Philosophy of Space and Time

And the Inner Constitution of Nature

Michael Whiteman

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And the Inner Constitution of Nature

MICHAEL WHITEMAN

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AN admirable statement of the aims of the Library of Philosophy was provided by the first editor, the late Professor J. H. Muirhead, in his description of the original programme printed in Erdmann's *History of Philosophy* under the date 1890. This was slightly modified in subsequent volumes to take the form of the following statement:

'The Muirhead Library of Philosophy was designed as a contribution to the History of Modern Philosophy under the heads: first of Different Schools of Thought—Sensationalist, Realist, Idealist, Intuitivist; secondly of different Subjects—Psychology, Ethics, Political Philosophy, Theology. While much had been done in England in tracing the course of evolution in nature, history, economics, morals and religion, little had been done in tracing the development of thought on these subjects. Yet "the evolution of opinion is part of the whole evolution".

'By the co-operation of different writers in carrying out this plan it was hoped that a thoroughness and completeness of treatment, otherwise unattainable, might be secured. It was believed also that from writers mainly British and American fuller consideration of English Philosophy than it had hitherto received might be looked for. In the earlier series of books containing, among others, Bosanquet's *History of Aesthetic*, Pfleiderer's *Rational Theology since Kant*, Albee's *History of English Utilitarianism*, Bonar's *Philosophy and Political Economy*, Brett's *History of Psychology*, Ritchie's *Natural Rights*, these objects were to a large extent effected.

'In the meantime original work of a high order was being produced both in England and America by such writers as Bradley, Stout, Bertrand Russell, Baldwin, Urban, Montague, and others, and a new interest in foreign works, German, French and Italian, which had either become classical or were attracting public attention, had developed. The scope of the Library thus became extended into something more international, and it is entering on the fifth decade of its existence in the hope that it may contribute to that mutual understanding between countries which is so pressing a need of the present time.'

The need which Professor Muirhead stressed is no less pressing today, and few will deny that philosophy has much to do with enabling us to meet it, although no one, least of all Muirhead himself, would regard that as the sole, or even the main, object of philosophy. As Professor Muirhead continues to lend the distinction of his name to the Library

of Philosophy it seemed not inappropriate to allow him to recall us to these aims in his own words. The emphasis on the history of thought also seemed to me very timely; and the number of important works promised for the Library in the very near future augur well for the continued fulfilment, in this and other ways, of the expectations of the original editor.

H. D. LEWIS

PHILOSOPHY OF SPACE AND TIME

and the Inner Constitution of Nature

A PHENOMENOLOGICAL STUDY

BY

MICHAEL WHITEMAN

LONDON · GEORGE ALLEN & UNWIN LTD
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It is hardly more than a pardonable exaggeration to say that the determination of the meaning of nature reduces itself principally to the discussion of the character of time and the character of space.

A. N. WHITEHEAD, *The Concept of Nature*, p. 33

The Greeks were accustomed to speak of the *peripeteia*, the change or reversal of fortune or circumstances which marked the turning-point in a drama. Such a *peripeteia* took place about the beginning of the present century, a reversal, a crisis, which, though many workers are hardly even now aware of the convulsion, shook the citadel of scientific thought to its foundations, and, as a philosophy, left it in ruins.

W. MACNEILE DIXON, *The Human Situation*, London 1938, p. 160.

The possibility always remains open to examine into the primal evidence of a tradition, for example, of geometrical or of any other deductive science, which works on through the centuries. If this does not occur, then the original activities, which are found within the fundamental concepts of this deductive science, and their foundation in pre-scientific materials remain undisclosed. The tradition in which these sciences are handed down to us is then emptied of meaning, and the basis of meaning to which these sciences refer, namely, the life-world, is forgotten.

ALFRED SCHÜTZ, *Phenomenology and the Social Sciences*, Philosophical Essays in Memory of Edmund Husserl, ed. Marvin Farber, Harvard, 1940.

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PREFACE

In this century we are seeing the decisive breakdown of a widely assumed though paradoxical world-view, which took firm hold in the eighteenth and nineteenth centuries. The supposition was that there is only one 'real' space and one 'real' time, all other impressions being mere accompaniments of imagination or thought. Space and time were the field of ordinary perception; and they were also abstract mathematical fields in which the results of calculations were represented. Everything that happens was then supposed fully accounted for in terms of the 'primary qualities'—positions, shapes, velocities, masses, etc.—precisely given in nature in the one all-inclusive space and the one inflexible succession of time-instants.

In Whitehead's phrase, everything was 'simply located'. As Einstein put it, space was an infinite 'box' or 'container' in which everything had its place, regardless of what might or might not be observable. What was overlooked was that location means measure; that every measure presupposes a theory and a technique; that measurement means interference and approximation; and that as soon as we look beyond the mere collection of isolated measures and deal with the orderly unfolding of actual experience, nature must be regarded as a *rational whole*, transcending the particular spatio-temporal measures and observations, yet in some way also *given* to us.

The theory of simple location and the mechanistic sciences built upon it were supposed to be pure empiricism—dependence on nothing but direct experience. We have now discovered, however, that they constitute an attempt to escape the complexities of direct experience by means of an over-simplified and misleading abstraction. A true empiricism will consist in the face-to-face contemplation of direct experience and understanding of what the scientist is doing, whether sensory or rational in character; and this is called *phenomenology*. Throughout this book I have tried to keep strictly to the face-to-face presuppositionless method of phenomenological analysis.

Scientists in general, though interested in problems of interpretation, are very often weak in philosophical thinking and knowledge. In quantum theory, for example, the term *phenomenological* has been taken over to indicate a dependence on visualized 'models' (picturing, for example, the atom as a planetary system), such 'models' being not matters of direct experience and possibly not even fully adequate to the theoretical situation. Because of the complete contradiction here with philosophical usage, I have substituted the term *model-motivated* for such a view. To avoid further confusion for quantum

theorists, I have then, in this context, called the phenomenological approach *analytic*.

Although Poincaré and Einstein, in their discovery of the special theory of relativity, can be said to have adopted the phenomenological method of face-to-face analysis, I believe it is safe to say that a thoroughgoing phenomenological approach to the subject of space, time and natural law, in all its aspects and primitive applications including those of the theory of relativity and quantum theory, has never been attempted. The power of the phenomenological method to clarify the foundations and solve problems of interpretation has not been realized.

Understandably in science there is strong inclination to build the house first, and to give thorough attention to the foundations only when the whole house begins to collapse.

In presenting now a comprehensive study of this many-sided subject, I feel it would be out of place to point to sections or conclusions regarded as particularly important. It has been my experience that what one person finds particularly helpful and stimulating may seem to another a scarcely relevant distraction. 'Minds are many, though truth be one.' What needs most emphasis, I think, is the unitary conception of the whole subject and the inadequacy of a *piecemeal* attitude, which would reject particular species of contemplation or analysis as being extraneous to 'science'.

Besides the technical term *phenomenology* there is another—*transcendental*—on which a few remarks seem called for in this preface. There is no need for anyone to look askance at this term, as if it signified a regrettable lapse from the scientific attitude. In my usage it is simply the term introduced to describe those features of face-to-face contemplation or analysis which come to light when one passes beyond 'simple location' in space or time. In mathematics and theoretical physics, group-structure is 'transcendental'; in the philosophy and psychology of perception, Gestalten (significant wholes) are 'transcendental'. Every physical law involves transcendence.

Many kind friends have read portions of the book at various stages of its composition and made valuable criticisms or suggestions, discussed difficulties with me, or otherwise given encouragement. Above all, I wish to record my very warm thanks to Professor H. H. Price for his interest in my work over many years, and particularly for his very thorough examination of the first complete draft and his numerous comments and searching questions on it, and for further detailed comments on a second draft of the material which ultimately went to form Part IV. It is impossible to say how much I owe to him in the way of encouragement, clarification and stimulation in the working out of the problems. He is not of course to be charged with faults that still remain.

I am deeply indebted and grateful to Professor Werner Heisenberg for his courtesy in reading my second draft of the long study of the interpretation of quantum theory in Chapter 20, and for being so kind as to say that he was in thorough agreement with what is now the middle portion of the chapter. I think I have profited considerably from advice he gave regarding the earlier portion, and having now based the discussions consistently on field theory and its established applicability I hope that this difficult chapter will be found generally acceptable and a considerable advance on interpretations limited to elementary quantum mechanics.

The discussions of quantum theory in various parts of the book have also been most kindly read by Professor Henry Margenau, of Yale University, and it is a pleasure to acknowledge the help and stimulus given by his comments.

I have also to thank Professor L. Jánossy, of the Hungarian Academy of Sciences, for kindly sending offprints of several of his important papers on Interference.

Others whom I should like to thank specifically for their help and interest are Dr Mary Hesse, who read the whole of Part I in its first draft and made valuable suggestions, and Dr G. J. Whitrow and Professor W. B. Bonnor, who kindly read relevant portions; also my colleagues Professor D. G. Parkyn and Professor W. E. Frahn, for the benefit of useful discussions.

Lastly, I have to thank the University of Cape Town most sincerely for giving me encouragement and special facilities for the writing of the book

Cape Town, October 1966

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Chapters 13 and 15, and a few other sections, make some use of concepts of elementary calculus or differential geometry. Readers may pass over these sections if they wish, without, it is hoped, being hampered in their understanding and appreciation of the rest of the book.

Readers specially interested in the status of parapsychological and mystical experience may find the following chapters most relevant: 1, 4, 8–10, 14 (beginning), 16, 19, 21, 22, 23 (pp. 374 ff.).

TERMINOLOGY

STUDIES

1. Practical and Commonsense
   (Matters of fact, development of skills.)
2. Scientific
   (The spiral of experiment, theory, refined experiment, refined theory, etc.; axioms, statistical confirmations.)
3. Philosophical
   (Direct analysis of intelligible structure and corresponding language uses in any field of experience.)

ANALYSIS

I. *Natural*

1. Linguistic
2. Scientific-theoretical
   i Outer operational (*a*) actual
   (*b*) ideal
   ii Postulational
3. Psychological

II. *Phenomenological*

4. Inner operational (*a*) quasi-temporal
   (*b*) absolute
5. Supraperspectival
6. Transcendental

POWERS OF ESSENTIAL ANALYSIS

1. Recollective Release (phenomenological suspension, epoché primordial recollectedness)
2. Analysis of the Simultaneous
3. Synthesis of the Extended

POWERS OF INNER TIME-CONTROL (Chapter 19)

I. Releasing *a*. Non-attachment, fixation-release (psychological suspension)
b. Recollective release (phenomenological suspension)
c. Unitary release and comprehension

II. Epimethic *d.* Reminiscent fantasy
e. Secondary remembrance, reproductive memory
f. Primary remembrance, primordial recollection

III. Promethic *g.* Expectation
h. Precognition
i. Protention, time-spot determination

MODES OF CREATIVITY

I. Actuality 1. Focal (of the intentional 'act')
2. Perspectival (of the total encounter)

II. Potentiality 3. Supraperspectival constitution (of 'apprésented' substructure)
4. Hierarchical constitution

STABILIZED LEVELS OF CREATIVITY

I. *The Natural Level*

(*a*) Real (actual or potential):
Ordinary Practical Experiencing [Erfahrung]
(*i*) outer (physically objective)
(*ii*) inner (psychological, subjective)

(*b*) Abstract: Thinking and Reasoning

II. *Imagination* (merged in the physical, or autonomous)

(*a*) 1. Fantasy and dream-states } *i* outer
2. Free imagination } *ii* inner
3. Constructive imagination

(*b*) 4. Involuntary error of belief ('feelings of self-evidence')
5. Voluntary uncertainty of supposition

III. *The Transcendental Level* (merged in other states, or autonomous)

'Experience C', beyond ordinary practical experiencing and thinking

(*a*) Real (actual or potential):
Comprising knowledge of given participation, intersubjectivity, empathy, universality, interconnections of significance, etc., in a 'total encounter' (Gestalt) not describable in terms of 'simple location' in space and time.

(*b*) Abstract:
Intuitive thought, essential thought and self-evidence of reason, as distinct from discursive and directed thinking.

PART I

INTRODUCTION

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CHAPTER 1

A MINIATURE 'HISTORY OF IDEAS' OF SPACE AND TIME

'The vision of a reality beyond the impressions of the senses . . .'

M. Polanyi, *Personal Knowledge*, London 1958.

From the beginning of history man has been drawn to reflect on the sense of a hidden significance and reason governing the orderly progression of natural phenomena—the seasons, solstice and equinox, youth and old age, the habits of animals, the motion and power of the elements. Even though the course of events might sometimes be influenced by 'magical' means, everything remained in the realm of law, partly natural and partly divine. The epoch-making developments in experimental science and mathematics after AD 1600 made possible a thorough systematizing of natural phenomena by their measurements and thus a confident demarcation of the realm of natural law. It is only in this century, however, that Space and Time themselves—the background 'form of phenomena'[1]—have become the subject of a vast and intricate study, stretching far beyond the confines of physics and mathematics as previously understood.

More than 4,000 years ago the ancient Babylonians, Egyptians and Chinese were making astronomical observations and reducing them to order. While this could be described as the beginnings of 'science', no appreciable advance towards the establishment of the modern exact sciences or towards the conceptual precision which they foster could be made till the complex art of measurement had become better understood and made more widely applicable according to reason. Hence the utmost significance and value must be attached to the achievement of the Greek and Alexandrian geometers, notably Thales (600 BC), Pythagoras (540 BC), Hippocrates of Chios (440 BC) and Euclid (300

[1] Kant's description of space and time (*Critique of Pure Reason*, trs. N. Kemp Smith, London 1933, pp. 66, 71, 77), with the term 'phenomena' standing for Erscheinungen (appearances) as in H. Weyl, *Philosophy of Mathematics and Natural Science*, Princeton 1949, p. 86.

BC),[1] in abstracting the ideal intuitive properties of space from the actual ones and exhibiting the former as a logical deductive system. After this, for more than 2,000 years, all measurement and study of natural law proceeded on the unquestioned basis of Euclidean geometry, supposed to be not merely ideal but actually constituting the 'form of phenomena', like a certain unique 'container'[2] with a variety of objects placed here and there inside.

Meanwhile, first religion and then philosophy had been attending to the problem of *reality*, as distinct from appearances, which plainly vary from person to person with every change of circumstances, cloaking purposes and significances which strike into the depths of our being.

To the seers who composed the poems of the Rig Veda, about 1000 BC, nature was a wondrous panorama, bodying forth a spiritual potency and structure which might be discerned by insight and revelation. For the same 'ordinances'[3] were to be found at work in man's spirit and manifested in the world around. 'Above was the (divine) intention [*prayati*], below was (the principle of creative) subsistence [*svadhā*]';[4] and the course of events was born, as it were, from 'the watery flood'.[5] In the *Timaeus* of Plato (380 BC) mind or spirit [νοῦς] is confronted with 'necessity' [ἀνάγκη] as an 'errant cause', concomitant and corporeal, and cannot therefore completely effect its purpose.[6] The most developed form of this teaching is found in Plotinos (AD 250) whose view of 'matter' as a principle giving externality with indefiniteness of measure, and of the 'sense-world' as 'a mixture of matter and reason',[7] corresponds remarkably with the standpoint of twentieth-century physics.

[1] Dates, in the case of persons, indicate the probable period of greatest mature productivity (perhaps between forty and sixty years of age). The *Elements* of Euclid, as a whole, was unknown to the western world till AD 1260. But the theorems of the most essential parts (Book I and parts of Books III and IV) were familiar through the *Geometry* of Boethius (AD 510), which was a standard text in the monastic schools.

[2] Einstein, Foreword to *Concepts of Space*, by Max Jammer, Harper, N.Y. 1960, p. xiv; Milton K. Munitz, *Space, Time and Creation*, Glencoe, Illinois, 1957, pp. 106ff. In precise terms, I take the 'container theory' to be the theory that shapes and separations in space and time are absolute, so that the whole universe is, so to speak, one all-inclusive pattern of three-dimensional shapes, given in that form uniquely.

[3] A. A. Macdonell, *A Vedic Reader*, Oxford 1917, i. 160.1, x. 90.16, v. 83.5, i. 1.8, etc.

[4] *Ibid.*, x. 129.5, p. 210, translation slightly modified.

[5] Indra 'forced out the waving of the waters' (i. 85.9), and Agni—'shining guardian of order', i. 1.8—was born from them.

[6] J. Burnet, *Greek Philosophy*, Part I, London 1924, pp. 341 f.

[7] Enneads iii. 2.2, i. 8.14, i. 8.8. According to the Copenhagen Interpretation (see Ch. 20) neither particles nor waves can be said to exist till an ('external') experiment is performed, and the uncertainty relations represent the indefiniteness of the applicability of the formalism ('reason') to the experimental means and givenness in nature ('matter').

The ancient world, in general, sought to explain nature in terms of spirit. The treasured lore of the time consisted of a blend of genuine mystical testimony, poetry and myth, theological speculation and rudimentary science, and the wise man was he who was at once seer, prophet, poet and philosopher.[1] In those days there seems to have been considerable communication of knowledge by word of mouth between east and west. Fruits of insight and trends of thought were disseminated, but the originators, and the original esoteric form of the teaching, remain most often unknown.

In this process of dissemination, moreover, true insights became rapidly coloured over by fanciful speculation or altogether transformed by people who did not understand the form in which they were expressed. The doctrine of the four (or five) elements, which in the Upanishads is presented as purely mystical, becomes in Empedocles a semi-mystical or philosophical doctrine and in Aristotle a theory of natural phenomena.[2] The doctrine of the 'celestial spheres', which the celebrated mathematician and astronomer Eudoxos (370 BC) developed under the stimulus of the 'wisdom of the magis', was further elaborated and modified by Kalippos and Aristotle (340 BC).[3] This remained influential till the time of Galileo and Kepler (AD 1620), alongside the purely mathematical system of epicycles devised by Ptolemy (AD 150) and intended only to 'save the appearances' as accurately as possible.

Other doctrines due to Aristotle or his predecessors, and resulting from premature attempts at physical or metaphysical analysis, were those of *topos* ('place', located bodily shape) and the 'void' or vacuum.[4] These were of influence as late as the seventeenth century. A more physical theory was that of 'natural motion'—for instance, the downward fall of unsupported objects. To Aristotle is also chiefly due the general acceptance throughout the scholastic period (AD 1150–1300) of the doctrine of 'matter' [ὕλη] as the potentiality or 'principle of individuation' of substantial forms. 'Everything is form, but form itself becomes the matter of a higher form.'[5]

Concurrently with all these speculations, and not, it appears, much influenced by them, the work of experimentalists in astronomical and terrestrial observation continued, measurements being made always in

[1] F. M. Cornford, *Principium Sapientiae, The Origins of Greek Philosophical Thought*, Cambridge 1952, chapters V and VI.

[2] Treatises on early Greek philosophy customarily regard Empedocles as the originator of the theory of the elements. There are, however, references in the earliest Upanishads, long before the time of Empedocles, e.g. B.A.Up. III. 7, Ch. VII. 7.1. For the interpretation, see my book, *The Mystical Life*, London 1961, pp. 237, 232.

[3] Concerning Eudoxos, see Werner Jaeger, *Aristotle*, Oxford 1934, many references in index.

[4] Jammer, *op. cit.*, pp. 7–10, 15–22.

[5] Jaeger, *op. cit.*, p. 382.

the space of Euclidean geometrical forms—the straight-line, circle, sphere, etc.[1]

Also resting on the assumption of an infinite Euclidean 'container of all' was the atomic theory broached by Leucippus (450 BC), elaborated by Democritus (420 BC), and taken over by Epicurus (290 BC). Here we find also the germ of a kinetic theory of matter, with a principle of conservation, and the theory of primary and secondary qualities. This 'new physics' remained something of a curiosity for 2,000 years, till experimental warrant could be found for its hypotheses.

A partial divorce was thus maintained between philosophy and early science. In this connection the importance of Galileo's study of the *Systems of the World* (1630) and Newton's of *Natural Philosophy* (1687) was twofold. Firstly, the mythical elements in Aristotelian cosmology and 'physics' were thus cleared away, and the rule of scientific method finally established. Secondly, dynamics was shown to determine a *preferred* set of reference-frames, each frame moving with uniform velocity relative to every other frame of the set. In this new 'container theory' the measure of acceleration became absolute, as were also the measures of spatial and temporal separation; but velocity was always relative. The science of Mechanics, which was to dominate man's scientific view of the world for 200 years, arose on this ground together with that of hypothetical laws relating changes in the measures of time to those of space, as made in a Euclidean reference frame of the class called 'inertial', i.e. requiring no 'correction' for forces due to absolute acceleration.

After the sixteenth century philosophy moved rapidly along with the development of the natural sciences till a culmination was reached in the nineteenth century. Then, to the majority of scientists and philosophers, it seemed as if everything that happened in the physical world was subject to known laws governing changes in Euclidean space and absolute time. After this it was a small step for many people to pass to what seemed the logical conclusion, that nothing remained unaccounted for. Mind was merely an *epiphenomenon* or 'colouring' of the physical mechanism.

At the very height of its success, however, this 'materialism' sowed the seeds of its own destruction. Experiments led conclusively to the special theory of relativity, according to which there is no one spatial 'form of phenomena', but an infinity of forms varying from person to person according to their movement. Spatial and temporal separation are no longer absolute. Nevertheless all the various 'forms' are equally 'objective'.

[1] On the contrast between the mathematical basis of medieval science and the metaphysical theories still associated with 'physics', see A. C. Crombie, *Robert Grossteste and the Origins of Experimental Science*, 1100–1700, Oxford 1953, Ch. V.

Moreover, one person's clock may go fast or slow relative to another's identically constructed; and events which are simultaneous for one person may not be so for another. Yet the new theory does not present itself as a paradox forced upon us by experiment and possibly capable of being reinterpreted when its terms are better understood. On the contrary, it arises from a clarification of our ideas on space and time and the removal of certain hidden assumptions which were unjustified and so led to error.[1]

Matter, also, had seemed to break down into minute particles whose precise mass, position, velocity and electric charge could be measured indirectly by means of the theories. Along with the clarification of the concepts of space and time entailed by the special theory of relativity, it was inevitable that the concept of mass should require modification too. But the enormous field of experiment on which quantum theory is based forced altogether more startling conclusions. Energy, hitherto conceived of in terms of mass and velocity or the ability to produce velocity in a mass, is now directly related to frequency of oscillation; and the limiting of energy interchanges as between one system and another to complete 'packets' or *quanta* imports into physics an operation which seems inconceivable except as the bare outcome of a piece of mathematical formalism.

In experiments in atomic physics, phenomena regarded as evidence of particles (scintillations, cloud-chamber tracks, etc.) or of waves (bright and dark bands, line spectra, etc.) are continually observed. They are well reconciled in the theory, which also obtains astonishingly accurate and varied experimental confirmation. Nevertheless the reconciliation is achieved at the price of relinquishing the concepts of particle *trajectory* and particle *ensemble*, as these would be ordinarily understood. We find that elementary particles can be taken to exist in a certain position only at the instant when the experimental effect appears; and the wave function characterizing any atomic system stands merely for a set of possibilities, one of which will be actualized, in an unpredictable way, under particular experimental circumstances. Thus the 'mechanism' of nature, if one may use this term, remains 'behind the scenes', and cannot be represented as anything definite in physical space.[2]

The general theory of relativity, which dispenses with certain mathematical limitations in the special theory and thus arrives at a new way

[1] On the meaning of the term 'Materialism' see Ch. 22 below. The special theory of relativity was not by any means the strongest force militating against 'materialistic' views. But it obviously requires a philosophical principle of 'manifestation upon occasion' (below, pp. 202, 50) instead of the 'container theory' in which shapes and durations are absolute.

[2] See Ch. 20, esp. pp. 332 ff.

of formulating the gravitational field, began by providing some remarkable astronomical confirmations, considered at the time to be within the limits of experimental accuracy. These confirmations had inevitably to be made by means of an interpretation of the formalism in terms of the Euclidean space and physical time of the laboratory. Once again the 'mechanism' has receded into the background, and interpretation imposes both metaphysical and practical difficulties. Today, research in general relativity is chiefly directed to the construction of formalisms unifying gravitational, electromagnetic and quantum mechanical laws, to the solving of field equations in specialized cases, or to the controversial subject of cosmology, in which a smoothed-out universe is considered instead of the actual one. The extreme difficulties encountered, not only in the mathematics of this work, but also, and most noteworthy for our present purposes, in its interpretation, may be seen as themselves evidence of fundamental obscurities still greatly clouding our understanding of space and time, to the complexities of which there seems no end.

The picture so far presented of the twentieth century upheaval in the philosophy of space and time has been one-sided, being concerned only with mathematical physics and its implications. We must now look briefly at the complementary side, which is that of *mind* or *spirit*. The study of nature indeed presupposes a mind or spirit to devise and assess theories, to interpret phenomena, to decide what is objective or true, and in short to give the substance of intelligible life to the shell of words and symbols.

We have to thank the progress of the natural sciences for a successively clearer distinction of the genuine phenomena and laws of physics from myth, superstition and baseless conjecture. Similarly we have to thank the accompanying growth of the scientific spirit for a successively clearer distinction of realities of the personal and inner life from confusions and counterfeits. A mature and wide-seeking scientific spirit could hardly make the one distinction without the other.

Large areas of research have been in this way covered by the various branches of psychology. More important to remark on, in an introductory survey, is the general movement to adopt a changed orientation to the foundations of the natural sciences themselves. We have now at last learnt the virtue of 'suspending' the presuppositions and techniques which have worked their way into almost automatic acceptance, and then examining by direct insight exactly what we are doing. Philosophy of Science is the broad name for an investigation of this kind, in which the study of the historical development of scientific concepts plays a leading part. A higher degree of detachment and insight, as well as a more comprehensive view of all the conjoint manifestations of life—not merely measurable effects—is claimed by Phenomenology.

This brings us finally to two specialized fields of knowledge, with quite different subject-matter from that of the natural sciences, but now also, in this century, open to scholarly investigation free from popular superstition and sectarian prejudice. Evidence of the kind now customarily called *mystical* passed quite beyond the ken of scientists and nearly all philosophers in the eighteenth and nineteenth centuries, while the name 'mysticism' was taken up as a synonym for emotional impressions lacking in scientific basis. At the beginning of the twentieth century, however, largely through the prestige of William James, the amenability of mystical experience (as it is now called) to scientific investigation, and its character and importance, became more widely appreciated.

Since that time there has been an enormous increase in information generally available concerning the development of various degrees and modes of 'contemplation' and intelligible insight, so much so that a new danger has arisen of unscientific popularization of the subject. Partly for this reason there is a sharp cleavage of opinion among scientists and philosophers. Many do not see their way to accepting an 'order of reality' prior to that of physical phenomena and the laws connecting them. Nearly all scientists and the majority of philosophers in the western world seem to accept an *unknowable* background of nature, that is, a background to which there cannot be any mode of direct access even as regards features of general structure (nature being separated from mind, any mode of *internal* access, even the most general, is automatically ruled out). Others, however, including some outstanding names both in science and philosophy, hold that the information and evidence now available and scientifically presented is such as to justify accepting that there are modes of access which could be described as the highest grades of phenomenological penetration into reality. A small minority of scientists and philosophers claim, to a varying extent and sometimes only in private, first-hand experience of their own to this effect.

William James's investigations into religious experience were in part prompted, no doubt, by his previous interests in parapsychology—otherwise known as psychical research, which was established in the latter part of the last century[1] for the purpose of maintaining scientific values in this field. The presupposition of the 'container theory' of space, and the peculiar difficulties inherent in any study of unusual mental faculties, still hamper its development. Nevertheless, an immense body of evidence, carefully checked by experienced and scientifically-minded investigators, has now been collected on so-called

[1] The (British) Society for Psychical Research was founded in 1882. William James was President of the Society in 1894–5. The term 'parapsychology' is often restricted, by authors writing in English, to the more experimental branches of psychical research.

'extra-sensory perception' of many kinds, precognition, hypnosis, and out-of-the-body experience; and research workers in more than sixty universities, as well as numbers outside, are engaged in interpreting and enlarging it. C. G. Jung was expressing a general conclusion when he declared that all attempts at orthodox explanation 'come to grief on the facts . . . It seems more likely that scientific explanation will have to begin with a criticism of our concepts of space and time on the one hand, and with the unconscious on the other.[1]

[1] C. G. Jung, *The Interpretation of Nature and the Psyche*, London 1955, p. 27.

CHAPTER 2

COMMONSENSE AND THE SCIENTIFIC ATTITUDE

> 'Science is a first-rate piece of furniture for a man's upper-chamber, if he has commonsense on the ground floor.'
>
> Oliver Wendell Holmes,
>
> *The Poet at the Breakfast Table*, Ch. 5

Whatever it may be that we decide to study, we must begin by provisionally accepting what may be called a 'commonsense' view. That is to say, we take a view which enables us to make immediate sense of what we find and which accordingly suggests to us ways of carrying out further simple investigations so as to arrive at some workable system or principles of behaviour in that regard. Commonsense, however, does not consciously devise any such system. Its aim, as distinct from that of studies proceeding from it, is to reach a state of *immediate intuitive comprehension* so that we are not continually being surprised by encountering what we could not have predicted and do not feel we understand. And there is always the practical test, 'Does it work?'—a test which is *necessary* for truth, but not sufficient.

By this commonsense we learn how to behave reasonably with other people, how to judge from tone of voice and gesture, and how to guide our limbs and predict the movements and reactions of material objects, without giving a thought to theories of human personality or mechanics. In the same way we obtain a good practical grasp of 'space' and 'time', without concerning ourselves with theories. We may come across some familiar problems, such as that of the apparent 'infinity of the world' and its 'creation in time', or that of indirect measurement by means of optical instruments and geometrical construction or physical formulae. But these seem a matter for specialist philosophers or scientists, without much bearing on life in general.

It is natural to suppose, therefore, that commonsense is merely a crude sort of science, growing in us more by accident than by intelligent investigation, open to all kinds of error, and limited to everyday experience. The next step is to discount commonsense as against a properly developed scientific theory, tested by experiment. And then, since every theory is hypothetical, and every test partial and approximate, we arrive at an epistemology in which there is no such thing as

sure truth. All that remains is the hoped-for course of progress towards more and more effective hypotheses—a course of progress in which nature, paradoxically, is able to act as a policeman, turning us back when we attempt to go in certain directions, but never having any particular terminus in view.[1]

But this line of thought, as I hope to show, is a dangerous over-simplification of the problem, arising principally from the failure to distinguish between scientific method (which consists chiefly in the experimental testing of the consequences of hypotheses) and the scientific attitude (which consists in the readiness to analyse presuppositions and renounce those which the whole state of affairs shows to be unwarranted).

Rival Explanations

In illustration, consider the famous renaissance controversy concerning the relative speeds with which objects of different sizes and weights fall under gravity. Those who believed with Aristotle that a heavy object must necessarily fall more quickly than a light one would seem to have adopted a theory generalized from experience with objects such as a cannon ball and a feather, simultaneously dropped. The theory appeared to them obviously true, because the generalization was done almost unconsciously, and so the results did not seem open to argument. Galileo, on the other hand, fastened his attention on logical flaws which had been discovered in the Aristotelian theory (by J. Buridan, *c.* 1340, and others); for instance, if a heavy ball is conceived of as divided in two parts which are dropped together but not separated, each half would be required to fall less rapidly than the whole. Thus direct intuitive analysis, renouncing all presuppositions and dependence on hypotheses, showed Galileo that the Aristotelian theory could not logically be accepted, and pointed the way to an alternative theory in which differences in acceleration were consistently accounted for by means of the concept of *air-resistance* according to the size and shape of the falling body. When therefore Coresio dropped cannon balls from the top of the leaning tower of Pisa in order to refute Galileo,[2] he demonstrated at once his adherence to scientific method and his departure from the scientific attitude, in not seeing that conceptual analysis (independent of theories, by commonsense) must take precedence over apparent confirmations of theories obscurely seen but rigidly adhered to.

Again, the Michelson–Morley experiment showed that light-waves

[1] See Addendum A to this chapter, p. 35.

[2] H. Butterfield, *The Origins of Modern Science*, London 1957, pp. 81–2. J. Bronowski and Bruce Mazlish, *The Western Intellectual Tradition*, Pelican 1963, pp. 145–9. Lane Cooper, *Aristotle, Galileo and the Tower of Pisa*, 1935.

could not behave like sound-waves in air, having their velocity relative to us increased or decreased according as the wind carries the waves forward or retards them. If indeed there were a medium in which the light-waves were carried, then it was wholly without effect on their velocity. Other experiments forced the same negative conclusion, while none confirmed the 'classical velocity addition formula'. But nothing of this established the new kinematics of the special theory of relativity, as against Lorentz's theory of electromagnetically-caused length contraction and time-scale dilatation, which perfectly reconciled the experimental results with the classical formula. What made the special theory of relativity preferable was the fact that closer analysis (by commonsense) of the method of correlating clocks using a reflected beam, as in radar, showed that, in consequence of the null results of the experiments, the classical approach involved a vicious circle. For to determine the time taken by the emitted beam in relation to the reflected beam we would need to know the velocity of each beam; and to find these velocities we would need to have already correlated the clocks. Hence, there being no evidence of departure from uniformity (i.e. no further experimental results which might favour one direction for light rather than another), we are obliged[1] to take the velocity the same both ways—which is of course what astronomers have always done.

The clear point of origin of the special theory of relativity was in fact Poincaré's insight that, if the failure of experiments to determine 'ether drift' was absolute, the postulation of an absolute space-frame would be unjustified.[2] And the modern theory of relativity is founded on the further insight, broached by Poincaré in 1904 and firmly taken up by Einstein in 1905, that the Lorentz transformation must then be interpreted as a new kinematics of space and time, and that such interpretation, far from being paradoxical, is a true analysis of the observational facts.

Here then we have a situation not uncommon in mathematical physics, where two or more theories fit the facts equally well. It might be, and often is, argued that we should then choose the 'simplest' or most 'convenient'.[3] But in point of fact (as our illustrations have shown) if a clear analysis of the commonsense basis is at all possible physicists will choose the theory which conforms with that analysis. This is indeed necessary, since a theory whose concepts conflicted with the concepts

[1] See Addendum B, p. 36. [2] See Addendum C, p. 36.

[3] This theory of *conventionalism* is usually traced back to H. Poincaré. By 'convenience' (commodité) Poincaré usually means 'simplicity' in the sense of mathematical handling or representative picturing (*Science and Hypothesis*, trs. W. J. G., London 1905, p. 146). But elsewhere (*ibid.*, pp. 152 ff.) he admits hypotheses of a 'natural and necessary' kind.

unconsciously involved in its observational basis would not in fact be a theory to account for those observational data.

The Analyses of 'Pure' Commonsense

Before proceeding any further, it is necessary to clarify further the distinction between commonsense and the investigations or theories which it may suggest. I propose therefore the term 'pure commonsense' to denote the *intuitive grasp* of what is actually so, renouncing all dependence on theories. 'Intuitive grasp' will be taken to imply the kind of comprehension that arises when the total state of affairs, as it meets us face to face, is held in mind objectively in that form, without addition of thought or imagination. The resulting 'commonsense view of the world' must be held (following G. E. Moore[1]) to be in a certain fundamental way wholly true, though our analysis of it progresses from one degree of clarity to another and at any stage may remain in certain respects doubtful.

In its 'pure' form commonsense is thus superior to all theories and expressions in language or symbol, being the touchstone of the applicability of these formulations to the actual situation. Since, however, the power of intuitive analysis is exceedingly difficult to cultivate and very rarely proceeds at once to that degree of face-to-face self-evidence and detailed precision and certainty called *phenomenological*, investigation normally requires the provisional trying out of theories, i.e. sophisticated guesses.

If now we are investigating some problem and have *unconsciously* assumed a theory (correct or incorrect, applicable or not-applicable), our opinions, following on that theory, will seem self-evident. Thus the impossibility of telepathy seems self-evident to those who unconsciously suppose that causal action occurs by motion or other space-time changes in an intervening medium. In such a case we call our opinions 'commonsense'. But this commonsense is *impure*, since the limitations of the theory of causal action by contact have never been recognized by intuitive analysis on the ground of experience (the theory is in fact obviously false for gravitational action, as on the tides). If, on the other hand, we *consciously* apply a theory, recognizing its limitations (by the light of pure commonsense), the theory and its consequences are better called 'science'. For the deliberate introduction of working-hypotheses, tested by experiment, is the most obvious feature of scientific method.

I am not suggesting that 'pure commonsense' is like a magic mirror or oracle revealing at once the true facts of the case. It is to be regarded not as a witness, but as the judge. It collects whatever evidence is available from every possible kind of investigation, and, if 'pure'

[1] *Contemporary British Philosophy*, 2nd Series, ed. J. H. Muirhead, London 1925, pp. 193–223.

enough, declares what presuppositions are hidden in our consideration of it and what conclusions we are entitled to draw.

We must also recognize that in science, as in the critical assessment of any art-work, musical performance or philosophical doctrine, the fully discerning and just judge on all points is rare in the extreme. Out of a thousand persons professionally-trained along the lines required, there may not be even one who is sufficiently knowledgeable, clear-sighted and emotionally balanced on that particular subject to fulfil the demands of Faraday, that, if he wishes, with 'industry', to 'walk within the veil of the Temple of Nature', he should first be 'willing to listen to every suggestion but determined to judge for himself. He should not be biased by appearances; have no favourite hypothesis; be of no school; and in doctrine, have no master.'

Perhaps alone among scientists prior to this century, Leibniz saw that space and time were the manifestation of relations essentially not spatio-temporal but belonging to an 'intelligible' realm of possibilities. Perhaps no one before Poincaré understood that measurement involves the implanting into the world of a geometry which, at least in its application to distant events, is to some extent arbitrarily chosen.

No limit should therefore be placed on the degree of open-mindedness required of us if our conclusions in science are to be always trustworthy and if we are to be capable of progress towards ever deeper insights. Great humility is demanded. The scientist must be a philosopher in the sense of Wittgenstein, in being prepared to say, 'I don't know my way about', and to tell himself, 'Don't think, but look!'[1] Hence the scientific attitude consists first of all in the recognition of the primacy of these intuitive analyses in the comprehension of nature and life in general, and secondarily in the readiness to use working-hypotheses as a heuristic technique and practical tool.

Positivist and other Dangers

We must now turn to a consideration of the dangers against which Faraday's demands are intended to guard us—dangers which are otherwise likely to cause a serious lapse from the 'scientific attitude' as I have just described it.

The first great danger to be recognized is that of giving too exclusive an attention to the *known facts*, and so failing to see that 'there always remain in the abyss of things slumbering parts, which have yet to be awakened'.[2] It may not occur to us that the world could present quite a different picture in the light of what we have still to discover. There is an inclination to deny what is inaccessible to our ordinary faculties. So

[1] *Philosophical Investigations*, Oxford 1953, pp. 123, 139.

[2] Quoted from Leibniz. W. Macneile Dixon, *The Human Situation*, London 1938, p. 340.

life becomes frozen, as it were, into something that appears to be given once for all and to require no further exploration, except as regards the rational organization of facts already known.[1]

Or we may readily accept that there are facts known and unknown, but fail to see that without some conceptual system to make them intelligible the facts cannot even be said to exist. All measurement, as Popper has well emphasized,[2] presupposes a theory. On the other hand, any theory which is adopted as the basis of measurement must be conformable to 'pure commonsense' as found in nature, as, for instance, special relativity (rather than Newtonian kinematics) is conformable to our actual concepts of time-measurement in homogeneous and isotropic space. Thus *in one way our conceptual constructions in physics are free, and in another way limited.*

It is easy to overemphasize either of these extremes, and virtually forget about the inevitability of the other. If we overemphasize the limitation to what is now or may in the future be provided by nature, we may form the view that scientific theories are merely ways of summarizing the experimental results,[3] which are taken to be 'facts' simply given. 'Verification and long usage' may breed in us what Einstein calls 'the positivist prejudice' that 'facts can and should yield scientific knowledge' without 'free conceptual construction'. But Einstein was apt to overemphasize the freedom of invention permitted us by nature. His claim that 'the axiomatic basis of theoretical physics cannot be abstracted from experience but must be freely invented'[4] cannot be considered very happily expressed. Only slightly better is Toulmin's, that we 'put forward a new way of looking at the phenomena, with the help of which *we can make sense* of the observed facts'[5] (my italics).

[1] Cf. A. Koestler, *The Sleepwalkers*, London 1959, p. 216: 'The science of the ancients was taken as Gospel truth, not because of any particular respect for the pagan Greeks, but because it was obvious that since they had come so much earlier they had harvested all there was to harvest in these fields and left nothing but a few stray stalks to pick in the way of tidying up. Since there was only one answer to every question, and the ancients had filled in all the answers, the edifice of knowledge was completed.'

[2] *British Philosophy in Mid-Century*, ed. C. A. Mace, London 1957, p. 189.

[3] This is positivism, as usually understood. Compare R. B. Braithwaite: 'The necessity of a scientific law consists simply in its being a true generalization associating the occurrence of one empirical property with that of another' (*Scientific Explanation*, Cambridge 1955, p. 96). He denies that this 'generalization' implies any 'features of logical necessity' or even anything that is 'closely akin' to it.

[4] *The World as I See It*, New York, p. 36. See Addendum B to this chapter for an example of how this attitude of mind led Einstein into error.

[5] *Philosophy of Science*, London 1953, p. 41. The suggestion seems to be that nature is quite senseless till the scientist, with his freedom to invent reasons, comes on the scene. Toulmin indeed recognizes that Einstein 'almost tips over backwards in his effort to rebut' Mach's positivist doctrine. But his own view seems to

It has to be agreed of course that, *in order of time* and as a matter of common practice, we come to a scientific theory nearly always by trying out possibilities arising in our subjective imagination. But the temptation then is to suppose that the theory, as an account of the *universal order of nature*, has the status only of a free subjective invention, and consequently all logic and reason reside in the subjective mind. Nature itself is denuded of all reason. This is the error of modern phenomenalism, and to it Ewing reasonably objects: 'I do not see how they [the "linguistic philosophers"] can avoid accepting a conclusion which they certainly do not want to accept, namely, that there must be necessary connections in the real if we are to apply logic or mathematics to it at all.'[1]

The problem here (which I shall attempt to solve in Part II) is to exhibit precisely how the 'logic or mathematics' got into nature in such a way as to leave us a certain conceptual freedom. *We do not solve the problem by imputing the logic wholly to ourselves or wholly to nature.*[2]

Besides the dangers just mentioned there is another and even greater danger, which ends up in a very similar 'frozen' attitude to life—but in this case frozen in regard to concepts, not facts. This other danger is not so much the outcome of a *positivist* tendency (i.e. limitation of scientific theories to supposed 'facts') as of a *realist* tendency (i.e. an over-credulous attitude to theories) also fostered in us by 'verification and long usage'.[3]

Suppose that some puzzling phenomenon of nature is at last brought within the compass of reason and practical prediction by means of a 'free conceptual construction', as gravitation was by Newton's inverse-square law of attraction. Whenever a question concerning this phenomenon arises, the theory or 'construction' is then assumed, because obviously 'it works' (in the sense of presenting things as comprehensible). The theory begins to be taken as more and more obviously true *in itself* (i.e. self-evident) instead of being a 'saving of the appearances'. Eventually it recedes into the background of all our thinking and becomes 'impure' commonsense.

Each age has its 'climate of opinion', its 'fossilized images', its theological and anti-theological dogmas, and its 'familiar scientific truths' dating usually from some 50 to 100 years previously. Hence

differ from Einstein's only in his insistence on the 'stiff training' of the imagination which the physicist needs.

[1] *Contemporary British Philosophy*, 3rd Series, ed. H. D. Lewis, London 1956, p. 153.

[2] Or by claiming, with Braithwaite (*op. cit.*, pp. 80 ff.), that it is legitimately 'side-stepped' by simply refusing to speak about *concepts* or *entities* as distinct from *terms* of the calculus which represents the theory mathematically.

[3] I am here following Dr Mary Hesse's broad distinction between 'realist' and 'positivist' views on scientific theories (*Forces and Fields*, London 1961, pp. 1 ff.).

arises what Professor Macneile Dixon has called 'the tyranny of the concept'. 'Every man', he said, 'fancies that what he now thinks he will continue to think. There is an air of finality about our present opinions. They seem secure and inevitable. And this in the face of the most obvious and overwhelming evidence.'[1]

Defence Mechanisms

It seems that this adherence to the familiar and to beliefs corporately held is, with most people, necessary as a ground for confidence in their own sanity. By a psychological 'defence mechanism' anything new, seeming to undermine this confidence, comes to bear, in Whitehead's words, 'a certain aspect of foolishness'. Or it may operate another way, showing itself in a reaction of emotional hostility, immediately bolstered up by an appeal to 'logic'. For the accepted theories are taken to be logic itself.

Everard Feilding vividly describes this sense of 'undermining' in his account of some psychical investigations:

'The effect of all this on my mind was singular. I appeared to lose touch with actualities. Once admit the possibility of such things—and the mere fact of investigating them implied such an admission—where could one stop? . . . I gradually began to feel that if a man seriously told me that the statue of the Albert Memorial had called in to tea I should have to admit that the question to be solved would not be the sanity of the narrator but the evidence for the fact.'[2]

It is remarkable that many scientists, even among those of the highest repute, have failed to recognize the nature and power of these instinctive reactions. Hence, without investigation, or with an investigation of the most perfunctory and prejudiced kind, they have dismissed what is new as 'clearly impossible' or even 'immoral'. After a study of numerous *dicta* of scientists on the subject of inventions, mathematical discovery, medical research, clairvoyance and telepathy, Tyrrell went so far as to declare:

'The major part of the scientific world . . . is not animated by a scientific desire to know the truth but is in the grip of a psychological urge to disallow what is distressingly unfamiliar . . . When the real test comes, the scientist proves himself to be an *a priori* theorist at heart.'[3]

[1] *Op. cit.*, p. 62.

[2] G. N. M. Tyrrell, *The Personality of Man*, Pelican 1947, p. 227.

[3] *Ibid.*, pp. 251 ff. See also the dicta of famous scientists quoted by Dr W. F. Prince in *The Enchanted Boundary*, Boston Society for Psychical Research, 1930.

It may be urged in mitigation that the scientist, as such, is forced to ignore or gloss over difficulties concerning fundamentals and to become to some extent an '*a priori* theorist', if he is to get ahead with his proper task, which is to elaborate the superstructure of a practical theory. Nevertheless, the history of theories of space and time has shown how great a need there is for the investigator to be on his guard against making subconscious appeal to customary opinions and world-views as if they had the status of self-evident truths. For thus the way is blocked against insights which have nothing against them except their novelty; and this in turn may very much hamper the progress of science in general.

ADDENDA TO CHAPTER 2

Addendum A: Nature's 'Policeman' (p. 28)

The 'policeman' who turns us back when we attempt to drive our research in certain directions would be behaving reasonably only if he was obeying 'higher authority'—a constitution given in nature, the existence of which is denied by positivists and phenomenalists. Scientific Realists accept a 'higher authority', but place it in the 'theoretical entities' of the science of the day.

K. R. Popper (in *Contemporary British Philosophy*, ed. H. D. Lewis, London 1956) takes a middle course. He claims that 'there is a reality' (p. 384) which scientific theories attempt to discover, but can never ultimately find (pp. 368, 383). His *third view* 'preserves the Galilean doctrine that the scientist aims at a true description of the world, or of some of its aspects, and at a true explanation of the facts; and it combines this doctrine with the non-Galilean view that though this remains the aim of the scientist, he can never know for certain whether his findings are true, although he may sometimes establish with reasonable certainty that a theory is false' (p. 382).

With this I would agree, provided the inability to find 'reality' is asserted only within the realm of hypothetico-deductive method. For if we say, to take an analogy, 'There is a square root of 2, but we can never ultimately find it', the implication is that our concept of $\sqrt{2}$, if we were to exhibit it clearly, would go beyond the limited notion of 'finding' which we have presupposed. The logical requirement is thus that we first exhibit what we mean by $\sqrt{2}$. Then it must become clear by what different process it is 'found'. Likewise, therefore, it seems to me, we must first exhibit what we mean by 'reality' in the field of nature; and that will show what 'finding' it could mean in that context.

The resolution of the apparent paradox is not by the hypothetico-deductive method, but by some other method—namely phenomenological analysis of a transcendental degree.

Addendum B: Necessity of Equal 'There-and-back' Velocity of Light (p. 29)

Einstein was evidently very impressed by the fact that the choice of the same velocity both ways seemed to him quite 'free'. He refers to it as 'a stipulation which I can make of my own freewill to arrive at a definition of simultaneity' (*Relativity, the Special and General Theories* London 1920, p. 23), and Reichenbach follows him in saying: 'Einstein's definition . . . is just one possible definition. If we were to follow an arbitrary rule restricted only to the form $t_2 = t_1 + \epsilon(t_3 - t_1)$, $0 < \epsilon < 1$, it would likewise be adequate and could not be called false' (*The Philosophy of Space and Time*, New York 1958, p. 127).

But any rule proposed must be a general one, consistent when applied in all possible circumstances. In other words, it must give rise to a *group* of transformations. This implies either homogeneity, with local velocity the same in all directions and at all points (as shown in Chapter 15, pp. 222 ff.), or else the admission of some independently measurable parameter such as the velocity with respect to air or ether. The postulation of significant variations in the formalism, when there are no associated variations in actuality, would be ruled out by *Weyl's Principle* (Ch. 9, p. 112), to which Einstein had already in effect made appeal in ruling out the possibility of velocities relative to an absolute frame of reference.

In an empty part of the world, therefore, where we can assert no velocity of light-waves relative to some detectable medium, a consistent space-time geometry or means of measurement requires that the velocity of light is the same in all directions.

Addendum C: Poincaré or Einstein as Originator of the Special Theory of Relativity

So long as Einstein was alive the question whether the 'intuitive spark' which originated the special theory of relativity came from Poincaré or Einstein was apt to be a somewhat emotional one. The facts, however, seem clear enough (E. T. Whittaker, *A History of the Theories of Aether and Electricity*, London 1953, pp. 30–31; G. H. Keswani, 'Origin and Concept of Relativity', *B. J. for the Phil. of Science*, XV, Feb. 1965).

A denial of the right to postulate an absolute space-frame or ether was in complete accord with Poincaré's strictly operational and 'conventional' approach to matters of science (he even pointed out that there was no *proof* that the earth was flat). But because such denial

meant the overthrowing of the 'container theory', 'absolute time', and in fact the re-casting of all the foundations of physics, it is not surprising that even Poincaré approached this unprecedented viewpoint very gradually.

Thus in 1899 he said tentatively, 'I regard it as very probable that optical phenomena depend only on the *relative* motions . . . and that this is true not merely as far as quantities of the order of the square or cube of the aberration, but *rigorously*'. In 1900 he said of the aether, 'Does it really exist? I do not believe that more precise observations could ever reveal anything more than *relative* displacements.' In 1902 he was emboldened to say outright, '*There is no absolute time* . . . we have not even direct intuition of the *simultaneity* of two events occurring at two different places'. By 1904 he was emphatic that 'we have not, and cannot possibly have, any means of discovery whether we are, or are not, carried along in such a motion [uniformly translated relative to a "fixed" observer]'. And he went on to suggest that 'we should construct a whole new mechanics . . . where, inertia increasing with the velocity, the velocity of light would become an impassable limit'.

Although Poincaré had thus arrived at the main conclusions of the special theory of relativity a year before Einstein's first paper on the subject, it was Einstein's firm acceptence of the 'new mechanics' and derivation of other results (aberration, Doppler Effect) from it that made the scientific world realize that a new view of nature was being seriously urged and would have to be reckoned with.

CHAPTER 3

SEVEN APPROACHES

'. . . a dim and perilous way'.

Wordsworth, *The Excursion*, Book 3

'When we wish to correct with advantage, and to show another that he errs, we must notice from what side he views the matter, for on that side it is usually true, and admit that truth to him, but reveal to him the side on which it is false.'

Pascal, *Pensées*, I. 9

'Our greatest future progress in Science may lie in the hands of men who are willing and able to ignore the artificial classifications which we have erected, men who will readily criss-cross these fields in the pursuit of an idea.'

Melvyn Calvin

In the previous chapters I have tried to show how natural science, by its own course of historical development and the open-minded application of 'commonsense', has been forced into recognizing a truth that would have been unpalatable in the extreme to earlier scientists. It has had to recognize that problems hitherto regarded as strictly scientific are so entangled with philosophical ones that isolation of the one type from the other is not possible. Conversely, many problems hitherto regarded as philosophical are now seen to require a foundation in scientific knowledge if their treatment is not to be superficial and quite possibly in serious error.

The Language of Science

In this situation the tendency naturally arises to enlarge the definition of 'science' so as to be able to say that the problems called by many people 'philosophical' are really 'scientific' ones, where they are not 'fictitious'. Then there is only one approach, namely the scientific one. This is a simplification of view the influence of which can be seen in many modern trends of thought—Logical Positivism, the 'Unity of Science' Movement, Logical Empiricism, and modern phenomenalism generally. It seems to be brought about by anticipating a state of affairs when all intuitive analyses are completed and precisely formulated in language, so as to be capable of being understood by everyone in exactly the same way. Then, by extending the existing language of

science till it includes (in imagination) these mythical formulations, it is clear that we shall have one language only, denominated 'scientific'; and instead of conceptual problems we shall have linguistic ones. More generally, we may have alternative languages all 'scientific' in this sense; but in some the 'conventions' may be more precisely specified than in others, so that they become better instruments for science.[1]

This supposed reduction of conceptual problems to linguistic ones exemplifies, I believe, an error which appears sometimes with catastrophic results in mathematics when we are tempted to place too much dependence on the formalism.[2] I shall call it the *chicken-counting* fallacy, because it may be said to consist in anticipating a completeness of analysis and definition which is not attained and may be actually unattainable. The remedy is to formalize nothing which is not supremely clear conceptually.

A linguistic 'simplification' of the kind just considered seems to be what Professor Coulson has in mind when he argues that if the results of experiments in extrasensory perception and precognition are as claimed they will merely become the basis on which scientists will build a 'revised model of time and space'.[3] To this argument one may answer, first, that in fact the analyses have not yet been made, and so cannot be expressed in a wider 'scientific' language. And secondly, any adequately analysed problems can be expressed in a suitable language, and we do not obliterate the distinction between types of problem by lumping together the languages used to express them.

No existing language, therefore, can be taken as a substitute for the intuitive analyses of commonsense. Nevertheless the various existing languages concerned with space and time—of physical science, neurophysiology and biology, psychology, philosophy of perception (in so far as this can be distinguished from the other studies), parapsychology, and mystical experience—may be taken as indicating, by their very differences, the existence of real distinctions of subject-matter correspondingly.

Moreover, there then arises the study of the relations between the various languages, including the critique of all approaches in the light of self-evident (i.e. phenomenological) insights such as those of Poincaré and Einstein already mentioned. These provide a higher ground, neither deductive nor inductive, from which conceptual systems and all other opinions and conclusions can be evaluated in respect of final validity (and not mere simplicity or convenience).

[1] A. J. Ayer, *The Foundations of Empirical Knowledge*, London 1940, p. 57. See also the discussion by A. C. Benjamin, *The Logical Structure of Science*, London 1936, pp. 238 ff.

[2] The numerous paradoxes of set-theory are familiar examples of this fallacy. For other instances, see below, pp. 40, 101.

[3] A. C. Coulson, *Science and Christian Belief*, London 1955, p. 27.

All this has to be expressed in an appropriate 'seventh language'. According to the scope and level of analysis, we may speak of Philosophy of Science, or Phenomenology in general, the highest grades of the latter being (in this book) termed 'mystical'.

Results of Measurement

To devote our attention to the first of these languages (that of physical science) means that we concentrate on problems of *measurement* and *natural law*, assuming that what we mean by perception of physical objects is sufficiently clear and does not require analysis. But there seem to be also various other vital assumptions, which are hidden, and must be brought to light by phenomenological analysis.

Foremost is the assumption that, once standards of length and time have been fixed with the help of measuring apparatus and an agreed procedure for 'correction' in accordance with certain theories, *no further analysis of measurement itself is needed*—only analysis of the *results* of measurement. In so far as the implication is that exact measures somehow *exist in nature*, and that we merely have to find them as accurately as we can, it is again 'chicken counting'. For no experimental procedure can actually give us such exact measure. The commonly made assumption that the givenness of nature consists in an exactitude of *measure* given everywhere in space and unceasingly in time is largely responsible, as I aim to show, for the prevalence of materialistic or other 'container-theory' views of nature, and for similarly mistaken theories of perception.

Closely associated with this is the assumption that *all exact knowledge of space and time is expressed in laws concerning such measurement* of physical objects or conditions by standardized apparatus. These laws, it is indeed customary to say, are 'irreducibly statistical' in character. Although they might predict an average value of $\sqrt{10}$, say, for a certain theoretical quantity, there is no guarantee that either this or some other particular value would be actually measured on any occasion (it would be absurd, of course, to require that $\sqrt{10}$ should ever be measured exactly). By such means there is a kind of reconciliation between the claim for exact knowledge and the fact of uncertain results of measurement. In some incomprehensible way, it might be said, the latter *approaches* the former. But this does not justify the assumption that exact knowledge is *reducible* to measurement. Such assumption of reducibility is a 'category mistake',[1] rather like that of Frege and *Principia Mathematica* in defining the cardinal number 3, for example, as the class of all particular triads. How do we know that three apples are three and not two, it might be asked. The answer is that we have a prior[2] understanding of what 'two' and 'three' mean, apart from the

[1] G. Ryle, *Proc. Aristotelian Soc.*, 1937–8, pp. 189 ff.

[2] In an epistemological, not temporal, sense.

existence of apples. The attempt to explain the universal 'three' in terms of particulars which involve it is therefore logically circular. In the same way, *we cannot explain the universal concepts of space and time in terms of measurements which presuppose them.*

The attempt to do so would be an example of a common error which I shall call the **Abacus Fallacy**. Explicitly stated, this consists in supposing that some more or less mechanical procedure can be the ultimate source of knowledge; or that because we arrive at comprehension of *a priori* truths by empirical procedures, those truths are *nothing but* those empirical procedures. It is the confusion of teaching and heuristic helps with the establishing of the knowledge itself by direct insight.

Measurement, in the usual narrow sense of the word, cannot therefore be the ultimate means of knowledge of truth or reality in the physical world. But it remains quite invaluable as a help to precision of thought and the discovery of intelligible relationships which are somehow hidden, in ideal quantitative (static) and creative (dynamic) form, within all that happens.

Perception of Physical Objects

The second approach to problems of space and time, by the philosophy of perception, becomes important as soon as we try to carry out the necessary further analysis of measurement, without assuming that what we mean by a physical object is not in need of analysis. In the philosophy of perception one of the chief problems is the very difficult one of distinguishing between what is to be called 'physical' and what is not, and of describing the characteristic features of the 'physical'. This is considered in the next chapter and need not be broached here.

Some present-day authors have strongly urged a *neurophysiological* approach to problems of physical perception,[1] and in the same line of approach we may perhaps put problems in related branches of experimental psychology. There seems indeed a large field here only recently becoming open to exploration, and what is so far known gives much food for thought. Nevertheless, as Professor H. H. Price pointed out in 1932,[2] these sciences, far from solving the primitive philosophical problems of the 'external world', presuppose a certain solution of them—as Sir Russell Brain has made clear: 'If we start with knowledge or experience, we start with the subject-object relationship already given.

[1] Lord (E. D.) Adrian, *The Physical Background of Perception*, London 1947. J. C. Eccles, *The Neurophysiological Basis of Mind*, Oxford 1953. Sir Russell Brain, *Mind, Perception and Science*, Oxford 1951; *The Nature of Experience*, London 1959. J. R. Smythies, *Analysis of Perception*, London 1956. R. J. Hirst, in *The Problems of Perception*, London 1959, refers often to 'nerve impulses' and the work of neurologists, though he occasionally differs from their conclusions on philosophical questions.

[2] *Perception*, London 1932, pp. 2, 27, 159.

We do not need to ask how we become aware of things outside ourselves because it is with that awareness that we begin.'[1]

Some of those neurophysiologists who realize that something is presupposed and that the subject-object relation needs analysis, take their stand on a Representative Theory of Perception. This is a variety of Scientific Realism which I describe in Chapter 22 as 'epistemological pragmatism' and which, further, presupposes the Sense-Datum Theory (dealt with in Chapter 4). As I understand it, the modern representative theory of perception maintains (A) that sensing is in some way a symbol which we have to interpret (by inference, 'logical construction', 'projection', settling of linguistic usages, etc.) in order to arrive at the concept and total impression of an 'external object'; (B) that this construction, projection, etc., are *entirely private*; yet (C) such 'objects', if not illusory, have a *public* character, because of the objective substructure of nature, claimed to be unknowable except through a certain uniformity of effects at different times and to different people (this uniformity being expressed in science as physical laws). An adherent of this theory will hold that the only justification for believing in these 'external objects' is that such an hypothesis provides 'the best explanation of the world'.[2]

There seem to be, however, at least five crucial problems which such a theory leaves unsolved:

(1) How there occurs the projection of 'objects' from the *private* activities in the brain on to a *common space*—after reversal, inversion, and entire spatial reorganizing and combination of the points of excitation, and followed by a private conceptual interpretation—so that at the end the whole is seen as if from the retina, where no visual sensation occurs.

(2) How, from the private conceptual interpretation of the projected images, we arrive at *natural law*, which is obeyed when no one is projecting the images.

(3) How our brain itself is 'projected' so that it can also be an object for us.

(4) What are the kinds of continuity of appearance which are sufficient to justify saying that 'an object' is really there (cf. the case of mirror-images, or rainbows), and why they justify it.

(5) How the respective modes of experiencing, by the physical senses and by the non-physical senses, are to be distinguished.

The modern representative theory of perception thus seems to be a combination of the container theory of space (holder of physical objects) and the container theory of mind (subject). The gap between the sharply divided subject and object is closed by a simple presumed inference, or perhaps by some other logical or linguistic device.[3]

[1] *The Nature of Experience*, p. 31.

[2] Eccles, *op. cit.*, pp. 280 ff.; Smythies, *B.J. for the Phil. of Science*, XI. 43, p. 224. See also the Addendum to this chapter, p. 44.

[3] A. J. Ayer, article in *British Philosophy in the Mid-Century*, ed. C. A. Mace, London 1956, pp. 222 ff.

Neurophysiology and experimental psychology are therefore to be ranked as epistemologically posterior to the philosophy of perception and can have only heuristic value as far as that is concerned. It is obvious in fact that if perception is 'happenings in the brain', then perception of those happenings must consistently be 'happenings' in another brain. Hence we are involved in a continual regress and never arrive at the study of perception itself.

Those who dispense with the theory of perception and explain the objective presentations of the world in terms of what goes on in the brain are amusingly referred to by Professor Macneile Dixon, as follows: 'Some philosophers, having done their best to cut off the branch upon which they are sitting, continue with superb confidence to sit upon it.'[1]

Similar remarks apply to the approach to problems of space and time through biological investigations,[2] which are also highly suggestive, but presuppose solution of the more primitive problems which are our concern here.

Parapsychology and Mystical Experience

The other two approaches—through parapsychology and mystical experience—by no means lead to the study of what may be called *concomitant mechanisms*, epistemologically posterior to the philosophy of perception. Mystical experience, objectively studied and reported as such, certainly does not presuppose any solution of more primitive problems. As regards parapsychology, the placing of objects in physical space is indeed taken for granted and left unanalysed; but the problems raised by extrasensory perception, out-of-the-body experience and precognition are not less primitive than those raised by physical perception.

A few remarks have already been made, in Chapter 1, on these two approaches; and the general question of 'stabilized levels of creativity' and their special kinds of 'reality' will be considered in due course in Chapters 8 and 21. Here it seems only necessary to add a brief explanation of some points of terminology.

In regard to the higher 'stabilized levels' of creativity and phenomenological insight, I shall adopt the scholastic distinction between two types of knowledge of intelligible self-evidence, namely: (1) *existences*, manifested in particular intelligible forms; and (2) *essences*, i.e. conceptual details or structures, known in their character as timeless and always 'the same', wherever and whenever they enter into 'existence'. The existences are intelligible because the archetypal essence (or 'reason') is clearly within them.

[1] *Op. cit.*, p. 62. The metaphor is no doubt borrowed from Price, *op. cit.*, p. 159, who also suggests (p. 311) the convenient term *physiologism* for the error represented.

[2] For example, Sir Alistair Hardy, 'Biology and Psychical Research', *Proc. S.P.R.*, 50, Part 183, May 1953, pp. 96 ff.

A similar distinction between existence and essence appears 'darkly' on the plane of ordinary reflection, as a distinction between the particular (e.g. three apples) and the universal (e.g. the cardinal number *three*). In either case it is important to note that we are dealing not with theories but with attempts to describe the findings of self-evident analysis, either 'darkly' or 'face to face'.

Error and Illusion

If imaginative and subjective elements blend with what is objectively given in any sphere of experience (physical or non-physical), it is also the business of phenomenological analysis to sort out the various factors and distinguish what is proper to each sphere. Illusion and error (including mistaken interpretation and imaginative additions) are thus explained as the confusion of different spheres of experience, through inadequate powers of analysis, and the 'cementing' of such confusion by personal 'belief'.[1]

The fact that one may have at some time attained to what is for oneself the 'absolutely faithful description' and the 'unshakable because self-evidencing conviction of truth'[2] imputed to the highest grades of phenomenological analysis does not preclude subsequent misinterpretation, confusions, and lop-sided judgements. This 'impure commonsense' may be even more dangerous than the impure commonsense which we have grown into unconsciously. Thus the ordinary degrees of detachment and phenomenological analysis, by which we are able to prevent a slipping back into distortion, darkening of view or total loss of comprehension, are certainly not less valuable than the ultimate analysis obtainable only in the highest grades.

[1] Compare Professor G. Ryle's excellent exposure of the view, 'which is accepted almost as an axiom', that all truths are inferred and hence errors are mistaken reasoning. 'Sensation', *Contemporary British Philosophy*, ed. H. D. Lewis, London 1956, pp. 436 ff.

[2] These are translations from Husserl. See pp. 108, 406.

ADDENDUM TO CHAPTER 3

R. J. Hirst's Theory of Perception (page 41, note 1)

Hirst puts forward a theory of 'three aspects' of perception—consciousness, hypothetical causal chain, correlation of the two. He explicitly rejects the representative theory of perception. But his own theory seems to be clearly *representative* in the sense defined on p. 42. Private consciousness symbolizes a presumed causal chain of inferred and otherwise unknowable objectivities in an already-posited external world.

The special feature of Hirst's theory is the 'correlation' by his 'Identity Hypothesis' (pp. 191 ff.). According to this, 'a ray of red light and a train of electromagnetic waves of 760 mμ are identical', 'only appearing to differ owing to the different modes of access to them' (compare Whitehead's distinction between the modes of 'presentational immediacy' and 'causal efficacy'). This seems to mean that the hypothetical causal chain and consciousness are, in the correlated view, *postulated* to be two aspects of the same 'whole event'. Neither the causal chain nor the correlation can be directly observed, because 'all conceivably relevant types of [empirical] test' are needed (p. 304) to establish 'real characteristics'; and such tests presuppose a set of hypothetical physical laws concerning 'objects' already posited in a presupposed external and public world. Hence all one can do is to postulate the coincidence of the observations and the hypothetical causal chain in the same 'whole event', till science adapts an improved hypothesis.

This is in fact exactly what has happened—with the result, as it seems to me, that Hirst's postulate (and Whitehead's likewise) is no longer tenable. For it has been found necessary in physics to accept that the 'hypothetical causal chains' are probabilistic, i.e. they belong to a sphere of potentialities, which *cannot* be identified with definite sensations such as 'a ray of red light'. There is a gap between the formalism and observation which renders the 'classical applicability' of the former necessarily imperfect. Even in electromagnetic theory the quantum 'functors' **E** and **H** of the waves are non-commuting operators, and *they*, along with the wave-equation (which is not picturable as events in space and time) as limited by the total boundary conditions, constitute the indivisible 'causal chain'—not any measured wavelength such as '760 mμ'. (These features of quantum theory are explained in Chapter 20.)

CHAPTER 4

DEFINITION OF THE 'PHYSICAL UNIVERSE'

'The thing and the world exist only in so far as they are experienced by me or by subjects like me, since they are both the concatenation of our perspectives; yet they transcend all perspectives because the chain is temporal and incomplete.'

M. Merleau-Ponty, *Phenomenology of Perception*, trs. Colin Smith, London 1962, p. 333

'Upon the great world's altar stairs
That slope through darkness up to God.'

Tennyson, *In Memoriam*, Canto 55

The terms *natural*, *physical* and *material* are all used to describe the things which affect our bodily senses, possess a continuing public character, and together constitute the spatio-temporal system which we call 'the world'. Of the three, the term *physical* is the most precise in meaning and least open to misunderstanding, as we can see by replacing it by one of the other terms in phrases such as, 'physical reaction', or 'the physical characteristics and appearance of a person'. For the sake of clarity I shall therefore adopt the term *physical*, and not use the other two terms except in a non-scientific and easily understood sense.

Unfortunately, owing to the manner of growth of the science of physics, the term *physical* has taken on two quite different connotations, and so perpetuates a 'category mistake' against which we need to be particularly on our guard in the philosophy of science. On the one hand it is used to refer to what is sensed here and now, in the ways just indicated. On the other hand it is used to refer to what is imaginatively pictured in terms of quasi-spatio-temporal 'theoretical concepts' and operations of some hypothetical conceptual system. Thus we might say: A piece of physical apparatus called the Geiger counter is able to detect the arrival of an electron in a small chamber', or, 'in the photoelectric effect the current flowing indicates the release of electrons through the absorption of photons from the incoming electromagnetic wave'. Here we use the word 'electron' as if it could replace the word 'pin' in the phrase: 'See a pin and pick it up'; and we use the words 'photon' and 'electromagnetic wave' as if they could replace the words

'leaf' and 'book' respectively in the phrase: 'A leaf is torn from this book.'

It is obviously a serious category mistake to imply that what is sensed here and now, and certainly, is the same kind of thing as what cannot be sensed in any way and whose very existence is hypothetical.

Theoretical 'Entities' of Physics

There are indeed two kinds of objection sometimes made to the distinction I am drawing here. According to the one line of thought (Whitehead, Hirst[1]), the causal substructure and what is consciously perceived by us must be put in the same category to avoid the 'bifurcation of nature' into two realms of actuality. According to the other line of thought, an 'entity' such as America is none the less 'real' because 500 years ago its existence for Europeans was hypothetical.[2]

But the word 'America' may suggest to us five very different things: (1) a class of imagined scenes and facts which we associate with a certain imagined place, according to what we have read or worked out for ourselves and come to believe in; (2) a class of memories which we possess because of having lived there; (3) the present impression of scenes and objects around us while we are living there; (4) the sense of a *common objectivity* in those scenes and objects, so that other people present, with whom we seem to be in communion, are sensed as perceiving something similar to us, though adapted to their different circumstances; or (5) the intellectual conception of a common objectivity, such as we have experienced in America, associated in our thought with the name 'America'.

As far as signification (1) is concerned, 'America' is no more real than Atlantis or Lilliput, and was certainly not a 'real entity' for Columbus in 1490, any more than phlogiston and the luminiferous ether were 'real entities' for eighteenth-century scientists. The significations (2)–(5) are commonly taken as merging together in such a way that one of them implies (not necessarily at the same time) the occurrence of the others. But, strictly, only (3) and (4) are 'real', (2) and (5) being 'thought-forms.'

Now in the case of photons, electrons, mesons, etc., we have only a signification of the type (1). Moreover, whereas there was a prospect that, in course of time, Columbus's ideas on a continent over the seas (he thought it would be part of Asia) might be found to have a certain correspondence with 'real' significations, there is virtually no prospect of this happening with photons, etc. (for these are *unobservable*). The only hope would be for a signification (4) to appear in some mystical state transcending physical observation but causally related to it.

[1] See Addendum to Chapter 3, p. 44. Whitehead's views are considered in Chapter 24, pp. 402 ff. See also Addendum A to this chapter, p. 62.

[2] This example was suggested to me by Dr Mary Hesse.

Altogether, there seems far too much optimism in speaking of photons and other 'particles' as if they were identifiable objects in space and time. Models of an electron, for instance, as a Euclidean point, as a particle carried by a probability wave, as a particle in an orbit, or as a distinct member of an ensemble, are no longer admissible except as heuristic makeshifts. The most acceptable view, at present, might be that the term 'electron' is used conventionally in connection with certain spinor-fields (not representable in any inertial frame of space and time), defined by linear wave-equations of a certain type, containing 'directional operators' of unknown significance, these equations and fields being related to other equations and fields (in ways not yet clear) and ultimately to certain macroscopic experimental measures.

