-i\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_0}^{x_1}[E-U(x)]^{\frac{1}{2}}\,dx\Big)\times \\ \times\exp\Big(\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_1}^{x_2}[U(x)-E]^{\frac{1}{2}}\,dx\Big) \tag{20}$$

† x_0 in (17) is a point at a great distance to the left of the barrier, i.e. $x_0 \ll x_1$

To join this function with the solution for $x > x_2$ we must again take a linear combination of the solutions representing oppositely directed waves,

$$\Psi_{x>x_2} = C[E-U(x)]^{-\frac{1}{4}}\left\{a_+ \exp\left(i\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_2}^{x}[E-U(x)]^{\frac{1}{2}}\,dx\right)+\right.$$

$$\left.+\,a_- \exp\left(-i\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_2}^{x}[E-U(x)]^{\frac{1}{2}}\,dx\right)\right\}, \qquad (21)$$

and the boundary conditions give

$$\left.\begin{aligned} a_+ &= \tfrac{1}{2}(1-i)b_+\exp\left(\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_1}^{x_2}[U(x)-E]^{\frac{1}{2}}\,dx\right); \\ a_- &= \tfrac{1}{2}(1+i)b_+\exp\left(\frac{\sqrt{(2m)}}{\hbar}\int\limits_{x_1}^{x_2}[U(x)-E]^{\frac{1}{2}}\,dx\right). \end{aligned}\right\} \qquad (22)$$

If, now, x_0' is some value of x lying at a great distance to the right of the barrier, the above solution may be written

$$\Psi_{x_2<x} = A_+\exp\left(i\frac{\sqrt{(2m)}}{\hbar}(E-U_{+\infty})^{\frac{1}{2}}(x-x_0')\right)+$$

$$+\,A_-\exp\left(-i\frac{\sqrt{(2m)}}{\hbar}(E-U_{+\infty})^{\frac{1}{2}}(x-x_0')\right), \qquad (23)$$

with

$$\left.\begin{aligned} A_+ &= -\tfrac{1}{2}iC[E-U(x)]^{-\frac{1}{4}}\exp\left\{\frac{\sqrt{(2m)}}{\hbar}\left(-i\int\limits_{x_0}^{x_1}[E-U(x)]^{\frac{1}{2}}\,dx+\right.\right. \\ &\qquad \left.\left.+\int\limits_{x_1}^{x_2}[U(x)-E]^{\frac{1}{2}}\,dx+i\int\limits_{x_2}^{x_0'}[E-U(x)]^{\frac{1}{2}}\,dx\right)\right\}, \\ A_- &= \tfrac{1}{2}C[E-U(x)]^{-\frac{1}{4}}\exp\left\{\frac{\sqrt{(2m)}}{\hbar}\left(-i\int\limits_{x_0}^{x_1}[E-U(x)]^{\frac{1}{2}}\,dx+\right.\right. \\ &\qquad \left.\left.+\int\limits_{x_1}^{x_2}[U(x)-E]^{\frac{1}{2}}\,dx-i\int\limits_{x_2}^{x_0'}[E-U(x)]^{\frac{1}{2}}\,dx\right)\right\}. \end{aligned}\right\} \qquad (24)$$

These two waves represent the incident and reflected beams of particles, they appear to have equal amplitude because we have neglected certain small-order terms (the second term in (18)) In actual fact a wave of small intensity has been transmitted by the barrier, and carrying out the calculations to the next order of approximation an expression representing the conservation of particles is obtained.

The coefficient of transparency of the barrier is given by the square of the ratio of the amplitudes of transmitted and incident waves, respectively. We have

$$G = \left|\frac{\Psi(-\infty)}{\Psi(+\infty)}\right|^2 = 4\left(\frac{E-U_{+\infty}}{E-U_{-\infty}}\right)^{\frac{1}{2}}\exp\left(-\frac{2\sqrt{(2m)}}{\hbar}\int\limits_{x_1}^{x_2}[U(x)-E]^{\frac{1}{2}}\,dx\right) \quad (25)$$

This is the formula which we shall employ in future calculations of the transparency of nuclear potential barriers. When this transparency is small ($2\sqrt{(2m)}\int_{x_1}^{x_2}[U(x)-E]^{\frac{1}{2}}\,dx \gg \hbar$), it is possible to show that (8), (15), the conditions of validity of the formula, are fulfilled.

We may remark, at this stage, that in certain special cases, the solution of the wave equation may be found directly in an analytical form. For the Coulomb potential barrier, for example, the solution may be expressed in the form of confluent hypergeometric functions This type of barrier is of great importance in the theory of nuclear disintegration, but here the analytical form of solution does not lead to any greater accuracy in calculation and we shall not further be concerned with it

We are now in a position to develop the general theory of the escape of a particle from a region surrounded by a potential barrier—which is just the process involved in the emission of an α-particle by an atomic nucleus We shall assume spherical symmetry for the distribution of potential energy around the centre of the nucleus and imagine that the appropriate function $U(r)$, starting from a certain finite value at the centre, increases to a large positive value at a relatively small distance and vanishes for very large distances, as indicated in the figure. The total energy of the particle considered is supposed to be smaller than the energy corresponding to the peak of the potential barrier, but larger than that corresponding to the bottom of the potential 'hole' in Fig 25 The wave equation for the particle may be written in spherical polar coordinates as follows

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+$$
$$+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}+\frac{2im}{\hbar}\frac{\partial\psi}{\partial t}-\frac{2m}{\hbar^2}U(r)\psi = 0. \quad (26)$$

If E is the energy of the particle, the solution of this equation will have the form

$$\psi = \Psi(r,\theta,\phi)\exp\left(-\frac{i}{\hbar}Et\right), \quad (27)$$

where Ψ satisfies the equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right)+$$
$$+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}+\frac{2m}{\hbar^2}[E-U(x)]\Psi = 0. \quad (28)$$

In so far as the potential energy of the α-particle in the neighbourhood of the nucleus is independent of the angular coordinates θ and ϕ, Ψ may be expressed in the form

$$\Psi = \frac{1}{r}P_j(\theta,\phi)\chi_j(r), \quad (29)$$

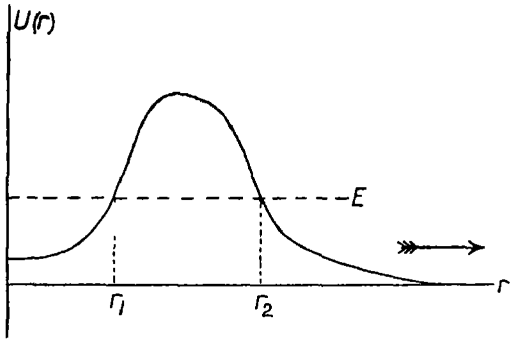


FIG. 25

where P_j is a spherical harmonic of order j ($j = 0, 1, 2, .$) and $\chi_j(r)$ satisfies the equation

$$\frac{\partial^2\chi_j(r)}{\partial r^2}+\frac{2m}{\hbar^2}\left[E-U(r)-\frac{\hbar^2}{2m}\frac{j(j+1)}{r^2}\right]\chi_j(r) = 0 \quad (30)$$

Here j represents the azimuthal quantum number and the last term within the brackets is to be interpreted as an additional potential energy corresponding to the centrifugal force In order that Ψ shall remain finite at the centre of the nucleus, $\chi(0)$ must be zero, as may be seen by inspection of (29)

We may thus obtain a general solution to (26), but it should be noted that not every particular case of such a solution represents the process of spontaneous ejection of an α-particle from the nucleus—which process we wish to describe. In order that any solution shall do this it is necessary first to satisfy certain boundary conditions appropriate to the physical interpretation in question. We may specify these conditions, formally, as follows Let us proceed to construct solutions of (30) for some arbitrary value of E, inserting, as above indicated, $\chi(0) = 0$. For large values of r we obtain in this way

$$\chi_{E,j} \underset{r\to\infty}{=} C_{+E,j}\exp\left(i\frac{\sqrt{(2m)}}{\hbar}E^{\frac{1}{2}}r\right)+C_{-E,j}\exp\left(-i\frac{\sqrt{(2m)}}{\hbar}E^{\frac{1}{2}}r\right), \quad (31)$$

where C_+ and C_- are the complex conjugate functions of E and j. Substituted in (27) and (29), the first and second terms of (31) represent, respectively, divergent and convergent waves passing through the origin Now the process of spontaneous disintegration must be described in terms of a diverging wave, only, thus

$$C_{+E,j} \neq 0, \qquad C_{-E,j} = 0 \quad (32)$$

At first sight it might seem that the conditions (32) were mutually contradictory, since C_+ and C_-, being complex conjugate functions, might be thought always to vanish simultaneously However, it can easily be shown that this need not be so, if we allow complex values for E, complex conjugate functions of the same complex argument in general give quite different amplitudes Postponing for a time the physical interpretation of this procedure, we write, therefore

$$E = E_0 - iE' \quad (33)†$$

We find now that the conditions (32) define, for each value of the azimuthal quantum number j, a discrete set of complex E values which we shall distinguish by different indices n ($n = 0, 1, 2, \ldots$) We may refer to these as the principal quantum numbers for this set of states Here, as in the ordinary problem of electron motion in the outer atom, we come upon a set of discrete proper values of the energy, the only difference being that the values are now complex quantities which on interpretation correspond to positive, rather than to negative, total energy.

Proceeding now to the interpretation which is necessary to complete the investigation, we must turn our attention to the time-dependence of ψ Substituting from (33) in (27), we obtain as the time-factor in the wave function

$$\exp\left(-\frac{i}{\hbar}Et\right) = \exp\left(-\frac{i}{\hbar}(E_0 - iE')t\right) = \exp\left(-\frac{i}{\hbar}E_0 t\right)\exp\left(-\frac{E'}{\hbar}t\right). \quad (34)$$

Then the probability, $\psi\bar{\psi}$, that the α-particle will be found in any

† It may be shown that a divergent wave requires a negative sign in (33), a convergent wave a positive sign

particular region of space, inside the nucleus or outside, becomes aperiodically dependent upon the time:

$$\psi\bar{\psi} \sim \left[\exp\left(-\frac{i}{\hbar}E_0 t\right)\exp\left(-\frac{E'}{\hbar}t\right)\right]\left[\exp\left(\frac{i}{\hbar}E_0 t\right)\exp\left(-\frac{E'}{\hbar}t\right)\right]$$
$$= \exp\left(-\frac{2E'}{\hbar}t\right) = \exp(-\lambda t). \quad (35)$$

In this expression the quantity λ $(= 2E'/\hbar)$ may be recognized as the disintegration constant which should appear as a result of our calculations. The complex values of the proper energies thus lead to exponentially decreasing probabilities of finding a particle, of given energy, in a finite region of space they are characteristic of the solution of any problem which deals with decay processes, the imaginary part of the energy being immediately connected with the rate of decay We may also regard the imaginary part of the energy from a slightly different point of view The ψ wave diverging from the nucleus will not be strictly harmonic, the aperiodic factor resulting in a damping of the wave In optics such a damped disturbance would produce a broadened line in a spectrograph, with moving particles we employ a mass spectrograph, or its equivalent, and here the broadening of the line will be interpreted as due to an uncertainty in the energy of the particles Developing the damped wave function by Fourier expansion it may be shown that the uncertainty in the energy (half-breadth of the line) has the value

$$\Delta E \sim \hbar\lambda \sim E'. \quad (36)\dagger$$

From this point of view the average energy of the emitted particles is given by the real part of the proper energy, whilst the probable deviation from the average is given by the imaginary part

Having carried the interpretation so far, it is necessary now to mention that the complexity of E introduces an exponential term involving the space coordinate also For large distances from the nucleus, where $U(r) \to 0$, we evidently have $(E' \ll E_0)$

$$\chi_{r\to\infty}(r) = C_+ \exp\left(i\frac{\sqrt{(2m)}}{\hbar}E^{\frac{1}{2}}r\right) = C_+ \exp\left(i\frac{\sqrt{(2m)}}{\hbar}\left[E_0 - i\frac{\lambda\hbar}{2}\right]^{\frac{1}{2}}r\right)$$
$$= C_+ \exp\left(i\frac{\sqrt{(2m)}}{\hbar}E_0^{\frac{1}{2}}r\right)\exp\left(\frac{\lambda}{2}\sqrt{\frac{m}{2E_0}}\,r\right). \quad (37)$$

† Remembering that the mean time, T, for an α-particle to remain in the nucleus is given by $T = 1/\lambda$, (36) may be written $\Delta E\ T \sim \hbar$, which is precisely Heisenberg's uncertainty relation, expressed in terms of time and energy

This implies that the amplitude of the divergent wave increases with increasing distance and tends to an infinite value at infinity; the same general result is obtained in all solutions of decay problems, for example in the classical solution for the radiation from a damped oscillator It need occasion no surprise, since it merely expresses the fact that the disturbance at a great distance from the origin, at any moment, was emitted from the origin at a correspondingly distant past time, when the rate of radiation was greater In the radioactive decay problem the flux of particles at a large distance r, at a given instant, must obviously correspond to the strength of the source at a time r/v earlier At that time the strength of the source was greater by the factor $\exp\left(\lambda\frac{r}{v}\right) = \exp\left(\lambda r\sqrt{\frac{m}{2E_0}}\right)$, which is precisely the factor to be explained in (37), when it is remembered that (37) gives the amplitude of the wave and the square of this amplitude enters into the calculation of the flux In order to appreciate the relative magnitudes of the various quantities involved, we may take the case of radium C′, for which E'/E_0 is larger than for any other element concerning which full data are available Here $E_0 = 1{\cdot}2\times 10^{-5}$ erg, $\lambda = 10^4$ sec^{-1}, thus $E = E_0 - i\frac{\lambda\hbar}{2} = 1{\cdot}2\times 10^{-5} - 5{\cdot}2\times 10^{-24}i$ The uncertainty in the energy-value is only $4{\cdot}3\times 10^{-17}$ per cent of the average energy, and, according to (37), the amplitude of the ψ wave is not doubled until a distance of 2·8 km from the nucleus is reached

Although the method of complex E values, proposed by Gamow, gives perhaps the most satisfactory representation of disintegration processes, the necessary calculations are impossible except in respect of very simple nuclear models, such as that represented by the rectangular potential barrier discussed in detail by Kudar † On the other hand, real nuclei must be treated on the basis of a potential-energy function for α-particles which at large distances is given by the Coulomb inverse-distance law and at small distances is modified by some unknown, rapidly varying, potential function which makes the resultant potential energy negative inside the nucleus In this case it is possible to obtain a relation between the real and imaginary parts of the proper energy-values—that is between disintegration energy and decay constant—without in the process determining the values of E_0 (energies of disintegration) to which the more

† J Kudar, *Zs f Phys* **53** (1929), 95, 134

complicated model leads These values are very sensitive to the form of the potential barrier inside the nucleus, but the E_0–λ relation may be obtained without exact knowledge of this form, as we shall presently discover. This relation may then very usefully be compared with the empirical relation which has already been discussed.

Suppose that we start with the expression, in terms of the wave function ψ, for the conservation of particles, it is

$$\frac{\partial}{\partial t}\int \psi\bar{\psi}\,d\omega = \frac{\hbar}{2mi}\int\left[\psi\frac{\partial\bar{\psi}}{\partial n}-\bar{\psi}\frac{\partial\psi}{\partial n}\right]d\sigma, \tag{38}$$

the integral on the left being taken throughout a certain region of space and that on the right over its boundary surface In the expression on the right, moreover, n is measured along the outward normal to the surface element $d\sigma$ In (38) ρ ($=\psi\bar{\psi}$) is the probability density relative to the particle in question and I_n $\left(=\frac{\hbar}{2mi}\left[\psi\frac{\partial\bar{\psi}}{\partial n}-\bar{\psi}\frac{\partial\psi}{\partial n}\right]\right)$ the normal component of the flux of probability across the surface. In the case of spherical symmetry, when we choose for the region to which (38) applies a sphere of radius R centred in the nucleus, we can, by using (27), (29), and (35), transform the conservation equation and obtain

$$\frac{\partial}{\partial t}\int\limits_0^R \chi\bar{\chi}e^{-\lambda t}\,dr = \frac{\hbar}{2mi}e^{-\lambda t}\left[\chi\frac{\partial\bar{\chi}}{\partial r}-\bar{\chi}\frac{\partial\chi}{\partial r}\right]_{r=R} \tag{39}$$

or

$$\lambda = \frac{\hbar}{2mi}\left[\chi\frac{\partial\bar{\chi}}{\partial r}-\bar{\chi}\frac{\partial\chi}{\partial r}\right]_{r=R}\Bigg/\int\limits_0^R \chi\bar{\chi}\,dr. \tag{40}$$

We have thus obtained a formula for the disintegration constant which may be interpreted simply as showing that this constant is just the ratio of the flux of particles across any large sphere to the number of particles remaining within the sphere at the instant in question A completely analogous relation is often employed for calculating the damping coefficient for any radiating oscillator in classical electrodynamics In using (40) in the radioactive case it is quite unimportant what value of R we take, provided that it is large in comparison with r_1, the inner radius of the potential barrier In fact we have seen that the rate of flow, as determined by

$$\frac{\hbar}{2mi}\left(\chi\frac{\partial\bar{\chi}}{\partial r}-\bar{\chi}\frac{\partial\chi}{\partial r}\right),$$

is fairly constant, increasing by a factor 2, only, over a distance which is almost always greater than 1 km. As concerns the denominator of (40), to a sufficient approximation we may take the integral here from $r = 0$ to $r = r_1$, since χ decreases so rapidly in the range $r_1 < r < r_2$ (within the barrier) that the portion of the integral corresponding to $r > r_1$ is negligibly small For the radioactive nuclei $\chi\bar{\chi}$ outside the nucleus is less than 10^{-16} of its value inside, so that for $R = 1$ cm the contribution of the extra-nuclear part of the integral will not be more than 0·01 per cent of its total value As we have seen above, the potential energy inside the nucleus may be regarded as approximately constant, consequently the solution for χ (for $r < r_1$) will correspond very nearly to harmonic oscillations of amplitude A (say) Then, with sufficient accuracy,

$$\int\limits_0^{r_1} \chi\bar{\chi}\, dr \sim \tfrac{1}{2}A^2 r_1. \tag{41}$$

Also, for large r values we have previously used

$$\chi_{r\to\infty} = C\exp\left(i\frac{\sqrt{(2m)}}{\hbar}E^{\frac{1}{2}}r\right), \tag{42}$$

so that the probability flux becomes

$$\frac{\hbar}{2mi}\left(\chi\frac{\partial\bar{\chi}}{\partial r}-\bar{\chi}\frac{\partial\chi}{\partial r}\right) = C^2\sqrt{\left(\frac{2E}{m}\right)} = C^2 v_e, \tag{43}$$

where v_e is the velocity of the particle outside the nucleus Substituting from (41) and (43) in (40) we obtain

$$\lambda = \frac{2v_e}{r_1}\left(\frac{C}{A}\right)^2 = \frac{2v_e}{r_1}G, \tag{44}$$

G being the transparency of the barrier, which may be taken over from (25) Then

$$\begin{aligned}\lambda &= \frac{2v_e}{r_1}4\frac{v_i}{v_e}\exp\left(-\frac{2\sqrt{(2m)}}{\hbar}\int\limits_{r_1}^{r_2}[U(r)-E]^{\frac{1}{2}}\,dr\right)\\ &= \frac{8v_i}{r_1}\exp\left(-\frac{2\sqrt{(2m)}}{\hbar}\int\limits_{r_1}^{r_2}[U(r)-E]^{\frac{1}{2}}\,dr\right),\end{aligned} \tag{45}$$

in which expression v_i is written for the velocity of the particle inside the nucleus Remembering that the de Broglie wave-length Λ_i corresponding to this velocity ($mv_i\Lambda_i = h = 2\pi\hbar$) must be an exact submultiple of the nuclear diameter ($k\Lambda_i = 2r_1$, where k is the radial quantum number), we have $v_i = \frac{\pi k\hbar}{mr_1}$, and may write the expression for the disintegration constant in the final form

$$\lambda = \frac{8\pi k\hbar}{mr_1^2}\exp\left(-\frac{2\sqrt{(2m)}}{\hbar}\int\limits_{r_1}^{r_2}[U(r)-E]^{\frac{1}{2}}\,dr\right) \tag{46}$$

3. Calculations for a simplified potential barrier

In order to apply the formulae already derived, we have clearly to make certain assumptions concerning $U(r)$. In actual fact, it must

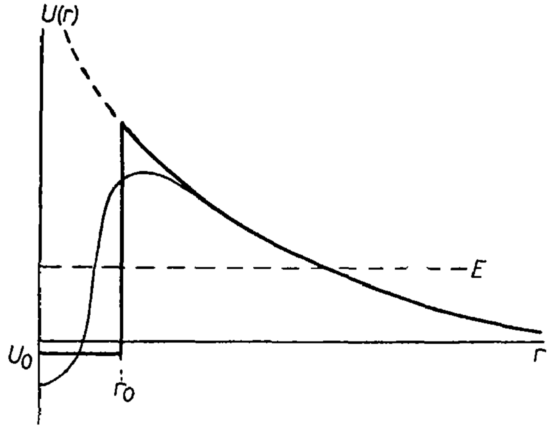


FIG 26

be supposed that the distribution of potential energy of a nuclear α-particle in the field of the rest of the nucleus is something like that shown by the smooth curve in Fig 26 At distances large compared with the dimensions of the nucleus we have the Coulomb inverse square law of force, and the potential energy of the escaping α-particle is given by $2(Z-2)e^2/r$, where Z is the charge number of the disintegrating nucleus. At a certain small distance from the centre of the nucleus attractive forces become important and the potential energy drops rather rapidly, inside the nucleus it has already been assumed to be practically constant For ease in calculation, and because the exact law of the attractive force is at present unknown, it is necessary to modify the potential distribution above described. A simple and natural approximation is to replace the curve inside the nucleus by a rectangular trough, as shown in Fig. 26, i.e. to assume

$$U(r) = \frac{2(Z-2)e^2}{r} \quad \text{for} \quad r > r_0,$$

$$U(r) = U_0 \qquad \text{for} \quad r < r_0, \tag{47}$$

with a discontinuous change from $2(Z-2)e^2/r_0$ to U_0 at the critical radius r_0 Thus our model of the nucleus is defined by two quantities, r_0 and the value of the internal potential energy U_0 Both of these clearly refer to the nucleus of the product element when calculations concerning decay constants are involved

We may notice here that the approximation which is to be adopted will scarcely affect the value of the coefficient of transparency of the barrier, since the chief part of the integral in formula (25) is taken over the region of the barrier where deviations from the Coulomb law are very small On the other hand, our model will give only very rough results concerning the energy-levels for α-particles within the nucleus, as the positions of these depend essentially upon the actual shape of the potential-energy curve With this warning, however, we may proceed to calculate the proper-energies involved The solution for $r < r_0$ is

$$\chi_{E,j} = Ar^{\frac{1}{2}}J_{j+\frac{1}{2}}\left(\frac{\sqrt{(2m)}}{\hbar}(E-U_0)^{\frac{1}{2}}r\right), \tag{48}$$

where $J_{j+\frac{1}{2}}$ is a Bessel function of order $j+\frac{1}{2}$ If $k_{n,j+\frac{1}{2}}$ are the roots of these functions, we have

$$E_{n,j} = \frac{k^2\hbar^2}{2mr_0^2}+U_0 \tag{49}$$

In the case of radial oscillations

$$\chi_{E,0} = A\sin\frac{\sqrt{(2m)}}{\hbar}(E-U_0)^{\frac{1}{2}}r, \tag{50}$$

and we have, simply,

$$E_{n,0} = \frac{n^2\hbar^2\pi^2}{2mr_0^2}+U_0 \tag{51}$$

In calculating the decay constant for our model we shall first examine the case of radial motion, this is precisely what has already been done in using (25) to obtain the expressions (45) and (46) of § (2) We may rewrite (46) with the new assumptions regarding $U(r)$ as follows

$$\lambda_{E,0} = \frac{8\pi k\hbar}{mr_0^2}\exp\left(-\frac{2\sqrt{(2m)}}{\hbar}\int\limits_{r_0}^{2(Z-2)e^2/E}[2(Z-2)e^2/r-E]^{\frac{1}{2}}\,dr\right).$$

The integral in the exponent of (52) may be simply evaluated by means of the substitution†

$$\cos^2 u = \frac{rE}{2(Z-2)e^2} = \frac{r}{r^*}. \tag{53}$$

We have

$$\lambda_{E,0} = \frac{8\pi k\hbar}{mr_0^2}\exp\left(-\frac{4e^2}{\hbar}\frac{(Z-2)}{v_e}(2u_0-\sin 2u_0)\right), \tag{54}$$

where

$$\cos^2 u_0 = \frac{r_0}{r^*} = \frac{r_0 E}{2(Z-2)e^2} \tag{54'}$$

In radioactive nuclei the value of r_0/r^* is small and the exponent of (54) may be developed in powers of this small quantity Taking the first two terms only we obtain a simple formula which is very convenient for the calculation of decay constants,

$$\lambda_{E,0} = \frac{8\pi k\hbar}{mr_0^2}\exp\left(-\frac{4\pi e^2(Z-2)}{\hbar v_e}+\frac{8e\sqrt{m}}{\hbar}[(Z-2)r_0]^{\frac{1}{2}}\right) \tag{55}$$

If the azimuthal quantum number differs from zero we must take count of the additional potential energy corresponding to the centrifugal force, and obtain the expression for the decay constant in the form

$$\lambda_{E,j} = \frac{8\pi k\hbar}{mr_0^2}\exp\left(-\frac{2\sqrt{(2m)}}{\hbar}\int\limits_{r_0}^{r^*}\left[\frac{2(Z-2)e^2}{r}+\frac{\hbar^2}{2m}\frac{j(j+1)}{r^2}-E\right]^{\frac{1}{2}}dr\right). \tag{56}$$

In radioactive nuclei ($Z = 80$ to 90, $r_0 \sim 10^{-12}$ cm.) the potential energy corresponding to centrifugal forces will, in general, be very small compared with the Coulomb potential energy, the ratio σ being given by

$$\sigma = \frac{\hbar^2}{2m}\frac{j(j+1)}{r_0^2}\bigg/\frac{2(Z-2)e^2}{r_0} \sim 0{\cdot}002j(j+1) \tag{57}$$

Thus, for calculation of the integral (56) we may develop the square root in powers of the small quantity σ. We get, finally,

$$\lambda_{E,j} = \frac{8\pi k\hbar}{m_0 r^2}\exp\left(-\frac{4\pi e^2}{\hbar}\frac{(Z-2)}{v_e}+\frac{8e\sqrt{m}}{\hbar}[(Z-2)r_0]^{\frac{1}{2}}(1-\tfrac{1}{2}\sigma)\right). \tag{58}‡$$

This shows that the azimuthal quantum number of the emitted particle must be assumed 'responsible' for a decrease (for the same energy of disintegration) of the disintegration probability, if this angular momentum amounts to several units the decrease may be quite appre-

† r^* is the so-called classical radius of the nucleus, i e the closest distance of approach of the α-particle according to classical mechanics

‡ In subsequent calculations we shall always take $k = 1$

ciable. As we shall see later, we may obtain interesting information concerning the changes of spin of decaying nuclei on this basis.

Formula (58), which can be written in the form

$$\log_{10}\lambda = \log_{10}\frac{8\pi\hbar}{mr_0^2} - \frac{4\pi e^2}{\hbar l}\frac{Z-2}{v} + \frac{8e\sqrt{m}}{\hbar l}\sqrt{(Z-2)}\sqrt{r_0}\left(1-\frac{\sigma}{2}\right), \quad (59)\dagger$$

or, putting in numerical values, as

$$\log_{10}\lambda = 21{\cdot}6693 - 1\ 191\times 10^9\frac{Z-2}{v} + $$
$$+4{\cdot}084\times 10^6\sqrt{(Z-2)}\sqrt{r_0}[1-0\ 001j(j+1)], \quad (60)$$

corresponds to the empirical relation found between the decay constants and the energies (or velocities) of α-disintegration for different radioactive bodies by Geiger and Nuttall (p. 86). We see from (59) that $\log_{10}\lambda$ depends actually not only on the velocity of the ejected α-particle but also on the charge-number Z and the radius r_0 of the nucleus, and that therefore it cannot be represented on a two-dimensional graph The reason why Geiger and Nuttall could get a smooth curve by plotting simply $\log_{10}\lambda$ against E (or v) is due to the fact that the variation of Z, E, and r_0 in a radioactive series is practically monotonic as we go down the series We should expect certain anomalies in the Geiger-Nuttall graph at the points where this regularity breaks down Fig 22 shows that such deviations actually occur, for example, for AcX the point does not lie on the curve—and this is due to the fact that the velocity of the α-particles ejected by this element is smaller than for the previous element RaAc, whereas, generally, the velocity increases as we go down the series. The values of $\log_{10}\lambda$ as calculated from (59) for a chosen constant value of r_0 give already a good representation of the Geiger-Nuttall graph because the small changes in radius from element to element (entering only in the last term) only slightly affect the results, a much better approximation can be obtained, however, if we assume that the nuclear radius varies proportionately to the cube root of the mass number of the nucleus (see Chap II)

4. Nuclear radii and the effect of spin

Using experimental data for decay constants and energies of disintegration we can calculate from (60) the values of

$$r_{\text{eff}} = r_0[1-0\ 001j(j+1)]^2 \sim r_0[1-0\ 002j(j+1)], \quad (61)$$

† Here $l = 2\ 303$ is the factor for converting natural logarithms to the base 10.

which we may call the effective radius of the nucleus. The effective radius is the same as the true radius when the emitted α-particle possesses no angular momentum, (61) shows that in other cases it is smaller than r_0. The values of the effective radii of the α-active elements are given in Table V† and are also plotted against mass number in Fig. 27.

TABLE V

Nuclear radii

Disintegrating nucleus	$Z-2$	*Effective velocity* $V_\alpha\left(1+\frac{4}{M}\right)\times 10^{-9}$ *cm/sec.*	*Decay constant* λ *sec* $^{-1}$	*Effective radius* $r_0\times 10^{12}$ *cm*	*Mass number* A
Th	88	1.41	1.3×10^{-18}	0.83	232
RaTh	88	1.63	1.16×10^{-8}	0.81	228
ThX	86	1.68	2.20×10^{-6}	0.79	224
ThEm	84	1.76	1.27×10^{-2}	0.77	220
ThA	82	1.83	4.95	0.75	216
ThC_{α_0}	81	1.75	1.5×10^{-5}	0.60	212
ThC′	82	2.10	$[7\times 10^{+6}]$	[0.72]	212
U_I	90	1.44	4.8×10^{-18}	0.89	238
U_{II}	90	1.53	$[2\times 10^{-13}]$	[0.87]	234
Io	88	1.51	2.9×10^{-13}	0.85	230
Ra	86	1.54	1.39×10^{-11}	0.81	226
RaEm	84	1.65	2.10×10^{-6}	0.79	222
RaA	82	1.72	3.78×10^{-3}	0.77	218
RaC_{α_0}	81	1.65	1.34×10^{-7}	0.57	214
RaC′	82	1.96	$3.5\times 10^{+3}$	0.73	214
RaF	82	1.62	5.88×10^{-8}	0.71	210
Pa	89	1.58	1.9×10^{-12}	0.76	231
RaAc	88	1.72	4.24×10^{-7}	0.73	227
AcX	86	1.68	7.14×10^{-1}	0.76	223
AcEm	84	1.84	1.77×10^{-1}	0.71	219
AcA	82	1.92	$4.74\times 10^{+2}$	0.73	215
AcC_{α_0}	81	1.81	4.79×10^{-3}	0.63	211
AcC′	82	1.91	$[1\times 10^{+2}]$	[0.72]	211

From Fig. 27 we see that effective nuclear radii vary in general smoothly, increasing with the mass number. However, the ratio $r/A^{\frac{1}{3}}$ is not strictly constant but increases by 15 per cent. from RaF

† In this table the relative velocity of α-particle and recoil-nucleus is used. In cases where 'fine structure' of α-spectra is present, nuclear radii are calculated using the *maximum velocity* of the α-particles and the *total* decay constant (the detailed discussion of effective radii for different components of α-spectra will be found in the next paragraph). For three elements for which the decay constants have not been measured the values given in brackets represent the result of calculation from formula (60) using the *interpolated* values of nuclear radii (also shown in brackets).

to U, showing that in the region of the radioactive elements we have a slight decrease of nuclear density with increasing mass, this effect might be connected with the formation of a new shell at this stage (see p. 50) It is very important that for all three C products, and also for most members of the actinium family, the values of effective radii are anomalously small This, as was indicated by Gamow,† may be due to the fact that in these cases α-particles are emitted with angular momentum different from zero In order to explain the deviation observed ($\sim$ 5 per cent) we should, according to

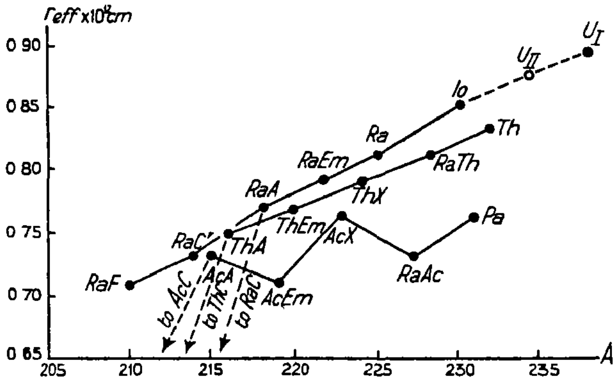


FIG 27

formula (61), suppose that in these cases the angular momentum of the emitted particles is about 4 or 5 quantum units, or, in other words, that the spins of the disintegrating and product nuclei differ by about as much

Later we shall see that the elements which show this anomaly are just the elements exhibiting the complex structure of α-spectra This association will lead us (Chap VI) to an understanding of the occurrence of intense structure for these elements in particular. In conclusion we may remark that there is known one α-decaying element which does not belong to the radioactive families It is Samarium, the α-activity of which was discovered by Hevesy, it has already been discussed from the point of view of the general stability laws in Chapter II The energy of the α-particles from Sm amounts to $2\ 0 \times 10^6$ e v (range 1·1 cm) and the decay period is $p \times (1 . 10^{12})$ years ($\lambda = (1/p) \times 3\ 10^{-20}$ sec $^{-1}$), where p is the percentage of active

† G. Gamow, *Nature*, **129** (1932), 470.

isotope. Accepting $p \sim 1$, we calculate from (60) the radius of Sm nucleus to be $r_0 = 0{\cdot}8 \times 10^{-12}$ cm , which is somewhat too large for its atomic weight ($A \sim 150$) In order to obtain the proper values for the radius we must suppose (as it seems also from other considerations) that the active isotope of Samarium is present in comparatively small quantities.

VI

γ-RAY EMISSION FOLLOWING α-DISINTEGRATION

1. Nuclear excitation by α-decay and the 'fine structure' of α-rays

In the process of α-emission the residual nucleus need not necessarily be left in its normal state If the energy released in the transformation is in excess of an excitation energy of this nucleus there is always the possibility that it will be found in an excited state, the energy

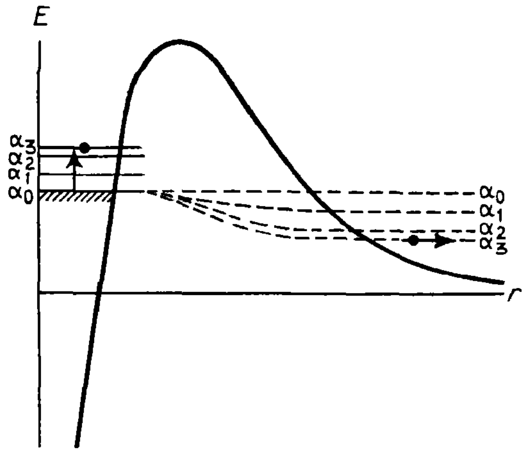


Fig 28

of the emitted α-particle being smaller just by this amount In these cases we may predict that the spectra of α-rays will show several discrete groups corresponding to the different quantum levels of the product nucleus, as represented schematically in Fig 28 The excited nucleus will emit this excess energy in the form of γ-radiation, which will follow immediately (due to the very short period of life of nuclear excited states) the emission of the α-particle. It is clear that the γ-rays originating in this way belong to the nucleus of the product element although they are experimentally observed together with the α-radiation of the original element It is also clear that the energies of these γ-rays must fit in the level scheme of the product nucleus—which may be obtained directly by turning upside-down a figure showing the energies of the various groups of α-

particles with respect to the group of smallest energy. We may notice here that, in general, the intensities of the slower α-components should decrease rapidly due to the increasing difficulty experienced by slower particles in penetrating the potential barrier.

Such weak groups of α-particles with smaller energies than the main group were first observed by Rosenblum† when investigating the deviation of the α-rays from ThC in strong magnetic fields. It appeared that there were actually five very close components referred to by Rosenblum as the components of 'fine structure' of the α-rays. The energy-differences and the relative intensities as measured by Rosenblum are given in the second and third columns of Table VI.

TABLE VI

'Fine structure' of the α-rays from ThC

α-group	Energy-difference $(E_{\alpha_0}-E_{\alpha_i})\times 10^{-6}$ e v	Relative intensities exp $I_{\alpha_i}/I_{\alpha_0}$		Relative intensities theor $I_{\alpha_i}/I_{\alpha_0}$
α_0		1 0	19 1%	1 0
α_1	0 040	3 3	77 0%	0 7
α_2	0 330	0 1	2 2%	0 03
α_3	0 477	0 01	0 2%	0 005
α_4	0 496	0 07	1 5%	0 004
α_5	0 626	..		..

The level scheme for the ThC″ nucleus obtained from these energies is shown in Fig 29, which also includes the γ-ray lines from the measurements of Ellis It will be seen that these fit well into the scheme Since the relative intensities of the different α-components evidently fix relative values for the partial decay constants corresponding to the different groups of particles, we see at once that the probability of disintegration does not vary so regularly (or, as we shall see later, so rapidly) with energy as might be expected from simple considerations; in particular the

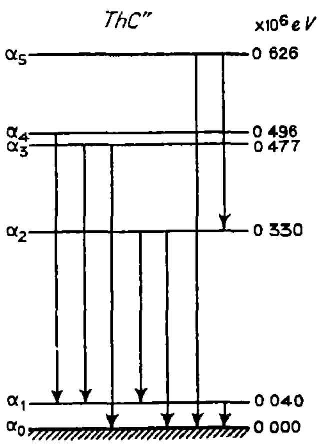


FIG 29

† S Rosenblum, *C.R.* **190** (1930), 1124, *J de Phys* **1** (1930), 438. For later references see Rosenblum in *Reports of the Solvay Congress*, 1933

intensity of the group α_1 is greater than that of the group α_0, although the energy of the α-particles is somewhat smaller

After the discovery of 'fine structure' with thorium C the same phenomenon was found with a number of other elements Rutherford and his collaborators,† using first the differential counter for range measurements, and later also the magnetic method, proved that with radium C and actinium C there are at least two components of comparable intensity, the corresponding level schemes and the γ-rays are shown in Fig 30, (*a*) and (*b*) Further investigations of

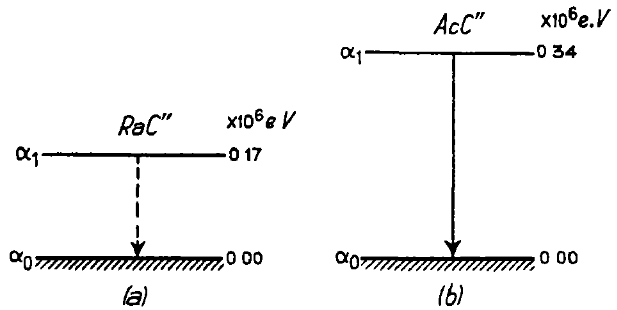


FIG 30

Rosenblum, by the magnetic method, showed the presence of more complicated and well-marked 'fine structure' with many members of the actinium family, he found three components with actinon, three components with actinium X, and eleven components with radioactinium Very probably, also, such 'fine structure' exists in the case of protoactinium, which, although not investigated by the magnetic method, is known to possess rather strong γ-radiation which has to be explained by excitation through α-decay

On the other hand, with all other elements so far investigated 'fine structure' is either not observed at all (ThX, Tn, ThA, Rn, RaA, RaF, AcA) or occurs in the form of an extremely faint component, corresponding to a γ-ray of small intensity (RaTh and Ra).

We shall investigate now in more detail the question of the relative intensities of different 'fine-structure' components in relation to the transparencies of the potential barrier for α-particles of the appropriate velocities Using the formula (55) of the previous

† E Rutherford, F A B Ward, and C E Wynn Williams, *Proc Roy Soc* A **129** (1930), 211, **139** (1933), 617.

chapter and accepting $j = 0$ (and $r_0 =$ const.) for all components of the structure, we can calculate the relative intensities which are to be expected The results of such a calculation for the groups of the ThC α-spectrum are given in the last column of Table VI. We see that in this case, and it is equally true in other cases of strong 'fine structure', the intensities of the components decrease much more slowly than would be expected theoretically—and notice even the inversion of order in the special case of the α_0 and α_1 groups of ThC. On the other hand, the faintness of the components with Ra

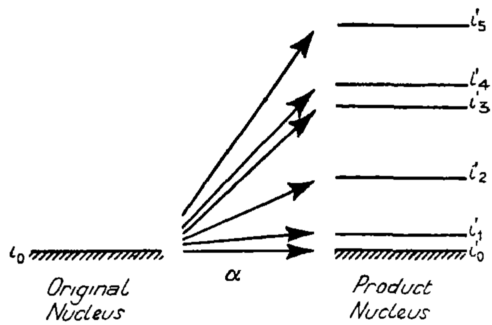


FIG 31

and RaTh is in good agreement with the theoretical expectation. In the same way we might say that the failure to detect any slow components with other elements must be due to their extreme weakness,† following the predictions of theory

These considerations lead us to the conclusion that we must consider the presence of strong 'fine-structure' components as an anomalous effect and try to find some explanation of it in terms of probabilities of different kinds of disintegration The explanation of this effect proposed by Gamow‡ is based on the dependence—already discussed—of the probability of disintegration on the angular momentum of the emitted particle Let us consider the process of α-disintegration of a nucleus possessing a spin i_0 when the product nucleus possesses, in its normal and excited states, spins i'_0, i'_1, i'_2, etc (Fig 31) If the spins of the normal states of the original

† We say this, rather than ascribe the absence of 'fine structure' to an energetic impossibility, in fact one can feel perfectly sure that in radioactive nuclei there are always levels with excitation energies less than the energy of α-particles amounting to at least 4×10^6 e v

‡ G Gamow, *Nature*, **131** (1933), 618

and product nuclei are the same ($i_0 = i_0'$) the probability of normal disintegration (i.e. without excitation) will be given by our previous expression (60) with $j = 0$, as in this case the α-particle can be emitted without angular momentum. Also, the intensities of disintegrations with excitation of different levels of the product nucleus will be considerably less than this on account of the smaller energy of the emitted α-particle and the effect of the additional 'potential barrier of centrifugal force' for those states with spin different from i_0. This represents just the above-mentioned case of very rapidly decreasing intensities of fine-structure groups characteristic of those elements for which a very faint structure or no structure at all has been observed. Let us imagine now that the normal state of the product nucleus possesses spin different from that of the original one. In this case the probability of normal α-decay will be considerably reduced as the emitted α-particle must take with it the spin-difference and escape from the nucleus with angular momentum $j = |i_0 - i_0'|$. On the other hand, it might now happen that some of the excited states of the product nucleus possess spin closer or equal to i_0, the spin of the original nucleus. For transitions to these states the decrease of disintegration probability due to the smaller energy of the emitted α-particle will be partially balanced on account of the absence of the additional 'centrifugal barrier' and the resulting intensity may become comparable with, or in certain cases even larger than, that of normal decay. These considerations, relating the existence of strong 'fine structure' with large spin-differences between the normal states of original and resulting nuclei, throw some light on the question why this effect is observed only for all three C products and for most members of the actinium family. For, as we have seen above (Chap. III), nuclear spins different from zero are to be expected only for elements with odd atomic number (C products) or odd mass numbers (actinium family). A glance at Fig. 27 shows us that it is just for the elements possessing strong 'fine-structure' components that the anomalously small values of the effective radius are observed, and this, as has been already mentioned, should be considered as indicating the emission of an α-particle with angular momentum different from zero. All these facts seems to prove, rather unambiguously, that the proposed explanation of the 'fine-structure' effect has the correct basis.

It is much more difficult, however, to give a quantitative treatment of the above effect, because the formula (60), used for calculating the effect of spin on the probability of disintegration does not take into account all the factors known to be concerned. In fact, in deriving this formula, we accepted a simplified model of the potential barrier with a *vertical fall at* $r = r_0$, and correspondingly cut off the potential barrier of centrifugal force at the same distance In reality the fall of potential near the nuclear boundary cannot be so abrupt, whilst the addition of centrifugal potential will somewhat change the whole distribution and slightly decrease† the value to be accepted for the nuclear radius, this effect cannot be taken into account until we know the exact form of the potential-energy curve near the nucleus The second effect neglected in the formula can be expressed as the change in the number of collisions of an α-particle inside the nucleus with the potential barrier. an α-particle possessing non-zero angular momentum will suffer fewer collisions with the 'wall' than a particle of the same energy, but without such momentum At present we can only say that both these effects will act in the same direction as the effect already taken in account, so that the actual decrease in intensity due to spin-differences should be larger than that given by our formula (60)

Anyhow we shall make the attempt to deduce nuclear spin-differences from the relative intensities of 'fine-structure' components and we shall proceed in the following way For each component we calculate the partial decay constant, using the experimental values of total decay constant and relative intensities (compare § 1) of the previous chapter) Substituting these, and the experimental values for the energies of the different components, in the formula (60), we obtain the effective radii corresponding to the components of 'fine structure'. These radii, as calculated by Gamow and Rosenblum,‡ are shown in Fig 32‖ plotted against the atomic weight of decaying nucleus We see that the points representing the effective radii for the different α-components of the same element are spread over a considerable region and that they show a tendency to accumulate in pairs, triplets, or other groups. If we agree, as stated above, that the reduction of effective radius is to be ascribed to the angular

† Because it is necessary to come closer to the nucleus in order that the stronger repulsion (Coulomb+centrifugal forces) may be compensated by nuclear attractive forces. ‡ G. Gamow and S Rosenblum, *C R* **197** (1933), 1620

‖ The values in Fig. 32 are slightly different from those given above (Fig 27), as in the new calculations somewhat improved experimental data have been used momentum of the emitted α-particle we must conclude that, for a given element, the disintegrations characterized by the same effective radius take place with the same change of angular momentum and those described by smaller radii with correspondingly large momenta. For example, the groups α_0, α_5, and α_6 in RaAc must have equal angular momenta and this momentum must be larger than that of the groups α_2 and α_3 In Fig 32 are included, also, the points representing the effective radii for different j's as calculated from the

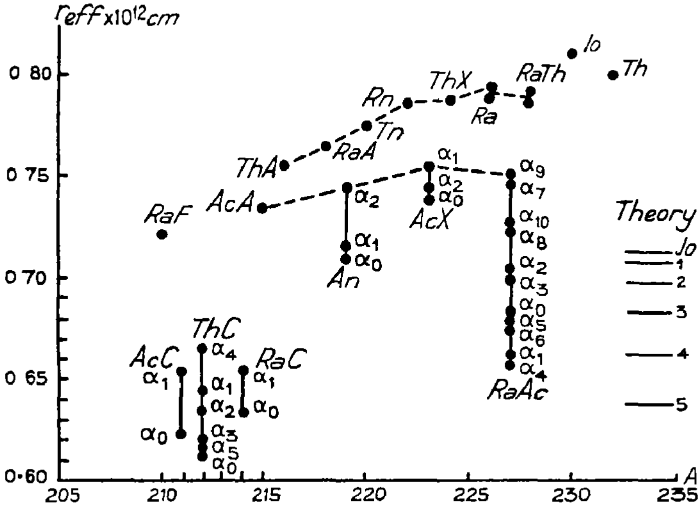


FIG 32

formula (60) We see that, although there is a close similarity between theoretical and experimental schemes, the experimental points are situated somewhat farther apart from each other than are the calculated points This can be accounted for by the roughness of the approximation used in obtaining formula (60)—as previously mentioned For this reason it is difficult with certainty to assign values of j to the different groups, for example, in the case of ThC we could accept $j(\alpha_4) = 0$, $j(\alpha_1 \text{ and } \alpha_2) = 1$,† and $j(\alpha_0, \alpha_3, \text{and } \alpha_5) = 2$, although the values 0, 2, and 3 or 1, 3, and 4 are also possible If, however, the above-mentioned correction can be taken into account it is not at all impossible that this method will permit us to give quite definite j-values to all components In conclusion we must mention

† The difference between the points for α_1 and α_2 in ThC is within the limits of experimental error

that our considerations lead only to the spin-differences ($j = i-i'$) between the nuclei, and, in order to assign values to the nuclear spin itself, a knowledge of at least one spin in each family is necessary. In the Th and U families it is very likely that the elements of the main α-decay sequence possess the spin $i = 0$, and for the Ac family we can use the spin of Pa, which, according to Schuler and Gollnow, is equal to $\frac{3}{2}$.

Another method of obtaining information concerning the relative spins of different excited states of a radioactive nucleus has been worked out by Ellis and Mott,† and is based on the difference between the internal conversion of dipole and quadrupole γ-radiation discussed in Chapter IV.

Although coefficients of internal conversion

$$\left(\frac{\text{number of secondary } \beta\text{-particles}}{\text{total number of transitions}}\right)$$

have not been measured directly for the elements exhibiting the fine structure of α-rays,‡ we can estimate these coefficients from the numbers of secondary β-particles and the numbers of α-particles in different groups—which give us directly the amounts of excitation of the corresponding nuclear levels.|| For example, for the case of ThCC″, Ellis gives the measured absolute intensities of different secondary β-groups (ejected from the K level) as shown in the third column of Table VII.

