MNEMOSYNE

BIBLIOTHECA CLASSICA BATAVA

SUPPLEMENTUM DUCENTESIMUM TERTIUM

JON SOLOMON

PTOLEMY *HARMONICS*

PTOLEMY *HARMONICS*

TRANSLATION AND COMMENTARY

BY

JON SOLOMON

BRILL
LEIDEN · BOSTON · KÖLN
2000

This book is printed on acid-free paper.

Library of Congress Cataloging-in-Publication Data

The Library of Congress Cataloging-in-Publication Data is also available.

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

[Mnemosyne / Supplementum]
Mnemosyne : bibliotheca classica Batava. Supplementum. - Leiden ;
Boston ; Köln : Brill
Früher Schriftenreihe
Teilw. u.d.T.: Mnemosyne / Supplements
Reihe Supplementum zu: Mnemosyne
203. Solomon, Jon: Ptolemy Harmonics. - 1999

Solomon, Jon:
Ptolemy Harmonics : translation ad commentary / by Jon Solomon. -
Leiden ; Boston ; Köln : Brill, 1999
(Mnemosyne : Supplementum ; 203)
ISBN 90-04-11591-9

ISSN 0169-8958
ISBN 90 04 11591 9

PRINTED IN THE NETHERLANDS

To my father, μουσικωτέρῳ

CONTENTS

PREFACE

It is the fortunate modern who dusts off this classic introduction to harmonics, for harmonics is a fascinating science that germinated among our stone age ancestors, took root in ancient Egypt and Mesopotamia, and blossomed here during the Greco-Roman epoch with Ptolemy as its greatest proponent. Far from abandoned today in anything but name, harmonics has now divided and reproduced itself amidst a very large array of scientists ranging from astro-physicists examining our universe, all billions of light years of it, to geneticists examining a phenomenon as microscopic as the nuclear structure of human DNA. It is the study of the harmony that can be found in mathematics, music, the human soul, and the cosmos which contains it all.

In a word or two, harmonics is the science of cosmic and psychic harmony. It starts with numbers and the relationship between numbers (mathematics). When applied to a taut string, such numerical relationships produce musical sounds, and these mathematical-musical relationships can be felt within ourselves. Certain musical sounds make us feel certain ways, and so music and our souls are harmonically related. Similarly, as the planets circle the heavens they stand in similar mathematical-musical proportions. Understanding all this, attempting to reduce it to a mathematical description, that is the domain of harmonics. Even more so than their widely heralded gifts to rational civilization—philosophy, tragedy, and politics—harmonics was ancient Greece's greatest attempt at rationalizing our existence.

And yet, harmonics was also ancient Greece's most beautifully proportioned and well-crafted creation, as intellectually stimulating to the mind as the newly completed Parthenon must have been to the eye. No Greek science was ever more exacting, but none of their arts was ever more ambitious. Harmonics was ancient Greece's most profound, comprehensive *logos*, for it stood on the shoulders of philosophy and mathematics and physics to tune in to the cosmic harmonies of the celestial spheres, and it organized and analyzed human habit and ethics and probed the psychic realms of the human soul.

I will tell you from the outset that the pages of this volume will hardly seem to hum any divinely inspired music as you page through them. Much of the text is mathematical proportion and geometrical proofs, and just when the author was about to detail the relationships between mathematical-musical proportion, the human soul, and the heavens, he...well, we cannot be certain, but Byzantine rumor has it that he died. Nonetheless, what the following pages do offer is the finest presentation of this cumulative science one of the most influential scientists the Western world has ever known was able to pen some indeterminate interval before his death a mere 1850 years ago.

History has played a lengthy, inconsistent game with Claudius Ptolemy. The astronomical fame and stereoscopic influence he enjoyed during the millennium of the Middle Ages waned finally only with the advent of Newtonian celestial mechanics. Then, after several centuries of relative dormancy, the stature of his greatest work, the *Almagest*, suffered considerably under the pressure of a fraudulence charge two decades ago. Suffering somewhat parallel fates, his *Geography* and astrological treatise, the *Tetrabiblos*, lost their dominance with the discovery of the new world and the development of a confirmed heliocentric astronomy. Sadly, the last complete copy of his *Optics* disappeared centuries ago.

His other great work, the *Harmonics*, also had great impact throughout fifteen of the first sixteen centuries of our era but was then for the most part ignored in the last few centuries. Now, though, as part of the renascence of interest in ancient Greek music theory, the *Harmonics* has received three complete translations in the twentieth century alone, the first into German by Ingemar Düring in 1934, the second into English by Andrew Barker in 1989; this is the third. Rarely consulted while remaining only in the original Greek or one of its late Renaissance Latin translations, the *Harmonics* is now available to many more scholars and interested readers to explore its forty-eight chapters on ancient music, acoustics, mathematics, physics, philosophy, astrology, or, in a word, harmonics.

No one agrees on a most correct type of translation. Reflecting in part the various artistic tastes of the last one hundred years, the archaizing translations of Greek and Roman works produced during the late nineteenth century gave way to more lyrical sorts a generation ago, and today we have a potpourri of styles ranging from the Latinate and literal to the free and imaginative. The present text demands something much closer to the Latinate and literal style, and for this I owe no apologies. Ptolemy, for all his number-crunching thoroughness and scientific importance, did not, to use the modern pedagogical parlance, write very well. He would be surprised to find the English version of his treatise on harmonics fluid and pleasant going, and I did not see my function as a translator to rewrite the treatise and make it seem fluid and pleasant going. I did regularly divide into digestible punctuated segments Ptolemy's lengthy, interdependent Mississippian sentences, and I did try to identify wherever possible or probable his vague antecedents. But at the same time I was reluctant to employ thirty words to translate an idea or process Ptolemy had described in ten. I used Richmond Lattimore's *Iliad* as my conceptual model, which means not that in some Nonnusian fashion I attempted the *Harmonics* in late-Greek hexameters, but that I kept before me the idea that the style of the Greek author should come through the English rendering. I love the ancient Greek language for some of its syntactical idiosyncrasies which by nature work beautifully in Greek but sound sophomoric in English. Rows of prepositional phrases remain, while omnipresent connectives expressing the result of one sentence which explained the cause of its predecessor and which will introduce a concession for the next sentence were toned down but not tuned out. The few passages into which Ptolemy inserted vivid vocabulary I also tried to enliven, and I then found it simple enough to parallel the formulaic prose of the geometrical demonstrations and the reconstructed last few chapters of the treatise. I will anticipate criticism by warning that if readers find my translation to be less than perfectly clear on occasion, I will in turn attempt to blame Ptolemy for not making his meaning clearer in the first place. Those who read German should not rely entirely upon Düring's translation; when he is not accurate he can be very inaccurate (although I do extend him the same sympathy which I invite others to have for me.)

I have followed Düring's 1930 text, to which the page and line numbers in the margins and the annotations of this book refer. Where Düring made (and in some cases later corrected) errors, where John Wallis, Bengt Alexanderson, or Carsten Höeg revised the text, and where I found manuscript readings in Düring's apparatus superior to those in his text, I abandoned Düring's printed text.

The annotations serve various purposes for a variety of audiences. For philologists there are entries discussing text and the translation, particularly of technical terms, for philosophers there are citations and responses to the work originated by Düring and Boll, and for historians of science there are references to an array of scholars from Heath and Neugebauer to (R. R.) Newton and Burkert. I make frequent cross references to other sections of the treatise. Where I felt the text required exegesis or summary I offer it, but where the text is self-explanatory or even obvious I keep silent. Although some *loci paralleli* are to be found, it would be a task for a computerized Sisyphus to cite all the passages in philosophical, astronomical, astrological, mathematical, and musicological works that are parallel to something Ptolemy discusses in the *Harmonics*; some different loci paralleli can be found in Barker's translation, but his parallels are not exhaustive either. They could not be. Ptolemy was a polymath, and harmonics is a consummate science which therefore does not allow for complete commentary, unless one could assemble and invite to make a contribution experts in each of harmonics' half-dozen subfields. The incomplete status of Ptolemy's surviving text does not warrant it, though; too much of Book III is abbreviated or lacunate, so what might have been an impressively comprehensive treatise leading from the mortar and bricks of pitch and tuning to the eternal, universal reflection of acoustical, psychic, and celestial harmonics instead has survived as a treatise useful mostly for understanding only ancient Greek methods of attunement.

In brief, my goals in providing a commentary were to expand the reader's opportunities for understanding and exploring Ptolemy's treatise by offering both internal and external guidance and providing references to contemporary scholarship. Ptolemy's *Harmonics* is not a work designed to be a simple and fundamental introduction to ancient Greek music; novices in the field would be more rapidly enlightened by reading Cleonides *Introduction to Harmonics* or even Winnington-Ingram's essay in *The New Grove Dictionary of Music and Musicians*. Ptolemy's *Harmonics* is a treatise that expects its readers to understand already the music theory promulgated by Aristoxenus, Archytas, and others, and when Ptolemy addresses their work, he becomes polemical. Ptolemy's *Harmonics* is a treatise that expects its readers to understand the essence of acoustical science, insofar as string lengths and ratios are concerned, and when Ptolemy addresses this science, he becomes idiosyncratic. In my commentary, I therefore assume that the reader is turning to Ptolemy as a dominant investigator in the composite field of harmonics.

This project has received some small but very much appreciated financial support in the form of a University of Minnesota Graduate School Grant-in-Aid and a Faculty Summer Research Appointment as well as an organological allowance from the Small Grants Program at the University of Arizona. Scholarly support I have received from several learned peers to whom I owe special gratitude for their encouraging and persistent questioning about the status of this project. At times I felt as if my two lustra of work would be repaid simply by their having a guilt-produced translation and commentary of Ptolemy's *Harmonics* in their private libraries. I thank particularly Thomas J. Mathiesen of Indiana University, Calvin Bower of Notre Dame University, and André Barbera of St. John's College, Annapolis, two of whom would have preferred a new text, the other of whom would have preferred a stemma for the diagrams. I owe special thanks as well to Claude V. Palisca of Yale University, the editor of the series under the aegis of which this book was originally conceived. I would also like to thank my children, who, though while growing from infancy allowed me only four or five hours each night between the harmonic rising of the full moon and the diapasonic cock's crow to prepare the present volume, are now old enough to read it. As in everything I am particularly indebted to my wife Lois who supported and encouraged me throughout the entire process.

Jon Solomon
Tucson, 1999

ABBREVIATIONS

Journals and Encyclopedias

The variegated nature of this treatise derives from both Ptolemy's breadth of knowledge and the nature of harmonics itself, a grand science subsuming a series of lesser sciences. The notes to the introduction and the annotations to the translation contain references to technical journals as disparate as *The Journal of Music Theory*, *The Journal of Hellenic Studies*, and *The Journal for the History of Astronomy*. To abbreviate each of these journals in the bibliographical shorthand normally used within each discipline would leave many readers unenlightened much of the time. As a result, I have for the most part left the names of journals unabbreviated; what this costs in space it makes up for in clarity. Those journals cited often enough to warrant such abbreviations, though not in every instance, are:

AfM	—	*Archiv für Musikwissenschaft*
AJPh	—	*The American Journal of Philology*
AM	—	*Acta Musicologica*
BCH	—	*Bulletin de correspondance hellénique*
CR	—	*Classical Review*
CQ	—	*Classical Quarterly*
JAMS	—	*The Journal of the American Musicological Society*
JHS	—	*The Journal of Hellenic Studies*
JMT	—	*The Journal of Music Theory*
RE	—	*A. Pauly, G. Wissowa, and W. Kroll, Real—Encyclopädie der klassischen Altertumswissenschaft*
REG	—	*Revue des études grecques*
TAPA	—	*Transactions of the American Philological Association*

Ancient Sources

I have used the standard abbreviations for the ancient sources cited. These standard abbreviations can be found in the preface to the second edition of the *Oxford Classical Dictionary* [N. G. L. Hammond and H. H. Scullard, eds., *The Oxford Classical Dictionary*² (Oxford 1970)], pp. ix–xxii, and in the introduction to *LSJ* [H. G. Liddell, R. Scott, and H. S. Jones, *A Greek-English Lexicon*⁹ (Oxford 1940; repr. 1968)], pp. xvi–xlv. On occasion some of the names of more obscure authors and treatises have been spelled out in full.

Of the ancient authors who wrote on music and musicology I have used the following editions most frequently:

Aristides Quintilianus [Aristid. Quint.]:
R. P. Winnington-Ingram, ed., *Aristides Quintiliani de musica libri tres* (Leipzig 1963).

Aristoxenus [Aristox.]:
Henry S. Macran, ed., *The Harmonics of Aristoxenus* (Oxford 1902; reprint: Hildesheim 1974).

Rosetta da Rios, ed., *Aristoxeni elementa harmonica* (Rome 1954).

Bacchius [Bacch.]:
Karl von Jan, ed., *Musici scriptores Graeci* (Leipzig 1892).

Boethius [Boeth.]:
Gottfried Friedlein, ed., *Boethius: De institutione musica libri quinque* (Leipzig 1867).

Calvin M. Bower, trans., *Ancius Manlius Severinus Boethius, Fundamentals of Music* (New Haven and London 1989).

Bryennius [Bry.]:
G. H. Jonker, ed., *ΜΑΝΟΥΗΛ ΒΡΥΕΝΝΙΟΥ ΑΡΜΟΝΙΚΑ* (Groningen 1970).

Cleonides [Cleon.]:
Karl von Jan, ed., *Musici scriptores Graeci* (Leipzig 1892).

Gaudentius [Gaud.]:
Karl von Jan, ed., *Musici scriptores Graeci* (Leipzig 1892).

Nicomachus *Enchiridion* [Nic. *Ench.*]:
Karl von Jan, ed., *Musici scriptores Graeci* (Leipzig 1892).

Pachymeres [Pach.]
Paul Tannery and R. P. E. Stéphanou, eds., *Quadrivium de Georges Pachymère* (Vatican 1940), (*Studi e Testi* 94).

Porphyry [Porph.]:
Ingemar Düring, ed., *Porphyrios Kommentar zur Harmonielehre des Ptolemaios* (Göteborg 1932; reprint: New York 1980).

Ptolemy *Almagest* [*Alm.*]:
J. L. Heiberg, ed., *Opera quae exstant omnia, I Syntaxis Mathematica* (Leipzig 1898–1903).

Ptolemy *Harmonics* [*Harm.*]:
Ingemar Düring, ed., *Die Harmonielehre des Klaudios Ptolemaios* (Göteborg 1930; reprint: New York 1980).

Ptolemy *Tetrabiblos* [*Tetr.*]:
F. E. Robbins, trans., *Ptolemy Tetrabiblos* (Cambridge MA 1940).

Theon of Smyrna:
Eduard Hiller, ed., *Theo Smyrnaeus Philosophi Platonici; Expositio rerum mathematicorum ad legendum Platonem utilium* (Leipzig 1878).

Robert and Deborah Lawlor, trans., *Theonos Smyrnaiou: Mathematics Useful for Understanding Plato* (San Diego 1977).

Books and Articles

Of the many works cited either in the introductory notes or in the annotations to the translation, some appear with such frequency that they warrant abbreviated names, titles, and publication information. These works are the following:

Abert
= H. Abert, *Die Lehre vom Ethos in der griechischen Musik* (Leipzig 1899).

Alexanderson
= Bengt Alexanderson, *Textual Remarks on Ptolemy's Harmonica and Porphyry's Commentary* (Göteborg 1969).

Barbera, "Divisions"
= André Barbera, "Arithmetic and Geometric Divisions of the Tetrachord," *The Journal of Music Theory* 21 (1977) 294–323.

Barbera, *Pythagorean Mathematics*
= André Barbera, *The Persistence of Pythagorean Mathematics in Ancient Musical Thought* (Ph.D. diss., University of North Carolina 1980).

Barker, *GMW I*
= Andrew Barker, *Greek Musical Writings: I, The Musician and His Art* (Cambridge 1984).

Barker, *GMW* II
= Andrew Barker, *Greek Musical Writings: II, Harmonic and Acoustic Theory* (Cambridge 1989).

Boll
= Franz Boll, *Studien über Claudius Ptolemaeus. Ein Beitrag zur Geschichte der griechischen Philosophie und Astrologie. Jahrbuch für classische Philologie Supplementband* 21 (Leipzig 1894).

Bowen, "Minor Sixth"
= Alan C. Bowen, "The Minor Sixth (8:5) in Early Greek Harmonic Science," *The American Journal of Philology* 99 (1978) 501–506.

Bowen, "Translator"
= Alan C. Bowen and William R. Bowen, "The Translator as Interpreter: Euclid's *Sectio Canonis* and Ptolemy's *Harmonica* in the Latin Tradition," in M. R. Maniates, ed., *Music Discourses from Classical to Early Modern Times*, (Toronto 1997) 97-148.

Bower, "Boethius and Nicomachus"
= Calvin Bower, "Boethius and Nicomachus: An Essay Concerning the Sources of the De institutione musica," *Vivarium* 16 (1978) 1–45.

Burkert
= Walter Burkert, (Edwin Minar, Jr., trans.), *Lore and Science in Ancient Pythagoreanism* (Cambridge, MA 1972).

Crocker, "Aristoxenus"
= Richard L. Crocker, "Aristoxenus and Greek Mathematics," in Jan La Rue, ed., *Aspects of Medieval and Renaissance Music. A Birthday Offering to Gustave Reese* (New York 1966) 96–110.

Crocker, "Pythagorean Mathematics"
= Richard L. Crocker, "Pythagorean Mathematics and Music," *The Journal of Aesthetics and Art Criticism* 22 (1963–64) 189–98 and 325–35.

da Rios
= da Rios, Rosetta, ed., *Aristoxeni elementa harmonica* (Rome 1954).

de Pace
= Anna de Pace, "Elementi aristotelici nell'*Ottica* di Claudio Tolomeo," *Rivista critica di Storia della Filosofia* 36 (1981).

Diels-Kranz[6]
= Hermann Diels, and Walther Kranz, eds., *Die Fragmente der Vorsokratiker*[6] (Berlin 1951).

Düring, *Ptolemaios und Porphyrios*
= Ingemar Düring, *Ptolemaios und Porphyrios über die Musik* (Göteborg 1934).

Düring
= Ingemar Düring, ed., *Die Harmonielehre des Klaudios Ptolemaios* (Göteborg 1930; reprint: New York 1980).

Gevaert, *Historie et théorie*
= F. A. Gevaert, *Histoire et théorie de la musique de l'antiquité* (Gand 1875).

Gohlke
= P. Gohlke, Review of Düring, *Die Harmonielehre des Klaudios Ptolemaios*, *Philologische Wochenschrift* 50 (1930) 1441–44.

Gombosi, *Tonarten*
= Otto J. Gombosi, *Tonarten und Stimmungen der Antiken Musik* (Copenhagen 1939).

Höeg, review of Düring
= Carsten Höeg, Review of Düring, *Die Harmonielehre des Klaudios Ptolemaios*, *Gnomon* 6 (1930) 652–59.

Höeg, review of Düring *Ptolemaios und Porphyrios*
= Carsten Höeg, Review of Düring, *Ptolemaios und Porphyrios über die Musik*, *Gnomon* 12 (1936) 152–57.

Levin, "*Plege*"
= R. Flora Levin, "πληγή and τάσις in the Harmonika of Klaudios Ptolemaios," *Hermes* 108 (1980) 205–29.

Levin, *Nicomachus*
= R. Flora Levin, *The Harmonics of Nicomachus and the Pythagorean Tradition* (State College PA 1975).

Lohmann, *Musike*
= Johannes Lohmann, *Musike und Logos* (Stuttgart 1970).

Mathiesen, *Aristides Quintilianus*
= Thomas J. Mathiesen, *Aristides Quintilianus: On Music in Three Books* (New Haven 1983).

Mathiesen, *Ancient Greek Music Theory*
= Thomas J. Mathiesen, *Ancient Greek Music Theory: A Catalogue Raisonné of Manuscripts* (= *Répertoire international des sources musicales* Bxi) (Munich 1988).

Michaelides
= Solon Michaelides, *The Music of Ancient Greece: An Encyclopaedia* (London 1978).

Monro
= D. B. Monro, *The Modes of Ancient Greek Music* (Oxford 1894).

Mountford, "Harmonics"
= J. F. Mountford, "The Harmonics of Ptolemy and the Lacuna in II, 14," *Transactions of the American Philological Association* 57 (1926) 71–95.

Neugebauer, *Exact Sciences*
= Otto Neugebauer, *The Exact Sciences in Antiquity* (New York 1962).

Newton, *Crimes*
= R. R. Newton, *The Crimes of Claudius Ptolemy* (Baltimore 1977).

Palisca
= Claude V. Palisca, *Humanism in Italian Renaissance Musical Thought* (New Haven 1985).

Richter
= Lukas Richter, "Antike Überlieferungen in der byzantinischen Musiktheorie," *Deutsches Jahrbuch der Musikwissenschaft für 1961* (1962) 86–105.

Solomon, *Tonoi*
= Jon Solomon, "Towards a History of *Tonoi*," *The Journal of Musicology* 3 (1984) 242–51.

Szabó, *Greek Mathematics*
= 'Arpád Szabó, (A. M. Ungar, trans.), *The Beginnings of Greek Mathematics* (Hingham MA 1978).

Toomer
= G. J. Toomer, *Ptolemy Almagest* (London 1984).

Vogel, *Enharmonik*
= Martin Vogel, *Die Enharmonik der Griechen* (Düsseldorf 1963).

van der Waerden, "Harmonielehre"
= Bartel Leendert van der Waerden, "Die Harmonielehre der Pythagoreer," *Hermes* 78 (1943) 163–199.

Wallis
= Johannes Wallis, *ΚΛΑΥΔΙΟΥ ΠΤΟΛΕΜΑΙΟΥ ΑΡΜΟΝΙΚΩΝ ΒΙΒΛΙΑ Γ* (Oxford 1682).

Wilson, *Scholars*
= Nigel Wilson, *Scholars of Byzantium* (Baltimore 1983).

Winnington-Ingram, "Aristoxenus and the Intervals"
= Reginald Pepys Winnington-Ingram, "Aristoxenus and the Intervals of Greek Music," *Classical Quarterly* 26 (1932) 195–208.

Winnington-Ingram, *Mode*
= Reginald Pepys Winnington-Ingram, *Mode in Ancient Greek Music* (Cambridge 1936; reprint: Amsterdam 1968).

Winnington-Ingram, "Greece"
= Reginald Pepys Winnington-Ingram, "Greece," *New Grove Dictionary of Music and Musicians* VII, 659–72.

INTRODUCTION

The Importance of the Harmonics

Claudius Ptolemaeus, better known today as Ptolemy, was without question one of the dominant figures in ancient science. Otto Neugebauer identifies him as one of the great mathematical astronomers, and Giorgio de Santillana adds that Ptolemy was "not only the greatest astronomer of antiquity and the author of its best textbook on optics, he was also its most comprehensive mathematical geographer."[1] From late antiquity through the Renaissance, European, Byzantine, and Islamic scientists inevitably found themselves confronted by Ptolemy's imposing treatises in a variety of different fields—astronomy, astrology, geography, optics, and harmonics. Ptolemy had already overshadowed his predecessors and almost eclipsed them entirely, for like Euclid and Aristotle in their fields of expertise,[2] Ptolemy so satisfied the late ancient and medieval quest for knowledge in his fields of expertise that almost all of the works of his very distinguished predecessors were ignored and thereby condemned to extinction.[3] The geographical work of Marinos of Tyre no longer exists, but Ptolemy's *Geography* and world map were still in use for the *Carte générale de l'Empire du Turc* in 1646. For

---

[1] Otto Neugebauer, *A History of Ancient Mathematical Astronomy* (New York 1975) I, 5; Giorgio de Santillana, *The Origins of Scientific Thought* (London 1961) 272.

As for the *Optics*, A. de Pace (123) believes that we have extant at best books 2, 3, 4, and fragments of 5. Albert Lejeune, *L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'emir Eugène de Sicile* (Louvain 1956) 13–15, also discusses its authenticity in his introduction to this Latin translation of the twelfth-century Arabic translation of the original, but Richard Lorch, "A Note on the Technical Vocabulary in Erastosthenes' Tract on Mean Proportionals," *Journal for the History of Arabic Science* 5 (1981) 166–73, presents a *caveat* on such a methodology.

[2] Wilson, *Scholars* 21, has found that in eleventh-century Byzantium the quadrivium was learned from Aristotle's *Logic* (Dialectic), Euclid's *Elements* (Geometry), Nicomachus' *Introductio arithmetica* (Arithmetic), and Ptolemy's *Almagest* (Astronomy). Later (266), however, he asserts that the *Harmonics* was used as well.

[3] Neugebauer, *Exact Sciences* 145, and Ivor Bulmer-Thomas, review of Toomer, *CR* n.s. 34 (1984) 299, make this observation. S. J. Goldstein, "Problems Raised by Ptolemy's Lunar Tables," *Journal for the History of Astronomy* 13 (1982) 201, offers an interesting alternative to this idea. He describes a scenario which involves the Caesarian fire near the harbor of Alexandria in 46 B.C.: If the books destroyed in that fire included a number of important astronomical works, they may have been in or near the harbor only because they were awaiting shipment to Rome; it may be that Julius Caesar had preselected a number of these important astronomical works because of the major calendar revision he was contemplating for the following year.

To the extent that Ptolemy preserves the tetrachordal divisions of Archytas, Eratosthenes, and Didymus, and that none of the original harmonic works of any of these authors otherwise survives, Ptolemy actually preserves the data of these mathematicians in the *Harmonics*.

more than a millennium his *Geography* served as the principal model for the known world.[4] The trigonometry of Menelaos no longer survives,[5] and the astronomical work of Hipparchus and Aristarchus survives in only one work,[6] but even amateur stargazers know of Ptolemy's *Almagest.*[7] The modern name of this work alone, which is an Arabization of the Greek word for 'The Greatest,'[8] testifies to its reputation among very learned scholars in medieval Islam.[9] Of similar impact, but in the related field of astrology, was *The Tetrabiblos.*[10]

[4] It contained specific data for topographical and geographical features and contours as well as lists of cities, rivers, and nations. Wilson, *Scholars* 196 and 234, discusses the survival of the treatise and its accompanying map found in Vat. gr. 177 (1295/6 A.D); cf. Edward L. Stevenson, *The Geography of Claudius Ptolemy* (New York 1932). The map may go back to Ptolemy; *contra*, see A. Diller, "The Oldest Manuscripts of Ptolemaic Maps," *Transactions of the American Philological Association* 71 (1940) 62–67. For the text of the *Geography* itself, see Charles Mueller, *Claudii Ptolemaei Geographia* (Paris 1883).

[5] Oskar Becker, *Das Mathematische Denken der Antike* (Göttingen 1957) 110–13. Lack of information has nurtured some controversy about the birth and use of trigonometry before Ptolemy. Marshall Clagett, *Greek Science in Antiquity* (New York 1955) 92–98, argues that in computing the lengths of chords for each angle from one-half a degree to 180 degrees Ptolemy lays the foundations for the discovery of trigonometry. Newton, *Crimes* 21–30, esp. 27–28, argues that such computations may well have been made centuries earlier in Hipparchus' (lost) *On the Chords in a Circle*, if not by Menelaos as well.

[6] On Hipparchus, see D. R. Dicks, *The Geographical Fragments of Hipparchus* (London 1960) 48–102, and Sir Thomas L. Heath, *A History of Greek Mathematics* (Oxford 1921) 254. Aristarchus' *On the Sizes and Distances of the Sun and Moon* is the only major extant astronomical work other than the *Almagest*. The rest of Aristarchus' astronomical works are lost, as are those of Hipparchus, Eudoxus, Heracleides Ponticus, and Apollonius of Perga. See Sir Thomas L. Heath, *Aristarchus of Samos* (Oxford 1913), esp. 317–414.

[7] Internal evidence proves that the *Almagest* is an early work, for Ptolemy mentions it in his *Geography* (8.2.3) and *Tetrabiblos* 1; cf. n. 20 The editio princeps of the *Almagest* was published in Basel in 1538, followed by the Greek and French edition of Halma in 1813–16, the Teubner edition in J. L. Heiberg, *Opera quae exstant omnia, I Syntaxis Mathematica*, 2 vols. (Leipzig 1898–1903), (which did not include the *Harmonics*,) and Manitius' German translation (*Des Claudius Ptolemaeus Handbuch der Astronomie* (Leipzig 1912). R. Catesby Taliaferro (trans.), *The Almagest by Ptolemy* (Chicago 1938), prepared an English translation for the Encyclopaedia Britannica, bettered by G. J. Toomer (1984); on the relative merits of the latter two, see the review by Bernard R. Goldstein, *Isis* 76 (1985) 117–18.

[8] The Greek title was *mathematike syntaxis* ('Mathematical Composition'), the name *Almagest* deriving from an incorrect Arabic form and a middle Persian translation of the Greek ἡ μεγίστη ('the greatest'). Cf. Bulmer-Thomas, review of Toomer, *CR* n.s. 34 (1984) 299–300.

[9] Ptolemy served as a significant or even major source for such theorists as Al-Kindi of the mid-ninth century and Al-Farabi and Ikhwan Al-Safa of the tenth, as well as for the tenth-century collections of Ibn 'Abd Rabbihi and Al-Mas'udi. See Henry George Farmer, "Greek Theorists of Music in Arabic Translation," *Isis* 13 (1930) 328; Olaf Pedersen, *A Survey of the Almagest* (Odense 1974) 11–25; and Karl Manitius, trans., *Des Claudius Ptolemaeus Handbuch der Astronomie* (Leipzig 1912) iii–xxiv. Although Ptolemy is not mentioned by name, Ishaq al-Mausili [i.e. of Mosul] already in the early ninth century establishes eight *majras*, a number possibly (but controversially) dependent on Ptolemy's discussion of the *tonoi* and represented by the *oktoechos* of the Syrian and Byzantine church; see I. Düring, "Greek Music," *Journal of World History* 3 (1956) 324.

[10] Boll (163–68) discusses the astrology and harmonics of the *Tetrabiblos*. G. E. R. Lloyd, *Greek Science After Aristotle* (New York 1973) 130–31, counters the tradition of doubt

In the field of harmonics, which was an ancient science comprising aspects of musicology, acoustics, mathematics, physics, philosophy and astrology, Ptolemy belongs with Aristoxenus and Aristides Quintilianus to the most important triad of extant ancient Greek scholars.[11] His work, the *Harmonics*, earned an incomplete but extant commentary by Porphyry even in Greek antiquity,[12] served as a major source for Boethius' *Institutio de Musica* in sixth-century Italy,[13] and had a significant influence as well on Al-

surrounding the *Tetrabiblos*' authenticity. See also the relatively recent edition and Italian translation by Simonetta Feraboli, *Claudio Tolomeo: Le Previsioni Astrologiche* (Milan 1985), and the standard English translation by F. E. Robbins, *Ptolemy Tetrabiblos* (Cambridge MA 1940). Both build upon J. Camerarius' edition and Latin translation of 1535, which appeared three years before the 1538 Basel edition of the *Almagest*); cf. Rudolph Pfeiffer, *A History of Classical Scholarship 1300–1850* (Oxford 1976) 139.

For the minor works, in *De iudicandi facultate et animi principatu* Ptolemy compares various effects of the soul and musical pitches created from various musical instruments; in this the treatise is similar to, but of a much smaller scope than, the *Harmonics*. Boll (99) categorizes this treatise (as well as the *Almagest*) as predominantly Peripatetic, in part Stoic; cf. A. A. Long, "Ptolemy *On the Criterion*: An Epistemology for the Practicing Scientist," in J. M. Dillon and A. A. Long, *The Question of 'Eclecticism': Studies in Later Greek Philosophy* (Berkeley 1988) 193-96. D.R. Dicks, *Early Greek Astronomy to Aristotle* (Ithaca 1970) 84–85, discusses Ptolemy's *Phaseis*. See also Bernard Goldstein, "The Arabic Version of Ptolemy's Planetary Hypotheses," *Transactions of the American Philosophical Society* n.s. 57 (1967) 3–55; D. R. Edwards, *An Annotated Translation and Commentary of the Peri Analemmatos* (Ph.D., diss., Brown University 1985)), and Neugebauer, *A History of Ancient Mathematical Astronomy* (New York 1975) II, 838–39, the latter for the *Planispherium*, *Planetary Hypotheses*, *Phaseis*, and *Canobic Inscription*.

[11] The earliest modern assertion of the triad I have found is Hugo Riemann, *Handbuch der Musikgeschichte: I Die Musik des Altertums* (Leipzig 1919) 20. More recently, see R. P. Winnington-Ingram, review of Mathiesen, *Aristides Quintilianus*, *Early Music History* 4 (1984) 376; John Scarborough, review of Toomer, Masi, and Mathiesen, *Classical Bulletin* 61 (1985) 88–89; and Mountford, "Harmonics" 74. For Aristoxenus, of course, we actually have only a small percentage of this prolific author's output, as can be seen in da Rios 95–96, n. 1; she offers (103–129) a general summary in an additional chapter ("Valore dell' opera musical di Aristosseno").

[12] As the pupil of Plotinus, Porphyry had great interest in the *Harmonics* and its tradition; cf. A. Smith, *Porphyry's Place in the Neo-Pythagorean Tradition: A Study in post-Plotinian Neoplatonism* (The Hague 1974), and Neugebauer, *Exact Sciences* 55–57. The latter also evaluates Ptolemy's contributions to the science: making the Dorian the central octave of the ametabolic system, eliminating the conjunct or Lesser Perfect System, and establishing the number of *tonoi* at seven; cf. Barbera, *Pythagorean Mathematics* 295.

The text is found in Ingemar Düring, ed. *Porphyrios Kommentar zur Harmonielehre des Ptolemaios*, and for a partial English translation, see Barker, *GMW* II, 229–44. Contemporary scholarship doubts the idea that Porphyry *ad* 1.1–4 was authentic; cf. F. Hultsch, *Pappi Alexandrini collectionis quae supersunt III* (Berlin 1876) xii; P. L. Schonberger, *Studien zum I. Buch der Harmonik des Claudius Ptolemäus* (Augsburg 1914) iv; and Boll 93, n. 3. Ruelle attributed this work instead to Theon Alexandrinus, who wrote an extant commentary to the *Almagest* (A. Rome, ed., *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste* (Vatican City 1936 and 1943).

[13] Boethius cites Ptolemy in nine different chapters of Book V of the *De institutione musica*. For the citations and the differences between the treatments by Ptolemy and Boethius, see Calvin M. Bower, trans., *Ancius Manlius Severinus Boethius, Fundamentals of Music* (New Haven and London 1989) xxvi–xxxviii; Bower, "Boethius and Nicomachus" 4–5, 30–31; Bowen, "Translator" 97–148; and Günther Wille, *Musica Romana: Die Bedeutung der Musik in Leben

Farabi's *Kitabu l'musiqi al Kabir* in tenth-century Aleppo and Damascus,[14] on such Byzantine scholars as Bryennius and Pachymeres,[15] on Leoniceno,

---

der Römer (Amsterdam 1967) 691–98. In turn, Bower, "The Role of Boethius' *De institutione musica* in the Speculative Tradition of Western Musical Thought," in Michael Masi, ed., *Boethius and the Liberal Arts: A Collection of Essays* (Berne 1981) 157–74, details the importance of Boethius to subsequent music theorists and philosophy. For the preservation by Hucbald, and knowledge of the diatonic and enharmonic, the monochord, and quotations from Pythagoras, Aristoxenus, Ptolemy, and Ambrose in Bede, see I. Düring, "Greek Music," *Journal of World History* 3 (1956) 324. A. K. Holbrook, *The Concept of Musical Consonances in Antiquity and its Application in the Earliest Medieval Descriptions of Polyphony*, (Ph.D. diss., University of Washington 1983) 122–46 and 169–96, offers a broader discussion on the musicological legacy of the late Empire to the Middle Ages.

[14] For Al-Farabi's debt to Ptolemy, see Baron Rudolphe d'Erlanger, *La musique arabe* (Paris 1930) *passim*. For the entire quadrivium, see Ali A. Al-Daffa and John J. Stroyls, *Studies in the Exact Sciences in Medieval Islam* (Dhahran (Saudi Arabia) 1984) 32f., who overlook music and harmonics for the most part in attempting to present a polemic proof that the Islamic scientists were scientists first, philosophers second. They (64) do remind us, however, that Ibn Sina, while in Isfahan in the early eleventh century, wrote his Shifa, "twenty percent of which concerned music."

[15] Christian Hannick, "Byzantinische Musik," in H. Hunger, ed., *Handbuch der Altertumswissenschaft: Die Hochsprachliche Profane Literatur der Byzantiner* II (Munich 1978) 183–8, discusses the sources of Pachymeres and Bryennius (including Nicomachus); cf. Carsten Höeg, review of Düring, *Porphyrios Kommentar zur Harmonielehre des Ptolemaios*, *Gnomon* 10 (1932) 322; and Janine Bertier (trans.), *Nicomaque de Gérase: Introduction Arithmétique* (Paris 1978) *passim*. Richter 92–103, describing the work of Bryennius, Pachymeres, Nicephorus Gregoras, Argyros, and Barlaam, specifies Ptolemaic sources for Pachymeres and Bryennius by chapter:

Ptolemy -	*Pachymeres*	*Ptolemy* -	*Bryennius*
I.1	2	I.8	2.6
I.15–16	5,7,8,9	I.15–16	1.7
I.6–7	10	II.5	2.3
I.10–11	13–14	II.8–10	2.4
I.12–14	15	III.12	2.7
I.16	16		
II.2	17		
II.4–6	18		
II.7	19		
II.10	52 (18)		
II.12	20		

Thomas J. Mathiesen, "Aristides Quintilianus and the Harmonics of Manuel Bryennius: A Study in Byzantine Music Theory," *The Journal of Music Theory* 27 (1983) 31–34, places these works into the historical perspective of the thirteenth and fourteenth-century Palaeologan Renaissance; Bryennius was tutor to the court of Andronicus Palaeologus II by 1303. Mathiesen also demonstrates (*contra* Richter) that because Bryennius uses typical Planudean editorial techniques he therefore rarely simply copied Pachymeres' text but in addition incorporated portions of the works by Aristoxenus, Cleonides, Aristides Quintilianus, and even Bacchius.

Egon Wellesz, *A History of Byzantine Music and Hymnography*² (Oxford 1961) 44–45, and 63, minimizes earlier influences of ancient Greek music theory and practice on the Byzantine oktoechos system. He seems to overstate his case, however, where Mesarites' criticism of contemporary education mocks the classical tradition which is not at all absent or unimportant at the time; the rest of the report seems to be equally ironic. Cf. Richter 86, and Roderick Beaton, "Modes and Roads: Factors of Change and Continuity in Greek Musical Tradition," *Annual of the British School at Athens* 75 (1980) 1–11.

Gogava, Bottrigari, Augio, Gaffurio, and Zarlino in fifteenth- and sixteenth-century Renaissance Italy,[16] and on Johannes Kepler's 'Janus-faced' *Harmonices Mundi* in seventeenth-century Austria.[17]

Ptolemy and the *Harmonics*

We have surprisingly little data about the life of this man whose legacy for more than a millennium was myriad bits of data about a variety of sciences.[18] We know that he lived in Alexandria, Egypt during the mid-second century A.D.[19] In the *Almagest* his latest recorded observation was probably on February 2, 141 A.D.[20] We do not know the date of his birth, whether it was in

---

[16] See Palisca, esp. 117–42 and 201. Gafurius owned two codices with Leoniceno's translation of the *Harmonics* which he then used for his *De harmonia musicorum instrumentorum* (1518). For additional detail, see Thomas J. Mathiesen, "Towards a Corpus of Ancient Greek Music Theory: A New Catalogue Raisonné Planned for RISM," *Fontes artis musicae* 25 (1978) 119, and Alberto Gallo, "Le traduzioni dal Greco per Franchino Gaffurio," *Acta musicologica* 35 (1963) 174.

Of course Ptolemy at the same time was having as profound an influence on Italian astronomy. Following Ptolemy's method (and with a knowledge of Euclidean geometry and Archimedean physics), Galileo tested mathematically derived hypotheses based on and supported by empirical data. He wrote a commentary on the *Almagest* in 1592, and he parted intellectual and scientific company with Ptolemy only as late as 1595–97—and ultimately disproved him in 1610). See Stillman Drake, "Ptolemy, Galileo, and Scientific Method," *Studies in the History and Philosophy of Science* 9 (1978) 99–115.

[17] E.g. Johannes Kepler, *Harmonices Mundi* (Linz 1619) esp. 249–54. See Théodore Reinach, "La musique des Sphères," *REG* 13 (1900) 432–49; and Jamie C. Kassler, "The 'Science' of Music to 1830," *Archives internationales d'histoire des sciences* 30 (1980) 114 and 121. Kepler also completed much of a Latin translation of Porphyry, and, like Ptolemy, also studied optics.

On the other hand, more contemporary historians of science, depending on their particular interests, have not always considered the *Harmonics* a major work. When Sir Thomas L. Heath, *A Manual of Greek Mathematics* (Oxford 1931) 412, discusses Ptolemy's "other" works, he omits mention of the *Harmonics* entirely. Such an omission probably depends in part on the fact that Ptolemy's influence in astronomy generally declined after Newton. Then, with the rise of syllogistic reasoning, the ancient quadrivium ceased to be a regular part of university and school curricula. Nonetheless, echoes of Ptolemy still appear in Milton's "how gird the sphere / With centric and eccentric scribbled o'er, cycle and epicycle, / orb in orb" (*P.L.* 8.82–84 [and cf. 3.575]); see A. W. Verity, ed., *Milton: Paradise Lost* (Cambridge 1910) 551 and 688–90 (Appendix K); and Thomas N. Orchard, *Milton's Astronomy* (New York 1913) 117–27. In general, see John Hollander, *The Untuning of the Sky: Ideas of Music in English Poetry 1500–1700* (Princeton 1961), and Hans Blumenberg (Robert M. Wallace; trans.), *The Genesis of the Copernican World* (Cambridge 1987) 209–220. The influence of Ptolemy's *Geography* seems to have waned in the same century.

[18] Boll (53–66) offers all the citations and copious discussion of the Greek and Arabic sources for Ptolemy's life. R. Catesby Taliaferro (trans.), *The Almagest by Ptolemy* (Chicago 1938), ix, offers a general biography for the lay reader; for more detail, see Bartel van der Waerden, "Klaudius Ptolemios," *RE* 48 (1959) 1788–1859.

[19] The earliest reference to Ptolemy's *Harmonics* is Nicomachus *Exc.* 4 (275.7–9 Jan), but this may be an interpolation, thinks Levin, "*Plege*" 206, n. 4.

[20] *Alm.* 9.7, 450 (Toomer), or at the latest 151. Norman T. Hamilton's work on the Canobic Inscription, however, suggests that it was written earlier than the *Almagest*, and we know that the

Pelusium or Ptolemais, the date of his death,[21] the names of his mentors,[22] what institutional position, if any, he held, or any other especially interesting or insightful information about him.[23] His personality and temperament reveal themselves only reluctantly and obscurely in his works; he was a moderately productive and energetic scholar[24] with a passion for collecting data already centuries old, data which he would then sift through, codify, and criticize.[25] Ultimately he would offer his own ideas, but quite often such ideas of his own production were neither revolutionary nor practicable—for instance his 12:11:10:9 tetrachordal division (*Harm.* I.16), his epicycle theory of planetary motion, and the triangulations in his geographical calculations—nor were they necessarily correct, as in his dependence on and proof of a geocentric system in *Almagest* I.5.[26] But more than a number-

inscription was erected in the tenth year of Antoninus (146/7 A.D.); cf. Toomer 1–2. In addition, if the *Almagest* has 141 A.D. as a *terminus post quem*, since Ptolemy refers to this work in both the *Geography* and *Tetrabiblos*, he probably did live to at the very least 146/7. If the Byzantines were correct in supposing the *Harmonics* to be Ptolemy's last work, for which cf. n. 21, then we might indeed posit a date as late as 155.

[21] R. Catesby Taliaferro (trans.), *The Almagest by Ptolemy* (Chicago 1938) I, offers the most accessible general discussion of the problem. The information that Ptolemy died before completing the *Harmonics* may derive from nothing more reliable than a supposition by a scholiast; cf. Mountford, "Harmonics" 78, and Mathiesen, *Ancient Greek Music Theory*, 525. Erwin Rohde, "*Γέγονε in den Biographica des Suidas*," *Rheinisches Museum* 38 (1978) 169, established that what was once thought to be Ptolemy's birthdate should actually be considered merely a *floruit*.

[22] Levin, *Nicomachus* 8–11, points out that Nicomachus (*fl.* 100 A.D.) does not mention either Ptolemy or Theon (*fl.* 115–40), which suggests an earlier date for Nicomachus. Curt Sachs, *The Rise of Music in the Ancient World* (New York 1943) 199, aptly categorized the early second century A.D. as a renascence in Greek mathematics and music theory.

[23] His dedications to Syrus in *Tetr.* 1.1 and *Alm.* 1.1 leave us equally unenlightened.

[24] That is, in his widespread pursuit of knowledge in various but related fields. There is a great question, of course, as to how meticulous he was in his accumulations of data. He did not publish nearly as voluminously as did Aristoxenus and Plutarch.

[25] Mountford, "Harmonics" 74, evaluates Ptolemy's preservation of data. Ptolemy refers to his predecessors' tetrachordal divisions, the tonoi, and to the contemporary use of the cithara. Against Archytas, for instance, Ptolemy's criticism (I.14) takes aim at Archytas' using non-superparticular intervals in his chromatic divisions and not having the smallest interval at the lowest position in his enharmonic.

[26] Ptolemy borrowed the concept of the epicycle from earlier Pythagorean models (a hint of which remains in Pl. *Tim.* 38C–D, supported by *Pap.Mich.* 149–6) for his *Almagest*; cf. B. L. van der Waerden, "The Motion of Venus, Mercury, and the Sun in Early Greek Astronomy," *Archive for the History of the Exact Sciences* 26 (1982) 99–113. Although Neugebauer, *Exact Sciences* 161, assumes that spherical trigonometry was not available to Hipparchus, (but cf. O. Neugebauer, *A History of Ancient Mathematical Astronomy* (New York 1975) I, 21–29), Newton, *Crimes*, believes most of Ptolemy's mathematical procedures were already known to Hipparchus. Marshall Clagett, *Greek Science in Antiquity* (New York 1955) 92, evaluates Aristarchus' use of angles and sides of triangles which precede trigonometry. In the *Geography* Ptolemy tried to reconcile a number of older sources but was not always successful in doing so. Cities and river mouths are often situated in two different places; cf. Francis J. Carmody, "Ptolemy's Triangulation of the Eastern Mediterranean," *Isis* 67 (1976) 601–605. In addition, his use of 180,000 stades for the circumference of the earth, although possibly dependent on an alternate

cruncher, Ptolemy respected his intellectual predecessors. He both lauded their discoveries and the coherence of their logic, e.g. his priase of Archytas in *Harm.* I.13 and he attacked their shortcomings, e.g. his attacks on the Pythagoreans , Aristoxenians (*Harm.* I.2 and I.5-12), and Didymus (*Harm.* II.13). Through modern eyes he can still be seen as a clear thinker but certainly not as a competent writer; very often what could have been a simplified and final explanation or solution he obfuscates.[27] His facility with data and mathematics led him to explore a variety of tangential fields, yet the homage he on occasion pays to more humanistic pursuits soon reveals itself to be short-lived.[28]

History has now balanced the tremendous influence and credibility Ptolemy enjoyed for a sesquimillennium with several centuries of doubt and a decade of indictment. As a human counterpart to the vicissitudes Thomas Kuhn associated with a scientific idea,[29] the once nearly deified Ptolemy has now been publicly accused of methodological fraud. An astrophysicist is his prosecutor,[30] and the rest of the scientific community serves variously as

size of the stade, may then also be erroneous; cf. Giorgio de Santillana, *The Origins of Scientific Thought* 272–75.

For a positive evaluation, see Neugebauer, *Exact Sciences* 226, who in discussing the *Optics* praises Ptolemy for taking that discipline from purely geometrical investigations to theoretical binocular vision and physiological optics and also for applying observable phenomena. Similarly, Frederick V. Hunt, *Origins in Acoustics* (New Haven 1978) 30–31, finds Ptolemy's intense diatonic tetrachordal division as his most enduring legacy to music; he also makes light of the criticisms of Burney (n. 51). Barbera, *Pythagorean Mathematics* 292–93, describes Ptolemy's dependence upon and innovations within the Pythagorean tradition.

[27] Winnington-Ingram, *Mode* 62, labels Ptolemy "a lucid thinker, if an ungainly writer." Levin, *Nicomachus* 30, laments the "ungainly, awkward, and abstruse stretches of composition"; cf. Levin, "*Plege*" 214, n. 30, where she mentions the "inadequacies of the language." E. K. Borthwick, review of Alexanderson, *CR* n.s. 21 (1971) 367, muses that Ptolemy produced "writings where the Muses seem to have been somewhat ungenerous in the distribution of the sweet dew that Hesiod says they pour upon the tongue." Even in the Byzantine period, Theodorus Metochites could look back and declare that "all writers educated in Egypt had a harsh style" and then cite Ptolemy as an example. It is the vague antecedents to which Ptolemy refers with his pronouns and his careless use of participles that make understanding his exact meaning a matter of educated guesswork. See also F. E. Robbins, *Ptolemy Tetrabiblos* (Cambridge MA 1940) xxiv, who details similar frustrations with Ptolemy's complex sentences.

[28] He often follows his mathematical instincts after approaching something we might attribute to common sense. While he sensibly criticizes the Pythagoreans for stubbornly excluding the diapason plus diatessaron from the list of consonances because of its 8:3 integer ratio, he nonetheless would rather exlude any other than a superparticular ratio from his own tetrachordal divisions. Also, when speaking of the Aristoxenians who insisted on using the ear and the senses as valuable tools for evaluating tetrachordal divisions, Ptolemy concentrates only on the spatial mathematics of their divisions.

[29] Thomas Kuhn, *The Structure of Scientific Revolutions* (Chicago 1962).

[30] Robert R. Newton, "The Authenticity of Ptolemy's Star Data - II," *Quarterly Journal of the Royal Astronomical Society* 24 (1983) 27–35, criticizes the method with which Ptolemy derived the rate of precession of the equinoxes and concludes that Ptolemy "tried to deliberately deceive his readers." Newton, *Crimes* 341–42, gives a summary list of thirty-six fabrications in the *Almagest*, with more at 344–46. See also his *Ancient Astronomical Observations and the Accelerations of the Earth and Moon* (Baltimore 1970). Newton is not by any account alone. Bernard R. Goldstein, "The Obliquity of the Ecliptic in Ancient Greek Astronomy," *Archives*

judge, jury, and defendant. No one doubts that some of Ptolemy's data is incorrect;[31] the question is whether he committed an intellectual crime, and, if he did, how extenuating are the circumstances.[32]

Internationales d'Histoire des Sciences 33 (1983) 3, for example, finds Ptolemy's "observation" of the angle between the celestial equator and the ecliptic (*Alm.* 1.12) to be in error. Hipparchus' correct observation (for his own epoch) probably derives from Babylonian work; cf. J. P. Britton, "Ptolemy's Determination of the Obliquity of the Ecliptic," *Centaurus* 14 (1969) 39f.

[31] Although Noel M. Swerdlow, "Ptolemy on Trial," *American Scholar* 48 (1979) 523–31, and John Phillips Britton, *Models and Precision: The Quality of Ptolemy's Observations and Parameters* (NewYork 1993), attempt to acquit Ptolemy of any wrongdoing, nonetheless one of Ptolemy's most impressive defendants, Owen Gingerich ("Was Ptolemy a Fraud?" *Quarterly Journal of the Royal Astronomical Society* 21 (1980) 253–66) freely admits that, for example, Ptolemy's "very careful" observation of the equinox of September 26, 139 A.D. is thirty hours late (because the observation was actually made by Hipparchus 278 years earlier.) In fact, Ptolemy erred as well in all his observations of the equinox and solstice, those of lunar eclipses, in his estimate of the obliquity of the ecliptic and the rate of precession, and also in the star catalogue. Most damaging in regard to the latter, as pointed out by Newton (*Crimes* 239) is that Ptolemy claims to have observed all stars to the sixth magnitude, but not one of the 1028 stars in his catalogue is visible south of the island of Rhodes; Hipparchus apparently made his observations in Rhodes, and Alexandria lies 5 degrees south. Cf. Heath, *Aristarchus of Samos* 131–32, n. 4, and 171–73; and Ivor Bulmer-Thomas, review of Toomer, *CR* n.s. 34 (1984) 302. Gingerich points out that a number of observational difficulties are bound to occur because of the refraction inherent in the use of a bronze equatorial ring.

Ptolemy's observational errors were noted as early as the late seventeenth century. Edmund Halley, "A Discourse concerning a method of discovering the true moment of the Sun's ingress into the tropical signs," *Philosophical Transactions* 19 (1965) 12–20, comments on Ptolemy's incorrect mean motion of the sun, "which principal Error in so Fundamental a Point does Vitiate the whole Superstructure of the *Almagest*, and serves to convict its Author of Want of Diligence or Fidelity or both"; Halley, it will be recalled, succeeded John Wallis, the seventeenth-century editor of the *Harmonics*, to the Savilian Chair of Geometry at Oxford. S. J. Goldstein, "Problems Raised by Ptolemy's Lunar Tables," *Journal for the History of Astronomy* 13 (1982) 195–200, reviews the history of criticisms of Ptolemy's accuracy, including P. C. Le Monnier (1757), J. J. F. Lalande (1792), J. B. J. Delambre (1817), Peters and Knobel (1915), P. Tannery (1893), and R. R. Newton (1977). Goldstein concludes that Ptolemy in fact made no lunar observations at all. Newton (*Crimes* 356–65) goes so far as to list well over a dozen aspects of astronomy for which he finds Ptolemy to have been not only fraudulent but even incompetent. Scholars in other disciplines have found the same sort of methodological deficiencies in Ptolemy's other works. Francis J. Carmody, "Ptolemy's Triangulation of the Eastern Mediterranean," *Isis* 67 (1976) 601–605, for instance, finds that Ptolemy in the *Geography* uses older sources which do not always agree, in which instances he made compensatory changes to account for the discrepancies. In other instances, to fit certain sites into the space available in his map, he altered the data.

[32] Owen Gingerich, "Was Ptolemy a Fraud?" *Quarterly Journal of the Royal Astronomical Society* 21 (1980) 253–66, avers that Ptolemy's theories surpass his data: he chose (or omitted) data to fit his theories, fitted his theory to a few, correct observations, or corrected his own observations to agree with his theories. Gingerich cites (263–64) several examples of reputable, even great, scientists "selecting" their data to fit their correct theories. Einstein, when learning that Eddington's eclipse expedition had provided data to prove his theory of relativity, remarked that he knew his theory was correct and that he simply would have doubted Eddington's findings if they had not corroborated it. Gingerich excuses Ptolemy's observational discrepancies since he wishes to evaluate and then praise Ptolemy as the first scientist in history to form parameters of planetary models from specific numerical data and then provide tables for precise computation of solar, lunar, and planetary positions.

Willy Hartner, "The Role of Observations in Ancient and Medieval Astronomy," *The Journal of the History of Astronomy* 8 (1977) 1, 3, and 5, attributes Ptolemy's incorrect values for the

Fortunately, none of this affects his *Harmonics*. Scholars have found little reason to accuse Ptolemy of falsifying any data here,[33] and where he offers us a collection of Aristoxenian divisions of the tetrachord, our extant chapters of the Aristoxenian corpus corroborate what Ptolemy preserves.[34] In fact, although Ptolemy does have important ideas of his own that he wishes to promulgate—the 12:11:10:9 tetrachordal division, superparticularity, the seven *tonoi*, the inclusion of the 8:3 ratio as a consonance, the downgrading of the synemmenon system, and the circularity of the ametabolic system—the *Harmonics* is equally useful to us in the twentieth century as an indispensable record of the science of harmonics as it developed from the late sixth-century B.C. Pythagoras through the fourth-century Archytas[35] and

---

solar parallax, the moon's maximum distance, and the planetary distances to Ptolemy's belief in the nested spheres (on which, see n. 69). He also argues, "We call it fraud, but Renaissance man... took it with greater lenience. If facts were at odds with an infallible theory, there was no expedient but to suppress or to change them."

Kritian Peder Moesgaard, review of Newton, *Journal for the History of Astronomy* 11 (1980) 133–35, attempts this justification of Ptolemy's incorrect observations: Ptolemy was synthesizing Greek and Babylonian astronomy; and in so doing he established new mathematical theories and tried to use them to support old, incorrect data or Hellenistic Greek (or Babylonian) parameters. Defending Ptolemy's general stance as well is Stanley E. Babb, Jr., "Accuracy of Planetary Theories, Particularly for Mars," *Isis* 68 (1977) 426-27. Very simply, he assumes that Ptolemy did not know observational astronomy.

Of interest methodologically is that in the wake of Newton's attacks, Ptolemy's apologists have not absolved Ptolemy of wrong-doing but have in the process of defending him attempted to find a whole new rationale and understanding of his working methods, assumptions, and therefore personality and reliability.

G. E. R. Lloyd, "Saving the Appearances," *CQ* n.s. 28 (1978) 202–22, (and Marshall Clagett, *Greek Science in Antiquity* (New York, 1955) 94), attempts to exonerate Ptolemy when he summarizes the instrumentalist nature of Greek astronomy and sciences in general, explaining that the Greeks were interested only in devices ('fictions') hypothesized without regard for the truth or physical reality and merely for the sake of calculations. Ptolemy knew that the epicycle theory did not represent the real heavens; his concern instead was with the geocentric universe and the proof of the sphericity of the heavens.

[33] Unlike data derived from astronomical observation, harmonic data does not vary from one geographical location to another, nor does it vary from one century to another or need to account for precession or significantly imprecise instruments. The canon predates Ptolemy by several centuries, the lyre, in one form or another, by several millennia. Geographical observations are much more difficult to make, so they are apt to be contradictory and inconsistent. On the other hand, it is not impossible that Ptolemy is guilty of misrepresentation in his description of how Archytas derived his tetrachordal divisions. See I.13, and Bowen, "Minor Sixth" 504–5. See also Burkert 380.

[34] Compare, for instance, Ptol. *Harm.* I.12 with Aristox. *Harm.* I.24–.27 (Macran (=30.17–35.8 da Rios)).

[35] See now, Carl A. Huffman, "The Authenticity of Archytas, Fr. 1," *CQ* 35 (1985) 344–48; and A. C. Bowen, "The Foundations of Early Pythagorean Harmonic Science: Archytas, Fragment 1," *Ancient Philosophy* 2 (1982) 79–83, who finds classical Pythagorean harmonic science to be dependent on "rigorous quantitative theory." G. S. Kirk and J. E. Raven, *The Presocratic Philosophers* (Cambridge 1957) 217–21, 242–50, and 307–313, offer background information on Pythagoras, his South Italian school, number theory, and Pythagorean individuals, for example Philolaus. The abundance of Pythagorean material reminds J. S. Morrison, review of Burkert, *Lore and Science*, *Gnomon* 37 (1965) 345, of an archaeological site in constant occupation.

Plato to the Hellenistic Aristoxenus and Eratosthenes and ultimately to the second-century A.D. Ptolemy.[36]

Unfortunately, however, the *Harmonics* has not fared so well as the *Almagest* or *Geography* in its state of preservation.[37] While Western, Islamic, and Byzantine scholars took great care to recopy and comment upon all three treatises throughout the first thousand years of their existence, the *Harmonics* as it survives today has several significant loci questionable as to whether Ptolemy ever completed them, how well they have been preserved, and even whether they are authentically Ptolemaic at all. Gregoras, the historian and polymath who lived from 1295 to circa 1359, certainly reworked the treatise in the fourteenth century,[38] and, much to the concern of his Calabrian peer, Barlaam, he wrote the necessary supplements for the final three chapters of Book III.[39] The tables and accompanying mathematical prose of the last three chapters of Book II are a bit more problematic simply because we have no direct information that Nikephoros or Isaac Argyrus or anyone else composed or reworked them.[40]

Most problematic of all is the last half if not almost the entirety of Book III. Negative lexical comparison may prove only that Ptolemy was describing different subject matter and relied upon different technical jargon, or that he wrote the final book some years after the first two. In fact, it was the Byzantine redactors who used this internal evidence to posit that the *Harmonics* was Ptolemy's final work and remained incomplete at the time of his death.[41]

The brevity of the final eight chapters, their relative lack of demonstration,[42] their relative lack of illustration,[43] the absence of polemics, and the simplicity of the skeletal comparisons made between the harmonics of music, the soul, and the celestial bodies, cannot give the reader confidence that Ptolemy labored hard or long on these passages. The conclusions are then inevitable but without the evidence which might lead to a positive differentiation: either Ptolemy never wrote these chapters, or he did and they did not survive well the intervening centuries, or he did and he did so only as incomplete sketches which his death prevented him from fleshing out.

The great harmonic scheme of things in its all-encompassing embrace by comparison makes such problems seem insignificant. After all, the *Harmonics* derives its importance not so much from being a single scientific treatise by an individual second-century A.D. scholar of controversial merit and abilities as from preserving harmonic data first subdivided by Archytas some five hundred years before the compiler's era and influencing harmonic hypotheses for the next fifteen hundred years. As a technical treatise, it was impossible for subsequent scholars to read and work through without commenting in the margins, and, as in many technical treatises, supplements or glosses made by subsequent technicians or philologists may not belong historically to the original work but might still belong technically. That is, a subsequent glossator might explain a technical passage (which is eternal) better than did Ptolemy (who is, reportedly by his own admission, ephemeral). Again, the purpose of a treatise like the *Harmonics* was to instruct its readers, and in doing so it inevitably became not just an ancient treatise but part of an ongoing educational and technical process.[44]

The Design of the *Harmonics*

Despite the problems surrounding the state of completeness of Book II and especially Book III, the framework of the treatise suggests that Ptolemy at least planned the entire work carefully.[45] In penning his contribution to the study of the science of harmonics, Ptolemy somewhat mystically designed the treatise to reflect the balance and numerical proportion of the subject matter at hand. The details have been aired elsewhere,[46] but Ptolemy's

[36] In I.12 (on Aristoxenus), I.13 (on Archytas), II.13 (on Didymus) and II.14 (on Eratosthenes). It is through Eratosthenes that we have the 'Delian' problem: when the people of Delos asked Apollo for relief from a plague, the god told them to double the size of their present altar; confused by this, they asked Plato for an explanation, and he told them Apollo was not nearly so desirous of a larger altar as he was for the people of Delos to pay greater attention to math and geometry. Cf. Robert and Deborah Lawlor, (trans.), *Theonos Smyrnaiou: Mathematics Useful for Understanding Plato* (San Diego 1977) 1–2.

While Ptolemy collects and discusses a variety of "ancient" data, he should not be considered an antiquarian, as, for instance, Aristides Quintilianus should be. Mathiesen, *Aristides Quintilianus* (1983) 1–3, has useful bibliography here.

[37] See Mountford, "Harmonics" 74-75, and Düring lxx–xcvii.

[38] See Mountford, "Harmonics" 77–78; Düring lxviii–xc; and my annotations to III.14. Biographical details are offered by Wilson, *Scholars* 266–67, and R. Guilland, *Essai sur Nicéphore Gregoras. L'homme et l'oeuvre* (Paris 1926).

[39] Barlaam's assumption was that Nicephorus had made emendations and added the supplements in a deceptive manner. See Wilson, *Scholars* 266.

[40] See Düring lxx–lxxxviii, and Mountford, "Harmonics" *passim*. One supposes Gregoras also composed the lengthy, intelligent scholia that appear occasionally, though Isaac Argyros is another likely candidate for authorship; cf. Mathiesen, *Ancient Greek Music Theory* 525.

[41] Wilson, *Scholars* 266. Boll 65 assumes that this idea derives from Nicephorus.

[42] III.9 contains a geometrical division of a circle into twelve sectors, but we might have expected a specific application of these twelve sectors to the twelve zodiacal divisions, and III.15 offers just too little information tempered only by a curious final apology, "More information we have relegated to more leisurely hours." (110.24)

[43] The only one appearing in III.9.

[44] This is true for any text or corpus which lends itself to perpetual use and study, be it Homer, the Aristotelian corpus, the "Library" of Apollodorus, or the culinary collection attributed to Apicius. Barlaam seems to have been unaware or at best unappreciative of such a process in accusing Nicephorus Gregoras of fraud by supplementing III.14–16; cf. Wilson, *Scholars* 266.

[45] Macrobius (1.19–20) apparently read three books of the *Harmonics*, and Mountford, "Harmonics" 78, discusses the scholiast's assertion that a table of contents for the entire *Harmonics* always existed; cf. Mathiesen, *Ancient Greek Music Theory* 525, 527; and Bowen, "Translator" 103.

[46] Jon Solomon, "A Preliminary Analysis of the Organization of Ptolemy's Harmonics," in André Barbera, ed., *Musical Theory and Its Sources: Antiquity and the Middle Ages* (Notre Dame 1990) 68–84.

division of the three books into sixteen chapters each is unique among his extant works.[47] The first book itself falls neatly into eight pairs of chapters,[48] and the first sentence, which contains the definitions of harmonics and sound, consists of clauses of fifteen and then eleven words in length. The diapason, which Ptolemy labels the most beautiful of the consonances,[49] contains eight notes, the double diapason system contains fifteen, and the conjunct or synemmenon system eleven.[50] Modern scientific thinking has absolutely no need and even less tolerance for such analogies, but the ancients did indulge in such "fooleries" quite regularly.[51] This very treatise, it should be remembered, culminates by likening the divisions of the diapason to those of the soul and making a scientific analogy between planetary motions and musical notes.[52]

---

[47] By contrast, the *Almagest* consists of an irregularly designed thirteen books of 16, 13, 9, 11, 19, 13, 5, 6, 4, 10, 12, 10, and 11 chapters. The *Geographica* consists of four books, the first of which contains 24 chapters. The *Almagest* manuscripts do contain the chapter headings, and although Toomer (who with Nigel Wilson has redated Heiberg's D source from the twelfth to the tenth century) believes that they are the work of later hands, he also believes that Ptolemy himself divided the work into thirteen books.

[48]

1) On Sounds	1—2
2) On Differences in Sounds	3—4
3) On Pythagorean Consonances	5—6
4) On More Accurately Defined Consonances	7—8
5) On Aristoxenian Consonances	9—10
6) Against Aristoxenian Integer Analysis	11—12
7) Against Archytas' Proportional Analysis	13—14
8) On the Proper Divisions of the Tetrachord	15—16

[49] I.5 (11.21).

[50] Although Ptolemy dismisses the synemmenon system in II.4, he does discuss its merits, function, and construction thoroughly in II.6. *Contra*, see Bowen, "Translator" 139.

[51] Walter Burkert 355, n. 27, uses this noun. Compare the statement by Charles Burney, *A General History of Music From the Earliest Ages to the Present* (London 1776–89; reprint: London 1935) I, 356: "[Ptolemy] passes suddenly from accurate reasoning and demonstration to dream analogies, and all the fanciful resemblences of the Pythagorean and Platonic Schools; discovers Music in the human soul and the celestial motions; makes the sciences and the virtues, some diatonic, some chromatic, and some enharmonic; turns the zodiac into a lyre, making the equinoctial the key note.... He seems to have been posssessed with an unbounded rage for constructing new scales...having a faculty and a pleasure in calculating, seems to have sported with the scale, and wantonly to have tried confusions by dissecting and torturing it in all possible ways."

[52] III.16; cf. also Burkert, *Lore and Science* 299–368. There are other interpretations, of course. Barbera, *Pythagorean Mathematics* 294, divides the chapters of the *Harmonics* as follows:

I.1–2:	Introduction and the scope of harmonics
I.3–4:	Acoustics
I.5–11:	Theory of intervals
I.12–II.1:	Genera and the divisions of the tetrachord
II.2:	The helicon
II.3:	Octave species
II.4–11:	The Greater and Lesser Perfect Systems
II.12–13:	The canon

To paint the broader picture, Book I defines harmonics and sound, describes two different types of sounds—discrete and continuous (I.4),[53]—and then focuses on the discrete, among which are musical sounds. These musical sounds should be described as mathematical ratios and derived on the scientific instrument called the canon (I.8).[54] Ptolemy finishes Book I by examining these ratios as divisions of the tetrachord described (incorrectly) by his predecessors Archytas (I.13)[55] and Aristoxenus (I.12), the two chief proponents of the opposing Pythagorean and Aristoxenian schools,[56] and by himself (I.15-16),[57] and contemporary musicians (I.16).[58]

II.14–16:	Tables of the genera
III.1–2:	Tuning the 15–stringed canon
III.3–7:	Musical intervals and the soul
III.8–13:	Musical intervals and the heavenly bodies
III.14–16:	Tables of the intervals and the heavens.

Rudolf Westphal, *Harmonik und Melopoie der Griechen* (Leipzig 1863) 237–38, offers this division:

I.3–11:	On notes
I.12–II.2:	On genus
II.3–11:	On systems and tonoi
II.11–16:	On all the tonoi.

See also, Richter 86.

[53] For a classic, twentieth-century investigation into the nature of sound, see Sir William Bragg, *The World of Sound* (New York 1920; reprint: New York 1968).

[54] *Alm.* 5.1 contains a parallel passage in which the astrolabe is constructed. Cf. also *Harm.* II.2.

[55] Archytas' predecessor and teacher Philolaus (c. 400 B.C.) described the diapason as a *harmonia*, asserted that it contained the diatessaron and diapente, and generated ratios for all three intervals. He defined the whole tone as the difference between the diatessaron and diapente and stated that the diapason had five of these, the diatessaron two, and the diapente three, plus remainders. He was also the first Pythagorean to commit his ideas to writing (in literary Doric). Cf. Levin, *Nicomachus* 2; Norman Gulley, review of Burkert, *Lore and Science in Ancient Pythagoreanism*, *CR* 14 (1964) 29; J. D. P. Bolton, review of H. Thesleff, *An Introduction to the Pythagorean Writings of the Hellenistic Period*, *CR* 14 (1964) 334; and Edward A. Lippman, "Hellenic Conceptions of Harmony," *Journal of the American Musicological Society* 16 (1963) 23.

Bowen, "Minor Sixth" 504, following Paul Tannery, *Mémoires scientifique* III (Paris 1915) 78–81, 110–14 and 234–37, and Winnington-Ingram, "Aristoxenus and the Intervals" 206–207, argues that Archytas then specified the genera by dividing the diapente into a minor third (6:5) and a major third (5:4), and the fourth (4:3) into a septimal third (7:6) and a major tone (8:7). Ptolemy I.13 does not give such an account, however.

[56] Levin, *Nicomachus* 46–50, details the possible accomplishments of the semi-legendary Pythagoras, namely that he added the eighth string to the lyre, discovered musical ratios, and divided the diapason into smaller intervals through mathematics. On the life of Pythagoras and his school in Southern Italy, see G. S. Kirk and J. E. Raven, *The Presocratic Philosophers* (Cambridge 1957) 217–21, and 242–50 on early Pythagorean number theory. Van der Waerden, "Harmonielehre" 197–98, offers a useful summary of Pythagorean intervallic theory beginning with Pythagoras' (or the Babylonian) tetraktys and continuing with Hippasus, Philolaus, Archytas, Eudoxos, Heracleides, Eratosthenes, Didymus, and finally Ptolemy. There were no major changes in our understanding of tuning until Galileo and Mersenne independently discovered in the seventeenth century that the period of vibration depends on a string's length, tension, and density—on which, however, see I.8—and then until Wallis and Sauveur independently discovered the cause of overtone vibrations. See Llewelyn S. Lloyd and Hugh

Book II then examines the larger consonant constructs of ancient Greek music, specifically the double diapason system (II.3-4) and the so-called conjunct system (II.6), and the species within them (II.5). The most important of these species, the *tonoi*, Ptolemy delimits according to the diapason (II.7-8), and attaches to them seven of the names derived from the traditional names associated with the archaic *harmoniai* (II.9-11). He concludes Book II, as he had Book I, with prose descriptions, tables listing the divisions of the diapason (II.14-16),[59] and an application of the book's data to contemporary music.