Suppose, however, that a certain amount of optimism concerning the popular physical reification of mathematical features of these theories is held to be justified. Would the objection to 'bifurcation', as between such entities and actually observed physical phenomena, remain? The objection might have force if some spatio-temporal 'actual entities' were put in one category and the rest in another category. But nothing of the kind is proposed in modern quantum theory. We are not suggesting 'two realms of physical actuality'. Broadly speaking, quantum theory introduces, on the contrary, a realm of *potentiality* actualized according to the occasion of a particular experience. Wave functions simply are not in the same category as observables, and it would be a fatal error to confuse the two.

Perceptual Analysis

In this and later chapters I shall try to keep clear the distinction just made between the two universes of physics, calling the one *the physical universe* and the other the *universe of mathematico-physical constructs.* The first thing to attend to then is the precise significance of the phrase 'what is sensed here and now', which was used above in characterizing the physical universe. This leads to consideration of *sense-data*, such as the yellow colour of a daffodil, the pitch of a musical note heard, or the rough feel of woollen material touched on some occasion. And at once we run up against further ambiguities.

Can these qualities really be abstracted from positions and times? It seems that they can be abstracted in two ways. The first way is by phenomenological analysis of a particular experience in space and time. In this case we can say (using the terminology introduced in Chapter 3) that the qualities, purely as such, are known in *essence.* They are abstracted from particular *existences* much as the number *three* can be abstracted from the sight of three apples while we are observing them. The qualities are thus known in essence (as if out of space and time)

and simultaneously in a particular given experience or existence. We can perhaps say that they are concept and percept in one. This is so even if the quality is a spatial or temporal one, such as straightness or equality of duration.

There is little reason in trying to put *qualities* in a certain category of 'sense-data' and forgetting about *relations*. Straightness is both a quality and a relation. Even the pitch of a musical note is generally recognized as a *relative* pitch; and the yellow of the daffodil is spread through different shades in different places and ceases to exist if a strict Euclidean point is taken. Let us therefore speak rather of 'details of perceptual analysis', understanding by this phrase that we include every detail of spatio-temporal organization, directly perceived, which helps to give the impression of objectivity, spatial position, solidity, and 'physical reality'.

Sense-Data

The second way of abstracting qualities of objects presented to us is to remove from them, in concept or imagination, every particularity of space and time, leaving merely 'universals of thought' represented by *names*. They become (in our thought) like a vocabulary of aids to memory and communication or to creation by ourselves (in imagination) or by nature (in what is given). It might be better therefore to call them 'universals of imagination'. In thinking of an object in terms of these universals, we imagine as it were a list of names—which in no way constitute the object, but may suggest to us a fuller conception of what the object might have been, with the missing synthesizing relations supplied by imagination.

Those who favour a 'sense-datum theory' seem usually to take sense-data as on the one hand abstracted and named in the way just explained, and on the other hand made into *particulars* by association with particular times, directions from the observing eye, touches with some part of the body, or other particularizing qualities thought of as subjective and private.[1] The spatial objectivity and externality of the object are then supposed to be *inferred* from these limited sense-data.

One of the reasons for this queer procedure, so far from common-sense, seems to lie in the adoption of an equally queer way of distinguishing between *physical reality* and *illusion*. In the former case, it is said,

[1] The conception of sense-data as a list of named qualities (at a given position and time) appears in Ayer's explanation that 'What the statement that material things consist of sense-data must be understood to designate is not a factual but a linguistic relationship' (*The Foundations of Empirical Knowledge*, p. 232). The fact that no such list can supply the total impression of intelligible objectivity seems to be admitted when he says, 'It is not by any verbal analysis in terms of sense-data that one can hope to make clear what is meant, for example, by "a pen" as opposed to "a pencil" ' (*ibid.*, p. 242).

we have *sensation* plus something which makes it into *perception* (i.e. an inferred spatial objectivity and externality), whereas in the latter case we have only sensation. But whether we see a table in the physical world, or in a dream, or in a vision, in each case there is an object. So the same argument would demand that we have not only physical perception, but dreamlike perception, and visionary perception, and so on. We shall then merely have adopted a pretentious way of re-stating the fact that one kind of perceptual object is 'physical', another 'dreamlike', and so on, without discovering in what the difference consists.

The view I am putting forward, in contrast to the sense-datum theory, is that the object or phenomenon, of whatever kind it may be, is given first, as a recognizable particular existence; and the *details of perceptual analysis* (as many as we have time and ability to identify) are then successively discovered in it, so that it becomes, with the help of our own powers of constructive thought and numerous intelligible relations clearly or dimly perceived, meaningful to us.

The Substructure

There is of course a substructure of the whole physical world, accounting for the objects being regarded as 'the same' on different occasions, for the diverse appearances of 'the same' objects to different people, and for the observations that are made when certain experiments are performed on those objects. The attitude we should take in this matter seems to me as follows.

If there are details of perceptual analysis considered to belong specifically to the substructure of a given experience, and each is for us associated with a particular physical position and time (inside or external to our physical body) we classify them as *physical*, along with the more obvious details of perceptual analysis, such as colours and touches. Otherwise, according to their character, we may classify them as various types of *non-physical* experience.

The extent to which this classification is conventional or, on the other hand, can be made absolute, is one of the chief matters to be inquired into. Shelving this question for the present, we may note that to demand (as opponents of Direct Realism seem to do) the addition of imagined or conjectural features of a causal substructure before we are prepared to admit the 'physical reality' or 'directness' of an encounter with a physical object hardly seems rational. For all those qualities giving the impression of 'physical reality' and 'directness' to our observations of a physical object are, as already stated, to be included in the details of perceptual analysis, discoverable in the encounter.[1]

[1] On the error of supposing that 'questions about perception are merely causal questions', see G. Ryle, *Sensation*, *op. cit.*, pp. 438 ff.

Assuming then, provisionally, that it is sufficiently clear whether or not a perceptual detail is associated with a particular *physical* position and time, we can reasonably define the *physical universe* as the totality of all physical percepts, perceptual relations, and other details of perceptual analysis. Putting it more loosely, the physical universe is what we can see, hear, smell, taste, touch, or sense in any other way in some spatio-temporal organization, while we are in our normal physical state of life. This does not imply that a tree disappears when we are not looking at it. The gap between one observation of the tree and another observation of 'the same' tree is bridged by *something given* (otherwise we should have no ground for calling the object 'the same'). But that something given is, by definition, *not part of the physical universe*; it must belong to a substructural or potential universe transcending the physical universe. How the details of this substructure may be disclosed to analysis is at present not in question. It is sufficient to characterize these circumstances by saying that the tree exists *in potentiality* (i.e. as a given origin of possible experience), and is not actualized in particulars of the *physical* universe, except when, and according to the manner in which, someone observes it.[1]

The difficulty which remains before us is how to distinguish *physical* perceptions from the many varieties of perception which cannot satisfactorily be described as 'physical'. It is customary among philosophers to restrict the term 'perception' to physical perception, assuming that an absolute or at least very workable distinction can be made (and is in fact made instinctively or by habit, as it were) between what is physical and what is not. The attempts made at analysis of this 'perception' then amount to attempts to uncover the distinctions in question, made by instinct or habit, especially those between a (physical) perception and a presentation which we would call 'illusory'.

Until this analysis is satisfactorily done, it will be misleading to restrict the term *perception* to what belongs to the *physical* universe, thus implying (what is not true) that the circumstances commonly used to rule out 'illusions' would also necessarily rule out all kinds of non-physical experience. I shall therefore always add the epithet 'physical' to the term *perception* if the reference is to the physical universe.

Criteria of 'Physical Reality'

Philosophers have proposed several criteria for distinguishing physical perception from 'illusion'. These, as we shall see, mostly reduce to an attempt to characterize an *objective continuity* which has the effect of a recognizable 'reality' amidst varying 'appearances'. A genuine physical

[1] See Addendum B, p. 63.

appearance (by which is to be understood an appearance belonging to a succession characterized by such continuity) will present to us some 'physical reality', while an illusory appearance represents no such reality.

The criteria proposed by scientists, on the other hand, have the chief purpose to establish the 'physical reality' of certain theoretical *constructs*. Every construct begins by being hypothetical; but it might require to be recognized as having some kind of existence, as the mass and charge of an electron are presumed to have existence and not to be mere figments. In accordance with the usual dualistic epistemology of science, any such existence is 'denuded of all knowable qualities',[1] and therefore not susceptible of perceptual analysis. For all sensory observations are explained by science as effects of an unobservable causal structure.

The scientist thus attempts to distinguish between existing and non-existing unknowables, whereas the philosophical distinction which primarily concerns us is between genuine physical appearances and non-physical ones, each being susceptible of perceptual analysis. Nevertheless both sets of criteria have the object of marking out something distinct and continuing within appearances. Hence it seems best to examine the scientific criteria of 'physical reality' first, to see if they may be accepted as reasonable in both types of problem.

The usual criterion is that if there is a technique for attaching a *measure* to a construct in certain experimental circumstances, then in those circumstances the construct is to be regarded as having an element of 'physical reality', or as referring to an existing 'theoretical entity'. Obviously, then, the perceptual presentation is 'physical', so *measurability* becomes also a criterion of 'physicality'.

In passing it must be noted that 'measurement' in this connection (and throughout this book) means the attaching of a number to a phenomenon, by applying to it a certain standard. It does not mean making a visual estimate of relative distance or relative angular extension from the eye; for this is obviously applicable also in non-physical states, even dream and imagination. As will be shown in Chapter 6, measurement requires (1) a *congruence technique*, using manipulable approximations to Euclidean geometrical structures, and (2) visual or tangible judgements of *coincidence*. If it is not the simple measure of a length, locally, then we require also (3) a physical *theory* of the quantity to be measured, so as to determine the manner in which the congruence technique is to be applied and the calculations to be made therefrom.

The Einstein Criterion

Instead of at once considering the broad criterion of measurability, as a test of physicality, it will be best to deal first with the much stricter

[1] A. G. Ramsperger, *Philosophies of Science*, New York 1942, p. 71.

criterion put forward as 'reasonable' in an important but controversial paper by Einstein, Podolski and Rosen:[1]

'If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.'

The three authors claim that 'regarded not as a necessary, but merely sufficient, criterion of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality'. It is not a definition. On the contrary, we must appreciate that for Einstein as for other scientific dualists, 'sense perception only gives information of this external world or of "physical reality" indirectly'; and therefore '*we can only grasp the latter by speculative means*' (my italics). The sure power of prediction, simplicity of theories, etc., are thus viewed as means by which we convince ourselves of the 'reality' of the 'picture' which the concepts suggest to us as corresponding to any given experimental situation.[2] Let us see whether the suggestiveness of prediction in this way is reliable.

Suppose we construct a right-angled triangle with the shorter sides 3 units and 1 unit in length, and the question arises, whether the hypotenuse contains 'an element of physical reality'. It might be said that the length of the hypotenuse is certainly $\sqrt{10}$. But $\sqrt{10}$ is a non-terminating decimal, and measurement cannot possibly yield a quantity of this kind. Far from being certain, the value predicted is impossible. Yet the hypotenuse must be granted a physical reality, whether regarded as a physical percept or as the causal substructure of the percept.

But possibly it will be maintained that 'certainty' in this context means 'verifiable by experiment to a conveniently specified degree of accuracy'. In that case, suppose that measurement yields 3·164 units instead of 3·162. Suspecting that something is wrong, we check the construction in various ways and discover that the right-angle is a trifle too large. Prediction has again failed, for the reason that it is impossible to construct an exact right-angle.

By such examples it becomes clear that accurate 'prediction' means the application of the rules of an ideal theory, *irrespective of what may be actually observed or may actually occur.*

It seems in fact that we must take the criterion as 'tailor-made' for collision problems in quantum mechanics. But there it is impossible

[1] *Phys. Review*, vol. 47 (1935), p. 777.

[2] Einstein, *The World as I See It*, New York 1934, pp. 60, 36. Also H. Margenau, *Einstein's Conception of Reality*, in *Albert Einstein, Philosopher-Scientist*, ed. Schilpp, New York 1949.

to know whether a system (say an atom) has been *disturbed* or not, in the usual sense of the term, i.e. by the state being found different after measurement from what it was before. For the process of measurement in quantum mechanics never reveals the state of the system, which is 'unobservable'. What the physicist means by 'disturbing the system' is simply that, in the formalism of his theory, allowance is made for interaction between the system (regarded as ideally abstracted) and some part or parts of the measuring apparatus (e.g. photons). So, as before, the physicist's predictions are within the limits of an ideal theory, irrespective of what actually occurs.

Theoretical Utility

It appears, however, that the criterion might be justified within the formalism of quantum mechanics in the following way. If something occurred, incompatible with what was predicted with certainty, the physicist would merely insert another interaction term or try out a different function as 'system'. The original interaction term or system having had to be abandoned, it presumably had no 'element of physical reality', and this is what the criterion correctly implies.

The criterion (in this reduced form) thus expresses the physicist's technique of fitting *theoretical terms* to an experimental situation. Are we to say then that this is the same thing as deciding the presence of an element of *reality* in the terms? The inappropriateness of this identifying of reality with theoretical utility is shown by consideration of what it would involve in particular cases.

Consider, for example, Fresnel's explanation of the dependence of the velocity of light in flowing water on the speed of flow (Fizeau Effect) in terms of the elastic ether theory. According to the theory we can deduce from the index of refraction of the water the ratio of the densities of the ether *in vacuo* and in water. The theory has been shown to possess complete 'utility' for the obtaining of a measure of this ratio in any particular experimental situation. So we are to conclude that there is an element of 'physical reality' in this calculated 'ratio of ether-densities'.

Once again, measurement has presupposed a theory; and the theory, being ideal, has in itself (before applied to any phenomenon) no necessary relation to 'reality'. What gives the character of 'reality' to measurement is, it seems, simply the fact that a theory *can be applied*; that is, the phenomenon interacts with physical objects, in such a way that some *congruence technique* can be applied to it. The measure of the interaction depends on the theory and is therefore of no importance in the consideration of 'reality'. Hence in place of measurability (in the wide sense of the term), it seems the criterion should require simply this kind of *interaction* between the phenomenon and other physical objects,

the phenomenon and objects being each assumed to have a distinguishable individuality. But there is also another condition, no doubt usually implied, namely, that the phenomenon and its interaction should be *public*, so that any trained observer suitably situated can make the observations in question.

One might of course take the view that a large number of 'experimental confirmations' regarded as 'independent' make the 'reality' of the concept *probable*. But we have only to consider the numerous confirmations of Ptolemaic Astronomy, or of Bohr orbits by the Old Quantum Theory (Hydrogen Fine-Structure, Zeeman Effect, etc.), to realize that the 'probable reality' which this view would establish (of epicycles, etc.) has virtually no relationship to 'reality' in any more usual sense of the term. In fact it is little more than 'utility in a formal system'.

Public Interaction

The conception of *interaction* between a phenomenon and physical objects, to which we were led in our search of a test for 'physical reality', is a peculiar one and requires clarification. For this purpose, consider the case of a mirage, with waves in it, or wave-like motion.[1] By the 'phenomenon' I understand here the visual appearance, and the verification that this appearance must count as *physical* might consist in trying to photograph it or in holding a small magnifying glass between the mirage and the eye and observing the altered appearance. There is, however, no *object*, in the sense of material substance with fairly well-defined outline. If by a 'physical reality' we mean the presence of such an 'object', then the waves have no physical reality. But they are *physical*. It seems better therefore, in order to avoid confusion, to drop the vague word 'reality' in connection with physical phenomena, unless we want to speak about an entity with some continuing identifiable individuality.

The 'interaction' in the case of the mirage might be of various kinds. We impute the effect on the photographic plate to light from the phenomenon of apparent waves, which thus act as cause. Conversely the interposition of the magnifying glass causes the phenomenon to change in our view. In the case of a graduated rule laid alongside an object or in front of a phenomenon for the purpose of measuring it, the interaction might be the displacing of parts of the object or phenomenon from our field of view, displacing the air-particles round the object, etc. In general, interaction with physical objects will mean some kind of compounding of the phenomenon (altered in the process) with the physical objects in one total manifestation. But it must be a

[1] This example was suggested to me by Professor H. H. Price.

properly *homogeneous* compounding—not a *mergence* such as that of a crystal vision in the crystal, or of an apparition in the room or other surroundings where it appears.

Reviewing all the proposals in regard to *the physical* and *physical* reality, our conclusion is that if the thing that is supposed to be measured is a theoretical concept (e.g. the mass of an electron) and not a phenomenon, no criterion of 'reality' offers itself as reasonable. We are trying it out provisionally, as if real.[1] On the other hand, if our intention is to distinguish *physical percepts* from non-physical ones (using the term 'physical' in the sense originally decided upon) the successful application of measuring rods or optical apparatus (without requiring any particular measure) would go a long way towards establishing *public interaction*, and therefore the physicality of the percepts. It would not go the whole way, without investigation into the 'homogeneity of the compounding'. For the position of an apparition can be measured, and in a 'public' way too, if the apparition is a collective one. The measurability criterion of physicality is therefore invalid, without a check on 'homogeneity'.

The Philosophical Problem of Perception

Having reduced the physicist's usual criterion of physical reality to questions of homogeneous interaction and publicity of *phenomena*, we have not solved our problem entirely. On the contrary, we have arrived at the *philosophical* problem of perception, i.e. of the perception of physical *objects*. This problem consists of two parts.

First, how do we decide when a phenomenon (variable as are all phenomena) has the kind of recognizable individuality which justifies our saying that it is evidence of the existence of one and the same *physical object*, instead of its being merely a succession of appearances, or perhaps something 'illusory' or 'imagined'? In the case of possible illusion or imagination it seems as if we need explicit rules for determining whether or not there is a 'homogeneous compounding' with other objects which we are assured are *physical*, and then, afterwards (if everything is physical), some way of determining what are *objects*, and not, say, mere spreads of light.

Secondly, how can we account for the *externality* of physical objects, seeing that our sensations are subjective and therefore, in certain respects, *internal*? This is the problem of dualism, which the representa-

[1] i.e. as if its formulation, symbolic or purely mental, represents exactly (or at least very closely) what is given in nature, as opposed to being imaginatively constructed by us, without any such representative truth.

tive theory of perception tries to solve. The solution which I offer in this book is a 'hierarchial' one, in which the distinction between *potentiality* and *actuality* plays an essential part. It is explained principally in Chapters 10, 14 and 22–24. For the present we must try to limit ourselves to the other part of the philosophical problem of perception, concerned with the identification of physical 'objects' without regard to their possible externality.

Whether we are dealing with successions of appearances known to be separately physical, or with appearances which might possibly be illusions or mental images, the usual view taken is that, in order to convince ourselves that a definite physical object is present, we depend on the observation of some special kind of *continuity*. Professor Ayer points to four factors involved in this continuity: (1) relations of resemblance; (2) the comparative stability of contexts; (3) the occurrence of systematically repeatable sense-data, thus constituting a 'permanent possibility of sensation' which (4) depends only on the movements of the observer, granted certain 'standing conditions' such as the state of the light or the nervous system of the observer. These, he says, are the conditions for 'a permanent possibility of sensation to be definitive of the continued and distinct existence of a material thing'.[1]

The first point that may strike us about these conditions is their imprecision. What kind of 'resemblance' is required, for example, to establish that a person I meet today is the same as one I saw yesterday, or that the electron operating a Geiger counter is the same as one emitted by a certain radioactive particle? How far and in what ways must the context be 'stable'? And what is the 'system' according to which the sense-data are 'repeatable' and dependent on the movements of the observer? In fact, without the indescribably complicated working out of these conditions in detail (on the ground of some *prior* conception of what a 'physical object' shall be) the conditions are so vague as to be useless in the case of any practical difficulty.

If therefore we feel satisfied with these conditions for 'physical reality', as providing some final kind of criterion, we shall have fallen once again into the *Abacus Fallacy*,[2] mistaking a more or less mechanical procedure for the operations of intelligence which that procedure presupposes.

In another way also it is plain that these conditions are inadequate. For, as they stand, they apply equally well to certain types of non-physical experience which cannot, at least for a considerable period of time, be distinguished from physical experience. Indeed, some apparitions have been mistaken for the physical person himself, and would have continued to be so mistaken but for evidence quite apart from the

[1] Ayer, op. cit., pp. 249–59.
[2] Above, p. 41

experience itself (as, for instance, evidence that the 'real person' was elsewhere at that very time).[1]

Intelligible Encounter

Thus the conditions seem to be not so much conditions of *physical reality* as broad characteristics of *objects* (physical or non-physical), which are judged to be 'objects' prior to all attempt to analyse their characteristics. The questions which we must set ourselves to answer are therefore: How do we first of all identify an *object*? And how do we then distinguish the object as physical?

The answer to the first of these questions has already been suggested, in the discussion of sense-data, and may now be somewhat extended. Since we cannot pick out the various characteristics of an object unless we have already identified it as an individual thing, it follows that the object must be first of all presented to us by the *total encounter* or impact of a certain intelligible structure, which commonsense is able to receive and begin to analyse. What happens then, to describe it in somewhat psychological terms, is as follows. There is a brief lapse of time; but memory can go back and continue the analysis of the same intelligible structure as at first, realizing possibly a great deal more of its essential nature. Meantime, however, the particular existence in which that structure was expressed has passed away and another taken its place; and this in turn can be analysed by commonsense. And it may be, then, that when memory compares the structure now with what it was a moment ago, there comes also the recognition of an underlying *sameness* (usually associated with a 'name'), although in various small details the structure will undoubtedly have altered.

In extreme cases, for instance, letting a drop of water fall into the sea, we no longer retain this impression of sameness, yet judge that the drop was none the less an 'object'. This seems to indicate that the criterion for determining that an 'object' is presented to us is principally that commonsense responds to a special kind of intelligible structure—affecting all the senses, including that of position, possessing that peculiarly forceful and independent quality that we call *objectivity*, and carrying with it also the kind of total significance which enables us to give it an appropriate *name*. To this it may be added that we should have the confidence that we could carry out observations and analysis of the 'same' object more or less at will, over at least a short period of time. And this possibility we express by saying that the object continues to exist *potentially* for us (and either actually or potentially for other people) at times when we are not actually observing it.

The use of the word 'potentially' in this connection seems to imply

[1] See, for example, G. N. M. Tyrrell, *Apparitions*, London 1953, cases 21 (p. 61), 30 (p. 66).

some awareness of or belief in the kind of *potentiality* (of a substructure transcending particular space-time presentations) briefly discussed on page 51 above, in connection with sense-data.

It may be noted that according to this account of the matter many cases regarded as 'illusion' remain *physical* without any philosophical difficulty arising. A coin, for example, can perfectly well look circular from one point of view and elliptical from another, and still be regarded as the 'same' coin. The meaning will be that the intelligible structure has sufficient sameness on the two occasions, especially in respects felt to be fundamental (e.g. the shape perceived by our sense of position as distinct from the sense of sight,[1] and its name, as that of a coin of a certain date) to be regarded as justifiably called the 'same'.

Non-Physical Objects and 'Stabilised Spheres of Creativity'

We now come, last of all, to the very difficult problem of distinguishing between 'physical' objects and the various types of 'non-physical' objects. Elsewhere I have proposed a distinction between 'imaginative' and 'separative' experience-types, with intermediate 'quasi-imaginative' types, on the ground of seven characteristics.[2] An independent distinction can also be made between so-called 'veridical' and 'non-veridical' experiences. In certain chapters of this book we have occasionally to touch on (1) imaginative types, (2) veridical and partly or wholly separative types, which I shall describe as 'parapsychical', and (3) the bare possibility of still other types called 'mystical'. Although it might be comfortably simple to take consensus of opinion as an adequate criterion in respect of what is 'physical' and to ignore the rest, such ostrich-like tactics are obviously neither good science nor good philosophy. In the present discussion, however, we need no more than the knowledge of the possibility of certain kinds of perceptual experience which obviously cannot be called 'physical' without causing a total misunderstanding of the situation.

In practice commonsense finds little difficulty in distinguishing what is physical from what is not, except in such rare and extreme cases as that of apparitions fully presented to all the senses, or out-of-the-body experience where the percipient mistakes the conditions for physical ones.[3] The difficulty arises when we try to analyse precisely how commonsense comes to its clear decision. Often, perhaps, it is by a vivid

[1] See Chapter 10. The fact that an object seen has different visual perspectives tends to confuse our ideas on space, which is essentially distinct from seeing. To the sense of position ('quasi-tactual function of spatial identification', in the terminology of Ch. 10), which we use when we feel an object with our eyes shut, the object has different possible orientations, but not different presented shapes.

[2] *The Mystical Life*, London 1961, pp. 104–5.

[3] Tyrrell, *op. cit.*, pp. 66, 70 (Cases 30, 32). Also *The Mystical Life*, pp. 75–6.

impression of *mental state*. This might be so, principally, in the case of mescaline phenomena, dreams (when thought over afterwards), apparitions in general, and mystical experience. But in case of doubt we always weigh up impressions of the behaviour of objects, the way the phenomena change, and so on, having learnt from experience that in the physical state only certain kinds of behaviour and changes of appearance are to be expected. To make all this precise, it seems we should have to analyse the behaviour of objects and changes of appearance in each kind of *state* and discover the precise differences.

There is, however, a preliminary question to be asked, namely, how far the distinction between physical and other states may be made to rest on a linguistic convention. Could we perhaps *define* the 'physical' to be that which interacts publicly? The flaw in this proposal is revealed by the fact that we must then ask, 'What kind of interaction, with what, and public to what?' For we must then answer, '*physical* interaction, with *physical* bodies, and public to *physical* people suitably placed in the *physical* world.' So the question is many times begged. A linguistic convention must of course be adopted; but it must rest on some objective distinction clearly discerned.

It is true that in practice simple forms of criteria of interaction and publicity effectively circumvent the need to carry out a full phenomenological analysis of behaviour in each state. In parapsychology, for example, we may use infra-red rays and photographic apparatus to establish the physical or non-physical character of phenomena. Nevertheless the distinction between the physical and the non-physical cannot *originate* in any simple principle of interaction. For it would be absurd to maintain that the difference between a levitation that can be photographed and one that cannot is *merely* that the one acts on a photographic emulsion while the other does not. On the contrary, the physical interaction on the emulsion and on our senses appears as one facet of a coherent system of physical behaviour, while the non-physical interaction on our minds and senses appears as one facet of a coherent system of non-physical behaviour. And we judge that the one interaction is physical and the other non-physical because we have prior knowledge of the two systems of behaviour.

Thus it seems clear that a final and absolute way of distinguishing physical experience from other kinds would be reached only when the phenomenological analyses of all the respective kinds of experience have been completed and compared with each other. And this we cannot hope to achieve in the present life. Popper, Braithwaite, and other 'agnostic dualists' would presumably assert that the finding of a final and absolute distinction is, in principle, impossible.[1] This, however, would seem to involve us in the conclusion that whether a phenomenon

[1] See Addendum A, Ch. 2, p. 35.

develops for us in a physical or a non-physical way depends not on any absolute distinction, but on the present state of scientific knowledge.

Allowing for difficulties contingent to present limitations in our knowledge and powers of analysis, the character of the primitive criteria by which different 'spheres of creativity' are distinguished may nevertheless be described with some confidence, in the light of the foregoing discussion. Physical experience must be ultimately distinguishable from the various kinds of non-physical experience, we may say, because of the power of analysis to reveal in it a *characteristic set of rational connections*—'physical laws', conceived of as a potentiality of *intelligible constructions* given in nature, not *measures*.

In Part II it will be shown how there is a conceptuality implicit in space-perceptions, not as a system of measures but as a potentiality for measurement of various kinds. For example, the existence of an approximate rectangle, 5×12, establishes the potentiality for a measurement of its area to give approximately 60 units, before any construction of 60 square-units has actually been carried out and the number counted.

Arithmetic is the human technique enabling us (because of its correspondence with geometry) to carry out, successively and on occasion, operations corresponding to those which in nature are carried out instantaneously and universally. Thus man *calculates*, but nature *constructs*.

By means, then, of each 'characteristic set' of nature-given constructive connections proper to a given 'sphere of creativity', there is constituted something distantly analogous to the mathematical concept of a *group*. Thus the infinite set of integers 1, 2, 3 . . . , with zero and their negatives, is established by the operations of addition and subtraction. The set of fractions is established also when we include the reciprocal operations of multiplication and division. In general, the rational coherence of interlocking conceptual operations, so to speak, establishes the existence of a 'universe' which is complete in respect of its sphere of operations. In the case which we are particularly considering in this chapter, the universe can be called 'the physical universe'. Then, similarly, with other 'characteristic sets' of given constructive connections we shall have the various imaginative or 'transcendental' universes.

It is possibly something of this kind that Eddington had in mind when he wrote:

'Physical reality is the synthesis of all possible physical aspects of nature. . . . How to make this synthesis has hitherto baffled concep-

tion. . . . Reality is only obtained when all conceivable points of view have been combined.'[1]

But he does not indicate how we are to demarcate the *physical* from the *non-physical*; nor does he distinguish between actual percepts and mathematico-physical constructs (which are hypothetical). He seems to be claiming an actuality for these constructs, without explaining how they could be seen, not as hypotheses, but as actual. It will be our purpose to repair these omissions by phenomenological analysis, not indeed complete, but serviceable enough for clarifying the problems into which we wish to inquire.

For whereas it seems that we have no hope of learning, in every detail and with the absolute finality of truth, exactly what may be the whole *corpus* of constructive laws characterizing even the physical sphere alone, nevertheless by our impression of *homogeneity*, learnt in a total intelligible encounter which may be subsequently analysed, we have adequate means of distinguishing one 'universe' from another and investigating its further properties.

It must be observed, however, that the analogy of laws of each universe with a mathematical group does not bear pressing too far. For the various universes are neither wholly apart from each other, nor is any one universe a subdivision of another, to which it is therefore 'reducible'. Against either of these mathematical possibilities, experience shows various phenomena of *blending*, *mergence* and *hierarchial influence*, to which some attention will be given in later chapters.

[1] *Space, Time and Gravitation*, Cambridge 1926, p. 182.

ADDENDA TO CHAPTER 4

Addendum A: Grover Maxwell's 'Three Views' on Theoretical Entities (p. 47)

The three views considered by Grover Maxwell in his paper on 'The Ontological Status of Theoretical Entities' (*Minnesota Studies in the Philosophy of Science*, III), are as follows:

(1) that electrons, protons, etc., 'Exist in the same full-blooded sense as chairs, tables, or sense impressions';

(2) that 'theoretical terms such as "electron" are useful in science only as calculating devices'; and

(3) that 'it is ontologically illuminating to remove such terms from the language of science by explicit definition . . .'

My thesis, on the other hand, is that science is groping towards genuine knowledge of the substructure of nature. We have not yet

reached the stage where enough confidence can be attached to any model, particularly of so-called 'particles', 'particle interaction' and 'ensembles', to justify reification of our ideas. Something may indeed be said to 'lie behind', but whatever it may be, to call it 'physical' as if it were the same kind of thing as sense impressions presented to us, would be grossly misleading.

Such 'terms', accordingly, do not stand for perceivable 'physical objects'; they are not mere calculating devices; and it is impossible to remove them from physics without putting in other terms more or less equivalent. Nevertheless, to the extent that our theories are in correct correspondence with the results of transcendental phenomenological analysis (as practicable now, or in the future), they stand for a kind of *potentiality* given in nature.

Addendum B: Conditional Statements and Potentiality

A conditional statement (e.g. 'If you were to count these cards, you would find there were 20 of them') may sometimes be useful to communicate to another person a hazy idea of potentiality. The cards really number 20, but neither of us knows this directly at the moment in question. Unfortunately it also tends to suggest three serious errors. The first is the assumption that 'the cards' are the same object in physical space and time, whether I am perceiving them or not, whereas the truth is that the presentation in space and time is completely indeterminate until the circumstances and mental and bodily state of some particular observer are given. The second is the phenomenalistic belief that, by manipulating suggestive verbal formulae with physical connotations, one can side-step the necessity of admitting a substructure of nature in a sphere transcending particular physical actualities. The third is the confusion between psychological *belief* in what might happen in future (as one may have deduced from supposed physical laws) with a *givenness* in nature transcending time.

The very complex and difficult subject of potentiality is dealt with continually in Parts II, III and IV.

CHAPTER 5

METHOD OF INVESTIGATION: THE MEASUREMENT OF TIME

'Since we admit that our universe is modelled on the intelligible world, we should so much the more recognize that the latter is the universal living organism, which constitutes all things because it consists of perfect essence.

'[The intelligible world] is true eternity, of which time furnishes a moving image in the sphere of the soul.'

Plotinos, *Enneads* vi. 7.12, v. 1.4

'Nothing has yet been said as to the measurement of time. Such measurement does not follow from the mere serial property of time; it requires a theory of congruence.'

A. N. Whitehead, *The Concept of Nature*, p. 65

In the preceding chapter we were led to replace the inadequate measurability criterion of physicality by a criterion of *homogeneous interaction* between the phenomenon and physical objects. We then remarked that even this was logically circular. The conclusion was that the ultimate distinction between physical and non-physical observations could not rest in any such criterion. It must rest in systems of laws or conditions of behaviour, each state of observation and life having its characteristic system. These laws were to be conceived of as a potentiality of intelligible constructions (by nature), or as a 'group of interlocking conceptual operations'. Let us now try to be a little more explicit and consider by what means and to what extent we might be able to recognize such 'laws'. For this purpose it will be convenient to take over, without evidence or considerable argument, some of the results of the detailed investigations in Part II.

Among the 'laws' characteristic of the physical state we must evidently include those of the local Euclidean geometry: at a 'higher level', ideal-operational; and at a 'middle level', actual-operational. The *ideal* is a matter of precise intelligible structures, and the *actual* of manifestation to sense in approximate form. The geometry becomes actual only with the help of subjective factors (memory, imagination, ideals and intentions, decisions of the will, persuasions of other people, etc.), which we can place at a 'lower level'. Let us speak of these three levels as being 'levels of physical potentiality' and any particular

observation as being the *actualization* of the three levels by an instantaneous 'manifestation upon occasion' of the totality of factors ('causes') operative on that occasion of observation.

In the case of a non-physical kind of experience, such as dream, we may similarly conclude that there must be factors belonging to a level of ideal or intelligible structure, factors belonging to a merely subjective level, and other factors belonging more properly to the level of the dream-state, all these factors being involved in the actualization at any instant of dream-awareness.

Since the actualization at any instant and in any state, physical or non-physical, comes about by integration of an immensity of past, present, future, near and distant influences, it is of course utterly beyond human reproduction. We may discern certain rules of correspondence by which, given a certain potentiality at the higher or lower level, there will be a tendency for the manifestation to take a certain corresponding form. But we cannot hope to discover a complete set of 'laws', of the same ideally exact kind, which shall be capable of determining, also exactly, a manifestation to a given person on a given occasion. For that would make identical the objective and the subjective, the public and the private, the *a priori* exact and the empirical approximate. It would also rule out the possibility of 'mergence' by which physical phenomena become occasionally subject to 'mental laws', as in muscular movement, imagination and will.

Mergence-Free Physics

For the purposes of drawing an absolute distinction between physical and non-physical states, however, it is not necessary to take into account the laws of actualization by correspondence. For physics (especially quantum theory) has clearly shown that we can set up, at the higher level of ideal and intelligible structure, a system of effectively mergence-free laws for the *potentiality* at that level. With the help of these laws, and the postulation of idealized 'boundary conditions' corresponding to the experimental conditions set up, we can proceed to deduce an indication of probable results of physical observation—this whole procedure copying, as it were, the *teleological integration* of 'causes' which we concluded must lie behind any physical actualization. Is it appropriate now to say that *further* 'laws' are necessary to specify the results of observation exactly? If so, then it appears that such 'laws' would be (1) laws proper to the subjective level, (2) modified laws of integration, including both ideal and subjective levels, and (3) laws of manifestation by correspondence. But in that case, what distinguishes the physical from non-physical levels is the system of laws of the ideal 'mergence-free physics' at the level of higher physical potentiality, and

not these other 'laws' whose function is to combine one level with another.

We have thus, it seems, some clear means of determining a system of laws which will sufficiently characterize the physical state. They will be laws, not of measurement, but of ideal 'constructions' such as spherical waves, rotations, vibrational frequencies, in abstract space or spaces. Measure does not come into being except on the occasion of and according to actualization. Nevertheless, *ideal congruence* must be admitted, in space, in time, and in space-time; otherwise there could be no universality and reproduction of intelligible structures such as would be necessary for a creation in space and time.

As regards the systems of 'laws' characterizing *non-physical* states, we evidently have no means of investigating these from the physical state by measurement. They would need to be directly discerned by an intuition developed in space or space-time fields more amenable to analysis. But to discuss this matter further would be to pass beyond the scope of the present book.

Ultimates

The next point to be considered is, what confidence we can have that the laws we have discovered, as we have formulated and conceived of them, go beyond the merely hypothetical and make a genuine approach to some ultimate truth. It must be admitted that every theory fitted to experimental results has the status of a hypothesis, if the closeness of the fit is its only claim to consideration. But *merely* striving to fit the facts somehow is not the way in which science progresses. The realization of an entire conceptual system is involved. We find, in fact, a strong element of *intuitive discovery* continually at work in scientific research, surveying what is given, utilizing judgements that go beyond mere convenience and bringing about a progress which we are induced to believe is from relative obscurity to greater truth. In other words a form of objective *analysis* is at work. Weyl puts this conclusion very strongly when he asserts:

'Nobody who really goes into the matter will deny that the world of perceptions determines the theoretical system in a virtually unambiguous manner, although no logical way leads to the principles of the theory.'[1]

This is perhaps too strong, because theoretical systems which still embody errors and arbitrary mathematical devices for convenient calculation can hardly be determined 'unambiguously'. Nevertheless, one must not underestimate the power of the mathematical intuition,

[1] Weyl, *op. cit.*, p. 153.

which surveys experimental results in the atomic or cosmical field and leaps to a new conception revealing a hitherto unimaginable order and reason in the changing phenomena of this world.

Given this means of progress in science—hypothesis guided by inspirational analysis and checked by experiment—there still remains the difficult problem, How is final confirmation of mergence-free physics actually to be obtained and recognized as such?

There are two attitudes that can be taken to this problem. One is absolute agnosticism. No final confirmation is possible, even in principle. The policeman who directs us[1] is obeying no higher authority and has no terminus in view. The other possible attitude to take is that we do have, in principle, a certain power of discerning an ultimate hierarchial structure of the universe but it needs development and can become effective only in favourable conditions. This power, which the *non*-agnostic accepts, is that of the transcendental grades of phenomenological insight and analysis, to be considered in chapters 10 and 21. It is not necessary to enter into further details at present.

If we can accept the possibility of various grades of analysis up to the transcendental, the following general conclusions in regard to method present themselves. The search for sure knowledge must be regarded as a progressive whole; and in this search the value of the study of measurement is certainly not to be underestimated. Measurement provides an indisputable basis for confirmations of conceptual relationships, often astonishingly accurate, and accessible to the large number of people who have undertaken the appropriate and not-too-difficult studies, without any need to transcend the state of ordinary physical observation. This knowledge begins by being hypothetical merely; but to most scientists it also has a flavour of 'truth in nature'.

In so far as scientific knowledge has this 'flavour', we may find ourselves able to proceed to certain insights into the 'operational' character of what we are doing in relation to what is given in nature. And these insights, so far as they are not counterfeit, indicate what is *not* hypothetical. We either know what we are doing, or we do not.

Lastly, if we allow, in principle, the power of analysis to resolve any obscurity or intellectual puzzle, it is possible to approach an *ultimate* knowledge of the distinction between physical and non-physical universes and of the 'characteristic set of rational connections' in each universe.

Three stages in the approach to ultimates are thus achieved successively by means of:

(1) The mathematical intuition, aimed at 'saving the appearances' most

[1] See Addendum A to Chapter 2, p. 35.

simply, elegantly, fruitfully, etc. For this purpose suitable axioms, symbolic techniques, and theorems are introduced, without examining the antecedent phenomenological analysis and justification of any of these. This stage is followed by:

(2) 'Ordinary philosophical' insight—to be described in Chapter 10 as 'inner operational analysis', or phenomenological analysis in its 'ordinary' grade. The insights so gained are confirmed (if correct) and extended by:

(3) Phenomenological insight in its transcendental grade. This is made possible by a 'release' from 'mergent fixation' and a transition to a transcendental 'stabilised level of creativity', where we have the power of apodeictic analysis in background-physical and non-physical universes.

For the investigations in this book the present achievements of the first stage are taken over and not enlarged. Our object becomes the phenomenological analysis which constitutes the second stage, and to some extent also that which constitutes the third stage.

The big task which invites attention first is the discovery of what is presupposed in measurement, (*a*) ideally, and (*b*) actually. The problems will be dealt with systematically in Parts II and III, beginning with the analysis of space-measurement. The preliminary question thus arises, whether perhaps we ought instead to begin with the analysis of time-measurement.

Is it possible to analyse the measurement of space apart from that of time? Is it possible to analyse the measurement of time apart from that of space? If both these are possible, which is it proper to take first? If neither is possible, how shall we set up the measurement of space and time conjointly?

Concepts of space obviously cannot be reduced to *concepts* of time; and likewise, concepts of time cannot be reduced to concepts of space. The questions just put are significant merely as suggesting how we should begin an analysis of *measurement*, having in view a deep phenomenological analysis of the concepts themselves.

The 'Elimination of Time'

Some authorities on the Theory of Relativity (Milne, Robertson, Synge) propose ways of establishing a system of time-measurement, and at a later stage introduce a derivative system of space-measurement. Synge takes this as the correct approach in his comprehensive treatise on special relativity,[1] and elsewhere asserts that 'the theory underlying' the measurement of time is 'the most basic theory of all'.[2] Whitrow quotes him with approval in justification of his own approach, and strongly argues against the 'elimination of time' implied by the

[1] J. L. Synge, Relativity: *The Special Theory*, Amsterdam 1956.

[2] *The New Scientist*, February 19, 1959, p. 410. Quoted by G. J. Whitrow in *The Natural Philosophy of Time*, London 1961, p. vii.

usual approach which begins with the study of the geometry of space.

But what does 'most basic' mean in this context? How is time supposed to be 'eliminated'? And why should it be bad to eliminate time, but not bad to eliminate space?

In trying to understand what Synge and Whitrow have in mind, I suggest we should first of all distinguish clearly between four broad aims:

(1) The aim to survey physical, psychological and traditional philosophical questions concerning time and time-measurement, concentrating on aspects which have been somewhat neglected hitherto and which are not obviously dependent on previous views concerning space and space-measurement. This is Whitrow's commendable aim in his book, *The Natural Philosophy of Time.*

(2) The phenomenological analysis of concepts of space and time and of measurement on the ground of these concepts. This is the basis of the investigations in the present book.

(3) The aim to produce the most convenient and elegant syntactical theory of time and space, beginning with well-chosen axioms and proceeding by the deductive method, with only incidental regard, where unavoidable, for the epistemological problem of the analysis of presuppositions.[1]

(4) The aim to establish actual physical techniques for setting up standards of time and space in the most accurate way, making the techniques as uniform in kind as possible. Thus, to quote from an article by Harold Lyons, recommending the use of atomic clocks for setting up the initial standard (of time):

'Atomic clocks would establish a more precise and invariant standard for the length of time units (e.g. the second) than the astronomical one. The right time could be checked instantaneously . . . The standard for distance could be related to the standard for time by means of an atomic clock coupled to an interferometer using microwaves. This would give the system of units greater coherence and logic, for length and time are now measured in independent ways.'[2]

The third of these aims, because of its initial dependence on axioms (i.e. propositions laid down concerning *undefined* terms) has only heuristic value in this book.[3] It is just these basic undefined terms whose

[1] cf. Synge, *op. cit.*, p. 1.: 'The foundations (of a scientific theory) are axioms principles or laws. . . . The accepted dogma consists of deductions from the foundations confirmed by observation and experiment, linking reason with nature in a satisfying way.' Also by the same author, *Science: Sense and Nonsense*, New York 1951, pp. 32 ff.

[2] 'Atomic Clocks', *Scientific American*, 196, February 1957, pp. 81–2.

[3] See below, p. 104.

conceptual structure we wish to analyse, thus establishing the axioms (if true) by apodeictic insight. The fourth aim needs further discussion, since it seems at first sight reasonable to suppose that what is found to be the most accurate practical way of setting up a standard will be also conceptually the most primitive.

What is a Clock?

To find out what substance there is in this supposition we must first decide what are the essential requirements in a clock which is to be used for the rational measurement of time-durations in the present or future. In the specification of such a clock we wish to avoid as far as possible all presuppositions of *spatial* congruence.

Three functions must be provided for. There must be a *repetitive* element, a *control* element, and a *counting* mechanism. Thus we may have a means for producing visible flashes intermittently. If no control is exercised, so that what happens in the future is the prey to countless accidental influences and wholly unpredictable, measurement in any rational sense of the term ceases to be possible. No law for the prediction of future measures would be valid, and 'past laws' would be no more than classifications of accidental occurrences. A pendulum hung in the breeze, from a soft plasticine support, is similarly useless for measuring because the number of swings occurring during some objective physical occurrence would depend largely on accidental and unpredictable features and would therefore provide no objective measure of the occurrence.

Controls are always provided, in an approximate way, by special cyclic phenomena in nature. The commonest types of control are obviously mechanical ones, depending on the prior establishment of space-congruence. Thus we must be sure that a pendulum remains exactly the same length, or we must impose 'corrections' such that the total effect, according to accepted theory, is that of an imaginary pendulum of *ideally* constant length. Indirect tests have to be imposed to verify that the corrections have the right effect. For instance, one can correct for temperature separately by investigating experimentally how temperature seems to affect length in relation to some standard of length already established. Then one can proceed to correct for other influences apparently at work. And the final time-keeping system can be checked against other time-keeping systems, such as the rotating earth, to make sure that a consistent scheme of time-measurement has been devised. Since all these methods make prior appeal to spatial congruence and standards, I pass over them here.

In atomic clocks the control is by the pure sinusoidal electromagnetic waves emitted or absorbed by atomic systems when transitions are

limited to two quantum-states only. The repetitive element, namely a vibrating quartz crystal, is still mechanical, but is said to be accurate (when tested in relation to other time-recording apparatus) to the extent of 1 second in 10 years. The fact that the inaccuracies of a clock of this kind can nevertheless be minutely tested by other apparatus and shown on a chart as a very wavy line, and that changes due to the age of the crystal, temperature, and so on, can also be accurately tested, indicates not only that spatial factors are involved (e.g. variable separation of the molecules) but also that the most accurate tests of regularity of time-keeping bring in the whole corpus of physical laws and the representation of effects in a spatial way. An entirely isolated and solipsistic time-keeper would be untestable and uncorrectable. In fact, its first application would destroy its isolation and bring into play physical laws depending on spatial congruence.

The physicist aims to overcome the influence of extraneous factors by means of the control mechanism, which, in the case of an atomic clock, is provided by vibrations within the ammonia molecule (Ammonia Clock, accurate to 1 part in 100 million) or within the caesium atom (Caesium Clock, accurate to at least 1 part in 100,000 million). But with even these clocks the deviations from regularity can be shown on a graph as a persistently wavy line at a more or less constant average level.[1]

It seems that, however accurate the repetitive and control elements of the clock may be, one can never arrive at a concept of standard time-duration without prior reference to space-congruence. In fact, the more accurate the clock, the more complex are the spatio-temporal physical laws which have to be known and utilized.

If we ask, then, what sort of standard time-interval is provided by atomic clocks, the answer seems to be that they do not directly provide any perceptible standard. They indirectly provide, in the first place, when conjoined with other checks involving spatio-temporal physical laws applied to nearly rigid physical structures and after reduction by an automatic counting mechanism, a succession of events which under these circumstances appears 'regular'. Then, if two given events are correlated respectively with events in this succession (e.g. positions of the pointer on the dial), we have an acceptable means of attaching a measure to the time-interval between those two given events. The *concept* of time-interval, meanwhile, has slipped out of sight.

A minor point which nevertheless deserves remark is that the repetitive and control elements of a clock are useless for measuring if no counting mechanism is provided. Here also, if the mechanical parts, dial and pointer are not rigid, but liable to accidental and unpredictable distortions in the future, we cannot have measurement of an objective

[1] Lyons, *op. cit.*, p. 74.

phenomenon, in any rational sense of the term. In principle, it might be urged, a superhuman being might count the cycles, attaching a number to each; instead of the counting mechanism, we then have the human memory. Even if this fantasy is admitted, we meet with a difficulty over subdivision of time-intervals, which in practice is done either on some rigid dial or by a frequency multiplier depending on spatio-temporal physical laws again.

There remains another kind of proposal for setting up, in principle, a system of standard clocks in various parts of the universe, without prior appeal to spatial congruence. This originated with Milne, and was subsequently taken up by the other authors mentioned. Here we are concerned not so much with its mathematical interest and value as with its phenomenological character, i.e. the presuppositions involved.

The Radar Technique

It is usual to begin by supposing that each observer is in possession of a clock with some kind of dial and pointer and provided with a revolution-counter. Milne says that all we need initially is something which moves in this way, 'no matter how irregularly'.[1] Synge, looking ahead to the development of the full four-dimensional theory, says, 'Let there be four observers, flying about in aeroplanes, not in any particular courses. . . . Let each observer carry a clock, not necessarily an accurate clock, but perhaps an old battered clock.'[2] The latter assumption is also implied by Robertson's method,[3] though initially he follows Milne in considering only pairs of clocks.

The stages in working towards a consistent system of time-measurement may now be summarized as follows:

(1) The reading t_1 on the clock of an observer A when he sends out a light signal, and t_2 when he receives the immediately reflected return signal from an observer B, and likewise the reading t_B on B's clock when the signal is reflected, are conceived of as made at every instant of time, past, present or future, so that functional relations of the form $t_2 = f(t_1)$, etc., can be determined for all time.

(2) Observers A and B then re-graduate their clocks so that there is complete symmetry as between observers A and B in this type of light-signal experiment (group-theory shows how to transform the variables so that the respective functions become identical). Clocks thus re-graduated are called *equivalent*.

[1] E. A. Milne, *Relativity, Gravitation and World-Structure*, Oxford 1935, p. 26.

[2] *Relativity: The Special Theory*, p. 7.

[3] H. P. Robertson, *Cosmological Theory*, included in *Jubilee of Relativity Theory*, Proceedings, Basel 1956, pp. 128 ff.

(3) We must now assume a one-dimensional manifold of observers $A, B, C, \ldots$ such that the clocks of A and B have already been made equivalent. We can make the clocks of A and C equivalent in the same way by re-graduation, but the further assumption has to be made that observer C and his clock are such that only C's clock has to be re-graduated to make it equivalent to A's (otherwise, if A's clock had to be re-graduated again, it would no longer be equivalent to B's).

(4) Given any event in the universe, any one of these observers can attach two co-ordinates to it, namely, the times when a signal would have to be sent off and would be received if its reflection occurred at the place and time of the event in question. If these times are t_1, t_2, Milne defines the time and distance of the event with respect to the chosen observer to be $\frac{1}{2}(t_2 + t_1)$, $\frac{1}{2}c(t_2 - t_1)$, respectively.[1]

These formulae are obviously what we would get by independently assuming the special theory of relativity, the chosen observers being at the origins of inertial frames whose relative velocities are purely in one direction (i.e. the observers themselves have uniform relative velocities along a straight line), and c being the velocity of light. Milne then extends this work to three dimensions with the help of a Euclidean map on which the paths of the light-rays are drawn in straight lines.[2]

Robertson proceeds quite generally, using group-theory, and obtains directly the most general differential invariant $ds^2 = d\tau^2 - \xi^2(\tau)\, d\sigma^2$, where $d\sigma^2$ is a quadratic form symmetrical in three space-like variables and $\xi(\tau)$ is a function of an invariant τ which is called the *cosmic time* since it is the same for all equivalent observers belonging to a given triply infinite family of observers whose world-lines do not cross. The formalism of the Special Theory then appears as the particular case when $\xi(\tau) = \tau$ and a rather complicated transformation is made from τ and the space-like variables in $d\sigma$ to the usual t and x of the Special Theory (i.e. the physicist's measures of time and space).[3]

Reviewing all this rather elaborate procedure, the following may be particularly noted:

(1) The fact that the functional relations between the clock readings t_1, t_2, t_B are supposed to be valid for all time (in order to cope with future measurement) implies that the pointer and dial are not liable to unpredictable distortion. That is, after due correction, they count as ideally rigid (i.e. independent of outside influence).

If, on the other hand, there is no pointer or dial, the clock being merely a means of originating a non-spatial series of flashes or other

[1] *Op. cit.*, p. 29.

[2] *Ibid.*, p. 48.

[3] If the σ-space has one variable u, the transformation is $\tau^2 = t^2 - x^2$, $u = \operatorname{argtanh}(x/t)$.

recognizable events (and the previously mentioned difficulties over counting and subdivision are ignored), again the series must not be liable to unpredictable distortion due to outside influences. But how can this be guaranteed, since corrections for outside influences cannot be made before the flashes or other events have occurred?

(2) Re-graduation of the future part of scales would not be possible if the allocation of numbers to future events was not similarly prescribed in some way similarly independent of outside influence.

(3) Milne's postulated formulae for the time and distance of a distant event imply that the clock-zero can be shifted any amount without altering the distances or relative times of distant events. This is the same thing as postulating time-congruence and space-congruence (as a ruler laid across two points may be used to find their distance apart, by subtracting the two readings, whatever the first reading may be). In Robertson's general method, covering all mathematical possibilities of consistent measuring by observers of a family of the kind described, we do not have any such measures of t and r characterized by time-congruence and space-congruence, respectively. Various kinds of congruence can be established, according to the kind of observer-family chosen and the relationship between the equivalent clocks. The warrant for Milne's formulae thus seems to be merely that this is what is true in the Special Theory (i.e. 'flat' space-time).

(4) The single infinity of observers with which Milne and Robertson begin could be observers who are not moving in a straight line relatively to each other in empty space. They could be moving in a curve of some sort, or through a region where there is a variable distribution of refracting or gravitating matter. In such cases equivalent clocks can still be established for each member of a family of such observers, exactly as before; for the method requires no postulate about such things. We thus obtain a symbolical description of flat, or possibly homogeneous, space-time, for a universe which is not flat or homogeneous. This system of measurement will have entirely failed to reveal the paths of the observers as non-straight and the universe as non-homogeneous, but will have presented it instead as homogeneous, with the paths of observers correspondingly regular.

The cause of the paradoxical state of affairs described in (4) is as follows. What Milne and Robertson have been developing is in each case an abstract geometry, to the symbols of which we may give any interpretation we please, *provided* we show that such interpretation is indeed operationally possible in the circumstances envisaged. Now if ordinary physical measures reveal that the universe is non-homogeneous, it is clear that interpretation of the 'space' and 'time' symbols of Milne and Robertson as ordinary physical ones is inadmis-

sible. But they do in fact claim to be describing an *actually physical* universe, admissibly. Hence we conclude that they have inadvertently assumed that there are ordinary physical measures of the universe and that these do not reveal it as non-homogeneous.

It appears, therefore, that the methods of Milne and Robertson actually presuppose some other (i.e. the usual) means of measuring time and space. There is indeed a way of operationally implementing the Robertson geometry in a non-homogeneous universe, such as ours obviously is. This will be explained in Chapter 18. But here again, the 'corrections' necessary require the previous operational implementation of *spatial* congruence in a local region.

All along, therefore, our conclusion is that the way to the analysis of primitive time-concepts does not lie through an analysis of time-measurement apart from that of space-measurement. Such analysis cannot be completed. Either one must discover the essential nature of time-concepts by more direct means, or one must analyse the measurement of time and space conjointly. But as we shall see in the next chapter, the analysis of local space-measurement *can* be carried through without a conjoint study of time-measurement. At a suitable later stage we can then bring in conjointly the analysis of time-measurement; and this may be of some help towards analysis of the *conceptual* character of physical time.

It must be kept in mind that in this approach by phenomenological analysis we are not applying presumed 'rigid' rods and presumed 'standard' clocks to the universe, tabulating the readings, modifying and correcting them according to presumed theories, and then systematizing them in other theories. We are trying to lay bare, by direct insight, indisputably self-evident, everything that is 'primordial' in the changing phenomena of the universe and in our attempts to measure them with the aid of some ideal system of geometry, suitably implemented operationally.

CHAPTER 2

'Commonsense' starts from *immediate intuitive comprehension.* By it we learn to behave reasonably in any ordinary human situation, to judge the positions and movements of objects, and so on, without appeal to theories.

The view of commonsense as a crude sort of science, growing in us by accident, is a misleading oversimplification of the problem of knowledge. For the hypothetico-deductive method of science is always under the direction of immediate intuitive comprehension, and must alter its course if this 'pure' commonsense discloses some inadequacy to the situation.

On the other hand, our analyses of 'pure' commonsense, by failing to go deep enough, may be supplemented by beliefs which are not warranted by analysis. If 'science' is the considered adoption of a theory, after due test, the unconsidered adoption of a theory as if it were self-evident may be described as 'impure' commonsense. The 'scientific attitude' then consists in the readiness to deepen our analyses, accept new ones, and redirect scientific method accordingly—in short, to be governed by 'pure' commonsense.

A lapse from the scientific attitude may occur from various causes:

(1) supposing that the chief *facts* are already known, and no new discovery could appreciably alter our world-view;

(2) supposing, on the one hand, that facts can be collected and constitute knowledge without a conceptual system to make them intelligible; or

(3) overemphasizing, on the other hand, our freedom to impose a conceptual system on nature;

(4) taking some principle or theory in the customary background of our thinking as self-evident and unchallengeable by new analyses;

(5) succumbing to the remarkable 'defence mechanism' whereby anything which threatens our settled world-view is felt as a threat to our sanity.

CHAPTER 3

Differences in language confirm the view that there are essential differences in subject-matter as between (1) physical science, (2) neurophysiology and biology, (3) psychology, (4) the philosophy of perception, (5) parapsychology, and (6) mystical experience. The correspondingly varied approaches to problems of space and time cannot therefore be reduced to a single 'scientific' one. But it still remains to assess them in their mutual relationships, from the higher ground of (7) phenomenological analysis, which is concerned with direct insights into essential character and validity. The highest grades of phenomenological analysis are termed 'mystical' because of their strongly apodeictic character.

In 'physical science' we study the *results of measurement*, after having decided on certain convenient techniques and theories which make those

results significant. We further propose laws connecting such results, thus enlarging our theories. Analysis of measurement itself is not undertaken.

Physical science and its methodology commonly strive to dispense with phenomenological analysis, using for this purpose a formalism which begins with undefined terms and develops itself by postulated syntactical rules. The belief that nothing more is required or possible in the study of nature is characterized as the *Abacus Fallacy*—the substitution of mechanical or rule-of-thumb procedures for direct insight into the conceptual state of affairs, and the belief that this circumvention of analysis is justified, not merely as an expedient, but absolutely and finally.

The philosophy of perception demands consideration in several later chapters. The approaches of neurophysiology, biology and experimental psychology, while highly suggestive, presuppose solution of the more primitive and general problems, and therefore have little place in this book. Parapsychology and mystical experience, on the other hand, provide evidence which strikes at the roots of both physical and philosophical questions.

Particular intelligible forms, whether physical or not, are called *existences*; conceptual details, known as timeless and always 'the same', are *essences*, and the analysis which reveals them in this character is called *essential*. *Error* is the confusion of different existential spheres, and the 'cementing' of such confusion by personal belief.

CHAPTER 4

The term *physical* is used in two quite different ways. It may be applied to the hypothetical 'constructs' of physical science. Or it may be applied to what is perceived here and now. The respective universes of discourse are distinguished as the *universe of mathematico-physical constructs* and the *physical universe*.

Discussion of the physical universe (in this sense of the term) is confused by the 'sense-datum theory', which treats details of analysis as if they had separate existence and in some way composed the object. Here we speak instead of 'details of perceptual analysis', on the understanding that no analysis can be complete or even intelligible when regarded as a mere compilation. The chief difficulty then is to distinguish a perceptual analysis which is *physical* from one which is in some category of the *non-physical*.

The criterion of *measurability* proposed by many physical scientists aims to establish the 'physical reality' of certain theoretical concepts. Presumably the phenomena measured are 'physically real'; so the criterion can be taken also as a test of the physicality of percepts. As a test of distinct existence corresponding to a concept, however, it fails; for a quantity measured presupposes a theory, which may 'save the appearances' without implying any distinct existence corresponding to the concepts used. For the same reason it fails as a test of the physicality of a percept, except so far as it requires *homogeneous interaction* and *publicity* in relation to objects and people already recognized as physical.

The philosophical problem of perception lies in the recognition of *physical*

objects, as being something more than mere changing appearances and also distinct from 'illusions'. The criteria of *continuity* commonly proposed to explain such recognition also rest on a prior means of identification of objects; otherwise we should not know what kinds of continuity objects do in fact manifest. The view adopted here is that the primitive identification of a physical object is by an *intelligible encounter*, and that analysis then follows.

An intelligible encounter may present to us not only an inexhaustible store of perceptual details which are physical, but also details of a *substructure* (described as 'causal') and a potentiality. The term potentiality has reference to conceptual elements which are inherent in our awareness of the continuing public character and recognizable identity of objects, physical or non-physical. It is conveniently expressed in a *name*, and so, in a certain way, transcends particular space-time presentations.

The distinguishing between the physical and the various non-physical universes must accordingly be by different kinds of intelligible encounter, and this implies different systems of behaviour of objects and changes of appearance in each state. The analysis of such behaviour and changes in each state will disclose the primitive distinctions. Thus each 'universe' must be supposed constituted in a characteristic set of 'interlocking conceptual operations'. Experience shows also that the various universes are not isolated from each other, but are subject to *blending*, *mergence*, and *hierarchial influence*.

CHAPTER 5

The manner of determination of 'laws' distinguishing the physical universe is first discussed. The standpoint taken is that a *mergence-free* physics can be effectively marked off, with a circumscribed indefiniteness in the abstract, but becoming definite when actualized by *teleological* and *integral* operations. We progress towards more and more detailed knowledge of the 'true state of affairs', whether physical or non-physical, by a combination of experiment, mathematical intuition, 'ordinary philosophical' insight, and phenomenological insight in its transcendental grade.

Beginning with the analysis of measurement, we have to decide whether measurement of time can be carried out without presupposing measurement of space, and *vice versa*. The proposals of Milne and others, to establish first the measurement of time, are shown to rest on presuppositions (1) that space-congruence is already established locally, and (2) that their ideal formalism can be implemented operationally with 'time' and 'space' measures understood in the usual physical sense, i.e. on the basis of space-congruence. Hence, epistemologically, we must begin with the analysis of space-measurement.

PART II

PRINCIPLES OF MEASUREMENT, AND ANALYSIS OF SPACE

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CHAPTER 6

PRIMITIVE CONCEPTUAL CONGRUENCE, AND THE BASIC TECHNIQUE OF SPACE-MEASUREMENT

> 'As to the mathematical arts which, as we were saying, have some apprehension of true being—geometry and the like—they only dream about being, but never can they behold the waking reality so long as they leave the hypotheses which they use unexamined, and are unable to give an account of them. For when a man knows not his own first principle, and when the conclusion and intermediate steps are also constructed out of he knows not what, how can he imagine that such a conventional statement will ever become science?'
>
> Plato, *The Republic*, 533, trs. Jowett

> 'The exact theory provides the framework for approximate verifications.'
>
> H. Weyl, *Philosophy of Mathematics and Natural Science*, p. 143

If we consider what is involved in any operation of measurement, and go back step by step in our analysis from the complex to the simple, we find first of all that measurement involves counting, then that this counting involves the congruence of a line AB, say, with a line CD; and lastly, since the 'shape' of the line is not an ultimately simple concept, there is involved the concept of *sameness* in the *relative placing* of points A, B on the one hand and points C, D on the other.

At this stage there may be a strong temptation for us, as formalistic mathematicians and physicists, to put the analysis aside as tending to what is vague and metaphysical, and to revert to the axiomatic method, making use perhaps of deductive systems already developed and conveniently applied to the problems of measurement. But if we do this we shall be throwing away what is, I think, the master-key to the problems of measurement and natural law. Let us therefore take courage and endeavour to continue the analysis further.

In thinking of *sameness of relative placing* it is very difficult to avoid confusing three separate conceptual strands. First we may consider *sameness of relative placing* in terms of the actualities of this world, being realized as such by us with the help of an *actual operation* of comparison. The actual operation is the carrying-out of an operation which is *ideally* conceived (just as Milne's basing of relativity theory on

time-measurement is ideally conceived, and still has to be applied, if and where possible, to the actual universe). Lastly, this ideal operation must be expressed in terms of the rational constructions and theorems of a purely conceptual geometry. These three strands may be called (in the reverse, more logical, order): the conceptual geometry, the ideal-operational geometry, and the actual-operational geometry.

The distinction just made is vitally important in all that follows.

Turning now to the conceptual space-geometry, it is clear that the possibility of measurement rests ultimately on a concept of *primitive sameness* in regard to relative spatial position, in a conceptual sense. Suppose, for example, that the spatial separation of A and B is declared to be 3 units, while that of C and D is 5 units. This implies that there is the determination of a *unit spatial separation* which can in some way be applied to all spatial separations for the purpose of comparison, without itself being subject to variation according to circumstances. This unit is then, by definition, the *same* in all different places and at all different times. Or if we conceive that the unit might vary, then the variations must be in accordance with some known determinate law, so that we could substitute some *absolute* unit with reference to which the variations of the other unit could be determined. Otherwise no definite measure of spatial separations could exist even conceptually (as we know it can, in ideal Euclidean Geometry, for example).

The term 'absolute', in connection with physical units, may have two different significations. A universal unit of distance (the metre, for example) may be called 'absolute' in the loose sense that we have devised some empirical operation for attaching measures to spatial separations, wherever and whenever the operation is applied. Two spatial separations, each of which has unit measure, are then 'special cases' of the absolute unit. If the unit is defined in this way—i.e. by some approximate and conventional procedure, depending in many cases on a physical theory which is, to say the least, not self-evident—it is *not* 'absolute' in the strict sense which provides us with a *conceptual* geometry. For the very first mention of 'approximation' presupposes that we have the concept of an *exact* measure towards which we are approximating; and it is to *this* concept, being exact, that we may expect the precise logical laws of some conceptual geometry to apply.

It is of course the exact conceptual 'sameness' (or 'congruence') with which the mathematician is primarily concerned. But in the present investigation what we are chiefly interested in is the relationship of *conceptual* congruence to *actual* observation.

The question we have to ask is, therefore, how we become conscious

of sameness or difference as regards spatial relation in actuality. How does the *concept* of a certain apartness become a *percept*?

The ways in which we may form an estimate of the actual separation of two points C and D as compared with that of two other points A and B may be classified under the following three heads, according as the impressions are given to us:

(1) Directly, by our sense of spatial position, as a person blind from birth might estimate the sameness or difference of the edges of a rectangular table which he stands near and explores by the hand. This sense is with most of us so bound up with the sense of sight that we hardly acknowledge its distinct existence. But apart from the obvious fact that people blind from birth must possess it in well-developed form without the aid of sight, it can also become astonishingly acute in certain parapsychical (e.g. hypnotically induced conditions. And moreover, the spatial presentation of a thing seen implies (by its being 'spatial') the fusing with it of a kind of conception different from that of seeing merely.[1]

(2) Indirectly, with the aid of other senses (seeing, hearing, touch) with which the sense of position and apartness is normally fused, in the way just described.

(3) Indirectly, with the aid of certain manipulations of physical objects, i.e. by measurement. This must evidently involve one or both of the previous modes of recognising actual spatial positions; otherwise our manipulations would not be ones in respect of actual spatial position, but would be merely conceptual or imaginary.

The verification by this third method that the spatial separation of A and B is the same as that of C and D is expressed in the more usual terminology as the verification that the figures A, B and C, D are *congruent*. Ultimately, it seems, every test of congruence is by attempted superposition. We conceive (in ideal-operational geometry) of a copy being taken of A, B and carried over *unchanging* so as to coincide with C, D. If it is then objected that in the process either C, D or the copy itself could have changed, we can answer as follows. To speak of 'change' implies, as we have seen, that there is some absolute way of distinguishing whether there is a change or whether there is not. The only ground on which such an absolute distinction can be asserted is that of the ideal-operational geometry which is the logical development of the primitive sameness and other spatial concepts or conceptual operations associated with it, on which the possibility of measurement ultimately rests. If then we find some inconsistency (or lack of fit) between the results of attempted superposition and the results of this

[1] This approach to space-*perception*, as the direct awareness of certain manifested space-*conceptions*, is studied in some detail in Chapter 10. My use of the term 'parapsychical' is explained below, p. 350.

ideal-operational geometry, that itself will be the criterion that there has been some change: in the copy of A, B, in C, D, or in both.

What then is the ideal-operational geometry developed from the concept of primitive sameness in respect of spatial apartness, that is, ideal *congruence*? As shown below,[1] it is simply Euclidean Geometry, not as developed step by step from axioms in a certain convenient order, with constructive rules and deductions, but as an ideal-operational geometry in the manner now to be described.

We first consider how a unique line between two points can be operationally obtained. In actuality no such line could be determined with absolute exactitude. For absolute exactitude is *ideal*, and all operations playing any part in actual measurement are *empirical*. The one exhibits its own self-evident analysis and thus presents itself as *intelligible*. The other involves the unanalysed unpredictabilities of all human actions and reactions in this world. Hence in actuality the operational obtaining of a unique line between two points can be done only by a method of successive approximation, at best. But in the ideal-operational geometry the method (explained in the next paragraph) may be conceived to yield, with all exactitude, the line required. For this geometry is ideal, and the approximation process can be ideally conceived of as continuing to any required degree of exactitude, i.e. to the limit.[2]

In practice we begin with a material structure (called an approximately 'rigid body') on which markings may be placed so that, always, to a high degree of accuracy, any figure made up of those markings remains congruent to itself—in the sense that copies could be taken and applied congruently regardless of the passage of time. Conceiving that the line we wish to construct will look and feel 'straight' (or as Euclid says, will 'lie evenly'), we make as good an approximation as we can to what we judge (by the first two methods of estimation) to be 'straight', making the body itself an approximation to what we call a 'straight-edge'. We then make, as far as possible, an exact copy of this, and lay the edges of the two bodies alongside in every possible orientation. If the edges do not coincide, we shave off portions where our visual and tactual judgement indicates it is necessary to do so, until we arrive at as close an approximation to a perfect fit as we desire. By this con-

[1] Chapters 10 and 11.

[2] The situation is analogous to the determination of $\sqrt{2}$, ideally and actually. The fact that, owing to the finite time available to us, $\sqrt{2}$ can be calculated only up to a certain degree of accuracy, and can be constructed in nature with an altogether lower degree of accuracy, is no argument at all against the assertion that ideally $\sqrt{2}$ is completely precise; for the *law of successive derivation* of the decimal places, i.e. the law of construction of $\sqrt{2}$, is completely precise.

structive method we obtain, as the ideally ultimate end, a uniquely defined line connecting any two points, although, in the *actual-operational* geometry, it is in principle impossible ever to obtain it absolutely.

Since now the ideal-operational geometry of these 'straight-lines' is Euclidean, it has appeared that in measurement we begin by *putting Euclidean geometry into the world*, so far as we can. And the world is capable of containing Euclidean geometry in the approximate way indicated because (1) this geometry, as Kant and Hilbert have said, expresses the true analysis of our intuitions of space,[1] and (2) these concepts (straightness, congruence, etc.) are capable of being represented and are in fact somehow represented in the world, in such a way that we can use the third method of estimating spatial apartness in order to attach measures to what we find.

The use of approximately rigid measuring rods may seem a most inconvenient and artificial limitation in the theory of measurement, in view of the far greater precision and serviceability of methods using light-rays. The limitations may seem all the more intolerable when we have under consideration the ultimate means of measurement to be adopted for all physics, in view of the molecular structure of matter, so that long before atomic dimensions are reached the wielding of measuring rods has become fantastically impossible. It may also be held that 'the naïve classical idea of absolute rigidity must be abandoned, since it implies that a disturbance can be propagated with infinite velocity through a body' (Whitrow[2]), and 'the size of measuring rods is determined by atomic interaction' (Bondi[3]) so that they cannot be taken as 'true primary standards'.

I submit, however, that in our present discussion of the foundations of measurement such objections are beside the point. In the first place we must avoid the category mistake of trying to combine, in the same account of measurement, statements about the *physical universe* (as defined in Chapter 4) with statements about the *universe of mathe-*

[1] D. Hilbert, *The Foundations of Geometry*, trs. E. Townend, Chicago 1910. Introduction, p. 1: 'This problem of the choice of axioms, etc., is tantamount to the logical analysis of our intuitions of space'. Unlike Kant, however, Hilbert would have agreed that our *intuitions of space* may also be utilised in the construction of non-Euclidean *geometries*, which are obtained by the theoretical relinquishing of one or more of the properties possessed by space in its full intuitive structure, or else reinterpreting the terms, notably the term 'straight line'. See, for example, his illustration of a geometry without five of the axioms of connection and one of the axioms of congruence (*op. cit.*, pp. 74 ff.) and the explanations in his 'Conclusion', pp. 126 ff.