TABLE VII

Intensities of ThC″ γ-rays

Transition	$h\nu \times 10^{-6}$ e v	*Absolute intensities of sec β (measured)* $\times 10^4$	*Calculated coefficient of internal conversion*		*Calculated absolute intensity of γ-ray*	
			dipole	*quadrupole*	*dipole*	*quadrupole*
$\alpha_4 \to \alpha_1$	0.456	2.2	0.0095	0.029	0.023	0.0076
$\alpha_3 \to \alpha_1$	0.437	2.2	0.0102	0.032	0.022	0.0069
$\alpha_3 \to \alpha_0$	0.477	0.9	0.0088	0.027	0.01	0.0033
$\alpha_2 \to \alpha_1$	0.290	28.0	0.0176	0.113	0.16	0.0250
$\alpha_2 \to \alpha_0$	0.330	6.1	0.0149	0.075	0.04	0.0081

† C. D. Ellis and N. F. Mott, *Proc. Roy. Soc.* A, **139** (1933), 369.

‡ The only direct measurements of these coefficients have been made by Ellis and Aston for RaCC′ and RaBC γ-rays.

|| As a matter of fact the excitation of each level is composed of a direct excitation produced by the escaping α-particle plus the excitation originating in transitions to the level in question from higher levels; however, due to a rapid decrease of excitation with increasing energy of the level, the second term can be neglected in most cases.

In the last four columns are given the coefficients of internal conversion for the energies in question calculated theoretically by Mott and Taylor (see Chap IV) both for dipole and quadrupole radiation and the expected intensities of γ-rays, obtained by dividing the experimental intensities of the secondary β-rays by these coefficients Now we can compare these results with the excitation of different levels of the ThC″ nucleus as given in the third column of Table VI. We see, for example, that there are two transitions from the level α_2 ($\alpha_2 \rightarrow \alpha_1, \alpha_2 \rightarrow \alpha_0$); if we suppose that both are dipole transitions, or one is dipole and the other is quadrupole, or, finally, that both are quadrupole, we obtain, using the data of Table VII, the following values for the total number of transitions to be expected 0 20 (d, d), 0·17 (d, q), 0 065 (q, d), 0·033 (q, q) Since the excitation of the level α_2, as may be seen from Table VI, is only 0·022, we must conclude, accepting a probable error of 50 per cent in the calculations of internal conversion, that both transitions are quadrupole From the level α_3 we have also two transitions ($\alpha_3 \rightarrow \alpha_1$ and $\alpha_3 \rightarrow \alpha_0$), the total intensities of which, corresponding to the four different possibilities, are 0 032 (d, d), 0·025 (d, q), 0 017 (q, d), and 0 01 (q, q), whereas the excitation of this level is only 0 002 Whether this disagreement (of 500 per cent !) is due to errors in experimental measurements or in interpretation is difficult to say. Finally, from the level α_4, we know of only one transition ($\alpha_4 \rightarrow \alpha_1$), which being considered as dipole gives to the total intensity the value 0 0023, and as quadrupole the value 0·0075, whereas the excitation of the level is 0 0015 Thus it is possible (again with an error of 50 per cent) that this transition is a dipole transition We see that (permitting rather large errors in the estimation of internal conversion) we must consider the transitions $\alpha_2 \rightarrow \alpha_1$, $\alpha_2 \rightarrow \alpha_0$, $\alpha_3 \rightarrow \alpha_1$, $\alpha_3 \rightarrow \alpha_0$ as quadrupole transitions and the transition $\alpha_4 \rightarrow \alpha_1$ as a dipole transition Remembering what was said in Chapter IV about the selection principle for γ-radiation we thus conclude that the results of the above considerations, although very uncertain, are not in contradiction with those obtained by the previous method,† and we can hope that a more detailed application of both methods, based on better experimental data, will give us definite knowledge of

† In their original paper Ellis and Mott accepted, in constructing the quantum scheme, that the fundamental level of the ThC″ nucleus is an S level This, as we have seen above, is not necessary and is probably wrong

the quantum numbers appropriate to the different excited states of nuclei.

2. The α-decay of an excited nucleus and the 'long-range' α-particles

We are now in a position to ask the question what happens to an excited nucleus when the next transformation is to be the emission of an α-particle It is clear that two different processes can take place (*A*) the γ-ray may first be emitted, bringing the nucleus to its normal state, and this may be followed by normal α-disintegration, or (*B*) the α-particle may be emitted, taking with it the excess energy of the excited nucleus Both processes are represented in Fig 33 The relative frequencies of the two processes will evidently be governed by the relative probabilities of γ-emission, and the ejection of an α-particle with the surplus energy Remembering that the probability of γ-emission ($\sim 10^{+16}$ sec $^{-1}$) is much larger than the usual decay constants even for high-energy α-particles, we shall conclude that process (*B*) will occur very infrequently and that we should look for such anomalously fast (or 'long-range') α-particles only with those elements having already, in the normal state, high disintegration probabilities In fact such very fast particles have been discovered in very small amount only with ThC′ ($\lambda \sim 10^{+6}$ sec $^{-1}$) and RaC′ ($\lambda \sim 10^{+3}$ sec $^{-1}$)—and these two are precisely the radioactive elements of shortest life The energies and relative intensities of the different 'long-range' groups of these elements have been investigated in much detail by Rutherford and his collaborators,† they are shown in Tables VIII and IX on the following page

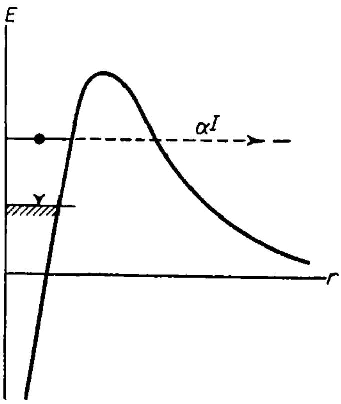


FIG 33

The energy differences listed in the third columns of these tables give us directly the positions of the excited levels for ThC′ and RaC′ and may be used as a term scheme for fitting in the various lines in

† E Rutherford, F Ward, and W Lewis, *Proc Roy Soc* A, **131** (1931), 684, **142** (1933), 347.

TABLE VIII

'Long-range' particles of ThC'

Name of the group	Energy of disintegration $\times 10^{-6}$ e v	Energy-difference $\times 10^{-6}$ e v	Number of particles per million disintegrations
Normal α_0	8 948		10^6
α^{I}	9 674	0 726	34
α^{II}	10 745	1 797	190

TABLE IX

'Long-range' particles of RaC'

Name of the group	Energy of disintegration $\times 10^{-6}$ e v	Energy difference $\times 10^{-6}$ e v	Number of particles per million disintegrations
α_0	7 829		10^6
α^{I}	8 437	0 608	0 43
α^{II}	9 112	1 283	0 45(?)
α^{III}	9 242	1 412	22 0
α^{IV}	9 493	1 663	0 38
α^{V}	9 673	1 844	1 35
α^{VI}	9 844	2 015	0 35
α^{VII}	9 968	2 139	1 06
α^{VIII}	10 097	2 268	0 36
α^{IX}	10 269	2 440	1 67
α^{X}	10 342	2 513	0 38
α^{XI}	10 526	2 697	1 12
α^{XII}	10 709	2 880	0 23

the γ-ray spectra of these elements These are best known for RaC', which possesses a very rich γ-ray spectrum (see Table D at the end of the book), giving us the possibility of filling up the level scheme defined by the 'long-range' α-particles Fig 34 represents the latest attempt of Ellis and F Oppenheimer to complete such a level scheme in this case † In the diagram values of the azimuthal quantum number are also indicated, having been obtained on the basis of the analysis of the internal-conversion coefficients for the corresponding γ-lines We see from this picture that the level schemes for nuclei may be at least as complicated as those for complex atoms

We can now turn our attention to the relative probabilities of ejection of a 'long-range' α-particle and emission of a γ-ray, as discussed in detail by Delbruck and Gamow ‡ In order to calculate the

† C D Ellis, *Report to the International Congress on Physics, London*, 1934.

‡ M Delbruck and G Gamow, *Zs f Phys* **72** (1931), 492

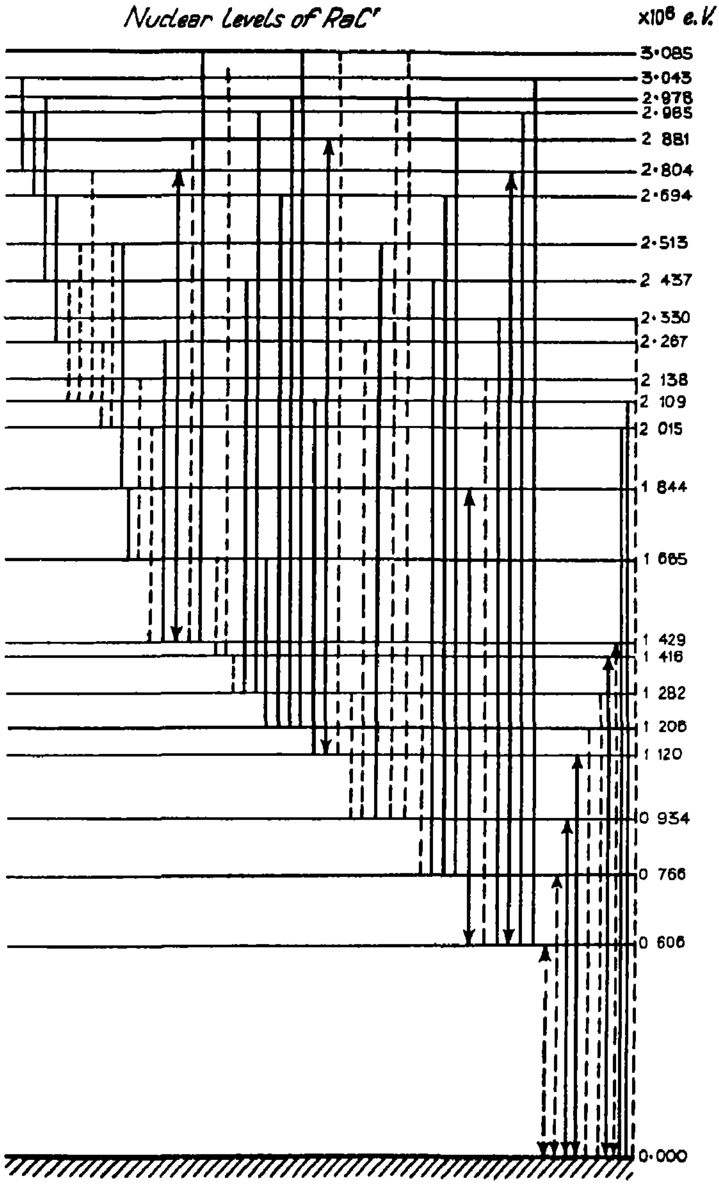

Nuclear Levels of RaC'
x10⁶ e.V.
3·085
3·043
2·978
2·965
2 881
2·804
2·694
2·513
2 437
2·330
2·267
2 138
2 109
2 015
1 844
1 665
1 429
1 416
1 282
1 206
1 120
0 934
0 766
0 606
0·000

Fig. 34

probability of the ejection of a 'long-range' α-particle we have merely to substitute the energy-values given by Tables VIII and IX in our decay-constant formula (60) of the previous chapter We can accept as a sufficient approximation that the radius of the excited nucleus is the same as that of the nucleus in the normal state. For the present, also, we shall neglect the effect of angular momentum For the calculation of the probability of γ-emission we must use the formulae (9) and (10) of Chapter IV for dipole or quadrupole radiation, respectively Choosing for example the first 'long-range' groups α^{I} of RaC′, we obtain for the decay constant of the excited state the value $\lambda \sim 2\times 10^{+5}$ sec $^{-1}$, whereas for the probabilities of γ-transition to the ground-level we have

$$\kappa_{\text{dip}} \sim 4\times 10^{+15}\ \text{sec.}^{-1} \quad \text{and} \quad \kappa_{\text{quad}} \sim 1\times 10^{+11}\ \text{sec.}^{-1}$$

Thus the theoretically expected ratio λ/κ is 5×10^{-11} for dipole and 2×10^{-6} for quadrupole radiation Table IX of this chapter and Table IV A of Chapter IV show us that in this case the number of 'long-range' particles per disintegration is $N_{\alpha^{\text{I}}} = 0{\cdot}43\times 10^{-6}$, and the number of γ-rays $N_\gamma = 0{\cdot}66$ Thus experimentally

$$\frac{N_{\alpha^{\text{I}}}}{N_\gamma} = 0\ 67\times 10^{-6},$$

which fits with the hypothesis of quadrupole radiation The same result may be obtained for any other level of RaC′, which leads to the conclusion that *all γ-transitions in radioactive nuclei possess approximately quadrupole transition probabilities* † It does not mean, of course, that there are no dipole transitions in the excited nucleus, but only that the probabilities of dipole transitions are largely reduced, as may happen, for example, if the dipole moment of the transition is small This is also in good agreement with the fact, discussed in Chapter IV, that the intensities of dipole and quadrupole γ-lines emitted by radioactive nuclei are entirely comparable.

Now we can easily explain why the γ-lines corresponding to the 'long-range' α-groups of ThC′ are so extremely faint In fact for the group α^{I} of ThC′ ($E_{\alpha^{\text{I}}} = 9{\cdot}67\times 10^6$ e v , $E_{\alpha^{\text{I}}} - E_{\alpha_0} = 0{\cdot}73\times 10^6$ e v.) we calculate for example $\lambda_{\alpha^{\text{I}}} \sim 3\times 10^{10}$ sec $^{-1}$ and $\kappa_{\text{quad}} \sim 2\times 10^{11}$ sec $^{-1}$,

† It is very improbable that this result is due to our neglecting the effect of azimuthal quantum number or the change of radius of the excited state, as the difference between dipole and quadrupole probabilities amounts to a factor of 40,000.

and the relation $N_\alpha/N_\gamma = \lambda/\kappa$ gives us for the number of γ-quanta per disintegration the value $N_\gamma = 34\times 10^{-6}\dfrac{2\times 10^{+11}}{3\times 10^{+10}} \sim 0{\cdot}0002$ †

In conclusion we may mention that combinations of the two effects discussed in this chapter ('fine structure' of 'long-range' α-groups and 'long-range' groups due to the excitations connected with 'fine structure' α-groups) are possible in principle although it has not yet been detected

† We may notice here that using the dipole probability $\kappa_{\text{dip}} \sim 6\times 10^{+15}$ sec $^{-1}$ we should expect almost one quantum per disintegration in contradiction with experimental evidence

VII

SPONTANEOUS β-DISINTEGRATION

1. General features of β-disintegration

In contrast to that of α-decay, the process of β-disintegration has, until quite recently, offered serious difficulties to theoretical understanding The fundamental difficulty concerns the continuous distribution of energy amongst individual β-particles emitted by a pure element—as first demonstrated by Chadwick † Fig 35 represents

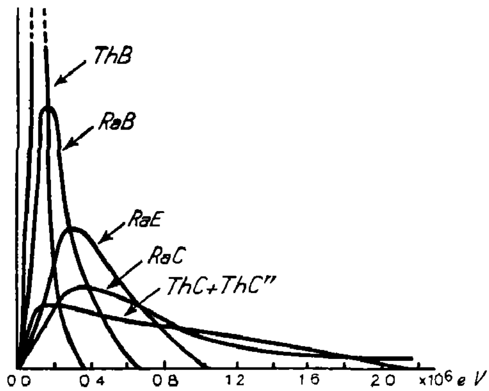


Fig 35

the energy-distribution in the β-ray spectra of the various radioactive elements investigated by Gurney‡ by the magnetic focusing method. We see that all the curves have the same characteristic shape, rather similar to that of a Maxwell velocity distribution, with the maximum at about one-third of the upper limit The question whether in fact there exists a sharp upper limit to these continuous β-ray spectra (which would be unnecessary if the hypothesis of non-conservation of energy in β-decay were correct) has been discussed for a long time with varying results, finally being settled in the affirmative by the work of Sargent,|| who also located the upper limits for a number of β-disintegrating elements The upper limits determined by him are given, together with the decay constants of the β-active elements in Table F at the end of the book

† J. Chadwick, *Verh d D Phys Ges* **16** (1914), 383

‡ R. W. Gurney, *Proc Roy. Soc.* A, **109** (1925), 541, **112** (1926), 380.

|| B. W Sargent, ibid. **139** (1933), 659.

Apart from the β-decaying members of the main radioactive families there have also been discovered two lighter elements which show the spontaneous emission of β-rays These are K and Rb, which emit moderately slow β-particles and have extremely long periods of life, the experimental data concerning these elements are given in Table F at the end of the book At first the understanding of the natural activity of these two elements presented considerable difficulties and the question was long undecided whether the activity should be ascribed to one of the known isotopes (K^{39}, K^{41}, Rb^{85}, Rb^{87}) of these elements or to unknown unstable isotopes present in small quantities.

This question was first investigated by v. Hevesy,† who showed that the β-activity of ordinary potassium decreases after 'ideal' distillation As the velocity of evaporation decreases with increasing atomic weight, in potassium obtained in this way the relative amount of the lighter isotope K^{39} must have been increased. This leads us to the conclusion that K^{39} is not responsible for the observed β-emission. However, the question whether the activity is due to the hypothetical isotope K^{40}, to the second known isotope K^{41} (abundance 7 per cent), or to a still heavier unknown isotope remained unanswered The situation was cleared by the observation of Fermi‡ that ordinary potassium being bombarded by neutrons shows induced β-activity (compare Chap X) of half-value period only 16 hours This was definitely proved by v Hevesy|| to be due to the potassium isotope ${}_{19}K^{42}$ formed by the reaction

$$ {}_{19}K^{41}+{}_{0}n^{1} \rightarrow {}_{19}K^{42}+h\nu \qquad (1)$$

We might expect that in such bombardment the reaction

$$ {}_{19}K^{39}+{}_{0}n^{1} \rightarrow {}_{19}K^{40}+h\nu \qquad (2)$$

would also take place, and explain the fact that the corresponding activity is not observed by assigning a very long period of life to the product nucleus ${}_{19}K^{40}$ This brings us to the natural conclusion that the activity of ordinary potassium is due to this isotope, ${}_{19}K^{40}$, present in very small quantities. Aston's failure to observe the corresponding line in the mass spectrum†† indicates an abundance of less than 0·3 per cent. As the value of the decay period usually given

† G v Hevesy and J Bronsted, *Zs phys Chem* **124** (1922), 22

‡ E Fermi, *Ricerca Scientifica*, 2 Dec 1934

|| G v Hevesy, *Nature*, 19 Jan 1935

†† F W. Aston, *Mass Spectra and Isotopes* (1933)

($1{\cdot}5\times10^{13}$ years) is based on the hypothesis that all the atoms of potassium are unstable we must reduce it by a factor ($<\frac{1}{300}$) in order to obtain the true period Thus we have $T<5\times10^{10}$ years ($\lambda>10^{-17}$ sec $^{-1}$) On the other hand, geological considerations definitely show that any potassium in the earth's crust was deposited not less than 6×10^{7} years ago, which gives as the upper limit for the decay constant $\lambda<10^{-14}$ sec $^{-1}$ Similar considerations may be made in respect of the activity of rubidium, for which the active isotope is probably $_{37}Rb^{86}$

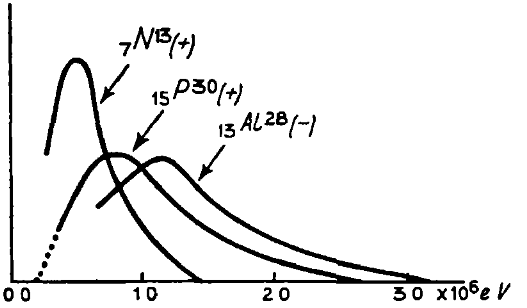


FIG 36

Our knowledge concerning β-decay was considerably increased by the discoveries of Joliot and Curie and of Fermi, who found that a large number of β-active elements, emitting positive and negative electrons, can be produced artificially by bombarding different stable nuclei with various projectiles (the nuclear reactions which give rise to artificial β-active elements will be discussed in detail in Chapters X and XI) In Fig 36 are given examples of the energy-distribution curves for some artificially produced elements it will be seen that they are rather similar to the β-spectra of ordinary radioactive elements Our present knowledge concerning the energies of disintegration and the corresponding decay constants of artificially produced radioelements of low mass-number are summed up in Table F at the end of the book

Inspecting the relation between the disintegration energies and the decay constants we find the same general regularity as we had in the case of α-decay the greater the energy of disintegration, the greater, also, the decay constant, although now the decay constant varies much more slowly with energy than in the case of α-decay This relation was first clearly demonstrated by Sargent,† who plotted the logarithm of the limiting energy in the continuous β-spectrum against

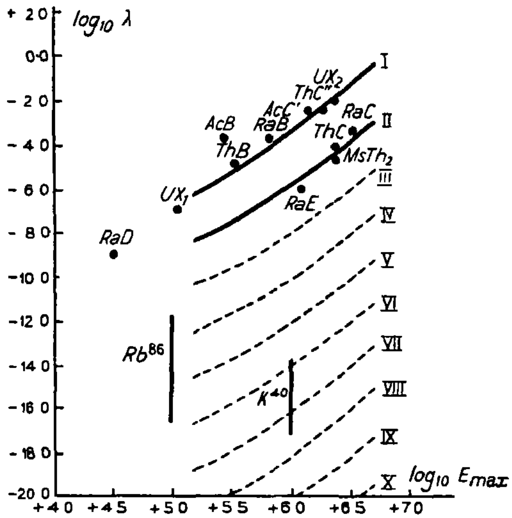


FIG 37 (*a*)

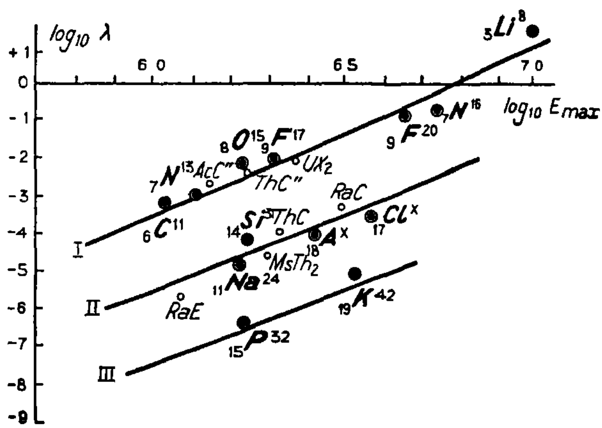


FIG 37 (*b*)

the logarithm of the decay constant The Sargent graph is given in Fig 37 (*a*), which also contains two points representing our previous conclusions regarding K and Rb The same graph for artificial β-active bodies is given in Fig 37 (*b*) We see that, at least for most

† Loc cit

of the elements, the representative points may be considered as distributed on two parallel curves separated vertically by two units (a factor of 100 in λ).

We naturally inquire what is the cause of the continuous distribution of energy in β-ray spectra, which represents such a striking departure from the discrete energies always observed in the spectra of α-particles That this cause is not to be found in any secondary effect of scattering or absorption of β-particles all originally of the same energy was proved by very careful experiments carried out first of all by Ellis and Wooster † These authors measured calorimetrically the total energy produced by a known amount of the β-disintegrating element RaE. As this element emits only very little γ-radiation, the total measured energy, divided by the number of atoms disintegrated, must give the average energy of disintegration per nucleus If the continuous β-spectrum were due to secondary scattering, in which β-particles lose energy in collisions with other electrons (so that the total energy remains inside the calorimeter), we should expect the average energy measured by the heating effect to be equal to the maximum energy observed in the continuous β-spectrum This, however, is not the case, the energy measured coincides exactly with the average energy of the electrons in the continuous spectrum, which proves that no energy is lost in collisions in the manner supposed, and that the *continuity of the β-spectrum is a property of the disintegrating nucleus* It is very interesting that in spite of large differences in the energy of the β-particles the decay constants of β-disintegrating elements possess such definite values‡ and are, as we have seen above, related, by what seems to be an exact law, with the upper limits of energy concerned We must also exclude another possibility the energy cannot in any way be distributed between two electrons which are simultaneously emitted, because the number of electrons in the spectrum of a β-decaying body has been shown by Gurney to be exactly one per disintegration (as may also be concluded from the location of the β-active element

† C D Ellis and W A Wooster, *Proc Roy Soc* A **117** (1927), 109 These experiments have been repeated by L Meitner and W Orthmann (*Zs f Phys* **60** (1930), 143), with the same result

‡ This can be shown, for example, by placing a source of RaE behind a screen which lets through only the fastest β-particles in the spectrum The relative decrease of these particles with time is found to be exactly equal to the relative decrease of the total number of electrons in the continuous spectrum

and its product in the periodic system of the elements). Finally, it is clearly impossible that different nuclei of a given element remain after β-emission with different amounts of internal energy (or possess different internal energies before β-emission), because in such cases other nuclear processes (α-decay, γ-radiation, etc.) would be different for different nuclei before or after β-decay, which is not observed

Thus we have the choice only between two possibilities *either, as proposed by Bohr, the energy-conservation law does not hold for the processes of β-disintegration, or, according to the hypothesis of Pauli, energy is taken away by some new kind of particle still escaping observation.*

We must notice here that parallel with the energy-troubles of β-decay we meet analogous difficulties in the problem of conservation of angular momentum In fact, as we have seen in Chapter III, experiment shows that nuclei with even mass-numbers possess spins which are even multiples of the quantity $\frac{1}{2}\hbar$, whereas the spins of nuclei with odd mass-numbers are odd multiples of the same quantity. This is just what we should expect if all nuclei were constructed of neutrons and protons each with spin $\frac{1}{2}\hbar$ (the orbital momenta, being whole multiples of $\hbar$, do not change the 'even and odd' properties of the total spin). In β-disintegration the mass-number of the nucleus does not change we must conclude, therefore, that the total change of nuclear spin amounts to an even multiple of $\frac{1}{2}\hbar$ Now it is impossible that this change of angular momentum should be balanced by that carried away by the emitted β-particle alone, as the β-particle itself, having a spin of $\frac{1}{2}\hbar$ and an orbital momentum which is an integral multiple of $\hbar$, can only contribute a total momentum which is an odd multiple of $\frac{1}{2}\hbar$

This discrepancy must be considered, according to the first of the above-mentioned hypotheses, as evidence for the non-conservation of angular momentum in the process of β-decay According to the second hypothesis, the hypothetical particle emitted must be responsible for this change of momentum, that is it must itself possess spin $\frac{1}{2}\hbar$ (or any odd multiple of this quantity)

The non-conservation hypothesis was proposed by Bohr, who indicated that, in so far as the behaviour of light particles in the nucleus cannot be described by means of modern wave mechanics but requires some new fundamental theory, there need be no *a priori* reason for retaining the conservation laws in this more developed theoretical formalism. During the last few years, however, it has become clear that this point of view encounters serious difficulties, both from the experimental and the theoretical sides The first difficulty consists in the existence of the sharp upper limits of β-spectra, which, although not in direct contradiction to the above hypothesis (which leads, as more plausible, to infinite tails for the continuous β-curves), still throws some weight on the side of the conservation laws The observation of Ellis and Mott (p 151), according to which the energy-conservation laws hold for the β-disintegration processes if we use the upper limits of β-spectra in the energy-balance formulae, gives still more important indication in favour of the general validity of these laws †

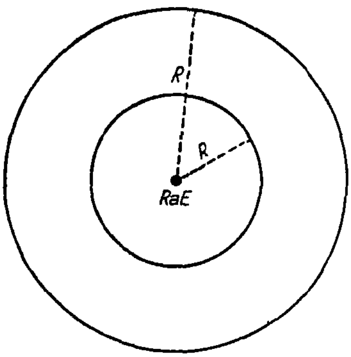


FIG 38

Another, purely theoretical, difficulty connected with non-conservation of energy was indicated by Landau,‡ who showed that, from the relativistic point of view, non-conservation of energy (which is equivalent to non-conservation of mass) would lead to a contradiction of the general laws of gravitation Let us consider a nucleus of RaE of mass M located in empty space and surrounded by two spheres of radii R and R' In the static case the total flux of gravitational force across each surface will be given by $4\pi M$ (Gauss formula) Let us imagine now that at a certain moment t_0 the nucleus emits a β-particle and that the conservation law does not hold Let the total mass of the system (mass of the product nucleus, electron, and kinetic energy) change to M' Since the corresponding changes in the gravitational field around the nucleus will be propagated with the velocity c, we may choose a time t, defined by the relation

$$R < c(t-t_0) < R', \tag{3}$$

and at this time the electron will still be within the inner sphere (R)

† The evidence given by upper limits of β-spectra permits the energy, if not conserved, only to decrease, thus still holding the non-conservation hypothesis, we should formulate the energy law in the form $\partial E/\partial t \leqslant 0$, in analogy to the entropy law $\partial S/\partial t \geqslant 0$

‡ L Landau, *Tchervony Gudok*, p 1001 (1932).

if R' and R have been suitably chosen. On the other hand, the field over the inner sphere will correspond to the new mass $\left(U_R = \frac{kM'}{R}\right)$ whereas the field over the outer sphere will still have the old value $\left(U_{R'} = \frac{kM}{R'}\right)$ Thus the total flux of gravitational force into the region between the two spheres is different from zero, which is inconsistent with the absence of mass in this space

According to the second hypothesis, proposed by Pauli, in the process of β-disintegration a second particle is emitted which carries away the excess of energy and angular momentum This particle can have no charge (as all the charge is already taken by the electron), and its mass must be small compared with that of the proton (as the mass-number does not change in β-disintegration). These two properties alone make this hypothetical particle, now called the *neutrino*, extremely difficult to observe, as, due to the absence of charge, it cannot produce sufficient ionization to be detected in a Wilson chamber (or in any other apparatus based on ionization measurement). Similarly, its small mass will exclude the possibility of observing the nuclei set in motion by collision with this particle (the method used in the detection of neutrinos) There remains only the possibility of observing electrons set in motion by impact of the neutrino, but, as was shown by Bohr, such collisions are extremely improbable

According to what has been said above, the neutrino should possess a spin at least $\frac{1}{2}\hbar$, but one can show that it cannot have a large magnetic moment. In fact the calculations of Bethe† have shown that the failure to observe the ionization along the track of a neutrino (according to Nahmias a neutrino may produce not more than 1 ion per $5 . 10^5$ km. path in the air) requires its magnetic momentum (if any) to be much smaller ($\leqslant 1/1{,}000$) than that of an electron

There are, however, certain other possibilities of proving or disproving the existence of neutrinos If, for example, we consider the behaviour of the recoil nucleus in β-emission we shall reach different conclusions according as we adopt non-conservation or the neutrino hypothesis. In fact we can make three different hypotheses as to the conservation of linear momentum in β-disintegration.

† H Bethe, *Proc Camb Phil Soc* 31 (1935), 108.

(1) Energy is not conserved (no neutrino), but linear momentum is conserved.

(2) Energy is not conserved (no neutrino), linear momentum is not conserved but is distributed at random.

(3) Energy is conserved (taking into account the neutrino) and linear momentum is also conserved between electron, neutrino, and recoil nucleus

In the first case the continuous spectrum of β-rays will give rise to a continuous distribution of the velocities of recoil nuclei (which will have the same shape as the β-spectrum In the second case the energy-distribution of recoil nuclei will be continuous but possibly of different shape from that of the β-spectrum In the third case the recoil nuclei will almost always† have larger velocities than otherwise (in the case of a slow electron a fast neutrino will be emitted) Thus the absence (or a small number) of slow recoil nuclei in β-decay would prove the existence of the neutrino Experiments in this direction have been undertaken by Lejpunskij‡ by the method of coincidence-counters for recoil nuclei and fast β-particles, the results of these experiments seem to be in favour of the existence of the neutrino

2. Theory of β-disintegration

It has already been mentioned in the first chapter that difficulties in reaching a theoretical understanding of the existence of electrons in atomic nuclei have led to a point of view according to which the process of β-disintegration is regarded as a 'creation' of a (negative or positive) electron as the result of transition of a nuclear 'heavy particle' from the state of neutron to that of proton or conversely This point of view|| draws a close analogy between the emission of an electron by an atomic nucleus and the emission of a light quantum by an atom The analogy appears closer when we remember that we never consider a light quantum as a constituent part of an atom although quanta are emitted and absorbed by the atom, and their mass, in the form of the energy of the electromagnetic field, makes an appreciable contribution to the total mass of the system The most important reason for this disregard is that light quanta do not

† Except in rare cases when the electron and neutrino are emitted in opposite directions with equal momenta

‡ A Lejpunskij, *Proc Camb Phil Soc* (in press)

|| Clearly emphasized, for example, by Heisenberg in *Zeeman Festschrift*, p 108 (1935)

obey the law of 'conservation of number', and, for example, considering a nucleus excited to the third quantum level, we can never know whether the excess energy will be emitted in the form of one, two, or three quanta Just in the same way it is not reasonable to speak about the existence of individual electrons in nuclei, because the existence of both negative and positive β-emitters would force us to admit the presence of electrons of both kinds in the nuclear structure —and this is hardly possible in view of the mutual annihilation of these particles We can extend the analogy still farther. We know, for example, that light quanta are never *ejected* from an atom in collision experiments in the same sense as electrons are, but their emission (by an atom excited in collision) occurs only after a period of time characteristic of the interaction between the atomic electrons and the virtual electromagnetic field, which has nothing directly to do with the process of collision itself In a quite analogous way nuclear electrons are never observed to be *ejected* immediately in the process of collision (as protons, neutrons, and α-particles are), such emission, when it occurs (Joliot-Curie and Fermi's reactions) always takes some time which is characteristic of the nucleus itself† having nothing to do with the nuclear reaction in which the β-emitting nucleus is produced The possibility of discriminating between the particles 'actually present' in the atom (electrons) and those 'created' in quantum transitions (γ-quanta) is due, according to the analysis of Bohr, to the smallness of radiative forces (leading to the emission of light) as compared with the Coulomb interaction (holding the atom together). In a similar way we should expect, in the case of β-emission, that the forces leading to neutron-proton transition (Fermi forces discussed later) should be small as compared with the exchange interaction between nuclear constituent particles (discussed in detail in Chapter II) This smallness would then account naturally for the long lifetimes of β-emitters, as compared with the periods of oscillation of nuclear particles ‡

A tentative theory of β-emission, following the lines of this analogy, has been worked out by Fermi|| on the basis of the simplest hypothesis concerning the interaction energy between heavy and

† The interaction of nuclear constituent particles with the virtual fields of the light particles to be created in the process.

‡ We notice here that modern β-decay theories are in quite a different category from the theory of α-emission discussed above

|| E Fermi, *Zs f Phys* 88 (1934), 161

light particles (neutrons and protons; electrons and neutrinos) which is responsible for $(n \rightarrow p)$-transformations and the creation of electron and neutrino. In this theory the existence of a neutrino† is formally accepted as otherwise it would be impossible, using the present formalism of the relativistic quantum mechanics, to write proper expressions for the interaction energy in question For the formal representation of transitions of a heavy elementary particle from the state of neutron to the state of proton and vice versa we may follow Heisenberg, who describes an elementary heavy particle, for the purpose of the general nuclear model, by five coordinates· three ordinary space coordinates, the spin coordinate, and a new two-valued coordinate ρ. This last coordinate has the value $+1$ for the neutron state and -1 for the proton state ‡ On this basis the transitions from the neutron state to the proton state, and conversely, may be formally represented by matrix operators,

$$Q = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \quad \text{and} \quad Q^* = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}, \tag{4}$$

acting on any function of the internal coordinate ρ (Q represents the transition $p \rightarrow n$ and Q^* the transition $n \rightarrow p$)

Along with these operators we must also introduce other operators acting on the coordinates of light particles in such a way as to describe the creation of the electron and neutrino conjugated to a $(n \rightarrow p)$-transition and the disappearance of the same two particles conjugated to a $(p \rightarrow n)$-transition || Such operators are represented as amplitudes a_s, a_s^* and b_σ, b_σ^* in the development of the wave functions of electron (ψ) and neutrino (ϕ) in series involving the different eigenfunctions of the problem

$$\left.\begin{aligned} &\text{electron.} \quad \psi = \sum_{s=1}^{\infty} a_s \psi_s, \quad \psi^* = \sum_{s=1}^{\infty} a_s^* \psi_s^* \\ &\text{neutrino.} \quad \phi = \sum_{\sigma=1}^{\infty} b_\sigma \phi_\sigma; \quad \phi^* = \sum_{\sigma=1}^{\infty} b_\sigma^* \phi_\sigma^* \end{aligned}\right\} \tag{5}$$

† A theory of β-decay permitting non-conservation of energy and not introducing a neutrino was published by G Beck and K Sitte (*Zs f Phys* **86** (1933), 105) somewhat before Fermi's theory It seems, however, that Fermi's theory comes closer to what actually happens, so that we choose it for discussion in this book

‡ In order to allow for the possible existence of negative protons it would be necessary to permit three different values for ρ

|| In Fermi's theory the emission of a positive electron (and an antineutrino) is formally treated as the disappearance of a negative electron (and a neutrino) This does not, however, affect the general character of the theory

One can prove that the amplitude a_s, considered as an operator, corresponds to disappearance and the amplitude a_s^* to creation of an electron in the sth state. In the same way b_σ and b_σ^* represent the disappearance and creation of a neutrino in the state σ.

Combining the two types of operator we see that $Qa_s b_\sigma$ describes a $(p \rightarrow n)$-transition with disappearance (absorption) of an electron (from the sth state) and a neutrino (from the σth state), and $Q^*a_s^* b_\sigma^*$ a $(n \rightarrow p)$-transition with creation (emission) of an electron (in the sth state) and a neutrino (in the σth state) Thus we can write the Hamiltonian expression for the interaction energy corresponding to such processes in the form

$$H_{\text{int}} = Q \sum_{s,\sigma} c_{s,\sigma} a_s b_\sigma + Q^* \sum_{s,\sigma} c_{s,\sigma}^* a_s^* b_\sigma^*, \tag{6}$$

where $c_{s,\sigma}$ and $c_{s,\sigma}^*$ depend on the dynamical variables of the problem.

On the basis of the above analogy between the 'creation' of light quanta by an atomic system and the 'creation' of electrons by a nucleus, we must assume that the coefficients $c_{s,\sigma}$ in expression (6) depend on the coordinates, velocities, and perhaps higher derivatives of these quantities for heavy particles only, just as in the case of light emission only the coordinates of atomic electrons (and the derivatives of such coordinates)—and not the coordinates of light quanta—enter into the expression for the interactions which result in the emission As the simplest hypothesis Fermi assumed that the interaction energy depends only on the wave-functions of the light particles (but not on their derivatives)† and, neglecting at first the condition of relativistic invariance, wrote (6) in the form

$$H_{\text{int}} = g[Q\psi(x)\phi(x) + Q^*\psi^*(x)\phi^*(x)], \tag{6'}$$

where $\psi(x)$ and $\phi(x)$ are the wave functions of electron and neutrino taken at the coordinates x of neutron and proton (just as in radiation theory we take the values of the electromagnetic field at the coordinates of the radiating electrons) and g is a numerical constant of dimensions cm ^{5} gm sec $^{-2}$ (= erg cm ^{3}) In order to obtain the relativistically invariant form of expression (6′) (which is necessary as the velocities of the emitted electrons are close to the velocity of light) we must remember that in relativistic wave mechanics we must use, instead of the simple functions ψ and ϕ, Dirac's four

† We shall see later that this simple hypothesis has to be changed to account for the observed shapes of β-spectra.

functions ψ_1, ψ_2, ψ_3, ψ_4 and ϕ_1, ϕ_2, ϕ_3, ϕ_4.† In extending his theory, therefore, Fermi chose, from the sixteen possible bilinear combinations of these functions, just those which behave, in space-time transformations, as the components of a polar four-dimensional vector This choice is again suggested by analogy with radiation problems, since there one introduces the polar four-dimensional vector of electromagnetic potential The four combinations in question are

$$\left.\begin{aligned} A_0 &= -\psi_1\phi_2+\psi_2\phi_1+\psi_3\phi_4-\psi_4\phi_3, \\ A_1 &= \psi_1\phi_3-\psi_2\phi_4-\psi_3\phi_1+\psi_4\phi_2, \\ A_2 &= i\psi_1\phi_3+i\psi_2\phi_4-i\psi_3\phi_1-i\psi_4\phi_2, \\ A_3 &= -\psi_1\phi_4-\psi_2\phi_3+\psi_3\phi_2+\psi_4\phi_1, \end{aligned}\right\} \tag{7}$$

they should be substituted in the Hamiltonian for heavy particles in the same way as the components of electromagnetic vector potential are introduced in the ordinary radiation problem

Unfortunately we do not know at present how to construct the relativistic wave equation for heavy particles and therefore can only tentatively write

$$H_{\text{int}} = g\{Q[A_0+(\boldsymbol{\alpha}_h \mathbf{A})]+Q^*[A_0^*+(\boldsymbol{\alpha}_h \mathbf{A})^*]\} \tag{8}$$

in analogy with the corresponding formula of radiation theory Here the vector $\boldsymbol{\alpha}_h$ is obtained by operating with Dirac's matrices on the spin of the heavy particle, whilst the vector $\mathbf{A}$ has components A_1, A_2, and A_3 However, since they are of the order of magnitude $\left(\frac{\text{velocity of heavy particle}}{\text{velocity of light}}\right)$, there will be no immediate use of the doubtful scalar products in (8), they can be neglected in a first approximation

Introducing the matrix

$$\delta = \begin{vmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{vmatrix} \tag{9}$$

and neglecting terms of the order v/c, we can rewrite (8) in the form

$$H_{\text{int}} = g[Q\bar{\psi}^*\,\delta\phi+Q^*\bar{\psi}\,\delta\phi^*] \tag{10}$$

Comparing (10) with (6) we find

$$c_{s,\sigma} = g\bar{\psi}_s^*\,\delta\phi_\sigma; \qquad c_{s,\sigma}^* = g\bar{\psi}_s\,\delta\phi_\sigma^* \tag{11}$$

† See, for example, P A M Dirac, *Quantum Mechanics* (1935), Clarendon Press, Oxford

If now we denote by N and P the energy-operators for neutron and proton, we can express the part of the Hamiltonian of our problem corresponding to heavy particles in the form

$$H_{\text{heavy}} = \frac{1+\rho}{2}N + \frac{1-\rho}{2}P, \tag{12}$$

which is equal to N for $\rho = +1$ (neutron) and to P for $\rho = -1$ (proton)

In a similar way for light particles we have

$$H_{\text{light}} = \sum_s h_s H_s + \sum_\sigma k_\sigma K_\sigma, \tag{13}$$

where H_s and K_σ are energy-operators for the stationary states of electron and neutrino, and h_s and k_σ the populations of these states (both h_s and k_s being able to assume only the values 0 or 1, since the Pauli principle holds for electrons and most probably also for neutrinos) We can now write the total Hamiltonian of our problem as follows

$$H = H_{\text{heavy}} + H_{\text{light}} + H_{\text{int}} \tag{14}$$

—and proceed with the calculations of the transition probabilities leading to the creation of electron and neutrino We shall carry out these calculations in the same way as in radiation theory, considering *the interaction part of the Hamiltonian* (H_{int}) *as a small pertubation* This is permissible because, as mentioned above, the periods of transformation are large compared with the periods of oscillation of heavy particles in the nucleus †

Now the undisturbed state of our system can be described by the following set of quantum numbers

$$(\rho, n, h_1, h_2, h_3, \quad k_1, k_2, k_3, \quad) \tag{15}$$

Here ρ may be $+1$ or -1 (neutron or proton), n ($= 1, 2, 3,$) characterizes the quantum state of the heavy particle, and h_s and k_σ ($= 0$ or 1) are the populations of the quantum levels for electrons and neutrinos, respectively The nth eigenfunctions for the heavy particle we shall denote by $u_n(x)$ or $v_n(x)$ according to the state of the particle, whether neutron or proton, and we shall use the old notations ψ_s and ϕ_σ for the states of electrons and neutrinos We must remember that the general form of the H_{int}-function (6) is

† In the same way the Hamiltonian in the radiation problem is written in the form $H_{(\text{atomic electron})} + H_{(\text{electromagnetic field})} + H_{(\text{interaction})}$ and the last member is considered as a perturbation because the period of light emission is large compared with oscillation periods of atomic electrons

arranged in such a way that $(n \rightarrow p)$-transformation automatically increases by one the numbers of electrons and neutrinos and conversely; in other words it gives rise to matrix elements different from zero only for such transformations. Using the previous results we find for these matrix elements (different from zero) the expression†

$$H^{+1,n,h_1,h_2,\ \ ,0_s,\ \ ,k_1,k_2,\ \ ,0_\sigma,}_{-1,m,h_1,h_2,\ \ ,1_s,\ \ ,k_1,k_2,\ \ ,1_\sigma,} = \pm \int v_m^* c_{s\sigma}^* u_n \, d\omega, \tag{16}$$

where the integral is taken over the whole coordinate space of the heavy particle (including ρ). Introducing the values of $c_{s,\sigma}^*$ from (11), we obtain

$$H^{+1,n,0_s,0_\sigma}_{-1,m,1_s,1_\sigma} = \pm g \int v_m^* u_n \bar{\psi}_s \, \delta\phi_\sigma \, d\omega \tag{17}$$

as the expression for the matrix element. Remembering that in all cases of β-decay the de Broglie wave-lengths of the emitted electron and neutrino are large compared with the nuclear radius (thus ψ and ϕ are almost constant inside the nucleus), we can write (17) in the form

$$H^{+1,n,0_s,0_\sigma}_{-1,m,1_s,1_\sigma} = \pm g[\bar{\psi}_s \delta\phi_\sigma]_{r=0} \int v_m^* u_n \, d\omega \tag{18}$$

Let us calculate now the transition probability for the definite case of an nth state neutron being transformed into an mth state proton with the creation of an electron and a neutrino in the states s and σ, which before the transformation were empty. At the initial moment of time $t = 0$ we have, for the probability amplitudes of original and final states, the values

$$a_{+1,n,0_s,0_\sigma} = 1 \quad \text{and} \quad a_{-1,m,1_s,1_\sigma} = 0. \tag{19}$$

According to the ordinary perturbation theory the probability of the second state will increase, due to the interaction H_{int}, at the rate

$$\frac{d}{dt}(a_{-1,m,1_s,1_\sigma}) = -\frac{i}{\hbar} H^{+1,n,0_s,0_\sigma}_{-1,m,1_s,1_\sigma} \exp\left\{\frac{i}{\hbar}(-W+H_s+K_\sigma)t\right\}, \tag{20}$$

where W is the energy-difference between the states of neutron and proton. Integrating and remembering that for $t = 0$, $a_{-1,m,1_s,1_\sigma} = 0$, we obtain

$$a_{-1,m,1_s,1_\sigma} = -H^{+1,n,0_s,0_\sigma}_{-1,m,1_s,1_\sigma} \frac{\exp\left\{\frac{i}{\hbar}(-W+H_s+K_\sigma)t\right\} - 1}{-W+H_s+K_\sigma} \tag{21}$$

† Representing evidently the transition of a neutron originally in the nth state to a proton in the mth state with the creation of an electron in the sth state ($h_s = 0 \rightarrow h_s = 1$) and a neutrino in the σth state ($k_\sigma = 0 \rightarrow k_\sigma = 1$)

and consequently, for the probability of transition (using (18)),

$$|a_{-1,m,1_s,1_\sigma}|^2 = 4g^2\left|\int v_m^* u_n\,d\omega\right|^2 \bar{\psi}_s\,\delta\phi_\sigma^*\,\bar{\phi}_\sigma^*\,\delta\psi_s \frac{\sin^2\frac{t}{2\hbar}(-W+H_s+K_\sigma)}{(-W+H_s+K_\sigma)^2}. \tag{22}$$

In order to calculate the total transition probability for creation of an electron in the sth state we must sum up the expression (22) over all possible states of the neutrino In this way Fermi obtained after some calculation

$$|a_{-1,m,1_s,1_\sigma}|^2 = t\frac{g^2}{2\pi\hbar^4}\left|\int v_m^* u_n\,d\omega\right|^2 \frac{p_\sigma^2}{v_\sigma}\left(\bar{\psi}_s\psi_s - \frac{\mu c^2}{K_\sigma}\bar{\psi}_s\beta\psi_s\right), \tag{23}$$

where p_σ and v_σ stand for the momentum and the velocity of the neutrino in the state σ, satisfying the condition

$$-W+H_s+K_\sigma = 0 \tag{24}$$

Here, also, β is the following matrix

$$\beta = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} \tag{25}$$

and μ the mass of the neutrino

The factor multiplying t in (23) evidently represents P_s, the transition probability per unit time We can further simplify the expression if we suppose, anticipating later discussions, that the mass of the neutrino is zero In this case $v_\sigma = c$, $K_\sigma = cp_\sigma$, and

$$p_\sigma = \frac{K_\sigma}{c} = \frac{W-H_s}{c},$$

then our expression becomes

$$P_s = \frac{g^2}{2\pi c^3\hbar^4}\left|\int v_m^* u_n\,d\omega\right|^2 \bar{\psi}_s\psi_s(W-H_s)^2 \tag{26}$$

This formula gives the transition probability for the case when the created electron is in the sth state It is, however, more interesting to obtain a formula representing the number of electrons of some definite velocity or momentum, in order to be able to make comparison with the experimental energy-distribution of the continuous β-spectrum To do this we must integrate (26) over those states to which correspond momenta between the limits ηmc and $(\eta+d\eta)mc$.