Book III begins with some final comments on the divisions of the diapason and their application to actual music and instruments (III.1), which lead into a discussion of harmony, reason, and the senses. Harmony exists in nature (III.3), and therefore Ptolemy can apply the science of harmonics to the human soul (III.4), the ecliptic (III.8), zodiac (III.9), fixed stars (III.10-12), and planets (III.13-16). Unlike the rest of the treatise, these latter passages are much abbreviated and without internal reference to the *Tetrabiblos*, *Almagest*, or any other astronomical or astrological works or authors.[60]

Although not entirely satisfying or complete in its parts, the *Harmonics* displays that breadth of learning which Plato had outlined five centuries earlier and which Boethius four centuries later labeled the quadrivium.[61] It

Boyle, *Intervals, Scales and Temperament* (London 1963) 2–4, who offer a historical summary from Pythagoras to Helmholtz.

[57] Frederick V. Hunt, *Origins in Acoustics* (New Haven 1978) 30–31, thinks Ptolemy's intense diatonic (I.15, 36.28f.), with just intonation as befits a true diatonic scale, is probably Ptolemy's most enduring legacy.

[58] And II.16. Thomas J. Mathiesen, "Problems of Terminology in Ancient Greek Theory: HARMONIA," in Burton Karson, ed., *Festival Essays for Pauline Alderman* (Provo 1976) 3, describes Ptolemy as a "Hellenic Medievalist" situated intellectually and chronologically between the Pythagorean and Aristoxenian disputes of the fourth century B.C. and the technical manual writers of the third and fourth centuries A.D.

[59] Neugebauer, *Exact Sciences* 534, provides an early parallel for the difficulties found generally in the manuscript transmission of tables and charts. For the *Almagest*, this involved a whole separate transmission complete with its own tradition of ancient commentary by Theon and Stephanus of Alexandria; cf. Wilson, *Scholars* 81.

[60] References to other, earlier scholars are also conspicuously lacking. In modern science, astronomy and astrology have become completely separate. The latter is considered a non-scientific cult and the former has become the costliest of high-tech scientific pursuits. In Ptolemy's epoch as well the distinction between astronomy and astrology had already become well established. The distinction may be traced even three centuries earlier to the Sibyline Books; cf. J. Gwyn Griffiths, review of Jack Lindsay, *The Origins of Astrology*, *CR* n.s. 24 (1974) 315–16; Alexander Jones, "A Greek Saturn Table," *Centaurus* 27 (1984) 311–17.

[61] It is with Boethius (*de Ar.* 1) that the term *quadrivium* (*v.l. quadruvium*) is established; cf. Michael Masi, *Boethian Number Theory* (Amsterdam 1983) 71. Plato (*Resp.* 530D–531A) had concluded his discussion of astronomy by making music the fifth discipline of study. He would describe the same collection and progression of scientific disciplines as well in *Leg.* 7 (817E), *Epin.* 990A–992E, and *Tht.* 145A. Francois Lasserre, *The Birth of Mathematics in the Age of Plato* (Larchmont NY 1964) 12–18, offers an historical background to mathematics at the Academy; cf. Edward A. Lippman, "Hellenic Conceptions of Harmony," *Journal of the*

includes mathematics, i.e. arithmetic and geometry, as well as astronomy and music. The order in which Ptolemy presents these four areas of learning differs from that progressive one described as the quadrivium,[62] however, for throughout the treatise Ptolemy makes music the assumption rather than the culmination of a scientific educational process.[63]

Some scholars have criticized Ptolemy for his treatment of music in this treatise. They balk because Ptolemy chose unnecessarily to make each of his ratios superparticular,[64] or they are disappointed that he gives too shallow a presentation of the actual music of his day,[65] or they say that his treatment of the philosophical and astronomical/astrological passages in Book III leaves the reader insufficiently enlightened.[66] None of these criticisms lacks substantial foundation. But Ptolemy had no intentions of writing a book about music. His work is on harmonics, and despite our familiarity with the modern derivative of that term, and despite that 'harmony' has quintessentially a musical meaning and only a secondarily a metaphorical meaning (as in *Harm.* 1.1), Ptolemy's conception of harmonics was one that was self-contained, well-established, highly scientific, and technical.

At the very outset, in the first sentence of the treatise, Ptolemy states that harmonics is a perceptive function of the difference between high and low sounds (1.3). His use here of the term 'function' (δύναμις - *dynamis*), the same term which will be so vital to him in II.5 where he creates the grandest construction in Greek music—a two-octave, shifting scale of limited but all-encompassing nature—suggests the scope that harmonics is given in Ptolemy's treatise. Wherever the perception of music is possible, and even where music is not audible, i.e. in the soul or in the heavens, there is harmonics, and harmonics is the perceptive function of the various intervals,

American Musicological Society 16 (1963) 15. Archytas had described the Pythagorean mathematical sciences—arithmetic and astronomy (spherics)—and music as "sisters"; cf. Olaf Pedersen, "Logistics and the Theory of Function: An Essay in the History of Greek Mathematics," *Archives Internationelles des Sciences* 24 (1940) 29.

[62] He describes the acoustics, that is, the scientific basis for music or harmonics, first, uses arithmetic and geometry to prove some basic assumptions about musical ratios, and then applies these ratios to astronomy/astrology. In general, these sciences were arranged differently in the late Hellenistic period, mostly because of the influence of Geminus of Rhodes.

[63] For Plato, of course, philosophy is the 'highest' goal. Philosophical investigation *per se* is actually quite lacking in the *Harmonics* and all the Ptolemaic works. Boll (109) describes Ptolemy as essentially Aristotelian (Peripatetic), but there are Stoic passages, and his soul divisions and harmonics are Platonic and Pythagorean. Boll rightly compares Ptolemy the philosopher to Ptolemy the scientist, neither entirely a compiler nor an innovative thinker.

[64] Cf. Barbera, "Divisions," 302–305.

[65] Düring, *Ptolemaios und Porphyrios* 201f.

[66] Some of this criticism derives from excessive rationality, e.g. the eighteenth-century comments of Charles Burney (n. 51); see Dayton Clarence Miller, *Anecdotal History of the Science of Sound* (New York 1935) 6. An alternative evaluation of Ptolemy is offered by Amy K. Holbrook, *The Concept of Musical Consonance in Greek Antiquity and its Application in the Earliest Medieval Descriptions of Polyphony* (Ph.D. diss., University of Washington 1983) 91–121. She argues that Ptolemy's lack of a musical aesthetic is the inevitable result of his astronomical training and approach.

primarily the consonant intervals,[67] which are direct, mathematically certain, and eternal differences.

Harmonics is not the science of music, and it is not the ability of the human ear to hear harmonics.[68] Harmonics is a function of nature that allows humans to perceive, accept, digest, feel, emote, intuit, study, compute, hypothesize, and theorize the differences between highness and lowness in sounds, whether they are heard (or hearable) or not. Perception functions to differentiate things, among them sounds, and of sounds by how much they differ in highness or lowness, this highness being anywhere within the edge of the then-known, (by today's standards) microscopic universe at the end of Saturn's nested shell,[69] the lowness being perhaps the 85:84 diesis, which Ptolemy so carefully avoids, played on a small lyre by a mere mortal musicologist here on earth. Harmonics begins when that proverbial tree falls in the forest and makes the air be beaten, and it ends when that abused bit of our atmosphere enters our ears, filters to the seat of reason, concords with the similarly calculable notes within us and in the distant heavens, and gives us pleasure in knowing that there is in the huge, sometimes audible, sometimes visible universe, a unified, predictable, divinely ordained order.[70]

[67] In Book III at least, Ptolemy emphasizes the consonant intervals for his astrological comparisons to the harmonics of music and the soul. Claudius Mamertus attributes to a thinker as early as Philolaus the thought that "the soul is connected with the body by means of number and harmony"; cf. F. M. Cornford, "Mysticism and Science in the Pythagorean Tradition," *CQ* 16 (1922) 146.

[68] Cf. the annotations to I.1, and see D. J. Furley, review of Ross, *Aristotle De anima*, *CR* n.s. 13 (1963) 46.

[69] Nested shells describe the distance from the edge of influence of one celestial body to the edge of the next (from the apogee of one to the perigee of the next), e.g. the maximum distance of the moon and the minimum distance of Mercury, the maximum distance of Mercury and the minimum of Venus, and the maximum of Venus with the minimum of the sun. Cf. R. C. Riddell, "Parameter Disposition in Pre-Newtonian Planetary Theories," *Archive for the History of the Exact Sciences* 23 (1980) 112, n. 71.

[70] In his introduction to the *Almagest* Ptolemy discusses ethos, proportion, divinity, natural beauty, and the soul; cf. G. E. R. Lloyd, *Greek Science After Aristotle* (New York 1973) 115. Historians well trained in the technical sciences often state the case even more firmly. Otto Neugebauer, in J. B. Brackenridge (and Mary Ann Rossi, trans.), "Johannes Kepler's On the More Certain Fundamentals of Astrology," *Proceedings of the American Philosophical Society* 123 (1979) 85, writes, "The Greek...universe was a well defined structure of directly related bodies. The concept of predictable influence between these bodies is in principle not at all different from any modern mechanistic theory." Ptolemy himself in opening the *Tetrabiblos* (3 (Robbins)), argues that astronomy is concerned with observing, recording, and predicting aspects of the heaven; astrology applies this knowledge to human affairs with less quantitative exactitude. In modern science, the two disciplines came to be studied separately only after Newton demonstrated the gravitational affect of planets on human affairs to be ineffective and excessively complex. In general, see Edward A. Lippman, *Musical Thought in Ancient Greece* (New York 1964) 1–44; E. G. McClain, *The Myth of Invariance* (New York 1976).

EPIGRAM

I know that I am by nature mortal and ephemeral.
But surrounded by celestial bodies,
When I track their ever-rushing spirals,
My feet no longer touch earth.
I stand before Zeus himself and take my fill
of ambrosia, divine fare.[71]

[71] See Franz Boll, "Das Epigram des Claudius Ptolemaus," *Socrates* 9 (1921) 2–12.

BOOK I

Summary

1. On the Criteria in Harmonics.
2. What is the Purpose of the Harmonicist?
3. How Highness and Lowness in Sounds Exists.
4. On Notes and Their Differences.
5. On the Pythagoreans' Positions Concerning the Hypotheses of the Consonances.
6. That the Pythagoreans did not Investigate About the Consonances Properly.
7. How the Ratios of the Consonances Could Be More Properly Defined.
8. In What Way the Ratios of the Consonances Will Be Demonstrated Con-fidently via the Monochord Canon.
9. That the Aristoxenians Incorrectly Calculated the Consonances by the Intervals and Not by the Notes.
10. That They Improperly Suppose the Consonance of the Diatessaron to Contain Two and One-half Tones.
11. How by means of the Octochord Canon the Diapason Could Be Shown to One's Perceptions to be Smaller than Six Tones.
12. On the Aristoxenian Division of the Genera and the Tetrachords in Each.
13. On Archytas' Division of the Genera and the Tetrachords.
14. Proof that Neither of These Divisions Preserves the Real Emmelic Interval.
15. On the Rational and Audible Tetrachordal Division by Genus.
16. How Many and Which Genera Are More Familiar to the Hearing.

1.1 - *On the Criteria in Harmonics*[1]

[3.1] Harmonics[2] is a perceptive function[3] of the differences in sounds between high and low,[4] and sound is a condition of beaten air[5]—the first and

[1] Part of the manuscript tradition dependent upon the twelfth-century M (Venetus Marcianus app. cl. VI/10 [=Mathiesen, *Ancient Greek Music Theory* #273]) mistook this chapter title ("On the Criteria in Harmonics" [Περὶ τῶν ἐν ἁρμονικῇ κριτηρίων]) for the title of the entire treatise (*Harmonics* [Ἁρμονικά]). The extant Ptolemaic corpus does include the work entitled Περὶ κριτηρίου καὶ ἡγεμονικοῦ [*De iudicandi facultate et animi principatu*, for which see the text and translation in Pamela Huby and Gordon Neal, eds., *The Criterion of Truth: Essays Written in Honour of George Kerferd Together With a Text and Translation (With Annotations) of Ptolemy's On the Kriterion and Hegemonikon* (Liverpool 1989) 179-230], but Macrobius (1.19–20) knew of a Ptolemaic *Harmonics* centuries before the medieval manuscript tradition; cf. Düring lxxi.

[2] The absence of a dedication or addressee in this first sentence is to be noted. Ptolemy addresses Syrus at the outset of the *Almagest* and *Tetrabiblos*, just as Aristides Quintilianus (1.2 [Winnington-Ingram]) addresses Eusebius and Florentius and Nicomachus (*Ench.* 237.17 [Jan]) initially addresses his patroness.

[3] Δύναμις ('function') has perpetually been mistranslated as "power." Gogava translated the term as "facultas," Wallis as "potentia," Düring as "Fahigkeit" (cf. Willy Theiler's review of Düring, *Göttingsche gelehrte Anzeigen* 198 [1936] 199; Theiler's reference to Quint. *Inst.* 2.15.2 is not helpful, although it should be remembered that Quintilian and Ptolemy are near contemporaries), and Barker as "power that grasps," none of which can be correct. Harmonics is not a "power" but a function (δύναμις) of perceiving (καταληπτική) differences in sounds. For details, see Jon Solomon, "A Preliminary Analysis of the Organization of Ptolemy's Harmonics," in André Barbera, ed., *Musical Theory and Its Sources: Antiquity and the Middle Ages* (Notre Dame 1990) 71; *contra* see Bowen, "Translator" 105-106. This accords with the most common technical use of the word δύναμις in the early empire; Galen (10.635 Kühn), for instance, distinguishes between physical, biological, and spiritual "functions." Szabó (17–21, 36–40, 44–46), and Maria Timpanaro-Cardini ("Two Questions of Greek Geometrical Terminology," in J. Mansfeld and L. M. de Rijk, eds., *Kephalion: Studies in Greek Philosophy and its Continuation Offered to C. J. de Vogel* (Assen 1975) 187) discuss δύναμις in the general mathematical sense; cf. the review of Szabó by Burkert in *Erasmus* 23 (1971) 103, and Lohmann Musike 31. For the classic view, see Louis Laloy, *Aristoxène de Tarente de la Musique de l'antiquité* (Paris 1904) xii. With these references include also the phrase ἀκουστικὴ δύναμις ('auditory function') in Sophonias *In libros Aristotelis de anima paraphrasis* 86.37 (Hayduck).

[4] Cf. Porph. 191.28. Aristoxenus (*Harm.* 1 (5.1–6.6)) and Cleonides (179.1–.2) define harmonics as a theoretical and practical science. Both the Ptolemaic and Aristoxenian definitions are recast in Bry. II.6 (172.25f.) and Pach. 100.1–2. For other definitions and parallels, see Michaelides 130. The scholiast offers this variant: "Harmonics is the theoretical knowledge of the nature of what is tuned or the theoretical skill of intervallic music and what is incidental to it; more accurately, it is what serves as the rather comprehensive foundation of this book." Harmonics is also a science which requires the acquisition of knowledge and which leads to usage. Usage, however, falls under the category of μουσική (*mousike*), which the all-embracing science of μέλος (*melos* - 'music') encompasses.

On harmonics in general, see Mathiesen, "Problems in Terminology in Ancient Greek Theory: *Harmonia*," in B. Karson, ed, *Festival Essays for Pauline Alderman* (Provo 1976) 3-17, and, for an aesthetician's viewpoint, see Andrew Barker, "Music and Perception: A Study in Aristoxenus," *JHS* 98 (1978) 9–16.

[5] Borrowed verbatim by Bry. I.4 (88.14), this phrase was once thought to derive from earlier Stoic definitions of sound, e.g. Diog. Laert. 7.55 (Zeno) and Eduard Zeller, *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung*[4] (Leipzig 1923) III.1.69, n. 1;

most basic[6] element of what is heard.[7] The criteria in harmonics[8] are hearing and reason[9], but not in the same way:[10] hearing is the criterion for matter and condition, while reason is the criterion for form and cause.[11] This is because, generally speaking, discovering what is approximate and accepting what is

but see now the citations of Peripatetic, Platonic, Pythagorean, and pre-Platonic sources in Andrew Barker, *GMW* I.206, n. 5 and II.276, n. 2, as well as Theon of Smyrna 34 and 40 (Lawlor). Ptolemy uses the same concept to describe the "sister" sense of vision later in III.3; cf. Albert Lejeune, *Euclide et Ptolémée; deux Stades de l'Optique géométrique grecque* (Louvain 1948) 23. Porph. ad loc. (7.8f.) distinguishes between the two Greek terms for sound, ψόφος (*psophos*) and φωνή (*phone*), the former referring to sounds from inanimate objects.

[6] For 'most basic' (γενικώτατον), cf. Ptol. *Judic.* 4.17; Friedrich Lammert, ed., *Claudii Ptolemaei opera quae exstant omnia*, vol. III (Leipzig 1961) 2, cites also Aristotle's discussions in the *Topics* of predicables (101^{b}18), of which *genus* is one type (102^{a}36), and Sen. *Ep.* 58.8 [not 5]. See also, F. Lammert, "Zur Erkenntnislehre der späteren Stoa; Ptolemaios Περὶ κριτηρίου καὶ ἡγεμονικοῦ 10.11–.13, 13H," *Hermes* 57 (1922) 186.

[7] As so often, Ptolemy's word order and lack of clear antecedents leave the meaning of this second clause open to some interpretation. Ptolemy surely calls beaten air, not sound, the first and most basic element of what is heard, and the latter phrase (ἀκουστῶν) is clearly "of what is heard" and not "of the hearing."

[8] Just one sentence into his treatise Ptolemy uses an alternate form, ἁρμονία (*harmonia*), for ἁρμονική (*harmonike*). Both words should be rendered as 'harmonics,' but Ptolemy's employment of two different, albeit related, words for the same technical concept immediately warns the reader and modern translator to be prepared for his characteristically negligent but occasionally intentional shifts in vocabulary and even technical terms throughout the treatise.

On the fundamental term ἁρμονία, see also Winnington-Ingram, *Mode* 3; Winnington-Ingram, "Greece" 666; E. A. Lippman, "Hellenic Conceptions of Harmony," *JAMS* 16 (1963) 3–5 and *Musical Thought in Ancient Greece* (New York 1964) 3; Jacques Chailley, "Le Mythe des modes grecs," *AM* 28 (1956) 148–9; I. Henderson, "Ancient Greek Music," in Egon Wellesz, ed., *Ancient and Oriental Music*, vol. I of *The New Oxford History of Music* (London 1957) 340–41; I. Düring, "Greek Music," *Cahiers d'histoire mondiale* 3 (1956) 311-12; F. Levin, *Nicomachus* 1–7; Francois Lasserre, *The Birth of Mathematics in the Age of Plato* (Larchmont NY 1964) 169–87; F. M. Cornford, "Mysticism and Science in the Pythagorean Tradition," *CQ* 16 (1922) 137–50 and 17 (1923) 1–12; and R. Falus, "*Harmonike phanere*," *Acta Antiqua Academiae Scientiarum Hungaricae* 29 (1972) 1–4.

[9] I have translated λόγος (*logos*) as 'reason' here knowing full well that 'logos' has a variety of important but different meanings within this treatise alone. Most important is its use as the word for 'ratio, proportion' (cf. Theon of Smyrna 2.18, but in these initial chapters Ptolemy for the most part avoids employing technical harmonic vocabulary. For a discussion of the term 'reason' (λόγος), see Andrew Barker, "Reason and Perception in Ptolemy's Harmonics," in R. Wallace and B. MacLachlan, eds., *Harmonia Mundi: Musica e filosofia nell'Antichità* (Rome 1991) 104–30.

[10] Ptolemy's use of the word τρόπον (*tropon*), which can mean generally 'way' or 'method' but also, technically, 'tonos' (hence 'mode' - cf. 80.15), here is further evidence of his non-use of technical vocabulary in the early chapters of the treatise.

[11] Ptolemy sacrificed clarity for succinctness here. He means, of course, that hearing is the criterion used for judging harmonic matter and condition, reason that used for harmonic form and cause. Ptolemy will begin a general comparison of all forms of perception and reason in the next sentence. For the basic Peripatetic term 'cause' (τὸ αἴτιον) Porph. (11.3–24) refers us to Aristotle's discussion of sensible substances in *Meta.* 1042^{a}1–1043^{a}21.

exact are characteristic of perception, while accepting what is approximate and discovering what is exact[12] are characteristic of reason.

[3.8] Since matter is determined and limited only by form, and since conditions are determined and limited only by the causes of motion, some of which belong to perception and some to reason, it will follow in all likelihood that the observations of perception will be determined and limited by those of reason, first suggesting to them the differences picked out roughly, insofar as they are known through perception, and then being processed by the latter into more accurate and confirmed differences in sound.[13]

[3.14] This is so since reason happens to be both simple and unmixed, wherefore also complete, fixed, and ever constant in relation to the same things. Perception, on the other hand, is of constantly mixed and fluctuating matter,[14] so that on account of its instability neither the perception of all men[15] nor that of the same men always observes the same thing in what has remained the same. Perception needs as its crutch, as it were, the educational assistance of reason.[16]

[4.1] Just as a circle described by one's eye alone has often seemed to be accurate until one drawn by reason redirects the eye to recognize what is truly accurate, if some specific difference of sounds is perceived by the hearing alone, it will then for a time seem neither to fall short nor to be excessive; [4.5] yet it will frequently be proved not to be so when one selected according to the appropriate reason is heard. By comparing them, our hearing recognizes the more accurate, as if comparing the genuine with the illegitimate.[17] [4.7] After all, judging something is generally easier than doing it, as judging wrestling is easier than wrestling, judging dancing is easier than dancing, judging aulos-playing is easier than aulos-playing, and

[12] For the phrase 'discovering what is exact' (τοῦ δῷ ἀκριβοῦς εὑρετικόν), cf. *Judic.* 15.8-9. with Boll 97–98.

[13] One of the premises of the *Harmonics* is that reason generally surpasses the senses and that, insofar as music and harmonics are concerned, reason surpasses the hearing. The logic in these last two sentences leads to a satisfactory conclusion only if this premise is accepted. On the phrase 'known through perception,' see Porph. 16.22--3.; cf. Boll 80, n. 1. Izydora Dambska, "L'épistémologie de Ptolémée," *Semaine de Synthese* 31 (1975) 35, puts this passage into a larger philosophical context.

[14] For an analysis of Aristotelian and Pythagorean differentiations between matter and reason, see Düring, *Ptolemaios und Porphyrios* 141–43, Barker *GMW* II.276, n. 4.

[15] The noun 'men' is an obvious but helpful supplement added in two manuscripts, Vaticanus gr. 186 (Düring's E [Mathiesen, *Ancient Greek Music Theory*, #210]) and Vaticanus gr. 198 (Düring's G [Mathiesen, *Ancient Greek Music Theory*, #218]).

[16] A restatement of the superiority of reason. The instability of our perceptions leads to false readings, not only when everyone views the same phenomenon at the same time but even when the same men observe the same phenomenon at different times, even if at the same place and under the same circumstances. Ptolemy does not specify clearly which of the variables remain constant. He makes similar statements at 6.22–23 and 8.27–9.1.

[17] For an elaboration, see Bowen, "Translator" 107.

judging singing is easier than actually singing.[18] [4.10] Therefore the aforementioned deficiency of our perceptions would not be far from the truth in ascertaining simply the difference or lack thereof in these things, nor again in observing the excesses of the differences, at least those excesses found in their greater parts of what they are.[19]

[4.13] In the comparison of smaller parts, more of the deficiency will be concentrated and now made detectable in them, and even more so in still smaller ones. The reason is that something which is just once quite short of the truth[20] and cannot make the amount of its shortness perceivable by few comparisons, will when a number of comparisons are made be most noticeable and comprehensible to all.[21]

[4.19] When a straight line is given, it is a very easy matter to find by eyesight a line smaller or larger than it—not only because it is wide-ranging,[22] but also because there is only one comparison. Halving and doubling it are still easy, even if not as easy, for there are only two comparisons.[23] [4.22] Tripartite divisions and tripling are more difficult since now three junctions[24] are involved; and it will always be proportionately

[18] Düring separated these clauses by creating a full stop here but placing only a comma after 'singing' (*infra*). I have followed in part the suggestions made by Alexanderson 8. Cf. Porph. 18.24–19.19, esp. 19.9–.11. At 19.15 he says "Perception is the instrument of reason."

Boeth. *De mus.* (5.2) offers an enlightened paraphrase of this passage in which he explains that the eye cannot sense a true circle because the senses are oriented toward matter (and imperfection). The wrestling metaphor is surely Platonic. Düring cites Eust. 1312.41: "See in these matters the unusual *skopsis* [mockery] just as Ptolemy's harmonic singing makes a song," but Eustathius need not be referring to this particular passage.

[19] Ptolemy has just established the greater ease in judging some activity rather than performing it. Now he is going to demonstrate how noticeable the difference is between what is established correctly (by reason) and incorrectly (by perception), particularly in large items and in larger bundles of smaller items. That is to say, in larger bodies our perceptions can quite satisfactorily observe and loosely determine the differences, but in smaller bodies the deficiency of our perceptions, their inability to distinguish what is correct from what is incorrect, becomes greater. The entire passage has harmonic implications foreshadowing Ptolemy's refutation of the Aristoxenian assertion that six whole tones equal a diapason.

[20] Alexanderson (8) expands "short of the truth" to "a deviation from exactitude."

[21] Ptolemy goes on to say that while our perceptions are more noticeably deficient in comparing smaller items, the bundling or extension of several or many of these smaller items can make the differences quite observable to everyone. Porph. (19.30–20.2) seems to suggest an imprecise parallel in the theater (or odeon), where the large ("artless") crowd hisses at a mistake by the aulete. Cf. *Alm.* 9.2 (208 Heiberg).

[22] In progressing from single division and multiplication to octuple and eightfold, Ptolemy wishes to demonstrate that differences in size are increasingly more difficult to perceive as the size of the items considered decreases but increasingly easy to perceive as the number of divisions increases. He states this, however, as if it were the number of the divisions which was more crucial. Albert Wifstrand, "Eikota: Emendationen und Interpretationen zu griechischen Prosaikern der Kaiserzeit," *Kungliga Humanistiska Vetenskapssamfundet* I 3 (Lund 1933) 67, includes this sentence in his collective discussions.

[23] Cf. Porph. 20.24, Düring (1934) 143, and Willy Theiler, rev. of Düring *Ptolemaios und Porphyrios über die Musik*, *Göttingenische gelehrte Anzeigen* 198 (1936) 199.

[24] Ptolemy uses the word 'junctions' (ἁρμογῶν) in a non-musical context; elsewhere, e.g. II.12 (66.14), it will have a strictly musical context. In these introductory chapters Ptolemy will

more difficult in observations made with more subdivisions when we take the quantity to be examined by itself, such as the seventh or the sevenfold.[25] [4.26] We should not seek an easier method, such as when the eighth is determined by first taking half, then half of that, then again half of that, or when the eight-fold is determined first by doubling and doubling the result and again doubling that. [4.28] For the eighth or the eightfold will not in this way have been found by using the basic unit, but by using the halves and doubles of several unequal parts.[26]

[5.2] Because circumstances are similar in the case of sounds and hearing, there is need, as there is for sight, of some rational criterion for them from the appropriate instruments, just as the straight edge has, so to speak, need of a ruler,[27] and the circle and the measurement of its parts have need of a compass. [5.6] In the same way, the hearing, which is, along with sight, for the most part a messenger for the theoretical and rational aspect of the soul,[28] needs some reasoned approach, too, for those things its nature cannot judge accurately, which it will not contradict but confirm as correct.

I.2 - *What is the Purpose of the Harmonicist?*

[5.11] The tool of this type of approach is called the harmonic canon,[29] which takes its name from the regular category[30] and from providing

use musical terms in a non-musical context. Cf. 'function' (δύναμις) in I.1 with its usage in II.5, as explained in my "A Preliminary Analysis of the Organization of Ptolemy's Harmonics," in André Barbera, ed., *Musical Theory and Its Sources: Antiquity and the Middle Ages* (Notre Dame 1989) 71. There is still another example here: Ptolemy's use of the number eight, the number of notes in a diapason, is hardly entirely coincidental.

[25] Seven is a prime number and must be "examined by itself." Ptolemy's use of seven, the number of intervals in a diapason, may or may not be coincidental. He continues in the next example with the number of notes in a diapason, eight, which is, of course, not a prime number.

[26] Ptolemy thus demonstrates that since eight is not a prime number, one need not rely on the monad to discover it. Musically, it means either that the series of eight notes in the diapason should not use a monad or basic unit to double, redouble, and redouble again, or that the monads used in such multiplication are unequal. The former would be a negation of Aristoxenian theory, the latter perhaps an affirmation of Pythagorean. Ptolemy seems to prefer the more difficult method which depends on the essential, elementary well-measured monad. If this were called a hemitone, the sentence would surely read differently. Boeth. *De mus.* (5.2), in fact, paraphrases and adds as an example that a near-tone, another near-tone, another, and a half tone might individually each sound like whole tones and one half tone, but combined they would not sound like a diapente. Similarly, he interprets the "eight-fold passage" to mean that the senses could double a line reasonably accurately, but they could not double it, double it again, and double it again accurately.

[27] Such a requirement is not unique to this discussion; cf. *Judic.* 4.12.

[28] Ptolemy uses the sense of sight as a visible analogy to the sense of hearing. Also, the sense of hearing, not sight, is the messenger of the theoretical, i.e. 'visible' (in the original meaning of the word), and the rational. Even Plato (*Resp.* 531A) treats the art/science of music as preliminary to philosophy, whereby hearing needs be a prelude to reason.

[29] The Greek word for 'tool' (ὄργανον) also serves as the word for any type of 'instrument,' including a musical instrument, but Ptolemy does not search here for a musical

measurement[31] where the perceptions are deficient in regard to the truth. [5.13] The purpose of the harmonicist[32] would be then to preserve in every way the reasoned hypotheses of the canon which do not in any way at all conflict with the perceptions as most people[33] interpret them, just as the purpose of the astrologer is to preserve the hypotheses[34] of the heavenly

tool, i.e. instrument, as much a technical measuring device. Charles-Émile Ruelle, "Le Monocorde, Instrument de musique," *REG* 97 (1897) 309–12, explores the question of whether the canon was played as an actual instrument. The obvious astronomical parallel for the canon as a scientific research tool would be the astrolabe which Ptolemy describes and employs in *Alm.* 5. (217–19 [Toomer]), for which, see Newton, *Crimes* 353; cf. also Ptol. *Optics* 238.5 (Lejeune).

Porph. (22.20–24) cites Ptolemais of Cyrenaica, who in *The Pythagorean Elements of Music* said, "The canon is the measure of accuracy in the difference realized in notes and contemplated in numerical ratios." Porphyry cites and quotes Ptolemais a number of times in these early passages; cf. Burkert 370, n. 6, and Konrat Ziegler, "Ptolemais," *RE* 48 (1959) 1867–68; for translations, see Barker *GMW* II.229–44.

Despite the importance of the canon for Pythagorean investigations of musical intervals, Burkert 372–75, follows van der Waerden, "Harmonielehre" 177, in demonstrating the improbability that a canon existed for Archytas or for anyone earlier than Aristotle. The extant works or fragments of and testimonia about Philolaus, Archytas, Plato, Aristotle, Heracleides Ponticus, and Aristoxenus make no mention of the word 'canon,' and *Sectio canonis* Prop. 19 (163.15 (Jan)) specifies the instrument only in the last propositions; cf. Mathiesen, "An Annotated Translation of Euclid's Division of the Monochord," *JMT* 9 (1975) 236–58, esp. 249–51. Most correctly ignore the ancedote in Aristides Quintilianus 32 (97.3–7 [Winnington-Ingram]) that says it was Pythagoras who on his death bed suggested using the monochord to test musical ratios. Van der Waerden supposes that the canon was invented by Straton of Lampsacus in the late third century B.C. Llewelyn S. Lloyd and Hugh Boyle, *Intervals, Scales and Temperament* (London 1963) 155–75, give instructions on how to construct and calibrate a monochord.

[30] Ptolemy uses the word 'category' not in reference to Peripatetic logic but simply to identify the name 'canon' as a normal instrument. This canon is normal in that, as any canon, it measures something, but its uniqueness is that it is a canon that measures truthful harmonic intervals.

[31] Ptolemy's verb here is κανονίζειν (*kanonizein*—'to measure'), which echoes the noun κανών (*kanon*—'kanon').

[32] One of the purposes of this chapter is to define the function, purpose, or task of the ἁρμονικός (*harmonikos*—'harmonicist') who is to investigate the science of ἁρμονική (*harmonike*—'harmonics'). The harmonicist does not spend all his research energies and time investigating harmonics, of course. Because of the mathematical, philosophical, and cosmic nature and history of the subject, the many ancient scientists who worked and wrote on the subject all necessarily worked and wrote in a number of other fields as well. In the fourth century B.C., however, the label *harmonikos* (in the plural) was one of derision when used by Aristoxenus (*Harm. passim*, e.g. 7.3 and 12.10) and by the author of the Hibeh papyrus, for which see Wilhelm Croenert, "Die Hibehrede über die Musik," *Hermes* 44 (1909) 503–21.

[33] The scholion in the thirteenth-century manuscript E (Vat. gr. 186 [Mathiesen, *Ancient Greek Music Theory*, #210]) identifies these "people" as Aristoxenians, but there is not yet need to do so. The scholiast adds, "Or the arithmetical models, for the symbol of the double ratio, for example, is two to four [sic], and the sesquialter 3 to 2." But when Aristoxenians use such "symbols," i.e. numbers, they are also apt to describe merely the relationships within intervals, e.g. that there are exactly two half tones in a whole tone and six whole tones in a diapason.

[34] "Preserving the hypotheses" was an important concept in ancient Greek science. Here it can be applied to *Alm.* 1.1, for which, see Porph. 24.27 (the only citation of the *Almagest* in

movements concordant with observable paths.[35] [5.17] Even these hypotheses are themselves assumed from what is clear and roughly apparent,[36] but with the help of reason they discover detail with as much accuracy as is possible.

[5.19] For in every subject it is inherent in observation and knowledge to demonstrate that the works of nature have been crafted with some reason and prearranged cause and completed not at all in random or as it happened, particularly in its most beautiful constructions which are simply those of the sense perceptions more closely related to reason—sight and hearing.[37]

[5.24] Although this is the purpose of the harmonicist, some seem altogether not to have taken it into consideration in that they applied themselves to manual application alone and to the simple, irrational practice of perception, while others pursued their goal more theoretically. These harmonicists are the Pythagoreans and Aristoxenians, and both were in error.

[6.1] The Pythagoreans, in not heeding the impressions of their hearing, something everyone ought to do, applied to the differences in sounds ratios often inappropriate for the phenomena.[38] The result was that they rendered

Porphyry) with Newton, *Crimes* 1–2. Cf. G. E. R. Lloyd, "Saving the Appearances," *CQ* n.s. 28 (1978) 202–22; and Marshall Clagett, *Greek Science in Antiquity* (New York 1955) 94.

[35] Ptolemy only rarely preestablishes what he will discuss at a later chapter; cf. 6.11 for one counterexample. Consequently he offers no specific reference here to his lengthy discussions comparing musical and astronomical harmonics (III.8–16). The scholiast does look ahead, however, in commenting that "there are two species of locomotion, which is also known as a carrying motion, and they are circular and direct. There are more species of the direct locomotion (which we do not need to discuss now), and two of the circular. The first is that movement which consists of a carrying motion from locus to locus, as, for example, with the movement of the sun, moon, and the other planets. The second is that which is stationary, as with the moving cones and with spheres moving around their own axis. It is in this form of movement that we say the celestial sphere revolves."

[36] Boll 100 inappropriately cites Arist. *De caelo* 271ᵃ33 to locate the Aristotelian differentiation between *dynamis* and *energeia*. Galen (6.21) specifies a physiological 'function' (*energeia*). Cf. 1.4 (9.18–19).

[37] This brief encomium of nature has near parallels elsewhere in Ptolemy, e.g. 85.15 here and *Tetr.* 1.1–3. For Ptolemy's distinguishing the senses of sight and hearing, cf. *Judic.* 23.8 and *Harm.* III.3, esp. 93.12.

[38] A. C. Bowen, "The Foundations of Early Pythagorean Harmonic Science: Archytas, Fragment 1," *Ancient Philosophy* 2 (1982) 101, discusses the Pythagoreans' neglect of the ear and their emphasis on number. Burkert, *Lore and Science* 369–400, offers an overview with copious bibliography, as do Robert W. Wallace, "Music Theorists in Fourth-Century Athens," in Bruno Gentili and Franca Perusino, eds., *MOUSIKE: metrica ritmica e musica greca in memoria di Giovanni Comotti* (Pisa 1994) 17-39, and W. K. C. Guthrie, *A History of Greek Philosophy* 1 (Cambridge 1967) 146–340. Amy K. Holbrook, *The Concept of Musical Consonance in Greek Antiquity and its Application in the Earliest Medieval Descriptions of Polyphony* (Ph.D. diss., University of Washington 1983) 1–40, offers a survey of the history of Pythagorean harmonics and mathematics through Euclid. The members of the Pythagorean circle would include Archytas, Eratosthenes, Didymus, all of whom Ptolemy discusses in some detail, the author of the *Sectio Canonis* (for the authenticity of which, see Barbera 112–13 and 151–55), Nicomachus, Theon, Gaudentius (10–17), Aristides Quintilianus (to a limited extent), and Boethius (for whom see Barbera, *Pythagorean Mathematics* 197–242), with considerable influence felt by Iamblichus, Proclus, Porphyry, Censorinus, Chalcidius, Macrobius, Martianus Capella, and Cassiodorus.

this criterion open to slander by those belonging to other schools of thought.[39] [6.5] The Aristoxenians, concerning themselves chiefly with data gathered by perception,[40] left reason to be mistreated as secondary, the result being contrary both to reason and to the evidence—contrary to reason since they applied the numbers, that is, the symbols of ratios,[41] not to the differences in sounds but to their intervals;[42] and contrary to the evidence since they compared the numbers in divisions improper for perception's approval.[43]

Ptolemy's sources may have included many of the same works Porphyry employed. Didymus had written a treatise "On the Differences Between Pythagorean and Aristoxenian Music Theory," quoted in part at Porph. 26.6–25; cf. Burkert, *Lore and Science*, 370 (and 386–400, where he discusses Philolaus). Porphyry also quotes Ptolemais, Heracleides Ponticus, Archytas, Aristoxenus, Democritus [not in Diels[4] II 10–140, as Düring points out at Porph. 32.10], Aelian, and Dionysus [*musicus*; cf. C. Scherer, *De Aelio Dionysio Musico qui vocatur* (Ph.D. diss., Bonn 1886) 50]. Although we have become accustomed to reading about the Pythagoreans almost entirely in the spheres of music, mathematics, and, to a very limited extent, astronomy, Ptolemy apparently has need to make reference to them in the *Optics* (16.9) as well.

[39] One of these schools of thought turned out to be the Ptolemaic. Ptolemy will discuss the various numerical ratios advanced by Archytas, Eratosthenes, and Didymus at II.13 and 14.

[40] Or 'perceptions' (mss.). Putting the work of Aristoxenus into historical perspective are Walther Vetter, "Die antike Musik in der Beleuchtung durch Aristoteles," *AfM* (1936) 1–41; Francois Lasserre, *The Birth of Mathematics in the Age of Plato* (Larchmont NY 1964) 183–87; Warren D. Anderson, "Musical Developments in the School of Aristotle," *Research Chronicle* 16 (1980) 78–98, esp. 87–91; Amy K. Holbrook, *The Concept of Musical Consonance in Greek Antiquity and its Application in the Earliest Medieval Descriptions of Polyphony* (Ph.D. diss., University of Washington 1983) 47–72; and E. A. Lippman, *Musical Thought in Ancient Greece* (New York 1964) 111–66. Cf. Louis Laloy, *Aristoxène de Tarente de la Musique de l'antiquité* (Paris 1904 (reprint: Genève 1973)) 1–42. Other Aristoxenians include Cleonides, Bacchius, Alypius, and to a certain extent Aristides Quintilianus, Martianus Capella, and Censorinus.

[41] Cf. *Harm.* 50.9–.25.

[42] Barbera, "Divisions" 299–300, finds significance in the use of the term διάστημα (*diastema* - 'interval') by both Aristoxenus and Euclid, for both investigators think of the term as describing physical space, while Szabó 107f., differentiates between διάστημα as a musical interval and as a line segment; cf. K. von Fritz, "The Discovery of Incommensurability by Hippasus of Metapontum," *Annals of Mathematics* 46 (1945) 250. Ptol. *Alm.* 12.9–10 employs the same term in regard to the distances, that is 'elongations,' between the planets and the sun. Cf. Ptol. *Geog.* 1.3.

[43] 'Approval' (συγκατάθεσις) is a Stoic term; see Boll 98 and 109.

As Ptolemy will demonstrate at I.10, if one plays six whole tones in succession, the interval formed between the highest and lowest notes will not sound to the ear like a perfect diapason. Ptolemy criticizes the Aristoxenians in two ways, the Pythagoreans only in the one. He leans more toward the latter school, but his wording at 6.1 assures the reader that he does not want to be associated with either. Useful here is Mountford, "Harmonics," 71–73. Barbera, "Divisions" 294–323, expanding on Neugebauer, *Exact Sciences* 148, describes the difference between the two schools of harmonics in mathematical terms alone, that is, that the Pythagorean was an arithmetic approach, the Aristoxenian a geometrical approach. Cf. Barbera, *Pythagorean Mathematics* 98f. Norman Cazden, "Pythagoras and Aristoxenus Reconciled," *JAMS* 11 (1958) 97–105, puts the differences between the two schools of musicology into some perspective as regards the history of aesthetics; *contra*, Barbera, *Pythagorean Mathematics* 7, n. 6. On Aristoxenus's purported empiricism, see Malcolm Litchfield, "Aristoxenus and Empiricism: A Reevaluation Based on His Theories," *The Journal of Music Theory* 32 (1988) 51–73.

[6.11] Each of these errors will become clear in the following discussions if matters preliminary to what follows are defined first.

1.3 - *How Highness and Lowness in Sounds Exist*

[6.14] With sounds,[44] like all other things, differing in both quality and quantity,[45] we will not be ready to show in which of these two categories the difference in highness and lowness should be placed until we examine the causes of such an occurrence which seem to me to be somehow similar to the variables found in the other types of beatings.[46] [6.19] Their conditions differ according to the force of that which does the beating, the material consistency of both the thing beaten and the thing by which the beating is effected, and again the distance from the thing beaten to the origin of the movement.

[6.22] Clearly each of the things mentioned has its particular effect on the condition when it becomes different in some way or other and when the other things remain constant.[47] [6.24] With sounds, the difference according to the consistency of the thing beaten either would not exist altogether or

[44] This chapter begins the genuine musicological, i.e. acoustical or pro-portional, analysis which Ptolemy will complete only in the second chapter of Book III. In order to explain the fallacies inherent in the Pythagorean and Aristoxenian approaches, he will need to examine the quintessence of intervals, that is, the comparison of and mathematical differences in sounds between high and low. This sounds very much like Ptolemy's definition of harmonics in the first chapter of Book I (3.1), so for his argument this and the subsequent chapters are crucial. It is not, however, the acoustics underlying ratios and intervals which will have the greatest bearing on his preferred scale. It will be the quantitative nature of acoustics, and this quantitative nature is what he establishes in this particular chapter.

For two perspectives on sound, one earlier, one later than Ptolemy, see A. C. Bowen, "The Foundations of Early Pythagorean Harmonic Science: Archytas, Fragment 1," *Ancient Philosophy* 2 (1982) 88, 92, and 101, who analyzes the Aristoxenian and Euclidean works entitled *Stoicheia* ('Elements') as indicative of a method in which propositions are preliminary to conclusions drawn from them (cf. André Barbera, *The Euclidean Division of the Canon* (Lincoln and London 1991) 115-117 with notes and parallels); and R. Bruce Lindsay, *Acoustics: Historical and Philosophical Development* (Stroudsburg, PA 1974) 21-24.

[45] On Ptolemy's concern with placing differences in sounds as subspecies of either quality or quantity, see Pl. *Tht.* 182A, where Plato apologizes for using the word ποιότης ('quality'), and Arist. *Cat.* 8^b26, for which cf. *Harm.* III.5.

[46] At 3.9 causes were subject to reason, matter to form. These most basic causes bring about sound itself, beatings of air. Levin, "*Plege*" (1980) 212-13, discusses these concepts and translates and summarizes the passage to 6.22.

[47] Ancient Greek scientific terminology lacked an adequate term for what we would label a 'variable.' Ptolemy's immediate concern is to determine which 'variable' might affect specifically the condition which brings about highness and lowness. He lists the three basic variables of all sound production—force (amplitude), quality (resonance), and distance (frequency), which produce respectively volume, timbre, and pitch. It is the latter, of course, that Ptolemy wants to identify as the cause of change in highness and lowness, but his prose loses its logical progression along the way. He does sufficiently make clear that the first two variables—material consistency and force of the beating—form one subgroup, distance the other.

would not be perceivable at least in that our perceptions in such a case sense only the variations in the air.[48] [6.27] On the other hand, the difference according to the force of the beater would be the cause only of magnitude and not of highness or lowness, for with this variable we do not see any such change in sound occurring.[49] [7.3] The sound becomes softer or louder in speaking, for example, or again lighter or more forceful or more robust in blowing and plucking, while a greater volume alone accompanies the greater force and a lesser volume the weaker force.[50]

[7.5] The variability in the object by which the beating is effected occurs then in the primary consistency of the material, that is through which each is porous or dense, thin or thick, and smooth or rough. [7.9] It occurs in the form as well, for what is there in common between the qualities of feeling—I mean smell, taste, and color—and beating? [7.10] It is through the form of the receptacles beating, such as tongues and mouths,[51] that the configurations, as if forms[52] for sounds are made through which we utter clatters, thuds, sounds, crashes, and a thousand other sounds like them. [7.14] To do so we imitate each of the configurations because we humans have the rational and technical command.[53]

[7.15] It is then through the quality of smoothness or roughness alone, wherefore certain things are similarly named smooth and rough, that these are also proper qualities. It is through the qualities[54] of porosity or density

[48] He seems to mean that the variable of material consistency in itself has no effect on our hearing. This is not true, of course, but Ptolemy attempts here to isolate the variable that produces differences in pitch, and since he needs to demonstrate that this depends on a quantitative difference i.e. distance, he needs first to eliminate the most observable qualitative variable, that of material consistency. Barker, *GMW* II, 280, believes the thing beaten is always the air external to the instrument, which, while acoustically sensible, does not explain what Ptolemy wrote here.

[49] Ptolemy's 'change in sound' (ἀλλοίωσιν περὶ τοὺς ψόφους) here refers to change in pitch (highness and lowness).

[50] The variable of force is, unlike the qualitative variable of material consistancy, one of quantity. It applies to singing, speaking, lyre or aulos playing, and in each the greater force produces greater volume, the lesser force the lesser volume.

[51] Cf. Aristotle at *Cat.* 9^a28-9^b9 (with Porph. 41.11-27).

[52] Ptolemy employs here the same technical term (σχηματισμούς) he will use in the astrological material of Book III, e.g. 108.6 (best translated there as 'aspects'). He also seems to make reference to a well-known musical 'form,' the *nomos*, which has a name originally signifying 'law, custom,' as at 99.10. In using these terms, Ptolemy connects the acoustical properties of the human voice, song, and the constellations.

[53] Material consistency in and of itself did not affect pitch, nor did force, which created only differences in volume. Ptolemy will now return to material consistency to suggest that it concerns quality and then quantity. Once there, he can then demonstrate that differences in pitch are a matter of quantity.

[54] Alexanderson (8) misrepresents Ptolemy's original construction in lines 10, 15, and 17. The definite article is erased in several manuscripts, but 'qualities' is added in others. He ignores 'the quality' here and that in 7.17. Ptolemy does not always make potentially or expectedly parallel expressions precisely parallel.

Porph. (42.24-43.6) again quotes from Arist. *Cat.* 10^a16, where Aristotle, using density and thinness in a series of examples, disassociates them from his four kinds of quality. Ptolemy will and thickness or thinness, where again we say some sounds are dense or loose and full-bodied or thin; and then in highness and lowness as well since each quality affected by the aforementioned consistencies is proportional to the amount of substance. [7.22] What is more dense with a constant bulk has more substance and what is thicker in a similar construction and with constant length has more substance. Also, denser and thinner produce higher sounds, sparser and thicker produce lower sounds. [7.25] Now even in other matters the higher is said to be such because it is thinner just as the duller is said to be such because it is the thicker, for thinner things beat more frequently because of quicker penetration, dense things because of greater penetration.[55]

[7.29] Because of this bronze makes a higher sound than wood, and a gut string makes a higher sound than a linen one, for they are more dense. [7.30] The thinner of bronze pieces of similar density and of the same bronze, and the weaker of strings of similar density and of the same substance make higher sounds; so do hollow substances rather than filled, and again so do the denser and thinner of pipes.[56]

[8.2] All of this happens surely not on account of density or thinness themselves, but on account of the tension,[57] for the higher sounds happen where there is such a greater tension. The more intense the beating, the stronger it is, the more frequent it is, and the higher it is.[58] [8.5] So, if otherwise something is more tensed, for instance if something is harder or utterly greater, it makes a higher sound since the proportional difference prevails where a potential similarity is present in both, as when bronze makes a higher sound than lead since bronze is harder than lead more than lead is denser than bronze. [8.10] And again if a bronze object happens to be greater[59] and thicker than another smaller and thinner, it makes a higher sound if its magnitude is proportionally greater than its thickness.

be more interested in their quantities, and Aristotle (10^{a}20–21) specifically refers to relative positions, e.g. harmonics.

[55] Penetration of the material itself. Plato (Pl. *Tim.* 67B) and Aristotle (*De an.* 420^{a}26) did not see this causal relationship between speed and highness; cf. Porph. 45.24–49.26.

[56] Ptolemy has been overly generous with his material examples. The reader should notice, however, that Ptolemy intentionally avoids for the most part giving any details about the strings themselves, for it is with strings of equal thickness and density that he will do his subsequent analyses and computations. The concept of tension he introduces in the next sentence (8.2). This provides Ptolemy with the step he needs from the quality of material to the quantity of tension and string lengths. It was not until the seventeenth century that the relationship between tension and frequency in a stretched string was satisfactorily explained.

[57] Levin, "*Plege*" (1980) 218, uses the term 'elasticity,' although I prefer various forms of 'tension' (i.e. τείνω); cf. Düring 150. I have benefitted much from Levin's careful translation of this section.

[58] Aristotle wrote a treatise on the matter; cf. *De audil.* 803^{b}18, as cited in Porph. 51.1n. One should look as well at the extensive passage in Porph. 67.15–77.18, which Jan (*Musici scriptores graeci* (Leipzig 1895) 53 and 135) attributed to Heracleides Ponticus. Düring fails to see why Jan or Ross deny Aristotelian authenticity.

[59] Alexanderson 8 (8.10) corrects Düring's orthography here.

[8.12] For sound is a continuous tuning of air which, after it is taken around inside the objects making the beatings, follows through to the outside and therefore, according to how much force there might be in the greater tension of those objects through which the beats are made, is made both smaller and higher.[60]

[8.15] Therefore the difference between highness and lowness in sounds seems to be a species of quantity and more a result of the inequality of the distances from what is beaten to what is beating. [8.18] For it[61] quite evidently exists by the quantity of these distances, with the highness following upon the smaller distances on account of the great force created by its nearness, and the lowness upon the greater distances on account of its faintness, created by its being farther away. The sounds are therefore in inverse relationship with the distances. As the greater distance from the origin of the sound stands in relationship to the smaller, so the sound from the smaller stands in relationship to the larger, just as in weighing scales: as the greater distance from the weight moves toward the smaller, so the scale moves from the smaller to that of the larger.

[8.25] And this is clear, evident in the sounds caused by some length of strings, for instance, or of auloi or pipes. When the other factors remain the same, the sounds will be without exception higher in strings where the sounds are produced by smaller rather than greater distances from the bridges,[62] [9.2] and in auloi, where the sound exits closer to the mouthpiece,[63] that is, that which beats the air, rather than falling out through the farther holes, and in the windpipe, where the sounds originate higher and nearer to that which is beaten than those closer to the bottom. [9.6] The human windpipe can be compared to a natural aulos, for in one aspect alone do they differ. With auloi the place of the beater remains constant, while that being beaten moves nearer or farther from the beater as a result of the manipulation of the holes. [9.9] With the windpipe it is the reverse since the location of the thing being beaten remains the same while that of the beater

[60] Some manuscripts have "more rapid and higher." For the definition of sound in other ancient treatises, cf. H. B. Gottschalk, "The *De audibilibus* and Peripatetic Acoustics," *Hermes* 96 (1968) 440f., Düring 151, and Michaelides, q.v. 'Sound.' Porph. (51.27) reminds us that a bull lows at a pitch higher than that of a cow because of the greater tension in its trachea. Arist. *Hist. An.* 4.9 (535^{a}28–536^{b}23) discusses voice and animal sounds, whereafter, at 538^{b}14, in his discussion of sexual differences, he then analyzes the higher pitch of the cow, the lower of the bull. See *Harm.* 10.11.

[61] I.e. pitch; cf. Levin "*Plege*" (1980) 215. This passage was adapted loosely by Bry. I.1 (68.8–11) and Pach. (100.27–31).

[62] For the term for 'bridge,' see Szabó 116.

[63] Levin, "*Plege*" (1980) 213, n. 28, answers K. Schlesinger's organological comments on this passage. The manuscripts offer the synonym *epiglossis* for this part of the instrument. Cf. Düring, *Ptolemaios und Porphyrios* 151. On aulos terminology, see Heinz Becker, *Zur Entwicklungs-geschichte der antiken und mittelalterlichen Rohrblattinstrumente*, Schriftenreihe des Musik-wissenschaftlichen Instituts der Universität Hamburg, Band 4 (Hamburg 1966). Porph. (55.15– 57.27) on this passage quotes extensively from Archytas fr. 1 (Diels^6 I.431–35 (=47.B1)), and later (61.17f.) from Theophrastus' lost *de Musica*.

moves nearer to and further from the thing beaten, for that which controls us is gifted with natural musical ability and amazingly and at the same time easily finds and seizes upon, [9.13] as if using bridges, the places in the windpipes from which the distances from the outside air are proportional to the differences forming the different sounds.

I.4 - *On Notes and Their Differences*

[9.16] How highness and lowness of sound[64] exist and that they are some species of quantity have been described in the last chapter. What we must consider in addition is how their increases, just as those of magnitude, happen to be boundless potentially but in reality bounded, and that there are two limits,[65] one particular to the sounds themselves, the other to hearing, which is broader.[66] [9.21] Those things which produce sounds vary increasingly in their construction,[67] but even if the distances in each were to differ from the lowest to the highest by no significant amount, both their boundaries[68] would differ much in many ways, some toward the lower boundary, some toward the higher. [9.25] But our hearing perceives the lowest of the lower and the highest of the higher however much we consider increasing such distances when we make our instruments.

[9.29] Once this has been established, it next becomes necessary to specify that some sounds are isotonic,[69] some anisotonic. The isotonic sounds do not change their tone, the anisotonic do. [10.2] With the former the so-called 'tone,'[70] understood as one species of tuning, would be the

---

[64] Cf. Porph. 78.9f., who cites Pl. *Phlb.* 24A (with 16C and 17B) on limits and extremes (of heat and cold). The Ibycus fragment to which Porphyry (79.5–.7) refers is now found in D. L. Page, *Poetae melici graeci* (Oxford 1967) 158 (#311).

[65] Szabó 106–107, and Lohmann Musike 11, discuss ὅρος (*horos*) in its geometrical sense of 'limit' or 'endpoint.'

[66] The spectrum of sounds, of course, covers a far greater range than is audible to the human ear. Cf. Aristox. *Harm.* 19.1–20.15.

[67] E.g. in length and thickness, as a manuscript gloss suggests.

[68] Ptolemy uses the two terms for boundaries, πέρας (*peras*) and ὅρος (*horos*), indistinguishably; Szabó 106–10 7, would prefer to understand the former as 'endpoint,' the latter as 'limit.'

[69] The scholiast offers the synonym 'homoeomeric,' i.e. of similar parts, and explains as follows: "They are equal in tuning to themselves from the beginning of itself to the end of itself. Each sound has endpoints and middles, for they are not non-intervalic as is a point. Some of the notes are arranged to taper from beginning to end, and others in the opposite direction. Both are anisotonic." The importance of these matters is attested by Bry. 1.4 (88.18–90.24) and 1.3 (86.2f.).

[70] The scholiast offers this: "A tone is also defined as the interval from tone to tone." This is another (albeit Aristoxenian—cf. Aristox. *Harm.* 46.1–2) definition of the Greek word *tonos*, but Ptolemy means that there is a certain kind of sound which does not vary in highness and lowness; it is always the same pitch. Ptolemy will have several uses for the term isotonic ('equally tuned, of equal tension') throughout the treatise. Here, however, it describes a hypothetical kind of static sound which has no variation in pitch. In the subsequent sentences of

common genus of highness and lowness—the limit of the end and the beginning. [10.5] Of the anisotonic sounds, some are continuous, some discrete. Continuous sounds have their places of transition into one another obscured, or they are sounds of which no isotonic part at all occurs in a perceivable distance, again like the colors of the rainbow. [10.8] Such sounds blend together during tightening and loosening movements, ceasing at the lower end with the lowing of cows[71] and at the upper end with the howling of wolves.

[10.11] Discrete sounds are those which have their places of transition evident whenever their isotonic parts remain at a perceivable interval, like the noticeable juxtaposition of unmixed, unblended colors.[72] [10.14] Continuous sounds are foreign to harmonic studies since they produce not at all one and the same thing and are thus able to be contained by neither boundary nor ratio, contrary to the specific aims of the science. [10.16] Discrete sounds are, on the other hand, germane to harmonic studies. They are limited by the boundaries of isotonic notes and measured by the arrangement of their differences.[73]

[10.18] And now let us call such sounds 'notes,' since a note is a sound which has one and the same tone.[74] Wherefore also each one alone is irrational,[75] for it is one and not different from itself, while a ratio is in

---

the text isotonic notes are a subspecies of anisotonic notes; cf. 10.12. Not understanding the multiplicity of the term led Schoenberger and Düring into difficulties; cf. Düring, *Ptolemaios und Porphyrios* 173 and 192 (*ad* 27.10).

[71] This term (βουκονισμοί) attracted the interest of Düring (1934) 175, citing Maximilian Hermann Vetter, *Additamenta ad H. Stephani thesaurum ex musicis graecis excerpta* (Programm-Zwickau 1867).

[72] Cf. Boethius *De mus.* 5.5. Ptolemy's differentiation between continuous and discrete sounds stems from a long peripatetic tradition that includes the work of Aristoxenus (*Harm.* 8.5–9.4 and 19.1–20.14) and Cleonides (180.11–181.11 Jan), and derives ultimately from Books 3 and 4 of Aristotle's *Physics*, as described in Crocker, "Aristoxenus" 99–100. If the *Optics* is genuine, it is Ptolemy who discusses the same issue of continuity (*color continuus* and *color terminabilis*) at 15.17 (Lejeune).

[73] Szabó 117, n. 43, discusses the Philolaus fragment from Porphyry (91.4–92.8 (= Diels-Kranz[6] 1.405 (44.25)), which introduces the concept of 'differences, excesses' between intervals.

[74] A note therefore turns out to be an isotonic structure. Porph. (86.1–87.19) offers the only *locus parallelus*. See also, Barbera, *Pythagorean Mathematics* 298–99.

[75] Szabó 33–36, following K. von Fritz, "The Discovery of Incommensurability by Hippasus of Metapontum," *Annals of Mathematics* 46 (1945) 242–64, following H. Hasse and H. Scholz, eds., *Zeno and the Discovery of Incommensurables in Greek Mathematics* (Charlottenburg 1928 (reprint: New York 1976)), discusses irrational numbers, the origins of incommensurability, and the Theaetetus problem; *contra*, R. Falus, "*Harmonike phanere*" *Acta Antiqua Academiae Scientiarum Hungaricae* 29 (1972) 20 –35, and Paul-Henri Michel, *De Pythagore a Euclide* (Paris 1950) 474–75. Iamblichus (*Vit. Pyth.* 88 (=52.2-9 (Deubner) and Diels-Kranz[6] 1.108.4)) records the legend that Hippasus of Metapontum was drowned for revealing the secret of the irrational. Von Fritz (242, n. 2) reports a personal letter from Otto Neugebauer in which he opines that it was Archytas in the fourth century who should be credited with its discovery. Cf. Burkert, *Lore and Science* 386, who cites Pl. *Resp.* 546B [prefaced by 534D].

relation to something and of two main[76] parts; [10.21] but sounds in relation to each other, when they are anisotonic, form some ratio from the quantity of differences. In these now appear what is ecmelic and emmelic. [10.23] Emmelic notes are those which happen to be manageable[77] for the hearing when fit together with each other; ecmelic are those which are not so. [10.25] Lastly, they say that they named consonant notes [sym*phônoi*] after the most beautiful of sounds,[78] the voice (*phônê*), which make a similar impression on the hearing,[79] while dissonant notes are those not doing so.[80]

I.5 - *On the Pythagoreans' Positions Concerning the Hypotheses of the Consonances*[81]

[11.1] Perception accepts the consonances, both those called the diatessaron and the diapente, the difference[82] between which is called a whole tone, as well as the diapason, the diapason plus diatessaron,[83] the diapason plus diapente, and the double diapason. For our discussion we are omitting any consonances greater than these.

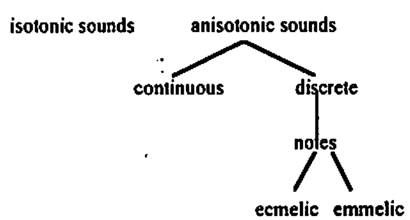
[76] Literally, 'two first numbers,' but not the technical, mathematical 'first' numbers; cf. 23.3.

[77] The manuscripts are divided between 'manageable' (εὔφοροι) and 'euphonic' (εὔφωνοι). The latter results from a correction early on and should be dismissed as the *facilior lectio*. Following Aristox. *Harm.* 37.8ff. ((da Rios); 29.10 (Meibom)), Düring ((1934) 174) subdivides the emmelic with consonant and dissonant, but it is not without risk to apply a fourth-century B.C. musicologist's terms to a second-century A.D. treatise. Ptolemy does not directly subdivide the emmelic here, or, if he does, he normally lists the emmelic as third after homophones and consonances, e.g. 15.3–10; but cf. 16.12f.

[78] Ptolemy does not identify the authors of this statement.

[79] Ptolemy recognizes the word φωνή (*phone* - 'voice') in the Greek word for 'consonance' (συμφωνία - *symphonia*). See here Karl von Jan, *Bericht über griechische Musik und Musiker von 1884–1899* (Strassburg 1900) 48.

[80]



[81] This entire chapter is translated (into German), given reference to Euclid *El.* 8.7–8, and discussed by van der Waerden, "Harmonielehre" 166–70. For the locus parallelus, see Bry. II.1 (126).

[82] Wallis (9) distinguishes properly between 'difference' and 'excess,' the former technically referring to the result of division, not subtraction. Ptolemy is not entirely consistent, Wallis is right to say, in distinguishing between these two terms, but cf. Porph. 91.3f. for the many earlier definitions of 'interval.'

[83] The hendecachord (schol.).

[11.5] The Pythagoreans in their calculations left only the diapason plus diatessaron off this list. This was in accord with their own precepts, which the leaders of this school devised from the following line of thinking: [11.8] Taking a most suitable premise for their method, wherein the equal numbers are associated with the isotonic notes[84] and the unequal numbers with the anisotonic notes,[85] they proposed that since of the anisotonic notes there are two prime species in relation to each other—the consonances and dissonances—and since that of the consonances is more beautiful, so of the unequal numbers as well there are two first differences in ratios. [11.13] The one kind is that called superpartient, or number to number.[86] The other consists of the superparticular and the multiple; the former is better than the latter because of the simplicity of its ratios, for the excess in the superparticular ratio is a simple part[87] in it, while in the multiple it is the smaller part of the greater number.[88]

[11.18] On account of this they apply the superparticular and multiple ratios to the consonances. That of the diapason they fit with the duple ratio (2:1),[89] and that of the diapente with the sesquialter ratio (3:2),[90] and that of the diatessaron with the sesquitertial (4:3).[91]

[84] Equal numbers were the most basic of the six types of ratios recognized by, among others, the Pythagoreans—the equal (3:3), multiple (3:1), superparticular [or 'epimore' in transliteration] (3:2), multiple-superparticular (7:3), superpartient [or 'epimere' in transliteration] (5:3), and multiple-superpartient (8:3). Equal numbers represent simply the ratios of the sides of squares. For greater detail, see Crocker, "Pythagorean Mathematics" 191.

Isotonic notes, i.e. singable notes not in relation to each other, a theoretical monophony, are calculated in equal numbers, that is, 1:1, 2:2, and the like. At Porph. 92.1f, 6:3 and 2:1 have the same ratio but with "unequal differences." Cf. Szabó 140.

[85] These anisotonic notes represent the ordinary notes of music and music theory, and they are represented by unequal numbers, i.e. 2:1, 3:2, 4:3, etc., in ratios. That Ptolemy labels them anisotonic does not mean each note has an unfixed pitch. It means they belong to the genre of notes which can be tuned, i.e. harmonized, in relation to one another, thus being distinguished from the theoretical isotonic notes.

[86] Any number to any number, with no specified relationship required.

[87] That is, one unit, a monad, or 'one.'

[88] Since Ptolemy considers only the relationships of one integer to another, this definition of a multiple ratio is not quite as unsatisfying as it at first might seem.

[89] Mountford, "Harmonics" 86, argued that such a ratio should be written as 1:2, not 2:1, because the numbers of the ratio refer to lengths of strings and not to vibrations per second. Bowen, "Minor Sixth" 502, however, has demonstrated that ancient Greek musical ratios should always be expressed as ratios of greater inequality, i.e. 2:1 rather than 1:2, which is the format used throughout this translation, specifically because this method reflects the string-length ratio and not its reciprocal; cf. Llewelyn S. Lloyd and Hugh Boyle, *Intervals, Scales and Temperament* (London 1963) 125. Barbera, *Pythagorean Mathematics* 299, distinguishes between ratio and interval, whereby 3:2 and 2:3 produce the same interval but are not the same *logos*.

[90] On the Greek terms for such superparticular ratios, διπλάσιος (*diplasios* - duple), ἐπίτριτος (*epitritos* - sesquitertian), and ἡμιόλιος (*hemiolios* - sesquialter), see A. Szabó, *Greek Mathematics* 130f. Ptol. *Tetr.* 1.13 (=72–73 (Robbins)) refers to the same two superparticular ratios as the most consonant.

[91] Of the two genera of notes, the realizable anisotonic and the theoretical isotonic, the anisotonic are described by unequal numbers (ratios not reducible to one) and the isotonic are

[11.20] Their approach was rational.[92] Of the consonances the diapason is the most beautiful, and of the ratios the duple is best, the diapason by its being nearly isotonic, the duple ratio by its being the sole ratio to make its remainder equal to its excess. [11.24] In addition, it happens that the diapason is composed from the two, successive first consonances, the diapente and the diatessaron, while the duple ratio is composed of the two, successive first superparticular ratios, the sesquialter (3:2) and the sesquitertian (4:3). [11.27] Moreover, the sesquialter ratio is greater then than the sesquitertian, and then the consonance of the diapente is greater than that of the diatessaron. [11.29] The resulting excess between these, that is the whole tone, is therefore described with the sesquioctave (9:8) ratio, the ratio by which the sesquialter ratio is greater than the sesquitertian.[93]

[12.1] Following these[94] are also the magnitude of the diapason plus diapente and also that of the two diapasons,[95] that is the double diapason. [12.4] They admitted these into the consonances—since it follows that the latter consists of the quadruple ratio, the former of the triple—but not the[96] diapason plus diatessaron, since it forms the ratio 8:3, which is neither superparticular nor multiple.[97]

described by equal numbers (ratios reducible to one). Of the unequal numbers, i.e. ratios, some are superpartient, some superparticular, some multiple. Superpartient describe dissonances while the superparticular and multiple can describe the consonances; [9:8 would be a counterexample]. Of these, the diatessaron and diapente belong to the superparticular ratios, the diapason to the multiple.

[92] Burkert, *Lore and Science* 384, n. 66, qualifies the term Ptolemy uses here.

[93] These last two sentences are incidental to the present argument. Ptolemy probably initiates the parenthesis by pointing out that as 2 is greater than 1, so is the sesquialter greater than the sesquitertian, and then so is the diapente greater than the diatessaron. Then he specifies the size of the difference between them even though this difference, a whole tone (described by a superparticular ratio) is not a consonance. Although this derivation of the whole tone is the kernel of Aristoxenian theory, it was also instrumental for the first Pythagorean theorist for whom we have any reliable information—Philolaus; cf. Edward A. Lippman, "Hellenic Conceptions of Harmony," *JAMS* 16 (1963) 12f.

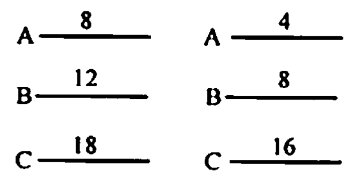
[94] The scholion misinterprets the pronoun reference 'these' (τούτοις) as if were to the Pythagoreans (τοῖς Πυθαγορείοις) instead of to the intervals themselves.

[95] In Greek the word for 'diapason' (διὰ πασῶν) is actually a prepositional phrase meaning 'through all.' The Greek plural of the word(s) διὰ πασῶν is formed therefore not by adding a plural ending but is merely signified by the preceeding, plural definite article (here, τῶν διὰ πασῶν). In English the odd neologism 'diapasons' will have to suffice even if it is rather unGreek.

[96] Düring 176 adds "magnitude of the."

[97] Düring's text here has a misprint (16 for 18 on the last line in the left column), which I have corrected.

This important passage has received much treatment, beginning with Boeth. *De arith.* 1.31 and Bry. III.9 (352–59 (Jonker)); Boethius correctly describes the 8:3 ratio as multiple superpartient. André Barbera, "The Consonant Eleventh and the Expansion of the Musical Tetractys: A Study of Ancient Pythagoreanism," *JMT* 28 (1984) 191–223, offers the most complete analysis of Ptolemy's methodology, sources, and influence here; see also his *Pythagorean Mathematics* 342–44. Cf. Palisca 85 and 60 [where 5.9 - 1.5].



[12.8] Exploring the same matter in this graphic fashion, [98] they said let there be a diapente AB and next to this another diapente BC in such a way that AC is a double diapente. Since the double diapente is not a consonance, and since AC is therefore not a duple ratio, then neither is AB multiple;[99] yet it is a consonance, and the diapente is therefore superparticular. [12.12] In the same way they show[100] also the diatessaron to be a superparticular ratio smaller than the diapente. [12.13] Again they say let there be a diapason AB and next to this another diapason BC so that AC is a double diapason. Therefore, since the double diapason is a consonance, AC is either superparticular or multiple; but it is not superparticular, for there is no middle ratio in a superparticular ratio, so AC is multiple, which means that also AB is multiple and therefore that the diapason is multiple. [12.19] It was clear to them from such reasoning that the diapason is a duple ratio and that within it the diapente is a sesquialter ratio and the diatessaron a sesquitertian. Since of the multiple proportions[101] the duple ratio alone is composed from the two greatest superparticular ratios, which means that two ratios composed from the other superparticular ratios will be smaller than the duple, [12.23] and, since there is no multiple interval smaller than the duple, accordingly, since the whole tone is shown to be a 9:8 ratio, they then declare that the half tone is ecmelic; no other superparticular ratio can be divided by a middle ratio, and superparticular ratios must be emmelic.[102]

1.6 - *That the Pythagoreans did not Investigate About the Consonances Properly*

[13.1] In spite of what the Pythagoreans hypothesize about the consonances, the diapason plus diatessaron is clearly a consonance in every way and

[98] Barbera, *Pythagorean Mathematics* 301–302, offers a critique of Ptolemy's demonstration; the diagrams in Düring are incorrect. Burkert 384 (n. 66), briefly discusses this passage to 12.27. Cf. [Euc.] *SC* 1 (Porph. 99.1–.26), for which see 150.4 (Jan).