[2] *Op. cit.*, p. 197.

[3] *Reports of Progress in Physics*, 22, 1959, p. 105.

matico-physical constructs. The vicious effects of this category mistake are seen chiefly in its blinding of us to the fact that the latter 'universe' presupposes measurement in the former 'universe', i.e. macroscopically, with the help of measuring rods. For a concept like that of atomic frequency or electronic mass is null and void unless at some point in the theory there is an operational rule relating such concepts to some macroscopic manner of measurement. Hence to discount the macroscopic manner of measurement ultimately depending on the use of rigid rules would be to take away from atomic theory the very ground on which it rests.

However, to resolve the difficulties clearly step by step, there are the following special points to note:

(1) Ideal 'rigidity', in the present context, means simply agreement with the Euclidean axioms, notably those of congruence; it has nothing to do with the properties of matter, which in any case could be measured only by first setting up a congruence geometry in the world.

(2) The basing of the theory of measurement on measurement by a rigid rule does not imply that an actual rigid body must always be used. Any physical phenomenon, such as a light-ray, can be used as an equivalent, provided we are able to establish, from that basis, that the ray can be consistently taken as describing a straight-line as we have operationally defined it.

(3) In order to test whether a light-ray is thus equivalent to a straight-line for purposes of measurement, we must always go back eventually to the operational definition in terms of approximately rigid bodies. *In vacuo* we do not ordinarily consider any test necessary; but this does not mean that it can be dispensed with in principle. For in the case of gravitational deflection, inconsistency would arise from supposing that a light-ray was 'straight' in the sense defined. We have to learn from experience under what circumstances we can reasonably expect consistency and so dispense with a test of straightness.

(4) Measurements of extreme accuracy, astronomical or atomic, are made by first setting up a rigid frame, suitably graduated by repeated subdivision into congruent intervals, and then by extending this frame with the help of light-rays treated as equivalent straight-line structures. The optical instruments by which this is done (if indeed it can be done consistently with the strict logical principles and developments of our ideal-operational geometry) present to us, accordingly, the measurements which would have resulted if ideal rigid rules had been used—even in regions where the use of actual rigid rules would be outrageously impossible.

(5) If this attempt at extending the local frame by light-rays, taken as equivalent straight-line structures, leads to inconsistency which

cannot be satisfactorily removed by 'correction', it is still open to us to investigate whether the extension can be made more satisfactorily by treating light-rays as equivalent to 'lines' of another kind, e.g. geodesics in some non-Euclidean or four-dimensional geometry.

(6) An objection may be made that measurement of atomic and astronomical distances thus proceeds in a different manner from that of local macroscopic distances, making therefore two separate techniques and an unsatisfactory dichotomy in the theory of measurement. To this, however, an answer can be given along the following lines.

It may well be that there is an ideal-geometry which is creatively expressed in all the varied phenomena of the world and which has the uniform character of construction by some kind of wave (e.g. electromagnetic, gravitational, quantum-mechanical). If so, it would express itself actually in the behaviour of rigid bodies by means of the same constructive operations and theorems as in the kinds of visible behaviour which we impute to the operation of atomic systems. To suppose something of this kind is merely to suppose that the world behaves, as far as its measurability is concerned, in a rational manner, and that all the rational principles in question can be grouped together and called an 'ideal-operational geometry'. But in becoming *actual* to us, and therefore *knowable in the phenomena*, this geometry must become subject to severe limitations. For, being *ideal*, it cannot be known as exactly expressed in the *empirical* phenomena. We can approach knowledge of it only by some technique of successive approximation. Such technique (which is to be regarded in this context as heuristic in intention) can be applied only where and how nature provides the means. Now the means nature provides us in this world are local and macroscopic, with structures (immensely massive and nearly rigid bodies) which already manifest the properties of a certain ideal-geometry in a wonderfully complete, but still necessarily approximate, way. The fact that, heuristically, we are compelled to begin our successive approximations with the 'first approximations' provided by nature (i.e. with experimenting on local and approximately rigid bodies) does not conflict with the claim that the possibility of measurement depends on the *same* ideal-operational geometry being found applicable to all phenomena, local and macroscopic, astronomical, or atomic.

Finally, far from this technique of measurement, derived originally from the making of material approximations to ideal Euclidean congruences, being 'inconvenient' or 'artificial' or 'an intolerable limitation', it is the very means by which measurements of the most extreme refinement are actually made, astronomically or atomically—in fact, it is the only means which scientists have been able to devise for consistent measurement, or which it seems in principle possible to devise.

CHAPTER 7

VICIOUS CIRCLES IN THE THEORY OF MEASUREMENT

'. . . we now realize that the foundations of a theory are actually the most elusive and confusing part of it.'

J. L. Synge, *Relativity, the Special Theory*, p. 2

One vicious circle has appeared already in our introductory discussions, when we attempted to find an absolute distinction between physical and non-physical phenomena. The tests for physicality resolved themselves into ones for '*physical* interaction, with *physical* bodies, and public to *physical* people suitably placed in the *physical* world'.[1] In practice this circularity causes no trouble. For we can proceed, in our tests, by successive stages from the obviously physical to the less obviously physical, judging the one by the other. The first stage, that is, the obviously physical, is judged to be so by 'pure commonsense', which provides for us a broad recognition of different 'systems of behaviour', physical or non-physical.

Two other vicious circles have now to be considered. Of these, one is also broken by a method of successive stages. But more is involved in this case than in the case already dealt with; for, as shown below, there is a presumption that another vicious circle, which refers to non-local measurements, has already been broken in some way. I therefore call it the *secondary* vicious circle. To break the other vicious circle (which I shall call *primary*), and thus to obtain an actual operational system of measurement over the whole universe, we need elaborate mathematical techniques together with an appeal to philosophical principles of considerable subtlety. Its significance in the study of space and time can therefore hardly be overestimated.

Temperature

The secondary vicious circle can be dealt with fairly briefly, provided we accept the necessity of beginning measurement (as explained in the previous chapter) by putting a local Euclidean geometry into the physical world by construction of successive approximations to ideal rigid bodies, ideal congruence, etc.

In the measuring of temperature, where perhaps the occurrence of

[1] Page 60 above.

this kind of vicious circle is most obvious, the position is clearly put by Lenzen:

'The definition of the standard of length requires specification of its temperature. But the definition of temperature presupposes the concept of length. . . . Thus we seem to have become involved in a vicious circle. In order to measure length, we must correct for the temperature of the rod, but, in order to measure temperature, we need to measure length.'[1]

To state precisely how the circle is broken in this case would take many pages of careful explanation; but the outlines may perhaps be sufficiently well conveyed as follows. Basically, the technique is to carry out measurements with the aid of presumed straight and rigid rods (or presumed straight-line equivalents, such as light-rays) with presumed congruent divisions, and, wherever there is a departure from what the ideal Euclidean geometry would require, to interpret it as due to (and so providing a measure of) the departure of the supposed straightness or congruence from the ideal, associated with a difference in 'temperature'. Obviously 'temperature' must mean for us something more than the departure itself from the ideal. Otherwise we would be doing no more than simply stating that there *is* a departure. Hence it is to be presumed that we have some independent means of recognizing what 'temperature' is and when two temperatures are the same or not. But we cannot presuppose an independent means of *measuring* temperature, since that would imply the prior establishment of spatial congruence, which is the very thing we are now trying to establish.

In order to carry out the correcting technique progressively we must begin with a small region where there are no 'temperature' variations of this kind, i.e. Euclidean geometry can be very accurately actualized there. The Euclidean geometry of this region is then extended to regions in which there are apparent departures, associated with the 'sense' of temperature (a similar procedure would be followed with departures associated with impressions of 'stress'). In the simple case of two originally identical rods, beside each other, one being enclosed in a hot-air jacket, the departure from coincidence provides at once a measure of the relative temperature. This is of course a very special and artificially simplified case. In general the procedure should be, in principle, to measure a distant object which we suspect to be at a different temperature, assuming Euclidean geometry, and then to try to account for the departure from that geometry in terms of some systematic alteration (e.g. proportionate increase) in the dimensions of the measured object or the supposedly rigid structures used to measure it. The complications are of course vastly reduced in practice by the fact

[1] *Procedures in Empirical Science*, Chicago 1938, p. 12.

that the paths of light-rays *in vacuo* can be supposed unaffected by temperature, being always straight-line equivalents. That is, we actually find that no special 'corrections' to the supposed straightness of light-rays *in vacuo* are to be made on the score of temperature.

Absolute Standards

It is to be noted that this secondary vicious circle, and the technique of successive approximation by which it is overcome, arise because we are trying to implant an ideal geometry in the world empirically. The vicious circle (which is broken only in an approximate sense) may be regarded as evidence that such implantation, if required to be exact, is impossible. The problem of *absolute standards*, to which Lenzen refers, may be said to arise from the same cause, but in a certain way goes considerably beyond the techniques just described.

To establish an absolute standard of temperature (and so also of length), we must first try to extend our actualization of Euclidean geometry, and hence our measures of *relative* temperature, over the universe or at least a considerable portion of it. If all goes well, the temperature of any one of the small local regions could be taken as standard. If, on the other hand, we find that in the course of time it is inconsistent with the supposed actualization of Euclidean geometry to take this temperature as unchanging, such inconsistency will itself be the criterion of change. The inconsistency provides a measure of the change, and so also of the 'correction' required to remove it.

The manner in which the correction is calculated might be excessively complicated, depending, perhaps, on a certain judicious choice as regards the manner of *extension* of the local Euclidean geometry—possibly by supposing idealized 'free-falling' rods coinciding end to end, or by postulating a uniform rectilinear velocity of light when suitably corrected for passage through or near matter. But we need not, at this stage, investigate possible means of extension. The conclusion is merely that inconsistency with the ideal geometry or system of measurement which we are trying to impose on the universe provides, in principle, a 'correction' by which we approach an ideally unchanging standard. These corrections are to be correlated with corrections or measures of temperature obtained in other and perhaps more convenient ways, so that possibly we shall find our choice of ideal geometry narrowed considerably. Thus by progressively applying corrections obtained in different ways, in different parts of the universe, and at different times, we progressively approach the determination of absolute standards of length and temperature.

This elaborate manner of approach, which is theoretically necessary, is again vastly simplified in practice by means of certain facts of experience conveniently supplied by nature. For example, there is the

very precise association of particular temperatures with other physical effects which are obvious, e.g. water changing into ice or steam. In consequence we can fairly confidently take the temperature of melting ice or of steam in contact with boiling water as our first approximation to a standard in the sense just explained, and can rely on the corrections still required (because of other observed departures from Euclidean geometry, or inconsistencies with other branches of physics) being very small and easily made.

A 'Logical Gap'

There seems to be some doubt as to which authors have recognized the occurrence of the *primary* vicious circle and the conclusions to be drawn. Many times, where reference seems to be made to some general difficulty of the kind, the language is too vague to give us confidence that *two* vicious circles and their contrasting implications were in fact distinguished.

In the following passage, Borel seems to be saying that geometry presupposes physics and physics presupposes geometry:

'To measure space and time we shall have to begin by reconstructing the whole of physics; and we thus find ourselves at the outset faced with a difficulty, since apparently we cannot even begin to make observations of physical phenomena if we do not know how to measure either space or time.'[1]

But his conclusion that successive approximations, provided that they 'converge', get over the difficulty, suggests that he had primarily in mind the vicious circle we have already discussed. Similar remarks might be made about the opinions expressed in passing by Bridgman and Ramsperger.[2]

Reichenbach is primarily interested in the problem of gravitation, in which, following Einstein, it is customary to say that a method quite different from that of measuring temperature must be adopted for the measurement of space and time. That is, Euclidean geometry is *not* actualized in successive stages; neither, therefore, are 'corrections' used to give the means of measurement of the phenomenon (gravitation, in this case) which is associated with departure from the Euclidean results predicted. But the 'logical gap' is closed a different way, by the adoption of an arbitrary *coordinative definition*.[3] This amounts, in fact, simply to

[1] *Space and Time*, London 1926, pp. 27 ff.

[2] P. W. Bridgman, *The Logic of Modern Physics*, New York 1928, p. 71. A. G. Ramsperger, *Modern Philosophies of Science*, New York 1942, p. 224.

[3] *The Philosophy of Space and Time*, English translation, New York 1958, pp. 14 f., 33 f.

the decision *not to make any corrections* for departures regarded as due to gravitation. Rigid rods, idealized by the usual local methods, are imagined as carried about the gravitational field, and whatever measures they yield (after being corrected for temperature, stress, etc. are to be the measures of length to be accepted by physicists as correct for the gravitational field.

This explanation of Reichenbach is easily seen to be a loose and incomplete way of stating the following ideal-operational *rule of correlation of scales* in different regions of space-time: (1) If one and the same ideal rigid rod (or 'standard distance-interval') is measured in each of various local Galilean frames, in each case at rest in the frame, the measures of length shall be the same; and (2) the velocity of light as calculated in each frame shall be a certain arbitrarily chosen constant (unity, if 'gravitational units' for time are required); this determines the units for time. Reichenbach omits condition (2), and assumes that an *actual physical* rod (corrected for temperature, stress, etc.) will agree precisely and always with the geometry built up from the ideal correlation rule. He forgets about the unavoidability of random deviations and possible deviations due to unknown causes, such deviations being measurable only by their relation to the *ideal* standard established by some prescribed geometry.

These confusions concerning 'physical geometry' and 'proper' distance and time will be further dealt with later in this chapter and in Chapter 12 below.

In the meantime two other points require to be noted:

(1) Nothing is said by Reichenbach to indicate how this approach may be made operational in all ways, and in particular how the system of space and time measurement may be actually set up—if indeed it can!

(2) Reichenbach is to be given the credit of clearly recognizing that the case of gravitational fields and other 'universal forces' which 'affect all materials in the same way' differs from that of temperature, and that a 'logical gap' or *operational hiatus* thus appears. We can at least be perfectly consistent if we decide to 'ignore the effect of universal forces' in deciding what 'corrections' to make and what not to make, whereas it would be hopelessly inconsistent to ignore the effect of 'differential forces' such as temperature. For then the distance between two points would depend on the material of which the measuring rod was made. But his further statement that 'our measurements will still contain some degree of inexactness which a progressive technique will gradually reduce but never overcome[1] reveals confusion between (*a*) the operational hiatus due to refusal to 'correct' for gravitation, and (*b*) the epistemological hiatus between the ideal and the empirical. Only the latter is bridged by a method of approach by successive stages.

[1] *The Philosophy of Space and Time*, pp. 29 ff.

The Primary Vicious Circle

Recognition of the primary vicious circle is the evident basis of various remarks by certain outstanding mathematical physicists and philosophers, that of Weyl being typical:

'A space that can serve as "form of phenomena" is necessarily homogeneous.'[1]

Likewise Whitehead: 'I doubt the possibility of measurement in space which is heterogeneous as to its properties in different parts. I do not see how the fixed conditions of measurement are to be obtained.'[2] Bertrand Russell, on similar grounds set forth at length, finds fault with Riemann's views in his famous thesis of 1851, 'Über die Hypothese, welche der Geometrie zu Grunde liegen':

'His philosophy is chiefly vitiated by the uncritical assumption that a metrical coordinate-system can be set up independently of any axioms as to space-measurement. . . . Riemann has failed to observe that unless space had a strictly constant measure of curvature, geometry would become impossible.'[3]

In order to show as clearly as possible how conclusions of this kind arise from the recognition of a vicious circle, I shall formulate Weyl's assertion explicitly with the help of three definitions, as follows.

Def. 1. 'Arithmetized'[4] *actual-operational geometry* consists in the acceptance of

(*a*) operational rules by which an observer in a local Galilean frame may actually and approximatively[5] allocate, in a one-one way, four measures x^1, x^2, x^3, x^4 to events in distant regions, using any one of a group of methods to be described as *orientations*. By an orientation is meant a set of values of the quantities x^i in the locality of the observer together with a set of values of the quantities $\frac{dx^i}{dx^j}$ in four objectively identifiable 'directions' there. These various values will be called the *zero-values of the orientation.*

[1] *Op. cit.*, p. 86.

[2] Quoted E. T. Whittaker, *History of Theories of Aether and Electricity*, London 1953, p. 174.

[3] *The Foundations of Geometry*, Cambridge 1897, p. 69.

[4] Weyl uses, apologetically, the term *coordinatization* (*The Classical Groups*, Princeton 1939, pp. 15 ff.). I prefer the term *arithmetization* partly for reasons of euphony, but chiefly to emphasize the fact that in operational geometry a symbol alone is insufficient. There must be means for attaching a numerical measure.

[5] Throughout this book the word *approximatively* implies the process of successive approximation of the empirical to the ideal by some operational technique.

(*b*) operational rules or formulae by means of which (utilizing the above measures) the observer may obtain, for two *end events* in a certain locality near or distant, a *correlation measure* ('interval') which is approximatively the same, whatever orientation he may adopt.

This may be described as a *one-observer limited* actual-operational geometry (arithmetized). A *general* actual-operational geometry would require also:

(*c*) operational rules showing how the coordinates obtained by one observer may be transformed approximatively into the coordinates allocated to the same event by any other observer, given the zero values of his orientation, so as to obtain in each case the same correlation measure for any given pair of end events.

The verification that a certain geometry is a general actual-operational one cannot, of course, be complete. It must be made with observers in as widely varying circumstances as possible. Prediction as to what will happen on future occasions and in other circumstances will then depend on an appeal to *uniformity in nature*, which of course cannot have absolute validity. There is no way of avoiding this appeal; for without it prediction is impossible. What remains then is only a collection of observations as and when made; and this does not constitute science.

It would be quite incorrect, I think, to regard 'uniformity in nature' either as an *a priori* principle justifying all inductive generalization or as a mere expression of human expectation arising from psychological causes. I think we should view it in terms of a principle which I shall discuss in the next chapter, under the name of Helmholtz's Principle.[1] Briefly, this expresses our consciousness of the fact that departures from regularity in any kind of experience cannot occur in complete isolation, physical and conceptual, from everything else in the universe. They must have some substructure, which we can denominate their 'cause'. Hence if, after the most thorough endeavours to discern whether there is any relevant difference in the total circumstances and their substructure, conditions appear the same as on other occasions when a certain regularity was observed, we are justified in supposing that that kind of regularity will be maintained. If it is not, then we merely proceed to isolate the new 'cause' and correct for it.

If the predictions of the geometry with regard to the measures that would be found by distant observers and at future times are sufficiently well confirmed for the deviations to be regarded as due to random error, the geometry may be described as *verified*. On the other hand it is possible to continue to apply a geometry which is not verified in this way, and to reduce the now considerable deviations to some kind of

[1] Pages 114 ff.

system. We must conclude that these deviations are systematic; otherwise the occurrence of varying phenomena in the universe will be haphazard, and science will again be impossible. The system, when found, may be regarded as providing a measure of some physical property (e.g. temperature, gravitational potential, etc.). In this case the applied ideal-operational geometry may be called *standard.*

A procedure of this kind is exactly what is followed—and *must* be followed, it seems—for the obtaining of measures of 'differential effects' such as temperature. For a geometry which varied according to the materials used in measurement would be quite unserviceable and would also give a misleading picture of the objective universe.

Since more and more systematic deviations are discovered in physics as research progresses, we have no right to expect a miraculous fit between some ideal-operational geometry and observation in all circumstances. In the case of the gravitational red-shift in the spectrum of the sun, for example, the variations over the sun's surface require to be accounted for by some physical theory taking as *standard* the value predicted by the geometry of the Schwarzschild solution, or perhaps some standard established some other way. In any case a *standard* must be established in distant parts by the application of an ideal-operational geometry, whether or not this geometry turns out to be 'verified' (in the sense explained).

With these qualifications in regard to the occurrence of 'differential effects', to be removed by 'correction', it is clear that we may continue to call the geometries under consideration *actual-operational* ones, in so far as they can actually be applied and then yield correct results except for small random deviations.

Prescribed Geometries

At this point we might proceed at once to the very plausible conclusion that the *standard* geometry which is applied to the universe cannot be influenced by measures in distant parts for which it is to provide the standard. But this conclusion is not quite correct, and it is better to proceed more strictly, somewhat as follows:

Def. 2. Operational rules are to be termed *prescribed* if they do not depend on the prior determination of the measure of any physical phenomenon (e.g. mass) in distant regions.

Def. 3. An actual-operational geometry is to be termed *prescribed* if either (1) all its operational rules are prescribed, or (2) its operational rules depend on the determination of a finite number of measures of some physical phenomenon in distant regions, e.g. the mass of the sun, or the 'radius of the universe'.

The second case has to be admitted here because an actual-operational geometry may be established in that case by leaving the measures as unknowns in the formalism, investigating the observable consequences and so obtaining a number of equations for the measures concerned. The equations can be solved if the unknowns are finite in number. The *standard* actual-operational geometry obtained by assuming that the measures in question remain constant may then be verified by further observations; or if not verified, then the deviations will provide a measure of the changes.

On the ground of these definitions and terms, Weyl's remarks may now be put in the form of a vicious circle argument: If any actual-operational geometry is *not prescribed*, we must know in what way the measures of physical phenomena in distant regions (other than the measures which may have been used in its prescription) require a modification of the operational rules. It is necessary therefore to attach a measure to those phenomena. But all measures in any locality are ultimately dependent on space and time measures there. Thus, concisely, in order to obtain space and time measures in distant regions we must first be able to obtain space and time measures in distant regions. The only escape from this vicious circle is by all actual-operational geometries being necessarily *prescribed.*

We cannot here proceed as in the second case covered by Definition 2, that is, determine the measures in distant parts indirectly, one by one. For by hypothesis (the geometry being not prescribed) they are infinite in number.

In later chapters[1] I shall illustrate how these conclusions are in every way borne out in the practical problems which General Relativity attempts to solve.

A prescribed geometry, as defined, may of course be subjected to an arbitrary 'distorting' transformation, perhaps depending on the orientation of some preferred observer. It need not therefore be 'homogeneous' in any customary sense of the word; but one such geometry, or group of geometries, may have *prior operational validity* in so far as the others are derived from it. This, and other aspects of the problem of geometrical representation will also be considered in later chapters.

[1] See, in particular, Chapter 13, pp. 197–8.

CHAPTER 8

OPERATIONAL AND PHENOMENOLOGICAL ANALYSIS

'No opinion is to be accepted as philosophical knowledge unless it is seen to be adequately established by observation of what is seen as itself given "in person". Any belief seen to be incompatible with what is seen to be itself given is to be rejected. Towards opinions that fall in neither class—whether they be one's own or another's—one is to adopt an "official" philosophical attitude of neutrality.'

D. Cairns, 'An Approach to Phenomenology' (from *Philosophical Essays in Memory of Edmund Husserl*, ed. M. Farber, Harvard 1940, p. 4).

Against the conclusion now reached, that the basis of measurement must be a prescribed operational geometry, it can of course be argued that physics has had to abandon the attempt to exhibit its fundamental concepts operationally. For example, wave-functions, whether of single particles or of systems of particles as in the atom or nucleus, are in principle unobservable. To claim that they might somehow be observed, even by as indirect a way as the 'observation' of energy-levels, would be to overthrow the formalism of quantum mechanics in its very origins, and thus to discount all its experimental confirmations.

Again, no elementary particle can be defined in terms of an operational technique. There are, indeed, certain definite contexts in which we consider it justifiable to speak of an 'elementary particle'. In physical experiment, a cloud-chamber track is supposed to originate from the heaping-up of particles of one composite kind (water-vapour) caused by another particle, which is rapidly moving; and the kinks in the track are supposed to be due to collisions or near collisions with still other particles. These picturings are in plausible analogy with macroscopic physics, and determine certain fairly consistent language usages for the term 'particle'. But as soon as we wish to establish the identity of such hypothetical 'particles' with 'entities' signified by the term 'particle' in other circumstances, notably 'within the atom' or 'in a beam', we find ourselves dealing with a most complex formalism, altogether beyond the scope of 'physical operations'.

It may therefore be urged that the fact that the coordinate systems

used in General Relativity are 'conceptual' is inevitable and no disadvantage, provided only that the experimental predictions agree sufficiently well with what is actually found. As Einstein has said of the conceptual systems proposed by modern physics: 'It is only necessary that enough propositions (of them) be firmly enough connected with sensory experiences.'[1]

In the case of General Relativity the experimental checks will be by measurements in some observatory. Hence, we may argue, only local measures of space and time need be defined operationally. And thus we may confirm ourselves in the accepted opinion that there is no reason for refusing to admit a non-Euclidean geometry which cannot be made 'operational' and that we *ought* in fact to admit such geometry because it is of great utility in physics.

Such a line of thought, however, misses two subtle but essential points; and it will be best to deal with these before discussing 'operations' more generally.

(1) How actually are our predictions to be verified? The argument in the last chapter says in effect: 'You *will not be able* to obtain a local verification without the aid of a geometry which is operationally prescribed. If you have doubts as to the validity of the argument, try out the alleged verifications, and analyse what is actually being done!'

The analysis of certain attempts at verification of non-prescribed geometries of space will be set out in detail in Chapter 13 below.[2] From this one may readily see how, in practice as well as in theory, it must similarly prove impossible to set up a non-prescribed geometry of space and time.

Even in the case of a geometry with prescribed *formalism* (metric, equations of motion) it is easy to see that purely local operational rules are inadequate. Thus, in the case of the gravitational deflection of light in passing near the sun, the prediction and verification of the magnitude of the deflection depends on measurement of the *mass* and *diameter* of the sun by the principles of Newtonian Mechanics in Galilean space-time. That is, operational definitions of measures in distant parts are in fact imposed, agreeing approximately with those of classical physics. Moreover, the deflection is calculated by comparing the trajectories of the light-ray on a Euclidean space-map in the cases when the sun is or is not present, supposing that the positions of the earth and the star remain the same on this map. These positions, it is assumed, are the ones given by astronomical observation under normal circumstances. The claim that a purely local verification can be made, without implying

[1] Article in *The Philosophy of Bertrand Russell*, ed. Schilpp, Chicago 1944, p. 289.
[2] Pp. 197–8.

any operational determination of measures and coordinates for what lies beyond, turns out therefore to be unsubstantiated.

The 'Living Skeleton' Fantasy

The second essential point is as follows:

(2) If only local measures are to be defined operationally, we must conclude that outside our local region physics reduces to a list of number-quadruplets, together with other measures associated with them and formulae by which one measure is related to another, all measures being classified according to certain suggestively-named types. In a strict sense we should not name the number-quadruplets 'events', because that would be to suppose that our concepts of space and time have been carried beyond our local region, though no means of doing so has been exhibited. Nor, for the same reason, should we speak of mass, velocity, or any other physical quantity, outside our local region. But in point of fact we shall always be speaking in such suggestive ways, without perhaps realizing what we are doing. Otherwise the universe beyond our local region would have no physical meaning for us.

To make quite clear what is the force of the objection here I shall first explain and illustrate the fantasy of the 'living skeleton', as I shall call it. We might incline to say that although of course there are no living skeletons in the world, it is not beyond us to imagine a skeleton walking about, moving its jaws, and so on. But if we think like this we shall have failed to notice that the very concept of a skeleton involves the fact that there are no muscles, nerves or blood vessels to make the movement and life possible.

Next suppose that a foreign mathematician is trying to master the English language and comes to the point where he cannot recall the respective meanings of the words *discrete* and *discreet*. He gets over the difficulty by writing *discr*, followed by a scrawl in which only the letter t is clear. In this way he hopes to trick others into thinking that he really knows which word he ought to write. And he may also subconsciously trick himself into thinking that the question which word is chosen is of no concern.

With regard to 'conceptual coordinates' we may draw similar conclusions. When we establish a local frame for physical measurement, and go on to postulate non-operational number-quadruplets in the rest of the universe, all that is strictly before us is a 'skeleton'. Nevertheless, by clothing it imaginatively (and vaguely) with space and time concepts, we trick ourselves and others into thinking that we really have produced a system of *physics*, or else that the vagueness is of no concern.

After these preliminary objections to conceptual vagueness, we turn

to a more thorough consideration of 'operational analysis', as a technique of conceptual analysis in general.

Concepts and Operations

A concept is not like a brick, sharply outlined and ready to be placed alongside others so as to form a larger structure. Whether it is the concept of an *apple*, a *right angle*, a *photon*, *physicality*, or *goodness*, our ability to think it depends on its having, within itself, an intelligible structure of which our thinking can catch hold. Or rather, we form the inflowing intelligible impressions into a fairly clearly marked and unvarying structure with the help of names, associations and other devices of thinking.

Starting thus with a 'conceptual region' which is identified somewhat roughly by name, our object in conceptual analysis is to *demarcate* it strictly and sort out the various interrelated *conceptual strands* in it clearly. These are not independent operations, to be done in succession. Demarcation itself depends on the internal structure of the 'region' in question, and also, even more obviously, on the contexts in which it is encountered. In the case of all but perhaps the most primitive and simple conceptual elements, demarcation, inner constitution and contexts cannot be entirely separated.

For the difficult task of clarification we need to proceed systematically, with aiding techniques of analysis, direct or indirect. There is only one type of analysis which I call 'indirect', and this will be considered in due course. All others, called 'direct', can be distinguished according to their 'grade' and general character or function. It will be convenient to distinguish two chief grades, to be described as *natural* and *phenomenological*, respectively. The *natural* types of analysis are those which assume certain matters to be sufficiently clear and settled, namely: a universe of discourse comprising words and language-structures of 'ordinary language'; the character and observed results of manipulations with physical objects and physical phenomena such as electromagnetic waves; and the ordinary psychology of stimulus and response, so far as this is considered not open to debate. The *phenomenological* types are those which go beyond all this, being free from presuppositions, hypotheses, inferences of discursive reason, and symbolic thinking in general. They are aimed simply at face-to-face self-evidence.

The types of natural analysis which now need further discussion are *linguistic* analysis and *outer operational* analysis—this last being understood as analysis in terms of physical or imagined-physical operations (other than symbols or words) which seem to carry with them the precise conceptual character of the 'region' being studied. Such operations cannot of course be identical with a conceptual structure; but we may think of them as equivalent to it.

Linguistic Analysis

Linguistic analysis is obviously important for demarcation. We need a *name* to act like a tab by which we can recognize the same thing and come back again and again to study it. Our use of a name must therefore be discerning, intelligent and systematic. In so far as the name is used in these ways, the corresponding conceptual region tends to be clearly demarcated; and it tends to become more and more clearly demarcated as we become able to allocate more and more subsidiary names in the same discerning, intelligent and systematic way, to subsidiary details which are discovered. But we must not mistake this exterior means which we use at various stages of the demarcation for the finally clarified conceptual region itself. To do so would be to slip into both the Abacus Fallacy and the fallacy of 'chicken-counting'.[1]

Linguistic or symbolic analysis consists in study of the interpretation of words or symbols in terms of concepts which are regarded as already sufficiently clear. The character of this 'study' or technique is conveyed by Wittgenstein's injunction: 'Don't ask for the meaning, ask for the use' (a well-defined use serving to fix a well-defined interpretation). The implication is that if the meaning is vague for us, the use is nevertheless clear; and the whole content which we dimly sense in the conceptual region in question or in our name for it corresponds exactly with the structure of simple and obvious concepts involved in the thought of its uses.

Let us suppose that the usage of a certain name or symbol is clearly established in the context of other concepts to which virtually no obscurity seems to attach (for instance, numbers, or sensory observations). Such a name or symbol then becomes like a material *counter* moved about in purely external relationship to other counters. If we are then presented with other concepts which appear still vague, the natural tendency is then to try to 'reduce' them to relations between the existing 'counters'. And the temptation may then follow, if they appear irreducible in this way, to declare them 'meaningless'. But hidden in such conclusion is the assumption that the existing 'counters' constitute the entire basic material for the whole of intelligible life, and we have nothing more to learn except the recognizing of their various combinations.

This assumption may be characterized as the Fallacy of the **Conceptual Closed Shop**. It has no place in any scientific or phenomenological inquiry. What *does* have place is the simple recognition that the process of bringing some conceptual detail to light must be accompanied by a process of *naming*, as appropriately as possible in regard to common

[1] Compare May Brodbeck's distinction between 'reference' and 'significance', *Minnesota Studies in the Philosophy of Science*, III, pp. 258 ff. For explanation of the fallacies mentioned, see above, pp. 41, 39.

usage, if we (and those with whom we wish to communicate) are to have rational control of the newly illuminated details.

Inner and Outer Operational Analysis

The nature of inner operational analysis may be simply illustrated by consideration of the mathematical concept of *continuity* of a curve or function $f(x)$. The concept of continuity as a 'smooth joining' is suggestive enough, but too vague either to play any part in a precise rational discussion of particular cases or to stimulate development of a theory. Closer thought, however, shows that the broadly-grasped idea of continuity of a curve or function involves two facts: (1) that every abscissa $x = c$ gives a point of the curve (or perhaps we can say that there are no gaps in the x-direction), and (2) that the variation in the y-direction can be made as small as we please by choosing a sufficiently small x-interval. The next step is to translate these facts into the language of *limits* by saying that $f(x)$ tends to $f(a)$ as x tends to a. Even if there are concepts of other kinds of continuity, we have thus produced at least the clear concept of one kind, possessing rationally precise connections with other mathematical concepts and fields of study.

It is clear then that by a kind of 'operational analysis' and conceptual 'focusing', aided by words and symbols, we have reduced a vague and little more than suggestive concept to something very precise. In this case the analysis can be described as an *integral one-level* analysis, since it is concerned only with the level of intelligible structures of thought and exhibits nothing more nor less than the concept itself with which we wish to deal.

Quite different is the analysis of a concept such as that of 'three apples'. Here we can carry out an *outer* analysis in terms of physical (or remembered physical) operations, including the colours, shapes, etc., which are seen and kinaesthetic impressions of which we are conscious in the process of counting the apples. We can also carry out an *inner* analysis of the abstract concept 'three' and of the laws of arithmetic in the context of which it has meaning. Lastly there is a *given* and *self-evident* connection or correspondence between the two levels of analysis, this correspondence being also open to analysis (of a higher phenomenological grade). In such a case we may call the analysis, regarded as a while, an *integral many-level* one.

Operational analysis of a concept such as that of the electron could be described as an *associative* or *defective* many-level analysis. For in such a case we are still struggling to obtain a clear and perfectly coherent conceptual structure (to be called 'the electron') by surveying a great variety of physical operations (experiments) each of which seems to suggest a substructure of a roughly indicated type. The 'outer' physical operations, considered as *merely* physical, can be subjected to a very

full analysis about which no two scientists are likely to differ. In addition, however, we have the *formalism* and a *model*, or imaginative interpretation of the formalism. The model puts a certain 'dress' on the bare symbols of the formalism, so that we can then interpret and manipulate the formalism in correspondence with particular physical operations or particular happenings which are supposed to occur in the model. Here, therefore, between the inner and outer levels of analysis there is a *hypothetical association*, not a *given correspondence*. The hypothetical association is of course also open to analysis, but the integral and self-evident character of the analysis of 'three apples' is lacking.

Lastly, on top of all the analyses mentioned, there is a *conventional linguistic association* between the concepts in which we are primarily interested and the visible or audible names which we attach to them. In this connection we have to note the further serious danger in using terms proper to the *outer* part of a defective many-level structure in an attempt to communicate a conceptual view of the whole. The terms are then said to be 'theory-loaded', and the inner part of the structure, if not also explicitly 'named', assumes the garb of (impure) commonsense. The safeguard to clear thinking and sound conclusions is then to submit *all levels*, as well as the connections between them whether given or imagined, to the phenomenological technique of conceptual focusing in a state of awareness free from all presuppositions.

Bridgman

In his classic treatise on *The Logic of Modern Physics* (1928) P. W. Bridgman set out to show the ever-present need for conceptual clarification through operational analysis, purely mental, ideal-operational, or actual-operational, as the situation may require: 'In general, we mean by a concept nothing more than a set of operations; the concept is synonymous with the corresponding set of operations' (p. 5). 'They [physicists] do find it useful to employ all sorts of constructs not defined by *purely physical* operations. Most of these non-physical operations are the operations of mathematics and logic.'[1]

In view of Bridgman's repeated references to 'mental operations', choosing even mathematical continuity as an illustrative example, it seems rather strange that contemporary opinion should have widely imputed to him a positivistic insistence on *purely physical* operations. Hence has arisen a non-Bridgman *Operationism*, well described by Popper as 'the doctrine that theoretical concepts have to be defined in terms of measuring operations'.[2]

[1] *Reflections of a Physicist*, New York 1955, p. 15.

[2] *Op. cit.*, 1957, ed. Mace, p. 189. Also *The Logic of Scientific Discovery*, London 1959, p. 440.

So Churchman calls Bridgman's a 'positivistic thesis' and, later in his book, writes: 'In terms of previous discussion, "operations" are only means to an end, and no single means is *necessary* for the pursuit of an end.'[1]

But operational analysis, in the broad sense of seeing the concept and its connections with other concepts clearly, is both means and end, and is the one thing which is unavoidably necessary.

Bridgman dissociates himself strongly (1955) from this 'Operationism', accepting instead the term operational analysis. At times, however, his phraseology still tends to emphasize the behaviouristic (i.e. psychological) aspects as against the intelligible ones, the linguistic as against the directly conceptual: 'To know the meaning of a term used by me it is evident, I think, that I must know the conditions under which I would use the term; and conversely to know the meaning of a term which my fellow uses, I must be able to reconstruct the conditions which compelled the use of the term by him. Analysis of the conditions attendant on the use of the term is an analysis of operations. From this point of view, *meanings are operational.*'[2]

From the point of view developed in this chapter this account of the matter confuses the art of communication with the thing to be communicated. Both may be necessary in practice; but the former consists in the proferring of helpful atomic clues, while the latter is a given organic structure.

Axioms

The *indirect* aid to analysis is *axiomatization.* Comparing, for example, our vague concepts of a straight line and a triangle, we observe a necessary connection in so far as, if a straight line is shown to be the shortest distance between two points, we are obliged to admit also that two sides of a triangle are greater than the third. That is, when the first fact is taken as established (i.e. set up as an *axiom*, or 'proved' from other axioms), the second fact is simultaneously established (or follows deductively, as we say). Such an analysis thus reveals something of the extraordinary generality contained in the concept of 'shortest distance' and also how closely the properties of a triangle are bound up in the concept of a single straight line. But we make here no attempt at *direct* analysis of the concept of a 'straight line'. On the contrary we take the straight line as an undefined concept, and introduce the constructive connections with other concepts (also undefined) one by one as we need them, i.e. as their necessary involvement in some other truth is disclosed to us.

[1] *Theory of Experimental Inference*, New York 1948, pp. 111, 215.
[2] *Op. cit.*, p. 4.

It is to be particularly noted, however, that a conceptual system such as Euclidean geometry may be subjected to innumerable axiomatizations, all hazy in different ways. In the example just given, for instance, we have passed over the illogicality of speaking about a 'shortest distance' without having previously considered any variety of lines and corresponding distances between two points. 'Distance' and 'line' are taken as already understood in a vague way. To avoid this vagueness we may prefer to begin with straight lines and polygons as undefined concepts, a line in general being the limit of a polygonal path. If we then impose an *axiom* that two sides of a triangle are greater than the third, we deduce the fact that there is a shortest distance between two points (found by repeatedly joining successive angular points of any given polygonal path) and that this shortest distance is the straight line joining them.

Thus, depending on the features to which initially we turn a blind eye (refusing to analyse concepts in those respects) we have deduced either of two propositions from the other. Like the human body, the conceptual system can be dissected in innumerable ways; but every way destroys something of its intelligible life, which is indivisible.

The epistemological service provided by axiomatization may perhaps be described as follows. When two conceptual regions, *A* and *B*, say, overlap, as do the concepts of 'shortest distance' and 'triangle', characteristics of the common region (e.g. 'line', 'additivity', 'change of direction'), set down as axioms, provide our initiation to a more accurate analysis. If then we search for other conceptual connections in *A* or *B* separately, and set down these also as axioms, it will appear that *A* and *B* have been *deduced* from our axiom system; for the connections are all 'necessary' to the concepts in question. We judge the validity of the deduction partly by mere quoting of the axioms written down, and partly (if we are investigating concepts and not merely occupied in 'the quoting game') by intuition. A keen mathematical intuition quickly senses a lack of connection at some point, by focusing on which a further characteristic is disclosed and set down as a further axiom. Or, more probably, the original axioms may have to be recast, because our original ignoring of the composite character of the 'primitive concepts' referred to in the first axioms has been found to cause avoidable complications and confusions.

The whole procedure is thus heuristic and not final. To believe it final would be to succumb to the living skeleton fallacy. In other words, we should have failed to see that axioms (if they are anything more than a pattern of signs to be copied) make sense only in a context, which other axioms partly supply, themselves also needing a context, thus riddling the whole with vicious circles.

Survey of Types of 'Natural' Analysis

As a preliminary to discussion of the deeper kinds of conceptual analysis we must review the position so far reached, with special reference to the states of mind and reflective techniques which have to be brought into play at each level.

There is first of all the *outer operational* analysis in which actual physical operations are envisaged and all the limitations of physical experience are taken into account. This may be suitably characterized as 'actual'. An illustration would be the construction of a straight-edge and congruent intervals on an approximately rigid body by successive approximations, which never, in actuality, attain to the ideal concept of a straight line or of equal intervals.

There is also, however, a type of outer operation which is *ideal*. That is to say, it is a conceptually precise procedure, but one which fails of becoming physically exact because of physical limitations such as those just mentioned, inevitably involved in what is empirical.

Thirdly, there is an *inner operational* analysis, i.e. one not concerned with actual or idealized physical objects, but which nevertheless involves conceptual focusing on details of procedure or structure in the abstract. I shall call this analysis *quasi-temporal* if there is an essential logical order in which the procedure is conceptually unfolded for us. The mathematical concept of continuity, already discussed, is of this kind. *First* we must stipulate that there are 'no gaps in x', and *afterwards* we must conceive of the possibility of taking *successively* smaller and smaller intervals round any given value of x in the range of continuity and then finding that the variation of $f(x)$ can (in principle) be made as small as we please.

On the other hand, inner operational analysis is *non-temporal*, or, as I shall say, *absolute*, if no particular order of unfolding of the conceptual connections is implicit in the concept, however much we may feel, psychologically, that one order of investigation of the conceptual structure is easier or more convenient than another. An illustration would be the inner operational investigation of the conceptual-content of the cardinal number *three* regarded as the abstract, timeless, conceptual 'essence' of 'three'—not three apples, three inches, three days, but *that* intelligible act of recognition by virtue of which we are justified in attaching the name 'three' to them all. Clearly our psychological habits and language conventions are not the all-sufficing reason why three apples and three days are both 'three' to us.

From this illustration it is also clear that an analysis of the temporal order and character of psychological responses, or of the structure of personal habits from which these seem to proceed, is quite different from a conceptual analysis of that which is recognized in the response, on various similar occasions, by us or by other people, which is not

temporal in the same sense, or which is perhaps not temporal in any sense. The mistaking of an analysis of psychological responses for an analysis of the more strictly intelligible structure which prompts them is the error of **Psychologism.**

Linguistic analysis can be regarded as a special type of psychological analysis of habits. In this case the habits are associations of verbal forms with conceptual structures, these associations being established by a certain degree of unconscious or conscious agreement with other people. Unless the conceptual structure is of a simple and easily recognized kind, already quite sharply demarcated from other concepts which lead us to it, assist its comprehension or are constructively linked with it, the correspondence of word and 'meaning' cannot be at all precise and leaves the door open to misconceptions. Analysis must then proceed at a higher level (perhaps again with the assistance of particular words, imprecise but nevertheless suggestive) in order to clear away these misconceptions and assist the disclosure of what has been overlooked.

I shall describe as 'natural' all the types of analysis just mentioned, except *inner operational* analysis. Being *quasi-temporal* or *non-temporal* this will be better included under the 'phenomenological' types now to be considered.

Phenomenological Analysis

The term 'phenomenological' is here used in a sense broadly corresponding to that of Husserl[1] and modern phenomenology. By 'phenomenon' is meant any object of recognition, understood to be directly observed without dependence on the 'apparatus of inference and proof'. All 'interpretations which transcend the given are kept at a distance'. The suffix '-logical' indicates that the kind of recognition ('intentionality') to be demanded reveals the object as *precisely intelligible*, and does not merely name it as a whole according to convention.

It is due principally to Husserl that there is now in western philosophy an influential school of thought which admits the need for a special kind of approach and a specially developed and open state of mind, if this 'immediate seeing' of intelligible characteristics and constitution is to be possible for us. It is to be remarked that Husserl's account of the method, character and degrees of phenomenological analysis agrees very strikingly, even in matters of detail and language, with what can be found in certain classics of eastern philosophy and psychology. Here I shall give only brief indications of Husserl's approach and method, further clarified, I hope, with the help of a few references to the methods and degrees of contemplation in Theravada Buddhism.

[1] Husserl's work is surveyed in detail in Chapter 24, pp. 404 ff.

According to Husserl, the 'natural attitude' which we tend to adopt to the world and problems of life conceals the essential character of things in a 'cloak of ideas' which we have grown to accept without question and which shapes all our opinions and theories. Among the unfortunate results to be seen are the dominance of 'physicalistic objectivism' and the loss of vital meaning (Lebensbedeutsamkeit) of the sciences.

In order to arrive at an adequate comprehension of any state of affairs or problem, 'the decisive factor lies in the absolutely faithful description of that which really lies before one in phenomenological purity, and in keeping at a distance all interpretations which transcend the given' (I.90).[1] We must put the naturalistic theses 'out of action'. We can contemplate them, but must do so in 'full freedom', without being bound to 'cognitive attitudes that would set themselves up as coordinate with the thesis' (I.31). Then, by a suspension of attachments, and a 'stoppage' (*epoché*), we come to a 'unique form of consciousness', characterized by 'a certain refraining from judgement which is compatible with the unshakable because self-evidencing conviction of truth' (*ibid.*).

The 'operation' of clarifying essential character by the transcendental *epoché* 'will split up into different steps of "disconnection" or "bracketing", and thus our method will assume the character of a graded reduction' (I.33). So we are enabled, step by step, to 'bring to the normal distance, to *complete clearness*, what at any time floats before us shifting and unclear and more or less far removed intuitionally' (I.67). This insight does not set before itself a succession of memory-images. That which we see has, on the contrary, the character that it 'already was before the gaze was directed to it'. *Secondary* remembrance reproduces memory-images; but *primary* remembrance ('primordial recollection', in my terminology) presents to us the originally-given changeless essence.[2]

The distinction between *inner operational* analysis and higher grades of phenomenological analysis to be described as *transcendental* will be of considerable importance in some later chapters of this book. I shall therefore complete this survey of types of analysis with a few remarks on the epistemology of Theravada Buddhism, a system whose entire teaching centres upon grades of 'release' and their attainment.

'The Only Way', it is said, 'leading to attainment of the Method (*Yoga*), to the realizing of Release (*Nibbāna*)', is by the fourfold practice

[1] Numbers here refer to sections in the first volume of *Ideen*.

[2] On the subject of 'primordial recollection', see Chapter 19, especially p. 295; for the term 'primordial' in general, see pp. 408 ff.

of 'setting up of recollectedness' (*sati-paṭṭhānā*).[1] The following explanation of 'recollectedness' in respect of *thought* (*citta*) evidently characterizes a kind of psychological and phenomenological suspension:

'In this method a brother is aware of a passionate thought that it is passionate: of a dispassionate thought that it is dispassionate: of a confused thought he is aware that it is confused, and of a clear thought he is aware that it is clear. . . . Of a thought composed or discomposed, of one that is liberated or bound, in each case he is aware that it is so. . . . He contemplates the rise of things in thought or the fall of things in thought, or the rise-and-fall of things in thought. . . . Thus does he abide detached and he grasps at nothing at all in the world.'[2]

Following on practice of the four kinds of recollectedness, there are four grades of 'contemplation' (*jhāna*). I render the descriptions of these somewhat freely from the Pali, exhibiting the correspondences with Husserl:

(1) Having established a psychological suspension by which we are 'detached from desire, ill-will, and ingrained beliefs', we enter upon a state (ordinary concentration) in which we are concerned wholly with 'directed and discursive thinking'.

(2) By the neutralizing (*vūpasama*, appeasing) of directed and discursive thinking we come upon a new subjective state 'born of unitary comprehension (*samādhi*), 'wholly tranquillized in heart', 'characterized by singleness of aim'. [We may describe this alternatively, perhaps, as the achieving of 'inner operational' or 'simply intuitive' analysis, and a resting in the resulting self-evidence.] This is the first stage of phenomenological analysis.

(3) By passing in a certain way beyond the subjective and taking on the universal and intersubjective, we come to a condition of stabilized unitary release and 'primordial recollectedness',[3] in clear relation to nature but not itself immersed in nature or in any way limited by it.

This may be described as 'pure consciousness'. As in Husserl: 'Radical discussions are necessary in order to penetrate to the knowledge that there is, indeed can be, any such thing as the field of pure consciousness, which is not a portion of nature itself' (I. 51).

(4) By transcending even that release we assume an 'ecstatically' transcendent condition characterized by a 'supreme purity of detached objectivity and primordial recollectedness (*upekkhā-sati-pārisuddhi*).

We may interpret this as a state in which fixation on space, time and bodily impressions is fully transcended. In other words, 'simple location' in space and

[1] *Dīgha Nikāya*, Sutta 22. F. L. Woodward, *Some Sayings of the Buddha*, Oxford 1925, pp. 72 ff.

[2] Woodward, *op. cit.*, pp. 78–9.

[3] See note 2 above, p. 108. In my terminology here, 'recollectedness' is a continuous, effortless state, while 'recollection' is the selective contemplation of one instant. The word *sati* is used for both in Buddhism. See my book, *The Mystical Life*, London 1961, pp. 8–11.

time is replaced by the free contemplation of everything, even of our bodies, as a structure of 'pure ideas' (*dhammā*).

The final release (*nibbāna*) is not the fourth contemplation. It is a confirmed transcendence of the attachments of 'self' (*atta*), with all its rigidities of thinking and belief, its physicalistic presuppositions and 'cloak of ideas', in such a way that all four contemplations can be freely exercised from a higher wisdom in the light of what is absolutely and finally 'uncompounded' (*asankhata*).

One more comment on the phenomenological method may be helpful. Some of those who have been brought up, academically, in the linguistic and axiomatic-deductive traditions of logical empiricism seem to have their view of the phenomenological method blocked by curious misconceptions. Thus even G. Nakhnikian, in his introduction to the translation of Husserl's 'The Idea of Phenomenology',[1] objects:

'The phenomenological method with its ultimate appeal to intuition, not the logic of language, makes *argument* impossible. As a way of *proving* anything, it is simply inadequate . . . the shifting of the burden away from language and upon the self-evidently given indicates a somewhat naïve view of the role of language.'

What is the 'proof' and 'argument' that is apparently supposed to dispense with analysis of primitive concepts and relations, and relies instead on the syntactical rules of some language or formalism? Surely, what constitutes a 'naïve view of language' is the psychologistic opinion (Abacus Fallacy) that one can thus dispense with intuitive analysis, deal with the undefined terms as if they were material counters, and equate truth with the end-results of the sign-manipulations.

The phenomenological method indeed admits 'proof' in the sense of 'awareness of a high degree of self-evidence', and also description and discussion to any extent that may be helpful. What it does not admit is the view of 'inference' and 'proof' as a means of establishing ultimate truth by a succession of formal or linguistic constructions *without any direct and self-evident analysis of the whole situation.*[2]

[1] The Hague, Nijhoff, 1964, pp. xxi, xxii.
[2] See also the quotation from Whitehead, below p. 157.

CHAPTER 9

OPERATIONAL PRINCIPLES AND THE NATURE OF SCIENTIFIC THEORIES

> 'An adequate insight into the techniques of scientific inquiry is necessary in order to understand what a powerful instrument for the analysis of basic philosophical questions has been created.'
>
> H. Reichenbach, *The Philosophy of Space and Time*, Introduction, p. xiii

If a scientific theory is to throw a true light on the constitution of nature—as we must surely believe possible to some extent—it is clear that we must try to ensure that our procedures and conclusions are epistemologically and ontologically warranted, and not merely put forward as a matter of present serviceability. Hence arises the importance of a clear understanding of the nature of the theory and the principles involved in its construction.

I call the principles with which we are to deal in this chapter 'operational' ones, since they are concerned with *procedures* in the building of theories in general, rather than with particular observations, hypotheses or techniques.

First it will be convenient to enunciate a General Principle (O) of Operational Implementation, summarizing (in part i) some of the conclusions drawn in Chapter 8 and (in part iii) some to be drawn in this chapter:

(O) i. All terms and 'inner' operations of any theory must have been clearly analysed phenomenologically, and the signs or words indicating them must be used consistently with that analysis;

ii. In the case of a physical theory, some terms or 'inner' operations of the formalism must be implemented in 'outer 'operations;

iii. Reasonable correspondence of a physical theory with givenness in nature is further subject to the principles of Weyl and Helmholtz (to be enuniciated presently).

Parts i and ii are sufficiently self-evident, I think, without further discussion. Besides the two principles mentioned in part iii, there are a number of fairly obvious requirements of procedure in regard to the propriety of a *representation* of any given theory. These will be dealt with in Chapter 18. Considerations (hardly meriting the title 'principles')

of 'simplicity', 'fertility', and similarly imprecise characteristics have been proposed as useful for guiding the construction of scientific theories. I ignore these here, on the ground that their value, if any, is purely pragmatic and I also reject the 'Verifiability Theory of Meaning', leading dogma of the Logical Positivists, now abandoned by universal consent. The above principle (O) takes its place.

Weyl's Principle

This is the principle on which we depend for the exclusion from physics of the concept of an ether, and also (guardedly) of some explanations described as *ad hoc*, the Lorentz–Fitzgerald 'contraction' interpreted as 'real', for example. Slightly different and vaguer formulations having a similar general purport are the *Principle of Economy* (Occam's Razor —assumptions should not be multiplied unnecessarily) and the principles of the *Identity of Indiscernibles*[1] and the *Elimination of Redundant Parts*.[2]

The principle which I shall call *Weyl's Principle* is given in Weyl's own words (translated) as follows: (W. 1) 'The objective image of the world may not admit of any diversities which cannot manifest themselves in some diversity of perceptions.'[3] Since, however, there are some obscurities here which seem to need clearing up, I suggest the following more precise form: (W. 2) 'We may not assert the "givenness in nature" of two different *entities* or *theoretical concepts* or *conditions* if, in terms of the formalism and geometry assumed and the class of experimental data admitted, such entities, concepts or conditions manifest themselves in identical ways.'

The existence of an absolute space frame (such as the ether) is ruled out by this principle, so long as we admit only phenomena within the limits of our galaxy. For it is implicit in the local space-time geometry of Special Relativity that our manner of measuring space and time and hence all other physical quantities cannot single out one inertial frame as *preferred* over the others. A similar state of affairs holds in General Relativity, regarded simply as a measurement tool for gravitational effects as well as mechanical ones. When, however, we admit the observational data concerning the distribution and recession of galaxies, an absolute frame is approximately determined (i.e. found to be given in nature). But this is a 'given absolute frame' only in a sense corresponding to that in which it appears in the formalism—i.e. it is the result of an empirical averaging-out of certain material positionings in the universe, and is not a characteristic inherent in the measurement-tool upon whose use all our representations of the universe are founded.

[1] Leibniz, *The Monadology and other Philosophical Writings*, Robert Latta, Oxford 1898, pp. 36 ff., 222, 377.

[2] *Ibid.*, p. 122.

[3] Weyl, *op. cit.*, 1949, p. 117.

Even when the observational data concerning galaxies are admitted, in fact, the definition of a 'cosmical' time and a corresponding preferred quasi-spatial picture of the universe involves principles going beyond strictly mathematical or physical theories. The manner in which we may reasonably arrive at such definitions is sketched out in Chapter 18, and it may be helpful to give a very brief summary of it here.

Corrections for local mass-concentrations, as in the neighbourhood of the sun, must first be made. A minute irremovable uncertainty in this may be ignored, since it is of no practical consequence. Then, for the choice of a *preferred* universal geometry and representation (the choices of formalism and of representation being interrelated) we must appeal to five distinct principles, if the choice is to be absolute and not in some ways arbitrary. The principle (O) of the present chapter would permit an arbitrary choice, provided such choice is capable of operational implementation. The principle (W. 2) requires an absolute choice, if the representation chosen is to have the status of something 'given in nature'.

Verifiability

The 'verifiability criterion of meaningfulness' goes far beyond both principles (O) and (W. 2) in requiring of any proposition a *physical* operation of verification here and now, or else the imagination of such operation, if the proposition is to be meaningful. It clearly gets into difficulties when the meaningfulness of such expressions as the following is being considered:

The interior of the sun.
The death of Julius Caesar.
The apparition was seen by both persons.
The ache in your limbs.
Virtue is its own reward.

There are also arbitrary conventions involved in the distinction between the *meaningless* and the *false*. Is the assertion, 'The square root of 2 is a rational fraction', meaningless, or false? We are tempted to say at once, 'false'. But if the square root of 2 is operationally defined by dividing all rational fractions into the classes according as their squares are greater or less than 2, the definition is not complete until we have decided whether any rational number has 2 as its square. And as soon as this point is decided, the assertion becomes self-contradictory and therefore meaningless. It is equivalent to the sentence, 'Black is white'. Alternatively, if we refuse to decide this point, the assertion includes a phrase whose meaning is not fully defined and which is meaningless, therefore, in a different sense of the term.

In the investigations of this book I abandon the verifiability criterion of meaning and appeal instead to the principle (O).

Helmholtz's Principle

The converse of Weyl's Principle is also of importance. In Helmholtz's form it reads: (H) 'We are justified, when different perceptions offer themselves to us, in inferring that the underlying real conditions are different.'[1]

If, for instance, we have established in a certain region the (approximative) applicability of Euclidean geometry, using (approximative) rigid rods or light-rays, and if, on carrying out measurements in this region on a later occasion, we find a departure from Euclidean geometry, we must, in terms of this principle, account for the departure on the ground of some physical difference—temperature, for example, or elastic stress, gravitation, presence of a refracting gas, etc.

The principle as it stands is obviously somewhat obscure, especially in its mention of 'real conditions', and in an endeavour to obtain greater clarity we shall be led into two matters of prime importance in the philosophy of science. The first of these will require rather lengthy discussion.

In the example just given, temperature, stress, etc., would be described as 'causes' of the differences in measure. Hence it is clear that Helmholtz's Principle could be called a Principle of Causality. It asserts the existence of a cause of every observed difference. Reference to 'causality', however, is in some ways misleading, since in the empirical tradition the concept of 'cause' is bound up in theories of logical atomism and deductive chains from particular initial conditions to particular conclusions, unexplained associations of necessity between events at different times, or inductive generalizations from experiments of the same class. To be precise, one could perhaps say that in science the word 'cause' designates a thing or condition symbolized by terms of a hypothetico-deductive theory and such that from those terms, in conjunction with others, the 'effects' in question (simultaneous with or subsequent to the 'causes') can be deduced by carrying out the operations of the theory.

The principle (H) as stated above, however, refers to 'real conditions' —a phrase which cannot reasonably stand for hypotheses which merely 'save the appearances'. The epithet *real* would seem to suggest either (1) that the conditions can be discovered by face-to-face phenomenological or operational analysis, or (2) the existence of the conditions as a givenness in nature, or as a common objectivity for many people, can be otherwise guaranteed in some way. In the former case the principle might be justified as follows. Any observed irregularity

[1] Weyl, *op. cit.*, 1949, p. 26.

is not like a brick added to or removed from the structure of physics without influencing anything else, or like a colour which can be put in or taken out at will. It is part and parcel of the total conceptual procedure which we have applied so as to make the situation intelligible to us and which has thus disclosed the irregularity. It will therefore be susceptible of further analysis.

One must face the fact, however, that in science the principle is used almost always in sense (2), that is to say, as a guiding principle of discovery, indicating the places where some new 'entity' or 'term' needs to be introduced in the theory in order to complete the explanation. Face-to-face intuitive analysis is hardly ever envisaged. Even temperature and elastic stress, as 'causes' of differences in measure, are concepts of a hypothetical theory. Their connection with the observed differences in measure is by the regularity of associated sensations, rather than by intuitive analysis of the process and presuppositions of measurement. It thus becomes necessary to ask whether Helmholtz's Principle can always be validly applied in the construction of scientific theories, thus establishing, in the case of deficient explanation, that 'hidden variables' must somewhere be introduced. This question becomes of outstanding interest in quantum theory.

The subject of the incompletely specifiable 'causation' (hypothetico-deductive or phenomenological) of particle-like effects, according to quantum theory, is a notoriously difficult one, and I propose to carry out an initial investigation here, with the help of two simple illustrations. Then in various places of Chapter 20 the problem will be reviewed and a few more technical points will be discussed. We turn now to the first illustration.

Let there be two concentric hollow spheres of conducting material, the inner one being supplied with a large positive charge while the potential of the outer one is gradually lowered. There comes a time when a spark jumps across, so as to meet the outer sphere at a certain point P. No scientist would then say that our inability to predict the position of P shows that electrical laws are irreducibly statistical; he would say that if we could examine the actual conditions on any occasion minutely enough we would find irregularities in the shape of the sphere or in its material or in the medium between, which would theoretically account for the point P being favoured on that occasion.

Similarly, we may consider why absorption of a photon occurs at some point on a screen rather than another, when light of very weak intensity is diffracted through a slit on to it. Our calculations cannot even begin without a theoretical averaging-out of local variations in the molecules on the screen, in the medium between, and in the waves passing through the slit. It is inevitable then that we can predict nothing except that the *average* number of absorptions per second in any small

region of the screen is proportional to the intensity of the wave there. An insistence on the irreducibly statistical nature of quantum laws thus begins to take on the aspect of an insistence that the actual physical circumstances, waves and apparatus, are to be represented by idealized mathematical forms.

'Spray Mechanics'

As an analogy with quantum theory, the above 'spark experiment' is obviously very inadequate. A rather better analogy might be as follows.

Consider a garden sprinkler hanging vertically downwards, like a bathroom spray, over uneven and very absorbent ground, possibly on a hillside. Given the amount of water flowing per minute through the pipe, we should be able to find a correlation between the amount of water absorbed per minute per square foot in any part of the ground and the gradient and location of that part with respect to the sprinkler. Moreover, if a steady wind arose, we should be able to find the laws of transition, due to this 'perturbation', from the original distribution to the new one. All this would be calculated on the average, ignoring the fact that absorption actually takes place one water-drop at a time. If we examine a small area closely we may be able to count the drops one by one as they fall; and with exceedingly accurate instruments we might even be able, in some particular case, to detect that a temporary diminution in the number of drops falling there per minute was associated with a local eddying of the air current as it passed over the inequalities of the ground. But while we might be able to discover this kind of explanatory correlation in the case of a few drops, or even perhaps a single drop, the exact accounting for the movement of *every* drop simultaneously is altogether impossible. Even the attempt to investigate the behaviour of a single drop in mid-air will disturb the conditions and bring about a result different from the one to be accounted for.

The conclusion seems justified, therefore, that the infinite multiplicity of causes and irregularities inherent in any physical actualization, and their inaccessibility without upsetting the experiment, renders impossible the prediction of more than averaged-out effects.

This argument against the possibility of exact prediction applies equally to classical and to quantum physics. The difference is as follows. In classical physics, although it is accepted that the given infinitely-complex conditions are replaced by something quite simple in the mathematics (so the result cannot then be more than an approximation to actuality), nevertheless for any precise (simplified) conditions the outcome is also precise as regards determination of all future positions, momenta, field-intensities, etc. In quantum physics, on the other hand, even when the given conditions are replaced, as they must be, by simple

mathematical structures, it still remains impossible within the formalism to predict precise values for *both* positions and momenta (whether on the same or on different occasions) as they would be classically measured directly or inferred from observation of frequency, etc. In all such cases the joint prediction is impossible.

As far as Helmholtz's Principle is concerned, however, the question whether there is a reason for observed differences, discoverable in the substructure of the experience, requires the same kind of consideration in each case. What still has to be made clear, in either case, is (1) whether the differences in substructure are to be determinable with absolute fullness and precision of detail, or with precision in at least one discoverable respect, or only in general terms; and (2) how the differences in substructure are supposed to be discovered. Without clarification of these points, Helmholtz's Principle will be vague in its application, and its validity or otherwise cannot be decided.

Helmholtz's Principle in Science and Phenomenology

In each of the above analogies we have tacitly accepted the existence of precise individualized trajectories, for the sparks or the particles as the case may be. In this respect the analogies are misleading. For in the free field, as shown below (Chapter 20),[1] no such trajectories can logically be admitted. We therefore first consider whether perhaps this feature of quantum theory would alone require us to abandon Helmholtz's Principle at atomic and sub-atomic levels, in every reasonable interpretation of it.

The kind of difficulty that arises is seen particularly clearly in experiments where scintillations, from a radioactive source, appear on a screen at unpredictable times and in unpredictable places, or, equivalently, photoelectrons are liberated on a screen as a result of diffraction or interference. One can arrange, in the latter case, that the intensity of light is so weak that less than one quantum of energy is in the apparatus at any time; then each quantum-absorption can be regarded as an isolated phenomenon in a succession of absorptions occurring now and then, and here and there, when the source is placed in position. What sort of specific formalized 'cause' or intuitively disclosed difference in substructure, we may ask, can be shown to 'explain' differences in the timing and position of the successive absorptions?

One might suppose, as suggested in discussion of the above 'spark' experiment, that the differences observed between the position of one absorption and that of another, and the precise timing of each, are to be explained by differences in the conformation of the molecules on the screen, in the medium, on the sides of the slits, and, above all, in the manner of release of energy from the source. The serious objection

[1] Pp. 315 ff.

then arises, however, that it would be impossible to determine these alleged differences of 'causal' conformation—other than, at best, very roughly and after the event—even in principle, if the experiment is not to be upset thereby. *The precise conditions of any specific observation cannot be determined even in principle.* Yet the postulates of the theory require that 'causal' differences of the kinds mentioned *must* affect the result. For some assumption (in practice, an idealized one) concerning their precise nature in the unobservable substructure of the experiment *must* be made if the theory is to be applicable.

Here we have, therefore, an absolute conflict between what the theory requires logically and what is experimentally possible. It is perhaps an illustration of the paradox, to be considered below,[1] that scientific theories are concerned with an 'unknown given'.

It is in fact quite general, in science, that we cannot directly observe or verify physically the difference in 'real conditions' which the theory requires us to admit as 'causing' the observed differences. Even in the case of a car failing to start because of a dirty sparking plug, we cannot observe that at a particular moment when a spark should occur it is prevented from doing so because of a piece of dirt on the plug. Behind our conclusion that a dirty plug is the 'cause' lies a complicated mental process of plausible generalization, imagination of unobservable substructure (in electric currents, potentials, etc.), and theories which have been found reliable in practice to a high degree of approximation. We do not say, however, in that case, that Helmholtz's Principle is not valid—that there is *no* cause or explanation. On the contrary, Helmholtz's Principle is the very ground on which we base our struggles to find a scientific explanation. We then regard it as sufficient if we can explain *in general terms*—not in terms of direct observation—what sort of 'causes' are reasonably to be supposed active in the unobservable substructure of the experience.

Quite different is the meaning and scope of Helmholtz's Principle in phenomenology, and it is possibly the great difference in meaning and scope of the principle as between science and phenomenology that has led to the widespread supposition that an element of 'pure chance' (a sort of 'lucky dip') enters into quantum theory.

In phenomenology, 'explanation' is proposed in terms of *face-to-face analysis*, not of *theory-fitting*. For example, to 'explain' phenomenologically the actually heard differences in tone-quality between a note played on the oboe and a note played on the clarinet, the tones on two specific occasions must be analysed by the ear and the differences in intensity of the various harmonics, as well as other sound-effects, must be directly discerned.

If a scientist were then to say that the 'true' explanation lies in the

[1] Pp. 123 ff.

wave-form of the air-waves reaching the ear-drum, the phenomenologist might reply as follows: 'An outside experimenter might certainly discover an effect of air-waves alongside the ear of the listener; and it is even conceivable that in some state of hyperaesthesia the vibrational character of these air-waves might be directly sensed. But *this* sensation would be quite a different experience from the hearing of the tone-quality of the sound. Likewise with other details asserted by acoustical theory, these are no part of any observation which has been directly made.'

The phenomenological view is thus that *when* any observation is made, the direct analysis of its differentiating characteristics constitutes its 'explanation' (or rather, *part* of its explanation). But if no observation is directly made, then the question of phenomenological explanation does not arise. Hence Helmholtz's Principle, *phenomenologically* interpreted as asserting the discoverability of differences in substructure corresponding to and serving to 'explain' observed differences, is not affected by arguments from quantum theory.

A certain difficulty remains, even with this form of the principle, namely, when the differences occur as between phenomena at different times or different places but otherwise very similar. To carry out a phenomenological analysis in such cases we would need to comprise the phenomena in a total intelligible view, as for instance the notes of a musical phrase can be comprised in a total intelligible view. Then if our powers of discernment in all ways can be made sufficiently deep and acute, while still retaining wholeness of view, we might sense (in the musical case, for instance) 'causes' lying in the understanding and intention of the performer, in the construction of the instrument, or in the acoustical conditions around (echoes, etc.). The phenomenological analysis of sensations may be guided by scientific analysis in terms of reliable theories, but its scope, in the end, will be overwhelmingly more extensive.

Since such phenomenological analyses are in general far beyond ordinary powers and conditions of mind, it becomes necessary to substitute the scientific interpretation of Helmholtz's Principle, namely, as providing a *general heuristic* indication, for the purpose of fitting a hypothetico-deductive theory. It then appears that the apparent failure of the Principle of (phenomenological) Causality in quantum theory is really an inevitable failure in *theory-fitting*—inevitable, because in any theory the total conditions of any experiment must be represented in idealized form, and further, because the classical concept of 'momentum' has ultimately to be replaced by a characteristic of the *field* in a finite region. *Ipso facto*, 'momentum' at a point no longer exists.

It may be noticed that I have avoided use of the word *determinism*.

This is because I can attach meaning to it only in the context of a theory, like Newtonian mechanics, which is supposed (quite incorrectly) to be capable of representing with complete and absolute precision the infinite givenness in nature. For the same reason the term *indeterminism*, or 'pure chance', should in my opinion be avoided.

We now turn to the other important matter for discussion arising in connection with Helmholtz's Principle, and incidentally in connection with Weyl's Principle too.

If we accounted for certain observed differences by supposing a cause which manifested itself *only in that way*, the explanation would be, as we say, *ad hoc*. We might, for instance, correlate the apparent departures from Euclidean geometry (when random effects have been removed) with the value of a certain world-tensor in that locality. On the hypothesis we are making, such a world-tensor would be merely a new symbol or name for the given perceptible condition, without reference to any other kind of experience whatsoever. Thus there would be no distinction (apart from the possible effects of arbitrary mathematical transformations) between the measures of the world-tensor and the measures of the given perceptible condition. Accordingly, by Weyl's Principle, we should not be justified in asserting the 'givenness in nature' of the world-tensor as distinct from the perceptible conditions.

We conclude then that the 'real conditions' referred to must manifest themselves in a variety of ways—not, for example, *merely* in the departure from Euclidean geometry. Experience confirms this. For wherever we find some apparent departure from Euclidean geometry we can decide from *other evidences* whether temperature, elastic stress, etc., or gravitation is the cause. In short, each of these causes, being a 'real condition', shows itself in characteristic physical effects of various kinds.

'Ad Hoc' Explanation: Types of 'Theory'

In this discussion we have treated an explanation as *ad hoc* if it contravenes Weyl's Principle—that is, although we may have legitimately introduced a new *naming* or *technique* in connection with a phenomenon, the further hypothesis that there is a 'givenness in nature' corresponding to such naming or technique is redundant. Classic instances of such *adhocness* are the postulate of the Lorentz–Fitzgerald contraction along with time dilatation (to account for the velocity of light coming out to the same measure in each of two inertial frames in relative motion), and the allied postulate of the ether as a substantial space-frame.