Thus we obtain.

$$P(\eta)\,d\eta = \frac{g^2m^2c}{2\pi\hbar^4}\left|\int v_m^* u_n\,d\omega\right|^2 \sum_{\eta}^{\eta+d\eta} \bar{\psi}_s\psi_s\{\sqrt{(1+\eta_0^2)}-\sqrt{(1+\eta^2)}\}^2. \tag{27}$$

Using the known expressions for the relativistic eigenfunctions for electrons of positive energy in the central Coulomb field, Fermi finally derived the following result

$$\sum_{\eta}^{\eta+d\eta} \bar{\psi}_s\psi_s = d\eta\frac{4m^3c^3}{\pi^2\hbar^3\{\Gamma(3+2S)\}^2}\left(\frac{2mcr_0}{\hbar}\right)^{2S}\eta^{2+2S}\times$$
$$\times\exp\left(\pi\gamma\frac{\sqrt{(1+\eta^2)}}{\eta}\right)\left|\Gamma\left(1+S+i\gamma\frac{\sqrt{(1+\eta^2)}}{\eta}\right)\right|^2, \tag{28}$$

where $$\gamma = \frac{Z}{137}, \quad S = \sqrt{(1-\gamma^2)}-1,$$

Z is the atomic number of the disintegrating nucleus, and r_0 the nuclear radius

Assuming as characteristic of a β-disintegrating element belonging to one of the radioactive families $Z = 86$ ($\gamma = 0·6$, $S = -0·2$) and $r_0 = 0·9\times 10^{-12}$ cm , we obtain numerically

$$P(\eta)\,d\eta$$
$$= 1·75\times 10^{+95}g^2\left|\int v_m^* u_n\,d\omega\right|^2(\eta+0·355\eta^2)\{\sqrt{(1+\eta_0^2)}-\sqrt{(1+\eta^2)}\}^2\,d\eta, \tag{29}$$

and for the total probability (integrating for all η from 0 to η_0)·

$$\lambda = P(\eta_0) = \int_0^{\eta_0} P(\eta)\,d\eta = 1·75\times 10^{+95}g^2\left|\int v_m^* u_n\,d\omega\right|^2 F(\eta_0), \tag{30}$$

where

$$F(\eta_0) = \tfrac{2}{3}\sqrt{(1+\eta_0^2)}-\frac{2}{3}+\frac{\eta_0^4}{12}-\frac{\eta_0^2}{3}+$$
$$+0·355\left[-\frac{\eta_0}{4}-\frac{\eta_0^3}{12}+\frac{\eta_0^5}{30}+\frac{\sqrt{(1+\eta_0^2)}}{4}\log\{\eta_0+\sqrt{(1+\eta_0^2)}\}\right]. \tag{31}$$

For small values of η_0 the function $F(\eta_0)$ is given by $\eta_0^6/24$, for larger η_0 $F(\eta_0)$ may be taken from Table X.

TABLE X

η_0	<1	1·0	2·0	3·0	7·0	5·0	6·0	7·0
$F(\eta_0)$	$\eta_0^6/24$	0·03	1·2	7·5	29	80	185	380

3. Comparison with experiment, and the consequent improvement of the theory

Three points should particularly be considered in comparing with the results of experiment the predictions of expression (30) regarding the disintegration probability in β-disintegration These points are as follows: (1) the numerical value of the interaction constant g (the physical significance and interpretation of which will be discussed in § 4), (2) the behaviour of the integral $\int v_m^* u_n \, d\omega$, and (3) the dependence of $F(\eta_0)$ on η_0—which should reproduce the Sargent relation, connecting disintegration constants and limiting energies, which has already been noticed

We shall pay attention first to the integral $\int v_m^* u_n \, d\omega$, which evidently depends only on the motion of the original neutron and the produced proton (or the other way round in the case of β^+-decay) and in particular on the azimuthal quantum numbers of the neutron state u_n and the proton state v_m One can easily see, from symmetry considerations, that this integral vanishes for all pairs of v and u states except those which have equal angular momenta (azimuthal quantum numbers), and that for these it has a value about unity. Remembering that the total angular momentum, or the spin, of the nucleus is the resultant of the angular momenta of its constituent parts, we can say that *the transition probability is different from zero only in the case in which the original and product nuclei possess the same spin*:

$$i = i' \tag{32}$$

This constitutes a selection rule for β-emission According to Gamow and Teller† this selection rule has to be modified if we take into account the possibility of the inversion of the spin of the heavy particle during β-transformation ($n\uparrow \rightarrow p\downarrow + \beta$) *In this case the permitted transitions are those for which $\Delta i = 0$ or 1* In the next chapter (pp. 155–7) we shall see that this modified selection principle fits better with experimental data than does the original principle of Fermi.

Neither selection rule, however, holds absolutely strictly, the formula (30) having been obtained using certain approximations. First of all, in (17, 18) we took the function $\bar{\psi}_s \, \delta\phi_\sigma$ outside the integral because the de Broglie wave-length, Λ, of electrons and neutrinos is much larger than the nuclear radius, r_0. However, in so far as the ratio Λ/r_0 is not strictly infinite, the result which is thus obtained

† G. Gamow and E. Teller, *Phys Rev* **49** (1936), 895.

must be considered only as an approximate one, since, as in the case of quadrupole radiation (see p 72), the corresponding probability will not vanish entirely, but merely be reduced by a factor $(r_0/\Lambda)^{2\Delta i}$, which, in the case of ordinary β-decay, is about $(\frac{1}{100})^{\Delta i}$ Here Δi is the spin change accompanying disintegration Furthermore, although the velocities of nuclear neutrons and protons are small compared with the velocity of light, the ratio v/c is not strictly zero and the corresponding relativistic correction† will also act in the direction of permitting the transition for $i \neq i'$, only reducing the probability by about the same amount as the first effect Thus we come to the conclusion that for $i \neq i'$ β-disintegration is still permitted but its probability is reduced by a factor $(\frac{1}{100})^{|i-i'|}$, or by a factor $(\frac{1}{100})^{\{|i-i'|-1\}}$, if we take into account the possibility of inversion of the spin of the heavy particle This makes our selection rule very similar to the corresponding rule for electromagnetic radiation. A look at the Sargent diagram (Fig 37 (a)) shows that the experimental points are distributed on two parallel curves two units apart on the logarithmic scale (factor $\frac{1}{100}$) It seems natural, therefore, to consider the transitions represented on the upper curve as permitted and those of the lower curve as non-permitted We do not have, in Fig 37 (a), any points on curves corresponding to higher spin-differences, but, as we shall see in the next chapter, it is possible that this is due simply to the insufficient number of β-elements investigated

Now we can calculate the value of the numerical constant, g, from the experimental decay constant for some element belonging to the upper curve, using the formula (30, 31), and putting $\int v_m^* u_n \, d\omega = 1$. In this way we obtain

$$g \sim 4\times 10^{-50} \text{ erg cm}^3 \tag{33}$$

This, then, is the empirical value for the coefficient in the interaction expression (6′) In the complete theory g should be expressed in terms of the fundamental constants (h, c, κ), we shall discuss this question somewhat later

Accepting this value for g we can now plot theoretical curves both for the permitted and also for various non-permitted β-transformations, as is done in Fig 37 (a). We see that the agreement with the experimental points is quite a fair one We see also, for example,

† The omitted term is of the type $\frac{1}{c}(u_x A_x + u_y A_y + u_z A_z)$

that the point for K is located about the 7th curve, which means that the long lifetime of this element can be explained by an exceptionally large spin difference between the normal states of the nuclei $_{19}K^{40}$ and $_{20}Ca^{40}$ Remembering that $_{20}Ca^{40}$ has most probably no spin (even mass, even charge), we are led to the not impossible hypothesis that the nuclear spin of the $_{19}K^{40}$ nucleus must be rather high Similar considerations hold also for the $_{37}Rb^{86}$ nucleus

As we have mentioned above, the points corresponding to artificially produced light β-active elements fit more or less into Sargent's curves for radioactive families (compare Fig. 37 (*b*)). This fact, however, must be considered rather as in disagreement with the theory, as one would expect according to the theory that the points for light elements would be considerably shifted downwards, due to the smaller value of electron density ψ_e inside the nucleus for smaller atomic number. The approximate coincidence between the curves for heavy and light elements, however, can be understood according to the proposal of Bethe if one makes the hypothesis that for light nuclei the values of matrix elements $\int u_n v_m^* \, d\omega$ are in general somewhat higher than in the case of heavy elements where, due to a large number of particles, especially symmetrical configurations of wave functions may exist Now we can approach the question from the other side, comparing the energy-distribution given by (27, 28) with the experimental evidence We rewrite the formula (27), expressing it in terms of energy and introducing new rational units. mc^2 for energy, $\dfrac{h}{mc^2}$ for time, and $\dfrac{\hbar}{mc}$ for length (mc for momentum). The new form is then†

$$P(E)\,dE = G^2\left|\int v_m^* u_n\,d\omega\right|^2 F(E,Z)(E_0-E)^2(E^2-1)^{\frac{1}{2}}E\,dE, \quad (34)$$

$$F(E,Z) = \left\{\frac{4}{[\Gamma(3+2S)]^2}\right\}(2\eta r_0)^{2S}\exp\left(\pi\gamma\frac{E}{\eta}\right)\left|\Gamma\left(1+S+i\gamma E\frac{1}{\eta}\right)\right|^2, \quad (34')$$

where the dimensionless constant G is related to the original (Fermi's) g by the formula

$$G = g\times\frac{m^2c}{\sqrt{(2\pi)}\hbar^3} \sim 3\times 10^{-14} \quad (35)$$

In (34) the factor $F(E,Z)$, which represents the effect of the nuclear Coulomb field on the probability of disintegration (through the density of electron wave functions near the nucleus), reduces to

† $E = \sqrt{(1+\eta^2)}$, $\eta = \sqrt{(E^2-1)}$

unity for $Z = 0$. The factor $(E_0-E)^2(E^2-1)^{\frac{1}{2}}E$, as was made clear by Uhlenbeck and Goudsmit, is common to any expression giving the statistical distribution of energy between two particles and is proportional to the volume in the phase space $(p_e^2 p_\nu^2 \, dp_e \, dp_\nu)$ or the number of states in which each particle may be found We may notice here that the shape of the β-spectrum curve depends, for

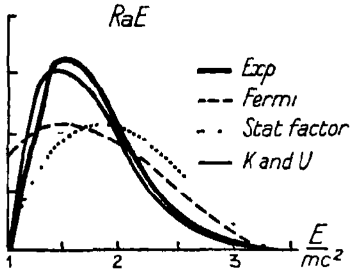


FIG. 39 (a)

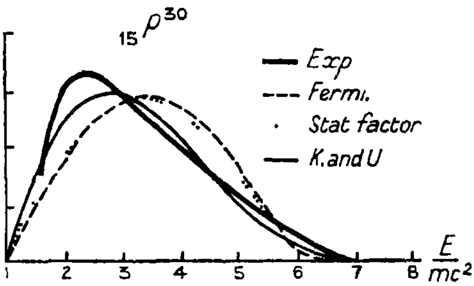


FIG 39 (b)

light nuclei (where $F(E, Z) \to 1$), mostly on this statistical distribution factor. In order, therefore, to explain the strong asymmetry of the observed β-spectra of light nuclei (maximum for about $\frac{1}{3}$ of the limiting energy) we must accept widely different values for the masses of neutrino and electron (otherwise the statistical factor will be symmetrical in the energies of these particles). The estimate of the mass of the neutrino can be given by inspection of the upper limit of the β-spectrum. It can be shown on the basis of the above theory that for $m_\nu \sim m_e$ the β-curve will show a vertical, and for $m_\nu \ll m_e$ a horizontal descent to zero near the upper limit. The observed shapes of β-spectra (compare Fig. 39) thus lead us to the conclusion that the mass of the neutrino is very small or zero.

In Fig 39 (a, b) are plotted, by heavy lines, the experimental

β-curves for RaE (Sargent†) and for the artificial light β^+-emitting element ${}_{15}P^{30}$ (Ellis and Henderson‡). The curves calculated according to Fermi's formula are indicated on each plot by a broken line whereas distributions determined only by the statistical factor (omitting $F(E, Z)$) are shown by dotted lines ‖ We see at once that the theoretical formula is in considerable disagreement with experiment, not giving enough asymmetry to the distribution curves. This indicates that some modification must be introduced into the basic assumptions used by Fermi for the calculation of transformation probabilities. Remembering the development of the calculation in the last section we immediately see that such modifications can be effected, without altering the fundamental ideas of the theory, by accepting some more complicated form of interaction (6′, 8) between the heavy and light particles This can be done, for example, by introducing into the interaction expression the derivatives of the wave functions of the light particles as was first proposed by Bethe and Peierls †† There are, of course, many possibilities of doing this, and in our choice we should be guided only by the necessity of better agreement with experiment This was the method of Konopinski and Uhlenbeck,‡‡ who indicated that greater asymmetry in the energy-distribution curve can be obtained if one introduces the derivatives of the neutrino wave functions instead of the functions themselves This introduction of $d\phi_i/dx$ instead of ϕ_i can also be supported by the consideration that it is probably actually only the momenta and not the coordinates of neutrinos which are observable quantities

Konopinski and Uhlenbeck have indicated that the only four-dimensional polar vector (analogous to (6′)) which can be constructed from ψ's and $\dfrac{\partial\phi}{\partial x}$'s can be written in the form

$$B_i = \psi^*\beta\frac{\partial\phi}{\partial x_i} \quad (i = 0, 1, 2, 3), \tag{36}$$

where β is defined by (25) Correspondingly the Hamiltonian for

† B W. Sargent, loc cit. p 122

‡ C. D. Ellis and W J Henderson, *Proc Roy Soc* A, **146** (1934), 206

‖ As mentioned above we see that the action of the Coulomb field ($F(Z, E)$) is of practically no importance for light elements although it makes a considerable contribution for heavy ones

†† H Bethe and R Peierls, *Reports of Int Conf. on Physics*, London, p. 66 (1934).

‡‡ K I Konopinski and G E Uhlenbeck, *Phys Rev* **48** (1935), 7

the interaction between light and heavy particles can be written in the form

$$H_{\text{int}} = \bar{g}\{[B_0+(\boldsymbol{\alpha}_h, \mathbf{B})]Q+Q^*[B_0^*+(\boldsymbol{\alpha}_h, \mathbf{B})^*]\}, \tag{37}$$

where $\bar{g}$ is the modified Fermi's constant having dimensions erg cm.^{4}

Proceeding in the same way as in the previous section we obtain for the transition probability in the above given rational units

$$P(E)\,dE = G^2\left|\int v_m^* u_n\,d\omega\right|^2 F(E,Z)(E_0-E)^4(E^2-1)^{\frac{1}{2}}E\,dE, \tag{38}$$

where G is given by

$$G = \bar{g}\frac{m^3c^2}{\sqrt{(2\pi)}\hbar^4} \tag{38$'$}$$

and has about the same numerical value as in (35), to fit experimental data.

This expression differs from Fermi's original expression (34) by the factor $(E_0-E)^2$, i.e. by the square of the energy of the neutrino, giving more asymmetry to the distribution curves. The thin lines in Fig. 39 (a, b) represent the results calculated from formula (38), we see that they form a much better representation of the experimental curves. We must remember, however, that (37) does not represent the only possible way of writing the interaction function. One could, for example, introduce also the derivatives of the electron wave functions to obtain the vectors

$$K_i = \frac{\partial\psi^+}{\partial x_\alpha}\gamma_i\frac{\partial\phi}{\partial x_\alpha} \quad \text{or} \quad L_i = \frac{\partial\psi^+}{\partial x_\alpha}\gamma_\alpha\frac{\partial\phi}{\partial x_i}, \tag{39}$$

or go to higher derivatives of ψ and ϕ if it should be found necessary in order to fit the details of β-spectrum curves. Integrating (38) we obtain an expression for the dependence of λ on the upper limit of the continuous β-spectrum giving more rapid variation of λ than the original relation of Fermi. If we introduce the derivatives of the wave function for the electron, or go to derivatives of higher order, we have still more rapid change of λ. Judging from the $[\log\lambda, \log E_{\max}]$ graph (Fig. 37 a, b) it would seem that Fermi's original hypothesis fits the empirical relation better than does any other, however, we must remember that the results can be considerably modified by the effect of the factor $|\int v_m^* u_n\,d\omega|^2$ (which has been assumed to be constant in our previous considerations) and that the steepness of the theoretical curves may be changed by the effect of an additional potential barrier 'of centrifugal force' in the cases in which the β-particle is emitted with angular momentum.

4. Physical meaning of the constant g and its connexion with the properties of elementary heavy particles

In the general theory of β-disintegration, discussed in the previous sections, the constant g was formally introduced in the interaction law governing the creation or absorption of an electron in a quantum transition between two states of the heavy particle—and its numerical value was empirically determined by a comparison of the results of calculations with the experimentally observed dependence of λ on $E_{\max}$ It is clear, however, that in a complete theory of such transformations the interaction constant g has to be expressed in terms of universal constants such as h, c, k,† and the elementary charge and masses.‡ One can try, of course, without having any theory of interaction, to construct g by means of purely dimensional analysis only, using the universal constants in question, but such juggling with numbers has never led to satisfactory results We can say, however, that the elementary charge e most probably must not be used for the construction of the constant g, and that consequently the interaction is not of a purely electromagnetic type. In fact, according to definition (7) of section 2, g depends linearly on the wave function of the electron, in contrast with the quadratic dependence of the corresponding constant in most interaction formulae of atomic theory || It is thus very plausible that, if the electronic charge were present in the expression for g, it would occur as the first power, which is very unlikely since it would give different signs to the interaction energy for positive and negative emission.

An idea was at one time considered by Bohr, that the emission of a neutrino, which has no charge and no mass, but just carries away energy, might be considered as some type of radiation of gravitational waves by a heavy particle in the process of transformation In this case the long periods of β-decay would be connected with the

† Constant of gravitation

‡ In the future complete system of theoretical physics e and m's also must be expressed in terms of h, c, and k

|| This depends on the fact that in Fermi's theory the electron exists in the final state and does not exist in the original state of the system (Creation), whereas most other formulae refer to cases in which the electron was present before and after the transition

small probability of such emission. The roughly estimated probability for the emission of gravitational waves is

$$\kappa \sim \frac{KM}{\hbar^4 c^5} r_0^2 (\hbar\nu)^4, \tag{39}$$

where K is Newton's constant of gravitation ($= 6{\cdot}6 \times 10^{-8}$ cm.^{3} sec.$^{-2}$ gm^{-1}), M the mass of the emitting particle, and $\hbar\nu$ the energy carried away by the wave (the energy of the neutrino). Accepting $M \sim 10^{-24}$ gm (heavy particle), $\hbar\nu \sim 1 \times 10^6$ e.v (ordinary energy of β-decay), and $r_0 \sim 10^{-12}$ cm. (nuclear radius) we obtain from (39) $\kappa \sim 10^{-23}$ sec.$^{-1}$, which would correspond to a period of life 10^{23} sec $\sim 10^{16}$ years! This result, not excluding the possible relation of β-transformations with gravitational phenomena, shows, however, that it cannot be obtained without a more complete understanding of the nature of the mass of elementary particles

We can now try a more formal method of reasoning in order to connect the probability of electron emission by a neutron or proton with other properties of these particles. Let us consider a proton, for example, subject to the action of an external periodic electromagnetic field (of light quanta) of continuously increasing frequency When the energy of the field reaches a value of the order $\hbar\nu > Mc^2$, i e the energy of the light quantum becomes comparable with the rest-energy of the heavy particle, the motion of our proton should be described by some kind of relativistic wave equation We have seen, however, in an earlier chapter that the only consistent wave equation which can be written for the relativistic case is Dirac's equation, and that it fails to describe the behaviour of heavy particles. In the case of electrons we got over the difficulty by saying that under such large interactions ($h\nu \gg mc^2$ for electrons) the formation of pairs (of positive and negative electrons) in collisions of light quanta with the original electron gives rise to so many new electrons that we cannot in fact any longer trace the motion of the original particle, consequently we do not need any equation for the description of this motion (all that we can describe in this case is some kind of statistical equilibrium between light quanta and appearing and disappearing electrons). In a similar way† we can say in the case of protons subject to such extremely strong fields (which do not exist, even in nuclei) that the transformation proton to neutron, and vice

† G. Gamow 'Negative Protons', *Nature*, **135** (1935), 858.

versa, with the continuous creation of positive and negative electrons, will occur with such frequency that there will be no sense in asking about the motion of a proton or a neutron for any long period of time. If we accept this hypothesis, we can formally write the condition for the life T of a p or n state of a heavy particle under such strong interaction (with energy $\sim Mc^2$) in the form

$$T_{E\sim Mc^2} \sim \frac{\hbar/Mc}{c}, \tag{40}$$

where on the right-hand side stands the time necessary for a proton moving with velocity c to cover a distance comparable with its diameter. On the other hand, for the much weaker interaction (of the order of several mc^2) inside an atomic nucleus the considerations of the last section give roughly†

$$T_{E\sim mc^2} = \frac{1}{\lambda} \sim g^{-2}\left(\frac{E}{mc^2}\right)^{-n} \tag{41}$$

(in $\hbar/mc^2$ units of time), where n is about 5 in Fermi's theory (interaction (8)), 7 in Uhlenbeck's theory (interaction (37)), and still larger if higher derivatives are introduced into the interaction expression Deliberately extrapolating formula (41) to the high-energy domain $E \sim Mc^2$, and comparing with (40) we obtain

$$\frac{\hbar}{Mc^2} \sim \frac{\hbar}{mc^2}\cdot g^{-2}\left(\frac{Mc^2}{mc^2}\right)^{-n} \tag{42}$$

or

$$g = \left(\frac{M}{m}\right)^{-\frac{n-1}{2}} = \left(\frac{1}{1840}\right)^{\frac{n-1}{2}}, \tag{42'}$$

and we see that g can be very small if n is large enough. As we have seen (pp 140, 144), formulae (34), (38), and others obtained by introducing higher derivatives, fit best with experimental data if we accept $g \sim 3\times 10^{-14}$ ‡ This can be obtained from (42′) if we take n about 9, corresponding to the introduction of the first derivatives both for electron and neutrino (39), or, better (in order to retain the asymmetry of β-spectra), the second derivative $\partial^2\phi/\partial x^2$ for the neutrino in the expression for the interaction. As we shall see later, other considerations also indicate the necessity of introducing higher derivatives into this expression.

† For $Z = 0$ or 1 $F(Z,E) \sim 1$

‡ These formulae give almost equal values of g because the comparison is made in the region where E expressed in mc^2 units is of the order unity (half-million volt β-particles)

We shall now attack the problem from the other side and, accepting the value of g as given (and, in future, due to be expressed in terms of h, c, k), we shall try to derive other information concerning the properties of heavy particles. In this way Wick† succeeded in reaching interesting conclusions concerning the magnetic moments of proton and neutron. Considering a heavy particle in free space (or in a field with not enough energy to induce β-decay), we may expect, according to the wave-mechanical point of view, that the process of ($\pm$) electron creation will still occur but that the created light particle, having not enough energy to get away, is swallowed back again immediately after creation, existing only for a very short time and having during its existence 'negative kinetic energy' ‡ During its short existence *in statu nascendi*, however, the electron must possess a magnetic moment of one Bohr magneton, $\frac{e}{m}\hbar$, and this must make its contribution to the observed magnetic moment of the heavy particle Adopting Fermi's interaction expression, Wick was able to treat in some detail this process of creation of 'half-born' electrons and evaluate their average contribution to the magnetic moment which has to be added to the proper moment of the heavy particle ($(e/M)\hbar$ for proton and 0 for neutron, according to Dirac's theory) With Fermi's original interaction one obtains values several thousand times smaller than the nuclear magneton, and only by introducing higher derivatives of the wave function into the interaction law can one increase these values to the necessary order of magnitude The existence of this additional moment throws more light on the general considerations of Chapter III concerning the anomalously high values of the magnetic moment of the proton as obtained by Stern and others In the case of the neutron the contribution of magnetic moment due to the virtual creation of electrons will be almost equal to the corresponding contribution in the case of a proton,‖ but if both kinds of transformation, $n \rightarrow p+e$ and $n \rightarrow \bar{p}+\partial$, are possible (see p 15) the average contribution in the case of the neutron will be zero Excluding the second, doubtful, possi-

† C Wick, *Rend R Nat Accad Lincei*, **21** (1935), 170

‡ This process of creation of electron, in spite of unfavourable energy-balance ($|M_n-M_p| < m_e+m_\nu$), is analogous to the partial penetration of a particle through a barrier which is too high to let it go through, as for example happens for electrons in the atom

‖ A slight difference may be due to the difference between the masses of neutron and proton bility, we should expect the spin of the neutron to be almost equal and opposite in sign to the spin of the proton This is in agreement with the fact that the observed magnetic moment of the deuteron is very small.

Another application of Fermi's interaction to the investigation of the properties of heavy particles would be to calculate the exchange forces between neutron and proton, in order to compare them with our knowledge obtained from the general nuclear models (Chap. II) This question was first considered by Ivanenko† and Tamm ‡ If we remember that the mutual potential energy of a neutron and a proton due to such exchange forces must be proportional to the square of the constant g in the expression for the interaction (7), and must include also the mutual distance r and the constants h and c (the only constants entering into the calculations), we can easily obtain, on the basis of dimensional analysis, for this potential energy the following form. $g^2h^{-1}c^{-1}r^{-5}$ || The calculations of Tamm actually give

$$I(r) = \frac{g^2}{16\pi^3\hbar c}\frac{1}{r^5} \sim 10^{-85}r^{-5} \text{ erg} \tag{43}$$

For distances $r \sim 10^{-13}$ cm this expression predicts for the exchange energy-values $\sim 10^{-20}$ erg $\sim 10^{-8}$ e v , and only for $r \sim 10^{-16}$ cm. energy-values around one million volts We see, thus, that the exchange forces calculated on the basis of Fermi's original expression (7) are too small to explain the attraction necessary to keep the nucleus together (Chap II)

It should be noticed, however, that the introduction of the derivatives of the wave functions of the light particles in the expression for the interaction will considerably increase the attractive forces due to exchange this procedure may give the correct order of magnitude necessary for the understanding of nuclear stability.

† D D Ivanenko, *Nature*, **133** (1934), 981

‡ I Tamm, ibid

|| We have $|g^2h^xc^yr^z| = \text{erg}$ or $\left(M\frac{L^2}{T^2}L^3\right)^2\left(M\frac{L^2}{T^2}T\right)^x\left(\frac{L}{T}\right)^y L^z = M\frac{L^2}{T^2}$, which gives us $x = -1$, $y = -1$, and $z = -5$

VIII

γ-RAY EMISSION FOLLOWING β-DISINTEGRATION

1. Excitation of nuclei by β-decay

JUST as in the case of α-decay, the product nucleus resulting from β-transformation may be formed in an excited state. In this case the total energy of transformation (upper limit of the spectrum) must be reduced by the amount of energy necessary for excitation,† this energy later being emitted in the form of γ-rays. We should thus conclude that β-decaying elements emitting strong γ-radiation must give rise to several groups of β-particles (in analogy with the 'fine structure' of α-rays) However, due to the continuity of β-ray spectra these β-groups with different upper limits will overlap and it is therefore, in general, impossible to distinguish them purely experimentally. Nevertheless, it should be possible to make such phenomena evident by a detailed study of the energy-distribution in the continuous spectrum, as in such cases it should be different from the 'simple spectrum' corresponding to the transition from normal state to normal state An attempt to treat existing data in this way was made by Ellis and Mott,‡ who were also the first to show that the total energy is conserved in the process of β-emission—as far as the upper limits of β-ray spectra are concerned These authors estimated the energy-balance for the two different disintegration sequences leading from ThC to ThD. The scheme for the branching in the thorium family is shown in Fig. 40, where the energies of the two α-disintegrations and the upper limits of the two β-ray spectra are also shown. With these data, for the

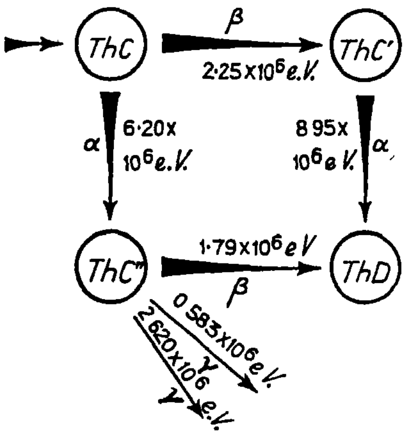


FIG 40

† This is not necessary, however, if we accept the non-conservational hypothesis so that the experimental proof of such an effect may be regarded as evidence against non-conservation of energy

‡ C D Ellis and N F Mott, *Proc Roy Soc* A, **141** (1933), 502.

energy-liberation for the sequence ThCC′D we have

$$2{\cdot}25+8{\cdot}95 = 11{\cdot}20\times 10^6 \text{ e.v.},$$

whereas for ThCC″D we have $6{\cdot}20+1{\cdot}79 = 7{\cdot}99\times 10^6$ e.v., giving, at first sight, the impression that energy is not conserved. But it must be remembered that the β-disintegration of ThC″ is accompanied by the emission of two very strong γ-rays, $0{\cdot}583\times 10^6$ e.v. and $2{\cdot}620\times 10^6$ e.v., both γ-lines having, according to measurements of Ellis, intensities almost one quantum per disintegration.† We must thus conclude that the observed upper limit of the ThC″ β-ray spectrum corresponds to excitation of the level $0{\cdot}583+2{\cdot}620 = 3{\cdot}20\times 10^6$ e.v. of the ThD nucleus (the fast β-group corresponding to the ground-to-ground transition being too weak to be observed) and add this amount to the energy-balance of the second branch. In this way we have now for ThCC″D $7{\cdot}99+3{\cdot}20 = 11{\cdot}19\times 10^6$ e v., in excellent agreement with the value obtained for the first branch These considerations not only show us that the conservation law holds for β-transformations but also prove an interesting fact that in certain cases the normal transitions are suppressed in favour of transitions with excitation, which fact, as it is easy to see, is connected with the above-discussed selection principle for β-transitions and constitutes a complete analogy with the appearance of strong 'fine-structure' components in the case of α-decay In so far as the different sub-groups of continuous β-ray spectra cannot be separated empirically, we must draw conclusions about their energies and relative intensities from the energy and percentage excitations of the different quantum levels of the resultant nuclei information about these levels being obtained from the analysis of γ-ray spectra and long-range particles. In Tables XI A, B, C, D the most important levels of ThC′, RaC′, ThC, and RaC nuclei are shown as adopted by Ellis.

TABLE XI A

ThCC′: $\lambda_{\text{total}} = 1{\cdot}23\times 10^{-4}$ *sec.*$^{-1}$; $E_{\text{total}} = 2{\cdot}25\times 10^6$ *e v.*

Energy of excited level $\times 10^{-6}$ *e v*	*Excitation* p_n	*Upper limit of group* $E_{\text{tot}}-E_n$	*Partial decay constant* $\lambda_n = p_n\lambda$ *sec* $^{-1}$	*Class of transformation*
0	0 75	2 25	$1\ 00\times 10^{-4}$	II
0 726	0 20	1 52	$2\ 46\times 10^{-5}$	II
1 800	0 05	0 45	$6\ 20\times 10^{-6}$	I

† There are also other γ-lines present for the elements in question, but they are comparatively weak and must give rise to feeble β-groups which are lost in the intense β-ray spectra of the main transitions.

TABLE XI B

RaCC′: $\lambda_{\text{total}} = 5{\cdot}92\times10^{-4}$ *sec.*$^{-1}$, $E_{\text{total}} = 3{\cdot}76\times10^{6}$ *e.v.*

Energy of excited level $\times 10^{-6}$ *e v*	*Excitation* p_n	*Upper limit of group* $E_{\text{tot}}-E_n$	*Partial decay constant* $\lambda_n = p_n\lambda$ *sec* $^{-1}$	*Class of transformation*
0	~ 0	3·76	$< 10^{-7}$	III ?
0·61	0·66	3·15	$3{\cdot}39\times10^{-4}$	II
1·67	0·065	2·09	$3{\cdot}31\times10^{-5}$	II
2·14	0·065	1·62	$3{\cdot}31\times10^{-5}$	II
2·70	0·14	0·88	$7{\cdot}24\times10^{-5}$	I
2·88	0·21	0·86	$1{\cdot}07\times10^{-4}$	I

TABLE XI C

ThBC. $\lambda_{\text{total}} = 1{\cdot}82\times10^{-5}$ *sec.*$^{-1}$, $E_{\text{total}} = 0{\cdot}60\times10^{6}$ *e v*

Energy of excited level $\times 10^{-6}$ *e v*	*Excitation* p_n	*Upper limit of group* $E_{\text{tot}}-E_n$	*Partial decay constant* $\lambda_n = p_n\lambda$ *sec* $^{-1}$	*Class of transformation*
0	~ 0	0·60	0	?
0·238	0·95	0·36	$1{\cdot}73\times10^{-5}$	I
0·299	0	0·30	0	?
0·414	0·05	0·19	$9{\cdot}1\times10^{-7}$	I

TABLE XI D

RaBC: $\lambda_{\text{total}} = 4{\cdot}31\times10^{-4}$ *sec* $^{-1}$; $E_{\text{total}} = 1{\cdot}00\times10^{6}$ *e v.*

Energy of excited level $\times 10^{-6}$ *e v*	*Excitation* p_n	*Upper β-limit* $E_{\text{tot}}-E_n$	*Partial decay constant* $\lambda_n = p_n\lambda$ *sec* $^{-1}$	*Class of disintegration*
0 (normal)	0	1·00	0	?
0·0529	0	0·95	0	?
0·2571	0·01	0·74	$4{\cdot}3\times10^{-6}$	II
0·2937	0·47	0·71	$2{\cdot}07\times10^{-4}$	I
0·3499	0·52	0·65	$2{\cdot}24\times10^{-4}$	I

The maximum energies of the separate β-ray groups and the corresponding partial decay constants are indicated in the third and fourth columns of these tables. In Fig. 41 the points for the separate β-components are plotted in a ($\log\lambda$, $\log E$)-diagram. We see that these points can, without much ambiguity, be ascribed either to the first or to the second Sargent curve, thus giving us some indication of the spin-differences between the respective states of the nuclei in question. We notice also that two points for RaC groups are located considerably lower than the second Sargent curve, presumably because they correspond to larger spin-differences.

Ellis and Mott† made a successful attempt to represent the β-ray spectrum of RaC as the sum of components representing the different excited levels of the product nucleus RaC′. The scheme showing the main excited levels of RaC′, as adopted by Ellis and Mott, and their percentage excitation (obtained from the absolute intensities of γ-rays) has been given in Table XI B. Just as in the case of ThC″D the probability of normal transition is here negligibly small, and the observed upper limit of the RaC β-ray spectrum ($3{\cdot}20\times10^6$ e.v.) must evidently correspond to the transition to the first excited level,

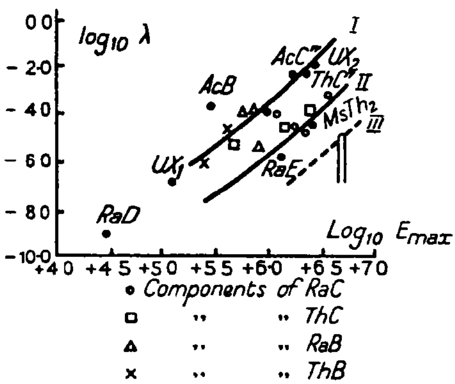


FIG 41

$0{\cdot}61\times10^6$ e v , thus the total liberation of energy in the RaCC′ transformation is $E_{\max} = 3{\cdot}81\times10^6$ e v. According to previous considerations we must expect the observed β-spectrum to be made up of components with upper limits $E_{\max}-E_n$ and relative intensities p_n. In order to construct the curve representing the result of the overlapping of these components we must, however, know the shape for each component In their original work Ellis and Mott made the assumption that all these partial β-components had the same shape (differing only in linear extension along the axis of energy), and took for the standard shape the energy-distribution for the RaE β-particles, which element, showing no γ-radiation, should possess an elementary spectrum. The results of such construction are shown in Fig. 42, together with the RaE-type curve used as a standard. The agreement between the constructed and the observed distributions is quite satisfactory. We must notice here, however, a fact which

† C. D. Ellis and N F. Mott, *Proc Roy Soc* A, **141** (1933), 502.

has been mentioned before: the energy-distribution is not the same for permitted and prohibited β-transformations. Consequently, in the construction just given, different shapes of energy-distribution should have been used for the different components. However, in the case of RaC, as may be seen from Fig. 41, many of the important components belong to prohibited transitions, therefore to use as standard the energy-distribution for RaE (itself belonging to the prohibited class) should give more or less correct results.†

As we have seen, the analysis of continuous β-ray spectra into components corresponding to the excitation of various levels, and

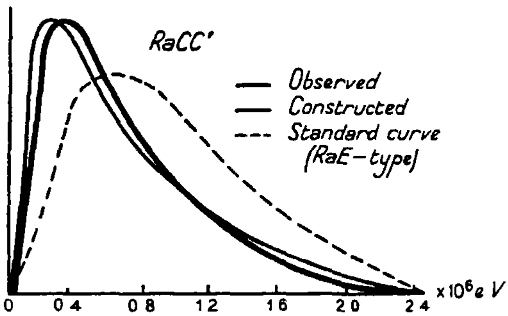


FIG 42

the relation between λ and $E_{\max}$ for these components, can give us some information concerning the spin-differences between the levels An attempt has been made‡ to use this information, together with the evidence supplied by the fine structure of α-rays (Chap VI) and the dipole or quadrupole character of γ-radiation (Chap IV), to construct a tentative scheme of nuclear spins for the normal and excited levels of a number of nuclei of the same radioactive family. The elements chosen for this purpose were ThB, ThC, ThC′, ThC″, and ThD—including those involved in the branching at the end of the thorium series The detailed scheme for this part of the disintegration series, including the energies and the excitations of different excited levels, is shown in Fig. 43 The arrows marked β indicate the various β-transitions leading to excited states of the product nuclei, the thickness of an arrow corresponding

† The analogous construction made by Ellis and Mott for the (ThCC′+ThC″D) β-ray spectrum failed because these elements give rise to a very strong permitted β-component, consequently the use of the RaE spectrum as standard is no longer correct.

‡ G Gamow, *Proc Roy Soc* **146** (1934), 217, G Gamow and E. Teller, *Phys. Rev.* **49** (1936), 895.

to the relative frequency of the appropriate transition. Of the two α-transformations (vertical arrows) the transition ThCC″ shows marked 'fine structure' (p 108), indicating that the spins of the normal states of the two nuclei in question differ by at least two, or even three, units. The second α-transformation, having no fine-structure components, we must similarly conclude that the spins of the normal states of ThC′ and ThD are the same

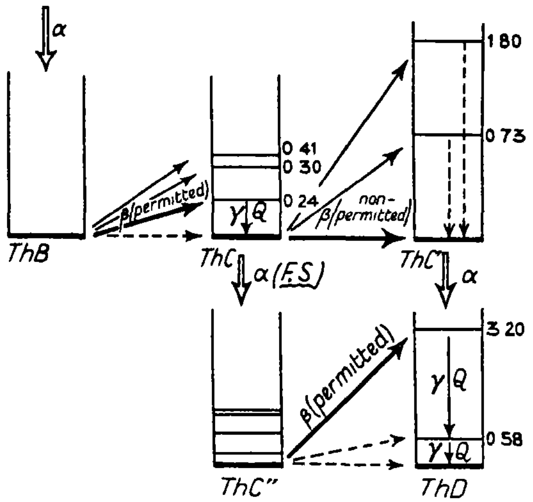


FIG 43

Suppose that we start from the ThB nucleus, for which we make the plausible assumption that its spin in the normal state is zero. This hypothesis is based on the absence of intense fine structure throughout the whole sequence of α-transformations

$$\mathrm{RaTh} \xrightarrow{\alpha} \mathrm{ThX} \xrightarrow{\alpha} \mathrm{Tn} \xrightarrow{\alpha} \mathrm{ThA} \xrightarrow{\alpha} \mathrm{ThB},$$

which shows that all these nuclei possess the same spin, and, in so far as all investigated nuclei of this type (even A and Z) possess the spin zero (compare Table C), it is more than likely that this constant spin value is zero Now, as we can see from Table XI c, the normal β-transition ThBC is not observed, but in 95 per cent of the transformations the product nucleus is formed directly with $0\ 238 \times 10^6$ e.v. excess energy. We must thus conclude, using Fermi's original selection principle, that

$$i(\mathrm{ThC}_{\mathrm{normal}}) \neq i(\mathrm{ThB}_{\mathrm{normal}}) = 0$$

and

$$i(\mathrm{ThC}_{\mathrm{excited}}) = i(\mathrm{ThB}_{\mathrm{normal}}) = 0,$$

as the corresponding β-group belongs to the class of permitted transitions. Concerning the γ-ray $0{\cdot}238 \times 10^6$ e.v., it was shown by Ellis (from the value of the coefficient of internal conversion) that this is emitted in a quadrupole transition—and we are forced to adopt, for the normal state of the radiating nucleus, the spin value $i(\mathrm{ThC_{normal}}) = 2$

If, according to p 139, we assume that permitted β-transitions correspond to $\Delta i = 0$ or 1, we may have $i(\mathrm{ThC_{excited}}) = 0$ or 1 and, consequently, $i(\mathrm{ThC_{normal}}) = 2$ or 3

The next β-transformation, ThCC′ gives a very small excitation of the ThC′ nucleus, as indicated by the presence of two long-range α-groups (p. 117) from levels 1·79 and $0{\cdot}72 \times 10^6$ e v, respectively, the main β-transition occurring between two normal states. Now we can see from Fig 37 that the thorium C β-spectrum belongs to the non-permitted class, we therefore have the relation

$$i(\mathrm{ThC'_{normal}}) \neq i(\mathrm{ThC_{normal}}).$$

The absence of fine structure for the ThC′ α-rays also gives us $i(\mathrm{ThC'_{normal}}) = i(\mathrm{ThD_{normal}})$, and the spin of ThD, according to general spectroscopic evidence, is most probably zero

We may compare this last result regarding $i(\mathrm{ThC'})$ with the above conclusions regarding $i(\mathrm{ThC})$ Moreover, since the latter conclusions depend upon the selection principle employed, we have the following alternatives.

Selection principle		
Permitted $\Delta i = 0$	$i(\mathrm{ThC_{excited}}) = 0$	$i(\mathrm{ThC'_{normal}}) = 0$
Prohibited $\Delta i = 1$	$i(\mathrm{ThC_{normal}}) = 2$	
Permitted $\Delta i = 0$ or 1	$i(\mathrm{ThC_{excited}}) = 0$ or 1	
Prohibited $\Delta i = 2$ or 3	$i(\mathrm{ThC_{normal}}) = 2$ or 3	

According to the first possibility, for the β-transition

$$\mathrm{ThC_{normal}} \rightarrow \mathrm{ThC'_{normal}},\quad \Delta i = 2$$

and, with the selection principle employed, the corresponding point should be located on the third Sargent curve With the second alternative Δi for this transformation might be 2 or 3, and the corresponding point be on the second ($\Delta i = 2$) or third ($\Delta i = 3$) curve. We see from Fig. 41 that the point representing the main β-groups of the ThC spectrum is in fact located on the second curve, which leads to the conclusion that *the modified selection principle should be employed* and the following spin values chosen:

$$i(\mathrm{ThB}_n) = 0;\quad i(\mathrm{ThC}_{ex}) = 0;\quad i(\mathrm{ThC}_n) = 2;$$
$$i(\mathrm{ThC}'_n) = 0;\quad i(\mathrm{ThD}_n) = 0.$$

We can further conclude that the normal spin of the ThC″ nucleus is 4 or 5, as required for the strong prohibition of the $\mathrm{ThC}''_n \rightarrow \mathrm{ThD}_n$ transition, evidenced by the 100 per cent. excitation of the high energy-level ($3{\cdot}13\times 10^6$ e v) of the ThD nucleus

We see from these considerations that present evidence is sufficient to make probable, though not absolutely certain, a number of conclusions about the spins of different radioactive nuclei; it also provides a check on the validity of the selection principles for β-transformations.

We may now ask whether, in the case of β-disintegration, it is possible to observe phenomena analogous to the emission of long-range α-particles, i.e. the β-disintegration of an excited nucleus. Remembering, however, that in the case of β-decay the decay constant changes much more slowly than is the case with α-decay (for β-disintegration the change of energy from 1 to 3×10^6 e v. increases λ only by a factor of 600, whereas in the case of α-decay the change in energy from 4 to 8×10^6 e v. increases λ 10^{21} times), we must conclude that, even for a high-energy excited state, λ_β is incomparably smaller than the corresponding probability of γ-emission and that, consequently, the 'long-range β-particle', although possible in principle, must be far outside the limits of experimental observability.

Cosmic ray showers

(*Note added in the proof*)

It was indicated on pp. 146–7 that if the energies involved in β-transformations are of the order Mc^2 ($\sim 10^9$ e v) one would expect many electron-neutrino pairs to be created in the period of time during which the heavy particle remains inside the limits representing the uncertainty of its position. This effect, which can be described as simultaneous creation of many pairs of particles, has recently been used by Heisenberg† to explain the showers of light particles produced by cosmic rays According to Heisenberg a cosmic ray proton, moving with the high energy E, will be affected by the Coulomb field of a nucleus and will discharge its energy in

† W Heisenberg, *Zs f Phys* **101** (1936), 533

the form of many negative and positive electrons and the corresponding number of neutrinos. The effective cross-section for the simultaneous creation of n electron-neutrino pairs is given by

$$\sigma_n \sim \left(\frac{Ze^2}{\hbar c}\right)^2 \frac{f^{8n}}{\Lambda^{8n-2}}, \tag{44}$$

where Λ is the de Broglie wave-length of the proton and f the new fundamental length of Fermi's theory defined by

$$f \sim \left[\frac{g}{\hbar c}\left(\frac{\hbar}{mc}\right)^s\right]^{1/(s+2)}, \tag{45}$$

s being the number of derivatives introduced in the expression for the interaction (compare p 143) We can easily see that for $\Lambda \sim f$ the simultaneous creation of many pairs becomes very probable In order to fit the experimental energy values for which shower production is observed it is necessary, however, to choose for s the value 2 or 3, which, as we have seen above, is also necessary for the explanation of the observed n-p exchange forces and of the magnetic moments of heavy particles

PART III

NUCLEAR TRANSFORMATIONS BY COLLISIONS

IX

COLLISIONS WITHOUT DISINTEGRATION

1. Inverse-square scattering

THAT the experimental investigation of collisions between charged particles and the resultant scattering provided the first data which led to the present nuclear model of the atom is well known.† On the basis of classical mechanics Rutherford obtained a formula for the scattering of α-particles in the central field of Coulomb forces, showing its dependence on the angle of scattering, the velocity of the incident particle, and the charge on the nucleus. Subsequent experiments demonstrated the general correctness of this formula, particularly as deviations from its predictions could be satisfactorily explained on the assumption that for very small distances of approach specifically nuclear forces between the particles also become appreciable.