[99] There is some manuscript support here for 'multiple' instead of 'duple'; either is acceptable, of course, since the duple ratio is a multiple ratio. Cf. [Euc] *SC* #10.

[100] As in Alexanderson (8–9) and Höeg, review of Düring *Ptolemaios und Porphyrios* 154.

[101] Düring (176) translates as "number," but the technical terms *analogos* and *logos* are thoroughly analyzed. Barbera, *Pythagorean Mathematics* 42 and 92, and Albert Riethmüller, "Logos und Diastema in der griechischen Musiktheorie," *AfM* 42 (1985) 18–20 and 33–36 (who labels *logos* "die spezifisch-griechische Denkform," both address Szabó directly. Cf. Lohmann *Musike* 1–2. St. Augustine *de Musica* 1.12 (16 Knight) renders *logos* as *proportio*.

[102] Cf. Burkert 384, n. 66.

brings shame upon the ratio they applied to it. [13.3] Generally, when the consonance of the diapason, since the notes which comprise it do not differ in function from one note, is joined to one of the other consonances,[103] it keeps its form unchanged, just as, for example, does the number 10 in relation to the lower numbers.[104] [13.7] And if some consonance should be added in the same way at the ends of the diapason, be it at the lower of two ends or at the higher, as it would stand in relation to the nearer of them, so it appears also to stand towards the farther, and it has the same function as that one.[105]

[13.10] The consonances of the diatessaron and diapente are sung in themselves in the nearer position[106] of the diapason; the diatessaron is sung with the diapason and again the diapente is sung[107] with the diapason in the more remote, [13.13] so that in all likelihood the same impact of the hearing takes place with the diatessaron plus diapason as with the diatessaron alone, and with the diapente plus diapason as with the diapente alone. [13.16] Wherefore it surely follows that since the diapente is a consonance, the diapason plus diapente is also a consonance, and that since the diatessaron is a consonance, the diapason plus diatessaron is also a consonance, that the impact of the diapente plus diapason in relation to the diatessaron plus diapason is similar to the impact of the diapente alone in relation to that of the diatessaron, in accordance with what is experienced in a plain experiment.[108]

[13.23] The association of consonance with only these superparticular and multiple ratios and not with others—by 'others' I mean the sesquiquartial (5:4) and the quintuple (5:1), which are of the same form

[103] 'Note' is not included in the original text, but a selection of manuscripts add it in suprascript. One scholion offers "just as the function of one note differs with itself." Cf. Porph. 105.23–27. Similarly, 'consonances' is supplied by various translators, whereas Ptolemy offers only a pronoun.

[104] Ptolemy means here that the diapason creates essentially the same note one octave higher; nothing within a consonance is changed when the the diapason consonance is added to it. Cf. III.2 and [Euc.] *SC* Prop. 7. The reference to the number 10 results from the way the teen numbers 13 and following are named in ancient Greek—by adding ten, e.g. 13 is 'ten plus three.'

[105] The reading in several manuscript families adds 'because' before this final clause, which mitigates the harshness of transition in subject, according to Alexanderson (9). The clause is now understood to refer to the farther extreme, which has the same function as the nearer.

[106] The nearer position refers to that locus which is at the 'bottom' of the diapason, and so when sung in a diapason plus diatessaron (or diapente), the two diatessarons or diapentes are parallel, i.e. homophones, an octave apart.

[107] This verb (ᾄδονται) is replaced by the verb 'to be' (γίνονται) in a number of manuscript versions. Certainly the verb is unique in this Ptolemaic context, for elsewhere (4.9 and 39.14) it refers specifically to singers singing.

[108] Boeth. (*De mus.* 5.10) supplements Ptolemy's arguments concerning the addition of a diapason to either end of the diatessaron and diapente by using the analogy of adding 10 to 2 or subtracting 10 from 12. See also Bry. III.9 (352.16f.). Other and later authors who accept the diapason plus diatessaron as a consonance include Theon of Smyrna, Gaudentius, Chalcidius, Martianus Capella, and Cassiodorus; cf. Barbera, *Pythagorean Mathematics* 300 and 305.

compared to others—created not insignificant difficulties for the Pythagoreans;[109] so did their selecting of consonances by whatever method they chose. [14.2] For taking away a unit from each[110] of the smallest numbers[111] which form their ratios, which they do because of the similarity[112] on both sides, and adding the remaining, dissimilar numbers, where they appeared to be smaller, they said these were more consonant, which is quite ridiculous. [14.6] For a ratio is not particular to only the smallest numbers which make it, but plainly to all which stand in a similar relation to each other, so that it would also be similar among these, at times composed[113] of the smallest amount and at other times of the greatest amount of the dissimilar numbers in the same ratios. [14.10] For if—and this seems most appropriate for our investigation—we substitute the same number in all the numerators—6, for instance, and divide the denominator by those equal to it, instead of the similarity, we would then estimate the remainders as partaking of the dissimilar. [14.14] In the duple ratio these will be 6, in the sesquialter 3, and in the sesquitertian 2; and there will be more dissimilar in the more consonant.[114]

[14.16] Completely in keeping with this methodology, the diapason plus diapente is shown to be the more consonant of those remaining after the diapason, with the two dissimilarities left in it, but more in all the others, three, for instance, in the diapente and in the double diapason, with the diapason plus diapente most clearly established as more consonant by far

[109] The scholion here correctly assumes Ptolemy's pronoun to refer to the Pythagoreans; cf. 13.5. Porph. (107.15) supplies support from Archytas (=Diels-Kranz[6] I.429 (47.17)) and Didymus.

[110] The manuscripts are divided between "a unit from each" and "each unit"; cf. Burkert, *Lore and Science* 383, n. 63 and 386, n. 77.

[111] 'First numbers' in Greek, or "bases" (scholion). Cf. Pl. *Resp.* 546C and Nicom. *Ar.* 1.21 and 2.19; Martin Luther D'Ooge, *Nicomachus of Gerasa. Introduction to Arithmetic* (New York 1926) 221–2 and 259–62, describes them as "antecedents."

[112] On the term 'similarity,' see Szabó 172.

[113] Düring's infinitive συνίστασθαι can stand; Alexanderson (9) proposes a participle συνιστάντων, which might be preferable, except the verb is so colorless that it is difficult to determine which conveys the better meaning.

[114] A sentence which has left a tradition of confusion in previous translations. If one subtracts one integer from both the numerator and denominator, i.e. both "sides" of the ratio in the ancient sense, e.g. 4/3, the resultant fraction composed of smaller integers, e.g. 3/2, will be not only larger but also more consonant.

Ptolemy's irregular use of the terms 'similarity' and 'dissimilar' have caused much of the confusion. Wallis (13), Schoenberger (77), and Düring (179–80) believe that the "base" plus a fractional equivalent of 1, the similitude, equal the dissimilarity. This is the technical and inverted way of expressing something basic to Pythagorean harmonics, that one is the most perfect number and that each number farther removed is less and less perfect. 1:1 is therefore perfect, with 2:1, 3:2, and 4:3 decreasingly so. But when Ptolemy takes this theory to task in the following sentence, he does not dispute the notion that 2:1 is more consonant than 3:2 but that 2:1 is more consonant than 4:2 or 8:4 or 16:8, which is indeed absurd.

than each of these[115] since the diapente is both simpler than the diapason plus diapente and more non-composite[116] as if of more pure consonance.[117] [14.23] The double diapason stands in this relationship with the diapente plus diapason—that is as the quadruple ratio to the triple, as the diapason alone to the diapente alone—that is as the duple ratio to the sesquialter, for if of a single number both the triple and quadruple and again sesquialter and duple ratios are taken, the quadruple makes a sesquitertian ratio with the triple as does the duple with the sesquialter, so that as the diapason is more consonant than the diapente, so is the double diapason more consonant than the diapason plus diapente.[118]

I.7 - *How the Ratios of the Consonances Could Be More Properly Defined*

[15.3] Such errors must be attributed not to the function of proportion but to those who hypothesize incorrectly about it. One must attempt to search out the true and natural ratios with anisotonic, separated notes divided primarily into three forms: first the homophones on account of their excellence, second the consonances, and third the emmelic.[119] [15.8] Clearly both the diapason and the double diapason differ from the other consonances just as the former differ from the emmelic, since these ought more properly to be called homophones.

[15.10] Homophones are defined by us as those which make an impact on our hearing in the conjunction[120] of one sound, for example, diapasons and intervals composed from them. Consonances are nearest the homophones, for example, diatessarons and diapentes and intervals composed from them and their homophones. [15.14] Emmelic are those nearest the consonances, for example, the whole tones and the rest of the

[115] For the diapason plus diapente, 3:1 [the base] less the "similitude" 1:1 becomes (3 –1) + (1–1) = 2, that is two dissimilarities; for the diapente, 3:2 = (3 - 1) + (2 -1) = 3; for the double diapason, (4 - 1) + (1 - 1) = 3.

[116] For the term ἀσύνθετον ('non-composite'), cf. Düring 180, Aristox. *Harm.* 21.21–.22, Aristides Quintilianus 10.20, Cleonides 188.3–.8, and Michaelides 37.

[117] The discussion of the diatessaron and diapente ends with "of more pure consonance," and the next clause will discuss the double diapason and diapason plus diapente. I have therefore put a full stop before the sentence, and then joined to it the causal ('for') clauses. Düring continues the sentence; cf. Alexanderson 9. The reader must understand that Ptolemy uses long, complex sentences; many contain five or six dependent clauses. Some of these dependent clauses consist of only unspecified pronouns and participles, and the confusion arises when one has to determine whether a dependent clause belongs to the previous or ensuing independent clause.

[118] That is, as 4:1 is more consonant than 3:1, so is 2:1 more consonant than 3:2.

[119] Cf. Boeth. *De mus.* 5.11.

[120] The scholiast offers "concurrence" as a synonym to 'conjunction,' but Ptolemy (17.17) elsewhere seems to use 'concurrence' (σύγκρουσις) to refer to percussion or (67.7) trilling. For homophones, cf. Düring 179 (with an uncorrected misprint), where he refers to the term 'antiphones' offered by Theon 2.5 (33–34 (Lawlor)), Gaud. 347.26, and Bry. 98.27–100.21.

intervals of that sort. The homophones are therefore in some way composed of consonances and the consonances of the emmelic.[121]

[15.18] Once these definitions have been established, we must move into the discussion pursuant to them by taking the same initial approach as the Pythagoreans, that is, assigning the equal numbers to the isotonic notes and the unequal numbers to the anisotonic notes. All of this is self evident.[122]

[15.22] Therefore, following this approach we measure the differences in anisotonic notes by their proximity to equality. It is immediately clear that the duple ratio is nearest this equality since its remainder is equal to and the same as its excess. [15.26] Of the homophones, most suitable and beautiful is the diapason, so the duple ratio accommodates it while the double duple ratio, that is, the quadruple ratio, clearly accommodates the double diapason, and whatever generally is measured[123] by the diapason and the duple ratio.

[15.29] Again, after the duple ratios, closest to equality would be those divided almost in two, that is, the sesquialter and sesquitertian; for what is almost in half is close to that of two equal parts. [16.2] After the homophones, the first of the consonances are those which divide the diapason nearly in two, that is, both the diapente and diatessaron, so that the diapente is, again, located by the sesquialter ratio, and the diatessaron by the sesquitertian. [16.6] Second are those which are composed from each of the first consonances with the first of the homophones—the diapason plus diapente with its ratio composed of the duple plus the sesquialter, i.e. the triple ratio, and the diapason plus diatessaron with its ratio composed of the double and sesquitertian, i.e. the 8:3 ratio. [16.10] Now, that this interval is not superparticular nor multiple does not trouble us at all since we proposed nothing of the sort beforehand.

[16.12] After the sesquitertian ratio next closest to equality would be those forming a ratio with symmetrical[124] differences, that is those smaller than these superparticulars and inferior to the consonances in respect to excellence, the emmelic intervals, for example, the whole tone and whatever makes up the smallest of the consonances, so that those superparticular ratios less than the sesquitertian fit them. [16.17] For the same reason, those ratios making their divisions nearly in half are more emmelic, as are those differences limiting greater, simpler parts than their excesses, since these,

[121] Only insofar as homophones are generally larger than the consonances which are in turn larger than emmelic notes. Ptolemy will of course allow that a diapason consists of the two consonances, the diapente plus diatessaron, but those are not composed from an equally divisible number of whole tones alone. He offers part of a definition at 16.12ff.

[122] Cf. 11.5ff.

[123] The scholiast suggests "the multiple numbers" here; Porph. (115.12–116.11) refers us to Pl. *Tim.* 35B.

[124] In Greek the term 'symmetrical' (συμμέτροις) can mean 'proportionate, suitable,' the meaning Ptolemy most often employs. Because the word is so well established in English, I have left it in the translation as a veritable transliteration.

too, are nearer equality just as is primarily the half, and then the third and each of the succeeding ratios.[126]

[16.21] In a word, the homophones would come from both the first multiple ratio and those measured from it. Consonances are the first two superparticular ratios and those composed from them plus the homophones, while the emmelic are those superparticular ratios after the sesquitertian.[127] [16.25] So the characteristic ratio of each of the homophones and consonances has been stated, while of the emmelic the sesquioctaval [9:8] ratio of the whole tone has been demonstrated through the difference between the two first superparticular and consonant ratios.

[16.28] Those remaining will have their particular divisions in their individually appropriate places. Now it would be well to show evidence for those already mentioned so that we could confirm without any doubt that which is agreed upon by the perception.[128]

I.8 - *In What Way the Ratios of the Consonances Will Be Demonstrated Confidently via the Monochord Canon*

[16.32] We must give up examining our subject by means of auloi and syrinxes or weights attached to strings,[129] for such demonstrations cannot bring us to accurate conclusions.

[17.1] Instead they invite slander upon their undertakings,[130] for the auloi and syrinxes[131] are difficult to examine, and even if their

[126] Cf. Barbera, *Pythagorean Mathematics* 292–93, on Ptolemy's Pythagorean dependency.

[127] That is, superparticular ratios smaller than the diatessaron but composed of larger integers, e.g. 5:4.

[128] These last two sentences serve to introduce the beginning of chapter 8.

[129] The "weights attached to strings" makes an unheralded reference to the very famous, thoroughly promulgated story about Pythagoras and the anvil at the blacksmith's shop which Nicomachus *Ench.* 245.19f. and a number of subsequent authors preserve and discuss with varying degrees of acoustical understanding. B. van der Waerden, "Harmonielehre" 170f., gives the account some historical perspective. Levin, *Nicomachus*, 67–74, gives the fullest, most logical account in proving that the story originates with Nicomachus. The acoustics of the story, that Pythagoras heard various consonances produced by an anvil struck with different sized hammers, is of course acoustically innacurate.

Despite its inaccuracy, the story can be traced from Nicomachus to Ptolemy, Gaudentius, Iamblichus, Censorinus, Diogenes Laertius, Chalcidius, for which, see J. H. Waszink, *Timaeus: a Calcidio translatus commentarioque instructus*, Vol. IV of Raymond Klibansky, ed., *Plato Latinus* (London 1975), Macrobius, Fulgentius, Boethius (for which, see Bower, "Boethius and Nicomachus" 37– 38), and Isidore. The charm of the story still has its influence; Lucas N. H. Bunt, P. Jones, and J. Bedient, *The Historical Roots of Elementary Mathematics* (Englewood Cliffs NJ 1976) 72, label it "the oldest example in history of a natural law found empirically." Mary Lefkowitz, *The Lives of the Greek Poets* (Baltimore 1981) viii–ix, demonstrates that the lives of ancient poets, i.e. ancient biographies, are all essentially fictional, so the literary legends surrounding Pythagoras do not need to be and in fact would be unique if they were historically accurate.

inconsistencies are corrected, their limits,[132] which are necessary for comparing lengths, are still only vaguely established. [17.5] In the majority of wind instruments there is in addition generally some irregularity as well in the flow of air. [17.7] With weights attached to strings, the strings are not kept unchanged in relation to each other. After all, it is difficult to do so for each string in relation to itself, so no longer will the ratios of the weights be able to fit those sounds realized by using them since with the same tension denser and thinner strings make higher notes.[133] [17.12] Even more, if someone assumed these things to be possible and also assumed equal length for the strings, the greater weight by its greater tension will increase the length of the string fastened to it [134] and increase its density as well, so that even in this arrangement there would be some difference in the sounds not in keeping with the ratios of their weights.

[17.16] Similar difficulties occur with sounds produced by percussion, sounds which one sees produced by hammers of unequal weight or disks and by full or empty vessels,[135] for it is indeed a great task to observe in all of these constancy in matter and form.[136]

[130] The phrase 'invite slander upon their undertakings' (διαβολῆς μᾶλλον ἐμποιεῖν ἀφορμὰς τοῖς πειρωμένοις) is characteristically vague. Levin, "*Plege*," 220–27, while arguing brilliantly for "slander on the investigators," thinks Ptolemy is attacking Nicomachus. Ptolemy does not seem to be associating these experiments with a particular school, however; elsewhere his frequent polemics at his multiple music theoretical opponents are well marked. On the other hand, the ancients do seem to use this particular Greek participle (πειρωμένοις) most comfortably in the middle voice. Burkert, *Lore and Science* 376, argues for "to the correct theory."

[131] See da Rios 31 (*trad.*); Barbera, *Pythagorean Mathematics* 313–14, surmises that Ptolemy experimented with the syrinx and panpipes. See also, Burkert 374, who also comments on auloi, incorrectly stating—following Aristox. *Harm.* 37.25f. (= 47.6f. [da Rios])—that their borings were not placed according to numerical ratios.

[132] Cf. Szabó 106–107.

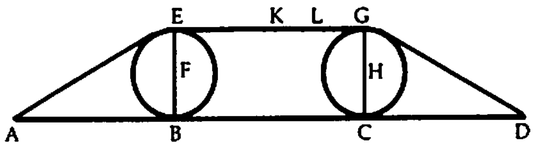
[133] Barbera, *Pythagorean Mathematics* 309–313, includes a translation of 27.14 –28.3 and a proof using t (tension), d (diameter), and p (frequency); he suggests that Ptolemy may have in fact known the very sophisticated inverse relationship between pitch/tension and pitch/diameter, but Ptolemy only approached discussing it. Nowhere does Ptolemy recognize the importance of the square root here, only that p:t::d:p.

[134] Alexanderson (9) argues for an emendation here because of the lack of evidence that the verb 'to fasten' requires a dative (αὐτῷ). The meaning is clear, however.

[135] With reference to the experiment most commonly attributed to Hippasus of Metapontum (cf. the scholium to Pl. *Phd.* 108D, translated in Jonathan Barnes, *Early Greek Philosophy* (New York 1987) 214), but originally performed by Lasus of Hermione, the teacher of Pindar; cf. Theon of Smyrna 39–40 (Lawlor), [Arist.] *Prob.* 19.50, Levin, *Nicomachus* 71–72, and Burkert, *Lore and Science* 377–78. Francois Lasserre, *The Birth of Mathematics in the Age of Plato* (Larchmont NY 1964) 172, discusses the improbability of the historicity of this experiment since squares of the required ratios are needed; no doubt the acoustical properties were discovered before these myths were invented. Jorgen Raasted, "A Neglected Version of the Anecdote About Pythagoras' Hammer Experiment," *Cahiers d'Institut du moyen age grec et latin* 31a (1979) 1–9, ingeniously proposes an early textual confusion in which the word *sphairai* ('bowls' or 'discs') was confused for *sphurai* ('hammers'). If this was in fact what happened between the sixth century B.C. and the second century A.D., the likelihood increases that there was in fact such an experiment performed by either Pythagoras himself or, more

[17.20] The string which is stretched across the canon, as it is called, will show us both more accurately and more easily the ratios of the consonances, not, of course, in random tuning, but first with some examination of the irregularity that would occur with the apparatus, [17.24] and then of the proper placing of the limits which are chosen so that the limits of those parts to be played in them,[137] by which the whole length is defined, might have their proper and clear points of origin.

[17.27] Imagine a canon along the straight line ABCD. At its ends on each side are bridges[138] equal and alike forming as much as possible spherical surfaces under the strings.[139] [17.29] F marks the center of the aforementioned surface on the line BE, and H similarly marks the center on CG, while the signs E and G are understood to mark the bisections of the two circular surfaces.



[18.2] Let the bridges be placed such that the lines through the bisecting E and G and through the centering F and H, that is, the lines EFB and GHC, be perpendicular to the line ABCD. If we stretch across a string of equal measure from A and D, that is AEGD, it will be parallel to ABCD since the bridges have equal height.[140] [18.7] And the beginning of the area to be played will be taken at E and G. The points of contact with the circular surfaces are made here since EFB and GHC are perpendicular to it as well.

likely, Hippasus; cf. Kurt von Fritz, "The Discovery of Incommensurability By Hippasus of Metapontum," *Annals of Mathematics* 46 (1945) 245. Raasted (1) and Georg Kinsky, *A History of Music in Pictures* (New York 1951) 108.2, contain the same illustrations of this experiment which were to be found in Gaffurio.

136 That is, material consistency and the arrangement of the sound-producing apparatus. Düring (181) makes the worthwhile reference to Aristox. *Harm.* 41.20 (=52.1–.8 da Rios), where he, too, criticizes the use of wind instruments.

137 The Greek term *apopsalmata*, here rendered as "those parts to be played in them," can refer either to the end of the strings in general or specifically the part which is played. This ambiguity often causes confusion or at best uncertainty; cf. 87.5 (with n. 35).

138 Ptolemy uses no less than five different terms for 'bridge.' This particular term (μεγάς) refers to a fixed, external bridge. The other four terms are not so clearly distinguishable.

139 Cf. Boeth. *De mus.* 5.12 and 4.18; and Wallis 18. At 'spherical' the scholion says, "We should recognize that Ptolemy speaks of 'a spherical body' incorrectly here because of both its length and bore. In the same context he speaks earlier of spheres and disks." Porphyry (121.24) suggests the singular "string," which Alexanderson (9) uses to focus on the use of the monochord. But the present sentence concerns the bridges, and the generic "strings" could in that case be the authentic reading.

140 Alexanderson (9), citing Porphyry 121.19–26 and Düring [1934] 182, says the bridges need to be cylindrical up to the height of F and H, then hemispherical on the upper surfaces.

[18.9] Fitting a calibrator[141] to the string and transferring the length EG to it so that we can make our measurements more easily, first to the middle point of the whole length, K, and then to the middle point of the half, L, we will set very thin and smooth supports, or, by Zeus, other bridges, a little higher than the others[142] but unchanged in position, equality, and similarity at the middle line of the curve.[143] [18.16] This support will be under the same middle point of the canon or again under the middle of the half so that if EK should be found to be an equal part of the string with KG, and also KL with LG, it will be clear to us that the string is of unchangeable consistency.

[18.19] Otherwise, let us change our examination to another part or to another string until the following is maintained, that is, an unchanging expanse which is similar, of equal ratio and length, and having one tension. [18.22] Then after some such thing is established and the calibrator is divided by the ratios appropriate to the consonances, we will find from the comparison at each portion marked by the bridge differences in their notes which accord very accurately with the hearing.

[19.1] For if the distance EK is assumed to be four parts and KG three, the notes from each of them will make the consonance of the diatessaron by the sesquitertian ratio. If the distance EK is assumed to be three and KG two, the notes from each of them will make the consonance of the diapente by the sesquialter ratio. [19.4] Again, if the whole length is divided so that EK consists of two parts and KG of one, their sounds[144] will make the homophone of the diapason by the double ratio. [19.8] If EK is composed of eight parts and KG of three, the consonance will be the diapason plus diatessaron since the ratio is 8:3.[145] [19.11] If EK consists of three parts and KG of the same one, the consonance will be the diapente plus diapason in the triple ratio. [19.13] If EK is composed of four parts and KG of the same one, there will be the double diapason homophone in the quadruple ratio.

141 The word κανόνιον - *kanonion* (cf. κανών - *kanon*) means a 'monochord, calibrator.'

142 The 'others' are the fixed bridges.

143 The "middle line of the curve" refers to the hemisphere of the movable bridge where contact will be made between bridge and string (the ἀποψάλματα - *apopsalmata*). Significant is the triad of qualities to be found in the bridges—position, equality (or uniformity), and similarity—especially at the point on the bridge where contact will be made between bridge and string (the *apopsalmata*). Cf. 17.28 and 18.2, Porphyry 122.15–19, and Alexanderson 9–10.

144 Produced from each endpoint or limit.

145 Reading with the supplement of Höeg, review of Düring, 656. Bower, "Boethius and Nicomachus" 38, cites the parallel with Boeth. *De mus.* 2.27.

1.9 - *That the Aristoxenians Incorrectly Calculated the Consonances by the Intervals and Not by the Notes*[146]

[19.16] In light of these demonstrations, one must criticize the Pythagoreans not for their discovery of the ratios in the consonances, since they were correct, but for their inquiry into the causes wherein they failed to achieve their goal. [19.18] One must on the other hand criticize the Aristoxenians since neither do they agree that those ratios are clear, nor when they reject them do they search for more secure ones despite their promise of a theoretical study of music.[147]

[20.2] It is necessary for them to agree that such conditions affect the hearing because of the relationships of notes to each other. And it is necessary to agree with them as well that of the same perceptions the

[146] The scholion, no doubt written by Nicephoros Gregoras or Isaac Argyros (cf. Mathiesen, *Ancient Greek Music Theory*, 525), attached to this chapter says, "The Aristoxenians did not define the differences between notes, neither did they define the differences found between various whole tones. They concentrated on the differences between intervals, and not very well, nor as it was necessary for them to do, but rather irregularly. When asked what a whole tone was, they said that a whole tone was that which was the difference between the diapente consonance and the diatessaron; and again, when asked as well what the difference between a diapente and a diatessaron was, neither did they render a definition of that because this is something, as they said, that involves two components, such as is the diatessaron with five. Again, asked next what are these five in relation to the diatessaron consonance, they said that they are just as the diapason to twelve. In all of these definitions they make reference to something else. They do this because they established improperly the diatessaron consonance of two and one-half tones. From that point they say that the one whole tone consists of two numbers, the other whole tone of two numbers, but the hemitone of the monad. From there they say the diatesssaron consonance consists of five monads, that the diapente exceeds the diatessaron by a whole tone, and that by the same reckoning the diapente necessarily consists of seven monads. This means the diapente consonance consists of three and one-half wholetones. Since the diapason is composed of the diatessaron and the diapente, they propose the diatessaron to have five monads, the diapente seven. Consequently, it follows that the diapason will have twelve."

Malcolm Litchfield, "Aristoxenus and Empiricism: A Reevaluation Based on His Theories," *The Journal of Music Theory* 32 (1988) 51–73, esp. 58, offers a general defense of Aristoxenus.

[147] Ptolemy uses the term μουσικῇ here in the phrase "the theoretical study of music," so this is not the same as the present study of harmonics. Cf. Jamie C. Kassler, "The 'Science' of Music to 1830," *Archives internationales d'histoire des sciences* 30 (1980) 113, who offers a general definition of *mousike* as the vocal art of declamation which incorporates and dominates the trivium's grammar, rhetoric, and poetry.

In considering the Aristoxenian tradition today, it is worthwhile to emphasize 1) that the extant *Elements* does not contain one complete work but parts of several; 2) that Aristoxenus himself (42.8–43.6) actually allows for the use of reason and hearing in tandem; 3) that half a millennium separates Ptolemy from Aristoxenus; and 4) that Ptolemy's attacks fall more poignantly on the extant works of Aristoxenus' successors; cf. Porph. 124.14, who cites Aristox. [*Harm.* 45.3 (55.17 [-56.19] da Rios)]. Annie Bélis, "Les 'nuances' dans le Traite d'Harmonique d'Aristoxene de Tarente," *REG* 95 (1982) 54–73, discusses the 'shades' of Aristoxenus, noting the discrepancy at 24.16–31 and 24.31–25.11 (Meibom; 31.5–.16 and 35.16–32.7 da Rios).

differences are defined and the same.[148] [20.5] But how the two formative notes relate to each other in each species they neither say nor inquire about, but, just as if these notes were immaterial and the intervals between material,[149] they compare only the distances of the species and so appear to be working somewhat with number and ratio. But it is entirely to the contrary.

[20.9] First, they do not define by their method what each species is in itself. For example, when someone asks what a 'whole tone' is, we should say that it is the difference of two notes which make up the 9:8 (sesquioctave) ratio, but they immediately digress to some other indefinable subject, such as when they say the whole tone is the difference between the diatessaron and diapente.[150] [20.14] And yet if the perceptions wish to tune a whole tone, they do not have need of the diatessaron or some other interval; the perceptions would in themselves be sufficient to define each of these differences.

[20.18] If we examine the size of this so-called difference, they do not show this difference isolated from another but they only would say, if there are two of some thing, as the diatessaron is 5, and this 5 is again as the diapason is 12, and so on until they come to speak of what two the whole tone consists.[151] [20.23] Second, they do not in this way define the

[148] Porph. 125.10 is a bit premature in citing Aristox. *Harm.* 15.25 [20.20 da Rios].

The scholion here says, "They said that they were defined, but just what is defined they do not say. They lead us instead to a different consideration. For just as Plato said that the Ideas were in a place, so does Aristoxenus say that certain Ideas [I omit the additional 'just as' as an error] of all the consonances are in a place to which the intervals of the strings bring them and establish their relationships, contrary to the criterion of the hearing."

[149] Ptolemy finds both inexcusable and incomprehensible the Aristoxenian treatment of intervals. They regard them spatially (essentially dividing the scale by an imaginary unit of measurement—the 1/12 tone) and not as ratios, i.e. differences, of string lengths. Mountford, "Harmonics" 72, shows a bit more tolerance in allowing that the human ear finds it 'not unnatural—no matter how inaccurate' to hear music as if part of a linear progression. R. P. Winnington-Ingram, "Aristoxenus and the Intervals" 195–208, (esp. 195) still provides one of the most thorough explanations and analyses of the Aristoxenian tetrachordal divisions. K. Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *CQ* 27 (1933) 88–91, rejects the suggestion that Aristoxenus' intervals were "tempered" since tempering implies a departure from an established system. If there was an attempt at tempering in ancient Greece, it was by the elusive Harmonicists (cf. Pl. *Resp.* 530C–531C) who divided their scale into twenty-eight consecutive quarter-tone dieses, for whom see Aristox. *Harm.* 28.1–29.1. Aristoxenus did equally subdivide the scale into half tones, although the half and whole tone he then subdivided by quarter, third and eighth tones; cf. 29.11.

[150] Ptolemy criticizes Aristoxenus' methodology as self-contradictory and inconsistent because he first defines an interval --the whole tone interval - as the difference between two notes, then he says the whole tone is the difference between two intervals - the diatessaron and diapente. Cf. 29.19 and Porph. 126.3–22, who cites Aristox. *Harm.* 21.22 [27.15 da Rios].

[151] The scholion explains: "From that point they say that the one whole tone consists of two numbers, the other whole tone of two numbers, but the hemitone of the monad. From there they say the diatesssaron consonance consists of five monads, that the diapente exceeds the diatessaron by a whole tone, and that by the same reckoning the diapente necessarily consists of seven monads. This means the diapente consonance consists of three and one-half whole tones. Since the diapason is composed of the diatessaron and the diapente, they propose the diatessaron

differences since they do not compare them to those things to which they belong. [20.24] There are a limitless number of them gathered at each ratio, but they are not yet defined even though they form the distances. [20.25] And on account of this neither do they observe the distances[152] which make the diapason, for example, in instrument-making, to be the same, but shorter distances are found in the higher tunings.[153]

[20.28] When equal consonances are compared with each other but measured against other boundaries, the distance of the excesses will not always be equal. [21.1] If they are harmonious with the notes higher than them, the distance will be greater than to the others; if they are harmonious with lower notes, the distance will be smaller.[154]

[21.3] Let us assume the distance AB, a diapason in which A is understood to be the higher limit, with two diapentes—one descending from A, that is AC, and the other ascending from B, that is BD. [21.6] The distance AC will be smaller than BD since it falls at the higher tuning, and

BC is a greater excess than AD.

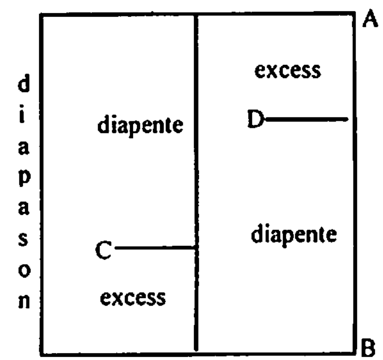
to have five monads, the diapente seven. Consequently, it follows that the diapason will have twelve." Cf. Aristox. *Harm.* 32.6–.10 and Cleon. 192.12–193.2.

[152] Albert Riethmüller, "Logos und Diastema in der griechischen Musiktheorie," *AfM* 42 (1985) 26–27, discusses this passage and the difficulties in interpreting Ptolemy's use of such terms as 'distance' (διάστασις), 'interval' (διάστημα), 'difference' (διαφορά), and 'excess' (ὑπεροχή).

[153] Ptolemy will correct these irregularities when he constructs his helicon at II.2.

[154] Originally Düring printed the Greek text as translated here. In his corrections ([1934] 18) he then reversed 'greater' and 'smaller' as had a number of manuscripts. Then in his commentary (184 *ad* 21.2) he admitted that his correction had been in error; cf. p. 11. Alexanderson (10) agrees, citing the similar confusion in Porph. 127.9. The problem lies in understanding the general sense, for the following line shows clearly how the diapente consonance AC is smaller than the diapente consonance BD while the difference or 'excess' AD is smaller than the excess BC, where A is higher and B lower. Alexanderson (ad loc.) believes the precision of Ptolemy's term 'distance' has created the confusion for the scribes, Düring, and himself. He is confused as to whether 'distance' here refers to the distance of the consonances (i.e. AC and BD) or to the distances of the excesses (i.e. AD and BC) with higher notes.

But no one seems yet to have observed this: the crux of the matter depends on how Ptolemy envisions the relationship between the notes themselves and the notes to which they are harmonious. If A is higher, then a diapente from A [AC] will have a smaller distance than a diapente from a (lower) B. But if A is higher, then a diapente harmonious with A will, I assume, be lower than it ['the others'] and therefore one of a greater distance. Similarly, if B is lower, a diapente harmonious to it and standing relatively higher will have a lesser distance because it is higher. At 20.28–21.1 Ptolemy establishes that he is comparing equal consonances with each other at other endpoints. With this understood, the text should read as printed originally in Düring and as translated here.



[21.9] It would seem altogether most foolish to consider the excesses worthy of some ratio which was not reckoned through those magnitudes which form them, and then to consider the magnitudes worthy of nothing even though it is through them that proportion can be immediately calculated. [21.11] If they did not say that comparisons existed between the differences in notes, they could not say where else they belonged. [21.13] For an empty distance and length are not only either consonant or emmelic, nor material, a matter of one certain thing, magnitude, but of two initial and unequal entities (that is, the sounds making them), so that it is consequently not possible to speak of quantitative comparisons of anything but notes and their differences. [21.18] They made none of this known[155] or equipped it with a common ratio by which it is shown to be one and the same thing—the relationship of sounds both to each other and to their difference.

I.10 - *That They Improperly Suppose the Consonance of the Diatessaron to Contain Two and One-Half Tones*

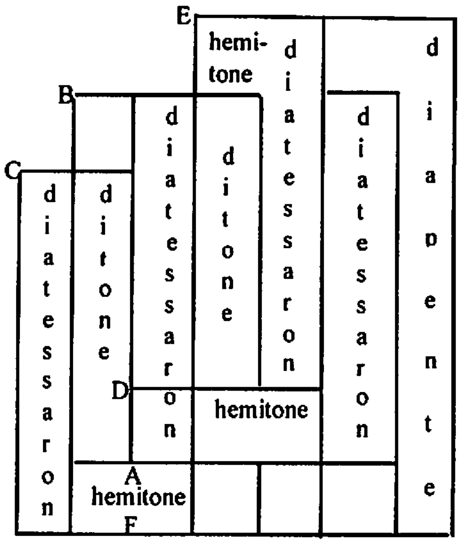
[21.21] They therefore err as well in measuring the shortest and first consonance, constructing it from two and one-half tones, so that the diapente is composed of three and one-half, the diapason of six tones, and each of the other consonances according to what follows from this.[156] [21.25] Reason, which has been shown to be more believable than our perceptions in the smallest intervals, proves this not to be the case, as will be shown. These, then, attempt to illustrate their proposition as follows:

[22.3] Let two notes AB stand as a diatessaron consonance. From A let there be an ascending ditone AC, and similarly from B let there be the descending ditone BD. Each interval AD and CB is therefore equal, and this interval is the difference between the ditone and the diatessaron. [22.6] Then

[155] Most manuscripts add a phrase that they are "conterminous with nature." Cf. Aristox. *Harm.* 15.25 (20.20–21.1 [da Rios]).

[156] Cf. Aristox. *Harm.* 62.14–65.20, Cleon. 190.12–193.2, and Bacch. #22–24 (298.7–.15).

let an ascending diatessaron DE be taken from D, and similarly a descending diatessaron CF from C. Since each of BA and CF is a diatessaron, BC is equal to AF, and in the same way AD is equal to BE. [22.10] Thus the four intervals are equal to each other, but the whole length FE forms the consonance of the diapente. Therefore, since AB is a diatessaron, and FE a diapente, the difference between them, FA and BE, makes up the remainder of a whole tone,[157] while each of them, that is, each of AD and CB, forms a hemitone,[158] and with AC a ditone, the diatessaron AB is composed of two and one-half tones.[159]



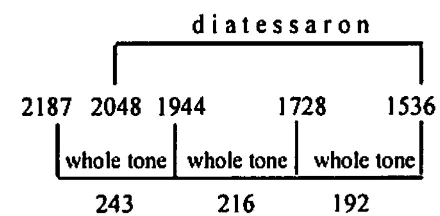
[157] By using this phrase here instead of the more Aristoxenian "equals a whole tone," Ptolemy reveals his dissatisfaction with this simplistic and unnaturalistic analysis of the diatessaron.

[158] The scholion offers, "The Aristoxenians call the diesis the fourth part of the whole tone. They divide the whole tone into two segments, of which the leimma is said to be the smaller segment, the apotome the larger. For the whole tone, considered to be in the sesquioctave ratio, is not able to be divided into two equal parts, yet they incorrectly label these unequal parts as 'half tones.'" Cf. 29.11.

[159] The manuscripts consistantly confuse these last symbols (AB) with DB and BC; cf. Alexanderson 10. The text at 22.11 introduces a result clause which should be taken with the two infinitives which I have translated as 'make up the remainder of' and 'composed of.' Intervening, however, are two causal clauses ['since...diapente'] and several asyntactic clauses ('each....ditone']. Included therein is the odd sequence "each of them [that is, FA and BE], that is, each of AC and CB." Porphryry tried to paraphrase by saying that AC and BC [i.e ditone and hemitone] consist of two and one-half tones. I might substitute 'and also' for 'that is'—one manuscript omits it—and then read DB for AB, as does another manuscript. "Therefore, since AB... while each of them and each of AD and CB [makes up the remainder of] a half tone, with AC being a ditone, and DB making a diatessaron compared with two and one-half tones." AD plus DB equals a diatessaron as would BC plus AC, FA and AC, and BE plus BD.

[22.16] Once the proportion of the whole tone has been shown to be 9:8 and that of the diatessaron 4:3, clearly then proportion makes the excess by which the diatessaron exceeds the ditone, the so-called leimma, smaller than a half tone.[160]

[23.3] Assume the first number capable of showing this proposition,[161] which is the 1536 units. The sesquioctave of this is 1728, and again the sesquioctave of this is 1944, which clearly will have with 1536 the ditone ratio. 2048 forms the sesquitertian ratio with 1536, so the leimma is in the ratio 2048:1944.[162] [23.7] But if we take the sesquioctave ratio of 1944, we will have the number 2187, and the ratio 2187:2048 is greater than that of 2048:1944. For 2187 exceeds 2048 by more than one fifteenth part but less than one fourteenth part. [23.12] 2048 exceeds 1944 by more than one nineteenth part but less than one eighteenth. [23.13] So the smallest section of the third whole tone[163] is within the diatessaron on the side of the ditone, so that the size of the leimma is smaller than the half tone and the whole diatessaron is smaller than two and one-half tones. The ratio of 2048:1944 is the same as that of 256:243.[164]



[23.19] We should not consider this a dispute between reason and perception but one of differing hypotheses, an error by the younger group,[165] who already were employing a compromise contrary to both criteria. [23.21] For perception all but shouts aloud when it recognizes clearly and without

[160] Because Ptolemy has been introducing the subject matter so gradually, this is actually his first mention of the leimma.

[161] The lowest common multiple.

[162] This one example will illustrate Ptolemy's methodology in this and subsequent demonstrations:

1) 1536 x 9/8 = 1728 x 9/8 = 1944
2) 1536 x 4/3 = 2048
3) 1728/1536 x 1944/1728 x 2048/1944 = 4/3

wholetone + wholetone + leimma = diatessaron
\ /
ditone

[163] A curious expression for the third note of the diatessaron which, unlike the other two, is smaller than a whole tone, but Ptolemy's purpose here is to treat this third note not as a hemitone but a leimma, the 'leftover part.' Cf. Aristox. *Harm.* 57.9–.12.

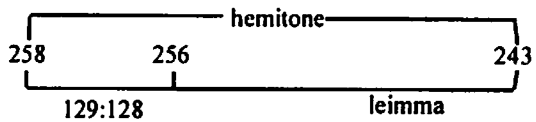
[164] The ratio widely accepted as the leimma. Cf. Theon of Smyrna 2.14 (43–47 [Lawlor]).

[165] The scholion identifies this "younger group" as the Aristoxenians. Düring corrected his own punctuation here.

doubt the consonance of the diapente, when the sesquialter ratio is demonstrated on the aforementioned monochord,[166] and the diatessaron when the sesquitertian ratio is demonstrated. [24.1] But they do not remain in agreement with this, from which assuredly it follows both that the difference between the aforementioned consonances is a whole tone in the sesquioctave ratio and that the diatessaron consonance is smaller than two and one-half tones.[167]

[24.4] In the intervals[168] in which perception is sufficient to judge, that is, in the larger intervals, they mistrust it altogether, yet in the differences in which perception is not sufficient to judge, that is in the smaller intervals, here they trust their perceptions and apply their judgments contrary to the first and more important excesses.

[24.8] We could see further the foolishness of their demonstration if we were to calculate the size of the discrepancy between the leimma and the half tone.[169] For since the sesquioctave ratio is not divided into two equal halves, nor is any other superparticular ratio,[170] the nearly equal[171] ratios 17:16 and 18:17 make up the sesquioctave ratio.[172] The half tone ratio should then exist between these two somehow, that is, it should be greater than 18:17 and smaller than 17:16. [24.14] Of 243, 15 is greater than one seventeenth and smaller than one sixteenth.[173] Therefore, with 243 and 15 added together, they would approximate a half tone ratio of 258:243. Yet the ratio of the leimma was shown to be 256:243, and therefore the half tone differs from the leimma by the ratio 258 to 256, which is 129:128.



[166] I follow the punctuation of Höeg, review of Düring, 657. Jacques Handschin, "The 'Timaeus' Scale," *Musica Disciplina* 4 (1950) 5, analyzes this (otherwise uncharacteristically vivid) statement.

[167] The Aristoxenians, like the Pythagoreans, are not entirely incorrect nor are they incorrect about everything, for 3/2 - 4/3 = 9/8. But the diatessaron has been shown to be slightly smaller than two and one-half tones.

[168] My supplement for the relative pronoun; Barker offers "cases" instead, Düring "decisions," Gogava "consonances." Cf. Alexanderson 10 and Porphyry 131.6.

[169] For the demonstration, see Barbera, *Pythagorean Mathematics* 308–309.

[170] Oskar Becker, "Frühgriechische Mathematik und Musiklehre," *AfM* 14 (1957) 156–62, demonstrates why no superparticular number can be rationally halved; cf. Boeth. *De mus.* 3.11.

[171] Before "nearly equal," the Byzantine version adds "two" for clarity.

[172] When added together, since one is larger than 9:8, the other smaller, but not by the same amount. In linear terms, a hypothetical musical "line" can have a mid-point, but in mathematical terms, irrational results, egg. those that contain square roots, prevent the calculation of a real half tone.

[173] Not one sixteenth and one seventeenth of 243, for one fifteenth is, of course, larger than both. 1/16 > 15/243 > 1/17.

[24.20] They themselves would not say that it is possible for the hearing to discriminate between such small differences.[174] If, then it is admitted that the perception hears wrongly once, much more would be admitted in the collection of more data. They erred, for instance, in the preceding demonstration with the diatessaron taken three times and the ditone twice in different positions, when it was not easy for them even once to make a ditone accurately. [24.26] They would rather make a whole tone than a ditone since this whole tone is emmelic and in sesquioctave ratio while the non-composite ditone is ecmelic, as in the ratio 81:64, and since the more symmetrical intervals are those easier for the perceptions to comprehend.[175]

I.11 - *How By Means of the Octochord*[176] *Canon the Diapason Could Be Shown to One's Perceptions to be Smaller Than Six Tones*

[25.1] This proposition would be most clearly refuted, as would the inability of the hearing in such matters, by using the homophone of the diapason. They show that the octave has six tones according to the reasoning that the diatessaron consists of two and one-half tones and that the diapason has the diatessaron twice and an additional whole tone.[177]

[25.5] If we ask some skilled musician[178] to produce six successive and self-contained whole tones, not taking into consideration previously tuned notes,[179] so reference might not be made to some other of the consonances, then the first note in comparison with the seventh will not make a diapason. [25.9] If such a thing were to happen, despite the weakness of our perception, he would show the falsity[180] of the statement that the consonance of the diapason consists of six tones. [25.10] But if our perception is not able to hear the whole tones accurately, then it will not be much more trustworthy

[174] No extant Aristoxenian treatise so specifies that a half tone sounds like the leimma, but the Aristoxenians apparently were not bothered by any perceived discrepancy. As a result, they could extend six whole tones and claim that it created a diapason, a first and most important interval.

[175] Cf. Cleon. 190.9–.12, where the ditone is simply assumed to be the size of two whole tones and not of the ratio 81:64 (or 243:192).

[176] Several of the manuscript families offer 'heptachord' as an alternative, surely because of Ptolemy's later insistance on seven as the number of *tonoi*.

[177] Cleon. 194.3–.9.

[178] Düring (189) thinks this to be a reference to Aristoxenus, but Düring generally underplays Ptolemy's willingness to cooperate with and respect for actual musicians; cf. 8.25f. One scholiast would similarly identify the subject ('he') of this sentence at 25.11 as Aristoxenus, but the question remains whether Ptolemy would refer to Aristoxenus as μουσικώτατος (*musikotatos* - 'skilled musician'); cf. II.13 on Didymus.

[179] Tuned according to non-Aristoxenian methods.

[180] Cf. Alexanderson 11.

in hearing the ditones from which he intends to find the diatessaron of two and one-half tones.[181]

[25.12] This is quite correct,[182] for not only does the diapason not occur, but neither does any other magnitude at all by means of an interval, whether the same produce it or whether it is always among the same ones.[183] [25.16] And yet if we take in the same way in succession the diatessaron and diapente, their endpoints form the diapason, since these are intervals more easily definable for the hearing. [25.18] Still, if we take six tones in succession of their ratios, the outer notes have a magnitude slightly greater than the diapason and always with the same difference,[184] that is the double

[181] The diapason is formed from the duple ratio; $(9/8)^6$ does not yield 2:1 but 531441:262144. Ptolemy arranges these statements in two parallel conditions. If our ear can grasp the difference between the first and seventh notes, the Aristoxenian proposition is proved false; if our ear cannot grasp the difference, then how can it grasp the ditone which is the basis of the two-and -one-half-tone diatessaron? cf. 24.26.

[182] Comparative for superlative; cf. Düring 190.

[183] A passage open to a variety of interpretations presented by Wallis, Düring, Höeg, Alexanderson, the scholiasts, and several manuscript variants. In sum, Düring interprets as "neither the diapason, diapente, nor diatessaron remains the same through repetition." Alexanderson opts for "not only the octave but generally speaking any interval is not the same even if the same persons do the harmonizing and always on the same chords." In this instance, Ptolemy's meaning could not securely survive manuscript corruption since every signifiant term is expressed vaguely, oddly, corrupted, or not at all.

Ptolemy's task here must be to explain that our perceptions cannot be relied upon for establishing the diapason of "six tones" or for any interval in two circumstances. Just what the roadblock is, what intervals are included, and what the two circumstances are present the reader with difficulties. Düring (190) calls this a *locus perplexus* but assumes "magnitude... the same" is to be taken attributively as through the same magnitude. He then correctly assumes this means that when an interval is repeated it may vary from the previous realization. He later takes his own emended reading 'among all' (ἐπὶ πάντων) as referring not to 'all instruments' (τῶν διαφόρων ὀργάνων), as the scholion suggests, but to "all magnitudes."

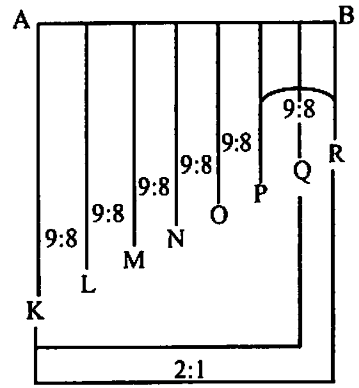
Alexanderson (11–13) points out that the verb 'produces' (ἁρμόζεσθαι) should have a personal subject and that therefore its subject, the adjectival pronoun 'all,' which he reads for Düring's "the same ones" (τῶν αὐτῶν), refers to "people," ie. musicians. He thinks "the same ones" then refers to 'always on the same chords' (sc. χορδῶν).

I find the (Greek) word order of Alexanderson's version very strained, but I also find Düring's emendations excessive, as did Höeg (656). I believe the sense to depend in part on the main verb and in part on whatever 'the same' originally meant. The main verb (γίνεται) means not necessarily 'is' but 'occurs.' Ptolemy has completed saying that the diapason does not consist of six whole tones, and he now implies that it is hard to construct at all. He will eventually say (25.16) that we should do this through the diatessaron and diapente (fourths and fifths), as complex tuning is often accomplished today, and therefore he says, "For not only does the diapason not occur [through six tones or the use of ditones] but neither does any other [consonance—cf. Pach. 14.461 and Boeth. *De mus.* 2.31] altogether by means of magnitude of difference [the Byzantine reading in one manuscript group (g) and Porphryry, e.g. a ditone, whole tone, or whatever] neither when the same [musician (manuscript group f)] reads them nor when they are always in the same [strings?]. The difficulties may have originated with specific glosses for 'magnitude' (μέγεθος).

[184] "Ptolemy," says the scholion, "has demonstrated that the diatessaron ought to be composed of two whole tones plus the leimma, and that this leimma should be the remainder of the hemitone by the 128th part of itself. And that since the diapason consists of two diatessarons plus a whole tone, we add the 128 of the two diatessarons, that is the reduction of the two

of the ratio between the leimma and the half tone, which, in keeping with earlier hypotheses,[185] is nearly the 65:64 ratio.

[26.3] This will be clear to us if we fit seven other strings onto the canon using the same preparation and positioning as we did with the one string. For if we arrange accurately the eight notes with equal tension and with equal lengths of strings, so that they are ABCDEFGH,[186] [26.6] then if through application of the calibrator it is divided into six successive sesquioctave ratios, let us set down at each note an equal bridge at its appropriate section, so that the distance AK is the sesquioctave of BL, and BL of CM, and CM of DN, and DN of EO, and EO of FP, and FP of GQ. [26.11] AK makes a double ratio with HR, and these notes will form precisely the homophone of the diapason, while QG will be a little and always by the same amount higher than HR.[187]



531,444:262,144

[26.15] That the strings, when there are more than one used, and if they are of equal tension and in equal lengths, are different will be shown in the following: there are three reasons for the difference between high and low sounds in them—the density, the diameter,[188] and the lengths of the strings. [27.1] The higher sound results from a denser and thinner string and a shorter length; tension here takes the place of density, for it creates greater

leimmata, and we make one 64. Since also this one 64 when divided makes two 128s, Ptolemy therefore says that the six whole tones exceed the true diapason by 64, that is two times 128."

[185] Cf. 24.19.

[186] Barbera, *Pythagorean Mathematics* 309–313, analyses this whole passage thoroughly.

[187] Just one of several passages which suggests that Ptolemy's original manuscript was intended to include tables and charts. This is not unexpected, however, considering the nature of the *Almagest* and *Geography*.

[188] The scholion offers "thickness."

tension and hardens also what would be similar if the string had a shorter length.[189]

[27.5] From all this it is clear that, with other variables remaining constant, as the greater tension relates to the smaller, so the sound produced by the greater tension relates to the smaller, and as the greater diameter stands in relation to the smaller, so the sound produced by the smaller diameter stands in relation to the greater. [27.8] I mean, in these instances, that with dissimilar strings, which have equal tension and are of equal lengths, any need the sound has for a greater diameter is compensated for by the prevalence of the greater tension. And the ratio of the greater diameter to the smaller is certainly the same as that of the greater tension to the smaller.

[27.14] Let there be two notes of equal pitch and equal lengths, A and B, of which the diameter and, clearly, the tension of A is greater than that of B. Let us take another note of equal length, C, which has the same diameter as B and the same tension as A. Therefore, since C differs from B in tension alone, as the tension of C is to that of B, so will the sound of C be to that of B. [27.19] Again, since C differs from A in diameter alone, the diameter of A will be to that of C as the sound of C is to that of A. But the sound of C will have the same ratio to each of those of A and B, for those of A and B are the same. [27.23] And so, as the tension of C is to that of B, so will the diameter of A be to that of C. And as the tension of C is to that of B, so will the tension of A be to that of B since the tensions of A and C are equal.

A	greater diameter	-	greater tension
B	smaller diameter	-	smaller tension
C	smaller diameter	-	greater tension

[28.1] Therefore, the diameter of A is to that of C as the diameter of A is to that of B since the diameters of B and C are equal. And so the tension of A is to that of B as the diameter of A is to that of B. This would be the case in these matters even if they were all in every way unchanged and undifferentiated.

[28.5] Again, if with the same conditions we make the lengths of AB and CD unequal and decrease the other to CE, the length AB will be to the length CE as the sound CE is to the sound AB. For since as the distance CD is to that of CE, so the sound CE is to that of CD, and since the distance AB is equal to CD and the sound of AB is equal to that of CD, the distance of AB is to that of CE as the sound of CE is to that of AB.

A---------------------------------------B
C---------------------------E----------D

[189] I chose not to use here Düring's emendation. Cf. 8.2ff.

I.12 - *On the Aristoxenian Division of the Genera and the Tetrachords in Each*

[28.15] Let our definitions about the greater differences in the notes stand as is.[190] We must move now to the smaller constructs, those which measure out the first of the consonances. These are found when the diatessaron is divided into three parts according to the method defined before, [28.19] so that the first homophone, being one, is constructed from two of the first consonances, while the first consonance is constructed from three emmelic notes up to the number which defines the ratio.[191] [28.21] But the division of the diatessaron does not happen to be the same in every case but is constructed differently in different instances. [28.23] While the two outer notes remain the same to preserve the established consonance, for which reason they call them 'fixed,' the two notes in between are called 'movable' since they make the diatessaron's remaining notes unequal.[192]

[28.26] Such movement is called 'modulation by genus,' and[193] 'genus' in harmonics refers to a certain placement of these notes comprising the diatessaron consonance in relation to each other. [28.28] The first division of genus is into two, that is, whether it is the soft or the intense. The soft is the more compact in ethos,[194] the intense more separated. [29.2] The second division is into three with the third part placed somehow between the aforementioned two and called 'chromatic.' Of the remaining two the softer is the enharmonic, the more intense the diatonic.[195]

[29.5] The so-called pyknon is particular to the enharmonic and the chromatic when the two lowest intervals are both smaller than the remaining

[190] Ptolemy here announces the conclusion of the second major division of Book I. The first was that in which he established through a series of definitions (chapters 1 and 2) the need to examine harmonics on a quantitative basis, specifically emmelic notes (chapters 3 and 4). To do so he concentrated on the Pythagorean and Aristoxenian methods of defining and determining the consonances (chapters 5–11). Now he will begin to concentrate on the divisions of the tetrachords, i.e. diatessarons, and this will consume the rest of Book I. This and the next four chapters provided the major source for Bry. I.7 (112.12–116.7) and Pach. 110.13f.

[191] The first homophone, the diapason, which is one construct, is itself constructed of the first two consonances, the diapente and the diatessaron. The diatessaron, then, is constructed of three intervals and four notes, three of which share varying intervallic relationships and make up the pyknon. Cf. 15.10–18 and 16.21–28; and Boeth. *De mus.* 5.16, contaminated with Aristox. *Harm.* 30.17–35.8.

[192] For the Greek terms signifying 'fixed' and 'movable,' which can vary widely from author to author, cf. Michaelides 136.

[193] Several manuscript groups change this conjunction to 'for,' which, although not original with Ptolemy, may be preferable to the author's parataxis.

[194] The first mention of ethos, i.e. effect, associated with harmonics. The effect of music, which varies according to the arrangement of intervals, will become increasingly important to Ptolemy in Book III, esp. chapters 3–7.

[195] Ptolemy does not explain these divisions very clearly. There are only six: the three chromatic, the soft and intense diatonic, and the enharmonic, which is relatively soft, i.e. more compact. Ptolemy's concentration here is on the soft vs. the intense genera.

one. The so-called apyknon is particular to the diatonic when neither one of the three ratios is greater than both of the remaining two.[196] [29.9] The progressive theorists make more divisions, but we, for the present, will discuss Aristoxenian theory as follows:[197] [29.11] He divides the tone first into two equal parts, then into three, then into four, then into eight. The fourth part of the tone he calls the enharmonic diesis, the third the soft chromatic diesis, the fourth plus the eighth the sesquialter chromatic diesis; the half tone belongs to both the tonic chromatic and the diatonic genera.

[29.16] From these he proposes the differences between the six unmixed genera, one of the enharmonic,[198] three of the chromatic—soft, hemiolic, and tonic—and the remaining two of the diatonic—soft and intense. [29.19] The bottommost interval of[199] the enharmonic genus, the 'following,' and the middle interval each make up an enharmonic diesis, and the remaining interval, the 'leading,'[200] consists of two whole tones. [29.21] Here if the

[196] The term 'pyknon' originally refers to being 'close together,' but it has become, like 'tonos,' a term to be transliterated rather than translated. Summaries of and references to other definitions of *pyknon* and *apyknon* can be found in Michaelides 279–80, and Mathiesen, *Aristides Quintilianus* 79, nn. 66–69. See also, Bry. I.6 (106.11) and 342.17 (*apyknon*). Düring's discussion (194–95) on the pyknon is marred by his understanding that the enharmonic is a purely theoretical genus, or at best relegated to use by the aulete. (M. L. West in his review of G. Panagua, *Musique de la Grece antique*, *Gnomon* 52 (1980) 698, understandably requested a recording of the *Orestes* papyrus in the enharmonic genus.)

Since in the last few chapters Ptolemy has made it very clear that he considers the Aristoxenian approach to defining intervals and constructing pitches to be by nature entirely inappropriate to the purposes of the music theoriest in search of accuracy, one should then inquire as to the reason Ptolemy begins his discussion of the divisions of the tetrachord with Aristoxenus' divisions. One reason might be that these divisions were the invention of Aristoxenus, who, despite his relatively irrational approach, was a great innovator and consolidator in the modernization of ancient Greek music. More likely however, Ptolemy probably wishes to eliminate the worst system first, move onto the Pythagorean thinkers, and finish with his own version.

[197] It is not clear who the "progressive" theorists are, but for the Aristoxenian divisions, cf. Porph. 137.13f., who cites Aristoxenus [*Harm.* 62.15f.]. Cf. Boeth. *De mus.* 5.16 (and Porph. 138.14, with Düring's note).

[198] Aristoxenus has only one enharmonic tetrachordal division, as will be the case for Eratosthenes, Archytas, Didymus, and even Ptolemy who proposes numerous versions of the chromatic and diatonic. Aristoxenus (*ap.* [Plut.] *De mus.* 1134F–1135B) reports that the enharmonic was growing out of fashion in his day, so there is no reason to suppose it would resurface centuries later. I would maintain that the old tribal *harmoniai* reported in Aristides Quintilianus I.19 (19.2–20.1) were the source of most enharmonic music, and that Aristoxenus' Greater Perfect System, while accommodating the enharmonic genus, effectively eliminated its use in so structured a two-octave system. See Solomon, "*Tonoi*" 242–51; John Thorp, "Aristoxenus and the Ethnoethical Modes," in R. Wallace and B. MacLachlan, eds. *Harmonia Mundi: Musica e filosofia nell'Antichità* (Rome 1991) 54–69; Otto Gombosi, "Key, Mode, Species," *JAMS* 4 (1951) 20–26; and Matthew Shirlaw, "The Music and Tone-Systems of Ancient Greece," *Music and Letters* 32 (1951) 131–39.

[199] Reading the genitive with Düring; cf. Alexanderson 13.

[200] Ptolemy here introduces the terms 'following' and 'leading' to describe the bottommost and highest interval, respectively, within the three of the tetrachord. This is the same terminology he uses in the *Almagest* referring to motion defined by and counter to the heavenly

number 24 is employed for the whole tone, each of the pyknon intervals consists of 6, the remainder of 48.[201]

[29.23] In the soft chromatic, each of the pyknon intervals consists of one-third of a tone, the remainder of one plus one-half plus one-third; thus each of the former is 8, the latter 44. In the hemiolic chromatic each of the two pyknon intervals consists of a fourth plus an eighth of a tone, the remainder of one plus one-half plus[202] one-quarter tone; each of the former is 9, the latter 42. [29.28] In the tonic chromatic, each of the two pyknon intervals consists of a half tone, the remainder of a whole tone and one half; each of the former is 12, the latter 36.[203]

[29.31] In the remaining two genera without pykna, the following interval in both preserves the half tone. Of the succeeding intervals in the soft diatonic, the middle interval consists of a half tone plus a quarter tone, the leading interval of a whole tone plus a quarter tone, for example 12 and 18 and 30. [29.35] In the intense diatonic the 'following' interval consists of a half tone, while the remaining intervals, the middle and the leading, each consist of the whole tone, for example, 12, 24, and 24.[204]

[30.2] The numbers are arranged in this way:[205]

enharmonic	soft chromatic	hemiolic chromatic	tonic chromatic	soft diatonic	ntense diatonic
48	44	42	36	30	24
6	8	9	12	18	24
6	8	9	12	12	12
60	60	60	60	60	60

motion from east to west, for which, see Toomer 20. [Arist.] *Prob.* 19.33, defines mese as "the leader"; cf. [Plut.] *De mus.* 1134F–1135A. Monro 45, discusses all three passages.

[201] Two enharmonic dieses of 6 each plus two whole tones (24 each) of 48 ("the remainder") equals the entire diatessaron of 60. This system of using integers to represent the diatessaron distance is not found in the extant writings of Aristoxenus, although it is found in the work of some of his imitators. Cleon. (192.12–193.2) uses 30 to signify the diatessaron, as does Ptolemy in the extant prose portion of II.14. Ptolemy here and Aristides Quintilianus(17.21–18.4) use 60, which eliminates the need for any fraction (4 1/2) in describing the pyknon of the hemiolic chromatic. Cf. Mountford, "Harmonics" 91, n. 39; Palisca 49, 214, and 271; *Anon. Bell.* 2.26–27 (Najock); and Porph. 137–38.

[202] For the reading, see Höeg, review of Düring, 658, n. 2.

[203] For alternate (Latin) translations by Gaffurio and Leoniceno, see Palisca 213–14.

[204] On the middle diatonic, a seventh possible division, cf. da Rios 38 and 40, n.1 (*trad.*).

[205] To summarize:

enharmonic	soft chromatic	hemiolic chromatic	tonic chromatic	soft diatonic	intense diatonic
1/4	1/3	3/8	1/2	1/2	1/2
1/4	1/3	3/8	1/2	3/4	1
2	1 5/6	1 3/4	1 1/2	1 1/4	1

I.13 - *On Archytas' Division of the Genera and the Tetrachords*

[30.3] Aristoxenus[206] seems here to have thought not at all of ratios but instead divided the genera only by what was between the notes and not by the relationships of their differences.[207] [30.5] He abandoned the causes of these differences as uncaused, nonexistent, or mere limits and categorized them as immaterial and empty. [30.7] It was for this reason that he was not concerned when he quite regularly divided the emmelic intervals in half, even though superparticular ratios do not admit of such treatment.[208] [30.9] On the other hand, Archytas the Tarentine, who of the Pythagoreans had the greatest interest in music, attempted to preserve what fit the ratios not only with the consonances but also the divisions within the tetrachords.[209] [30.11]

He assumed that measuring the differences was appropriate to the nature of emmelic intervals.[210]

[30.13] Nevertheless, in applying this thesis he seems to have erred in several important facets. For the most part he was in full command of his study, yet he clearly strayed away from those things which agree openly with the perceptions. We will now observe this by examining his division of tetrachords.

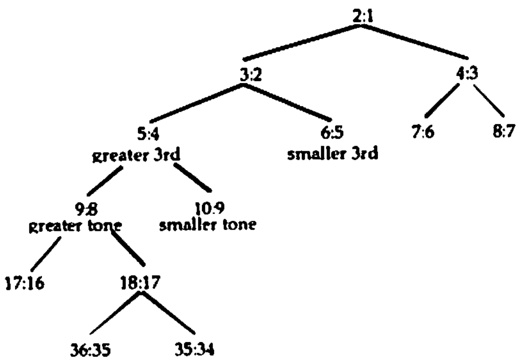
[30.17] He proposed three genera—the enharmonic, chromatic, and diatonic. He divided each of them in this way: [30.19] the 'following' ratio in the three genera he made the same, 28:27;[211] the middle interval in the enharmonic 36:35, in the diatonic 8:7, so that the leading interval in the enharmonic genus was 5:4,[212] in the diatonic 9:8.[213] [31.2] The second from

---

[206] The scholion supports identifying Ptolemy's pronoun as Aristoxenus. Cf. Aristox. *Harm.* 20.20–21.1.

[207] Ptolemy criticizes Aristoxenus for employing intervals, instead of notes derived from differences. Also, Aristoxenus is criticized for concentrating on only the internal divisions of the tetrachord. Once an Aristoxenian establishes that the fixed limits of the diatessaron could be equated to the lowest common integer 60 (or 30), he proceeds to subdivide the tetrachord by considering only the various integer relationships within that number, i.e. what was "between the notes." Da Rios 37, n. 2 (*trad.*), offers a brief analysis.

[208] Barbera, *Pythagorean Mathematics* 303, analyzes the three relevant propositions in *SC* 3 (that no geometric mean divides a superparticular ratio), 13 (that 9:8 is the wholetone), and 16 (that therefore the whole tone cannot be halved). See also Theon of Smyrna 36 (Lawlor). Hermann Koller, *Musik und Dichtung im alten Griechenland* (Bern 1963) 182, in establishing his analogies between the building blocks of harmony and poetry, demonstrates how these ratios cannot be subdivided:



[209] Pythagoreans previous to Archytas were satisfied to derive the consonances from the four basic integers of the tetractys (2:1, 3:2, 4:3, 3:1, 4:1), and then to subdivide the consonance of the diatessaron in (what would be known as) the diatonic division, the same scale most familiar both to the ancients and to us as "the Timaeus scale." The evidence is very slight that Philolaus, the teacher of Archytas, may have been first to derive the sesquioctave whole tone from the difference between the sesquitertian diatessaron and the sesquialter diapente; cf. Jacques Handschin, "The Timaeus Scale," *Musica Disciplina* 4 (1950) 3–42, and Crocker, "Pythagorean Mathematics" 332–33.

Archytas then made (or at least 'systematized' (Crocker, "Aristoxenus" 102) the two additional tetrachordal divisions (chromatic and enharmonic) for the first time. It may also be that Archytas was responsible for devising the 'quadrivium' as it would be promulgated in Plato, his associate in Sicily in the 380's B.C. Eva Keuls,"The Apulian 'Xylophone': A Mysterious Musical Instrument Idenitified," *The American Journal of Archaeology* 83 (1979) 476–77, summarizes arguments and attempts to verify the "rattle" depicted on several South Italian vases and thought to be the *platage* invented by Archytas.

Despite his polemical tone, Ptolemy reveals his respect for Archytas in the next few sentences. Francois Lasserre, *The Birth of Mathematics in the Age of Plato* (Larchmont NY 1964) 175–80, and Paul Tannery, "Sur les Intervalles de la musique grecque," *REG* 15 (1902) 348–50, put Archytas into historical and mathematical perspective. Sir Thomas L. Heath, *A Manual of Greek Mathematics* (Oxford 1931) 51, discusses Archytas' work on means; cf. Burkert 384. For biographical information on Archytas, see van der Waerden, "Harmonielehre" 181–87; Heath 213–26; and Diels-Kranz[6] I.421–39 (#47). Barbera, *Pythagorean Mathematics* 23–29, offers the most recent and complete summary of the literature.

[210] And so established a method one step superior to that of Aristoxenus, although Ptolemy only tacitly confirms Archytas' assumption that the nature of emmelic intervals lends itself to measurement of differences (excesses). Ptolemy organizes his discussion of the history of tetrachordal divisions here along theoretical, not chronological considerations, for Aristoxenus postdates Archytas by over a generation. This passage serves as the sole surviving source for Archytas' tetrachordal division; it is paraphrased by Boeth. *De mus.* 5.17. A summary can be found at Palisca 214–16 (although the reference in fn. 87 should be to 2.17).

[211] Archytas is unique in maintaining the 28:27 (or any) ratio throughout the three genera. 28:27 is approximately a quarter tone. Crocker, "Pythagorean Mathematics" 193, places intervals of this approximate size (28:27, 36:35 and 39:38) into some musical, mathematical and historical perspective. Paul Tannery, "Du rôle de la musique grecque," *Bibliotheca Mathematica* 3 (1902) 161–75, believes Archytas derived his tetrachordal divisions by determining harmonic means.

[212] The Timaeus system, which existed previous to Archytas' divisions, contains two consecutive sesquioctave ratios, which would seem to reflect the acceptance of the ditone at that period. Considering that most later theorists make the large interval of the enharmonic a ditone (81:64), it is possible that Archytas configured his as a major third (5:4) to escape resemblance to and improve upon the Timaeus diatonic. Cf. Barbera, "Divisions" 296.

Paul Tannery, "Sur le spondiasme dans l'ancienne musique grecque," *Revue archéologique* 4/17 (1911) 41–50 (=*Mémoires scientifique* III 209–309) attempts to equate Archytas' enharmonic lichanos with Aristoxenus' soft chromatic (29.23f.); Winnington-Ingram, "Aristoxenus and the Intervals" 203–204, refutes the attempt but has an equally interesting interpretation.

the highest note in the chromatic genus he takes from the same position as in the diatonic, for he says that the second from the highest ratio in the chromatic with the parallel ratio in the diatonic forms the 256:243 ratio.[214]

[31.6] Such tetrachords [215] are, according to the aforementioned ratios , composed with these whole numbers: If we assume the highest numbers in the tetrachords to be 1512, the lowest by the sesquitertian ratio to be 2016, then this number is 28:27 of 1944, and the second from the lowest in these three genera will have this number. [31.11] 1890 will be the second from the highest in the enharmonic genus, for this in proportion to 1944 makes the 36:35 ratio, and in proportion to 1512 makes the 5:4 ratio.[216]

[31.13] 1701 will be the second highest in the diatonic genus for this with 1944[217] makes the 8:7 ratio, and to 1512 the 9:8 ratio.[218] [31.15] The

[213] J. M. Barbour, "The Principles of Greek Notation," *JAMS* 13 (1960) 12, correlates Archytas' diatonic (28:27 x 8:7 x 9:8 = 63 + 231 + 204 cents) with Aristoxenus' soft chromatic (and "stretched diatonic"—1/3 + 7/6 + 1 = 67 + 233 + 200 cents).

[214] That is, 8:7 x X = 256:243. Martin Vogel, "Über die drei Tongeschlechter des Archytas," in *Bericht über den Internationalen Musikwissenschaftlichen Kongress Hamburg 1956* (Kassel 1957) 233–35, analyzes Archytas' chromatic in reference to the diatonic synemmenon and diezeugmenon tetrachords. Chronologically, of course, this would be impossible unless it could be shown that the Greater Perfect System significantly predated Aristoxenus.