I shall describe theories of this kind as 'definitively' *ad hoc*, it being understood in such a case that a certain settled body of evidence is admitted, and if we wish to admit further relevant evidence which may

have come to light, then we must consider that we are dealing with a new theory. Thus so long as the Lorentz–Fitzgerald contraction was introduced merely to explain the null result of the Michelson–Morley experiment and some others to the same effect, the explanation was definitively *ad hoc*. But the later experiment by Kennedy and Thorndyke,[1] in which the interfering beams of light were made as unequal as was practicable, showed that the formal technique of 'contraction' gives incorrect results in that case. The 'givenness in nature' of contraction alone (without time dilatation) was therefore ruled out, and the theory of the contraction, far from being definitively *ad hoc*, became *false*. But the theory of contraction combined with time dilatation was then seen to be definitively *ad hoc*, and has remained so ever since. It *could* be significant (since the formalism fits the observations); but, on the evidence admitted, it is redundant.

It is clear that in a case of definitive *adhocness* we always have to distinguish between the *applicability* of a formalism as such and the *reified significance* of the formalism. There are, however, broader and very confused uses of the term *ad hoc*, in connection with formalisms whose reification may or may not be directly a matter for consideration. The term may be used, for instance, simply as a term of disapprobation for a theory which seems far-fetched and only explains information which is already known. Rather often it seems to be supposed that to characterize an explanation or theory as *ad hoc* in this sense is at once to rule it out of court.

Yet every scientific theory is 'fitted to what is given', since its object is to account for certain evidence; and most scientific theories seem far-fetched to begin with. What then is it, precisely, which induces the feeling of disapprobation? It will be a useful exercise in 'conceptual focusing' to try to disentangle the various conceptual strands suggested by the term *ad hoc* and to settle on a definite and reasonable *modus operandi* for it.

First, if we survey any large and varied collection of theories and explanations, I think we shall find that, in regard to the majority of them, it would be unreasonable to attempt to decide whether or not they are *ad hoc*. The term is simply not applicable. I suggest that the term 'theory' may be taken to include (1) two-level 'speculative' explanations, of 6 types (2) two-level 'self-evident' or 'settled' explanations of 5 types, and (3) non-explanatory theories. The term *ad hoc* is then applicable to two of the 'speculative' types of theory, and to no other type.

In the case of these two speculative types of theory which may or may not be *ad hoc*, the two 'levels' (sometimes called *explanans* and *explanandum*) might be aptly described as a 'guess worked out' and its

[1] *Phys. Rev.*, **42**, 1932, p. 400.

contemplated 'verification'. In the case of the other four speculative types and the 'settled' or 'self-evident' types the levels could be suitably described as the outward 'facts' and their 'analysis'. If the analysis is directly and decisively confirmed the theory is 'settled'. Otherwise the analysis has to be 'guessed objectively', and the theory then comes in the speculative category. We thus distinguish between 'guesses' of two kinds: (1) guesses at discoverable 'inward facts', i.e. at the analysis, and (2) guesses worked out for the purpose of 'fitting', without any claim that the 'true' self-evident analysis is discoverable. These subtleties, unavoidable in any thorough discussion of *adhocness*, will be better understood by taking illustrations of each type.

The five types of *settled* theories are as follows:

(A) Factual Physical. Columbus's 'guess', for instance, at 'a continent over the seas' is today directly confirmed and 'settled'. But in his time it was a 'speculative' theory in which the 'true' facts were reasonably regarded as discoverable.

(B) Factual Non-Physical. For instance: the Obsession Theory of Disease (especially mental illness), i.e. the theory that diseases are accompanied (not necessarily *caused* in the popular sense of the term) by corresponding 'monadic influence' from a non-physical sphere of existence. From the point of view of Swedenborg and other seers who have emphatically claimed that it is the result of direct intuitive analysis, the theory is 'settled'. From the point of view of those who have no such direct evidence but who admit the possibility of non-physical experience, the theory is a 'guess at discoverable facts'. From the point of view of those who refuse to admit the possibility of 'non-physical experience', the theory is a mere 'saving of appearances'.

(C) Given Intelligible. For instance: the Laws of Arithmetic, as applicable to the counting of objects in actuality. Or Euclidean Geometry, as seen to be put approximatively into the world for the purpose of measuring it.

(D) Part-conventional. For instance, the method described above for operationally determining a cosmical time in the universe.

(E) Directly Refuted. For instance: the Cantorian Theory of Ordinals of the Second Class, the assumption of which leads to the Burali–Forti Paradox. In a strict sense 'direct refutal' means that one must see by clear self-evident analysis that the definition of $\aleph_1$, $\aleph_2$ etc, is non-operational and in fact embodies self-contradictory ideas.[1]

Leaving these theories, to none of which the term *ad hoc* is applicable, we come to the six 'speculative' types, and first, the four which can be described as *discoverable*, because they guess at discoverable facts. In the terminology above, they 'guess objectively'. Here (A) and (B) are *macroscopic* and (C) and (D) are *substructural*:

[1] This category (E) is intended to comprise all 'theories' which are better described, not as theories, but as imagined errors and inconsistencies. It would therefore also include *factual* theories which are directly refuted; for example, our view today of Columbus's theory that the land he discovered was part of Asia.

(A) Temporal. For instance, the theory, when lights go out, that it is due to a fuse; or when a car engine stops, that it is because the petrol has run out, or the plugs are dirty. The 'cause' here is usually supposed to precede the effect; or at least it acts at a given moment of time.

(B) Non-Temporal. For instance, Columbus's theory, before 'America' was discovered.

(C) Theories of Existent Substructure. For instance, astrological influence, in the sense of a connection between the month of birth and certain qualities of character; or the obsession theory of disease, from the point of view of those who admit the possibility of non-physical experience, but have not had such experience of the kind required.

(D) Theories of Intelligible Substructure or Solution of Problems. For instance, the theory that it is possible to establish a cosmical time throughout the universe, no 'solution' of this problem having already been accepted as valid.

Before considering the other two speculative types of theory we can deal with the 'non-explanatory' theories in a few lines. Examples might be the non-Euclidean geometries (regarded as formalisms) or Tensor Calculus. Of such theories a variety of 'interpretations' or applications is possible. We do not get an 'explanatory' theory till the specific field of experience to be 'explained' is decided.

'Open-Deductive' Theories, and the 'Unknown Given'

It remains to deal with the two *non-discoverable* speculative types of theory, described as a 'guess worked out' and its 'verification'. They are (E) the definitively *ad hoc* theories already discussed, and (F) theories to be described as *open-deductive* ones. These last may possess a degree of 'conditional *adhocness*', and a complementary degree of 'scientific character'. Practically all theories called 'scientific' come into this category; and the fact that they are open-deductive and also non-discoverable raises peculiar difficulties, which we may approach best by further analysis of the concept of *adhocness*.

It is probably safe to say that when we call a theory *ad hoc* the general impression at the back of our mind is that it is 'artificial' in some way, 'lacking in substance', not corresponding with 'reality', merely a 'fitting' of the formalism to facts, a 'saving of appearances'. The conclusion is that, as in the case of definitively *ad hoc* theories, we have consciously or subconsciously assumed that scientific theories must have a 'reified significance' and have gained the impression somehow that the theory in question lacks this significance.

Now in the case of the nine types of explanatory theory dealt with above, the 'significance' of the theory is either actually discovered or reasonably asserted to be discoverable, i.e. it is *operationally exhibited* (in the wide sense of the word 'operation' adopted in this chapter). Hence the 'significance' must be taken as something which *exists* (or is

'real', or 'given', if we prefer these terms). Moreover, it must be taken to exist in some sufficiently well-defined form, so that it could be recognized when discovered, or otherwise so that the guessed 'form' could be seen to be different from what is discovered. It is true that if we make a wrong guess the 'significance' which we thought would correspond to it does not exist. But to say this is to say also that there is a right significance which *does* exist and with which we can compare our guess.

In the case of definitively *ad hoc* theories, the formalism of the theory and the evidence admitted are specified in such a way that there is no logical ground whatever for asserting that a certain further hypothetical 'significance' exists. So far as the theory goes, this 'significance' is nothing but a form of words.

In the case of *open deductive* theories, on the other hand, we claim that there is a 'significance' which is 'given', or 'exists' in some way behind the formulations, but is *not* discoverable by analysis. In fact we claim that there is nothing whatever knowable about the 'significance' except that it is a 'givenness', an 'existence', or 'truth' of some kind, supposed to correspond somehow with the formulations. Hence we may not inappropriately characterize this vacuous kind of 'significance' as the Pseudo-Concept of the Unknown Given.[1]

There may of course be some scientists who hold that it may some time become possible to obtain *direct* confirmation of the 'existence' of entities corresponding to our present conceptions of elementary particles. In that case we must ask them to declare the precise manner of observation or analysis which would permit us to discover whether or not our present conceptions correspond with what 'exists' or is 'given'. If they can declare this, we have at once a new theory, of 'discoverable' type.

Expansion of Theories

We must now consider in some detail the nature, difficulties, and possible *adhocness* of *open-deductive* theories.

Obvious characteristics of most theories called 'scientific' are (1) that they admit a 'working out', using new techniques, (2) that they may be combined with other theories to form a more comprehensive theory, and (3) that applications of the theory may thus be devised to fit (or 'predict') experimental observations which are not yet 'explained' by any theory or which perhaps are envisaged as possible but have not actually been made.

These theories are therefore not closed, finished and complete, in

[1] The belief that our knowledge of nature can go no further than the development of open-deductive theories and the postulation of this 'vacuous actuality' is what I describe in Chapter 22 as the 'agnostic' variety of Scientific Realism.

the sense that Euclidean geometry is closed, finished and complete, because it is not possible to add a single new hypothesis or postulate or type of construction. Nor are they closed in the same sense as that in which the 'dirty plug theory' to explain the stopping of a car engine is closed, namely, because there is just one point at issue and when that is decided there is nothing more to say. On the contrary, these theories are always subject to *trial additions*. They are indeed constructed by deduction from hypotheses and are tested by approximative fitting; but the hypotheses and techniques are all the time growing, as it were, organically.

Thus the Atomic Theory, used to explain gas laws, was combined with Dalton's Atomic Theory for the explaining of chemical laws. Each theory was then held to confirm the other, and the combined theory became much less *ad hoc* than either theory separately. An illuminating instance of an expanded theory is Galileo's theory that all bodies fall with the same acceleration, expanded with the help of the hypothesis of air resistance. Here the original theory is obviously false as it stands, and is 'saved' by the additional hypothesis. Yet no one would regard this hypothesis as *ad hoc*, since it has applications to countless other types of experience and explains them all so naturally that we can hardly avoid regarding the fact of air-resistance as obvious. 'Saving' of theories, by itself, is therefore no criterion of *adhocness*.

Other instances of expansion would be the trial addition of the terms $-\frac{e}{c}\mathbf{A}$ to the momentum operator in Schrödinger's Wave Equation so as to cover also the case when an electromagnetic field is present, and the subsequent factorizing of the relativistic form of this equation with the help of a sedenion algebra, so as to obtain Dirac's linear wave-equation. It is to be noted, in passing, that expansion of a theory generally entails some modification in the signification of the terms and operations.

What now are the grounds for the belief that certain of the formulations in a theory of this kind correspond to an 'unknown given', so that the theory, regarded as a whole and at a specific stage of its development, is not *ad hoc*? To be more precise, I believe the correspondence should be taken as between the 'unknown given' and entities in the *model*, contemplation of which guides us in constructing these formulations. For the formulations are mere arrangements of symbols unless we have in some way pictured objective happenings to which the symbols and operations of the formalism apply. The grounds for belief in an 'unknown given' rest largely, therefore, in the usefulness of the model; and our judgement of the usefulness of the model rests in the usefulness and other virtues of the formalism built on it. This is necessarily so, because the model is purely a work of imagination, in many

cases obviously crude, and there is no means in these theories for its appropriateness or resemblance to 'reality' to be judged directly.[1]

I suggest that the justifiability of the affirmation of an 'unknown givenness' in a theory is not for us a simple matter of *yes* or *no*, but is a matter of degree. We judge it, I think, from the extent to which the formalism is

(1) extensive in its applications (*a*) to existing evidence, and (*b*) to the making of 'new' predictions, i.e. ones not easily explicable by other theories or already explained by a closed form of the theory;

(2) 'natural and inevitable', not 'artificial'; promoting admiration and joy, as at some given intellectual beauty rather than some ingenuity of man.

These indications are of course vague and uncertain. They are very far from the direct intuitive analysis which reveals once for all the character and status of the *discoverable* theories. The 'scientific intuition' is like a guiding voice to people in the dark. Yet it can be decisive enough, I think, to those who have extensive knowledge, an open mind, and a true insight into the nature of scientific discovery—a feeling for the way in which nature works 'behind the scenes'.[2] As already quoted from Weyl: 'Nobody who really goes into the matter will deny that the world of perceptions determines the theoretical system in a virtually unambiguous manner, although no logical way leads to the principles of the theory.'[3]

Thus at any stage of our knowledge, if several possible steps are considered, the step which will lead us most sure-footed to 'expansion' and 'inevitability' can usually be judged quite confidently.

The 'Conventional Index of Scientific Character'

Our conclusion is thus that the open-deductive theories of science are not inaptly described as a strange kind of ignorant groping.[4] Such a theory is *ad hoc* if the groping seems to get nowhere, that is to say, our guesses lack substance. To make our judgements as definite as possible we may allot points for *adhocness* to each theory, out of a maximum of 5; for example (first column):

[1] Compare Popper's view that science 'consists of *doxai* (opinions, conjectures), controlled by critical discussion as well as by experimental *technē*'. *Three Views Concerning Human Knowledge*, p. 365.

[2] The characteristic 'style' or 'hallmarks' of nature is the theme of several chapters in *The Faith of a Physicist*, by H. E. Huntley, London 1960.

[3] Above, p. 66.

[4] 'At Harwell a layman realizes with apprehension what in his own awe of science he can well forget, that scientists are not omniscient or prophetic, but groping in the dark like anyone else.' A. Sampson, *Anatomy of Britain*, London 1962, p. 523.

Contraction and Time Dilatation (today)	5	0
Ptolemaic Epicycles (AD 200)	4	1
Elastic Ether (AD 1850)	3	2
Dalton's Atomic Theory (AD 1820)	2	3
The Inverse Square Law of Gravitation (AD 1850)	1	4
Arithmetic, in counting of objects	0	5

The second column, giving differences from 5, may then be perhaps usefully defined as the Conventional Index of Scientific Character. As a check, the figures in this column should then give an estimate of the possibility of 'natural extensions' to the theory. This again makes it clear, I think that the scientist is working in the dark. The scientific character of his theories depends on their connection with an *unknown given*. There is thus a fundamental illogicality or hiatus in them, if not an actual lapse into the fallacy of the Living Skeleton. Rationality is restored to our view of such theories only by admitting the possibility of a direct intuitive discovery of the 'givenness', 'truth' or 'reality' for which the theory is groping.

CHAPTER 10

THE LOCAL EUCLIDEAN GEOMETRY OF THE PHYSICAL WORLD: PERCEPTUAL AND OPERATIONAL BASIS

'We have been forced to recognize that spatial perception is a structural phenomenon and is comprehensible only within a perceptual field which contributes in its entirety to motivating the spatial perception by suggesting to the subject a possible anchorage. The traditional problem of the perception of space and perception generally must be reintegrated into a vaster problem.'

'The transcendental attitude is already implied in the descriptions of the psychologist, in so far as they are faithful ones.'

M. Merleau-Ponty, *Phenomenology of Perception*, trs. Colin Smith, London 1962, pp. 280, 59

'We may acknowledge the existence of a *space of intuition* and maintain its metrical structure of necessity satisfies Euclidean geometry . . . intuitively of unobscured clarity, though in the objective construction the state of affairs can only be represented as a compromise.'

H. Weyl, *Philosophy of Mathematics and Natural Science*, p. 135

In this chapter and the next we aim to investigate how far a geometry is 'given in nature', and thus to arrive eventually at a detailed working out of the broad conclusion of Chapter 6, that 'in measurement we begin by *putting Euclidean geometry* into the world, so far as we can'.

It is particularly to be noted that this conclusion concerns the *beginning* of measurement. There are of course many ways of measuring phenomena in the world resulting in 'geometries' in which the theorems of Euclidean geometry are no longer true. We could refuse to correct for temperature and stress in our measuring rods; we could take light rays as always, by definition, 'straight', distances being measured along their trajectories from one point to another; or we could adopt a new formula for the 'distance' between two points, depending on the definition of a unique path joining them in some pre-established co-ordinate system somehow set up in the world.

What we are concerned with now, however, is something quite different and conceptually far more primitive. We wish to make clear precisely what is meant by saying that the physical world is locally Euclidean; and for this purpose we must approach the problems of geometry from another angle, giving close attention to certain matters

fundamental in all spatial observation, thinking and operations, but so far passed over or only briefly noticed. These are:

(1) The *perceptual basis* of spatial concepts. What is there, if anything, which is essentially a space-conceptual structure, but which is nevertheless 'given in nature'?

(2) The *operational basis* of the theorems of Euclidean geometry. We wish to show how these theorems are justified by the operational character of the space-structures concerned, without having recourse to deduction from assumed axioms containing undefined terms.

(3) *Arithmetizations* of Euclidean geometry, ideal or actual; their origin and significance.

The first of these topics will require rather lengthy discussion, without however pretending that more than an introduction to a very difficult subject has been offered. The last topic is specially important for the study of generalized and analogistic geometries in Chapter 12.

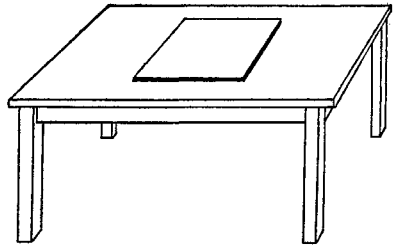
We shall not be concerned at all with uninterpreted formalisms of pure mathematics, even if these are described as 'geometries'. Our concern is with space-concepts and with formalisms which are somehow implemented in space-concepts. It is to be understood, therefore, that in these investigations the term 'operation' is used in the restricted sense of 'outer operation'—*ideal* if the implementation is in ideally-conceived operations, and *actual* if the implementation is in actual physical observations. 'Operationism' in the positivist sense, as requiring a *definition* in terms of *actual physical* operations, can be accepted here only in the weakened form which requires actual physical implementation of *certain elements* of the formalism, as a check on the conceptual clarity and actual applicability of the whole.

THE PERCEPTUAL BASIS

A *perception* of particular spatial relationships in the physical world or in a remembered or imagined spatial form, is intelligible to us as such by reason of its space-*conceptual* character. Conversely, a space-conceptual structure, in itself a matter of timeless essence only, is nothing if not manifested or capable of manifestation in particular space-percepts of some kind. In reply to the question, 'What exactly are these space-percepts?' I shall consider two illustrative experiments and various recorded experiences, drawing certain conclusions from each.

Experiment 1. This is to be performed in three stages. (1) Suppose we are seated, with our eyes shut, before a table on which a friend has placed a cardboard lamina cut to a certain shape (actually a square of side 4 inches). On being asked to discover its shape, we reach forward with the hand and eventually touch the far right-hand corner with the

tip of our middle finger. We then slide the tip of the finger along the far edge, and on reaching the far left-hand corner we keep the finger-tip down and slide the thumb down the left edge till it comes to the near left-hand corner. Suspecting then that the shape is square, we raise the thumb a little, slide the tip of the middle finger back, and when it reaches the far right-hand corner again we lower the thumb, and find it touching at the near right-hand corner, where we had mentally conceived that it would touch. (2) We open our eyes, and see that the visual shape is one of those which we are accustomed to think of as the appearance of a square. (3) We shut our eyes again, and the friend moves the lamina about 6 inches to one side, rotating it a little also. We carry out the same operation with finger and thumb, observing again the squareness of the lamina.



The first conclusion to be drawn from this simple experiment is that our apprehension of the spatial form of a square, reduced to essentials, is not dependent on visual appearances. The function of visual appearance is to provide a rapid and accurate *clue* to spatial form in cases where we cannot discover that form by the sense of touch alone. This means also that, in case of doubt, tactual and kinaesthetic evidence must normally take precedence over visual evidence (as, for instance, in the case of a straight stick which appears bent when partly immersed in water).

A second conclusion is that although we can judge, by the feeling in the arm, whether the square has been rotated or translated, nevertheless, so far as the sense of spatial form obtainable through the finger is concerned, the perception of the lamina as square does not depend on its position or orientation relative to other objects. In judging the lamina to be square we are not concerned with the question whether it is the lamina or our hand which has been shifted relative to the surrounding objects. The recorded fact is simply that what may be described as the 'conceptual hand-space' is fitted to the actual space.

From this we pass to the further important conclusion that, when we do in fact consider the square as capable of having an orientation

relative to other objects, the various possible orientations (which I shall call perspectives) must be conceived *as a whole.*

For instance, we cannot conceive of the square as seen from a point due above without also conceiving of the possibility of its being seen from below, and equally well appearing square (to our *space* perception) from either position. A square lamina with only an upper face, or with no aspect for certain angles of inspection, is an impossibility.

The totality of perspectives is conceived of in one way (as a kind of potentiality), while the one perspective which is in the forefront of our perception on any occasion is conceived of in another way (as a 'central' actuality). But the potentiality and the central actuality must be thought of as forming a given indivisible whole, over which the attention passes so as to bring into focus, as it were, first one aspect and then another. For the one central aspect presupposes the other possible aspects. The whole then assumes for us the status of a 'total' actuality.

Gestalt Psychology emphasizes the tendency of the mind to supply, subjectively, material not perceived, in order to make a comprehensive whole. Here, on the contrary, I am pointing to the epistemological necessity for material not 'centrally' perceived to be subconsciously or 'peripherally' perceivable if an object is to be comprehended as the spatial form which it really is. Subjective impressions may provide clues, correct or false, but cannot provide a real, i.e. given, totality.[1]

In my usage of the term, therefore, a *Gestalt* will be a *given* totality constituting an object which is intelligibly encountered. This totality is then analysable into particular percepts and concepts of spatial and temporal form. But I may also speak, by qualification, of an *imaginative* or *subjective* Gestalt, imposed on or blended with the given (unqualified) Gestalt. As will be explained later in this chapter, these two Gestalten belong to different 'levels of objectivity', and discovery of the *given* Gestalt requires what Merleau-Ponty calls a 'second-order reflection'.[2]

Another point requiring special emphasis is that in the case of a given Gestalt the concepts and conceptual relations which are discovered in it by analysis are also given in nature—as well as the percepts and perceptual relations. The first are *essences*, and the second are

[1] According to Gestalt Psychology, 'the form always has a "whole-character" that transcends the characteristics of the parts' (Floyd H. Allport, *Theory of Perception and the Concept of Structure*, New York 1955, Chapter 5). Most Gestalt psychologists, however, view this 'whole-character' in terms of bodily reaction, through 'brain-fields' with their 'forces', 'stimulus-patterns' and 'specific energies', and postulate that by the physiological and psychological elements in his organism man is enabled to make sense of his environment. Quite different is the phenomenological analysis of space-concepts—which are, of course, presupposed in any talk about 'brain-fields', etc.

[2] See p. 133, note 3; *op. cit.*, p. 63.

existences; and the existences are intelligible to us by virtue of their essential structure.

So accustomed are we to using concepts in our subjective thinking or imagination, that we are apt to view everything which is intelligible as subjective and private, as if taking place in our individual brains. Yet it is clear that the natural numbers 1, 2, 3, . . . (for instance) are concepts which are given *in things* as well as appearing *in thought*. A given Gestalt, inasmuch as it is something to be comprehended, is similarly conceptual, while at the same time it constitutes the thing presented. Even qualities of things such as colours, so far as they are understood in relation to each other and to other qualities of the thing, are conceptual and also given (provided they are genuinely physical).

I suggest, therefore, that we should recognize that when an object is 'perceived', as we say, the perceptual details have two aspects: (1) a sensory aspect, or aspect of *pure existence*, as if void of intelligible quality, and (2) a conceptual aspect, or aspect of *pure essence*, as if void of sensory quality, easily blended with imagined forms and so taking on the character of subjective thinking. Any detail may have predominantly one or the other aspect, according to the type of attention which we give it; but in either case it is 'perceived'. These conclusions may be summed up as a Principle of **Perceived Conceptuality**.

In this particular discussion I have used the term 'perception' always with reference to the apprehending of a *physical* object. It may be convenient, however, sometimes to speak of *non-physical* objects as 'perceived'—dream-objects, for instance, or mental images. We may also speak of them as 'given', in a corresponding sense of the term, i.e. given in that non-physical universe or 'sphere of creativity'. In exactly the same way we conclude that there are 'sensory' and 'conceptual' aspects of non-physical objects. And the above-mentioned Principle is correspondingly extended in applicability.

It is now necessary to examine more deeply and systematically our manner of perceiving 'the world' as an 'external reality'.

The Spatial World of the Unseeing

A sighted person, when asked to determine the spatial form of an object, would never in ordinary circumstances shut his eyes and feel its shape. He would judge the spatial form by the visual appearance, even though he knows that these may differ very greatly. Hence he may come to regard the spatial form of an object as a *quality* deduced from, suggested by, or projected on to its visual appearance. One who subconsciously thinks of space in this way is likely to be very puzzled by the achievements of certain celebrated blind persons.

One of the most notable was Nicholas Saunderson, who, on Newton's strong recommendation, became his successor as Lucasian Professor

of Mathematics at Cambridge University and held this post for nearly 30 years. Saunderson was particularly admired for his popular lectures on Optics. Yet he had lost his sight completely when only one year old.[1]

Even more remarkable is the career of John Metcalf, blinded when six years old:

'When he was nine years old, he could go anywhere without a guide . . . At nineteen, Metcalf ventured farther afield, walking two hundred miles to London, visiting Windsor Castle and Hampton Court on the way . . . He got about so freely that few thought of him as blind. Asked to take a stranger to Harrowgate, he agreed to do so on condition that the man was not told that he was without sight.'[2]

Subsequently Metcalf set up as a builder of roads, sometimes over difficult terrain. His work and innovations were highly esteemed, and he obtained contracts totalling £65,000. He was also a proficient horseman, and in his earlier years was active in hunting.

The French philosopher Merleau-Ponty (d. 1961) has considered the nature of space psychologically and transcendentally with great thoroughness.[3] His view is that there is

'a function much deeper than vision, deeper too than touch conceived as a collection of given qualities. It appears to concern the subject's vital area: that opening upon the world which has the effect of making objects at present out of reach count notwithstanding for the normal person; they exist for him as touchable things and are part of his world of movement' (p. 117).

'This phenomenal field is not an "inner world", the "phenomenon" is not a "state of consciousness", or a "mental fact" . . . Once the prejudice of sensation has been banished, a face, a signature, a form of behaviour cease to be mere "visual data" whose psychological meaning is to be sought in our inner experience, and the mental life of others becomes an immediate object, a whole charged with immanent meaning' (pp. 57–8).

Our spatial perceptions of the world are centred on the 'body image', which 'is not confined to contents actually and fortuitously associated in the course of our experience, [but] is in some way anterior to them and makes their association possible' (p. 99).

[1] G. Farrell, *The Story of Blindness,* Harvard 1956, p. 10.

[2] *Ibid.*, p. 11.

[3] M. Merleau-Ponty, *Phenomenology of Perception,* trs. Colin Smith, London 1962.

'As far as bodily space is concerned, it is clear that there is knowledge of place which is reducible to a sort of co-existence with that place' (p. 105).

This 'body image' can then be 'dilated' by the 'function' just mentioned, through the mediation of sight or in various other familiar ways:

'My eye for me is a certain power of making contact with things, and not a screen on which they are projected. The relation of my eyes to the object is not given to me in the form of a geometrical projection of the object in the eye, but as it were a hold taken by my eye upon the object' (p. 279).[1]

'A woman may, without any calculation, keep a safe distance between the feather in her hat and things which might break it off. She feels where the feather is just as we feel where our hand is . . . To get used to a hat, a car or a stick is to be transplanted into them, or conversely, to incorporate them into the bulk of our own body. Habit expresses our power of dilating our being in the world' (p. 143).

More generally, Theodor Lipps described (in 1903) the enormous heightening of this 'function' which may occur in aesthetic experience and the existential significance which may then be disclosed in it:

'This is what is meant by Empathy: that the distinction between the self and the object disappears or rather does not yet exist. I see a man making powerful, free, light, perhaps courageous motions of some kind, which are objects of my full attention. If I am entirely absorbed in contemplation of the movement, by that very fact I am entirely distracted from what I am doing . . . In a word, with my feeling of activity I am absolutely incorporated in the moving body. I am even spatially in its position, so far as the self has a spatial position; I am transported into it. So far as I thus feel myself active in the observed object, I at the same time feel myself free, light, proud, in it. Here all emphasis must be laid on the "identity" which exists for my consciousness. This must be taken quite strictly.'[2]

By pondering such experiences we may come to appreciate better the

[1] The medieval theory of vision was possibly intended to describe this psychological impression of 'dilation' and 'contact', along with the fact that vision occurs for us only when and according to the manner in which this contact is made. The theory pictured the eye as sending out (instantaneously) rays (of recognition?) to its objects, which are then actualized accordingly.

[2] *Archiv für die gesamte Psychologie i* (1903), passage condensed. Quoted by E. F. Carritt, *Philosophies of Beauty*, Oxford 1931, pp. 253 ff. Further on 'empathy', see Chapter 16 below.

dictum of the blind teacher and psychologist, Pierre Villey, that 'Sight is long-distance touch, with the sensation of colour added. Touch is near sight minus the sensation of colour and with the sense of rugosity added. The two senses give us knowledge of the same order.'[1]

All this evidence, I suggest, is to the following effect. Within our seeing of objects of the world around, and within all other clues to spatial position and form which we may receive, are the substantial intimations of a *quasi-tactual sense of space*, by which we are in some way identified spatially with the perceived object. The character of these 'intimations' is that individually they have the character of a self-identified 'substance', while the relations between them are what we call the perceived relations of 'space'—depth, apartness, orientation, etc. By saying that these intimations are 'within' seeing and other clues, I mean that they are 'discoverable causes', that is, they belong to the category of perceptual details discoverable by intuitive analysis of the object and objective relations in question.

There is indeed for us, even when we shut our eyes and consider the positions of our limbs or of objects in the room around, a link-up of spatial configurations with a conceptual kind of seeing. The end of the feather has, for the woman in whose hat it is fixed, a space-orientation and depth rather as if she were looking at it. But whether the visual impressions are conceptual in this way, or are presented with all the fullness of sense, the sense of sight is one function and the sense of space is another.

The term 'sense', however, can mislead by suggesting a subject-object dualism with the subject sharply localized in the physical body. I shall therefore usually prefer to follow Merleau-Ponty and speak of the *function* of spatial identification.

The conclusions now drawn as to the virtual autonomy of this function are not affected by the fact that persons who have obtained sight by an operation, after having been blind from birth, may learn with practice the spatial correspondences of seeing, and may then find seeing far more comprehensive, detailed, precise and reliable than touching;[2] nor are they affected by the fact that blind people (as well as the sighted) evidently vary enormously in the degree of development of this sense of space and may sometimes possess it only in rudimentary form. We grant that every 'clue' has its special uses and demands both natural ability and practice. But it remains nevertheless a clue, and

[1] *The World of the Blind*, New York 1930, p. 18. Quoted by Farrell, *op. cit.*, p. 93.

[2] Merleau-Ponty, *op. cit.*, pp. 222 ff., references to Von Senden, *Raum- und Gestaltauffassung bei operierten Blindgeborenen, vor und nach der Operation*, Leipzig 1932.

cannot be equated to that to which it is a clue—namely, the conceptual and perceptual space-field which constitutes the world for us.

The other opinion, that the objects of the world are formed for us from the projections of visual and other sense-data, suffers from the fatal objection that there is no space upon which to project these data unless space already exists for us.

Imagination and Error in Spatial Judgements

The sense of touch is usually regarded as one of the least dependable of our senses, and many experiments have been devised to estimate the magnitude of the 'error of localization' which might occur in various circumstances. There is also the familiar 'illusion of Aristotle', which occurs when the first and second fingers are crossed and a pencil is placed between (appearing then double). Are we to conclude that there can be no approach to 'actual' or 'real' properties of space by means of touch, and that we are therefore obliged to use either the much more reliable sense of sight or more abstract means such as measurement on the basis of scientific theories?

On this question the following preliminary comments seem necessary:

(1) The purpose of the first illustrative experiment was not to show that the sense of space is resident in the sense of touch, but that the sense of space can be exercised in complete independence of the sense of sight (we could also show that it can be exercised in complete independence of the sense of rugosity).

(2) Sensations of rugosity, in themselves, are as little indicative of spatial form as sensations of colour. It is not till these sensations are *formed*, either by the occurrence of several such sensations simultaneously or in succession, or by their combination with other sensations which may carry impressions of form (e.g. muscular sensations associated with convergence of the optical axes of the two eyes), that an impression of spatial form arises.

(3) As an example of the paramount importance of *context*, consider the following. The points of two pins, one millimetre apart, touching the skin, normally seem to be one point only. But if we slide the finger-nail slowly across the points we can easily distinguish the two, even when they are only a very small fraction of a millimetre apart.

(4) Measurement provides a most valuable context. It is to be noted, however, that the first mention of 'measurement' implies the previous recognition of a spatial 'world' in which we can manipulate graduated measuring devices. This 'world' is not *given* by measurement.

(5) In deciding what observations are the 'true' ones, referring to what is 'real' or 'actual', we avail ourselves of every possible clue, testing our findings also in every possible way for consistency, until, superior to particular surface impressions of seeing and touching, we come upon a three-dimensional configuration which is indeed physically *actual* for us and not *imagined*. For we can manipulate it, or move bodies in relation to it, with the greatest confidence, knowing it as objectively 'physical'. Yet although we may approach

this confidence with the help of reason, we cannot account for what is 'real' and 'actual' by an explanation *merely* in terms of what we do with our subjective reason. That to which we apply our subjective reason must exist for us prior to our subjective reason.

Error, now, according to the view maintained in Chapter 3 above,[1] is the confusion of spheres of awareness, particularly between the sphere of imagination and the sphere of the given. We overcome it by the piling up of evidence, clues, insights, etc., and by consideration of rational coherence, till the confusion is dispelled and control is obtained over the different spheres of awareness and mental functions which we possess.

It will be convenient to distinguish between the *presentation* of some Gestalt given to us physically in a total encounter, and a *representation* which is, in some respects at least, created by our imagination or thought. I take the term *presentation* to include what may be discerned in the initial encounter, and also all perceptual details which may then be discovered in the *primordial recollection*[2] of it. A *representation*, on the other hand, will be one of the following: (1) a spontaneously imagined whole closely resembling the original (the extent of the resemblance being testable by primordial recollection); i.e. it is a new production of the mind and not a primordial recollection of the original; (2) a blend of the primordial recollection with contributions of imagination or thought, in such a way that we involuntarily impute to the original the characteristics which we imagine or think that it has; and (3) a deliberate creation of our imagination or thought, devised so as to suggest characteristics of the original presentation which we have in mind.

The primordial recollection of a total encounter is exceedingly difficult to maintain or to restore after a few seconds of time have elapsed. Almost inevitably, imaginative elements begin to blend with it and it becomes a *memory* in the usual sense of the word, i.e. a representation of types (1) or (2). Nevertheless we can call this memory 'reliable' if, as regards characteristics which are relevant to a certain matter under consideration, there is agreement with the original, sufficient to avoid error of belief. Likewise, we can call a representation of the third kind 'reliable' if it agrees with the intention of the creator in the respects relevant to the matter under consideration.

It is necessary that we should be able to form representations of any given presentation, in order to be able to think about the presentation

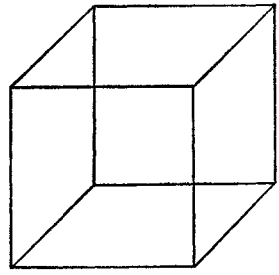
[1] See p. 44. This 'error, is error of belief, not zero truth-value of a proposition. I regard the latter as merely incorrect syntactical procedure, i.e. failure to obey the rules decided on for the use of signs, in reference to a factual situation about which it is presumed there is no longer any error of belief.

[2] This term is explained in Chapter 19, p. 295.

afterwards and discuss it with other people. Without such representation, science and reasonable living would clearly be impossible. We must therefore be prepared to make a copy (in our memory, with pen and paper, or by some other material means) and be able to produce or recall this copy when needed. If the copy is seen or imagined as seen from some particular position or angle, we must again recognize, as in the case of the original, that the appearance is only a clue to the totality which constitutes the spatial form of the object represented.

This kind of 'clue', it is important to remark, differs very much from the clues which we have so far been discussing. It does not contain, as a discoverable cause, the spatial actuality of the object represented, but only *suggests* a memory or thought-structure of it. I shall therefore distinguish such 'clues' as *representational.*

In practice, the separation of the sphere of imagination and the sphere of the physically given is not as difficult as might be supposed, if due care and intuitive reflection are exercised. The second illustrative experiment, as follows, is intended chiefly to illustrate this.



Experiment 2. This is to be performed with a three-dimensional structure in the form of a cube seen from above (or below, if preferred) with edges made of fluorescent tubing. The percipient is to be seated before a large sheet of frosted glass, behind which the structure is placed, and he is not informed in advance whether the view is from above or from below. Taking the visual appearance as a clue, he will then try to reach actuality by conceiving of the two possible spatial forms in three dimensions, and somehow determining which of these fits better with what he sees. He may find it quite easy to impose either conception on what he sees, and the spatial effect may be quite vivid; but without further investigation he may be unable to decide which is *actual.* Indeed, insofar as he is conscious that each of these conceptions is voluntarily imposed, neither is actual. All he was hoping for was a *correspondence* with actuality, as determined in some other way.

If, however, we now allow him to reach with his hand behind the screen, he will at once obtain another conception of spatial form,

knowing it as *actual*, and different in an indefinable but nevertheless very definite way from the trial conceptions—which were also each a totality of three-dimensional aspects, but not actual.

Now let us vary the experiment, and instead of allowing the percipient to reach behind the screen, we remove the streen, so that he can see clearly the edges of the tubing and judge distances by muscular feeling in the eyes, effects of perspective, shadows, etc. As his attention passes from one corner to another he vividly perceives the three-dimensional spatial form in actuality, receiving the same kind of impression as if he were touching the various points, except for the absence of roughness or smoothness, pressure, temperature, and other such qualities.

On looking at parts of the structure with too concentrated a gaze, or if a shadow is momentarily cast over the whole, he might temporarily lose contact with the actual three-dimensional form. In that case a concept of the whole as it is in actuality might be retained imaginatively in the memory and be imposed on what he sees. But if asked to check whether the cube is actually viewed from above or from below, he need only make a quick survey covering a large number of clues, and complete confidence will be again established.

We must admit that wherever there is dependence on sensation a mistake is possible. But if on any occasion we happen to mistake the imagined for the physically given, our error must be adjudged due simply to inadequate observation. For plainly there is a context of clues determining which of the two possibilities is actual, and only by passing over essential parts of this context could the imagined possibility be supposed actual.

The 'Exhibit' Theory of the External World

The distinction just drawn between imagined spatial worlds and the physically real spatial world could be at once followed up by the elaboration of distinctions between the many non-physical worlds—each to be recognized partly by the general character of the spatial identification experienced and partly by the total context of the sensory clues in a presentation provisionally judged to be 'homogeneous'.[1] These further distinctions do not seem to need detailed consideration at present, so I proceed to examine the impression of *externality* which is conveyed to us in each of these worlds, but nowhere more sharply presented as a dualism—subjective mind versus external matter—than in the physical world.

The theory of externality which I am developing in this book may be

[1] See above, p. 56.

called the 'Exhibit Theory', and is to be contrasted with the 'Message Theory' which has prevailed during the last 300 years or more.[1]

In the exhibit theory it is recognized that, as far as perception on a given occasion is concerned, we are not interested in previous history. We have before us an 'exhibit' whose *given* character is to be directly discerned here and now. Part of that given character is the 'reality' of the exhibit, as impressed on us in the exercise of our quasi-tactual function of spatial identification. To forestall possible confusion of terms, I shall call this kind of reality the P-reality (P for 'perceived' or 'participated').

Theories and evidence referring to what other observers may or may not have experienced on other occasions are here irrelevant. No interpretation is therefore desired, since this would introduce concepts which are *not given*. In particular, we shall not demand knowledge or imagination of a supposed substructure or time-succession of previous 'causes', before agreeing that we may be in touch with P-reality.

Essential to the exhibit theory is the principle of the Total Encounter, along with the views already developed concerning phenomenological analysis, 'discoverable causes', perceived conceptuality, etc., and further views to be presented in due course concerning potentiality and actualization.

By saying that the P-reality is 'directly' known in the spatial identification we do not deny that there is a substructure to which it is appropriate to impute a U-reality (U for 'unperceived'). But we point out that those philosophers and scientists who insist that objects are not 'directly' known seem to be running counter to commonsense in their use of the term 'direct'. They proceed, correctly on a *subconscious* reaction to the P-reality of spatial identification, to describe sense-appearances as 'indirect'. At the same time, however, they omit to tell us what is 'direct'. For the U-reality, being unperceived, could not in commonsense be described as 'directly' known; and the 'directness' of a P-reality is not recognized by them.

This situation points to the need to distinguish between a *strict* P-reality which is consciously or half-consciously observed in a total encounter, and a *loose* P-reality in which we can be said to participate without observing it consciously in any way. Perhaps we have not reached the stage of acknowledging that there is any such 'function', and believe that positions are inferred and projected outwards from our brains. Or, in the case of an encounter with a distant star, there is a

[1] And also with the 'Prehensive Theory' as described by C. D. Broad in 'Dreaming, and Some of its Implications', *Proc. S.P.R.*, 52, 1959, pp. 63 ff. I do not deal here with this theory or with any other existing variety of Direct Realism, because I think it will be seen that these are fruitless attempts to justify the commonsense view of the Exhibit Theory without recognizing any function of spatial-identification.

wholeness of the encounter which seems unmistakably 'real' (i.e. given in nature), but it seems that the function of spatial identification must fail on account of the enormous separation. In either case, colours and other sensory attributes are also in a certain way *given* to us and not magined. Further language conventions in regard to the terms 'real' and 'direct' seem therefore needed.

The subject of the 'reality' of very distant objects, or objects seen through the telescope or microscope, is dealt with at length in Chapter 16. The status of colours and other sensory attributes needs brief consideration here.

It seems that we must recognize that colours do in fact appear very convincingly located on the surface of objects near at hand; but whenever the function of real spatial identification fails and an imaginative one takes its place, the localization of the colours takes on a similarly imaginative or subjective quality. The same applies to perceptual details belonging to the substructure, provided these are of the kind which we may call 'immediate', i.e. they are discoverable causes. For instance, the conductor of an orchestra, hearing a general blend of sound from the instruments, is nevertheless able to separate the individual contributions and even the structure of the harmonics and formants making up the tone-quality of each instrument, and those details are confidently localized in the instrument producing them. Certainly it would be absurd to localize them elsewhere.

If, on the other hand, we hear a train approaching, and are not sure whether the sound comes from the north after having been reflected by a wall, or directly from the south, any localization of the harmonic structure of the sound must seem as imaginative or doubtful as our localization of the train itself. Also, if the 'causes' are not 'discoverable' but 'contingently inaccessible' (as, for instance, the structure of the iron molecules in the train) or 'distant' (as, for instance, the velocity possessed by the train one second before), then they are not, on that occasion, anything to which we should consider applying the terms 'direct' or 'real'. For they are not an 'exhibit'.

According to these analyses I suggest accepting a Principle of **'Real' Localization** of Perceptual Details and Discoverable Causes, which localization occurs with colours and other details of objects which are 'directly' known by spatial identification. Along with this must go a Principle of **'Imaginative' Localization** of Perceptual Details and Discoverable Causes, which occurs when the objects are imaginatively localized. In the case of a distant object seen through red glass (to take a rather involved example) there is the blending of a spatial identification in the position of the glass, accompanied by the real localization of the colour and other details there, with an imaginative localization of the distant configuration and also of the colour at the imagined

place of the object. The blending may be sorted out by a reduction of the encounter to more restricted encounters, with the help of eye-focusing and other clues, so far as our powers of sense and mind permit.

Greater complexities still, perhaps, arise when distant objects are seen through a mist, or a refracting atmosphere or lens.[1] But the general standpoint of this theory is that we must describe each 'exhibit' just as we find it and restrict the terms 'reality' and 'directness' to

(1) experiences of spatial identification of the 'real' kind (i.e. given, and not imaginative);
(2) sense appearances 'really' localized at the position so identified;
(3) any details of the immediate substructure likewise 'really' localized at that position.

The 'Message' Theory of Externalization

In contrast to the exhibit theory, the Message Theory denies the propriety of calling our knowledge of any external object or of any of its attributes 'direct'. Nothing that we observe in it is to be described as 'reality', but all is 'appearance', which needs to be interpreted.

The message theory begins by provisionally accepting that there are *actual objects* each of which exists as a U-reality in a certain place in the world, which is somehow imaginatively conceived as an 'external' container of them. From each actual object as cause there proceed physical effects such as those which ultimately will be recognized as light, sound, smell, etc., when the causal chain is completed by physiological effects in the sense-organs, nerve-currents, and finally the conversion of brain-activity into conscious sensation. The conscious impressions constitute the 'appearance' of the object. The postulate of externality with which the theory began is held to be justified by the coherence of the interpretations. It is to be noted that the postulating of an externality is the *imagining* of one. But the full postulate is to the effect that, corresponding to the imagined externality, there is an *actual* kind of externality which is unperceived. Although it is not in fact 'given', it is supposed to have the status of something given.[2]

The operation of the causes just mentioned is also admitted, with qualifications, in the exhibit theory. This is not the point at issue. The objection to the message theory is partly that it tries to account for a given externality in terms of an imagined one, and partly the related linguistic objection that it attaches (by implication at least) the epithet 'indirect' to certain kinds of observation without telling us what would be 'direct'. Involved also in the outlook which a theory of this kind

[1] A discussion of telescopic and microscopic vision will be found in Chapter 16.
[2] An instance of the Living Skeleton Fallacy (p. 99 above).

inculcates is the belief in the carrying of a spatially-formed 'message' step by step through a 'container', instead of in the manifestation or actualization of a transcendent potentiality on the occasion of a total encounter.

There is indeed a more sophisticated kind of message theory, in which potentialities or probability fields in space-time take the place of actual objects. But the first two objections apply equally strongly to this, and the third almost equally strongly.

The Time-Lag Argument

The finite velocity of light, it is sometimes said, provides an invincible argument against any theory of Direct Realism—a denomination which would seem to apply to our account of the function of spatial identification. Obviously we cannot see the 'original' object when we look at it, because what we see is actually the product of an occurrence at a previous time and not the object as it is now. The argument may be made clearer and more plausible, perhaps, when presented analogously in terms of sound-waves. If we hear what we think is a clock ticking in a dark room, we know that the sound is a local effect of air-movement and cannot give immediate knowledge of its distant origin. To obtain 'direct' knowledge of its origin we must trace the sound to the clock, which we then recognize by going closely up to it and touching it. In the same way, it is argued, light-waves cannot give immediate knowledge of distant objects.

It may be remarked first that the exhibit theory is purely ostensive—it plainly exhibits what is to be called 'real' and 'direct'. A judgement as to its feasibility cannot therefore be affected by any theoretical argument. The most that the time-lag argument can achieve is to persuade us that our proposed use of these terms 'real' and 'direct' is inappropriate or to be deprecated for some reason.

Secondly, this is an argument concerning properties of *light* and *seeing*. Now the exhibit theory does not claim that light carries to us a 'direct' knowledge of the distant object. The claim is that seeing and other clues are like the setting in order of the numbers or symbols of a combination lock, so that a door then opens to the new 'world' of spatial identification. Consequently, if the time-lag argument is to have any relevance to the exhibit theory, it must be carefully recast so as to bring in some presumed temporal relation between light and the spatial identification to which it is a clue.

Let us therefore try to set forth the argument with perfect precision, without slurring over any presupposition, and linking it with spatial identification in the way most favourable to its success. We must think in terms of a substructural picture, in the proper inertial frame of a certain observer O, an object being at position P_1 at time t_1. Times are

understood to be marked with mathematical exactitude in this ideal geometry.

At the time t_1 a light-wave is proceeding from the various points in the ideally-marked configuration of the object at P_1, and every point of the wave then travels with a velocity of 186,000 miles per second towards the eyes of the observer O at a position P_2, arriving at time t_2. After further physical and physiological changes initially induced by the wave, there arises in the consciousness of O a visual configuration, along with certain muscular impressions in or around the eyes. As a result, the object is *recognized* and placed *externally* at some 'depth', in some direction, and with some orientation relative to the body-image of the observer. Whether this 'placing' is imaginative or, in our terminology, 'real', is not yet considered.

It will be no limitation of the argument to suppose that the object is less than ten miles away from the observer. For the exhibit theory is not suggesting that beyond this distance there is any hope, in a normal physical state, of placing the object by a 'real' spatial identification. An exponent of the theory will readily agree that the placing of the object is then only 'apparent' and 'imagined'.

We must also allow for the well-recognized fact that a certain finite physical time-duration is required for a total encounter. Various perspectives in the three-dimensional Gestalt and a considerable number of perceptual details must be surveyed and recognized as forming a developing objective unity. This is done by the faculty of primordial recollection ranging over such a time-duration as may be necessary and presenting the configuration to us instantaneously (in a phenomenological sense) in relation to our body-image and the other objects in the locality. If an attempt is made, by physical methods, to estimate this duration—called the 'specious' or 'mental' present[1]—we obtain a measure of the order of one twentieth of a second. But, if, as we are presuming, the object is less than ten miles away, the mental present constituting the observer's total encounter 'at time t_2', and the mental present constituting an observation at P_1 'at time t_1', would have an overlap of more than 99·9 per cent.

At this juncture let us give the argument its only chance of success by postulating an exact and necessary correspondence between, on the one hand, the emission and arrival of the light-waves, and on the other hand, the manifesting of the two-term relation called 'spatial-identification'. More explicitly, we suppose that, when a certain light-wave is a clue to a spatial-identification, the theoretical representation of the overlapping mental presents just mentioned is also in some way an exact and perfectly appropriate theoretical representation of the two

[1] This is discussed in Chapter 19, p. 297. See also Whitrow, *op. cit.*, pp. 77 ff. The term 'mental present' is due to Whitrow.

apparently simultaneous mental presents (near and distant, respectively) which constitute the two-term spatial-identification. We can then indeed deduce that there must be a minute lack of temporal concidence between the supposed theoretical representations of the two terms in the spatial-identification. But because these are mental presents, the difference is unobservable. Thus it cannot be said that there is a time-hiatus between the observation at P_2 and the identification at P_1; and the argument that the relation must be called 'indirect' because of a hiatus altogether fails. The relation *may* be sometimes of a kind appropriately called 'indirect' (as when the spatial-identification is imagined). But the argument does not compel it to be so regarded; nor does it even plausibly suggest such a thing.

As regards the analogy with sound-waves, the underlying assumption seems to be that coincidence of the sense organ with the object in space as well as in time is necessary for 'direct' knowledge of the object. In that case the only 'direct' means of knowledge of distant objects is by a spatial-identification in which our power of observation is genuinely transferred to (or genuinely discloses) the place concerned, without any observable time-lapse. Hence the usage of the term 'direct' in the Exhibit Theory is justified—the only means of escape being the flat denial that a *real* spatial-identification is possible, as against an imagined one, and the arbitrary limitation of the term 'direct' to cases of touch.

An Extended Aristotelian Theory of 'Actualization'

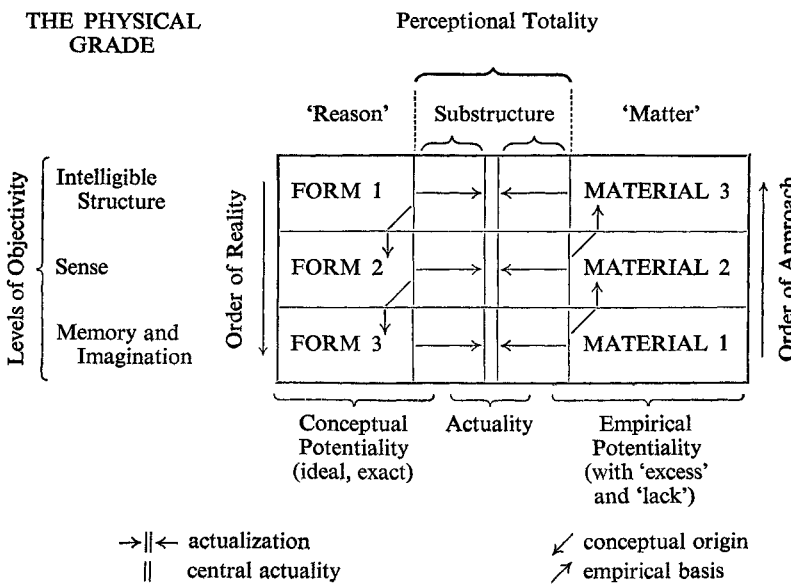
One of the subsidiary uses of our analysis of the time-lag argument has been to emphasize the impossibility of any exact correspondence between the theoretical structure of nature and empirical actuality. There is a kind of parallelism, but the two operate in different 'worlds', so to speak. Similarly, imagination and memory contribute to our sensory recognitions, partly by a filling in of gaps (e.g. providing an impression of 'depth' when spatial identification fails), and partly by interpretative guidance (e.g. inducing, from the clues of muscular feeling in the eyes and other associated effects, the recognition of a certain familiar three-dimensional shape); and again there is no exact correspondence.

In any physical perception, it thus appears, we can discern by intuitive analysis a certain co-operation of three 'levels'. I shall call these, respectively, the level of *intelligible structure*, the level of *sense experience*, and the level of *memory and imagination*. At each level there is a 'central' sphere of consciousness, which expresses the general effect of what is in the 'subconscious' at that level. In hearing the vowel sound ēē, for example, we are not usually conscious of the high-pitched formant (or slight whistling sound) by which, in fact, we are enabled to recognize it. Such a perceptual detail or 'discoverable cause' has been described in Chapter 4 as belonging to the physical *substructure*, which

we can now specify as part of the 'immediate' subconscious. There will also be 'distant' parts of the subconscious, which have effect only in special circumstances. What we observe most vividly on any occasion, whether an effect of the whole or a certain perceptual detail of it, may be called the 'central actuality' for us then.

In any contributory detail of the substructure (such as the formant in the vowel ēē) there will be a conceptual or formative aspect, and an empirical and objective, yet 'fluidic' or indefinite aspect. These may be regarded as conceptual and empirical aspects of *potentiality*, respectively, so far as they are not 'central'. A specially important kind of potentiality, at the level of sense experience, is that which consists of the totality of three-dimensional perspectives implied by the central perspective and constituting the Gestalt.

It would be a mistake, I think, to attempt too rigid an account of the various relationships so far mentioned and of others which might still be uncovered. Instead, I offer a suggestive diagram which I hope will represent, in spite of obvious inadequacies, a considerable advance on the over-simplified ontologies usually assumed in philosophy and the natural sciences. This diagram is of the 'physical grade' only, as if in isolation. It is clear that various parapsychical and mystical grades, as well as imaginative or dream-like states, must also be recognized, and each of these will be, in my view, analysable somewhat on the same pattern. And I think we must also accept (following Swedenborg) two



kinds of causal connection between grades, one immediate and perpetual, the other mediate and occasional.[1]

In Aristotle, one stage of being is the realized meaning or *form* of the stage next below and the *matter* of the stage next above. In our extended theory of actualization there are similar relationships, which I suggest calling 'creative connections'. Four of them are indicated in the diagram by vertical arrows, and can be expressed in words as follows:

(1) intelligible structure provides the form of sense experience;
(2) sense experience provides the form of memory and imagination;
(3) memory and imagination provide a fluidic basis for sense experience;
(4) sense experience provides a fluidic basis for intelligible structure.

The upward movement, shown on the right-hand side of the diagram, is the *order of approach*, by which we come to knowledge and understanding. The downward movement, shown on the left-hand side of the diagram, is the *order of reality*, from the greater reality of intellligible structure to the lesser reality of imagination which results 'causally' from it.[2] There is also a 'causal' relationship (according to my use of the term) between potentiality and actuality, and this is shown by the horizontal arrows. The diagram does not show the effects of *error* in confusing the various levels, aspects, grades, and orders.

The scheme now set forth in outline is offered as a straightforward attempt to represent and thus clarify certain important insights; in short, this evidence and not hypothesis. We have then to consider what insights we may have in regard to the ontological status of Euclidean geometry, and to locate these, as it were, in the diagram.

In accordance with the views expressed above on the possibility of a three-dimensional knowledge of the given Gestalt of objects, I assume that we can (and do, continually) make an extremely rapid empathic survey of our spatial surroundings, thus obtaining a *total environmental impression.* This may be likened to seeing a map all at once, but simultaneously drawing our 'central' attention to a focus at different points successively in whatever way it occurs to us to do so, and noting their relations to the whole.

In this total environmental impression, the structures and relations and distances sensed by our function of spatial identification agree, so far as we can distinguish, with the laws of Euclidean geometry. There are straight lines, planes, spheres, etc., with the properties theoretically possessed by these structures. Visual perspectives are also accountable in terms of the Euclidean theorems on projection. If we doubt whether

[1] *Heaven and Hell*, 297 (also 37, 208). *Arcana Coelestia*, 6472.

[2] The terms *order of approach* and *order of reality* are due to Boyce Gibson. See Chapter 23 below, p. 372 and note 3.

the structures are exactly as we think they are, we can investigate them by local measurement. But then at once the impossibility of reconciling exactly the levels of intelligible structure and sense experience comes centrally into evidence.

The level of sense experience is therefore the level of actual-operational geometry, whether the operations are carried out with material apparatus or directly by the judgements of sense. At the level of memory and imagination we have similar appearances of Euclidean geometry, but a check by measurement using apparatus is not possible, the 'fluidity' being too great. The chief functions of this level appear to be to fill out the little that we can discern of the 'given' and to enable us to direct voluntary operations (analysis, conceptual construction, measurement) at the higher levels.

Finally, at the level of intelligible structure we have the conceptual tool of ideal-operational Euclidean geometry, to be developed and applied to the problem of non-local measurement in any of the numerous ways discussed in Chapters 11–13 below.

The Powers of Māyā

I feel it is important now to emphasize certain features of this theory of actualization which have so far remained rather in the background.

According to the view I am presenting, the intelligible structure behind any particular observation, whether it is of an everyday kind or made in some elaborate scientific experiment, is to be understood as transcending space and time. No part of it is to be located 'out there'. To speak metaphorically, the intelligible structure is *operative in* and *analyzable out of* the total experience (subconscious potentialities included).

The difficulty is that we are apt to think of tangible objects, elementary particles, sun, moon and stars as being actualities placed somewhere external to us and therefore possessing a substantial nature which is absolutely beyond the reach of intuition. I am maintaining, on the other hand, that this way of thinking is an error which arises from inability to see that externality, however 'real' it may be to us, comes into evidence only with actualization as described above, and previously *did not exist as such*. And thus, I maintain, we are led to reify a container of space, to actualize what is potential, and to materialize what is conceptual.

The doctrine that the whole nature of externality is conceptual and occasional, but nevertheless objective, is a difficult mystical one, scarcely to be found in western philosophy apart from Leibniz and Berkeley. I append therefore a brief interpretation of the Vedic and

Vedantic doctrine of *māyā*, the 'trick' of creation, in the hope that this alternative presentation may help to make the subject clearer.[1]

Two powers or functions are possessed by *māyā*. By the power of 'projection' (*vikṣepa*), relations which in themselves are conceptual or spiritual become convincingly presented as a three-dimensional localization of perceptual details in relation to each other. Obviously there are different kinds of localization, such as those which I have described as 'physically real' and 'imagined'. Also, spatial identification is the primary localization. Colours, sounds, and all other perceptual details acquire secondary kinds of localization in the place of the objects reputedly 'causing' them, although reason shows that the sense effect is a result of the *total* circumstances and mental condition of the observer. All this is to be regarded as a law of creation, operating at each grade of experience.

By the power of 'enclosure' (*āvaraṇa*), there is a partial cutting off of participation except in the region of the body-image. The effect is to give objects an imagined non-mental or unparticipated quality, while yet we are somehow forced to realize that imagination alone cannot account for them. As a more primitive working of this power of enclosure we may include the formation of 'objects', which continue more or less the same because of a shielding from disintegrating 'causes'.

Specially to be noted in this description of 'enclosure' is the self-contradictory epithet, 'imagined non-mental'. The non-mental quality which attaches itself to what is 'enclosed' may perhaps be identified with 'the Other' of Plato or with *materia prima* of Aquinas. It is part of the 'trick' that we should forget that even this 'otherness' and 'matter' is the deliverance of conceptual analysis, and that we thus tend to fall into a mind-matter dualism and all the philosophical troubles that stem from it.

Two senses of the term 'projection' must be carefully distinguished, according to our reaction to the trick. If we discern the projection and enclosure in their true nature, our view of the world around us is somewhat as follows:

(1) We have experiences described as 'total encounters', and discern perceptual details in them; but we do not locate these 'in the mind' or even in the body-image. Localization is discerned, but it is only another perceptual detail.

(2) Thus among the perceptual details are those of spatial identification. Viewed as a whole, these constitute the substantial form of the 'world' for us on that occasion.

(3) Colours, etc., are convincingly located on objects, as already explained.

(4) Accessible within the total encounter (by means to be mentioned presently) are elements of a *transcendent* substructure, which operates in the

[1] S. Dasgupta, *A History of Indian Philosophy*, Cambridge 1922–55.

total effect, not actually and locally, but in an 'as if' way, by general laws of correspondence.

If, on the other hand, we succumb to the trick, we become subject to what I shall call the *Projection Complex*, or *sensory fixation*. Our view is then correspondingly as follows:

(1) We think of sensation as located in us, and therefore necessarily private.

(2) We think of a theoretically postulated or inferred space-frame existing external to us, like a container of non-mental objects; or we take it for granted that there is such an external 'container'.

(3) We plant our subjective sensations on the inferred non-mental objects in the external space-frame.

(4) The substructure of nature, postulated or inferred, is also externally localized and therefore quite inaccessible. It is thought of as a non-mental substratum, on which the subjective impressions of the mind are causally superposed.

This full scheme of belief, which the Representative Theory of Perception tries to justify, is the *Scientific Dualism* whose historical development is studied at length in Part IV. In a general way, acceptance of any one of the four constituent beliefs may be said to make our world-view *dualistic* to a corresponding extent.

There are many other essential points to be discussed in connection with the perceptual basis of spatial concepts. These, however, are concerned with the intersubjective and transcendental substructure, and are therefore deferred till Chapter 21.

Berkeley's 'New Theory of Vision'

To conclude the present discussion of the 'physical grade' of perception and its three 'levels of objectivity', a few comments on Berkeley's 'New Theory of Vision' (1709) seem called for. Between the two theories, Berkeley's and the one I am developing here, there are strong points of resemblance and strong points of difference, and neither ought to be overlooked.

The first difficulty is in discovering what Berkeley intends to convey by his frequently used phrase, 'ideas of touch'. Evidently contact and roughness are of little or no concern. He seems to be thinking, almost exclusively, of spatial relations, movement, etc., of the kind associated with touch rather than any other sense. This 'touch' is for him the primary and 'real' knowledge of space. It can perhaps be regarded as the awareness of a positional background, or perhaps a quality or form, of touch in the narrow sense, these things being apprehended by us also through movement.

This might seem to go a long way towards admitting a knowledge of

space or a 'function of spatial identification' ontologically prior to both seeing and touch (in the narrow sense). But Berkeley, on many occasions, plainly and forcefully denies that there is any perception of space through 'ideas of sight'. There are visual impressions of distance, magnitude, shape and orientation; but the connection of these with the impressions proper to tactual space is by habitual association only. The visual impressions 'suggest' impressions of the primary tactual space. Thus:

'The extension, figures and motions perceived by sight are specifically distinct from the ideas of touch, called by the same names; nor is there any such thing as one idea, or kind of idea, common to both senses.' (127)

'[The proper objects of Vision] do not suggest the things signified by any likeness or identity of nature, but only by an habitual connexion that experience has made us to observe between them.' (147)

'Visible figure and extension [are] of a nature entirely different and heterogeneous from tangible figure and extension.' (137)

The important question now is whether anything of the three-dimensional world 'suggested' by sight is to be regarded as 'real'. Since touch provides what is 'real', it seems that sight, being 'entirely heterogeneous', cannot ever provide what is real. But it will be best to proceed more cautiously.

Among 'ideas of sight' Berkeley includes the sensation of the 'turn of the eyes', i.e. variable convergence of the optic axes, and other muscular effects. These, he says, are 'immediately perceived' (18), as are colours and visual shapes, as well as touches. All such 'ideas of sense' are to be called *real*, because not 'excited in ourselves at pleasure':

The sun that I see by day is the real sun.

Or at least they have *more reality* than 'ideas of imagination'.[1] What then is the status of the spatial features which we apprehend in the visible sun? And what is the status of the spatial features which we apprehend in an object close at hand?

Berkeley's answers to these questions show some vacillation between the feeling that something spatial is 'apprehended' by the eye (and is therefore 'real', because not 'excited at pleasure'), and the theory that spatial effects *suggested* by visual appearance must be 'ideas of imagination'. He does not realize, apparently, that there is a third alternative—the spatial impressions might be prompted by visual 'clues' and still be 'real'. Thus he maintains:

[1] *Principles of Human Knowledge*, 33, 36. A. C. Fraser, *The Works of George Berkeley*, Oxford 1901, vol. I, p. 275.

'There are two sorts of objects apprehended by the eye—the one primarily and immediately, the other secondarily by intervention of the former. Those of the first sort neither are nor appear to be without the mind,[1] or at any distance off . . . they do not, cannot approach or recede from us. Whenever we say an object is at a distance, whenever we say it draws near, or goes farther off, we must always mean it of the latter sort, which properly belong to the touch, and are not so truly perceived as suggested by the eye, in like manner as thoughts by the ear.' (50)

He also speaks of a 'Universal Language of Nature' (147) and a 'strict connexion' between the 'secondary objects' suggested by sight and the 'proper objects' of the sense of sight, namely colours, visual shapes, etc. (51). If the connection is 'strict' and 'universal', how can it also be nothing but a result of the vagaries of personal experience? In spite of these slight inconsistencies, however, the theory that the spatial effects suggested by sight are wholly and always imagined finally triumphs:

'The ideas of sight . . . do not suggest or mark out to us things actually existing at a distance, but only admonish us what ideas of touch will be imprinted in our minds at such and such distances of time, and in consequence of such and such actions.[2]

That is to say, we think of a certain visible object as being, for example, 100 feet away, *because* we have previously felt sensations in the eyes from an object which, by pacing or otherwise, we knew to be tactually 100 feet away; and we remember or imagine this 'tactual' distance. The 'seen' distance is therefore always imagined, as is also the *suggested* tactual distance. But the tactual distance as originally experienced is real.

Here, surely, is an instance of the Abacus Fallacy, insidious trap for those prone to psychologism. Pacing or imagined pacing could indeed be a means helping us to develop our powers of perceiving 'real' spatial relations; but it cannot *be* that perception itself. Moreover, it can

[1] Here, 'without the mind' means 'outside the body', the 'mind' being customarily thought of in those times as localized in the body. Thus in his Theory of Vision Berkeley implies that 'ideas of touch' are genuinely 'without the mind', i.e. they are 'needed to infuse true reality into our conceptions of distance or outness' (*Fraser*, p. 100). In his later *Principles*, 'extension and figure', even 'tangible objects' (44), 'exist only in the mind' (49). But in that case 'in the mind' seems to mean *mental* in character, i.e. not a property of an extended unknowable *matter*. Both visible and tangible objects are then 'in the mind', but the tangible ones are for Berkeley 'immediately' real, the visible ones only 'mediately' so.

[2] Principles, 44.

hardly be a sufficient means for development. For we never actually learn the positions of objects in the world around us by such cumbersome means as pacing. It would be truer, perhaps, to say that we have an instinctive means of judging them (i.e. another 'function') and so are enabled to extend our 'body-image' and gradually build up a more and more 'real' total environmental impression.[1]

In spite of defects, scarcely avoidable in those times and in a very youthful philosophical essay, the New Theory of Vision may well be accounted a discerning and courageous piece of work, chiefly notable for its demonstration that in our judging of spatial form from visual appearances the work of the imagination plays an essential part.

THE STRUCTURES OF GEOMETRY

Our next task is to try to determine precisely what is meant by saying that, in the physical world, a lamina is square, or a three-dimensional figure is a cube. As now shown, it is not on sight that we finally depend in order to decide that a figure is square or cubic, but on the function of spatial identification. Without exercise of this function there is, strictly speaking, no perception of size or shape. But we can form an ideal structure in conceptual space; or we can go further and fit physical objects closely (though not exactly) to preconceived ideal structures. This was done in Chapter 8 for the straight line, which we then judge from direct perception to be a straight line in the physical sense, to the extent that it is perceived to copy closely the ideal concept of a straight line.

As with all geometric structures, the concept of a straight-line is excessively complex—too complex for any ordinary present-day analysis to be regarded as satisfactory. We depend therefore on the devising of some operational technique (first ideal and then actual) to assure us that the concept in question is well defined (in terms of space-perceptions, not words or symbols) and that we comprehend reasonably well the connections with related concepts such as translation, order, etc. Our comprehension would seem to be adequate if the operations as we have conceived of them ideally can be carried out actually, and up to the limit of precision, were it not for limitations which are empirical in nature and not *a priori*.

The straight line, so constructed, is one example of what I shall call

[1] Fraser, *op. cit.*, p. 113, refers to the view of Dr Porterfield (*Treatise on the Eye*, vol. II, Edinburgh 1759, pp. 299 ff.) that our judgements of 'the situation and distance of visible objects depend, not on custom and experience, but on original instinct'. But this also falls short of truth. These judgements depend on an 'original instinct' only so far as we use the function of real spatial identification. If that fails, as it is bound to do very often while we are developing it and also with very distant objects, imagination comes in to help and fill in the gaps.

a *self-congruent* structure. Other examples are the plane, the circle, and the sphere. Each of these is congruent to (or can be fitted exactly over) a copy of itself, displaced in certain ways which I shall describe here as forming a corresponding *displacement group*.[1] For instance, the plane may be turned over, and still fits with its copy; but the sphere may only be rotated about its centre. An important fact which the operational construction of a straight line makes clear is that it is impossible to separate the concept of a straight line from the concept of its corresponding displacement group.

The question to be asked is therefore, more explicitly: 'How do we recognize a straight line as straight, or a circle as circular, without any actual testing of it by displacements in relation to physical apparatus?' It would seem that we must in some way copy conceptually the displacements which are needed to construct the figure in actuality. This 'copying' is indeed, as we have seen, just the kind of thing done by the function of spatial-identification when a perceptual-totality is surveyed and various spatial aspects are successively discerned as actual but nevertheless implicit in each other. In the case of a straight line, therefore, it appears that our 'point of view' will undergo displacements exactly analogous to those of the displacement group needed to define it operationally—i.e. relative translations along its length and relative rotations when two points on it are fixed. If, upon making those mentally-conceived displacements, we find the perceptions remaining with a quality of *sameness* (i.e. congruence), we adjudge the line to be straight.

Thus if a line has a kink in it, we may be said to recognize that it is not straight because obviously a translation of a copy of the line along its length would move the kink with it, and the line would not remain *the same*.

Self-Evidence of Axioms and Theorems

The next problem is how to see the truth of the axioms and theorems of Euclidean geometry as self-evident from their *operational* meaning. It would obviously be unsatisfactory if, after having exhibited the concept of the straight line or the circle as an ideal-operational structure recognizable in a certain way by our function of spatial-identification, we depended thereafter on axioms which were plausible but not self-evident from that structure, or if we proceeded to 'prove' theorems by a chain of deductions from such axioms or from theorems which were equally lacking in self-evidence. In any case, if we wish to show in what sense the geometry of the physical world is Euclidean, it is clearly necessary to abandon the axiomatic approach (in which the

[1] The more usual term is Isometry Group.

axioms are arbitrarily laid down) and adopt instead an operational approach which is concerned with what is actually possible.

The principle most fundamental to all operational geometries seems to be that figures can be constructed statically (i.e. remaining the same) and can then be copied, so that corresponding parts can be identified and recognized as the same (i.e. congruent). The recognition of such sameness can only be by a conceptual fitting together of the two figures such as we have seen is possible by abstracting impressions of orientation relative to surrounding objects. Hence Euclid's 'Principle of Superposition' correctly expresses the conceptual analysis of what is involved in congruence, while axiomatic methods of avoiding this principle, such as by introducing the axiom of 'the rigidity of a triangle with a tail',[1] present themselves as mere formalistic tricks begging the question at issue.

Our principle of congruent repeatability is indeed fundamental in operational geometries, not as a mere matter of fact, but from necessity. For if we cannot say whether two figures are the same or different, geometry, as a science of logical relations in space, becomes impossible. Even in Projective Geometry, which does not admit the concept of measure, it is presumed that the structures called 'points' and 'lines', whatever they may be, can be recognized in some way—namely, by the criterion that two 'points' determine a 'line' and two 'lines' determine a 'point', and by such additional (unstated) criteria as will suffice to make these structures fully recognizable. Otherwise this 'geometry' is a mere syntactical system, and does not concern anything definite which can be characterized as 'spatial'.