The wave-mechanical theory of scattering was considered by Mott,‡ who was the first to show that Rutherford's classical scattering formula remains unchanged by the application of wave mechanics The general expression for the scattering in any central field of force can be developed in the following way || Using the coordinate system in which the centre of gravity of the two particles is at rest, we can write the wave-equation in the form

$$\nabla^2\Psi+\frac{2\overline{M}}{\hbar}[\tfrac{1}{2}\overline{M}v^2-U(r)]\Psi = 0, \tag{1}$$

where $\overline{M} = \dfrac{M_1 M_2}{M_1+M_2}$ is the reduced mass, v the relative velocity, and $U(r)$ the potential energy of interaction between the two colliding particles

For an incident plane wave moving parallel to the z-axis we must evidently write

$$I(z) = e^{ikz}, \qquad k = \frac{\overline{M}v}{\hbar}, \tag{2}$$

the wave being normalized to unit density. Introducing polar

† E. Rutherford, *Phil Mag* **21** (1911), 669

‡ N. F Mott, *Proc Roy Soc* **118** (1928), 542

|| We give here only a short account on the theory of scattering A detailed mathematical study of the question can be found in the book by N. F Mott and H. S W Massey, *Atomic Collisions*, Clarendon Press, Oxford (1933)

coordinates (θ being the angle between the radius vector and the z-axis), we can, as is well known, expand the plane wave (2) in the series

$$e^{ikz} = e^{ikr\cos\theta} = \frac{1}{r}\sum_{l=0}^{\infty}(2l+1)i^l P_l(\cos\theta)\chi_l^0(r), \tag{3}$$

where $P_l(\cos\theta)$ is a spherical harmonic of order l and χ_l^0 can be expressed in terms of Bessel functions,† with asymptotic form

$$\chi_l^0(r) \underset{r\to\infty}{=} \frac{1}{k}\sin(kr-\tfrac{1}{2}\pi l). \tag{4}$$

The general solution of (1) for an arbitrary potential function $U(r)$ can be written in the form

$$\Psi(r,\theta) = \frac{1}{r}\sum_{l=0}^{\infty} A_l P_l(\cos\theta)\chi_l(r) \tag{5}$$

Here A_l ($l = 0, 1, 2, ..$) are arbitrary coefficients and the various $\chi_l(r)$ satisfy the equations

$$\frac{d^2\chi_l}{dr^2}+\frac{2\overline{M}}{\hbar^2}\left[\tfrac{1}{2}\overline{M}v^2-U(r)-\frac{\hbar^2}{2\overline{M}}\frac{l(l+1)}{r^2}\right]\chi_l = 0 \tag{6}$$

with the boundary condition $\chi_l(0) = 0$ (in order to have $\Psi \sim \frac{1}{r}\chi_l$ finite at the origin) Integrating equation (6) from $r = 0$ to $r = \infty$ we evidently obtain as an asymptotic expression for the solution for large r's

$$\chi_l(r) \underset{r\to\infty}{=} \frac{1}{k}\sin(kr-\tfrac{1}{2}\pi l+\delta_l), \tag{7}$$

where the phases δ_l are determined in the process of integration by the special form of the potential function $U(r)$. The difference between this solution (7) and the expression (3) for the incident wave will clearly represent the scattered wave, and, as this must be divergent, the difference

$$\Psi - I = \frac{1}{r}\sum_{l=0}^{\infty} P_l(\cos\theta)[A_l\chi_l-(2l+1)i^l\chi_l^0] \tag{8}$$

must contain only terms of type e^{ikr} (not those of the type e^{-ikr}). Using the asymptotic expressions for χ_l and χ_l^0 ((4), (7)), we have for

† $\chi_0^0(r) = \frac{\sin kr}{k}$, $\chi_l^0(r) = \left(\frac{\pi r}{2k}\right)^{\frac{1}{2}} I_{l+\frac{1}{2}}(kr)$.

the bracket terms in (8)

$$\frac{e^{i(kr-\frac{1}{2}\pi l)}}{2ik}[A_l e^{i\delta_l}-(2l+1)i^l]-\frac{e^{-i(kr-\frac{1}{2}\pi l)}}{2ik}[A_l e^{-i\delta_l}-(2l+1)i^l] \tag{9}$$

and the condition of divergence gives

$$A_l = (2l+1)i^l e^{i\delta_l}. \tag{10}$$

The general solution now becomes

$$\Psi(r,\theta) = \frac{1}{r}\sum_{l=0}^{\infty}(2l+1)i^l e^{i\delta_l}P_l(\cos\theta)\chi_l(r), \tag{11}$$

and for the scattered wave only

$$S_{r\to\infty} = \frac{1}{r}f(\theta)e^{ikr} \tag{12}$$

with
$$f(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)[e^{2i\delta_l}-1]P_l(\cos\theta). \tag{12'}$$

The scattered intensity in the direction θ now becomes

$$|f(\theta)|^2 = \left[\frac{1}{2k}\sum_{l=0}^{\infty}(2l+1)(\cos 2\delta_l-1)P_l\right]^2+\left[\frac{1}{2k}\sum_{l=0}^{\infty}(2l+1)\sin 2\delta_l P_l\right]^2 \tag{13}$$

and the total cross-section effective for elastic scattering

$$\sigma = \int\limits_{\phi=0}^{2\pi}\int\limits_{\theta=0}^{\pi}|f(\theta)|^2\sin\theta\,d\phi d\theta = 2\pi\int\limits_0^{\pi}|f(\theta)|^2\sin\theta\,d\theta$$

$$= \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l \tag{14}$$

If the potential energy $U(r)$ is the ordinary Coulomb energy, $Z_1 Z_2 e^2/r$, the phases δ_l are given, according to Gordon,† by

$$\delta_l = \arg\Gamma(i\gamma+l+1), \qquad \gamma = Z_1 Z_2 e^2/\hbar v; \tag{15}$$

then the cross-section for scattering is obtained from

$$|f(\theta)|^2 = \left(\frac{Z_1 Z_2 e^2}{2\bar{M}v^2}\right)^2\operatorname{cosec}^4\tfrac{1}{2}\theta, \tag{16}$$

which is exactly Rutherford's classical result

If the mass of the incident particle, M_2, is small compared with the mass of the scattering particle, M_1 (scattering in heavy elements), the reduced mass becomes equal to M_2, the relative velocity is the original velocity of the incident particle, and the coordinate system

† W. Gordon, *Zs. f. Phys.* **48** (1928), 180

used in the above calculation is one in which the heavy scattering particle is initially at rest. In this case formula (16) can be directly employed for comparison with experimental data If the masses of the colliding particles are comparable, however, we must first make the transformation to a coordinate system at rest with respect to the observer. The result of this transformation may easily be obtained; it is

$$I(\theta) = \left(\frac{Z_1 Z_2 e^2}{M_2 v^2}\right)^2 \operatorname{cosec}^3\theta \frac{[\cot\theta \pm \sqrt{\{\operatorname{cosec}^2\theta-(M_2/M_1)^2\}}]^2}{\sqrt{\{\operatorname{cosec}^2\theta-(M_2/M_1)^2\}}}. \tag{17}$$

Clearly, when $M_2 \ll M_1$, equation (17) goes over into (16), as would be expected.

A notion of some importance at this stage is that of the distance of closest approach This distance has immediate significance in classical theory, in the wave theory it represents the beginning of the rapid exponential decrease of the wave function. For this distance we obtain simply

$$D = \frac{Z_1 Z_2 e^2}{v^2}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)(1+\sec\chi), \tag{18}$$

where χ is the angle of recoil

In their early experiments on the scattering of α-particles in different materials Rutherford and his collaborators demonstrated the validity of these formulae for the heavier elements In Fig 44 the fit between theory and experiment is shown for the case of α-particle scattering in gold On the other hand, for light elements deviations from inverse square-law scattering have been found, especially for very fast α-particles and large angles of scattering. This is due to the fact that in these cases the incident particles penetrate very close to the nucleus and are influenced by the nuclear exchange forces. A detailed discussion of this 'anomalous scattering' will be given in the next section.

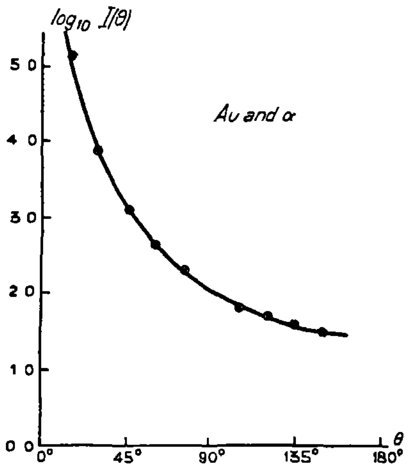


Fig 44

We must refer here to one case for which the wave-mechanical treatment gives results considerably different from those of the classical formula (8). As was first shown by Mott,† this happens for collisions between identical particles (for example, with α-particles in helium) and is due to a special type of interference between the waves representing the scattered and recoil particles respectively. The situation can be briefly stated as follows According to classical mechanics, in the collision between identical particles (equal masses), those particles observed in a given direction θ to the incident beam belong to two classes, incident particles 'really scattered' in this direction and recoil particles corresponding to other incident particles scattered in the direction $\pi-\theta$ (in the coordinate system bound to the centre of mass). From this point of view the intensity in the direction θ should be proportional to

$$\operatorname{cosec}^4\tfrac{1}{2}\theta+\operatorname{cosec}^4\tfrac{1}{2}(\pi-\theta) \tag{19}$$

On the other hand, from the point of view of wave mechanics, in which the streams of scattered and recoil particles are represented by waves of amplitudes proportional to $\operatorname{cosec}^2\frac{1}{2}\theta$ and $\operatorname{cosec}^2\frac{1}{2}(\pi-\theta)$, there is a possibility of interference if the two particles are identical. The total intensity in the direction θ is then given in terms of

$$\operatorname{cosec}^4\tfrac{1}{2}\theta+\operatorname{cosec}^4\tfrac{1}{2}(\pi-\theta)+2\operatorname{cosec}^2\tfrac{1}{2}\theta\operatorname{cosec}^2\tfrac{1}{2}(\pi-\theta)\cos\Delta\alpha, \tag{20}$$

where $\Delta\alpha$ is a phase-difference which can be estimated from the complete solution of the wave equation. According to Mott, the flux of particles in the solid angle $\sin\theta\,d\theta d\phi$ is

$$\left(\frac{Z^2e^2}{Mv^2}\right)^2\Big\{\operatorname{cosec}^4\theta+\operatorname{cosec}^4(\tfrac{1}{2}\pi-\theta)-$$
$$-\operatorname{cosec}^2\theta\operatorname{cosec}^2(\tfrac{1}{2}\pi-\theta)\cos\left(n\log\frac{1+\cos 2\theta}{1-\cos 2\theta}\right)\Big\}2\sin 2\theta\,d\theta d\phi, \tag{21}$$

when the particles possess spin $i=\frac{1}{2}$, and, when they do not,

$$\left(\frac{Z^2e^2}{Mv^2}\right)^2\Big\{\operatorname{cosec}^4\theta+\operatorname{cosec}^4(\tfrac{1}{2}\pi-\theta)+$$
$$+2\operatorname{cosec}^2\theta\operatorname{cosec}^2(\tfrac{1}{2}\pi-\theta)\cos\left(n\log\frac{1+\cos 2\theta}{1-\cos 2\theta}\right)\Big\}2\sin 2\theta\,d\theta d\phi. \tag{22}$$

Here n is defined by
$$n=\frac{Z^2e^2}{\hbar v} \tag{23}$$

and the angle θ is measured in the observer's space.

† N Mott, *Proc. Roy Soc* **126** (1930), 259.

In Fig. 45 (a, b) is shown the ratio of the 'wave-mechanical scattering' to the 'classical scattering' in its dependence on velocity and the angle of scattering Fig. 45 (a) refers to collisions between two particles possessing spin $\frac{1}{2}$ (two electrons or two protons), Fig 46 (b) to those of particles without spin (say, two α-particles). We see that in both cases quite appreciable deviations from the classical result are to be expected.

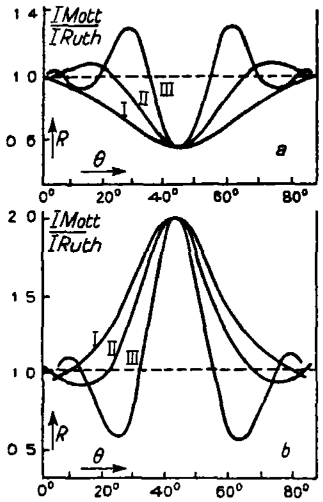


FIG. 45

The experimental results of Chadwick† for the scattering of α-particles in helium are given in Fig 46 This figure refers to observations made at 45° to the incident beam We see that for comparatively small energies of the incident particles the scattering is about *twice as large* as would be expected from classical theory. This is in full agreement with Mott's prediction For larger energies considerable deviations are found These deviations are evidently due to the action of intranuclear exchange forces when the distance of closest approach

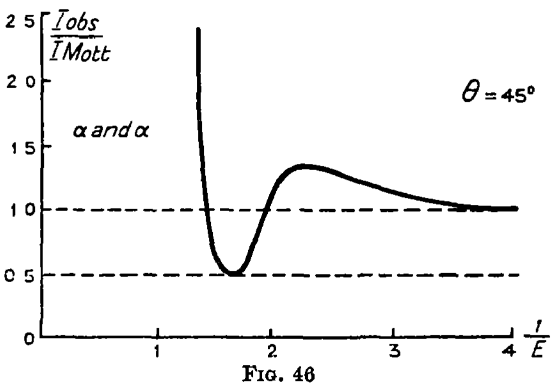


FIG. 46

becomes of the order of magnitude $\sim 10^{-12}$ cm This effect will be discussed separately in the next section

For particles possessing spin $\frac{1}{2}$ the scattering at 45° should be one-

† J Chadwick, *Proc Roy Soc* **128** (1930), 114

half of that given by Rutherford's formula Again, this is in agreement with the observations of Williams, who investigated (by the expansion-chamber method) the scattering of electrons by electrons. He found the scattering at 45° about half the expected amount. A much better illustration of the quantum-mechanical laws of scattering, however, is provided by the recent experiments of Tuve, Hafstad, and Heydenburg on proton scattering in hydrogen † Their results for different proton energies and different angles of scattering are represented in Fig. 47. We see that, for small energies, the ratio

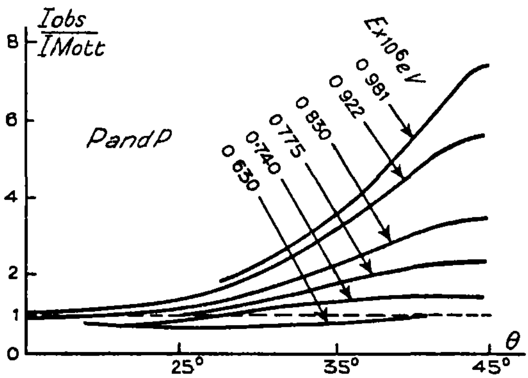


FIG 47

of the observed scattering to the value given by Mott (formula (21)) approaches unity very closely For larger energies and larger scattering angles deviations from Mott's formula again take place These have to be accounted for by a strong interaction (attraction, as we shall see later) between two protons, effective at small separations

2. Influence of nuclear exchange forces on inverse-square law scattering

As we have already mentioned, the scattering of fast particles by light nuclei cannot be explained entirely in terms of Coulomb forces when the closest distance of approach is small The curves of Figs 46 and 47 show this in the case of α-α and p-p collisions Another example is furnished by Fig 48, which gives the results of α-particle scattering in hydrogen Already on the basis of classical considerations‡ it was concluded that such results provide evidence for attractive forces of range $\sim 10^{-12}$ cm.

† M Tuve, L Hafstad, and N Heydenburg, *Phys Rev* **50** (1936), 800.
‡ E Bieler, *Proc Camb Phil Soc* **21** (1923), 686

The wave-mechanical treatment of the deviations from inverse-square law scattering can be carried out along the broad lines indicated in the last section. The general solution is again

$$\Psi'(r,\theta) = \frac{1}{r}\sum_{l=0}^{\infty} A'_l P_l(\cos\theta)\chi'_l(r) \tag{24}$$

with asymptotic form

$$\Psi'_{r\to\infty} = \frac{1}{k}\sin(kr - \tfrac{1}{2}\pi l + \delta_l + \Delta\delta_l) \tag{25}$$

The change in phase $\Delta\delta_l$ (relative to the inverse-square law case) is

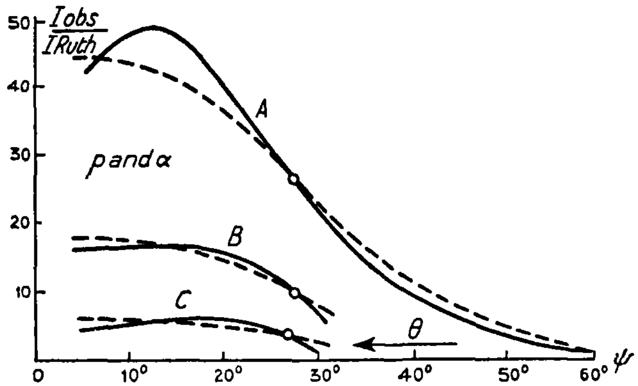


FIG 48

to be estimated by integration of equation (6) for the new potential distribution Comparing the solution (24) with the inverse-square law solution (11), we must again choose the coefficients A'_l so that the difference $\Psi'-\Psi$ contains only terms of the type e^{ikr}. This gives us

$$A'_l = (2l+1)i^l e^{i(\delta_l+\Delta\delta_l)} \tag{26}$$

and the deviation from Coulomb scattering becomes

$$\Psi'-\Psi = \frac{1}{2ikr}\sum_{l=0}^{\infty}(2l+1)(e^{2i\Delta\delta_l}-1)e^{2i\delta_l}P_l(\cos\theta)e^{ikr}. \tag{27}$$

For the ratio of intensities of the new scattering and inverse-square law scattering we now have

$$R = \left|1+\frac{i}{\gamma}\sin^2\tfrac{1}{2}\theta e^{i\gamma\log\sin^2\frac{1}{2}\theta}\sum_{l=0}^{\infty}(2l+1)(e^{2i\Delta\delta_l}-1)e^{2i(\delta_l-\delta_0)}\right|^2, \tag{28}$$

where γ is defined by (15)

A detailed study of 'anomalous scattering' in respect of its connexion with the shape of the nuclear 'potential hole' has been carried out by Taylor.† As an example we consider his results for the collision between an α-particle and a proton. The experimental data for this case (scattering of α-particles in hydrogen) are shown in Fig. 48, where the ratio R is plotted against the angle ϕ of proton recoil (in these experiments protons and not α-particles were counted owing to the greater range of protons with the same energy). Neglecting terms for which $l > 0$ and introducing the recoil angle ϕ instead of the scattering angle θ, we can rewrite (28) in the form

$$R = \left| e^{-i\gamma \log\cos^2\phi} + \frac{i}{\gamma}\cos^2\phi(e^{2i\Delta\delta_0}-1)\right|^2. \tag{29}$$

This expression now contains only one unknown phase $\Delta\delta_0$, which is, of course, a function of the velocity v (but does not depend on θ or ϕ). Using the observed values of R for one recoil angle ϕ (indicated in Fig. 48 by the circles) Taylor estimated the values of $\Delta\delta_0$ for three different velocities. The broken curves in Fig. 48 show the dependence of R on ϕ as calculated, with these phases, from the formula (29). The good agreement with the experimental curves shows that to neglect higher spherical harmonics ($l > 0$) is to work to a sufficient approximation.

Assuming for the potential energy near the nucleus the rectangular model

$$\left.\begin{aligned} U(r) &= \frac{Z_1 Z_2 e^2}{r} \qquad (r > r_0), \\ U(r) &= U_0 = \text{const} \quad (r < r_0), \end{aligned}\right\} \tag{30}$$

already used by us in the theory of α-disintegration, Taylor adjusted (graphically) the values of r_0 and U_0 in such a way as to obtain the phase-shifts $\Delta\delta_0$ already calculated for three different velocities. In this way, for the α-p interaction he obtained

$$[\alpha, p] \quad \begin{aligned} r_0 &= 0.45\times 10^{-12}\,\text{cm} \\ U_0 &= -6\times 10^{6}\,\text{e.v.} \end{aligned}$$

in good agreement with other estimates of nuclear dimensions.

Similar calculations have been carried out by Taylor for α-α collisions with the following results

$$[\alpha, \alpha] \quad \begin{aligned} r_0 &= 0.35\times 10^{-12}\,\text{cm} \\ U_0 &= -15.6\times 10^{+6}\,\text{e.v.} \end{aligned}$$

† H. M. Taylor, *Proc. Roy. Soc.* **134** (1931), 103, **136** (1932), 605.

The curves for *p-p* collisions obtained by Tuve, Hafstad, and Heydenburg have been analysed theoretically by Breit and Condon (on the same lines as indicated above). Breit was able to show that the observed deviations can be explained by short-range attractive forces and that the depth of the potential hole is approximately two-thirds of the mutual potential energy in the case of *n-p* interaction, as estimated, for example, from the binding energies of deuteron and α-particle (compare Chap II) We may notice here that the observed interaction between two protons might be expected theoretically since, as indicated in Chapter III, the radius of the proton must be larger than $\hbar/mc \sim 1{\cdot}3\times 10^{-13}$ cm, whereas in the above-mentioned experiments deviations of 20 per cent from Mott's formula enter only when the apsidal distance between the colliding protons is $\sim 2{\cdot}7\times 10^{-13}$ cm.

3. Scattering by pure exchange forces (neutron scattering)

That the observed scattering of neutrons by atomic nuclei is to be ascribed entirely to nuclear exchange forces is evident since here the force of Coulomb repulsion is completely absent Graphically the distribution of potential energy may be represented roughly by a hole with rather steep walls

Exchange forces between elementary particles being directly responsible for the scattering of neutrons in hydrogen, this case is especially interesting It has been treated theoretically by Wick,† Wigner,‡ and, in more detail, by Bethe and Peierls ‖ The latter authors carefully investigated the difference in the distribution of scattered particles for the case of 'ordinary' central forces (used by Wigner) and that of exchange forces of the Heisenberg-Majorana type

As we have seen in § 1, the effective cross-section for scattering between the angles θ and $\theta+d\theta$ is given by

$$d\sigma = \frac{1}{4\bar{k}^2}\left|\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)(e^{2i\delta_l}-1)\right|^2 \sin\theta\, d\theta d\phi, \tag{31}$$

where δ_l is the phase to be estimated from the corresponding wave equation

$$\frac{d^2\chi_l}{dr^2}+\frac{2\bar{M}}{\hbar}\left[E-U(r)-\frac{\hbar}{2\bar{M}}\frac{l(l+1)}{r^2}\right]\chi_l = 0 \tag{32}$$

† G Wick, *Zs f Phys* **84** (1933), 799

‡ E. Wigner, *Zs f Phys* **83** (1933), 253

‖ H Bethe and R Peierls, *Proc Roy Soc* **149** (1935), 176 with the condition $\chi_l(0) = 0$ For the case of n-p collisions

$$\bar{M} = \frac{M_n M_p}{M_n + M_p} \sim \tfrac{1}{2} M_p$$

and
$$\bar{k} = \frac{\bar{M}v}{\hbar} = \frac{\sqrt{(2\bar{M}\bar{E})}}{\hbar} = \frac{\sqrt{(\frac{1}{2}ME)}}{\hbar} = \tfrac{1}{2}k$$

For the potential energy of interaction we now take over the model represented by a potential hole with a rather small radius, $r_0 \sim 10^{-13}$ cm, which, as we have seen, has been fairly satisfactory in the theory of the deuteron (p 19) Then, for not very fast neutrons the de Broglie wave-length $\Lambda = \dfrac{2\pi\hbar}{\sqrt{(2\bar{M}\bar{E})}}$ is large compared with the dimensions of the hole and only the component $l = 0$ of the incident wave is of importance (other components have vanishingly small amplitudes for $r \sim r_0$ due to the potential barrier of centrifugal force) In such a case the scattering is necessarily spherically symmetrical in a coordinate system bound to the centre of gravity of neutron and proton Neglecting terms with $l > 0$, we obtain for the cross-section effective for scattering (formula (14))

$$\sigma = \frac{4\pi}{\bar{k}^2} \sin^2\delta_0 \tag{33}$$

In order to estimate the phase δ_0 we must remember that equation (32) possesses a stationary solution representing the fundamental state of the deuteron This solution corresponds to an energy $-\epsilon$ ($= -2{\cdot}2 \times 10^6$ e v) and extends to an average distance

$$r_d = \frac{\hbar}{\sqrt{(M\epsilon)}} \sim 0{\cdot}43 \times 10^{-12} \text{ cm}$$

(the radius of the deuteron) For distances larger than the range r_0 of nuclear forces we have

$$\chi_0 \sim ce^{-\alpha r}, \qquad \alpha = \frac{\sqrt{(M\epsilon)}}{\hbar}. \tag{34}$$

Thus for $r > r_0$ the expression $\dfrac{1}{\chi}\dfrac{d\chi}{dr}$ is negative and of the order of magnitude $-\alpha$

It can be shown that for the solutions of (32) corresponding to positive E

$$\left(\frac{1}{\chi_0}\frac{d\chi_0}{dr}\right)_{r>r_0} \sim -\alpha - (\tfrac{1}{2}E + \epsilon)\gamma\frac{Mr_0}{\hbar}, \tag{35}$$

where γ is a numerical constant of the order of magnitude unity. If the energy satisfies the condition

$$E < \sqrt{\left(\epsilon \frac{\hbar^2}{Mr_0^2}\right)} \sim 20\times 10^6 \text{ e v },\tag{36}$$

the second term on the right-hand side of (35) is small compared with α, and, as χ_0 must have the asymptotic form

$$\chi_0_{r\to\infty} = \frac{1}{\bar{k}}\sin(\bar{k}r+\delta_0),\tag{37}$$

we obtain

$$\frac{\bar{k}\cos(\bar{k}r+\delta_0)}{\sin(\bar{k}r+\delta_0)} \sim -\alpha\tag{38}$$

This gives

$$\delta_0 = \tfrac{1}{2}\pi+\arctan\left(\frac{\alpha}{\bar{k}}\right)-\bar{k}r_0\tag{38'}$$

or, neglecting $\bar{k}r_0 \sim \frac{r}{\lambda} \ll 1$,

$$\cot\delta_0 = -\frac{\alpha}{\bar{k}}\tag{38''}$$

Thus we see that, for energies satisfying the condition (36), the phase $\delta_0 < \frac{1}{2}\pi$, and that it becomes $> \frac{1}{2}\pi$ for larger energies We may notice here that these results do not depend on any special hypothesis regarding the type of forces acting

For the phase of the next harmonic, which becomes important only for fast neutrons, Bethe and Peierls give

$$\delta_1 \simeq \pm\frac{\beta}{18}(\bar{k}r)^3,\tag{39}$$

where the positive sign must be taken in the case of 'ordinary' forces and the negative sign in the case of exchange forces (β is a numerical coefficient of the order unity)

For the scattered intensity in the direction θ we now obtain

$$d\sigma = \frac{\pi}{2\bar{k}^2}(4\sin^2\delta_0+12\delta_1\sin 2\delta_0\cos\theta)\sin\theta\, d\theta d\phi\tag{40}$$

Remembering what has been said about the phases δ_0 and δ_1, we find the results concerning the angular distribution of the particles resulting from the collision summarized in Table XII. This table shows that by investigating the angular distribution of recoil protons we ought to be able to draw definite conclusions concerning the type of force acting between neutrons and protons Unfortunately the asymmetry expected for $E < 20\times 10^6$ e v. is much too small to be easily observed ($\sim$ 1 per cent. according to Bethe and Peierls) On

TABLE XII

'Ordinary' forces

	Small energies	$E_0 < 20\times 10^6$ e v	$E_0 > 20\times 10^6$ e v
Neutrons scattered mostly	uniformly	at right angles	forwards
Recoil protons thrown mostly	uniformly	forwards	at right angles
		Exchange forces	
Neutrons scattered mostly	uniformly	forwards	at right angles
Recoil protons thrown mostly	uniformly	at right angles	forwards

the other hand, present experimental means are not available for the study of neutrons with energies as high as would be necessary to observe the much stronger deviations which take place for $E > 20\times 10^6$ e v , according to our theory

For the total cross-section effective for scattering we obtain from (33) and (38″)

$$\sigma = \frac{4\pi}{\alpha^2+\bar{k}^2} = \frac{4\pi\hbar^2}{M}\frac{1}{\epsilon+\frac{1}{2}E}. \tag{41}$$

Using the experimental value $\epsilon = 2\ 2\times 10^6$ e v , we calculate for $E = 4\ 3\times 10^6$ e v , $\sigma = 12\times 10^{-25}$ cm ^{2}, and for $E = 2\ 1\times 10^6$ e v., $\sigma = 16\times 10^{-25}$ cm ^{2}, in satisfactory agreement with the experimental values 5–8×10^{-25} cm and 11–15×10^{-25} cm obtained by Chadwick† for these energies

For very *slow neutrons* ($E \ll \epsilon$) (compare p. 176) the formula (41) gives a constant cross-section

$$\sigma = \frac{4\pi\hbar^2}{M\epsilon} = 23\ 5\times 10^{-25}\text{ cm}^2, \tag{42}$$

representing the maximum scattering predicted by the theory Experimentally‡ much larger values for the collision of slow (thermal) neutrons have been observed, the most probable value being $\sigma = 300\times 10^{-25}$ cm ^{2}

In order to understand such large cross-sections for slow neutrons Wigner|| has indicated that the results so far obtained hold only

† J Chadwick, *Proc Roy Soc* **142** (1933), 1

‡ C Westcott and T Bjerge, *Proc Camb Phil Soc* **31** (1935), 145, J Dunning, G. Pegram, G Fink, and D Mitchell, *Phys Rev* **47** (1935), 970

|| E Wigner, H Bethe, R Peierls, E Teller, *Proceedings of 'S S Aquitania'* (1935)

for collisions in which neutron and proton have parallel spins as in the deuteron nucleus. The binding energy of this nucleus was used in the calculations. For that part of the scattering due to particles with anti-parallel spins we must use the binding energy of the (excited) state of the deuteron nucleus in which the spins of neutron and proton are oppositely orientated, but the orbital momentum remains zero Introducing such a hypothetical excited state, and supposing that its binding energy is rather small, we can obtain very large cross-sections for scattering. The total scattering cross-section will now be

$$\sigma = \frac{4\pi\hbar^2}{M}\left(\frac{3}{4}\frac{1}{\epsilon+\frac{1}{2}E_0}+\frac{1}{4}\frac{1}{\epsilon'+\frac{1}{2}E_0}\right), \tag{43}$$

where ϵ' is the binding energy for the excited state, and the factors $\frac{3}{4}$ and $\frac{1}{4}$ occur because the probability of parallel spins is three times greater than that of anti-parallel spins In order to get the experimentally observed cross-section for slow neutrons ($E_0 \rightarrow 0$) we must assume $\epsilon' = 0{\cdot}046\times10^6$ e v , and correspondingly the amazingly large value $\hbar/\sqrt{(M\epsilon')} \sim 3\times10^{-12}$ cm for the radius of the deuteron in this state of excitation

These considerations brought the above authors to the conclusion that the exchange forces between a neutron and a proton must be considerably smaller in the case of anti-parallel spins, in fact, the pure magnetic interaction between protonic and neutronic magnetic momenta amounts in the case of the deuteron only to about 100 v and thus cannot possibly account for the shift of energy-level from $-2{\cdot}2$ mv to almost zero It was thus necessary to accept that the attraction due to exchange forces between a neutron and a proton is considerably smaller in the case of anti-parallel spin, which means that one should mix the Majorana forces (independent of spin-direction) with a certain amount of Heisenberg forces (corresponding to repulsion in the case of anti-parallel spins). If the state of a deuteron with the binding energy near zero really exists, one should be able to prove the validity of the formula (43) also for the neutrons of small but not yet thermal velocity The experiments in this direction were carried out by Goldhaber,† who used the neutrons of $0{\cdot}2\times10^6$ e v energy (photo-neutrons from deuterium illuminated by the γ-ray $h\nu = 2\ 62\times10^6$ e v). It was found by him that for this energy the scattering cross-section is about three times smaller than

† M. Goldhaber, *Nature*, **137** (1936), 824

one would expect from formula (43). One must notice, however, that careful more recent experiments carried out by Tuve and Hafstad† lead to results consistent with the above theory.

We must mention here a specific effect of the scattering of very slow neutrons, in hydrogen It was indicated by Fermi that the effective collision cross-section will be increased by a factor 4 if one of the colliding particles is not free, but must remain at the same place after the collision This is, however, just the case which we have in the experiments with the scattering of thermal neutrons: the bombarded protons are bound in the molecules of hydrogen-containing substance (water or paraffin), the kinetic energy of incident slow neutrons being not enough to kick the proton out of the molecule. Thus one should observe a much smaller scattering cross-section for slow neutrons in hydrogen gas, a proof of which, however, evidently represents very grave experimental difficulties

The scattering of neutrons by different complex nuclei has been very carefully investigated by Dunning and Pegram,‡ who estimated the effective cross-sections for scattering from the measured absorption coefficients || This, of course, involves the hypothesis that the absorption of fast neutrons is entirely due to scattering The results of these measurements are given in Table XIII, showing a gradual

TABLE XIII

Element	$\sigma \times 10^{24}$ *cm* ^{2}	*Element*	$\sigma \times 10^{24}$ *cm* ^{2}
H	1 6	Cu	3 2
Li	1 6	Zn	3 3
C	1 7	Sn	4 3
N	1 8	I	4 6
Al	2 4	W	5 3
S	2 7	Hg	5 8
Fe	3 0	Pb	5 7

increase of effective cross-section with atomic weight, i e with the geometrical cross-section of the nucleus The wave-mechanical calculation of scattering on the basis of the generally accepted

† M Tuve and L Hafstad, *Phys Rev* **50** (1936), 890

‡ J Dunning and G Pegram, *Phys Rev* **43** (1933), 497, 775, J Dunning, ibid. **45** (1934), 586

|| The decrease of intensity of a neutron beam passed through a thickness d of the investigated substance varies as $e^{-\alpha d}$, where $\alpha = N\sigma$ Here N is the number of nuclei per cm ^{3} and σ the effective cross-section for scattering

'rectangular hole' model of the potential distribution inside the nucleus† is somewhat complicated in this case as the wave-length of the incident neutrons is of the same order of magnitude as the nuclear radius, and the higher harmonics must be taken into account We may say, however, that for non-resonance neutron energies the effective cross-section for scattering must be approximately given by four times the geometrical cross-section $\sigma \sim 4\pi r_0^2$

In Fig 49 the values of σ are plotted against atomic weight We see that these values are in agreement with the curve representing the squares of nuclear radii as estimated on the hypothesis of constant nuclear density (compare formula (36) of Chapter II).

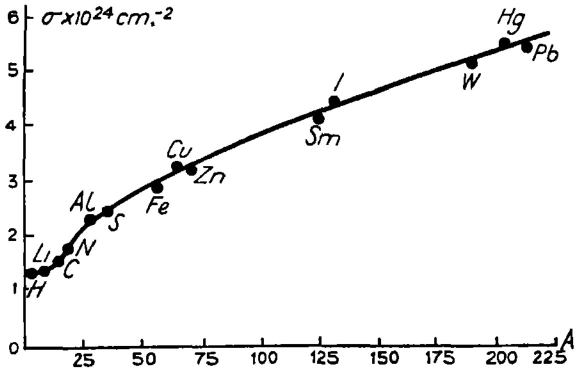


FIG 49

After it became clear that neutrons traversing layers of hydrogen-containing substances are slowed down to thermal velocities by collisions with hydrogen atoms (for a more detailed discussion see Chapter XI), it also became interesting to investigate the scattering of such slow neutrons Experiments in this direction have been carried out mainly by Dunning, Pegram, Fink, and Mitchell,‡ who again used the method of estimating scattering cross-sections from

† An analysis of these data on the basis of the 'rigid sphere' model was carried out by I Rabi (*Phys Rev* **43** (1933), 838), who compared the observed cross-sections with the wave-mechanical formula for scattering on the hypothesis that very strong repulsive forces come into play between the nucleus and the neutron when the distance between them becomes equal to the sum of their radii $R_{(\text{nucleus})}+r_{(\text{neutron})}$ Estimating the values of $R+r$ for different elements in this way and subtracting the nuclear radius R deduced on the hypothesis of constant nuclear density (with the value $R = 0\ 78\times 10^{-12}$ cm for Pb), Rabi obtained for the radius of the neutron the value $r = 0\ 13\times 10^{-13}$ cm fairly constant for all elements investigated

‡ J Dunning, G. Pegram, G Fink, and D. Mitchell, *Phys. Rev* **45** (1934), 586 **47** (1935), 416

absorption measurements. These authors found that the effective cross-section obtained in this way varied very irregularly from element to element, as can be seen from the third column of Table XIV.

TABLE XIV

Scattering of slow neutrons

Element	Z	$\sigma\times 10^{24}$ *cm*^{2} (*obtained from absorption measurements*)	$\sigma\times 10^{24}$ *cm*^{2} (*obtained from 'direct' scattering measurements*)
C	6	4 1	3 2
Mg	12	3 5	2 8
Al	13	1 5	0 9
S	16	1 4	0 8
Cr	24	4 9	1 3
Mn	25	14 3	2 0
Fe	26	12 0	9 9
Ni	28	15 4	17 0
Cu	29	7 5	7 7
Zn	30	4 7	3 4
Ag	47	55!	5 9
Cd	48	3,300!	1 2
Sn	50	4 0	3 8
Hg	80	380!	4 4
Pb	82	8 6	7 2
Bi	83	8 2	9 5

As we can see from this table, the cross-section for the scattering of slow neutrons (determined from absorption measurements) is, in general, somewhat larger than the corresponding cross-section for fast neutrons, but still of the same order of magnitude as the geometrical cross-section of the nucleus For a few elements, however (Ag, Cd, Hg), the cross-section is enormously greater than the geometrical cross-section of the nucleus In view of the fact that slow neutrons, as we shall see later, possess a very large probability of being absorbed directly by nuclei with the emission of γ-rays, we must be careful in considering these data as giving the *real* scattering cross-section. The direct determination of the scattering of slow neutrons was carried out by Mitchell and Murphy,† who measured directly the intensity of neutron beams scattered by different materials. The data of these authors are given in the fourth column of Table XIV; we can see that they show no trace of the extraordinary large cross-sections for Ag, Cd, or Hg, although the scattering in some cases (such as Ni) is somewhat larger than the average.

† A Mitchell and E Murphy, *Phys Rev* **48** (1935), 653, **49** (1936), 400.

Thus we must conclude that the large cross-sections for slow neutrons obtained by the 'absorption method' are actually due to nuclear transformations produced by such neutrons and that the *elastic scattering of neutrons does not show any large anomalous effects for small velocities.*

The wave-mechanical calculations of the scattering of slow neutrons by an atomic nucleus, considered as a potential hole with very steep walls, have been carried out by Bethe.† Taking count only of the first spherical harmonic component of the incident wave Bethe obtained for the cross-section for elastic scattering the expression

$$\sigma = 4\pi(\Lambda_0 \cot\phi_0 + r_0)^2, \tag{44}$$

where r_0 is the nuclear radius, Λ_0 the de Broglie wave-length of the neutron inside the nucleus (for slow neutrons $\Lambda_0 = \dfrac{\hbar}{\sqrt{(2MU_0)}}$, where U_0 is the depth of the nuclear potential hole), and $\phi_0 = \phi(r_0)$ the phase of this wave at the boundary of the nucleus

Substituting $r_0 = 0{\cdot}7\times 10^{-12}$ cm and $\Lambda_0 = 0{\cdot}24\times 10^{-12}$ cm, we have

$$\sigma = 0{\cdot}7\times 10^{-24}(\cot\phi_0+3)^2 \tag{44'}$$

We see that the cross-section may become fairly large in the resonance case, when $\phi_0 \sim n\pi$, far from resonance conditions, however, the magnitude of this cross-section must be $\sim 4\pi r_0^2 = 6\times 10^{-24}$ cm ^{2} (for $r_0 = 0{\cdot}7\times 10^{-12}$ cm), i e four times larger than the geometrical dimensions of the nucleus. Not having an exact nuclear model one cannot calculate ϕ_0 for each special case. Thus only a statistical estimate may be made as to how often the resonance increase in the scattering cross-section is to be expected with the known elements. On the basis of such statistical considerations Bethe concludes that possibly in 55 per cent of cases $\sigma \leqslant 10\times 10^{-24}$ cm ^{2}, in 35 per cent. of cases $10\times 10^{-24} < \sigma < 100\times 10^{-24}$ cm ^{2}, and in 10 per cent. of cases $\sigma \geqslant 100\times 10^{-24}$ cm.^{2} It may seem at first sight that these conclusions fit with experimental evidence, since very large absorption coefficients have been deduced for certain elements (Ag, Cd, Hg). The results of Mitchell and Murphy, however, show that this agreement is entirely illusory. As we have seen (Table XIV, column 4), the strong absorption by these elements is due not to elastic scattering but to a nuclear reaction (radiative capture—which will be discussed later), the scattering being normal throughout. This fact constitutes

† H Bethe, *Phys Rev* **47** (1935), 747

an important point of disagreement with theoretical calculations based on the ordinary model of the nucleus which considers the motion of each separate constituent particle (in particular the incident neutron after penetration inside the nucleus) as determined by the average field of other particles We shall see later (Chap. XI, § 1) that the absence of the selective phenomena for scattering for the energies for which the radiative capture shows distinct selective effects can be accounted for on the basis of new views on the nature of nuclear processes proposed recently by Bohr† in his critical investigations concerning the behaviour of systems of particles bound together by very strong interactive forces.

4. Nuclear excitation by collision

Now we can turn our attention to the case of inelastic collisions in which the incident particles are reflected after having lost energy

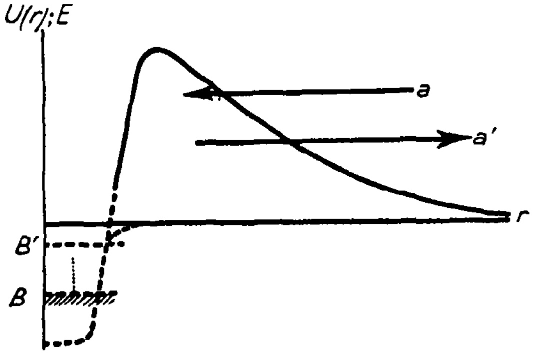


Fig 50

and the bombarded nuclei are brought to an excited state, from which they return later, emitting the excess energy in the form of γ-rays. The general wave-mechanical theory of inelastic collisions between charged particles has been developed by Landau‡ and its application to the case of nuclear collisions gives for the cross-section effective for excitation the value

$$\sigma = A\,e^{-[I(E')-I(E)]} = A\,e^{-\frac{2\pi e^2 Z z}{h}\left(\frac{1}{v'}-\frac{1}{v}\right)} \tag{45}$$

Here A is a coefficient of the order of magnitude Λ^2 ($\Lambda =$ the de Broglie wave-length of the incident particles), slowly changing with

† N Bohr, *Nature*, **137** (1936), 344

‡ L. Landau, *Zs Sov Phys* **1** (1932), 88.

energy, and $e^{I(E)}$ and $e^{I(E')}$ are the transparencies of the nuclear potential barrier for the original and final energies of the bombarding particles, as given in Chapter V. Thus we see that the probability of excitation by collision depends essentially on the *ratio* of the transparencies for the reflected and incident particles $\left(\frac{e^{-I(E')}}{e^{-I(E)}}\right)$, and thus reaches the largest possible value when this ratio approaches unity, i e when the excitation energy $E-E'$ is small Formula (45) tells us also that this probability rapidly decreases with decreasing energy of the incident particles (for a constant excitation energy) and with increasing charge-number of the bombarded and bombarding nuclei For light nuclei and comparatively small excitation energies the effective cross-section is, however, sufficiently large and one should easily observe the γ-rays due to such collisions It is very probable that many γ-rays observed during the bombardment of light nuclei by fast α-particles are of this origin, but it is difficult to distinguish this radiation from other γ-rays connected with the processes of nuclear transformation which also occur quite frequently in these experiments (p 183) It is clear, however, that the γ-radiation of energy $h\nu \sim 0\ 6 \times 10^6$ e v observed in the case of lithium bombarded by slow α-particles is to be ascribed to such excitation by collision in this particular case no other nuclear transformation takes place Similarly, there are some indications of such process of γ-emission in the α-bombardment of $_7N^{14}$, $_9F^{19}$, and $_{13}Al^{27}$, as stated by Savel †

† X Savel, *C R* **198** (1934), 1404

X

NUCLEAR REACTIONS

1. Substitutional reactions

If the incident particle possesses enough energy, it can, in the process of collision with a nucleus, give a part of its energy to one of the nuclear constituent particles, throwing it out and so producing artificial transformation in the bombarded substance. Such artificial nuclear transformations were first discovered in 1919 by Rutherford,† who, bombarding nitrogen atoms with the fast α-particles of RaC′, observed the ejected protons which had evidently originated in nitrogen nuclei which had suffered violent collisions The Wilson chamber photographs of this process taken by Blackett‡ (see Plate I) show the tracks of the bombarding particle, the ejected proton, and the recoil nucleus—but no track which would correspond to the original α-particle reflected after collision This led to the conclusion that the α-particle itself is captured by the nitrogen nucleus Later, this suggestion was substantiated by an investigation of the balance of energy and momentum in the collisions Thus the nuclear reaction is

$$ {}_7N^{14}+{}_2He^4 \rightarrow {}_8O^{17}+{}_1H^1+Q, \tag{1}$$

where ${}_8O^{17}$ is the heavy isotope of oxygen (later found in very small amounts in the atmosphere) formed in the process of transformation and Q the energy-balance in the reaction Q may be estimated by measurements of the kinetic energies of the particles involved in the process For the particular case of nitrogen $Q = -1{\cdot}3\times10^6$ e v Transformations of this type, resulting in the substitution of one particle in the nucleus by another, we shall call *substitutional reactions* The schematic representation of the energetics of such reactions is given in Fig 51, where E_a and E_{a_0} represent the original energy of the incident particle and the quantum level on which it is captured and E_{b_0} and E_b the original level of the disintegration particle and the energy with which it is ejected after the transformation ‖ We see

† E Rutherford, *Phil Mag* **37** (1919), 581

‡ P Blackett, *Proc Roy Soc* **107** (1925), 349, P Blackett and D Lees, ibid **136** (1932), 325

‖ We must remember that in such diagrams as Fig 51 the quantum levels for captured and emitted particles belong actually to different nuclei (${}_8O^{17}$ and ${}_7N^{14}$ in our case).

that the energy of the ejected particle will be larger or smaller than the energy of the incident particle according as the level B is above or below the level A.

Investigating other elements, Rutherford and his collaborators were able to observe the emission of protons from most of the light elements and with some of them found that the energy-balance was positive and rather large For example, in the reaction

$$_{13}\mathrm{Al}^{27}+{}_2\mathrm{He}^4 \rightarrow {}_{14}\mathrm{Si}^{30}+{}_1\mathrm{H}^1+Q \quad (2)$$

we have $Q = +2\ 3\times 10^6$ e v

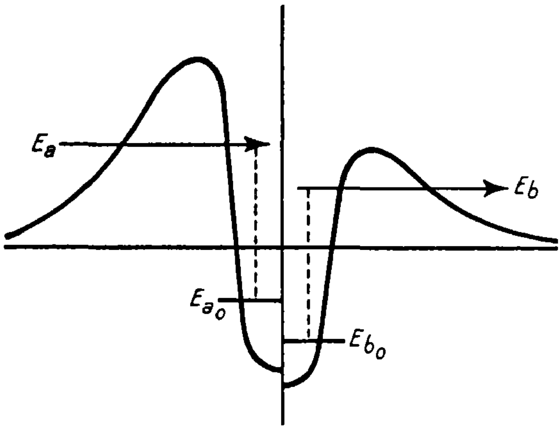


FIG 51

As the atomic number of the bombarded element increased, the yield of protons rapidly decreased, and practically no disintegrations were observed for the elements heavier than argon For a given element the yield of protons decreases exponentially with the energy of the bombarding particles, and for energies less than $\sim 4\times 10^6$ e v , in most cases, the number of ejected particles becomes too small to be observed An explanation of these facts was first proposed by Gamow† in terms of the transparency of the potential barriers surrounding the nucleus for the incident and ejected particles respectively We shall discuss this question in more detail in the next section.

Now we must consider the possibility that the incident particle, on entering the nucleus, can be captured, not on the fundamental level, but on a level corresponding to an excited state of the nucleus.

† G. Gamow, *Zs f Phys* **52** (1928), 510.

In such a case the emitted particle will have an energy less than normal by the amount of excitation—and we may expect to observe several groups of emitted protons,† in complete analogy with the phenomenon of 'fine-structure' in the case of the radioactive emission of α-particles. It is also clear that those transformations which are characterized by several groups of emitted particles must likewise be accompanied by γ-radiation, corresponding to the transition of the product-nucleus to the normal state after the reaction.

In the case of nitrogen (reaction (1)) only one group of protons has been found This is evidently due to the fact that, because of the very low energy-balance in this reaction (loss of energy $1\ 3\times10^6$ e v.), excitation is either energetically impossible ('negative energy' for the ejected protons) or possesses a very small probability (small transparency of the potential barrier for very slow protons) On the other hand, in the case of aluminium (reaction (2)) a short-range group with an energy-balance $Q' = -0\ 1\times10^6$ e.v. has been observed‡ corresponding to the formation of the product nucleus with the extra energy $Q-Q' = 2{\cdot}4\times10^6$ e.v This energy should be emitted subsequently in the form of one or more γ-rays Actually the experiments of Bothe|| have shown that the bombardment of Al by α-particles is accompanied by a rather intense γ-radiation

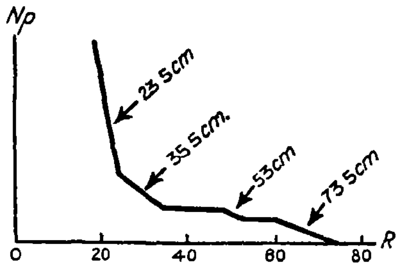


FIG 52

In the case of boron (consisting of two isotopes of mass numbers 10 and 11 with the relative abundance 1 4) Bothe and Franz†† observed two groups of protons each giving a positive energy-balance and one group corresponding to negative balance.

More detailed investigation of proton groups from boron was carried on by Paton,‡‡ who found that the protons emitted from boron by α-rays of RaEm consist of four homogeneous groups (Fig 52) with the energy-balance $+3{\cdot}1$, $+0{\cdot}35$, $-0{\cdot}78$, and $-1{\cdot}86\times10^6$ e v. It seems reasonable to ascribe the observed fast proton group to the

† J. Chadwick and G Gamow, *Nature*, **76** (1930), 54.