Barbour, "The Principles of Greek Notation," *JAMS* 13 (1960) 12–13, finds Archytas' chromatic (28:27 x 243/224 x 32:27 = 63 + 141 + 294 cents) roughly equivalent to Aristoxenus' soft chromatic (1/3 + 2/3 + 3/2 = 67 + 133 + 300 cents), in addition to the similarities between Archytas' diatonic and Aristoxenus' soft chromatic, and Archytas' diatonic with Ptolemy's tonic diatonic. Noticing the frequency with which the one-third tone appears here, Barbour posits that Archytas may have been the man responsible for creating the supine position of the letters in the instrumental notation to designate this one-third tone (=67 cents). Any similarity between the music theorized by Archytas and Aristoxenus could depend to some extent on the fact that these men were nearly contemporary and from the same city in southern Italy. For an elaboration of this idea, see infra, n. 267.

Burkert, *Lore and Sciences* 384 and 388, discusses the tetrachordal divisions of Archytas, particularly the combination of the 28:27 interval (parhypate-hypate) with the 9:8 (hyperhypate) which yields 7:6. This interval occurs in the late fifth-century *Orestes* papyrus, the literature for which is summarized in Jon Solomon, "*Orestes* 344–45: Colometry and Music," *Greek, Roman, and Byzantine Studies* 18 (1977) 71, n. 2.

[215] To summarize:

diatonic	*chromatic*	*enharmonic*
28:27	28:27	28:27
8:7	243:224	36:35
9:8	32:27	5:4

[216] 5:4 is, of course, the major third, 6:5 the minor third. The failure of the ancient Greeks to recognize 5:4 as a consonance has provoked much reaction, not the most sober of which was the prediction and accusation made by John Redfield, *Music: A Science and an Art* (New York 1930) 69–70, that if the Greeks had not been so inept so as not to discover the M3 and major triad, they could have developed the classical symphony two thousand years ago!

[217] Following Höeg, review of Düring, 658, n. 2.

[218] Winnington-Ingram, "Greece" 664, derives this division from the original Timaeus diatonic (9:8 x 9:8 x 256:243 as described by the Pythagoreans, whole tone plus whole tone plus half tone as described by the Aristoxenians). An enlarged major third (5:4) yields 9:7 (9:8 x 8:7) which forms the diatessaron with 28:27. Aristoxenus' 1/3 + 1 1/6 + 1 closely resembles this

same tone in the chromatic genus will be 1792, for this has the same ratio to 1701, that is 256:243. Below the table of these numbers is displayed:

enharmonic	diatonic	chromatic
1512 - - -		
5:4	9:8	32:27
1890 - - -		
36:35	8:7	243:224
1944 - - -	1944 - - - -	1944 - - - -

I.14 - *Proof That Neither of These Divisions Preserves the Real Emmelic Interval*

[32.1] As we said, Archytas constructed the chromatic tetrachord contrary to theory,[219] for the number 1792 stands in a superparticular ratio neither to 1512 nor to 1944. And he constructed his chromatic and enharmonic tetrachords contrary to the clarity of perception. [32.4] We find the following ratio of the normal chromatic to be greater than 28:27, and again Archytas makes the following ratio of the enharmonic genus,[220] which seems much smaller than parallel intervals in the other genera, equal to them. [32.8] Besides this, he makes the middle interval smaller than the lowest in the 36:35 ratio, even though this is clearly an ecmelic interval.[221] With such

configuration, so Winnington-Ingram's conclusion is that the septimal tone (8:7) must have been employed in Greek music from Archytas' day to Ptolemy's. Burkert, *Lore and Science* 389, presupposes a scale derived from the addition and subtraction of intervals and the ratio 256:243 and all derived from Philolaus.

Bowen, "Minor Sixth" 504–505, follows up on the progress made in understanding this chapter by Tannery and Winnington-Ingram ("Aristoxenus and the Intervals" 203–204). He describes as persuasive their arguments that Ptolemy's description of how Archytas derived these tetrachordal divisions is incorrect. If this is the case, then we may have another, albeit minor, example of Ptolemy misrepresenting to a certain degree calculations made by earlier scientists. Cf. Introduction, n. 33.

[219] Contrary to Pythagorean theory; see 11.5–21, and cf. Boeth. *De mus.* 5.18. Düring (197) sets Ptolemy in opposition to the actual music of his day. No doubt he bases his premise on Winnington-Ingram's assumption that Archytas tetrachordal divisions reflect actual musical practice of his (and Ptolemy's) day. Ptolemy aims his present argument against Archytas, who precedes him by some four centuries.

[220] See Burkert, *Lore and Science* 385, n. 71.

[221] Cf. 16.21f.

reasoning the size of the bottommost interval becomes greater than the size of the middle interval.[222]

[32.10] Archytas' calculations appear to discredit any proportional criterion since they do not preserve the emmelic in dividing the canon according to the previously proposed ratios.[223] For the majority of the aforementioned ratios and those calculated by nearly all others do not conform to ethe[224] generally agreed upon.[225]

[32.15] It seems also that the number of the genera, as Archytas reckons them, fall short of the proper amount; not only his enharmonic but also both his chromatic and diatonic are shown in one form. [32.18] With Aristoxenus,[226] on the other hand, the number is too great in the chromatic since the soft and hemiolic dieses differ by one twenty-fourth part of a tone which provides the hearing with no noticeable change, and too small in the diatonic where the musical variety seems clearly more plentiful, as will be possible to observe from the discussion below.[227] [32.23] Still, neither was Archytas correct in making the magnitudes of the two following intervals in the pyknon equal to each other with the middle being greater in size always,

[222] Archytas ignores the guidelines established by Aristoxenus that the lowest interval of the tetrachord should be the smallest in magnitude. Of course, Archytas predates Aristoxenus. Winnington-Ingram, "Aristoxenus and the Intervals" 197, n. 2, relates Archytas' enharmonic to Aristoxenus' (52–53 (Macran; = 64–66 (da Rios)) "aharmonic" 3/8 + 7/24 + 1 5/6.

[223] Burkert, *Lore and Science*, 380, questions the authenticity of Ptolemy's reporting of Archytas' divisions but ultimately believes them to be genuine.

[224] The term 'ethe' should surprise the reader at this point because elsewhere, with the exception of the brief mention at 29.1, Ptolemy does not so abruptly associate harmonics, i.e. tunings, with ethos. Several manuscripts which are glossed with "in the melodies" preserve an earlier attempt to put ethe into context. Judging by what is now known of Ptolemy's treatment of the calculations of his predecessors, I feel comfortable in attributing this obtrusive and undeveloped mention of ethos to Ptolemy's direct source, whether that was a collection of the works of Archytas himself or an intermediary and indirect source. Archytas and Plato were not only contemporaries but in close contact, and what else we know of fourth-century music theory frequently and almost compulsively turns to discussions of ethos. Cf. Pl. *Resp.* 398E; Arist. *Pol.* 1340B; Warren D. Anderson, "The Importance of Damonian Theory in Plato's Thought," *TAPA* 86 (1955) 88–102, who notes (89, n. 5) Philolaus' suggestion that ethical considerations began even with Pythagoras; and Robert W. Wallace, "Damone di Oa ed i suoi successori: un'analisi delle fonti," in R. Wallace and B. MacLachlan, eds. *Harmonia Mundi: Musica e filosofia nell'Antichità* (Rome 1991) 30–53. Ethos became such a major concern in fourth-century and much subsequent music theory that the author of the Hibeh papyrus (13–17), Sextus Empiricus (e.g. 143 Greaves (#16)), and Philodemus felt compelled to attack the entire notion. On the latter, see L. P. Wilkinson, "Philodemus on *Ethos* in Music," *CQ* 32 (1938) 174–81.

Aristides Quintilianus has a much greater concern than does Ptolemy for traditional ideas of ethos. For Aristides each individual note, let alone whole musical systems and national musics, has an ethical quality and effect; cf. Mathiesen, *Aristides Quintilianus* 40–47; Solomon, "*Tonoi*" 248–50; and W. G. Anderson, *Ethos and Education in Greek Music* (Cambridge MA 1955) 98–99.

[225] In fact, the *ethe* were rarely "generally agreed upon." Pl. *Resp.* 339b labels the Phrygian harmonia as sober and resigned, Arist. *Pol.* 1341A as ecstatic and purgative. Cf. L. P. Wilkinson, "Philodemus on *Ethos* in Music," *CQ* 32 (1938) 177.

[226] Aristox. *Harm.* 63.2–.16.

[227] For a summary, see Palisca 214–16.

nor again in making the intervals at the bottom note of the intense diatonic and tonic chromatic equal, while [the diatonic is][228] greater than the chromatic.

I.15 - *On the Rational and Audible Tetrachordal Division By Genus*

[33.1] Now since they[229] have not divided the first genera of the tetrachords in a manner agreeable to the perceptions, let us ourselves attempt here to complete the research into what concurs with the theory about the emmelic intervals and what appears to the perceptions. And let us do this in agreement with the first and natural assumptions about divisions.[230] [33.5] We assume for positions and arrangements of the magnitudes in accordance with the original hypothesis and reason, something in common for all the genera, that contiguous notes in the tetrachords always form superparticular ratios with each other,[231] ratios cut into no more than two or three nearly equal parts which define the differences between the first consonances, are valid up to three in number, and[232] are thereby also capable of producing all its intervals.

[33.11] We start with the homophone of the diapason and of the double ratio, wherein the difference between the outer limits is equal to their remainder; in the division into two equal parts, the sesquialter ratio of the

[228] Düring (197) offers several possibilities here, the most expeditious of which is this supplement.

[229] One group of manuscripts (the G manuscripts) offers "the Pythagoreans" as a supplement here, but the reference is to a broader group of music theorists.

[230] After fourteen chapters of definition and critical analysis of his predecessors, Ptolemy now advances his own examination of the homophones, consonances, emmelic notes and the division of the tetrachord. Cf. I.7 (15.3–.18), Boeth. *De mus.* 5.19, and Bry. III.8 (334.4f.). For the version of Franchino Gaffurio (and Nicolò Leoniceno), cf. Palisca 218–23.

[231] Ptolemy does not contradict his arguments or his criticisms outlined in chapter six of Book I. There he took issue with the Pythagoreans because they insisted that the consonances, e.g. the diapason plus diatessaron, be described by a superparticular ratio. Moreover, that was an instance in which a different principle, the juxtaposition of consonant intervals, overcompensated for the arbitrary dictum that the compound interval had need of a superparticular ratio. Ptolemy's insistence on superparticularity has evoked some modern criticism, e.g. Bower, "Boethius and Nicomachus" 4–5.

Interestingly, Annie Bélis, "Auloi grecs du Louvre," *BCH* 108 (1984) 14–22, has recently investigated a double aulos, the two pipes of which contain fourteen different intervals, all but two of which are superparticular:

32:27	11:10	9:8	8:7	7:6	16:15	9:8	7:6
32:27	9:8	14:13	9:8	8:7	18:17		

The last interval, 18:17, is very close to the leimma of 256:243.

[232] *Contra* Alexanderson (13), who overlooked Ptolemy's tripartite division of this final clause ('and...and...and'). 'And are thereby' does not refer to "in the three first consonances," as A. claims, but to the fact that these propositions belong to the first consonance, 'and' are also three in number, 'and' therefore are capable of effecting all the intervals.

diapente consonance is taken, wherein the difference between the outer limits embraces half of the remainder; [33.15] the sesquitertian ratio of the diatessaron consonance, wherein the difference between the outer limits embraces a third part of the remainder; and in moving away from equality the triple ratio is taken from the diapason plus diapente consonance, [33.18] wherein the difference between the outer limits makes two remaining parts (the reciprocal of the half part); and the quadruple ratio of the double diapason homophone, wherein the difference between the outer limits makes three remaining parts (the reciprocal again of the third part).[233]

[33.22] Agreeing with our sense perceptions, we assume as an equally common element in the three genera that the following intervals of the three magnitudes are smaller than each of the remaining intervals; [33.24] and it is particular to the genera which have pykna that the two bottommost intervals be smaller than the topmost interval, while it is particular to the apyknon genera that no interval be greater in magnitude than that of the two remaining intervals.[234]

[33.28] Once we have established this, we then divide first the sesquitertian ratio of the diatessaron consonance, as much as is possible, into two superparticular ratios. [33.29] Again we can do this only three times, taking three lesser superparticular ratios in succession: 5:4, 6:5, and 7:6, for these fill the sesquitertian ratio when 16:15 is added to 5:4, 10:9 to 6:5, and 8:7 to 7:6; after these last two, we would not find the sesquitertian ratio constructed from any others.[235]

[34.5] In the genera which have the pyknon, since the leading ratios in them are greater than both the remaining ratios, we apply[236] to their leading ratios the greater ratios of those pairs described above, that is 5:4, 6:5, 7:6, and the remaining and smaller ratios, that is 16:15, 10:9, and 8:7, to both the remaining intervals.[237] [34.10] The division of each of these two 'following' intervals is accomplished in three parts since the tetrachords in this case are also divided into three parts; the differences are observed to be equal but the ratios nearly equal (since they cannot be equal).

[34.14] We will take the first numbers which make the ratios 16:15, I mean by this 15 and 16, and we will triple them to make 45 and 48, and make middle numbers between them with equal excesses, 46 and 47. [34.17] Since 47 makes no superparticular ratios with either of the outer numbers,

[233] 2/1 - 1 = 1; 3/2 - 1 = 1/2; 4/3 - 1 = 1/3; 3/1 - 1 = 2 (x 1/2 = 1); 4/1 - 1 = 3 (x 1/3 = 1)

[234] Cf. 29.5ff.

[235] Where 5:4 x 16:15 = 6:5 x 10:9 = 7:6 x 8:7 = 4:3. Ptolemy has already used up the first three superparticular ratios to construct the fixed notes, so he now proceeds with the next lowest available triad, 5:4, 6:5, and 7:6. The next ratio, 8:7, would have to use 7:6 again, which is larger; 9:8 would require 32:27, which is not superparticular. This passage describes the divisions made by the "ancients" to whom Bry. (III.8 (334.19f.)) refers; cf. 336.19f.

[236] I follow Höeg in reading the improved verb form (ἐφαρμόσομεν), also adopted by Düring in 1934.

[237] 16:15 is the major half tone, 10:9 the minor tone (or minor whole tone), and 8:7 the septimal tone.

only 46 in relation to 48 makes the ratio 24:23, while in relation to 45 it makes the ratio 46:45; according to our earlier hypothesis, the greater ratio 24:23 is added to 5:4 while the remaining 46:45 fills out the following ratios.[238]

[34.21] Again we will take the first numbers which make the ratio 10:9, that is 10 and 9, and triple them to make 27 and 30, while the middle numbers of these with equal excesses will be 28 and 29. [34.23] But 29 does not form a superparticular ratio with the two outer numbers, while 28 in relation to 30 makes the 15:14 ratio and to 27 the 28:27 ratio, so that here the 15:14 ratio is added to the 6:5, and 28:27[239] is left for the following interval.

[34.27] Similarly we will take the first numbers making the ratio 8:7, that is 7 and 8, and triple them to make 21 and 24, and their middle numbers with equal excesses will be 22 and 23, where 23 does not form a superparticular ratio to the two outer numbers, [34.31] but 22 alone in relation to 24 makes the 12:11 ratio and with 21 the 22:21 ratio, so the 12:11 ratio will be added to the 7:6 ratio, and 22:21 will hold the following locus.

[34.33] And since the enharmonic is the softest of all the genera, there is almost a path through increases in the intervals from it to the more intense genera, first through the soft chromatic, then to the more intense chromatic up to the succeeding apykna of the diatonic. [35.1] Generally speaking, those genera which have the greater leading interval are the softer, while the more intense have it smaller.[240]

[35.3] We will apply the tetrachord constructed from the 5:4, 24:23, and 46:45 ratios to the enharmonic genus; that composed from the 6:5, 15:14, and 28:27 ratio to the soft chromatic; that constructed from the 7:6, 12:11, and 22:21 ratio to the intense chromatic.[241] [35.7] The first numbers which make up the outer limits of these three tetrachords are the same for each,

[238] This and the subsequent paragraphs all describe different divisions of the tetrachord (and the diatessaron consonance). Barbera, "Divisions" 304–305, offers a logical clarification: Ptolemy attempts to improve on the work of his predecessors by having a consistent and rational method of deriving the tetrachordal subdivisions. After selecting the lowest remaining superparticular ratio (5:4) and that ratio which added to it will yield 4:3, the pyknon 16:15, he subdivides the pyknon by multiplying each of its components by three (16 x 3 = 48; 15 x 3 = 45) and then forming a superparticular ratio with the tripled original denominator as denominator (46:45). This is the lowest interval. The middle interval he then derives simply by subtracting this lowest (and smallest) interval (46:45) from the entire pyknon 16:15 (=24:23). Düring (199), following H. Riemann, *Handbuch der Musikgeschichte I: Altertum und Mittelalter* (Leipzig 1923) 237, finds these calculations quite impractical, yet he admits of their profound and centuries-long influence.

[239] The pseudo-Aristotelian *Problems* 19.3 verifies the reality of such microtones: the author there admits how difficult these intervals are to sing because of "compression of the voice."

[240] This passage, *contra* Düring 199, preserves typical Ptolemaic prose.

[241] Mountford, "Harmonics" 82, discusses the reasons for the discrepancy between Ptolemy's and Archytas' chromatic divisions, to which, again, should be added that there is a four-century gap between the two theorists.

106,260, and 141,680.[242] Belonging to the second from the leading intervals are 132,825, 127,512, and 123,970. For the third are 138,600, 136,620, and 135,240, as the table shows:[243]

enharmonic	soft chromatic	intense chromatic
106,260 - - -	106,260 - - -	106,260 - - -
5:4	6:5	7:6
132,825 - - -	127,512 - - -	123,970 - - -
24:23	15:14	12:11
138,600 - - -	136,620 - - -	135,240 - - -
46:45	28:27	22:21
141,680 - - -	141,680 - - -	141,680 - - -

[35.13] In the genera without pykna, consequential to the previous analyses, the smaller of the ratios from the primary division into two parts in the sesquitertian ratio must be placed again in the leading locus, while those contiguous greater ratios[244] must be divided in the same way into the two following loci; the 16:15 ratio is not found to be capable of standing in the leading locus. [35.18] For if we again triple the numbers which comprise the remaining interval, the 5:4 ratio, that is 4 and 5, so as to get 12 and 15, and if numbers which fit between them with equal excesses are 13 and 14, since 13 makes no superparticular ratio with either of the two outer numbers, [36.3] and since 14 in relation to 12 makes the 7:6 ratio, and to 15 the 15:14 ratio, we can place neither of these in the following position. They will be greater

[242] For the capability of the Greeks to generate symbols for such larger numbers, see W. F. Richardson, "The Greek Number System," *Prudentia* 9 (1977) 15–26; O. Neugebauer, *Exact Sciences* 25–27; Newton, *Crimes* 17–20; and Lucas N. H. Bunt, P. Jones, and J. Bedient, *The Historical Roots of Elementary Mathematics* (Englewood Cliffs NJ 1976) 66–68. Wilbur Knorr, "Techniques of Fractions in Ancient Egypt and Greece," *Historia Mathematica* 9 (1982) 133–71, discusses the use of unit fractions and relevant computing techniques, which one can then use with interest in reading Robert R. Stieglitz, "Minoan Mathematics or Music?" *Bulletin of the American Society of Papyrologists* 15 (1978) 127–32, who claims to have found a second-millennium B.C. monochord symbol and a Timaeus-like progression in a graffito from Hagia Triada. See also, Marcus N. Tod, "The Alphabetic Numeral System in Attica," *Annual of the British School at Athens* 45 (1950) 126–39 (reprint in Tod, *Ancient Greek Numerical Systems* (Chicago 1979) 84–97).

[243] Düring (51) has a misprint in the table of his German translation: read '136620' for his "132620."

[244] Reading the accusative τοὺς δὲ μείζονας; cf. Alexanderson 13–14. The greater ratios would be 5:4, 6:5, and 7:6; the smaller would be 16:15, 10:9, and 8:7. For when they are contiguous, see 34.4.

than that in the leading position, that is 16:15,[245] and this would be contrary to clarity and to our original reasoning.

[36.6] Once 8:7 is arranged as the leading position, the first numbers embracing the remaining interval 7:6, 6 and 7, when tripled make similarly 18 and 21, with the numbers between them of equal excesses being 19 and 20. [36.9] Once more, 19 does not form a superparticular ratio with either of the outer numbers, while 20 in relation to 18 is 10:9 and to 21 is 21:20. Of these similarly the greater, 10:9 is added to the 8:7, and the smaller, 21:20, fills out the following ratio.[246]

[36.13] Once 10:9 is arranged in the same way in the leading position, if the numbers embracing the remaining interval 6:5, 5 and 6, when tripled make 15 and 18, and if numbers which fall between them with equal excesses are 16 and 17,[247] [36.17] 17 does not form a superparticular ratio with both outer numbers, while 16 in relation to 18 is 9:8, and to 15 is 16:15, so that the greater, 9:8, is added to 10:9, and we apply the remainder, 16:15, to the following position.

[36.20] But before all these ratios, 9:8 is found by itself from the difference between the two first consonances to encompass the whole tone. It ought, by reason and necessity, to hold down the leading position of those which are added next to it since in conjunction with it no superparticular ratio fills up the sesquitertian ratio. [36.25] The 10:9 ratio is added to it in the preceding division, but 8:7 is not, wherefore we add this to it in the middle position, and what remains of the sesquitertian ratio 4:3, that is 28:27, we will allocate to the following position.[248]

[36.28] Thus in accordance with the size of the leading positions we will assign the tetrachord constructed from 8:7, 10:9, and 21:20 to the soft diatonic,[249] that constructed from 10:9, 9:8, and 16:15 to the intense diatonic,[250] that constructed from the 9:8, 8:7, and 28:27 to that between the

[245] Cf. 33.28–34.4.

[246] Ptolemy would normally have made his lowest interval 19:18, but subtracting this from the pyknon (7:6) produces 21:19, not a superparticular ratio. See Barbera, "Divisions" 305–306.

A number of manuscripts have 'locus' (τόπον) for 'ratio' (λόγον), a substitution often found in Ptolemy; cf. Düring (199) but also Höeg, review of Düring, 656.

[247] Cf. Höeg, review of Düring, 657.

[248] Ptolemy changes his method a bit for the tonic diatonic because multipying the parts of the pyknon (4:3 - 9:8 = 32:27) by three would yield a superparticular ratio (85:84) too small for musical practicality. David Kraehenbuehl and C. Schmidt, "On the Development of Musical Systems," *JMT* 6 (1962) 51, explain the theoretical background.

[249] K. Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *CQ* 27 (1933) 95, finds Ptolemy's soft diatonic (in cents: 85 + 182 + 204) quite similar to Aristoxenus' (102 + 153 + 255), and identifies each as an aulos scale.

[250] 16:15 is somewhat larger than a half tone. Crocker, "Pythagorean Mathematics" 193f., discusses it and its neighboring intervals (18:17 and 17:16). Vogel, *Enharmonik* 82–83, compares this Ptolemaic division with Archytas' enharmonic and discusses (48–49) Ptolemy's diatonic divisions in general.

soft and intense which is logically called 'tonic' on account of the leading position's being of just that size.[251]

[36.35] Encompassing the limits of these three tetrachords are the first common numbers 504 and 672; particular to the second from the leading interval are 576, 567, and 560; and to the third are 640, 648, and 630. The chart shows this:

soft diatonic	tonic diatonic	intense diatonic
504 - - -	504 - - -	504 - - -
8:8	9:8	10:9
576 - - -	567 - - -	560 - - -
10:9	8:7	9:8
640 - - -	648 - - -	630 - - -
21:20	28:27	16:15
672 - - -	672 - - -	672 - - -

[37.5] That these divisions of the genera are not only in accord with reason but are also consonant with our perceptions will be again recognizable with the help of the eight-stringed canon which encompasses a diapason. [37.7] The notes will be accurate, as we said,[252] so long as the strings are of equal quality and equally tuned. Once the movable bridges are set up alongside the divisions marked by the calibrators we are using in a manner which accords with the ratio in each genus, the diapason will be tuned in such a way that not even the most skilled musicians would be disturbed. [37.13] They would wonder at the natural coordination of what was tuned,[253] for reason by nature configures it as if it were giving form to the differences which preserve the music.[254] [37.15] The hearing obeys its authority as much as possible since the hearing is so disposed to proportional

[251] The size is that of the tonos or 'whole tone.' To summarize:

soft enharmonic	Intense chromatic	soft chromatic	tonic diatonic	ditonic diatonic	intense diatonic	even diatonic	diatonic
46:45	28:27	22:21	21:20	28:27	256:243	16:15	12:11
24:23	15:14	12:11	10:9	8:7	9:8	9:8	11:10
5:4	6:5	7:6	8:7	9:8	9:8	10:9	10:9

For cents values for all of these divisions, see Barbera, "Divisions," 311–13.

[252] In I.8 (17.20f.).

[253] 'What was tuned' is a translation based on Cleon. 179.4, where 'what is tuned' (ἡρμοσμένον) signifies "that composed of both notes and intervals having some arrangement."

[254] Ptolemy's first use of the word 'melos'; (Düring's citation in the index has a misprint: 37.5 for 37.15).

arrangement and recognizes what is particularly suitable in each. And it will condemn those who subscribe to this approach but are either not able to apprehend the proper proportional divisions or do not deem worthy of discovery those[255] which perception reveals.[256]

I.16 - *How Many and Which Genera Are More Familiar to the Hearing*

[38.2] Of the genera we have treated, we would find all the diatonic familiar to the hearing, but this would not be so for either the enharmonic or the soft chromatic since [our ears] do not delight[257] in the genera which are too slack in character. It is enough for them in crossing over[258] towards the soft to reach the intense chromatic. [38.6] For the pyknon,[259] by which the nature of the soft in comparison to the intense is more or less defined, has its upper limit with this genus.[260] It begins here in the path towards the softer and ends here again in the path to the more intense.

[38.9] And furthermore, when the entire tetrachord is cut into two ratios, the intense chromatic is divided by the contiguous ratios nearest equality, that is 7:6 and 8:7, and they divide into two the whole excess between the extremes of the ratio. Wherefore on account of what has been said before this genus seems most suitable to the hearing.

[38.13] Another genus is suggested[261] to us when we begin with the emmelic interval standing near equality and search for some suitable arrangement of the diatessaron by dividing it now into three almost equal ratios with, once again, equal excesses. [38.17] Such a genus is composed from the ratios 10:9, 11:10, and 12:11.[262] As before, when the first numbers

[255] Düring (200) renders merely as "divisions."

[256] A final criticism of the Aristoxenians and Pythagoreans, respectively. J. G. Landels, "Music in Ancient Greece," *Proceedings of the Classical Association of Greece* 77 (1980) 24, reports an attempt at testing these divisions.

[257] The subject of "delight" is not actually given in the Greek. Reference is again made to it, however, in the initial pronoun of the next sentence.

[258] Cf. 34.34f.

[259] Reading with Höeg, review of Düring, 656.

[260] Cf. Aristox. *Harm.* 51.20 (64.4–.7 [da Rios]).

[261] Ptolemy's purpose here has branched to a discussion of the diapente consonance and its divisions. He digresses somewhat simply to point out the superparticularity of the potential diapente formed with the 10:9 x 11:10 x 12:11 tetrachord when combined with a disjunct 9:8. Bower, "Boethius and Nicomachus" 22, correctly describes this last division as "almost an afterthought."

[262] Winnington-Ingram, "Aristoxenus and the Intervals" 204, equates the (relative) pitch resultant from the (undecimal) 12:11 ratio with that from the geometrical 3/4 tone of the hemiolic chromatic and the soft diatonic. The former can be calculated to 151 cents, the latter to 148. Because this interval represents the old Spondeiasmos (cf. Aristides Quintilianus 28.1–7 and [Plut.] *De mus.* 1135A–B) and is to be found again in Aristoxenus and Ptolemy, Winnington-Ingram proposes that the interval "is unlikely to have been pure invention by Ptolemy." Cf. also, Winnington-Ingram, *Mode* 21–30 and 64, but *contra*, Burkert, *Lore and*

indicating the 4:3 ratio are tripled, they make both the successive numbers 9, 10, 11, and 12 and the aforementioned ratios.

[38.21] Then[263] with the greater ratios arranged first there appears a tetrachord more similar to the intense diatonic in itself and still more in the way it fills out the diapente. [38.23] A disjunction added before the leading note forms a sesquioctave ratio, and it produces its characteristic equality not only with the three excesses but also with the four which are encompassed by the successive ratios from 9:8 to 12:11. [38.27] When the disjunction is placed in the middle, making up the resulting diapason are the first numbers 18, 20, 22, 24, 27, 30, 33, and 36.[264]

[38.30] When the division arranged according to these numbers is set up on the equally-tuned strings, the character will seem rather strange and boorish but otherwise mild and more appealing for the hearing, so that it would not properly be overlooked on account of the particularity of the music and the arrangement of the division. [38.33] Still, when tuned by itself it does not offend the perceptions, and this is true of almost solely the middle diatonic genus.[265] The others by themselves, meanwhile, fitted together by force, can proceed in mixing with the aforementioned diatonic when the tetrachords[266] softer than it are used below the disjunctions, the more intense above them. [39.5] Let this genus be called the 'even diatonic' because of its characteristics.[267]

[39.6] Now we proceed with the examination of the other, more familiar genera. The middle, that is, tonic diatonic, when arranged by itself and unmixed, fits at the sterea in the lyre and to the tuning of the tritai and

---

Science 385. Similarly, M. I. Henderson, "The Growth of Ancient Greek Music," *The Music Review* 4 (1943) 4–13, attempts to reconstruct diatonic forms of the fourth-century scales preserved in Aristides.

K. Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *CQ* 27 (1933) 90, discusses briefly the 3/4 tone found in aulos borings, and then identifies this particular Ptolemaic tetrachordal division as a "four quanta" aulos scale still used in both the Islamic and Christian traditions of Anatolia; cf. John Tzetzes, *Über die altgriechische Musik in der griechischen Kirche* (Munich 1874) 30, 53, 77, 89, 93, etc.

263 Cf. Lohmann Musike 85.

264

18		20		22		24		27		30		33		36
	10:9		11:10		12:11		9:8		10:9		11:10		12:11	

Ptolemy's purpose here has branched to a discussion of the diapente consonance and its divisions. He digresses somewhat to point out the superparticularity of the potential diapente formed with the 10:9 x 11:10 x 12:11 tetrachord when combined with a disjunct 9:8.

265 The scholiast points out that this is "the tonic diatonic."

266 The chromatic and the soft diatonic. (scholion).

267 The Greek word ὁμαλόν (*homalon*) means 'even, uniform, equal.' Düring (200–201) demonstrates the usefulness of this scale as a "ground scale" on which the thetic and dynamic notations are based and from which the thetic then varies (in pitch) from scale to scale. In the Dorian, of course, both are identical. M. L. West, *Ancient Greek Music* (Oxford 1992) 170, offers the even diatonic and Ptolemy's six other divisions of the tetrachord in cents and "modern notes with appropriate modifiers."

hypertropa in the cithara.[268] [39.10] That aforementioned, self-contained mixing of the intense chromatic fits the soft in the lyre, the tropika in the cithara. [39.11] The mixing of the soft diatonic with the tonic[269] fits to the parhypatai in the cithara, the mixing of the intense diatonic with the tonic fits the modulating ethe, which the citharodoi call Lydian and Iastian, except when they sing according to the already demonstrated intense diatonic, as will be possible to examine by comparing its own ratios.[270]

---

268 The scholion adds "the fixed and immovable" to describe the word 'sterea.' All three of the terms used in this paragraph—'sterea,' 'tritai,' and 'hypertropa'—are organological in origin and function. They appear in the extant Greek literature only rarely, for ancient Greek music theorists did not regularly apply their science to the actual production of music; at least this was not their primary interest. Appearances of organological terms are so uncommon, in fact, that scribes at times confused the term 'tritai' for the word for the number three and at times for the word 'tropai'; cf. Monro 102. I concur with the evaluation by Düring 203 and Barbera, *Pythagorean Mathematics* 335, that the text must be corrupt here or that Ptolemy badly represented the "actual" musical practices of his day; *contra* Barker, *GMW* II, 357–60, who praises Ptolemy for having professional musicians evaluate his theoretical systems.

M. L. West, *Ancient Greek Music* (Oxford 1992) 171–72, offers the six variations of tetrachords, with some appended discussion, in modern notation. Ptolemy will return to these terms in Book II.

269 Winnington-Ingram, "Aristoxenus and the Intervals" 202, observes the significance in Ptolemy's mixing the tonic diatonic, particularly with its 8:7 ratio, with other tetrachordal divisions. It may reflect the persistence of that interval in actual musical practice from the era of Archytas in the fourth century B.C. to the era of Ptolemy in the second century A.D. He reminds us elsewhere ("Greece," VII 664–65) of our relative unfamiliarity with all the septimal intervals—the septimal minor third (7:6), the septimal semitone (28:27), and this septimal tone (8:7).

Similarities between the divisions of Archytas and Aristoxenus must be kept in historical perspective, however, since our mistaken tendency is to divorce entirely members of one tradition from another tradition. Nonetheless, as different as the methodology of deriving and stating tetrachordal divisions and methodologies may have been between Archytas and Aristoxenus, our evidence is that Aristoxenus' father, Spintharus, a scholar of music, was born in Tarentum and may well have known Archytas; Aristoxenus is also reported to have written, among his over-400 books, a biography of Archytas. After pointing this out, Flora R. Levin, "'*Synesis*' in Aristoxenian Theory," *TAPA* 103 (1972) 211–34, restores Aristoxenus to a position of some respect as a music theorist after the lambasting he has taken from the Pythagoreans, Ptolemaians, and their modern annotators and advocates. Similarly, I. Henderson, "Ancient Greek Music," *Ancient and Oriental Music*, ed. Egon Wellesz (London 1957) 342–43, points out that the perception of musical pitches as linear points along a spatial line originates not with Aristoxenus but with the *Harmonikoi* whose work he refutes. Crocker, "Aristoxenus" 96, in addition makes the argument (supported by Barbera, *Pythagorean Mathematics* 98f.) that Aristoxenus was not an empiricist ignorant of mathematics but an exponent of a revolutionary, exciting, and influential new theory which would (in the non-musicological field) be soon recognized as every bit as rigorous as the Pythagorean approach—Euclidean geometry. It should also be remembered that Aristoxenus developed his findings despite the Pythagorean tradition to which he must have been exposed in Tarentum, including his early training with the Pythagorean Xenophilus. "Aristoxenus' new kind of music theory was designed—like the new geometry—to handle intervals without reference to numbers or ratios" (99).

270 See Burkert, *Lore and Science* 387, n. 7. Düring (201–215) offers a thorough discussion of the various usages on lyre and cithara of the genera just described. He also compares the descriptions in I.16 [misprint on p. 201 @ I], II.16, and II.1 and 2.

[39.16] They[271] assemble another genus[272] resembling that one,[273] yet clearly different, for they make two leading whole tones and the remaining interval, as they reckon it, a half tone, but as reason posits, that known as the leimma. [39.19] Their method succeeded for them because they did not differentiate by any substantial amount either in the ratio in the leading positions, 9:8 from 10:9,, or that in the following positions, 16:15 of the leimma.

[39.22] For if we take of the number 72 the 10:9 and 9:8 ratios, the former yields 80 and the latter 81. And 9:8 of 10:9 will be 81:80. This very ratio is that of the ditone, that is of twice 9:8 in ratio to 5:4, which is the highest of the enharmonic genus. [39.26] For of the number 64, the 5:4 ratio makes 80, while twice 9:8 makes 81. Similarly, since the ratio of the leimma is 256:243, 16:15 of 243 is 259, and the ratio of 16:15 to the leimma is 259:256. The same again is 81:80, and this is because the 5:4 ratio is equal to both the 9:8 and 10:9.[274] [40.2] Wherefore, in neither of the genera set out does any significant offense occur when they misuse in the intense diatonic the 9:8 ratio instead of the 10:9 for the leading position and the leimma instead of the 16:15 for the following position, and in the enharmonic the doubled 9:8 ratios instead of the 5:4 for the leading position and the leimma again instead of the 16:15 for both following positions.[275]

[40.9] Let us now allow also this genus on account of the ease of modulations from the tonic genus to a mixture with the tonic,[276] and because the ratio of the leimma has some relationship to the diatessaron and the whole tone in contrast to the other non-superparticular[277] ratios, since by necessity it follows upon the two 9:8 ratios occurring in the 4:3 ratio.

[271] The transition here is quite abrupt. The scholiast points out to us that Ptolemy has returned to criticism of the Aristoxenians, but the positions criticized here, particularly at 40.2f., are not orthodox.

[272] The ditonic diatonic (scholion); cf. 29.11. See here also, B. van der Waerden, "Harmonielehre" 187-91.

[273] The intense diatonic (*syntonon diatonon*).

[274] When "added" together, i.e. 5/4 = 9/8 x 10/9.

[275] Winnington-Ingram, "Aristoxenus and the Intervals" 200, n. 2, discusses the resulting scale as most preferable for practical use because of its avoidance of major and minor thirds.

28:27 x 8:7 x 9:8 x 9:8 x 256:243 x 9:8 x 9:8

For the (partial) versions of this passage treated by Gaffurio and Leoniceno, cf. Palisca 221.

[276] Ditoniaion (Düring [215]).

[277] The negative in 'non-superparticular' is omitted in a later part of the manuscript tradition; cf. Düring 215. Where there is a relationship between the leimma (256/243), the diatessaron (4/3), and two whole tones (9/8), each of these ratios is superparticular except the leimma. Therefore in the equation 9/8 + 9/8 + 256/243 = 4/3, the leimma has a relationship to these superparticular ratios in contrast to other non-superparticular ratios, which do not fit well into such or similar equations.

[40.14] The leimma just like the whole tone will be found somehow by itself and through the consonances, the latter from the excess between the first two consonances, the former from the excess between the ditone and the diatessaron consonance. [40.17] This genus is described by the first numbers 192, 216, 243, and 256. It would rightly be called ditonic diatonic since it has the two leading ratios of a whole tone.[279]

[279] With its two whole tones, the ditonic diatonic is then similar to at least that part of the Aristoxenian diatonic tetrachord. By containing the leimma it differs from the Aristoxenian model, of course, but it also strays from Ptolemys guideline (33.5f.) that all divisions of the tetrachord must be superparticular. Cf. Gombosi, *Tonarten* 100-10, and Gevaert, *Histoire et théorie I* 325-27.

BOOK II

Summary

1. How the Ratios of the Customary Genera Might Be Recognized as Well By the Senses.
2. On the Use of the Canon in Comparison With the Instrument Called the Helicon.
3. On the Species of the First Consonances.
4. On the Perfect System and That It Alone Contains the Double Diapason.
5. How the Names of the Notes Are Derived By Position and By Function.
6. How the Conjunct Magnitude, the Diapason Plus Diatessaron, Has the Reputation As a Perfect System.
7. On the Modulations of What Are Called *Tonoi.*
8. That It Is Necessary That the Limits of the Tonoi Be Defined By the Diapason.
9. That It Is Necessary For Only Seven Tonoi To Be Proposed, a Number Equal to the Species of the Diapason.
10. How the Differences Between the Tonoi Are To Be Taken Properly.
11. That Tonoi Ought Not To Be Increased By Means of the Halftone.
12. On the Difficulty of Using the Monochord Canon.
13. On What Additions the Musician Didymus Thought To Make For the Canon.
14. Table of the Numbers Making the Division of the Diapason In Both the Ametabolic Tonos and the Genera in Each.
15. Table of the Numbers Making the Divisions of the Customary Genera in the Seven Tonoi
16. On Realizations Made on the Lyre and Cithara.

II.1 - *How the Ratios of the Customary Genera Might Be Recognized As Well By the Senses*[1]

[42.1] Let us next examine but in a different way the same measurements of those genera which are customary[2] and easily manageable[3] for the hearing. We will not do this as we just did,[4] producing the differences from the proper ratios alone and then testing them on the canon against the evidence of what appears to the hearing, [42.4] but in the opposite manner, first setting forth the tunings from the senses alone and then demonstrating from them the ratios appropriate to the equalities or excesses of the notes of each genus.[5] [42.8] We assume here only those ratios agreed upon utterly by all, that the consonance of the diatessaron consists of the sesquitertian (4:3) ratio, the whole tone of the sesquioctave (9:8).[6]

[42.10] Of the tetrachords realized by cithara players,[7] first let the diatessaron from nete to paramese, which they call 'tropai,'[8] be made as

---

[1] Düring reinterprets this chapter heading as "Genera which are Pleasing to the Ear Because of their Character," which is excessive. Ptolemy uses the adjective συνήθης throughout the treatise to describe the 'usual/regular/customary' genera used by practicing musicians, e.g. at 80.11f. These genera are indeed pleasing to the ear, as Ptolemy intends to demonstrate, but that fact is not implicit in the title. In the first sentence (42.2) Ptolemy specifies that they are "genera easily manageable by the ears" with different vocabulary (εὐμεταχειρίστων ταῖς ἀκοαῖς γενῶν).

[2] By citing the customary genera Ptolemy designates here the intense chromatic, tonic diatonic, the ditonic diatonic, intense diatonic, and soft diatonic. At I.16 (38.2f) he eliminated the enharmonic and soft chromatic. Porph. (152.1–6) refers us to Arist. *De sensu et sensibili* 439b30, an interesting *locus parallelus* describing the *logos* of color blending in 3:2 and 4:3 ratios.

[3] It is not clear whether the word 'manageable' (εὐμεταχειρίστων), literally, 'handy,' is intentionally ironic.

[4] In I.8.

[5] Notice that Ptolemy has been careful not to say he will begin with the actual sounds, i.e., that he will pluck them, as it were, out of mid air, but that he takes the 'tunings.' He does this by using the genera customarily used by cithara players, whose terminology he introduced in I.16 (39.6).

[6] As established in I.5.

[7] Traditionally *citharodoi* both sang and accompanied themselves on the cithara, while *citharistes* played without singing; cf. Michaelides 171–2. Ptolemy uses the term *citharodoi* here, which suggests that his actual musical experience included more than instrumental recitals. He was a contemporary of Mesomedes, several of whose choral hymns survive in a manuscript tradition.

[8] Tropai ('turning') is a term used in playing the cithara; cf. II.16. The scholiast explains that the term is derived from the 'turning' of the diezeugmenon tetrachord into the synemmenon and vice versa, "the result of turning the tetrachord of these with disjunction [i.e. diezeugmenon tetrachords] into that of conjunction [i.e. synemmenon] and back again into that of the disjunctions"; Cf. II.6 and Wallis 47; *contra*, Barker, *GMW* II, 360. Porphyry (154.2) adds here that this form of modulation was the most popular, and we indeed have examples of such modulationd in the Second Delphic Hymn, *Pap. Wien* 29825 a/b recto, *Pap. Zenon* 59533, *Pap. Oslo.* 1413, and *Pap. Oxy.* 2436.

Ptolemy here assumes the tuning of the *citharodi* to be valid as actual music, and we are grateful to him for preserving some of what little information we know about them. But can one

ABCD, with A positioned at the nete. [42.13] I say that contained within it is the aforementioned intense chromatic genus,[9] and first that the ratio of AB is 7:6, that of BD 8:7—those of BC and CD will be shown below—[42.15] and then that each of AB and BD will be found to form a magnitude greater than a whole tone, that is, greater than the 9:8 ratio, and that of AD will be 4:3.[10]

[43.1] No two ratios[11] greater than 9:8 fill out the 4:3 ratio except 7:6 and 8:7, so of the ratios AB and BD one will be 7:6, the other 8:7. Let there be H equal in pitch to B, and from it let there be a higher tetrachord EFGH similar to ABCD. [43.5] A will therefore be found to be higher than F—B and H were of equal pitch—and so the ratio of AB will be greater than FH. But the ratio FH is assumed to be the same as BD; therefore the ratio AB is greater than BD, so AB is 7:6, BD 8:7.[12]

[43.9] Again, with the tetrachord ABCD remaining, let F be of equal pitch to B, and with this pitch fixed let there be a diatessaron of the sterea[13] from paramese to the chromatic lichanos, that is EFGH, with H positioned at paramese.[14] [43.12] I say that contained within it is the genus of the tonic

---

assume any consistency in the tunings of various professional *citharodi*? Ptolemy suggests a consistency in their relative-tuning—not any absolute pitch—which turns out to be the eqivalent of the *syntonon* ('intense') *chroma*(tic.); cf. Gombosi, *Tonarten* 105. Gevaert, *Histoire et théorie II* 267-70, comments on possible playing techniques by the *citharodi*.

[9] *Chroma syntonon.*

[10] The other divisions of the tetrachord accepted by Ptolemy consist of 9:8 and 32:27 (tonic diatonic and ditonic diatonic), 10:9 and 6:5 (intense diatonic and even diatonic), and 8:7 and 7:6 (soft diatonic). In the intense chromatic it is the 8:7 ratio which is subdivided; in the soft diatonic it is the 10:9.

[11] Wallis (48) specifies and adds *superparticulares*.

[12] Both FH and BD are 8:7. Using Wallis' chart, with A (nete) = 90, and D (paramese) = 120: (120/90 = 4/3), 8/7 = 120/x, where x = 105 = B, 7/6 = 105/y, where y = 90 = A.

If the distance AB were less than or the same as FH, then either AD would not form a diatessaron or the internal ratios of EFGH would not be superparticular and similar to ABCD. Wallis' whole numbers show how B = 105 and how BA (=15) > H = 13 1/8 (105-91 7/8).

Intense Chromatic (=Tropai)

				E------(78 3/4)	
	nete	=	A------(90)		7:6
				F------(91 7/8)	
7:6					
				G------(100 5/22)	
					8:7
	paranete	=	B------(105)	H------(105)	
	trite	=	C------(114 6/11)		
8:7					
	paramese	=	D------(120)		

[13] For sterea, cf. I.16 (39.6). Scholion: "He calls 'sterea' the fixed notes and those not turned as those called 'tropai' ('turning')." Wallis (48) adds that 'sterea' are tetrachords [in succession] which contain a disjunct whole tone. They do not produce modulations as do the 'tropai.'

[14] Gogava rendered *a paramese chromaticum solidorum diatessaron*; Wallis *a Paramese in genera Epichromatico (quod vocant) Solidarum diatessaron*; Düring *ein Tetrachord der Sterea von der Paramese bis zur Chromatike*; Barker "a tetrachord from *paramesē* to *chromatikos*

diatonic, and the ratio EF is the 9:8 ratio, FG 8:7, and GH 28:27. [43.14] The ratio EF forms a whole tone precisely, that is the 9:8 ratio, and G will be found to be of equal pitch with D, so also the ratio FG will be the same as

---

belonging to the *sterea*." The problem that arose here was in understanding Ptolemy's use of χρωματική (*chromatike*). Wallis guessed the existence of an epichromatic genus labeled such by the citharodi, while Gogava supposed the sterea themselves to be chromatic. Düring confuses the issue by highlighting the Hypodorian tetrachord in the *chroma syntonon* (intense chromatic) (204 from the table on 94) to illustrate the tropai, i.e. intense chromatic. There the ratios are 7:6 x 12:11 x 22:21, not as here 9:8 x 8:7 x 28:27. (There is also an unfortunate supplement in the tabular abbreviation 'Hyp.' (for Hyperbolaion) on the last two lines of p. 203.) Düring also had to correct his text here (*chromatike* for *chromatikon* and E for H) in the commentary.

At II.16 Ptolemy says these sterea are encompassed within the tonic diatonic. This can be seen by the chart after II.14 in which the tonic diatonic contains the ratios 9:8 x 8:7 x 28:27, as here. Archytas' chromatic contains the 28:27 ratio but also the 32:27 and 243:224.

By the completion of his lengthy discussion, Düring leaves us with a tonic diatonic tetrachord stretching from paramese to nete with the ratios 9:8 x 8:7 x 28:27.

E nete	
	9:8
F paranete	
	8:7
G trite	
	28:27
H paramese	

Ptolemy's text, however, describes the sterea tetrachord as that "from paramese to the chromatic." Düring correctly albeit late (205) identified *chromatike* as Cleonides' (184-5) (and others') terminology for any lichanos or paranete which is 'movable' to become part of diatonic, chromatic, and enharmonic tetrachords and to identify them specifically. But there are only two intervals between paramese and chromatic lichanos (paramese, mese, chromatic lichanos) and only two between paramese and chromatic paranete (paramese, trite diezeugmenon, chromatic paranete diezeugmenon). Ptolemy says it must equal B (at 105). Could he mean that we should stretch the tetrachord from H (paramese) to F (at chromatic paranete diezeugmenon) and then continue on to nete diezeugmenon?

Düring adds in his comments the phrase ἐπὶ τὸ ὀξύ ('ascending'). The textual corrigendum (H for E) at 43.11 makes H the paramese instead of E. It must be so; at least E cannot be at paramese while F is at chromatic paranete. Perhaps Ptolemy's insistence that F be 'fixed' (ἰσοτῶτες) at B's pitch is of more significance than we have previously given it. Perhaps it is E which is to be equal to B (at 105) and then the diatessaron reaches (ἐπὶ τὸ βαρύ) from (dynamic) chromatic paranete diezeugmenon to mese. After all, the tonic diatonic does contain the whole tone 9:8 between mese and paramese.

Thomas F. Mathiesen, in private correspondence, proposed as an alternative solution not to emend the textual E to H. With E as paramese, F at mese will then properly be 9:8 to E, G will be chromatic lichanos, and H will be parhypate, a fixed note at least as a result of its position in this tetrachord. Using Aristoxenus [34.25-31] as precedent, he uses the term 'chromatic' to refer to the parhypate as well as to the lichanos.

Tonic Diatonic (= Sterea)

nete	= E	
		9:8
paranete	= F	
		8:7
trite	= G	
		28:27
paramese	= H	

the ratio BD, that is 8:7. The ratio GH will be found to be 28:27, which with the 9:8 and 8:7 ratios fills out the 4:3 ratio.[15]

[43.19] Next let there be the diatessaron of what is called the Iasti-Aeolian from trite to diatonic lichanos, ABCD, with A positioned at trite.[16] I say that contained within it is the genus of the ditonic diatonic, in which each

---

15

	nete	=	A------(90)		
7:6				E-----(93 1/3)	
					9:8
	paranete	=	B------(105)	F-----(105)	
	trite	=	C------(114 6/11)		8:7
8:7					
	paramese	=	D------(120)	G----(120)	
					28:27
				H-----(124 4/9)	

[16] Here again the description of the diatessaron is confusing. As with the sterea, the Iasti-Aeolian diatessaron must include four notes, three intervals, but trite diezeugmenon ἐπὶ τὸ ὀξύ ('ascending') to diatonic paranete diezeugmenon is only one interval, to diatonic paranete hyperbolaion is five intervals (trite diezeugmenon, paranete diezeugmenon, nete diezeugmenon, trite hyperbolaion, diatonic paranete hyperbolaion). However, trite ἐπὶ τὸ βαρύ ('descending') to diatonic lichanos meson (trite diezeugmenon, paramese, mese, diatonic lichanos meson) does contain three intervals, two of which need and can be of the 9:8 ratio (trite diezeugmenon—paramese, paramese—mese) required here. With A positioned at trite, the tetrachord descends.

Düring (207-209) defends the text here and then observes that the designations of trite and diatonos are of the dynamic nomenclature. Shifting to the thetic, he reads nete—paramese in the Hypophrygian tonos. The extended Iastian octave (9/8 x 9/8 x 256/243 x 9/8 x 9/8 x 28/27 x 8/7) from dynamic lichanos mese to paranete hyperbolaion parallels exactly the Aeolian (9/8 x 256/243 x 9/8 x 9/8 x 28/27 x 8/7 x 9/8) from mese to nete hyperbolaion one interval lower.

nete hyperbolaion		
	9:8	
paranete hyperbolaion		paranete hyperbolaion
	8:7	
trite		trite hyperbolaion
	28:27	
nete diezeugmenon		nete diezeugmenon
	9:8	
paranete diezeugmenon		paranete diezeugmenon
	9:8	
trite diezeugmenon		trite diezeugmenon
	256:243	
paramese		paramese
	9:8	
mese		mese
	9:8	
lichanos meson		

The cithara player could therefore shift back and forth between these two tunings, whence the name 'Iasti-Aeolian.' Wallis translates, "*a Trite, in genera Diatonico*," suggesting perhaps a conventional tetrachord from trite to trite, i.e., trite hyperbolaion to trite diezeugmenon. In his diagram, however, he illustrates a tetrachord from trite (diezeugmenon) to lichanos.

of the leading[17] ratios is 9:8, the remaining ratio a leimma.[18] [44.1] And this is evident in itself, for the cithara players tune it so that both AB and BC form whole tones, that is, the 9:8 ratio, and that the remaining CD is in the ratio 256:243, which, being smaller than the 19:18 but greater than 20:19, along with the two 9:8 ratios, fills out the 4:3 ratio.[19]

[44.6] But if we form the aforementioned tetrachord while maintaining the precise ethos and without a proclivity for modulation,[20] BC will again complete the whole tone and the 9:8 ratio while AB will be a little smaller than a whole tone, its ratio being the greatest of those smaller than the 9:8 ratio, that is, 10:9. [44.10] CD will be of the ratio 16:15, which along with the 10:9 and 9:8 ratios fills out the 4:3 ratio, comprising the intense (*syntonon*) diatonic genus.

[44.13] Again with the diatessaron ABCD remaining—I say in the ditonic tuning[21]—let H be of equal pitch with D, and let a diatessaron EFGH from mese to hypate in the parhypatai[22] be tuned above it with G being the

[17] The 'leading' ratio is the highest.

[18] 256:243. This scale is the Timaeus scale; cf. I.10, I.13, and II.14. It is impossible to overestimate the importance of this scale in antiquity and the middle ages; see J. H. Waszink, ed., *Timaeus: A Calcidio translatus commentarioque instructus*, vol. IV of *Corpus platonicum medii aevi* (Leiden 1962), esp. VI.

[19]
trite = A---------(90)
9:8
paramese = B---------(101 1/4)
9:8
mese = C---------(113 29/32)
256:243
lichanos = D---------(120)

[20] Ptolemy demonstrates here how closely related the Iasti-Aeolian (9/8 x 9/8 x 256/243) and intense diatonic (9/8 x 10/9 x 16/15) are. He prefers the latter because it consists of superparticular ratios—Wallis adds the phrase *ex solis superparticularibus*—and yet Ptolemy is failing to demonstrate what he proposed at the outset. The citharodists's tuning here is not demonstrated by the intense diatonic. As for Ptolemy's use of ethos, Barker (*GMW* II.318 and 360) has a tempting alternative, identifying this 'modulation' with the *metabolika*. Mathiesen assigns Ptolemy's "proclivity for modulation" to that between thetic Aeolian and the dynamic Iastian which accommodates the tetrachords in the previous paragraph of the text; this would account for "maintaining the ethos."

[21] Each of these three diatonic tunings (tonic, ditonic, and intense) have at least one 9:8 interval (9/8 x 8/7 x 28/27; 9/8 x 9/8 x 256/243; 10/9 x 9/8 x 16/15). Only the ditonic has two such whole tones, whence its name διτονιαίον (*ditoniaian* - 'of the ditones.')

[22] The codices read 'tritai,' not 'parhypatai,' which Düring (209, followed by Gohlke 1443) printed. Citing I.16 and II.6, he reminds us that the tritai are connected with the tonic diatonic.

A---90 90---------mese = E
9:8 8:7
B---101 1/4
102---------lichanos meson = F
9:8 10:9
C---113 29/32
114 2/7----parhypate meson = G 7:6
256:243 21:20
D---120 120---------hypate meson = H

parhypate. [44.16] I say that contained within it is the soft diatonic genus, wherein we found the leading ratio to be 8:7, the middle ratio to be 10:9, and the remainder 21:20[23]. That the ratio EF is 8:7 has been shown with the sterea, for neither of the pitches has been moved in this genus. [45.2] But it must be shown also that the ratio FG is in the ratio 10:9 and GH in 21:20. Therefore C will be found to be a little higher than G, so the ratio GH will be smaller than CD, that is, 19:18.[24] [45.5] FG make a ratio smaller than a whole tone, so the ratio FG is also smaller than 9:8, and the ratio FH is 7:6 since EF is 8:7. [45.7] No other two ratios of which one is smaller than 9:8, the other smaller than 19:18, fill out the 7:6 ratio except 10:9 and 21:20. The ratio GH is smaller than 19:18, and this will then be 21:20, while FG will be 10:9.[25]

[45.11] Finally, with the tetrachord EFGH remaining, let C be of equal pitch with G, and with this pitch established, let there be a diatessaron ABCD of the next chromatic tetrachord[26] with A again positioned at the highest pitch so that BD forms the ratio 8:7. [45.14] It must be shown that the ratio BC will form the ratio 12:11, CD 22:21. D will be found therefore to be a little higher than H so that the ratio CD will be smaller than GH, that is, 21:20. [45.17] B is clearly lower than F so that the ratio BC is smaller than FG, that is, 10:9. Again, no ratios of which one is smaller than 10:9, the other smaller than 21:20, fill out the 8:7 ratio except 12:11 and 22:21, and the ratio CD is smaller than 21:20, so that it will be 22:21 with the remaining BC ratio being 12:11. This was what we proposed to demonstrate.[27]

Because the tetrachord descends from mese to hypate, the lichanos, parhypate, and hypate all belong to the meson tetrachord.

Cf. 37 (tab.), 39.12, and 80.14. The confusion on the part of some early hyparchetype is quite remarkable in this context. It cannot have been due to a mere copying error but probably someone who in thinking that Ptolemy had made an error, emended the correct text. The same motivation no doubt caused the substitute of 'greater' for 'smaller' in several manuscript groups later (at 45.8 and 45.19).

[23] Barker, *GMW* II, 318, has 20:19 (19:20) instead of 21:20 (ἐπι ʹκ).

[24] Düring offers the alternative 256:243, the size of the CD ratio.

[25] Porphyry (156.8-10) adds that citharodoi most often use four *tonoi*—Hypolydian, Iastian, Aeolian, and Hyperiastian.

[26] Ptolemy has come full circle to the intense chromatic which has ratios equivalent to those of the tropai with which he started.

[27]
----89 39/49 = A
E---90
8:7 7:6
F---102 6/7
----104 16/21 = B
10:9 12:11
G---114 2/7 ----114 2/7 = C 8:7
21:20 22:21
----119 107/147 = D
H---120

II.2 - *On the Use of the Canon in Comparison With the Instrument Called the Helicon*

[46.2] We have established in these ways[28] the differences between the genera of tetrachords through the examination and comparison of isotonic[29] notes. [46.4] The application of the diapason to the octachord canon might be made also in another way—with the instrument called the helicon,[31] which was devised by theoreticians[32] for displaying the consonant ratios as follows:

[46.7] They propose a rectangle ABCD, halve AB and BD at E and F, connect AF and BGC, and parallel to AC they lead EHK through E and LGM through G.[33] [46.10] In such a configuration AC will be double each of BF and FD, as will each of these be double EH since also AB is double AE, so that also AC will be quadruple EH and sesquitertian of the remainder HK.

In his table (in the German translation), Düring (60) has $119^{101/147}$ for $119^{107/147}$. Gombosi, *Tonarten* 25, offers a listing of notes and intervals within the Hyperiastian ditonic diatonic.

[28] The first is calculating the intervals by ratio, then testing them on the canon; the second is assuming the actual intervals in common use by cithara players, then demonstrating the ratios which describe them. Cf. 42.2

[29] The manuscripts are divided here between ἰσοτόνων (isotonic - lit. 'of equal tuning') or ἀνισοτόνων (anisotonic). Düring cites 10.1 and 11.10 (and Porph. 215) to support his reading 'anisotonic'; cf. Barker, *GMW* II, 319: "notes of unequal pitch." In general, of course, isotonic sounds, as opposed to anisotonic sounds, are non-musical. But at 10.3-.19 Ptolemy makes clear that isotonic notes are a sub-species of anisotonic sounds. He could mean here notes which simply have different tuning (anisotonic), but in order to create differences between genera the notes are necessarily different. I would suppose that Ptolemy originally wrote 'isotonic' to refer to the parallel tetrachords he established in the previous chapter.

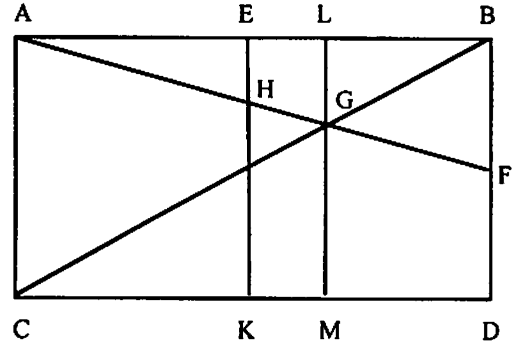
[31] Aristides Quintilianus 3.3 (98.22-99.25) describes the helicon as well. Mathiesen, *Aristides Quintilianus* 163-64, n. 33, offers a clear diagram of the intervals, as does M. L. West, *Ancient Greek Music* (Oxford 1992) 242. See also, C. E. Ruelle, "Hélicon," in C. Daremberg and E. Saglio, eds., *Dictionnaire des Antiquités Grecques et Romaines* (Paris 1900) III.1.59-60. Barbera, *Pythagorean Mathematics* 337-40, poses the question of the alleged theoretical vs. actual design of a helicon.

[32] We do not know who these theoreticians might be. We have no evidence other than the references here and in Aristides Quintilianus 3.3. Mathiesen, *Aristides Quintilianus* 11, discusses the possibility that Aristides derived his information not from Ptolemy himself but from Porphyry's commentary, which would make Aristeides' *floruit* quite late. The possibility of a lost common source should be maintained as well, since Ptolemy does attribute the instrument to earlier theoréticians. Cf. Düring 216 *ad* 46.7.

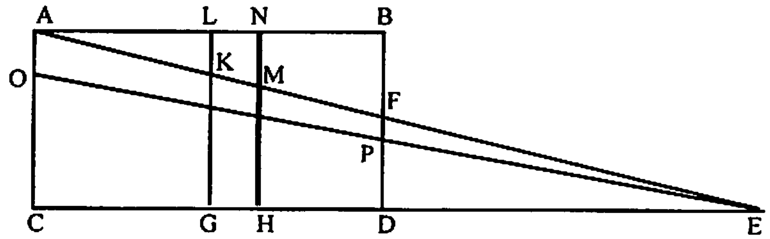
[33] E falls in the center of AB. In the next sentence Ptolemy adds that AB is double AE.

[34] Düring's HM=6 [capital Eta, capital Mu] in the accompanying diagram is incorrect; see Höeg, review of Düring, 658, n. 2. Barker, *GMW* II, 320, omits several phrases here.

[46.13] It is also shown that MG is double GL since, as DC is to CM, so is DB to GM, and as BA is to AL, so is BF to LG. Therefore also as BD is to GM, so is BF to LG, and, alternately, as BD is to BF, so is MG to LG.



[47.1] AC is therefore three-halves of GM and triple GL. This means that four strings of equal tension fastened at the same positions as the straight lines AC, EK, LM, and BD, with the bridge placed under them at their positions along the straight line AHGF, will provide the numbers 12 for AC, 9 for HK, 8 for GM,[34] 6 for each of BF and FD, and again 4 for LG and 3 for EH. [47.7] All the consonances and the whole tone are revealed: the diatessaron and the 4:3 ratio is formed by AC and HK, GM and FD, and LG and HE; the diapente and the hemiolic ratio by AC and GM, HK and FD, and BF and LG; [47.11] the diapason and the double ratio by AC and FD, GM and LG, and BF and HE; the diapason plus diatessaron and the 8:3 ratio by GM and HE; the diapason plus diapente and the triple ratio by AC and LG; the double diapason and the quadruple ratio by AC and EH; and then the whole tone of the 9:8 ratio by HK and GM.



[47.18] If alongside[35] this instrument we place simply a parallelogram ABCD, and if we consider AB and CD to be boundaries of the strings and

[35] Or, 'in addition to.' Ptolemy is about to offer another ingenious alternative to the theorists' helicon, an alternative which he admits has one advantage and one disadvantage. He does not give this other instrument or its arrangement a name.

AC and BD to be the outer notes of the diapason,[36] then extending a line DE equal to CD, let us divide opposite the bridges the side CD with the ratios appropriate to the genera.[37] [48.4] At E we place the higher limit. Beginning with this division let us stretch strings parallel to AC and of equal tension to each other, and once this is done let us place a common bridge under the locus joining the symbols A and E, that is AFE. [48.8] Again, let us maintain the same ratios among all the lengths of the strings so as to accept the examination of the ratios realized in the genera.[38]

[48.10] We do this because, as the strings taken from E are in relation to each other at CD, so also will those parallel to AC led up from their end points to DF; as EC is to ED, for instance, so is CA to DF. Wherefore, these form the diapason since their ratio is duple.

[48.15] Again, if from CD we take away CG (a fourth of EC) and CH (a third), and then establish strings through G and H, GKL and HMN which are of equal tuning to the first set,[39] so that AC will be the sesquitertian of GK and the sesquialter of HM, [48.19] and so that HM will be the sesquitertian of DF and GK the sesquialter, and again so that GK will be 9:8 of HM, then these as well in relation to each other make the consonances fitting the ratios.

[48.21] Similar results are found in the sections between the tetrachords with ratios appropriate to our analysis.[40] [48.23] The first method[41] has an advantage over the latter, that there is no need to move the distances of the strings from each other;[42] the second method has these advantages over the former, both that it has the one common bridge at one position,[43] [48.26] and further that all the notes can be raised an entire whole tone when the bridge through E is moved down, that is, at OPE, while the characteristics of the genus remain unchanged.[44]

[36] Since Ptolemy is trying to reveal the similarities and differences between the helicon and this variation of it, here he shows as a similarity that the strings reach only to B and D—he will extend one to an E further out—and that AC and BD are the farthest horizontal extension. He does not seem to consider AB and CD as 'notes.' If AC and BD are isotonic, of course, they have the same pitch; cf. 48.5, where LG and NH are specifically described as being of equal tension.

[37] Where CD is 2:1 of CG and 3:2 of CH, and where CH is 4:3 of CG. The key advantage found in this variation on the helicon is its absence of bridges. Cf. Düring (216 *ad* 48.23).

[38] As in 46.7f.; cf. Pach. 17 (143.1-145.29 (Tannery)).

[39] AC. To be isotonic, i.e. of equal tuning or tension, with AC, they should also be parallel to AC, where L and N correspond along AB to the G and H along CD.

[40] Ptolemy will explain this after he states that his variation has this advantage, that it is variable to such an extent that even the other pitches, i.e. those "between the tetrachords," can be shown to have the 12:9:8:6 ratios.

[41] The use of the helicon.

[42] Actually, this advantage limits the helicon to the tuning established initially; once accomplished, it cannot be shifted. Cf. II.16.

[43] At F and extending from A through K and M to E; cf. 68.6.

[44] Ptolemy could have made his point clearer if he had said that the bridge moved "from E" or "pivoted at E." The phrase "while the characteristics of the genus remain unchanged" offers relevant yet entirely neglected evidence against the argument that ethos was a function of relative pitch.

[48.28] For example, as CA is to FPD, so is OC to PD; the others work similarly.[45] Again, the former method is disadvantageous to this one in that the former requires that several bridges be moved for each retuning while with the latter method the disadvantage is that all the strings need to be moved and that they no longer remain at the same distances. [49.3] Often the positions of contact are changed considerably.[46]

II.3 - *On the Species of the First Consonances*[47]

[49.4] Up to this point in our considerations we have theorized about consonance and the emmelic in notes lying at the endpoints,[48] and along with the consonances we have taken up homophones.[49] [49.7] A discussion of systems will follow here,[50] but it is necessary first to define the differences in what are called the 'species' of the first consonances as follows.[51] A species is a particular positioning of the ratios characteristic for each genus within their appropriate boundaries.[52] [49.11] These[53] would be disjunct whole tones for the diapente and the diapason, and for the diatessaron whichever of the two leading ('highest') notes effect the shift between the softer and more intense genera.

[45] Ptolemy includes the P here to save the preliminary step in which he would have said that P is the bridge point along F(P)D.

[46] The shifting bridge and variability of the second method make tuning difficult, a serious liability when one considers the sole purpose of the instrument. Ptolemy apparently had constructed such a model and found it difficult to employ.

[47] Barbera, *Pythagorean Mathematics* 333-34, discusses this and the following chapter in detail. For an interesting discussion of the influence of this chapter, cf. Palisca 246, and Boeth. *De mus.* 4.14. There is a scholion attached to the entire chapter as well: "Tetrachord is a generic term. It is divided into differential names and those of species, which at one time he labels species, at another genera. For you might see in the setting out of the canon how the diatessaron is thought of only in the lower notes while the diapente and diapason they hear both at the whole tone near mese and through the notes disjoined to it."

[48] Cf. at 17.26.

[49] The helicon and its Ptolemaic variation provided for the diatessaron and diapente consonances, the diapason homophone—Ptolemy's isolation of the term here is unexpected—and the emmelic 9:8 ratio configured from the endpoints (*apopsalmata*) of the strings.

[50] Ptolemy refers to the next chapter (II.4). The Greek word σύστημα means 'system' or 'scale'; contemporary musicologists generally prefer to translate as 'scale,' but the word 'system' remains when used in reference to the Greater Perfect and Lesser Perfect Systems, which Ptolemy discusses in Book II.

[51] This is described as a necessity because the species of the consonances and homophones are components of the larger and perfect, i.e. 'complete,' system.

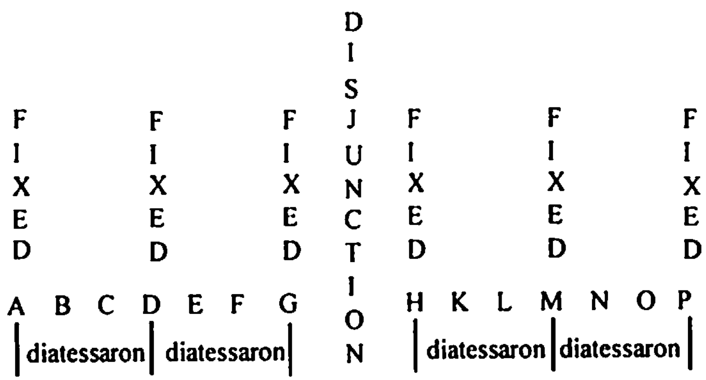
[52] The wording resembles that of the definition of harmonic genus (28.27) as noted in the f-family of manuscripts. Cf. Boeth. *De mus.* 337.22 (Friedlein). For loci paralleli, see Michaelides 90, s.v. "*Eidos.*"

[53] That is, the most characteristic ratios.

[49.13] We commonly label it the first species when the characteristic ratio occupies the leading ['highest'] position since the leading position is the first, the second species when it is in the second from the leading position, the third when it is in the third position, and so on.[54] [49.17] Wherefore there are as many species for each consonance as there are loci of the ratios—three for the diatessaron, four for the diapente, seven for the diapason. [49.19] And it happens that only one species of the diatessaron, the first, is bounded by the fixed notes, for the diapente only two—the first and the fourth, and for the diapason only three—the first, fourth, and seventh.

[49.22] For if we propose a diatessaron ABCD in which we consider A to be the highest note, and if we will add below this another and similar diatessaron DEFG, and then similarly a whole tone GH, [49.25] and then again a diatessaron HKLM, and still another diatessaron MNOP,[55] the standing notes will be A, D, G, H, M, and P, and the first species of the diatessaron is MP, the second LO, and the third KN. Clearly the first species MP alone is bounded by fixed notes.

[50.3] Of the diapente the first species is GM, second FL, third EK, and fourth DH, and clearly of these only the first species GM and the fourth species DH are bounded by fixed notes. [50.6] And of the diapason the first species is GP, second FO, third EN, fourth DM, fifth CL, sixth BK, and seventh AH, and clearly of these again only the first species GP, the fourth DM, and the seventh AH are bounded by fixed notes.



[54] Cf. Cleon. 190.6-192.11.

[55] This structure is, of course, a generic perfect system. Characteristically, Ptolemy has introduced a specific musical concept by offering first a generic description. He did this with notes in I.4. This paragraph marks the first section in which Ptolemy concentrates on the "fixed notes."