Various kinds of repeatability have next to be recognized as operationally and therefore conceptually possible, beginning with those that give rise to self-congruent structures such as the plane and the straight line. In the construction of the straight line, as described in Chapter 6, two points on the original may be marked and kept unvaried while the operation of successive approximation to straightness is carried out. This involves rotation, as estimated by the position of a third point. When a third point is similarly marked and kept unvaried while adjustments for the constructing of a plane are carried out (by sliding the copy over or beneath the original), it is clear that we are thereby carrying out *some* of the displacements necessary for the construction of a straight line through the original two points. Hence it is impossible for such straight line to lie at any point outside the plane; for if it did so lie, the displacements necessary for its construction could not all have been carried out. Thus it is self-evident (ideal-operationally) that the straight line joining two points of a plane must lie wholly in it.

The theorems concerning congruence of triangles are nothing more

[1] H. S. M. Coxeter, *Introduction to Geometry*, New York 1961, p. 5.

than verifications that a unique copy of the whole triangle has been made when certain specified features of the original have been copied. The verification depends on the fact that when two points are given, the straight line constructed through them is unique.

The theorem known as 'Pons Asinorum' (the angles at the base of an isosceles triangle are equal) raises the question of symmetry, which is a kind of self-congruence. The test to show the theorem as operationally self-evident would therefore be to copy the triangle in a *turned-over* form and verify that it fits with the original. Coxeter refers to this proof as posing a 'pedagogical difficulty' and being 'merely a short cut'.[1] Operational analysis, however, presents it as the correct 'proof' (i.e. most directly demonstrating self-evidence), while Euclid's construction is required only if we wish to establish something about the axis of symmetry.

Passing over many other theorems which are easily seen to be self-evident, we come to the problem of the Axiom of Parallels. The investigation will be too lengthy and troublesome to be included in the present chapter, and is therefore better deferred to the next chapter, where it can be studied together with arithmetization.

[1] H. S. M. Coxeter, *Introduction to Geometry*, New York 1961, p. 6.

CHAPTER 11

THE LOCAL EUCLIDEAN GEOMETRY OF THE PHYSICAL WORLD: PARALLELS, AND ARITHMETIZATION

'Mathematicians wish to treat matters of intuition mathematically, and make themselves ridiculous, wishing to begin with definitions and then with axioms, which is not the way to proceed in this kind of reasoning.'

Pascal, *Pensées*, I. 1

'There is very little large-scale understanding, even among mathematicians. There are snippets of understanding, and there are snippets of connections between these snippets . . . At the best there is a vague memory of details which have recently been attended to. This succession of details of self-evidence is termed "proof".'

A. N. Whitehead, *Modes of Thought*, p. 64

Two parallel lines constitute a self-congruent figure. We cannot, however, immediately assert that a copy shifted in the direction of their length remains in the position of the originals. For we have first to show that it is *possible* to construct two straight lines, simultaneously or in succession, which are self-congruent in this way; and it is not (at this stage of our analysis) self-evident that the possibility is implied by our operational definition of a single straight line.

One way (fruitless) of attempting to show this possibility is to begin with the figure for the 'Pons Asinorum' and to make the vertex tend to infinity along the axis of symmetry. In the limit we obtain two straight lines which do not intersect on the side towards which the vertex has moved. But we have still to establish that (1) if the triangle were reflected about its base and the vertex tended to infinity in the opposite direction, the same pair of lines would be obtained, or (2) the sum of the interior angles on one side of a transversal is two right-angles, or (3) the figure is self-congruent. Any one of these circumstances implies the others; but none seems self-evident from the definition of a straight-line. The axiom of parallels accordingly seems to import into geometry an entirely new property of straight lines, not involved in their definition, and therefore to some extent arbitrary and dispensable. Indeed, as is well known, consistent axiom systems can be produced by replacing this axiom by an axiom of another kind.

In this illustration, however, the mention of a limiting process to

'infinity' has confused the issue. Contrary to what is sometimes stated, it is not necessary to define a parallel in terms of the possibility of continuation to infinity without the lines meeting. To see this clearly, let us endeavour to construct a square by means of five straight lines perpendicular to each other in succession (figure 1). The crucial question

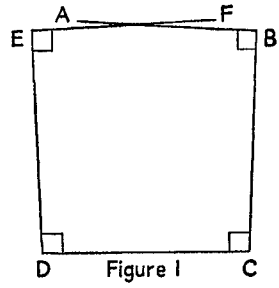


Figure 1

is: 'How can we be sure that the figure "joins up", *AB* and *EF* being the same straight line?' If it does, then clearly the pattern may be repeated indefinitely in either direction of *CD*, producing two straight lines which do not meet however far produced either way, and which are such that, given one of them and the distance apart, the other is uniquely defined.

The difficulty, now isolated without any mention of 'infinity', is therefore that while we feel sure that the square will 'join up' if the straight lines are constructed operationally as we have described (i.e. as self-congruent structures for translation along their length and rotations when two points are kept unvaried in original and copy), we cannot see that the joining up is self-evident, i.e. logically necessary.

Nevertheless, if it is necessary that the square should join up, then the circumstances which make it necessary must be somehow presupposed in our operational construction of straight lines and right-angles. We proceed, therefore, to a more detailed analysis.

The 'Rotation Test'

Consider the construction of a straight line, at a stage before the approximation to straightness has become very close. The object which is to be thought of as carrying the copies unchanged will have an edge which is markedly curved:

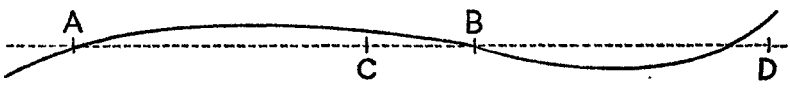


Figure 2

This curve has to be copied (actually or conceptually) on another object, which for convenience may be regarded as the same object or curve translated or rotated in various ways relative to the original. In endeavouring to make the copy coincide with the original after making suitable adjustments in shape, we may carry out two distinct types of operation. First we may rotate the original while keeping A and B fixed, and make repeated adjustments as required to bring the copy more and more nearly into concidence with the original. Then we may extend the portion AB in either direction so that whatever point D is taken (to the right of B, say, as in the above figure), there is a portion CD congruent to AB. We then say that AB has been 'translated' to CD.

The consistency of the two procedures is self-evident; for if ABD is self-congruent for rotations, then *ipso facto* AB is also. The extension to D could indeed be defined in terms of rotation of a curve of wider extent.

It is to be noted, however, that the operation of 'translation', in a more limited and derivative sense of the word (self-congruent displacement of points of the curve *along the curve*) can be carried out without any use of rotation in the case of great-circles on a sphere, if a certain sphere is already constructed: all great-circles are self-congruent in this way, and congruent to each other. Now parallels are not possible on the sphere. The relinquishing of the 'rotation test' has therefore done away with the axiom of parallels; and as soon as we admit the (three-dimensional) rotation test, producing straight lines in the accepted sense of the word, we rule out the possibility of great-circles being taken as 'straight lines', and so, it seems, admit the axiom of parallels. The rotation-test and the axiom of parallels seem therefore in some way equivalent.

The next step in our investigation is to try to make *self-evident* the dependence of the axiom of parallels on the rotation-test, by a detailed examination of what is involved in the latter. Strictly speaking, as already emphasized, the whole three-dimensional rotation and translation group is involved, and no one limited kind of displacement can be considered in isolation once it is decided to use rotations for the construction of single straight lines through any two points in space. But for the purpose of establishing self-evidence, particular figures and displacements may be studied.

What follows is therefore to be regarded as one among many possible illustrative devices for making these conclusions self-evident and thus establishing that a local three-dimensional operational geometry is necessarily Euclidean.

Self-evidence of the 'Joining-up of Rectangles'

Let us first consider the following alternative operational definition (or construction) of a plane. A straight-line OA is constructed by the

rotation-test, and in one of the positions as it is rotated a perpendicular *OP* is erected at *O*. As we rotate this structure about *OA*, the line *OP* describes a surface which clearly possesses all the properties of a plane according to our previous suggested operational definition: (1) it can be 'slid-over' itself by a rotation about a point *O*, and is self-congruent for such displacement; (2) it can be 'turned over' by a reversal of the direction of *OA*; (3) a straight line *OP* can be continued in either sense and still lies in the surface; (4) if two positions *OP*, *OR* be taken, the straight line *PR* lies in the surface (for if not, the surface could not be turned over and remain congruent to itself).

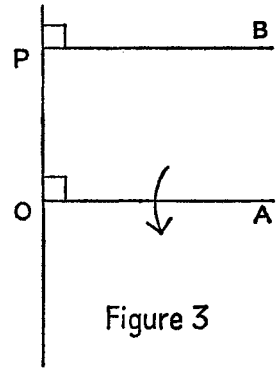


Figure 3

Now by displacing the plane so that *P* concides with the original position of *O* and the new position of *PO* lies along *PO* produced, we conclude that there is a straight line *PB* such that the plane could equally well be generated by the rotation of *PO* about *PB*. Therefore *PR* is perpendicular to *PB*, and we have concluded that at each point *P* of the plane there is a 'normal' *PB* perpendicular to all lines in the plane through *P*.

The rotation-test presupposes that any three-dimensional figure remains the same (i.e. congruent to its original) when rotated while two points *A*,*B* of it are fixed. The straight line operationally constructed by means of this 'test' is then the axis of rotation. In figure 4 we take

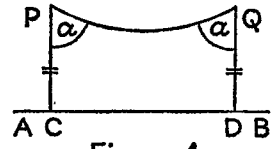


Figure 4

the quadrilateral *CDQP* with *C*,*D* on the axis, *PC* and *QD* at right-angles to it and equal; *PQ* is shown curved to guard against a too hasty

assumption that the angles at P and Q are right-angles by self-evidence. By 'turning over' the quadrilateral $CDQP$ (i.e. rotating about the line midway between CP, DQ) we verify that $\angle CPQ = \angle DQP$.

Next we rotate this figure a certain angle, producing a congruent quadrilateral $CDQ'P'$ (figure 5). We can also regard this whole figure

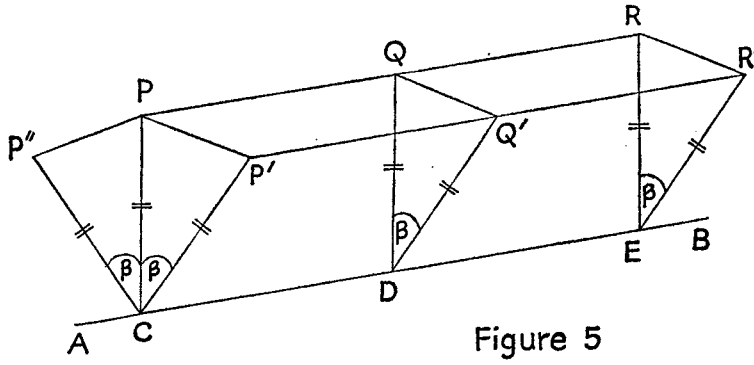


Figure 5

as produced by a rotation of $CDQ'P'$ backwards, i.e. by a clockwise rotation looking from B to A instead of by a clockwise rotation looking from A to B. Hence if $QQ' > PP'$, by merely changing our viewpoint we deduce $PP' > QQ'$. Hence $PP' = QQ'$. This type of argument is admissible since the rotation test is relative, and takes no cognisance of any absolute position in space (this is one of the things meant by saying that the displacements considered constitute a *group*, operating on a certain field but capable of being regarded in the abstract).

The next step is to show that $PQQ'P'$ is a plane. This can be done in various ways, one being as follows. From the congruence of triangles PCP', QDQ', now established, it is clear that we can consistently speak of rotation through a certain angle (β in figure 5). When this angle is a right-angle we have two planes intersecting at right-angles, so that the normal to one at any point on the line of intersection lies in the other. Now rotate the plane $CDQP$ back an angle β to the position $CDQ''P''$. The planes $CDQP$ and $CP'PP''$ are perpendicular; and since it is easily shown (by congruence of triangles) that $P'P''$ is perpendicular to PC and bisected by it, and similarly with $Q'Q''$, we deduce that the three-dimensional figure $P'PCDQ'Q$ is the *reflection* (or 'inside-out' congruence) of $P''PCDQ''Q$. Hence if QP' lies above $Q'P$, then QP'' lies above $Q''P$. But since the reflected figure is also the result of rotating the original backwards, we deduce that if QP' lies above $Q'P$, then $Q''P$ lies above QP''; which is a contradiction. Therefore QP' and $Q'P$ intersect, and $PQQ'P'$ is a plane.

Lastly, construct a congruent repetition of the whole figure by rotating the plane $DERQ$, which is the continuation of the plane $CDQP$, the same angle β. Suppose then that the continuation of the plane $PQQ'P'$ towards B lies above the plane $QRR'Q'$; then by viewing the figure from B instead of A we find that the continuation of plane $QRR'Q'$ lies above the plane $PQQ'P'$. These two results are inconsistent since the planes cross over at their line of intersection QQ' (our figure 5 shows that complete intersecting planes are obtained by rotation of a 'half plane' through angles β, two right-angles, and two right-angles plus β).

Thus $PQQ'P'$ and $QRR'Q'$ are in the same plane, and PQR, being the intersection of two planes, is a straight line. Therefore the two angles adjacent at Q, PQD and RQD, together making a 'straight angle', are each right-angles. So the rectangle $CDQP$ has 'joined up'.

Conceptual Homogeneity and Isotropy

The full setting-out of this argument has necessarily been elaborate, since the required result could not be seen as rigorously true without careful consideration of the group properties of rotations and translations. The substance of the argument, however, may be expressed in a more direct though less precise way by saying that the geometry of our operationally-constructed straight lines must be *homogeneous* and *isotropic*; for the construction in question recognizes no absoluteness of position or orientation. A quick glance at figure 5, bearing this in mind, assures us that $PP' = QQ' = RR'$ and that $PQRR'Q'P'$ is plane. It thus becomes immediately self-evident that PQR is a straight line, and that the space of the plane $CDQP$ extended in all directions can be mapped out in rectangles. And this is the necessary and sufficient condition for Euclidean geometry.

The homogeneity and isotropy we are speaking of here may be called *conceptual*, because it has no reference to anything physical. The space is homogeneous not because it is physically empty or because the distribution of mass is uniform. It is homogeneous, one can perhaps say, because it is the *means* of measurement and not any *object* measured or any measurement resulting. This does not imply, however, that these rotations and translations are *merely* conceptual, without any actuality. For they are actualized in things just as the natural numbers are actualized in things. We can see two objects in dream, and then three, making a total of five. So also, in dream, clairvoyance, or out of the body, we can see a table and recognize that its top is plane, and that its four edges are straight lines meeting at right-angles.

In asserting the all-pervasiveness of Euclidean geometry, therefore, we are most certainly *not* implying any 'container' theory of space, with absoluteness of position and infinity of extent with no room, as it were,

for any kind of space other than the physical. We have merely been analysing our concepts of spatial form; and the conclusion is that wherever and in whatever state of consciousness we recognize spatial form perceptually and conceptually, the same conceptual analysis holds. Even if we are conscious in two spaces at once, as in most clairvoyance, each space is Euclidean—neither infinite nor finite in actuality, but presented according to the occasion.

ARITHMETIZATION

The exact correspondence which holds between ideal spatial structures and arithmetical operations or formulae is of fundamental importance for the following chief reasons:

1. It enables us to clarify in our minds the concepts and conceptual operations in space, particularly those concerning rotation and translation groups, by dealing with the corresponding arithmetical operations.
2. We can assure ourselves of the consistency of the various spatial operations we have introduced, because no one can seriously doubt the consistency of arithmetical operations.
3. Where certain characteristic spatial (or spatio-temporal) forms appear in nature, as with light-waves, we are nearly always totally unable to carry out the construction *actually* (full-scale, or in a model). But arithmetization allows us to calculate the result, even though nature may have effected the construction, as it were, instantaneously.

What follows is a very brief outline of the subject, covering essentials only.

Addition, and the Euclidean Point

If a segment AB of a straight line is repeated ideally by translation along the line or congruent copying so as to give segments BC,CD, etc., the points A, B, C . . . , may be numbered 0, 1, 2, . . . , just as if we were counting them; displacements to the left are consistently indicated by the negative sign. Subdivision of the segment AB into equal parts gives the rational fractions. Each number so found is considered to correspond to a precise *point* on the ideal line because a well-defined operation exists for finding it. Even if the numbers were to differ by 10^{-15}, AB being 1 cm, the corresponding points must be regarded as different. Otherwise we should fall into error when the picture is magnified 10^{15} times. We can likewise conclude that a *section* of the rational numbers (by taking, for instance, those whose squares are greater than 2, on the one hand, and those whose squares are less

than 2) corresponds to a precise point on the ideal line because a well-defined operation exists ideally.

Arithmetically there is a difference in *kind* between the operation to determine $\frac{3}{4}$, say, and the operation to determine $\sqrt{2}$. But in terms of spatial form each operation involves a process of successive approximation. The difference is merely that $\frac{3}{4}$ can be constructed by a primitive adjustment of congruence, whereas $\sqrt{2}$ requires the construction of right-angles, circles, or something more elaborate still.

We have to recognize that there is a large number of kinds of arithmetical operations determining real numbers, and a large number of kinds of geometrical operations determining points. Any real number has an unlimited number of formulations which are conceptually equivalent (e.g. $\frac{3}{4} = \frac{6}{8} = 0{\cdot}75 = 0{\cdot}74\dot{9} = \sin^2(\pi/3)$). Some are simpler, or more manageable arithmetically, than others. Corresponding to these different formulations there may be different constructions which fix the same point in a straight line in relation to a given zero and unit length. The point itself may be regarded as a single position (because the operations determining it are equivalent); but no operation, for number or point, is uniquely 'correct'.

The 'Euclidean Point'

Certain authors, taking a psychological or physical view, have argued that the 'Euclidean point' is an inadmissible abstraction, without any possible existence in actuality. There is always some *blur* or *uncertainty*. Similarly with time, there must be a *specious present* of finite extent, so that it is meaningless to talk of precise instants of time.

The terms *blur* and *uncertainty* are presumably intended to express something about which we can talk intelligibly, not a mere emotional state of vagueness or confusion. Accordingly, they must have a definite conceptual structure, which we must clearly analyse if we are to engage in profitable discussion. What, then, is it that is *blurred* or *uncertain*? What is the extent of the *blur* or *uncertainty*? By what means are we to calculate that extent?

It seems that the only possible answer is to the following effect: *Position*, *timing* and all other kinds of *measure* are 'blurred' or 'uncertain' in so far as there is not, and cannot be, an exact fit between conceptual measures (which are logically precise) and experimental ones (which involve practical limitations beyond every prescribed conceptual system). Now 'position' and 'measure' imply a metrical space, with all the (conceptual) properties which we have been investigating. And indeed, when we examine how the magnitudes of the *nuclear range* and *chronon* are calculated (from wave-equations, using space and time derivatives, velocity of light-waves visualized as crossing the nucleus, etc.) it becomes obvious that the whole apparatus of ideal-operational

Euclidean geometry, including the axioms of order and continuity, is presupposed.[1]

Multiplication

Let us suppose we have in some way constructed approximations to right-angles and rigid straight-edges with equal graduations, and with their help construct a rectangle 53 units by 129. We know then that a subsequent dividing of the rectangle into rows and columns of unit squares *must* yield a total of 6837, this being the number we find by the process of 'long multiplication'. The reason why this must be so, by self-evidence, can be seen by taking a simpler case. The product of 3 and 4 is not *merely* the figure that we read off from a multiplication table. The tables are drawn up from consideration of the primitive meaning of 3 times 4, which is '3 rows of four objects in line' (or 4 rows of 3, the result being the same by the rotational properties of spatial form). We see then that '3 times 4' is a definite number, found to be 12 by counting, and then learnt in a table of similar results. By deciding to use the notation 12, we have adopted a method of counting 'in batches', and in more elaborate examples such as 13 times 4 all we have to learn is a technique for dealing with the various 'batches' efficiently. The self-evidence of the laws of arithmetic thus reduces to the self-evidence of the principles of spatial grouping and regrouping of congruent objects.

The feature to be specially remarked in this example is that the product 6837 is *potentially created* (as we may perhaps say) as soon as we have outlined the rectangle, before we have divided it up into unit squares. We could say therefore that Nature has made the calculation implicitly and instantaneously because of the conceptual properties of straight lines, right-angles and planes, man having merely to put these separate structures together. Nature has acted as an *instantaneous arithmetical computer*. From another viewpoint, however, the calculation is never made by Nature. Man has made use of the conceptual properties of space by means of which Nature permits him to actualize particular structures approximatively, and the possibility of a corresponding calculation is thus implied.

In this illustration the computation seems to be exact because we have ignored the fact that the constituent squares are only roughly the same shape and size and have counted them as if they were the same

[1] On the 'specious present', see Chapter 19 below, p. 297. Also Whitrow, *op. cit.*, pp. 78 ff, 309, 312. On physical objections to the identifying of world-points by precise coordinates, see the discussion on a paper by E. Wigner, *Jubilee of Relativity Theory*, Basel 1956, pp. 224 ff. (Remarks by van Dantzig, F. Hoyle, M. Born and P. G. Bergmann). The subject of psychological, physiological and physical obstacles to the precise observing of positions is further discussed in Chapter 17.

exactly. Accurate measurement, however, involves the repeated subdivision and application of exactly congruent figures. This is so, whether we are concerned with the simple measurement of a given length or with more elaborate results like that of Pythagoras's Theorem. Hence the verification in nature, in such cases, can never be exact.

Our conclusion, therefore, is that if two *rational* numbers $\frac{p}{q}$ and $\frac{r}{s}$ are to be multiplied, and the rectangle with these magnitudes as sides is subdivided into squares of side $\frac{1}{qs}$, which are then counted, the product will be found exactly. But if either the subdivisions are not actually made and counted, or the numbers are irrational, we must say that the product is only *potentially* what would be found by the laws of arithmetic, and that if the product were found by some other construction the two results could not be expected to agree exactly. But they would agree more and more closely, the more accurately we implanted the conceptual constructions into the world.

Implicit Fine-Structure

This is an appropriate place to introduce a principle of fundamental importance, which I shall call the Principle of **Implicit Fine-Structure.** The example just discussed may be taken as a particularly simple illustration of this principle. A general statement might be as follows: Certain macroscopic structures may come into actuality, or be voluntarily actualized, in any sphere of conscious life (the physical world, thought, dream, clairvoyance, out-of-the-body, mystical levels), in such a way that the whole and each of its macroscopic parts is clearly recognized. But there will always remain, in the cognitive nature of each part and of the whole, an inexhaustible *fine-structure*, the finding of which provides, as we say, the *explanation* of the macroscopic observations.

Applied to Psychology, this principle might be taken as providing a definition of 'the unconscious'. For example, we recognize the vowel-sounds when we hear them because, unconsciously, we hear the various harmonics and formants which explain their different characters. In terms of our Principle, this means that there is a fine-structure in vowel-sounds, capable of being discerned in certain conditions. And when we do so discern them, we recognize that this fine-structure is the 'explanation' of their quality.

A certain form of the 'doctrine of the determinative whole',[1] in Gestalt Psychology, is also included in the Principle. The *whole* is to be regarded as having teleological priority over the parts (i.e. the fine-

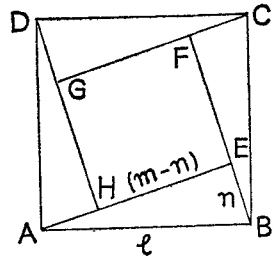
[1] Allport, *op. cit.*, pp. 113, 141.

structure). For we freely construct the figure as a whole, and the fine-structure of the whole is then prescribed by the conceptual fine-structure of the separate parts when unified by us or by nature in a recognizable configuration.

Pythagoras's Theorem

It is of special interest to see how a particular 'fine-structure' in the field of geometry—namely, Pythagoras's Theorem—arises inevitably, as the principle asserts, because of the conceptual character of the separate constructive parts.

Given two sides of a right-angled triangle, the third side evidently has determinate length. The problem is to find what is the arithmetical formula relating its length to the lengths of the two given sides.



In the figure, the sides containing the right-angle are marked as of lengths m and n, and the hypotenuse AB is of length l.

$$\begin{aligned} \text{Evidently } l^2 &= (m-n)^2 + 4(\tfrac{1}{2}mn) \\ &= (m^2 - 2mn + n^2) + 2mn \\ &= m^2 + n^2 \end{aligned}$$

Consistency

The consistency of the system of translations and rotations which characterizes Euclidean geometry now follows from the following facts:

1. Points are in one-one correspondence with coordinate-triads x^i ($i = 1, 2, 3$).

2. Planes are in one-one correspondence with coefficient-tetrads (A_1, A_2, A_3, A_4), where not all of A_1, A_2, A_3 are zero and the sum of their squares is 1.

3. Translations are in one-one correspondence with coordinate transformations $\bar{x}^i = x^i + a^i$, forming a group.

4. Rotations are in one-one correspondence with coordinate transformations $\bar{x}^i = x^j\gamma^i_j$, where the orthogonal matrices γ^i_j form a group,

with combinatory operation that of matrix multiplication $\gamma^i_j \beta^j_k$ (summed for $j = 1, 2, 3$).
The compound group of translations and rotations is given by $\bar{x}^i = x^j \gamma^i_j + a^i$ (summed for $j = 1, 2, 3$) and the group property ensures that however we impose transformation on transformation, or reverse transformations, the total result is a uniquely determined translation and rotation of the group.

This formalization of translations and rotations also makes clear what may be called their *dual function*, subjective and objective. We may regard the *field of objects* as remaining the same, while the axes (orientation of observer) shift. This gives us the various *spatial-aspects* of a given field of objects. Or we may regard the observer as fixed, while the objects are *repeated* or *multiplied* in any part of the field and in any required orientation with respect to the observer. This gives us the multiplicity property of a space-field, whereby the *same* object (as regards its abstract form) may appear in various positions and orientations. We conclude therefore that this *subjective* and this *objective* are inseparable.

CHAPTER 12

CLASSIFICATION OF GEOMETRIES

> 'Undoubtedly the most striking feature in the development of geometry during the last 2000 years is the continued expansion of the concept of "geometric object".'
>
> Karl Menger, *Theory of Relativity and Geometry* (Albert Einstein, ed. Schilpp, p. 466)

What is 'geometry'? Probably no scientific term has such varied significations as the term 'geometry', yet, deceptively, we seem to know at once what it means. Correspondingly there will be varied significations of geometrical terms such as *straight line, Euclidean, distance, triangle, curvature*, in fact of almost every term in use.

The extent of the mathematical and philosophical confusion that has thus arisen can be estimated only when we struggle to make the numerous distinctions clear in our minds—by no means a simple task when the presuppositions and techniques in one kind of geometry seem to be in a certain way embodied or utilized in several other kinds of geometry and vice versa, thus continually shifting in meaning.

In the previous chapters an attempt has been made to bring some important distinctions to light. The attempt is continued more systematically in the present chapter.

The Myth of 'Physical' Geometry

We begin with the consideration of a certain fundamental difference of outlook on systems of measurement, or, as we may say, of *applied* geometry. According to the one view, applied geometry must be 'prescribed'; according to the other it is 'physical', as a table of properties of substances (density, index of refraction, etc.) is 'physical'. Various shades and combinations of opinion are found, and it will be best to illustrate the broad differences by two contrasting sets of quotations.

Taking proponents of 'physical' geometry first, these may be said to hold to a Machian or positivistic view. For them, it seems, applied geometry begins with an orderly catalogue of data, so to speak, read off from the physical world with the help of standard pieces of apparatus. The geometry itself then consists of relations discovered between these data and presented in generalized form. I now give six quotations showing this trend of thought. The first two, though likening geometry

to the empirical science of mechanics, are cautious in their phraseology; but the other four assert an unyieldingly 'physical' view of geometry:

'Therefore geometry is founded on mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of measuring.' (Newton, Introduction to *Principia*.)

'I am coming more and more to the conviction that the necessity of our geometry cannot be demonstrated, at least neither by, nor for, the human intellect. Perhaps in some other life we may arrive at other insights into the nature of space that are at present inaccessible to us. Until such time geometry should be ranked, not with arithmetic, which is purely aprioristic, but with mechanics.' (Gauss, Letter to Olbers, 1817.)

'Let us agree that by *length* and *distance* you always mean a quantity arrived at by measurements with mathematical and optical appliances. You have studied experimentally the laws obeyed by these *measured lengths*, and have found the geometry to which they conform.' (Eddington, *Space, Time and Gravitation*, p. 2.)

'The concept "equal length" is defined by reference to a physical object, a solid rod, whose transport lays down equal distances.' (Reichenbach, article in *Albert Einstein: Philosopher-Scientist*, p. 294.)

'The only possible attitude to this question (whether astronomical space is Euclidean) is that it is entirely for experiment to decide, and that we have no right to form any prescribed notion whatever.' (Bridgman, *The Logic of Modern Physics*, p. 68.)

'The discoveries of the experimental physicists . . . are formulated in laws which are in no sense imposed by the mind of the discoverer.' (Stebbing, *Philosophy and the Physicists*, p. 209.)

I suggest that in all these opinions one can detect the error of *psychologism*, verging in some cases on that extreme psychologism which I have called the *abacus fallacy*. They assume that an *a priori* exact and universally applicable conceptual system can be determined merely by examining the tabulated results of empirically uncertain observations by means of 'tools' whose manner of prefabrication is irrelevant. Or at least they confuse the 'order of approach' with the 'order of reality'. It is true that our minds first become open to the geometrical concepts of straightness and measure by manipulating graduated straight-edges. But no graduated straight-edges could exist and be intelligible to us, and no measures

for distant objects could be found, if there were not an elaborate conceptual system which we have made use of to construct and manipulate such physical objects and to determine measures by their means. We begin to perceive something of this unconsciously prescribed system only when we set to work to analyse intuitively what it is that we have been doing.

As this analysis deepens we are led to take quite other views of geometry than the 'physical' one; for example:

'This *objective* world is of necessity relative; it can be represented by definite things (numbers or other symbols) only after a system of co-ordinates has been arbitrarily carried into the world.' (Weyl, *Philosophy of Mathematics and Natural Science*, p. 116.)

'The space-time frame is therefore something overlaid by the observer on the external world. . . . We must endeavour not to lose sight of its ficitious and arbitrary nature.' (Eddington, *The Mathematical Theory of Relativity*, pp. 8–9.)

'The axiomatic basis of physics cannot be abstracted from experience.' (Einstein, *The World as I see It*, p. 36.)

'Measurements presuppose theories.' (Popper, *British Philosophy in the Mid-Century*, ed. Mace, p. 189.)

'Taken alone, the statement that a certain geometry holds for space is therefore meaningless.' (Reichenbach, *The Philosophy of Space and Time*, p. 35.)

It would be absurd, of course, to claim that *everything* concerning measure is put into the physical world by man. When we see the sun shining through gaps in a cloud and lighting up straight paths through the rain falling from it, the straightness is given in nature and is immediately recognizable as such. But when we wish to *place* these straight lines in relation to ourselves, allowing possibly for refraction (and, in principle, also for aberration, velocity of light, elasticity of measuring apparatus, etc.), our findings are not by any means uniquely given in nature. Perhaps we could say that in a certain generalized sense of the word the 'position' of the lines must be uniquely given in nature, because they are where they are, and nothing we can do can alter their 'position'. But in the process of converting what is thus *constructively* given into something which can be conceptually handled by the symbolic tools of mathematics and physics, we have the liberty to choose any one of innumerable different procedures. Possibly one of these may be

epistemologically preferable, as being closest to the presuppositions which will be inevitably involved in an ultimately complete and all-discerning investigation. But even in those very few specialized circumstances where we feel that most of those presuppositions have come to light, it remains far more true to say that we have applied a prescribed geometry (whether prescribed by us or by nature is of little concern) to nature in order to measure it, than to say that the measures are read off from nature as uniquely *given*.

This discussion is intended to clear the air, so that in future we may consider a 'geometry' to be always some *prescribed* conceptual system.

Characterization of 'Geometry' in General

Although it it impossible to define the meaning of the term 'geometry' because its uses are too varied, one may attempt to indicate broadly what are the features of certain conceptual systems which make us feel that 'geometry' would be a suitable name for them, whereas other conceptual systems would not suitably be named 'geometry'. Opinions may differ in matters of detail, but possibly there would be general agreement that a 'geometry' should concern itself with:

(1) A field of 'objects' in 2, 3, 4 . . . 'dimensions'. The 'objects' are vaguely thought of as 'lines', 'surfaces', points on lines, points on surfaces, etc.; but it is sufficient if the 'objects' are symbols or sets of symbols, to which we attach, by a *general visualizing or space-imagining power*, ideas of varied position, direction and extent, without any consideration of measure or operational possibility. Even the logical coherence of what we vaguely imagine in this way does not come in question, this question only arising when the 'geometry' is fully formalized. In the same way a 'dimension' suggests vaguely a different 'direction' from those in the 'space' or 'dimensions' already considered. Hence one may imaginatively pile one dimension on another, as new sets of objects are brought into consideration, without any regard for the operational possibility of the new dimensions in terms of space, time or other manifestations.

(2) The composition of 'objects' of certain primitive types into larger structures, according to principles laid down.

(3) Multiplicity, i.e. the repetition of structures, otherwise to be regarded as the *same*, in different parts of the 'space', together with relationships (called *transformations*) between each structure and any repetition of it. The features which are taken as establishing the *sameness* of structures are at our free choice, and may be few or many.

In further clarification we need to add that in nearly all geometrical discussions the *general visualizing or space-imagining power* may become

so general and abstract that it recedes into the subconscious and we deal instead, consciously, with the formalism or 'arithmetization'.

It is clear that the process which gives us varied 'geometries' is one of abstraction, analogy, and generalization. The vagueness which necessarily goes with this abstraction has been remarked on by Borel in his comparison between Geometry of Position and Metric Geometry: 'Geometry of position embraces all those properties which can be described by means of a roughly approximate drawing... but even from the beginning a metrical element enters into it, that is to say, a certain notion, more or less vague, of the relative value of distance.'[1]

By 'metric geometry' Borel seems to mean an *operational* geometry characterized by the attaching of the name 'distance' to some specific operation. This is hardly the most usual meaning of the term 'metric geometry'. Moreover, we shall find it inconvenient and confusing to include all geometries not 'metrical' in this sense under the title 'geometry of position'. Quite a different classification will therefore be adopted here.

Complete and Incomplete Geometries

The term *complete* is to be taken as synonymous with *operational* (ideal or actual) in the sense explained at the beginning of Chapter 10. In an *incomplete* or non-operational geometry we are unable to exhibit anything definite corresponding to the symbols and operations of the formalism yet distinct from them. But we nevertheless depend on our 'general space-imagining power' to give us the impression that something of 'spatial' character is in fact represented by the formalism. This impression then assists and guides our thought in manipulating the formalism. In a *complete* geometry, on the other hand, we plainly exhibit the spatial concepts and operations with which the geometry is concerned and their corresponding symbolic formulations, if any.

A definition of *arithmetized actual-operational geometry* was given in Chapter 7. The epithet *actual* was inserted to indicate that the operations in question are being applied to physical objects. If we regard the geometry as, in itself, a prescribed conceptual system, the application to physical objects merely has the function of convincing us that the concepts are clearly seen. This conceptual aid may therefore be abstracted, and the definition becomes one of arithmetized ideal-operational geometry. Arithmetization also, as we have seen, is a device for helping us to clarify spatial concepts and to determine the ideal results of measurement without carrying out any construction; so this also may be abstracted. After the double abstraction, the same definition stands as one of ideal-operational geometry, simply. To describe this I shall

[1] *Space and Time*, London 1926, pp. 39–40.

usually prefer, in the present context, the term *complete*, because of the variable meanings which are attached to the term *operation*.

I shall also assume that a complete geometry has the following characteristics which are indicated in Definition 1 (p. 93) and remain after the arithmetization has been abstracted:

(1) There is a local Galilean (or Euclidean 1, 2 or 3 dimensional) frame for each given observer; and

(2) there is a correlation of magnitudes of local intervals (or distances) as between one locality and another.

Although a kind of geometry might be said to exist if the characteristic (2) were dropped, it hardly seems appropriate to call such geometry 'complete'.

It will now be convenient to present a classification of prescribed geometries in the form of a table. The terms and new distinctions introduced can then be explained and discussed in due order.

TABLE OF PRESCRIBED GEOMETRIES

Complete	Universal	Euclidean		CA
		Complete Homogeneous Non-Euclidean		CA
		Specific Riemannian (1)		A
	Occasional Specific Riemannian (2)			A
Incomplete	Riemannian		Metric	A
	Homogeneous Non-Euclidean		Metric	A
	Non-Symmetrical Unified			A
	Five- or Six-dimensional Unified			A
	Affine (general)			A
	Affine (classical)			CA
	Projective			CA
	Merely Topological			C

Universal and Occasional Geometries

Let us now return to the consideration of complete geometries when the arithmetization has been abstracted. It will be seen that there are then, for any given pair of observers making observations at particular times, two distinct local Galilean geometries, but *no spatial or spatio-temporal representation of what lies between*, unless there is some spatial or spatio-temporal *construction* linking the two localities in some representation common to both. Let us first ignore the possibility of such common representation. The corresponding philosophical view of experience is then that wherever there is an observer a corresponding manifestation is space and time springs forth; but where there is no

observer no definite manifestation in space and time can be specified. The world, we may say, is manifested *upon occasion.*

It would be a false criticism to say that this 'occasional' theory compels us to say that an object ceases to exist when there is no one to observe it. The manifestations for all possible observers must be regarded as potentially determinate; but it is clearly impossible to specify which shall become actual unless the full circumstances of some particular observer (his motion, direction of vision, condition of eyesight, etc.) are given.[1]

There is, however, another objection to occasional geometries, namely, that the links between one observer and another, and between the manifestations for one and the manifestations for another, are purely arithmetical. Now arithmetical computations require time for their successive operations, unless they are done automatically by means of a construction (as we have seen is the case in Euclidean geometry). Hence it seems clear that in nature some actual construction, such as idealized electromagnetic or gravitational waves, must actually relate the manifestations for the one observer to those for another.

But these constructions must obviously in some way transcend each particular local representation in space and time, expressing, as it were, a potentiality for all. Questions concerning such transcendental potentialities are beyond the scope of this chapter. On the other hand, we may reasonably take the view that the connections between the various local geometries *ought* to be represented in some way, even if there is no uniquely right way for all observers. And there is, in fact, a way of representing such connections, namely, by choosing a *preferred family of observers* filling all space (an 'observer gas'), of such a kind that the various local maps (somewhat distorted in size or shape, if unavoidable) *join together* in a universal representation of the world at a given 'cosmical time'. The total techniques by which this is done can then be described as constituting a *universal geometry.*

The symbols in the last column of our table may now be explained. The symbol A, standing by itself (as for occasional geometries) means that arithmetization is essential. The symbol C means that the geometry can be conceived of in purely constructive form (i.e. without arithmetization; as, for instance, Euclidean geometry). The combined symbol CA (as for universal geometries) means that arithmetization is possible, but not essential.

Though occasional and universal geometries are thus sharply distinguished, the corresponding occasional and universal views of experience are perfectly compatible. We have only to note the fact that the universal geometry must change if we choose a different 'observer gas'.

[1] A thorough discussion of 'manifestation upon occasion' is deferred to Ch. 14, esp. pp. 211 ff.

The only questions undecided are (1) what kinds of preference are there for one observer gas over another, and (2) is there any particular observer gas, and corresponding universal geometry, to which we must allow some *absolute* kind of preference, so that the resulting 'representation of the world' can be said to be, to that extent, the *correct* one.

The subject of universal geometries is dealt with in the next chapter, and will be illustrated later in special problems. In the meantime a few further general remarks seem called for.

The technique of arithmetization for all complete geometries is, in general, that of Riemannian geometry,[1] which, in itself (before being made operational), in incomplete. In the special case of Euclidean geometry the techniques become specially simple and may be presented in different forms of 'algebraic geometry'.

The various Riemannian geometries that can be made operational must be *prescribed* (in the sense of Chapter 7). If no measure determining the geometry is to be correlated with physical conditions in the manner there explained, the geometry will be Euclidean in three dimensions or Galilean (flat space-time) in four dimensions. If *one* measure is to be correlated with physical conditions it could be, among other possibilities, (1) the Gaussian curvature, uniform throughout the space; in which case the geometry will be the so-called 'absolute' (homogeneous) non-Euclidean geometries of Gauss, Bolyai and Lobachevsky; or (2) the 'mass' of the central particle in the Schwarzschild problem, i.e. gravitational field with spherical symmetry. In such cases the geometry is called here 'specific Riemannian', and will be of type 1 if associated with a *universal* representation, or type 2 if it is interpreted as *occasional*.

Classification of Incomplete Geometries

The most elaborate of modern geometries are the numerous Unified Field Theories, designed to include in one and the same formalism both gravitational and electromagnetic equations, and sometimes quantum mechanical formulas as well. They utilize tensor operations made familiar in Riemannian geometry, but introduce additional techniques suggested by Affine Geometry and other fields of mathematics.

At the other extreme come geometries from which the qualities required to make them operational (as embodied in the Euclidean Axioms of Incidence, Congruence, Parallels, Order and Continuity)

[1] Riemannian geometry is a geometry determined by a formula giving the separation (ds) of two neighbouring points in terms of the coordinate increments dx^1, dx^2, etc., thus: $ds^2 = g_{ij}dx^i dx^j$, where summation for $i = 1 \rightarrow n$, $j = 1 \rightarrow n$, is understood, and the multipliers g_{ij} vary with the position. This formula is the result of Pythagoras's Theorem being true in each separate locality, although the space may not be Euclidean as a whole. The geometry of a sphere is Riemannian.

have been successively abstracted. This kind of generalization leaves them operationally vague, referring not to anything definite such as a Euclidean straight line but rather to *classes* of structures imagined as possessing certain general properties (such as the ability to intersect with others in a 'point').

A different kind of generalization, the reverse of abstraction, results in Riemannian geometry. This may be described as the *analogical* extension, to any number of dimensions, of the geometry of surfaces in Euclidean space, using a coordinate network on the surface (therefore not referring to any points outside). Hence it imports into the formalism of those higher dimensions certain techniques analogous to the determination of properties of curvature in surfaces which are constructed in Euclidean space. But these techniques have, initially, *no operational significance* apart from the formalism. To give them operational significance we should first have to show how the coordinate network is set up; then the coordinates, at least, have definite meaning. Afterwards we can consider whether there are any physical properties which we can measure with the help of the new mathematical concepts of 'curvature', 'geodesics', etc., so as to make these concepts operational in the spatio-temporal sense.

Further discussion of the operational implementation of incomplete geometries will require a chapter to itself. In the meantime, to conclude this brief survey, a few remarks on Affine Geometry are called for because of the prominent part it plays in unified field theories such as those of Weyl, Eddington and Schrödinger, and in non-symmetric field theory.

Classical affine geometry is 'geometry' in the broad sense explained above, being concerned with a 'field of objects', vaguely presented to the imagination as if in space, but otherwise determined by an axiom system which can be regarded as an abstraction, generalization or modification of the Euclidean system. The peculiarity of this geometry is that distances in different directions cannot be compared. The 'lines' are not, of course, Euclidean straight lines (which satisfy all the axioms of Euclidean geometry); nevertheless, through each point there is a unique 'parallel' to a given line, so that the 'direction' of a line at any one point can be taken as definitely either the same as or different from the direction of a line at another point. Given any direction, equal lengths can be laid down along a line in that direction according to some standard or unit; but the standards for different directions cannot be compared.

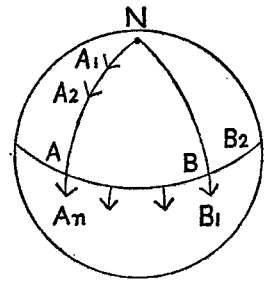
It is as if configurations were marked on some material substance subject to homogeneous strain (which deforms spheres into ellipsoids and rectangles into parallelograms) and we are unable to control, measure or allow for the deformation. Hence in this geometry a sphere and an ellipsoid cannot be distinguished. Likewise we cannot make any

distinctions between a rectangle and a parallogram, or between two shapes of a parallelogram unless the respective directions of the sides are the same in each case. For if right-angles could be determined, as well as parallels, we should have Euclidean geometry, and comparison of lengths in different directions would be possible.

The theorems which are true in classical Affine Geometry are those which hold of a figure regardless of how it is deformed in homogeneous strain. This fact gives it a certain amount of practical interest in spite of its obvious incompleteness.

General Affine Geometry

General affine geometry embodies a broader conception of parallels, such as appears in the geometry of the curved surface of a sphere. In brief, there is no unique parallel when the 'lines' are well separated, but a parallel becomes more and more accurately determinable, so to speak, as the lines approach each other. This, however, is an obscure and paradoxical kind of statement, since in Affine Geometry as such (before being operationally interpreted) there is no measure in the usual sense of the term. To make things clear we must have recourse to an illustration and to a brief account of the formalism.



Consider a right-angled equilateral spherical triangle NAB, N being the north pole and AB part of the equator. We can transfer an infinitesimal vector NA_1 in its own direction to A_1A_2 and so eventually to AA_n. Passing then by parallel displacement along AB we arrive at BB_1. But by successive parallel displacement along NB instead of along NA and AB we arrive at B_2B, which is perpendicular to BB_1 for this triangle. Thus there is no unique parallel at B to NA_1 at N.

In Riemannian geometry the angular separation of BB_1 and BB_2 is measurable. But now we generalize these properties to all kinds of surface, in any number of dimensions, and *abstract the concept of measure*, except so far as the infinitesimal parallel displacement of a vector may be said to keep it 'the same'. What results is general affine geometry. We can carry a vector along a path always in its own direction

(as NA_1 along NA in the figure), making off equal 'steps'. But if the path joins up we cannot say that the 'steps' at the end will be the same as those at the beginning; for the vector at the end will not in general coincide in direction with what it was at the beginning, so there is no comparison possible. Also, the choice of the vector to displace is quite arbitrary and there is no standard of comparison as between the 'steps' on one path and those on another.

In the formalism a set of n^3 functions Γ^i_{jk} of 'points' $(x^1, x^2 \ldots x^n)$ is supposed given. If then the vector A^i undergoes a parallel displacement so as to become $A^i + dA^i$ when the point is displaced to $x^i + dx^i$, the rule is that $dA^i + \Gamma^i_{jk} A^j dx^k = 0$. For consistency, the functions Γ^i_{jk} must obey a certain transformation law when we change from one system of coordinates x^i to another system $\bar{x}^i$. Equations for 'curves' along which a vector is carried in its own 'direction' can then, in theory, be determined.

From this brief outline it can well be imagined that the operational application of general affine geometry is obscure in the extreme. At some stage in the working out of the formalism a metric $ds^2 = g_{ij}dx^i dx^j$ must be introduced, because only by measure can any kind of physical test be imposed. This brings another concept of parallelism into the geometry, incompatible with the first—actually a minor consideration, because the first kind of 'parallelism' appears to have no spatial significance, as such, in these theories.

What is more important is that no attempt is or can be made in general to set up the coordinate system or to determine either the functions g_{ij} or the functions Γ^i_{jk}. For the field equations are differential ones; and even if we knew how to set up the coordinate system in a way common to all observers, we are unable to determine boundary conditions for their solution.

The object of such theories seems to be to build the familiar local equations of electromagnetic theory and Einstein's law of gravitation (both of which are checked by ordinary measurements in Euclidean space) into a coherent formalism. The fact that this synthesis can be done in very many different ways, quite elegant perhaps but never quite satisfactory, is surely an indication of the enormous resources of mathematics and the comparative ease of interlocking different physical theories, rather than of any essential correspondence with the hidden structure of nature. It seems at least very unsatisfactory that in these theories the only links with experience are in familiar equations framed in terms of Euclidean geometry, while the whole of the substructure of the theories (the coordinates, the functions, the transformations) consists of uninterpreted symbols, with no indication how they could be numerically specified.

CHAPTER 13

THE OPERATIONAL IMPLEMENTATION OF FORMALLY NON-EUCLIDEAN GEOMETRIES

'The concept does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled in an actual case.'

A. Einstein, *Relativity, The Special and the General Theory*, London 1920, p. 26

'The relativity of geometry is a consequence of the fact that different geometries can be represented on one another by a one-to-one correspondence. For certain geometrical systems, however, the representation will not be continuous throughout, and there will result singularities in individual points and lines.'

H. Reichenbach, *The Philosophical Significance of the Theory of Relativity*, essay in 'Albert Einstein: Philosopher-Scientist', ed. P. A. Schilpp.

An incomplete geometry—one which is not fully operational in the ideal-spatial sense—is in some respects an abstraction and generalization. As a *geometry*, it purports to be concerned with space-concepts. We may presume also that it is precisely expressed in symbols or words related to each other by axioms and theorems; otherwise it would be nothing definite. But these symbols or words and the mathematical operations referring to them will remain a formalism capable of being interpreted spatially in many different ways, unless we attach some particular ideal-spatial interpretation which removes the abstraction and generality.

Thus a metrical geometry determined by $ds^2 = dx^2 + dy^2$ is customarily described as Euclidean. But the theorems thence obtained, concerning formulas which we speak of as denoting 'straight-lines', 'right-angles', 'parallels', and so on, will not in fact be about any such things unless the coordinate network for x and y is set up so that the coordinate lines are Euclidean straight lines in the full operational sense (implying all the theorems of Euclidean geometry) and unless the units for measuring x and y are equal, and distance, in the Euclidean sense of the term, is measured by ds using the same units as for x and y. Without an ideal-spatial interpretation of this or some other kind, the geometry remains incomplete.

Some incomplete geometries are, by themselves, too abstract and general to be completed by the mere addition of ideal-operational rules of application. Others can be completed with the help of quasi-physical concepts such as idealized light-rays and atomic frequencies accredited with the properties which the formalism requires; the resulting ideal-operational geometry is then made approximatively actual by means of experiments which may be actually carried out. If the incomplete geometry is Riemannian, with $ds^2 = g_{ij}dx^i dx^j$, I shall speak of it as *formally Euclidean* or *formally non-Euclidean* according as the metric is or is not reducible to the form $\delta_{ij}dx^i dx^j$. It will be *representatively Euclidean* if there are rules for the construction of figures representing, on a Euclidean map, structures and formulas derivable from the metric or expressible in terms of it. Any such map, in two or three dimensions, will be a *representation* or *model*. The latter term will be preferred if there is another representation, of which the 'model' is regarded as in some way a copy, or if for some other reason the representation in question seems to lack full validity.

It is to be noted that an incomplete geometry, formally Euclidean or non-Euclidean, may be operationally implemented (1) in a local representation using Euclidean constructions, (2) in local *measurements* of physical phenomena, accompanied by a representation if we please, or (3) in astronomical or other large-scale measurement. In the last case there may or may not be a representation regarded as appropriate and acceptable. A Euclidean operational geometry, in some locality, is always necessary in a complete geometry; otherwise there would be no way of initiating measurement. If, while accepting this, we do not accept any Euclidean representation or class of Euclidean representations of the wider universe, then the geometry is *occasional*, though still operational in the sense of Chapter 7.

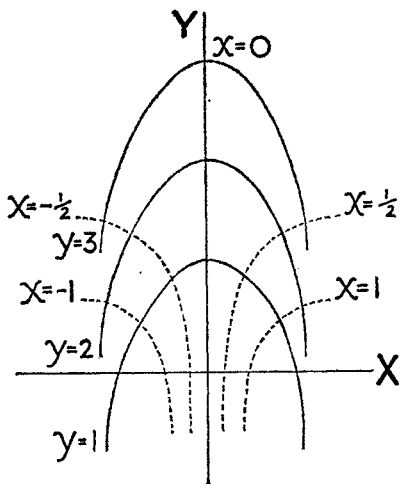
In saying that in this case (Euclidean) representations[1] of the wider universe are 'not accepted', a certain ontological standpoint is implied. Universal representations, whatever are possible, may be admitted as mathematically serviceable. But it can be held that no representation or class of representations is *ontologically appropriate*, having in view certain topological and other requirements of correspondence between a thing and its representation. In other words, to accept a representation or class of representations, Euclidean yet referring to non-local phenomena, amounts to planting on nature limitations which arise within our own minds. It would not therefore give a valid picture of the universe; or at least the picture would be misleading. Here we see how the subject of complete geometries inevitably brings in the question of validity of representation.

[1] All 'representations' spoken of in this chapter are understood to be Euclidean, so the epithet will hereafter be dropped.

The various distinctions we have now made are necessary to avoid useless disputes as to whether 'the geometry of the world' is Euclidean or not. The question depends, as we have seen, on the force of various epithets—formal, complete, incomplete, ideal, actual, local, universal, occasional—and also on how the formalism is implemented, in a representation, a model, or neither. One or more of these epithets or clarifying conditions must be imposed on the term 'geometry' if the discussion is to be about anything definite.

A 'Model' of Euclidean Geometry

Formally Euclidean geometry has what may be called a *natural* representation in operationally constructed straight lines, etc. This we may define as one in which equal 'distances' *ds*, in the formal sense, are represented as equal distances operationally constructed by the usual Euclidean means. The following simple illustration will show how easy it is to construct other representations, which must be regarded as in various ways distorted and inappropriate.



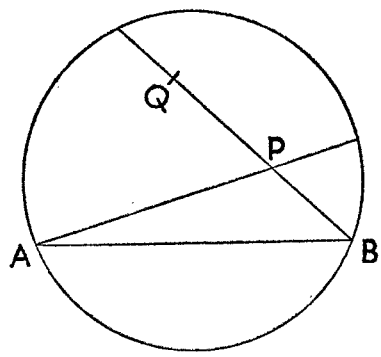
Let the coordinate lines $y =$ constant be operationally implemented in the parabolas $X^2 = y - Y$ on the X–Y Euclidean plane. Let the coordinate lines $x =$ constant be operationally implemented in the curves $X = xe^Y$, which are the orthogonal trajectories of the parabolas. Then every number-pair (x, y) in the formalism is represented, in one-one correspondence, by a point (X, Y) constructible on the X–Y plane.

The reasons for regarding this manner of operational implementation as a *model* will be studied more thoroughly in Chapter 18. For the

present we may merely note the lack of symmetry as between x and y, the lack of uniformity in the representation of distance ds, and the consequent failure to represent congruent figures as congruent.

The Beltrami Model of Hyperbolic Geometry

Passing now to the consideration of formally non-Euclidean geometries in more detail, the simplest are those with uniform Gaussian curvature K, namely 'elliptic' geometry (as on the sphere) with positive K, and 'hyperbolic' geometry, with negative K. The latter is given by the metric $ds^2 = dx^2 + \sinh^2(x/a)dy^2$, where x, y are coordinates which are initially absolutely devoid of operational significance (except in the purely mathematical sense of the term). Alternatively, the geometry may be formally defined by axioms of connection, order, congruence, and continuity, as with Euclidean geometry, omitting only the axiom of parallels (and introducing instead the hyperbolic axiom of parallels, which admits more than one line, through a given point, not intersecting a given line). The topological properties embodied in certain of these axioms are understood in the metrical definition, but not explicitly stated.



Assuming the metrical definition we obtain Beltrami's model by writing, first $x = r$, $y = a\theta$, so that

$$ds^2 = dr^2 + a^2 \sinh^2(r/a)d\theta^2 \tag{1}$$

then by putting $R = R_0 \tanh(r/a)$, and representing (R, θ) as polar coordinates on the Euclidean plane. The geodesics of metric (1) are:

$$A \sinh(r/a) \cos \theta + B \sinh(r/a) \sin \theta + C \cosh(r/a) = 0 \tag{2}$$

which, on making the substitution for R, become the straight lines

$$AX + BY + C' = 0$$

where A, B, C, C' are arbitrary constants, and $X = R\cos\theta$, $Y = R\sin\theta$.

One can now verify that the representation obeys all the axioms of hyperbolic geometry. Since $R < R_0$, the figure is bounded by the circle $R = R_0$, which is more and more nearly attained as r is increased. If AB is any hyperbolic line and P any point, there are two parallels PA, PB separating lines which intersect AB from those that do not. On the strict understanding that space within the circle is *open* in the mathematical sense, i.e. the points actually on the circle are not included, the axioms of order are also obeyed. Thus, given distinct points P and Q, there is another point S on PQ beyond Q (more concisely, $[PQS]$).

Nevertheless there are some unsatisfactory features of the representation which make one feel that it should be described as a *model* rather than a direct and valid operational implementation of the formal system (Riemannian metric, or Axiom System) of hyperbolic geometry. Some of these features appear from the metric in the R–θ coordinate system, which may be written:

$$ds^2 = c^2\gamma[\gamma dR^2 + R^2 d\theta^2]$$

where $\gamma = (1 - R^2/R_0^2)^{-1}$ and $c = a/R_0$. Here ds is the measure we must take for *distance* if the axioms of congruence are to be obeyed. It is clear therefore that congruent figures are not represented as congruent. Angles of intersection of linear structures are not correctly represented (the representation is not conformal). As we approach the boundary $R = R_0$, the scale on which we are representing distance tends to zero, producing a kind of singularity. Lastly, although the formal system is homogeneous, in the sense that the axioms make no reference to any particular origin or direction and the metric can be put into the same form by a transformation when another point is taken as origin, the representation of distance is violently inhomogeneous and makes the point represented by the centre of the circle specially privileged.

This last objection is very much weakened by postulating a *class* of representations such that any arbitrary point (r, θ) can be placed at the centre of a representation, and an operational method is exhibited for transforming a figure on one representation into the corresponding figure on another; no point is then privileged. The other objections however remain.

One further objection, very serious, has to be mentioned. There is no distinction, operationally, between an open and a closed area. Operationally the lines PA and BA do actually meet in a point on the circular boundary. The hyperbolic lines therefore come to an end at A and B, a fact which is at variance with the axiom system, especially the axiom of order mentioned above. We feel that the supposed removal

of the points on the circle from the space is merely a formalistic trick to make the lines AB appear to be unending, when in fact they do come to an end. Altogether, we are likely to decide quite confidently that this operational implementation of hyperbolic geometry is not strictly accurate, and must be called a *model* only.

Plane Representations in General

It is evident that there is no limit to the number of models of hyperbolic geometry which can be constructed within a circle of radius R_0 as with the Beltrami model. We have only to substitute $R = f(r)$ in metric (1), where $f(r)$ is increasing and bounded above by R_0. One of these alternative representations is the conformal Poincaré Model, in which the geodesics are circles intersecting the boundary orthogonally.[1]

There is also no limit to the number of models of hyperbolic geometry which can be constructed so as to cover the whole Euclidean plane. Models of this kind are obtained by likewise substituting $R = f(r)$ in metric (1), but taking $f(r)$ steadily increasing, or decreasing, and tending to infinity as r tends to infinity. The special advantage of the Beltrami and Poincaré models is that in their case the hyperbolic lines are represented by easily constructed 'curves'. But whatever substitution is made (or even *no* substitution) preparatory to representation, there is no possibility of congruent figures being represented as congruent. For this would imply that some transformation of coordinates would reduce the metric (1) to the Euclidean form $ds^2 = dx^2 + dy^2$, which is not so. The best one can do is to produce a conformal representation, preserving angles.

The legitimacy of imposing coordinate transformations and regarding the geometry as still 'the same' is a question which can be decided only in the light of some convention in the use of the epithet 'same'. Thus we may agree to call geometries 'the same' if (1) their metrics are identical; or if (2*a*) the curvature tensor R_{ijkl} is the same when a suitable coordinate transformation is made; or, in the case of a homogeneous geometry, if (2*b*) the Gaussian curvature K is the same or (2*c*) the sign of K is the same, or K zero in each case; or if (3), in addition to R_{ijkl} being the same, the geometries have the same class of representations for a given class of coordinate systems.

Hyperbolic geometry is customarily distinguished by the convention (2*c*), asserting merely that K is negative, and accordingly permitting any coordinate transformations we please. Nevertheless, if the geometry is to be complete, the representations (and correspondingly the choice of coordinate system) are drastically limited.

For instance, with the original form for the metric,

$$ds^2 = dx^2 + a^2 \sinh^2(x/a)dy^2$$

[1] H. S. M. Coxeter, *Introduction to Geometry*, New York 1961, p. 289.

a representation in which x and y are Cartesian would show an infinity of points along the line $x = 0$ for arbitrary values of y; but according to the metric these points are zero distance apart, i.e. they are one and the same point. The difficulty is not avoided by choosing a representation in which the coordinate lines $x =$ constant, $y =$ constant are not straight but are the coordinate lines of a new system of coordinates x', y' in one-one correspondence with x, y.

There are three methods of escape from a dilemma of this kind. One is to admit representations in which an infinity of points (x, y) corresponds to a single point operationally constructed; this is regarded as reasonable on the sphere if the duplication arises merely because zero displacement from a pole is in no particular direction, or because directions in radian measure can increase $2n\pi$ (n integral) without altering the situation operationally. As the original metric stands, this method is inapplicable. A second method is, accordingly, first to abandon the original metric and substitute metric (1) as determining hyperbolic geometry. We can then admit further metrics in coordinates r', θ' provided the points (r', θ') are in one-one correspondence with the points (r, θ). The third method is to reinterpret ds in such a way that an infinity of distinct points operationally constructed can have zero ds between any pair; this method is adopted in geometries of space-time, in which the concept of invariant distance is discarded and replaced by something analogous.

Again we see how questions of representational validity cannot be separated from questions of operational implementation of geometries.

Models, and Measurement-Tools

The models just considered have been constructed on the understanding that Cartesian coordinates and other structures of Euclidean geometry can be marked out on the plane surface of a rigid body. Even though in certain regions of this surface no coordinate lines may be at present marked, and the coordinates of a given point cannot therefore be read off, it is understood that a simple Euclidean construction can be carried out so as to provide the values of the coordinates required. Thus, in effect, there is an *omnipresent labelling* of points with coordinates *omnisciently available*. The formalism is manipulated on the understanding that this is so.[1]

We now have to consider the implications of the fact that, outside a very restricted region in a laboratory, there is on the contrary no omnipresent labelling and omniscient availability. Indeed, of the one chief purposes of geometry is to enable us to carry out measurements

[1] Compare Weyl, *op. cit.*: 'How is it possible to assign to the points of a point-field marks or labels which could serve for their identification or distinction? The labels are supposed to be self-created. . . .'

in distant regions where there is no possibility of omnipresent labelling omnisciently available. We hope to do this by devising a technique for *fitting* a formal geometry here and there so accurately to the phenomena that we feel reasonably safe in assuming that it will fit similarly in places which we cannot reach. The justification for this assumption is not in a vague Principle of Uniformity, but in the fact that, after making our provisional fit, it is usually possible to verify the fit in places which we were at the time unable to reach. After a number of successful verifications of this kind have been made we feel the same kind of confidence, justified in practice, as we do in the truth of the assertion that 'the sun will rise tomorrow. Something might just conceivably 'go wrong'. But if so, we feel sure that it could be reasonably accounted for in terms of some exceptional cause.

Use of Light-Rays

To make contact with distant regions, so as to record some measurement depending on physical phenomena there, we seem to have only one means—electromagnetic waves. On the earth, and particularly in the laboratory, we have continually the most comprehensive and convincing verification that light-rays are normally equivalent to Euclidean straight lines. It is clear that effects of refraction must be regarded as a local distortion of the 'normal' path which would be taken *in vacuo*; for they are differential effects depending on the character of the medium interposed. In many cases, also, the refracting material (e.g. a lens) can be alternately interposed and removed while the surrounding objects remain unchanged in position, so that the distorting effect can be clearly observed and measured. Hence if the light-rays we are using for measurement pass through an intervening refracting medium it is entirely in accordance with reason to impose corrections.

We may also correct for diffraction, though not by anything like the same means. Where diffraction bands or rings are observed we may replace the actual bands or rings by an ideal undeviated line whose position is clearly fixed relative to the apparatus by which the bands or rings are observed; or we can suppose ideal straight lines from points of the slit (or whatever it may be that produces the effect) to points of a band, so that the effect is precisely accounted for by means of an optical theory on the ground of an operational Euclidean geometry.

Within the solar system, also, astronomers find no great practical difficulty in setting up a Euclidean frame of coordinates in which the sun is at rest. They have merely to record the angular position of astronomical objects allowing for the rotation of the earth, and to correct for atmospheric refraction, aberration due to the earth's motion, and time-lapses due to the transit of light from the objects in question to the earth. The barely observable gravitational deflection of

light in passing near the sun is detected by comparison with the undeflected position, in close analogy with the experiment in which a lens is alternately interposed between two objects and removed. So we can reasonably consider this as removable by 'correction'. We can even argue that it is a differential effect which *must* be removed by correction; for, in principle, the mass and size of the body interposed could have varied from one occasion to the next.

Straightness

The amazing accuracy with which the theorems of Euclidean geometry are verified astronomically, using light-rays which cover countless millions of miles without any detectable deviation from straightness (or so it works out), needs to be fully appreciated. Any philosophy of space and time must regard this remarkable fact as of the utmost significance. Nevertheless there is danger of reading too much into it. No astronomical system of coordinates can be Euclidean in the sense that the straightness of the corrected light-rays can be verified by testing of self-congruence in the translation and rotation of a copy made on an idealized rigid body; nor can we directly test straightness by inspection from different standpoints throughout the universe. All such tests are ruled out. In fact we have abandoned the operational definition of straight lines in terms of an actualized transformation-group, and have substituted a quasi-physical definition in terms of corrected light-rays.

Obviously the two definitions are in some respects equivalent; the same theorems appear to hold, and the same kinds of correction are applied. But consider how the positions and shapes of objects in one inertial frame differ from their positions and shapes in another, because of the Lorentz–Fitzgerald contraction. Each frame has been set up by light-rays, represented as straight lines, and is in that sense Euclidean. The positions and shapes of objects are given a Euclidean representation. But if some physical structure or phenomenon is represented as a straight line in one frame, it will be represented in other frames as a multiple infinity of straight lines, one to each frame at each instant of time. Moreover, the angle between two such 'straight lines' in a given frame will not in general be the same as the angle between their representations in another frame at a given time, though, if parallel, their representations in any other frame at any given time will also be parallel. In these features we have something like an operational implementation of affine geometry, in which a class of figures takes the place of a fully operational single figure.

A more serious difficulty arises if we are trying to implement a formally non-Euclidean geometry, i.e. one with curvature tensor $R_{ijkl} \not\equiv 0$, corresponding in General Relativity to the case of an irreducible gravitational field. Consider, for simplicity, a single frame of

reference, at a given t (or x^4), so that $dt = 0$. If we carry a small vector, by parallel displacement, round a closed path back to its starting-point, the final orientation will not in general coincide with the initial orientation. We can conclude therefore that a natural (Euclidean) representation in this frame is not possible. On the usual understanding that ds is local Euclidean distance, it is *impossible to construct straight lines* in the full operational sense. So also it is impossible to construct planes. The converse of these propositions is more readily self-evident: If in any space-frame it is possible to construct straight lines and planes the metric of the space-coordinates (obtained by putting $dt = 0$) must be formally Euclidean.

The essential part played by representations in the determination of straightness is shown more clearly if we ask, How do we test whether a light-ray is 'straight'? There seem to be only two possible kinds of answer. It is 'straight'

(1) if, when its course is marked by pins on a rigid body, we can carry out the usual operational tests of translation and rotation; or

(2) if, when we sketch its path as a straight line on a Euclidean diagram (on the same or a smaller scale), accepting that such diagram is a valid representation of the spatial properties of the structure being considered, or when we imagine its path on such a diagram for the purpose of mathematical calculations, there is agreement (on the principles of correspondence laid down) with what is actually observed. This would be the case, for instance, when we check the lengths of shadows, heights of towers, and so on, using trigonometric formulae.

On all accounts, therefore, it appears we must consider the term 'straight' to be void of meaning except in terms of some representation. In other words, the course of a linear structure which is to be distinguished as straight or not must be shown on a Euclidean diagram or model or in the imagination of such diagram or model, the full operational definition of a Euclidean straight line being assumed; and the decision will be strictly in reference to the diagram or model chosen.

'Flat' Space-Time

Although in this section we are trying to limit our investigations to properties of space, our study of straightness has made unavoidable the consideration of the trajectories described by light, in particular the uniform-velocity trajectories of light *in vacuo*.

To account for the non-conformality between the space-representations of such trajectories in one inertial frame and those in another by regarding the geometry as affine in a single space would leave the geometry incomplete. Instead, let us keep to a straightforward account of the state of affairs, as follows. Along with the introduction of changes with respect to time, which definitely occur with light trajec-

tories, the definition of a straight line has had to be enlarged. The primitive linear structures of the 'space' become then uniform-velocity trajectories, and the geometry of these is such that their space-tracks have a natural representation as Euclidean straight lines in frames called *inertial*. The properties of non-conformality, etc., are simply properties of this wider operational geometry of inertial frames, determined either as a Riemannian geometry or by means of an axiom system without use of coordinates. The conditions in the former case are (1) ds^2 is reducible to the 'pseudo-Euclidean' form $-dx^2 - dy^2 - dz^2 + dt^2$, where x, y, z are Euclidean measures of distance and t is a measure of time set up by taking the velocity of light as unity; (2) ds is zero for neighbouring events on a light-ray and invariant otherwise; and (3) the uniform-velocity trajectories are given by the geodesic condition, $\int ds =$ minimum.

It is customary to regard conditions (2) and (3) as postulates. But a postulate can be either a summary of the results of intuitive analysis or an attempt to circumvent analysis by putting forward assumptions from which the experimental or theoretical results under consideration can be deduced. We reject this latter view here, and must therefore in effect derive the postulates from the analysis of measurement.

Recalling therefore that an operational geometry is here being applied to the universe for the purpose of measuring phenomena in it, and that such geometry must be *prescribed*, we first choose our formal system (Riemannian or axiomatic) and then show how it may be operationally implemented. The implementation will at first be *ideal*, but afterwards we can endeavour to apply it to the actual world in regions where we would expect deviations to be small or fairly easily corrected.

In the Riemannian formulation the choice of metric to begin with will be governed by our conception of the physical conditions to which we ultimately wish to apply the geometry. These conditions cannot be perfectly general if the geometry is to fit accurately, since the prescription of the geometry permits only a finite number of unknowns. We must therefore decide on some fairly simple kind of ideal physical conditions, involving not more than a finite number of physical measures to be determined, and must measure everything else relative to this 'form of phenomena'.

The simplest ideal physical conditions will be those which do not involve the determination of even a single physical measure; and in this case it can be shown by group theory that the metric must be reducible to the form $ds^2 = -dx^2 - dy^2 - dz^2 + dt^2$, where x, y, z have a natural representation as measures of distance in a Euclidean space-frame, and clocks in that frame are correlated so that the velocity of light (given by $ds = 0$) is unity. With this metric there is also a *class*

of frames, obtainable by coordinate transformation, such that all points of any one frame have the same velocity with respect to any other frame, i.e. the frames may be taken as *rigid*; and again by group theory it can be shown that this is the only metric with such a property.

The next step is to implement this geometry in some *actual* operational procedure, bearing in mind that in actuality there will certainly be departures from this 'form of phenomena'. Dealing first with a single inertial frame set up in the way explained, we note that ordinary physical experiment indicates that free particles, in the Newtonian sense, move very accurately in uniform-velocity trajectories; and since such trajectories (regarded as ideally exact) are equivalent to the equation $\int ds =$ minimum, we conclude that they can be identified with geodesics. In particular, therefore, light-rays can be used to set up the coordinate network or to find coordinates by triangulation, given some rigid body to serve as base. Deviations from the ideally exact trajectories (due, for instance, to atmospheric refraction, gravitational attraction, or acceleration of the base) will be revealed by inconsistencies in supposing that the chosen geometry fits the physical situation exactly. These inconsistencies may then lead to a measure of the deviation in question, and hence also, perhaps, to a measure of some independently manifested physical condition (e.g. temperature of the measuring apparatus) regarded as the *cause* of the deviation.

We have also to show how to implement this geometry in two inertial frames conjointly. The presumed formal equality of ds in the two frames must be implemented by some operational rule of correlation. If ideal standard rods and clocks at rest in one frame are carried by a smooth motion into another frame where they are likewise at rest, the rule is that they be taken as establishing standards for *proper* distance and time in each frame. That this rule of implementation makes the geometry fit experience is verified by the Michelson–Morley experiment, carried out at different times of the year. We find that in frames with various directions of motion, the velocity of light can be consistently taken as the same in all directions, provided the same ideally rigid standards are used. From this fact alone, the Lorentz Transformation would follow. But we can also use recognizable atomic periods (such as that of the H_β line) as equivalent to clocks, and verify the invariance of ds indirectly, as in the Ives–Stilwell experiment. The accuracy of the 'verification' is in all such cases a measure of the accuracy of the fit between our prescribed geometry and the physical conditions to which it is being applied.