‡ E Rutherford and J Chadwick, *Proc Camb Phil Soc* **25** (1929), 186.

|| W Bothe and H Becker, *Zs f Phys* **66** (1930), 289

†† W. Bothe and H Fränz, ibid **49** (1928), 1

‡‡ R Paton, ibid **90** (1934), 586.

disintegration of the lighter isotope of boron according to the equation

$$_5B^{10}+{_2He^4} \rightarrow {_6C^{13}}+{_1H^1}+Q \tag{3}$$

In fact, substituting in (3) the values of the nuclear masses, we obtain for energy-balance of transformation without excitation

$$Q = 10{\cdot}0161+4{\cdot}0039-13{\cdot}0073-1{\cdot}0081 = 0{\cdot}0046 = +4{\cdot}6\times10^6\text{ e.v.},$$

which is, considering the possible errors, in reasonable agreement with the observed maximal energy-balance. If the reaction responsible for the fast group were

$$_5B^{11}+{_2He^4} \rightarrow {_6C^{14}}+{_1H^1}+Q, \tag{3'}$$

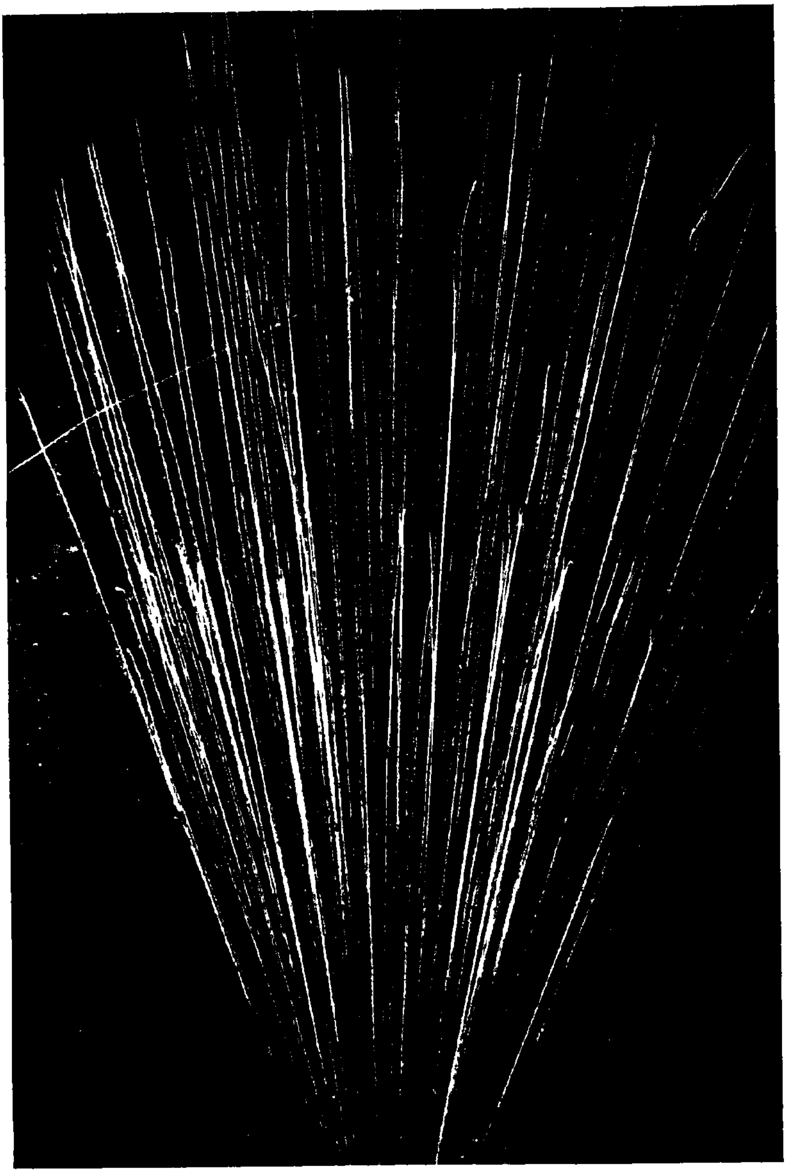
the mass of $_6C^{14}$ would be 14·0055 and the ordinary $_7N^{14}$ (mass 14·0073) would be unstable, transforming into $_6C^{14}$ with the emission of a positive electron. It is, however, not impossible that some of the slower observed proton groups have their origin in reaction (3′).

Bombarding beryllium nuclei with the α-particles of Po, Bothe† observed a very penetrating radiation which was at first interpreted as γ-rays. This radiation was carefully investigated by Joliot and Curie,‡ who found it to possess the property of throwing out protons of high velocity from hydrogen-containing compounds (e.g. paraffin) through which it passed. This fact alone might have been considered as providing evidence against the 'γ-ray interpretation', as the effective cross-section for the Compton effect of γ-rays of these energies scattered by protons should be much smaller than the observed value. The problem was cleared up by Chadwick,‖ who investigated the recoil of nitrogen nuclei under bombardment by the new radiation and compared it with the recoil of protons previously observed. According to the γ-ray hypothesis, E_r, the maximum energy of recoil of a nucleus of mass number M_r, after scattering of a light quantum of energy $h\nu$, should be

$$E_r = 2h\nu\Big/\left(2+\frac{M_r c^2}{h\nu}\right). \tag{4}$$

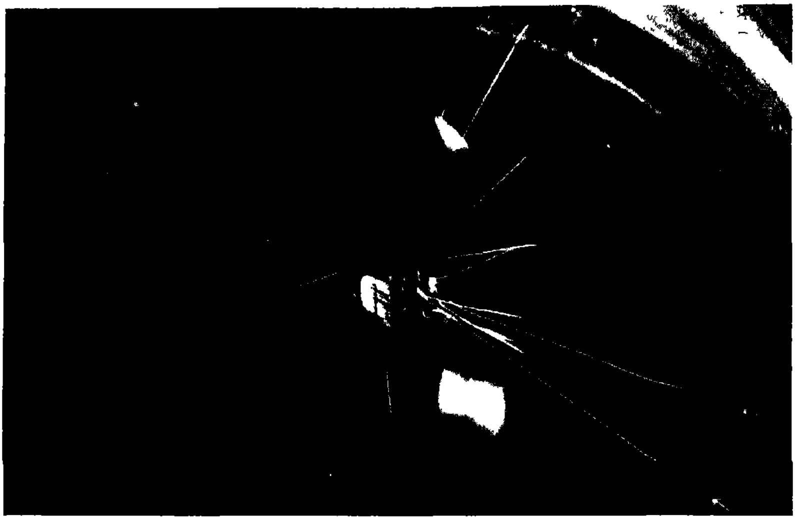
Substituting in (4) the value $E_r = 5{\cdot}7\times10^6$ e.v., found with hydrogen by Curie and Joliot, we have $h\nu = 55\times10^6$ e.v. On the other hand, with nitrogen Chadwick observed $E_r = 1{\cdot}6\times10^6$ e.v. This gives, according to (4), $h\nu = 110\times10^6$ e.v. The discrepancy here shows

† W. Bothe and H. Becker, *Zs. f. Phys.* **66** (1930), 289.
‡ F. Joliot and I. Curie, *C. R.* **193** (1931), 1412.
‖ J. Chadwick, *Proc. Roy. Soc.* **136** (1932), 692.

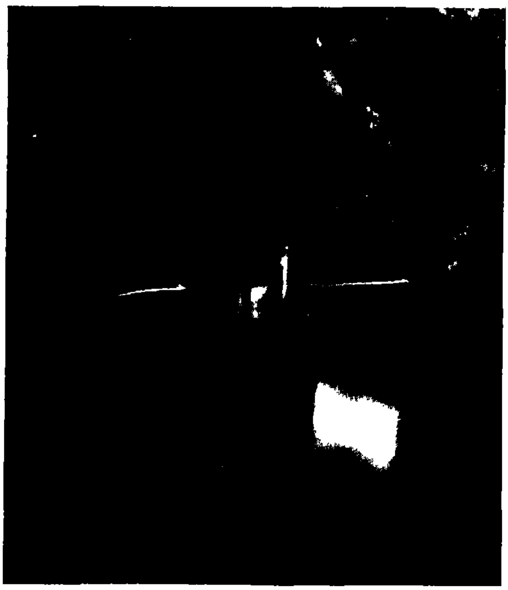


Reaction ${}_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1$

PLATE II



(a)



(b)

(a) Reaction ${}_3Li^7 + {}_1H^1 \rightarrow 2\,{}_2He^4$ (b) Reactions ${}_3Li^6 + {}_1H^2 \rightarrow 2\,{}_2He^4$ and ${}_3Li^6 + {}_1H^2 \rightarrow {}_3Li^7 + {}_1H^1$

conclusively that the γ-ray hypothesis is untenable. To explain the experimental facts Chadwick made the alternative suggestion that the radiation emitted by beryllium as the result of bombardment by α-particles in fact consists of material particles carrying no electric charge (otherwise these particles would give—contrary to the findings of experiment—ionization tracks in a Wilson chamber). These are the so-called neutrons, the possibility of the existence of which, in atomic nuclei, had long before been emphasized by Rutherford.† If the mass number of the neutron is M_n and the velocity with which it is emitted from the Be nucleus is V_n, the maximum velocity V_0 of the recoil particle (of mass number M_r) will evidently be given by

$$V_r = \frac{2M_n}{M_n+M_r}V_n \tag{5}$$

Using this equation for the cases investigated ($M_r = 1$, $V_r = 3{\cdot}3\times 10^9$ cm /sec , $M_r = 14$, $V_r = 4{\cdot}7\times 10^8$ cm /sec) Chadwick obtained $M_n \sim 1$ and $V_n \sim 3\ 3\times 10^9$ cm /sec The reaction leading to the formation of the neutron is now

$${}_4\text{Be}^9+{}_2\text{He}^4 \rightarrow {}_6\text{C}^{12}+{}_0n^1+Q \tag{6}$$

The discovery of the neutron as an important constituent part of the atomic nucleus gave a strong impulse to the development of the general theory of nuclear structure as set forth in previous chapters.

It was shown as a result of further experiments that neutrons are emitted from many light elements under bombardment by α-particles (Table G at the end of the book) Using the known masses of nuclei entering into such reactions, many estimates of the exact value of the neutron mass have been made, ranging in wide limits due to inexact knowledge of nuclear masses The best estimate, however, has since been obtained from the experiments on the disruption of the deuteron nucleus by γ-rays (see § 4 of this chapter).

Investigating the emission of neutrons caused by α-bombardment, Joliot and Curie‡ observed that in a number of cases (B, Al, Si, P) positive electrons are emitted after the source of bombarding α-particles is removed This induced positive β-activity behaves quite similarly to the ordinary β-activity of the radioactive elements. Thus, the number of positrons which are emitted decreases exponentially with time, and we are able to introduce the notion of a

† E Rutherford, *Proc Roy. Soc* **97** (1920), 374

‡ I Curie and F Joliot, *C.R.* **198** (1934), 254.

decay constant Also, the emitted positrons have a continuous energy distribution, similar to those which characterize ordinary β-disintegrating elements. These observations suggest that in the corresponding nuclear reaction the product nucleus is not stable, but, possessing more protons than would correspond to the minimum energy for a given mass-number (see Chap II), is subject to positive β-transitions The reactions with boron and aluminium can be written

$$_5B^{10}+{}_2He^4 \rightarrow {}_7N^{13}+{}_0n^1+Q\dagger \tag{7}$$

and

$$_{13}Al^{27}+{}_2He^4 \rightarrow {}_{15}P^{30}+{}_0n^1+Q \tag{8}$$

These are followed by the transformations

$$_7N^{13} \rightarrow {}_6C^{13}+\partial \quad (T_{\frac{1}{2}} = 10 \text{ min }) \tag{7'}$$

and

$$_{15}P^{30} \rightarrow {}_{14}Si^{30}+\partial \quad (T_{\frac{1}{2}} = 3{\cdot}2 \text{ min }) \tag{8'}$$

It was shown by Joliot and Curie that if one mixes activated boron (or aluminium) with ordinary nitrogen (or phosphorus) and then makes a chemical separation of the added substance, the ∂-activity separates with it and no trace of activity is left with the original boron (or aluminium) ‡ This proves, without ambiguity, that the observed activity is really due to the species produced according to the above equations The discovery of the artificially produced ∂-active elements is of very great importance for nuclear theory, first because it gives new arguments in favour of the neutron-proton model of the nucleus ($p \rightleftarrows n$-transformations), and, secondly, because it gives us (when considered together with Fermi's discovery of similar e-emitting elements) much new material for the study of the processes of nuclear transformations (see Chap VII)

According to Gamow's‖ theory of artificial disintegration the probability that an incident particle will penetrate inside the nucleus is given essentially by the transparency of the potential barrier††

$$G \sim e^{-\frac{\pi\sqrt{m_a}\, z_a Ze^2}{\hbar\sqrt{E_a}}}, \tag{9}$$

where m_a, $z_a e$, and E_a are the mass, charge, and energy of the incident particle, and Z is the charge-number of the bombarded nucleus From this formula we conclude that the probability of penetration increases when the charge and mass of the bombarding particle

† The lighter isotope must be responsible for the reaction since the isotope $_5B^{11}$ would give the stable nucleus $_7N^{14}$

‡ F Joliot and I. Curie, *C R* 198 (1934), 559

‖ G Gamow, l c. 193 ff.

†† Compare Chap. V.

decrease. If we use, for example, protons instead of α-particles the factor $\sqrt{(m_a)}z_a$ in the exponent will be four times smaller. This means that, if we use protons of the same energy as the α-particles previously employed, the limit at which disintegration is still observable will be shifted to the element with four times larger atomic number. On the other hand, for a given element disintegration by proton bombardment should be observed for sixteen times smaller energy than is the case for α-bombardment This theoretical conclusion has been verified by the experiments of Cockcroft and Walton,† who first succeeded in producing artificially intense beams of fast protons and effecting nuclear disintegration by means of them Bombarding a lithium target, Cockcroft and Walton observed the emission of α-particles of $8{\cdot}5\times10^6$ e v. energy, evidently produced in the reaction

$$_3\mathrm{Li}^7+{}_1\mathrm{H}^1\rightarrow 2\,{}_2\mathrm{He}^4+Q \tag{10}$$

The yield of α-particles rapidly decreases with the energy of the bombarding protons, but it has been traced for energies as low as $0{\cdot}02\times10^6$ e v. The observed energy-balance fits very well with the value ($17\ 04\times10^6$ e.v) calculated from equation (10) ($_3\mathrm{Li}^7 = 7{\cdot}0147$, $_1\mathrm{H}^1 = 1{\cdot}00778$, and $_2\mathrm{He}^4 = 4\ 00216$) A Wilson chamber photograph of the disintegration of lithium‡ by protons is given in Plate II A and shows clearly the pairs of α-tracks of equal length in opposite directions, each such pair representing the two α-particles formed from a lithium nucleus in this transformation.

Cockcroft and Walton observed also the presence of short-range α-particles emitted by lithium bombarded by protons These were investigated in detail by Oliphant, Kinsey, and Rutherford,‖ who showed that they formed two groups of 11 5 mm and 6–8 mm. range respectively They suggested that these groups are due to the disintegration of the lighter lithium isotope

$$_3\mathrm{Li}^6+{}_1\mathrm{H}^1\rightarrow {}_2\mathrm{He}^4+{}_2\mathrm{He}^3+Q, \tag{11}$$

with $Q = 3{\cdot}6\times10^6$ e v. Thus a new, hitherto unknown, isotope of helium (with mass $3{\cdot}0163\pm0{\cdot}0004$) was found

The disintegration of other light elements by artificially accelerated protons has been investigated by Cockcroft and Walton and by many other authors, the important results are shown in Table G at the end of the book

† J Cockcroft and E Walton, *Proc Roy Soc* **137** (1932), 229

‡ P. Dee and E Walton, ibid **141** (1933), 733

‖ M Oliphant, B Kinsey, and E. Rutherford, ibid **141** (1933), 722.

After the discovery of the heavy isotope of hydrogen by Urey† it became interesting to study the reactions produced by the bombardment of different elements by deuterons (nuclei of heavy hydrogen). The first experiments in this direction were carried out by Lawrence,‡ who found that the isotope $_3Li^6$, when bombarded by deuterons, gives rise to two α-particles according to the equation

$$_3Li^6+{}_1H^2 \rightarrow 2\,{}_2He^4+Q, \tag{12}$$

with $Q = 22{\cdot}08\times10^6$ e v Another possible reaction following this collision is, according to Lawrence,

$$_3Li^6+{}_1H^2 \rightarrow {}_3Li^7+{}_1H^1+Q, \tag{13}$$

with $Q = 4{\cdot}8\times10^6$ e v. A Wilson chamber photograph providing evidence of both modes of disintegration is given in Plate II B ‖ The two heavy tracks in opposite directions are due to two α-particles formed in reaction (12), and the thin track in the foreground is most probably that of a proton formed in reaction (13) (the range of the recoil $_3Li^7$ nucleus is not great enough for it to pass from the target vessel into the expansion chamber)

Especially interesting are the experiments on the collision between two deuterons which have been carried through by Oliphant, Harteck, and Rutherford †† In this case it is found that two different reactions may take place

$$_1H^2+{}_1H^2 \rightarrow {}_1H^3+{}_1H^1+Q, \tag{14}$$

with $Q = 4\ 0\times10^6$ e v , or

$$_1H^2+{}_1H^2 \rightarrow {}_2He^3+{}_0n^1+Q \tag{15}$$

The first reaction gives rise to a hitherto unknown isotope of hydrogen of mass-number 3, in the second is produced the light helium isotope $_2He^3$, already figuring in reaction (11) For the mass of $_1H^3$ the present estimate is 3 0151 A photograph of this reaction, (14), taken by Dee‡‡ is shown in Plate III A, where the longer tracks are those of ordinary protons and the shorter tracks are due to the isotope $_1H^3$ For the calculation of the energy-balance and the mass of $_2He^3$ from reaction (15) we must know the velocity of the emitted neutrons This can be obtained by observing the energy of protons projected by these neutrons In Wilson chamber photographs‡‡ one

† H Urey, F Brickwedde, and G Murphy, *Phys Rev* **39** (1932), 164, 864

‡ E Lawrence, *Phys Rev* **44** (1933), 55

‖ P Dee and E Walton, l c

†† M Oliphant, P Harteck, and E Rutherford, *Proc Roy Soc* **144** (1934), 692

‡‡ P Dee, *Nature*, **133** (1934), 564

can sometimes observe, apart from the two groups of particles due to reaction (14), the tracks of such recoil nuclei From a number of such photographs Dee estimated that the neutrons from (15) constitute a homogeneous group with an energy close to $2{\cdot}0\times10^6$ e.v. This gives for the energy-balance in reaction (15) the value $Q = 2{\cdot}6\times10^6$ e v and for the mass of ${}_2He^3$ the value which is in good agreement with that obtained previously from reaction (11).

We must notice here that substitutional reactions may sometimes result in the emission of several particles simultaneously. This is probably the case in the bombardment of ${}_3Li^7$ by deuterons from which Oliphant, Kinsey, and Rutherford† observed a continuous distribution of ejected α-particles with ranges up to 7·8 cm and neutrons with energies up to 6×10^6 e v. The reaction most probably is

$$ {}_3Li^7+{}_1H^2 \rightarrow 2\,{}_2He^4+{}_0n^1+Q, \tag{16}$$

the available energy being distributed at random between two α-particles and a neutron The largest energy for an α-particle will be obtained if this particle is emitted in a direction opposite to that of the other α-particle and the neutron, the latter escaping in directions parallel to one another. In this case the first α-particle takes nearly five-ninths of the total energy. Assuming that the observed upper limit of the emitted α-particles can be explained in this way, we obtain for the energy-balance the value $Q \sim 14{\cdot}6\times10^6$ e v , in satisfactory agreement with the value $14\ 9\times10^6$ e v , calculated from the known masses of the nuclei taking part in the reaction Another case, in which the two 'emitted particles' and the 'residual nucleus' are of the same kind, is that of boron Bombarding boron with protons we may expect the reaction

$$ {}_5B^{11}+{}_1H^1 \rightarrow 3\,{}_2He^4+8{\cdot}7\times10^6 \text{ e v.} \tag{17}$$

The fact that this reaction really does take place (at least much more often than the alternative reaction ${}_5B^{11}+{}_1H^1 \rightarrow {}_4Be^8+{}_2He^4$) has recently been shown by Dee and Gilbert‡ by the direct Wilson chamber method, one of their photographs showing three α-particles diverging almost symmetrically from the point of collision is given in Plate III B

When neutrons are used as bombarding particles, due to the absence of electric charge, they should penetrate without difficulty

† M Oliphant, B Kinsey, and E Rutherford, l c

‡ P Dee and C Gilbert, *Proc. Roy Soc* **154** (1936), 279

inside the nucleus. However, one must not, on this account alone, expect all nuclei to be disintegrated in this way.

As we have mentioned already, the probability of disintegration is governed by the *product* of the transparencies for the entering and escaping particles; if the first factor is unity when neutrons are used as bombarding particles, the second factor, corresponding to the ejected α- or other nuclear particle, will still rapidly decrease with increasing charge-number of the bombarded nucleus The first case of disintegration by neutron bombardment was registered by Feather,† who observed in a Wilson chamber filled with nitrogen the paired tracks of the particles originating in the reaction

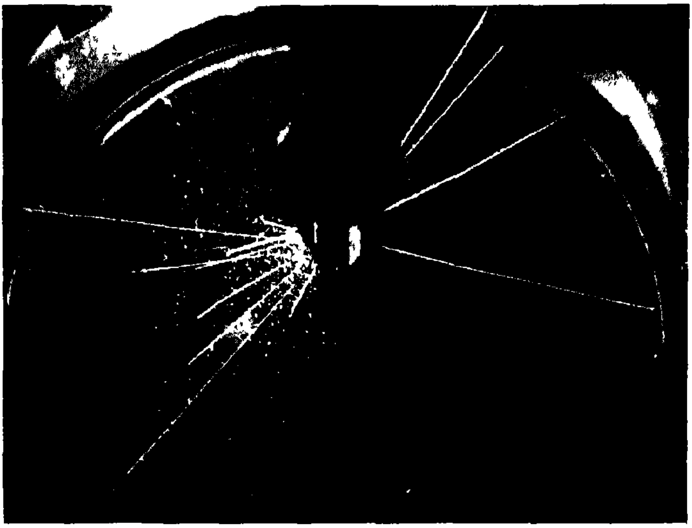
$$_7N^{14}+{}_0n^1 \rightarrow {}_5B^{11}+{}_2He^4+Q \tag{18}$$

One of such photographs is shown in Plate III c Here the long track belongs to the emitted α-particle and the short one to the recoil nucleus The energy-balance Q, estimated from about thirty photographs, did not appear to be constant This was taken as evidence that very often the $_5B^{11}$ nucleus is formed in an excited state Amongst other elements emitting α-particles under neutron bombardment we may mention $_6C^{12}$, $_8O^{16}$, and $_{10}Ne^{20}$, from which result the stable nuclei $_4Be^9$, $_6C^{13}$, and $_8O^{17}$.

It was first established by Fermi and his collaborators‡ that in many cases the substitution of an α-particle by a neutron leads to a nucleus which, having an excess of neutrons relative to protons, is therefore radioactive, emitting a negative electron (Fermi's reactions of the first type). In one sense these are the reverse of the Joliot-Curie reactions (see p 185) in which substitution of a neutron by an α-particle leads to the emission of a positive electron The activity of the product nucleus makes it very easy to study such reactions, investigating the β-activity 'induced' in the bombarded element, after removing the source of neutrons Making chemical tests on the irradiated substance, one can find with which element the induced activity is precipitated, establishing in this way the chemical identity of the product of the transformation. Thus, for example, phosphorus irradiated with fast neutrons shows a β-activity with a decay period 2·3 min If one takes phosphorus in the form of phosphoric acid and, after irradiation by neutrons, neutralizes the solution with sodium

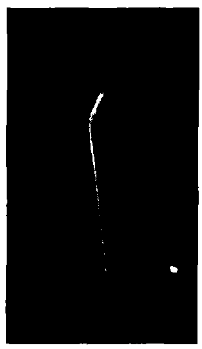
† N Feather, *Proc Roy Soc* **136** (1932), 709

‡ E Amaldi, O D'Agostino, E Fermi, B Pontecorvo, F Rasetti, and F Segré, *Proc Roy Soc.* **146** (1934), 483, **149** (1935), 522



(a)

(b)



(c)

(a) Reaction $_1H^2 + {}_1H^2 \rightarrow {}_1H^1 + {}_1H^3$. (b) Reaction $_5B^{11} + {}_1H^1 \rightarrow 3\,{}_2He^4$, (c) Reaction $_7N^{14} + {}_0n^1 \rightarrow {}_5B^{11} + {}_2He^4$

carbonate and adds aluminium chloride, the activity is found to be concentrated in the precipitated aluminium. In this case, therefore, the active element is an isotope of aluminium. In so far as the active element has an atomic number two units less than that of the bombarded element, we must conclude that its formation is due to the ejection of an α-particle† and write the reaction in the form

$$_{15}P^{31}+_0n^1 \rightarrow {}_{13}Al^{28}+{}_2He^4, \tag{19}$$

$$_{13}Al^{28} \rightarrow {}_{14}Si^{28}+e \text{ (2·3 min)} \tag{19'}$$

With some other elements Fermi has found, by analogous chemical tests, that the active element has an atomic number only one unit less than the original one, in these cases we have evidently the substitution of a nuclear proton by the incident neutron The active product with the period $\sim$10 min obtained by neutron bombardment of aluminium, for example, must have its origin in the reaction

$$_{13}Al^{27}+_0n^1 \rightarrow {}_{12}Mg^{27}+{}_1H^1, \tag{20}$$

$$_{12}Mg^{27} \rightarrow {}_{13}Al^{27}+e \text{ (}\sim\text{10 min.)} \tag{20'}$$

We may notice here that, in reactions of this type, the final stable nucleus is identical with the original nucleus, the complete process being equivalent to the splitting of the incident neutron into a proton and an electron.

Finally, there are many cases in which, according to Fermi, the active product is an isotope of the original species; in such cases we must assume that the neutron is directly captured by the nucleus, without ejection of any nuclear particle, the excess energy being emitted in the form of γ-radiation ‡ For example, aluminium irradiated by neutrons shows, apart from the activity already discussed (active product with the period 10 min), another activity decreasing more rapidly (period 2·3 min) and precipitating in chemical tests with aluminium itself We must suppose that in this case we have the reaction

$$_{13}Al^{27}+_0n^1 \rightarrow {}_{13}Al^{28}+h\nu, \tag{21}$$

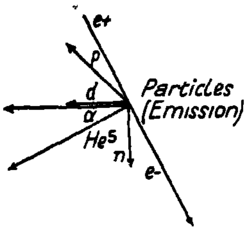
$$_{13}Al^{28} \rightarrow {}_{14}Si^{28}+e \text{ (2·3 min)} \tag{21'}$$

We see that the active product here is identical with the product in reaction (19), a fact which is supported by the consideration that

† Such α-particles have been reported by H Klarmann, *Zs f Phys* **95** (1935), 221

‡ Another hypothesis that the incident neutron knocks out a nuclear neutron ($_{13}Al^{27}+_0n^1 \rightarrow {}_{13}Al^{26}+2{}_0n^1$) must be excluded on the basis of energy-balance considerations

both activities have the same period. We see that the possibility of several kinds of reaction for a given bombarded species with the



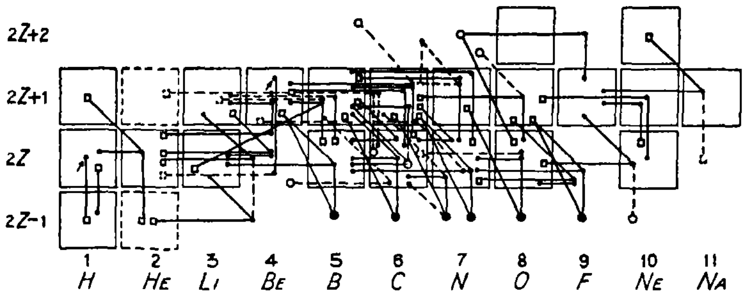


Fig 53 (*a*)

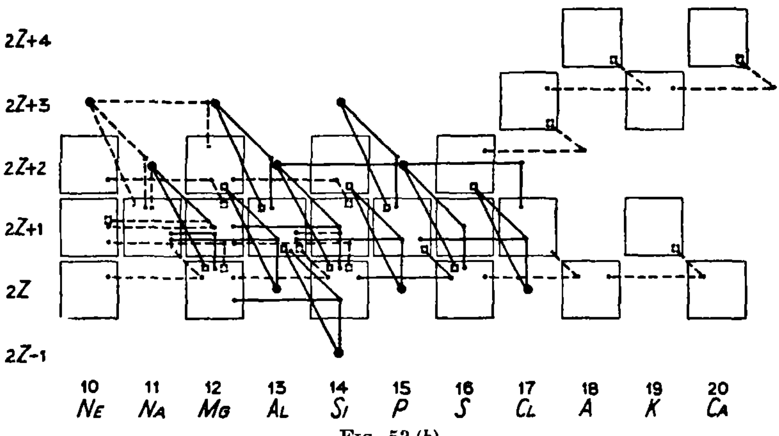


Fig 53 (*b*)

additional circumstance that many elements are mixtures of several isotopes makes the complete picture rather complicated However, chemical tests and the comparison of periods of the different induced activities usually permits us to classify the observed phenomena

The different kinds of nuclear reaction in the region of light elements are summarized in Figs. 53 (*a*) and 53 (*b*).†

† According to D Evans and M Livingston, *Rev Mod Phys* 7 (1935), 229.

We must notice here that these phenomena (of induced activity by neutron bombardment) were observed by Fermi for many very heavy elements also In these cases, however, the chemical test almost always shows that the process is that of radiative capture of the incident neutron, which is quite natural, as for heavy elements the potential barriers are too high to permit the ejection of any charged particle from the nucleus (except perhaps with radioactive elements, for which compare p 201) The process of radiative capture of neutrons will be discussed in detail in the next chapter

The types of reaction mentioned above represent the complete set of substitutional reactions involving bombarding and emitted particles of small charge ($z \leqslant 2$) † Reactions involving heavier particles, as, for example,

$$ {}_3\mathrm{Li}^7 + {}_3\mathrm{Li}^6 \rightarrow {}_4\mathrm{Be}^9 + {}_2\mathrm{He}^4, \tag{22} $$

or

$$ {}_3\mathrm{Li}^7 + {}_3\mathrm{Li}^7 \rightarrow {}_4\mathrm{Be}^{10} + {}_2\mathrm{He}^4, \tag{22'} $$

$$ {}_4\mathrm{Be}^{10} \rightarrow {}_5\mathrm{B}^{10} + e, \tag{22''} $$

although possible in principle, possess very small probabilities, due to the small transparencies of nuclear barriers for particles with larger z These reactions can hardly be expected to be observed by present experimental means.

2. Probabilities of substitutional reactions

The naive description of a substitutional reaction would consist in the hypothesis that the incident particle, upon entering the nucleus, gives its energy by direct collision to some other particle previously existing in the nucleus, thus throwing it out However, it seems that such a simple picture cannot correspond to reality, as, for example, if the ejected particle is an α-particle it seems doubtful whether we may treat it as 'existing' in the nucleus before the transformation. In fact, we have seen in Chapter II that, due to the strong interactions inside the nucleus, we should rather speak of saturated α-shells (as of electronic shells in an atom) without attributing to the α-particle the properties of an individual unit Thus it seems much nearer to reality to consider any substitutional reaction as a *splitting* of the system formed after the entrance of the incident particle into two (or more) parts, one of which might be a nucleus of helium, i e. an α-particle. The result of such splitting is governed, not by the

† With the exception of reactions leading to the emission of deuterons

presence of such and such complex particle in the original nucleus, but by the relative probabilities of disintegration, defined in terms of the penetration of the potential barrier † However, in the present stage of our knowledge, the naive picture of 'direct substitution' may be used to a certain extent and we may attempt more elaborate calculations of transformation probabilities, assuming that the process can be considered as the transition between two states of equal energy for the system, consisting of the incident particle a and the ejected particle b in the central field of the rest of the nucleus The schematic representation of such a model is given in Fig 51, where E_a and E_b are the energies of incident and emitted particles, and E_{a_0} and E_{b_0} the stationary levels of negative energy on which the particle a is captured and from which the particle b is ejected The potential distribution is naturally different for the two particles (left and right halves of the diagram) The curves in Fig 51 are specifically drawn for the reaction of substitution of a nuclear proton by an incident α-particle—and correspond to a negative energy-balance The two states between which the transition takes place are (1) a-particle coming from outside with positive energy E_a, and b-particle on the stationary level with negative energy E_{b_0}, (2) a-particle on the stationary level with negative energy E_{a_0}, and b-particle going out with positive energy E_b The solutions of the wave equation for the bound state can evidently be written in the form

$$\Psi_{a_0} = \frac{1}{\sqrt{\Omega}}\,\omega_{a_0}, \tag{23}$$

$$\Psi_{b_0} = \frac{1}{\sqrt{\Omega}}\,\omega_{b_0}, \tag{23'}$$

where $1/\sqrt{\Omega}$ is the normalization coefficient ($\Omega \sim \frac{4}{3}\pi r_0^3 =$ nuclear volume) and the functions ω_{a_0} and ω_{b_0}, representing the solution of the wave equation inside the nucleus, can be approximated to (in the case of a nuclear model deviating only slightly from a rectangular potential hole) by harmonic functions of unit amplitude

The incident wave, normalized to unit flow across unit cross-section, can be written in the form

$$\Psi_{a\,(r>r_0)} = \frac{1}{\sqrt{v_a}}\,e^{ikz} = \frac{1}{\sqrt{v_a}}\,e^{ikr\cos\theta}, \qquad k = \frac{\sqrt{(2ME_a)}}{\hbar}. \tag{24}$$

† Thus, for example, the reaction $_4Be^9 + {}_2He^4 \rightarrow {}_3Li^7 + {}_3Li^6$ is unknown, not because the Li nucleus 'does not exist' in the Be nucleus, but because the potential barrier between these two product nuclei is too high

As mentioned in § 1 of the last chapter, e^{ikz} can be developed in the series†

$$e^{ikz} = \sum (2l+1)i^l \frac{1}{r} P_l(\cos\theta)\chi_a(r), \tag{25}$$

which means that the incident plane wave can be considered as represented by the superposition of spherical waves with relative intensities

$$\frac{(2l+1)^2}{r^2k^2} = \frac{(2l+1)^2}{r^2}\frac{\hbar^2}{2ME} = \frac{(2l+1)^2}{r^2}\Lambda^2, \tag{26}$$

where $2\pi\Lambda$ is the de Broglie wave-length of the incident particles.

For the spherical waves representing the emitted particle we evidently have

$$\Psi_{b\,(r>r_0)} = \sqrt{\left\{\frac{2l'+1}{4\pi v_b}\right\}}\,\frac{1}{r}P_{l'}(\cos\theta)\chi_b(r), \tag{27}$$

where the different l''s correspond to particles ejected with different amounts of angular momentum The functions χ_a and χ_b in (25) and (27) represent the radial solutions of the corresponding wave equations, and are, at infinity, represented by harmonic waves of unit amplitude

When there is no resonance‡ the amplitudes of the waves (25) and (27) will be reduced inside the nucleus by the penetration factor of the potential barrier, which, according to Chapter V, is given by

$$\sqrt{G} = \sqrt{\frac{v_e}{v_i}} \times e^{-\frac{1}{2}J(E,l)};$$

$$J(E,l) = 2\frac{\sqrt{2M}}{\hbar}\int\limits_{r_0}^{r^*}\sqrt{\left(U(r)+\frac{\hbar^2}{2M}\frac{l(l+1)}{r^2}-E\right)}\,dr \tag{28}$$

Here v_i and v_e must be replaced by $v_{a'}$ and v_a for the a-particles and by $v_{b'}$ and v_b for the b-particles Thus the wave functions for incident and emitted particles, inside the nucleus, can be written in the form

$$\Psi_{a\,(r<r_0)} = \sum_{l=0}^{\infty}(2l+1)\frac{\hbar}{\sqrt{(2mE_a)}}\frac{1}{\sqrt{v_{a'}}}\frac{1}{r_0}\omega_{a'}e^{-\frac{1}{2}J(E_a,l)} \tag{29}$$

and

$$\Psi_{b\,(r<r_0)} = \sqrt{\left\{\frac{2l+1}{4\pi v_{b'}}\right\}}\,\frac{1}{r_0}\omega_{b'}e^{-\frac{1}{2}J(E_b,l')}, \tag{29'}$$

where the functions $\omega_{a'}$ and $\omega_{b'}$ are again closely represented by harmonic functions of unit amplitude

† Here as $r \to \infty$, $\chi_a(r) \sim \frac{1}{k}\sin(kr-\frac{1}{2}\pi l)$.

‡ Resonance phenomena will be considered in § 4

The effective cross-section, corresponding to the transition from the original state $[\Psi_a, \Psi_{b_0}]$ to the final state $[\Psi_{a_0}, \Psi_b]$ is given, according to the general theory of perturbations, by

$$\sigma = \frac{1}{\hbar^2}\left|\int \Psi_a \Psi_{b_0} V \Psi_{a_0} \Psi_b \, d\Omega d\Omega'\right|^2, \tag{30}$$

where V is the mutual potential energy between particles a and b. In so far as Ψ_{a_0} and Ψ_{b_0} rapidly vanish for $r > r_0$, the integral in (28) can be taken only inside the nucleus. Substituting (23), (23′), (29), and (29′) into (30), we obtain for the effective cross-section the expression

$$\sigma = \sum_{ll'} \Lambda^2(2l+1)^2 \frac{2l'+1}{4\pi} W_{ll'}, \tag{31}$$

where $W_{ll'}$, which can be called the *probability of disintegration per collision*, is given by

$$W_{ll'} = \frac{1}{\hbar^2}\left|\frac{1}{\sqrt{v_{a'}}} \frac{1}{\sqrt{v_{b'}}} \frac{\Omega}{r_0^2} e^{-\frac{1}{2}J(E_a,l)} e^{-\frac{1}{2}J(E_b,l')} \bar{V}\right|^2, \tag{32}$$

with
$$\bar{V} = \frac{1}{\Omega^2}\int V \omega_a \omega_{a_0} \omega_b \omega_{b_0} \, d\Omega d\Omega' \tag{32'}$$

representing the *average* energy of interaction of the particles a and b in the nucleus. The summation in (31) must be taken only for such pairs (l, l') as (when the spins of original and final nuclei are considered) satisfy the law of conservation of angular momentum.

Remembering that $\Omega \sim r_0^3$, we can write

$$W_{ll'} \sim \frac{1}{\hbar^2} \frac{r_0^2}{v_{a'} v_{b'}} \bar{V}^2 e^{-J(E_a,l)} e^{-J(E_b,l')}, \tag{33}$$

which allows us to estimate the order of magnitude of this probability per collision. Actually we know that $v_{a_0} \sim \frac{\hbar}{m_a r_0}$, $v_{b_0} \sim \frac{\hbar}{m_b r_0}$; thus $\frac{r_0^2}{\hbar^2} \sim \frac{1}{m_a v_{a_0} m_b v_{b_0}}$. The coefficient before the exponential terms in (33) now becomes

$$\frac{r_0^2}{\hbar^2} \frac{1}{v_{a'} v_{b'}} \bar{V}^2 \sim \frac{\bar{V}^2}{m_a m_b v_{a_0} v_{a'} v_{b_0} v_{b'}} \sim \frac{\bar{V}^2}{\sqrt{(K_{a_0} K_{a'} K_{b_0} K_{b'})}}, \tag{34}$$

where K_{a_0}, $K_{a'}$, K_{b_0}, $K_{b'}$ are the kinetic energies of a- and b-particles inside the nucleus before and after transformation. Because the kinetic energies of nuclear particles must be of the same order of magnitude as their mutual potential energies, the expression

$\dfrac{\overline{V}^2}{\sqrt{(K_{a_0} K_{a'} K_{b_0} K_{b'})}}$ must be of the order of magnitude unity. This means that (due to strong interaction) the probability of disintegration per collision is given simply by exponentials expressing the transparencies of potential barriers—and the effective cross-section can reach values comparable with the square of the wave-length of the incident particle, if both incident and ejected particles possess energies corresponding to the tops of the appropriate potential barriers, or possess greater energies

Now we are in a position to compare the results of our calculations with the experimental evidence In most of the experimentally investigated cases of α- and p-bombardment the emitted particle has an energy larger than that corresponding to the top of the potential barrier,† so that only the transparency for the incident particle need be taken into account Furthermore, the additional potential barrier due to centrifugal force makes the terms corresponding to incident particles with finite angular momentum small compared with the term for 'head-on' collisions (except for those cases in which the main term is prohibited by spin conditions), so that only this term in the probability formula remains to be considered

In comparing our theoretical formula with experimental evidence we must remember that in most experiments thick foils of the bombarded substance are used, so that the increase in the observed yield of ejected particles with increasing energy of the bombarding particles is due, not only to the greater transparency of the nuclear potential barriers, but also to the deeper penetration of the incident particles into the bombarded target (and, consequently, the larger total number of collisions) In particular, after the energy of the incident particles reaches values higher than that corresponding to the top of the potential barrier of the bombarded nucleus, the increase in the observed yield must become proportional to the extra range of the incident particles (which may be given by the $\frac{3}{2}$ power of the incident energy). Thus when thick targets have been used one must either differentiate the experimental curve, constructing a graph of the

† In the case of $A+\alpha \rightarrow B+p$ transformations the potential barrier for the ejected particle (p) is only half as high as for the incident α-particle, so that, if the energy-balance has not a very large negative value, the proton goes above the barrier For most of the observed reactions $A+p \rightarrow B+\alpha$ the energy-balance is usually so large (due to the formation of a new α-shell in the nucleus) that the α-particle now goes above the barrier

derivative $\dfrac{dW}{dR}=\dfrac{dW}{d(E^{3/2})}$ against the incident energy, or integrate the expression (31), taking into account the change of energy of the incident particles in their passage through the target material.†

We must further remember that the exponential factor representing the transparency of the potential barrier, and giving the main dependence of disintegration probability on the energy of the incident particles, is very insensitive to the shape of the potential distribution inside the nucleus On the other hand, the coefficient multiplying this factor can be calculated only very roughly and depends markedly on the law of interaction between nuclear particles The value of this coefficient is also considerably affected if the incident or emitted particles possess angular momentum. Thus, in further comparison with experiment (Figs 54, 55, 56), we shall adjust this factor so as to fit the experimental curve in one point

In Fig 54 is reproduced the first test of the potential-barrier theory for artificial nuclear transformations The crosses represent the results of early measurements by Rutherford and Chadwick‡ on the yield of protons by disintegration of the Al nucleus by α-particles of different energies (as abscissae are plotted the corresponding ranges, R, of these particles) and the full curve gives the relative disintegration probability as calculated by Gamow‖ on the basis of wave theory We see that the agreement is quite good

Much more exact measurements of disintegration yields have been made for the reactions occurring under proton bombardment, these also are in good agreement with the theoretical calculations In Fig 55 are given the results of measurements by Rutherford and Oliphant†† of the yield of α-particles from very thin Li foils bombarded with protons of various velocities Fig 56 gives the analogous results of Lawrence‡‡ for thick foils of F (CaF_2) The theoretical

† Such integration has been carried out by J R Oppenheimer for comparison with Lawrence's experimental results for F^{19} For the total number of disintegrations in the thick foil Oppenheimer obtains

$$N = KE\left(1+\frac{E^{1/2}}{6}+\frac{E}{24}+\frac{E^{3/2}}{72}+\quad\right)\exp\left\{\frac{2\pi e^2}{\hbar}\left[Z\left(\frac{M_p}{2E}\right)^{1/2}+2Z'\left(\frac{M_\alpha M}{2(M_\alpha+M)}\frac{1}{Q+E}\right)^{1/2}\right]\right\},$$

where E is the energy of the incident proton, Q the energy-balance in the reaction, and K a numerical coefficient depending upon the interaction inside the nucleus

‡ E Rutherford and J Chadwick, *Phil Mag* **43** (1921), 816

‖ G Gamow, *Zs f Phys* **52** (1928), 510

†† E Rutherford and M Oliphant, *Proc Roy Soc* **141** (1933), 259

‡‡ E O Lawrence, *Phys Rev* **46** (1934), 38

curve in Fig 55 has been calculated directly from the expression (31), whereas, for the case of F (Fig 56), integration over all ranges of protons has been carried out We see that in both cases the theory is in excellent agreement with experimental data

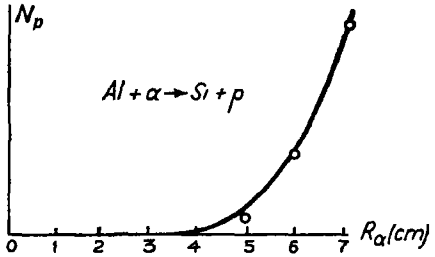


FIG 54

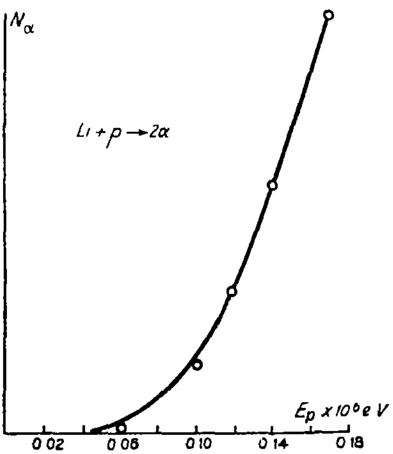


FIG 55

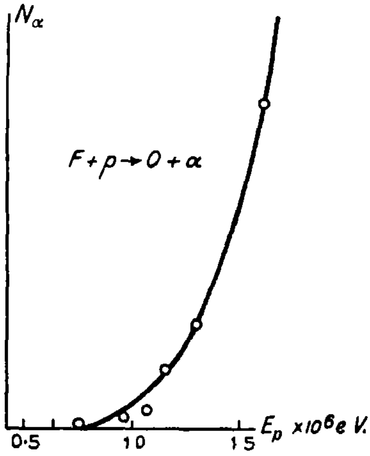


FIG 56

Somewhat elaborate calculations for the case of substitutional reactions have been carried out by Breit,† who developed exact mathematical expressions for the probability of such reactions for the simplified rectangular-hole model of potential distribution inside the nucleus on the heuristic hypothesis that the probability of disintegration is proportional to the density $\psi\bar{\psi}$ of the incident beam of particles inside the nucleus (the coefficient of proportionality P being constant for each individual case but different for different nuclei).

† G Breit, M Ostrovsky, and D Johnson, *Phys Rev* **49** (1936), 22

Comparing his calculations with the experimental evidence concerning the $_3Li^7 + _1H^1$ reaction, Breit was able to estimate the depth of the corresponding potential well Assuming for the radius of the Li nucleus the probable value $r_0 = 0\ 35 \times 10^{-12}$ cm , he concluded that the depth of the potential well must be $U_0 = -35 \times 10^6$ e v , if one supposes that the incident protons with $l = 0$ are responsible for the reaction ('head-on' collisions), or, alternatively, $U_0 = -21 \times 10^6$ e v , if the protons producing the disintegration are those with $l = 1$. It seems that we should rather adopt the second possibility ($l = 1$), particularly as the value -21×10^6 e v fits nicely with the approximate estimate of this depth from the mass defects of the light nuclei We may, however, notice here that one must be careful in estimating the accuracy of such 'exact' mathematical calculations, as they are based on very rough physical hypotheses concerning the potential distribution and the laws of interaction forces inside the nucleus All this makes the mathematical rigour of the calculations rather illusory.

As we have seen, when an element is bombarded by *neutrons* substitutional reactions leading to the emission of protons or α-particles may take place (radiative capture of neutrons will be discussed in the next chapter) In all these cases the incident neutrons penetrate without difficulty inside the atomic nucleus This does not mean, however, that such reactions can take place for all elements in the periodic table as, with increasing charge-number, the potential barrier preventing the ejected particle from escaping becomes higher and higher and its transparency rapidly decreases We must take account of the energy released in each type of reaction which may occur

Now the average energy of binding of a neutron in the nucleus can be estimated (from a comparison of the mass defects of isotopes) to be about 10×10^6 e v For an α-particle we have only about 8×10^6 e v. This shows that the substitutional reactions ($n \rightarrow \alpha$) must have, on the average, a positive energy-balance of $\sim +2 \times 10^6$ e v. In the case of substitutional reactions in which protons are emitted, on the other hand, similar considerations show that in general the energy-balance will be negative (and approximately equal to the energy of the β-particles subsequently emitted if the product is unstable, i e to a value between 1 and 5×10^6 e.v). Thus, using fast neutrons with energy $E_n \sim 7 \times 10^6$ e v , we may expect α-particles having energies $E_\alpha \sim 9\times 10^6$ e v and protons with energies from 2 to 6×10^6 e.v. For very slow neutrons ($E_n \sim 0$) the energy of the ejected α-particles should be $\sim 2\times 10^6$ e v and the emission of protons generally excluded on energy-balance conditions The transparencies, G, of nuclear potential barriers for such α-particles and protons are given in Table XV †

TABLE XV

Element	B	N	Ne	P	Ca	Mn	Zn	Zr	Sn
Z	5	7	10	15	20	25	30	40	50
G_α $E_\alpha = 9\times 10^6$ e v	1	1	1	1	0 4	0 1	0 01	10^{-4}	2×10^{-6}
G_α $E_\alpha = 2\times 10^6$ e v	0 05	0 004	5×10^{-5}	2×10^{-8}	2×10^{-10}	10^{-13}	2×10^{-17}	10^{-24}	10^{-31}
G_p $E_p = 4\ 5\times 10^6$ e v	1	1	1	1	0 7	0 4	0 2	0 04	0 01

Assuming that for $G < 0{\cdot}1$ the process cannot be detected, we come to the conclusion that with fast neutrons α-emission should stop for $Z \sim 25$ and p-emission for $Z < 35$ For slow neutrons the cross-section for collision is greater by a factor of about 5,000 than that for fast neutrons and 2×10^6 e v α-particles should be emitted. Thus the effect of α-emission with very slow neutrons should extend to those elements for which the transparency of the potential barrier begins to be smaller than $0{\cdot}1\times\frac{1}{5000} = 2\times 10^{-5}$, i e up to $Z \sim 10$

We must notice here what was first indicated by Gamow,‡ that we may expect the substitution $(n \rightarrow \alpha)$ in the case of radioactive elements emitting α-particles spontaneously, because in such cases the nuclear α-particle is already originally located on a high positive level and, in the case of capture of a fast neutron, will have enough energy to get out through the potential barrier with sufficient probability Such transformations of the nuclei of a radioelement of one family will give rise to nuclei belonging to an element of another family Thus elements of the uranium family $(4n+2)$ will be transformed into elements belonging to the actinium family $[(4n+2)+ +1-4 = 4(n-1)+3]$, and elements of the actinium family into elements belonging to the thorium family $[(4n+3)+1-4 = 4n]$. The elements of the thorium family, on the other hand, will be transformed by $(n-\alpha)$ substitution into elements of the yet unknown radioactive family of the mass-type $4n+1$ $[4n+1-3 = 4(n-1)+1]$, as, for example,

$$_{90}\mathrm{Th}^{232}+{}_0n^1 \rightarrow {}_{88}\mathrm{Ro}^{229}+{}_2\mathrm{He}^4. \tag{35}$$

† Bethe, l c ‡ G Gamow, *Uspekhi Phisicheskikh Nauk* (1934).

The occurrence of the last reaction was probably observed by Hahn and Meitner† and Curie, Halban, and Preiswerk,‡ who were able to show that the element ${}_{88}\mathrm{Ro}^{229}$ of the new family undergoes two subsequent β-transformations according to the scheme

$$ {}_{88}\mathrm{Ro}^{229} \xrightarrow[\text{(1 min)}]{\beta} {}_{89}\mathrm{RoAc}^{229} \xrightarrow[\text{(11 min)}]{\beta} {}_{90}\mathrm{RoTh}^{229}. \tag{36} $$

These β-disintegrations are probably followed by a sequence of α-transformations, but the latter have not as yet been detected.