II.4 - *On the Perfect System And That It Alone Contains the Double Diapason*

[50.12] Once these preliminaries have been set out, a system can now be labeled simply as a magnitude composed of consonances.[56] Just as a consonance is a magnitude composed of emmelic intervals,[57] a system as well is essentially a consonance of consonances.[58] [50.15] When a system contains all the consonances with their particular species, it is said to be perfect, for, in general, what contains all its parts is perfect.[59] [50.17] Therefore, by our first definition[60] the diapason is a system—this at least seemed sufficient to the ancients—as are the diapason plus diatessaron, diapason plus diapente, and the double diapason. Each of these is bounded by[61] two or more consonances.

[50.21] But by our second definition the only perfect system will be the double diapason since it alone contains all the consonances with their aforementioned species. Anything larger would contain nothing more than what can be grasped in the functioning of the double octave,[62] and anything smaller would lack some of it, wherefore it is not proper to label the system composed of the diapason plus diatessaron[63] 'perfect.' [51.2] It will never contain the seven species of the diapason, and it will not always contain the four species of the diapente. When it is arranged so that a whole tone disjoins two conjunct tetrachords from a third, it will contain four species of the diapente but only four again of the seven of the diapason—those from either extreme. [51.7] When, on the other hand, it is arranged so that the whole tone stands at one of the endpoints,[64] the three conjunct tetrachords

[56] Copied by Bry. I.6 (110.14f.); cf. Porph. 162.31f., Boeth. *De mus.* 341.22 (Friedlein), and Pach. 150.1-13 (Tannery).

[57] E.g. I.12 (28.15-.21). Cf. Cleon. 180.2 (that composed of one or more intervals [unspecified]), with Aristox. *Harm.* 21.6-21.7.

[58] C. A. Keys, "The Word Symphony," *Classica et Mediaevalia* 30 (1969) 578-94, discusses the varied history of the Greek term συμφωνία (*symphonia*) for 'consonance.' See also, Johannes Lohmann, "Die griechische Musik als mathematische Form," *AfM* 14 (1957) 153; Will Richter, "*Symphonia*; zur Vor-und Frühgeschichte eines musikologischen Begriffs," in Heinrich Hüschen and Dietz-Rüdiger Moser, eds., *Festschrift für Wolfgang Bötticher zum sechzigsten Geburtstag am 19 August 1974* (Berlin 1974) 264-90, esp. 266-70; and Carl Dahlhaus, "Ein vergessenes Problem der antiken Konsonanztheorie," in L. Finscher and Christopher-Helmut Mahling, eds., *Festschrift für Walter Wiora zum 30. Dezember 1966* (Kassel 1967) 167.

[59] 'Perfect' is the standardized English, albeit Latinate, translation for τέλειον, which means specifically 'complete,' not 'immaculate.'

[60] That a system is a magnitude composed of consonances.

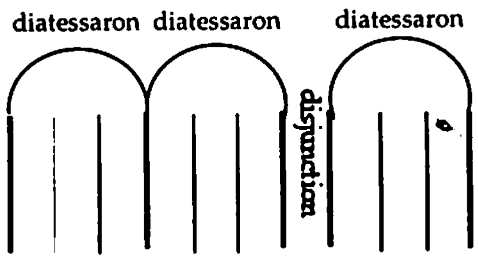
[61] Or, 'contains.'

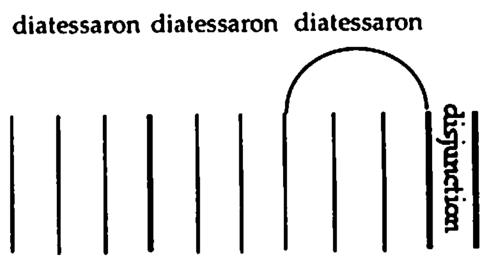
[62] The smaller consonances (diatessaron, diapente, diapason) would simply repeat.

[63] The system usually called the 'Lesser Perfect System.' Düring (219) refers us to the brief reference in Ion fr. 6 [Diels-Kranz[6] 381 (36.5)]; for which, cf. Ulrich von Wilamowitz, "Lesefruchte," *Hermes* 37 (1902) 305-307.

[64] Similar to the position of the disjunction of the prosiambanomenos in the Greater Perfect System.

will contain only one species of both the diapente and diapason,[65] either the first or last of each.[66] One can see this from the following table where one similar tetrachord is fitted onto the arrangement at each endpoint.





[51.12] Now we will find in the double diapason composed of two uniform, similarly arranged diapasons, no matter where the beginning of the disjunctions might be, all the species of the diapason and of the diapente and diatessaron as well.[67] [51.15] But we will find nothing additional in units larger than the double diapason.

[65] The diapente and diapason both need the internal disjunction.

[66] Alexanderson 14 (51.7) repunctuates and supplies the verb εἶναι ('to be') to render this as, "When it is arranged so that the whole tone is at the endpoint and the three tetrachords are conjunct, it will contain only one species... " Düring originally read it this way but added a comma in the corrections listed in his volume of 1934 (18). Cf. the brief note in Winnington-Ingram, *Mode* 64, n. 1.

[67] In the diapason plus dia tessaron system, there is no disjunction, i.e. area or space in which to shift the diapasons up or down. Such a disjunction does exist, of course, in the double diapason system. Now that Ptolemy has established this need for the double diapason magnitude he can proceed in II.5 with its construction as the double diapason, also known as 'ametabolic,' system.

II.5 - *How the Names of the Notes Are Derived By Position and Function*[68]

[51.19] Whence the diapason plus diatessaron system is joined to the double diapason will be scrutinized in the following chapter. The notes of the real, perfect, double diapason system—it consists of fifteen notes since one note, the middle note, is common to the lower and higher diapasons[69]—we name sometimes according to their position, that is, simply by criteria of being higher or lower.[70] [52.2] We name[71] mese the aforementioned note common to the two diapasons, proslambanomenos the lowest note, and nete hyperbolaion the highest note.[72] [52.4] We then name the notes ascending from proslambanomenos to the mese: hypate hypaton, parhypate hypaton, lichanos hypaton, hypate meson, parhypate meson, and lichanos meson, and similarly those ascending from above the mese to nete hyperbolaion: paramese, trite diezeugmenon, paranete diezeugmenon, nete diezeugmenon, trite hyperbolaion, and paranete hyperbolaion.

[52.10] At other times we name the notes according to their function, that is their relationship to something. But let us first fit to these positions the functions of the double diapason of that which is known as the ametabolic system.[73] [52.12] We will do this to make in it common categories of

[68] There is another scholion clearly parallel to that at 49.4 (at chapter 3) attached to chapter 5 describing the term δύναμις (*dynamis* - 'function.') "He calls 'function' the ratio which one string has to another, such as the duple or sesquialter or otherwise higher and lower. Labels by position (thesis) he uses in such names as mese, paramese, first, and third; those by function (*dynamis*) he uses in such terms as disjunct, proslambanomenos ('taken in addition'), lichanos ('forefinger'), and the rest."

Although a minor point, the manuscripts' title here does not have the 'both... and' parallelism found in the initial summary of chapter headings.

[69] Ptolemy avoids the term 'conjunction,' so he needs to add this causal clause explaining why there are not sixteen notes. See also, Bry. III.1 (284.15f.).

[70] For the text, see Höeg, review of Düring, 657; for the theory, see Lohmann, *Musike* 45, n. 1.

[71] Düring (227) seems to find something of significance in Ptolemy's speaking in the first person here, but the names of the notes are well-established centuries before Ptolemy, and Ptolemy elsewhere, e.g. 51.15, speaks in the first person.

[72] Ptolemy here presents the names of the notes in the Greater Perfect System for the first time in this treatise. There was no need for such a list as this in Book I since Ptolemy there discusses and establishes tetrachordal arrangements only; in Book II thus far he has had first to establish the importance of using the double diapason system.

[73] The scholiast explains that the meaning of the term ἀμετάβολον (*ametabolon* - 'unchanging') results "not because there is no transfer from genera into other genera and from species into species, for there is transfer in all, either by diesis, diesis, ditone, as in the enharmonic... but because of the function of the whole tone which disjoins the two diapasons and establishes in each of its parts the perfect system, and does not [reading οὐ for οὗ] change each from the perfect system, or because the whole tone in breaking evenly [the two diapasons] somehow makes some movement from one scale into another and does not give a clear perception of modulation."

position and function. Later we can transfer these functions to other positions.[74]

[52.14] If we take out one whole tone of the two in the double diapason from the mese (by position) and add to it on either side two of the four conjunct tetrachords in the whole system, and then add the second whole tone to the remaining and lowest interval, we label in light of its new position, the lower note of the higher disjunction mese by function.[75] [52.19] We label the higher note paramese.[76] Proslambanomenos and nete hyperbolaion we label the lower note of the low disjunction,[77] and hypate hypaton the higher. [52.21] Then we label hypate meson the note common to the two lower conjunct tetrachords above the lower disjunction, nete diezeugmenon the note common to two [higher][78] conjunct tetrachords above the higher disjunction, [53.1] and again parhypate hypaton the second lowest note of the tetrachord after the lower disjunction, lichanos hypaton the third note, parhypate meson the second lowest note of the tetrachord below the higher[79] disjunction, and lichanos meson the third note. [53.6] Trite diezeugmenon we then label the second lowest note of the tetrachord above

[74] E.gg. 2.8 and 2.10. A valuable collocation of early Latin translations of this passage with some commentary can be found in Palisca 125-28. These pages include work by Giovanni Battista Augio (1545); Leoniceno's (1499) is given on 120-22, and Gogava's (1562) on 135-39.

[75] Or, 'dynamic mese.' Ptolemy here constructs the double diapason system from the inside (i.e. from mese) out.

Besides the discussion by Düring (219-27), who recommends, R. Issberner, "Dynamis und Thesis," *Philologus* 55 (1869) 541-60, see also *Bell. Anon.* 21-23 (6.21-7.6 Najock); Francisque Greif, "Études sur la musique antique," *REG* 24 (1911) 270; Lohmann, *Musike* 32-33; Gevaert, *Histoire et théorie* I, 256-58; and Palisca 45. Winnington-Ingram, *Mode* 65, n. 1, observes that Ptolemy's use of the thetic names brought ancient Greek string nomenclature full circle since originally such terms as mese ('middle'), lichanos, and hypate ('highest') were thetic. Düring (222) makes a similar simplification, comparing Greek dynamic notation to our own dominant, subdominant, and tonic (and solfeggio). He (224) also points out that thetic notation is used only for strings, not for 'notes.'

[76] Three consecutive whole tones (at the central disjunction) mark the mese. I. Henderson, "Ancient Greek Music," in Egon Wellesz, ed., *Ancient and Oriental Music*, vol. 1 of *The New Oxford History of Music* (London 1957) 355-57, believes any loss of paramese will cause the ethos to be altered because of the shift to the synemmenon tetrachord.

[77] The proslambanomenos and nete hyperbolaion are now of the same pitch; cf. 53.14. Ptolemy does not apparently supply a label for this fifteenth pitch. Ernest G. McClain, *The Pythagorean Plato: Prelude to Song Itself* (Stony Brook NY 1978) 150-54, observes that by thus bending the Greater Perfect System into a circle, each part becomes 1/12 of that circle and thus the ametabolic system contains, geometrically at least, equal temperament.

[78] Wallis added a great number of supplementary words and phrases throughout the treatise, most of which help complement Ptolemy's otherwise too often vague pronouns and participles. Why Düring felt comfortable incorporating (in brackets) only this supplement, 'higher,' is not clear to me since the tetrachords in question are identifiable without Wallis' supplement.

The scholiast pointed out here, "One should know that when [Ptolemy] intends to make clear the proximity of the disjunction he uses the preposition 'after' [μετά], as in 'the first tetrachord after the lower disjunction'; when he wishes to make clear the tetrachord next to the former, then he uses the preposition 'before' [πρό]." In this translation I prefer 'higher' and 'lower.'

[79] The note in Düring (228) is misprinted as '54.4.' (Similarly (229), for '53, 57' read '53, 27.')

the higher disjunction, paranete diezeugmenon the third note, trite hyperbolaion the second lowest note of the tetrachord below[80] the lower disjunction, and paranete hyperbolaion the third note.

[53.10] Only with this terminology, that is, the functional, would these tones legitimately be called 'fixed' in the modulations of the genera:[81] proslambanomenos, hypate hypaton, hypate meson, mese, paramese, nete diezeugmenon, and nete hyperbolaion, the latter being one and the same[82] as proslambanomenos. All remaining notes would be called 'movable.'

[53.15] The limits of the 'fixed' and 'movable' notes no longer fit the same loci when the functions are moved about in position.[83] [53.17] It is clear that even the first species of the octave in the above system called 'ametabolic' for the aforementioned reason[84] is bounded by the paramese and hypate hypaton, the second by trite diezeugmenon and the parhypate hypaton, the third by the paranete diezeugmenon and the lichanos hypaton, the fourth by the nete diezeugmenon and the hypate meson, [53.22] the fifth by the trite hyperbolaion and the parhypate meson, the sixth by the paranete hyperbolaion and the lichanos meson, the seventh by the nete hyperbolaion or the proslambanomenos and the mese. To make the comparison easier, here is an table[85] demonstrating the ametabolic system:[86]

[80] Gohlke (review of Düring, 1443) misinterprets the passage and would emend the manuscripts to read "above the higher disjunction," but Ptolemy's circular description of the double diapason system obviates the need for this.

[81] This occurs once a pitch originally movable (by position) becomes renamed (by function) 'fixed.' Winnington-Ingram (*Mode* 64, n. 3) and Höeg (review of Düring *Ptolemaios und Porphyrios* 156) discuss Düring's mistranslation of this passage.

[82] In function (Düring 229).

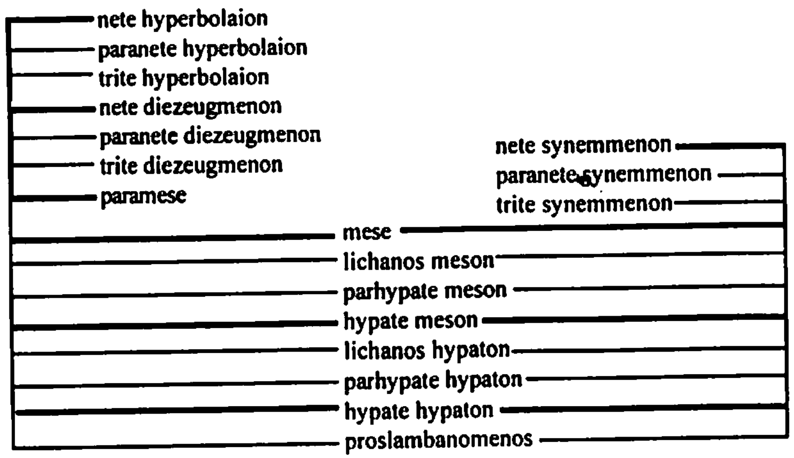
[83] When positions, i.e. loci, have shifted, then so do the terms 'fixed' and 'movable.' Some of the surviving pieces of ancient Greek music offer actual examples of how this shift occurs, for example the Seikilos Inscription, for which see Jon Solomon, "The Seikilos Inscription: A Musicological Analysis," *AJPh* 107 (1986) 455-79.

R. P. Winnington-Ingram, review of Vogel, *Enharmonik*, in *Die Musikforschung* 18 (1965) 60-64, discusses the merit of equating the movable notes described by Ptolemy here and those labeled hypatoeides, mesoeides, and netoeides by Aristides Quintilianus [9.20-.26].

[84] The ametabolic, 'non-modulating' system by definition does not contain any tonal shifts. It is a pitch-less system; cf. I. Henderson, "Ancient Greek Music," in Egon Wellesz, ed., *Ancient and Oriental Music*, vol. 1 of *The New Oxford History of Music* (London 1957) 352). K. Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *CQ* 27 (1933) 93, describes the ametabolic system as "a frame, consisting of a series of recognized steps or degrees, the nomenclature of which, indefinite as to the pitch and function of notes, bore only an indication of the relative sequence of positions," as Ptolemy demonstrates here. Because it bears no relation to actual pitch, Ptolemy has no need of (and makes no mention whatsoever of) notation. Schlesinger suggests therefore that notation was devised only for the old *harmoniai*. This would have to be true if the use of notation predated Aristoxenus' development of the ametabolic system. Fifty years later her assumption can now be supported by Annie Bélis ("Un Nouveau Document Musical," *BCH* 108 (1984) 99-109), who has identified solfeggio notation on several early fifth-century painted pottery fragments (Museum of Eleusis, inv. #907).

[85] Ptolemy 'announces' his tables in 30.2, 31.18, 35.32, 37.4, 53.26, 69.10, and elsewhere. The tables in Wallis at II.11 are taken from the sixteenth-century J manuscript [Codex Collegii S. Johannis Bapt. Graecus 30 = Düring #41, Mathiesen, *Ancient Greek Music Theory* #153].

AMETABOLIC DISJUNCT PERFECT SYSTEM CONJUNCT SYSTEM



II.6 - *How the Conjunct Magnitude, the Diapason Plus Diatessaron, Has the Reputation as a Perfect System*

[53.30] This system is also called the disjunct system in contrast to the previously discussed[87] conjunct magnitude of the diapason plus diatessaron, which is called the 'conjunct' since it substitutes another, higher, conjunct tetrachord for the disjunction at mese. [54.3-4] Because of this, the tetrachord is labeled 'conjunct,'[88] and above it, similar to the 'disjunct' labeling, is trite synemmenon - the note above mese, paranete synemmenon - the next note, and nete synemmenon - the leading and fixed note of the tetrachord.

[54.7] The ancients seem to have developed such a system for a second species of modulation, as if from some metabolic ['changing'] system to the ametabolic ['unchanging'], since an ametabolic system is so called not

Mathiesen (790-91n and 828) lists all the Ptolemy manuscripts that contain tables; there are seven of them. Palisca (134) evaluates them.

[86] Rudolf Westphal, *Harmonik und Melopöie der Greichen* (Leipzig 1863) 103-110, offers copious tables.

[87] At 51.1; cf. the discussion in Bry. III.9 (352.17f.). Ptolemy makes reference here, of course, to the perfect, double diapason system.

[88] The term συνημμένον (*synemmenon* – 'conjunct') applies to both the diapason plus diatessaron system and the tetrachord above (and conjunct with) mese which distinguishes that system. Cf. Bry. 356.1 (and Pach. 148.10f.). Ptol. *Geog.* 1.1.5 employs the term in the very first sentence of that treatise to help define to what the subject of the discipline is 'connected.'

because of a non-modulation by genus, since all genera can modulate, but because of the lack of modulation in the function of the tonos.[89]

[54.12] There are in respect to this usage of the word 'tonos'[90] two primary types of modulations, the first whereby we play through an entire melody at a higher or lower pitch while maintaining the succession of intervals in every species, [55.1] the second whereby the entire melody is not transposed in pitch but only some part of it moves contrary to the previous succession. The latter should be labeled a modulation of melody (*melos*) rather than a modulation of tonos, [55.4] for in the former case it is not the melody but the tonos which is transposed entirely, while in the latter the melody is shifted away from its original arrangement.[91] Its pitch serves not so much as pitch as for the sake of the melody.[92]

[55.7] Modulation of tonos, then, does not impress the perception as a change in function, which determines the ethos,[93] but a change only in highness or lowness. [55.9] Modulation of melos, however, makes the perception stray from the accustomed and expected melody when the sequence proceeds at length and then at some point changes to another species either by genus or by pitch.[94] [55.12] For example, after being continuously diatonic, the genus moves over at some point to the

[89] Since Ptolemy begins here a lengthy exegesis on his conception of the phenomenon known as τόνος (*tonos*), it will be well to refer to Winnington-Ingram, *Mode* 62-69, who makes it very clear (66, n.1) that τόνος here is not the whole tone disjunction at mese but the series of notes, i.e. the system, to which Ptolemy refers. Düring (229-30) argues the opposite point extensively, as he does again for the preposition (παρὰ) I have translated as 'in respect to' at 54.12. Ptolemy's modulation of tonos signifies a shift from one tonos, e.g. Dorian, to another, e.g. Lydian, whereupon the sequence of intervals would not be different after the modulation. Ptolemy labels modulation by tonos a shift in pitch within the ametabolic double diapason sliding system which necessitates that actual modulation of the intervallic sequence.

[90] Ptolemy only rarely distinguishes which technical meaning of the word 'tonos' he is using.

Leoniceno's Latin translation of the following sentences is offered (and translated) at Palisca 289-90. Giovanni Battista Doni's is presented and compared in Claude V. Palisca, "Giovanni Battista Doni's Interpretation of the Greek Modal System," *The Journal of Musicology* 15 (1997) 8-9.

[91] On modulation of tonos or melos, see Aristides Quintilianus 22.11-.24, with ample references and *loci paralleli* in Mathiesen, *Aristides Quintilianus* 88-89 (nn. 114-19). In a modulation by melos there is a shift to an entirely different form of the ametabolic system, e.g., to use Ptolemy's examples, from diatonic to chromatic or from disjunct (diezeugmenon) to conjunct (synemmenon).

[92] Ptolemy means that the change in pitch does not simply carry the old succession of notes at a different pitch; in fact, the pitch is almost irrelevant, especially compared to the importance of pitch in modulation by tonos. The modulations in modulation by melos are so radical as to change the sequence (and actual relative pitches) of the notes used. The pitch "is not turned in its entirety through all the strings but in part." (Scholion)

[93] Ptolemy here clarifies that for a change in ethos to occur there must be a change of system or interval sequence; cf. Winnington-Ingram, *Mode* 67. Gevaert, *Histoire et théorie* I, 255, offers a translation of this passage.

[94] Monro 67, n. 1, finds particular interest where Ptolemy changes the pitches since he associates pitch change with change in 'mode' and ethos. Agreeing with Monro, but more recent, is E. V. Gertsman, "The Appreciation of High and Low Sound Registers in Ancient Musical Thought," *Vestnik Drevnej Istorii* 118 (1971) 194.

chromatic;[95] or when from a melody accustomed to having its transitions made at diapente consonances there is some transfer at the diatessaron consonances, as in the aforementioned systems.[96]

[55.15] With the melody moving up to the mese,[97] when it goes not as it is accustomed to the diezeugmenon tetrachord which stands the consonance of the diapente away from the meson tetrachord,[98] [55.18] but twisted around as if it were abducted[99] towards the tetrachord conjunct at the mese[100] so as to form a diatessaron and not a diapente with the notes below mese,[101] there is a change[102] and a deviation from what the senses expected to happen. [55.21] Such a deviation is pleasing when the transition is well-measured and

[95] There seem to be several examples of modulation between the chromatic and diatonic in the extant fragments of Greek music, including in all likelihood *P.Oslo* 1413 (Egert Pöhlmann, *Denkmäler altgriechischer Musik* (Nürnberg 1970) 114-17) at line 11. *P.Mich.* 2958 (Pöhlmann 130-5) may contain one as well, although this depends on an insecure reading of N in line 5. The third and fourth lines of Mesomedes' first *Hymn to the Muse(s)* (Pöhlmann 14-15) contain a borrowed or *leiterfremde* note which belongs not to the Lydian diatonic but to the Lydian chromatic. The purpose and historical basis of such a note is discussed in Nancy Sultan, "New Light on the Function of 'Borrowed Notes' in Ancient Greek Music," *The Journal of Musicology* 7 (1988) 387-98.

[96] The Seikilos inscription and the first Mesomedes hymn both begin with a striking leap of a diapente. The former then moves at the end of the first line of transcription to a sustained note at the diatessaron. See Jon Solomon, "The Seikilos Inscription: A Musicological Analysis," *AJPh* 107 (1986) 463-64.

[97] Although 'melody' is the most appropriate and standard English translation of μέλος (*melos*), the word does not have to mean only 'melody'; it can also refer to a melodic line or to simply a realized sequence of notes. On the breadth of its meanings, see Mathiesen, *Aristides Quintilianus* 16-17, n. 111.

[98] Many of the extant fragments of ancient Greek music lack modulation, e.gg. *P.Berlin* 6870 (Pöhlmann 94-5), *P.Oxy.* 1786 (Pöhlmann 106-107), and Mesomedes' fourth hymn. *P.Wien* 29825 a/b verso contains no modulations between the conjunct and disjunct systems even though it appears to modulate between four different *tonoi* (chromatic Hyperlydian, chromatic Hypolydian, diatonic Lydian, and diatonic Hypophrygian).

[99] The uncustomary, rich vocabulary here results from Ptolemy's attempt to describe the audible effect of a modulation at the half tone step (mese - trite synemmenon) instead of at the whole tone (mese - paramese). In a monophonic sense, this would be similar to what we hear in modulating from major to minor key and thus would be significantly audible. The extant fragments of ancient Greek music preserve numerous examples of this type of modulation, e.gg. *P.Wien* 29825e (Pöhlmann 92), *P.Zenon* 59533 (Pöhlmann 110), *P.Oslo* 1413 (Pöhlmann 114-17), *P.Oxy.* 2436 (Pöhlmann 126-27), and both Delphic Hymns.

[100] Or, 'by the mese.'

[101] I see no reason to repunctuate as suggested by Alexanderson 14, although the resultant detailing of Ptolemaic prepositions he offers is useful.

[102] The scholion comments here, "It should be known that in the system of disjunctions the term 'disjoined' is used in regard to position, but the term 'exchanged' one might say more properly in reference to the production of music. For since such a mediating, disjoined whole tone is not added in the conjunct [synemmenon] system, there is an exchange and unexpected wandering." The assumption here, as usual, is that the disjunct system, otherwise known as the ametabolic or Greater Perfect System, is the primary one, the conjunct or Lesser Perfect secondary. Our ear misses the disjunction and is confused by the 'wandering' into the next and third conjunct tetrachord.

emmelic[103] but displeasing when not. Wherefore the most beautiful and nearly the only one functional is the one similar to that just discussed which takes the anticipated[104] whole tone transition wherein lies the difference between the diapente and diatessaron. [55.25] Insofar as the whole tone is common to the genera, it is able to make the modulation in all[105] of these; insofar as it is apart from the ratios that form the tetrachords, it is able to alter the melody.[106] Insofar as it is commensurate (as if it is first of the emmelic intervals), it makes the progressions of the melody neither too great nor too small, both of which are difficult for the hearing to judge.[107]

[56.1] There are therefore three tetrachords joined consecutively which are suitable for such a modulation; they derive from some partial mixture of two disjunct systems when they[108] differ entirely in tonos from each other by a diatessaron. [56.4] But the ancients, since they had not progressed to increasing these tonoi—they knew only the Dorian, Phrygian, and Lydian[109] which differ from each other by one whole tone and do not reach to the tonos standing higher or lower by a diatessaron—[56.7] and since they did not have a way of making three consecutive tetrachords from the disjunct, embraced the conjunct with the name 'system' so that they would have the aforementioned modulation available.[110]

[56.10] In general, in fact, regarding tonoi differing from each other by[111] the diatessaron, if, of those tetrachords below a disjunction similar in each, that of the higher tonos is fitted above that of the lower tonos, it forms in the lower tonos three conjunct tetrachords of which the transferred member is the highest; [56.14] and if, of those tetrachords above a similar disjunction, that of the lower tonos is fitted below that of the higher tonos, it

[103] It is not clear whether 'emmelic' is the technical term referring to intervals other than homophones and consonances (II.3 [49.4]), or simply a descriptive adjective meaning 'euphonic.' See also, III.14 with n. 228.

[104] The anticipation would make the most beautiful and emmelic modulation, *contra* Alexanderson 15 (55.24).

[105] I read ἐν ἅπασιν with Düring here and omit the ἐμφανῆ ('clear') added by several manuscript groups and incorporated into the text by Wallis.

[106] These are simply the modulations by tonos (pitch) and melos (melody) restated.

[107] The modulation should not occur at too large or too small an interval. The diapente is the largest intervallic leap one finds with any regularity in the extant pieces of ancient Greek music.

[108] The whole tetrachords.

[109] These are the ones listed in Plato *Rep.* 398E-399A, along with the Ionian.

[110] Winnington-Ingram, *Mode* 53, labels such a synemmenon/diezeugmenon modulation a modulation by 'key.'

Ptolemy may be correct in assuming that "the ancients"—and this would be most likely Aristoxenus—contrived this system in order to accommodate a different kind of modulation. The Greater Perfect System is no doubt a contrivance, a modernized system to replace such scales as we find in the Spondeion scales; cf. Winnington-Ingram, *Mode* 21-30. For the growth and replacement of such systems, see Solomon, "*Tonoi*" 248-51, and John Thorp, "Aristoxenus and the Ethnoethical Modes," in R. Wallace and B. MacLachlan, eds., *Harmonia Mundi: Musica e filosofia nell'Antichità* (Rome 1991) 56-61.

[111] For the text, cf. Alexanderson 15 and Höeg, review of Düring, 656.

again forms in the higher tonos three conjunct tetrachords of which the transformed member is the lowest.[112]

[56.18] Beginning with the highest note A,[113] let there be a descending tetrachord AB, another BC conjunct to it, then a whole tone of disjunction CD, and again two conjunct tetrachords DE and EF beneath it. [56.21] Let a disjunction[114] GH, similar to CD, be taken from the tonos higher by a diatessaron, two more tetrachords HK and KL conjunct to and beneath it, and a disjunction MN similar to CD but from the tonos lower than the first by a diatessaron, and two tetrachords NO and OP conjunct ~~to~~ and above it.

	A	
	diatessaron	
G	B	P
H disjunction	diatessaron	diatessaron
diatessaron	C	O
K	disjunction D	diatessaron
diatessaron	diatessaron	N
L	E	disjunction M
	diatessaron	
	F	

[56.26] Since the note H is similar to D—H will be higher than it by a diatessaron—it is higher than K also by the same amount; and so D and K are of the same pitch so that it will be possible for the tetrachord HK to be fitted in above D thereby making three consecutive tetrachords in the tonos A - F, of which HK will be the highest, that is FE, ED, and DH. [57.4] Again, since the note N is similar to C—it will be lower than C by a diatessaron—it will also be lower than O by the same amount. C and O are therefore of equal pitch so that the tetrachord ON will be able to be added below C thereby making three consecutive tetrachords in the tonos AF, of which ON is the lowest, that is AB, BC, and CN.[115]

II.7 - *On the Modulations of What Are Called Tonoi*[116]

[57.10] Let it be evident from this that the conjunct system, beyond its not having the nature of the perfect system, as we have said, is superfluous when

[112] Ptolemy seems to be transferring or adding an extra tetrachord to either side of the disjunction, thus shifting the double diapason system to the conjunct in a way similar to his shifting of the mese in II.5; see Lohmann, *Musike* 62-67.

[113] Ptolemy offers a geometrical demonstration of the formation of conjunct systems from disjunct. This will lead him to the discussions in the next chapter on the tonoi and the conclusion (57.10) that the conjunct system is superfluous.

[114] Düring deleted the 'diatessaron' inserted into the manuscripts, as read by Wallis. Perhaps it derives from the abbreviation for 'diatessaron' (διὰ δ) used frequently in the manuscripts.

[115] HK is renamed DH, and ON is renamed CN. Several manuscripts add, "These, then, are what are called the three tetrachords"; cf. Wallis 64.

[116] Winnington-Ingram, *Mode* 66-68, discusses tonoi in this and the last chapter.

compared with the perfect disjunct systems[117] at the diatessaron. [57.13] But we must specify again, since throughout all the structures[118] which we call properly tonoi[119] there are modulations occurring—the differences exist by virtue of the tunings[120]—that by function they are infinite in number, as were the notes. [57.16] After all, the only differences between that which is called 'tonos' and a 'note' is that a tonos is composite, the note non-composite. [57.17] Similar is how a line differs from a point if without any obstruction we extended the sole point or the whole line along continuous loci. But in reality the number of tonoi is limited by our perception, as are the notes.

[57.21] And so there will be three determinations within the realm of tonic theory, as with each of the consonances:[121] first, what is the ratio formed by the extremes; second, what is the number of those between the extremes; and third, what is the measure of difference between each consecutive tonos.[122] [57.24] This is similar to the diatessaron, to cite one example, since its extreme notes form the 4:3 (sesquitertian) ratio, since there are three ratios only which form the whole diatessaron,[123] and since the differences between the ratios are such and such.[124]

[57.27] There is a difference,[125] however, in to what extent each of these determinations of the diatessaron has its own purpose. In the tonoi the second and third determinations follow the first and are dependent upon one and the same limitation.[126] [57.30] Most have ignored the implications of

[117] See 56.10 and 56.18. Cf. also, Palisca 45-46 and 307- 308. The plural 'systems' is odd.

[118] The Greater Perfect System, the Lesser Perfect System, and any others larger than a diapason, e.g. the diapason plus diapente.

[119] The Greek word τόνος (*tonos*) derives from the verb τείνω (*teinô* - 'to stretch, tune'), whence Ptolemy's simplification of the term here. Szabó 120, thinks that the "musical tone was above all the tone of a stringed note." Insofar as tonos is derived from τείνω and applies to the single whole tone as well as to the system of tones, this assessment is correct. But it should be remembered that the most basic word for a musical note is φθόγγος (*phthongos*). On the differences and similarities between Ptolemy's tonoi and Boethius' *modi*, see the classic account in Antoine Auda, *Les modes et les tons de la musique et spécialement de la musique médiévale* (Brussels 1930; reprint: Hildesheim 1979) 67-70. Vaticanus gr. 198 (Düring's G [Mathiesen, *Ancient Greek Music Theory*, #218]) offers the variant τρόπους, i.e. 'tropoi,' the term used in Alypius *passim* to describe this same structure. The scholion comments on this term as well, adding that Ptolemy at the outset of the second book [42.12] had used the term, but these are not the same type of tropoi.

[120] A tonos is composed of a certain number of notes, as the double diapason is composed of a certain number of consonances. *Contra*, Bacch. 304.1-.5 and Cleon. 180.4-.5.

[121] Ptolemy has just demonstrated that the number of tonoi is theoretically limitless; he needs now to limit the number with these three parameters.

[122] Ptolemy will elaborate on each of these three questions consecutively between here and II.11.

[123] In one tetrachord. Again, the number could be limitless theoretically, but Ptolemy limits the number of diatessarons to three, as he will the number of tonoi to seven.

[124] As elaborately explained and then detailed in the second half of Book I; cf. 39.14.

[125] Unfortunately, this is the last sentence for which Porph. (174.1-.27) has a comment.

[126] That all-important limitation is the precise magnitude of the tonos. Ptolemy's tone, no pun intended, begins to be quite polemical here.

this in disposing of each determination differently. Some have made them smaller than the diapason, some the same as, and some greater than the diapason. [58.1] The more recent writers have continually searched for essentially some such increase beyond those of previous writers.[127] [58.3] This is not in keeping with the nature of harmonics and its periodicity[128] which alone are necessary to determine what the distance will be between the outer tonoi, since neither a modulation by voice nor by any producer of sound is able to have one and the same limit.[129] [58.7] It is not by means of higher or lower voices that we would find constituted a modulation by tonos. The raising or again the lowering of whole instruments produces this difference, and no change of melody occurs as long as[130] the lower-voiced and higher-voiced singers continue the melody without changing it.[131] [58.13] It is rather by means of one voice that modulation by tonos exists. The same melody begins at some point at higher loci, then at lower loci, and makes some shift in ethos.[132] [58.15] In performing such modulations of tonos, the range of the voice no longer fits each of the ranges of the melody but stops consistently short; while the limit of the voice stops short of that of the melody, in the opposite case the limit of the melody stops short of the voice. [58.18] The result is that what at the outset had concurred with the

[127] Rather than the specific number of tonoi, Ptolemy here is discussing the size of each tonos (which is not unrelated to their number, of course). The spondeion scale (see R. P. Winnington-Ingram, "The Spondeion Scale," *CQ* 22 (1928) 83-91), for example, is not exactly a diapason, nor are the other (fourth-century) "Platonic" scales preserved in Aristides Quintilianus 19.2-20.1; cf. Mathiesen, *Aristides Quintilianus* 85-86 and J. F. Mountford, "The Musical Scales of Plato's *Republic*," *CQ* 17 (1923) 125-36. The diapason is the limitation.

[128] An unexpected but apt term (ἀποκατάστασις), and one which will reappear (81.21) as a basic concept for the astrological comparisons in Book III; cf. also 58.22. The Hypermixolydian "repeats" [the period of] the Dorian (one octave higher).

[129] The actual instrument or voice with which tonoi are produced has no bearing on the parameters of the tonoi. This is particularly true of modulations of tonoi. Düring (233) uses this as evidence against the theory advanced by D. B. Monro, (*Modes*) that pitch determines tonos. Cf. I. Henderson, "Ancient Greek Music," in Egon Wellesz, ed., *Ancient and Oriental Music*, vol. I of *The New Oxford History of Music* (London 1957) 348 and 385, where she translates the last six sentences of this chapter.

Ptolemy spends the rest of the chapter discussing modulation by tonos as a preliminary to the discussion of the size and number of tonoi.

[130] Or, "when." Winnington-Ingram, *Mode* 67, n. 1, would differentiate the two.

[131] Ptolemy can effectively and dogmatically limit the number of tonoi to seven because a shift in tonos must include a shift in intervallic arrangement, not just a change of pitch. A mere change of pitch would produce the same sounds only higher or lower but could do so indefinitely.

[132] Here the effect of ethos is again only loosely tied to the concept of pitch. Cf. Lohmann, *Musike* 72, n. 1; and Gombosi, *Tonarten* 137, for whom it is the sound of changing interval sequences, not the change in pitch itself, that "makes some shift in ethos, or gives the hearing an impression of another ethos." This expands upon but does not contradict what Ptolemy had said about modulation by tonos in II.6.

For Latin translations, see Palisca 290-91 (Leoniceno) and 308-309 (Girolamo Mei).

distance of the voice at some point in the modulation ceases to do so and then at another succeeds gives the hearing an impression of another ethos.[133]

II.8 - *That It Is Necessary For the Limits of the Tonoi To Be Defined By the Diapason*

[58.21] Let the first and most important periodicity[134] of harmonic similarity exist again in the first of the homophones, that is, the diapason. The notes bounding it, as we have demonstrated,[135] act as one. [58.24] Just as those of the consonances composed with it do this --as they would if they were isolated[136]—so every melody which lies at the distance of simply the first homophone or at some interval composed from it is able to take[137] as its starting point one of the limiting notes and then run through similarly.[138] [58.28] Wherefore in the permutations of the tonoi as well, whenever we wish to modulate into a tonos higher or lower by a diapason, we do not move any of the notes.[139] We always move some notes in other kinds of modulations, but this tonos remains the same tonos as it had been from the beginning.[140]

[133] Pach. (151.38 (Tannery)) helps us verify the correct reading here. Düring (233-35) deserves credit for applying the fragments of ancient Greek music—the *Orestes* papyrus, Seikilos inscription, two Delphic Hymns, and Mesomedes Hymns (those discovered by 1934)—to this passage. Ancient music theory and the extant fragments of Greek music both belong to the same musical tradition, and the study of one can shed much light on the other. But Düring's discussion, attempting to prove that second-century A.D. music already limited melodies to two octaves, did not necessarily provide the best occasion to apply this methodology. It is rare that any piece of vocal music strains its singer's (singers') voice(s) to a range greater than two octaves, and there is no evidence that before Ptolemy's day music had a greater range. In fact, the opposite must be true if one judges by the number of strings on the early lyres and phorminges; cf. M. L. West, "The Singing of Homer and the Modes of Early Greek Music," *JHS* 101 (1981) 113-29. Winnington-Ingram (*Mode* 68, n.1) has other criticisms of Düring's interpretations of these last few sentences. On ethos in this context, see Abert 6-7.

[134] The Greek ἀποκατάστασις (*apokatastasis*) term refers to 'recurrence' or 'cycle' or 'period.' Cf. n. 126. As Ptolemy explained in II.6 and II.7, the various sub-structures of the double diapason recur with such a periodicity. For a brief analysis, see Palisca 291.

[135] At I.7 (15.10f.); cf. also 53.14.

[136] I.e., as if they were not included in the double diapason system.

[137] Reading the neuter participle (λαβὸν) instead of the masculine (λαβών); cf. Alexanderson 15.

[138] The interior notes then become the 'starting points.' Winnington-Ingram, *Mode* 11-16, offers a most intelligent and thoughtful analysis of Aristoxenus' octave species.

[139] One changes only the starting pitch. Gevaert, *Histoire et théorie* I, 253, offers a French translation of this sentence. Barker, *GMW* I 280-84, offers a modern English translation of Athenaeus *Deipnosophistae* 623D-626D with copious notes on the various tonoi.

[140] This paragraph serves to establish for the following paragraph the principle on which Ptolemy proves that the magnitude of the tonos is a diapason.

[59.2] Again[141] it follows that the tonos standing a diatessaron away from the original tonos will be the same tonos as that standing a diapason plus diatessaron away from the original tonos; [59.4] and that the tonos standing a diapente away from the original tonos will be the same tonos as that standing a diapason plus diapente away from it, and so on.

[59.6] Therefore, those establishing the boundaries of the tonoi as smaller than the diapason would not produce the harmonic periodicity since some tonos will be dissimilar and lay beyond all the others.[142] On the other hand, when tonoi are established beyond the diapason, rendered superfluous are those tonoi which lie beyond the diapason itself. [59.10] They will always be the same as those used previously, whether they lie a diapason from the original or whether they lie an amount away from the diapason equal to its distance from the original diapason.[143]

[59.12] Those who proceed up to only the diapason do not reckon correctly[144] among the tonoi that which stands a diapason from the original.[145] They will be seen to make the same mistake as those who exceed the prescribed limit except that where the former differ by one tonos the latter differ by many. [59.16] They will rightly be criticized by those who argue about the existence of the excesses. They will say that they began and provided the purpose for employing an excess, and once one employs some tonos similar to that of the original diapason, what is to prevent anyone from adding those parallel to the others in succession? [59.20] And yet in the species bounded by the diapason we have a most natural example of how we do not need to measure its functions in the number of boundaries of the diapason but in the number of ratios constituting it.[146] [59.23] All of us surely posit only seven of these, even if there are eight notes which form them, and no one would say that a diapason, for example, taken down from the lowest note would make a species different from the first, or similarly from the highest notes,[147] [59.27] since universally anything beginning in the

[141] Cf. I.6 (13.16f.) and II.4.

[142] When one then attempts the periodic progression he will find the one-diapason tonos to be different and of greater magnitude than his.

[143] Again, this depends on the axiom Ptolemy established at 59.2.

[144] "For," says the scholiast, "Ptolemy demonstrated earlier that the diapason had to have less than six complete whole tones. And if he also fills out the diapason with up to eight strings it will not be entirely in sesquioctave distances. For in between he places the leimmata to fill out the sesquioctave [distances] which amount to less than the six whole tones."

[145] E.g. mese - nete hyperbolaion from proslambanomenos - mese.

[146] Because it is the difference in intervallic succession that matters, not the succession of pitches.

[147] Whether higher or lower Ptolemy does not make clear because the difference is of little consequence (except that lower would be superimposed on an established tonos). The scholion offers: "The one diapason, the first consonance, occurs twice, that is to say, that which the mese makes with the nete hyperbolaion and that which the same mese makes with the proslambanomenos."

same way from either of the boundary notes of the diapason will have the same function.[148]

II.9 - *That It Is Necessary For Only Seven Tonoi To Be Proposed, a Number Equal to the Species of the Diapason*[149]

[60.1] Our investigations have now led us to considering the number of the tonoi.[150] It would be best to make them as numerous as the species of the

[148] In this way Ptolemy satisfies the first determination; cf. 57.21. This last paragraph also introduces the argument of II.9.

[149] For the entire chapter, see Gombosi, *Tonarten* 100-101; Rudolf Westphal, *Harmonik und Melopöie der Griechen* (Leipzig 1863) 167; cf. Palisca 46 and 291. In addition, this lengthy, learned scholion, no doubt written by Nicephoros Gregoras or Isaac Argyros [cf. II.14 and I.9], accompanies chapter 9:

"Since the double diapason is the perfect system, it is necessary—if it is going to be tuned in the best ways—to encompass within itself all the species of the diapason. But this will happen, not only if the nete hyperbolaion by position is consonant with the mese and again the mese with the proslambanomenos by the diapason, but also if each of the notes after the nete hyperbolaion forms a diapason with each of those after the mese and eighth from it, be it the paranete hyperbolaon to the lichanos meson or the trite hyperbolaion to the parhypate meson, and so on in succession.

But this would not happen if one species of the diapason were encompassed by the nete hyperbolaion and mese, the other by mese and the proslambanomenos. For if, for example, nete hyperbolaion and mese encompass the first species of the diapason and mese and proslambanomenos the second, each of those notes after the nete hyperbolaion will not make a diapason to each of those after the mese and eight from itself. Therefore it is necessary, if the double diapason system is going to be tuned in the best way, that each of the diapasons in succession be of the same species. Since there are seven species of diapason, the best arrangement of the double diapason will have to occur in sevens, and these seven arrangements the ancients named the seven tonoi. For what scale each of the diapasons in succession makes, the first consonance they called the Hypolydian tonos, the second the Hypophrygian, the third the Hypodorian, the fourth the Mixolydian, the fifth the Lydian, the sixth the Phrygian, and the seventh the Dorian.

Since the sound normally dwells and abides around the middle melodies for the most part, only occasionally moving out to the limits on account of the difficulty and force of loosening and tightening contrary to measure, the ancients took the diapason at those loci more or less between the perfect system in each of the tonoi, that is those from the hypate meson by position to the nete diezeugmenon at its arrangement, which it holds for the species of the diapason, and for those arranging the tonoi, from which, since in the Mixolydian tonos the first species happens to be that said to be the middle of the diapason, in the Lydian second, the Phrygian third, the Dorian fourth, the Hypolydian fifth, the Hypophrygian sixth, and the Hypodorian seventh, Ptolemy arranged the Mixolydian as the first tonos in this segment, Lydian second, Phrygian third, Dorian fourth, Hypolydian fifth, Hypophrygian sixth, and Hypodorian seventh. It is clear from this that it is impossible for another tonos to be placed in the double diapason perfect system which he himself will recognize as different from these. By necessity it will happen that this will turn out to be the same as one of the aforementioned seven, probably differing only in its arrangement. For if it were otherwise, so that the diapason encompassed by the nete diezeugmenon and hypate meson would be the first species, it will be likewise in the Mixolydian tonos, if it is second to the Lydian and so on in order, so that it would be impossible for there to be more than seven tonoi."

[150] Cf. 57.21.

diapason, for the species of both the first two consonances are of the same number,[151] [60.4] if we take them in succession by the ratios in each.[152] Their nature permits neither more nor less to be posited.[153]

[60.6] Now if someone should wish these divisions to consist of more parts, e.g. more than three in the diatessaron, or, by Zeus, in that many divisions but with imprecise excesses, or again well defined[154] but different from those described by harmonic proportion, immediately he is opposed by reason and natural appearance. [61.1] Similarly we cannot condone those who propose tonoi bounded by diapasons—tonoi which conform to the nature of the consonances and have their origin in them so that even entire systems might contain consonant differences—[61.4] to be either more in number than the seven species of the diapason and their ratios, or to have equal excesses between one another.[155]

[61.6] They do not have a worthy explanation[156] for either the equality of all the increments—in harmonics such an assumption is altogether unacceptable— or for all the differences to be, for example, whole tones, [61.9] or again half tones or dieses,[157] which they arrange and then calculate the number of the tonoi according to the amount of component[158] parts in the diapason.

[61.11] Why should they prefer to produce these particular magnitudes[159] when the consonance admits of whole tones and half tones and dieses and other intervals in the arrangements of both genera and

[151] Three of the diatessaron, four of the diapente; cf. I.6.

[152] Cf. 59.21f. Wallis (68) completes the sentence with, "that is, three in the diatessaron, four in the diapente, equal to the number of ratios, and so many for each [consonance] and in each genus as the nature of each genus requires."

[153] Mountford, "Greek Music and its Relation to Modern Times," *JHS* 40 (1920) 13-42, approaches the passage pragmatically by suggesting that application of the seven tonoi to the lyre would obviate the need or desire for the eighth 'mode.' Curiously, however, most of the extant fragments of notated ancient Greek music require the use of eight or more pitches despite the universality of the seven-string lyre and cithara.

[154] That is, with precise differences and three in number.

[155] Cf. Cleon. 190.6-192.11; the Aristoxenians state that the diapason consists of six whole tones, (the diatessaron of two whole tones plus one half tone, the diapente of three whole tones plus one half tone), and so they are more likely to allow a greater number of tonoi. Cleon. (203.4-204.8) proposes 13.

[156] The scholion clarifies this attack on the Aristoxenians this way: "That is to say that there should be no condoning of those established with more than seven species and ratios of the diapason, nor for those established where there are equal differences or purely unmixed sesquioctaves."

[157] As offered by the Harmonicists, who, as Aristoxenus (28.1-29.1) suggests, established twenty-eight consecutive dieses; cf. I.9, n. 147.

[158] The sense is that each succeeding gradation in pitch, be it by diesis, half tone, or whole tone, becomes the bottom pitch of the next higher *tonos*. The manuscript tradition is confused here, but Düring's reconstruction of the text here is quite serviceable.

[159] Düring (238) supplements 'differences, excesses' for 'magnitudes.' The Greek has only a pronoun (αὐτὰς), and adjective (τηλικαύτας).

intervals?[160] [61.13] They cannot say that this magnitude divides the diapason exactly while that one divides it inexactly, or that this one divides the diapason into virtually equal parts but that one divides it into unequal parts. [61.16] But if a whole tone[161] divides a diapason into six parts, the half tone into twelve, the third tone into eighteen, and the quarter tone into twenty-four, and thus none of these has an imperceptible difference,[162] which difference,[163] someone might ask, should separate[164] the seven tonoi, for the diapason is not divided into seven equal ratios, [62.1] nor is it a simple matter, assuming the parts to be unequal, to determine which of them are proper to employ?[165] [62.2] One would need to discuss the ratios found accordingly from the first consonances, that is, those left from the addition[166] on both sides[167] of the diatessaron within the diapason which is the corollary to the diapente taken as its opposite.[168]

[62.6] For when a note lower than another note by a diatessaron is higher by a diapente than its lower homophone, the note higher than it by a diatessaron is also lower by a diapente than its higher homophone. [62.9] It is necessary not only here but in general that the homophones precede and be established before the consonances and the consonances before the emmelic intervals. [62.11] It is therefore necessary to take first also the consonances of the tonoi and then those found from the differences between these, whatever they might be, since not as pleasant is a transition creating a modulation to the next tonoi as one creating a modulation by the differences between the first consonances.[169]

[160] The term Ptolemy uses here for 'intervals' (διαστάσεων) is actually more regularly rendered as 'distances.' It refers to more than the Aristoxenian, standardized and equalized diesis, half tone, and whole tone. It is intentionally vague, allowing for all the non-standard intervals he generated in Book I.

[161] Ptolemy infelicitously uses the term tonos here. Düring (238) cites the parallel from Aristides Quintilianus 25.6 [Meibom; 22.17-.21 Winnington-Ingram] and suggests that Aristoxenus was their common source.

[162] When any of these intervals are compiled in succession, they still amount to six whole tones.

[163] Or, 'ratio.'

[164] Or, 'define.'

[165] Ptolemy's argument is circular, for once again he makes basic assumptions (which he considers proven in Book I) about consonances. He reveals this at 62.6f. Here also he assumes seven *tonoi*, even though the purpose of the present chapter is to prove just that. Other tonoi systems could and apparently did exist dependent on entirely different assumptions.

[166] The addition would be of a second diapason and/or a disjunct whole tone. Combined, they form the diapente he labels the 'corollary' (τῇ συνισταμένῃ).

[167] The scholiast argues convincingly that 'on both sides' (ἐφ᾽ ἑκάτερα) is only the Ptolemaic idiom for 'the other side.' Cf. Alexanderson @ 62.3.

[168] Düring's rewritten phrase is needed on account of the two participles οὔσης and συνισταμένης. Lacking a connective, they should be in different cases, and the dative matches the case of the article found in part of the manuscript tradition.

[169] Which would be a smoother, more emmelic modulation. Cf. Cleon. 205.6-206.2.

II.10 - *How the Differences Between the Tonoi Are To Be Taken Properly*

[62.16] Those advancing the notion of the eight tonoi[170] by superfluously adding another to the seven seem to have almost hit upon, even if they did not use the required method, the proper differences of the tonoi.[171] [62.18] They simply put forth the three most ancient tonoi, called the Dorian, Phrygian, and Lydian after the names of the peoples from whom they derive[172]—others have other etiologies—which differ from each other by a whole tone, whence perhaps they named them equally[173] 'tonoi.' [62.22] Then they make the first consonant modulation from the lowest of the three, the Dorian, to one diatessaron higher. This tonos they labeled the Mixolydian[174] from its proximity to the Lydian because it no longer differed from the Lydian by a whole tone but by the remainder of the diatessaron less the ditone lying between the Dorian and Lydian.[175]

[170] Cf. 59.23. We have no other passage in which an ancient theorist proposes eight tonoi. Admittedly this is an *argumentum ex silentio*, but since Ptolemy took such care to detail the tetrachordal divisions of Archytas, Aristoxenus, Eratosthenes, and Didymus, it is doubtful that any of them proposed eight. There is always the possibility that we must attribute the number to one of the lesser, non-extant music theorists surely known to Ptolemy, e.g. Theophrastus, and we must consider as well anonymous musicians (as opposed to music theorists).

Düring (238-39) has a useful discussion here of tonic terminology. Summarizing such useful passages as Heracleides Ponticus *ap.* Ath. 625A and D, Plutarch *Plat. Quaest.* 1008E-F (on the relative harmonics of the soul)—Pollux *Onomasticon* 4.75 (223.17f. Bethe) is not as helpful, but cf. 4.78)—he suggests that the 'hyper/hypo' terminology must have resulted from the systematization by a theorist (or practicing musician). Gray areas include the Aeolian (=Hypodorian) and Iastian (=Hypophrygian).

[171] Cf. 57.21.

[172] It is not common for musicologists—ancient or modern—to connect these tonic names with the peoples of the same names, but these nations had very disparate musics in the eighth through sixth centuries B.C. By the early fourth century, however, these national musics had become simplified, reduced to mere scales of different magnitudes and intervallic successions; cf. Aristides Quintilianus 19.2-20.1, for the so-called "Platonic" scales. Two generations later, Aristoxenus could label diapason scales Dorian, Phrygian, and Lydian even though they were of similar magnitude and regularized intervallic successions. In Ptolemy's day they were well established as such, so much so that in the first few centuries A.D. a number of authors, including Aristides Quintilianus, the pseudo-Plutarchian author of the *De musica*, and Bacchius, had a considerable antiquarian interest in them. I have discussed this in "Ekbole and Eklusis in the Musical Treatise of Bacchius," *Symbolae Osloensis* 50 (1980) 111-26 and in "*Tonoi*" 248-51.

[173] For the correct text (ἴσως), see Ernest Frederick Bojesen, "De tonis sive harmoniis graecorum commentatio," *Indbydelsesskrift til Sorö Academies Skoles aarlige hovedexamen i Juli 1843* (Copenhagen 1843) 12. Théodore Reinach, "Zu Ptolemaios Harmonika II,10," *Hermes* 43 (1908) 478, delineates the argument, a clear case of reading a technical term (ἰσοτόνους - 'isotonic') for a common expression (ἴσως τόνους - 'equally tonoi'); cf. M. L. West, *Textual Criticism and Editorial Technique* (Stuttgart,1973) 18, on this type of textual corruption.

[174] Cf. Bry. II.3 (158.31-159.4), and Pach. 198.7f. Barker, *GMW* I.221-22 (*ap.* [Plut] *De mus.* 1136C-F) offers a number of primary references.

[175] Notice Ptolemy's careful, non-committal wording here. He avoids using his own calculations or referring to the half tone of the Aristoxenians. Palisca (305-306) offers parallel translations of Ptolemy and Mei (*De modis* 74-5).

[63.1] Then, because the Dorian was a diatessaron lower, in order to add to the other tonoi the tonoi a diatessaron lower, they named that under the Lydian the Hypolydian,[176] that under the Phrygian the Hypophrygian, and that under the Dorian the Hypodorian. [63.5] They labeled the tonos similar to it but one diapason higher than this the Hypermixolydian[177] in light of its being established 'above'[178] the Mixolydian. (They misused the term 'Hypo-' to indicate the lower, 'hyper' for the higher.) [63.9] There is a whole tone difference between the Hypodorian and Hypophrygian, as was appropriate for the first tonoi,[179] and so it is between the Hypophrygian and the Hypolydian. [63.11] From the Hypolydian to the Dorian, however, is the leimma, which they[180] prefer to make a half tone. [63.12] But, as we said,[181] one ought not to take the consonances from the emmelic intervals but the opposite, the emmelic intervals from the consonances, since the consonances are easier to reckon and more important for a number of things, particularly modulations.

[63.15] This would occur properly if in putting forth a higher tonos, which we shall call A, we take the first tonos lower than this by a diatessaron, that is B, and that lower than this by a diatessaron[182] but still within the diapason, which we shall call C. [63.18] Then, since the tonos lower than this by a diatessaron falls beyond the diapason, taking the tonos which functions like it, that is the tonos higher than C by a diapente, which we shall call D, let us again take the tonos lower than this by a diatessaron, which we shall call E; and again instead of taking the tonos lower than E by a diatessaron, [63.22] since it, too, would fall beyond the diapason, let us make a tonos F higher than E by a diapente and another, G, lower than this by a diatessaron.

[63.25] If we take the tonoi in this way by the continual lowering from the first consonance, the diatessaron, which is, as we said, the same as the

[176] [Plut.] *De mus.* 1141B, for which consult Barker (*GMW* I.222, n. 114), offers the information that Polymnestus established this *tonos*. Cf. Düring, *Ptolemaios und Porphyrios* 6.

[177] Without fanfare Ptolemy finally specifies the unnecessary *tonos*. Still maintaining his method, however, he treats the problem theoretically first, applies it to a specific instance or series of instances, and then, as he does more often than not, gives a geometrical demonstration, as he does here at 63.25 - 64.11. On Ptolemy and the Hypermixolydian, see Winnington-Ingram, *Mode* 18-19. Bower, "Boethius and Nicomachus" 32-35, discusses the misconception by Kunz ("Die Tonartenlehre des Boethius," *Kirchenmusikalisches Jahrbuch* 31 (1936) 5-24) and U. Pizzani ("Studi sulle fonti del 'De Institutione musica' di Boezio," *Sacris erudiri* 16 (1965) 5-164) that Boethius believed Ptolemy had in fact added this eighth 'mode.'

[178] The prefix 'hyper' means 'above.' Cf. Bry. II.3 (160.7-9).

[179] The Dorian and Lydian.

[180] The Aristoxenians.

[181] 62.9-.12. Düring (239) here specifies Didymus as the pre-Aristoxenian systematizer of the seven (or six) tonoi / harmoniai.

[182] The reading in f-group of manuscripts (τὸν τούτου τῷ διὰ τεσσάρων βαρύτερον) is clearer than that used by Düring (τὸν τούτῳ διὰ τεσσάρων ἐπὶ τὸ βαρύτερον) may be the result of a two-step error—first conflation and then expansion.

rise by increments of diapente,[183] surely it will follow that whole tone differences exist between CE, GE, BD, and DF, while those between GB and FA will be of what is called the leimma. [63.33] Since the tonos D lies higher than E by a diatessaron, and C by a diapente, the difference between C and E will be a whole tone. Similarly since the tonos F is higher than G by a diatessaron and E by a diapente, also the difference between E and G will be a whole tone. [64.2] Again, since C is lower than G by a ditone and B by a diatessaron, the difference between B and G will be the leimma. Finally, since BC, DE, FG, and BA are all the diatessaron, so that the difference of EC is the same as that of DB, that of EG the same as that of FD, and BG that of AF, the differences of both DB and FD will be whole tones and that of AF the leimma. [64.7] And if we should take some tonos a diapason from C or from A, clearly it also will have an difference of a whole tone from the next tonos since AC forms a double diatessaron and misses a diapason by a whole tone.

Mixolydian	A------------------
	leimma
Lydian	F------------------
	tonos
Phrygian	D------------------
	tonos
Dorian	B------------------
	leimma
Hypolydian	G------------------
	tonos
Hypophrygian	E------------------
	tonos
Hypodorian	C------------------

[64.11] A is in the Mixolydian , F Lydian , D Phrygian , B Dorian ,[184] G Hypolydian, E Hypophrygian, and C Hypodorian.[185] And so their differences, once handed down carelessly, will be discovered by reason.[186]

---

[183] E.gg. 60.1f. and 62.1f.

[184] The Byzantine scholion adds here, "One should know that each of the aforementioned which makes a difference in echos is able as well to fold out its own echos into eight tonoi or into the diapason system, as we say of the Mixolydian echos and tonos, the Dorian, Phrygian, and so on. This will be demonstrated more clearly in chapter 15." Cf. E. Wellesz, *Byzantine Music and Hymnography*² (Oxford 1961) 300f.

[185] Ptolemy's notational capital letters have nothing (except for indirect ancestral origin) to do with our alphabetical notation system. Nonetheless, for those who would place the seven *tonoi* on a modern staff, J. Murray Barbour, "The Principles of Greek Notation," *JAMS* 13 (1960) 1-17, has collected certain guidelines: If one uses the Hypolydian as the natural tonos (as did Bellermann, Gombosi, Macran, Reinach, Sachs, and Westphal), then there will be from 0 to 6 flats; if one uses the Hypodorian (as did Auda, Greif, Paul, and Vincent), then there will be from four sharps to two flats; if the Dorian (as Abert, Düring, and Riemann), there will be from six sharps to one flat.

II.11 - *That Tonoi Ought Not To Be Increased By Means of the Half Tone*[187]

[64.16] Now that we have established these tonoi, it is clear that one particular note of the diapason belongs as the dynamic mese for each tonos since the tonoi and the species of the diapason have the same number.[188] [64.18] If we take the diapason at the loci in the middle, so to speak, of the perfect system, that is, the loci from the thetic hypate meson to the nete diezeugmenon—[65.3] this is because the voice gladly moves and remains at the middle of melodies, less frequently moving out to the limits, since it involves laboring and forcing to sing higher or lower than the middle[189]—the dynamic mese of the Mixolydian is realized at the locus of the [65.6] paranete diezeugmenon so that the tonos might make the first species of the diapason we have established;[190] the dynamic mese of the Lydian is realized at the locus of the trite diezeugmenon for the second species;[191] [65.10] the dynamic mese of the Phrygian at the locus of the paramese for the third species; that of the Dorian at the locus of the mese which makes the fourth and middle species of the diapason; that of the Hypolydian at the locus of the lichanos meson for the fifth species; [65.13] that of the Hypophrygian at the locus of the parhypate meson for the sixth species; and that of the Hypodorian at the locus of the hypate meson for the seventh species.

[65.15] We do this so some notes in the system are able to remain unmoved[192] and preserve the magnitude of the voice[193] in the transitions of

---

[186] It is worthwhile to reflect here upon how consistent and therefore effective Ptolemy's method has been to this point. As in Book I, he reports or assumes what music and music theory his predecessors have produced, attempts to organize and justify it through rational, predictable, logical, 'natural' analysis, and then attempts a geometrical proof or at least demonstration of it. He did this with sound, notes, the division of the tetrachord, the Greater Perfect System and Lesser Perfect System, and now the tonoi.

[187] The placement of this chapter is a bit odd. Ptolemy has just finished detailing his three determinatives of the tonoi (II.7-10). He now will discuss another school which maintains improperly that there are more than seven tonoi; perhaps the chapter was an afterthought in this sense. On the other hand, he adds the discussion of dynamic mese to II.10 to make the demonstrations here and there valid and comprehendible (and practicable). On the entire chapter, see Lohmann, *Musike* 36-38, 41-45, and 55f. Palisca (46) offers a slightly different summary; there follow (140-42) Gogava's translation of the entire chapter and Leoniceno's (291-92); see also 303-304, and Winnington-Ingram, *Mode* 68-69.

[188] Ptolemy distinguishes practical, usable music (dynamic) from mere analytical species. Cf. I.11 (25.5f.). Augio's translation (1545) of this passage is given in Palisca 124-5.

[189] Cf. [Arist.] *Problems* 19.20f. (919ᵃ13).

[190] Ptolemy begins describing the sequence here with (thetic) paranete diezeugmenon. Winnington-Ingram, *Mode* 68-69, describes each tonos thereafter as itself a Greater Perfect System, a veritable species of the disdiapason [=double diapason].

[191] At the turn of this century such a shifting of dynamic mese suggested to followers of Helmholz that the Greek mese was analogous to a keynote or tonic. C. F. Abdy Williams, "The Notes Mese and Hypate in Greek Music," *CR* 12 (1898) 98- 100, preferred to think of mese as a Gregorian dominant.

[192] Especially in the Dorian.

[193] This phrase refers to the range of the pitch itself. Cf. 65.23-24.

the tonoi since similar functions in the different tonoi never fall in the loci of the same notes.[194]

[65.19] But if more tonoi are posited beyond these, which is what those who increase the differences between them by half tones do,[195] it will be necessary to realize mesai of two tonoi at the locus of one note. [65.22] Whole systems are moved as a result of the transition of these two tonoi into one, and no longer preserved is any common initial pitch by which the characteristic of the voice is measured. [65.24] If, for example, the dynamic mese of the Hypodorian is joined to the thetic hypate meson and that of the Hypophrygian is joined to the parhypate meson, it will be necessary for the tonos taken between them—they call it the lower Hypophrygian (as opposed to the higher)[196]—to have its mese at the hypate, as in the Hypodorian, or at the parhypate as in the higher Phrygian.

[65.31] In this instance, when we make a transition between tonoi having a common note, this note will be moved up or down by a half tone; and by virtue of its having the same function in each of the tonoi, that is, as mese, the raising or lowering of all the other notes will follow so as to preserve the same relationships[197] with mese for those notes taken before the modulation in the genus common to both tonoi. [66.1]As a result this tonos will not be thought of as different in species from what it was before but is again Hypodorian or the same Hypophrygian only tuned upwards or downwards. As far as it goes, let this be both a rational and sufficient explanation of the seven tonoi.

II.12 - *On the Difficulty In Using the Monochord Canon*[198]

[66.6] Since there remains the task of dividing up the harmonic canon to demonstrate with all the evidence the agreement between reason and perception —not in only one tonos, for example in the ametabolic system, nor in one or two genera as has been done by earlier theorists, [66.9] but in every single tonos and in each of the genera realized so that we might have laid out simultaneously the common loci of the notes—let us first briefly run through the deficiencies of this monochord canon.[199] [66.13] Nothing up to

[194] By definition each note must change in function when the tonos has been changed.

[195] As does Alypius, for instance, who employs 15 tropoi (=tonoi).

[196] Gombosi, *Tonarten* 99, offers a translation and discussion of this description of the lower Hypophrygian.

[197] He means intervallic relationships.

[198] This entire chapter has previously been translated into French and commented on by Charles-Émile Ruelle, "Le Monocorde, Instrument de musique," *REG* 10 (1897) 309-312, and into German by S. Wantzlöben, *Das Monochord als Instrument und als System* (Halle 1911) 4-30. Düring (241-3) treats both harshly. See also Théodore Reinach, "La guitare dans l'art grec," *REG* 8 (1895) 371, with Nic. *Ench.* 243.14-.15, Pollux *Onomasticon* 4.60 (219.5-6 Bethe), and Ath. 183F, where the monochord is called φάνδουρος or πανδούρα.

[199] I read the sentence as a statement; Düring takes it as a question.

now seems to have been invented for the perceptions to judge clearly the tunings derived by reason in all their realizations,[200] yet this instrument seems to have faded from use for both practical playing and for speculation about the results of realized ratios.

66.17 To others, each of these purposes I just mentioned did not seem appropriate. It was only a theoretical instrument among acousticians, a practical instrument among lyres, cithara, and the like, even if in these instruments the emmelic intervals properly composed by reason are found but not demonstrated by them.[201] [66.21] Nor can such perfection be pinpointed either on auloi or syrinxes, instruments more appropriate for both demonstrations since differences between notes correspond to pipe lengths.[202]

[66.24] The monochord would be shown to be more deficient than the others since each of them suffices in one aspect[203] but it in neither; first the regularity of the string is not assured, nor are the positionings of the limits, neither are the given ratios of part of the strings correct.[204] [66.28] By no means will the divisions be made according to proportion but simply by tightening the string, then moving the bridge until each of the desired notes strikes the hearing. [66.31] There they show the appropriate length but disregard[205] the purpose for which it exists[206]—just as with those who manufacture wind instruments. [66.32] And then if the framework is properly fitted and the bridge moved slowly, the notes might be able to be compared reasonably well, but when the bridge is moved more hurriedly, forced by the progression of the melody or the rhythm,[207] [67.1] no longer will the result be similar, with none of the proper indicators being accurately taken and the bridge on account of the haste not well placed.

[67.3] This instrument would be last of all and the weakest insofar as actual use is concerned not only because the hands tune and play it separately,[208] so as to be deprived of the most beautiful of techniques

[200] Cf. Alexanderson 16.

[201] I would prefer the genitive plural participle (δεικνυμένων) here.

[202] Cf. supra, I.3, esp. 8.25f.

[203] The wind instruments just mentioned have only relatively accurate, but preestablished (and therefore inflexible) tuning.

[204] Cf. Alexanderson 16 (66.28) for the full stop.

[205] Düring (245) mentions G's (Vaticanus gr. 198) gloss here (ἀμελήσαντες τοῦ λόγου) with a misprint: 66.29 for 66.31; also, read 66.33 for 66.23, where the "framework" resembles the arms of the lyre.

[206] Ptolemy allows for some accuracy of the hearing, but achieving the proper tuning by this "sliding bridge" method is neither scientific nor commendable. Cf. Burkert 374.

[207] E.g. at a faster tempo.

[208] Düring (245) overemphasizes this adverb, for Ptolemy does not make a general statement about playing the lyre or cithara—striking the strings with the right hand, stopping them with the left—but that with this awkward monochord one hand, presumably the left, would be busy retuning (not stopping) the strings. Following Karl von Jan, "Die griechische Saiteninstrumente," *Programm des Gymnasiums Saargemund* (Leipzig 1882), he summons up a rarely cited passage, Cic. *II Verr.* 1.20 (53), which mentions, he claims, a left-handed virtuoso.

[67.6]—playing on instruments,[209] striking together,[210] weaving up, weaving down,[211] dragging,[212] and playing together through the separate leaps[213]—since the playing hand, being one hand only, is not easily able to jump to the greater intervals,[214] nor to play at two different loci at the same time;[215] [67.10] but also since there necessarily will follow in most instances a continuity of sounds which provide the most ecmelic species and render no fixed or limited note.[216] [67.12] It makes such sounds by moving the bridges which are dragged forth and rub the string, since they cannot leap and, so to speak, jump to different loci, whereby they are not easily able to be used in swifter rhythms. [67.16] On account of this, I think those using such an instrument and knowing its failings in realizing notes[217] should by no means use it alone, where it can be evaluated by the senses, but play it with the aulos or syrinx so that by their accompaniment the monochord's deficiencies can escape our attention.[218]

II.13 - *On What Additions the Musician Didymus Thought To Make For the Canon*

[67.21] The musician Didymus first attempted to add some improvements to the canon.[219] He did not achieve what was necessary, however; he applied

Cicero's phrase, however, is merely that the citharist *omnia intus canere*, i.e. "sings inside" (so Verres removed him to his own home).

[209] Our best guess is that *epipsalmou* (ἐπιψαλμοῦ) and the following terms describe various methods of lyre playing. Judging by the etymology of this word and συγκρούσεως (*sugkrouseos* - 'strike together'), one supposes, as does Düring (246-47), that this is open playing, probably with two hands. This passage on lyre-playing techniques also tempts us to understand more completely the art of ancient *melopoiia* or, literally, 'melodic composition.' Aristoxenus wrote an entire treatise on the subject, but it is lost. Its echoes can be found in Cleon. 206.19f., Aristides Quintilianus 28.8-.31.2, and Bry. III.3, 5, and 10. See also my discussion in "The Seikilos Inscription: A Musicological Analysis," *AJPh* 107 (1986) 472-77.

[210] The definition in LSJ is "rapid alternation of two notes, trill."