Spheres of Implementation

It is safe to say that, without corrections of various kinds, the formalism of 'flat' space-time *never* fits with experimental observations in any

considerable region of the universe, even allowing for the imperfections of measuring instruments and of human perception. If the corrections are ones which we feel are most appropriately made in such a way as preserves flat space-time, we accept flat space-time as the 'form of phenomena'. If, on the other hand, the adoption of another geometry results in some class of deviations (imputed to a corresponding physical cause) falling away, so that no correction is then needed for the effects of that cause, it can be argued that this other geometry should be accepted instead as the 'form of phenomena' in the region where it is being applied.

In this chapter we do not consider the precise grounds on which it may be maintained that one geometry is more appropriate or more *valid* (as I shall say) than another as form of phenomena. The problem we have set ourselves is to determine how a formally non-Euclidean geometry of space may be operationally implemented, without restricting ourselves to models in the laboratory. This inevitably leads (as we have seen) to the consideration of flat and non-flat geometries of space-time. And thus we are led to investigate the related problem of determining the circumstances in which a formally Euclidean space-geometry or a flat geometry of space-time can be operationally implemented, and the circumstances in which the attempt to implement such geometries fails.

It is evident that the class of operations in which the formal geometry may possibly be implemented changes as we shift our attention from our immediate and tangible surroundings to objects farther off, and ultimately to objects conceived of as at distances of 10^{20} miles or more away. In fact four chief regions of application can be distinguished, each with its manner of implementation and corresponding space-concepts. These may be called *spheres of implementation*, the term *spheres* being used to suggest not merely differences of measurable extent but differences, rather, in conceptual character. I shall also loosely refer to a sphere of implementation as a 'space'.

(1) STRICTLY LOCAL space comprises everything whose position is manifested to us by physical *space-perceptions*—stimulated perhaps by sensations of touch and movement, visual impressions of various kinds, effects of binocular vision, and sensations of muscular adjustment in eye-focusing, but not imaginatively suggested by these. We presume also that a Euclidean base-frame can be approximatively set up with the help of nearly rigid bodies corrected for temperature, stress and any other subjective or differential effects, or effects variable with the occasion. A measuring rod corrected in this way will be called a *standard rod*. For consistency of language and thought, we must not describe the apparent placing of a mirror-image at a certain point in physical space as due to a *physical* space-perception of the image there;

it might be better described as a fragmentary and compulsively imaginative placing. Nothing but experience can teach us the essential difference between a direct physical space-perception and an imaginative one.

It may also be convenient to distinguish subdivisions as follows: (*a*) gravitationally local space, which is a strictly local space, further restricted so that trajectories of free particles approximate very closely to uniform-velocity ones in the base-frame set up; and (*b*) gravitationally non-local space, in which the trajectories of free particles are not treated as approximately uniform-velocity ones.

By 'free particles' here we mean particles which are allowed to move without contact with other bodies, particles or gases and without influence by electromagnetic fields. In gravitationally non-local space, therefore, these will not be 'free' in the Newtonian sense, because their motion is affected by gravitation; but they are 'free' in the relativistic sense.

The distinction (*a*) or (*b*) is relevant only if we wish to account for the curved or accelerated paths of particles in a field of force by taking them as geodesics in a non-Galilean space-time, the Euclidean base-frame being set up by light-rays. In purely space-geometries it is irrelevant. Also, the question of 'correcting' for effects of gravitational potential does not arise in this space, since by hypothesis a Euclidean frame can actually be set up without any such correction being necessary.

To give a very rough idea of the extent of this sphere of implementation we can characterize it as 'the laboratory'.

(2) MANIPULATIVE space is a sphere of implementation where it is possible to carry out accurate measurements by manipulation of standard rods or surveyor's bands or rod-chains pulled tight. The latter can be said to give a mechanical construction of geodesics. To give an idea of the extent of this sphere, in space-geometries, we can characterize it as 'the garden' (a very rough estimate might be about 200 feet across at most, depending on the accuracy required and the way in which the geodesics are constructed).

In geometries of space-time, where the introduction of the time variable would lead to results different from those obtainable with a purely space-geometry, I propose to take the manipulative methods under consideration as quite impracticable, if not actually meaningless. We cannot lay links of a chain along the path of a projectile so that each link is moving with the velocity of the projectile and also the total number of links is a minimum; hence it is impossible to verify by this method that the trajectory of a free particle is a geodesic. The verification must be done with the help of light-rays.

(3) OPTICALLY LOCAL space is a sphere of implementation in which measurement by light-rays, from a strictly local base-frame and *represented* as uniform-velocity trajectories on a Euclidean diagram

after having been corrected for refraction and any other subjective or differential effects or effects variable with the occasion, is not disconfirmed (within the limits of experimental error) by any other observation yet made. It is assumed that very numerous observations, covering all chief types, have already been made and tested.

I propose to say that light-rays are *presented* as straight only in cases where their course over some rigid body is shown or can be marked and verified as straight in the strictly local sense. Otherwise I shall not regard it as correct to say that the light-rays are *actually straight*. For they are not directly presented as such. All we know is that the *representation* works out correctly and gives a reliable impression of what strictly local presentations would be if we changed our base-frame. Hence we may call them only *representatively straight*.

To give an idea of the extent of this sphere of implementation we can characterize it as 'the earth'.

(4) A COSMICAL sphere of implementation is such that occasional inconsistencies arise from representing light-rays (corrected as usual) as uniform-velocity trajectories in some Euclidean diagram. It is to be understood that even if the inconsistencies are very small and could be fairly easily removed by further 'correction', nevertheless there is good reason for adopting another geometry than that of flat space-time. In particular, since the geodesic equation $\int ds =$ minimum needs to be implemented in trajectories of *all* velocities, we incline to reject a geometry which does not account for planetary orbits on the same (geodesic) principle as that on which the trajectories of light-rays are accounted for.

The minimum extent of this sphere of implementation can be normally characterized as 'the solar system'; there is no maximum extent, since the cosmos as a whole is dealt with by the same techniques. The application of these techniques to the case of projectiles on 'the earth' and fluid motion in 'the laboratory' raises philosophical and mathematical difficulties which cannot be considered at this stage. For the present I propose to ignore such application and treat it as a merely theoretical device or alternative derivation of tensor laws of motion in flat space-time. Indeed, it is in flat space-time that all our perceptions and verifications of such motion occur.

Non-Euclidean Geometries of Space

We are now ready to survey systematically the possibilities of implementation in the four 'spheres'. In each sphere we must distinguish two cases: (1) where the metric is given and we arrange matters physically in such a way that the geometry is implemented, the arrangement being presumed ideally exact; and (2) where the physical circumstances are to be explored, using a prescribed geometry as a measurement tool, or

else trying (unsuccessfully, as Chapter 7 predicts) to determine the geometry from measurement of the physical conditions at distant points.

(1) STRICTLY LOCAL SPACE, *metric given.* This space is Euclidean, in a natural representation. Non-Euclidean implementation is therefore possible only by changing our definitions of 'distance' and 'straight-line', as Poincaré recognized when he considered such implementation:

'We now interpret them [our experiments] on the non-Euclidean hypothesis. This is always possible, only the non-Euclidean distances of our different bodies in this new interpretation will not generally be the same as the Euclidean distances in the primitive interpretation.'[1]

The term 'distance' in Euclidean geometry, however, is variable in meaning. Normally it means distance measured by congruent intervals along a straight line. It can also mean distance measured by infinitesimally congruent intervals along a given curve. Let us call the two concepts distance_1 and distance_2. It is with the concept distance_1 that implementation of a formally non-Euclidean geometry is impossible; for the geometry of straight lines in the full operational sense of the term has been shown to be Euclidean. With the concept distance_2 implementation of a formally non-Euclidean geometry might be possible, in a natural representation (equal ds represented as equal infinitesimal distances_1) if through any two points of the space a certain non-straight curve can be uniquely constructed.

There are two ways of supposing such a curve constructible. First, we may introduce a subsidiary metric, non-Euclidean, with an interval ds' different from the Euclidean ds, and expressed in terms of coordinates marked throughout the material. The curves in question can then be defined as geodesics $\int ds' =$ minimum, sketched out by examining the marks on the material. It is to be noted, however, that what is implemented here is the non-Euclidean geometry of the subsidiary interval-measure ds', in a non-natural representation, while all the time the implementation of the Euclidean geometry of ds has to be presupposed. I therefore consider it unnecessary to deal further with this attempted mode of implementation.

Secondly, we may construct the required curves as geodesics on a two-dimensional curved surface in three Euclidean dimensions, thus retaining the natural representation of ds while relinquishing the definition of distance_1. Since every curved surface has its metric we could take any such metric and regard the corresponding formal geometry as having a natural representation on the surface in question. If topological difficulties are ignored, there may be sometimes alterna-

[1] *Science and Hypothesis*, p. 76.

tive surfaces, each implementing the same metric (as a plane and a cylinder each implement two-dimensional Euclidean geometry if geodesics, instead of straight lines, are taken as the primitive lines of the space).

In particular, if it is required that the Gaussian curvature K be a constant, i.e. the geometry is homogeneous and isotropic, there is an implementation on the sphere, if K is positive, and on the pseudosphere, if K is negative. In each case there are topological difficulties. On the sphere the space continually repeats itself as the geodesic lines (great circles) are produced; the Axioms of Order taken over from Euclidean geometry are therefore not all obeyed. On the pseudosphere, given by $r = \operatorname{sech} u$, $z = u - \tanh u$, in cylindrical coordinates r, θ, z, with $ds^2 = \tanh^2 u\, du^2 + \operatorname{sech}^2 u\, dz^2$, geodesic lines which are not meridians ($\theta =$ constant) all come to an abrupt end at the cuspidal edge.[1]

There are two kinds of non-natural representations of non-Euclidean geometries. The Beltrami model, for instance, associates a number ds with increments in the variables at a certain point, its value being found by substituting the given coordinate values and coordinate increments in the metric. To describe this substitution as an 'operational implementation' in the spatial sense (as well as in the pure mathematical sense) would be contrary to the distinction we have made between the two types of operation.

The difficulty may be said to be surmounted if we arbitrarily associate a certain temperature with each point such that this ds would be the apparent distance measured by a supposedly standard rod which actually expands according to the usual law of temperature expansion. This is done in models proposed by Poincaré and Barankin.[2]

Here also, however, the term *model* is appropriate, because (1) the implementation is necessarily local, on a Euclidean basis; (2) it is not an exhibiting of the geometry as a measurement-tool; and (3) there are likely to be topological difficulties which render the implementation imperfect. Thus Barankin implements hyperbolic geometry by keeping the long sides of an elongated rectangle at different fixed temperatures; the geodesics are then cut off at the boundary.

It remains to consider what could be meant by implementing a strictly local geometry as a measurement-tool. By saying that the space is strictly local, we are presuming that physical circumstances are in fact such that Euclidean geometry can be implemented in a natural representation by the use of standard rods. It would be inconsistent to suppose that a non-Euclidean geometry could also be implemented in a natural representation by the same standard rods. Consequently

[1] Coxeter, *op. cit.*, pp. 377 ff.

[2] H. Poincaré, *op. cit.*, pp. 65 ff. E. W. Barankin, 'Heat Flow and Non-Euclidean Geometry', *Am. Math. Monthly*, vol. 49, 1942, pp. 4 ff.

there is nothing to discover concerning the metric and its implementation, and the question of implementation of a non-Euclidean geometry as a measurement-tool cannot arise.

(2) MANIPULATIVE SPACE. The methods of measurement used here are either consistent with those used in strictly local space, resulting in Euclidean geometry ('the laboratory' extended into 'the garden'), or inconsistent. Having already dealt fully with strictly local space, we need only deal here with the case of inconsistency between the two methods of measurement.

Manipulative space being contained in optically local space, it is naturally Euclidean. If therefore the metric (non-Euclidean) is given, we conclude as before that a natural representation of such metrical geometry is impossible. Any physical arrangement devised so as to implement the geometry must count as a *model*. Again, if we are to have an operational implementation in the spatial sense (and not merely the pure mathematical sense) we shall need some physical deforming influence, such as temperature, for which the observer must *not* correct. The shortest path between two points, as found manipulatively with uncorrected rods, should then coincide with the path as calculated from the given metric and plotted on the material with the help of a coordinate-network marked out on it; otherwise the model will have been incorrectly devised or implemented. It is difficult to see what useful purpose such a model might serve.

Those who propose manipulative methods of measurement seem to have chiefly in mind the use of non-Euclidean geometry as a *measurement-tool*. Thus Robertson imagines a flat metal plate irregularly heated by a Bunsen burner.[1] By rod-chains, laid along the perimeter of (what appears to be) a circle, and not corrected for temperature expansion, one obtains a value for the average Gaussian curvature K over the region of the circle. So by taking a small circle at each point one approximates to K at all points of the plate. We thus obtain what may be called an 'uncorrected-temperature geometry'; and similarly there will be an 'uncorrected-pressure geometry' in fluids, an 'uncorrected-gravitational-potential geometry', and countless other kinds.

We are now supposing, however, that such a geometry is to be used as a measurement-tool. One of its chief purposes, therefore, is to enable us to find the 'distance' between two points *without going there*. If the answer to the question, 'What is the distance between A and B?' were merely, 'Take a standard rod and go there to see', geometry would cease as a conceptual system and become an empirical record, of no predictive value.

[1] H. P. Robertson, *Geometry as a Branch of Physics*, pp. 327 ff. (included in *Albert Einstein, Philosopher Scientist*, ed. Schilpp, New York 1949).

It is true that with any measurement-tool we must be able in some way to make contact with the distant parts which we wish to measure. How is it then, precisely, that this particular 'geometry' fails as a measurement-tool? For this is the conclusion to which we are being drawn.

The use of Euclidean geometry as a measurement-tool may perhaps be said to lie in the fact that, given a few measurements of a certain figure, we can use the theorems and formulae of the geometry to calculate what every other measurement would be if we were in a position to make it. Can we do this in Robertson's 'geometry'? Can we, for instance, go out along geodesics in different directions to B and C, measure AB, AC and angle A, and then deduce the geodesic distance from B to C without laying a rod-chain between these points?

To calculate the geodesic path and distance we must know the metric tensor g_{ij} corresponding to some coordinate-system set up throughout the space. An obvious method is to pull our rod-chains tight in various directions from A, and so establish polar coordinates r, θ. In order then to determine the quantities g_{ij} at each point P we must suppose the coordinate directions marked on the material at P, and a small standard rod laid down in those directions, comparing the coordinate increment with the 'distance'. If ϕ is the angle between the coordinate directions at P, we then have (using two dimensions, for simplicity):

$$g_{11} = \left(\frac{\partial s}{\partial u}\right)^2, \; g_{22} = \left(\frac{\partial s}{\partial v}\right)^2, \; g_{12} = \frac{\partial s}{\partial u}\frac{\partial s}{\partial v}\cos\phi.$$

Now, if we wish to calculate the geodesic path between A and B, we shall have to find the components g_{ij} at *every point of the region between*; and this, as we have seen, requires going there with a standard rod, and also an omnipresent labelling of the material.

It follows that a Riemannian geometry cannot be used as a *measurement-tool* unless we are told in advance what the metric is for our given coordinate-system—i.e. the geometry is *prescribed.*

(3) In OPTICALLY LOCAL SPACE (on 'the earth') Non-Euclidean geometry can be likewise implemented only in a non-natural representation. The same remarks as above about models apply, so we consider only the attempt to use a non-Euclidean geometry as a measurement-tool.

The only difference between this 'space' and manipulative space is that here the geodesics are taken to be constructed at once by light-rays without any need to 'go there' with a rod-chain. But the argument that the distance between B and C cannot be calculated without knowing the components g_{ij} at every point still applies, so the geometry must again be prescribed.

The simplest case will be when K is the same everywhere, giving, for instance, three-dimensional hyperbolic geometry, with:

$$ds^2 = dr^2 + \sinh^2(r/a)\{d\theta^2 + \sin^2\theta\, d\phi^2\} \ldots K = -1/a^2$$

The geodesics $\theta = \pi/2$, $\phi =$ constant, form a surface which is plane at the origin. On this surface, formed by light-rays starting out from the origin in directions lying in a plane, the metric reduces to the two-dimensional form (1) of hyperbolic geometry. Hence it seems we may have some means of testing how accurately such a geometry may be implemented on 'the earth', and for what value of K the fit is closest. What such a test would give us, however, is the *limits of optically local space*. For we have defined optically local space as a sphere of implementation such that the representation of light-rays as Euclidean straight lines is not disconfirmed by any observation yet made. A well-known experiment by Gauss, using the formula for the integral curvature of a geodesic triangle:

$$\int_{ABC} K dS = A + B + C - \pi$$

applied to the triangle formed by points on three mountains of Germany, showed accordingly, by its zero result for K, that optically local space extends at least to 100 miles.[1] Later investigations, of course, show that it extends in fact to the immediate neighbourhood of the sun.

In this respect we can say that the natural geometry of space is determined by nature up to the limits of optically local space, and that we have no freedom to choose anything but Euclidean geometry except by artificially representing equal Euclidean distances as unequal, thus producing a geometry of derived or secondary character, in short, a *model*.

(4) COSMICAL 'space' is a space-time of four variables, with ds no longer standing for 'distance' in any of the senses so far considered. Only in the 'static' case would it reduce to a geometry which can in any way be regarded as purely spatial. If such a geometry is thought desirable, we must accordingly limit our view to the case when the metric of our prescribed geometry can be put in the form:

$$ds^2 = -d\sigma^2 + g_{44}dt^2$$

where $d\sigma^2$ stands for a metric in the three space-variables x^1, x^2, x^3 and g_{44} is independent of time. The tracks of the null-geodesics (light-rays) are then independent of time, and can be taken as the primitive lines of the geometry.

[1] Milton K. Munitz, *Space, Time and Creation*, Glencoe, Illinois, 1957, pp. 118 ff. The greatest side of the triangle was 197 kilometres.

CHAPTER 14

THE SIXFOLD SPACE-CONCEPTUALITY (I)

'Our universe is modelled on the Intelligible World . . . which constitutes all things because it consists of perfect essence.'

Plotinos, *Ennead* vi. 7: 12

In the previous chapter the various possible spheres of implementation of formal geometries were considered chiefly as regions, of small or great extent, each requiring its own technique of measurement. We turn now to a much deeper study of space-concepts, not limiting ourselves to the theory of measurement but giving chief attention to epistemology and psychology in each of the different spheres of observation and thought relating to space and time. In other words, we shall follow a stricter phenomenological method.

For this purpose a different classification is required, omitting 'manipulative space' and associating 'cosmical space' with a 'space' (or rather, space-time) of similar general character but concerned with occurrences which are conceived of as too *small* to be dealt with in normal ways. In brief, we first distinguish between *personal* and *communal* space-time conceptual spheres, deferring the consideration of time in the personal sphere till Chapter 19. The personal and communal spheres are then each divided into *real* and *artificial* spheres, the term 'artificial' being used to indicate the presence of some conceptual activity which is imposed by the deliberating mind rather than given in nature, but which, nevertheless, does not distort what is given or replace it by something else. Manipulative space is omitted because it clearly comes under the heading of the 'artificial personal' and its technique merely duplicates, very inefficiently, what is supposed to occur in optically local space.

The 'real personal' will be considered to comprise two different conceptualities, termed *strictly local* and *extended.* Lastly, a strict and detailed phenomenological examination of the real communal conceptuality, whether cosmical, normal or atomic, would require perception of a *transcendental-analytic* constitution, relating the communal to the personal, and showing it as it 'really' is. Thus there are four 'real' space-time conceptualities: strictly local, extended, communal, and transcendental-analytic, in the last two of which time must be treated

in an integral way with space. These four are supplemented by 'artificial conceptualities', personal or communal.

Elaborate though this classification may seem, it oversimplifies a state of affairs which is still more involved and abstruse. It is, in fact, somewhat arbitrarily limited to conceptions of space and time which are more or less directly involved in physical and 'paraphysical'[1] phenomena; and the physical phenomena, at least, are limited to those which can be effectively regarded as 'mergence-free'.[2] The subjective constitution of individuals and its detailed effect on the phenomena is beyond our scope, but nevertheless, in principle, ought to be included in the table somewhere.

Likewise excluded is the study of non-physical experience which might be described as wholly or partly autonomous. An analysis of this, right up to the mystical grades, covering also its manner of occasional part-mergence in physical experience, would involve us in an enormous digression, far beyond what would be practicable here, even in outline.

TABLE OF SPACE AND SPACE-TIME CONCEPTUALITIES

I. PERSONAL
1. *Real* (A) Strictly Local.
(B) Extended: (a) telescopic, (b) microscopic, (c) paraphysical.
2. *Artificial* (imaginatively added): Local, non-local, or extended. Harmonious and assisting, misleading or conjectural.

II. COMMUNAL
1. *Real:* Cosmical, normal, or atomic.
2. *Artificial:*
Harmonious and assisting, misleading, or conjectural.
Cosmical, or atomic.

III. TRANSCENDENTAL-ANALYTIC (always real).
Transcendental Levels of Creativity, disclosed in the physical by higher degrees of phenomenological analysis.

It will be convenient to treat the topics in the following order: I.1A, I.2, II.1, II.2, I.1B. After this, the analysis will be applied, in Chapter 17, to problems of infinity and infinite subdivision. Topic III, except for various remarks or explanations made in passing, is deferred to Part III. The discussions in the present chapter and the two im-

[1] This term is intended to cover all possible means of knowledge of things in physical space and time otherwise than by the usual means of touching them, seeing them directly or through an optical instrument, or observing them by the function of spatial identification. Most instances come under the category of 'extra-sensory perception'. They are classified here only with the 'extended' conceptuality.

[2] This term is explained in Ch. 5, pp. 65 ff.

mediately following will be concerned almost exclusively with normal and large-scale spheres.

THE PERSONAL SPHERES

The Strictly Local Conceptuality

This sphere of 'real personal' conceptuality comprises all that makes the 'physical reality' of here-and-now perception spatially intelligible to us. We reach it through manifestations to the physical senses, the function of spatial identification being fundamental, while the other senses (including the sense of muscular action and movement) serve as prompting aids. By these means we obtain a *comprehensive* (but strictly local) *presentation* of the world around us in space and time. That is, we do not merely locate isolated external objects in their relationship to our own body-image; but we perceive (by a partial transcendence of physical space and time) *all* objects, including our body-image and its various parts, as a constituted *whole*, in which the function of spatial identification reveals the possibility of translations and rotations and associated differences of perspective.

In particular, our actual perspective of any specific object in this 'world' on any occasion is thus intelligibly related to its other possible perspectives. That is, the other perspectives, in potentiality for us, are implicit, or subconsciously present, in the actual perspective. They constitute, with the actual perspective, the strictly-local spatial-Gestalt of the object for us. There is also a strictly-local time-Gestalt of each object. By means of these two conjoint Gestalten of a given object, it comes to have for us, in the personal and strictly local sphere, a three-dimensional solidity in relation to our body-image and a rational continuity in time.

The Artificial Personal Conceptuality

Through the sense of sight we receive presentations of objects beyond the strictly local sphere. Then the function of spatial identification, exercised imaginatively, usually comes in to provide a comprehensive three-dimensional space-presentation of the object. What results then is a spatial character very similar to that which the object would have if it were in the strictly local sphere—so similar, in fact, and so intimately blended with the visual phenomena, that a high degree of phenomenological analysis may be needed to disclose the difference between what is 'real' and what is 'imagined'. Indeed, we do not normally make any distinction.

Thus none of the ancients seems to have doubted that the moon is a sphere. We do not, of course, sense that the moon is a sphere quite as we

sense that a tennis ball on the table in front of us is a sphere. But by studying the shadows on its surface, and recalling similar effects on the surface of the tennis ball or other solid objects we readily impose on our visual sensations the imagination of spatial perceptions which would give it the character of a comprehensive three-dimensional space-presentation. We feel that this *must* be done because, after all, the moon is an object in three-dimensional space; so instinct guides us almost instantaneously to the most likely form. In such a case it may be said that we have a *quasi-comprehensive* space-presentation. In other cases the space-presentation may be partly real and partly 'quasi'.

The essential activity of the imagination in the formation of our concepts of astronomical objects has been remarked on by Reichenbach: 'We frequently say that we can visualize the terrestrial globe; but this seems to be a mistake. . . . What we visualize, when we speak of the possibility of visualizing the earth, is a small sphere which we believe to be similar to the terrestrial globe.'[1]

It is very necessary to avoid falling into the fallacy of psychologism in this connection, believing that the visual sensation is *all* that is ever given in nature, even in the strictly local sphere, and that the comprehensive space-conceptuality is always an imaginative one, imposed by us on very near objects exactly as it is in the case of very distant ones. Even Poincaré falls into this mistake when he writes:

'It is said . . . we have the sense of the *direction* of each movement, and that this is an integral part of the sensation. . . . If this were so, geometrical space would certainly be a form imposed on our sensitiveness. But I do not see this at all when I analyse my sensations. What I do see is that the sensations which correspond to movements in the same direction are connected in my mind by a simple *association of ideas*.'[2]

Apart from the impossibility of thus reducing the exact *concept* of direction to an *empirical* association, this kind of view leads to an infinite regress. The moon appears spherical (it is said) because of association of ideas with the tennis ball. The tennis ball appears spherical because of association of ideas with a ball held in the hand. The ball held in the hand appears spherical because of association of ideas—with what? But in any case the question is begged when we speak of a ball held in the hand, since this form of words presupposes that the ball is objectively presented to us as an actual three-dimensional sphere, and is not merely a visual pattern which we have imagined to be a sphere.

[1] *The Philosophy of Space and Time*, pp. 44–5.
[2] *Science and Hypothesis*, pp. 55–6.

THE REAL COMMUNAL CONCEPTUALITY

On account of the complexity of this subject and the break it makes with the older epistemology of space and time, I shall highlight certain of the conclusions in the form of principles, continuing this plan also in later chapters where similar difficulties occur. The principles are put forward on the ground that they are attested partly by the confirmations of the Theory of Relativity and Quantum Theory, and partly by direct phenomenological analysis of objective experience, especially by the exercise of the function of spatial identification and 'primordial recollection'.[1] But first we must consider what can be retained of the 'classical scientific' view, and what must be rejected.

In the classical scientific (or 'Newtonian') view there is a one-level space and time, such that events are conceived to occur each at a definite position and time-instant, the position being fixed by three coordinates x^i ($i = 1, 2, 3$) in Euclidean space relative to a given origin and axes while the time-instant is fixed by a standard clock anywhere, allowing for the velocity of light in transit from the event to the clock. The space-coordinates for different choices of origin and axes are related by transformations of the Euclidean isometry group, i.e. translations and rotations.

If we represent the transformation from one set of space-coordinates (x_a^i) to another, (x_b^i), by the formula: $(BA)(x_a^i) = (x_b^i)$, we have also, for a further transformation to (x_c^i): $(CB)(x_b^i) = (x_c^i)$. Hence $(CB)(BA)(x_a^i) = (CA)(x_a^i)$. The *group* property of the transformations is shown by the law: $(CB)(BA) = (CA)$. That is, we can pass from one system of coordinates to another either directly or in two stages, and the final results are the same. This must obviously be so, for physical consistency.

The same facts may be described phenomenologically, in terms of the function of spatial identification, as follows. As we recognize spatial identification with various points in the real personal sphere around us, we perceive also (as implicit in their Gestalten as points) mutual relations between them, namely, distances and directions, possible rotations, different perspectives of the set of points conjointly, etc. All these perceptions have a very elaborate (but observable) structure; and the logical consistency of this structure is expressed by the transformations of the Euclidean isometry group. The mathematical expression, of course, contains much more than is immediately observable by the function of spatial identification, on account of the reference to specific origins and axes. But the intelligible constitution of spatial identification (which is the original foundation of all our concepts of position, distance, etc.)

[1] See p. 135 above. The terms 'primordial recollection' and 'primordial' are explained below on pp. 295, 408.

nevertheless includes in itself the *possibility* of choosing such origins and axes. Otherwise we should not be able so to choose them.

Precisely conceived and elegant though the mathematical formulation undoubtedly is, it partly obscures the state of affairs in a way that actually proved dangerous for later developments. What the observer deals with directly is neither a coordinate system nor a map but a succession of *sensory presentations* in his real personal space-time. The classical supposition (justified in a strictly local sphere and with objects having speeds small in relation to the velocity of light) is that the positions of the objects can always be marked on a Euclidean map, and that the various presentations to an observer as he moves about and turns in different directions can all be found (calculated or constructed to scale) by the device of supposing the map unchanged while the observer is appropriately displaced on it; alternatively the map and the observer may be displaced in any way relative to each other, and a corresponding presentation will always be deducible.

The reason why this device serves its purpose perfectly in the strictly local sphere, when the objects have small speeds, is that the map does indeed copy a *static* or *instantaneous* presentation of objects around us by the function of spatial identification (the form of this being Euclidean). The difficulty arises when there is movement, of object or observer.

In the classical scientific view it was thought to be consistent to suppose that the effect of the velocity of an observer on the presentations of objects which he observes (and vice versa) could be likewise obtained by showing the movement (or rather, the varying displacements) on the map. Scientists prior to this century failed to realize that measurement from a series of displaced positions P, P', P'' . . . each at rest on the map, might not be the same as measurement from positions Q, Q', Q'' . . . momentarily coinciding with P, P', P'' . . . but *moving* with respect to them. It is obvious, of course, that the results of measurement on any occasion, and the relationship of these results to the actual presentation, depend on the operational techniques employed. But, till this century, the operational techniques suitable for the case when the object and observer are in relative movement were not thought to require closer examination.

The truth—logically and experimentally established in the Special Theory of Relativity—is that *no communal map* can be constructed so as to show, by a simple displacement on it, what the perspective of a given observer would be, except in the very limited case when the space-time is flat and the observers are all at rest in some chosen map corresponding to an 'inertial frame'. To indicate what the presentations to observers would be, whatever motion they have, we need a triple infinity of maps, corresponding to the triple infinity of their possible velocities.

The objective conditions must therefore be taken as represented by this triply-infinite set of maps, and the determination of the presentation to a given observer is effected by a double process. First, the map in which the observer is at rest is selected, and the observer himself is placed and oriented on the map in his 'proper' configuration. Secondly, corresponding to this choice of map, modified positions and shapes of objects, light-rays, etc., and calculated and placed on the map. We can then 'read off', with further calculations if desired, the characteristics of the corresponding presentation.

If there are intermediate objects such as a prism or refracting gas, the mathematical procedure is much more complicated. The manner of emission of light waves from an object is determined with reference to a frame *A* in which the object is at rest. To determine the effect of an intermediate object, a frame *B* in which that object is at rest must likewise be selected, and the light waves, correctly transformed, must be placed in it. Then, finally, to determine the presentation to some observer, we must transform to a frame *C* in which the observer is at rest and orient the observer suitably at a certain position on the map. If there are 1,000 intermediate objects all affecting the course of the light waves, the operation of each one is in the first place determined by its 'proper' characteristics, i.e. those which it possesses in its own rest-frame.

We have now an outline of the mathematical techniques required in principle when subjective space-time appearances have to be related to a communal objectivity, and intermediate influences, also objective, have to be allowed for. The next question to ask is the phenomenological one: whether some of these stages might disappear, and the whole process becomes more intelligible, if we recognize an extension of the function of spatial identification to a function of *spatio-temporal* identification.

This will mean that we have, at the root of our sensitivity and comprehension of intelligible relations in space and time, a power to survey the time-field as a whole just as we have a power to grasp the geometrical structure of the spatial world around us as a whole; and furthermore, that time-configurations and spatial-configurations in this holistic quasi-space affect each other mutually, according to relations which are outwardly represented to us as relative velocity.

One may reasonably suppose that some power of this kind is at work, to account for the physical effects observed and to satisfy the demands of logical coherence. But one may also suspect that this 'function of spatio-temporal identification', implying a transcendence of ordinary time as well as of ordinary space, may operate in a far more abstruse way than we can at present conceive—perhaps even the term 'spatio-temporal' is misleading.

In a general way our conjectured function of spatio-temporal identification would provide the necessary ontological basis for the communal objectivity which the mathematics seems to establish (after allowing for time-lags, effects of refraction, etc.). Another difficulty, however, remains to be mentioned.

In accordance with the character of spatial identification as set forth in Chapter 10, a function of spatio-temporal identification would establish direct participation in the objective givenness of an *event*, while the visual appearances would be merely 'clues' to this participation. We would then still have to account for the special form taken by the 'clues', obviously as the result of a virtual infinity of intermediate and subjective factors. However puzzling it may be, it seems that we must admit that there is in nature a power of instantaneously integrating effects from the entire objective universe with our subjective constitution, and by the same operation creating a 'symbolical' manifestation, which (like the expressions on the human face) contains the contributing factors as a subconcious background and 'explanation' of its quality.

Potential Group-Objectivity (in Flat Space-Time)

In spite of these overwhelming complexities to which a deeper study of *objectivity* seems to lead, we may perhaps venture to formulate some general conclusions as follows.

The Principle of the **Total Object-Gestalt** (I suggest) asserts, first, that each object 'carries' a conceptual space-time (with Euclidean space-constitution) which is called its *proper* frame or space-time field, meaning that the object is at rest in it (we pass over for the present the fact that the object may have parts which are in relative movement). In addition, the object carries the possibility of infinitely many other space-time fields in each of which it has a corresponding (potential) configuration. These other space-time fields, in motion (so to speak) relative to the proper field, present the configuration of the object in relation to other objects.

The possibility of all these space-time fields and configurations is implicit in the total conceptuality of the object, rather as other visual perspectives of an object are implicit in the one actual perspective presented to us on any occasion. For the Poincaré Group, representing the properties of flat space-time, is a conceptual whole. It should be remarked, however, that under ordinary circumstances the Gestalt of which we are conscious does not reveal to us any evidence of variation in shape (Lorentz–Fitzgerald contraction) or in time (Einstein time-dilatation) as a result of relative motion.

The Principle of the **Total Subject-Gestalt** asserts that every observing subject must similarly be conceived to carry a proper space-time field

and infinitely many other potential space-time fields and corresponding configurations of itself. 'Incoming presentations' of objects around are actualized for the subject in his proper space-time field; but the other fields are presupposed as possibilities of a changed constitution in relation to other objects.

The Principle of **Intermediate Transformations** asserts that for the actualization of any object to any subject an elaborate process of selection between the space-time fields of given object, intermediate objects and subject, must occur, with an integration or synthesis, the final result of which is an actualization for the subject according to the total circumstances on each occasion of observation.

Because of their intimate connection we may comprise these three principles in a single principle of **Potential Group-Objectivity.** The term 'potential' is introduced here in order to emphasize the fact that there is no actualization of a particular perspective of an object except when a particular observation is made by a particular subject. The epithet 'group-' is attached in order to emphasize the fact that the possibility of operations of rotation, translation and relative velocity, consistently combinable, is implicit in the concept of an object or subject.

Mathematical and physical analysis, confirmed by experiment, and assisted with phenomenological analysis, has thus shown how to reduce the concept of an 'object' to definite and mathematically precise form. The analysis has been made in the case of flat space-time. As regards the concept of an object in a more general space-time—if it turns out that a more general space-time which is not 'artificial' is really provided by nature—all we can say is that this will certainly not be simpler.

Transcendentally Operative Objectivity

We must now try to take a more consistently phenomenological view of the matters just discussed. The chief difficulty is as follows.

We have proposed an Object-Gestalt which has formal characteristics different from those of its presentations to particular observers. Yet how can we come to know anything objective except from our standpoint as a particular observer, thereby limiting or influencing it in some way? What kind of abstraction is it, by which we are supposed to pass from subjective variations to an objective constitution?

In arriving at the four principles just enunciated we followed a two-fold track. We depended *inferentially* on the mathematical formulations of the Euclidean isometry group of translations and rotations and the Poincaré Group which includes also the Lorentz transformations corresponding to uniform relative velocity of observer and object; and we depended *phenomenologically* on the mental power which we possess to perceive the three-dimensional Gestalt of an object (say, a cube) by the function of spatial identification in such a way that there appears

no essential preference for any one perspective over the others. The final step was to identify the inferred mathematical objectivity (quite devoid of subjective variability) with a Gestalt postulated as having a *spatio-temporal* constitution but otherwise similar to the perceived Gestalt which consists of the infinity of potential *spatial* perspectives also constituted without essential preference for any one over the others.

In view of the fact that we seem forced both by direct evidence and rational inference to adopt this unifying procedure, as well as other facts to be considered presently and also the whole trend of these discussions, I suggest we are justified in accepting a Principle of **Transcendentally Operative Objectivity,** to the effect that we must acknowledge, and can in principle find, a *givenness* common to or operative in the various perceived effects of an object identified and observed by various persons. Such givenness must have a constitution of *ontological* and not merely *psychological* character; and since it is not subjectively limited, it must be transcendental in character and therefore found by transcendental means.

In accepting this principle we shall also be accepting one of the chief applications of a broader Principle of **Subject–Object Transcendence.** This, I suggest, asserts that there is a perceptive condition in which what is perceived is not strictly to be denominated either 'objective' or 'subjective', but is constituted in a *subject–object middle ground.* The most important features of this 'middle ground' are that (1) all structures in it are conceptual, exact, and apodeictic, (2) it is 'out of' ordinary space and time, in the sense that it is not tied conceptually to spatial positions in the physical world or to the fixed succession of instants in the onward urge of physical time, and (3) it is contemplated by a 'released' and 'universalized'[1] consciousness, whose perceptions, in their essential character, are quite independent of the state or tendencies of the 'personal' consciousness.

This 'middle ground' may be identified with the *intelligible world* of Plato and Leibniz (or rather, the part of it which is in direct causal relation with physical phenomena). For its 'contents' are in principle accessible *in the same intelligible form* to all observers who have adequate powers of discernment and have orientated themselves suitably to the object in question. The middle ground may be said to 'pass into subjectivity' when the conditions of observation of a certain observer at a certain time actualize it as the perception of a *primary perspective.* On the other hand, it may be said to manifest a 'real communal objectivity' when a certain structure of it is recognized by the 'released' mind as springing from a potentiality independent of the particular subjective standpoint taken.

[1] See pp. 218, 343, 352 below.

Cohesion

The term *cohesion* is useful here to express the integrating activity in nature and humanity, whereby the effects of objectivities throughout the universe (in contemplated object, intermediate objects, and observing subject) are combined to produce a manifestation on any particular occasion. It is clear that there must be some intelligible connection or mutual participation of objects in each other, so that all the conceptual potentialities can be rationally combined and conjointly actualized. (We cannot suppose that a material product of one object, such as a wave emitted by it, is later combined with material products elsewhere; for there is no particular actualization till the observation occurs, everything being in the 'middle ground' till then.)

The Principle of **Universal Cohesion** may be taken to assert the real intelligible relationship of every Object-Gestalt to every other Object-Gestalt, prior to their conjoint manifestation in space and time. It normally appears to us, of course, that objects influence each other in a container of space and in the inflexible onward march of time. But the truth, to which this principle testifies, is that the background of nature and humanity is indivisible, and all is taken into account when a manifestation is presented to us.

It will be helpful later on to distinguish three kinds of cohesion. By *general intersubjective cohesion*, I shall mean the intelligible connection (transcending space and time) between different observers who observe the same object at the same proper time for the object. The fact that they agree between themselves that they are observing 'the same' object indicates that the two observers and the object are somehow in the same 'intelligible world'. For to suppose that the judgements of 'sameness' were mere psychological or physiological reactions would be the Abacus Fallacy (since the meaning of these reactions for us presupposes comprehension of a prior ontological sameness). Nevertheless our conclusion, in such a case, is a kind of inference.

By *particular intersubjective cohesion*, I shall mean the communality of one person observing and participating in another by 'empathy', a function of the mind which will be briefly considered in section IV of this chapter. An *interobjective cohesion* will be the cohesion between objects which proceed according to natural law and influence each other when no one is observing them. The regularity of the changes between one observation and another indicates that the conceptual connection represented must be taken as operative in the interim.

Elementary quantum mechanics provides a good analogical exemplification of interobjective cohesion and the process of actualization in a particular observation. An individual space-time frame of coordinates x, y, z, t must first be chosen. We then have a function $\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, t)$ which expresses the cohering potentialities of the system under con-

sideration. The coordinates $r_1, r_2, \ldots$ are associated with one or more 'particles', each of which has thus a space-time field in which corresponding 'particle-events' can be placed (the various particle-fields must at this stage be kept distinct). The various space-time fields cohere in two ways: (1) their coordinates appear combined in a certain way in the wave-function, and (2) ultimately each is to be identified with the actualized space-time in which the observation is made. This ultimate identification is, however, necessarily an idealized mathematical one; for the space-time of the observations is empirical, while the space-time of the particle-fields is conceptually precise.

The wave-function is specified by various procedures. First we require a characteristic mathematical form which has been found appropriate for the type of objective system with which we suppose we are dealing. Then terms of a certain form are added to express interaction between the 'particles' (this is a special kind of interobjective cohesion). Boundary conditions, expressing in a simplified way the effect of the wider world beyond the region immediately around the system we have singled out, must also be imposed. A simple form of these is necessary to make the calculations manageable. The wave-equation finally specifying Ψ is an 'operational equation' which also determines *possible energies* of the system when it is observed or gives rise to manifestation in any way; and any measurement of a manifestation actually observed is the 'proper' value (eigenvalue) of a 'differential operation' corresponding to the type of measurement applied. The actualization (i.e. observation) on any occasion is completed by the appearance, unpredictably, of one of the possible eigenvalues.

THE SUBJECTIVE ENCOUNTER

Our attempt in the foregoing section to clarify the concept of a communal object left out of account the question of the subjective recognition of such an object, as distinct from the study of its spatial and other features *once it has been recognized.* The manner of recognition of objects and their properties has been incidentally discussed in Chapters 4 and 10. Here a more systematic presentation must be attempted, with attention given also to some subsidiary problems which will turn out later to be of great significance.

The Principle of **Perspectival Manifestation upon Occasion** states the fact (obvious, if the term 'actual' means 'personally observed') that no space-time perspective is actual for an observer till his personal standpoint is brought into relation with the Object-Gestalt. For the present, we take the term 'Object-Gestalt' as sufficiently clear from the previous discussions or from commonsense. It should be noted that the term 'actual' has no application to occurrences which are imagined to happen

between one observation and another. If therefore we set up in our thought a one-level container of space, with a single flowing time for all places in it, and then (imaginatively) set objects and other physical structures at exactly determinate positions and moments of time in this container, we shall have formed an entirely false picture of the workings of nature. Neither physical method nor phenomenological analysis warrants such beliefs.

Quantum theory provides a clear exemplification of 'manifestation upon occasion' in the Principle of the Indivisibility of the Phenomenon, which asserts that neither a wave nor a particle (in the classical sense of these terms) can be said to exist till an experiment is set up and considered as a whole[1]; then observation may result in an appearance of one or the other. The puzzlement which many people feel at this and related peculiarities of quantum theory may accordingly be imputed to the complete break which the deeper metaphysics of *potentiality* and *actuality* makes with the 'classical scientific view' inadvertently assumed by scientists even in quantum theory.

Now comes the question of *what* exactly is encountered on any occasion. Regarded operationally, how does our mind proceed when we recognize an object?

The Principle of the **Total Encounter** asserts that an 'object' is recognized as a totality and that we can then proceed to discover perceptual details, each of which becomes a *focal actuality* as we bring it to the forefront of our attention; that there is also a *primary perspective* which is 'focal' in a less strict sense, consisting (as it appears) of a continuous field of details all of which have a *present vividness* of immediate presentation (in spite of the fact that only one detail at a time can be strictly focal); and also, that we can transcend the limitations of the primary perspective by *supraperspectival insight*, which presents other perspectives which are 'in potentiality', and which thus make each three-dimensional object intelligible to us as such.[2]

The last of these conclusions was drawn in Chapter 10 from considering, in particular, the perception of a straight line and the necessity for a *Gestalt* if it is to be intelligible. The Principle as a whole was also implied in Chapter 4 by the recognition that to regard objects as made up of sensea-data, first given separately, and then pieced together and interpreted subjectively, is to fall into the error of psychologism.

The essential character of such an 'encounter', however, cannot be made very clear until light has been thrown on two subsidiary problems. It must be admitted that, as a matter of everyday experience, we *can* focus our attention on some details or intelligible feature of a certain object which we have recognized, and that other details are at

[1] See p. 332 below.
[2] See p. 131 above.

the time perceived faintly, so to speak, in the periphery.[1] It still remains, however, to consider (1) how we decide that two focal details, although observed at *different times*, belong to the *same unchanging* Gestalt; and (2) how we decide that two focal details *differently located* belong to the same Gestalt, and not to two different Gestalten fundamentally distinct (in the sense that different objects are constituted).

The only way in which these questions can be rationally answered, I suggest, is by admitting again the possibility of an intelligible *transcendence* of physical space and time. Then what occurs in physical space and time can be contemplated from a 'higher' level of quasi-time and quasi-space. A *hierarchical* view, in short, must be adopted. The apprehended apartness of physical objects in physical time and space becomes then accountable as the result of a shifting of attention (at the physical level) over an *always present* givenness of constitution (at a higher level) with countless interwoven ties of rational unification, potentially spatial and temporal. The possibility of such transcendence has already been discussed in the context of the 'middle ground' and considered in Chapter 8 as a fact of experience.[2] It will also be examined more thoroughly under various heads in Parts III and IV. In the meantime, to complete the present consideration of the Total Encounter, it seems best to indicate, very concisely, the broad lines along which solution of the two subsidiary problems just mentioned might be found, in accordance with the hierarchical view.

First, an *unchangingness* of the object encountered, in spite of change in *physical* time, becomes knowable by 'primordial recollection',[3] which takes us to another time-level, where durations and other time-relations in the constitution have a somewhat space-like character. That is, one event is not destroyed as another is created in its stead; but we can return to contemplation of the former event in all its identifiable intelligible character. It is not then an imaginative reproduction of an event which is itself now past and gone, but a re-contemplation, from a freer time-level, of the *same* event.

The 'Name'

The second problem is how we are able to single out an object as a certain thing encountered, recognizable as a unity, even though the focus shifts continually and the details are differently located. This is possible, I suggest, because of our participation in an intelligibly unifying or creative power, which expresses itself outwardly to us not only in the observable appearances but also in *names*. Each of the perceptual details has a 'name' in so far as it can be recognized, and the same applies

[1] Husserl's theory of focus and periphery (Horizont) is dealt with in Ch. 24, p. 406.
[2] See pp. 108 ff.
[3] See footnote on p. 204 above.

to every grouping of details recognized as a whole. The intelligible structure flows, so to speak, into some 'outer name' (thought, heard, spoken, read) which is for us the concrete groundwork of its recognition. Then, when we abstract from that groundwork, we rest in an 'inner name' which is the more perfect intuitive presentation and purer recognition of the corresponding intelligible structure.

The 'inner name', according to the view I am presenting, is the necessary link between the *will* and intelligible structures regarded as pure essences in complete abstraction from the individual mind. 'Inner naming', if we follow Husserl's terminology, is the *intentional act*[1] of recognition, on the side of the subject. But this 'will' or 'intention' has for us no character of *effort* or *purpose worked out*. Anything in the nature of effort would impose a false psychological colouring. It is, on the contrary, nothing but a *direction* of the pure consciousness[2] inspired by the needs of its developing personal structure and the objectivities available for observation.

The 'name' functions in a manifold way and at various levels in the hierarchy of intelligible life. We extend and deepen our analysis of any state of affairs, presented to us objectively or subjectively, by discovering subsidiary 'names' which are intelligible constituents in the greater 'names' of the features more easily recognized.

A broadly recognizable but complex structure like that of the human body and mind may be likened to a musical composition. At a first glance we can do little more than attach a single name and take note of a few general impressions. With more familiarity, and a desire to understand, we may note rational connections of innumerable kinds, elaborately interwoven with each other—reminiscences of motives occurring in an orderly way, a tendency to fall into distinct phrases each of which has a subtly logical course of development considered in relation to the whole, a concurrent 'vertical' structure of harmony, which also helps to guide the horizontal flow, the whole also taking specific form only in accordance with the instrumental possibilities.

Several important conclusions may be drawn from this analogy. First, a given conceptual structure cannot become even passably clear in our minds unless we study it as we would study a musical composition in order to understand it. We must realize that every detail of it posesses its significance by virtue of its rational connections with the whole and also with what lies 'outside' that whole—for example, the structure and tendencies of the human mind, either absolute or determined by an historical and social background.

Next, we can note that although intermingled rational connections are everywhere to be found, it is possible to mark off (but usually in an

[1] See pp. 405, 408 below.
[2] Pages 109, 406 below.

imperfect or somewhat conventional way) particular *unities*, such as phrases, chords, instrumental techniques prescribed, etc. The apparent isolation of any such unity obviously results from a *clouding over* a large class of its conceptual connections, with the result that in extreme cases (such as the perception of a physical object) the unity may seem to lose its conceptual character and become a non-mental 'substance'. Thus the unity of a conceptual structure seems to be an *externally imposed* one, brought about, as it were, by a creative process of obscuration, 'dying to spirit', or enveloping with 'primordial matter'.[1]

Finally, we may conclude that if there are distinctly 'nameable' structures in the background givenness of physics, each is to be considered rather as a nexus of potential relations in a general conceptual field. The 'simple location'[2] of a supposed self-contained entity dropped into a deconceptualized void, and progressing from point to point and time to time, is an irrational myth, a 'living skeleton'.[3]

EMPATHY

An account of communal objectivity cannot be complete without some consideration of the fact that 'communality' presupposes the existence of *different observing minds*, able mutually to identify phenomena as referring to 'the same' real object.

The difficulty which meets us first here is in clarifying the expression 'different minds'. An *inference* that a mind other than ours exists would have meaning for us only if we already possessed the concept of 'another mind'. We cannot infer something of which we can form no conception. But if our concept of 'another mind' is purely a figment of our imagination, arising privately within us, it would not be the concept of another mind in the sense which is demanded by our experience of the real communality of objects. How can we say that the same objects are perceived by various *real* people who are different from us and at the same time suppose that these 'people' are only inventions of our imagination? The question to ask is, therefore, What is the *really* given concept of another *really existing mind*, and how, precisely, do we distinguish it from 'our own mind'?

To frame the problem this way is tantamount to saying that it is one that can be solved only by the phenomenological method. We must set before ourselves, in recollection, a certain familiar type of experience which every normal person would call 'real'—namely, the experience of

[1] 'Matter' is to be understood here in the sense of Plotinus and the Scholastics. It is both the principle of individuation, and a principle of obscuration or limitation of the intelligible. See, for example, Ennead i. 8.

[2] On the 'Fallacy of Simple Location', see A. N. Whitehead, *Science and the Modern World*, Cambridge 1926, p. 72. Also below, p. 289.

[3] Above, p. 99.

'communion' (of feeling and thought) with an intimate friend. And we must set ourselves to penetrate to its essential substance and nature, as distinct from sensory appearances and impressions of spatial position in the physical world. For evidently the 'reality' which the other person has for us is more than just these outward showings.

As a preliminary step, all confusions of thought arising from the postulation of a 'container of space' and the 'simple location' of 'entities' in it must be thrown out. This means that we must also suspend belief in the Causal Theory of Perception, which postulates a temporal succession of simply located causes, proceeding according to strict formal laws, in the container of space. After freeing our thoughts from these prepossessions, our attention must be focused, as keenly as possible, on those varieties of direct experience just mentioned, in which we recognize a division between what we call 'our own' and what we conceive of as *not* our own but proper to another person.

Scheler has drawn useful distinctions between fellow-feeling (Mitgefühl), projective 'in-feeling' (Einfühlung), contagion (Gefühlsansteckung), and genuine participation (Einsfühlung).[1] For the purposes of the present investigation the first three of these varieties have interest only so far as the experience of participation can be abstracted from them. Our conclusions concerning participation will be very similar to those of Scheler.

Before entering upon the main part of the discussion, it may be best to deal with a certain curiously delusive argument—namely, that since all our experience is 'our own', *ipso facto* it cannot be the experience of another person. This might have weight if we had previously decided, with good reason, that the experiencing consciousnesses of two people *cannot* blend and become, in a real sense, one consciousness. But the possibility of such blending is the very point which we are aiming to establish. Hence the argument obviously begs the question.

Now a blending of consciousnesses can be reasonably proposed only if, from another level of contemplation, a certain distinction comes to light. What I shall maintain, therefore, is that when we achieve a certain high degree of detached observation we find that a perception (most commonly of a feeling or intention) appears, from one standpoint, as if 'qualified' by a personal constitution which we call 'our own', while from another standpoint it appears qualified by a personal constitution which is *not* properly called 'our own'; and furthermore, that such perception, when limited to *act* and *object* (and so deprived of all personal 'qualification') is seen to be apodeictically and really *the same*, whatsoever personal 'qualification' may at other times be attached to it.

[1] Max Scheler, *Wesen und Formen der Sympathie*, 3rd ed., Bonn 1931, section A. II.

If there were not, within each individualized consciousness, this possibility of a single 'pure consciousness' or Universal Mind, it seems we should have to adopt a world-view in which all mental powers—the ability to form concepts, perceptions of objects, and so on—were duplicated in every one of all the numberless multitudes of conscious beings in such a way that each set of powers was absolutely sealed off from every other set, with no possibility of knowing whether the powers were or were not really duplicates of each other. We could never know whether 'blue' for one person might not be the same as 'green' for another, or indeed whether 'seeing' for one person might not be a kind of 'hearing for another', or the essential quality of the feeling of 'love' in one person might not be the same as the essential quality of the feeling of 'hate' in another. Questions concerning 'essential quality' would in fact be meaningless, because there would be no absolute criterion of 'sameness' or 'difference' of quality for different people. Yet the order constituted by the associated actions and bodily effects demands some intelligible linkage; and if we may not admit a single 'pure consciousness', it seems we must *postulate* an unobservable *duplication*—a duplication, as it were, without any primordial pattern to duplicate.

But there are numerous arguments indicating the absurdity of such a pieced-out world-view. Let us therefore proceed to the claimed evidence of experience.

We are all familiar with the experience in which we seem to project our own feelings or sensations of movement into the imagined situation of another person. He may be a cricketer wielding a bat, or a friend who has been made the subject of a cruel practical joke. If we feel that there is *distance* between ourselves and him, or that we, in our usual personality, have been transported into another situation, we are not experiencing primordial participation. We are still *subjectively-fixed* in our own personality; we have not adequately transcended the spatial separation of the other person from us. We are still tied to a 'lower self' or 'personality' which presents itself as nothing very definite, but, rather, as a shifting ground of localized identifications thought to be with parts of our physical body or occurring in our heads. We feel the swing of the cricketer's arms in our own body, so to speak; and we feel in our own heart the hurt of our friend and his courageous resolve to 'put a good face on things'.

But now we must realize that the swing of the arms which we think we feel in our own body is not really the swing of our arms;[1] the imagined location of the movement in our body is a sheer delusion. The impression of movement, as such, is not located anywhere in the physical world.

[1] Unless we are 'contagiously' induced to cooperate to the extent of making actual movements of our body.

Likewise our friend's courageous resolve is not our own courageous resolve. We have nothing to be courageous about.

Putting aside, therefore, all *location* and *appropriation* of movements and feelings, and supposing that this is done to such an extent that it is impossible to say whether we are in or out of the body, what remains is a 'pure consciousness' which can take on various personal qualifications and appear in various personal situations, but which (in its own pure essence) transcends appearances in space and time, being primordially *released* and *universalized.*

The released and universalized consciousness, if we consider it in the light of direct experience, is by no means a colourless and static Absolute. It is the real core of our consciousness, opening out into universality when the binding accretions of personality are cut away. Moreover, it acts according to a higher 'I-direction', which (because the intelligible world is the pattern for everything) must even be identified, in a certain way, with an intelligibly unique or *supreme* 'I-direction'. In consequence, when we speak of 'release' and 'participation', the terms 'self', 'personality', 'I', 'mine', etc., are all in various degrees inappropriate and misleading. For this higher 'I-direction' can, 'at will', recognize a temporary personality associated with a specific physical body ('its own', for the time being); or it can abstract from this and recognize a temporary personality which appears associated with another physical body.

But in our ordinary state of mind, the social presences around, and other conscious trends not properly identified with our physical body, may be blended into a whole, and then to some extent fixated in 'our own' physical body, so that the power of separating the ocean of pure consciousness into its personal sources is lost.

Thus Scheler seems to me correct in stating[1] that this ocean of pure 'unselfed' consciousness is given 'first' (in the order of reality[2]), and that 'then' (at a lower level of manifestation) a separation is made between, on the one hand, that which belongs properly to our 'own personality', and, on the other hand, impressions which belong properly to 'another personality'. These impressions are then in essence genuinely 'the same' as those experienced by the other individualized 'I-direction', but are covered over by the fixations in which its personality subsists distinct from the universal ocean of pure consciousness.

[1] *Op. cit.*, Section C. III.

[2] See p. 372 below, and the reference (note 3) to A. Boyce Gibson.

CHAPTER 15

THE SIXFOLD SPACE-CONCEPTUALITY (II): THE ARTIFICIAL COMMUNAL CONCEPTUALITY OF SPACE-TIME

'When we speak of *place*, it is only in relation with our visible world. But the Universal has no need of being in a place, nor in anything whatever . . . What has been built on the Universal, being other than it, participates in it and approaches it, receives strength from it, not by dividing it, but because it finds it in itself.'

Plotinos, *Ennead* vi. 4.2.

'The space-time frame is therefore something overlaid by the observer on the external world . . . We must endeavour not to lose sight of its fictitious and arbitrary nature.'

Eddington, *The Mathematical Theory of Relativity*, pp. 8–9

Up to this point our analysis of space-conceptuality has been outwardly concerned with phenomena in 'strictly' or 'optically' local space. We have been led to the view that there is a communal and non-preferential multiplicity of conceptual but 'real' spatio-temporal fields (conceived of as with different relative velocities), in which all objects and observing subjects participate (according to their situation, state of motion, and subjective constitution). Out of this 'cohesion', there arises for each observer, by 'integration', a corresponding physical manifestation.

The bed-rock origin of the Theory of Relativity can be said to lie in the recognition of the necessity for such a non-preferential set of fields, and the discovery of a precise mathematical expression of the relations in it by the formalism of group-theory. In the case of local spheres (flat space-time), the group in question is the Poincaré group of transformations.

Although, so far as the local spheres are concerned, we have thus found objectivity in nature to be expressed naturally and inevitably in a certain group-formalism, when we turn to consideration of the entire cosmical sphere (astronomical universe) we shall find that whatever non-preferential set of 'fields' there may be, it is *not* naturally and inevitably expressible in any particular and comparatively simple mathematical formalism. We shall find that the creative ingenuity of the human mind must cooperate with nature in producing a mathematical and representa-

tional conception adjudged most 'appropriate' in the circumstances to which we have had to limit ourselves.

The Cosmical Non-Preferential 'Field'

The first question is whether we can say anything useful about the supposed cosmical non-preferential 'field' constituting cosmical 'objectivity', and might thus be guided in the construction of an appropriate mathematical and representational formalism. We must of course keep as closely as possible to a strict phenomenological analysis of what is given in nature and what we conceptually impose on it by free or partly free choices.

In physical practice, virtually all astronomical objects, as we conceive of them at their calculated distances, are beyond doubt inaccessible even in the distant future. The nearest star is nearly 4 light-years away; the radius of our galaxy is said to be of the order of 6,500 light years; and the farthest astronomical objects whose distances can be estimated are said to be at a distance of 2×10^9 light-years. We must admit that all these objects are in a certain way *given in nature*, their manifestation to us being, as always, limited by our physical condition. Yet we cannot, by nearer approach, know them by any kind of spatial identification in a comprehensively grasped Euclidean space.

An attempt is made in the next chapter to consider the nature of such 'givenness' as an 'extended conceptuality'. For our present purposes what has to be noted is that, with the falling away of the possibility of spatial identification there falls away also the strict meaningfulness of Euclidean geometry as a characteristic of the world. Its derivative meaningfulness in optically local space, as a method of representation, also falls away, unless we can show that this is the most 'appropriate' geometrical formalism and representation to adopt.

The view to which astronomical investigations have led is that an attempt to force observations into a formalism and representation according to flat space-time is an uncalled-for restriction of view which impoverishes research and deprives us of remarkable unifying explanations. We have only to approach the problems more generally to see the epistemological correctness of freeing ourselves from such restriction.

Taking the more general view, therefore, we must accept that the givenness of astronomical events is not 'spatial' in any local sense; and because, beyond the local sphere, space and time are interrelated, this givenness is also not 'temporal' in any local sense. From this we infer that astronomical events are not the manifestation of any spatio-temporal field in the usual sense of the term. The Cosmical Non-Preferential Field which is to be regarded as constituting cosmical 'objectivity' must be a *potentiality for local spatio-temporal manifesta-*

tion given the particular locality concerned, and not anything spatio-temporal in itself.

A further conclusion is that such cosmical field is, in a normal state of life, quite indiscernible, whether as perceptual details or as substructure, or even by any ordinary degree of phenomenological insight. We must conceive of it as essentially beyond measure, but nevertheless expressible in local spatio-temporal measures. But what we are now aiming to do, if possible, is to find an expression in *cosmical measures.*

Our problem therefore seems to reduce to finding some appropriate mathematical and spatially representational means by which local spatio-temporal representations may be linked and unified, although there is not given in nature any such *spatio-temporal* unification. In other words, we have to devise an *occasional geometry*, introducing many mathematical operations not given in nature, or other redundancies, and chosen from among a variety of possible prescribed geometries which in turn may be implemented in various ways.

The corresponding conceptual view of the universe thus embodies *imagined* variations and additions, beyond what is actually observed outwardly or as substructure. It must accordingly be described as 'artificial'. And any Euclidean representation which we make of it, or of widely separated portions of it, will be a 'model' with obvious deficiencies.

It must be kept in mind, also, that this work will be concerned only with the objective elements in manifestation. The necessary 'integration' with subjective physiological and psychological elements is totally ignored.

Universal Representations

The view of the universe corresponding to an *occasional* geometry may be described, in a general way, as a 'patchwork' one; for the presumed conceptual field uniting the local 'patches' is neither spatio-temporal nor even conceivable by us in any normal state. But as we have seen, it may be possible to set up in imagination a preferred class of observers, one at each point like particles in a fluid, in such a way that the representations of the strictly local spaces fit together in a universal Euclidean representation. This is even more decidedly an 'artificial' way of conceiving of space and time; for in addition to reducing the non-preferential space-time field to particular models by prescribing and arithmetizing a suitable geometry, we have exercised another choice in deciding on a particular 'observer gas'.

What is the justification for forming such a universal (Euclidean) representation? It can be urged that the cosmos is not in fact a patchwork of local views, with connections which are abstract merely and unrepresentable in any map of the universe. If we were to travel in a space-

ship from one part to another we would actually perceive a continuity of change from one local view to another. It is even possible to imagine a network of *stations* at each of which our space-ship stops in turn, taking a record of the local presentation of the universe at each station, and then (after necessary adjustments of scale, etc.) piecing these records together in a representation of the universe as a whole. Something of this kind may be not only *possible*. From the practical point of view it is *necessary* if we are to form some conception of what is to be expected in various parts of the universe and how our research is to be guided. Otherwise we should be like explorers without map or compass, our knowledge being limited to such regions as we have somehow come to investigate.

Derivation of the Metric

The necessary postulates concerning universal representation will be formulated *seriatim* in Chapter 18. In the present chapter we are concerned with exhibiting the technique by which a family of universal Euclidean representations of the kind just described can be constructed, corresponding to a suitably contrived measurement-tool.

There will be one member of the family for each observer of the chosen 'observer gas', without preference for any particular one. It is reasonable to require that the family of universal representations should give no preference to any particular observer or observers of the 'gas'; otherwise the representations would embody preferences which are absent in the conceptual space-time field which is to be represented. The observers of the 'gas' are, of course, preferred over other observers; but this is merely because it would be impossible to construct any universal representation without limitation to some particular 'observer gas'.

It is also reasonable to require a conformal representation, because any map which distorts shapes locally, by introducing differences of scale in different directions and so altering angles would be misleading and not in fact a composition of local views. But the technique we are led to devise brings about conformality without our having to introduce a special postulate to that effect.

Since we are using some geometry as a measurement-tool, there will be a coordinate system x^1, x^2, x^3, x^4 which we are using throughout space-time, each coordinate being operationally defined in such a way that the geometry may be applied consistently to any phenomenon in the universe. The difficulty of checking consistency is one that we must pass over, on account of the lack of a variety of means for estimating stellar distances and for observing angles from different positions in the universe. So far as our knowledge goes, we suppose that the definitions, along with any other necessary physical theories, give consistent results.

Geometrical assumptions cannot be avoided in drawing up operational definitions for measures referring to distant parts of the universe; hence the finalizing of these definitions must be left till, on prior grounds, we have decided on the type of formalism required; then definitions and formalism will be finalized together.

It must be remembered that we are now proposing to construct a universal representation and not merely a formalism which relates local measures but is not itself spatially interpreted. We must therefore show three coordinates on a Euclidean map, supposed to apply for a given 'cosmical time' t. Let these coordinates be called x, y, z and represented as orthogonal Cartesian ones. Each is a one-valued space-like function of the four general coordinates x^i required to fix an event in space-time. It is usual to represent x^1, x^2, x^3 as spherical polar coordinates r, θ, ϕ on the Euclidean map, on the ground that non-preference among the various directions at the origin is ensured by choosing a metric which is spherically symmetric in form. Further consideration, however, will show that this complicates the argument unnecessarily.

It is clear that our choice of a representation must be guided by certain principles. To begin with, the geometry must be prescribed if it is to be operationally implementable as a measurement tool; and secondly it would be absurd to choose a representation whose geometrical properties were needlessly in conflict with the properties of the space-time field to be represented. For the present let us speak of the choices simply as having to be made in an 'appropriate' way. Stricter justification is deferred to Chapter 18.[1]

If our representation is to be appropriate to a non-preferential space-time field, the metric should take the same form whatever origin is chosen and whatever directions are chosen for axes at a given origin. We cannot say that the metric will take the same form whatever the velocity of the observer, for, as shown above, we must specify a particular observer gas in order to obtain a universal representation. The choice of observer gas is part of the method of representation, and is in fact already made when we stipulate that the 4-dimensional metric preserves its form when we change from the coordinates x^i used by one observer to the coordinates $\bar{x}^i$ used by any other belonging to the 'gas'. It being understood that each observer is placed at the origin of his coordinate system, a change of origin means a change of observer and a change of coordinate system.

We next have to show that the geometry which is being represented in this way must be Riemannian characterized by a differential quadratic form which is invariant (the 'metric'). Assuming the possibility of implementing a flat space-time formalism in any locality sufficiently small (i.e. optically local space or smaller), for the reasons given in previous

[1] 'Prescribed' in the sense of Ch. 7, pp. 95 ff.

chapters, the coordinates x^1, x^2, x^3, x^4 must be such that, by transformation to any one of a set of local reference frames, with coordinates X, Y, Z, T, we establish the invariance of the quantity $ds^2 = -dX^2 - dY^2 - dZ^2 + dT^2$ for all such local frames. Substituting for X, Y, Z, T in terms of x^1, x^2, x^3, x^4 we obtain the invariant quadratic form $ds^2 = g_{ij}\, dx^i dx^j$, where the functions g_{ij} express the correlation of units for ds in the different localities, and hence ultimately determine the prescribed geometry necessary for finalizing our method of measuring the universe. We can now transform from x^i to other systems $\bar{x}^i, \bar{\bar{x}}^i, \ldots$ one of which will be the system x, y, z chosen for Euclidean representation, together with a cosmical time variable t giving the 'time' at which the map is applicable.

The metric for these variables may now be put down in the form:

$$ds^2 = -d\sigma^2 + \psi(r, t)\, dt^2 + 2\,\chi(r, t)\, dr\, dt \qquad (1)$$

where $d\sigma^2$ is a quadratic form homogeneous in dx, dy, dz; for the following reason. The formula for ds^2 must not be affected by interchange of the directions chosen for the axes x, y, z; otherwise the representation will embody preferences which were absent in the non-preferential spacetime field which it is supposed to represent, and so will be 'inappropriate'.

The next step is to show that the term in $dr\, dt$ must (and can) be removed by a substitution. Consider a local 'inertial' frame (i.e. with $ds^2 = -dX^2 - dY^2 - dZ^2 + dT^2$ for local Euclidean variables X, Y, Z) momentarily at rest in the representation, i.e. $dx/ds, dy/ds, dz/ds$ are all zero. Then the proper time in this frame, $dT = \psi^{1/2}dt$, and the proper distance squared is $dT^2 - ds^2 = \psi dt^2 - ds^2 = d\sigma^2 - 2\chi.dr\, dt$. The proper distance is therefore not given by a metric in three space variables, with the time variable appearing at most in the coefficients (causing a slow change of scale in the representation). It cannot therefore be represented on any Euclidean map, but a complicated calculation must be made, involving a non-zero dt for the observation. If, however, $\chi \equiv 0$, a representation immediately becomes possible. Tolman has shown[1] how the appropriate mathematical substitution may be determined if the functions ψ and χ are known; so that it is no limitation of the formalism, as regards its tensor properties, to write

$$ds^2 = -d\sigma^2 + \psi(r, t)\, dt^2.$$

The equations of geodesics give the world-lines of ideal particles in the geometry whose positions x, y, z and 'velocities' dx/ds etc., may be specified in any way we wish and whose 'accelerations' d^2x/ds^2, etc., are then determined by the metric. Let us take such a particle as coinciding, at a certain cosmical time t, with one of the observers of the 'gas', as

[1] *Relativity, Thermodynamics and Cosmology*, Oxford 1934, p. 365.

regards both position and velocity. This means that at time t both x, y, z and dx/ds, dy/ds, dz/ds are zero. If the acceleration components were then not all zero, we should have a lack of isotropy resulting from the metric, which therefore would be 'inappropriate' to represent a non-preferential space-time field. We can therefore show, again as in Tolman, that the metric must be reducible to the form

$$ds^2 = -d\sigma^2 + dt^2 \tag{2}$$

It is to be noted, however, that in this deduction we have made no appeal whatsoever to dynamics, nor are we supposing the existence of any mass-particles in the universe. We have merely been constructing a prescribed geometry which shall be appropriate for use as a measurement tool in application to the non-preferential space-time field which, we have found reason to believe, must exist at the root of our real space and time conceptuality.

A further conclusion from the necessary symmetry of the metric in the coordinates x, y, z is that

$$d\sigma^2 = A(dx^2 + dy^2 + dz^2) + 2B(dy\,dz + dz\,dx + dx\,dy),$$

where A, B are functions of r, t. If we reverse the sign of x in this formula, keeping the axes x, y, z in the representation the same, the effect is to represent a reflection of the universe about the yz-plane. For the reasons already given the metric must remain the same in form; so we conclude that $B \equiv 0$.

For an appropriate representation, again, the proper distances $d\sigma$ in the respective coordinate directions x, y, z should be not merely the same for equal represented distances dx, dy, dz at the origin. Their fractional rates of change with proper time, at any point, should be the same whether r increases or decreases. Writing $d\sigma^2 = e^{\mu(r,t)}(dx^2 + dy^2 + dz^2)$, this gives $\partial^2\mu/\partial r\,\partial t \equiv 0$, from which we find that the metric must in fact take the form:

$$ds^2 = -\frac{f^2(t)}{F^2(r)}(dx^2 + dy^2 + dz^2) + dt^2 \tag{3}$$

To complete the specification of the geometry and representation so far as 'appropriateness' of the representation to the non-preferential space-time field requires, we may consider the proper components of the curvature tensor R^i_{jkl}. Most vanish identically, R^a_{aab} and R^a_{abc}, for instance. But R^1_{212} will be found to contain terms symmetrical in x, y, z together with $z^2(rF_{rr} - F_r)F/r^3$. Similarly, R^1_{313} contains the asymmetrical term $y^2(rF_{rr} - F_r)F/r^3$. The element R_{11} of the contracted curvature tensor is then found to contain symmetrical terms together with the asymmetrical term $-(y^2 + z^2)(rF_{rr} - F_r)F/r^3$. Since, by hypothesis, there is no preference in this measurement-tool for one

direction over another, we have $R_{11} = R_{22} = R_{33}$. It follows that $rF_{rr} - F_r = 0$, which gives $F = A + Br^2$, in accordance with the accepted formula if we include the arbitrary constant A in $f(t)$ and write $F = 1 + r^2/4R_0^2$.

Operational Implementation

The metric now obtained must be implemented, first in an ideal-operational geometry and then in an actual-operational one. In each case we need 'material' and 'rules'.