We may notice here that when neutrons bombard radioactive nuclei the process of radiative capture (see p 217) is highly probable Bombarding uranium with neutrons Fermi observed several periods of induced β-activity and was able to show by chemical tests that at least two of the active bodies so produced have atomic numbers greater than 92 The reactions in this case most probably are

$$ {}_{92}\mathrm{U}^{238}+{}_{0}n^{1} \rightarrow {}_{92}\mathrm{U}^{239}, \tag{37} $$

$$ {}_{92}\mathrm{U}^{239} \xrightarrow[\beta]{} {}_{93}\mathrm{X}^{239} \xrightarrow[\beta]{} {}_{94}\mathrm{X}^{239} \tag{37'} $$

3. Influence of nuclear spin on disintegration probability

In the previous section we have already remarked that, just as in the case of α-disintegration, the transparency of nuclear potential barriers for the incident and ejected particles can be considerably reduced if the particle in question possess rotational momentum (non-zero azimuthal quantum number) As the rotational momenta of the particles taking part in any transformation are connected with the spin-differences between the original and product nuclei, it is clear that studying the absolute probabilities of disintegration we can draw certain conclusions as to the spins of the nuclei entering into the reaction, as was first done by Goldhaber ||

Let us first consider the disintegration of lithium by protons In this case, as we have seen above, two different reactions have been observed corresponding to the two known isotopes of this element:

$$ {}_{3}\mathrm{Li}^{7}+{}_{1}\mathrm{H}^{1} \rightarrow {}_{2}\mathrm{He}^{4}+{}_{2}\mathrm{He}^{4}, \tag{38} $$

$$ {}_{3}\mathrm{Li}^{6}+{}_{1}\mathrm{H}^{1} \rightarrow {}_{2}\mathrm{He}^{3}+{}_{2}\mathrm{He}^{4} \tag{39} $$

The number of α-particles emitted in the first reaction (long-range

† O Hahn and L Meitner, *Naturwiss* **23** (1935), 320

‡ I Curie, H Halban, and P Preiswerk, *J de Phys* **6** (1935), 361

|| M. Goldhaber, *Intern Conf on Physics*, London (1934), 163

group) is about two or three times smaller than the number of α-particles emitted in the second one (short-range groups) Remembering that the relative abundance of the lithium isotopes is, according to Aston, ${}_3\mathrm{Li}^7 : {}_3\mathrm{Li}^6 = 11 . 1$, we must conclude that the probability of the first reaction is about thirty times smaller than the probability of the second one

In so far as the potential barriers due to Coulomb repulsion must be practically identical for the two isotopes of lithium (except perhaps for a small difference due to slightly different nuclear radii), we see that the small probability of the ${}_3\mathrm{Li}^7$ reaction must be explained either in terms of the additional potential barrier of centrifugal force or by the small 'internal probability' discussed in the last section Goldhaber has pointed out that this 'internal probability' may be very small if, in the process of transformation, a proton or a neutron has to reverse the direction of its spin This conclusion is based on the fact that the interaction between the magnetic moments of these particles,

$$U \sim \frac{\mu^2}{r^3} \sim 10^3 \text{ e v}, \tag{40}$$

which might be responsible for the change of spin-direction, is very small compared with other forces acting in the nucleus

Let us consider now the spin conditions governing the ${}_3\mathrm{Li}^7 + {}_1\mathrm{H}^1$ transformation The most probable model of the ${}_3\mathrm{Li}^7$ nucleus† consists of an α-particle, a neutron-shell of two neutrons with opposite spins, and an extra proton with orbital momentum $l = 1$ and spin parallel to it, so that the total spin of the nucleus has the experimental value $\frac{3}{2}$ The spins of the incident proton and the ejected α-particle are $\frac{1}{2}$ and 0 respectively

Now, due to the symmetry of the wave functions describing α-particles, the total angular momentum of the two α-particles which are produced in the reaction must necessarily be an *even* number (0, 2, 4,.) and, from the law of conservation of angular momentum, we conclude that the total angular momentum of the proton and ${}_3\mathrm{Li}^7$ nucleus before the collision must also be even

If before the collision the proton possesses zero angular momentum relative to the nucleus (head-on collisions), this condition can only be satisfied if the spin of the proton is originally parallel to the spin

† See, for example, A Landé, *Phys Rev.* **44** (1933), 1028.

of the nucleus ($\frac{3}{2}+\frac{1}{2} = 2$), that is, to the spin of the nuclear extra proton. Remembering, however, that in each of the α-particles formed in the transformation the two protons must possess opposite spins, we see that in this case the reaction must involve the inversion of protonic spin and, therefore, be very improbable. On the other hand, considering an incident proton with spin opposite to that of the $_3Li^7$ nucleus, we must, in order to satisfy the above condition for the total angular momentum, assume that the orbital momentum about the nucleus is one unit† ($\frac{3}{2}-\frac{1}{2}\pm 1 = 0$ or 2) This, again, makes the probability of the reaction very small, due to the additional potential barrier of centrifugal force Thus we see that the spin conditions in the transformation (38) will always lead to a smaller probability than would be expected from a Coulomb potential barrier only On the other hand, if one of the α-particles is formed in an excited state with spin 1 (containing protons with parallel spins), the reaction can happen for head-on collisions also We shall return to this question in the next section

In what follows we shall be dealing chiefly with 'probable' transformations If we make the assumption that in no case is there reversal of particle spin in these reactions, we may consider that the spin components and the orbital components of angular momentum are separately balanced in them We shall treat these two components apart, therefore, throughout the discussion

From the fact that the reaction (39) is much more probable than (38) we must conclude that in this case no such prohibitions as previously considered are effective, and that the relative angular momentum of the two particles is zero both before and after the transformation As the orbital momentum of $_2He^4$ is also zero, we conclude that the orbital momenta of $_3Li^6$ and $_2He^3$ have the same value

In order to obtain more information concerning the $_3Li^6$ nucleus we now consider the reaction

$$_3Li^6+{}_1H^2 \rightarrow {}_2He^4+{}_2He^4, \tag{41}$$

which shows a large disintegration probability For the deuteron nucleus we adopt (see p 57) a model consisting of a neutron and a proton with parallel spins ($n\uparrow$, $p\uparrow$) and zero orbital momentum From the large probability of the reaction (41) we must again con-

† Or $1+2n$, where n is a whole number

clude that the relative angular momentum of the two particles is zero before and after† collision, i e. that the orbital part of the $_3Li^6$ spin is zero

Now, in order to be able to obtain a closed shell for the α-particle $(n\uparrow n\downarrow p\uparrow p\downarrow)$ which is formed we must assume that the extra neutron and extra proton in the $_3Li^6$ nucleus have parallel spins Thus we come finally to the conclusion that the spin of $_3Li^6$ must be 1 $(\alpha+n\uparrow+p\uparrow$, with no orbital momentum) and that its magnetic moment must be of the same order as that of the deuteron (i e very small) A similar conclusion regarding the magnetic moment is necessary to explain the failure to observe hyperfine structure in the optical spectrum of this isotope

We shall consider next the reactions

$$_1H^2+{}_1H^2 \rightarrow {}_1H^3+{}_1H^1, \tag{42}$$

$$_1H^2+{}_1H^2 \rightarrow {}_2He^3+{}_0n^1, \tag{43}$$

which are about equally probable The experimental evidence seems to show that one or other transformation takes place in almost all collisions, from which we must conclude that, just as in the case of (39) and (41), we are dealing with zero angular momentum before and after transformation Remembering that the orbital momentum of $_1H^2$ is zero, we deduce that the same is true for $_1H^3$ and $_2He^3$ In so far as the two neutrons in $_1H^3$ and the two protons in $_2He^3$ are assumed to have opposite spins, we should expect that the total spin of each nucleus is $\frac{1}{2}$, whilst the magnetic moments are of the same order as for a free proton and a free neutron respectively

Finally, let us consider the reactions

$$_5B^{10}+{}_1H^2 \rightarrow 3\,{}_2He^4, \tag{44}$$

$$_5B^{11}+{}_1H^1 \rightarrow 3\,{}_2He^4, \tag{45}$$

both of which show high disintegration probability Remembering that the spins of neutrons and protons in $_2He^4$ constitute two anti-parallel pairs, we must conclude that the spin momenta of $_5B^{10}$ and $_5B^{11}$ are 1 and $\frac{1}{2}$ respectively Since the velocities of the α-particles formed in the reaction (45) are not very large, the large probability of disintegration indicates that all orbital momenta are zero, from what we have already said, therefore, the total spin of $_5B^{11}$ is $\frac{1}{2}$. We cannot draw similar conclusions for (44) as here the energy-balance

† Due to the identity of the two α-particles, the angular momentum after collision must be even The value 2, however, would give a large decrease in probability

is too great, and the α-particles are liberated with large velocities. The results of Goldhaber's analysis are summarized in Table XVI.

TABLE XVI

Nucleus	Spin momentum	Orbital momentum	Total	Order of magnetic moment
$_1H^3$	$\frac{1}{2}$	0	$\frac{1}{2}$	same as p
$_2He^3$	$\frac{1}{2}$	0	$\frac{1}{2}$	,, n
$_3Li^6$	1	0	1	,, $_1H^2$
$_5B^{10}$	1	(0)?	(1)?	
$_5B^{11}$	$\frac{1}{2}$	0	$\frac{1}{2}$	same as p

This method of estimation of nuclear spin is of great importance for unstable β-active nuclei produced by artificial transformations, especially where the same nucleus can be produced in many different ways (for example $_{13}Al^{28}$ which, as can be seen from Table G, can be produced in five different ways) Unfortunately, in most cases experimental data are not complete enough to permit such estimates

4. Resonance disintegration

It was first indicated by Gurney† that in cases in which the energy of the bombarding particles is close to one of the nuclear virtual energy-levels (of positive energy) the *phenomenon of resonance* may be expected, resulting in a large increase of the reaction probability We can get a simple idea as to how such phenomena can occur by considering the simplified rectangular model of potential barrier (Fig 57) for which calculations can be easily carried through ‡ The general solution of the wave equation can, as usual, be written in the form

$$\psi = \frac{1}{r}\chi(r)P_l(\cos\theta)e^{i\frac{E}{\hbar}t}, \tag{46}$$

where $\chi(r)$ satisfies the equation

$$\frac{d^2\chi(r)}{dr^2}+\frac{2M}{\hbar^2}\left[E-U-\frac{\hbar^2}{2M}\frac{l(l+1)}{r^2}\right]\chi(r) = 0 \tag{47}$$

We shall consider, for simplicity, only the solution for $l = 0$ This can be written in the form

$$\chi(r) = A_+\sin\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r+A_-\cos\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r \quad (\text{for } r_0 < r < r_1), \tag{48}$$

† R W. Gurney, *Nature*, **123** (1929), 565

‡ R H Fowler and A. H Wilson, *Proc Roy Soc* **124** (1929), 493

$$\chi(r) = B_+ e^{+\frac{\sqrt{(2M)}}{\hbar}\sqrt{(U_0-E)}r} + B_- e^{-\frac{\sqrt{(2M)}}{\hbar}\sqrt{(U_0-E)}r} \quad (\text{for } r_1 < r < r_2), \tag{48'}$$

$$\chi(r) = C_+ \sin\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r + C_- \cos\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r \quad (\text{for } r_2 < r), \tag{48''}$$

and we have to satisfy the condition of finiteness of ψ at $r = 0$, which gives $\chi(0) = 0$ or $A_- = 0$ Satisfying next the conditions of continuity of $\chi(r)$ and $d\chi(r)/dr$ at the boundaries $r = r_1$ and $r = r_2$, we find that for $r > r_2$ we shall have two waves (divergent and convergent) of equal amplitude and representing the incident and

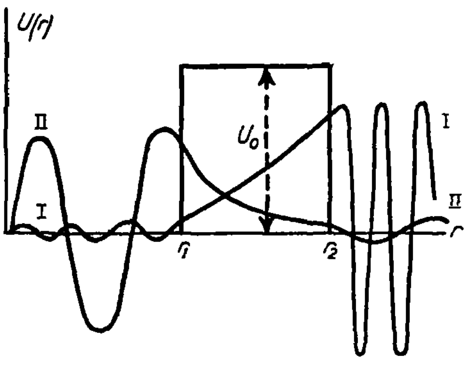


FIG 57

scattered particles with zero angular momentum For the ratio of intensities inside and outside the nucleus we obtain

$$\left|\frac{A}{C}\right|^2 = 16\Big/\left[\delta\left\{1+i\sqrt{\left(\frac{U_0-E}{E}\right)}\right\}\left\{\sin\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r_1 + \right.\right.$$
$$\left. +\sqrt{\left(\frac{E}{U_0-E}\right)}\cos\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r_1\right\} + \delta^{-1}\left\{1-i\sqrt{\left(\frac{U_0-E}{E}\right)}\right\}\times$$
$$\left.\times\left\{\sin\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r_1 - \sqrt{\left(\frac{E}{U_0-E}\right)}\cos\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r_1\right\}\right]^2, \tag{49}$$

with
$$\delta = e^{\frac{\sqrt{(2M)}}{\hbar}\sqrt{(U_0-E)}(r_2-r_1)}, \tag{49'}$$

which shows that, in general, the density inside the nucleus will be $e^{\frac{2\sqrt{(2M)}}{\hbar}\sqrt{(U_0-E)}(r_2-r_1)}$ times smaller than outside If, however, for certain values of E

$$\sin\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r_1 + \sqrt{\left(\frac{E}{U_0-E}\right)}\cos\frac{\sqrt{(2M)}}{\hbar}\sqrt{E}\,r_1 = 0, \tag{50}$$

the density inside becomes by the same factor larger than outside, which means that, for those energies, the transparency of the barrier becomes unity The two types of possible solution—for arbitrary and for resonance energies—are indicated in Fig 57 by curves I and II. It is easy to show from (49) that the breadth of resonance is given by

$$\frac{\Delta E}{E} = \frac{1}{\delta^2} = e^{-\frac{2\sqrt{(2M)}}{\hbar}\sqrt{(U_0-E)}(r_2-r_1)}. \tag{51}$$

These considerations show us that, in cases in which the energy of the bombarding particles is close to one or other resonance energy, the effective cross-section for disintegration may increase up to a value $\sim \Lambda^2$ (Λ is the de Broglie wave-length of the incident particles)

A somewhat more elaborate study of the theory of resonance disintegration has been given by Mott,† who has shown, in particular, that the breadth of the resonance maximum depends on the azimuthal quantum number of the resonance-level Thus a detailed study of resonance disintegration may give us some indication about the quantum numbers of nuclear energy-levels One point should be particularly stressed at this stage the calculations given above can only be used to get a general idea as to the process of resonance in nuclear transformations, the real process being much more complicated For we have considered in the above calculations the penetration of the incident particle inside the nucleus as independent of the process of ejection of the disintegration particle This should not actually be done, on account of the *very strong* interaction between the two particles in question One result of such a treatment is to give rise to the wrong idea that there can, in fact, be two kinds of resonance-level one corresponding to the entrance of the incident particle and the other to the exit of the nuclear particle ejected in the process Due to the very strong interaction of particles inside the nucleus, however, the distinction between the levels of the separate particles becomes rather illusory, and one should better speak about the energy-levels of the excited nucleus as a whole This will not considerably change our results concerning the efficiency and the breadth of resonance, but the conditions necessary for resonance disintegration must now be formulated in the following general way *If we consider the nuclear reaction $A+a \rightarrow C \rightarrow B+b$, the phenomenon of resonance will take place if the nucleus C, representing the intermediate state of transformation, possesses energy corresponding to one*

† N F Mott, *Proc Roy Soc* **133** (1931), 228.

of its quantum levels. From this it follows, for example, that if for certain velocities of the incident α-particle resonance takes place, it will also take place if we bombard the product nucleus with protons having the same energy as those emitted in the first reaction (we neglect here the recoil energy).

We shall now turn our attention to the experimental evidence for resonance phenomena The first indication of a large increase in the yield of emitted particles for certain discrete energies of the bombarding particles was found by Pose† in his experiments on

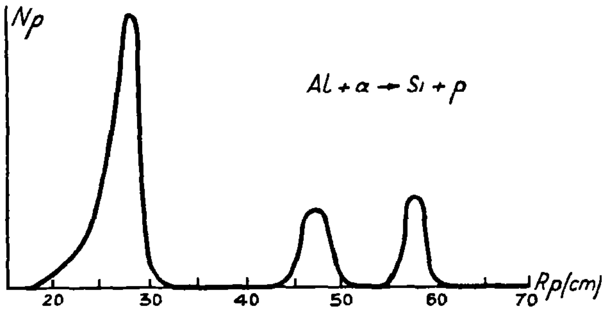


FIG 58

the artificial transformation of aluminium under bombardment by α-particles In these experiments Pose used thick layers of Al ($\sim$ 0·04 mm), so that the incident α-particles were gradually losing energy and were finally stopped before escaping on the other side of the target In such a case we should expect that the protons produced by collision of α-particles with Al nuclei at different depths should have different velocities, so that their observed energy-distribution would be in the form of a continuous spectrum extending from an upper limit determined by the original energy of the α-particle to much smaller values The measurements indicated, however, that the ejected protons belonged to several more or less discrete groups, as shown in Fig 58, where the number of ejected protons is plotted against their range. This observation can be explained only on the hypothesis that the observed groups of protons were produced at a number of particular depths in the Al target, evidently the depths at which the α-particles (continuously slowing down) have energies just corresponding to resonance penetration.

† H. Pose, *Zs f Phys.* **30** (1929), 780, **64** (1930), 1.

A somewhat more detailed study of the resonance phenomena with aluminium was carried out by Chadwick and Constable,† who found a still larger number of discrete groups than was originally observed by Pose It was shown that to each resonance-level correspond several proton groups—as if the α-particle entering the nucleus with a particular resonance-energy might be captured on different inside levels and consequently give rise to an ejected proton of one of a number of different energies ‡ The results of more recent investiga-

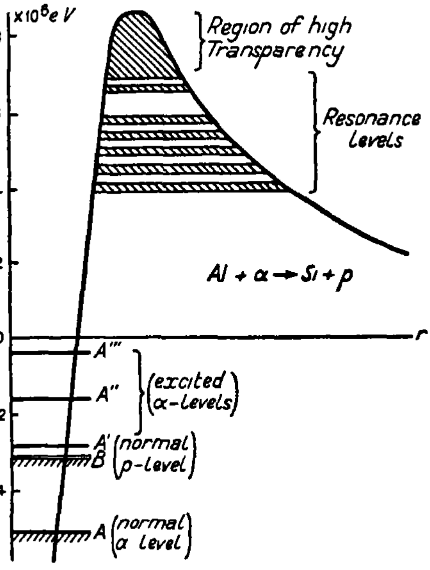


FIG 59

tions by Duncanson and Miller|| are summarized in Fig 59, where the resonance-levels and the stationary levels for the α-particle are shown, together with the fundamental level of the proton before its ejection We see that an incident α-particle may enter the nucleus through any one of six resonance-levels and that in each case four distinct modes of capture are possible—the particle may, first of all, occupy any one of the levels α_0, α_1, α_2, or α_3 Thus, with a beam of α-particles bombarding an aluminium target, four discrete groups of protons are produced for each of six resonance-energies of the α-particles. The shaded region near the top of the potential barrier in

† J Chadwick and J Constable, *Proc Roy Soc* **135** (1932), 48 ‡ Compare p 183.

|| W Duncanson and H Miller, *Proc Roy Soc* **146** (1934), 396.

Fig. 59 is the region of high transparency through which an α-particle may enter the nucleus for a wide range of energies: the corresponding feature of the protons ejected from a thick aluminium target bombarded by α-particles is a more or less continuous energy spectrum.

In the case of disintegrations produced by proton bombardment no definite indications of an increase in the yield of α-particles for definite discrete values of proton energy have as yet been reported. This simply means that, with the nuclei investigated, no resonance-

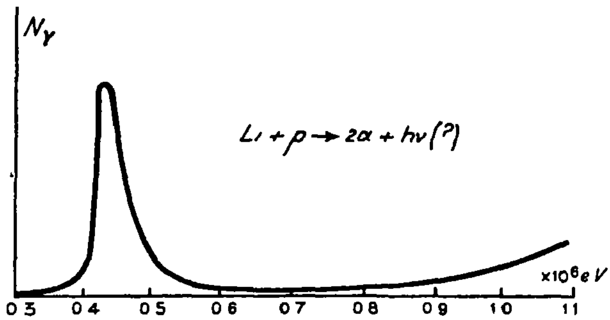


Fig 60

levels are located in the range of proton energies used in the experiments It remains probable that further experiment will bring to light resonance phenomena in this case also Meanwhile, we should notice an effect which, though not yet quite clear, may be due to resonance penetration of the incident protons As we have already mentioned, the yield of α-particles from Li increases smoothly with the energy of the incident protons, showing no trace of resonance phenomena in the range investigated The α-particle reactions are, however, according to Lauritsen,† accompanied by strong γ-radiation, and this, according to Tuve and Hafstad,‡ shows quite a definite increase for certain discrete proton energies (Fig 60). However, the origin of these γ-rays is not yet quite clear They may be due to direct radiative capture of a proton by the bombarded nucleus (which is certainly the case for carbon, as will be discussed in the next chapter), but it is also very likely that they are due to the formation of an α-particle in an excited state, according to the equation

$$_3\mathrm{Li}^7 + {_1\mathrm{H}^1} \rightarrow {_2\mathrm{He}^4} + {_2\mathrm{He}^4}(\mathrm{exc.}). \tag{52}$$

† C Lauritsen, H Crane, L Delsasso, and W Fowler, *Phys Rev* **46** (1934), 531

‡ L Hafstad and M Tuve, ibid **48** (1935), 306

It was indicated by Feenberg† that the excitation energy of an α-particle is probably of the order of magnitude 16×10^6 e v., and this, being subtracted from the energy-balance of the reaction ($17 \cdot 1 \times 10^6$ e.v.), would give very small kinetic energies to the two α-particles formed according to (52) (This would account for the fact that these α-particles have not been observed experimentally) If this explanation be accepted, we naturally conclude that the observed maxima of γ-ray intensity are due to the resonance penetration of the incident protons But we are left to account for the fact that resonance does not occur in the reaction leading to the formation of two normal α-particles, but does occur if one of these α-particles is formed in an excited state This might be done on the basis of spin considerations discussed in the last section Regarding the reversal of particle spin as entirely excluded, we have already seen that the normal reaction is produced only by protons with spin axes oppositely directed to the axis of spin of the $_3Li^7$ nucleus, and that from this it follows that only incident P protons (with angular momentum $l = 1$) are responsible for the reaction We do not know the relative directions of the spins of constituent particles in an excited α-particle, but if we make the hypothesis that in α_{exc} the two protons have parallel spins, it will follow immediately that the reaction leading to the formation of an excited α-particle is due solely to incident protons with spins *pointed parallel* to the spin of the $_3Li^7$ nucleus The spin part of the total angular momentum being now even ($\frac{3}{2}+\frac{1}{2} = 2$), we must conclude that incident S protons (with angular momentum 0) alone can be responsible for the reaction

Thus the absence of resonance phenomena in the normal reaction and their presence in the 'reaction with excitation' may now be interpreted by saying that in the range of proton energies investigated is located an S resonance-level but no P resonance-level

Analogous considerations may be applied to the F+H reaction, which, according to Tuve and Hafstad,‡ also shows some sharp maxima in the intensity of the γ-rays which are emitted

We must remember, however, that all these considerations remain purely hypothetical until the real origin of the observed γ-rays has been definitely established

† E Feenberg, *Phys Rev* **49** (1936), 328

‡ M Tuve and L Hafstad, l c.

5. Disintegration '*en passant*' and 'exchange reactions'

In previous sections we have discussed the most frequent modes of nuclear transformation, in which the incident particle, entering through the potential barrier into the nucleus, stays there, giving all its energy to another nuclear particle which is consequently ejected

There are, however, other types of nuclear reaction in which the incident particle is not necessarily captured We shall first consider the case in which the energy of the incident particle is greater than

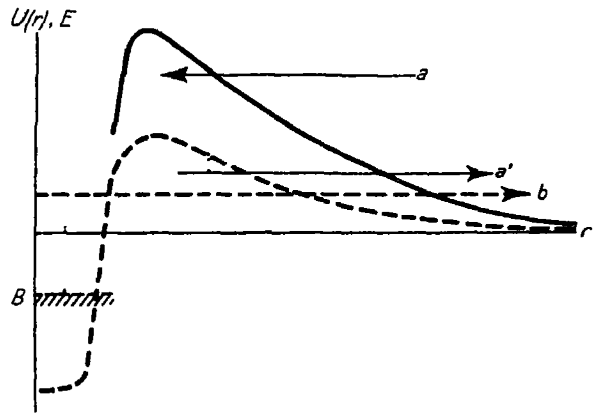


FIG 61

the energy necessary to remove the nuclear particle In such cases it is always possible that the nuclear particle will be ejected from the nucleus in the process of collision and the incident particle escape with the rest of the energy (Fig 61) Such reactions, which are of course always characterized by a negative energy-balance, we shall call disintegrations '*en passant*' This process is in many ways analogous to the strong excitation by collision, discussed in the last section of Chapter IX, if excitation is followed by the emission of a particle —with this difference, that in this case the energy can be randomly partitioned between ejected and inelastically reflected particles

The probability of the process can be roughly estimated as the product of the excitation probability given by formula (45) on p 179 and the transparency of the potential barrier for the ejected particle Thus the probability of the process depends essentially on the factor

$$e^{-[I(E_a)-I(E_{a'})]}\times e^{-I(E_a-E_{a'}-\Delta E_l)}, \tag{53}$$

where E_a and $E_{a'}$ are the energies of the exciting particle, a, before and after collision, and ΔE_b is the energy necessary to extract the particle b from the nucleus Remembering that the average binding energies of nuclear particles in light nuclei is usually about 10×10^6 e.v. and that the fastest projectiles (α-particles from ThC′) have energies not above 9×10^6 e v , we should conclude that disintegrations of this type are in most cases either energetically excluded or, because they involve reflected and ejected particles of very small energy, are characterized by extremely small disintegration probabilities according to (53) In fact, nuclear reactions of this type have never yet with certainty been observed

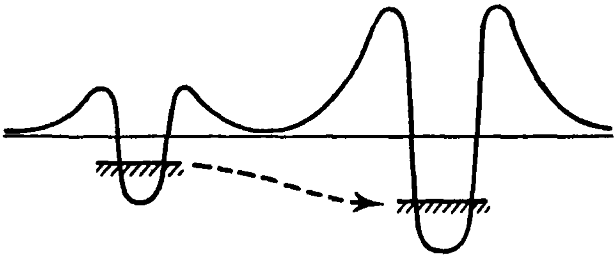


FIG 62

Another, somewhat peculiar, type of nuclear reaction has been discussed by Oppenheimer,† it may be said to take place with 'partial capture' of the incident particle Suppose that the incident particle is a complex particle (such as a deuteron or an α-particle) One can emphasize the possibility that, in the process of collision, one of the constituent parts of the incident particle may be detached from it and captured by the bombarded nucleus—'leaking' through the barrier separating its normal state in the incident particle from the corresponding normal state in the bombarded nucleus (Fig 62) Such an exchange process will give rise to two product nuclei which will then move apart with an energy determined by the energy-balance appropriate to the reaction Exchange processes of this type may be of particular importance if the particle to be exchanged is a neutron, as in this case there are no additional potential barriers of Coulomb force preventing the transition For example, if the nucleus ${}_Z X^M$ is bombarded by deuterons (${}_1H^2 = n+p$), the neutron can be captured by the nucleus and the proton escape with an excess energy determined by the difference of binding energy for a neutron in ${}_Z X^M$ and in ${}_1H^2$ respectively We may notice that, from the point

† R Oppenheimer and M Phillips, *Phys. Rev* **48** (1935), 500

of view of the product and the energy-balance in the reaction, such a process can be considered as an ordinary substitutional reaction in which a deuteron enters the nucleus and is captured, and a nuclear proton is emitted. However, there is an important difference in the process itself which leads to a different value for the disintegration probability. Qualitatively, we may understand this difference as follows According to the 'exchange' point of view, transformation becomes possible when the neutron in the deuteron comes within range of the nuclear exchange forces, a substitutional reaction is impossible until the centre of mass of the whole deuteron enters the nucleus. Clearly, owing to the finite size of the deuteron, the former condition is frequently fulfilled when a substitutional reaction would still be impossible In the quantitative treatment of the exchange process we should notice that the velocity of the incident deuteron is small compared with the velocities of particles inside the nucleus, so that the effective time of collision is *large* compared with the periods of oscillation of the intranuclear particles This consideration allowed Oppenheimer to treat the transition of a neutron from one of the colliding nuclei to the other as an adiabatic process, i e. to consider the transition as taking place whilst the distance apart of the deuteron and bombarded nucleus remained constant After rather troublesome calculations Oppenheimer finally obtained the following expression for the effective cross-section

$$\sigma \sim \Lambda^2 e^{-\frac{4e^2 z}{\hbar}\left(\frac{M}{\epsilon}\right)^{\frac{1}{2}} \overline{F\left(\frac{E}{\epsilon}, \frac{\epsilon r}{ze^2}\right)}}. \tag{54}$$

Here F is the average taken for different relative distances, r, between the deuteron and the nucleus, E and Λ are the energy and the de Broglie wave-length of the incident deuteron, ϵ is the deuteron internal binding energy ($\epsilon = 2\ 2 \times 10^6$ e v.), and the function $F(x, y)$ is defined by

$$F(x,y) = yA\left(\frac{l}{y}-1\right)+\frac{1}{\sqrt{x}}B(x,y), \tag{55}$$

where
$$A(y) = \frac{1+y}{\sqrt{y}}\cot^{-1}\sqrt{y}-1,$$

$$B(x,y) = B(z) = \cos^{-1}\sqrt{z}-[z(1-z)]^{\frac{1}{2}} \qquad (z > 1)$$

$$= [2(1-z)]^{\frac{1}{2}}-1+\frac{1+z}{\sqrt{z}}\left[\tan^{-1}\sqrt{\left(\frac{2z}{1-z}\right)}-\tan^{-1}\sqrt{z}\right] \qquad (0 < z < 1)$$

$$= [2(1-z)]^{\frac{1}{2}}-1+\frac{1+z}{\sqrt{(-z)}}\left[\tanh^{-1}\sqrt{\left(\frac{-2z}{1-z}\right)}-\tanh^{-1}\sqrt{(-z)}\right] \quad (z < 0).$$

According to Oppenheimer these expressions can be evaluated numerically in a finite time. In Fig. 63 (*a*, *b*) the excitation curves for the reactions of this type with Al and Cu are shown as obtained by Lawrence † The dotted curves represent the rise of probability as expected from the ordinary theory of potential barriers (formula

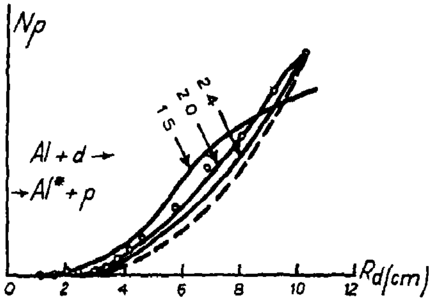


FIG 63 (*a*)

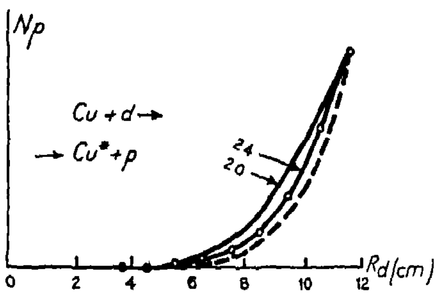


FIG 63 (*b*)

(28)), we see that they give too rapid an increase of disintegration probability The curves calculated from (54) with different values for the internal energy of the deuteron, however, fit the experimental points much better In the case of Al the best fit is for $\epsilon = 2{\cdot}0\times10^6$ e v, and for Cu for $\epsilon = 2{\cdot}4\times10^6$ e v Taking into account the approximate character of the calculations we may say that general agreement can be reached in all cases if we use the currently accepted value $2{\cdot}2\times10^6$ e v for the binding energy of the deuteron nucleus

† E O Lawrence, E. McMillan, and R Thornton, *Phys Rev.* **48** (1935), 493

XI

NUCLEAR REACTIONS ESSENTIALLY INVOLVING RADIATION

1. Radiative capture

IF an incident particle penetrates inside a nucleus it need not necessarily eject another nuclear particle (substitutional reaction), since the excess energy can also be got rid of by the emission of a γ-quantum. We shall call such a process *radiative capture*

In the cases where the bombarding particles are neutrons the process of radiative capture becomes very important especially for heavier elements where, due to the height of potential barriers preventing the ejection of charged particles, the emission of a γ-ray is the only way to get rid of the extra energy brought in by the incident neutron

The first, and the most simple, example of such a process was reported by Lee,† who observed that hydrogen bombarded by neutrons emits a strong, hard γ-radiation and interpreted this result as due to formation of deuterons according to the reaction

$$_1\mathrm{H}^1 + {_0n^1} \rightarrow {_1\mathrm{H}^2} + h\nu. \tag{1}$$

The energy of such reaction must be $h\nu = \frac{1}{2}E+\epsilon = \frac{1}{2}E+2\ 2\times 10^6$ e v. (where E is the energy of incident neutrons) According to Lee the effective cross-section for this process is of the same order of magnitude as the geometrical dimensions of the nuclei.

It was later shown by Fermi and his collaborators‡ that practically all elements, especially heavy ones, being bombarded with neutrons capture the incident neutrons, thus forming the heavier isotope, and emit the surplus energy in the form of γ-radiation, for all such processes the effective cross-sections turn out to be also of the order of magnitude of the nuclear cross-section. The understanding of these comparatively high capture probabilities presented for a period of time serious difficulties, as the theoretically calculated cross-section for the velocities of neutrons used in these experiments lead to considerably smaller values ($\sigma \sim 10^{-29}$ cm.^{2}).

The important step towards the understanding of the large probability of radiative capture of neutrons was made by Fermi's‡

† D Lee, *Nature*, **133** (1934), 24.

‡ E Fermi and others See reference p 190.

discovery that, in those cases in which activation can only be due to radiative capture, the intensity of activation is usually considerably increased if during irradiation the bombarded substance is surrounded by a layer of hydrogen-containing material (such as water, paraffin wax, etc.) This fact, together with Lee's observation of the unexpectedly large effect of neutrons in hydrogen itself, led Fermi to the important conclusion that radiative capture is very much more probable in collisions of atomic nuclei with *very slow* neutrons, the observed increase of activity when irradiation takes place in paraffin being due to the slowing down of neutrons by collisions with hydrogen atoms. In fact, due to the almost equal masses of neutron and proton, each such collision will reduce the kinetic energy of the neutron by a factor $\sim 1/e$, so that neutrons with initial energies of 4×10^6 e.v., after 10 collisions each, will retain only $\frac{1}{20{,}000}$ of this, i.e. about 200 e.v., and less than 20 collisions will be necessary to bring them into *thermal equilibrium* with materials at ordinary temperatures. In so far as the effective cross-section increases rapidly with the increasing de Broglie wave-length, which is very large for thermal velocities, it is plausible to assume that the cross-section for radiative capture may become very large for such slow neutrons. Parallel with the increase in activation is the increase in absorption coefficient for slowed-down neutrons, thus, for example, in the case of boron the absorption of slow neutrons is 1,000 times larger than that of fast neutrons. The corresponding collision cross-section has the surprisingly large value $\sigma = 3{,}000\times10^{-24}$ cm.^{2} Actually, still larger values have been recorded for some other elements, as, for example, $\sigma = 7{,}000\times10^{-24}$ cm.^{2} for yttrium and $\sigma = 10{,}000\times10^{-24}$ cm.^{2} for cadmium. We must notice here, however, that the radiative capture of slow neutrons varies very capriciously from element to element, several metres of lead, for example, absorb *less* than a millimetre of boron. If we assume, then, that slow neutrons (of thermal velocities) are especially effective in the process of radiative capture by atomic nuclei, just because the large cross-sections can, in this case, be understood theoretically in terms of large values of the de Broglie wave-length, it seems natural to suppose that the radiative capture processes observed with the ordinary fast neutrons are actually due to an admixture of slow neutrons in the admittedly unhomogeneous radiation used in 'fast-neutron' experiments.

In the case of the reaction $(H+n)$ the explanation based on the hypothesis that the large capture-probability is due to neutrons slowed down in the previous collisions with other hydrogen nuclei, meets another serious difficulty in the considerations concerning the conditions governing angular momenta of colliding particles. This difficulty may be stated as follows As we have seen, it is usually accepted that the observed spin ($i = 1$) of a deuteron arises from neutron and proton with parallel spins but no orbital momentum (S state). If, then, particle spin is not reversed in any nuclear reaction (cf. § 3 of the last chapter), it would follow from this assumption that the incident neutrons which are radiatively captured by protons are those having unit angular momentum (in respect of the proton) before the collision (P neutrons), the transition between an S state of the continuous spectrum and an S state of the stable deuteron being prohibited for electric radiation One can easily understand, however, that for very slow neutrons of angular momentum different from zero the probability of collision is vanishingly small (the wave functions describing such states of the continuous spectrum vanish rapidly for $r \to 0$), so that we should not expect to observe any capture effect in this case It was first pointed out by Fermi† that these conclusions no longer hold if we take into account the possibility of *magnetic radiation* connected with the inversion of spin of one of the two colliding particles, in fact, the incident S neutron, having originally its axis of spin antiparallel to that of the proton, can be captured into the stable S state of the deuteron nucleus, if in the process of capture the direction of neutron—or proton—spin is reversed and energy is liberated in the form of magnetic radiation Straightforward electromagnetic theory gives for the inverse mean life of a neutron moving with a velocity v in a medium containing n protons per unit volume the following expression

$$\frac{1}{\tau} = n\frac{128\pi^5 h\nu^3}{c^3m^2v^2}\mu_0^2(g_p-g_n)^2\left|\int f\phi r^2\,dr\right|^2, \tag{2}$$

where ν is the emitted frequency (of magnetic dipole radiation) and $\mu_0 g_p$ and $\mu_0 g_n$ are the magnetic moments of proton and neutron respectively (μ_0 is the nuclear magneton) The wave function $f(r)$ corresponds to the fundamental state of the deuteron nucleus and $\phi(r)$ to the state in the continuous spectrum representing the incident

† E Fermi, *Phys Rev* **48** (1935), 570 Compare also E Wigner, H Bethe, R Peierls, and E Teller, see reference p 173

neutron. Evaluating the integral in (2), Fermi obtains

$$\frac{1}{\tau} = \frac{2\epsilon^{\frac{1}{2}}}{\hbar^3 c^3 m^{\frac{1}{2}}} \frac{\mu_0^2(g_p - g_n)^2}{l}, \tag{3}$$

where ϵ is the binding energy of the deuteron ($\epsilon = 2\cdot 2 \times 10^6$ e v.) and l is the free path for elastic scattering of slow neutrons in hydrogen. Inserting numerical values in (3), we have for the inverse mean life of a neutron passing through hydrogen $\frac{1}{\tau} = 5\cdot 2 \times 10^{+3}$ sec $^{-1}$, in good agreement with the experimental estimate of this quantity:

$$\sim 10^4 \text{ sec}^{-1}$$

Turning our attention to the process of radiative capture of slow neutrons by heavier nuclei we find that capture with the emission of electric dipole radiation is in general possible—except in some cases (analogous to that of capture by protons, already discussed) in which considerations of angular momentum make such capture impossible † The general theory of the capture of slow neutrons by heavier nuclei—each considered as a potential hole with steep walls—was developed independently by Perrin and Elsasser‡ and by Bethe ||

The effective cross-section for capture is given in this case by the ordinary formula

$$\sigma = \frac{4(2\pi\nu)^3}{3\hbar c^3}\left|\int \phi f R \, d\tau\right|^2, \tag{4}$$

where again ϕ and f are the wave functions for the initial and final states and R is the electric moment of the system with respect to the centre of gravity If the nucleus in question possess mass M and charge Ze (the charge of the neutron is 0), we have

$$R = -e'r, \qquad e' = e\frac{Z}{M+1}, \tag{5}$$

where r is the distance between the centre of the nucleus and the neutron. Developing the incident normalized wave into spherical harmonics, and taking only the first term (corresponding to head-on

† The necessary condition for the possibility of electric dipole capture of a slow neutron by a nucleus possessing total orbital angular momentum L is that the product nucleus shall possess a state with orbital angular momentum L or $L \pm 1$ (except $L = 0 \rightarrow L = 0$) and an energy lower than that of the original nucleus and the neutron at rest

‡ F Perrin and W Elsasser, *J de Phys* **6** (1935), 194

|| H Bethe, *Phys. Rev* **47** (1935), 747

collisions), one can bring the expression (4) into the form

$$\sigma = 4\pi^2\frac{e'^2}{\hbar c}\frac{[\hbar(2\pi\nu)]^2}{Mc^2}\Lambda\Lambda_0\frac{f}{\{\sin^2\phi_0+(\Lambda_0^2/\Lambda^2)\}}, \tag{6}$$

where Λ and Λ_0 are the de Broglie wave-lengths of the neutron outside and inside the nucleus, ϕ_0 is the phase at $r = r_0$, and f is defined by

$$f = \frac{4\pi\nu M}{\hbar}|z|^2, \tag{7}$$

z being the matrix element of the distance between neutron and nucleus. After making a number of approximations and inserting numerical values, Bethe finally obtained

$$\sigma = \frac{\Lambda\Lambda_0}{2300}\frac{1}{\{\sin^2\phi_0+(\Lambda_0^2/\Lambda^2)\}}. \tag{8}$$

For slow neutrons $\Lambda_0 \ll \Lambda$, and the term $(\Lambda_0/\Lambda)^2$ can in general be neglected as compared with $\sin^2\phi_0$ Then (in so far as Λ_0 is almost constant) the cross-section σ varies as Λ, i e. as the inverse velocity of the incident neutrons For neutrons of thermal velocities we have $E = kT = \frac{1}{40}$ volt (at room temperature), and for the corresponding Λ we obtain $\Lambda = 2\ 9\times10^{-9}$ cm The value of Λ_0 is of the same order of magnitude as the nuclear radius, Bethe accepts $\Lambda_0 = 0{\cdot}24\times10^{-12}$ cm. With these values we have

$$\sigma = 0{\cdot}3\times10^{-24}\frac{1}{\sin^2\phi}\ \text{cm}^2 \tag{9}$$

Thus we see that for slow neutrons the cross-section for electric capture is generally of the same order of magnitude as the geometrical cross-section of the nucleus (for $\sin^2\phi \sim 1$), but can reach much larger values if $\phi \sim 0$ The probability of such a resonance effect has already been discussed in the section dealing with elastic scattering of neutrons (Chap IX, § 3), and we have seen that according to an estimate of Bethe in about 10 per cent. of all cases the cross-section may be larger than 100×10^{-24} cm ^{2} This is in good agreement with experimental evidence, as can be seen from Table XIV (Chap. IX), where two elements out of sixteen (namely, Cd and Hg) have a very large absorption coefficient, which, as has been indicated, must be due to high probability of radiative capture in these cases.

We must remember, however, that these large values of capture cross-sections are not accompanied by corresponding increase of scattering (compare the last two columns of Table XIV, Chap. IX), which is in direct contradiction to the theory which requires that $\sigma_{\mathrm{scatt}}/\sigma_{\mathrm{capt}} = 2$.

It was indicated by Bohr† that this fundamental discrepancy is most probably due to the failure of ordinarily used methods of calculation of nuclear processes in which the incident particle is considered as moving in the average field of the rest of the nucleus (Hartree method) In fact, due to very strong interaction of intranuclear particles, we shall be much closer to the truth if we consider the energy of the incident neutron as being rapidly distributed between all nuclear constituent parts even before this neutron is able to cross the diameter of the nucleus From such point of view the scattering of the incident neutron must rather be considered as the 're-emission' taking place when the originally dissipated energy is again accumulated in a single particle If the energy of the incident neutron is small compared with its binding energy in the nucleus (slow neutrons), such reaccumulation of energy in a single particle becomes very improbable and in most cases the radiative capture (and not re-emission) takes place

Experimental evidence for radiative capture of a charged incident particle by a bombarded nucleus was first furnished by Cockcroft,‡ who noticed that a carbon target bombarded by an intense beam of protons becomes positron-active, the activity being evidently due to the formation of $_7N^{13}$ by the reaction||

$$_6C^{12}+{}_1H^1 \rightarrow {}_7N^{13}+h\nu, \tag{10}$$

$$_7N^{13} \rightarrow {}_6C^{13}+\partial \tag{11}$$

The chance that a charged particle will be captured by atomic nucleus depends on two factors the probability that it penetrated the nuclear potential barrier due to Coulomb forces, and the probability that it will emit γ-quanta during its stay inside the nucleus The chance of penetration has already been discussed in the previous chapter, one would expect that such penetration will happen with appreciable probability only for light elements The chance that the

† N Bohr, *Nature*, **137** (1936), 344

‡ J D Cockcroft, C Gilbert, and E Walton, *Nature*, **133** (1934), 328

|| The fact that the active product is an isotope of nitrogen has been proved by Cockcroft by chemical methods It is evidently identical with the active product obtained by Joliot and Curie in the reaction $_5B^{10}+{}_2He^4 \rightarrow {}_0N^{13}+{}_0n^1$, as both have the same decay constant The same active product may also be formed by the reaction $_6C^{12}+{}_1H^2 \rightarrow {}_7N^{13}+{}_0n^1$

particle having penetrated inside the nucleus shall get rid of its energy in the form of γ-quantum evidently depends on the relative values of the period for γ-emission and the time during which the incident particle stays inside the nucleus.

The period τ for γ-ray emission is given, according to Chapter IV, by the expression†

$$\tau_d = \frac{1}{\kappa_d} \sim 5 \times 10^{-4}(h\nu)^{-2} \tag{12}$$

in the case of dipole radiation. We have seen, however (Chap. IV, p 73; Chap VI, p 120), that in all cases so far investigated the actual probability of γ-emission is about 10,000 times smaller than corresponds to the periods given by (12), this means either that we are dealing with quadrupole radiation or with dipole radiation the intensity of which is considerably reduced on account of certain symmetry properties of the nucleus For quadrupole radiation we have†

$$\tau_q = \frac{1}{\kappa_q} \sim \frac{1}{\kappa_d}\left(\frac{\Lambda}{r_0}\right)^2 \sim 5 \times 10^{+12}(h\nu)^{-4}. \tag{13}$$

For the time, T, that the particle stays inside the nucleus (when there is no resonance) we evidently have

$$T \sim \frac{r_0}{v_i} \sim \frac{10^{-12}\text{ cm}}{10^9\text{ cm /sec}} \sim 10^{-21}\text{ sec}, \tag{14}$$

where r_0 is the nuclear radius and v_i the velocity of the particle inside the nucleus Thus we see that the probability that the particle will be captured by a radiative process after entering the nucleus is rather small ($T/\tau \ll 1$) Then the effective cross-section of capture evidently becomes

$$\sigma \sim \Lambda^2 e^{-I(E)} \frac{T}{\tau}, \tag{15}$$

where Λ is the de Broglie wave-length of the incident particle and $e^{-I(E)}$ the transparency of the potential barrier

In the case of resonance (i e. when upon entering the nucleus the incident particle forms, together with the bombarded system, an excited state of the resultant nucleus) the probability of capture may become considerably larger.

The yield of the reaction (10) for a proton energy about $0{\cdot}5 \times 10^6$ e.v.

† $h\nu$ expressed in e v.

is, according to Cockcroft, one capture per 5.10^9 incident protons corresponding to effective cross-section $\sigma \sim 10^{-31}$ cm^2 Assuming that the emitted radiation possesses the energy $h\nu \sim 7.10^{-6}$ e.v., we obtain from the formula (13) $\tau_q \sim 3.10^{-15}$ sec., and the chance that the proton will be captured after penetrating inside the carbon nucleus is $\dfrac{T}{\tau} \sim 10^{-5}$. With this value of T/τ the formula (13) gives the value for the effective cross-section for capture which is much smaller (about 1,000 times) than the experimentally observed cross-section This large discrepancy—and also the fact that no similar effect has been found for other neighbouring elements—suggests

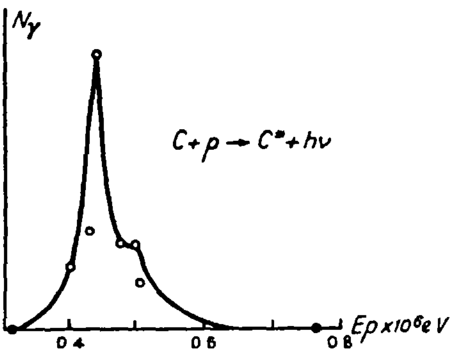


FIG 64

that we may have here a case of resonance-capture In the case of resonance the transparency of the barrier becomes unity and the expression (15) gives a result in agreement with the experimental value In fact, the detailed investigations of Tuve and Hafstad† have shown that the process of proton-capture by the C nucleus takes place only for incident protons of quite definite energies ($\sim 0{\cdot}45 \times 10^6$ e v), as can be seen from Fig. 64, in which the reaction yield is plotted against the energy of the protons. These experiments also indicate that the resonance maximum may have a complex structure, although the measurements are not sufficiently exact to prove this with certainty as yet Thus we see that the experimental evidence is in this case in satisfactory agreement with results of rough theoretical estimates.