[211] Cf. Aristides Quintilianus 16.20-16.21, 207.3-.4.

[212] The term is σύρματος (*surmatos*), which seems to refer to a bending of the string. At 67.13 ἐπισυρόντων (*episuronton*) refers to slurring notes.

[213] Ptolemy refers to broader, non-composite intervals, e.g. those that begin the Seikilos inscription and Mesomedes' *Hymn to the Muse*, which would require greater hand movement.

[214] Because of the awkwardness of tuning the canon.

[215] Ptolemy provides evidence here for a simple organum. The lyre or cithara, of course, but not the monochord canon, could provide simultaneous, multiple pitches.

[216] Cf. I.4 on continuous sounds.

[217] Alexanderson (16) translates: "They are aware that the instrument misses, fails to arrive correctly at, the notes of the harmony."

[218] The canon produces accurate tones correctly but slowly. Wind instruments produce them more quickly but more inaccurately. Düring (247) compares Plut. *De recta ratione audiendi* 41C, but Plutarch says only that the impressive delivery of those who sing to the flute's accompaniment makes the audience ignore many mistakes.

[219] Didymus (fl. 60 A.D.), for whom see L. Cohn, "Didymus #11," *RE* V,1 (1903) 473-74, wrote a treatise, no longer extant, *On the Difference Between the Aristoxenians and*

himself only to placing the bridge more conveniently. [67.23] He was not able to discover any solution for the other difficulties greater and more numerous, which we have detailed.[220] For he takes the distances between the notes not from only the one endpoint but also from the opposite;[221] [67.26-7] at such positions the extreme lengths are unequal to each other and each forms its own ratio of some note in relation to the entire length.

[67.28] When, for example, there are two parts standing in a duple ratio to each other, it is evident that in relation to the whole the greater is in the sesquialter ratio, the diapente, the smaller in the triple ratio, the diapason plus diapente. [68.2] With the entire stretch tuned as the proslambanomenos, the greater of the portions, two-thirds of it, will make the hypate meson, the lesser, one-third of it, the nete diezeugmenon,[222] and so on with the other strings that are similar.

[68.6] Such an improvement, I will concede, addresses the deficiency caused by the continuous[223] bridges since very often the bridges can remain during several different strikings at loci common to two notes. [68.9] When this happens the beating is transferred from them to each section.[224] But it makes the method [of playing][225] more difficult when the melody does not join common notes; [68.11] because the loci of the same notes are different, one looks to see which string he must use, but continuous playing does not permit any time for such deliberations. It would be handier not to choose most of the loci but to attack in succession at one, unchanging locus.

68.15 In his calculations of divisions, he did not pay any attention at all to what appears to the perceptions. Even he established three genera, diatonic, chromatic, and enharmonic, but he made the divisions of only two genera, the chromatic and diatonic, and only in the ametabolic system, nor even here were the calculations made correctly. [68.20] He placed the leading tones of the tetrachords in the 5:4 ratio to the third tones from them in both genera, to the second highest in the chromatic in the 6:5 ratio, and in 9:8 to that of the diatonic.[226] [68.24] Therefore the following intervals in other genera encompass the 16:15 ratio, the middle 25:24 in the chromatic, 10:9 in the diatonic, contrary to what our perceptions sense.[227]

Pythagorean Music Theory, which Porphyry cited frequently, e.gg. @ 5.11., 25.5., 26.6., 27.17., and 107.15. Cf. Palisca 216; and Burkert 370.

[220] In II.12.

[221] Didymus uses the canon differently by calculating intervals and ratios from both endpoints. In this sense, Didymus uses the canon in much the same way Ptolemy uses the helicon; cf. II.16.

[222] Cf. Wallis 86.

[223] And movable. Cf. 67.11 (and 48.23).

[224] Of the strings.

[225] My supplement.

[226] Cf. the tables in II.14.

[227] Llewelyn S. Lloyd and Hugh Boyle, *Intervals, Scales and Temperament* (London 1963) 38, identify Didymus' diatonic (9:8 x 10:9 x 16:15) as our modern major mode, that is:

[68.27] For he made the following ratio of these bounding the pyknon in the chromatic genus greater than the middle ratio;[228] this is by no means emmelic.[229] [68.29] In the diatonic also he made the leading ratio greater than the middle, the opposite of what is required in the simple diatonic.[230] In addition, he made nonetheless the following intervals of two genera equal, but those of the diatonic need to be smaller.[231]

[68.31] Many have hypothesized such untested ratios, and the cause for this is their not considering beforehand the use to which these interval will be put. [68.34] Only from such considerations are they able to be compared by our sense perceptions.[232] On account of this, even the ratios of the consonances which are able to be compared on one string seem to have been taken by dividing the string into two, [69.3] but the ratios of the emmelic intervals which can be scrutinized only in conjunction with the whole system—which is impossible to see accurately on one string—they miscalculated entirely. [69.5] They would be clearly refuted if someone would form the appropriate lengths on the eight equal strings we discussed earlier.[233] These would suffice to demonstrate to the hearing the series of the melody. Then the hearing might distinguish the genuine and the false.

[69.8] And in order to make the comparison easier between our own generic divisions and those discussed above, at least all the ones we have encountered, we will display a partial collation of them in the middle, that is, Dorian, tonos to clarify the aforementioned differences in the scale only.[234]

[69.13] We have not at all used the same methods for division used by our predecessors. They divided the entire length into the ratios designated for each note, but this is a laborious and difficult method of measurement. [69.16] We instead will at the outset place our calibrator along the strings and divide the length remaining from the sounding portion at the highest limit to what will be the sign under the lowest note at lengths equal and symmetrical in magnitude. [69.20] To these we add the numbers, beginning

C		d		e		f		g		a		b		c
24		27		30		32		36		40		45		48
	9:8		10:9		16:15		9:8		10:9		9:8		16:15	
	264		297		330		352		396		440		495	

[228] Cf. the tables in II.14.

[229] Aristoxenus (*Harm.* 65.25) describes such an arrangement of notes and intervals as 'aharmonic' (ἀνάρμοστοι). Winnington-Ingram, "Aristoxenus and the Intervals" 197, however, estimates that such a tetrachordal division as 1/2 + 1/4 + 1 3/4, which closely resembles this Didymean chromatic, may represent an actual chromatic scale used by contemporary musicians.

[230] 9:8.

[231] For a discussion of Gaffurio's misinterpretation of these lines, see Palisca 217.

[232] Later in the textual tradition the adverb (ὑγιῶς) 'properly' was added here or after (ἐξειληφότες) 'compared.'

[233] In II.8.

[234] Cf. II.14.

with the highest end point, and these numbers will include whatever fractions are permitted.[235]

[69.21] Then, stretching out those constituting in the proper ratios each of the notes from the aforementioned common connections, we can always and easily place the connections of the moveable bridges under the places designated by the calibrator. [69.24-25] Since it happens that the continuous numbers [i.e. integers] describe the common differences between genera in tens of thousands, we will use whole, complete units with the nearest fractions to the sixtieth of that unit. The comparisons in the division of the canon will not then differ by more than one sixtieth of one part.[236]

[69.30] In addition, so that the length of the diatessaron beneath the disjunction consists of 30 parts, as Aristoxenus prescribed,[237] and in considering by the same numbers the division of the tetrachord and also the divisions in it on a larger scale, we divide the length from the common endpoint to the lowest note of the diapason into 120 parts; [69.35] the 3:4 ratio into 90, the note higher than it by a diatessaron; so that the note higher than the lower of them by a diapente will have 80 parts in the sesquialter ratio;[238] the highest of the diapason 60 parts in the duple ratio; and those moveable notes in between have as many parts as are fitting for the ratios in each genus.

II.14 - *Table of the Numbers Making the Divisions of the Diapason in Both the Ametabolic Tonos and the Genera in Each*[239]

[70.7] We have drawn up three tables of eight lines each.[240] The first has five columns, the second eight, the third ten, and the order of notes is arranged in the first column of each.[241]

[235] Alexanderson 16 (*ap* 69.20f.) proposes "at an interval of as many parts (sections) as is convenient."

[236] Düring (259-60), citing Moritz Benedikt Cantor, *Vorlesungen über die Geschichte der Mathematik* (Leipzig 1907), reminds us that Ptolemy's use of the sexigesimal denominator should be attributed to the influence of Babylonian astronomy; to this one can add Toomer 6-7, and Neugebauer, *Exact Sciences* 2-28, esp. 17-23. This helps to corroborate what little evidence we have that Ptolemy came to the study of harmonics after his astronomical education was well established.

[237] At I.12 (29.21).

[238] Düring (85) incorrectly reports the ratio here as 4:5.

[239] The scholiast here defines ametabolic as resulting from "the whole tone at 9:8, which when placed between makes the modulation from one tetrachord to the next so similar as to seem to be 'unchanging.'"

[240] Citing and thanking Winnington-Ingram ("Aristoxenus and the Intervals" 195-208), Düring (248-59) gives a lengthy discussion on the tables of which this chapter is almost entirely comprised. He offers both ratios and cents with accompanying discussion. A list of simple equivalencies of ratios, cents, and pitch relationships follows:

[70.10] The first table contains the enharmonic genera.[242] In the first column is that according to Archytas in the ratios 5:4, 36:35, and 28:27; in the second that according to Aristoxenus in distances of 24, 3, and 3 parts;[243] in the third that according to Eratosthenes in the ratios 19:15, 39:38, and 40:39;[244] in the fourth that according to Didymus in the ratios 5:4, 31:30, and 32:31; and in the fifth ours in the ratios 5:4, 24:23, and 46:45.[245]

(256:243 = 90.225)

10:9	=	182	=	c-c#
9:8	=	204	=	c-d
6:5	=	316	=	c-d#
5:4	=	386	=	c-e
4:3	=	498	=	c-f
3:2	=	702	=	c-g
2:1	=	1200	=	c-c'

Llewelyn S. Lloyd and Hugh Boyle, *Intervals, Scales and Temperament* (London 1963) 226-41, supply a full conversion table for cents, decimal ratios, and monochord string lengths, where the ratio x 10,000 equals the reading in the third column. See also, John W. Link, Jr., *The Mathematics of Music* (Baltimore 1977) 30-32, and Vogel, *Enharmonik* 33-35.

The scholion here says, "He describes 'lines' as those encompassing the sexigesimals in order and those moving in parallel to them. The columns, he says, go at the bottom where the ratios are calculated in relation to each other." He apparently mistakes Ptolemy's summaries for his five parallel columns.

[241] In all Ptolemy described 23 tetrachordal divisions. Of course it is due to coincidence, but John Curtis, "Reconstruction of the Greater Perfect System," *JHS* 44 (1924) 10-23, also found the number of total scales available within the G.P.S. (including the fifteen Alypian tropoi, the Mixolydian (Pl. *Resp.* 398E), Charalolydian (Pl. *Resp.* 399A), Syntonolydian (Ath. 624F), Hypermixolydian (Cleon. 204.14), the Gregorian Hypomixolydian, and then a hypothetical Hypersyntonolydian, Hypocharalolydian, and Hyposyntonolydian) to be 23 in number as well.

[242] Blanket statements about the use or survival of the enharmonic genus in the late fourth century are *argumenta ex silentio*. Winnington-Ingram, ("Greece" 664), observes that all these enharmonic divisions assume a large interval (whether the ditone (81:64) or major third (5:4) [or Didymus' 19:15]) and two approximately equal microtones which are falling into disuse in the fourth century. He notes their absence in the Delphic Hymns. Düring's (249-54) analysis of the "enharmonic Lydian" in the *Orestes* papyrus, however, depends on fitting the music of the fragment into the Greater Perfect System, or the upper segment thereof. One should fit it instead to the Dorian/Phrygian scales preserved in Aristides Quintilianus (19.2-20.1); cf. Vogel *Enharmonik* 109-113; and Jon Solomon, "A Diphonal Diphthong in the *Orestes* Papyrus," *The American Journal of Philology* 97 (1976) 172-73, and "*Orestes* 344-45: Colometry and Music," *Greek, Roman, and Byzantine Studies* 18 (1977) 71-83.

[243] For the Aristoxenian integers and the substitution of 30 for 60, cf. I.12 (29.21), with note 199. See also, da Rios 37, n. 2 (*trad.*).

[244] On Eratosthenes (c. 240 B.C.), librarian in Alexandria under the Ptolemies Euergetes and Epiphanes, the same scientist who measured quite accurately the circumference of the earth and the mathematician who is credited with creating the "sieve" with which to derive the prime numbers, see G. E. R. Lloyd, *Greek Science After Aristotle* (New York 1973) 49-50, and Lucas N. H. Bunt, P. Jones, and J. Bedient, *The Historical Roots of Elementary Mathematics* (Englewood Cliffs, NJ 1976) 97-103. Levin, *Nicomachus* (1975) 27, discusses the degree to which Eratosthenes is known to Ptolemy's contemporary mathematician and harmonicist, Nicomachus. Ptolemy, of course, knew Eratosthenes' non-harmonic work quite thoroughly; cf. C. M. Taisbak, "Eleven Eighty-Thirds; Ptolemy's Reference to Eratosthenes in *Almagest* I.12," *Centaurus* 27 (1984) 165-66. K. Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *CQ* 27 (1933) 93-94, traces the intervals of Eratosthenes' enharmonic (which Paul

The Enharmonic Genera[246]

Archytas	*Aristoxenus*	*Eratosthenes*	*Didymus*	*Ptolemy*
60	60	60	60	60
75	76	76	75	75
77 1/7	78	78	77 1/2	78 6/23
80	80	80	80	80
90	90	90	90	90
112 1/2	114	114	112 1/2	112 1/2
115 5/7	117	117	116 1/4	117 9/23
120	120	120	120	120
$\frac{5}{4}\times\frac{36}{35}\times\frac{28}{27}=\frac{4}{3}$	24+3+3=30	$\frac{19}{15}\times\frac{39}{38}\times\frac{40}{39}=\frac{4}{3}$	$\frac{5}{4}\times\frac{31}{30}\times\frac{32}{31}=\frac{4}{3}$	$\frac{5}{4}\times\frac{24}{23}\times\frac{46}{45}=\frac{4}{3}$

Tannery, "Sur les Intervalles de la musique grecque," *REG* 15 (1902) 343-44, labels "une fiction mathematique") into medieval Islamic theory, specifically the *Kitabu L-Musiqi Al-Kabir* of Al-Farabi; cf. Baron Rudolphe d'Erlanger, *La Musique Arabe* I (Paris 1930) 218f.

[245] We have intervallic computations from Archytas, Eratosthenes, Didymus, and Aristoxenus here in addition to Ptolemy's own, and elsewhere we have those divisions found in Plato (*Tim.* 35B) and Boethius (*De mus.* 4.6 and 3.8, the latter those of pseudo-Philolaus. Winnington-Ingram, "Aristoxenus and the Intervals" 195, and Paul Tannery, "Sur les Intervalles de la musique grecque," REG 15 (1902) 345-51, put them into context. See also Barbera, "Divisions" 306-307, on the divisions made by Thrasyllus and Gaudentius.

The scholion reminds us here that "Aristoxenus made the divisions of the harmoniai not from the ratios, by which the excesses of the notes are in relation to each other, but by the notes in between the intervals, as was shown in the twelfth chapter of Book I."

[246] I use here the sexagesimal fractions preferred by Ptolemy (69.24-25) and found in the manuscripts. The decimal fractions originate with Düring. The Aristoxenian column in this and the following two (chromatic and diatonic) tables are misconceived as proportional and not spatial. Particularly because of the lacuna in the text here, one is unable to assume that it was Ptolemy himself who so misrepresented the Aristoxenian view of tetrachordal divisions. In the prose portions of I.11 and I.12 Ptolemy understands thoroughly (and therefore stands in opposition to) the Aristoxenian method of using constant string lengths to measure out the tetrachord, so why here would he not only apply proportional calculations to Aristoxenian space but even calculate whole diapasons rather than characteristic Aristoxenian tetrachords? Since the Aristoxenian enharmonic tetrachord is divided by quartertone diesis plus quartertone diesis plus ditone, then, assuming with the author of these presumably contaminated pseudo-Ptolemaic tables that the diapason (5 whole tones and 2 half tones) is 60 units in length, the whole tone will consists of 10 units, the ditone of 20, and the quartertone diesis of 2 1/2. The column would then read: 60, 62 1/2, 65, 85, 95, 97 1/2, 100, 120.

[71.7] The second table contains [the chromatic genera.[247] In the first column is that according to Archytas with the ratios 32:27, 243:224, and 28:27; in the second that according to the Aristoxenian 'soft' chromatic[248] of 22, 4, and 4 parts; in the third that according to the Aristoxenian hemiolic chromatic of 21, 4 1/2, and 4 1/2 parts; in the fourth that according to the Aristoxenian tonic chromatic of 18, 6, and 6 parts; in the fifth that according to Eratosthenes in ratios of 6:5, 19:18, and 20:19;[249] in the sixth that according to Didymus in ratios of 6:5, 25:24, and 16:15;[250] in the seventh our 'soft' chromatic in ratios of 6:5, 15:14, and 28:27;[251] and in the eighth our syntonic chromatic in ratios of 7:6, 12:11, and 22:21.[252]

[247] The bracket I have placed here marks the first major lacuna in the manuscript tradition of the treatise. Mountford, "Harmonics" 82, n. 30, argues persuasively that this lacuna cannot be attributed to folium loss; cf. Düring lxxi and lxxxix. Ironically, Ptolemy himself (*Tetr.* 1.21 (=102-103 (8)) complains about using a text the concluding tables of which were damaged.

Wallis constructed his supplement by correcting the tables and deriving information from the prose of the preceding chapters. The tables display the divisions of 23 different tetrachords—5 enharmonic, 8 chromatic, and 10 diatonic. To this point in the chapter there are the five enharmonic genera (of Archytas, Aristoxenus, Eratosthenes, Didymus, and Ptolemy) described in prose, so 18 have to be supplied. Archytas' chromatic and diatonic he derived from I.13; Aristoxenus' three chromatic and two diatonic from I.12, Didymus' chromatic and diatonic from II.13, Ptolemy's 2 chromatic and 5 diatonic in I.15 and 16. The chromatic and diatonic of Eratosthenes had to be supplied from the tables. Mountford (82-95) describes the process in detail as well as the corrections and supplements made independently by Isaac Argyrus in the mid-fourteenth century. (At 85, n. 35, Mountford observed that in the tables the Aristoxenian divisions have been converted into approximate ratios and posits that Ptolemy himself might have calculated these.) For Düring's summary, see 254-55. The tables are also incomplete in the manuscript tradition; cf. Palisca 123.

[248] Because Ptolemy's original text is lost, the texts which exist—those of Wallis and Düring and that in Vaticanus gr. 176 (known as A for the redaction by Argyros)—all differ. For instance, Düring prefers the simple neuter (χρῶμα - *chroma*), Agyros the neuter with suffix (χρωματικόν - *chromatikon*), and Wallis the genitive of the simple neuter (χρώματος - *chromatos*).

[249] Barbera, "Divisions" 302, demonstrates how, like Boethius and Didymus (and Aristoxenus' 1/4 + 1/4 = 1/2), Eratosthenes figured the lowest interval of his chromatic (20:19) to be equivalent in magnitude to the pyknon, i.e. the bottom two intervals of his enharmonic (40:39 x 39:38). Bowen, "Minor Sixth" 505, presents a technical demonstration that both Eratosthenes' chromatic and Archytas' enharmonic contain the 8:5 (minor sixth) ratio.

[250] In Didymus' chromatic divisions the lowest ('following') interval (16:15) is larger than the middle interval (25:24), which is unique; cf. 33.22f. and Aristox. *Harm.* 65.2-.4.

At During 72 (Didymus' chromatic): read 112 1/2 for 112 1/9; cf. Höeg (1930) 658, n. 2.

[251] 28:27 is a correction of Düring's text. Winnington-Ingram ("Greece" 664) uses the similarity between Ptolemy's soft chromatic (6:5 x 15:14 x 28:27) and Archytas' chromatic (32:27 x 243:224 x 28:27) to demonstrate, as he did elsewhere (e.g. n. 227), that where Ptolemaic, Pythagorean, and Aristoxenian (here 1/3 + 2/3 + 1 1/2) divisions correspond we can approximate actual Greek music of antiquity.

[252] This collection of microtonal chromatic divisions left its mark on medieval Islamic theory as well. Although I. Düring, "Greek Music," *Journal of World History* 3 (1956) 326, has no reason to assume that it must have been specifically Ptolemy whom the Arab theorists followed, the fact remains that the Arabic treatises label their microtonal system *malekos*.

The Chromatic Genera

Archytas[253]	*Aristoxenus* soft chromatic	*Aristoxenus* hemiolic chromatic	*Aristoxenus* tonic chromatic
60	60	60	60
71 1/9	74 2/3	74	72
77 1/7	77 1/3	77	76
80	80	80	80
90	90	90	90
106 2/3	112	111	108
115 5/7	116	115 1/2	114
120	120	120	120
$\frac{32}{27}\times\frac{243}{224}\times\frac{28}{27}=\frac{4}{3}$	22+4+4=30	21+4 1/2 + 4 1/2=30	18+6+6=30

Eratosthenes	*Didymus*	*Ptolemy* soft chromatic	*Ptolemy* intense chromatic
60	60	60	60
72	72	72	70
76	75	77 1/7	76 4/11
80	80	80	80
90	90	90	90
108	108	108	105
114	112 1/2	115 5/7	114 6/11
120	120	120	120
$\frac{6}{5}\times\frac{19}{18}\times\frac{20}{19}=\frac{4}{3}$	$\frac{6}{5}\times\frac{25}{24}\times\frac{16}{15}=\frac{4}{3}$	$\frac{6}{5}\times\frac{15}{14}\times\frac{28}{27}=\frac{4}{3}$	$\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}=\frac{4}{3}$

[72.6] The third table contains the diatonic genera. In the first column is that according to Archytas in ratios of 9:8, 8:7, and 28:27; in the second is that according to Aristoxenus' soft diatonic in distances of 15, 9, and 6 parts; in the third is that according to the Aristoxenian syntonic ('intense') diatonic with 12, 12, and 6 parts; in the fourth that according to Eratosthenes in ratios of 9:8, 9:8,[254] and the leimma; [72.11] in the fifth that according to Didymus

[253] The sexigesimals in the Archytas column would be 71 7, 77 9, and 115 43. The scholion points out, "The reason why we have not displayed all the ratios in Archytas' chromatic genus one can learn accurately in the thirteenth chapter of Book I, which describes the division of the tetrachords according to Archytas."

[254] Nicomachus (*Ench.* 260.15 [Jan]) states that Eratosthenes' diatonic was "false," that in the *Timaeus* "proper." This suggested to K. Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *CQ* 27 (1933) 94, that the original Eratosthenian diatonic may have been 20:19 x 19:17 x 17:15; in cents, the Timaeus scale would be 90 + 204 + 204, Erastosthenes' 89

in the ratios of 9:8, 10:9, and 16:15;[255] in the sixth is our soft diatonic in the ratios 8:7, 10:9, and 21:20; in the seventh is our tonic diatonic in the ratios 9:8, 8:7, and 28:27; in the eighth is our ditonic diatonic in the ratios 9:8, 9:8, and the leimma; in the ninth is our syntonic ('intense') diatonic in the ratios 10:9, 9:8, and 16:15; and in the tenth is our 'even' diatonic in the ratios 10:9, 11:10, and 12:11.]

The Diatonic Genera

Archytas	*Aristoxenus*[256] *soft diatonic*	*intense diatonic*	*Eratosthenes*	*Didymus*
60	60	60	60	60
67 1/2	70	68	67 1/2	67 1/2
77 1/7	76	76	75 15/16	75
80	80	80	80	80
90	90	90	90	90
101 1/4	105	102	101 1/4	101 1/4
115 5/7	114	114	113 15/16	112 1/2
120	120	120	120	120
$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}=\frac{4}{3}$	15+9+6=30	12+12+6=30	$\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}=\frac{4}{3}$	$\frac{9}{8}\times\frac{10}{9}\times\frac{16}{15}=\frac{4}{3}$
	$\frac{7}{6}\times\frac{38}{35}\times\frac{20}{19}=\frac{4}{3}$	$\frac{17}{15}\times\frac{19}{17}\times\frac{20}{19}=\frac{4}{3}$		

+ 192 + 216. Of particular interest to her is that she can identify this sequence on two auloi in the British Museum and in Cairo. Following Barbera, "Divisions" 302, one observes that this is the only scale of his that is not constructed entirely of superparticular elements; cf. Francis M. Cornford, *Plato's Cosmology* (London 1937), 142-49.

Höeg (1930) 658, n. 2, calculates the entry in the table (Düring (1930) 73) to 113 29/32.

[255] Didymus' diatonic and indeed all his divisions are entirely of superparticular ratios:

enharmonic	*chromatic*	*diatonic*
32:31	16:15	16:15
31:30	25:24	10:9
5:4	6:5	9:8

Winnington-Ingram ("Greece" 664) derives Didymus' diatonic from the Timaeus scale by first exchanging the two equal 9:8 whole tones for the major third (5:4) in which the two intervals are now the unequal 9:8 x 10:9, the major and minor whole tone. These with 16:15 then complete the division.

[256] Düring (1930) 73, preserves these alternatives which maintain the disjunction between 80 and 90: Soft diatonic: 60, 69 22/60, 76 18/60, 80, 90, 104 4/60, 112 52/60, 120; intense diatonic: 60, 67 35/60, 75 52/60, 80, 90, 101 15/60,112 56/60, 120.

For Eratosthenes, 75 56 and 113 54; for Ptolemy's soft diatonic, 68 34, 76 11, 102 51, and 114 17; for the tonic diatonic, 77 9 and 115 43; and for the ditonic diatonic, 75 56 and 114 54.

Ptolemy

soft diatonic	*tonic diatonic*	*diatonic diatonic*	*intense diatonic*	*even diatonic*
60	60	60	60	60
68 4/7	67 1/2	67 1/2	66 2/3	66 2/3
76 4/21	77 1/7	75 15/16	75	73 1/3
80	80	80	80	80
90	90	90	90	90
102 6/7	101 1/4	101 1/4	100	100
114 2/7	115 5/7	113 29/32	112 1/2	110
120	120	120	120	120
$\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}=\frac{4}{3}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}=\frac{4}{3}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}=\frac{4}{3}$	$\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}=\frac{4}{3}$	$\frac{10}{9}\times\frac{11}{10}\times\frac{12}{11}=\frac{4}{3}$

II.15 - *Table of the Numbers Making the Divisions of the Customary Genera in the Seven Tonoi*

[74.4] We happen to have designed these divisions solely, as we said,[257] for the preliminary investigation of the differences between genera. Now, so that we use all the modulations, we have in the same way taken the numbers constituting each of the seven tonoi and the genera displaying the customary type of melody.[258] [74.8] Moreover, as each of them is naturally connected through its whole range (that is, it can form a melody with the numbers taken from the same[259] ratio), others can form a partial mixture with this, unless one wishes it to be forced, where the notes of the mixed ratios are sounded at places appropriate for the mixture.[260]

[74.13] As a result, even we forgot ourselves and stepped beyond the bounds of what is necessary by spending too much time on the divisions of the uncustomary genera. [74.15] We then arranged fourteen tables, double the number of the seven tonoi, each equally with eight lines (which is the number of notes in the diapason), and each with five columns (for the number of the customary genera.)

[75.1] The first seven tables include the numbers forming that from the thetic nete diezeugmenon to an octave lower, the second seven include those forming that from the thetic mese or nete hyperbolaion to an octave lower.

[257] E.g. at 69.8.

[258] At 39.1-3.

[259] The scholiast explains, "By 'same ratios' he means those in others and the same ones maintained in other numbers, as, for example, 15:12 makes the 5:4 ratio, but also 75 to 60 makes the same sesquiquartial ratio; again, 90 to 80 makes the 9:8 ratio, but so does 630:560 make the same sesquioctave ratio. And so on for the others likewise."

[260] Barbera, *Pythagorean Mathematics* 324-32, discusses this passage; cf. 54.12f. and Cleon. 205.16f. Ptolemy offers details about these mixtures at 75.9f.

We do this to be able to form our tunings from whichever starting point we choose.[261]

[75.6] Moreover, the first two tables contain the Mixolydian tonos, the second the Lydian, the third the Phrygian, the fourth and middle the Dorian, the fifth the Hypolydian, the sixth the Hypophrygian, and the last the Hypodorian. [75.9] The first of the columns in each tonos makes the mixture of the intense (syntonic) chromatic and the tonic diatonic, the second the mixture of the soft diatonic and the tonic diatonic, the third tonic diatonic by itself and the unmixed, [75.13] the fourth the mixture of the tonic diatonic and the ditonic diatonic, and the fifth of the tonic diatonic and the intense (syntonic) diatonic.

[75.15] Again the number of the position of the notes is arranged in the first columns with the suprascripts for each tonos and genus arranged above in the proper places. [75.17] We then added a table of ten lines and eight columns including all the differences between numbers for each note so that clear to us would be the number of places and the magnitude of the distances calculated for each of the notes in all the modulations under consideration.[262]

---

[261] Cf. 69.8, which was for convenience.

[262] Barbera, *Pythagorean Mathematics* 325, in discussing this passage points out two misprints in Düring's charts on p. 91, chart 2, column a, and on p. 97, chart 13, col. a, but the reading should indeed be 94 17/21 (even though the hexigesimal fractions (94 49) seem consistent with the other fractions which read 94 22/27).

Ptolemy's belabored details here in this chapter, which are unnecessary since they would be obvious enough from the tables themselves, and in the preceding chapter (II.14) are not written in un-Ptolemaic diction; they help confirm my suspicions that Ptolemy intentionally divided each book into 16 chapters each. Otherwise there would not have been a need for separating chapters II.14, 15, and 16 except that they progress in succession from the more theoretical to the more practical.

Barbera (326-31) gives a clear derivation of the 'virtuosic arithmetic display' in these tables; cf. also Wallis 102f. Höeg, review of Düring, 658, n. 2, points out the misprint in the Mixolydian table (Düring 76) in the third column: read 28/27 for 28/72; similarly on 77 (Phrygian) third column, 28/27 for 9:8; to these add on 76 (Lydian) column 3, 121 19/21 for 121 17/21, and on 78 (Hypolydian) first column, 12/11 for the second 28/27; cf. Düring lx. These tables include cents (249, 254, and 256); cf. Schlesinger "Further Notes on Aristoxenus and Musical Intervals," *CQ* 27 (1933) 88, and Winnington-Ingram, "Aristoxenus and the Intervals" 195. Paul Tannery, "Sur les Intervalles de la musique grecque," *REG* 15 (1902) 337-39, offers full, non-fractional displays of all these divisions.

Although Ptolemy offered 14 tables, he makes it quite clear that each table has a duplicate:

from nete		from mese
Mixolydian	-	Hypolydian
Lydian	-	Hypophrygian
Phrygian	-	Hypodorian
Dorian	-	Mixolydian
Hypolydian	-	Lydian
Hypophrygian	-	Phrygian
Hypodorian	-	Dorian

These seven tonoi fall into two sets, one with the Mixolydian, Lydian, and Phrygian from nete (and the corresponding Hypolydian, Hypophrygian, and Hypodorian from mese), the other with Dorian, Hypolydian, Hypophrygian, and Hypodorian from nete (and the corresponding

1) *Mixolydian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
67 1/2	67 1/2	67 1/2	67 1/2	67 1/2
78 3/4	77 1/7	75 15/16	75 15/16	75 15/16
85 10/11	85 5/7	86 11/14	86 11/14	86 11/14
90	90	90	90	90
101 1/4	101 1/4	101 1/4	101 1/4	100
115 5/7	115 5/7	115 5/7	113 29/32	112 1/2
120	120	120	120	120
$\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}$

Mixolydian, Lydian, Phrygian, and Dorian from mese). The order of the two sets is reversed in moving from the 'from nete' group to the 'from mese' group. In sequence, the disjunct whole tone (9:8) appears in the first position for the (from nete) Mixolydian, the second for the Lydian, the third for the Phrygian, the fourth for the Dorian, the fifth for the Hypolydian, the sixth for the Hypophrygian, and the seventh for the Hypodorian. The progression of ratios proceeds accordingly (all from nete):

Mixolydian:	*9/8* x 7/6 x 12/11 x 22/21 x 9/8 x 8/7 x 28/27
Lydian:	28/27 x *9/8* x 7/6 x 12/11 x 22/21 x 9/8 x 8/7
Phrygian:	8/7 x 28/27 x *9/8* x 7/6 x 12/11 x 22/21 x 9/8
Dorian:	9/8 x 8/7 x 28/27 x *9/8* x 7/6 x 12/11 x 22/21
Hypolydian:	22/21 x 9/8 x 8/7 x 28/27 x *9/8* x 7/6 x 12/11
Hypophrygian:	12/11 x 22/21 x 9/8 x 8/7 x 28/27 x *9/8* x 7/6
Hypodorian:	7/6 x 12/11 x 22/21 x 9/8 x 8/7 x 28/27 x *9/8*

The same method is used for each of the five columns, all of which include the disjunct whole tone initially in the Mixolydian (from nete):

intense chromatic:	7/6	x	12/11	x	22/21
tonic diatonic:	9/8	x	8/7	x	28/27
soft diatonic:	8/7	x	10/9	x	21/20
tonic diatonic:	9/8	x	8/7	x	28/27
tonic diatonic:	9/8	x	8/7	x	28/27
tonic diatonic:	9/8	x	8/7	x	28/27
ditonic diatonic:	9/8	x	9/8	x	256/243
tonic diatonic:	9/8	x	8/7	x	28/27
intense diatonic:	10/9	x	9/8	x	16/15

2) *Lydian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60 20/21	60 20/21	60 20/21	60	59 7/27
63 17/81	63 17/81	63 17/81	63 51/243	63 17/81
71 1/9	71 1/9	71 1/9	71 1/9	71 1/9
82 26/17	81 17/63	80	80	80
90 1/2	90 170/567	91 3/7	91 3/7	91 3/7
94 17/21	94 22/27	94 22/27	94 22/27	94 22/27
106 2/3	106 2/3	106 2/3	106 2/3	105 85/243
121 19/21	121 19/21	121 19/21	120	118 14/27
$\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}$

3) *Phrygian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	59 7/27
68 4/7	68 4/7	68 4/7	67 1/2	66 2/3
71 1/9	71 1/9	71 1/9	71 1/9	71 1/9
80	80	80	80	80
93 1/3	91 3/7	90	90	90
101 9/11	101 37/63	102 6/7	102 6/7	102 6/7
106 2/3	106 2/3	106 2/3	106 2/3	106 2/3
120	120	120	120	118 14/27
$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}$

4) *Dorian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
67 1/2	67 1/2	67 1/2	67 1/2	66 2/3
77 1/7	77 1/7	77 1/7	75 15/16	75
80	80	80	80	80
90	90	90	90	90
105	102 6/7	101 1/4	101 1/4	101 1/4
114 6/11	114 2/7	115 5/7	115 5/7	115 5/7
120	120	120	120	120
$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$

5) *Hypolydian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60 1/3	60 1/5	60 20/21	60 20/21	60 20/21
63 13/63	63 21/100	63 17/81	63 17/81	63 17/81
71 3/28	71 89/800	71 1/9	71 1/9	70 170/729
81 13/49	81 27/100	81 17/63	80	79 1/81
84 52/189	84 7/25	84 68/243	84 68/243	84 68/243
94 17/21	94 163/200	94 22/27	94 22/27	94 22/27
110 11/18	108 36/100	106 2/3	106 2/3	106 2/3
120 2/3	120 2/5	121 19/21	121 19/21	121 19/21
$\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}$	$\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$

6) *Hypophrygian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
62 2/9	60 20/21	60	60	60
67 29/33	67 137/189	68 4/7	68 4/7	68 4/7
71 1/9	71 1/9	71 1/9	71 1/9	71 1/9
80	80	80	80	79 1/81
91 2/7	91 3/7	91 3/7	90	88 8/9
94 22/27	94 22/27	94 22/27	94 22/27	94 22/27
106 2/3	106 2/3	106 2/3	106 2/3	106 2/3
124 4/9	121 19/21	120	120	120
$\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}$	$\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}$

7) *Hypodorian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
70	68 4/7	67 1/2	67 1/2	67 1/2
76 4/11	76 4/21	77 1/7	77 1/7	77 1/7
80	80	80	80	80
90	90	90	90	88 8/9
102 6/7	102 6/7	102 6/7	101 1/4	100
106 2/3	106 2/3	106 2/3	106 2/3	106 2/3
120	120	120	120	120
$\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}$

8) *Mixolydian (from mese, = Dorian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
67 1/2	67 1/2	67 1/2	67 1/2	66 2/3
77 1/7	77 1/7	77 1/7	75 15/16	75
80	80	80	80	80
90	90	90	90	90
105	102 6/7	101 1/4	101 1/4	101 1/4
114 6/11	114 2/7	115 5/7	115 5/7	115 5/7
120	120	120	120	120
$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$

9) *Lydian (from mese; = Hypolydian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60 1/3	60 1/5	60 20/21	60 20/21	60 20/21
63 13/63	63 21/100	63 17/81	63 17/81	63 17/81
71 3/28	71 89/800	71 1/9	71 1/9	70 170/729
81 13/49	81 27/100	81 17/63	80	79 1/81
84 52/189	84 7/25	84 68/243	84 68/243	84 68/243
94 17/21	94 163/200	94 22/27	94 22/27	94 22/27
110 11/18	108 36/100	106 2/3	106 2/3	106 2/3
120 2/3	120 2/5	121 19/21	121 19/21	121 19/21
$\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}$	$\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$

10) *Phrygian (from mese; = Hypophrygian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
62 2/9	60 20/21	60	60	60
67 29/33	67 137/189	68 4/7	68 4/7	68 4/7
71 1/9	71 1/9	71 1/9	71 1/9	71 1/9
80	80	80	80	79 1/81
91 2/7	91 3/7	91 3/7	90	88 8/9
94 22/27	94 22/27	94 22/27	94 22/27	94 22/27
106 2/3	106 2/3	106 2/3	106 2/3	106 2/3
124 4/9	121 19/21	120	120	120
$\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}$	$\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}$

11) *Dorian (from mese; = Hypodorian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
70	68 4/7	67 1/2	67 1/2	67 1/2
76 4/11	76 4/21	77 1/7	77 1/7	77 1/7
80	80	80	80	80
90	90	90	90	88 8/9
102 6/7	102 6/7	102 6/7	101 1/4	100
106 2/3	106 2/3	106 2/3	106 2/3	106 2/3
120	120	120	120	120
$\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}$

12) *Hypolydian (from mese): [= Mixolydian]*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
56 136/729	56 136/729	56 136/729	56 136/729	56 136/729
63 17/81	63 17/81	63 17/81	63 17/81	63 17/81
73 181/243	72 136/567	71 1/9	71 1/9	71 1/9
80 400/891	80 1360/5103	81 17/63	81 17/63	81 17/63
84 68/243	84 68/243	84 68/243	84 68/243	84 68/243
94 22/27	94 22/27	94 22/27	94 22/27	93 1409/2187
108 68/189	108 68/189	108 68/189	106 2/3	105 85/243
112 272/729	112 272/729	112 272/729	112 272/729	112 272/729
$\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}$

13) *Hypophrygian (from mese; = Lydian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60 20/21	60 20/21	60 20/21	60	59 7/27
63 17/81	63 17/81	63 17/81	63 51/243	63 17/81
71 1/9	71 1/9	71 1/9	71 1/9	71 1/9
82 26/27	81 17/63	80	80	80
90 1/2	90 170/567	91 3/7	91 3/7	91 3/7
94 17/21	94 22/27	94 22/27	94 22/27	94 22/27
106 2/3	106 2/3	106 2/3	106 2/3	105 85/243
121 19/21	121 19/21	121 17/21	120	118 14/27
$\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}$

14) *Hypodorian (from mese; = Phrygian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	59 7/27
68 4/7	68 4/7	68 4/7	67 1/2	66 2/3
71 1/9	71 1/9	71 1/9	71 1/9	71 1/9
80	80	80	80	80
93 1/3	91 3/7	90	90	90
101 9/11	101 37/63	102 6/7	102 6/7	102 6/7
106 2/3	106 2/3	106 2/3	106 2/3	106 2/3
120	120	120	120	118 14/27
$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}$

[Düring derived the previous tables by multiplying the integers by the reciprocal of the ratio. The following represent the same tables, based on sexigesimal fractions, as found in a number of the codices; cf. Wallis 95f.]

1) *Mixolydian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
67 30	67 30	67 30	67 30	67 30
78 45	77 9	75 56	75 56	75 56
85 55	85 43	86 47	86 47	86 47
90	90	90	90	90
101 15	101 15	101 15	101 15	100
115 43	115 43	115 43	113 54	112 30
120	120	120	120	120
$\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}$

2) *Lydian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60 57	60 57	60 57	60	59 16
63 13	63 13	63 13	63 13	63 13
71 7	71 7	71 7	71 7	71 7
82 58	81 16	80	80	80
90 30	90 18	91 26	91 26	91 26
94 49	94 49	94 49	94 49	94 49
106 40	106 40	106 40	106 40	105 21
121 54	121 54	121 54	120	118 31
$\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}$

3) *Phrygian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	59 16
68 34	68 34	68 34	67 30	66 40
71 7	71 7	71 7	71 7	71 7
80	80	80	80	80
93 20	91 26	90	90	90
101 49	101 35	102 51	102 51	102 51
106 40	106 40	106 40	106 40	106 40
120	120	120	120	118 31
$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}$

4) *Dorian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
67 30	67 30	67 30	67 30	66 40
77 9	77 9	77 9	75 56	75
80	80	80	80	80
90	90	90	90	90
105	102 51	101 15	101 15	101 15
114 33	114 17	115 43	115 43	115 43
120	120	120	120	120
$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$

5) *Hypolydian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60 20	60 12	60 57	60 57	60 57
63 13	63 13	63 13	63 13	63 13
71 7	71 7	71 7	71 7	70 14
81 16	81 16	81 16	80	79 1
84 17	84 17	84 17	84 17	84 17
94 49	94 49	94 49	94 49	94 49
110 37	108 22	106 40	106 40	106 40
120 40	120 24	121 54	121 54	121 54
$\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}$	$\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$

6) *Hypophrygian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
62 13	60 57	60	60	60
67 53	67 43	68 34	68 34	68 34
71 7	71 7	71 7	71 7	71 7
80	80	80	80	79 1
91 26	91 26	91 26	90	88 53
94 49	94 49	94 49	94 49	94 49
106 40	106 40	106 40	106 40	106 40
124 27	121 54	120	120	120
$\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}$	$\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}$

7) *Hypodorian (from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
70	68 34	67 30	67 30	67 30
76 22	76 11	77 9	77 9	77 9
80	80	80	80	80
90	90	90	90	88 53
102 51	102 51	102 51	101 15	100
106 40	106 40	106 40	106 40	106 40
120	120	120 120	120	
$\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}$

8) *Mixolydian (from mese, = Dorian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
67 30	67 30	67 30	67 30	66 40
77 9	77 9	77 9	75 56	75
80	80	80	80	80
90	90	90	90	90
105	102 51	101 15	101 15	101 15
114 33	114 17	115 43	115 43	115 43
120	120	120	120	120
$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$

9) *Lydian (from mese; = Hypolydian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60 20	60 12	60 57	60 57	60 57
63 13	63 13	63 13	63 13	63 13
71 7	71 7	71 7	71 7	70 14
81 16	81 16	81 16	80	79 1
84 17	84 17	84 17	84 17	84 17
94 49	94 49	94 49	94 49	94 49
110 37	108 22	106 40	106 40	106 40
120 40	120 24	121 54	121 54	121 54
$\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}$	$\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}$

10) *Phrygian (from mese; = Hypophrygian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
62 13	60 57	60	60	60
67 53	67 43	68 34	68 34	68 34
71 7	71 7	71 7	71 7	71 7
80	80	80	80	79 1
91 26	91 26	91 26	90	88 53
94 49	94 49	94 49	94 49	94 49
106 40	106 40	106 40	106 40	106 40
124 27	121 54	120	120	120
$\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}$	$\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}$

11) *Dorian (from mese; = Hypodorian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
70	68 34	67 30	67 30	67 30
76 22	76 11	77 9	77 9	77 9
80	80	80	80	80
90	90	90	90	88 53
102 51	102 51	102 51	101 15	100
106 40	106 40	106 40	106 40	106 40
120	120	120	120	120
$\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}$

12) *Hypolydian (from mese):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
56 11	56 11	56 11	56 11	56 11
63 13	63 13	63 13	63 13	63 13
73 45	72 14	71 7	71 7	71 7
80 27	80 16	81 16	81 16	81 16
84 17	84 17	84 17	84 17	84 17
94 49	94 49	94 49	94 49	93 39
108 22	108 22	108 22	106 40	105 21
112 22	112 22	112 22	112 22	112 22
$\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{256}{243}$	$\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}$

13) *Hypophrygian (from mese; = Lydian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60 57	60 57	60 57	60	59 16
63 13	63 13	63 13	63 13	63 13
71 7	71 7	71 7	71 7	71 7
82 58	81 16	80	80	80
90 30	90 18	91 26	91 26	91 26
94 49	94 49	94 49	94 49	94 49
106 40	106 40	106 40	106 40	105 21
121 54	121 54	121 54	120	118 31
$\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}$	$\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}$	$\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}\times\frac{9}{8}$

14) *Hypodorian (from mese; = Phrygian from nete):*

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	59 16
68 34	68 34	68 34	67 30	66 40
71 7	71 7	71 7	71 7	71 7
80	80	80	80	80
93 20	91 26	90	90	90
101 49	101 35	102 51	102 51	102 51
106 40	106 40	106 40	106 40	106 40
120	120	120	120	118 31
$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{7}{6}\times\frac{12}{11}\times\frac{22}{21}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{8}{7}\times\frac{10}{9}\times\frac{21}{20}\times\frac{9}{8}$	$\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{256}{243}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{9}{8}$	$\frac{9}{8}\times\frac{16}{15}\times\frac{9}{8}\times\frac{9}{8}\times\frac{8}{7}\times\frac{28}{27}\times\frac{10}{9}$

15) The codices include this table of all the numerical differences calculated for each note:

56 11	63 13	70 14	79 1	84 17	93 39	105 21	112 22
59 16	66 40	71 7	80	88 53	94 49	106 40	118 31
60	67 30	72 14	80 16	90	100	108 22	120
60 12	67 43	73 45	80 27	90 18	101 15	110 37	120 24
60 20	67 53	75	81 16	90 30	101 35	112 30	120 40
60 57	68 34	75 56	82 58	91 26	101 49	113 54	121 54
62 13	70	76 11	85 43	93 20	102 51	114 17	124 27
	76 22	85 55				105	114 33
	77 9	86 47				115 43	
	78 45						

In order, these derive from the following tables, each of which is assigned a number (1 - 14) to designate the table (*tonos*) and a letter (a - e) to designate the column (genus): [The fractions are given in both forms.]

56 11 (136/729)	: 12a-e
59 16 (7/27)	: 2e, 3e, 13e, 14e
60 00	: 1a-e, 2d, 3a-d, 4a-e, 6c-e, 7a-e, 8a-e, 10c-e, 11a-e, 13d, 14a-d
60 12 (1/5)	: 5b, 9b
60 20 (1/3)	: 5a, 9a
60 57 (20/21)	: 2a-c, 5c-e, 6b, 9c-e, 10b, 13a-c
62 13 (2/9)	: 6a, 10a
63 13 (17/81)	: 2a-e, 5a-e, 9a-e, 12a-e, 13a-e
66 40 (2/3)	: 3e, 4e, 8e, 14e
67 30 (1/2)	: 1a-e, 3d, 4a-d, 7c-e, 8a-d, 11c-e, 14d
67 43 (137/189)	: 6b, 10b
67 53 (29/33)	: 6a, 10a
68 34 (4/7)	: 3a-c, 6c-e, 7b, 10c-e, 11b, 14a-c
70 00	: 7a, 11a
70 14 (170/729)	: 5e, 9e
71 7 (1/9)	: 2a-e, 3a-e, 5a-d, 6a-e, 9a-d, 10a-e, 12c-e, 13a-e, 14a-e
72 14 (136/567)	: 12b
73 45 (181/243)	: 12a
75 00	: 4e, 8e
75 56 (15/16)	: 1c-e, 4d, 8d
76 11 (4/21)	: 7b, 11b
76 22 (4/11)	: 7a, 11a
77 9 (1/7)	: 1b, 4a-c, 7c-e, 8a-c, 11c-e
78 45 (3/4)	: 1a
79 1 (1/81)	: 5e, 6e, 9e, 10e
80 00	: 2c-e, 3a-e, 4a-e, 5d, 6a-d, 7a-e, 8a-e, 9d, 10a-d, 11a-e, 13c-e, 14a-e
80 16 (1360/5103)	: 12b
80 27 (400/891)	: 12a
81 16 (17/63)	: 2b, 5a-c, 9a-c, 12c-e, 13b
82 58 (26/27)	: 2a, 13a
85 43 (5/7)	: 1b
85 55 (10/11)	: 1a
86 47 (11/14)	: 1c-e
84 17 (52/189)	: 5a-e, 9a-e, 12a-e
88 53 (8/9)	: 6e, 7e, 10e, 11e

90 00	: 1a-e, 3c-e, 4a-e, 6d, 7a-d, 8a-e, 10d, 11a-d, 14c-e
90 18	: 2b, 13b
90 30 (1/2)	: 2a, 13a
91 26 (3/7)	: 2c-e, 3b, 6a-c, 10a-c, 13c-e, 14b
93 20 (1/3)	: 3a, 14a
93 39 (1409/2187)	: 12e
94 49 (17/21)	: 2a-e, 5a-e, 6a-e, 9a-e, 10a-e, 12a-d, 13a-e
100 00	: 1e, 7e, 11e
101 15 (1/4)	: 1a-d, 4c-e, 7d, 8c-e, 11d
101 35 (37/63)	: 3b, 14b
101 49 (9/11)	: 3a, 14a
102 51 (6/7)	: 3c-e, 4b, 7a-c, 8b, 11a-c, 14c-e
105 00	: 4a, 8a
105 21 (85/243)	: 2e, 12e, 13e
106 40 (2/3)	: 2a-d, 3a-e, 5c-e, 6a-e, 7a-e, 9c-e, 10a-e, 11a-e, 12d, 13a-d, 14a-e
108 22 (36/100)	: 5b, 9b, 12a-c
110 37 (11/18)	: 5a, 9a
112 30 (1/2)	: 1e
113 54 (29/32)	: 1d
114 17 (2/7)	: 4b, 8b
114 33 (6/11)	: 4a, 8a
115 43 (5/7)	: 1a-c, 4c-e, 8c-e
112 22 (272/729)	: 12a-e
118 31 (14/27)	: 2e, 3e, 13e, 14e
120 00	: 1a-e, 2d, 3a-d, 4a-e, 6c-e, 7a-e, 8a-e, 10c-e, 11a-e, 13d, 14a-d
120 24 (2/5)	: 5b, 9b
120 40 (2/3)	: 5a, 9a
121 54 (19/21)	: 2a-c, 5c-e, 6b, 9c-e, 10b, 13a-c
124 27 (4/9)	: 6a, 10a

The same ratios for each tonos can be applied within standardized outer limits of 60 and 120:

Lydian (from nete):

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
62 2/9	62 2/9	62 2/9	63 51/243	64
70	70	70	71 1/9	72
81 2/3	80	78 3/4	80	81
89 1/11	88 8/9	90	91 3/7	92 4/7
93 1/3	93 1/3	93 1/3	94 22/27	96
105	105	105	106 2/3	106 2/3
120	120	120	120	120

Phrygian (from nete):

tonic diatonic intense diatonic
60
67 1/2
72
81
91 1/8
104 1/7
108
120

Hypolydian (from nete):

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*
60	60	60	60	60
62 6/7	63	62 2/9	62 2/9	62 2/9
70 5/7	70 7/8	70	70	69 11/81
80 40/49	81	80	78 3/4	77 7/9
83 17/21	84	82 26/27	82 26/27	82 26/27
94 2/7	94 1/2	93 1/3	93 1/3	93 1/3
110	108	105	105	105
120	120	120	120	120

Hypophrygian (from nete):

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*
60	60
65 5/11	66 2/3
68 4/7	70
77 1/7	78 3/4
88 8/49	90
91 3/7	93 1/3
102 6/7	105
120	120

Hypolydian (from mese):

intense chromatic tonic diatonic	*soft diatonic tonic diatonic*	*tonic diatonic*	*tonic diatonic ditonic*	*tonic diatonic intense diatonic*

[same as Mixolydian from nete]

II.16 - *On Realizations Made on the Lyre and Cithara*

[80.6] Let these tables demonstrate the ease of division and the display of both the ratios and the commonalities[263] the tables are able to reveal. [80.8] What are called 'sterea'[264] on the lyre are bound in any tonos by the numbers of the tonic diatonic of the same tonos, the soft by the numbers in the mixture of the intense (syntonic)[265] chromatic of the same tonos.

[80.11] When notes are realized on the cithara, those numbers from the nete of the tonic diatonic of the Hypodorian tonos define the tritai.[267] Similarly the Hypertropa are defined by numbers of the tonic diatonic of the Phrygian, [80.14] the Parhypatai by the numbers from the mixture of the soft diatonic of the Dorian, the tropai by the numbers of the mixture of the intense (syntonic) chromatic of the Hypodorian; [80.16] that which they call the Iasti-Aeolian by the numbers of the mixture of the ditonic diatonic of the Hypophrygian, and the Lydian by the numbers of [the mixture][268] of the tonic diatonic of the Dorian.[269]

[80.18] Since the highest of all the notes from the common endpoint was seen to be approximately 55 parts,[270] the lowest approximately 125,[271] it is then necessary that there be in addition to this segment some interval up to the opposite endpoint. It should be half the width of the fixed and movable

[263] A term (κοινοτήτων) suggestive of modulation (between columns) and found (albeit with a different noun suffix) in Cleon. 205.17 and 205.20, Aristides Quintilianus 20.23, and elsewhere.

[264] Cf. II.1 (43.9f.).

[265] This is a translation of Düring's emendation, for which he cites 39.10; cf. Gohlke 1443. The codices have μαλακοῦ ('soft'). The use of the chromatic and diatonic here for the same sterea necessitate this reading.

[267] Cf. II.1, for all of these organological terms. Here Ptolemy applies the work of his two now completed volumes to actual tunings on the cithara. That is, he applies at long last the correct tunings in the correct seven tonoi with the correct genera.

[268] Düring's supplement, for which he cites [D. B.] Monro [*The Modes of Ancient Greek Music* (Oxford 1894)] 84, [P. Leander] Schoenberger, [*Studien zum I. Buch der Harmonik des Claudius Ptolemäus* (Augsburg 1914)] 107, and Ptolemy 39.12; cf. also, Höeg (1930) 656-57. The supplement is not really necessary, however, for Alexanderson (16-17) is quite correct in pointing out that Ptolemy did not necessarily repeat every word in formulaic passages, e.g. the omission of the verb six times at 64.11-.13.

[269] This makes the seventh tonos. M. L. West, *Ancient Greek Music* (Oxford 1992) 171, lists all six varieties of cithara tunings.

[270] E.g. the 56 136/729 in the Hypolydian.

[271] E.g. the 124 4/9 in the Hypophrygian.

bridge, [81.1] and it should take from the whole length what is equal to or even greater than both the aforementioned widths. The remainder we divide into the 25 parts of five parts each. The subdivision into parts will be necessary for only the fourteen groups of five bounding the 70 parts[272] between the outermost notes, that is, those from 55 to 125.

[81.5] It will be useful as well to establish others equal in number to the pegs at the opposite endpoints of the canon so we can easily change their lengths during the testing of the strings by tightening and loosening the pegs containing them.[273]

[81.9] It will also be useful to make them movable on the angled[274] side along the width of the canon for another use, that being by which with one wide bridge placed under the strings the sideways movements of the strings make the proper tunings. [81.12] For with two canons[275] equal in length to the fixed bridges divided again into the parts lying between the outermost notes,[276] and with the bridges of each placed with equal, similar numbers opposite to the other, those sideways movements of the strings will be shown by them to those who are able to tune it.[277] [81.17] With the pegs established there, their notes will maintain the same tunings, but if they remain they will need again a new isotonic resetting[278] for the strings loosened or tightened from the sideways movement.[279]

[272] Düring (99), has 80 for 70.

[273] Ptolemy is perhaps indebted for this idea to Didymus; cf. 67.21f.

[274] The phrase ἐν τῇ πελεκήσει means literally 'ax-shaped.'

[275] κανονίων is the correct reading; cf. Düring 263.

[276] Cf. 81.4-.5. Düring (263) is perhaps correct in suggesting "equal in length of the fixed bridges..."

[277] Cf. 15.4.

[278] Or, 'periodicity;' cf. 58.4 (n. 126).

[279] Ptolemy thus devises a much more flexible and variable arrangement of the canon, its strings, and its bridges.

BOOK III

Summary

1. How Through the Whole System We Might Both Use and Judge the Ratios By Means of the Fifteen-stringed Canon.
2. Methods for Dividing Up To the Double Diapason With Only the Eight Notes.
3. In Which Genus Must We Place Harmonic Function and Our Knowledge of It.
4. That the Function of the Harmonic Exists in the Natures of All the More Perfect Things and That it Appears Particularly In Both Human Souls and Heavenly Motions.
5. How the Consonances Are in Accord with the First Differences of the Soul and Their Species.
6. A Comparison Between the Harmonic Genera and Those of the First Virtues.
7. How Harmonic Modulations Resemble the Circumstantial Modulations of Souls.
8. On the Similarity of the Perfect System and the Circle Through the Middle of the Zodiac.
9. How the Harmonic Consonances and Dissonances Resemble Those in the Zodiac.
10. That Succession in Notes Resembles the Longitudinal Movement of the Stars.
11. How the Stellar Movement in Altitude Compares with the Harmonic Genera.
12. That Modulations of *Tonoi* Are Like Stellar Crossings in Latitude.
13. On the Similarity of the Tetrachords and the Aspects of the Sun.
14. By What First Numbers Might the Fixed Notes of the Perfect System Be Compared to the First Spheres in the Cosmos.
15. How the Ratios of Their Motions Are Calculated By the Numbers.
16. How the Combinations of the Planets Should be Compared to Those of the Notes.

III.1 - *How Through the Whole System We Might Both Use*[1] *and Judge the Ratios By Means of the Fifteen-stringed Canon*

[83.1] Sufficient for our previous demonstration[2] was the use of intervals up to only the diapason, which is the first [interval] able to contain within itself every musical form.[3] [83.3] In fact, it is probably for this reason that it is named the 'diapason' and not 'diaocto' just as the diapente and diatessaron are named from the number of the notes surrounding them.[4]

83.5] But if someone wished for the sake of[5] surplus to fill out the canon with a double diapason system to meet every sort of variety so that he would add to these eight notes the seven which are lacking from the double diapason magnitude for the fifteen in the lyre,[6] [83.9] it will be possible within our methodology to accommodate such an addition so that the short segments left from the highest notes do not make the pitches cacophonous[7] and so that the bridges, once they have been put in place, do not take their divisions up to the double diapason.[8]

---

[1] Gogava rendered 'use' as 'judgment' (κρίσις for χρῆσις).

[2] Ptolemy may be referring specifically to the discussion in II.16, but he may also be referring generally to the progression of his treatise with its culmination in II.16, where the reason applied to harmonics (λόγος ἁρμονικός) encounters, reexamines, and decribes perfectly realized music (μουσικὴ μελῳδουμένη).

[3] Here in Book III Ptolemy begins to use technical terms from the philosophical schools in more traditional and non-musical meanings. This last phrase refers to 'every idea of *melos*' (τὴν πᾶσαν τοῦ μέλους ἰδέαν) which rings Platonic. This standardized translation should not be misleading, though, since in modern terminology a 'musical idea' involves melodic lines; (jazz motifs are usually *melos*, not *melodia*). Nor does this mean, as a second literal translation might suggest, 'every form,' i.e. genre or compositional type, 'of music.' Ptolemy means here essentially 'every possible harmonic, proportionally derived tuning.'

[4] Διὰ πέντε (*diapente)* means literally 'through five,' διὰ τεσσάρων (*diatessaron*) 'through four,' δι' ὀκτώ (*di' okto*) 'through eight,' and διὰ πασῶν (*diapason*) 'through all,' cf. da Rios 14., n. 4 (*trad.*). Ptolemy seems to be unaware that the consonances previously had different names - συλλαβή (*syllabe)* for the diatessaron, διοξεῖα (*dioxeia)* for the diapente, and ἁρμονία (*harmonia*) for the diapason. Szabó, *Greek Mathematics* 109, offers an attempt at etymologies and cites the original source (preserved by Nic. *Ench.* 252.17f.) in Philolaus (=Diels-Kranz[6] 1.408–410 (44.B.6)). Erich Frank, *Plato und die sogenannten Pythagoreer* (Halle 1923) 273, believes *syllabe* is late; see also, Burkert, *Lore and Science* 390. [Arist.] *Prob.* 19.32 (920ᵃ24) claims that διὰ πασῶν *(diapason)* was more appropriate for instruments with seven strings; δι' ὀκτώ (*di' okto*), of course, would not be.

[5] This prepositional phrase (ἐκ περιουσίας) occurs frequently enough in philosophically oriented treatises, as this one is now about to become in Book III. It does not make Ptolemy's meaning perfectly clear, however. The greatest likelihood is that it means one would add the additional strings just to 'make some extra.' It almost means, 'just for the sake of experiment.'

[6] By Ptolemy's era the fifteen-string lyre had become widely available, so he now contemplates constructing a fifteen-string canon.

[7] 'Segments' and 'strings' are my supplements. They would be cacophonous (δυσήχους - *dysechous*) because of their very short length and great tension; the precise pitches would be very difficult for the ear to ascertain

[8] Ptolemy's precise meaning in 'take their divisions' (λαμβάνῃ τὰς διαιρέσεις (Wallis' *divisiones recipiant*). is not clear. Barker (262) rewrites with 'marked with divisions,' and Düring simplifies by writing merely *einteilen* ('divides.') It is also not clear whether the phrase

[83.12] We can do this if we separate each of the outer two diapasons[9] by the tension and thinness of the strings, if we maintain with equal tension to each other the eight thinner notes[10] from the middle to the highest in the tuning most suitable for the mese, [83.16] and if we maintain the remaining, fuller, seven notes again with equal tension to each other but now in the tuning for the proslambanomenos. With their counterparts they form a diapason, by which interval the proslambanomenos is lower than the mese.[11]

[83.19] Because of this the segment of only one of the diapasons will fit both arrangements,[12] for it makes the ratio of a diapason in each of what are necessarily homophones.[13] 83.22 If we think of two notes in equal distances of length, AB and CD, where AB is one diapason higher than CD,[14] then let us take equal lengths, AE and CF. AE will be a diapason higher than CF.

```
A---------------------------E-----------B
C---------------------------F-----------D
```

[84.2] For, generally, since as the distance AB is to AE so is the sound of AE to that of AB, and as the distance CD is to CF, so is the sound of CF to that of CD. And as the distance AB is to that of AE, so is CD to CF. And as is the sound of AE to that of AB, so is the sound of CF to that of CD; [84.4] and the opposite, as the sound of AE is to that of CF, so is the sound of AB to that of CD,[15] so that, since the sound from AB lies one diapason higher than that from CD, that from AE will also be one diapason higher than that from CF, which will be true for all the notes surrounding the seven intervals on this instrument, if the same arrangement of the canon is used for both diapasons.[16]

'up to the double diapason' (μέχρι τοῦ δὶς διὰ πασῶν) is to be applied to the placement of the bridges or the resulting divisions.

The discussion of the use of the 15-stringed canon now yields to that of how one can best tune it. Ptolemy will propose that the lower octave be tuned to one note, the upper octave a note one octave higher. This will keep the bridges from being placed too close to the end of the strings, which would dull and obscure the pitch that Ptolemy wants to be able to hear. He offers proof in this chapter that if two strings of equal density are tuned an octave apart, any equivalent segment of those two strings will also be an octave apart. This concept is necessary for Ptolemy to rely solely upon the one central octave and its seven tonoi.

[9] These "outer two diapasons" are those from thetic proslambanomenos to nete hyperbolaion

[10] Ptolemy must mean the strings, but the manuscript tradition does not offer an appropriate variant

[11] On the canon this is the original diapason, and all others should be of precisely the same magnitude.

[12] The lower and upper diapasons.

[13] This uniformity of the diapasons is both the reason the canon can be theoretically extended indefinitely and the reason Ptolemy finds no good reason to do so; this demonstration is merely (ἐκ περιουσίας) 'for the sake of surplus.'

[14] Presumably AB is thinner than CD, or under greater tension.

[15] The metathesis of AB and AE in Düring (104) is not to be undone nor despised.

[16] My supplement to the adjective.

[84.11] Even he who is able to grasp only equally tuned notes would tune this way,[17] but it will be possible for him who has thoroughly investigated also the differences which belong between each note for each species to do the opposite, [84.14] that is, whatever tuning the notes have, to establish the bridges at the division for one certain genus and tonos, then to tune by his hearing according to our suppositions.[18] [84.16] When this is done once, the bridges are then transferred into the loci of another genus or tonos, and this, too, will be tuned as will all the others in the same way, since the first tuning establishes notes of equal tension again with the same lengths.

[84.21] Let there be two notes similarly, AB and CD, and let unequal segments AE and CF be marked off from each. They should be tuned so that the sound of section CF is in proportion to that of AE as the length of AE is to CF. I maintain that the equal segments of notes will be equally tuned.

```
A---------------------------E-----------B
C------------F-------------G-----------D
```

[84.25] For let CG, equal in distance to AE, be marked off.[19] Since as the distance AE, that is CG, is to CF as the sound from CF is to that of both AE on account of its being tuned thus and to that of CG on account of its being so at the outset, [84.28] the sound from CF both to that of AE as well as to that of CG will have the same ratio, for the sections of the notes AE and CG will be equally tuned when they are of equal lengths.[20]

[85.3] What has been demonstrated will be again self-evident once the bridges have been transferred back, after, as we said, the notes have been tuned to the previous loci with all the distances equal.[21] [85.6] For we will find that each of the arrangements in it is equally tuned and also that both ranges in the double diapason are equally tuned to each other, just as we posited previously.[22]

[17] Cf. I.9 and 10. The scholiast offers this explanation: "In the first segment he shows the use of the monochord via two notes analogous to the two systems of diapason equal in length but unequal in thickness; in the second he shows it via two notes again but now equal in thickness and unequal in length because of a decrease in the bridges." For this, see 66.18 and 81.17.

[18] That one can hear the homophones correctly; cf. II.13 (69.5–.8).

[19] This passive verb makes one take notice of how Ptolemy's diction throughout this demonstration differs a bit from the earlier demonstrations both lexically and syntactically. That more philosophical jargon and astrological vocabulary as well fill the following pages of Book III makes this book stand apart from the previous two.

[20] Cf. the scholiast's remarks about string thickness and length (n. 17 supra).

[21] The Greek text does not convey Ptolemy's meaning clearly here since each of the last seven (and nine of the last ten) words offered is in the same (accusative) case (τοὺς φθόγγους ἐπὶ τοὺς ἀπολαμβάνοντας τόπους πάσας τὰς διαστάσεις ἴσας). This translation is as literal as possible.

[22] At 83.19 and 62.6.

[85.8] And the abundance[23] of the notes should not move anyone when they do not differ in function nor, according to our common proposition, from the one; if it were not the case for them all, the whole would be incorrect.

[85.11] This was not the purpose of the canon,[24] to reveal through strings one or more in number (but having a limited amount) the ratios of the emmelic notes, but simply, through whatever number of equally tuned strings [85.13] (so long as[25] they are revealed to be no different than from one string), to tune by means of ratio alone that which the very musically inclined can tune by their hearing.[26]

[85.15] Particularly for the sake of juxtaposing the works of nature and such an incomparable art as this, but also for the consequences of its use, this must be presupposed for both the discovery and demonstration of the ratios which make what is tuned accurate.[27]

[85.19] In the one method of using the canon, I mean that in which one bridge is brought under each of the strings, there is no error if the whole scale is divided into two similar segments[28] so we can tune all the proposed differences. [85.23] In the other method, wherein it will be necessary for only two bridges to be placed under the two arrangements, it will often happen that the strings at the ends of the bridges in the middle of the canon will, [85.26] in the transfer to the sides of the retunings, take hold of the

[23] A less extraordinary term (τὸ πλῆθος) than that used earlier at 83.5 (ἐκ περιουσίας - 'for the sake of surplus.')

[24] As introduced at 5.13.

[25] It might be that Düring's text, which omits the various supplements 'only' (μόνον) and 'so long as' (δι' ὁσωνοῦν) in manuscript groups fgA, is correct, but the supplements do help to make Ptolemy's (or the tradition's) awkward phrasing more coherent. The f-group (21 manuscripts grouped with Monacensis gr. 361a [=Mathiesen, *Ancient Greek Music Theory* #22]) adds μελῶν ('music' - *melos*), which is wrong, and A (Vaticanus gr. 176 [=Mathiesen, *Ancient Greek Music Theory* #208]) adds just 'only'; the g-group (29 manuscripts grouped with Vaticanus gr. 198 [=Mathiesen, *Ancient Greek Music Theory* #218]), which I follow here, offers μόνον χορδῶν... ('strings only/so long as') which makes the meaning clearer. The number of the equally-tuned strings which are used for demonstration is not important. What is essential is 'only' that these equally-tuned strings, however many there are, show no difference from the one string. This meaning can be derived from 'whatever number' (δι' ὁσωνοῦν) alone, but the supplements make it clearer.

[26] In these two sentences Ptolemy summarizes the conclusions reached in Books I and II. Cf. also Aristides Quintilianus 131 [108.25–110.9 (Winnington-Ingram)].

[27] A prelude to III.4 and to the conclusion of the treatise. It is very important to this end for Ptolemy to establish both the naturalness of the correct ratios and the accuracy of their generation and description via proportional analysis and the use of the eight-stringed canon. If the tunings and notes were not natural, then one would not find "inner" harmony in hearing them, nor would they be repeated eternally and identically in musical systems, the human soul, and the planetary systems.

[28] The 'whole scale' (τοῦ ὅλου συστήματος) is the double diapason; the 'segments' (κατατομὰς) must then be the two outer diapasons.

opposite ends of the bridges and will no longer be able to maintain their proper lengths.[29]

[85.28] For this reason it is possible only by this method to complete scales in which one of the designated notes holds one and the same locus in the retuning.[30] This happens most frequently in music realized on the cithara, and only in this music does the use of the continuous bridges discussed above suffice. [85.32] This means that even the pegs of the notes which are common and unmoved in them are able to remain without shifting to the side.

III.2 - *Methods for Dividing Up To the Double Diapason With Only the Eight Notes*

[86.1] The division of the double diapason with only the eight originally established notes can be investigated in this way: Let us consider a monochord[31] which fits the entire string length AB, and let it be divided at the symbol C so that the segment AC is double that of CB. [86.4] Within each segment let CD be measured off from C towards B and CE towards A, so that the whole of DE takes away the width of one of the movable bridges, or a little more, and so that EC is double CD whereby the remainder AE is still double the remainder DB.

A----------------------------E---------C------D------------B

[86.8] If we divide each of the segments BD and AE into the parts extending to the lowest note, taking the first numbers from A and B, and if we then make a double placement of the bridges in a juxtaposition at each endpoint of the diapason, [86.12] the positionings having the same numbers for each note will again preserve the duple ratio for the segment towards A in proportion to that towards B. Therefore also the whole diapason towards B will stand higher than that towards A by a diapason.

[86.16] Let the monochord be divided in this way. When the eight notes are stretched with equal tension, it is inevitable that the highest of the two diapasons, those which are taken at the halves of AE and DB, will sound cacophonous,[32] especially the nearer B one approaches,[33] on account of the shortness of the segments producing them. [86.20] We will therefore take

[29] Their lengths will be in effect shortened because the opposite end of the bridge will stop the strings.

[30] The first method obviates the need for such excessive retuning.

[31] κανόνιον here clearly means 'monochord,' but its meaning is not consistant,. As a diminutive of κανών. it can also mean a table, as at 75.18.

[32] Δυσήχους - *dysèchous*) ; cf. 83.9.