As 'basic material' in the ideal case we need (1) local and astronomical objects; (2) rods and clocks which exist in a local region and which are ideal standards in the sense to be explained presently; (3) light rays *in vacuo*. The scales of the rods and clocks are to be established and correlated on the principle that measures of distance are those of a local and proper Euclidean space established as explained in Chapters 6 and 10, and that the local velocity of light is to be unity (in relativistic units). Among the standard clocks we may include frequencies of recognizable lines in standard (ideal) recognizable spectra. We also need (4) some means of communication of distant measures of interval to a centrally coordinating observer who can be imagined at the origin; in the case of spectra this is provided by frequencies or wavelengths of light rays, it being assumed that the ideal space traversed is a vacuum.

When we come to introduce our operational rules, we shall find that the basic material (1), (2) and (3) is sufficient for an observer at the origin to fix the coordinates x, y, z, t of a distant event, using the parallax method, i.e. by constructing geodesics from the given distant event to two local points—provided the metric is fully specified. The basic material (4) should give results consistent with what is derived from the other basic material, because of the consistency of the postulates by which our measurement-tool is constructed. It provides what is to some extent an alternative to the basic material (1), (2) and (3), but is essential when we wish to find grounds for specifying $f(t)$.

The 'subsidiary material' posited, again in the ideal case, is a homogeneous lattice of identical nebulae, in particular with the same diameter and the same distribution of brightness among the component stars. It is usual to make some centre point in each nebula coincident with a particular observer of the observer gas chosen; the coordinates are then, as we say, co-moving. This convention is introduced as a *practical* necessity, and could be regarded as by no means 'appropriate'. For if we decide eventually that the lattice is 'expanding', then the representation must be imagined to expand too; i.e, the length on the representation indicating a proper distance of 1 metre now, will indicate in the future more than 1 metre. The alternative, however, of keeping the ratio of proper distance to represented distance the same for all time

obviously requires a static metric. And this we shall dismiss later as definitely 'inappropriate', in view of the experimental evidence.

The 'practical necessity' is partly mathematical (to avoid introducing a nebula gas in addition to and distinct from the observer gas) and partly operational. For, light being received at the origin from distant nebulae coincident with ideal observers, it is as if such observer communicated to the observer at the origin the readings of his standard clock. Hence we have a means of dispensing to some extent with live and intelligent observers in distant parts of the universe (in so far, of course, as the lattice actually exists as posited).

The operational rules needed to implement the geometry finally are as follows: (1) The Geodesic Postulate, (2) the Postulate of Proper Time, and (3) a postulate determining the measure of the radial coordinate r for a given distant event. All these involve conceptual difficulties of some magnitude.

The **Geodesic Postulate** asserts, in its narrow version, that the world-lines of light rays are to be represented as null geodesics. In its wider version it asserts that the world-lines of ideal 'free particles' (i.e. not in contact with other particles or influenced by electromagnetic fields) are to be represented as geodesics. Let us suppose there is a certain strictly-local region or frame R in which, over a considerable period of time, Euclidean space and standard clocks can be set up, and in which, further, free particles have uniform-velocity trajectories. We can take $ds^2 = -dX^2 - dY^2 - dZ^2 + dT^2$ in this frame, throughout the given period, consistently with both versions of the postulate. But the metric cannot be of this 'pseudo-Euclidean' form throughout all space-time. We may use the strictly-local frame to set up a system of measurement in optically-local space; then although (by definition of such space) light-rays have uniform velocity trajectories within the expected limits of accuracy, free particles will in general be accelerated. We say then that there is a 'gravitational field', and the condition for a gravitational field is obviously a small departure from the pseudo-Euclidean form of metric, although a set of Euclidean coordinates X, Y, Z can indeed be set up very accurately by the use of light-rays.

In terms of the postulate, which does not mention 'mass', the acceleration must be the same for all particles in a sufficiently small region of optically-local space; and this fact is so accurately confirmed in actuality, after obvious corrections have been made where necessary, that we feel, on this score, the postulate is unobjectionable.

Now in the frame R, over a considerable period, particles are (by hypothesis) not accelerated, so there is no effect of a 'gravitational field'. All particles at rest in the frame, if free, remain at rest, and the gravitational field seems to have disappeared. In this analysis it is to be noted that we have said nothing about accelerated frames, whatever they

may be, or about forces which may act on particles fixed in a frame when the frame is accelerated with respect to a frame in which $ds^2 = -dX^2 - dY^2 - dZ^2 + dT^2$. We are still concerned only with implementing a measurement tool in a consistent way.

The fact that strictly local frames can be set up as explained, and that the Geodesic Postulate then holds independently of 'mass', can be taken as asserted by a principle, which must hold with extreme accuracy in nature if this implementation is to be practicable (and of course it does so hold). The so-called Principle of Equivalence asserts what is required; but the usual presentations of it, in my opinion, confuse the relativistic requirements with irrelevant formulae of Newtonian dynamics. Inertial mass is the m in Newton's second law, $F = ma$, while gravitational mass is the m and m' in the inverse-square law of gravitational attraction, $F = m'/r^2$. It is only within the context of Newton's theory, therefore, that the equivalence of gravitational and inertial mass can be taken as the cause of the fact that all free particles have the same acceleration in a space-time field which is not strictly 'flat'.

The problem is confused further by the fact that what we regard as inertial frames in the laboratory are not in fact such, because free particles fall in them with the acceleration of gravity. These frames should be regarded as optically-local frames (although of strictly local extent), because their seeming 'flatness' holds only for light-rays. In the above discussion, gravitational fields are discerned in optically local space; and outside those limits we are obliged to construct some 'artificial' representation before we can decide questions concerning 'gravitational' or 'cosmical' effects on movement.

To make the **Postulate of Proper Time** clear we must introduce the concept of an ideal 'free-falling clock gas'. The term 'free-falling' means that, although we have a *system* of particles mechanically connected, the whole can be regarded as at rest in an inertial frame (in which free particles do not accelerate); it is as if the clock were placed in a freely falling lift. The requirement that we should be able to construct identical standard clocks in various inertial frames is then a reasonable one.

If, now, an observer is moving in any way, possibly accelerated, with respect to other observers, we are to imagine him finding himself alongside any number of standard clocks moving with various velocities and all free-falling. He singles out one which, for the period of the observation he wishes to make, keeps step with him exactly; naturally the period must be very short if he has acceleration relative to inertial frames. The postulate then asserts that he must take this clock as recording his 'proper time', i.e. his dt, or ds when his local coordinates x, y, z are fixed. If he is himself free-falling, then we can imagine him carrying his own clock, which belongs also to the 'clock gas'. If, on the other hand, he is accelerated and carries a clock which was standard in

the unaccelerated state, we thus have a standard with which to measure the irregularities of his clock brought about by or associated with the acceleration. In the case of atomic clocks, there is reason to believe that no irregularities enter; nevertheless it is clear that we must have a standard by means of which we may judge whether they do or do not enter, and this is the one adopted.

For determining the radial coordinate r of an astronomical object, the only available method, using the basic material and postulates (1) and (2), is that of parallax—the object being marked at the intersection of light-rays from it arriving at two points in the locality of the origin. For this to be effective, the terms $f(t)$ and R_0 must be decided, and the parallax must not be too small to be measurable. We must consider, therefore, (1) how it might be 'appropriate' to fix $f(t)$ and R_0 in some way, and (2) whether there are any other reasonable postulates which might be brought in, so as to provide consistently another method for determining r in cases where the parallax is too small to be measurable.

It is customary to write $f^2(t) = e^{g(t)}$, where $g(t) = 2(kt + lt^2 + mt^3 \ldots)$, so that we know $k, l, m, \ldots$ are exceedingly small. We could of course elect to take $g(t) = 0$ for our measurement-tool, or else $g(t) = 2kt$ ('steady-state' universe). The constants l, m are usually retained in the formula, in the hopes of getting a close fit with the average motion of concentrated masses in the universe and the supposed average mass-density and pressure of radiation in the universe. The latter are related to the metric by means of somewhat dubious dynamical theories, generalized from the tensor formalism of the motion of material fluids (without radiation) in optically local space.[1] When we add to these considerations the fact that present observational data are not of such a kind as can do more than suggest very wide limits for these constants and for R_0, the conclusion, I think, is that it would *not* be 'appropriate' to make our measurement-tool thus vaguely dependent on doubtful physical theories.

If we ignore such theories, so far as the choice of measurement-tool is concerned, there appears to be no great objection to adopting the 'steady-state' form of measurement-tool—in fact there are considerable arguments in its favour. We shall come to these presently, and in Chapter 18. One other point (also dealt with more systematically in Chapter 18) first needs attention.

There is an objection to taking $k = 0$, namely, that the representation of the nebular lattice would then expand relative to our chosen 'observer gas'; and it can therefore be maintained that such a choice of observer gas is 'inappropriate' seeing that the 'expansion' is a cosmical phenomenon evidencing what is the *natural* state, regarded as 'rest' there, in

[1] Outside 'optically-local' space the coordinate system could not be set up, because the geometry is not prescribed.

each region of the universe. After all, a choice of observer gas was originally intended to combine together in a universal view a system of local views each of which would be among those most naturally to be expected in the locality concerned. In fact, the observers in those distant regions would have no reason to regard themselves as moving in any particular direction, while some other observer was at rest; so it is 'inappropriate' to represent them as so doing.

I shall therefore accept that it is appropriate to fit the observer gas closely to the nebular lattice found in nature and to represent both as at rest. There are, however, strong arguments for taking $f(t)$ to be of the specially simple form e^{kt}, even if the fit is not always close. I am again not concerned with dynamical theories; the arguments deal purely with the requirements of a cosmical measurement-tool.

The 'Steady-State' Geometry

We have not yet referred to another non-preferential characteristic which we can reasonably demand of our geometry, in view of the fact that it is to represent a conceptual space-time field in which all observers participate and which all are trying to measure by some common means. We are proposing to devise and implement this geometry as the measurement tool most 'appropriate' to this space-time field and the actual structures in it. It seems therefore altogether reasonable to demand that the operational rules for implementing our measurement-tool should not depend on the time when they are to be applied.

To be more explicit, if we wished to decide what variation should be made in the rules by an observer living 10,000 years hence, we should need first to have some independent means of measuring the 10,000 years. But for accurate measurement of time over such a long period we must ultimately depend on astronomical processes, since local cyclic mechanisms (even the most accurate atomic clock) would soon require replacement and adjustment. Even if it were regarded as satisfactory to suppose that future generations would set up a temporal chain of atomic clocks, each one calibrated by comparison with the preceding one before it becomes unusable, we should still require a more universal check, to verify the consistency of time-measures found in this way in various parts of the universe with different gravitational potentials and at various times when the 'expansion' might have differently affected the measures recorded by local cyclic mechanisms of the kind in question.

In fact it will be impossible to set up, *in isolation*, time-standards for all circumstances in the universe and for the indefinite future. There will always be tests and corrections needed for the influence of other parts of the universe and for cosmical expansion (apart from more accidental effects which might be detected and allowed for locally), if a consistent universal measure of time is to be obtained. Such influences

could not be tested and allowed for by correction unless we had already completed the construction of an ideal geometry, to serve as standard. Here, then, we have an obvious vicious circle.

It is true that in Chapter 7 we saw that a fully operational geometry might be prescribed by, first, admitting a finite number of unknown quantities in the formalism, and subsequently, deciding values for these in agreement with certain observations. A general functional form, $f(t)$, could not be determined. If one initially undetermined constant is allowed, we could try $f(t)$ in such forms as e^{kt}, t^n, $\log(1 + at)$. And now it might be urged that we could, in principle, eventually determine forms with 2, 3, 4 . . . constants, the work being limited only by observational facilities and accuracy. But here another serious objection arises.

What must be questioned is, in fact, the entire policy of fitting an elaborate functional form, involving several constants, to empirical phenomena and hoping to use this satisfactorily as a measurement-tool in the universe. There can never be even an exceedingly close fit with empirical observations under all circumstances, but there will also necessarily be departures from the chosen form, due to specific causes. When, for example, we apply the methods of classical physics to the measuring of solar phenomena, we find systematic irregularities such as the gravitational red-shift and deflection of light; we then conclude that there must be some specific cause of these irregularities and that a corresponding scientific theory would enable us to account and 'correct' for them. But this conclusion is justified only because our original measurement-tool was so 'appropriate' in principle and in practice that the departures from regularity were genuinely 'shown up' as additional features, systematic, possessing their own logic, and therefore reasonably to be imputed to some specific additional cause. If we had devised a system of measurement which lacked 'appropriateness' through its being founded on accidental features rather than sound epistemological principles—for instance, if we carried out our measurements at the top of a growing tree, or used as standard the span of our hands—the deviations would not 'show up' as systematic and the causes could not be uncovered and allowed for. In the same way, therefore, if we try to fit a geometry too closely to what are largely accidental variations among phenomena in the universe, we shall risk obscuring the underlying *rationale* of nature and so may make it impossible for further *systematic* deviations to be discovered and explained. These considerations are, I think, at the root of any justifiable demand for 'simplicity' in physical theories.

In the problem of cosmical geometry, therefore, it seems that we should first try to find the soundest possible epistemological reasons for choosing some particular form or class of forms; and among such reasons, I can think of none stronger than that the measurement-tool

should be (if possible) the same for all time. Let us see if this is indeed possible.

I do not take this requirement as implying that the metric must be a static one; for the following reason. A gradual and uniform increase in the measures of all proper-distances locally has no operational meaning. A metre-stick will continue to be regarded as a metre-stick, because everything with which it might be compared will have increased similarly. Such increase has significance only by reason of the fact that when a universal representation is artificially imposed, with an associated non-static metric, the value of ds may work out to be increasing or decreasing with time, and other cosmical phenomena (e.g. the red-shift) may thus be more satisfyingly fitted by the formalism. From the local point of view, the increase or decrease is illusory.

What the local observer requires of his measurement-tool is not that the absolute values of his cosmical ds shall be the same for all time when dx, dy, dz, dt are given, but that the astronomical configurations corresponding to his local observations shall be the same *in relation* to the representation of his local surroundings. Then, at whatever time he makes his observations, he will know that the same relative configuration is implied.

The condition to impose is therefore that substitution of $t + t_0$ for t in the metric does no more than alter the scale of representation of the spatial quantities dx, dy, dz, R_0. This gives $f(t)\,dx = f(t + t_0)\,d\bar{x}$, and similarly with y, z, where $\bar{x}$, $\bar{y}$, $\bar{z}$ are the coordinates in the new representation, and are required to be simply multiples of x, y, z respectively. The ratio of corresponding coordinates therefore depends only on t_0, so that

$$\frac{f(t)}{f(t + t_0)} = \phi(t_0), \text{ say.}$$

Differentiating logarithmically with respect to t, we find that $d/dt \log f(t)$ is constant. Writing this equal to k, we obtain $f(t) = Ae^{kt}$.

The ideal determination of the value of k, and (assuming for the moment R_0 given) of the radial variable r and cosmical time t for any visible event, is now straightforward, the ideal 'material' (light-rays, free particles, nebular lattice) being given. We could, ideally, use the parallax method, the apparent diameter method, measurements of 'red shift', or, assuming also a certain application of Quantum Theory and other physical theories, comparisons of luminosity; in addition, as a rough check, there is the distribution of nebulae as shown by a 'nebular count'. Because of the way in which our representation has been constructed, so as to be consistent with the ideal definitions of the 'material', the results must be consistent, however obtained, to the extent that the *actual* nebular lattice agrees with the ideal.

In actuality, the parallax method is of no service by itself, except for finding the value of r for the nearer stars within our own galaxy. All methods except the nebular count require correction for effects due to the earth's atmosphere, and the nebular count is useless by itself. Even when these corrections are reasonably made, we must expect to find other deviations from the ideal, both systematic ones and ones that are 'random' in the sense that they are to be imputed to subjective or accidental causes. If our epistemology has been correct, the systematic deviations should be reasonably accounted for as due to some specific physical condition or influence. This conclusion is drawn from a broad form of Helmholtz's Principle (Chapter 9) in which the 'conditions' from which a 'difference' is observed are those under which an actual-operational measurement-tool of assured epistemological truth, such as ideal-operational Euclidean geometry for strictly local space, has been constructed and is accordingly applicable. The epistemological 'truth' is to be tested by mature intuitive analysis and the consistency of continual and varied applications.

Only a few comments can be made on the choice of a value for R_0 (real or imaginary). Present attempts nearly all rely on the formalism of relativistic mechanics, generalized for averaged-out cosmical masses and radiation. Even so, only a possible range for $(R_0)^2$ can be found. A better way would be to investigate geodesic paths or other observational results dependent on the form of the metric, with such accuracy that the closeness of fit for various values of R_0 could be investigated. But this seems far beyond the possibilities of research for many years to come. For the time being, epistemological preference indicates only some finite and real value (see Chapter 17, p. 262). In what follows I shall assume that some suitably large finite and real value has been provisionally allocated.

It should be added that in the steady-state universes of Bondi, Gold and Hoyle, R_0 is taken as infinite. As a 'measurement-tool', however, a geometry with $f(t) = Ae^{kt}$ obviously need not be restricted in this way, and I therefore take the term 'steady-state' as applicable to *any* such geometry, whatever the value of R_0.

Bondi also argues that a great virtue of his steady-state theory is that it is a hypothesis which might in principle be disconfirmed by observation, and which is thus an objective one. From the phenomenological standpoint, however, even if systematic departures of actual nebulae from the system of ideal nebulae fundamental to the geometry were finally established, it might still be more appropriate to use a steady-state geometry for measuring the universe, and to account for the systematic departures as being due to some special cause. We cannot

read off the geometry of the universe from observations; we can only fit the most appropriate geometrical formalism to it and choose the most appropriate representation in terms of our local spatio-temporal concepts.

Cosmical 'Pseudo-Distance'

A few further remarks seem called for on the interpretation of universal representations, as now constructed. It has been emphasized in the earlier part of this chapter that our perception of a *real* spatial field is normally limited to a 'strictly local' region around us; we then customarily represent these perceptions to ourselves by drawing or imagining a Euclidean map. If the events are not located in the strictly local region, we can with some plausibility extend the map so as to help us to imagine, by a reduction of scale, spatial properties in the optically local region analogous to those actually perceived in the strictly local region. If the events are not located even in an optically local region, we can conceive of no spatial or temporal relation between them except by constructing a representation which 'artificially' imports into nature spatial connections such as it does not possess. For all such representations are, at every stage of their construction, imagined—it being quite impossible to *perceive* astronomical 'distances' as local distances are perceived. What we perceive, when observations concerning astronomical 'distances' are made, is a local visual Euclidean presentation, with the objects radially identified and as if 'at infinity'. Everything else of space is imaginatively added.

According to this analysis, therefore, and what has been said above on the *occasional* character of cosmical geometry, the apartness of astronomical events should not be conceived of as 'distance'; we should think, instead, in terms of *participation in the roots of objective knowledge*, or perhaps we can say, in terms of the *degree of likeness of ontological state* in some respect, presented locally as 'distance' or 'visible separation' on any occasion of observation.

Only two definitions of *cosmical* 'distance' (better called 'pseudo-distance') seem to me open to serious consideration. I shall call these, respectively, (1) the *simultaneity-distance* $\int d\sigma$, taken over a given represented path at a given cosmical time; and (2) the *geodesic-distance* $\int dt$, taken over a null-geodesic, i.e. light-ray *in vacuo*, connecting the two events whose 'distance' apart corresponding to that time of observation is to be recorded. In the first case it is usual to imagine, though the thing is utterly impracticable, rigid measuring rods laid simultaneously along the given path like a chain. In the second case there could also be an imaginary chain, but of free-falling space-ships momentarily at rest in the representation. The times of transit of the chosen light-ray between each space-ship and the next (every clock

reading cosmical time) are to be measured; some outside observer must be then notified of the various measures and must add them together. Since we are using relativistic units for time (velocity of light = 1), each local time-lapse is a measure of proper distance; so the end-result is a total of proper distances, each made at a determinate time.

These imaginations have a vividness and plausibility which tempts us to suppose that they really show, on a reduced scale, something which might ideally happen in the universe. The next step, logically, would be to say that the universe can be mapped out by ideal rod-chains placed radially and providing a radial measure of distant events. We then arrive back at a comfortable 'container' view of the universe, the container being moreover Euclidean, though the calculations and changes to alternative pictures of the container and its contents (for other observers) must be made with the help of a metric which is formally non-Euclidean.

The flaw in this reasoning is as follows. The operational definition of a straight line holds, as we have seen,[1] only in strictly local or manipulative space. Outside manipulative space *it is impossible to construct straight lines* in the full operational sense. If it were possible to do so, the space would be Euclidean, and time and the movement of the observer would be irrelevant. There would be no change of angle between light-rays when we change from one inertial frame to another. We should have illegitimately equated straight lines constructed in three space dimensions with light-rays given in four dimensions, and so become unable to apply any Riemannian technique in four dimensions to the numberless physical problems by which Special Relativity is confirmed and gravitational effects consistently accounted for.

To put the objection another way, the construction of a cosmical straight line in the full operational sense means the construction of a cosmical rigid frame. This in turn means the infinite transmissibility of physical effects; and on this rock the whole of modern physics will founder.

If, therefore, we are asked to lay a chain of rods, in actuality, along the path in the universe corresponding to a straight line or any other curve in the Euclidean representation, the answer must be, 'It cannot be done, in practice or in principle'. By showing the straight line or curve in a Euclidean diagram we have been led to imagine the circumstances incorrectly, importing into nature an *imagined* connection which cannot in principle be made an *actual* one. The true view of the universe remains *occasional*.

In the case of 'geodesic distance' the argument against possible actuality in the way implied must proceed differently. For it might be possible to locate the space-ships as required—say, along a radial line

[1] Above, pp. 188 ff.

from the origin, and with radial coordinates of suitably increasing magnitudes—and an observer in each has then only to carry out a local measurement. The sum of the local measures then gives us the cosmical 'distance' to the event in question. But it does *not* give us another distance having the space-conceptuality of the strictly local field. What it gives us is a number to which we proceed to attach an imagined space-conceptuality, working out the details of our picture (in imagined Euclidean space) according to our chosen principles of representation. In short, the separate manifestations upon occasion to the observers in the various space-ships do not actually combine to produce the real presentation of a greater distance.

If we nevertheless take either of these 'pseudo-distances' as providing an acceptable conventional measure of the spatial apartness of nebulae, on that understanding we can say that the separation is increasing with time, or more loosely, the universe is expanding. But even so, I think, it has to be borne in mind that it is not appropriate to think of the average nebula as *moving* away from us; for to an observer in that locality it is we who would seem to be moving away from him, while he is at rest. We can perhaps see the origin of all such confusions in the simple fact that there is no cosmical 'distance' of a spatial character except what is 'artificial' and imagined.

The Clock Paradox: Accelerated Frames

Almost the only paradoxical thing about the Clock Paradox these days, perhaps, is the fact that the difficulties still worth considering are primarily *spatial* ones.[1] Four forms of the 'paradox' can be distinguished, and I shall deal with each in turn as briefly as possible.

First, we may suppose a standard clock travelling a distance d from a point A to a point B at a constant velocity v relative to a given inertial frame Σ, both points being fixed in Σ. As it reaches B it passes another standard clock proceeding from B to A, also at a constant velocity v relative to Σ. The sum of the times recorded by the two clocks is $(2d/v)\sqrt{1 - v^2}$ in relativistic units, whereas the lapse of time on a clock stationary at A is $2d/v$. There is no paradox here, and experimental evidence gives every reason to believe the results correct.

In the second form of the paradox there is only one clock supposed to travel relative to Σ. As it arrives at B it is supposed to suffer an instantaneous reversal. At the speeds which have relevance here, any humanly-made clock would be entirely disintegrated, and no problem remains. If we take an atomic system as our 'clock' and suppose its velocity reversed in direction by an atomic collision, the case is equivalent to that just considered, except that no human being can travel with the clocks to record the passage of time. Effects of frequency become

[1] For history and references, see Whitrow, *op. cit.*, pp. 215 ff.

actualized when an observation is made on the system; but except imaginatively, or to observers in their local systems, there is no passage of time.

In the third form of the paradox there is again only one clock supposed to travel relative to *A*. Before arriving at *B* it suffers a gradual deceleration, comes to rest at *B*, and then gains speed again for the journey back to *A*. The paradox consists in the fact that Einstein and other scientists following him seem to have supposed that because no observer is 'preferred', the decelerating clock could have been regarded as at rest while *A* moved. In that case *A*'s clock would have lost time. relative to the other, contradictory to the former result. The paradox, in this form, is resolved, first, by noting that the situations of the two clocks are not reciprocal; one is at rest in an inertial frame, while the other is not and therefore suffers effects equivalent to those of a gravitational field. The Lorentz transformation is applicable to the measures of one but not to those of the other. Secondly, one can nevertheless devise a system of coordinates suitable for an *accelerated frame* and verify that the difference between the proper-time lapses for the two clocks turns out the same, whichever coordinate system is used.

This second stage of resolution was achieved in a simple way by Tolman[1] and more elaborately by Møller.[2] In brief, the accelerated frame must be defined indirectly, in terms of measurements in an inertial frame; also, the measure of time taken to be standard in such a frame is that of a neighbouring clock of the free-falling 'clock gas' having momentarily the same velocity as the accelerated point in question (i.e. the Postulate of Proper Time is adopted). There is reason to believe that an accelerated atomic system needs no 'correction' for its frequency to conform with this postulate. The observed frequency of such a 'clock' is affected by its velocity and by gravitational potential, but not, apparently, by acceleration or gravitational forces. Macroscopic clocks, on the other hand, will obviously need correction because of the forces resulting from acceleration or gravity. Biological time, likewise, can hardly be expected to conform; but how it differs probably depends on numerous factors, some of them psychological or environmental.

The difficulty of this form of the paradox thus resides in the definition of an accelerated frame; for in such a frame any material body will suffer elastic distortion or worse, and some standard is needed to measure the distortion. Møller shows[3] how a 'rigid' standard frame with rectilinear acceleration can be constructed mathematically—that is, the proper distance $d\sigma$ between two neighbouring reference points of the

[1] *Op. cit.*, pp. 194 ff.
[2] C. Møller, *The Theory of Relativity*, Oxford 1952, pp. 255 ff, 233 ff.
[3] *Ibid.*, p. 252.

frame is constant in time. If X, Y, Z, T are the coordinates of a given inertial frame, the coordinates x, y, z, t of the accelerated frame are given by

$$X = \int_0^t \sinh\theta\, dt + x\cosh\theta,$$

$$T = \int_0^t \cosh\theta\, dt + x\sinh\theta,$$

where $\theta(t)$ is any function of t, $Y = y$, $Z = z$. It must be remarked, however, that this 'rigidity' expresses merely a local first-order coincidence of velocity with a free-falling 'standard-rod gas', and does not imply that any rigid body, in the full Euclidean sense, could have the non-flat metric which an accelerated frame is required to have. The *representation* is indeed Euclidean; but if the coordinate network really consisted of straight lines in the full operational sense, the space-time field would be flat and the frame would be an inertial one moving with *constant* velocity relative to the given inertial frame. Hence these accelerated frames are imaginary constructions of mathematics, and it will not be possible to implement them in an actual rigid frame.

The fourth and last form of the paradox is one suggested by Whitrow[1] in order to do away with the need for acceleration. In the static Einstein universe ($k = 0$, R_0 real) a free particle can theoretically proceed in a path represented as a straight line and described with constant velocity, and will eventually arrive back at its starting-point. A small region round such a point can therefore be regarded as an inertial frame, with respect to which a clock is at rest. On arriving back at its starting-point, the moving clock will have lost time in relation to a clock which had remained there, at rest with respect to the non-preferential 'observer gas' of the Einstein universe.

This 'circumnavigation' of the universe is not, perhaps, too far-fetched a conception, since what we have to imagine is a succession of viewpoints which, for a short distance, lie along a straight line, but which are such that the addition of all these straight-line segments does not make a structure which we can actualize in space alone, as it were. Therefore we must not imagine the complete path as straight; it is a succession of occasions, the concluding ones of which join up smoothly with the beginning ones, because the formalism works out that way.

The paradox in this case, if there is one, will arise by alternatively taking the 'moving' clock as marking the position of an observer belonging to another observer gas, equally appropriate for description of the universe. Then the clock which was at the origin of the original representation will be the one which 'moves' and in consequence runs slow. The formulae of relativistic fluid-mechanics, applied to the nebular

[1] *Op. cit.*, pp. 219 fi.

lattice, indeed show that the two descriptions are not reciprocal; only one can apply to the actual nebular lattice and give the correct motion for the masses in it. So the other proposed method of measurement falls away. Not wishing to assume these formulae here, I prefer to take the view that the final implementation of a measurement-tool requires that either an actual observer gas or an actual nebular lattice be given (otherwise k and R_0 are not determinate). But when one of these is given and the metric decided according to accepted principles of 'appropriateness', there is only one geometry which can be said to 'fit' the actual universe. To change to another observer gas would be equivalent to changing to another universe. Hence there is no reciprocality of the kind which could bring about a paradox. In any case the argument breaks down for the actual 'expanding' universe.

CHAPTER 16

THE SIXFOLD SPACE-CONCEPTUALITY (III): THE EXTENDED CONCEPTUALITY

> 'The system of experience is not arrayed before me as if I were God; it is lived by me from a certain point of view. I am not the spectator, I am involved, and it is my involvement in a point of view which makes possible both the finiteness of my perception and its opening out upon the complete world as a horizon of every perception.'
>
> M. Merleau-Ponty, *Phenomenology of Perception*, trs. Colin Smith, London 1962, p. 304.

In each of the four conceptual spheres now discussed we have in effect a different language for problems of measurement and scientific law. Terms such as 'distance' and 'time' change their meaning as their operational implementation changes; other terms are abandoned and replaced by new terms. Correspondingly, the view we take of space and time varies from sphere to sphere. Pertinent questions to ask are, Where and how does the transition occur, from the one view to the other? Does the one view of space and time fade out and disappear as another gradually takes over? What could be meant by 'proceeding to infinity'?

The 'Hazing of Concepts'

Before attempting any answer let us examine these opinions of Weyl and Bridgman:

'The relation of the intuitive to the physical space becomes the vaguer the farther one departs from the ego centre.'[1]

'Mathematics does not recognize that as the physical range increases, the fundamental concepts become hazy, and eventually cease entirely to have physical meaning, and therefore must be replaced by other concepts which are operationally quite distinct.'[2]

In the first of these quotations, Weyl seems to be saying that we have concepts of 'intuitive space' (direct awareness?) and concepts of

[1] *Philosophy of Mathematics and Natural Science*, p. 135.
[2] *The Logic of Modern Physics*, p. 63.

'physical space' (theoretical structure?), and that whereas the two are closely fused around the 'ego centre', the relationship becomes indefinite as we contemplate things farther away. Bridgman seems to say that we have two sets of 'fundamental concepts' for space, that the one set is used for local phenomena and the other for distant phenomena, and that the former have to be abandoned when distance somehow causes the former to become to indefinite. Weyl seems to be thinking chiefly of psychological limitations, Bridgman of inadequacies of measuring procedures.

Perhaps one should not attempt to see in these opinions anything more than passing references to a certain point of obvious difficulty. It can hardly be said that they offer any analysis or explanation. Our aim here, however, is to discover an analysis and explanation, and in fact to do away with the 'haze', which cannot attach to *concepts* except so far as *we* are confused over them. We have therefore to reduce the seeming haze to effects of blending, transition, substructure, etc.—effects in which what is near and what is far off, what is visually perceived, what is prompted, and what is imagined, may all play their part.

The Visual Transition

In Chapter 10 it was necessary to make a very definite contrast between 'real' and 'imagined' localizations. I want now to emphasize that this contrast is in fact a contrast of extremes, and that we must very often admit a kind of intermediate 'reality' or 'givenness' where before it may have seemed there was only imagination.

Consider first the changing spatial impressions as we gradually walk up to a tree originally in the far distance, watching it all the while. At first the tree appears as in a painted landscape, and we imagine or think out its three-dimensional shape and distance away. When we are close to it, perhaps touching it, it is vividly 'real', its position and shape being known by spatial identification, with such absolute conviction and immediacy of presence that it seems absurd to say that anything else connected with the tree is *more* real than what we know then. *That* is the real tree, as we can know it in this world. Even if we do not touch it, sight has opened the door to its reality.

Is there any definite intermediate position then—50 yards away, 100 yards away—where sight for the first time ceases to be able to open the door to that reality? If on some occasion this spatial identification is prompted when the tree is 53 yards away, and on another occasion it is not prompted when the tree is 54 yards away, can we say that a barrier exists between? Any such kind of thinking is of course mistaken here. The function of spatial identification responds subjectively as it will or can, while the objective cause remains unaffected by any barrier. It seems evident then that whatever, in the

objective tree or its appearances, is a cause of the prompting, it must be taken as potentially present even when, because of our variable subjective powers, the prompting does not occur. It cannot suddenly cease to be in the tree when we happen to be 53·5 yards away from it. It must remain like a *seed* of reality, which may bear fruit in us if the soil is favourable.

But perhaps it will be said that this 'seed' operates on us according to a strict physical law, so that as the distance increases greatly the power of sight to prompt spatial identification in us is so weakened that nothing of that kind can actually become perceptible; therefore an imagined space must take the place of the real one. Or it might be said that at great distances there will necessarily be a lack of overlap between the mental present of the observer and the mental present in the place of the distant object, if the two mental presents coincide physically; so spatial identification is impossible under such conditions, and all that is possible to us is a flat visible presentation, filled out with imagination. I have already maintained many times throughout this book that such mechanistic ideas, however persuasive, are mistaken; and I hope the analysis which follows will make their superficiality still more evident.

'Real Spatial Knowledge'

By way of introduction, consider the various ways in which we may come to have knowledge of the spatial characteristics of the moon, not necessarily in relation to other objects in the universe, but as a spatially-formed presentation which we recognize and call by the name of 'the moon'. There is, first of all, the sight of the moon by the naked eye and the exploration of visible details by varying the direction of optic axes of the eyes. We can carry out a similar examination through a refracting telescope; here certain three-dimensional effects may be suggested to us by shadows, etc. We may look at the reflection of the moon in a mirror or pond. We may discover, after looking fixedly at the moon, that we have an after-image of it. We may examine a photograph of it. And there is always memory at work in enabling us to correlate details seen at different times.

Reviewing such diverse means of knowledge, it seems clear that there are *degrees of internality of relationship* between a given presentation said to be of 'the moon' and the givenness in nature of the moon as an astronomical object round which, for instance, a satellite could pass. The photograph, we can perhaps say, is fully *external* to the actual moon. We can describe it as an *artefact* copying a certain appearance of the moon on a certain occasion. There is, as it were, nothing actually in common between the photograph and the moon. Much the same applies to the after-image. In the case of the other means of knowledge

(naked eye, telescope, mirror) it would be absurd to say that the knowledge provided is wholly the work of imagination or of a surviving physiological disposition. In the very knowledge itself, something must be *given* objectively, because 'the same thing' (as we say, allowing for differences of perspective, etc.) can be seen by other people. It is reasonable to say, therefore, that in such a case the relationship is *internal* and that we have a degree of *real spatial knowledge* of the object, although real spatial identification (if it means awareness of the object in its physical spatial relation to our body-image) has not been established.

The closest internal relation and the most direct 'real spatial knowledge', outside the range of touch and spatial identification, occurs with the naked-eye vision. Probably no one will deny that the moon which we see with the naked eye is 'the real moon'—not, of course, as regards substructure or physical theory, but as regards observation and givenness. That, at least, is the accepted language convention. The moon which we then see is certainly not the work of imagination, though imagination may supply impressions additional to what is given. A mirror-reflection of the moon also presents 'the real moon', in much the same sense; but we feel that it has not the same *kind* or *degree* of 'reality' because, with mirror images, it is impossible to obtain, by going up more closely, a real spatial identification with the apparent position of the image. The telescope presents us with still another kind or degree of 'reality'.

Each kind or degree is *sui generis*, and should therefore be investigated and described as it is, without trying to explain it as in fact some other kind of 'reality' or 'imagination'. Explanation is nevertheless necessary, in terms of more primitive insights, if the whole subject is to be made rationally coherent; and in this connection there is one general problem demanding attention first. It may be put in the form of a question: If it be granted that everything of visual form and appearance is always according to the observer and the occasion, then in the case when there is no real spatial identification with a distant object, what precisely is 'real' or 'given' in the presentation? Rule out from the visual presentation all colours, visible shapes, directions from the eye—what then remains to be 'given'? To solve this problem we must try to uncover the manner in which 'prompting' occurs.

Mechanism of 'Prompting': the 'Seminal Core'

As a beginning, I state the obvious fact that the prompting of spatial identification with an object is not automatic; it does not occur by exact correspondence with appearances, or in obedience to mergence-free physical laws. The 'function' needs to be cultivated, from birth

onwards. It finally presents itself as a power of response, helped by experience and understanding, but not imaginative. It may be likened to learning to hear the quality and component sounds making up the recognizable character of each spoken word; or to getting to know people, and so coming to love them. All is vague at first. But something is *really there*, and knowledge of the cipher opens the appropriate door. The 'five senses' must be understood to prompt a knowledge of the reality of spatial identification rather as, on opening an oven door, we receive evidence, through a sense till then inactive, of the temperature inside. Our impressions will be affected by the size of the door, by our own state of temperature, and so on. But the fact will remain that the temperature is not an imaginative imposition on our visual or tangible impressions of the open door.

But what is 'knowledge of the cipher'? Here further analogies may be useful. When we open our eyes in a certain direction, an entire visual scene appears. Rather similarly, when through subconscious habits we recognize a general visual pattern and become dimly aware of muscular feeling in the eyes, an entire three-dimensional spatial form comes to our attention. But we cannot say that the prompting is merely the result of fixing a general direction of the eyes or other organs towards an object in space; for in that case it would be independent of experience and comprehension. Moreover, in hearing, the sounds heard in the two ears have no direction, only relative loudness. This is a *clue* to spatial direction, but not spatial direction itself. Hence we must conclude that the prompting is through a *reasonable structure of association*, and not by anything perceivable as being actually in physical space. An awareness of physical space could not prompt an awareness of physical space.

Since, however, the prompting seems closely *analogous* to the visual givenness following on the opening of the eyes in a certain general direction, I propose to use the word 'direction' metaphorically, and to set forth the 'mechanism' of prompting explicitly as follows:

(A) **Direction** means conceptual focusing on a certain existing unity or 'meaningful object', in such a way as to bring to our attention a spatio-temporal presentation of the object in relation to ourselves.

(B) **Prompting** of spatial identification means the *opening* of the function of spatial identification, together with the establishing of a *direction* to a reasonable structure of associations or *rational unity*, which is internal to the 'object' in question. The *opening* is by psychological reactions dependent on experience and comprehension. The prompting is intelligently comprehended in so far as it is followed by an *analysis* of totalities and the following out of rational connections.

I shall say that any detail of spatial identification thus discerned is *directly* presented to us.

The 'existing unity' or 'meaningful object', which initiates the prompting, is not limited to any particular spatio-temporal presentation. I shall call it the *seminal core* of the sensory object in question. We have thus been led to a deeper view of what constitutes an object or recognizable continuing existence in the physical or any other 'world'. The conclusion now is that such object is not to be thought of merely as in correspondence with a mathematical group-field. Having a recognizability transcending all spatio-temporal presentations, it is essentially a matter of *significance*, from which, as seed, comes the flowering forth in space and time.

The Telescope

I now wish to show that this account of the prompting of real spatial identification by a seminal core of significance in the object must be supplemented by a very similar account of the prompting of any kind of 'real spatial knowledge' such as we have of the moon. But this is a difficult subject, and we must come to our conclusions in slow stages with the help of a series of simple 'test cases'.

First suppose that the lens in one of our eyes is removed and replaced by an exact equivalent made of plastic, while the other eye is left as it is, and that we then look with binocular vision at an object in strictly local space. It is surely unreasonable to maintain that the contribution of the right eye to the total encounter is 'real' but that of the left eye is 'artificial' because the interposition of a foreign body makes the presentation no longer an immediate encounter and thus necessitates a subjective interpretation. For consistency, therefore, it appears we must agree that we still apprehend (through either eye or both together) the *real personal* conceptuality of a strictly local space. Focusing of the visual image and directing of the optic axes remain as before, so the conditions for prompting of real spatial identification are not affected. We also have, as before, real spatial knowledge, gained through the eye, of distant objects.

Suppose next that the lens in each eye is removed and lenses are placed in front, so as to produce the same optical effect on the retina. Presumably the visual observations will be the same as when the eyes were in their natural state, and muscular sensations around the eyes will again be correlated with distance so as to make real spatial identification possible as before. It seems we must conclude, therefore, that whether the lenses are inside or outside the eyes, of natural growth or made of plastic, there is no essential difference in the manner of prompting of real spatial identification and knowledge.

Instead of single lenses in front of the eyes, let us now arrange pairs of lenses in front so as to make a binocular telescope. Or we may include prisms as well, and make prismatic binoculars in which the

rays of light suffer many reflections before reaching the eye. The same reasoning shows that even this optical apparatus prompts, in a very similar manner, a degree of real spatial knowledge. For if the spatial impressions were entirely imposed by our subjective imagination they could not be veridical.

In confirmation of the view towards which we are working, further thought-experiments can be proposed. If we look at an object through opera glasses held backwards, it appears more distant. Let us then place another pair in front, the correct way, so as to cancel out the effect of the first pair and restore the naked-eye appearances. With the nearer objects a real spatial identification is still possible, and with the distant ones a real spatial knowledge.

Interposed lenses and prisms are thus of no concern in the *manner of prompting*, either of real spatial identification or real spatial knowledge. They affect the visual form taken by the real spatial knowledge. But the prompting must still be conceived of as *initiated by participation in the seminal core* of the object, guided beforehand by the 'directional' means, and visually formed by the same means and by our own subjective circumstances in the process of its creative presentation to us.

Finally, consider the case when an object such as a tree 200 yards away is first looked at without real spatial identification, as if on the backcloth of a stage, and is then seen through a powerful telescope so that it seems only some 20 feet away. The focus can again be varied by action of the eye-muscles; and if the telescope is binocular we have the same stereoscopic dependence on the distance between the eyes and the angle between the optic axes. Thus we obtain a strong impression of three-dimensional form, as if we and our body-image had been brought up close to the tree. In this case the presentation imparts a real spatial knowledge, with a new kind of spatial identification—one whose form is very largely controlled by the means adopted and by our own habitual reactions, but which must nevertheless, in consistency, be termed *real* in some degree. For there is no impression whatever, if the perceptions are clear, that we have filled out a flat presentation with an imagined three-dimensional imposition.

Because of ingrained habits of thought the temptation to believe these spatial impressions a mere *interpretation* subjectively added may be almost irresistible. For we say to ourselves (assuming the 'classical scientific' view) something like this: 'My only contact with the distant object is by light-rays refracted and reflected on the way; therefore all spatial impressions must come solely from me.' As we have seen, however, the spatial impressions cannot be wholly imagined in the case when single lenses are in front of the eyes. It is scarcely reasonable, therefore, to suppose that they must become wholly imagined when

two or more lenses are in front. The belief mentioned could not, in fact, have arisen if it had been properly appreciated that according to both present-day physical science and philosophical analysis our objective experience occurs by a *manifestation upon occasion* in a *total encounter*, and that it is illegitimate to break up the experience into parts, set out one part (light-rays) on a spatial map, and place the other part in the brain, as it were.

In fact to place within our brain the objective world as we know it outside the brain is plainly an absurd proceeding. As we have already shown, all space relations are conceptual, neither inside nor outside the brain. As such they belong to a *subject-object middle-ground* of cohering individual abstract spaces. The eye therefore prompts spatial impressions not as if it were forming them from causes which are light-rays into effects which are nerve impulses carried from point to point, but in the sense that the conceptual 'machinery' by which our spatial impressions come into being on any occasion is limited by the total psychological, optical, tangible or other means available to us—or rather, by the seminal core and potential group-objectivity corresponding to such means.[1]

Extensional Devices

Among the multitude of means which nature provides for thus prompting spatial impressions of objects otherwise inaccessible or indiscernible by us may be mentioned (besides mirrors and telescopes) the microscope, drugs to heighten sensitivity, inspirational states when our powers of sensory discernment are greatly enhanced, and extreme hyperaesthesia established under hypnosis. From this we pass with hardly a break to the vast field of 'transcendent' or 'parapsychical' means, voluntarily cultivated and experimentally verified, as outlined below in Chapter 21.

The means used may accordingly be classified broadly under three heads: (1) *actualized physical* (as with the telescope), in which case the relevant mental reactions appear automatic, each observed detail in exact correspondence with some veridical fact, in a way which can be confirmed by all normal observers; (2) *physiologico-mental* (as with drugs, or hypnosis by an outside persons), in which case the general state of hyperaesthesia seems to be established automatically, i.e. without cultivation or control by the subject himself; and (3) *transcendental*, as with the voluntarily cultivated and controlled higher degrees

[1] The potential group-objectivity (p. 208 above) is the detailed 'mathematical' conceptuality in regard to spatio-temporal possibilities, revealed by a deeper phenomenological analysis of the simple unity called the 'seminal core'.

of phenomenological analysis[1] or parapsychical experience.[2] In each case where an 'extensional device' is used, one must expect some change of scale or other change of relation to the physical body image. This must not be interpreted as meaning that we are not 'really' in touch with the object. The meaning is that the potentiality of the object (transcending space and time), together with that of the means and all the other contributing circumstances, has manifested itself in a way different from that of 'normal' physical experience.

By the Principle of **Prompting Extensional Devices** I point to the fact that such means are provided in nature, enabling us to obtain, in a way that is *essentially direct* but variable according to circumstances, veridical spatial impressions of objects excessively far away or excessively small. The impressions are not, of course, 'direct' in the sense that there is a unique 'true' presentation of the object and these means give us something of that presentation without the interposition of 'intermediate causes'. There is *no* such presentation, and the effects of single causes cannot be abstracted from the whole 'manifestation upon occasion'. Thus we must not look askance at the objects seen either with the naked eye or with a telescope, as if they were in some way 'unreal' or 'subjective'. Nothing more *actual* and *physically real* could have been presented to us under those total circumstances.

The same argument as we have developed through a series of 'test cases' for the sense of sight could have been developed for the sense of touch. We can begin by feeling the form of an object with newly-grown soft skin on the finger; then with hardened and nearly dead skin; then with the finger-nail, as in our previous illustration in which pin-points 1/10 mm apart were distinguished by sliding the nail across them. The impressions of position and extent are judged from a combination of touch sensations all round the nail, resulting in the sensing of a single point located where the contact is made. Since in the first of these cases the spatial impressions are by definition 'real', they are real also in the last case. We conclude that impressions of touch prompt impressions of spatial position and form in a similar way to that in which visual impressions prompt them.

Proceeding with these test cases, we can imagine the finger-nail allowed to grow, and eventually being replaced, as a space-exploring tool, by a short match-stick held in the fingers. If, finally, we use a long stick held in the hand, the spatial impressions gained by touching objects with the far end (such impressions being located at the far end of the stick where the contact is made) will be likewise 'real', though we must expect some admixture with 'artificial' impressions as the stick is made longer and our sensitivity with that kind of prompting

[1] Ch. 8, pp. 107 ff. [2] Ch. 21, pp. 348 ff.

extensional device begins to fail. For imagination always tends to supply portions that are missing from a desired *Gestalt* because our objective sensitivity has failed.

We conclude that in every such case the spatial impressions are substantially 'real' and essentially 'direct', and that nothing more *actual* and *real* could have been presented to us under those total circumstances.

Presentations and Representations

Let us introduce the term, *causal presentational device*, to signify some means, physical or mental, which brings to light some specific detail of substructure ('cause'). A *prompting extensional device* will be then a particular kind of causal presentational device, whereby direct contact is made with the Gestalt of an object in an otherwise inaccessible part of space.

The instances just given of prompting extensional devices must be carefully distinguished from *representational devices* such as drawings of objects seen, or photographs, whose relationship to the original is 'external'. These representations may well be veridical; but there is a clear difference between devices of this kind and 'prompting' devices. In these devices *an actualized physical object* is in fact carried from place to place; also, the 'occasion' of observation of a drawing or photograph is not an occasion of observation of the phenomenon which is copied. Hence there is justification for saying that an 'intermediate actualization' is interposed and that the whole presentation is thus 'artificial'. It may, however, be more satisfactory to put such presentations in a new category, neither 'real' or 'artificial', but called 'representational'; this would perhaps make clear in what sense they can still be 'veridical'.

The microscope, being very similar in construction to the telescope, must presumably be classified as a prompting extensional device. One important difference, however, has to be carefully considered. Whereas the spatial form revealed by the telescope can often be verified by moving close to the object, there is no similar means of verification of the spatial form of what is seen in the microscope. The electron microscope provides an extreme case. Must we admit an absolute impossibility in principle of ever verifying the spatial forms photographed with the help of the electron microscope, otherwise than by the satisfying coherence of our interpretations in a scientific theory? What of atomic and sub-atomic forms? Are they absolutely beyond the reach of all extensional devices, physical or mental? These questions have received some attention in previous chapters, and will also be considered more thoroughly in Part IV, in the context of the entire investigation into dualistic world-views. But to conclude the present chapter a discussion of certain outstanding points seems to be needed.

The Dilemma of 'Agnostic Dualism'

The agnostic dualism[1] commonly accepted in scientific thinking today seems to be an outgrowth of the 'phenomenalism' which holds that physical 'objects', 'entities' and 'terms' are all *logical constructions* out of sense-data. In other words, statements about them are reducible to statements about sense-data.[2] By 'sense-data' a phenomenalist will understand data of the kind which I have called 'physical',[3] *not*, however, including any direct perception of the position of objects by the 'function of spatial-identification'.[4] For positions in 'outer space' are taken by him to be 'constructed' in our thought or imagination and *not* perceived.

If, now, on some particular occasion, a specific kind of logical construction from sense-data is thought of as corresponding to some specific *givenness in nature* on that occasion, and this 'givenness' is considered to be (by reason of its abstraction from sense) otherwise *unobservable*, we have, not phenomenalism, but *agnostic dualism*. For it is the purpose of phenomenalism to do away with such dualism and to account for nature in terms of a monism (viz. of structures of sense-data) plus the organizing logical powers of the individual mind. Agnostic dualism overcomes the absurdity of supposing that a rational givenness in the *communality* of nature can be supplied by the thought and imagination of *individual* minds assumed to be locked in their private worlds. On the other hand, it does not see how it can admit that the rational givenness in nature of elementary particles, wave-functions, and so on, can possibly be knowable by the individual minds, since these minds are (presumably) limited to the scope of their bodily senses.

We now have to face up to the fact that a claim is here being made (by the agnostic dualist) that the *unknowable* is nevertheless *knowable* in a certain way, namely, through the study and manipulation of sense-data which are not themselves descriptive of the 'entities' or causal structures in question.

The needs of the present discussion may be met by drawing a simple analogy between an 'entity' or 'structure' such as a wave-function, made significant by a 'logical construction' from experimental data, and the arithmetical structure called $\sqrt{2}$—described as 'irrational' by the ancient Greeks. On the basis of the clear-cut laws of addition, subtraction, multiplication and division in simple arithmetic, it could be held that $\sqrt{2}$ is unknowable in principle, since there is no number

[1] Ch. 22, p. 362.

[2] A. J. Ayer, article in *British Philosophy in the Mid-Century*, ed. C. A. Mace, pp. 223, 228.

[3] Above, p. 49.

[4] Ch. 10, pp. 135 ff.

whose square is 2. And it might also be held that we have knowledge of $\sqrt{2}$, nevertheless, as a logical construction obtained by extending the 'square-root operation' to all numbers.

But what precisely is signified by this 'logical construction'? To make things clear we must *exhibit* the operation by which we arrive at a clear concept to be denominated '$\sqrt{2}$'. It is by no means sufficient to introduce the phrase 'square-root operation', as a magician's wand, producing something meaningful in circumstances as yet in complete haze and confusion. When we do in fact define $\sqrt{2}$ operationally, as a law of successive approximation, we find that what seemed at first 'unknowable' and 'irrational' has in the process become knowable and rational.

We conclude that if a process of discursive thinking or formal deduction from postulates is to lead to some concept, e.g. $\sqrt{2}$, which is to be for us *valid* (i.e. not, probably, a mere haze or confusion of ideas) and *given* in the state of affairs considered, then that concept can and must be brought to the light of direct ostensive knowledge. Likewise, if anything put forward as a 'logical construction' (e.g. a meson) is to be for us a *valid* conceptual structure (i.e. not, probably, a mere haze or confusion of ideas) which is truly *given* on the occasion when we are supposed to have constructed it from its effects, then that structure can and must be brought to the light of direct ostensive knowledge. A clearly conceivable form of structure of the kind supposed must be present 'potentially' and be in some way discoverable in the total circumstances on that occasion. Otherwise our supposition has no substance.

Thus we cannot postulate, at the same time, the existence of a clear identifiable conceptual structure given in nature, and the impossibility of ourselves ever being in a position to cognize such givenness. To do this would be to fall into a Fallacy of **Merely Intensive Definition**[1]—that is, thinking that a definition in terms of desired effects or properties is sufficient to establish existence without a corresponding *existence theorem.*

'Discoverable Causes'

The conclusion, therefore, is that there can be no limit to the power of causal presentational devices. Nothing can be in principle unknowable. Or rather, if we postulate something as in principle unknowable, it can be no more than an empty or self-contradictory form of words. A view of this kind differs sharply from current expressions of agnostic

[1] 'Intensive', as opposed to 'ostensive'; not exhibited, but supposed to exist and be sufficiently defined by virtue of certain named properties which the thing or concept (as the case may be) must possess. The paradoxes of the mathematical Theory of Sets all result from this fallacy.

dualism, but not from practice in current scientific research. Suppose, for instance, that some phenomenon appears to us not fully analysed (for instance, the difference between vowel sounds). Scientific procedure is to apply some device, such as a harmonic analyser or a trained ear, which is likely to disclose or give the presentation of a substructure in correspondence with the broad differences which the senses, in their normal state, detect. For things which are discernible one from another cannot have identical substructure.[1]

What I call the Principle of **Discoverable Causes** (Subconscious Rationality) could be regarded as a more explicit version of Leibniz's Principle of Sufficient Reason. It asserts the *potential presence* in a phenomenon of elements which serve to 'explain' it—that is, these elements can be disclosed, either by phenomenological analysis in some degree, or with the help of material presentational devices (such as microscopes) the interpretation of whose evidences is reasonably judged to be sufficiently guaranteed by phenomenological analysis of the total circumstances in which such devices are used. It is not to be supposed, however, that the presentational devices always present something sensory which has to be interpreted in the light of other information, or that the analysis when found is in itself sensory. Being an 'explanation', it is always conceptual, as the Euclidean geometry which is put into the physical world in order to measure phenomena in it, and which is implied in the measurements which we thus make, is conceptual.[2]

It is also not to be supposed that there is only one kind or degree of analysis of any phenomenon. We may be exercising a high degree of reliance on material presentational devices and a low degree of analysis by *primordial recollection*,[3] or a low degree of reliance on material presentational devices and a high degree of primordial recollection. The use here of the term 'primordial recollection' is intended to emphasize the fact that intuitive analysis of an elaborate substructure (for example, the constituent notes of a chord struck on the piano or the timing and grading of notes in a rapid scale) cannot be carried out while the phenomenon strikes us in the ordinary sensory way. We must stop time, as it were, and analyse the primordially recollected form. So long as we are carried along by time, analysis is impossible.

These conclusions can be embodied in a Principle of **Degrees of Phenomenological Analysis,** to the effect that the discoverable causes at the back of any object of attention, serving to 'explain' its intelligible structure, can be found and contemplated only by establishing a

[1] This is Helmholtz's Principle (above, p. 114).

[2] On 'discoverable causes' and the view of logic here presented, see further in my book, *The Mystical Life*, pp. 111 ff.

[3] Explained above, Ch. 8, p. 108; also more fully, Ch. 19, p. 295.

degree of phenomenological suspension and insight appropriate to the particular discoverable causes in question.

We must defer till Chapter 21 all consideration of the way in which enormously heightened degrees of primordial recollection, suitably directed, might disclose even a substructure accounting for the success of the formalisms concerning 'elementary particles'. As the study of the Quantum Theory of Fields indicates,[1] these 'particles', so far as anything conceptually precise is meant by the term, are not by any means entities localized in space and time, but represent the possibility of certain operations *transcending* space and time. Therefore, to form even a passably clear idea of them we must to some extent achieve a phenomenological analysis which also transcends space and time.

[1] Ch. 20, esp. pp. 322 ff.

CHAPTER 17

INFINITE SUBDIVISION AND EXTENSION

> 'Providence equally embraceth all things together, though diverse, though infinite; but Fate puts into motion all things, distributed by places, forms and times.'
>
> Boethius, *Consolations of Philosophy*

In general the term 'infinite' means 'unending'. We must consider the possibilities of an 'unending' in (1) space, (2) time, or (3) non-temporal process. We must also consider non-temporal processes which potentially create quantities successively smaller, and those which potentially create quantities successively bigger and bigger.

The surface of a sphere is unending in the sense that there is no boundary curve; but we would not call it infinite. *Infinite space* is usually understood to be space such that, whatever extent we measure, we can find an extent with a greater measure. The surface of a sphere, as normally measured, does not fulfil this condition. We could of course introduce a new scale of area measurement, say $dS = (\sin\theta)^{-1}\, d\theta\, d\phi$, so that this 'area' S tends to infinity as we approach the pole $\theta = 0$. But in the process of setting-up the coordinate system θ, ϕ we shall have defined an 'area' of *prior operational validity*,[1] which is finite for the sphere. The other measure of 'area' then appears as derivative merely—the result of applying a mathematical formula to a surface which we already know has a finite 'area' in a prior operational sense of the term. We may agree, therefore, to call a mathematically defined curve, surface or volume *infinite* only if the measure of the extent of a portion of it, in a prior operational sense, can be made as great as we please by suitably choosing the portion. Later we shall need to consider regions of space not mathematically defined but actually *given in nature*.

A claim that time is infinite would similarly imply that whatever duration we measure, we can find a duration with a greater measure. These measures must also be understood to have a certain prior operational validity. For instance, if we introduced a time measure $T = T_0 \tanh t$, where t is in seconds, the T-time never exceeds T_0. But this measure of time is obviously derivative and artificial.

The subject of the infinity of non-temporal process is of fundamental

[1] This term is explained in Chapter 18.

importance in the theory of measurement, and a very careful consideration of it is now called for, before discussing the infinity of space and time in actuality.

Infinite Subdivision

There is no exact science of physics without measurement, and there is no measurement without repeated subdivision into congruent intervals.[1] Measurement may be defined as the association of a number with a physical phenomenon, according to some recognized procedure. The question then arises, 'What kind of number?' We are thus involved in consideration of the properties of the *mathematical continuum*, a brief sketch of which must now be given.

Real Numbers are of two kinds: (*a*) those defined by a finite number of arithmetical operations and called *rational*, e.g. ($2\frac{3}{4} + \frac{15}{17} \div 4{\cdot}02$; and (*b*) those defined by infinite process, i.e. by successive approximation. Taking again $\sqrt{2}$ as an example of the second kind of real number, we can obviously form an increasing sequence of rational numbers whose squares are less than 2, and a decreasing sequence of rational numbers whose squares are greater than 2, making the terms of the two sequences eventually approach each other as closely as we please (e.g. 1·4, 1·41, 1·414, . . . and 1·5, 1·42, 1·415 . . .). In this way we have also a sequence of *intervals*, (1·4, 1·5), (1·41, 1·42), . . ., each 'boxing in' the later intervals in the sequence, with the possibility of making all intervals after a certain one (I_n, say) as small as we please by taking n large enough. An inexhaustible variety of sequences could be taken as determining $\sqrt{2}$ in this way. The important fact, however, is that any one of them provides the theoretical possibility of allocating every rational number to a *left* or *right* class, according as its square is less than or greater than 2. Any such method of dividing the rational numbers into two classes, or of forming convergent sequences, can thus be regarded as a *law of formation* determining a real number.

The mathematical continuum comprises real numbers of both kinds. Although the two kinds are very different as originally defined, both can in fact be specified by this device of 'sections'. Hence they can be *simply ordered* in a single set of real numbers, visualized, if we please, as successive points along a line. Great caution, however, is needed in speaking of a 'set of real numbers', and paradoxes are apt to arise when we think of numbers or points as entities to be counted.

[1] Measurement by a *discrete manifold*, as by counting teeth on a ratchet, presupposes distinctly recognizable objects in space, with positions carefully checked by repeated subdivision if the measure is not to vary according to constructional irregularities.

The Non-enumerable

The rational numbers can be counted one by one; for the symbols and operations permitted are finite in number and can be introduced and combined one by one in turn. For example, if we introduce 0, 1, 2 first, and permit two figures alongside, we have the positive or negative numbers:

$$0, 1, 2, \tfrac{1}{2}, 10, 11, 12, 20, 21, 22, \tfrac{1}{10}, \tfrac{1}{11}, \ldots \tfrac{1}{22}, \tfrac{2}{11}, \ldots \tfrac{2}{21}.$$

We may then introduce the symbol 3 and allow combinations of three figures. Having reached the symbol 9, with combinations of nine figures allowed, we proceed to list the combinations of ten figures, then eleven, and so on. Real numbers of the second type, as Cantor showed in an epoch-making piece of work,[1] cannot be listed in any such way. Any list that we may produce, though extending to infinity (i.e. unending), remains *unfinished* in a more radical sense.[2] Through the inexhaustible power of the mind to invent new non-temporal laws of formation, we can always add, even to an infinite set of real numbers, new real numbers not included in it.

Real numbers of the second kind are introduced in *pure mathematics* because operations such as those implied in $\sqrt{x}$, arctan x, $dy/dx = y$, continually arise. We admit them also in calculations concerning *physical measurement* for one and only one reason. The conceptual system adopted for the purpose of putting measure into the world usually requires that operations such as those mentioned be performed on the measurement initially made, in order to obtain derivative measures which are more useful; or such operations are performed on certain fundamental measures assumed, in order to implement the system operationally and so obtain predictions. If, in place of the square-root operation rigorously defined, we permitted ourselves an approximation, either the errors would pile up, or the manipulation of 7 or more decimal places (to ensure sufficient accuracy) would be an intolerable burden. Thus real numbers of the second kind are admitted in physics for convenience only. Measurement itself is concerned only with the *rational* numbers, and the 'Dedekindian continuity' of the set of real numbers (as it has been called) is wholly irrelevant.

In regard to the 'boxing-in of intervals', as used in pure mathematics, there are two essentials to be noted. First, the 'boxes' require

[1] See, for example, A. A. Fraenkel, *Abstract Set Theory*, Amsterdam 1953, pp. 64 ff. Cantor's paper is dated 1892.

[2] L. E. J. Brouwer describes any dense set of real numbers in an interval as *denumerably unfinished.* It must be denumerable because only a finite number of signs can be used to specify it, and it is unfinished because new real numbers of the interval can always be added. See, for example, 'Intuition and Formalism', *Bull. of the Amer. Math. Soc.*, **20** (1913), pp. 93 ff.

laws of formation for their end-points; there must be infinite non-temporal processes precisely and clearly laid down, so that, no matter how far on we go in the sequences of boxes, there is a rule to determine the end-points of the next box. Otherwise the process is conceptually indeterminate. There is no convergence to a definite number. Secondly, the reason for introducing 'boxes' is that we are obliged to define real numbers of the second kind by means of sequences of real numbers of the first kind. In the measurement of a physical phenomenon this reason falls away, since real numbers of the second kind (i.e. laws of formation of infinite sequences) cannot be observed.

Measurement of 'Physical Points'

It is commonly said that the extensionless Euclidean point cannot actually exist. Likewise the pointlike time-instant cannot actually occur. The psychologist E. R. Clay,[1] and William James following him, introduced the term *specious present* for a 'finite segment of time' which seems to us wholly *present*. On analogy, there grew up in the 1920's a movement, led by Whitehead[2] and Russell,[3] to regard the mathematical point in physical theory as an abstraction from physical sense-experience, to which a point was always presented as a 'blur'. Again on analogy, this time with the pure mathematical definition of real numbers of the second kind, Whitehead took over the method of the 'boxing-in of intervals' (calling it the Method of Extensive Abstraction) in order to define a mathematical point from a set of observed position-blurs.

To avoid the fairly obvious circularity inherent in any attempt to define points in terms of intervals, an axiom system is laid down, in which 'events' are credited with the properties necessary for convergence, without any explicit mention of boundaries. Thus concepts of 'extending over', 'intersection', 'formative condition', and many others, are brought in so that, in a formal sense, it can be asserted that an abstractive class which is an 'absolute antiprime' converges to an 'extensionless moment of time' throughout all nature.[4] At this stage (although neither any geometrical structures nor coordinates have been mentioned) Whitehead assumes 'the electromagnetic theory of radiation' (i.e. the Special Theory of Relativity).[5] The 'intersection' of moments is then considered; and in view of the fact that specifying a value of the

[1] For this reference to Clay I am indebted to G. J. Whitrow, *The Natural Philosophy of Time*, where further useful references on this subject will be found, *op. cit.*, pp. 78 ff.

[2] *An Enquiry Concerning the Principles of Natural Knowledge*, Cambridge 1919; *The Concept of Nature*, Cambridge 1920.

[3] *The Analysis of Matter*, London 1927.

[4] PNK, pp. 101 ff., 106, 110 ff.

[5] *Ibid.*, p. 115.

time variable in each of two frames limits the spatial possibilities, these intersections are defined as instantaneous planes ('levels'), so that eventually, by multiplying intersections, we obtain a 'punct' as the common intersection of four moments.[1] These 'levels', 'rects' and 'puncts' are assumed to have all the conceptual Euclidean properties of planes, straight lines and points—properties which in fact must be presupposed before any measurement or physical theory can be begun.

For anyone accustomed to philosophical analysis, it is not difficult to see in this programme the taint of psychologism, the 'cart before horse' logic of non-operational idealism, and the misuse of the axiomatic method to evade analysis instead of clarifying it.

Till interpreted, an axiom system is void of content. It remains a 'quoting game' with symbols. Here, if we begin by asking what is the interpretation of the terms 'event' and 'extending over', it is clearly inadmissible to give an answer referring in any way to measurement; for the structures of geometry, congruence, and the numbers attached to phenomena signifying their position, etc., are the very things which are supposed to be deduced. To bring them into the first axiom would therefore be circular in the most obvious way.[2] Presumably we are supposed to receive some psychological or idealistic impression of one event extending over another. In that case we must ask how, out of this empirical material, a system of exact conceptual relations arises. It appears, in fact, to be put in surreptitiously, first when 'formative conditions' necessary to convergence are assumed (so that a *form* already exists), and secondly when 'the electromagnetic theory of radiation' is assumed, thus presupposing a Euclidean space-frame and time-instants in a simply ordered set. These and other confusions seem to have arisen from the erroneous psychologistic view that measurements and conceptual systems are somehow 'read off' from the physical world, instead of being approximatively fitted to it.

What I conceive to be the true analysis of the nature of 'physical points' is a comparatively simple matter, depending on the fact that objects are presented to us in a *total encounter* and perceptive details are associated with rational numbers or a range of rational numbers, according to circumstances and an agreed technique of measurement.

Suppose it is required to find the angular separation of two stars, with the help of a low-power telescope with graduated circle. On five separate occasions, on each of which the observer tries to make as accurate a reading as possible, the following measures (in degrees) are recorded, respectively:

$$1{\cdot}73,\ 1{\cdot}73,\ 1{\cdot}74,\ 1{\cdot}74,\ 1{\cdot}76.$$

[1] PNK, p. 117.

[2] Cf. J. L. Synge, *Relativity, the Special Theory*, p. 3: 'The appeal to undefined elements, essential in an axiomatic treatment . . .'

The measures will differ owing to a variety of causes. Different portions of the graduated circle (departing in different ways from perfect exactitude) may be used; the light from the star will produce a slight physiological haze in the nerve cells of the retina; diffraction produces another kind of haze; difficulties of psychological control, when repeated subdivisions are estimated, cause a slight confusion of judgement. The measure eventually recorded has in every way, and of unavoidable necessity, the characteristics of a *guess*.

Nevertheless, accepting this set of measures, it is customary to summarize the experimental state of affairs by recording two derivative measures, (1) the arithmetic mean, which is here 1·74, and which the Theory of Errors indicates is the 'most probable' value, assuming the disturbing causes operate equally strongly towards an increase or a decrease; and (2) the standard deviation, which is here 0·011, approximately, and which gives a rough idea of the deviation which is to be expected, on the average, if further observations were to be made. Now what has particularly to be remarked is that the original 'guesses', the mean, and the standard deviation, are all taken from the dense set of rational numbers in the mathematical continuum, thus brought approximately into correspondence with the total encounter on the various occasions when it is measured.

Next, suppose that a telescope of greater magnifying power is used, and a series of observations yields a mean of 1·742, with standard deviation 0·002. From the fact that the accuracy of observation has thus been vastly improved we might be tempted to suppose that a limit of *perfect* accuracy (say $1{\cdot}7405\dot{2}\dot{7}$) ideally exists, i.e. that nature contains or presents an exact measure of the separation, even though it is categorically beyond all possibility of being attained by measurement. If, further, it were found that in a cluster of five or six stars the angular separations were all multiples of a certain smaller measure, say 0·58, we might be tempted to suppose that clusters were constituted out of a certain indivisible *quantum* of separation. But none of these conjectures would alter the facts, that (1) no measures of a total encounter, whether found directly or derivatively, can be absolutely exact, correct, or all-inclusive of the possibilities, and (2) all measures refer to an everywhere dense mathematical 'space' of rational numbers.

Curves and Trajectories

Physical curves and trajectories are good examples of Gestalten known in a total encounter. By *points* of the curve we may mean either a perceptive detail seeming to be localized more or less in one position (a 'physical point') or the coordinates which we attach to such point when we carry out measurements on it. No assemblage of such points, in either sense, can constitute the curve, or can make it continuous, or

can create a gradient. All these things (curve, continuity, gradient) involve, conceptually, the *denumerably unfinished*, and observationally, the possibility of ever-new observations according to circumstances objective and subjective. Like all created phenomena, these cannot be contained in particular human experiences or measures; The infinite potentiality of the creative substructure of the world lies within them.

To speak of running through 'all the points' of a curve would therefore be inadmissible. Nevertheless, there are exact concepts of *continuity* and *gradient*, which are applicable to conceptually defined curves, and thence are applicable approximately to physical curves. Thus in any particular case a physical curve may be tested for continuity, and an approximate measure of its gradient may be found. Likewise a physical trajectory does not consist of isolated events. Nevertheless, by selecting points, the exact concept of velocity may be applied so as to obtain an approximate measure of velocity at any physical point of the trajectory.

Infinity of Space

A non-temporal process in which numbers increase without limit may be meaningfully introduced in pure mathematics, and thus also, for manipulative convenience or as a 'form of phenomena', in physics. The set of natural numbers 1, 2, 3, . . . is a meaningful concept because the law of formation is clear; given any term n, the next is $n + 1$. There is no limit; but the sequence may be used to determine limits, e.g. the sequence a, $a^{1/2}$, $a^{1/3}$, . . . has the limit 1. A sequence of real numbers without any law of formation, or a function of such numbers, is nothing definite, being given only so far as the numbers have been allocated. In the same way, an infinite sequence of *objects*, without any law of formation showing with absolute exactitude how each later object in the sequence is to be obtained from the previous ones, is a contradiction in terms, and void of meaning. Milne offends against this logical principle when he writes: 'Though some theories profess to give a figure for the total number of galaxies, my own belief is that the number is infinite.'[1]

It appears therefore that, in measuring the physical universe, we may choose only such 'form of phenomena' as implies that the number of objects capable of being manifested is finite. There are here two matters requiring close examination: (1) what distinguishes an 'object' in this context, and (2) what kind of implication this is.

If we take an 'object' as a *rational unity* or *seminal core*, there is by implication the possibility of focusing attention on particular parts

[1] *Relativity, Gravitation and World-Structure*, p. 2. The logical principle concerned may be described as the Principle of *Non-Actual Infinity*.

of it—such parts being, it might seem, necessarily 'objects' also, of more restricted scope. For to speak of a 'unity' implies the existence of parts which are united and so give the whole its recognizable character. In that case, these more restricted objects would be further analysable in the same way. Is then an object capable of subdivision into an infinity of objects?