The exact calculations concerning the radiative capture-probability

† M. Tuve and L. Hafstad, *Phys. Rev.* **48** (1935), 306.

for a schematized nuclear model have been carried out by Breit.† In these calculations it was assumed that the capture of a proton is due to a transition from the P state of free motion to the bound S state in the nucleus. Taking reasonable values for the nuclear radius and for the depth of the potential well, Breit was able to locate the resonance-level at the correct place. However, for the probability of capture he obtains values about 3,000 times larger than experimentally observed. This is most probably due to the fact that in these calculations the probability of γ-emission (corresponding to transition $P \rightarrow S$) was automatically considered as that of dipole radiation, whereas, as we have seen above, there are many reasons to expect that the probability of nuclear radiation usually fits with values calculated for a quadrupole (comparep. 120). Adding to Breit's calculation this additional assumption one obtains a very good fit with the experimental results of Cockcroft and more recent data on this subject by Tuve and Hafstad.

2. Nuclear photo-effect

By analogy with the excitation and ionization of atoms by light, we may expect that the atomic nucleus can be excited or 'ionized' by absorbing a γ-ray quantum of suitable energy For excitation the energy of this quantum must be larger than the energy of the first nuclear quantum level, for 'ionization', i e for the ejection of one of the nuclear constituent parts, it is necessary that the quantum energy of the incident γ-rays should be greater than the binding energy for that constituent which is ejected If we employ the γ-rays $h\nu = 2{\cdot}62 \times 10^6$ e v , which are the most energetic γ-rays available in sufficient intensity, we might expect the process of excitation to occur, at any rate in most of the heavier elements, as, according to our general knowledge, we should conclude that the energy-difference between the fundamental and first excited levels is, for these elements, often much smaller than the quantum energy of the radiation employed. Such nuclear excitation by γ-radiation, however, has not as yet been observed, due no doubt mainly to the difficulties of detecting the very weak 'fluorescent' radiation (the only evidence of excitation) in these cases. On the other hand, the energy conditions which have to be satisfied before the ejection of a nuclear particle by the action of γ-radiation is

† G. Breit and M. Ostrovsky, *Phys Rev.* **49** (1936), 352.

possible (nuclear photo-effect) are fulfilled only for very light nuclei, such as ${}_1H^2$ and ${}_4Be^9$, or for the radioactive nuclei which emit α-particles.†

The disintegration of the deuteron into a neutron and a proton, according to the scheme

$$ {}_1H^2+h\nu \rightarrow {}_0n^1+{}_1H^1, \tag{16}$$

was first reported by Chadwick and Goldhaber,‡ who observed that heavy hydrogen, being irradiated by an intense beam of γ-rays (from ThC″ or RaC), emits protons and neutrons which evidently originate in the splitting of the ${}_1H^2$ nucleus in the manner indicated. The emitted protons were directly observed by their ionization effect and their energy was estimated to be 0.24×10^6 e.v. The neutrons, which should have about the same energy, were first of all detected by the activation of silver, and, in later experiments, by the reactions with Li and B,|| in which fast—and thus easily detectable—particles are produced (on account of the considerable positive energy-balance in each case). Subtracting the total energy of the resulting proton and neutron ($\sim 0{\cdot}5\times10^6$ e.v.) from the original energy of the γ-quantum ($2{\cdot}6\times10^6$ e.v.), we obtain for the binding energy of the ${}_1H^2$ nucleus the value $2{\cdot}1\times10^6$ e.v. or 0·0023 in mass units. Combining this value with the masses of the nuclei ${}_1H^1$ and ${}_1H^2$, obtained from other nuclear reactions (Chap. X, § 1), Chadwick and Goldhaber deduced for the mass of the neutron the value 1·0084. This was the best value available at that time.†† Investigating the angular distribution of the emitted particles Chadwick and Goldhaber were able to show that in this case spherical symmetry did not exist. Instead, a distinct maximum was found in a direction at right angles to the axis of the γ-ray beam—an observation in good agreement with what one would expect if the disintegration were due to the action of the electric vector of the incident electromagnetic wave. Finally, for the effective cross-section for this nuclear photo-effect they give

† The chances of observing the increase in α-activity induced by γ-rays are still very small, for, as we have seen in Chapter VI, the transparency of the potential barrier of a heavy nucleus is quite small even for α-particles with a considerable excess of energy (long-range particles).

‡ J. Chadwick and G. Goldhaber, *Nature*, **134** (1934), 237; *Proc. Roy. Soc.* **151** (1935), 479.

|| ${}_3Li^6+{}_0n^1 \rightarrow {}_2He^4+{}_1H^3$ and ${}_5B^{10}+{}_0n^1 \rightarrow {}_3Li^7+{}_2He^4$.

†† The currently accepted value for the binding energy of the deuteron is 2.26×10^6 e.v. and for the mass of the neutron 1.0090.

about $6{\cdot}6\times 10^{-28}$ cm ^{2}, which, as we shall see later, is again in good agreement with theoretical calculations

The disintegration of $_4\mathrm{Be}^9$ by γ-rays was first observed by Szilard and Chalmers,† who used the γ-radiation of a sealed Ra source and were able to observe the 'photo neutrons' by means of the induced radioactivity in iodine. A more detailed study of this case of photo disintegration was made by Chadwick and Goldhaber,‡ who were able to show that the neutrons previously observed originate in the reaction

$$_4\mathrm{Be}^9+h\nu \rightarrow {_4\mathrm{Be}^8}+{_0n^1} \tag{17}$$

(and not in the reaction $_4\mathrm{Be}^9+h\nu = 2\,{_2\mathrm{He}^4}+{_0n^1}$) and that the energy required to remove a neutron from the $_4\mathrm{Be}^9$ nucleus is about $1{\cdot}6\times 10^6$ e v.

Except for these two cases ($_1\mathrm{H}^2$ and $_4\mathrm{Be}^9$), however, no further evidence of a nuclear photo-effect has been obtained. Twenty-six elements, of small and great atomic number, have been investigated, but without success This fact supports the suggestion previously made that the binding energy of a neutron in a heavy nucleus is in general greater than $2\,6\times 10^6$ e v.

The wave-mechanical calculation of the probability of the nuclear photo-effect has been carried out by Bethe and Peierls|| on the basis of the deuteron model discussed in § 1, Chapter II The effective cross-section for the nuclear effect is given, just as in the case of the ordinary photo-effect in the outer atom, by

$$\sigma = \frac{8\pi^3 e^2\nu}{c}|z_{0E}|^2, \tag{18}$$

with

$$z_{0E} = \int \psi_0\,\tfrac{1}{2}z\psi_E\,d\omega \tag{18'}$$

Here $\frac{1}{2}z$ is the z-component of the distance of the proton from the centre of mass, ψ_0 is the wave function for the proton in its normal state in the deuteron nucleus, and ψ_E the wave function representing the emitted proton, which with sufficient accuracy can be considered as a free particle. We have seen (p. 171) that ψ_0 for the accepted deuteron model is given by

$$\psi_0 = \sqrt{\left(\frac{\alpha}{2\pi}\right)}\frac{1}{r}e^{-\alpha r}, \tag{19}$$

† L Szilard and T. Chalmers, *Nature*, **134** (1934), 494 ‡ l.c.
|| H Bethe and R Peierls, *Proc Roy Soc.* **148** (1935), 146.

where†
$$\alpha = \frac{\sqrt{(\epsilon M)}}{\hbar}. \tag{19$'$}$$

Now, the wave function ψ_E can evidently be written in the form
$$\psi_E = a\, e^{ikr}, \tag{20}$$
where
$$k = \frac{\sqrt{\{M(h\nu-\epsilon)\}}}{\hbar} \tag{20$'$}$$
and a is the coefficient of normalization Inserting (19) and (20) in (18′) Bethe and Peierls obtain, after some calculation,
$$|z_{0E}|^2 = \frac{2}{3\pi}\frac{M}{\hbar^2}\frac{\alpha k^3}{(\alpha^2+k^2)^4}, \tag{21}$$
and the effective cross-section becomes
$$\sigma = \frac{16\pi^2}{3}\frac{Me^2\nu\alpha k^3}{\hbar^2 c(\alpha^2+k^2)^4}. \tag{22}$$
Substituting for α and k, using (19′) and (20′), we get finally
$$\sigma = \frac{16\pi^2}{3}\frac{\hbar^2 e^2\nu(h\nu-\epsilon)^{\frac{3}{2}}\epsilon^{\frac{1}{2}}}{Mc(h\nu)^4} \tag{23}$$
or
$$\sigma = \frac{8\pi}{3}\frac{e^2}{\hbar c\alpha^2}\frac{(\gamma-1)^{\frac{3}{2}}}{\gamma^3} \sim 1{\cdot}25\times 10^{-26}\frac{(\gamma-1)^{\frac{3}{2}}}{\gamma^3}\,\text{cm}^2, \tag{24}$$
where
$$\gamma = \frac{h\nu}{\epsilon}. \tag{24$'$}$$

This expression has a maximum for $\gamma = 2$ ($h\nu = 2\epsilon = 4{\cdot}2\times 10^6$ e.v.) and has then the value $\sigma_{\max} = 1{\cdot}6\times 10^{-27}$ cm ^{2}

For the γ-ray $h\nu = 2{\cdot}62\times 10^6$ e.v used by Chadwick and Goldhaber formula (24) gives $\sigma = 8\times 10^{-28}$ cm ^{2} As previously mentioned, this is in satisfactory agreement with the experimental value $\sigma = 6{\cdot}6\times 10^{-28}$ cm.^{2}

These results, however, are somewhat changed if we take into account the magnetic dipole radiation introduced by Fermi‡ to explain the reverse process of radiative capture of a neutron by a proton As was shown by Fermi, the effective cross-section for the photoelectric disintegration of the deuteron due to the interaction of the incident light quantum with the magnetic moments of neutron and proton is
$$\sigma = \frac{2\pi a^2\alpha^2\mu_0^2(g_p-g_n)^2}{3hc}\frac{(\gamma-1)^{\frac{1}{2}}}{\gamma[1+a^2\alpha^2(\gamma-1)]}, \tag{25}$$

† ϵ is the binding energy of the deuteron

‡ E Fermi, see reference on p 219.

where γ and α have the same meaning as above and $a = 1/\pi nl$. This cross-section must now be added to that previously calculated for pure electric forces.

Bethe and Peierls have also discussed the probability of the splitting of the deuteron by the impact of a fast electron—the process being completely analogous to that just discussed, the splitting by a γ-quantum. For the effective cross-section they obtain

$$\sigma_{\mathrm{el}} = \frac{2\pi}{3\alpha^2}\left(\frac{e}{\hbar c}\right)^2\left\{\log\frac{E^2}{\epsilon mc^2}-1{\cdot}432\right\}$$

$$= 2{\cdot}3\times 10^{-29}\left\{\log\frac{E^2}{\epsilon mc^2}-1{\cdot}432\right\}, \tag{26}$$

where E is the energy of the electrons employed. This formula shows, for example, that electrons with an energy $E = 10^6$ e v. passing through heavy water will produce one disintegration per 0·8 km of their path. the process will evidently be rather difficult to observe.

XII

RELATIVE ABUNDANCE AND ORIGIN OF THE ELEMENTS

1. Relative abundance of different nuclei

IN discussing the relative abundance of different nuclei in the universe we may start with a rather striking experimental fact: the remarkable constancy in isotopic constitution of the elements found on the earth The investigations of Aston† have shown that the relative amounts of the different isotopes of a given element remain very nearly constant for samples obtained from very different places on the earth's crust. This is probably also true on the astronomical scale, as, for example, the ratio of the amounts of the lithium isotopes Li^6 and Li^7 in the chromosphere of the sun (determined by a spectroscopic method) is exactly the same as the corresponding ratio for the ordinary lithium used in our laboratories Furthermore, a chemical analysis of the earth gives for the relative abundance of the different elements results differing only very slightly from those found for stellar bodies Fig 65 represents the relative numbers of different atoms in the earth. Inspecting this diagram we first notice that the rare gases (He, Ne, A, Kr, Xe,) are really very rare—but, since this may be explained on purely chemical grounds, it need not be stressed too seriously in further discussion The chemical explanation is as follows the rare gases, being chemically inert, do not readily enter into stable compounds, they cannot easily be held in solid matter, simply on this account

A second point in comparing the data for the earth's crust and for the meteorites is that the latter contain relatively larger amounts of elements possessing small atomic volumes, i e large densities. This again can be understood as a subsidiary difference if we remember that meteorites are probably formed in the breaking up of larger stellar bodies and consequently may be expected to have a constitution more approaching that of the interior of the earth, where, generally, denser elements are known to predominate

If we wish to understand theoretically the relative abundance of the different nuclei in the universe we must evidently take into

† Aston, *Mass Spectra and Isotopes* (1933)

account two different factors: (1) the relative stability of different nuclei in respect of several possible nuclear reactions; (2) the physical conditions existing in different places in the universe where transformations of nuclei can take place. Although our present knowledge, especially regarding the second point, is far from being complete it is still possible to reach some interesting conclusions concerning the creation and transformation of elements in different parts of the universe.

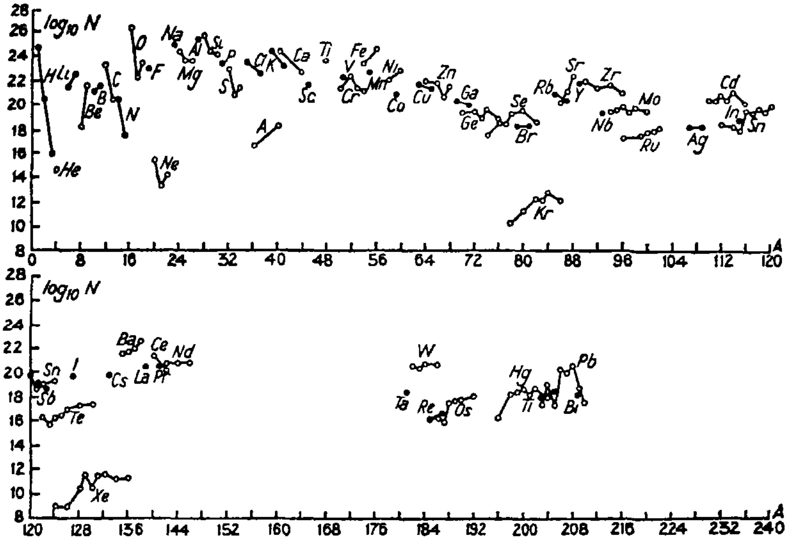


FIG 65 Relative abundance of elements in the earth's crust

2. Thermal transformation of elements in stars

As we have seen in previous chapters, the efficiency of nuclear transformations produced by the bombardment of different materials by beams of fast charged particles is never very great, the yield being sometimes one disintegration per million incident particles—and frequently much less The reason for this is that such fast charged particles lose most of their energy passing through the electronic shells of atoms on their way so that only very few of them have the chance of hitting a nucleus and producing disintegration. Only in the case of neutron beams has each particle this chance of producing nuclear transformation, but, as there are no free neutrons in nature, we have first to produce them by ejecting neutrons from stable nuclei by α- or proton-bombardment. We are again brought back to a process which has a very small efficiency.

One can, however, instead of using artificially accelerated beams of charged particles, expect nuclear transformations to take place as the result of thermal collisions if the temperature of the reagent substances is high enough. It is clear that if we keep a mixture of, say, lithium and hydrogen at a very high temperature for a sufficiently long time the reactions of nuclear transformation between colliding nuclei will go on until we shall have our vessel filled with pure helium—and we should expect that, if such a reaction once starts, the energy developed in the $Li+H \rightarrow 2He$ transformation will raise the temperature of the mixture to such an extent that the process will be accelerated and may even have the character of an explosion, with the liberation of an immense amount of energy per gramme of substance. However, the temperatures necessary to keep such reactions going are themselves extremely high, as we shall see, they can be expected to obtain only inside our sun and other stars. The first calculations concerning thermal nuclear reactions were carried out by Atkinson and Houtermans,† who considered the process of 'thermal' penetration of protons through the potential barriers of different elements for temperatures of the order of 10^7 degrees (usually believed to occur inside the sun)

In calculations of the transparency of the nuclear barriers for the thermal protons in this connexion one should not take for the proton energy the average energy kT (k, Boltzmann constant, T, absolute temperature), as there are always, according to Maxwell's law, fast protons present in small number and they are much more efficient in their disintegration effect

In Fig. 66 is given a schematic representation of the number of particles N and their effective cross-section for disintegration σ as a function of the velocity We can see that the total number of disintegrations, given essentially by the product $N\sigma$, reaches a sharp maximum for certain velocity well above the average velocity corresponding to a given temperature In order to estimate the total number of disintegrations one should integrate $N\sigma$ over all different velocities from 0 to ∞, the most essential part, however, being given by the particles possessing the optimum velocities. The results

† R Atkinson and F. Houtermans, *Zs f Phys* **54** (1929), 656, G Gamow and L. Landau, *Nature*, **132** (1933), 567.

obtained in this way are shown in Table XVII, which gives the average life of different elements in contact with hydrogen at a temperature of 60×10^6 °C on the assumption that each penetration gives rise to disintegration

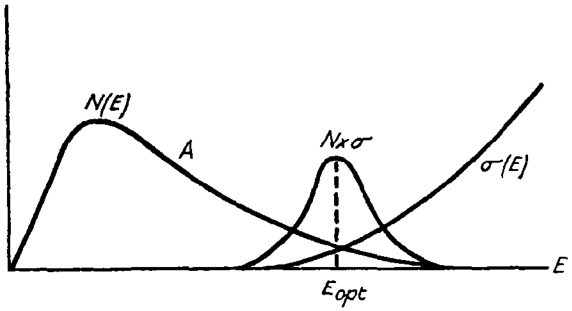


Fig 66

Table XVII†

Element	He	Be	C	O	Ne	Pb
Average life	8 sec	5 days	100 years	5 10^5 years	10^9 years	10^{61} years

We see that only the lightest elements may be appreciably transformed at this temperature during a time equal to the estimated present age of the stars ($\sim 10^9$ years) We must notice, moreover, that these comparatively short disintegration periods for the lightest elements themselves refer only to the case of the hydrogen reactions; if we try to calculate the lifetime corresponding to thermal collisions with helium atoms, we get extremely large values, even for the lightest elements, owing to the much smaller penetration ability of the α-particles Atkinson and Houtermans considered particularly those thermal reactions involving radiative capture of protons and the gradual building up of heavier nuclei As we have seen in the last chapter, the probability of reactions of this type is only $T/\tau \sim 10^{-5}$ per nuclear penetration, which means, for example, that the average life for the reaction between carbon and hydrogen (${}_6C^{12}+{}_1H^1 = {}_7N^{13}+h\nu$) will be almost 10^7 years This lifetime can, however, be largely reduced if the nucleus possesses a resonance-level in the region of thermal protons

† This table is taken from the article of Atkinson and Houtermans, the periods being calculated on the assumption that the cross-section of collision is given by r_0^2; assuming the cross-section Λ^2, we get somewhat shorter periods

The discovery of neutrons started a new stage in the discussion of the thermal transformations of elements in stars. It is not impossible that, by some process taking place inside a star (as for example the photoelectric nuclear disintegration by very hard γ-rays emitted during radiative capture of protons), a considerable amount of neutrons can be produced On the other hand, we have seen that neutrons (especially slow neutrons) possess a very large probability of being radiatively captured by nuclei, especially heavy nuclei, in reactions such as

$$_{Z}X^{A}+{}_{0}n^{1}\rightarrow {}_{Z}X^{A+1}+h\nu, \tag{1}$$

forming heavier isotopes which, by the process of β-emission,

$$_{Z}X^{A+1}\rightarrow {}_{Z+1}Y^{A+1}+e, \tag{2}$$

may give rise to elements located higher up in the periodic table Thus we see that thermal transformations inside the star (assuming a sufficient supply of hydrogen, which seems, however, always to be present in large quantities) may be effective throughout the whole range of elements—and, being considered along with the relative stability of different types of nuclei, may lead to an understanding of the relative amounts of the various elements in the universe

We may also notice at this point that reaction-chains of the type (1)(2), corresponding to the building up of heavy nuclei, must in general involve the emission of large amounts of energy—probably enough, in fact, to provide a source for the radiation of the stars, which for a long time presented an inexplicable puzzle to astrophysicists

3. Nuclear state of matter in the interior of a star

We can now approach the very interesting problem of the physical state of matter under the immensely high temperatures and pressures existing inside a star It is quite clear that such extreme conditions will not only completely dissociate all chemical compounds (which already happens on the surface of the stars), but will also produce complete ionization of the atoms, tearing away, in vigorous collisions, the electrons even from the most tightly bound shells surrounding the heaviest nuclei The resulting mixture of free electrons and bare nuclei can be considered as an ideal gas until the density reaches such a high value that the average distance between particles becomes of the order of magnitude $\hbar/mc \sim 10^{-10}$ cm. (corresponding to a density $\rho \sim 10^{+6}$ gm /cm ^{3}) For still higher densities electrons will probably be absorbed by the nuclei (an inverse β-decay process) and the mixture will tend to a state which can be described very roughly as a gas of neutrons. For densities of the order of magnitude $\rho \sim 10^{+12}$ gm./cm.^{3} (average density of atomic nuclei) nuclear exchange forces between the gas particles will come into play and the conditions in the 'gas' will become analogous to the conditions inside an atomic nucleus Such an extreme state of matter we shall call the 'nuclear state' and the region of the star occupied by such nuclear matter the 'stellar nucleus'. We must consider now the conditions under which such stellar nuclei can really be formed. In order to see whether such high densities can in fact be reached inside a star we shall compare the gravity pressure tending to decrease the star's volume with the pressure of the gas inside the star opposing it, as was first done by Landau† and Chandrasekhar ‡ As is well known, the gravitational energy of a sphere of mass M and radius R is given by

$$U_g = \frac{3}{5}k\frac{M^2}{R} = \frac{3}{5}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}} kM^{\frac{5}{3}}\rho^{\frac{1}{3}}, \tag{3}$$

where k is Newton's constant of gravitation and V is the volume of the star

For the pressure due to gravitation we thus obtain

$$P_g = -\frac{\partial U_g}{\partial V} = \frac{3}{20\pi}k\frac{M^2}{R^4} = \frac{1}{5}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}} kM^{\frac{2}{3}}\rho^{\frac{4}{3}}. \tag{4}$$

Of course, at sufficiently high temperatures the gas-pressure can always be large enough to compensate the pressure of gravity, but, if we wish to know anything about the *final state of the star* we must compare (4) with the gas-pressure at the *absolute zero* of temperature, when in the course of time all the kinetic energy originally available has been radiated The kinetic energy of a gas consisting of particles subject to the Pauli principle at the zero-point temperature (degenerate Fermi gas) can be calculated in the following way. The kinetic energy of a single molecule inside a volume l^3 is evidently given by

$$K = \frac{1}{2m}p^2 = \frac{1}{2m}\frac{4\pi^2\hbar^2}{4l^2}(n_x^2+n_y^2+n_z^2), \tag{5}$$

where n_x, n_y, and n_z are the three quantum numbers describing the motion of the particle. The number of particles possessing a definite

† L. Landau, *Zs f Sov Phys* **5** (1932), 285

‡ S. Chandrasekhar, *Astroph. Journal*, **74** (1931), 81.

value of K is similarly the number of combinations of quantum numbers corresponding to the same value of n $\{=\sqrt{(n_x^2+n_y^2+n_z^2)}\}$ and is given (for large n) by n^2 Thus the total energy of the gas is

$$E=\sum_{n=1}^{n_{\max}} Kn^2=\frac{\pi^2\hbar^2}{2ml^2}\sum_{1}^{n_{\max}} n^4 \tag{6}$$

Replacing the sum by an integral, we have

$$E=\frac{\pi^2\hbar^2}{2ml^2}\int\limits_0^{n_{\max}} n^4\,dn=\frac{\pi^2\hbar^2}{10ml^2}n_{\max}^5 \tag{7}$$

Now $n_{\max}$ is given by the condition that all levels are occupied.

$$\sum_{n=1}^{n_{\max}} n^2=N \quad \text{or} \quad \tfrac{1}{3}n_{\max}^3=N, \tag{8}$$

where N is the total number of particles Combining (7) with (8), we obtain

$$E=\frac{3^{\frac{5}{3}}\pi^2\hbar^2}{10ml^2}N^{\frac{5}{3}}. \tag{9}$$

The pressure of the gas is now given by

$$P_F=-\frac{\partial E}{\partial V}=-\frac{\partial E}{3l^2\partial l}=\frac{3^{\frac{2}{3}}}{5}\frac{\pi^2\hbar^2}{m}\frac{1}{l^5}N^{\frac{5}{3}} \tag{10}$$

and, substituting the density $\rho=\dfrac{N}{l^3}m$,

$$P_F=\frac{3^{\frac{2}{3}}}{5}\pi^2\hbar^2\frac{1}{m^{\frac{8}{3}}}\rho^{\frac{5}{3}} \tag{11}$$

This may be taken as the final expression for the pressure of the degenerate Fermi gas Comparing (11) and (4), we notice that P_F increases with a higher power of the density than does P_g, and therefore, for sufficiently high densities, we conclude that equilibrium is always possible

However, the formulae which have just been developed hold only when the velocity of the particles of the gas is everywhere much smaller than the velocity of light For sufficiently high densities, however (small l), the classical expression $k=\dfrac{1}{2m}p^2$ must be replaced by the relativistic expression $k=cp$ The formula for the kinetic energy now becomes

$$E=\sum_{n=1}^{n_{\max}} c\frac{\pi\hbar}{l}nn^2=\frac{\pi\hbar c}{4l}n_{\max}^4=\frac{3^{\frac{4}{3}}\pi\hbar c}{4l}N^{\frac{4}{3}} \tag{12}$$

and for the pressure

$$P_{F,R} = -\frac{\partial E}{\partial V} = -\frac{\partial E}{3l^2\partial l} = \frac{3^{\frac{1}{3}}\pi\hbar c}{4l^4}N^{\frac{4}{3}} = \frac{3^{\frac{1}{3}}\pi\hbar c}{4m^{\frac{4}{3}}}\rho^{\frac{4}{3}}. \tag{13}$$

We see, in particular, that the pressure of the 'relativistic Fermi gas' depends on the total number of particles per unit volume (N/l^3), but not on their mass m. This makes it more or less immaterial whether we consider, inside the stellar nucleus, an electron gas or a gas of heavy particles The graphical representation of the (P, ρ) relation is given in Fig. 67. Again, comparing (13) and (4), we see that now

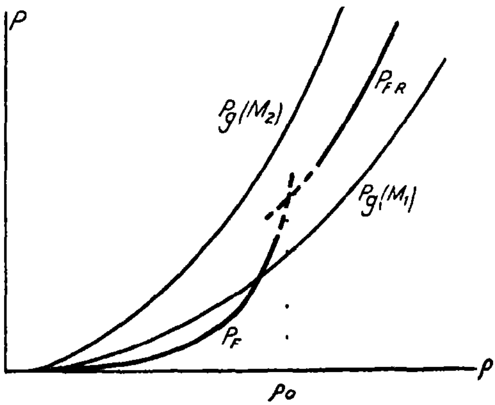


Fig 67

both pressures increase with the same power of ρ, so that if originally P_g is larger than $P_{F,R}$ equilibrium will never be possible for larger densities and the compression will proceed without limit The condition for such unlimited contraction is evidently

$$\frac{1}{5}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}}kM^{\frac{2}{3}} > \frac{3^{\frac{1}{3}}\pi\hbar c}{4m^{\frac{4}{3}}}, \tag{14}$$

which gives†

$$M > M_0 \sim \left[\gamma\frac{\hbar c}{km^{\frac{4}{3}}}\right]^{\frac{3}{2}} \sim 3.10^{+33}\,\text{gm.} \sim 1\ 5 \text{ sun-masses} \tag{15}$$

We may notice here that unlimited contraction may start already for smaller masses than M_0, if we take into account the exchange attractive forces between particles discussed in previous chapters. Thus we see that most of the stars, and possibly all stars, if the limiting mass M_0 is lowered by intranuclear forces, are subject to

† $\gamma = \frac{5\ 3^{\frac{2}{3}}}{4^{\frac{4}{3}}}\pi^{\frac{2}{3}}$.

the formation of matter in the nuclear state in their interior at some period of their existence. The question whether most stars at present actually possess such nuclei cannot, however, be answered definitely until the relevant astronomical evidence has been thoroughly examined, but there seems to be no reason why they should not The theory of stellar nuclei gives us another aspect of the question of the creation of the elements and the liberation of energy in the stars In fact, the surface of a stellar nucleus must not be considered as a smooth mathematical sphere, since eruptive processes of different types may go on continuously over the boundary between a large stellar nucleus and the surrounding matter in the ordinary gaseous state Such processes give innumerable occasions for the tearing away of portions of the nuclear substance which, coming into a region of smaller pressure, immediately split into small fragments, thus forming the nuclei of different elements As to the liberation of energy, one can easily see that pure gravitational energy liberated in the contraction to such immense densities will already be quite enough to secure the life of the star for a very long period of time

APPENDIX

In the investigation of nuclear phenomena it is always very important to know the energies of the various particles and of the electromagnetic radiations emitted in the process of nuclear transformation. The most direct and precise method of energy-measurement for the charged particles emitted in spontaneous and artificial disintegration is the study of the curvature of their paths in a magnetic field (*magnetic spectra*) This method led, for example, to the discovery of the 'fine structure' of α-rays and it is also very important in the study of the shapes of the continuous spectra of electrons emitted in β-transformations However, it cannot always be applied, especially in the study of artificial nuclear transformations in which, due to the comparatively small yield of particles, it meets with serious technical difficulties In such cases conclusions regarding the energies of emitted particles are generally made on the basis of measurements of their *absorption coefficients* in various substances In the case of heavy particles (such as α-particles or protons) the distance travelled in a given substance is, except for small deviations, completely determined by the original energy, and it is customary, therefore, to express the energy of such particles by quoting this length, which is called the *range* of the particles. The presence of several discrete energies in the original beam can then be distinctly recognized by the occurrence of a number of rapid increases in the intensity of the transmitted beam with decreasing thickness of the absorber

In the case of electrons the definition of range cannot be made so exactly as with α-particles because the intensity decreases more or less uniformly with increasing thickness of the absorber. This fact, also, makes very difficult the analysis of absorption curves when electrons of several velocities are thought to be present In particular, the absorption method fails almost completely to give any reliable indication of the distribution of energy amongst the electrons emitted in natural β-decay As we have seen already, these electrons possess a continuous energy spectrum, which makes such analysis almost impossible. Thus, in this case, absorption measurements can give only a rough idea as to the maximum energy of the emitted electrons and the magnetic method must always be preferred.

In the case of electromagnetic radiation, as we have seen, measurements by the method of 'internal conversion' are able to provide very accurate data concerning the energies of γ-ray quanta, but this method also requires large intensities and, practically speaking, is inapplicable to the γ-radiation accompanying artificial nuclear transformations. In some cases the (Skobelzyn's) method of investigating the Compton electrons can then give us valuable information, but in very many experiments this γ-radiation is studied by the ordinary absorption methods. We shall, therefore, at this stage include a short account of the energy-dependence of the absorption of various kinds of radiation by matter.

I. Absorption of heavy particles (range-energy relation)

The absorption of heavy charged particles, such as α-particles or protons, in their passage through matter is mainly due to the loss of energy in collisions with atomic electrons. The first detailed calculations of the stopping power of matter for such particles were made by Bohr,† who, assuming that the atomic electrons could be represented by harmonic vibrators with different frequencies ν_n, obtained for the loss of velocity per unit length of path the expression

$$\frac{dV}{dx} = -\frac{4\pi e^4 Z^2}{MmV^3} N \sum_1^n \log \frac{KmMV^3}{2\pi\nu_n Ze^2(M+m)} \tag{1}$$

Here N is the number of atoms per unit volume of the absorbing substance, M and Ze are the mass and charge of the moving particle, and the summation is to be taken over all electron vibrators present in the atom. The numerical constant K was found by Bohr to have the value 1.123. This formula remains essentially unchanged when, instead of the vibrator-model of the atom used in the original calculations of Bohr, the quantum picture is used. Such calculations have been carried out successfully by various authors, we do not discuss them here, as this problem has no direct connexion with the questions of nuclear structure.

For very large V, when one can neglect the variation of the logarithmic term in (1), direct integration gives

$$V^4 - V_0^4 = A(x - x_0), \tag{2}$$

showing that the distance which the particle can go before its velocity

† N. Bohr, *Phil. Mag.* **24** (1913), 10, **30** (1915), 581.

becomes zero (the *range* of the particle) is proportional to the fourth power of its initial velocity. For particles of smaller velocities, such as are ordinarily used in our laboratories, the logarithmic term varies approximately as V, and we have

$$V^3 - V_0^3 \sim B(x - x_0) \tag{3}$$

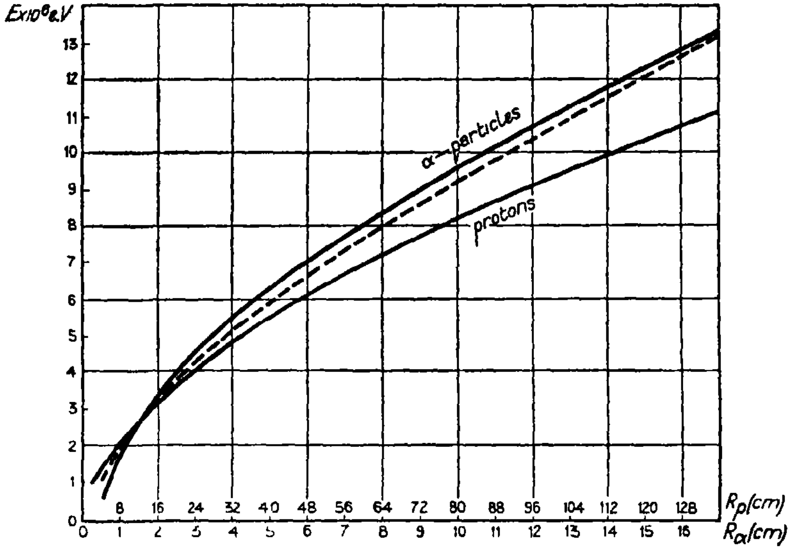


FIG. 68

This is the well-known Geiger rule established experimentally, it provides a very good approximation for the velocities of the α-particles emitted by most radioactive bodies

Furthermore, we can easily see from formula (1) that the range of a particle in a given substance is, for a given velocity, proportional to M/Z^2 and for a given energy proportional to $1/Z^2M^{\frac{1}{2}}$ (this proportionality is only approximate, as again it is derived neglecting the variation of the logarithmic term). Thus, for example, comparing α-particles with protons, we must conclude that for equal velocities they have approximately equal ranges, whereas for equal energies the ranges of protons will be about eight times longer than the ranges of α-particles.

We see, from what has been said above, that the detailed theoretical calculation of the range-velocity relation in the general case is rather difficult and that only approximate laws can be deduced in this way Therefore, for practical purposes, i.e for estimating energies from the measured ranges, it is better to use the empirical curves obtained by the measurement of range and energy (by magnetic deflexion) directly, for several groups of particles Such curves as used in the Cavendish laboratory for α-particles and protons are given in Fig 68. The dotted curve represents the Geiger rule which, as can be seen, leads to considerable errors except in a restricted interval of energy. One can also see that the above-mentioned dependence on the mass and charge of the particles is very rough; for example, for $E = 6\times 10^6$ e v. we have $R_\alpha = 4{\cdot}5$ cm and $R_p = 47$ cm., differing by a factor 10 instead of the expected factor 8.

II. Absorption of β-rays

The absorption of fast electrons can be calculated in the same way as has already been done in the case of heavy particles, with the difference that, due to the large velocities of the β-particles, relativistic corrections have to be taken into account In his original paper Bohr, applying these corrections, obtained the following expression for the range of an electron with initial velocity βc.

$$R = \frac{m^2c^4}{2\pi e^4 SN}[(1-\beta^2)^{\frac{1}{2}}+(1-\beta^2)^{-\frac{1}{2}}-2], \tag{4}$$

where
$$S = \sum \log\left[\frac{K^2\beta^2c^2}{4\pi\nu_n^2}NZR-\log(1-\beta^2)-\beta^2\right]. \tag{5}$$

The general notation is the same as in the corresponding formula for α-particles The coefficient K, which again had the value $1{\cdot}123$ according to Bohr's calculation, is slightly changed in the quantum calculation, in much the same way as for heavy particles

If, as is usual for β-particles, we express the range in grammes per cm ^{2} (G), we can write (4) in the form

$$G = \frac{m^2c^4}{2\pi e^4\left(\frac{Z}{A}\right)N_A}\,\frac{(1-\beta^2)^{\frac{1}{2}}+(1-\beta^2)^{-\frac{1}{2}}-2}{\left[\log\frac{K^2}{4\pi}c^2N_A\left(\frac{Z}{A}\right)+\log G-\log\frac{1-\beta^2}{\beta^2}-\beta^2-2C\right]}, \tag{6}$$

where N_A is Avogadro's number and C stands for $\frac{1}{Z}\sum \log \nu_r$, being a characteristic constant for each substance Since one cannot easily calculate this constant C in the general case, its value has to be found empirically.

In order to illustrate the agreement between this formula and experiment we give the ranges G of electrons of various velocities in mica as measured by White and Millington † These results are shown in Fig. 69 by the small circles For comparison the full curve gives the results of calculation from the formula (6) with $C = 11{\cdot}2$.

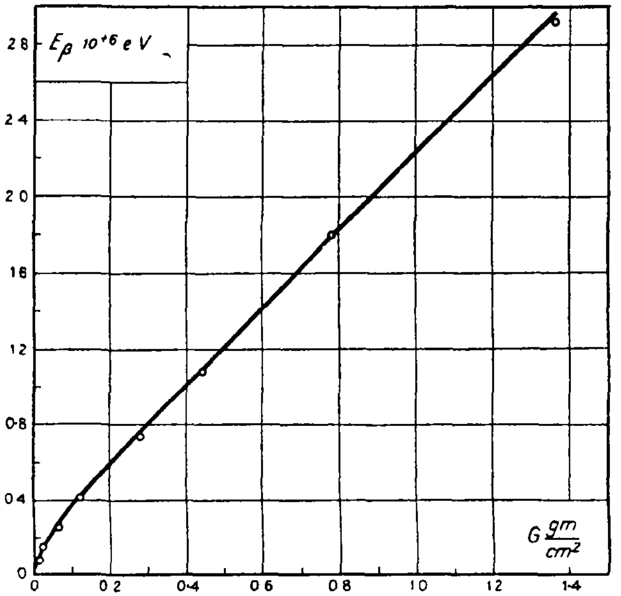


FIG 69

However, as we have already noticed, this curve is of no great use for the purpose of estimating disintegration energies for various β-transformations, as in all cases the emitted electrons have a continuous energy-spectrum

Besides the ordinary losses of energy due to collisions with atomic electrons, the beam of charged particles can lose energy by radiation due to strong accelerations during passage close to atomic nuclei These radiative losses become of particular importance when the velocity of particles is very close to the velocity of light Thus, though they can be neglected for the heavy particles emitted by radioactive bodies, they must be taken into account for high-energy β-rays For a more detailed account of the energy-losses of fast electrons in matter the reader may refer to Heitler's monograph on this subject

† P. White and G Millington, *Proc Roy Soc* **120** (1928), 701.

III. Absorption of neutrons

We have already seen that, due to negligibly small interaction of neutrons with electrons, the absorption of neutrons in matter is mainly due to elastic and inelastic collisions with atomic nuclei, which have been discussed in detail in Chapters IX and XI. It has recently, however, been suggested by Bloch† that when the absorber is a magnetic substance an additional scattering of slow neutrons may be expected, due to the action of the internal inhomogeneous field on the magnetic moment of the neutron, of which the value at present accepted (p 60), is about 2 nuclear magnetons The effect must be particularly large in strongly ferromagnetic substances and, according to Bloch, may then reach values comparable with nuclear scattering (for example $\sigma_{\text{magn}}/\sigma_{\text{nucl}}$ should be about 1·4 for iron).

A glance at Table XIV shows us that the observed scattering cross-sections for Fe and Ni are actually exceptionally high, which is in agreement with Bloch's suggestions This additional magnetic scattering should disappear when the substance is heated above the Curie point, experiments to test this prediction are now in progress

It may be emphasized here that the investigation of the magnetic scattering of neutrons may prove to be a valuable method for the direct estimation of the magnetic moment of the neutron

IV. Absorption of γ-rays

The absorption of γ-rays in their passage through matter can be due to three different causes

(1) First of all the decrease in intensity of a beam of γ-rays may be due to Compton scattering in collisions with atomic electrons. The effective cross-section per atom for such scattering has been calculated by Klein and Nishina‡ on the basis of Dirac's relativistic wave equation and is given by the expression

$$\sigma = 4\,96\times 10^{-25}\left[\frac{1+\alpha}{\alpha^2}\left\{\frac{2(1+\alpha)}{1+2\alpha}-\frac{1}{\alpha}\log(1+2\alpha)\right\}+\right.$$
$$\left.+\frac{1}{2\alpha}\log(1+2\alpha)-\frac{1+3\alpha}{(1+2\alpha)^2}\right]Z, \quad (7)$$

where
$$\alpha = \frac{\hbar\omega}{mc^2} \quad (\omega = 2\pi\nu) \quad (7')$$

† F Bloch, *Phys Rev* **50** (1936), 259

‡ O Klein and Y Nishina, *Zs. f. Phys.* **52** (1929), 853

and Z the number of electrons in the atom This formula is in very good agreement with experiment for light elements, but for heavier elements there appears to be some additional scattering or absorption (increasing, when estimated in terms of the cross-section per electron, with atomic number)† which has to be ascribed to the next process.

(2) If the energy of the γ-quantum is greater than $2mc^2 \sim 1 \times 10^6$ e v., the process of formation of a pair of positive and negative electrons in the Coulomb field of the nucleus becomes possible, the incident γ-quantum in such cases completely disappears and energy in amount

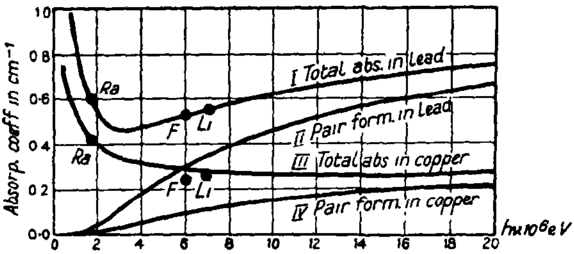


FIG 70

$h\nu - 2mc^2$ is distributed as kinetic energy between the two created electrons (a very small part of the energy is also given to the recoil nucleus) The effective cross-section per atom for such pair-formation is given by‡

$$\sigma = \left(\frac{e^2}{mc^2}\right)\frac{Z^2e^2}{2\pi\hbar c}\left(\frac{28}{9}\log\frac{2\hbar\omega}{mc^2}-\frac{218}{27}\right) \tag{8}$$

In so far as this expression contains Z^2, whilst the Klein-Nishina formula does not, we see why the additional absorption due to pair-formation becomes especially important for heavier elements

The absorption coefficients for γ-rays in lead and copper, due to Compton scattering (7) and pair-formation (8), are shown in Fig 70 for different quantum energies We see, in particular, that the sum of these absorption coefficients, first decreasing with increasing energy, begins to increase again for higher energies, so that the determination of $h\nu$ from absorption measurements in only one absorber is ambiguous

(3) Finally, we can expect some absorption of γ-rays due to

† C Y Chao, *Proc Nat. Acad Amer* **16** (1930), 431, G Tarrant and L Gray, *Proc. Roy Soc* **128** (1930), 345, L Meitner and H. Hupfeld, *Zs f Phys* **67** (1931), 147

‡ R. Oppenheimer and M Plesset, *Phys Rev* **44** (1933), 53, H. Bethe and W Heitler. *Proc Roy Soc* **146** (1934), 83.

scattering by atomic nuclei (coherent scattering) or to the disintegration or excitation of the nuclei if the energy of the γ-quanta is great enough. It can be shown, however, that the ordinary scattering of electromagnetic radiation by nuclei is negligibly small and that the disintegration processes due to the action of such radiation (nuclear photo-effect) take place with a few of the lightest elements only, and then possess rather small probabilities Thus we may conclude that the contribution to the total absorption due to strictly nuclear processes is very small indeed. It can properly be neglected in estimating $h\nu$ from the absorption measurements.

TABLES

TABLE A
PERIODIC SYSTEM OF THE ELEMENTS

I	II	III	IV	V	VI	VII	VIII	0
1H 1 00807								2He 4 0030
3Li 6 9373	4Be 9 0123	5B 10 8050	6C 12 0108	7N 14 0071	8O 16 0000	9F 19 0001		10Ne (20 183)
11Na (22 997)	12Mg (24 32)	13Al 26 9847	14Si (28 06)	15P 30 975	16S (32 06)	17Cl (35 457)		18A 39 952
19K (39 096) 29Cu (63 57)	20Ca (40 08) 30Zn (65 38)	21Sc 44 958 31Ga (69 72)	22Ti (47 90) 32Ge (72 60)	23V (50 95) 33As 74 917	24Cr (52 01) 34Se (78 96)	25Mn (54 93) 35Br 79 909	26Fe (55 84) 27Co (58 94) 28Ni (58 69)	36Kr 83 766
37Rb (85 44) 47Ag (107 880)	38Sr (87 63) 48Cd (112 41)	39Y (88 92) 49In (114 76)	40Zr (91 22) 50Sn (118 70)	41Nb 92 905 51Sb (121 76)	42Mo (96 0) 52Te (127 61)	43— — 53I 126 903	44Ru (101 7) 45Rh 102 90 46Pd (106 7)	54Xe (131 3)
55Cs 132 902	56Ba (137 36)	57La (138 92)						
58Ce (140 13) 59Pr (140 92)	60Nd (144 27) 61— —	62Sm (150 43) 63Eu (152 0) 64Gd (157 3)						
65Tb (159 2) 66Dy (162 46)	67Ho (163 5) 68Er (167 64)	69Tm (169 4) 70Yb (173 04) 71Lu (175 0)	72Hf (178 6) 82Pb (207 22)	73Ta 180 886 83Bi (209 00)	74W (184 0) 84Po —	75Re (186 31) 85— —	76Os (191 5) 77Ir (193 1) 78Pt (195 23)	86Rn (222)
79Au (197 2)	80Hg (200 61)	81Tl 204 402						
87— —	88Ra (226 05)	89Ac —	90Th (232 12)	91Pa (231)	92U (238 14)			

‡ The atomic weights given in this table are calculated wherever possible from the data of Table B reduced to the basis *mixed oxygen* = 16 0000, and are based on determinations by the mass spectrograph of relative abundances of isotopes The values given in brackets are taken from *International Atomic Weights* 1936 and are used for elements for which complete data are not available in Table B

TABLE B

STABLE ISOTOPES†

Nucleus	*M* (*exact atomic mass*)	*I per cent.* (*relative percentage*)
$_{0}n^{1}$	1 0091	100
$_{1}H^{1}$	1 0081	99 98
$_{1}H^{2}$ (D)	2 0147	0 02
$_{1}H^{3}$ (T)	3 0171	$\sim 10^{-7}$
$_{2}He^{3}$	3 0171	
$_{2}He^{4}$	4 0039	100
$_{3}Li^{6}$	6 0167	7 9
$_{3}Li^{7}$	7 0180	92 1
$_{4}Be^{8}$	8 0078	$\sim$ 0 05
$_{4}Be^{9}$	9 0149	99 95
$_{4}Be^{10}$	10 0164	
$_{5}B^{10}$	10 0161	20 6
$_{5}B^{11}$	11 0128	79 4
$_{6}C^{12}$	12 0036	99 0
$_{6}C^{13}$	13 0073	$\sim$ 1 0
$_{7}N^{14}$	14 0073	99 7
$_{7}N^{15}$	15 0048	0 3
$_{8}O^{16}$	16 0000	99 80
$_{8}O^{17}$	17 0046	0 03
$_{8}O^{18}$	18 0065	0 17
$_{9}F^{19}$	19 0045	100
$_{10}Ne^{20}$	19 9986	90 00
$_{10}Ne^{21}$		0 27
$_{10}Ne^{22}$		9 73
$_{11}Na^{23}$		100
$_{12}Mg^{24}$		$\sim$ 78
$_{12}Mg^{25}$		$\sim$ 11
$_{12}Mg^{26}$		$\sim$ 11
$_{13}Al^{27}$	26 9909	100
$_{14}Si^{28}$	27 9860	89 6
$_{14}Si^{29}$	28 9864	6 2
$_{14}Si^{30}$		4 2
$_{15}P^{31}$	30 9825	100
$_{16}S^{32}$		$\sim$ 97
$_{16}S^{33}$		$\sim$ 0 8
$_{16}S^{34}$		$\sim$ 2 2
$_{17}Cl^{35}$	34 983	75
$_{17}Cl^{37}$	36·980	25
$_{17}Cl^{39}$		$\leqslant 1$
$_{18}A^{36}$		0 33
$_{18}A^{38}$		0 05
$_{18}A^{40}$	39 9754	99 62
$_{19}K^{39}$		93 4
$_{19}K^{41}$		6 6
$_{20}Ca^{40}$		96 76
$_{20}Ca^{42}$		0 77
$_{20}Ca^{43}$		0 17
$_{20}Ca^{44}$		2 30
$_{21}Sc^{45}$	44 968	100
$_{22}Ti^{46}$		8 5
$_{22}Ti^{47}$		7 8
$_{22}Ti^{48}$		78 3
$_{22}Ti^{49}$		5 5
$_{22}Ti^{50}$		6 9
$_{23}V^{51}$		100
$_{24}Cr^{50}$		4 9
$_{24}Cr^{52}$	51 948	81 6
$_{24}Cr^{53}$		10 4
$_{24}Cr^{54}$		3 1
$_{25}Mn^{55}$		100
$_{26}Fe^{54}$		6 5
$_{26}Fe^{56}$		90 7
$_{26}Fe^{57}$		2 8
$_{27}Co^{59}$		100
$_{28}Ni^{58}$	57 942	67 5
$_{28}Ni^{60}$		27 0
$_{28}Ni^{61}$		1 7
$_{28}Ni^{62}$		3 8
$_{29}Cu^{63}$		$\sim$ 70
$_{29}Cu^{65}$		$\sim$ 30
($_{30}Zn^{63}$)		
$_{30}Zn^{64}$	63 937	50 4
($_{30}Zn^{65}$)		
$_{30}Zn^{66}$		27 2

† Taken essentially from tables by O Hahn (*Ber Chem Ges* **69** (1936), 5), exact atomic masses are based on combined evidence of mass-spectra and nuclear reactions. In this table O^{16} = 16 0000 by definition. The values in Table A which are derived from those here have therefore been divided by the conversion factor 1 00023 to reduce them to the chemical scale for which by definition 'mixed oxygen' = 16 0000

Nucleus	M (exact atomic mass)	I per cent (relative percentage)
$_{30}Zn^{67}$		4 2
$_{30}Zn^{68}$		17 8
$_{30}Zn^{70}$		0 4
$_{31}Ga^{69}$		61 5
$_{31}Ga^{71}$		38 5
$_{32}Ge^{70}$		21 2
$_{32}Ge^{72}$		27 3
$_{32}Ge^{73}$		7 9
$_{32}Ge^{74}$		37 1
$_{32}Ge^{76}$		6 5
$_{33}As^{75}$	74 934	100
$_{34}Se^{74}$		0 9
$_{34}Se^{76}$		9 5
$_{34}Se^{77}$		8 3
$_{34}Se^{78}$	77 938	24 0
$_{34}Se^{80}$	79 941	48 0
$_{34}Se^{82}$		9 3
$_{35}Br^{79}$	78 929	50
$_{35}Br^{81}$	80 926	50
$_{36}Kr^{78}$	77 926	0 42
$_{36}Kr^{80}$	79 926	2 45
$_{36}Kr^{82}$	81 927	11 79
$_{36}Kr^{83}$	82 927	11 79
$_{36}Kr^{84}$	83 928	56 85
$_{36}Kr^{86}$	85 929	16 70
$_{37}Rb^{85}$		72 7
$_{37}Rb^{87}$		27 3
$_{38}Sr^{86}$		10 0
$_{38}Sr^{87}$		6 6
$_{38}Sr^{88}$		83 4
$_{39}Y^{89}$		100
$_{40}Zr^{90}$		48
$_{40}Zr^{91}$		11 5
$_{40}Zr^{92}$		22
$_{40}Zr^{94}$		17
$_{40}Zr^{96}$		1 5
$_{41}Nb^{93}$	92 926	100
$_{42}Mo^{92}$		14 2
$_{42}Mo^{94}$		10 0
$_{42}Mo^{95}$		15 5
$_{42}Mo^{96}$		17 8
$_{42}Mo^{97}$		9 6
$_{42}Mo^{98}$	(97 946)	23 0
$_{42}Mo^{100}$	99 945	9 8
$_{43}$ (?)		