[33] Where the highest notes are to be tuned.

care again that the top four notes[34] be thinner and of equal tension with each other, a diapente higher than the lower four,[35] which are also of equal tension to each other.

[86.23] In this way the division in both tetrachords from the lowest to the highest up to only the diatessaron will make the diapason composed from both the increase of the length in the diatessaron and the increase in tension in the diapente.

[86.28] For let us consider in one of the tetrachord segments the common endpoints at ABCD. Of these notes of equal lengths, AE is the highest, BF the fourth from this, and fifth is CG, eighth is DH, and in tuning AE and BF are a diapente higher than CG and DH. [87.3] And let equal segments AK and CL be marked off so that BF and DH stand in a sesquitertian (4:3) ratio to them. Once the edges[36] of the bridges have been brought under the symbols H, L, F, and K, then AK will clearly be a diatessaron higher than BF, and CL will clearly be a diatessaron higher than DH.

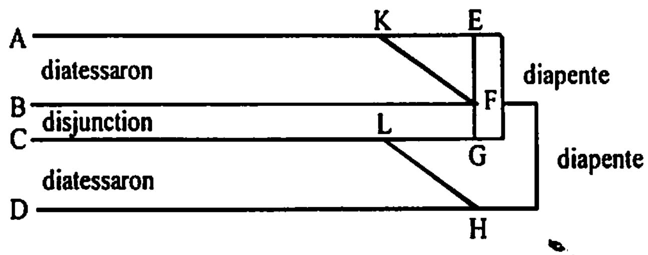
[87.7] Since also both BF lies a diapente higher than DH and AK than CL, since the whole of AE is such to CG as is BF to DH, it is clear that also BF will be higher than CL by a whole tone, and AK than DH by a diapason. Similar occurrences happen in those segments falling in between. [87.11] This is because generally the segment of the four higher notes is brought down by the sesquialter (3:2) ratio of the diapente, contrary to what they were, so that, if all were unequally tuned, as much as they are increased upwards by tension, so much are they brought downward by length[37] into restoration of the proportional quantities they originally had.

[34] It would be preferable if Ptolemy had used the word 'strings,' (as Barker (367) translates), but he uses 'notes' (φθόγγους).

[35] Ptolemy proposes here to tune these strings a diapente higher. This will eliminate the poor sound by increasing the length of the part of the string to be played. Each note, of course, will stand a diapente above one of the lower four notes, which spans the one diatessaron plus the disjunction.

[36] Cf. 17.24 (I.8) with n. 135; here the *apopsalmata* refer to 'the part of the string that the musician plays.'

[37] The strings are tuned a diapente higher, so their lengths remain essentially unaltered. Tuning them higher, though, allows the segments to be longer. Düring's printed text would necessarily mean, "so much are they brought downward by decrease in length," but Alexanderson (17) comments that decreasing the length would not bring the notes down in pitch. Ptolemy's expressions 'by tension' and 'by length' are meant to be parallel; he means that greater tension creates increase in pitch, while increase in length creates decrease in pitch. Cf. 88.1.



[87.17] For this reason, we must be careful, when we take the loci of the higher tetrachords as the sesquialter of the numbers designated by their positions [in the table],[38] that we bring them to the segments taken for each side of the monochord, [87.20] which we will extend to 130 11/60[39] parts so that we can take out from the remaining number of the lowest of the four notes from the highest 86 47/60 parts,[40] the sesquialter.

[88.1] Increased still more will be the lengths of the higher notes if we make the four notes previously mentioned[41] higher by a whole diapason than those under them, so that it no longer happens as previously that each of the two diapasons consists of both tetrachords but instead each of each, that is, the higher tetrachord consists entirely of the higher, the lower of the lower, with the same division used for each.[42]

[88.8] For let us consider the aforementioned schema encompassing the entire length of one of the tetrachords, and let us propose to mark off the lower four notes of the diapason at the ends ABCD, the higher at EFGH, with DH divided into both the lowest and the highest of the diapason, and

[38] My supplement. Surely Ptolemy's autograph was accompanied by or was at least meant to have been accompanied by graphs and figures; cf. 53.22 (with n. 84).

[39] Düring has written this number as the closest sesquialter number to 86 47/60, and he cites the table (p. 80) from II.15 (cf. here at p. 71), where 86 47/60 is found at the bottom of column 4. The closest sesigesimal equivalent of the sesquialter of 86 47/60 is 130 11/160, the exact being 130 21/120. The manuscript readings in mF[1] (131), F (31), g (132), and A (132 or 131) seem to be simply rounded off and probably represent a confusion early on in the manuscript tradition. The latter conjecture also accounts for the variants in the subsequent note. On the sexagesimal system, see II.13 (69.29 with n. 234). Gohlke 1441–44, takes issue with Düring's transcription of the sexigesimals; he points out, for instance, that ριδ' λγ' should be 114 33 rather than 114 6/11.

[40] The manuscript group f has 86 47/60, but MWE have 85 47/60, V 88 44/60, and gA 88 99/60 [=89 33/60]. The text's correct reading should match that cited in the previous note; cf. Düring p. 80, column 4), but to MWE's 85 47/60 the sesquialter would be 128 41/60 [= ρκη' μα']. mAg's 131 is the sesquialter of 87 20/60, not an offered variant. The 132 in F[1]gA makes the sesquialter of 88 (πη'). This is clearly one redactor's educated guess at simplifying a complex numerical relationship.

[41] At 86.21 and 87.7f.

[42] Ptolemy now expands the higher limits of his construction by an additional consonance. The method of tuning will be accordingly slightly different, as stated by this scholion: "The high tetrachord thinned and increased in juxtaposition is established for the higher sound in a sesquialter relationship with the higher tetrachord, for formerly from the thinness and brevity of the length it was able to be higher than the thicker tetrachord, but now also from the greater tension it had been thinned even more and became higher in pitch (and again the opposite)."

with the next, [88.12] that is, CG divided into the two second from the aforementioned, with BF into the two which are third from them, with AE into the two fourth from the highest, so that the arrangement from the highest to the lowest through HGFE and ABCD is encompassed in a circle.[43]

[88.16] If on the aforementioned monochord we apply in succession to the notes for each part of the aforementioned lengths in the duple ratio only that segment which is consistently greater, so that in the first four numbers the beginning of the parts fit the endpoints HGFE, [88.19] with the smaller taken from H, and in the subsequent four they fit the endpoints ABCD, with the smaller of these again taken from A, let us then move the bridges under the segments designated by the numbers. [89.2] Clearly the note HK will make the highest in the diapason, GL will make the second from it, FM the third, EN the fourth, and again AO the fifth, BP the sixth, CQ the seventh, and DR the eighth.[44]

```
A------------------------O--------N------------------E
B------------------------- P--------M----------------F
C----------------------------Q--------L--------------G
D-----------------------------R--------K------------H
```

[89.5] And if we join the second tetrachord to this by taking in it the section constituted by the same numbers, we will make two diapasons. Both of its tetrachords are equally tuned and are tuned equally to each other, virtually double, [89.9] for they differ by the tuning of the diapason, and they differ by the same magnitude and are conjoined in a double diapason.[45]

[89.12] It is evident from this first that after the length HK there is no further reduction into higher loci, as happened in the previous disposition[46] in which other notes were arranged as the highest.[47] [89.15] It is also clear that only the former use of the bridges[48] is able to be of advantage with this method and not that employing the common bridges.

[89.17] When the same distances in width were necessarily maintained through the entire string length, that arrangement maintained for all the endpoints the same ratios (bound by the same notes) appropriate to the similarity of their distances in width. [89.21] After all, it was presumed that all should form the diapason with the opposite parts. But this other

[43] A similar circular progression occurs in the unfolding of the octave species in the second book. One is of course reminded of the circle of fifths.

[44] In most instances Ptolemy pre-letters his figures; in this instance, however, KLMNOPQR have to be assumed. (In Düring's Greek text (88.11 tab.), the upper line should read left to right alpha, xi, nu, epsilon; cf. Höeg, review of Düring, 658, n. 2.)

[45] They could hardly be in a more 'perfect' arrangement.

[46] The Greek term here is ἀγωγῆς (*agôgê(s)*), which, although a regular part of peripatetic vocabulary, elsewhere in music theory refers to a succession of notes, as at Cleon. 207.2–.3, Aristides Quintilianus 16.19, *Anon. Bell.* 3.78 Najock; cf. da Rios 44, n. 1 (*trad.*).

[47] E.g. at 87.15f.

[48] My supplement (after Wallis 227) to the noun is suggested, in fact demanded, by the discussion of the two methods of bridge placement in III.1.

arrangement, since it necessitates altogether unequal ratios bound by the same notes and the same distances in width on both sides, is no longer able to include what is appropriate in the differences to the similarities through the whole length.[49]

[89.25] These should be the most reliable methods by which we divide up systems of doubles[50] in the half notes of numbers. [89.27] Generally for usage of the diapason one must have the segments of the aforementioned numbers from nete diezeugmenon so that the melody is played in the middle tunings,[51] of the double diapason those from nete hyperbolaion or mese so that it will be able to be tuned from both endpoints and in a similar way.[52] [89.33] Also, one must be aware, if the width of the movable bridges is smaller than that of those remaining at the ends, which is also appropriate lest they take away a great part of their length, that the curves of all make arcs of equal circles. [90.3] Also, there should not be any variation in the lengths between the edges since the movable bridges do not need to have a place higher than the endpoints.[53]

[90.6] Let us consider the base of the canon along the straight line AB, and let ACD and BEF be brought up to it at right angles. And from the centers C and E let segments of circles be drawn at the curved edges of the bridges,[54] that is GD and HF, so that BF is greater than AD. [90.9] Let the straight line at G and H continue touching the edge, that is, HG, and let GC and HE be joined,[55] GH be cut by the extension of CD at K, and similarly by the extension of EF at L.

[49] This latter phrase "appropriate in the differences to the similarities through the whole length" (τὸ ἀκόλουθον τῶν ὑπεροχῶν ταῖς δι' ὅλου τοῦ μήκους ὁμοιότησι) echoes that at 89.20 ("appropriate to the similarity of their distances in width" - ἀκολούθως τῆς κατὰ πλάτος αὐτῶν ἀποχῆς), but Ptolemy does manage to phrase the two quite differently.

[50] Or 'duples.'

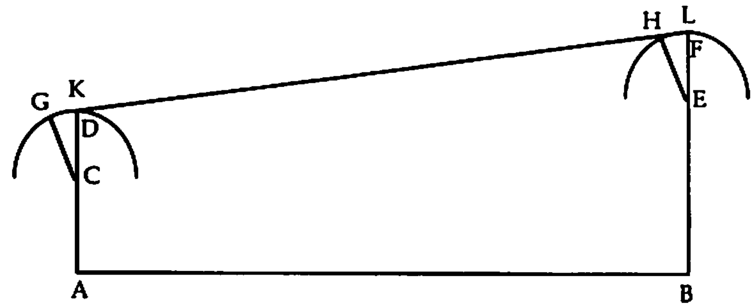
[51] At II.11 (64.16f.) Ptolemy makes similar reference to using the middle of the disdiapason system.

[52] Similar to each other, since one (the upper diapason) repeats the other (the lower).

[53] The original width of these movable bridges would have had to have been considerable for them to have interfered with the string lengths so decidedly. Düring, *Ptolemaios und Porphyrios* 266, assures us that the pegs are cylindrical.

[54] Ptolemy incorporates within an otherwise undistinguished tabular demonstration an interesting shift in vocabulary. Thus far in the treatise Ptolemy has had entirely linear concerns. At 88.15f. he explained that the two diapasons could be described circularly, and now he subtly uses the same word for 'arc' (περιφερείας) as he had used previously (and frequently) for 'segment'; they refer to the same concept, of course, except that one refers to a segment of a circle, the other to a segment of a line. Here Ptolemy uses the same word for 'arc' as he will for 'periphery' quite frequently in the subsequent chapters of Book III.

[55] Alexanderson (17) finds the passage hopelessly corrupt and would prefer AD and BF to be equal. Ptolemy wants the line GH to be maintained, which he states clearly enough, but he confuses the issue, or the manuscript tradition is indeed corrupt at 90.11, where he states that GC and HE be joined. It would be tautological for him to say that they are joined by GH since he has just said that in the previous clause; and the text as it stands shows little evidence of having once said that the two radii CG and EH should be parallel; cf. 91.1–.2.



[90.13] With straight lines shown falling through the middle of the width of the bridges at the symbols D and F, where if AD and BF have been extended, the points of contact with the strings also occur at the symbols D and F and the edges. [90.16] It is clear also that the application of the calibrator to the endpoints K and L will indicate the length KL, and that between the true contact points and the edges will make GH. [90.18] And the triangle CGK is similar to EHL since CK is parallel to EL, and GC to EH, wherefore also as EH is to CG, so is HL to GK. [91.3] With CG and EH being equal, that is, with the arcs FH and GD making segments of equal circles,[56] HL will be equal to GK, and KL to all of GH, so that by no means does the distance measured by the calibrator differ from the truth. [91.7] If they are not equal, this[57] cannot be maintained, but some other segment will be shown though that division which exists according to nature.

[91.9] And if it were to happen that such a variation were to occur in simply all the notes with the same ratio, which would have happened if all the bridges had had the same distances from the endpoints, there would have been no error in actual usage since the ratios in each of the strings would be increased or decreased by the same parts. [91.13] Yet since it is altogether necessary that the placings of the bridges make unequal lengths, by which it follows that the excesses in the greater distances are of lesser difference, and, on the other hand, in the lesser distances of greater difference, [91.16] not an insignificant error would occur in the lengths of the segments in such a tuning if we did not make both the positionings of both the fixed and movable bridges and their placements according to the method which we have delineated.[58]

[56] That these curves are arcs belonging to equal circles is what he meant to demonstrate after 90.3.

[57] Namely, that the distance on the calibrator or monochord will not differ from the truth.

[58] It is because the use of circular geometrical demonstration will be so vital to the psychic and cosmic harmonies described later in this book that Ptolemy takes great care here to emphasize the precision of the arrangement and the equality of the circles.

III.3 - *In Which Genus Must We Place Harmonic Function and Our Knowledge of It*

[91.22] That the nature of what is tuned,[59] even up to the emmelic intervals,[60] has appropriate ratios and what they are in each, I think that I have demonstrated more than sufficiently. And so because of the proper reason of our suppositions and in light of the scrutiny of actual performance, [91.25] that is, their use on the canon which we have described, my opponents should not be doubtful since they can recognize through all the species the approval of their perceptions.[61]

[92.1] Since it would follow for one considering these things to wonder straight away whether he had found something other than what is most beautiful—whether the harmonic function was the most rational possible, whether it was as completely accurate as possible, [92.3] and whether they had made the differences in the appropriate species — and then to desire by some divine[62] longing as if to observe its genus and to what other matters of those included in this cosmos this applies, [92.6] we will attempt chiefly, as much as it is possible, to investigate this remaining part of our present examination in order that we might demonstrate the magnitude of such a function.[63]

[59] The scholiast adds, "He demonstrated in the fourth chapter of Book I that [the harmonic function] is derived by quantity. Now he demonstrates, saying that the harmonic function is a species of the cause in proportion, the theoretical [function] the road to this mathematical knowledge."

[60] Ptolemy begins with the homophones, proceeds to the consonances 'up to' (μέχρι) the emmelic and superparticular ratios. Cf. I.7 (15.10f.).

[61] Cf. 85.11. Because the next paragraph and the remainder of this chapter serve as Ptolemy's introduction to the more philosophical aspects of harmonics, this is the appropriate place to survey Ptolemy's philosophical sources. His philosophical sources will be different, of course, from the music theorists he has studied in the first two books; cf. Boll *passim*. Düring, *Ptolemaios und Porphyrios* 266–68, was unsuccessful in pinpointing a specific source for Book III, for Ptolemy was widely read and quite the eclectic. We know this from his familiarity in the musical and mathematical disciplines alone with Aristoxenus, Archytas, Didymus, Eratosthenes, et al., not to mention his familiarity with a variety of sciences—astronomy, geography, optics, mathematics, and harmonics. Also, Düring overlooked the copious fragments (and even more plentiful lost works) of authors subsequent to Plato and Aristotle, particularly the Stoics. In every case Ptolemy selected from various sources what he thought instructive or correct; he followed no particular dogma consistently. Useful comparisons can be made with *Alm.* 1.25 (Heib.).

[62] The introduction of such a non-technical adjective (θείου) is striking but appropriate for the subject matter soon to be at hand.

[63] Ptolemy allows for disagreement with his hypotheses and demonstrations, but he finds his proof and explanation of such matters to be convincing and thorough enough to proceed to the next and more important step—the generic category and cosmic implications of "the harmonic function." The latter part of this paragraph lends credence to the theory that Ptolemy had the scope of his treatise mapped out from the outset. Ptolemy also discusses, albeit briefly, many of the same concepts—ethos, proportion, divinity, natural beauty, and the soul—at the very outset of the *Almagest* (1.1)

[92.9] Since all things initially use material, motion, and form,[64] where material is the basic subject matter and from which,[65] motion is the cause and by which, and form is the end and on account of which, it is necessary to accept harmony neither as the basic subject matter, [92.12] for it is something which produces and not something which is acted upon, nor as the end, since on the contrary harmony achieves some end, [92.14] such as fair melody, rhythm,[66] law, or cosmos, but as the cause which achieves for the basic subject matter the proper form.[67]

[92.16] Since the highest causes are understood as three in number, one concerned with nature and being only, one with reason and being well only, and one with god and always being well,[68] [92.18] one must not place that of harmony with nature, for it does not achieve being for the basic subject matter, not with god, since neither is it the first cause of what always is, [92.21] but it is clear that it should be placed with reason, which falls between the aforementioned causes and produces what is good in each.[69] It is also always present with the gods, [92.23] as if with what always exists, but not with all things that are natural nor altogether their opposite.[70]

[92.24] Since of the cause concerning itself with reason,[71] one is like the mind and with divine form, one like an art and with reason itself, and one as custom and with nature, we might find harmony in all of them accomplishing its own end.

[64] This passage, too, can be compared to that in the preface to the *Almagest* (35 Toomer, 5 Taliefero). It clearly has Aristotelian origins, as observed not only by de Pace 137, but also as early as Boll 102. See Düring, *Ptolemaios und Porphyrios* 268, Eduard Zeller, *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung*[4] (Leipzig 1921) II 2 (3) 328, and also E. R. Goodenough, "A Neo-Pythagorean Source in Philo Judaeus," *Yale Classical Studies* 3 (1932) 117f. [esp. 132f., with n. 67].

[65] Cf. Arist. *Meta.* 1032^{a}13–1034^{b}10, on the modes of generation.

[66] This second product is otherwise not addressed in the treatise and is therefore indicative of what the study of harmonics does not include.

[67] Because harmony is "the cause and by which," it must by definition be connected with motion, not matter or form. This seems surprising since the harmony Ptolemy has been employing in the previous two books constantly produces proper form. But it is not harmony's connection with form but its production of form that causes Ptolemy to attribute harmony to the emotion and cause category.

[68] These are given in ascending order from nature and being, to reason and being well, to the divine and being well forever. All three are subdivisions of cause, which Ptolemy established as the essence of harmony in the previous paragraph. Cf. Boll 102 and Ptol. *Judic.* 23.17f.

[69] Harmony again falls in the middle category as it did as the motion (or cause) between material and form.

[70] Although the "cause" of harmony belongs not in essence to what is natural or what is divine but to the rational, it still produces the better results in nature and is innately connected with the divine. I say 'better results' because Ptolemy points out that it is present with some, i.e. good, natural things but not with what is not good, i.e. 'their opposite.'

[71] All things use initially material, motion, and form. Harmony uses motion and is the cause of something. Of causes, then, there are three kinds—the natural, the rational, and the divine, of which harmony belongs most properly to the rational. This is because harmony functions in each of the three types of cause concerning the rational—the intellectual and divine, the technical and rational, and the customary and natural.

[92.27] Reason, you see, is simply and generally creative of order and symmetry, but what is harmonic is characteristically creative of things relevant to the genus of hearing, as the apparent is of sight and the critical of what is thought.[72]

[93.1] It directs the order of what is audible, which we call characteristically emmelic through both the theoretical, mental discovery of what is symmetrical, through the manual, technical demonstration of them, as well as through the subsequent, familiarizing experience.[73]

[93.4] This is because reason universally discovers by theorizing what is well, presents the result in operation, and assimilates the basic subject matter by becoming accustomed to it.[74] [93.6] The result is naturally that the common, rational understanding of the species of proportion, characteristically called the mathematical,[75] is not encompassed within the contemplation of the beautiful alone, [93.8] as some would profess, but also by the simultaneous demonstration and practice of what is subsequently created by it.

[93.11] For in serving this function it employs like instruments and ministers the highest and most admired of the senses, sight and hearing, which tend to lead the other senses[76] and judge their basic subject matter not by pleasure alone but, more importantly, by the beautiful.[77]

---

[72] This paragraph proceeds logically from the last in that reason (or the rational) itself is now generally defined and then applied specifically to harmonic logos. The scholion reminds the reader here that "harmonics is the efficient, rational, and critical, the technical, and the aesthetic cause of that pertaining to the genus of hearing."

[73] The scholion adds, "In the beginning of the first book he said that the criteria of harmonics were hearing and reason, but now he refers to what is mental and technical, stating that the technical replaces hearing and that the mental replaces reason," albeit inaccurately. It is the technical demonstrations on the canon that supplement, not replace, the hearing.

[74] A restatement, although this one is even more professorial than the one in the previous paragraph. On the punctuation, see Höeg (1930) 658, n. 2. Andrew Barker, "Reason and Perception in Ptolemy's *Harmonics*," in R. Wallace and B. MacLachlan, eds. *Harmonia Mundi: Musica e filosofia nell'Antichità* (Rome 1991) 104–130, offers a general discussion of this passage.

[75] The scholiast relates this passage to what we generally recognize as the seven liberal arts, four of mathematics, three of grammar: "He says the understanding of the four parts of mathematics shares something in common with the three species of speech, and astronomy more so. For it, too, is thought of by means of technical reason whenever we obtain knowledge through speech or grammatical demonstration, and similarly it is thought of by means of intellectual reason whenever by thinking about grammatical symbols we consider matters beyond perception. Third, it is thought of by means of the usual [reason], that is to say that which is conducted by attempting to examine and scrutinize each, for example the sun, moon, and stars."

[76] Boll 98, n. 1, analyzes the Stoic terminology; cf. *Judic.* 23.8 on sight and hearing.

[77] Ptolemy has returned to a notion he introduced in Book 1.2, which is the parallelism of the senses and the mind. His scope has expanded greatly in the intervening pages, and his nouns have become quite profound here in this encomium of reason, its practical applications, and the instruments, the senses, with which it achieves and judges its applications.

In the rest of this chapter Ptolemy will then concentrate on the senses themselves and how they not only enjoy proportional harmony and find it pleasant but also how these senses are derivative from a much higher source, the beautiful. He begins in this and the following

[93.15] For in each of the senses someone would find in what is sensible differences appropriate to each, for instance, the white and black in regards to the visible, the high and the low in the audible, the aromatic and ill-scented in the sense of smell, [93.18] the sweet and bitter in the sense of taste, and in the sense of touch the soft, for instance, and the hard; and, by Zeus, each of the differences is suitable or it is not. [93.20] No one would predicate the beautiful or shameful in the senses of touch, taste, or smell; this is so only in the senses of seeing and hearing, in regard, for instance, to form and melody, or again heavenly movement and human actions.[78] [93.23] These alone of all the senses therefore administer frequently in turn to the rational part of the soul the perceptions of each other, as if they were genuinely sisters.[79]

[94.1] The hearing reveals the visible entirely through interpretations, while the sight announces the audible entirely through description, and often each of these does this more distinctly than if one sense alone were to interpret the same things. [94.4] For example, what is given by reason becomes both more teachable and better remembered by us with diagrams and figures, but what is recognized by sight is made clear and better represented through poetic interpretation, [94.7] as with the sights of waves, topographical descriptions, battles, and circumstances of passions, so as to familiarize completely the souls with the sight of what has been stated aloud, as if it had been seen.

[94.9] And so not only because of their perceiving what is characteristic in each but also in that they somehow share with each other in the learning and contemplation of what is brought to its end in accordance with the appropriate reason, [94.12] these senses[80] and the most rational of

---

paragraph by distinguishing sight and hearing as the only two senses which administer frequently to the rational part of the soul.

[78] Ptolemy's argument is that each of our senses has certain differences to perceive; hearing hears high and low notes, seeing sees black and white, and tasting tastes sweet and bitter. Either a phenomenon, which Ptolemy would call "that which is perceived," is appropriate to be perceived by a certain sense (by Zeus) or it is not.

The beautiful and its opposite, αἰσχρὸν - best translated literally as 'shameful' but which means in effect 'what is not beautiful' - are not perceived by any of our senses except sight and hearing. The examples Ptolemy uses are, of course, purposely chosen. One can compare here Arist. *Pol.* 1340ᵃ12. Düring, *Ptolemaios und Porphyrios* 269 also applies [Arist.] *Prob.* 19.27 and 29, but there the author, not Aristotle himself, ignores the sense of sight.

[79] Hearing and seeing are almost sisters in their close relationship to each other as the only two senses with access, as it were, to the rational part of the soul. This idea was introduced already in Book 1 (n. 5); cf. Albert Lejeune, *Euclide et Ptolémée: deux Stades de l'Optique géométrique grecque* (Louvain 1948) 23. Their closeness or similarity seems to be emphasized in the presently illusive phrase τὰς ἀλλήλων καταλήψεις ('the perceptions of each other'), the meaning of which will be made clear in the ensuing sentences. In essence, what the ear seems to hear the sight supports with what it can see. Cf. Pl. *Resp.* 530D (with Theon *De arith.* 1.7). Düring, *Ptolemaios und Porphyrios* 269 allows for the probability of the use of the dual here, as at 14.14, 48.22, and 67.5; *contra*, Gohlke 1443. Cf. Carl von Jan, "Die Harmonie der Sphaeren," *Philologus* 52 (1893) 16.

[80] My supplement to the pronoun.

the sciences pertaining to them reach into more of the beautiful and the useful;[81] as to sight and the locomotion[82] of those things only seen—that is, heavenly bodies— [94.15] there is astronomy; as to hearing and, again, the locomotion of those things only heard—that is, sounds, there is harmonics.[83] Both use instruments indisputable, both arithmetical and geometrical, for the quantity and quality of the first movements,[84] [94.18] but they are themselves first cousins, born from the sisters sight and hearing, nurtured as close as possible to their stock by arithmetic and geometry.[85]

III.4 - *That the Function of the Harmonic Exists in the Natures of All the More Perfect Things and That it Appears Particularly In Both Human Souls and Heavenly Motions*

[94.21] With this we have now outlined that the harmonic function is a species of the cause in reason,[86] which concerns the symmetry of movements,[87] and that the theoretical knowledge of this is a species of mathematics,[88] which concerns the ratios of audible differences, [95.2] and

[81] A flexible prepositional and genitival phrase in Greek (ἐπὶ πλέον τοῦ τε καλοῦ καὶ τοῦ χρησίμου), the end of this sentence would most normally mean "into more than the beautiful and the useful." But this interpretation becomes philosophically difficult; astronomy and harmony, marvelous as Ptolemy has and will continue to make them to be, are not greater than beauty or utility.

[82] I.e. movement of loci.

[83] One great science very neatly applies to the hearing, the other to seeing. And just as for Ptolemy hearing and seeing are "sisters," so are harmonics and astronomy traditionally considered to be cousin sciences, even as early as Plato (*Resp.* 530D), where he claims dependence on Pythagorean teachings.

[84] A lengthy scholion: "When the numbers are considered according to themselves, for example 2, 4, and 8, it is a matter of quantity; when in relation to each other, it is a matter of quality. It is similar with the magnitude of what is written, where straight or tetragonal is a matter of quantity, but their relationship to each other is a matter of quality. This is because they are ratios of quality.

We refer to quantity in astronomy when the movements of the stars, sun, moon, and other bodies are considered in and of themselves, and we refer to quality when we consider them in multiple and superparticular ratios with one another. We refer to quantity in harmonics when, as we see the strings on the musical monochord, they are in certain quantities, such as four, five, or eight, and also when the lengths of the strings are of the same thickness; quality becomes a concern when we compare the notes occurring from the strings in relation to one another."

[85] Ptolemy continues the metaphor of sisterhood by pairing up as well arithmetic and geometry, and quantity and quality. Düring, *Ptolemaios und Porphyrios* 260, compares to Nicomachus *De arith.* 1.5. Boll (78 and 103) offers more Platonic and Aristotelian background, including the important passage in Pl. *Resp.* 530D. See also, A. Bouché-Leclerq, *L'Astrologie Grecque* (Paris 1899) 82, n. 1.

[86] At 92.16f; see Philodemus *De Mus.* 57 and 77 (Kemke).

[87] Συμμετρία (*symmetria* - 'equal measurement') here means essentially 'due proportion,' which is how it will be translated hereafter. Although 'symmetry' in this sense is a term hitherto unused in the treatise, motion was associated with cause already at 92.10–11.

[88] At 93.4f.

this extends to what becomes the accustomed[89] and proper arrangement created from contemplation and its consequences.[90]

[95.4] It must be said in addition that it might be necessary for such a function to be present in all things which contain the beginning of motion,[91] even to a small extent, just as the others,[92] particularly and most extensively in those which share a more perfect and rational nature on account of the individual conditions of their creation. [95.8] In these alone can it[93] be shown universally and clearly to preserve as much as possible the similarity of those ratios which make what is suitable and harmonious in the different species.[94]

[95.11] For generally, whatever is governed by nature shares in reason to a certain extent both in motions and the basic subject matter.[95] [95.12] And where reason is able to be guarded with due proportion, here can be found creation, nurturing, preservation,[96] and everything which is spoken of as superior.

[95.15] But when it is deprived of its own function,[97] wherever this occurs, all is the opposite of what I just described and is on the lower side of the balance. In those transformative motions of the same basic material it is not seen, [95.18] since neither its quality nor quantity can be defined on account of its inconsistency; it is found mostly in those which are concerned with the species.[98]

[95.20] These are the natures of the more perfect, as we said, and more rational, as those of heavenly nature are of the divine, and those of particularly human souls are of the mortal, since only the others just

[89] That nature has established the proper musical ratios and now humankind has become accustomed to them so that they not only *are* proper but indeed 'sound' proper is an idea which Ptolemy has stated or at least suggested in passing a number of times, e.gg. at 93.3–4 and 93.6. That idea will be expanded in this and the following chapters as Ptolemy begins to demonstrate that the harmonic ratios in the universe are the same as those in our souls.

[90] These consequences one would assume to be the experimenting with various ratios, building and tuning the canon, judging the resultant intervals with the ears, and then choosing the correct ratios for general and repeated use.

This will at this point become a transitional chapter (as was its couplet, III.3) which will move the treatise from the analysis of musical harmonics to that of heavenly and bodily harmonics.

[91] Cf. 92.9f., for the assumption that motion and cause are in tandem.

[92] E.gg. the cosmic and corporeal.

[93] That is, the harmonic function, defined at III.3; cf. n. 58.

[94] These different species must refer not to the species of the musical variety discussed so thoroughly in Book I but to the various species of the corporeal and cosmic variety.

[95] Cf. 92.9.

[96] Aristotle (*De mundo* 397^{b}1–7) speaks of generation, decay, and preservation.

[97] When whatever is governed by nature is deprived of due proportion, it almost ceases to be as it was because of its inconsistency and inability to be preserved.

[98] This probably refers to the specificity in the subdivisions or in the less important varieties. The term 'species' (εἶδος) seems to have a less technical, less musicological meaning in this chapter; cf. at 95.27 but also III.5.

discussed with the first and most perfect motion, that is, locomotion, still happen to be part of the rational.[99]

[95.24] This makes it apparent and demonstrates to what extent a human can grasp the governance in the harmonic ratios of the notes, since it is possible for those chosen[100] to examine in part each species and primarily that concerning human souls.[101]

III.5 - *How the Consonances Are in Accord with the First Differences of the Soul and Their Species*[102]

[95.28] There are three first parts of the soul — the intellectual, aesthetic, and habitual;[103] and there are three first species of homophones and consonances, both the homophone of the diapason and the consonances of the diapente and the diatessaron,[104] [96.1] so that that of the diapason accords with the intellectual, for mostly in each is what is simple, equal, and not

[99] The first half of this sentence presents the first hierarchical distinction between the harmonies of cosmic motions and the human body, the former being divine, the latter mortal; cf. 92.1f. The validity of the second half depends on that same definition, whereby motion, reason, and cause are linked. De Pace 134–36, discusses the Aristotelian background of the eternal and this passage in general. Cf. Theon of Smyrna 2.38 (62f. Lawlor).

[100] An extremely vague participle (διελομένοις), the antecedent of which is unspecified. One assumes Ptolemy refers to those chosen to study harmonics properly, although with the addition of 'in part' (ἐν μέρει) he could be putting a limitation on just how much even rational humans can 'grasp' (λαβεῖν). Cf. Théodore Reinach, "La musique des Spherès," *REG* 13 (1900) 438.

[101] F. M. Cornford, "Mysticism and Science in the Pythagorean Tradition," *CQ* 16 (1922) 146, cites Pl. *Phd.* 92B on the *harmonia* of the soul and lyre; cf. Abert 6–7. Philolaus is incorrectly credited (by Claudius Mamertus 2.3 = Diels-Kranz[6] 1.418 (44.22)) with saying that, "The soul is connected with the body by means of number and incorporeal harmony [*conventiam*]."

[102] Gogava includes this fifth chapter as part of his fourth chapter. It is adapted thoroughly by Gaffurio in his *De harmonia* (iv.7)—cf. Palisca 166–78, esp. 177 and 224—and Zarlino in his *L'Institutione harmoniche* (Venice 1589) 21–22, for which, see Werner Friedrich Kummel, *Musik und Medizin* (Munich 1977) 108. Valla's use of this and subsequent chapters is codified in Palisca 72.

[103] Aristides Quintilianus (58.6–60.9 [=Mathiesen, *Aristides Quintilianus* 121–22]) offers a similar discussion. These terms derive from both Platonic and Stoic vocabulary, but the "near parallels" found in Arist. *De an.* 407^b and 413^b are not such. In fact, there the argument is that the soul is not harmonic and shares only "movement," not construction, with harmony. Instead, the progression within the three terms seems to be from those grasped by the human mind, to those grasped by the human senses, to those somehow otherwise internalized in the human soul. The final term (ἑκτικόν) is an adjective derived from the noun (ἕξις) which is derived in turn from the verb 'to have' (ἔχω) and often refers to 'habit,' whence my translation 'habitual.' The Peripatetics and Stoics regularly use the noun ἕξις (*hexis*) to refer to inorganic nature. David E. Hahm, *The Origins of Stoic Cosmology* (Columbus OH 1977) 165f., discusses both ἕξις and tonos;. I do not see the relevance of Boll's (104) citing Cic. *De nat. deor.* 2.12.33; closer would be Arist. *Pol.* 1288^{b}15–16, but this passage involves organic nature.

[104] Ptolemy states his comparison as fact, but none of our extant sources say so boldly that the human soul consists of the intellectual, aesthetic, and habitual.

different,[105] the diapente to the aesthetic,[106] and the diatessaron to the habitual. [96.3] For the diapente is nearer the diapason than the diatessaron, since it is more consonant in having its excess nearer equality,[107] and the aesthetic is nearer the intellectual than the habitual on account of its sharing some of the same perceptions.[108]

[96.7] Since just as where there is possession there is not altogether perception, neither where there is perception is there altogether intelligence.[109] [96.8] Also, where there is perception, there, too, is possession, and where there is intelligence there will also be possession and perception, just as where the diatessaron is found there is not altogether the diapente, nor where the diapente is found is the diapason.[110] [96.12] But where the diapente is found, there, too, is the diatessaron, and where the diapason is found, there are also the diapente and diatessaron, since these are characteristic of the imperfect emmelic and composite intervals, while others are characteristic of the perfect.[111]

[96.15] One would say that there are three species of the habitual part of the soul, the same number as there are diatessarons: growth, zenith, and decline, for these are its primary functions.[112] [96.17] There are four species of the aesthetic, the same number as the consonance of the diapente: sight, hearing, smell, and taste (if we can establish that the species of touch is

[105] Ptolemy's reasons for equating the intellectual part of the soul with the diapason, other than that the diapason is the most perfect consonance, ie. a homophone, and that the intellectual is considered the most rational and therefore highest ranking part of the philosophical soul, are not stated very clearly. He labels them as 'simple' (τὸ ἁπλοῦν) because of their proximity to perfection, 'equal' (ἴσον), I assume, because they are equivalent to each other or because their parts are equivalent, and 'not different' (ἀδιάφορον) again because they are always the same in the nearness to perfection.

[106] Ptolemy's analysis differs from that of Aristides Quintilianus 3.14 (113.15–114.28), who equates the five senses with the five consonant tetrachords of the ametabolic system. Cf. Arist. *De an.* 424^{b}22 and 435^{a}17, passages which prove that the identification of five senses dates back to Aristotle.

[107] A simple matter of proportion and mathematics, where the 3:2 of the diapente is closer to the diapason's 2:1 than is the diatessaron's 4:3.

[108] Ptolemy does not elaborate on this. The next sentence offers details, but only from the musical perspective. Clearly the progression from habitual to intellectual is as hierarchical as that from diatessaron to diapason. [The note in Düring, *Ptolemaios und Porphyrios* 270, should read 96.5 (for 95.5)].

[109] Ptolemy offers these three terms as the noun-equivalents for the (adjectival) generic categories offered at the beginning of this chapter, where, in reverse order, 'possession' (ἕξις) refers to the habitual (ἑκτικόν), 'perception' (αἴσθησις) to the aesthetic (αἰσθητικόν), and 'intelligence' (νοῦς) to the intellectual (νοερόν).

[110] Wallis' corrections are necessary to employ here, as I have.

[111] The diatessaron includes the emmelic 9:8 whole tone, and the composite diapente includes the disjunct emmelic whole tone plus the diatessaron. The diapason consists of the nearly (or at least 'more') perfect 2:1 homophone.

[112] This completes Ptolemy's entire explanation of the habitual part of the soul. Judging by these subspecies, one would have to conclude this part of the treatise to be largely biological in nature. Indeed, Düring, *Ptolemaios und Porphyrios* 270–71, found the parallels in Arist. *De an.* 411^{a}30–411^{b}1 and *De mundo* 397^{b}2.

common to all, since they make their impressions by touching the sensible in some way).[113] [96.21] Again, there are seven differences among the intellectual, the same number as the species of the diapason: appearance for the presentation of the sensible, intelligence for the first impression, thought for the retention and memory of the impression, understanding for reconsideration and inquiry, [96.25] opinion for the likeness of the superficial, reason for the correct judgment, and knowledge for the truth and direct comprehension.[114]

[96.27] Then in another way our soul can be divided into the rational, emotional, and cupidinous.[115] The rational, for the sake of an equality similar to what we have previously discussed, [96.28-9] we equate properly to the diapason, the emotional somehow approaching it, to the diapente, and the cupidinous, arranged below it, to the diatessaron.

[96.32] All other matters concerning values and what they encompass are taken similarly from these, [97.1] and we would find the differences more evident in the individual virtues[116] to be of equal number to those of each species of the first consonances, since the emmelic is the virtue of notes, the ecmelic the vice, and in the opposite way, [97.5] virtue is the emmelic of souls, vice ecmelic, and, in common in both genera,[117] what parts of each are tuned are in accord with nature and what are not tuned are against nature.[118]

[97.9] In the cupidinous, the three species of virtue in the consonance of the diatessaron would be self-control in the contempt of pleasures, continence in enduring wants, shame with circumspection in what is disgraceful. [97.12] In the emotional there are four species of virtue as in the diapente consonance, gentleness in not being moved by anger, courage in being intrepid before impeding terrors, manliness in despising dangers, and patience in enduring toils.[119]

[113] Ptolemy (93.15f.) previously had counted five senses, where he differentiated the senses of sight and hearing as being more closely related to the rational than smell, taste, and touch. His point here is nonetheless well taken.

[114] These seven species of the intellectual lead again in a hierarchical progression from the most 'sensible' to the most intellectual and perfect. The corresponding Latin terms (used by Gaffurio, Gogava, and Zarlino) are given in Palisca 180.

[115] Such a division of the soul can be found as early as Plato (*Resp.* 442C). There the hierarchy within the division is already well established. That Ptolemy can use two completely different divisions of the soul does not disprove or undercut his comparison between the harmony in the soul and the harmony in music. Ptolemy might have preferred that the human soul were as magnficently divisible by whole numbers as the Greater Perfect System, but since it is not, any reputable or defensible division of the soul will help to illustrate his comparison.

Düring, *Ptolemaios und Porphyrios* 271 found a series of marvelously close parallels in Speusippus (F. W. A. Mullach *FPG* III 75), for which see now as well Leonardo Tarán, *Speucippus of Athens* (Leiden 1981) 374 and 383, n. 196.

[116] They were not evident at all in the previous division.

[117] Not the musical genera discussed in Book I but the emmelic and ecmelic, or perhaps the soul and the note.

[118] That is, 'tuned properly' or 'in accordance with the harmonic.'

[119] Cf. Boll 106–107, who cites Pl. *Resp.* 411E–414B.

[97.16] The seven species in the rational would be sharpness in being quick thinking, cleverness in being wily, readiness of mind in being discerning, good judgment in decision making, wisdom in the theoretical, thoughtfulness in the practical, and experience in what is practiced.

[97.19] Again, just as in what is tuned, it is necessary that accuracy of the homophones be of prime consideration, then that of the consonances and emmelic intervals,[120] since a small discrepancy impedes the melody not so much in the smaller ratios as in the chief, greater ones.[121] [97.24] So also with souls the intellectual and rational parts control the others which are subordinate to them, and more accuracy is needed for what is guarded by reason, since they have all or at least most of their errors in them.

[97.27] And so the soul's mightiest disposition, justice, is a veritable consonance of the parts among one another according to the prime ratios in the more important ones.[122] Those of good will and right reason represent the homophones; [97.30-1] and others, of right perception and good condition[123] or manliness and self-control, represent the consonances; and others, productive and sharing in the species of the tunings,[124] represent the emmelic intervals. [97.33] The whole philosophical disposition represents the whole tuning of the perfect system with the comparison of its parts[125] arranged according to the consonances and virtues, [97.34-98.1] the most perfect[126] being composed of a certain consonance and virtue drawn from all the consonances and all the virtues, as if of the virtues and consonances[127] both musical and spiritual.[128]

[120] Cf. 96.11.

[121] Cf. I.1 (4.13–5.2).

[122] Another adaptation from Plato's *Republic* where in the larger construct, the ideal republic, justice was found to be a harmony of the three segments of society—rulers, guardians, workers—while within the individual justice was the harmony of the three parts of the soul—the mind, the senses, and the appetite. Here, of course, Ptolemy has intensified and yet simplified Plato's mathematical basis for this conclusion.

[123] The root of this compound noun (εὐεξίων) is the 'possession' Ptolemy used elsewhere in this chapter as part of the 'habitual.' Cf. n. 102.

[124] Ptolemy uses here the simple noun *harmonia* (τῶν ἁρμονιῶν); cf. III.3.

[125] Cf. Alexanderson 17.

[126] That is, the most perfect of all the consonances and virtues.

[127] Cf. Alexanderson 17 – 18, who expedites matters by removing the comma from 98.3. But he does not separate 'a certain consonance' (συμφωνίαν τινὰ) and 'a virtue' (ἀρετήν) with their respective limiting genitives. The Greek syntax here surprises the reader now quite accustomed to Ptolemaic syntax because Ptolemy is again borrowing from Platonic method and now syntax as well. He writes here of not just the comparison between the homophone/consonance/emmelic-note construct known as the 'perfect system' and the whole arrangement of virtues but even a more perfect entity, almost a Platonic Form of harmony. This causes his syntax to change (in part because Ptolemy believes in such a perfect harmony only to a limited extent.)

[128] By 'spiritual' (ψυχικῶν) Ptolemy contrasts those harmonies or consonances of the soul with those of music. Also, his use of 'consonance' in this sentence becomes a bit lax. For the sense, see Boll 108 and Pl. *Resp.* 443C.

III.6 - *A Comparison Between the Harmonic Genera and Those of the First Virtues*

[98.6] Now there are in each of the two kinds of virtue,[129] that is the theoretical and the practical,[130] three genera. In the theoretical are the physical, mathematical, and theological; in the practical are the ethical, domestic, and political.[131] [98.9] They do not differ in function, for the virtues of the three genera have something in common and are connected to one another. They do differ, however, in magnitude, worth, and the compass of their conditions.[132]

[98.11-12] One could properly compare to each triad[133] the three genera named after 'harmony'[134]—I mean the enharmonic, chromatic, and diatonic — by some magnitude, and these take on their differences in mass[135] when raised up or lowered. [98.15] For in such a way both the pyknon and the apyknon among them are affected by position and function.[136]

[98.17] The enharmonic must be compared to the physical and ethical because of the common shrinking in magnitude in relation to other things, the diatonic to the theological and political because of the similarity of arrangement and magnificence, [98.20] and the chromatic to the mathematical and domestic because of the commonality of the middle towards the extremes.[137]

[98.21] For the mathematical genus is involved for the most part in the physical and theological; the domestic participates in both the ethical, insofar

[129] 'Of virtue' (ἀρετῶν) is found in the manuscript tradition (fA), which may have confused ἀρχήν with ἀρετῶν early on. Düring, *Ptolemaios und Porphyrios* 272 finds a precise Aristotelian parallel at *Meta.* 1026^a[19–20], but not at *Pol.* 1341^b[25f.].

[130] For a similar distinction between theoretical and practical, cf. Cleon. 179.1–.2.

[131] The three theoretical genera are given, as usual, in increasing order of importance from the physical to (what we would label today as) the theoretical to the divine. The three practical genera are given in expanding order from the personal, the familial, and then the social or political. Cf. 98.17 and Theon Alexandrinus, *Commentary to Ptolemy's Almagest* 2.321.14 (Rome).

[132] It is difficult to determine whether Ptolemy distinguishes or equates these three terms. To 'magnitude' (μεγέθει) he refers in the next sentence for identifying how the three harmonic genera can be distinguished, but he does not use the other two terms again. 'Worth' and 'compass of condition' do describe the differences between the genera of virtues in the non-mathematical sense. The political has a broader compass of condition and greater worth in the hierarchical arrangement presented by Ptolemy; in mathematical terms, this would be a greater magnitude.

[133] The word is a *hapax* in Ptolemy,. This caused some confusion in the manuscript tradition, for the scribe of g wrote 'three genera' (τριῶν γενῶν) instead of 'triad' (τριγενῶν).

[134] Only one of the genera, the enharmonic, is eponymous of the term *harmonia*.

[135] A unique description if in fact he describes the shifts in harmonic genera here.

[136] Cf. II.5.

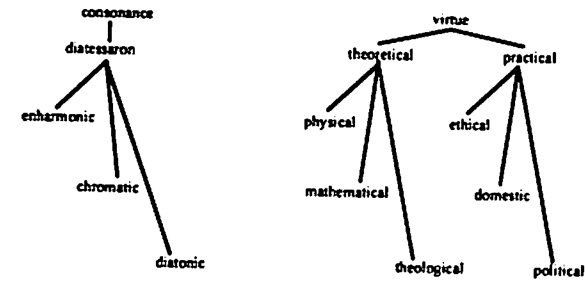
[137] This sentence depends again on hierarchical assumptions that the diatonic and divine/political are superior, the enharmonic and physical/ethical inferior, with the chromatic and mathematical/domestic in between. The terms 'common' and 'similarity' refer not to the genus by itself but to the musical and (both) philosophical genera in comparison.

as it is personal and subordinate, and the political, insofar as it is general and governmental; [98.24-5] the chromatic somehow fits both the enharmonic in loosening and slackening and the diatonic in turning and tension.[138] Each stands in relation to the other as mese is a diapason higher than the proslambanomenos but a diapason lower than the nete hyperbolaion.[139]

III.7 - *How Harmonic Modulations Resemble the Circumstantial Modulations of Souls*

[99.1] We might similarly coordinate the scalar modulations in the tonoi with the modulations of the soul through life's circumstances.[140] For just as with harmonic modulations, even if the same genera are maintained,[141] [99.4] some melodic change occurs when there is a change of loci, those through which they are effected, whether or not[142] they take on changes which are in

[138] This chart illustrates the sequence of comparisons:



The description of the chromatic here is not consistent with that in Book I.16 (38.2f.).

[139] The analogy is not intended to be precise insofar as the distance of the diapason is concerned.

[140] See Lohmann *Musike* 64. This last of the comparisons between the harmonic function of music and the human soul concentrates on the larger constructs, the tonoi. Ptolemy does not present this comparison in as neat or mathematical a fashion as he has the last few. Düring, *Ptolemaios und Porphyrios* 272, boldly declares the source as Peripatetic, whereas there are in fact only early examples of the comparisons between harmonics and the state to be found at Arist. *Pol.* 1340^{a}1–1340^{b}20 and *Eth. Eud.* 1241^{b}10–40. The latter includes κοινωνία, for which see on modulation at II.6 [55.25]) and III.4 (95.7–.11), and see also Vianney Décarie, *Éthique a Eudème* (Paris 1978) 180–81. In contrast, see David E. Hahm, *The Origins of Stoic Cosmology* (Columbus OH 1977) 165–77. Aristides Quintilianus also discusses the harmonics of the soul in Book II (from chapter 14) through Book III of his *De musica*, for which see Mathiesen, *Aristides Quintilianus*, esp. 143–48.

[141] Here referring to the enharmonic, chromatic, and diatonic genera in the musical side of the comparison, the physical/ethical, mathematical/domestic, and theological/political genera in the psychic. Musical genera can of course remain unchanged during modulations in tonos; similarly, the scope of the psychic genera can remain the same—that is, personal, social, or in between—while the soul modulates in other ways soon to be described.

[142] Düring's emendation (ἢ μή) must be correct since a) a negative is necessary, and b) it would be unlike Ptolemy to omit a second definite article before a second infinitive.

succession and composite,[143] [99.6] so also in life's modulations the same species of the spiritual[144] dispositions are somehow turned to dissimilar pastimes, drawn opportunely by those customs of the citizens towards conditions more suitable for themselves.[145] [99.9] Something similar happens as well even with legislation, for very often laws are readopted for the administration of justice suitable for prevailing fortunes.[146]

[99.11] In this way conditions of peace turn the souls of citizens toward the more tranquil and equitable, while conditions of war on the contrary turn them toward the more rash and contemptuous. [99.14] Again, need and scarcity of necessities turn them toward the more temperate and frugal, while conditions of abundance and plenty[147] turn them towards the more free and intemperate, and so on with other conditions.

[99.17] In the identical way in harmonic modulations as well the same magnitude in the higher tonoi is turned toward the more rousing,[148] in the lower toward the more depressing[149] since what is higher in notes is more tense, what is lower is more loosened, [99.21] with the result that naturally here the middle tonoi, those around the Dorian, are to be compared to the moderate and settled ways of life, the higher, those around the Mixolydian, to the motion-filled and vigorous, the lower, those around the Hypodorian, to the relaxed and sluggish.[150]

[99.25] Accordingly, our souls evidently experience the same effects as the melody, as if they recognize the kindred relationship of the ratios of each state and are modeled by some movements appropriate to individual musical

[143] Ptolemy adds this clause because, theoretically at least, we usually think of modulations in tonos as moving stepwise up or down in an orderly, textbook progression. They need not modulate that way in actual music, of course, nor does our soul modulate in mood or circumstance in such a predictable, regular pattern.

[144] Or, 'psychic' (ψυχικῶν).

[145] Again, the political analogies have a long philosophical history, which was detailed in the annotations to III.5. Ptolemy's view of the public will depends in part on the tradition inspired by Plato to which he has made allusion repeatedly in Book III, but also in part to his equating musical and public "free will." That is, the musician modulates because he wants to or because it produces music more suitable to the ethos of the occasion, so public, social or psychic modulation must occur in an analogous way.

[146] The scholion here makes reference to "the law prohibiting the Lacedaimonians from making a military expedition against the Persians at Marathon during the full moon. The Athenians were victorious by themselves."

[147] The same term (περιουσίᾳ) he used awkwardly at III.1 (83.6).

[148] A term (διεγερτικώτερον) commonly employed in descriptions of ethos, as in Sext. Emp. *Against the Musicians* 142.5–6 (Greaves).

[149] Cf. Ptol. *Tetr.* 172, where Saturn joins Mars and Venus to produce licentiousness and impurity; and Sext. Emp. *Against the Musicians* 129.4 (Greaves).

[150] Ptolemy now makes the analogy slightly more organized and predictable by equating tension in strings with a livelier soul, relaxation with a more depressed soul. This makes the ascending sequence of tonoi from Hypodorian to Mixolydian ever increasing in liveliness, with the central Dorian providing the 'norm.' He does not remind us of his earlier consideration, albeit in passing, of ethos when discussing Archytas at I.14; cf. Abert 5–7. Gombosi, *Tonarten* 137, offers a translation of this paragraph.

forms.[151] [100.1] As a result, at certain times they are led to pleasures and merriment, at others to lamentation and depression, at certain times deep sleep and rest, others excitement and rousing, [100.4] and at certain times turned to some leisure and relaxation, at others to frenzy and inspiration, and so on, with the music itself changing and inducing the souls to dispositions consisting of similar ratios.[152]

[100.7] I think even Pythagoras was thinking this when he advised rising at dawn and, before beginning some work, employing music and gentle melody so that the confusion in the soul felt at awakening from sleep,[153] [100.10] first converted by modulation into a pure state and an ordered mildness, then prepares the soul to be harmonized and consonant for its daily activities. [100.12] It seems to me also that to invoke the gods with certain kinds of music and melody, for instance, with both hymns and auloi or Egyptian trigona,[154] shows that we are eager for them to hear our prayers with a gentle temper.[155]

[151] Here and at the end of this paragraph Ptolemy suggests that there are or might be something resembling mathematical ratios by which the human soul could be analyzed, but he refrains from stating that in so many words and does not elaborate. Düring (273) refers us here to Aristides Quintilianus 95 [80.23–81.7 (Winnington-Ingram)] and Pl. *Resp.* 401D.

[152] Ptolemy mentions here six different conditions or states—pleasure, depression, deep sleep, excitement, relaxation, and frenzy. They do not seem to be presented in any particular order. If we can add 'the norm,' that would make seven, the same number as the tonoi. In 'scalar' order from highest, i.e. most tense, to lowest, these might be:

frenzy	-	Mixolydian
excitement	-	Phrygian
pleasure	-	Lydian
[normal]	-	Dorian
depression	-	Hypophrygian
relaxation	-	Hypolydian
deep sleep	-	Hypodorian

Düring, *Ptolemaios und Porphyrios* 273, following Abert, refers us to a number of sources, to which add Plato *Euthydemus* 290A, some more relevant than others; the reader should recall as well that many Roman sources, e.g. Cic. *Tusc.* 4.2 (and probably even Quint. *Inst.* 1.10.32), predate the Greek Plut. *De virtute morali* 441E.

[153] Cf. Düring, *Ptolemaios und Porphyrios* 273, with references, to which add Porph. *Vit. Pyth.* 32; cf. Quint *Inst.* 9.4.12. Plut. *De Iside et Osiride* 384A, compares a sixteen-ingredient sacrifice (κῦφι) to the effect lyre playing had on Pythagoras in the evening, for which see as well Iamblichus *Vit. Pyth.* 114 and Seneca *De ira* 3.9. At an early date our sources seem to have become confused as to whether Pythagoras used lyre music to lull himself to sleep at night or to ease his anxieties in the morning. Iamb. *Vit. Pyth.* 65 includes both. On this anecdote often commented upon, see also Boll 109–110; Isidore Lévy, *Recherches sur Les Sources de la Légende de Pythagore* (Paris 1926); and Burkert, *Lore and Science* 109–120.

[154] On the trigonon, see Walther Vetter, "Trigonon," *RE* XIII,2 (1939) 142. Juba ('Ιόβας) in the fourth book of his *History of the Theatre* (*ap.* Ath. 175D) says it was a Syrian invention, but the ensuing discussion at 182E–F and 183E demonstrates little more than mass confusion. At 636A–B, however, the trigonon (with pectis and magadis) are used to worship Tmolian Artemis in Asia Minor.

[155] He concludes again with the divine, as at 92.1f.; cf. Aristides Quintilianus 57.23f.

III.8 - *On the Similarity of the Perfect System and the Circle Through the Middle of the Zodiac*[156]

[100.18] Let the affinity[157] of human souls to the harmonic be scrutinized in this way. Since, in a word,[158] the homophones and consonances are found to be arranged like the first parts of souls, the emmelic sort being like those of the virtues,[159] [100.22] the differences in the genera of tetrachords are arranged like these genera of the virtues in worth and magnitude;[160] and the modulations in tonoi are arranged like the variations of dispositions in life's circumstances.[161]

[100.24] It remains to demonstrate as well that the heavenly hypotheses are perfected in accord with the harmonic ratios,[162] which we will do first

[156] The Greek phrase τοῦ διὰ μέσων τῶν ζῳδίων κύκλου meaning (literally, 'of the circle through the middle of the zodiac') serves simply as a periphrasis for 'the ecliptic' even as early as Aristotle (*Mete.* $343^{a}24$–25); cf. Ptol. *Tetr.* 1.10 (58–59 (Robbins)). In this chapter, however, Ptolemy describes the purpose, origin, and characteristics of this circle qua circle, so I have left the periphrasis unchanged, as in 101.8–10. Although not of immediate importance for the study of harmonics, it is in part the problems associated with Ptolemy's estimate of the obliquity of the ecliptic that has cast grave doubts on Ptolemy's accuracy, reliability, and scientific reputation. Cf. Introduction, pp. xxv-xxvi, and Sir Thomas L. Heath, *Aristarchus of Samos* (Oxford 1913) 131–32.

William J. Tucker, *Ptolemaic Astrology* (Sidcup UK 1962) 4 and 21, points out that Ptolemy never investigates, neither here nor in the *Tetrabiblos*, the signs of the zodiac nor the house divisions. Instead, Ptolemy works only with the sun, moon, five planets (Mercury, Venus, Mars, Jupiter, and Saturn), and the fixed stars. Ptolemy offers no horoscopes (nor did any astrologer of his era), and he typically begins calculating the zodiac from the vernal equinox, not from Spica as did the Babylonians.

[157] To the codices' οἰκειώσις (*oikeiosis* - 'affinity') Düring in the apparatus criticus prefers συνοικειώσις (*synoikeiosis* - 'combinations'), a well-established astronomical and rhetorical technical term which Ptolemy uses at *Tetr.* 50. Ptolemy himself uses this term in the heading of III.16, an astrological context. This is not necessary here, however, where Ptolemy simply summarizes the similarities between harmonics and the human soul which he has been discussing for the last four chapters. There is precedent, however, for such a premature introduction of a technical term, e.g. at II.7 (n. 126).

[158] I follow Alexanderson 18 (100.19–.24) here, but not in placing the comma before ἐπειδήπερ ('since') at 100.19.

[159] E.g. 95.28f.

[160] E.g. 98.6f.

[161] The subject of III.7. Jamie C. Kassler, "The 'Science' of Music to 1830," *Archives internationales d'histoire des sciences* 30 (1980) 111, discusses the "fusional nature" of music, science, astrology, astronomy, and philosophy, while Olaf Pedersen, "Logistics and the Theory of Functions: An Essay in the History of Greek Mathematics," *Archives internationelles d'histoire des sciences* 24 (1974) 29, traces back to Archytas the view that arithmetic and astronomy (specifically, sphaerics) are "sisters." See also, Olaf Pedersen, *A Survey of the Almagest* (Odense 1974) 26–32; and Amy K. Holbrook, *The Concept of Musical Consonance in Greek Antiquity and its Application in the Earliest Medieval Descriptions of Polyphony* (Ph.D. diss., University of Washington 1983) 110–21

[162] Heaven is a harmony and a number. The idea can be traced back to the *Republic* 530D, where astronomy and harmonics are "sister" sciences, and the *Timaeus* (47D–E), where the distribution of our souls among the stars creates a harmony between human soul and the

where one is in common with all pathways or at least most[163] [100.27] and then in the subdivisions found in each;[164] the first one[165] comes first and is in common for those making their beginning from it.

[100.28] First, therefore, is that both notes and the motions of the heavens are brought about by intervallic movement alone, since none of the modulations altering the substance are attendant upon it; this proves what I had proposed. [100.32] Then also is that all the periods of the celestial bodies are circular[166] and ordered and that similar are the states of the harmonic systems. [101.1] Since the arrangement and tension of the notes would seem to progress as if in a straight line, the functions and the interrelationships which are peculiar to them are both perfected and closed in by one and the same period in the ratio of circular movement. [101.5] Here by nature there is no beginning; it is only by position that each at various times is taken to the succeeding locus.[167] [101.6-7] If then one cuts a circle through the middle of the zodiac by a ratio calculated at one of the

Worldsoul. Aristotle (*Meta.* $986^{a}1$–7) discusses Pythagorean, not Aristotelian ideas. The concept is in fact quite scientific, from the historian's point of view. Otto Neugebauer, (*Exact Sciences* 171), as cited in J. B. Brackenridge (and Mary Ann Rossi, trans.), "Johannes Kepler's On the More Certain Fundamentals of Astrology," *Proceedings of the American Philosophical Society* 123 (1979) 85), for instance, explains that "the Greek...universe was a well defined structure of directly related bodies. The concept of predictable influence between these bodies is in principle not at all different from any modern mechanistic theory." Nicomachus (*Ench.* 241.12–242.18) discusses the music of the spheres, which, according to Iamb. *Vit. Pyth.* 65 (36.18–.21 (Deubner)) only Pythagoras could actually hear. See also Ptol. *Alm.* 9.2 (420–21 (Toomer)), and Bernard R. Goldstein, review of Toomer, *Isis* 76 (1985) 118.

The association of number and the universe was of importance throughout the Pythagorean tradition. Speucippus discussed just this in his *On the Pythagorean Numbers*; cf. Francois Lasserre, *The Birth of Mathematics in the Age of Plato* (Larchmont NY 1964) 46f. Of interest as well are Burkert, *Lore and Science* 299–368; Leo Spitzer, *Classical and Christian Ideas of World Harmony* (Baltimore 1963); and Gustav Junge, "Die Sphaeren-Harmonie und die Pythagoreisch- platonische Zahlenlehre," *Classica et Mediaevalia* 9 (1948) 183–94, who cites relevant but not extensive passages at Arist. *Meta.* $992^{a}32$ and $1090^{a}24$. Book III of the *De musica* of Aristides Quintilianus contains a different approach to the subject. Mathematics, astronomy, and music were all necessary parts, of course, of the Boethian quadrivium, for which, see Michael Masi, *Boethian Number Theory* (Amsterdam 1983) 71.

Because Ptolemy's treatment of astrology and astronomy in the latter half of Book III of the *Harmonics* represents a mere shadow of what is offered in the *Tetrabiblos* (and *Almagest*), this commentary will not cite *loci paralleli* for every chapter. For a modern summary of ancient sources, see G. P. Goold, trans., *Manilius Astronomica* (Cambridge MA 1977) cxv–cxix; typically, there is mention of Ptolemy's *Almagest* and *Tetrabiblos* but not the *Harmonics*.

[163] E.g. III.8. Analogies between the soul, mathematical ratios, and the heavens occur as early as the *Timaeus*. Ptolemy refers to them earlier in his career in *Alm.* 1.7.11 (Heiberg); cf. Düring, *Ptolemaios und Porphyrios* 273–74.

[164] III.9.

[165] The antecedent of πρώτη (*prôtè* - 'first') is not identifiable. Ptolemy has not used a feminine singular noun recently, and if this is intended to be a general construction, it is uncharacteristic.

[166] Theon of Smyrna 3.5–.10 (86–89 (Lawlor)), has a fuller discussion of celestial circles; cf. Johannes Kepler, *Prodromus dissertationum cosmographaphicarum* (Tübingen 1596) ch. 12.

[167] E.g. II.5. I follow Alexanderson 18 (101.1f.) in placing a full stop here and balancing the subsequent ἐάν ('if') with that in line 12.

equinoctial signs and as if for sake of explication fits it in equal lengths to the double diapason perfect system, [101.9] that of the equinoctial[168] which is uncut would be at mese, one of the endpoints of the cut parts at proslambanomenos, the other at nete hyperbolaion.[169]

[101.12] And if one bends the double diapason into a circle by function and fitting the hyperbolaion to the proslambanomenos reduces two notes to one,[170] such a union[171] will clearly be in opposition to the mese and will form with it the homophone of the diapason. [101.15-16] The fine ratio of the aforementioned comparison exists because of the similarity between the diametrically opposite position in the circle and what is revealed in the diapason.[172] [101.18] For the double ratio of the whole circle to the semicircle is contained within it, as is, for the most part, equality to the other parts, both because in a circle it is necessary that the diameter fall only through the center, [101.21] which is the beginning of the equality of the figure, and because whatever is carried through otherwise, even if it divides the whole circumference into however many equal parts, but not also the whole plane, the diameter divides both it and the circumference similarly. [101.24] From which those asterisms of the stars in the zodiac are more productive than the others, as also are those notes which form the diapason with others.[173]

III.9 - *How the Harmonic Consonances and Dissonances Resemble Those in the Zodiac*

[101.27] Again, just as the musical consonances up to the division into four are constructed with the greatest, the double diapason being four times greater than the smaller, [101.29-102.1] and the smallest or diatessaron exceeding the smaller by a fourth part of itself, in the same way also the up-to-four parts of the circle complete what we consider to be the consonances in the zodiac and the effective positions.[174]

[168] Cf. *Alm.* 2.9, 1.16, and *Tetr.* 31 (1.11). For a useful glossary to most of these astronomical terms, see William J. Tucker, *Ptolemaic Astrology* (Sidcup UK 1962) 220f. This term (ἰσημερινῶν), when used with 'circle,' usually refers to the equator; cf. Toomer 19.

[169] Cf. 98.21f., for the same disposition of the extremes.

[170] Cf. II.5 (52.14f.).

[171] The term synaphe (συναφή - *synaphe*) often means 'conjunction,' e.g. *Tetr.* 52 (1.24), where Robbins translates as "application."

[172] Implicit here is the similarity and equality of the seven tonoi in the double diapason. Cf. II.10.

[173] Productivity, motion, and cause have all been linked in III.3. Ptolemy uses a different Greek term here - ἐνεργητικώτατοι - for 'productivity,' however. Germaine Aujac, "Le zodiaque dans l'astronomie grecque," *Revue d'Histoire des Sciences et de leurs Applications* 33 (1980) 3–32, provides some general background.

[174] That is, 'aspects' (στάσεις), but this is not the technical term for 'aspects,' which is σχηματισμοί. See also, Aristides Quintilianus III.23 (123.23–125.20), Cic. *De div.* 2.89, and Ptol. *Tetr.* 1.9 (46–47, n. 1 (Robbins)).

[102.4] For if we describe a circle AB[175] and divide it from the same point, that is A, into two equal parts with AB, into three equal parts with AC, into four equal parts with AD, and into six equal parts with CB, the arc[176] AB will make the diametrical position, AD the tetragonal, AC the triagonal, and CB the hexagonal. [102.8] The ratios of those arcs taken from the same point, that is again A, will contain both those of the homophones and consonances and also the whole tone.

[102.11] It will be possible to see this if we propose a circle of twelve segments since this is the first number[177] of those having the half, third, and fourth parts, for of these segments the arc ABD is 9, the arc ABC is 8, and again AB, the hemicircle, 6, the arc ADC 4, and the arc AD 3.

[102.15] Those segments make the double ratio of the first homophone, that is the diapason, three times: the twelve of the whole circle to the six of the hemicircle, the eight of the arc ABC to the four of AC, and the six of ACB to the three of AD. [102.19] They make the sesquialter ratio of the greater of the first consonances, that is the diapente, again three times: that of the whole circle twelve to the eight of the arc ABC, the nine of the arc ABD to the six of AB, and the six of the arc AB to the four of AC. [102.23] They make the sesquitertian ratio of the smaller of the first consonances, that is the diatessaron, again three times: The twelve of the whole circle to the nine of the arc ABD, the eight of the arc ABC to the six of AB, and the four of the arc AC to the three of AD.

[102.27] Moreover, it forms the triple ratio of the consonance of the diapason plus diapente twice: the twelve of the whole circle to the four of the arc AC and the nine of the arc ABD to the three of AD. It makes the quadruple ratio of the homophone of the double diapason once: the twelve of the whole circle to the three of the arc of AD.[178] [102.31] It makes eight to three, that is, the consonance of the diapason plus diatessaron, once: the eight of the arc of ABC to the three of AD. The sesquioctave ratio of the whole tone it again makes once: the nine of the arc of ABD to the eight of ABC. [103.3] These differences between the numbers described here are shown in the following chart:

[175] This begins Ptolemy's first geometrical demonstration since III.2 (90.6f.).

[176] Düring uses 'segment' for 'arc' (περιφέρεια - *peripheria*), but he uses the same translation for τμήματα (*tmemata*) shortly after this.

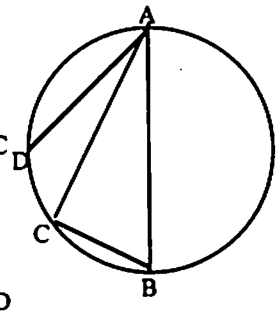
[177] Or lowest common denominator. Aristides Quintilianus 3.23 (123–25 (Winnington-Ingram); 193–94 (Mathiesen)) has a similar but more complex twelve-part division. For the historical perspective of this division, see Árpád Szabó, "Winkelmessung und die Anfänge der Trigonometrie," *Acta Antiqua Academiae Scientarum Hungricae* 24 (1976) 163–72.

[178] Several codices fill in Ptolemy's description here and elsewhere. At 102.26 they add "or the semicircle," which more accurately describes the segment AB. At 102.29 they add "and the 6 of the semicircle to the 2 of the arc CB"; since Ptolemy himself (102.6) described the segment BC as 1/6 of the whole circle, this would create three examples of the triple ratio, the diapason plus diapente. Still another is at 102.31, "And the 8 of the arc ABC to the 2 of the periphery CB," which makes two examples of the quadruple ratio, the homophone of the double diapason.

Diapason
ABCD to AB
ABC to AC
AB to AD

Diapente
ABCD to ABC
ABD to AB
AB to AC

Diatessaron
ABCD to ABD
ABC to AB
AC to AD



Diapason plus Diapente
ABCD to AC
ABD to AD

Double Diapason
ABCD to AD

Diapason plus Diatessaron
ABC to AD

Wholetone
ABD to ABC

Circle = 12	ABC = 4
ABD = 9	AD = 3
ABC = 8	CB = 2
AB = 6	DC = 1

[103.5] From the same points[179] also the diapente of the first consonances[180] can be arranged in the triangular position, the diatessaron in the quadrangular, and the whole tone in the duodecangular.[181] [103.8] For this reason the circle to the hemicircle AB makes the double ratio, this to the arc AC of the triangle makes the sesquialter,[182] and this then to the arc AD of the quadrangle makes the sesquitertian. [103.10] The difference between these is like a wholetone, the arc CD, consisting of a twelfth part of the circle. And therefore with fair reason nature organized the circle of the zodiac[183] in twelve parts since also the perfect system of the double diapason consists of nearly twelve whole tones,[184] and the whole tone interval fits the twelfth of the circle.

[179] My noun supplement to Ptolemy's adjective αὐτῶν.

[180] Although one should not forget the uncertainties about the state of authorship or relative date of these last chapters, Ptolemy does not contradict himself by labeling the diapente the first consonance here. He mentions it, along with the diatessaron, as one of the first consonances. He lists the diapente first here because in the duodecangular conception the diapente with four species is represented by the triangle of three sides, and the diatessaron with three species is inversely represented by the quadrangle of four. Cf. 104.12–17 and Barbera, *Pythagorean Mathematics* 335–36.

[181] A common term associate with the zodiac for the astronomer Ptolemy; cf. Toomer 20–21.

[182] ἡμιόλιον ('sesquialter'); following Höeg (1930) 658, n. 2, Düring corrects the ἡμικύκλιον ('semicircular') of his original text.

[183] "The circle of the zodiac" is the ecliptic.; cf. n. 154.