Nucleus	M (exact atomic mass)	I per cent. (relative percentage)
$_{44}Ru^{96}$		5
($_{44}Ru^{98}$)		
$_{44}Ru^{99}$		12
$_{44}Ru^{100}$		14
$_{44}Ru^{101}$		22
$_{44}Ru^{102}$		30
$_{44}Ru^{104}$		17
$_{45}Rh^{103}$	(102 92)	100
$_{46}Pd^{102}$		c
$_{46}Pd^{104}$		a
$_{46}Pd^{105}$		a
$_{46}Pd^{106}$		a
$_{46}Pd^{108}$		a
$_{46}Pd^{110}$		b
$_{47}Ag^{107}$		52 5
$_{47}Ag^{109}$		47 5
$_{48}Cd^{106}$		1 5
$_{48}Cd^{108}$		1 0
$_{48}Cd^{110}$		15 2
$_{48}Cd^{111}$		15 2
$_{48}Cd^{112}$		21 8
$_{48}Cd^{113}$		14 9
$_{48}Cd^{114}$		23 7
$_{48}Cd^{115}$		0 8
$_{48}Cd^{116}$		5 9
($_{48}Cd^{118}$)		
$_{49}In^{113}$		4 5
$_{49}In^{115}$		95 5
$_{50}Sn^{112}$		1 07
$_{50}Sn^{114}$		0 74
$_{50}Sn^{115}$		0 44
$_{50}Sn^{116}$		14 19
$_{50}Sn^{117}$		9 81
$_{50}Sn^{118}$		21 48
$_{50}Sn^{119}$		11 02
$_{50}Sn^{120}$	119 912	27 04
$_{50}Sn^{121}$		2 96
$_{50}Sn^{122}$		5 03
$_{50}Sn^{124}$		6 19
$_{51}Sb^{121}$		56
$_{51}Sb^{123}$		44
$_{52}Te^{122}$		2 9
$_{52}Te^{123}$		1 6
$_{52}Te^{124}$		4 5
$_{52}Te^{125}$		6 0
$_{52}Te^{126}$	.	19 0
($_{52}Te^{127}$)		.

Nucleus	*M* (*exact atomic mass*)	*I per cent.* (*relative percentage*)
$_{52}\mathrm{Te}^{128}$		32 8
$_{52}\mathrm{Te}^{130}$		33 1
$_{53}\mathrm{I}^{127}$	126 932	100
$_{54}\mathrm{Xe}^{124}$		0·08
$_{54}\mathrm{Xe}^{126}$		0 08
$_{54}\mathrm{Xe}^{128}$		2 30
$_{54}\mathrm{Xe}^{129}$		27 13
$_{54}\mathrm{Xe}^{130}$		4 18
$_{54}\mathrm{Xe}^{131}$		20 67
$_{54}\mathrm{Xe}^{132}$		26 45
$_{54}\mathrm{Xe}^{134}$	133 929	10 31
$_{54}\mathrm{Xe}^{136}$		8 79
$_{55}\mathrm{Cs}^{133}$	132 933	100
$_{56}\mathrm{Ba}^{135}$		5 9
$_{56}\mathrm{Ba}^{136}$		8 9
$_{56}\mathrm{Ba}^{137}$		11 1
$_{56}\mathrm{Ba}^{138}$	137 916	74 1
$_{57}\mathrm{La}^{139}$		100
$_{58}\mathrm{Ce}^{140}$		89
$_{58}\mathrm{Ce}^{142}$		11
$_{59}\mathrm{Pr}^{141}$		100
$_{60}\mathrm{Nd}^{142}$		36
$_{60}\mathrm{Nd}^{143}$		11
$_{60}\mathrm{Nd}^{144}$		30
$_{60}\mathrm{Nd}^{145}$		5
$_{60}\mathrm{Nd}^{146}$		18
$_{61}$ (?)		
$_{62}\mathrm{Sm}^{144}$		3
$_{62}\mathrm{Sm}^{147}$		17
$_{62}\mathrm{Sm}^{148}$		14
$_{62}\mathrm{Sm}^{149}$		15
$_{62}\mathrm{Sm}^{150}$		5
$_{62}\mathrm{Sm}^{152}$		26
$_{62}\mathrm{Sm}^{154}$		20
$_{63}\mathrm{Eu}^{151}$		50 6
$_{63}\mathrm{Eu}^{153}$		49 4
$_{64}\mathrm{Gd}^{155}$		21
$_{64}\mathrm{Gd}^{156}$		23
$_{64}\mathrm{Gd}^{157}$		17
$_{64}\mathrm{Gd}^{158}$		23
$_{64}\mathrm{Gd}^{160}$		16
$_{65}\mathrm{Tb}^{159}$		100
$_{66}\mathrm{Dy}^{161}$		22
$_{66}\mathrm{Dy}^{162}$		25
$_{66}\mathrm{Dy}^{163}$		25
$_{66}\mathrm{Dy}^{164}$		28
$_{67}\mathrm{Ho}^{165}$		100
$_{68}\mathrm{Er}^{166}$		36
$_{68}\mathrm{Er}^{167}$		24
$_{68}\mathrm{Er}^{168}$		30
$_{68}\mathrm{Er}^{170}$		10
$_{69}\mathrm{Tm}^{169}$		100
$_{70}\mathrm{Yb}^{171}$		9
$_{70}\mathrm{Yb}^{172}$		24
$_{70}\mathrm{Yb}^{173}$		17
$_{70}\mathrm{Yb}^{174}$		38
$_{70}\mathrm{Yb}^{176}$		12
$_{71}\mathrm{Lu}^{175}$		100
$_{72}\mathrm{Hf}^{176}$		5
$_{72}\mathrm{Hf}^{177}$		19
$_{72}\mathrm{Hf}^{178}$		28
$_{72}\mathrm{Hf}^{179}$		18
$_{72}\mathrm{Hf}^{180}$		30
$_{73}\mathrm{Ta}^{181}$	180 928	100
$_{74}\mathrm{W}^{182}$		22 6
$_{74}\mathrm{W}^{183}$		17 3
$_{74}\mathrm{W}^{184}$		30 2
$_{74}\mathrm{W}^{186}$		29 9
$_{75}\mathrm{Re}^{185}$		38 2
$_{75}\mathrm{Re}^{187}$	186 981	61 8
$_{76}\mathrm{Os}^{186}$		1 0
$_{76}\mathrm{Os}^{187}$		0 6
$_{76}\mathrm{Os}^{188}$		13 4
$_{76}\mathrm{Os}^{189}$		17 4
$_{76}\mathrm{Os}^{190}$	189 98	25 1
$_{76}\mathrm{Os}^{192}$	191 98	42 5
$_{77}\mathrm{Ir}^{191}$		33
$_{77}\mathrm{Ir}^{193}$		67
$_{78}\mathrm{Pt}^{192}$		d
$_{78}\mathrm{Pt}^{194}$		b
$_{78}\mathrm{Pt}^{195}$		a
$_{78}\mathrm{Pt}^{196}$		a
$_{78}\mathrm{Pt}^{198}$		c
$_{79}\mathrm{Au}^{197}$		100
$_{80}\mathrm{Hg}^{196}$		0 10
$_{80}\mathrm{Hg}^{197}$		0 01
$_{80}\mathrm{Hg}^{198}$		9 89
$_{80}\mathrm{Hg}^{199}$		16 45
$_{80}\mathrm{Hg}^{200}$	200 016	23 77

Nucleus	*M* (*exact atomic mass*)	*I per cent* (*relative percentage*)
$_{80}Hg^{201}$		13 67
$_{80}Hg^{202}$		29 27
$_{80}Hg^{203}$		0 006
$_{80}Hg^{204}$		6 85
$_{81}Tl^{203}$	203 037	29 4
$_{81}Tl^{205}$	205 037	70 6
($_{82}Pb^{203}$)		
$_{82}Pb^{204}$		1 52
($_{82}Pb^{205}$)		
$_{82}Pb^{206}$		28 03
$_{82}Pb^{207}$	.	20 40
$_{82}Pb^{208}$		50 05
($_{82}Pb^{209}$)		
($_{82}Pb^{210}$)		
$_{83}Bi^{209}$	.	100

TABLE C

MECHANICAL AND MAGNETIC MOMENTS OF NUCLEI†

(Mechanical moment, $\imath$, in $\hbar$ units, magnetic moment, μ, in nuclear magnetons $\mu_n = e\hbar/2M_p c$)

Z, even, M, even

Nucleus	$\imath$	μ
$_2He^4$	0	0
$_6C^{12}$	0	0
$_8O^{16}$	0	0
$_{16}S^{32}$	0	0
$_{48}Cd^{110}$	0	0
$_{48}Cd^{112}$	0	0
$_{48}Cd^{114}$	0	0
$_{48}Cd^{116}$	0	0
$_{56}Ba^{136}$	0	0
$_{56}Ba^{138}$	0	0
$_{80}Hg^{198}$	0	0
$_{80}Hg^{200}$	0	0
$_{80}Hg^{202}$	0	0
$_{80}Hg^{204}$	0	0
$_{82}Pb^{204}$	0	0
$_{82}Pb^{206}$	0	0
$_{82}Pb^{208}$	0	0

Z, odd, M, even

Nucleus	$\imath$	μ
$_1H^2$	1	0 75
$_7N^{14}$	1	$<$0 2

Z, even, M, odd

Nucleus	$\imath$	μ
$_4Be^9$	$\frac{1}{2}$	
$_{30}Zn^{67}$	$\frac{3}{2}$	
$_{36}Kr^{83}$	$\geqslant\frac{7}{2}$	
$_{48}Cd^{111}$	$\frac{1}{2}$	$-$0 67
$_{48}Cd^{113}$	$\frac{1}{2}$	$-$0 67
$_{50}Sn^{117}$	$\frac{1}{2}$	
$_{50}Sn^{119}$	$\frac{1}{2}$	
$_{54}Xe^{129}$	$\frac{1}{2}$	
$_{54}Xe^{131}$	$\sim\frac{3}{2}$	·
$_{56}Ba^{137}$	$\frac{5}{2}$	1 0
$_{80}Hg^{199}$	$\frac{1}{2}$	0·55
$_{80}Hg^{201}$	$\frac{3}{2}$	$-$0 62
$_{82}Pb^{207}$	$\frac{1}{2}$	0 60

Z, odd, M, odd

Nucleus	$\imath$	μ
$_1H^1$	$\frac{1}{2}$	2 5–3 5
$_3Li^7$	$\frac{3}{2}$	3 29
$_9F^{19}$	$\frac{1}{2}$	$\sim$3
$_{11}Na^{23}$	$\frac{3}{2}$	1 4
$_{13}Al^{27}$	$\frac{1}{2}$	2 1
$_{15}P^{31}$	$\frac{1}{2}$	
$_{17}Cl^{35}$	$\frac{5}{2}$	
$_{19}K^{39}$	$\frac{3}{2}$	$-$0 4
$_{21}Sc^{45}$	$\frac{7}{2}$	3 6
$_{25}Mn^{55}$	$\frac{5}{2}$	
$_{27}Co^{59}$	$\frac{7}{2}$	
$_{29}Cu^{63}$	$\frac{3}{2}$	2 5
$_{29}Cu^{65}$	$\frac{3}{2}$	2 5
$_{31}Ga^{69}$	$\frac{3}{2}$	2 01
$_{31}Ga^{71}$	$\frac{3}{2}$	2 55
$_{33}As^{75}$	$\frac{3}{2}$	0 9
$_{37}Rb^{85}$	$\frac{5}{2}$	1 3
$_{37}Rb^{87}$	$\frac{3}{2}$	2 7
$_{41}Nb^{93}$	$\frac{9}{2}$	3 7
$_{49}In^{115}$	$\frac{9}{2}$	5 4
$_{51}Sb^{121}$	$\frac{5}{2}$	2 7
$_{51}Sb^{123}$	$\frac{5}{2}$–$\frac{7}{2}$	2 1
$_{55}Cs^{133}$	$\frac{7}{2}$	
$_{57}La^{139}$	$\frac{5}{2}$–$\frac{7}{2}$	2 5
$_{59}Pr^{141}$	$\frac{5}{2}$	
$_{73}Ta^{181}$	$\frac{7}{2}$	
$_{79}Au^{197}$	$\frac{3}{2}$	
$_{81}Tl^{203}$	$\frac{1}{2}$	1 8
$_{81}Tl^{205}$	$\frac{1}{2}$	1 8
$_{83}Bi^{209}$	$\frac{9}{2}$	4 0

† These values are taken from tables by N Dallaporta, *Nuovo Cim* **12** (1936), 576.

TABLE D†

γ-RAY SPECTRA

(a) $MsTh_1 \rightarrow MTh_2$ (β); $MsTh_2 \rightarrow RaTh$ (β)‡

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 0581	L_1, L_3, M, N	250
0 0795	L_1, L_3	15
0 129	L_1, M, N	100
0 184	K, L_1, M	50
0 250	K, L_1	18
0 319	K, L_1	16
0 338	K, L_1	8
0 408	K, L_1	3
0 462	K, L_1, M	8
0 914	K, L_1	6
0 970	K, L_1	3

(b) $RaTh \rightarrow ThX$ (α)‖

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 0848	L_1, M_1	

(c) $ThB \rightarrow ThC$ (β)††

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 1147	L_1, L_2, M, N	16
0 1757	K, L_1	0 4
0 2379	$K, L_1, L_2, L_3, M_1, N_1$	165
0 2494	K, L_1	0 3
0 2990	K, L_1, M_1	6 0

(d) $ThC \rightarrow ThC'$ (β)†† ‡‡

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 726	K, L	0 3
0 864	K, L	0 2

† In these tables are included only those γ-rays of which the existence is proved by conversion in at least two atomic shells

‡ D Black, *Proc Roy Soc* **56** (1925), 632 ‖ L Meitner, *Zs f Phys* **52** (1928), 637.

†† C D Ellis, *Proc. Roy Soc* **138** (1932), 318

‡‡ C D Ellis also reports the weak γ-lines 1 623 and 1 802 converted only in the K level and fitting the energies of long-range α-particles

(e) $ThC \rightarrow ThC''$ (α)† ‡

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 0406	L_1, L_2, L_3, M, N, O	strong
0 287	K, L_1	0 65
0 298	K	
0 327	K	0 14
0 432	K	0 05
0 451	K	0 05
0 471	K	
0 617	K	

(f) $ThC'' \rightarrow ThD$ (β)†

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 2765	K, L_1, M_3	3 2
0 5100	K, L_1, M_1	1 7
0 5823	K, L_1, M_1	1 5
2 620	K, L_1, M_1	0 3

(g) $Ra \rightarrow RaEm$ (α)

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 189	K, L, M	

(h) $RaB \rightarrow RaC$ (β)††

$h\nu \times 10^6$ e v	*Conversion observed in*	*Number of γ-rays converted in the first level* $\times 10^{-4}$	*Coefficient of internal conversion*	*Number of γ-quanta per disintegration*
0 0529	$L_1, L_2, L_3, M_1, M_2, M_3, N, O$	240 (L_1)		
0 2406	K, L_1, M_1	425	0 364	0 115
0 2571	K, L_1	21		
0 2937	K, L_1	480	0 186	0 258
0 3499	K, L_1, M_1, N_1	530	0 117	0 450

† C D. Ellis, l c.

‡ In this case also several weak lines converted only in the K shell are included because they fit with the fine structure of α-rays

‖ L Meitner, *Zs f Phys* **26** (1924), 161

†† C D Ellis, *Proc Roy Soc* **143** (1934), 350

(i) $RaC \rightarrow RaC'$ (β)† ‡

$h\nu \times 10^6$ e.v	*Conversion observed in*	*Number of γ-rays converted in the first level* $\times 10^{-4}$	*Coefficient of internal conversion*	*Number of γ-quanta per disintegration*
0 6067	K, L_1, M_1, N	40	0 0061	0 658
0 766	K, L_1	3 2	0 0048	0 065
0 933	K, L_1	4 1	0 0061	0 067
1 120	K, L_1, M_1	12 8	0 0062	0 206
1 238	K, L_1	3 6	0 0057	0 063
1 379	K, L_1	0 9	0 0014	0 064
1 414	K, L_1, M_1	25 2		(0 000)
1 761	K, L_1	4 2	0 0016	0 258
2 193	K, L_1	0 95	0 0013	0 074

(k) $RaD \rightarrow RaE$ (β)‖

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 0472	L_1, L_2, L_3, M_1, N_1	

(l) $Pa \rightarrow Ac$ (α)††

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 0954	L_1, M_1, N_1	6
0 294	K, L_1, M_1	6
0 324	K, L_1, M_1	4

(m) $RaAc \rightarrow AcX$ (α)‡‡

$h\nu \times 10^6$ e v	*Conversion observed in*	*Relative intensity*
0 032	$L_1, L_2, M_1, M_2, N_1, N_2$	2
0 0437	L_1, L_2, L_3, M_1	4
0 0533	L_1, M_1	4
0 0614	L_1, M_1, N_1	9
0 1007	L_1, M_1, N_1	7
0 1491	K, L_1, M_1	8
0 1954	K, L_1, M_1	3
0 2539	K, L_1, M_1	4
0 2821	K, L_1	2
0 3002	K, L_1	2

† C D Ellis, *Proc Roy Soc* **143** (1934), 350

‡ Apart from the γ-lines listed here there are also many lines converted only in the K level (and therefore uncertain) which are, however, used in the construction of nuclear-level schemes

‖ C D Ellis, ibid **21** (1922), 125

†† L Meitner, *Zs f Phys* **50** (1928), 5

‡‡ L Meitner, ibid **34** (1925), 807.

(n) $AcX \rightarrow AcEm$ (α)†

$h\nu \times 10^6$ *e v*	*Conversion observed in*	*Relative intensity*
0 143	K, L_1, M_1	8
0 153	K, L_1, M_1	10
0 157	K, L_1, M_1	4
0 200	K, L_1	4
0 269	K, L_1	10

(o) $AcC \rightarrow AcC''$ (α); $AcC'' \rightarrow AcD$ (β)

$h\nu \times 10^6$ *e v*	*Conversion observed in*	*Relative intensity*
0 353	K, L_1, M_1	10
0 460	K, L_1	5
0 480	K, L_1	3

† L Meitner, l c

TABLE E†

α-DECAYING BODIES

(a) *Thorium family*

Nucleus	λ *sec* $^{-1}$ (T)	*R cm.*	$E_\alpha \times 10^6$ *e v*	*Relat Int*
$_{90}Th^{232}$	1 33 10^{-18} (1 65 10^{10} y)	(2 90)	4 23	
$_{94}RaTh^{228}$,,	1 16 10^{-8} (1 90 y)	(4 02)	5 420 5 335	5 1
$_{88}ThX^{224}$	2 20 10^{-6} (3 64 d)	(4 35)	5 6825	
$_{86}ThEm^{220}$	1 27 10^{-2} (54 5 s)	4 967	6 2832	
$_{84}ThA^{216}$	4 78 (0 145 s)	5 601	6 7759	
$_{83}ThC^{212}$,, ,, ,, ,,	6 65 10^{-5} ( ) ‡	4 693 (mean)	6 0837 6 0445 5 7621 5 6202 5 6012	27 2 69 8 1 80 0 16 1 10
$_{84}ThC'^{212}$,, L R ,, L R		8 533 9 687 11 543	8 7788 9 4912 10 5418	10^6 34 190

(b) *Radium family*

Nucleus	λ *sec* $^{-1}$ (T)	*R cm*	$E_\alpha \times 10^6$ *e v*	*Relat Int*
$_{92}U_{I}^{238}$	4 9 10^{-18} (4 5 10^9 y)	(2 70)	4 05	
$_{92}U_{II}^{234}$	$\sim$ 2 10^{-14} ($\sim 10^6$ y)	(3 28)	4 63	
$_{90}Io^{230}$	2 9 10^{-13} (7 6 10^4 y)	(3 19)	4 54	
$_{88}Ra^{226}$,,	1 373 10^{-11} (1 590 y)	2 26 3 08	4 793 4 612	. .
$_{86}RaEm^{222}$	2 097 10^{-6} (3 825 d)	4 014	5 48798	
$_{84}RaA^{218}$	3 79 10^{-3} (3 05 m)	4 620	6 1124	.
$_{83}RaC^{214}$,,	1 78 10^{-7} ( ) ‡	4 039 3 969	5 5068 5 4458	94 113

† The data for these tables are essentially taken from W B Lewis and B V Bowden, *Proc. Roy Soc* **145** (1934), 235

‡ In the case of C-bodies, where the phenomenon of branching takes place, the partial decay constants (see Chap V) only are given, the 'partial lifetimes' have evidently *no physical meaning*

Nucleus	λ *sec* $^{-1}$ (T)	*R cm.*	$E_\alpha \times 10^6$ *e v*	*Relat Int.*
$_{84}$RaC′^{214}	3 5 10^{+3} (1 9 10^{-4} s)	6 870	7 6830	10^6
,,			8 280	0 43
,,			8 941	0 45
,,		9 00	9 069	22 0
,,			9 315	0 38
,,			9 492	1 35
,,			9 660	0 35
,,			9 781	1 06
,,			9 908	0 36
,,			10 097	1 67
,,			10 149	0 38
,,			10 329	1 12
,,		11 47	10 509	0 23
$_{84}$Po^{210}	5 886 10^{-8} (136 3 d)	3 80	5 3003	.

(c) *Actinium family*

Nucleus	λ *sec* $^{-1}$ (T)	*R cm*	$E_\alpha \times 10^6$ *e v*	*Relat Int*
$_{91}$Pa^{231}	1 80 10^{-12} (1 25 10^4 y)	(3 67)	5 01	
$_{94}$RaAc^{227}	4 24 10^{-7} (18 9 d)	(4 68) (mean)	6 051	80
,,			6 019	15
,,			5 990	100
,,			5 968	15
,,			5 924	5
,,			5 870	10
,,			5 817	5
,,			5 766	80
,,			5 744	15
,,			5 719	60
,,			5 674	10
$_{88}$AcX^{223}	7 16 10^{-7} (11 2 d)	(4 37)	5 719	6
,,			5 607	4
,,			5 533	1
$_{86}$AcEm^{219}	0 177 (3 92 s)	5 655	6 826	10
,,		5 308	6 561	1
,,		5 147	6 436	1
$_{84}$AcA^{215}	3 5 10^{+2} (2 10^{-3} s)	6 420	7 368	
$_{83}$AcC^{211}	5 35 10^{-3} (.) ‡	5 392	6 611	100
,,		4 947	6 262	19
$_{84}$AcC′^{211}		6 518	7 437	

(d) *Lighter elements*†

Nucleus	λ *sec* $^{-1}$ (T)	*R cm*	$E_\alpha \times 10^6$ *e.v.*
$_{62}Sm^{(?)}$	$2\cdot2\ 10^{-20}$ ($1\ 10^{12}$ y)	1 13	2 05

† H Hoseman, *Zs. f Phys.* **99** (1936), 405 Decay constant calculated on the assumption that all isotopes of Sm are active

TABLE F

β-DECAYING BODIES

(a) *Thorium family*†

Nucleus	λ_β (*sec* $^{-1}$) (*T*)	E_{max} $\times 10^6$ *e v*
$_{88}MsTh_1^{228}$	3 28 10^{-9} (6 7 y)	
$_{89}MsTh_2^{228}$	3 14 10^{-5} (6 13 h)	2 05
$_{82}ThB^{212}$	1 82 10^{-5} (10 6 h)	0 36
$_{83}ThC^{212}$	1 23 10^{-4} ( ) ‖	2 25
$_{81}ThC''^{208}$	3 61 10^{-3} (3 20 m)	1 79

(b) *Radium family*†

Nucleus	λ_β (*sec* $^{-1}$) (*T*)	E_{max} $\times 10^6$ *e v*
$_{90}UX_I^{234}$	3 275 10^{-7} (24 5 d)	(0 13)
$_{91}UX_{II}^{234}$	1 013 10^{-2} (1 14 m)	2 32
$_{91}UZ^{234}$	2 87 10^{-5} (6 7 h)	
$_{82}RaB^{214}$	4 31 10^{-4} (26 8 m)	0 65
$_{83}RaC^{214}$	5 92 10^{-4} ( ) ‖	3 15
$_{81}RaC''^{210}$	8 75 10^{-3} (1 32 m)	
$_{82}RaD^{210}$	$\sim$ 1 37 10^{-9} ($\sim$ 25 y)	(0 035)
$_{83}RaE^{210}$	1 60 10^{-6} (5 0 d)	1 22

(c) *Actinium family*†

Nucleus	λ_β (*sec* $^{-1}$) (*T*)	E_{max} $\times 10^6$ *e v*
$_{89}Ac^{227}$	1 64 10^{-9} (13 4 y)	
$_{82}AcB^{211}$	3 21 10^{-4} (36 0 m)	(0 30)
$_{83}AcC^{211}$	1 6 10^{-5} (.) ‖	
$_{81}AcC''^{207}$	2 43 10^{-3} (4 76 m)	1 40

(d) *Lighter elements*‡

Nucleus	λ_β (*sec* $^{-1}$) (*T*)	E_{max} $\times 10^6$ *e v*
$_{19}K^{40}$	1 5 10^{-21} (1 5 10^{13} y)	(0 4), (0 7)
$_{37}Rb^{86(?)}$	5 0 10^{-20} (4 3 10^{11} y)	(0 1), (0 25)

† The upper energy limits are taken from B W Sargent, *Proc Roy Soc* **139** (1933), 659

‡ O Klemperer, ibid **148** (1935), 638 Decay constants are calculated on the hypothesis that all isotopes are equally active

‖ In the case of C-bodies, where the phenomenon of branching takes place, the partial decay constants (see Chap V) only are given, the 'partial lifetimes' have evidently no physical meaning

(e) *Fermi elements*†

(*e-emitters*)

Nucleus	T (λ *sec* $^{-1}$)	E_{max} $\times 10^{6}$ *e v*
$_{3}\mathrm{Li}^{8}$	0 5 s (1 38)	10 0 [11 2]
$_{5}\mathrm{B}^{12}$	0 02 s (34 6)	11 0 [13 0]
$_{7}\mathrm{N}^{16}$	9 s (7 7 10^{-2})	6 0 [6 5]
$_{8}\mathrm{O}^{19}$	40 s	
$_{9}\mathrm{F}^{20}$	12 s (5 8 10^{-2})	5 0 [5 9]
$_{10}\mathrm{Ne}^{23}$	40 s	
$_{11}\mathrm{Na}^{24}$	15 h (1 2 10^{-5})	1 7 [1 95]
$_{12}\mathrm{Mg}^{27}$	10 m	
$_{13}\mathrm{Al}^{28}$	137 s (5 10^{-3})	3 05
$_{13}\mathrm{Al}^{29}$	11 m	
$_{14}\mathrm{Si}^{31}$	145 m (7 7 10^{-5})	1 8 [2 05]
$_{15}\mathrm{P}^{32}$	14 d (5 8 10^{-7})	1 8 [2 15]
$_{17}\mathrm{Cl}^{x}$	37 m (3 1 10^{-4})	4 8 [1 5, 6 1]
$_{18}\mathrm{A}^{x}$	108 m (1 1 10^{-4})	2 7 [1 5, 5 0]
$_{19}\mathrm{K}^{42}$	16 h (1 1 10^{-5})	3 5 [1 4, 4 4]
$_{20}\mathrm{Ca}^{x}$	3 m, 3 h	
$_{23}\mathrm{V}^{52}$	3 75 m	
$_{25}\mathrm{Mn}^{56}$	2 5 h	
$_{29}\mathrm{Cu}^{x}$	5 m, 10 h	
$_{31}\mathrm{Ga}^{x}$	20 m, 23 h	
$_{32}\mathrm{Ge}^{x}$	30 m	
$_{33}\mathrm{As}^{76}$	26 h.	
$_{34}\mathrm{Se}^{x}$	35 h	
$_{35}\mathrm{Br}^{x}$	18 m, 255 m, 36 h	
$_{40}\mathrm{Zr}^{x}$	4 h	
$_{42}\mathrm{Mo}^{x}$	30 m, 36 h	
$_{45}\mathrm{Rh}^{104}$	44 s, 4 m	
$_{47}\mathrm{Ag}^{x}$	22 s, 2 3 m	
$_{49}\mathrm{In}^{x}$	13 s, 54 m, 3 h	
$_{51}\mathrm{Sb}^{x}$	2 5 d	
$_{52}\mathrm{Te}^{x}$	45 m	
$_{53}\mathrm{I}^{128}$	25 m	
$_{55}\mathrm{Cs}^{134}$	1 5 h	
$_{56}\mathrm{Ba}^{x}$	80 m	
$_{58}\mathrm{Ce}^{x}$	5 m	
$_{59}\mathrm{Pr}^{142}$	19 h	
$_{60}\mathrm{Nd}^{x}$	1 h	
$_{62}\mathrm{Sm}^{x}$	40 m	
$_{64}\mathrm{Gd}^{x}$	8 h	
$_{72}\mathrm{Hf}^{x}$	2 m	
$_{74}\mathrm{W}^{x}$	1 d	
$_{75}\mathrm{Re}$	20 h	
$_{77}\mathrm{Ir}$	19 h	
$_{78}\mathrm{Pt}$	50 m	
$_{79}\mathrm{Au}^{198}$	2 7 d	
$_{90}\mathrm{Th}$	1 m, 24 m	
$_{92}\mathrm{U}$	10 s, 40 s, 13 m, 100 s	

(f) *Curie-Joliot elements*†

(*ə-emitters*)

Nucleus	T (λ *sec* $^{-1}$)	E_{max} $\times 10^{6}$ *e v*
$_{5}\mathrm{B}^{9}$	1 m (1 1 10^{-2})	0 3
$_{6}\mathrm{C}^{11}$	20 m (5 8 10^{-4})	1 15 [1 3]
$_{7}\mathrm{N}^{13}$	10 m (1 1 10^{-3})	1 3 [1 5]
$_{8}\mathrm{O}^{15}$	126 s (5 5 10^{-3})	1 7 [2 0]
$_{9}\mathrm{F}^{17}$	1 16 m (1 0 10^{-2})	2 1 [2 4]
$_{13}\mathrm{Al}^{26}$	7 s	
$_{14}\mathrm{Si}^{27}$	8 m	
$_{15}\mathrm{P}^{30}$	195 s (3 5 10^{-3})	3 7
$_{17}\mathrm{Cl}^{34}$	40 m	
$_{21}\mathrm{Sc}^{x}$	3 h	

† The data for these tables are collected from various publications of Joliot and Curie, Fermi, and other authors, and many of them may not be quite correct, as the chemical identity of products has been established only in a few cases The energies of β-spectra are taken mainly from publications of W A Fowler, L A Delsasso, and C C Lauritsen (*Phys Rev* **49** (1936), 561), and N D Kurie, J R Richardson, and H C Paxton (ibid 368), and represent the observed upper limits of the continuous β-spectra These authors have also compared the observed shape of the spectrum with the formula of Uhlenbeck and Konopinski (p 144) and have thus obtained the 'extrapolated' upper limits which should better represent the actual energy of disintegration, these values are given, together with the observed limits, in square brackets

TABLE G

NUCLEAR REACTIONS†

(a) *Substitutional reactions*

(α) *Bombardment with α-particles*

Equation of reaction	*Energy liberated or Energy-balance* $Q \times 10^6$ *e v*	*Probability of reaction*		*Notes*
		Energy of incident particle $E_\alpha \times 10^6$ *e v*	*Yield per incident particle*	
${}_{3}Li^{6} + {}_{2}He^{4} \rightarrow {}_{5}B^{9} + {}_{0}n^{1}$				
${}_{3}Li^{7} + {}_{2}He^{4} \rightarrow {}_{5}B^{10} + {}_{0}n^{1}$	−3	1	10^{-9}	
${}_{4}Be^{9} + {}_{2}He^{4} \rightarrow {}_{6}C^{12} + {}_{0}n^{1}$	6	5	5×10^{-5}	
${}_{5}B^{10} + {}_{2}He^{4} \rightarrow {}_{6}C^{13} + {}_{1}H^{1}$	3 2, 0 2, −1 4			
${}_{5}B^{10} + {}_{2}He^{4} \rightarrow {}_{7}N^{13} + {}_{0}n^{1}$				
${}_{5}B^{11} + {}_{2}He^{4} \rightarrow {}_{7}N^{14} + {}_{0}n^{1}$	−0 5			
${}_{7}N^{14} + {}_{2}He^{4} \rightarrow {}_{8}O^{17} + {}_{1}H^{1}$	−1 3			
${}_{7}N^{14} + {}_{2}He^{4} \rightarrow {}_{9}F^{17} + {}_{0}n^{1}$	−4 5			
${}_{9}F^{19} + {}_{2}He^{4} \rightarrow {}_{10}Ne^{22} + {}_{1}H^{1}$	+1 5	5	3×10^{-7}	
${}_{9}F^{19} + {}_{2}He^{4} \rightarrow {}_{11}Na^{22} + {}_{0}n^{1}$	−2 5			
${}_{11}Na^{23} + {}_{2}He^{4} \rightarrow {}_{12}Mg^{26} + {}_{1}H^{1}$	+1 9			
${}_{11}Na^{23} + {}_{2}He^{4} \rightarrow {}_{13}Al^{26} + {}_{0}n^{1}$				
${}_{12}Mg^{24} + {}_{2}He^{4} \rightarrow {}_{13}Al^{27} + {}_{1}H^{1}$	−1 1, −1 9, −2 9			
${}_{12}Mg^{24} + {}_{2}He^{4} \rightarrow {}_{14}Si^{27} + {}_{0}n^{1}$				
${}_{12}Mg^{25} + {}_{2}He^{4} \rightarrow {}_{13}Al^{28} + {}_{1}H^{1}$				
${}_{12}Mg^{26} + {}_{2}He^{4} \rightarrow {}_{13}Al^{29} + {}_{1}H^{1}$				
${}_{13}Al^{27} + {}_{2}He^{4} \rightarrow {}_{14}Si^{30} + {}_{1}H^{1}$	+2 07, −0 16, −1 52, −2 67	5	2×10^{-6}	
${}_{13}Al^{27} + {}_{2}He^{4} \rightarrow {}_{15}P^{30} + {}_{0}n^{1}$		7	2×10^{-7}	
${}_{14}Si^{29} + {}_{2}He^{4} \rightarrow {}_{15}P^{32} + {}_{1}H^{1}$				
${}_{15}P^{31} + {}_{2}He^{4} \rightarrow {}_{16}S^{34} + {}_{1}H^{1}$	−0 1			
${}_{15}P^{31} + {}_{2}He^{4} \rightarrow {}_{17}Cl^{34} + {}_{0}n^{1}$				
${}_{16}S^{32} + {}_{2}He^{4} \rightarrow {}_{17}Cl^{35} + {}_{1}H^{1}$	−2 3			
${}_{19}K^{x} + {}_{2}He^{4} \rightarrow {}_{21}Sc^{x+3} + {}_{0}n^{1}$				

(β) *Bombardment with deuterons*

${}_{1}D^{2} + {}_{1}D^{2} \rightarrow {}_{1}F^{3} + {}_{1}H^{1}$	3 97			
${}_{1}D^{2} + {}_{1}D^{2} \rightarrow {}_{2}He^{3} + {}_{0}n^{1}$	3 2			
${}_{3}Li^{6} + {}_{1}D^{2} \rightarrow 2\,{}_{2}He^{4}$	22			
${}_{3}Li^{6} + {}_{1}D^{2} \rightarrow {}_{3}Li^{7} + {}_{1}H^{1}$	5			
${}_{3}Li^{7} + {}_{1}D^{2} \rightarrow 2\,{}_{2}He^{4} + {}_{0}n^{1}$	14 6	0 8	5×10^{-8}	
${}_{4}Be^{9} + {}_{1}D^{2} \rightarrow {}_{4}Be^{10} + {}_{1}H^{1}$	4 7			
${}_{4}Be^{9} + {}_{1}D^{2} \rightarrow {}_{4}Be^{8} + {}_{1}T^{3}$				
${}_{4}Be^{9} + {}_{1}D^{2} \rightarrow {}_{5}B^{10} + {}_{0}n^{1}$	4			
${}_{5}B^{10} + {}_{1}D^{2} \rightarrow 3\,{}_{2}He^{4}$	19 4			
${}_{5}B^{10} + {}_{1}D^{2} \rightarrow {}_{5}B^{11} + {}_{1}H^{1}$	9			
${}_{5}B^{10} + {}_{1}D^{2} \rightarrow {}_{6}C^{11} + {}_{0}n^{1}$		0 55	5×10^{-9}	
${}_{6}C^{12} + {}_{1}D^{2} \rightarrow {}_{7}N^{13} + {}_{0}n^{1}$				

† These data are taken from tables by Giorgio Fea, *Nuovo Cim.* **12** (1935), 369.

		Probability of reaction		
Equation of reaction	*Energy-balance* $Q \times 10^6$ *e v*	*Energy of incident particle* $E_\alpha \times 10^6$ *e v*	*Yield per incident particle*	*Notes*
${}_{6}C^{12} + {}_{1}D^{2} \rightarrow {}_{6}C^{13} + {}_{1}H^{1}$	2 6	1 26	8×10^{-8}	
${}_{6}C^{12} + {}_{1}D^{2} \rightarrow {}_{7}N^{13} + {}_{0}n^{1}$	-0 5	1	2×10^{-9}	
${}_{7}N^{14} + {}_{1}D^{2} \rightarrow {}_{6}C^{12} + {}_{2}He^{4}$	13			
${}_{7}N^{14} + {}_{1}D^{2} \rightarrow {}_{7}N^{15} + {}_{1}H^{1}$	8 5			
${}_{7}N^{14} + {}_{1}D^{2} \rightarrow {}_{8}O^{15} + {}_{0}n^{1}$	7 3	0 55	5×10^{-10}	
${}_{8}O^{16} + {}_{1}D^{2} \rightarrow {}_{8}O^{17} + {}_{1}H^{1}$	1 9	1 26	4×10^{-8}	
${}_{11}Na^{23} + {}_{1}D^{2} \rightarrow {}_{12}Mg^{24} + {}_{0}n^{1}$		2 15	3×10^{-7}	
${}_{11}Na^{23} + {}_{1}D^{2} \rightarrow {}_{10}Ne^{21} + {}_{2}He^{4}$	7	2 15	1×10^{-7}	
${}_{11}Na^{23} + {}_{1}D^{2} \rightarrow {}_{11}Na^{24} + {}_{1}H^{1}$	4 5	2 15	9×10^{-7}	
${}_{13}Al^{27} + {}_{1}D^{2} \rightarrow {}_{13}Al^{28} + {}_{1}H^{1}$	5 3	2 2	8×10^{-7}	
${}_{13}Al^{27} + {}_{1}D^{2} \rightarrow {}_{14}Si^{28} + {}_{0}n^{1}$			2×10^{-7}	
${}_{13}Al^{27} + {}_{1}D^{2} \rightarrow {}_{12}Mg^{25} + {}_{2}He^{4}$	6 6		6×10^{-7}	

(γ) *Bombardment with protons*

${}_{3}Li^{6} + {}_{1}H^{1} \rightarrow {}_{2}He^{4} + {}_{2}He^{3}$	3 5			
${}_{3}Li^{7} + {}_{1}H^{1} \rightarrow 2\ {}_{2}He^{4}$	17	0 45	8×10^{-8}	
${}_{4}Be^{9} + {}_{1}H^{1} \rightarrow {}_{3}Li^{6} + {}_{2}He^{4}$	2	0 5	1×10^{-8}	
${}_{4}Be^{9} + {}_{1}H^{1} \rightarrow {}_{4}Be^{8} + {}_{1}D^{2}$				
${}_{5}B^{11} + {}_{1}H^{1} \rightarrow 3\ {}_{2}He^{4}$	8 7	0 16	$6\ 5 \times 10^{-8}$	
${}_{5}B^{11} + {}_{1}H^{1} \rightarrow {}_{4}Be^{8} + {}_{2}He^{4}$	8 64			
${}_{7}N^{14} + {}_{1}H^{1} \rightarrow {}_{6}C^{11} + {}_{2}He^{4}$				
${}_{9}F^{19} + {}_{1}H^{1} \rightarrow {}_{8}O^{16} + {}_{2}He^{4}$	8	0 5	$1\ 2 \times 10^{-7}$	
${}_{11}Na^{23} + {}_{1}H^{1} \rightarrow {}_{10}Ne^{20} + {}_{2}He^{4}$				
${}_{19}K^{39} + {}_{1}H^{1} \rightarrow {}_{18}A^{36} + {}_{2}He^{4}$				

(δ) *Bombardment with neutrons*

${}_{3}Li^{6} + {}_{0}n^{1} \rightarrow {}_{2}He^{4} + {}_{1}T^{3}$	4 3			
${}_{5}B^{10} + {}_{0}n^{1} \rightarrow {}_{3}Li^{7} + {}_{2}He^{4}$	2 3			
${}_{6}C^{12} + {}_{0}n^{1} \rightarrow {}_{4}Be^{9} + {}_{2}He^{4}$				
${}_{7}N^{14} + {}_{0}n^{1} \rightarrow {}_{5}B^{11} + {}_{2}He^{4}$				
${}_{7}N^{14} + {}_{0}n^{1} \rightarrow {}_{6}C^{13} + {}_{1}D^{2}$				
${}_{8}O^{16} + {}_{0}n^{1} \rightarrow {}_{6}C^{13} + {}_{2}He^{4}$				
${}_{9}F^{19} + {}_{0}n^{1} \rightarrow {}_{7}N^{16} + {}_{2}He^{4}$				
${}_{10}Ne^{20} + {}_{0}n^{1} \rightarrow {}_{8}O^{17} + {}_{2}He^{4}$				
${}_{12}Mg^{24} + {}_{0}n^{1} \rightarrow {}_{11}Na^{24} + {}_{1}H^{1}$				
${}_{13}Al^{27} + {}_{0}n^{1} \rightarrow {}_{12}Mg^{27} + {}_{1}H^{1}$				
${}_{13}Al^{27} + {}_{0}n^{1} \rightarrow {}_{11}Na^{24} + {}_{2}He^{4}$				
${}_{14}Si^{28} + {}_{0}n^{1} \rightarrow {}_{13}Al^{28} + {}_{1}H^{1}$				
${}_{15}P^{31} + {}_{0}n^{1} \rightarrow {}_{13}Al^{28} + {}_{2}He^{4}$				
${}_{15}P^{31} + {}_{0}n^{1} \rightarrow {}_{14}Si^{31} + {}_{1}H^{1}$				
${}_{16}S^{32} + {}_{0}n^{1} \rightarrow {}_{15}P^{32} + {}_{1}H^{1}$				
${}_{17}Cl^{35} + {}_{0}n^{1} \rightarrow {}_{15}P^{32} + {}_{2}He^{4}$				
${}_{20}Ca^{42} + {}_{0}n^{1} \rightarrow {}_{19}K^{42} + {}_{1}H^{1}$				
${}_{21}Sc^{45} + {}_{0}n^{1} \rightarrow {}_{19}K^{42} + {}_{2}He^{4}$				
${}_{24}Cr^{52} + {}_{0}n^{1} \rightarrow {}_{23}V^{52} + {}_{1}H^{1}$				
${}_{25}Mn^{55} + {}_{0}n^{1} \rightarrow {}_{23}V^{52} + {}_{2}He^{4}$				
${}_{26}Fe^{56} + {}_{0}n^{1} \rightarrow {}_{25}Mn^{56} + {}_{1}H^{1}$				

Equation of reaction	*Energy-balance* $Q \times 10^6$ *e v*	*Probability of reaction*		*Notes*
		Energy of incident particle $E_\alpha \times 10^6$ *e v*	*Yield per incident particle*	
${}_{27}Co^{59} + {}_0n^1 \rightarrow {}_{25}Mn^{56} + {}_2He^4$				
${}_{30}Zn^x + {}_0n^1 \rightarrow {}_{29}Cu^x + {}_1H^1$				
${}_{35}Br^{79} + {}_0n^1 \rightarrow {}_{35}Br^{78} + 2{}_0n^1$ (?)				

(b) *Radiative capture*

${}_0n^1 + {}_1H^1 \rightarrow {}_1D^2 + h\nu$
${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + h\nu$
${}_{11}Na^{23} + {}_0n^1 \rightarrow {}_{11}Na^{24} + h\nu$
${}_{12}Mg^{26} + {}_0n^1 \rightarrow {}_{12}Mg^{27} + h\nu$
${}_{13}Al^{27} + {}_0n^1 \rightarrow {}_{13}Al^{28} + h\nu$
${}_{14}Si^{30} + {}_0n^1 \rightarrow {}_{14}Si^{31} + h\nu$
${}_{17}Cl^x + {}_0n^1 \rightarrow {}_{17}Cl^{x+1} + h\nu$
${}_{19}K^{41} + {}_0n^1 \rightarrow {}_{19}K^{42} + h\nu$
${}_{23}V^{51} + {}_0n^1 \rightarrow {}_{23}V^{52} + h\nu$
${}_{25}Mn^{55} + {}_0n^1 \rightarrow {}_{25}Mn^{56} + h\nu$
${}_{29}Cu^x + {}_0n^1 \rightarrow {}_{29}Cu^{x+1} + h\nu$
${}_{31}Ga^x + {}_0n^1 \rightarrow {}_{31}Ga^{x+1} + h\nu$
${}_{33}As^{75} + {}_0n^1 \rightarrow {}_{33}As^{76} + h\nu$
${}_{34}Se^x + {}_0n^1 \rightarrow {}_{34}Se^{x+1} + h\nu$

${}_{35}Br^x + {}_0n^1 \rightarrow {}_{35}Br^{x+1} + h\nu$
${}_{47}Ag^x + {}_0n^1 \rightarrow {}_{47}Ag^{x+1} + h\nu$
${}_{48}Cd^x + {}_0n^1 \rightarrow {}_{48}Cd^{x+1} + h\nu$
${}_{49}In^x + {}_0n^1 \rightarrow {}_{49}In^{x+1} + h\nu$
${}_{51}Sb^x + {}_0n^1 \rightarrow {}_{51}Sb^{x+1} + h\nu$
${}_{53}I^{127} + {}_0n^1 \rightarrow {}_{53}I^{128} + h\nu$
${}_{56}Ba^{138} + {}_0n^1 \rightarrow {}_{56}Ba^{139} + h\nu$
${}_{72}Hf^{180} + {}_0n^1 \rightarrow {}_{72}Hf^{181} + h\nu$
${}_{74}W^x + {}_0n^1 \rightarrow {}_{74}W^{x+1} + h\nu$
${}_{75}Re^x + {}_0n^1 \rightarrow {}_{75}Re^{x+1} + h\nu$
${}_{77}Ir^x + {}_0n^1 \rightarrow {}_{77}Ir^{x+1} + h\nu$
${}_{79}Au^x + {}_0n^1 \rightarrow {}_{79}Au^{x+1} + h\nu$
${}_{90}Th^{232} + {}_0n^1 \rightarrow {}_{90}Th^{233} + h\nu$
${}_{92}U^{238} + {}_0n^1 \rightarrow {}_{92}U^{239} + h\nu$