[184] Approximating the Aristoxenian view; cf. 25.1f.

[104.2] But remarkable also is that the signs through one twelfth part of those in the zodiac are not consonant but only in the genus of the emmelic,[185] and yet those of five of the twelfth parts on the contrary are in the genus of the ecmelic. [104.5] They are 'unconnected' in name and indeed in function.[186]. Wherefore to the two arcs under the straight line subtending the twelfth part the circle makes the 12:1 ratio or 12:11, which are foreign to the consonances but no longer to the emmelic. [104.8-9] To the two arcs from the straight line subtending the five twelfth parts, the circle makes the 12:5 or 12:7, being foreign to both the consonances and the emmelic; for each is neither superparticular nor multiple, nor is it composed from anything appropriate to the consonances. [104.12-13] In all the congruences through the twelfth-part signs of the circle, only three species of quadrangles are rendered, the same number as those in the consonance of the diatessaron, and only four of the triangle, the same number as in the consonance of the diapente, for these consonances alone happen to be non-composite.

III.10 - *That Succession in Notes Resembles the Longitudinal Movement of the Stars*

[104.18] Let this terminate our observations of that circular movement in both harmonic entities[187] and what are commonly[188] called both consonances and dissonances of the figures. [104.20] What we must look at next are the first differences of the celestial movements. There are three of these, that of forwards or backwards in longitude,[189] by which differences are completed from east to west and the opposite, [104.23-4] that of falling or rising and altitude, by which movements are in greater apogee or perigee, and then that in the transverse direction and latitude, by which the passings[190] happen to be more northerly or southerly.

[185] As are those in the 9:8 ratio.

[186] Four of the twelve parts comprise the diapason plus diapente, and six of the twelve comprise the diapason (102.15f.). The ratio containing five of the twelve (8:7) is ecmelic; cf. 104.8f.

[187] Simply 'harmonies' (τὰς ἁρμονίας) in Greek, this noun refers to both the musical and cosmic variety. The scholiast perhaps more precisely labels them the 'musical and spherical' (τήν τε μουσικὴν καὶ τὴν σφαιρικήν).

[188] Or, 'in common' (to both).

[189] Ptolemy uses this and the following two chapters to investigate each of these three differences in celestial movements. The scholiast refers us to Arist. *De caelo* 2.2 [284b6-286a2], where Aristotle discusses a more astrophysical question, the existence of a heavenly top, bottom, left, and right.

[190] This term 'passing' (τὰς παρόδους) when used with 'circle' normally refers to the meridian. It has its origins in non-technical geography, e.g. Xen. *An.* 4.7.4.

[104.27] The first, of longitude, we might quite reasonably compare to the transition into simply higher or lower notes, for succession[191] is similar in each kind of movement. That of eastern and western parts we might compare to the lowest tunings, and those at the meridian to the highest.[192]

[105.4] This is because the risings and settings[193] contain the beginning and end of appearance, the former as from the invisible, the latter as to the invisible; the lowest and highest tunings contain the beginning and end of the voice, the former as from silence, the latter as to silence, [105.8] since the lowest is nearest the disappearance of the voice, the highest is farthest from it.[194] Wherefore those who exercise their voices begin to make music from the lowest notes and come to a halt by ending on them.[195]

[105.11] Those near the meridian, since they are farthest from their disappearances, should be arranged at[196] the highest notes since these also stand farthest away from silence. [105.13] And then, since the low loci make the lowest sounds, the high the highest, for this reason we say the lowest tunings are carried from the sides,[197] the highest from the temples.

[105.17] The lowest are also the risings and settings, the highest those at the meridian. The latter are properly compared to the highest notes, the former to the lowest, [105.19] so that the stellar movements at the meridian are closely related to the transitions of notes from the lower tunings to the higher, while those again moving away from the meridian are related to the transitions of notes[198] from the higher to the lower.

[191] That is, the means of progression. The Greek term here is ἐφεξῆς, but *agôgê* (n. 45) is more commonly employed to signify stepwise musical movement.

[192] While this analogy makes some sense and Ptolemy elaborates upon it in the rest of the chapter, it is interesting that he viewed the arrangement of the heavenly canopy in this way rather than locating the Dorian mese at the meridian and the Hypodorian and Mixolydian at the extremes. Cf. 101.10f. and III.12 (106.23ff.)

[193] The text of this phrase has survived with many variations, but the variant I have translated here (ἀνατολαὶ καὶ δύσεις), derived from a second hand in Vaticanus gr. 191, seems quite logical and works well.

[194] Unlike Düring's text, a number of manuscripts have, "while the lowest [and highest] tunings contain the beginning and end of the voice, the former as from silence, the latter as to silence, since the lowest is nearest the disappearance of the voice, the highest is farthest from it." The complex comparison Ptolemy attempts is that between a) the rising of heavenly bodies over the horizon, which is the beginning of appearances (*from* the invisible), and the lowest tunings, which bring the voice from silence; and b) the setting of heavenly bodies over the horizon, which is the end of appearance (*to* the invisible), and the highest tunings. Düring's text does not make the comparison complete, and it would leave the reader with the impression that Ptolemy believed the voice could "disappear" only at the lower end of the system.

[195] This exercising with ascending and descending scales (but probably not arpeggios) seems to be remarkably similar to modern usage. Cf. the surviving ancient "exercises" in the *Anon. Bell.* 1.2 and 3.77-104 (Najock); cf. (Egert Pöhlmann, *Denkmäler altgriechischer Musik* (Nürnberg 1970) 36–38). But these are not necessarily "warm-up" exercises. See Annie Bélis, "Un Nouveau Document Musical," *BCH* 108 (1984) 99–109.

[196] 'Or, comparable with.'

[197] 'The diaphragm,' in our understanding and parlance; cf. Bry. II.6 (168.31f.).

[198] Any mention of the succession or progression—cf. n. 190—is absent.

III.11 - *How the Stellar Movement in Altitude Compares with the Harmonic Genera*

[105.23] The second of the differences, that of altitude, we will find similar to[199] what are called the genera in harmonics,[200] for the latter again contains three kinds—the enharmonic, chromatic, and diatonic — separated by the quantity of the ratios in the tetrachords, [105.27] while the latter has three kinds of distances—the smallest,[201] medium, and greatest—measured by the quantity of courses.[202] [106.2] Those passages in the middle position which hold quite close to the middle courses might properly be compared to the chromatic genera since in them the lichanoi divide the middles of tetrachords.[203]

[106.5] Those with the smallest movements, whether they follow distances of greater apogee or whether they follow distances of greater perigee, might be compared to the enharmonic since the two intervals of the remainder make an interval smaller in the figure of the so-called pyknon.[204] [105.8] Those with the greatest movements, again whether they follow distances of greater apogee or whether they follow distances of greater perigee, might be compared to the diatonic since the two intervals of their remainder are in no way smaller in the figure of the so-called apyknon. [105.12] Lastly, in general the enharmonic genus and the smallest of the courses is the systaltic, the former in melody, the latter in speed. The diatonic genus and the greatest of courses is the diastaltic,[205] and the

[199] The codices have ἐν ('in') here, but Düring, following Wallis, correctly deletes. Perhaps at some point it was confused with the ἐν before ἁρμονία (as in 'enharmonic'); this latter preposition is omitted in several branches of the manuscript tradition.

[200] Ptolemy's use of the singular form *harmonia* (ἐν ἁρμονίᾳ) is unexpected here; cf. Düring 276, but Ptolemy uses the same prepositional phrase in the chapter title; cf. III.10 (104.21f.). Theon of Smyrna 3.11 (89 (Lawlor)) offers a different description of the harmony of the stars. D. R. Dicks, "A Mistranslation in Manitius," *JHS* 103 (1983) 137, points out that eight of the *Almagest*'s thirteen books concern the distances of the sun, moon, and fixed stars in relation to the earth.

[201] The scholiast explains, "The star moving along the epicycle at apogee moves the shortest distance, that at perigee the greatest."

[202] This passage should be compared to that at III.6, where Ptolemy compared the harmonic genera with the first virtues.

[203] Cf. I.16; the chromatic is the middle genera—both musically and then with the soul and virtues. It is rare in any ancient Greek music theory for a generic analysis to begin with the chromatic.

[204] The scholiast explains, "It was demonstrated in the fifteenth chapter of the first book on the species of the pykna that in the tetrachords the one ratio is greater than the two remaining and successive ones." Cf. 33.22f. and 68.27f. This seems to be the sole reason for Ptolemy's comparison here.

[205] Most of the codices and Düring have 'diastatic' instead of 'diastaltic,' but the correct reading is the latter. For this and for the systaltic and diastaltic as larger concepts in ancient Greek music theory (particularly as related to ancient medicine), see Jon Solomon, "The Diastaltic Ethos," *Classical Philology* 76 (1981) 93–100, and cf. Egon Wellesz, *A History of*

chromatic genus and the middle courses have a place more or less between these extremes.[206]

III.12 - *That Modulations of Tonoi Are Like Stellar Crossings in Latitude*

[106.18] The third and remaining difference of heavenly movements, I mean that of latitude,[207] we must apply to modulations in tonoi, since with a change in tonoi there is then no shift at all in the genera,[208] [106.21] nor, on the other hand, is any perceptible anomaly in the courses sensed with the crossings in latitude.[209]

[106.23] In connection with these movements, the Dorian tonos, being the most central of the tonoi, we must compare to the middle crossings in latitude, those positioned towards the equinox, as it were, in each sphere.[210] [106.25-107.1] The Mixolydian and Hypodorian, the extremes, must be compared to those thought of as the most northerly[211] and southerly towards the solstice. [107.2] The remaining four tonoi, which lie between those just mentioned, must be compared to those falling in between the parallels of the solstices and equinox; these four exist in the division into twelve of the ecliptic[212] comparable to the twelve parts of the zodiac.

[107.6] For each one of the solstitial signs makes as it were a parallel. But the two signs standing equally apart from each of these to the other make again one and the same, so that five syzygies exist in the twelve-part distances as do five parallels of these. [107.10] All of these with the solstices[213] are seven in number, the same number as the modulations of tonoi.

[107.12] Those tonoi which on account of their higher melody are higher than the Dorian are arranged as if in summer, with the crossings at the

*Byzantine Music and Hymnography*² (Oxford 1961) 51. I find now that I was anticipated in part by Winnington-Ingram, *Mode* 54, n. 1.

Ptolemy does not elaborate here, which is a disappointment. This would have provided the opportunity for him to compare not just musical harmony with the cosmos or the soul but with both. Warren D. Anderson, "Musical Developments in the School of Aristotle," *Research Chronicle* 16 (1980) 94f., discusses Theophrastus' (lost) writings on this and related subjects.

[206] The comparison here is somewhat forced, for Ptolemy offers no technical term to describe what lies between the systaltic and diastaltic.

[207] Cf. III.10 (104.21f.), with n. 188, and Toomer 19.

[208] Cf. II.6 (55.7f.).

[209] Cf. *Tetr.* 109 (232 (Robbins)) and Vettius Valens 1.21 (37.9 (Kroll)), who employs this term in the sense of 'rotation' to calculate the influence of the sun and stars.

[210] The spheres of music and the heavens; cf. n. 186.

[211] For 'most northerly' (βορειοτάταις), Düring mistakenly printed 'lowest' (βαρυτάταις), a *facilior* musical term. Cf. Höeg (1930) 658, n. 2.

[212] In the Greek text this word (κύκλων) is given in the plural, but Düring (in the apparatus) is correct to prefer the singular κύκλου. (His reference to λοξός (136) is incorrect: read 5 for 15. His reference to this (277) has itself a misprint: 108.5 for 107.5.)

[213] Ptolemy mentions the two solstices without fanfare at 106.4. Cf. also the divisions in III.9.

raised pole, that is, where the North Pole rises,[214] and with those at the arctic, where the south is at the opposite. [107.15] Those which on account of their lower melody are lower than the Dorian are arranged as if in winter, with the crossings at the invisible pole, that is, where the South Pole rises towards those at the arctic,[215] where the North is at the opposite.

III.13 - *On the Similarity of the Tetrachords and the Aspects of the Sun*

[107.19] The remaining arrangement of both the tetrachords and whole tones in the perfect system stands clear for the remaining arrangement of the aspects of the sun.[216] The whole tones of disjunction fit the distances from occultations to appearances,[217] [108.1] and to oppositions[218] or full moons, while the notes conjoining the syzygies[219] of the two tetrachords, the hypate of the meson tetrachord and the nete of the diezeugmenon tetrachord, fit the quadrangular positions[220] in each of these, as with the moon in its half phase. [108.5] Therefore, the aspect from each rising and at the first lunar crescent can be compared to the hypaton tetrachord, for it is a common beginning of the rising and the lowest notes.[221]

[108.8] What comes next after this at the first gibbous phase can be compared to the meson tetrachord. Again, that which is from the opposite rising, as with Mercury and Venus, or from the opposition with the three remaining planets, or from the waning, [108.11] as with the moon at the second gibbous phase,[222] is compared to the diezeugmenon tetrachord,

[214] Cf. *Alm.* 2.11 (156.16 (Heiberg)).

[215] For the polar regions, cf. Manilius *Astronomica* 1.444–55.

[216] Not unexpectedly, Ptolemy progresses in scope from comparing the harmonic genera (III.11), the tonoi (III.12), and now the perfect system. Cf. *Tetr.* 1, Philodemus *De mus.* 73 (Kemke), and *Alm.* 8.4 (II.186 H). Newton, *Crimes* 208–209, summarizes (without polemics) the state of knowledge about the sun (and moon) in the second century A.D.

[217] The reference here is to the heliacal risings; cf. *Alm.* 8, esp. 186.11 and 186.14, and *Tetr.* 1.8 (44 (Robbins)).

[218] Cf. Arist. *Mete.* 342ᵇ34, Nic. *Ther.* 122 (on the rising of the Pleiades), *Tetr.* 212 (210 (Robbins)), and *Alm.* 8.6. The scholiast adds, "He speaks of the position of opposition in the three controlling planets, Saturn, Jupiter, and Ares, when they appear at sunset opposite the eastern horizon. He misuses the term when describing [at 108.8] the opposition of Mercury and Venus in saying that they themselves appear at sunset opposite them [the controlling planets] on the western horizon, for these two planets are never in a position in opposition to the sun."

[219] E.g. the conjunction of the meson and diezeugmenon tetrachords.

[220] Cf. 103.5f.

[221] As stated in 105.4f, although here the lowest notes which Ptolemy described vaguely in III.10 now belong specifically to the hypaton tetrachord of the perfect system.

[222] With Mercury it can cause blindness, with Venus sterility; cf. *Tetr.* 149 (322 (Robbins)). Columella *Rust.* 2.10.10 (II.162 (Boyd)), in describing the planting of lentils, inserts this Greek term for 'waining' (ἀπόκρουσιν) into his Latin treatise. On ἀμφίκυρτος ('gibbous'), cf. Arist. *Cael.* 291ᵇ20.

forming[223] both the opposite position and the diapason homophone at the first crescent and the hypaton tetrachord.[224]

[108.15] The next aspect after those up to the occultation at the second crescent is compared to the hyperbolaion tetrachord, it, too, making at the first gibbous phase and the meson tetrachord both the opposite position and the diapason homophone. [108.18] The distances from the occultations to the appearances and in the oppositions from the evening risings to the dawn settings or in the fullness of the moon are nearest[225] the twelfth part, just as the whole tones of the disjunction.

[109.2] Of the rest,[226] those in each of the four aspects are approximately 2 plus 1/2 twelfth parts, as also each of the four tetrachords is closest to 2 and 1/2 tones. Finally, with the moon those two aspects of opposition both fill out one in the appearance of the whole, [109.6] just as also the notes of the diapason create a unity since they make a similar impression.[227]

III.14 - *By What First Numbers Might the Fixed Notes of the Perfect System Be Compared to the First Spheres in the Cosmos*

[109.8] We now know by such comparisons what particularly applies in common to the differences between the emmelic[228] and the heavenly motions.[229] It remains to examine as well what is plausibly observed in each in regard to the existent [109.12] [numbers[230] and the ratios found in them.

[223]J. F. Mountford, in his review of Düring, *CR* 44 (1930) 242–43, found that Vat. gr. 197 supports the reading ποιῶν. In this cumbersome syntax this 'forming' is absent in the codices, although it has a not unanimously attested parallel in line 16.

[224] The entire chapter to this point has been one Greek sentence.

[225] For the reading ἔγγιστα, cf. 109.2 and Düring (1934) 279.

[226] Kepler in the appendix to Book V of *Harmonices Mundi* (Linz 1619) 249–54, takes issue with these last few chapters of Ptolemy's treatise.; see the text in Düring, *Ptolemaios und Porphyrios* 279–80.

[227] An alternate description of the heavenly proportions is offered by Plutarch (1028F), who, like Ptolemy in this chapter, compares the consonances to the four seasons, where the spring forms a diatessaron with autumn, a diapente with winter, and a diapason with summer; cf. O. Neugebauer, "On Two Astronomical Passages in Plutarch's *De animae procreatione in Timaeo*," *AJPh* 43 (1942) 455–59.

[228] 'Emmelic' here is not the technical term referring to intervals less consonant than those strictly labeled as consonances and the homophones but a more general term equivalent to 'musical.' Cf. II.6 (55.22) with note 102.

[229] The concept of the harmony of the "spheres" was not possible before Eudoxus' mathematical description of the heavenly bodies; see Burkert 351, n. 1, and Sir Thomas L. Heath, *Aristarchus of Samos* (Oxford 1913) 193–211.

[230] The remainder of the treatise is a supplement offered by Nicephorus Gregoras (1295–c. 1359), the Byzantine historian and polymath. Nicephorus' credentials for supplementing Ptolemy were substantial. As a result of his training by Theodorus Metochites, he had proposed several calendar reforms and written two libelli on the construction of the astrolabe.

For when the whole circle is cut into 360 parts, when the moon is diametrically opposite the sun or some of the planets, [109.14] then the distance between them consists of 180 parts along the circular arc we are considering, for if these are doubled they yield the number of the whole circle, that is 360.[231]

[109.17] But when they stand in triangular aspect to each other, then we say that they are at an interval of 120 from each other, for when they are tripled they yield the number of the whole circle, that is 360. [109.19] And when again they stand in quadrangular aspect to each other, then we say they are at a distance of 90 along the circumference, for again four times the 90 are likewise 360; [109.22] and when again in hexangular, then we say they are at a distance of 60 parts, for six times the 60 is once more 360.[232]

[109.23] When these are compared to the musical perfect system, the fixed notes will be compared to the position of these arithmetical intervals as follows: the proslambanomenos to the position of the 180 parts, the hypate meson to the position of the 120 parts, [109.27] the nete diezeugmenon to the position of the 90 parts, the nete hyperbolaion to the position of the 60 parts, and the two fixed whole tones which contain the disjunction to the position from which begin the aforementioned distances,[233] [102.29] or where the position is established for either the sun or some other planet, from

There is ample evidence for assuming that these chapters did exist originally. Macrobius (1.19–20) refers to a Ptolemaic work on harmonics in three books, and Vat. gr. 176 contains a version of a scholion saying that Nicephorus corrected, supplemented, and glossed its version of Book III. Another scholion says that an independent table of contents existed. In addition, Mountford, "Harmonics" 77–79, the first to pay attention to the passage in this century—cf. Boll 100–101—observed that Ptolemy had undoubtedly originally planned these last three chapters; at least the beginning of III.14 exists in twelfth and thirteenth-century manuscripts.

As a pupil of Metochites, Nicephorus had thorough training in mathematics and astronomy, yet his efforts were ridiculed by his long-standing, Italian adversary Barlaam; cf. Christian Hannick, "Byzantinishce Musik," in H. Hunger, ed., *Handbuch der Altertumswissenschaft: Die Hochsprachliche Profane Literatur der Byzantiner* II (Munich 1978) 191–92. (Barlaam's statements, which confound Mountford, "Harmonics" 79, n. 23, were no doubt ironic.) J. Franz, *De musicis graecis commentatio* (Berlin 1840) 1–23, discusses the infamous letter (as found in Neap. gr. III.C.3); and Hans-Veit Beyer, "Eine Chronologie der Lebensgeschichte des Nicephorus Gregoras," *Jahrbuch für Österreichisches Byzantinistik* 27 (1978) 134–35, discusses the feud of the summer of 1332 and the following winter.

Lukas Richter, "Antike Überlieferungen in der byzantinischen Musiktheorie," *Deutsches Jahrbuch der Musikwissenschaft für 1961* (1962) 92–93 and 104–105, puts Nicephorus into historical perspective along with Bryennius, Pachymeres, Metochites, Argyros, and Barlaam. Wilson, *Scholars* 266–67, offers bibliography. See also, Düring lxxxiiif., Wallis *ad loc.*, and Palisca 86 and 122–23 (because Giovanni Augio's source ended here).

[231] Cf. 109.4 for the non-arithmetical description of the same angle. For the angular divisions in general, cf. Ptol. *Tetr.* 1.13. That the author makes no mention of the "nested shells" here is notable and typical of the lack of sophisticated astronomical detail throughout these last eight chapters; cf. R. C. Riddell, "Parameter Disposition in pre-Newtonian Planetary Theories," *Archive for the History of the Exact Sciences* 23 (1980) 112, n. 71.

[232] Cf. 103.5f.

[233] Cf. 108.15 and III.13 *passim*. Levin *Nicomachus* 65–67, would relate Ptolemy's ratios here to her tripartite divisions of the diatonic octave, the acoustical octave (2:1), and the metaphysical octave (12:9:8:6).

which the disjunction occurs for each of the measurements of the distances of the circle.]

III.15 - *How the Ratios of Their Motions Are Calculated By the Numbers*[234]

[110.3] Since these things are so, the number of the quadrangular distance, 90, taken halfway between 120 parts of the triangular distance and the 60 of the hexagonal makes two intervals of ratios, both the sesquialter and the sesquitertian according to the similarity of the two first consonances of the harmonia, the diapente and diatessaron. [110.7] Just as the two first consonances in music, the diapente and diatessaron, when combined form the diapason homophone, so also then the intervals of the two aforementioned ratios, the sesquialter, of course, and the sesquitertian, when combined make the duple ratio analogous to the diapason homophone.[235]

[110.11] When the number of the 360 parts of the whole circle is compared to these, it makes with the 90 the quadrangular ratio, that which is analogous to the double diapason of the perfect system in music.[236] [110.13] In another way, from those twelve parts of the zodiac one might find in analysis the similar analogy, for 120 parts contain an interval of four twelve parts, 90 an interval of three, and 60 of two. [110.15] Of these, the three lying in the middle in relation to the four make the sesquitertian ratio, and to the two they make[237] the sesquialter ratio, and from both of these the duple ratio is composed, that is 4:2.

[110.17] The number of the twelve zodiac signs, the whole period of the circle, compared to these makes itself with three the quadruple ratio necessarily concordant with the double diapason of the perfect system in music. [110.20] When we think of polygons—triangular, of course, quadrangular, and hexangular figures—it will surely follow also both from the angles themselves and from whatever else is at hand that we can show similarly the ratios characteristic in harmonics. [110.23] I think for the most pressing aspects of the subject the preceding should suffice. More information we have relegated to more leisurely hours.[238]

---

[234] The entire chapter is again a supplement; cf. n. 227.

[235] As stated earlier at I.5 (11.21f.)

[236] Cf. 101.27f. and 103.5f.

[237] Cf. Alexanderson 18; Düring omitted ten words supplied here from Wallis.

[238] An odd statement to make in a supplement; it is perhaps derivative of (or even evidence for) Ptolemy's original sketches. Düring, *Ptolemaios und Porphyrios* 280–81, in a valiant albeit speculative attempt at a reconstruction of chapters 14 and 15, assumes they included important charts or tables. Cf. Karl von Jan, "Die Harmonie der Sphären," *Philologus* 52 (1892) 13.

III.16 - *How the Combinations of the Planets Should be Compared to Those of the Notes*[239]

[110.25] No one should think that the note of Jupiter forms a consonance with each of the lights, but Venus to the light of the moon alone since the whole tone does not belong to the ratio of the consonance.[240] For this belongs to the lunar condition,[241] [111.1] while that of Jupiter is considered in relation to the solar. Since each note of the Destroyers[242] forms a consonance of the diatessaron with each of the Benefactors—that of the nete hyperbolaion of Saturn to that of the nete diezeugmenon of Jupiter, [111.4-5] and that of the nete synemmenon of Mars to the mese of Venus[243]— [111.6]

---

[239] Many manuscripts include this chapter as part of chapter 9. Citing Macrobius 1.19–20, J. F. Mountford, review of Düring, *CR* 44 (1930) 242–43, thinks this chapter to be genuine; cf. Düring (*Ptolemaios und Porphyrios* 281–4. Although they are quite tempting, Macrobius' comments prove only that Ptolemy did indeed write three books on harmonics, not specifically that he wrote this chapter, or that these are his words, or, if they are, that he completed the chapter. The arguments by Luigi Scarpa, ed., *Macrobii Ambrosii Theodosii Commentariorum in Somnium Scipionis Libri duo* (Padua 1981) 447, that Ptolemy and Macrobius refer to the same pairs of celestial bodies, is irrelevant since Macrobius cites Ptolemy only as support for the harmonic nature of the universe and not as the source for these particular configurations.

A scholion referring to all of chapter 16 parallels information given in Ptol. *Tetr.* 7 (42 (Robbins)): "The astrologers make the sun and Jupiter the day-stars on account of the heat of the day, their effectiveness, and their masculinity; they make the moon and Venus night-stars on account of the coolness of night, their slack, and their femininity. The two destructive ones, that is Saturn and Ares, they set apart from the day and night stars, for there is good and bad mixed in with them so as to cause some decrease in their evil. Wherefore they have allotted Saturn, as cold-producing, to the heat of the day and made it more or less a colleague of the sun and Jupiter. Ares the dry they give to the cold of the night and make him with Mercury a participant in day and night." Ptol. *Tetr.* 1.4-7 (34-43 (Robbins)) bears a close resemblance. Cf. also, Aristides Quintilianus 3.21 (120-23 (Winnington-Ingram; 190-2 (Mathiesen)).

[240] The abruptness with which "Ptolemy" introduces this type of technical material (and the new term, 'lights' - φώτων), particularly in contrast with the care he took in introducing technical material in Books I and II and even earlier in Book III, leads one to observe, among other possibilities, how the Byzantine supplements in III.14 and 15 are woefully lacking. Barlaam complained to Nicephorus Gregoras about this, and Wallis again recognized in the seventeenth century that the diction in this chapter differed from that in the others. H. L. Mead, "The Methodology of Ptolemaic Astronomy," *Laval Theologique et Philosophique* 31 (1975) 65, forming a model tangential to the process he observes in Aquinas, describes three modes in Ptolemy's process, those of observation, instrumentation, and reason resulting in hypothesis. All three are clearly lacking here.

[241] Or 'sect'; cf. *Tetr.* 1.7 (42, n. 1 (Robbins)). Vettius Valens 1.1 (1.13 (Kroll)) in introducing the nature of the heavens uses this term (αἱρέσεως) to describe the sun; the lunar description comes just a few phrases later.

[242] Cf. Ptol. *Tetr.* 1.5 (38 (Robbins)).

[243] A. Bouché-Leclercq, *L'Astrologie Grecque* (Paris 1899) 109, explains that the sun in the median position lies between cold and hot, and therefore it has the positive benefits of Jupiter and Venus, the negative of Saturn and Mars. Theon of Smyrna 3.12-25 (89-100 (Lawlor)), following Alexander of Aetolia, presents a different arrangement:

it follows that Saturn belongs more to the solar condition, Mars to the lunar. Wherefore it happens that of the aspects of Saturn toward Jupiter all are beneficial while those of Saturn toward the sun are only triangular, so that they are more consonant than the rest. [111.10] Similarly, both those of Mars to Venus and the moon are not all good but only the triangular.[244] On the contrary, those of Saturn toward the moon and Venus[245] are all mean, those of Mars towards the sun and Jupiter all unstable.[246]

stars	-	nete
Saturn	-	
Jupiter	-	
Mars	-	
Sun	-	mese
Venus	-	
Mercury	-	
moon	-	
Earth	-	hypate

Cf. T. H. Martin, *Theonis Smyrnaei Platonici Liber de Astronomia* (Groningen 1971). Nic. *Ench.* 241.18f., however, has Saturn (Kronos) as hypate, the sun again at mese, but the moon at nete. Boethius uses almost the same arrangement except that he reverses the interior planets; his order is the same as that in Pliny *NH* 2.20.84, except that he adds the zodiac at the greatest apogee. The versions in Martianus Capella (*De nupt.* 2.169-199) and Censorinus (*De die natali* 13.3-5) stem from the same traditions. Karl von Jan, "Die Harmonie der Spharen," *Philologus* 52 (1893) 13-37, discusses these and other celestial harmonic arrangements. M. L. West, "The Midnight Planet," *JHS* 100 (1980) 206, discusses harmony in relation to the heptachord, for which see also Théodore Reinach, "La Musique des Sphères," *REG* 13 (1900) 432-49.

244 Ptolemy elsewhere considers triangularity particularly essential because it represents geometrically the most consonant formation; cf. *Tetr.* 1.18 (82-83 (Robbins)).

245 O. Neugebauer, "On the Allegedly Heliocentric Theory of Venus by Heracleides Ponticus," *AJPh* 93 (1972) 600-1 (reprint: *Astronomy and History: Selected Essays* (New York 1983) 370-1), puts this passage into a more scientific and mathematical perspective, thus helping us maintain awareness of what Ptolemy would have known.

246 With Saturn the nete hyperbolaion, Jupiter the nete diezeugmenon, Mars the nete synemmenon, the sun paramese, Venus the mese, and the moon hypate meson; cf. Wallis (1699) 273. On Mars as nete synemmenon, see Höeg, review of Düring, 657, and Jan, "Die Harmonie der Spharen," *Philologus* 52 (1893) 30-35. Robert Lachmann, *Musik des Orients* (Breslau 1929) 54-64, offers a parallel from India, where each raga is associated with a particular god or goddess. For Ptolemy, however, the planets have eclipsed any particular divine, mythical, or anthropomorphic characteristics. Had Ptolemy completed this passage, one might expect here greater detail relating to heat and cold, dry and moist; cf. J. B. Brackenridge (and Mary Ann Rossi, trans.), "Johannes Kepler's *On the More Certain Fundamentals of Astrology*," *Proceedings of the American Philosophical Society* 123 (1979) 95-99, where of the extreme planets Saturn is excessively humid and deficient in heat, Mars excessively hot and dry. Egon Wellesz, *A History of Byzantine Music and Hymnography*[2] (Oxford 1961) 48, tries to equate the Pythagorean tetraktys with these same opposites inherent in Empedocles' four elements; cf. F. M. Cornford, "Mysticism and Science in the Pythagorean Tradition," *CQ* 16 (1922) 137-38, who suggests that Philolaus had a number system which accommodated the system of Empedocles.

BIBLIOGRAPHY

Abdy Williams, C. F. "The Notes Mese and Hypate in Greek Music," *Classical Review* n. s. 12 (1898) 98–100.

Abert, H. *Die Lehre vom Ethos in der griechischen Musik.* Leipzig 1899; reprint: Wiesbaden 1968.

Al-Daffa, Ali A., and John J. Stroyls. *Studies in the Exact Sciences in Medieval Islam.* Dhahran [Saudi Arabia] 1984.

Alexanderson, Bengt. *Textual Remarks on Ptolemy's Harmonica and Porphyry's Commentary.* Göteborg 1969.

Anderson, Warren D. *Ethos and Education in Greek Music.* Cambridge MA 1955.

____. "Musical Developments in the School of Aristotle." *Research Chronicle* 16 (1980) 78–98.

Auda, Antoine. *Les modes et les tons de la musique et spécialement de la musique médiévale.* Brussels 1930; reprint: Hildesheim 1979.

Aujac, Germaine. "Le zodiaque dans l'astronomie grecque." *Revue d'Histoire des Sciences et de leurs Applications* 33 (1980) 3–32.

Babb, Jr., Stanley E. "Accuracy of Planetary Theories, Particularly for Mars." *Isis* 68 (1977) 426–34.

Barbera, André. "Arithmetic and Geometric Divisions of the Tetrachord." *Journal of Music Theory* 21 (1977) 294–323.

____. "The Consonant Eleventh and the Expansion of the Musical Tetractys: A Study of Ancient Pythagoreanism." *Journal of Music Theory* 28 (1984) 191–223.

____. *The Persistence of Pythagorean Mathematics in Ancient Musical Thought.* Ph.D. diss., University of North Carolina 1980.

Barbour, J. M. "The Principles of Greek Notation." *Journal of the American Musicological Society* 13 (1960) 1–17.

Barker, Andrew. *Greek Musical Writings: I, The Musician and His Art.* Cambridge 1984.

____. *Greek Musical Writings: II, Harmonic and Acoustic Theory.* Cambridge 1989.

____. "Ptolemy's Pythagoreans, Archytas, and Plato's Conception of Mathematics." *Phronesis* 39 (1994) 113-35.

____. "Reason and Perception in Ptolemy's Harmonics." In R. Wallace and B. MacLachlan, eds. *Harmonia Mundi: Musica e filosofia nell' Antichità* (Rome 1991) 104–30.

Barnes, Jonathan. *Early Greek Philosophy.* New York 1987.

Beaton, Roderick. "Modes and Roads: Factors of Change and Continuity in Greek Musical Tradition." *Annual of the British School at Athens* 75 (1980) 1–11.

Becker, Heinz. *Zur Entwicklungsgeschichte der antiken und mittelalterlichen Rohrblattinstrumente*, Schriftenreihe des Musikwissenschaftlichen Instituts der Universität Hamburg, Band 4. Hamburg 1966.

Becker, Oskar. *Das mathematische Denken der Antike*. Göttingen 1957.

____. "Frühgriechische Mathematik und Musiklehre." *Archiv für Musikwissenschaft* 14 (1957) 156–62.

Bélis, Annie. "Les 'nuances' dans le Traite d'Harmonique d'Aristoxene de Tarente." *Revue des Études Grecque* 95 (1982) 54–73.

____. "Un Nouveau Document Musical." *Bulletin des Correspondance Hellénique* 108 (1984) 99–109.

____. "Auloi grecs du Louvre." *Bulletin de correspondance Hellénique* 108 (1984) 14–22.

Berkowitz, Luci, and Karl A. Squitier. *Canon of Greek Authors and Works*[2]. New York and Oxford 1986.

Bertier, Janine (trans.). *Nicomaque de Gérase: Introduction Arithmétique*. Paris 1978.

Beyer, Hans-Veit. "Eine Chronologie der Lebensgeschichte des Nikephoros Gregoras." *Jahrbuch für Österreichisches Byzantinistik* 27 (1978) 134–35.

Blumenberg, Hans (Robert M. Wallace, trans.). *The Genesis of the Copernican World*. Cambridge 1987.

Bojesen, Ernest Frederick. "De tonis sive harmoniis graecorum commentatio." *Indbydelsesskrift til Sorö Academies Skoles aarlige hovedexamen i Juli 1843* (Copenhagen 1843) 12.

Boll, Franz. "Das Epigram des Claudius Ptolemaus." *Socrates* 9 (1921) 2–12.

____. *Studien über Claudius Ptolemaeus. Ein Beitrag zur Geschichte der griechischen Philosophie und Astrologie. Jahrbuch für classische Philologie*. Supplementband 21. Leipzig 1894.

Bolton, J. D. P. Review of H. Thesleff, *An Introduction to the Pythagorean Writings of the Hellenistic Period. Classical Review* n.s. 14 (1964) 333–35.

Borthwick, E. K. Review of Alexanderson, *Classical Review* n.s. 21 (1971) 367–68.

Bouché-Leclerq, A. *L'Astrologie Grecque*. Paris 1899.

Bowen, A. C. "The Foundations of Early Pythagorean Harmonic Science: Archytas, Fragment 1." *Ancient Philosophy* 2 (1982) 79–104.

____. "The Minor Sixth (8:5) in Early Greek Harmonic Science." *The American Journal of Philology* 99 (1978) 501–506.

____ and William R. Bowen, "The Translator as Interpreter: Euclid's *Sectio Canonis* and Ptolemy's *Harmonica* in the Latin Tradition," in M. R. Maniates, ed., *Music Discourses from Classical to Early Modern Times*. Toronto 1997.

Bower, Calvin M., trans. *Ancius Manlius Severinus Boethius, Fundamentals of Music*. New Haven and London 1989.

____. "Boethius and Nicomachus: An Essay Concerning the Sources of *De institutione musica*." *Vivarium* 16 (1978) 1–45.

____. "The Role of Boethius' *De institutione musica* in the Speculative Tradition of Western Musical Thought." In Michael Masi, ed. *Boethius and the Liberal Arts: A Collection of Essays*. Berne 1981.

Brackenridge, J. B. (with Mary Ann Rossi, trans.). "Johannes Kepler's On the More Certain Fundamentals of Astrology." *Proceedings of the American Philosophical Society* 123 (1979) 85–116.

Bragg, Sir William. *The World of Sound*. New York 1920; reprint: New York 1968.

Britton, John Phillips. *Models and Precision: The Quality of Ptolemy's Observations and Parameters*. New York 1993.

____. "Ptolemy's Determination of the Obliquity of the Ecliptic." *Centaurus* 14 (1969) 29–41.

Bulmer-Thomas, Ivor. Review of Toomer, *Ptolemy Almagest*. Classical Review n.s. 34 (1984) 299–302.

Bunt, Lucas N. H., P. Jones, and J. Bedient. *The Historical Roots of Elementary Mathematics*. Englewood Cliffs NJ 1976.

Burkert, Walter (Edwin Minar, Jr., trans.). *Lore and Science in Ancient Pythagoreanism*. Cambridge 1972.

Burney, Charles. *A General History of Music From the Earliest Ages to the Present*. London 1776–89; reprint: London 1935.

Cantor, Moritz Benedikt. *Vorlesungen über die Geschichte der Mathematik*. Leipzig 1907.

Carmody, Francis J. "Ptolemy's Triangulation of the Eastern Mediterranean." *Isis* 67 (1976) 601–609.

Cazden, Norman. "Pythagoras and Aristoxenus Reconciled." *Journal of the American Musicological Society* 11 (1958) 97–105.

Clagett, Marshall. *Greek Science in Antiquity*. New York 1955.

Cohn, Leopold. "Didymus #11." *RE* V,1 (1903) 473–74.

Cornford, Francis M. *Plato's Cosmology*. London 1937.

____. "Mysticism and Science in the Pythagorean Tradition." *Classical Quarterly* 16 (1922) 137–50, and 17 (1923) 1–12.

Crocker, Richard L. "Aristoxenus and Greek Mathematics." In Jan La Rue, ed. *Aspects of Medieval and Renaissance Music. A Birthday Offering to Gustave Reese*. New York 1966.

____. "Pythagorean Mathematics and Music." *Journal of Aesthetics and Art Criticism* 22 (1963–64) 189–98 and 325–35.

Croenert, Wilhelm. "Die Hibehrede über die Musik." *Hermes* 44 (1909) 503–21.

Curtis, John. "Reconstruction of the Greater Perfect System." *Journal of Hellenic Studies* 44 (1924) 10–23.

Dahlhaus, Carl. "Ein vergessenes Problem der antiken Konsonanztheorie." In L. Finscher and Christopher-Helmut Mahling, eds. *Festschrift für Walter Wiora zum 30. Dezember 1966*. Kassel 1967.

Dambska, Izydora. "L'épistémologie de Ptolémée." *Semaine de Synthese* 31 (1975) 31-37.
Décarie, Vianney. *Éthique a Eudème*. Paris 1978.
Dicks, D. R. *Early Greek Astronomy to Aristotle*. Ithaca 1970.
____. *The Geographical Fragments of Hipparchus*. London 1960.
____. "A Mistranslation in Manitius." *Journal of Hellenic Studies* 103 (1983) 137.
Diels, Hermann, and Walther Kranz, eds. *Die Fragmente der Vorsokratiker*[6]. Berlin 1951.
Diller, A. "The Oldest Manuscripts of Ptolemaic Maps." *Transactions of the American Philological Association* 71 (1940) 62–67.
Dillon, John M. and A. A. Long, *The Question of "Eclecticism": Studies in Later Greek Philosophy*. Berkeley 1988.
D'Ooge, Martin Luther. *Nicomachus of Gerasa, Introduction to Arithmetic*. New York 1926.
Drake, Stillman. "Ptolemy, Galileo, and Scientific Method." *Studies in the History and Philosophy of Science* 9 (1978) 99–115.
Düring, I. "Greek Music." *Cahiers d'histoire mondiale* 3 (1956) 302–29.
____, ed. *Die Harmonielehre des Klaudios Ptolemaios*. Göteborg 1930; reprint: New York 1980.
____, ed. *Porphyrios Kommentar zur Harmonielehre des Ptolemaios*. Göteborg 1932; reprint: New York 1980.
____. *Ptolemaios und Porphyrios über die Musik*. Göteborg 1934.
Edwards, D. R. *An Annotated Translation and Commentary of the Peri Analemmatos*. Ph.D. diss., Brown University 1985.
d' Erlanger, Baron Rudolph. *La musique arabe*. Paris 1930.
Falus, R. "Harmonike phanere." *Acta Antiqua Academiae Scientiarum Hungaricae* 29 (1972) 20–35.
Feraboli, Simonetta. *Claudio Tolomeo; Le Previsioni Astrologiche*. Milan 1985.
Frank, Erich. *Plato und die sogenannten Pythagoreer*. Halle 1923.
Franz, J. *De musicis graecis commentatio*. Berlin 1840.
Friedlein, Gottfried, ed. *Boethius: De institutione musica libri quinque*. Leipzig 1867.
Fritz, K. von. "The Discovery of Incommensurability by Hippasus of Metapontum." *Annals of Mathematics* 46 (1945) 242–64.
Furley, D. J. Review of Ross, *Aristotle De anima*. *Classical Review* n. s. 13 (1963) 46–49.
Gallo, Alberto. "Le traduzioni dal Greco per Franchino Gaffurio." *Acta musicologica* 35 (1963) 172–74.
Gertsman, E. V. "The Appreciation of High and Low Sound Registers in Ancient Musical Thought." *Vestnik Drevnej Istorii* 118 (1971) 194.
Gevaert, F. A. *Histoire et Théorie de la Musique de l'antiquité*. Gand 1875.
Gingerich, Owen. "Was Ptolemy a Fraud?" *Quarterly Journal of the Royal Astronomical Society* 21 (1980) 253–66.

Gogava, Antonius. *Aristoxeni Musici Antiquiss. Harmonicorum elementorum libri III. Cl. Ptolemaei Harmonicorum, seu de musica libri III. Aristotelis de obiecto auditus fragmentum ex Porphyrij commentarijs*. (Venice 1562).
Gohlke, P. Review of Düring, ed., *Die Harmonielehre des Klaudios Ptolemaios*. *Philologische Wochenschrift* 50 (1930) 1441–44.
Goldstein, Bernard. "The Arabic Version of Ptolemy's Planetary Hypotheses." *Transactions of the American Philosophical Society* n.s. 57 (1967) 3–55.
____. "The Obliquity of the Ecliptic in Ancient Greek Astronomy." *Archives Internationales d'Histoire des Sciences* 33 (1983) 3–14.
____. Review of Toomer, *Almagest*. *Isis* 76 (1985) 117–18.
Goldstein, S. J. "Problems Raised by Ptolemy's Lunar Tables." *Journal for the History of Astronomy* 13 (1982) 195–205.
Gombosi, Otto. "Key, Mode, Species." *Journal of the American Musicological Society* 4 (1951) 20–26.
____. *Tonarten und Stimmungen der Antiken Musik*. Copenhagen 1939.
Goodenough, E. R. "A Neo-Pythagorean Source in Philo Judaeus." *Yale Classical Studies* 3 (1932) 117–64.
Goold, G. P., trans. *Manilius Astronomica*. Cambridge MA 1977.
Gottschalk, H. B. "The *De audibilibus* and Peripatetic Acoustics." *Hermes* 96 (1968) 435–60.
Greaves, Denise Davidson. *Sextus Empiricus: Against the Musicians*. Omaha and London 1986.
Greif, Francisque. "Études sur la musique antique." *Revue des Études Grecque* 24 (1911) 233–86.
Griffiths, J. Gwyn. Review of Jack Lindsay, *The Origins of Astrology*. *Classical Review* n.s. 24 (1974) 315–16.
Guilland, R. *Essai sur Nicéphore Gregoras, L'homme et l'oeuvre*. Paris 1926.
Gulley, Norman. Review of Burkert, *Lore and Science in Ancient Pythagoreanism*. *Classical Review* n. s. 14 (1964) 28–29.
Guthrie, W. K. C. *A History of Greek Philosophy I*. Cambridge 1967.
Hahm, David E. *The Origins of Stoic Cosmology*. Columbus OH 1977.
Halley, Edmund. "A Discourse concerning a method of discovering the true moment of the Sun's ingress into the tropical signs." In *Philosophical Transactions* 19 (1965) 12–20.
Handschin, Jacques. "The Timaeus Scale." *Musica Disciplina* 4 (1950) 3–42
Hannick, Christian. "Byzantinishce Musik." In H. Hunger, ed. *Handbuch der Altertumswissenschaft: Die Hochsprachliche Profane Literatur der Byzantiner II*. Munich 1978.
Hartner, Willy. "The Role of Observations in Ancient and Medieval Astronomy." *Journal of the History of Astronomy* 8 (1977) 1–11.
Hasse, H., and H. Scholz, eds. *Zeno and the Discovery of Incommensurables in Greek Mathematics*. Charlottenburg 1928; reprint: New York 1976.
Heath, Sir Thomas L. *Aristarchus of Samos*. Oxford 1913.

____. *A History of Greek Mathematics*. Oxford 1921.
Heiberg, J. L., ed. *Opera quae exstant omnia, I Syntaxis Mathematica*. Leipzig 1898–1903.
Henderson, I. "Ancient Greek Music." In Egon Wellesz, ed. *Ancient and Oriental Music*, vol. I of *The New Oxford History of Music*. London 1957.
Henderson, M. I. "The Growth of Ancient Greek Music." *The Music Review* 4 (1943) 4–13.
Hiller, Eduard, ed. *Theo Smyrnaeus Philosophi Platonici Expositio rerum mathematicorum ad legendum Platonem utilium*. Leipzig 1878.
Höeg, Carsten. Review of Düring, *Die Harmonielehre des Klaudios Ptolemaios*. *Gnomon* 6 (1930) 652–59.
____. Review of Düring, *Porphyrios Kommentar zur Harmonielehre des Ptolemaios*. *Gnomon* 10 (1932) 318–26.
____. Review of Düring, *Ptolemy and Porphyry über die Musik*. *Gnomon* 12 (1936) 152–57.
Holbrook, Amy K. *The Concept of Musical Consonance in Greek Antiquity and its Application in the Earliest Medieval Descriptions of Polyphony* (Ph.D. diss., University of Washington 1983).
Hollander, John. *The Untuning of the Sky: Ideas of Music in English Poetry 1500–1700*. Princeton 1961.
Huby, Pamela, and Gordon Neal, eds. *The Criterion of Truth; Essays Written in Honour of George Kerferd Together With a Text and Translation (With Annotations) of Ptolemy's* On the Kriterion and Hegemonikon. Liverpool 1989.
Huffman, Carl A. "The Authenticity of Archytas, Fr. 1." *Classical Quarterly* 35 (1985) 344–48.
Hultsch, F. *Pappi Alexandrini collectionis quae supersunt III*. Berlin 1876.
Hunt, Frederick V. *Origins in Acoustics*. New Haven 1978.
Issberner, R. "Dynamis und Thesis." *Philologus* 55 (1869) 541–60.
Jan, Karl von. *Bericht über griechische Musik und Musiker von 1884–1899*. Strassburg 1900.
____. "Die griechische Saiteninstrumente." *Programm des Gymnasiums Saargemund*. Leipzig 1882.
____. "Die Harmonie der Spharen." *Philologus* 52 (1893) 13–37.
____, ed. *Musici scriptores graeci*. Leipzig 1892.
Jones, Alexander. "A Greek Saturn Table." *Centaurus* 27 (1984) 311–17.
Jonker, G. H., ed. *ΜΑΝΟΥΗΛ ΒΡΥΕΝΝΙΟΥ ΑΡΜΟΝΙΚΑ*. Groningen 1970.
Junge, Gustav. "Die Sphaeren-Harmonie und die Pythagoreisch-platonische Zahlenlehre." Classica et Mediaevalia 9 (1948) 183–94.
Kassler, Jamie C. "The 'Science' of Music to 1830." Archives internationales d'histoire des sciences 30 (1980) 111–36.
Kepler, Johannes. Harmonice Mundi. Linz 1619.
____. Prodromus dissertationum cosmographicarum. Tübingen 1596.
Keuls, Eva. "The Apulian 'Xylophone:' A Mysterious Musical Instrument Idenitified." The American Journal of Archaeology 83 (1979) 476–77.

Keys, C. A. "The Word Symphony." Classica et Mediaevalia 30 (1969) 578–94.
Kinsky, Georg. A History of Music in Pictures. New York 1951.
Kirk, G. S., and J. E. Raven. The Presocratic Philosophers. Cambridge 1957.
Knorr, Wilbur. "Techniques of Fractions in Ancient Egypt and Greece." Historia Mathematica 9 (1982) 133–71.
Koller, Hermann. Musik und Dichtung im alten Griechenland. Bern 1963.
Kraehenbuehl, David, and C. Schmidt. "On the Development of Musical Systems." Journal of Music Theory 6 (1962) 32–65.
Kuhn, Thomas. The Structure of Scientific Revolutions. Chicago 1962.
Kummel, Werner Friedrich. Musik und Medizin. Munich 1977.
Lachmann, Robert. Musik des Orients. Breslau 1929.
Laloy, Louis. Aristoxène de Tarente de la Musique de l'antiquité. Paris 1904; reprint: Genève 1973.
Lammert, Friedrich, ed. Claudii Ptolemaei opera quae exstant omnia, vol. III (Leipzig 196 1).
____. "Zur Erkenntnislehre der späteren Stoa; Ptolemaios Per‹ krithr€ou ka‹ ≤gemonikoË 10.11–.13, 13H." Hermes 57 (1922).
Landels, J. G. "Music in Ancient Greece." *Proceedings of the Classical Association of Greece* 77 (1980) 24.
Larsen, Mogens Esrom. "On the Possibility of a Pre-Euclidean Theory of Proportions." *Centaurus* 27 (1984) 1–25.
Lasserre, Francois. *The Birth of Mathematics in the Age of Plato*. Larchmont NY 1964.
Lawlor, Robert, and Deborah Lawlor, trans. *Theonos Smyrnaiou: Mathematics Useful for Understanding Plato*. San Diego 1977.
Lefkowitz, Mary. *The Lives of the Greek Poets*. Baltimore 1981.
Lejeune, Albert. *L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'emir Eugène de Sicile*. Louvain 1956.
____. *Euclide et Ptolémée; deux Stades de l'Optique géométrique grecque*. Louvain 1948.
Levin, R. Flora. *The Harmonics of Nicomachus and the Pythagorean Tradition*. State College PA 1975.
____. "πληγή and τάσις in the *Harmonika* of Klaudios Ptolemaios." *Hermes* 108 (1980) 205–22.
____. "'Synesis' in Aristoxenian Theory." *Transactions of the American Philological Association* 103 (1972) 211–34.
Lévy, Isidore. *Recherches sur Les Sources de la Légende de Pythagore*. Paris 1926.
Lindsay, R. Bruce. *Acoustics: Historical and Philosophical Development*. Stroudsburg, PA 1974.
Link, John W., Jr. *The Mathematics of Music*. Baltimore 1977.
Lippman, Edward A. "Hellenic Conceptions of Harmony." *Journal of the American Musicological Society* 16 (1963) 3–35.
____. *Musical Thought in Ancient Greece*. New York 1964.

Litchfield, Malcolm. "Aristoxenus and Empiricism: A Reevaluation Based on His Theories." *Journal of Music Theory* 32 (1988) 51–73.

Lloyd, G. E. R. *Greek Science After Aristotle*. New York 1973.

____. "Saving the Appearances." *Classical Quarterly* n.s. 28 (1978) 202–22.

Lloyd, Llewelyn S., and Hugh Boyle. *Intervals, Scales and Temperament*. London 1963.

Lohmann, Johannes. "Die griechische Musik als mathematische Form." *Archiv für Musikwissenschaft* 14 (1957) 147–55.

____. *Musike und Logos*. Stuttgart 1970.

Lorch, Richard. "A Note on the Technical Vocabulary in Erastosthenes' Tract on Mean Proportionals." *Journal for the History of Arabic Science* 5 (1981) 166–73.

Macran, Henry S., ed. *The Harmonics of Aristoxenus*. Oxford 1902; reprint: Hildesheim 1974.

Manitius, Karl, trans. *Des Claudius Ptolemaeus Handbuch der Astronomie*. Leipzig 1912.

Martin, T. H. *Theonis Smyrnaei Platonici Liber de Astronomia*. Groningen 1971.

Masi, Michael. *Boethian Number Theory*. Amsterdam 1983.

Mathiesen, Thomas J. *Ancient Greek Music Theory; A Catalogue Raisonné of Manuscripts* (=*Répertoire international des sources musicales* Bxi). Munich 1988.

____. "An Annotated Translation of Euclid's Division of the Monochord." *The Journal of Music Theory* 9 (1975) 236–58.

____. "Aristides Quintilianus and the Harmonics of Manuel Bryennius; A Study in Byzantine Music Theory." *Journal of Music Theory* 27 (1983) 31–47.

____. *Aristides Quintilianus: On Music in Three Books*. New Haven 1983.

____. "Problems of Terminology in Ancient Greek Theory: HARMONIA." In Burton Karson, ed. *Festival Essays for Pauline Alderman*. Provo UT 1976.

____. "Towards a Corpus of Ancient Greek Music Theory: A New Catalogue Raisonné Planned for RISM." *Fontes artis musicae* 25 (1978) 119–34.

McClain, Ernest G. *The Myth of Invariance*. New York 1976.

____. *The Pythagorean Plato; Prelude to Song Itself*. Stony Brook NY 1978.

Mead, H. L. "The Methodology of Ptolemaic Astronomy." *Laval Theologique et Philosophique* 31 (1975) 55–74.

Michaelides, Solon. *The Music of Ancient Greece; An Encyclopaedia*. London 1978.

Michel, Paul-Henri. *De Pythagore a Euclide*. Paris 1950.

Miller, Dayton Clarence. *Anecdotal History of the Science of Sound*. New York 1935.

Moesgaard, Kritian Peder. Review of Newton, *The Crimes of Claudius Ptolemy*. *Journal for the History of Astronomy* 11 (1980) 133–35.

Monro, D. B. *The Modes of Ancient Greek Music*. Oxford 1894.

Morrison, J. S. Review of Burkert, *Lore and Science in Ancient Pythagoreanism*. *Gnomon* 37 (1965) 344–54.

Mountford, J. F. "Greek Music and its Relation to Modern Times." *Journal of Hellenic Studies* 40 (1920) 13–42.

____. "The Harmonics of Ptolemy and the Lacuna in II 14." *Transactions of the American Philological Association* 57 (1926) 71–95.

____. "The Musical Scales of Plato's *Republic*." *Classical Quarterly* 17 (1923) 125–36.

____. Review of Düring, *Die Harmonielehre des Klaudios Ptolemaios*. *Classical Review* n. s. 44 (1930) 242–43.

Mueller, Charles. *Claudii Ptolemaei Geographia*. Paris 1883.

Neugebauer, Otto. *The Exact Sciences in Antiquity*. New York 1962.

____. *A History of Ancient Mathematical Astronomy*. New York 1975.

____. "On the Allegedly Heliocentric Theory of Venus by Heracleides Ponticus." *American Journal of Philology* 93 (1972) 600–601 (reprint: *Astronomy and History; Selected Essays*. New York 1983. 370–71.

____. "On Two Astronomical Passages in Plutarch's *De animae procreatione in Timaeo*." *American Journal of Philology* 43 (1942) 455–59; reprint: *Astronomy and History; Selected Essays*. New York 1983. 273–77.

Newton, R. R. *Ancient Astronomical Observations and the Accelerations of the Earth and Moon*. Baltimore 1970.

____. *The Crimes of Claudius Ptolemy*. Baltimore 1977.

____. "The Authenticity of Ptolemy's Star Data – II." *Quarterly Journal of the Royal Astronomical Society* 24 (1983) 27–35.

Orchard, Thomas N. *Milton's Astronomy*. New York 1913.

Pace, Anna de. "Elementi aristotelici nell'*Ottica* di Claudio Tolomeo." *Rivista critica di Storia della Filosofia* 36 (1981).

Page, D. L. *Poetae melici graeci*. Oxford 1967.

Palisca, Claude V. "Giovanni Battista Doni's Interpretation of the Greek Modal System." *The Journal of Musicology* 15 (1997) 8-9.

____. *Humanism in Italian Renaissance Musical Thought*. New Haven 1985.

Pedersen, Olaf. "Logistics and the Theory of Function: An Essay in the History of Greek Mathematics." *Archives internationelles d'histoire des sciences* 24 (1974) 29–50.

____. *A Survey of the Almagest*. Odense 1974.

Pfeiffer, Rudolph. *A History of Classical Scholarship 1300–1850*. Oxford 1976.

Pöhlmann, Egert. *Denkmäler altgriechischer Musik*. Nürnberg 1970.

Raasted, Jørgen. "A Neglected Version of the Anecdote About Pythagoras' Hammer Experiment." *Cahiers d'Institut du moyen age grec et latin* 31a (1979) 1–9.

Redfield, John. *Music: A Science and an Art*. New York 1930.

Reimann, H. *Handbuch der Musikgeschichte I: Altertum und Mittelalter*. Leipzig 1923.

Reinach, Théodore. "La guitare dans l'art grec." *Revue des études grecque* 8 (1895) 371–78.

____. "La musique des Sphères." *Revue des études grecque* 13 (1900) 432–49.

____. "Zu Ptolemaios Harmonika II,10." *Hermes* 43 (1908) 478.

Richardson, W. F. "The Greek Number System." *Prudentia* 9 (1977) 15–26.

Richter, Lukas. "Antike Überlieferungen in der byzantinischen Musiktheorie." *Deutsches Jahrbuch der Musikwissenschaft für 1961* (1962) 86–105.

Richter, Will. "Symphonia; zur Vor- und Frühgeschichte eines musikologischen Begriffs." In Heinrich Hüschen and Dietz-Rüdiger Moser, eds. *Festschrift für Wolfgang Bötticher zum sechzigsten Geburtstag am 19 August 1974*. Berlin 1974.

Riddell, R. C. "Parameter Disposition in Pre-Newtonian Planetary Theories." *Archive for the History of the Exact Sciences* 23 (1980) 87–157.

Riemann, Hugo. *Handbuch der Musikgeschichte: I Die Musik des Altertums*. Leipzig 1919.

Riethmüller, Albert. "Logos und Diastema in der griechischen Musiktheorie." *Archiv für Musikwissenschaft* 42 (1985) 18–36.

da Rios, Rosetta, ed. *Aristoxeni elementa harmonica*. Rome 1954.

Robbins, F. E., trans. *Ptolemy Tetrabiblos*. Cambridge MA 1940.

Rohde, Erwin. "Γέγονε in den Biographica des Suidas." *Rheinisches Museum* 38 (1978) 161–220.

Rome, A., ed. *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste*. Vatican City 1936 and 1943.

Ruelle, Charles-Émile. "Hélicon." In C. Daremberg and E. Saglio, eds. *Dictionnaire des Antiquités Grecques et Romaines*. Paris 1900.

____. "Le Monocorde, Instrument de musique." *Revue des Études Grecque* 97 (1897) 309–312.

Sachs, Curt. *The Rise of Music in the Ancient World*. New York 1943.

de Santillana, Giorgio. *The Origins of Scientific Thought*. London 1961.

Scarborough, John. Review of Toomer, *Almagest*; Masi, *Boethian Number Theory*; and Mathiesen, *Aristidies Quintilianus*. *Classical Bulletin* 61 (1985) 87–89.

Scarpa, Luigi, ed. *Macrobii Ambrosii Theodosii Commentariorum in Somnium Scipionis Libri duo*. Padua 1981.

Schlesinger, K. "Further Notes on Aristoxenus and Musical Intervals." *Classical Quarterly* 27 (1933) 88–96.

Schmidt, C. "On the Development of Musical Systems." *Journal of Music Theory* 6 (1962) 32–65.

Schonberger, P. L. *Studien zum I Buch der Harmonik des Claudius Ptolemaus*. Augsburg 1914.

Scott, John F. *The Mathematical Work of John Wallis*. London 1938; reprint: New York 1981.

Shirlaw, Matthew. "The Music and Tone-Systems of Ancient Greece." *Music and Letters* 32 (1951) 131–39.

Smith, A. *Porphyry's Place in the Neo-Pythagorean Tradition; A Study in post-Plotinian Neoplatonism*. The Hague 1974.

Solomon, Jon. "A Diphonal Diphthong in the Orestes Papyrus," *American Journal of Philology* 97 (1976) 172-73.

____. "Ekbole and Eklusis in the Musical Treatise of Bacchius." *Symbolae Osloensis* 50 (1980) 111–26.

____. "Orestes 344–45: Colometry and Music." *Greek, Roman, and Byzantine Studies* 18 (1977) 71–83.

____. "A Preliminary Analysis of the Organization of Ptolemy's Harmonics." In André Barbera, ed. *Musical Theory and Its Sources: Antiquity and the Middle Ages*. Notre Dame 1990.

____. "The Seikilos Inscription: A Musicological Analysis." *American Journal of Philology* 107 (1986) 455–79.

____. "Towards a History of Tonoi." *Journal of Musicology* 3 (1984) 242–51.

Spitzer, Leo. *Classical and Christian Ideas of World Harmony*. Baltimore 1963.

Stevenson, Edward L. *The Geography of Claudius Ptolemy*. New York 1932.

Stieglitz, Robert R. "Minoan Mathematics or Music?" *Bulletin of the American Society of Papyrologists* 15 (1978) 127–32.

Sultan, Nancy. "New Light on the Function of 'Borrowed Notes' in Ancient Greek Music." *Journal of Musicology* 7 (1988) 387–98.

Swerdlow, Noel M. "Ptolemy on Trial." *American Scholar* 48 (1979) 523–31.

Szabó, 'Arpád (A.M. Ungar, trans.). *The Beginnings of Greek Mathematics*. Hingham MA 1978.

____. "Winkelmessung und die Anfänge der Trigonometrie." *Acta Antiqua Academiae Scientarum Hungricae* 24 (1976) 163–82.

Taisbak, C. M. "Eleven Eighty-Thirds; Ptolemy's Reference to Eratosthenes in Almagest I.12." *Centaurus* 27 (1984) 165–66.

Taliaferro, R. Catesby, trans. *The Almagest by Ptolemy*. Chicago 1938.

Tannery, Paul. "Du rôle de la musique grecque." *Bibliotheca Mathematica* 3 (1902) 161–75.

____. *Mémoires scientifique* III. Paris 1915.

____. "Sur le spondiasme dans l'ancienne musique grecque." *Revue archéologique 4/17* (1911) 41–50 [=*Mémoires scientifique III* 209–309].

____. "Sur les Intervalles de la musique grecque." *Revue des Études Grecque* 15 (1902) 336–52.

Tannery, Paul and R. P. E. Stéphanou, eds. *Quadrivium de Georges Pachymère*. Vatican 1940. (*Studi e Testi* 94).

Tarán, Leonardo. *Speucippus of Athens*. Leiden 1981.

Theiler, Willy. Review of Düring, *Die Harmonielehre des Klaudios Ptolemaios*. *Göttingenische gelehrte Anzeigen* 198 (1936) 196–204.

Thorp, John. "Aristoxenus and the Ethnoethical Modes." In R. Wallace and B. MacLachlan, eds. *Harmonia Mundi Musica e filosofia nell'Antichità*. (Rome 1991) 54–68.

Tod, Marcus N. "The Alphabetic Numeral System in Attica." *Annual of the British School at Athens* 45 (1950) 126–39 [reprint: *Ancient Greek Numerical Systems*. Chicago 1979.]

Toomer, G. J. *Ptolemy Almagest*. London 1984.

Tucker, William J. *Ptolemaic Astrology*. Sidcup UK 1962.

Tzetzes, John. *Über die altgriechische Musik in der griechischen Kirche*. Munich 1874.

Verity, A. W., ed. *Milton: Paradise Lost*. Cambridge 1910.

Vetter, Walther. "Die antike Musik in der Beleuchtung durch Aristoteles." *Archiv für Musikwissenschaft* (1936) 1–41.

____. "Triganon." *RE* XIII.2 (1939) 142.

Vogel, Martin. *Die Enharmonik der Griechen*. Düsseldorf 1963.

____. "Über die drei Tongeschlechter des Archytas." In *Bericht über den Internationalen Musikwissenschaftlichen Kongress Hamburg 1956*. (Kassel 1957) 233–35.

van der Waerden, Bartel Leendert. "Die Harmonielehre der Pythagoreer." *Hermes* 78 (1943) 163–199.

____. "Klaudius Ptolemios." *RE* 48 (1959) 1788–1859.

____. "The Motion of Venus, Mercury, and the Sun in Early Greek Astronomy." *Archive for the History of the Exact Sciences* 26 (1982) 99–113.

Wallace, Robert W. "Damone di Oa ed i suoi successori: un'analisi delle fonti." In R. Wallace and B. MacLachlan, eds. *Harmonia Mundi: Musica e filosofia nell'Antichità*. (Rome 1991) 30–53.

____. "Music Theorists in Fourth-Century Athens." In Bruno Gentili and Franca Perusino, eds., *MOUSIKE: metrica ritmica e musica greca in memoria di Giovanni Comotti* (Pisa 1994).

Wallis, Johannes. *ΚΛΑΥΔΙΟΥ ΠΤΟΛΕΜΑΙΟΥ ΑΡΜΟΝΙΚΩΝ ΒΙΒΛΙΑ Γ*. Oxford 1682.

Wantzlöben, S. *Das Monochord als Instrument und als System*. Halle 1911.

Waszink, J. H. *Timaeus: a Calcidio translatus commentarioque instructus*. Vol. IV of Raymond Klibansky, ed. *Plato Latinus*. London 1975.

Wellesz, E. *A History of Byzantine Music and Hymnography*[2]. Oxford 1961.

West, M. L. *Ancient Greek Music*. Oxford 1992.

____. "The Midnight Planet." *Journal of Hellenic Studies* 100 (1980) 206.

____. "The Singing of Homer and the Modes of Early Greek Music." *The Journal of Hellenic Studies* 101 (1981) 113–29.

____. *Textual Criticism and Editorial Technique*. Stuttgart 1973.

Westphal, Rudolf. *Harmonik und Melopoie der Griechen*. Leipzig 1863.

____. *Die Musik des griechischen Altertums*. Leipzig 1883.

Wifstrand, Albert. "Eikota: Emendationen und Interpretationen zu griechischen Prosaikern der Kaiserzeit." *Kungliga Humanistiska Vetenskapssamfundet* I 3 (Lund 1933) 59–89.

Wilkinson, L. P. "Philodemus on Ethos in Music." *Classical Quarterly* 32 (1938) 174–81.

Wille, Günther. *Musica Romana: Die Bedeutung der Musik in Leben der Römer*. Amsterdam 1967.

Wilson, Nigel. *Scholars of Byzantium*. Baltimore 1983.

Winnington-Ingram, Reginald Pepys, ed. *Aristides Quintilianus; De musica libri tres*. Leipzig 1963.

____. "Aristoxenus and the Intervals of Greek Music." *Classical Quarterly* 26 (1932) 195–208.

____. "Greece." *New Grove Dictionary of Music and Musicians* VII. 659–72.

____. *Mode in Ancient Greek Music*. Cambridge 1936; reprint: Amsterdam 1968.

____. Review of Mathiesen, *Aristides Quintilianus*. *Early Music History* 4 (1984) 376.

____. Review of Vogel, *Die Enharmonik der Griechen*. *Die Musikforschung* 18 (1965) 60–64.

____. "The Spondeion Scale." *Classical Quarterly* 22 (1928) 83–91.

Zaminer, Frieder. "Hypate, Mese, und Nete in frühgriechischen Denken." *Archiv für Musikwissenschaft* 41 (1984) 1–25.

Zeller, Eduard. *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung*[4]. Leipzig 1921.

Ziegler, Konrat. "Ptolemais." *RE* 48 (1959) 1867–68.

INDICES

Index of Greek Terms

Index of Manuscripts and Papyri Cited

General Index

SUPPLEMENTS TO MNEMOSYNE

EDITED BY J. M. BREMER, L. F. JANSSEN, H. PINKSTER
H. W. PLEKET, C. J. RUIJGH AND P. H. SCHRIJVERS

Recent volumes in the series:

180. Günther, H.-C. *Ein neuer metrischer Traktat und das Studium der pindarischen Metrik in der Philologie der Paläologenzeit.* 1998. ISBN 90 04 11008 9
181. Hunt, T.J. *A Textual History of Cicero's* Academici Libri. 1998. ISBN 90 04 10970 6
182. Hamel, D. *Athenian Generals.* Military Authority in the Classical Period. 1998. ISBN 90 04 10900 5
183. Whitby, M. (ed.). *The Propaganda of Power.* The Role of Panegyric in Late Antiquity. 1998. ISBN 90 04 10571 9
184. Schrier, O.J. *The* Poetics *of Aristotle and the* Tractatus Coislinianus. A Bibliography from about 900 till 1996. 1998. ISBN 90 04 11132 8
185. Sicking, C.M.J. *Distant Companions.* Selected Papers. 1998. ISBN 90 04 11054 2
186. Schrijvers, P.H. *Lucrèce et les sciences de la vie.* 1999. ISBN 90 04 10230 2
187. Billerbeck M. (Hrsg.). *Seneca.* Hercules Furens. Einleitung, Text, Übersetzung und Kommentar. 1999. ISBN 90 04 11245 6
188. Mackay, E.A. (ed.). *Signs of Orality.* The Oral Tradition and Its Influence in the Greek and Roman World. 1999. ISBN 90 04 11273 1
189. Albrecht, M. von. *Roman Epic.* An Interpretative Introduction. 1999. ISBN 90 04 11292 8
190. Hout, M.P.J. van den. *A Commentary on the Letters of M. Cornelius Fronto.* 1999. ISBN 90 04 10957 9
191. Kraus, C. Shuttleworth (ed.). *The Limits of Historiography.* Genre and Narrative in Ancient Historical Texts. 1999. ISBN 90 04 10670 7
192. Lomas, K. & T. Cornell. *Cities and Urbanisation in Ancient Italy.* ISBN 90 04 10808 4 *In preparation*
193. Malkin, I. *History of Greek Colonization.* ISBN 90 04 09843 7 *In preparation*
194. Wood, S.E. *Imperial Women.* A Study in Public Images, 40 B.C. - A.D. 68. 1999. ISBN 90 04 11281 2
195. Ophuijsen, J.M. van & P. Stork. *Linguistics into Interpretation.* Speeches of War in Herodotus VII 5 & 8-18. 1999. ISBN 90 04 11455 6
196. Tsetskhladze, G.R. (ed.). *Ancient Greeks West and East.* 1999. ISBN 90 04 11190 5
197. Pfeijffer, I.L. *Three Aeginetan Odes of Pindar.* A Commentary on *Nemean* V, *Nemean* III, & *Pythian* VIII. 1999. ISBN 90 04 11381 9
198. Horsfall, N. *Virgil,* Aeneid 7. A Commentary. 2000. ISBN 90 04 10842 4
199. Irby-Massie, G.L. *Military Religion in Roman Britain.* 1999. ISBN 90 04 10848 3
200. Grainger, J.D. *The League of the Aitolians.* 1999. ISBN 90 04 10911 0
201. Adrados, F.R. *History of the Graeco-Roman Fable.* I: Introduction and from the Origins to the Hellenistic Age. Translated by L.A. Ray. Revised and Updated by the Author and Gert-Jan van Dijk. 1999. ISBN 90 04 11454 8
202. Grainger, J.D. *Aitolian Prosopographical Studies.* 2000. ISBN 90 04 11350 9
203. Solomon, J. *Ptolemy* Harmonics. Translation and Commentary. 2000. ISBN 90 04 115919