This difficulty disappears, I suggest, in the light of the following considerations. Firstly, a *seminal core* (a preferable term, here, to *rational unity*) constitutes a potentiality for the 'manifestation upon occasion' of an inexhaustible variety of perceptual details intimately cohering in space and time. There is no more objection to admitting an infinity of possible perceptual details than there is to admitting an unending possibility of new conditions of observation as time progresses and people and circumstances change. Secondly, it is not inevitable that the seminal core shall be divisible into other seminal cores. The subdivision of the original core into its parts may be like that of a total encounter into perceptual details, none of which have separate *existence*; or like that of a living organism into its separate organs and members, so that one category of existence (sentient existence) is replaced by another category (dead matter); or like the analysis of an atom into nucleus and electrons, the individuality of which disappears when they are regarded as parts of the atom. More generally, a structure may be divisible in *essence* but not in *existence*.

What are the smallest 'objects' capable of separate existence? It seems to me, from the evidence of the electron microscope, that it is reasonable to regard molecules, and probably atoms also, as separately recognizable in their effects. Elementary particles, however, have observable effects only when there is some energy-transfer by which an atomic system changes its character (at least that is how the formalism is interpreted). The nature of elementary particles is bound up in the nature of *fields*, the quanta of which are not localizable or separated from one another, but merely express the rule of increase or decrease of energy and its associated measures, in exchange with some other field or system. It seems to me in accord with present knowledge to say that fields (electromagnetic, fermion or boson) are our way of representing *potential structures implicit in an objective phenomenon*, and that our concepts of elementary particles are brought in to express the changes that can take place within a field or system which we recognize to be composite.

The conclusion seems to be, accordingly, that there is a limit to the divisibility of identifiable 'objects', although the potentialities inherent in their field-substructure are infinite. And the argument may still be accepted, that the number of identifiable objects in the physical universe is finite.

The Radius of the Universe, R_0

Let us now try to see what meaning can be reasonably given to the phrase 'infinity of space'. Consider first the ideal-operational Euclidean geometry, which serves as 'form of phenomena' in the strictly local region. This is a conceptual system to be applied to phenomena when we observe them. If we see a flexible string drawn tight in a straight line by a hanging weight, it is true that the substructure of nature must account for that actual appearance, thus putting into the world, so to speak, an approximation to a straight-line. But we recognize it as a straight line only because we compare it with the ideal straight line which we have in our minds. And such comparison is possible if we can see or feel the straight line, in a strictly or optically local region; so the question of the infinity of *actual*-operational Euclidean space does not arise. By the infinity of *ideal*-operational Euclidean space, on the other hand, we mean merely that Euclidean conceptual structures can be, ideally, enlarged or extended indefinitely in relation to other conceptual structures taken as standard.

In the same way, a measurement-tool which utilises the Robertson–Walker metric tells us nothing about the infinity or finiteness of the universe, so long as it is ideal. But we have seen that in order to implement it actual-operationally we need either to decide the constants (k,R_0, and others if desired) arbitrarily, or determine them on certain operational principles, e.g. the principle that the nebulae of a certain *actual* lattice coincide, on the average and when regarded as particles, with observers of the ideal observer-gas presupposed in deriving this form of measurement-tool. Thus if we wish to speak precisely about the form and size of the universe we must build our conclusions on the observed existence of the nebular lattice. The nebulae of the lattice are presumably 'objects' in the sense just considered. Hence it seems to follow conclusively that the number must be finite, and therefore R_0 will need to be a finite real number.[1] The universe is then represented to us as in a certain way finite in extent and in the number of 'objects' in it, but unbounded.

This inquiry has once again illustrated the fact that questions concerning the actualities of space and time cannot be merely 'read off' by observation. The answers depend on considerations of operational and representational 'appropriateness'—or *prior validity*, as I shall term it in the next chapter, when we have placed our decisions on a stricter logical ground.

[1] Tolman, *op. cit.*, p. 468; formula 181.2 for dN, integrated.

CHAPTER 18

THEORY OF REPRESENTATIONS

> 'And lo, looking over his shoulder, or rather in the very recesses of his observations, the man of science found the companion he most desired to avoid. Do what he could, he could not rid himself of the society of his own mind . . . He had regarded thought as "a mere pensioner of outward forms". Now into the broad sunlight emerged the menacing incontrovertible truth that in the picture he drew of nature he was invariably himself present, and that his presence gave to the picture features which virgin nature never had, nor could have.'
>
> P. Macneile Dixon, *The Human Situation*, p. 161

Every formal system, chosen as a basis for measurement, is capable of many interpretations, between which a choice must be made. The stipulation that the system of measurement shall be self-consistent is, by itself, insufficient to determine either the system or its interpretation. Consequently, in our attempts to comprehend the physical world around us in terms of space and time, instead of as a mere list of measures, we cannot proceed without an appeal to principles of preference. Some of these principles are so compulsive that we fail to realize when we have subconsciously appealed to them. Others are less compulsive, but seem nevertheless natural and inevitable. There are also vague principles, if we may call them such, of mathematical or practical 'convenience', 'simplicity', 'fertility', etc., and 'cosmological principles', to which appeal is sometimes made.[1] These may have strong persuasive power one way or another, although we see no clear epistemological or ontological ground for accepting the persuasion.

Thus every cosmologist speaks of the possibility of the universe being 'expanding', of its probable size, age, and so on. It is clear therefore that he has adopted, or is considering the adoption of, some special geometry for measuring the universe and some special interpretation of the measures in terms of concepts of 'distance', 'time' and so on. What are the grounds for his preferences? Even when, on very rare occasions, an attempt is made to state these explicitly, the impression made is one of vagueness and confusion.

There have been various occasions in the previous chapters[2] when

[1] Studies of the concept of *Simplicity*, by R. S. Rudner, M. Bunge, N. Goodman, R. Ackermann and S. Barker will be found in *Philosophy of Science*, April 1961.

[2] Pp. 175 ff., 181 ff., 222 ff.

appeal had to be made to what I hope were reasonably acceptable principles of preference or 'appropriateness'. The meaning of the term 'appropriate', however, was left to be sensed more or less intuitively. In this chapter I propose to review some of the questions already discussed and to introduce some new ones—endeavouring now to explain the principles concerned in such a way as to make clear how they contain an epistemologically compulsive element.

What is a 'Representation'?

In the widest possible sense a *representation* is an interpretation in terms of concepts of space and time. What is interpreted may be (1) something not itself spatial or temporal, e.g. a purely formal 'geometry', or measures regarded simply as numbers associated with a phenomenon or process not regarded as already possessing some kind of spatio-temporal measure; or (2) something which already possesses spatio-temporal measure or operationally defined spatio-temporal properties. The first of these kinds of interpretation could be distinguished, when necessary, as a *presentation.*

Specifically, representations of either general type fall into two classes, which may be described as (1) Euclidean or Galilean mappings, and (2) topological space-time conceptual-schemes; and each of these may be worked out or applied either regionally or universally, i.e. comprehending the entire astronomical universe. The first of these descriptions explains itself; it is to be noted, however, that a 'mapping' might be on a material substance, or it might be a 'conceptual-scheme' enabling our memories of sensory appearances and measures to be thought of in spatio-temporal terms (in loose language, the 'mapping' may be imagined).

The second description is intended to suggest the kind of view we take of the universe or part of it when we recognize that a light-ray, although mappable along a straight line in a given local frame, is not operationally the same as a straight line (having quite a different group-objectivity). It might therefore eventually arrive (after continuing always in the same direction) at the region from which it started out. In such a case the geometry assumed, though complete, is occasional. Although *analogous* with the geometry of great-circles on the sphere, the analogy is a very unsatisfactory one, since the great-circles are not mapped as straight lines. A 'topological' space-time conceptual-scheme is in fact incompatible with the imagination of a Euclidean or Galilean mapping. We can conceive of it only by thinking of the universe as of an extent far beyond the reach of any ordinary telescope, and manifested 'upon occasion', yet with potential qualities of continuity between each local region and its neighbouring ones.

Illustration of Difficulties

Beginning with a difficulty which is in practice trivial, but which may also be illuminating in certain ways, let us suppose that we have decided to take as a standard of time duration the period of a certain spectral line, say the chief component of the H_β line. The question to ask first is, 'How do we recognize this line?' We obviously cannot recognize it by measuring it, if it is the very thing which is to provide the standard. Presumably we must produce the whole Hydrogen spectrum, being able to identify Hydrogen by its qualitative properties; and then we must be able to recognize the *general form* of the spectrum sufficiently well so as to be able to pick out one part of it (the line in question) recognizably. In the case of the chief component of the H_β line, all this is easily done. But the epistemological difficulty remains, that to recognize a type of object for the purpose of measurement, we cannot depend on measurement but must depend in the first place on metascientific intuitions.

Similarly, in investigating the gravitational deflection of light-rays passing near the sun, one cannot simply measure the position of one star before and after the apparent approach of the sun to it. A certain star-field has to be *recognized* on each occasion, ignoring minute differences of conformation, taken to be irrelevant. One of the stars in this field has then to be recognized by its spatial relations to the others (or by its spectrum) again ignoring minute differences. The differences in position, which do not affect recognition, then measure the deflection.

Such examples illustrate what I shall call the Principle of (Unbroken) **Gestalt Recognition**—the principle that a conceptual system or interpretation which breaks up the Gestalt necessary for recognition is not admissible. Any hypothetical distinction made between two or more elementary particles with identical internal structure accordingly seems meaningless, unless some enduring Gestalt (such as presumably occurs within the atom) can be established.

Problems of Rotation

On looking out through the window of a moving train we see the scenery appearing to move in the opposite direction. Likewise, whenever we turn our heads, the surroundings will appear to move across our field of vision; but through long practice we have schooled ourselves to conceive of them as 'really at rest'. The question arises, 'With what justification do we make this assumption that they are *at rest*?' One reason appears to be that we move our heads voluntarily, and it is inconceivable that a voluntary decision on our part could move our whole environment. Another is that if two persons turned their heads

in opposite directions, and the visual appearance of movement was in each case taken as objective, there would be obvious contradiction. Essential to both these reasons is the recognition that the environment is an enduring Gestalt, substantially *the same* while the various experiments are performed, and therefore with elements of objectivity when dissociated from the *subjective* variations.

It may thus be said that our conclusion that apparent motions correlated with the turning of the head or the moving of a train are to be accounted *subjective*, and must be abstracted from the objective picture of the world, follows from principles of **Consistent Causality.** These may involve (1) the consistent application of a conceptual measuring system and its interpretation in terms of spatio-temporal concepts, (2) consideration of voluntary action and independent choices by different observers, and (3) considerations of topology.[1]

The starry sky is considered to rotate relatively to the earth because, ignoring very small deviations (due to aberration and refraction), the *same* constellations are recognized from time to time in different positions, implying a rotation. Let us first consider in what difficulties we should find ourselves if the deviations, arising from excessively rapid and erratic motion of the earth and from the presence of very highly refracting gases, were considerable and lacking in obvious system. The stars would nevertheless be presumed to have definite objective positions, but no method for determining those positions might present itself better than taking an average over a certain number of observations. Such averaging would have virtually no scientific status, being little more than a guess, using a familiar mathematical technique, while all the time we are quite ignorant of the circumstances.

Fortunately the deviations are in fact systematic, and can be separated into what I shall call *subjective* ones, imputed to the motion of the observer (on the earth), and *intermediate* ones, imputed to atmospheric refraction. Consistent theories are devised to 'correct' for these, and what remains is then taken as an *objective* appearance of the astronomical universe, in effect from the position of the sun,[2] but nevertheless still appearing to rotate because of the earth's rotation, which

[1] Reichenbach has drawn attention to the fact that *causal anomalies* may be associated with topological peculiarities and with the infinite. (*The Philosophy of Space and Time*, pp. 65 ff., 79, 279.) The example on which he chiefly depends, however, is open to objection in so far as the Euclidean construction of spherical shells is assumed along with topological circumstances incompatible with Euclidean geometry (p. 64 f.).

[2] In a lapse of time sufficient to determine an average position the star in question may have moved relative to the solar system. The only way to obtain what would in principle be a correct view of the astronomical universe from the position of the sun would be by a general relativistic transformation, which we cannot carry out because the two-body problem in Relativity has not been solved.

counts also as *subjective*. The next step is therefore to 'correct' for this rotation and produce a finally *objective* conception of the universe.

An Experiment

For this purpose consider two neighbouring space-ships A and B, each at rest in a certain inertial frame. This implies, among other things, that if there is a gravitational field in their neighbourhood each space-ship is 'free-falling' and the two space-ships have zero relative velocity and zero rotation relative to each other. Such a state of affairs can be ideal-operationally defined because of the fact that acceleration due to a gravitational field can be 'cancelled out' locally by allowing the reference frame to fall freely without rotation. The condition for this is that all other free bodies then move with zero acceleration relative to it (this is a fundamental experimental fact, usually formulated as the Principle of Equivalence).

Accordingly we introduce, as a further determining condition of our ideal-operational geometry, the concept of an 'ideal free-particle', whose trajectories are given by $\int ds =$ minimum, i.e. straight lines described at uniform velocity if the geometry is pseudo-Euclidean, as supposed. The operational implementation is normally in particles without material contacts of any kind and correlated for all other disturbing influences except gravitation. If this implementation were not valid, to a quite indescribably high degree of accuracy, the amazing confirmations of classical astronomy (which depend on the truth of Newton's Laws of motion) could not have been found. All this amounts to the Geodesic Postulate of Chapter 13.

We now have to make sure that there is a satisfactory operational test of non-rotation. In a conceptual analysis such as we are attempting, it will be sufficient if the test is ideal only.

Accordingly, let a pendulum be set up with electric (or magnetic) force in place of gravity. Thus the bob can be negatively charged, and the bob and supporting string can be made to swing in a plane through the point of support and a powerful positive charge which draws the string tight. Both support and positive charge are to be fixed in the given frame. So long then as the frame remains inertial, the bob will continue to swing in the same plane relative to the frame. If the plane of swing changes we shall know (by definition) that the frame can no longer be regarded as inertial. Consequently, over any short period of time it is possible to choose a frame in which an 'electric pendulum' (as I shall call it) shows no rotation, and the frame can be assumed inertial.

It is to be noted that a free-falling frame in which zero rotation is demonstrated by this means might develop, over a longer period of time, an apparent rotation due to the fact that the cancelling out of a

gravitational field can be done only locally in space-time. For the present we ignore this consideration.

The space-ship *B* is now set rotating relative to *A*, at an angular velocity which can be varied according to the free decisions of the commander.

Consistent Causality then requires that we class the variations in apparent angular velocity of the starry sky seen by the occupants of *B* as subjective. Let us consider, however, the state of affairs if *B* maintained a perfectly steady rotation relative to the frame of *A*. It might be urged that *A* is then rotating steadily relative to *B*, and that either might be taken as a standard non-rotating frame in that locality. The arguments against this are as follows, (1) the frame of *A* is Galilean, without any need for correction due to forces affecting the measuring apparatus, whereas the frame of *B* can be made Galilean only if corrections for the effects of centrifugal forces are introduced; (2) in frame A free particles move in straight lines and a Foucault pendulum swings always in the same plane, whereas in frame B free particles move generally in obvious curves and a Foucault pendulum swings in a rotating plane; (3) the corrections to be made in frame B and the behaviour of free particles depend on which particular angular velecity is chosen. To single out one of these motions as 'absolutely non-rotating' would require (in terms of Weyl's Principle, somewhat broadened in scope) corresponding experimental data attaching an operationally definitive character to such motion. This condition is satisfied by frame A but not by frame B.

Prior Validity

The broadened version of Weyl's Principle, just mentioned, might be stated as follows:

(W.3) We may not assert the 'givenness in nature' of a certain standard condition, as distinct from other conditions of the same class, if, in terms of the formalism and geometry assumed and the class of experimental data admitted, this standard condition cannot be defined by the internal properties, given in nature, of that condition.[1]

If we cannot define the given condition except by reference to another condition of the class, independently defined, then that condition counts as *derivative*. Alternatively this principle can be regarded as a special case of the following two principles:

(V.1) If, among various possible measures which can be attached to a given measurable condition, each obtained by the consistent operational

[1] The assumption of the givenness in nature of an absolute velocity (relative to a luminiferous ether or other absolute frame) and an absolute time are alike ruled out by this form of the Principle.

implementation of a conceptual system, there is one which is presupposed in the calculation of the others, that measure shall be said to have *prior operational validity* in relation to the others.

For example, the convenient relativistic scale of velocity, $V = \text{argtanh}\,(v/c)$, where v is dx/dt in Galilean coordinates, has not prior operational validity, since the Galilean measure of velocity is presupposed. Or, in the example concerning the space-ships, we have to construct a consistent operational method of implementation of a conceptual system for the measuring of angular velocities in the given region, and the most primitive standard for rotation which we can construct is the inertial frame A.

(V.2) If, among various possible representations of the same state of affairs, there is one which is required as an initial stage in the operational construction of the others, that representation shall be said to have *prior representational validity* in relation to the others.

For example, if we represent distances along a line by the cube of their Euclidean distances from a given point, our representation has not prior representational validity, since a Euclidean representation is presupposed. Or, in the experiment with space-ships, if we represent (to our minds) space-ship B as at rest and A as rotating, in an absolute sense (locally), our representation has not prior representational validity since the condition of space-ship A has to be operationally constructed (in the mind) first.

These principles (and later ones also) might be described as *definitions* of the terms introduced, with the implication that what has been accordingly preferred has a special kind of *value*.

On the minor problem of the strictly local implementation of a formal geometry, discussed in Chapter 12, it is perhaps sufficient to remark that a *natural* representation is one which has prior operational and representational validity over other kinds of representation, and that topological restrictions laid upon a representation might be included under the category of principles of Consistent Causality (using the term 'cause' in the sense explained above).

Returning again to the example of the space-ships, a difficulty might conceivably arise, namely, that although we have decided that space-ship A is to be regarded as absolutely non-rotating, the starry sky might nevertheless appear to its occupants to rotate. In that case the apparent rotation could be made to vanish, on the average, relative to space-ship B if its rotation relative to A is suitably chosen. But we must make up our minds as to what definition of non-rotation (during the limited time of this experiment) has prior validity, and keep to it. In my opinion the arguments already given would still apply in this

conjectured eventuality, and we should have to say that observation revealed an objective appearance of rotation of the starry sky.

The epistemological analysis, however, by which we derived the Robertson–Walker metric in the last chapter does not admit any such apparent rotation of the nebular lattice; and none is disclosed by observation after the effects of local and intermediate causes have been removed by correction according to principles of consistent causality. So I think this conjectured possibility can be safely ignored.

Absolute Non-Rotation

We must now turn to the further difficulty that while the experiment is in progress the plane of the electric pendulum in frame A may be found to be undergoing a very slow rotation. In that case our operational definitions, in conjunction with Helmholtz's Principle, show that some gravitational effect is present and operating as 'cause'. Our operational construction of an inertial frame then fails in this region of the universe when the time-duration is of the extent being considered.[1] This effect, while theoretically predictable, is minute and in fact irrelevant to astronomical observation in practice, even if an electric pendulum could be maintained in motion accurately enough over such a period of time. In principle, however, if the effect occurred it would be impossible to say whether or not frame A was rotating. Nor could we apply a universal geometry, using observations of this kind; for we would not know to what extent our observations should count as objective.

We can possibly get over this theoretical difficulty by the device of choosing for our space-ship a locality of the universe where no rotation is ever observed—i.e. where there is a gravitational neutral point or the gravitational field is negligible. This must be done in numerous places of the universe, and then it is to be hoped that a prescribed geometry can be fitted to the objective observations thus obtained, so that in the other regions (where the gravitational field is not negligible) the gravitational conditions can thereby be measured.

There are thus immense theoretical difficulties, which could easily have arisen in the astronomical universe and blocked all scientific investigation of it, had certain of the fundamental magnitudes (gravitational constant, velocity of light) been different from what they are. Fortunately, again, the difficulties have been in nature reduced to such small proportions that in practice they are of no account. It remains true, however, in the cosmic as in the atomic sphere, that a picture of objective phenomena, with precisely accurate measures, is a contra-

[1] On this problem, see also F. Pirani, *On the Definition of Inertial Systems in General Relativity* (Article in 'Jubilee of Relativity Theory', Basel 1956, pp. 198 ff.).

diction in terms. The empirical world of total personal encounters cannot contain the exact relations of *measure*.

Theory of 'Corrections'

It is time to look more closely into the epistemology of the 'corrections' needed to fit a prescribed geometry to the universe and so to enable objective measurements and predictions to be made, to the extent that nature permits.

An ideal-operational geometry assumed as 'form of phenomena' in the universe or any part of it will never fit exactly. If it did, the *a priori* would be made identical with the empirical. The deviations may be small and random, in which case we impute them to subjective causes (personal, or due to inadequacies of the apparatus) and deal with them by the technique of averages and standard deviations. Or they may be systematic and (by Helmholtz's Principle) correlated with some physical phenomenon otherwise observable. Science allows for the possibility of small random deviations, but it cannot predict them. On the other hand, what is systematic it hopes to allow for in such a way that prediction is possible. There are two ways of attempting this.

(1) Normally the deviation is associated with some physical effect to which a measure (say, A) can be independently attached. The measure of the deviation can be taken as a measure (B) of that effect, and a correlation between the two effects may be found. Conversely, when the effect is measured (A), measure (B) can be calculated and imposed as a 'correction', thus restoring consistency to the given 'form of phenomena'. This technique has to be used in practice for systematic subjective deviations, such as distortions of the graduated rule under stresses or temperature, and for intermediate deviations such as those due to a refracting gas interposed. Where, however, the disturbing effect is not in the measuring apparatus itself, or confined to a very small region, or resulting from the accumulation of disturbing effects confined to small regions (as in refraction), the technique fails or becomes highly artificial. In short, when the disturbing influence is of the nature of a *field*, a holistic technique seems epistemologically more correct, as well as being far more manageable mathematically.

(2) Let us suppose that in physical conditions of enormous variety a certain 'form of phenomena' (e.g. flat space-time) is found to fit accurately when corrections are made only for subjective effects and intermediate effects such as refraction, but that systematic deviations are found corresponding to another kind of physical effect, in the nature of a field (e.g. gravitation). In that case we may be successful in finding another prescribed 'form of phenomena' and an associated interpretation (operational definitions, 'laws', etc.), so that an accurate fit is obtained without the need for correction for that effect.

Consider now the following imaginary experiment. The occupants of a space-ship are exploring the neighbourhood of a dark star, with the help of light-rays, projectiles and atom-clocks. They find it possible to implement a non-flat geometry with metric, $ds^2 = g_{ij}dx^i dx^j$, the coordinates x^i of any recognizable point being operationally determinable with the help of light rays received from it, the trajectories of light-rays and free projectiles being presumed given by the geodesic condition. Landing on the dark star the cosmonauts arrange for nuclear explosions to take place round an equatorial belt. After they have gone to a safe distance the star explodes at a pre-arranged time, thus becoming two smaller stars, which separate slightly and eventually revolve about their common centre of mass. On further investigations the cosmonauts find it possible to implement another geometry, metric, $ds^2 = \bar{g}_{ij}d\bar{x}^i d\bar{x}^j$, using other coordinates $\bar{x}^i$, operationally defined, and assuming again the geodesic condition.

Presently the two portions of the star fall together, and the proposal is put forward now to divide the mass of the star not into equal parts but in the ratio three to one. One of the cosmonauts, being a specially ingenious mathematician, deduces a form of metric which might then be found operationally implementable. The experiment is carried out, and the form of metric verified.

It is clear, I think, that each of these metrics, and therefore also the corresponding measurement-tool or method, has a *provisional* character. The stars around, at distances too great to be appreciably affected by the events described, must be conceived of as remaining in their objective positions or moving according to the conditions acting on them. There must be some objective basis if we are to be able to correlate the various appearances one with another and make predictions, while one stage of the experiment is in progress, concerning a later stage. It is difficult to see what this could be unless it is something so vast or so distant that, on the average at least, it can be taken to be unaffected by any happenings of an accidental or voluntary kind. Here then is an unavoidable appeal to the existence of a cosmical structure such as the nebular lattice. This will determine the objective 'form of phenomena', or, in other words, the most valid measurement-tool for the universe. The metrics just mentioned, being implemented according to the (possibly voluntary) placing of certain masses, thus have the status of a new sort of correction-device.

In the case of a single mass-point in space-time with parameters R_0, k, it is easy to make the 'correction' in question. One has only to superpose the Euclidean representation of the coordinate network assumed in the Schwarzschild metric (or better, the isotropic metric, if conformality is desired) on the Euclidean representation of the homogeneous universe. Thus in a chart of the neighbourhood of the sun we

shall have to remove the straight lines representing geodesics if the sun were absent, and insert the curved geodesics which result from the gravitational action of the sun. It goes without saying that this is the rational procedure in the case of planetary geodesics; we must therefore follow the same procedure with the null-geodesics which are light-rays.

Our emphasis on the need for metrics of two kinds, one to show the basic geometry and the other to show superposed gravitational effects, the former being *preferred* in a certain way, may call to mind views put forward by Nordstrøm,[1] Rosen,[2] Whitehead,[3] Fock,[4] and others. It is to be noted here, however, that the basic geometry which I am maintaining is 'preferred' (i.e. has prior validity over other geometries for the purpose of a universal representation) is not flat space-time. Consequently a Lorentz Transformation applied to the universal representation in which a certain observer of the 'gas' is at the origin does not result in another universal representation of the same kind, with another observer of the 'gas' at the origin. It results, of course, in a representation, but not one of equal validity. For the only way in which we can obtain other representations of equal validity is to transform the cordinate system of one observer of the 'gas' into that of another such observer; and this transformation is not a Lorentz Transformation.

The Universal Representation: Principles of Preference

I propose finally to make more definite the justification, such as it is, for each of the steps taken in Chapter 13 towards the ultimate decision that the steady-state geometry is the most 'appropriate' measurement-tool for the universe as a whole. It will assist clarity, I think, if the claimed justification is set out in the form of principles, of various kinds, but all with some degree of epistemological force—not merely with the advantage of convenience, simplicity, or fertility in results or applications. Besides the principles of Consistent Causality which are

[1] Reference given in N. Rosen, 'General Relativity and Flat Space', *Physical Review*, Vol. 57, p. 150.

[2] *Op. cit.* Rosen remarks: 'Riemannian space in Relativity is not a geometrical *space* till it is mapped in this way.' Also: 'We may suppose that we map the Riemannian space with metric $g_{\mu\nu}$ on a flat space with metric $\gamma_{\mu\nu}$ and assign the same values to the coordinates of corresponding points. Or we may say that the two metrics side by side represent a comparison of the given space with the space one would have if the gravitational field were removed.'

[3] *The Principle of Relativity*, Cambridge 1922. Whitehead's Theory is much more elegantly presented by J. L. Synge in 'Orbits and Rays in the Gravitational Field of a Finite Sphere according to the Theory of A. N. Whitehead', *Proc. Royal Soc. London* A, 211 (1952), p. 303.

[4] 'Sur les systèmes de coordonnées privilégiés dans la théorie de gravitation d'Einstein', *Jubilee of Relativity Theory, Proceedings*, Basel 1956, pp. 239 ff.

required to justify methods of correction, seven additional principles may be listed; but several of these could be taken as narrower applications of the broad principles of operational or representational validity already laid down. What follows should perhaps be regarded as a suggestive exploration rather than an exhaustive analysis.

(1) The Principle of **Measurement-Tool Space-Invariance.** We must begin, I think, by granting that a universal representation is to be constructed by piecing together, so to speak, the local views of each of the observers belonging to a chosen class (the 'observer-gas'). We cannot construct a universal representation from the views of all observers, so a selection must be made. This principle then asserts that a representation which is constructed by certain general rules of procedure *independent of the choice of observer* from among the chosen class, and independent also of the choice of any axes of reference which may be used for him, has prior validity over a representation in which this is not the case.

If, contrary to this principle, we made our measurement-tool vary from observer to observer, the variations would need to be specified by means of some previously devised measurement-tool or else arbitrary. In the former case, the other measurement-tool would have prior operational validity. In the latter case, we should offend against Weyl's Principle by introducing features not corresponding to any physical condition.

(2) The Principle of **Riemannian Implementation.** I take it that the existence of an invariant $ds^2 = g_{ij}dx^i dx^j$ is necessitated by the demand for such a universal geometry, as argued in Chapter 13. The operational implementation then requires further a consistent correspondence between structures of the formalism and trajectories of ideal light-rays and free particles. Since these trajectories are given *locally* by the geodesic condition $ds =$ minimum, and zero for a light-ray, it seems impossible, with consistency, to choose any other condition for the general case. Also we need an operational rule of correlation between the units for ds in different localities; and again, since ds is a *local* measure of proper time, and we cannot make the scale vary from locality to locality without (as before) offending against Weyl's Principle, it seems impossible to choose for our primary measurement-tool any other consistent rule of correlation than the Postulate of Proper Time.

This Principle No. 2 then asserts that on account of the mathematical and epistemological compulsion to use these postulates to implement the geometry, it is to be expected that any systematic deviations of the *actual* trajectories from the ideal ones will be explicable (according to Helmholtz's Principle) in terms of some other physical cause, which will be measurable by some other means and so can be 'corrected'.

We shall not have to say that the systematic deviations occur because we have chosen the wrong geometry, and there is in fact no special physical condition causing them.

(3) The Principle of the **'Co-moving' Representation.** This has not the same force as the other principles, and is introduced primarily so that one definite type of representation shall be agreed upon and accordingly it will be easy to specify, in the same way for all observers, the rules for application of the measurement-tool corresponding to it. According to this principle the observers of the chosen 'gas' are to be represented as at rest, on the ground that the representation is intended to be a synthesis of local views, and no observer has reason to suppose that *he* is moving while others are at rest.

By virtue of principles, to be introduced subsequently, it will result that the proper distance between observers belonging to the 'gas' increases with time, although the represented distance remains constant. This is perhaps not a serious objection, since there is no local means of observing such an 'expansion' of proper distances (human beings and measuring apparatus would expand together with all their surroundings). The imputed expansion is in fact only the result of a calculation when the measurement-tool finally decided on is applied to the universe as a whole.

(4) The Principle of **Observer-Gas and Nebular-Lattice Coincidence.** From the above principles we can deduce the Robertson–Walker metric for any ideal observer-gas capable of being the basis for a universal representation of the kind required. The next step is to decide which of the ideally possible observer-gases has prior validity for measurement of the actual universe. The present principle No. 4 asserts that if observation clearly indicates a distribution of large masses in the universe, such as would justify the addition of an idealized nebular lattice (to which the actual distribution approximates) to the 'material' of our measurement-tool, then prior validity will attach to a representation and measurement-tool in which the nebulae of the lattice coincide with observers of the observer-gas.

In support of this principle it can be urged that the only way of establishing a definite basis for cosmical measurement, and thus deciding between the infinity of possible observer-gases, is to make use of the average mass-distribution in the universe. Since this is given in nature, for all observers, it should somehow be embodied in the geometry.

It will be convenient to defer consideration of how the average is to be made till we have made an attempt to justify the steady-state form of geometry, by Principle No. 5.

(5) The Principle of **Measurement-Tool Time-Invariance.** It is not necessary to repeat the arguments given in Chapter 13 in favour of a

measurement-tool the manner of operation of which does not vary from time to time; but other aspects of the proposal now need careful discussion.

It is quite possible to implement a 'flat' measurement-tool throughout the universe, by regarding all gravitational effects and effects of cosmical 'curvature' as imposed, just as electromagnetic effects are regarded as imposed. The red-shift from distant nebulae is then interpreted purely as a Doppler Effect due to their velocity of recession in flat space-time. In a similar way, but far more 'appropriately' in terms of Principles 1–4, we can implement the steady-state measurement-tool throughout the universe, by regarding regional deviations as due to the gravitational effect of concentrated masses. One of the pleasing features of this kind of measurement-tool is that a Riemannian technique is used both in the application of the tool itself and in the correction for the effect of the concentrated masses. This uniformity of technique makes it seem a truer, or less artificial, account of the laws of manifestation in nature than that which is given by the method of correction on the basis of a flat space-time.

In the steady-state geometry an exact linear relation appears between the ideal red-shift and the coordinate $\bar{r}$ of the nebula observed.[1] In actuality, there are deviations which seem, up to a proper distance of about 30 million parsecs, quite evidently random. At the greatest distances there seems a slight systematic increase in the gradient. This must then be interpreted, as in the case of the random deviations, as due to non-coincidence of the actual nebulae with observers of the ideal observer-gas. The very distant nebulae must be supposed (if the calculations are correct) to have a slight tendency to recession greater than that of the steady-state observer-gas.

Are we then obliged, by Helmholtz's Principle, to find some physical cause of this? Or should we, on the other hand, abandon all hope of finding such cause and build the average deviations into a new geometry, no longer of the steady-state kind? The chief difficulty here, it seems to me, is that the 'causes', whether random or systematic, must be in a certain sense *outside the physical universe*. How, we may ask, did any given nebula come to have its particular random departure from the cosmical pattern? How did the cosmical pattern come to be established in the first place? It does not seem to me a satisfactory answer to say that these regularities might have come about by chance explosions or creations *ex nihilo* within a universe initiated as a formless existence. On the other hand, to assert the action of physical causes will be to assume that these things were already in existence. Since then it seems we can know nothing about *physical* causes of the structure and devia-

[1] This $\bar{r} = \left(\frac{r}{1 + r^2/4R_0^2}\right)$, where r is the isotropic radial measure.

tions in question, it is pertinent to ask, 'What advantage is there in trying to build the slightly irregular recession of very distant nebulae into a new geometry, at the price, moreover, of abandoning the epistemological appropriateness of the steady-state measurement-tool?'

Nevertheless there is a crucial test which might be applied, in theory but hardly in practice. The geodesics in the steady-state geometry are of course different from those in other kinds of geometry. If we find, by repeated observations and experiments, that there are systematic deviations of the trajectories of free-particles and light-rays (after correcting for local mass-concentrations), but that these can be accounted for in terms of some specific physical cause, the prior validity of the steady-state measurement-tool and geometry can be taken as confirmed. But if there are systematic deviations which cannot be accounted for in this way, but which *can* be accounted for (or rather, brought within the scope of a single coherent theory) by the adoption of another measurement-tool and geometry—then it seems there is a strong argument for making the change.

At present there is no evidence of the kind just contemplated. The Principle No. 5 may therefore perhaps be taken to assert that, on account of the epistemological preferability of a time-invariant measurement-tool, it is to be expected that deviations from the ideal structures of the geometry, where they are systematic, can indeed be accounted for as due to specific physical causes. With regard to the slightly greater recession of very distant nebulae, if indeed the evidence claimed for this is not mistaken and not to be accounted for in some other way, there are still advantages in keeping to the steady-state measurement-tool—unless at any time more direct evidence concerning actual trajectories forces us to reconsider the question.

(6) The Principle of the **Epistemological Average.** This principle is required for the determination of the parameters k and R_0 from empirical data. There seem to be two possible viewpoints. First, we may accept, in principle, that random deviations may occur cosmically in a similar way to that in which they occur in any laboratory experiment —that is, from the accumulation of a vast number of weak causes tending symmetrically to increase or decrease the ideal 'mergence-free' measure. In that case, the ideal measure is found by the mean-square technique of the Theory of Errors. On the other hand, we may abandon all thought of explaining random deviations by accumulation of weak causes, and assert merely that nature provides a mean value, and that this is accordingly the most appropriate value to take. Whichever viewpoint is adopted, the plain fact is that an average must be taken; otherwise we should have no definite geometry, representation or measurement-tool.

(7) The Principle of **Conformality.** This principle is not needed for

the deduction of the Robertson–Walker metric, since the other principles automatically bring about conformality of the representation. We need a separate principle of conformality only in the correction for the effect of concentrated masses, in order to specify precisely how the correction is to be made. This principle No. 7 asserts that, in accordance with the principles of prior operational and representational validity, the correction for the gravitational effect of a mass-point must be made by means of the isotropic metric. For local structures (measuring rods, geometrical figures, apparatus, space-ships, etc.), to be shown in the representation, may first be constructed in the local Euclidean space; and a representation which shows them undistorted in shape has therefore prior validity over one which distorts them.

From the above discussions it will be clear how artificial and unsatisfactory in many ways is this attempt to form a representation of the entire physical universe. In particular, the method of correction, reducing actual experience to the idealized experience of selected observers, can hardly be supposed to resemble at all closely the methods by which a 'manifestation upon occasion' is created by nature for each observer. Also, to obtain a truer conception of how such manifestation occurs, we must cast out all thought of arithmetization and think instead of conceptual constructions like those of Euclidean geometry, but in time as well as space. In short, we must take this 'artificial' conceptuality as a feeble and often misleading guide to some 'real communal' conceptuality in space and time, actualizing itself for us in a 'real personal' conceptuality. Yet in a certain sense it is *given in nature*, because a strict line of thought, on the basis of what is observed, leads to a certain view which, for all its deficiencies, we are impelled to regard as the best of its kind.

SUMMARY

CHAPTER 6

The possibility of measurement rests on the concept of primitive *sameness of spatial separations*. With the help of this concept we devise ideal operations of measurement. These are then applied to the actual world. Thus we have three strands of spatial thinking—the conceptual space-geometry, the ideal-operational geometry, and the actual-operational geometry.

Actual spatial separation is recognized (1) directly by our sense of spatial position, (2) indirectly through seeing, touching, etc., and (3) indirectly by manipulations. Measurement makes use of manipulations, and depends on the construction and verification of congruences by superposition. Any *change* supposed to occur while superposition is being carried out, or between one time and another, implies inconsistency with the (timeless) ideal-operational geometry developed from the concept of unchanging congruence. It is therefore discoverable only when such geometry is presupposed as unchanging basis.

Measurement in practice begins with the construction of straight lines using a technique of successive approximation. The technique is first devised as an ideal-operational one, and then applied to actuality so far as is convenient or possible. The actual construction of a straight line and of congruent intervals on it is necessarily incomplete. Thus we put Euclidean geometry into the world approximatively for the purpose of measuring it.

Actual nearly-rigid bodies are used in measurement merely because we have no other means of actualizing the ideal congruent superposition with which measurement must begin. The ideal 'rigidity' established means simply agreement with the ideal-operational axioms of Euclidean geometry, and has nothing to do with properties of matter (the measurement of which can proceed only on this ground).

CHAPTER 7

Local and varying conditions, such as temperature and stress or the interposition of refracting gases, affect the numerical measure recorded for an objective phenomenon. To obtain an objective measure we must remove these effects by *correction*. To do this we must first measure the disturbing cause. Thus a 'secondary' vicious circle arises whenever measurement is made, and most obviously when standards need to be established. The circle is partly broken by successive approximations; but it remains inadmissible to assume *conceptually* exact measures of *empirical* phenomena.

The 'primary' vicious circle arises when we use a geometry to measure phenomena in distant regions, and the metrical structure of that geometry is supposed to depend on those very phenomena which must therefore first be measured). The view of some authorities that the geometry used for measure-

ment must therefore be homogeneous is considered too drastic. The geometry must be *prescribed*, in the sense that at most a finite number of measures may need to be determined approximatively by *fitting* to observations, and the geometry then becomes determinate. Some prescribed geometries may be more *valid* than others in given circumstances.

CHAPTER 8

A concept becomes precise by virtue of a *universe of constructive connections* with other concepts and within itself. There are direct and indirect means by which the structure of a roughly chosen concept becomes clear to us and clearly marked off from other concepts. These we describe as techniques of *analysis*.

The direct means may be of *natural* or *phenomenological* grade. In the natural types of analysis certain matters—notably the forms and usages of 'ordinary language'—are presupposed and assumed to be sufficiently clear. In the phenomenological types we do not build on presuppositions, or with inferences, 'logical constructions', or symbolic thinking of any kind; but we aim simply at face-to-face self-evidence in what is given.

An indirect means of clarification is *axiomatization*, which is a setting down of certain interrelated features of analysis in words or signs, in an order which makes their rational dependence clear. Such procedure spotlights particular constructive connections in a conceptual system one by one. The very first axiom must refer to undefined concepts (e.g. 'line', 'point'). The various possible axiom-sets and 'proofs' present different series of rational connections in the whole. The procedure is therefore heuristic, and cannot be considered either complete or finally correct.

The great danger in linguistic analysis is the Fallacy of the Conceptual Closed Shop. This is the belief that it is correct to limit our researches to the manipulation of particular words or symbolic forms (as if they were material 'counters'), taking them as equivalent to corresponding concepts. Essentially new concepts are then reduced to relations between the existing 'counters', or else are declared 'meaningless'.

Outer operational analysis is the discovery and clear specification of physical or imagined physical operations by which the concept is applied to physical phenomena or is to be found expressing itself. This assists mental 'focusing' on the precise intelligible character of the concept. *Inner* operational analysis (regarded here as of *phenomenological* grade) is the discovery and clear specification of the constructive connections internal to the given concept or relating it to other concepts, these connections being specified in a strictly conceptual way and not in terms of physical operations.

Bridgman's thesis is that the results of operational analysis (inner or outer), clarifying a concept, are to be presented in the form of an *operational definition*, strictly conceptual or physical as the case may be. This is a reasonable safeguard to clear thinking. By *Operationism*, on the other hand, is commonly understood the insistence that all entities introduced in physics should be defined in terms of outer, i.e. physical, operations. This 'opera-

tionism', equivalent to the *verifiability theory of meaning* (see Ch. 9), is rejected today.

Phenomenological analysis keeps 'at a distance all interpretations which transcend the given'. It requires a 'stoppage' (*epoché*), a 'refraining from judgement', and a 'unique form of consciousness' in which what was shifting and unclear is brought to the 'complete clearness' of self-evident truth. What we then discover has the character that it 'already was before the gaze was directed to it', and so transcends the successive acts of analysis. Truths and intelligible qualities known in this absolute and timeless way, by 'primordial recollection', are called *essences*.

Detailed expositions of the manner of arriving at the 'unshakable because self-evidencing conviction of truth' in the 'field of pure conciousness' are found in Patanjali's Yoga Sutras and in Theravada Buddhism. Hence the levels of phenomenological analysis more strictly called *transcendental* may be distinguished from the preliminary level of *inner operational* analysis.

CHAPTER 9

It is necessary to understand the various nature of theories and the principles involved in their construction. A General Principle of *Operational Implementation* requires that (i) all terms and inner operations of a theory shall have been analysed phenomenologically, (ii) in the case of a physical theory, some terms or inner operations must be implemented in outer operations, and (iii) a physical theory is subject to the principles of Weyl and Helmholtz.

Weyl's Principle declares scientifically invalid the postulation of differences given in nature but always manifested in identical ways. (This is the ground for rejecting the postulate of a 'luminiferous ether' in physics.)

Helmholtz's Principle declares that manifest differences must be accounted for by 'real' differences in substructure. Any element of the substructure revealed by analysis is termed a 'cause'.

Twelve types of theory are distinguished, special attention being given to difficulties in the 'open deductive' theories arising from their postulation of an 'unknown given' and the 'expansion' to which they are subject. Approximate ratings are proposed for *ad hoc* character (or, conversely, scientific character) of theories of the 'non-discoverable speculative' type.

CHAPTER 10

In discussing whether some particular geometry is to be imputed to the physical world locally, we have to consider (1) the perceptual basis of space-concepts, (2) the operational basis, and (3) arithmetization of geometrical structures.

The concept of a spatial figure recognizably perceived in the world is something distinct from visual perspectives and is constituted in a *whole* (a Gestalt) with a *potentiality* for various spatial aspects; otherwise it would not be comprehensible as a figure of a given kind. Any one appearance is a clue to the spatial totality which transcends the appearances. By the Principle

of *Perceived Conceptuality* we point to the fact that a concept may thus be objective and not merely subjective.

The nature and scope of the *function of spatial identification* are investigated, and the 'real' deliverances of this function are contrasted with 'imagination' and 'error'. In accordance with the principles of *Manifestation upon Occasion* and the *Total Encounter*, an 'Exhibit Theory' of the external world takes the place of the more usual 'Message Theory'. To make clear the nature of potentiality and its manner of operation at three 'ontological levels', an Extended Aristotelian Theory of Actualization is worked out.

The operational construction of a straight line (and therefore the concept of it) cannot be separated from the ideal-operational concept of the *Euclidean Isometry Group*—the rationally unified set of possible displacements. In recognizing a straight line the mind must somehow conceive of displacements applied to it or in relation to it. When the straight line is thus operationally defined the usual axioms and theorems of Euclidean geometry are seen to be self-evident.

CHAPTER 11

Using the operational definition of a single straight line in terms of the Euclidean Isometry Group, we show the self-evidence of the 'joining-up of rectangles'. Thence, without any mention of 'infinity', the axiom of parallels becomes self-evident. The ideal-operational geometry of fully-defined straight lines is therefore necessarily Euclidean.

Arithmetization is the laying down of correspondences between ideal spatial structures and arithmetical operations or formulae. It enables us to clarify our thinking, to assure ourselves of consistency, and to make predictions. The ideal geometrical and arithmetical operations of addition and multiplication are seen to correspond exactly, and Pythagoras's Theorem at once follows. A *point* in the ideal geometry corresponds to a recorded measure of position, whether or not in actuality there has been difficulty in coming to a decision as to what measure to take.

CHAPTER 12

The myth of 'physical' geometry is the unreflecting belief that measures of space and time, and hence geometrical theorems, are *read off* mechanically from the world. One geometry may indeed be epistemologically preferable to others; but in any case it must be first chosen and then put into nature by successive approximations, before any measure can arise.

'Geometry' in general is characterized by *objects* to which our *general space-imagining power* attaches spatial relationships. Geometrical objects have a recognizable structure, repeatable according to principles laid down. The spatial relationships characteristic of a geometry are between one object and another, or between constituent parts of a single object.

By a *complete* geometry is meant one which is *operational*, ideal or actual. Complete geometries may be *universal* or *occasional*. In some geometries,

complete or incomplete, arithmetization may be essential, in others unessential or impossible. Twelve chief varieties of geometry are thus distinguished.

CHAPTER 13

An incomplete geometry may be operationally implemented (1) in a local Euclidean representation, (2) in local measurements of physical phenomena, with or without a representation, or (3) in large-scale measurement. A *model* is a representation which is derivative or lacking full validity. The Beltrami model and other models of Hyperbolic Geometry are discussed and linked with questions of representational validity,

In the implementing of a prescribed geometry as a measurement-tool, we need either (1) an omnipresent labelling of points with coordinate-numbers, constructed so as to be in agreement everywhere with the prescribed formula for distance ds, in which case measurement of ds is a mere matter of substitution in this formula, thus verifying the construction; or (2) some means of connection with distant regions, e.g. light-rays, with trajectories determined from the prescribed geometry in such a way that ds can be calculated for neighbouring points in any distant region, given only the initial directions of trajectories to those points. In this case the agreement between calculated and known measures of ds must be sufficiently well verified in a sufficient number of regions for the geometry to be reasonably taken as the 'form of phenomena' everywhere.

A light-ray going beyond local regions is not equivalent to a Euclidean straight line as operationally defined, although the mapped path in any given frame may be so. Hence the second kind of implementation is compatible with the use of prescribed geometries of four dimensions, Galilean or non-Galilean. Galilean ('flat') geometry is the simplest 'form of phenomena' in space and time; no physical measure has to be determined by 'fitting'. It is implementable in the universe, in one frame, or in pairs of inertial frames conjointly, by imposing corrections for time of transit, bending of light-rays, etc. Other 'forms of phenomena' may be adopted if thereby the need for certain types of correction falls away, and it is therefore considered that such form of phenomena is more *valid* under those circumstances.

In the discussion of the *validity* of a geometry, four spheres of implementation need to be distinguished: (1) strictly local, (2) manipulative, (3) optically local, and (4) cosmical. Both in the strictly local sphere and in the manipulative sphere, the only *natural* representation possible is that of Euclidean geometry.

Optically-local space is a space in which light-rays *in vacuo* are consistently represented as straight-lines. It has therefore a Euclidean representation. In cosmical space this is not so, because gravitational deflections are not 'corrected'. A prescribed geometry of space and time must then be used.

Manipulative space is contained in optically-local space, and is therefore also naturally Euclidean in its representation. If, however, in manipulative space the measuring apparatus is uncorrected for some local disturbing influence (e.g. temperature), it becomes possible to use a (non-natural)

Riemannian geometry as a measurement-tool—but only if the geometry is prescribed as 'form of phenomena'. This verifies the conclusions of Chapter 7.

CHAPTER 14

Personal and *communal* space-time conceptual spheres are distinguished, each of these being *real* or *artificial*. The real personal conceptuality may be *strictly local* or *extended*. The link between personal and communal conceptualities is called *analytic*.

The strictly local conceptuality is that which is involved in here-and-now perception, as a Gestalt. The artificial personal conceptuality may be imaginatively added or constructed when space-time perceptions, of a kind which reason indicates should be present, do not actually occur.

The real communal conceptuality is characterized by a *potential group-objectivity*, whose existence and general character are known first by reasoning from observational data and by philosophical analysis of what is involved in communal observation. The direct mode of knowledge of the real communal conceptuality is by analysis of the *total encounter* with an object. This reveals a perspectival *manifestation upon occasion*, and also (if we proceed to *supraperspectival analysis*) a *subject-object middle-ground* which comprises the potentiality for all perspectives and also serves to introduce us to the substructure. All this constitutes the Gestalt. In this middle-ground, transcending particularities of physical space and time, analysis reveals also a universal *cohesion*, inter-subjective and inter-objective. The egocentric predicament falls away, because in principle all details of cohesion (out of physical space and time) can be made immediately accessible.

CHAPTER 15

The artificial communal conceptuality provides a representation of the universe as a whole, imagined as on a Euclidean map. The Robertson–Walker metric, and eventually the steady-state geometry, are derived in a series of steps determined by the need to choose the most 'appropriate' measurement-tool. As an application of the Postulate of Proper Time and the concept of an accelerated frame, the Clock Paradox is discussed.

CHAPTER 16

There may be *real spatial knowledge* of objects outside the strictly local sphere. Prompting, both of real spatial identification and real spatial knowledge, originates in a *seminal core* to which we may be 'opened' when there is a conceptual focusing or 'direction' established. Real spatial knowledge is therefore regarded as essentially 'direct', but presented on a certain scale and with a certain appearance according to circumstances. *Prompting extensional devices* such as the telescope prompt a veridical *extended conceptuality* which is a real spatial knowledge of objects otherwise inaccessible. The eye and finger are also devices prompting to knowledge of a spatial

Gestalt distinct from seeing and touching. All perception can thus be said to have a clairvoyant character.

The Dilemma of Agnostic Dualism is the predicament of those who hold that there are specified features of a causal substratum (e.g. electrons), somehow given in nature, but nevertheless beyond every possibility of verification that they are given in the form supposed. Agnostic dualists thus postulate a givenness whose givenness cannot be known as a givenness, and an inference which cannot be justified by an exhibiting of the thing inferred. We escape this dilemma by admitting no limit to the power of *causal presentational devices*. Every constituent 'cause' or detail of substructure, in so far as we are justified in saying that it is really present, may be discovered by an appropriate degree and kind of phenomenological analysis.

CHAPTER 17

The infinite is the unending, in space, time, or non-temporal process. Putting aside the meaning as 'unbounded', the infinite is such that, whatever stage we reach, there is a stage beyond.

The mathematical continuum of numbers arises from repeated subdivision of number-intervals, by a finite number of operations (for rational numbers) or by infinite non-temporal process (as with $\sqrt{2}$). In the latter case a *law of formation* is required. Numbers of this latter kind are admitted in calculations prior or subsequent to the determination of a measure, for convenience only; for all measures recorded directly are rational numbers.

A 'physical point' is distinguished in science by measures of position, i.e. coordinates which are rational numbers. The method of 'extensive abstraction' for defining points by the 'boxing-in of intervals' is rejected in physics as being non-operational idealism, psychologistic and circular. All measures are definite, but in the nature of a *guess*. Curves and trajectories containing an 'infinity of points' are mathematically given by non-temporal process, or physically given in a total encounter.

A principle of *Non-Actual Infinity* can be applied also to space as a whole, when objects are considered in it. An infinite sequence of objects or observations of objects, without a law of formation, is void of meaning. In this connection, 'objects' are recognized by their *rational unity* and *seminal core*, manifested upon occasion. The potential infinity of perceptual details in an object is contrasted with the non-infinity of seminal cores. It is concluded that the 'radius of the universe', R_0, must be finite.

CHAPTER 18

A representation is an interpretation in terms of concepts of space and time, either of something not itself spatial or temporal, or of something already with spatio-temporal measures or properties. It may be a Euclidean or Galilean mapping, a topological space-time conceptual-scheme, regional or universal. Underlying all representations is the principle of Gestalt Recognition, by which things to be represented are first recognized. Appeal must

also be made to principles of *Consistent Causality* in establishing what is to be taken as subjective, intermediate, or objective, as the case may be.

To establish an 'absolute non-rotation' a local inertial frame can be set up, over a reasonably short period of time, being defined by the fact that Euclidean geometry can be implemented and the trajectories of light-rays and 'free particles' are straight. To justify the absoluteness of this frame, appeal is made to a broadened version of Weyl's Principle, or alternatively, to principles of *Prior Operational* and *Prior Representational Validity*. It is, possible that such an inertial frame could not be set up in a certain locality if the period of time is considerable (because of the presence of a gravitational field). A theoretical way of possibly surmounting this difficulty is suggested. In practice the difficulty does not arise.

Two methods of 'correction', for the obtaining of what is objective, are considered: (1) for systematic 'subjective' deviations, such as those due to temperature affecting the measuring apparatus, and most 'intermediate' kinds; (2) for deviations corresponding to a 'field'. In the second case, as with gravitation, the metric introduced is shown to be *provisional*, since the objects (isolated from accidental or voluntary conditions) must be referred to a universal geometry.

In deciding on a definite form of universal representation, and in particular on the steady-state form, we must be guided by principles of Consistent Causality, and also by principles of (1) Measurement-Tool Space-Invariance, (2) Riemannian Implementation, (3) Co-moving Representation, (4) Observer-Gas and Nebular-Lattice Coincidence, (5) Measurement-Tool Time-Invariance, (6) the Epistemological Average, and (7) Conformity.

PART III

TIME AND SUBSTRUCTURE

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CHAPTER 19

THE PHENOMENOLOGY OF TIME

'What is that which always is and has no becoming? And what is that which is always becoming and never is? That which is apprehended by intelligence and reason always is, and is the same; but that which is conceived by opinion with the help of sensation and without reason, is always in a process of becoming and perishing and never really is.'

Plato, *Timaeus*, 27, trs. Jowett

'[The intelligible world] is true eternity, of which time furnishes a moving image in the sphere of the soul.'

Plotinos, *Ennead*, v. 1. 4

THE FALLACY OF SIMPLE LOCATION

Forty years ago A. N. Whitehead laid his finger on what he considered to be the original error of the 'mechanistic theory of nature', at that time —and even today, very largely—'the orthodox creed of physical science'. He denominated it the Fallacy of Simple Location, and described it as the assumption that

'as soon as you have settled, however you do settle, what you mean by a definite place in space-time, you can adequately state the relation of a particular material body to space-time by saying that it is just there, in that particular place; and, so far as simple location is concerned, there is nothing more to be said on the subject'.[1]

Adopting this 'anti-rationalistic' creed, 'physicists took no more interest in philosophy'. Nevertheless there was 'foisted onto philosophy' a dualism of *matter* and *mind* by which 'modern philosophy has been ruined'.[2]

Little attention seems to have been paid to Whitehead's protests, perhaps because philosophers, on the one hand, knew too little of modern mathematical physics and continued therefore to think in terms of an ingrained seventeenth-century world-view, while on the other hand scientists in general understood too little of philosophy to

[1] *Science and the Modern World*, Cambridge 1926, p. 72.
[2] *Ibid.*, pp. 74, 82.

realize that the world-view implicit in modern science was not what they supposed it to be.

One of the chief concerns of the previous chapters in this book has been to demonstrate in detail how the theory of simple location in *space* presents itself today as an oversimplification so gross that only its simplicity and practical utility can justify its provisional acceptance instead of the complex and abstruse state of affairs which careful phenomenological analysis discloses. The concern of the present chapter is to show that the theory of simple location in *time* is an even more gross oversimplification, responsible for world-views as fantastically inadequate as those inspired by the theory of simple location in space.

'Simple location' implies not merely a 'container' of space and an absolute succession of time but also the absence of any *internal* (i.e. conceptually inherent) relation between one event and another. Some of the extreme difficulties created by belief in simple location were pointed out by Whitehead:

'If in the location of configurations of matter throughout a stretch of time there is no inherent reference to any other times, past or future, it immediately follows that nature within any period does not refer to nature at any other period. Accordingly, induction is not based on anything which can be observed as inherent in nature. . . . In other words, the order of nature cannot be justified by the mere observation of nature. For there is nothing in the present fact which inherently refers either to the past or to the future. It looks, therefore, as though memory, as well as induction, would fail to find any justification within nature itself.'[1]

An unrecognized presupposition of simple location in time, once established in us, is powerfully self-defending. Any evidence which is likely to expose it as an oversimplification cries out to be 'explained away' by some philosophical theory contrived for the purpose, no matter how paradoxical this may be.

Thus it could be held that Phenomenalism—the theory that we ourselves, subjectively, put into nature its rational order, by means of our 'logical constructions'—stems chiefly from a subconscious belief in simple location as regards time. For when we have ruled out the possibility of any rational order in nature itself, the only explanation of the appearance of rational order in nature is that we have put it there ourselves. The Phenomenalist then plausibly consolidates his position by concentrating on the fact that we do indeed put 'logical constructions' into nature. But he conveniently forgets the qualification that we could not put into nature the particular logical constructions which constitute

[1] *Science and the Modern World*, Cambridge 1926, p. 75.

physics unless the givenness of nature permitted those kinds of construction and not others.

'Physical' and 'Psychological' Views of Time

Given a world-view based on the subconscious assumption of simple location in time, it follows, as we have seen, that the varieties of time-awareness have *no inherent conceptual structure*. They exist either as subjective psychological reactions of the individual (impressions of duration, rate of time-flow, attention to the present, memory, thought of the future, etc.) or else as an abstract mathematically represented background to changing measures, when the measures in some specially provided apparatus change regularly, the 'regularity' being tested indirectly by other measurements found by the use of certain other pieces of apparatus and workable theories concerning them.[1]

In the first of these cases we have what I shall call the 'psychological' view of time, and in the second, what I shall call the 'physical' view of time. The first rests on the fallacy of simple location in the sense that we then think of events as simply-located *stimuli* of a non-conceptual nature, and the responses considered are also virtually devoid of conceptual structure; that is to say, they are subjective accidents, and the exact logical relationship between universals does not enter into consideration. The second rests on the fallacy of simple location in the sense that the events measured are again simply-located stimuli of a non-conceptual nature, but the responses considered are *adjustments of apparatus* synchronized with the sensory appearances of the events. In neither case is any attempt made to analyse the conceptual structure of time-awareness, because it never enters our minds that there could be any such structure.

In the *phenomenological* approach to time-awareness, on the other hand, we accept, as with every kind of awareness in every sphere of human perception and knowledge, that there is a possibility of *explanation*, and that explanation consists in the laying bare of particular conceptual structures as creations of the universal *intelligible constitution* of nature and life. Accordingly, we shall not feel any satisfaction that we have understood time or how we are responding to time, until we have discerned how the varieties of time-awareness *do* manifest an intelligible constitution.

The 'tree' of phenomenological analysis is 'known by its fruit'. If we are unable to discern an apodeictically self-evident intelligible constitution, we are not yet phenomenologists. But in so far as we expose the theory of simple location in time as a self-evident fallacy, we have adopted a 'phenomenological' view of time.

[1] See the remarks on atomic clocks, pp. 70 ff. above

THE VARIETIES OF TIME-AWARENESS AND CONTROL

The Time-Structure of Gestalten

As was shown in Chapter 10, an encounter with a physical object is an encounter with an *intelligible whole* (Gestalt, configuration), in which we subsequently recognize perceptual details, each playing its part in the intelligibility and manifest effect of the object for us. What holds our attention first of all is the *spatial* configuration, which comprises details which are differently located in space but are nevertheless spatially coherent by virtue of the universal conceptual character of the Euclidean spatial relations which gives them spatial meaning. Thus the opposite corners of a square are recognized as two distinct points, connected by straight lines in a particular way, because we have conceptually mapped out, so to speak, the spatial field as a field of Euclidean translations and rotations and have seen the points and lines as particular actualizations in that field.

In attending to the spatial configuration of an object we overlook the fact that there is a lapse of physical time while we make our survey of the various perceptual details. We also overlook the fact that when, finally, with greatly improved comprehension, we again view the encounter as a whole, the original physical observation has receded to the past and we are dependent on something like memory.

If we adopt a physical or psychological view of time (based on the theory of simple location), we must accept that each detail which we recognize recedes at once into the past and is no longer verifiable except by plausible inference. Moreover, any Gestalt of which we may be conscious will be a new psychological response either to psychological processes or to the new physical appearance of the object; and such Gestalten are not analysable into intelligible details, because in the very first focusing of attention on some perceptual detail and relating it to some other detail so that it becomes intelligible, there is a lapse of time and the first detail is lost. Consequently, genuine analysis of the character of an object as it is at any time, and genuine comprehension of a time-series of events in an intelligible whole, are both impossible. And yet to speak of these psychological responses at successive moments of physical time as being *not* 'genuine' implies that we recognize some other method of recognition which *is* genuine.

These arguments demonstrate nothing more than the confusions and contradictions inevitably consequent on the presupposition of simple location as regards time. What is wanted now is a thorough phenomenological study of actual *experience* of time-Gestalten, aimed at disclosing exactly how we perform the various analyses and syntheses concerned in it.

Synthesis of the Extended

The study of Time-Gestalten is most easily introduced by examining certain very clear and definite evidence provided by the exercise of developed musical faculties. Sensitivity to time-duration and tempi is often very rudimentary, even with music students of many years' standing. Nevertheless, the general truth of the statements I wish to set forth here can be understood, I think, by all who have some small comprehension of melody, rhythm and harmonies as made up of constituent notes. Those who have little interest in music can discover similar evidences in the hearing of spoken words; but in that case attention must be given not so much to the meaning of the words as to the precise timing of each syllable in relation to the other syllables in the sentence regarded as a whole.

In music, then, the first fact to establish is that there is a melodic (or *horizontal*) time-Gestalt, and a harmonic (or *vertical*) time-Gestalt. In the former the 'wholeness' consists in the union of successive sounds; in the latter it consists in the simultaneous cooperation of the sounds of various distinguishable notes, instrumental tones, harmonies, overtones, formants, etc., in the total *tone-quality* at the physical time in question. There are also Gestalten which combine both horizontal and vertical elements, namely qualities of chord-progressions, significances depending on contrasts of tone and tone-quality, development of contrapuntal ideas, etc.

Considering first the horizontal Gestalt, I shall call the kind of intuition by which this is discerned the *synthesis of the extended.* It is easy to see, in a general way, that without some power of synthesis, at some one instant, of what is extended over many instants in physical time, a melody would not make sense to us. For 'making sense' means, in this context, recognizing the parts (occurring at different times) and their relations to each other; and a recognition of the relation between two things which are not *both* recognized in that relation is a contradiction in terms. But we must try to describe this power in some detail.

A musician with properly developed powers of aural discernment can easily hear (in his mind, so to speak) a melodic phrase consisting of a few notes, holding it 'in an act of unified comprehension', 'like water in the hollow of the hand', in a physical moment, before it is followed by other sounds. The same can be done with spoken or imagined word-sounds by anyone who possesses some degree of aural and positional imagination. The word 'unified', for instance, can be grasped as a whole, and at the same time the succession of different vowel sounds separated by the consonants *n* and *f* and the succession of different shapings of the mouth and lips can be discerned as constituting that whole while still being a succession. A musician with particularly well-developed powers can occasionally hear much longer phrases as a whole (I

remember one overwhelmingly vivid experience of a phrase more than 5 seconds long, grasped as an intelligible whole, with its details of timing also vividly recognized, in perhaps half a second of physical time).

There is one particularly important feature, generally overlooked, to notice about this kind of 'synthesis'. Time-durations, tempi, etc., can be perceived, in that physical moment, in their true and precise character, *without any compression or distortion of the phrase*. We have entered upon a new state of awareness, released from fixation on the inward urge of physical time, which normally binds us, as it were, to a revolving wheel. And thus we come to see that time-durations and tempi are *conceptual*, like features on a spatial map which can be explored backwards and forwards without in the least altering their conceptual character or the relations between them.

This insight is the evident origin of statements by mystics that there is no such thing as time (i.e. physical time) in the higher spiritual states. The condition is called, instead, eternity; and what is known in it is eternal concepts, essences, or Ideas. Plotinos, for example, has already been quoted as declaring:

> [The intelligible world] is true eternity, of which time furnishes a moving image in the sphere of the soul.[1]

This is such an important truth that I propose to formulate it as a **Principle of the Conceptuality of Durations and Tempi.**

In the comprehension of a musical phrase, the structure of time-intelligibility knowable in a state of *recollective release* (as I shall call it) has two kinds of correspondence with the physical time-scale. There is an *identity of intelligible content* with the phrase as it sounds in physical actuality; and there is a correspondence of *physiological correlation* between the period of recollective release and a certain period in the physical time-scale when we can say that 'it happens' physically. By means of the latter correspondence we can attach a rough physical measure to the period of recollective release, although this release occurs at a different level of time-awareness.

Although the most vivid experiences of synthesis of the extended, in which we are 'rapt' to the level of *eternal* configurations, are usually brief (in physical measure) this power of synthesis can clearly be exercised in a certain way over a time-duration which may extend even for an hour or more. Thus when a musical work is being composed, the composer frequently goes back to survey, in a few momentary reflections, the general impressions conveyed by the themes and structure of what has gone before, either in the previous few bars or in the whole movement. To a certain extent (as asserted, for instance, in a well-

[1] Ennead v. 1.4, trs. K. S. Guthrie, London 1918.

known letter attributed to Mozart) an entire movement can be grasped as a whole, without distorting the conceptual durations or tempi, which may be discerned precisely and with full intelligible significance. Moreover, when a movement or portion of a movement has been grasped in this way, we may spend a fraction of a second of physical time, or a much longer period if we please, exploring the recollected structure forwards and backwards.

One of the chief signs of a composer's mastery is his ability to judge exactly how long each section should be continued and at exactly what point of time changes of a certain kind must be made—neither one bar too soon nor one bar too late—in order that the structure of the whole shall be most perfectly balanced logically. Thus the proportions and character of material in the last act of an opera have to be weighed up minutely in relation to all that has gone before, without losing the true impression of the time-durations and tempi of each part.

Primordial Recollection

Those who are deeply imbued with the fallacy of simple location may object that the synthesis of the extended, as described above, is merely a fitting together of memory-images and imaginations of physiological impressions associated with various durations and tempi. *Imagination* is being studied (they may assert), not the actual physical occurrence, which is lost once the moment of its occurrence is past.

To think along these lines, however, is to misunderstand fundamentally the character of the experience in question. Labelling it 'imagination' is a mere lapse into the Fallacy of Reduction, i.e. trying to dismiss one type of experience from consideration by insisting that it is 'really' a variety of a more familiar type. But the difference between the experience in question and ordinary memory is indeed subtle, not to be grasped without the experience itself, and not easy to put into words.

To return to our musical example of 3 or 4 notes in a melodic phrase, it is to be remarked that as soon as the fourth and last note sounds, the whole may flash through our minds in a 'split second'. A moment later we may be asked what the notes, intervals and time-durations are. Then, usually, there is need for a certain amount of deliberation. Was the second note a quaver or a dotted quaver? Was it *E* or *F*? Here we weigh up possibilities which are *imagined*, trying to compare them with a certain elusive consciousness of what the constitution of the phrase *really* is. Reasoning does not enter into this deliberation, in general, nor does any consideration of possibilities. The distinction to be made is simply

[1] For explanation of the term *primordial* in translations of Husserl, see pp. 408 ff. below. 'Primordial recollection' is my term, following mystical usage of the term *recollection*. Husserl's equivalent is *primäre Erinnerung* (primary remembrance) or *retention*.

between the *imagined* and the *real*. By the 'real' in this connection is meant the 'eternal' structure, i.e. the structure as knowable in a state of release from the urge of physical time, that is, as knowable conceptually backwards and forwards while remaining identical with itself because universal in character. Physical time ceases, as it were, when we know such structures; and the fact that the *physiological* correlation changes in physical time is seen to be irrelevant to the unchangingness of the *intelligible* constitution of that which is thus 'eternally' known.

Analysis of the Simultaneous

Primordial recollection is most easily exercised in respect of a single instant of physical time, for example, when a single chord is struck on the piano, or a single word spoken, or a single-geometric figure glimpsed. Time then seems to stop as we grasp the intelligible wholeness of that which is presented.

To grasp the intelligible wholeness of a thing is to perceive the intelligible relations between its constituent details, so that we 'recognize' it and its essential character and qualities. In the case of a chord struck on the piano, we have no adequate comprehension of its essential character unless we can distinguish at least the separate notes, allowing our attention to pass from one to another while the chord is contemplated in its timeless identity. Again there is no question of the chord changing while we carry out our analysis. For what we contemplate is not in physical time, yet it is the timeless intelligible content of what did achieve actualization in physical time.

It is true that we do not contemplate the physical and physiological correlations which completed or supplemented the actualization, nor can we reverse physical time in order that our bodily senses and the world around shall go through the same actual processes again. But every intelligibly recognizable feature constituting the knowable existence and essential character of the thing we contemplate remains timelessly accessible to primordial recollection. To suppose that some feature became inaccessible would be to suppose that it is not in fact part of the eternal structure of the intelligible object whose encounter is before us.

Imaginative Reproduction, 'Ordinary Memory'

One of the chief obstacles to conveying a true idea of primordial recollection and of the powers of analysis and synthesis to which it opens us is the fact that unless we have voluntarily initiated and maintained a state of *recollective release* the thought of remembering something leaves us still entangled in fixation on the onward urge of physical time; the eternal structure corresponding to the event which we are thinking of is then replaced by a *new mental image*. This mental image

of a past experience, lacking the character of *timeless identity*, is the kind of thing customarily described as a 'memory'. I shall therefore describe the following through of such mental images as 'ordinary memory'. Not till we have found that it is possible to transcend ordinary memory by 'stopping time' and resting in the timeless identity of a thing are we in a position to recognize the possibility of primordial recollection.

Memory normally proceeds in a more or less automatic way, because of the absence of recollective control. That is to say, having somehow caught a memory image of a past event, a series of scenes or events is presented to us likewise in the form of memory images, appearing to proceed more or less at the same tempo as that in which they originally occurred. But just as in synthesis of the extended and the subsequent analysis of the constituent events we can cover any interval between events in a short or long period of physical time, so in ordinary memory we may sometimes seem to move through the events more quickly than they actually occurred and at other times more slowly. I shall call these types of recollection *quickened memory* and *slowed memory*, respectively, while characterizing the normal rate as *automatic memory*.

'Automatic Observation' of the Present

Considering again the case of a chord struck on the piano, we must note that we do not immediately hear the constituent notes separately, knowing at once that they are (say) B, C and F. Analysis of the simultaneous can be applied only to the primordially recollected form—not to the physical occurrence as we encounter it in a state of bondage to the onward urge of physical time. But every act of recognition is a kind of analysis in which we focus our attention on some intelligible characteristic or detail of structure which is the object of our recognition. It seems therefore that a complete state of bondage to physical time would be complete unconsciousness. What we have to admit, therefore, as happening when we have ordinary but unreflecting attention to what is going on, is a small and somewhat variable degree of release and of *automatically acting* primordial recollection of each recognized feature, this recollection occurring in a very short period (as physiologically correlated) after the physically measurable time of occurrence of the event in question. The duration of this period—called by some authors the 'specious present'[1]—obviously depends on the degree of reflection actually imposed (although we may not be conscious of any voluntary effort at reflection), and this depends on such factors as interest, previous experience of primordial recollection, ability to approach a state of release, age, physical state, etc.

As we have seen, a musical phrase occupying 5 seconds of physical

[1] For references and discussion, see G. J. Whitrow, *The Natural Philosophy of Time*, Edinburgh 1961, pp. 78 ff., 85 ff.

time can be held in the mind as a whole, 'like water in the hollow of the hand', in a vivid comprehension occupying a quarter of a second or less of physical time. On the other hand, a rapid scale of 8 notes in half a second can be primordially contemplated in such a way that the time-intervals between the individual notes, and their various grading of tone, equal or unequal, can be recognized—thus implying that the mind can appreciate differences of time-duration of 1/100th of a second or less, without needing to pass to any very exalted degree of release.

The mistake into which we must avoid falling is to suppose that our acquaintance with time-instants and durations is a mere physiological or psychological reaction to which we are limited by the constitution of our physical bodies or minds (without the possibility of developing rational control)—that 1/25th of a second, say, is the limit of our power of distinguishing time-instants, and that all events within 1/25th of a second of each other must appear simultaneous to us, while all events more than 1/25th of a second apart must appear distinct.

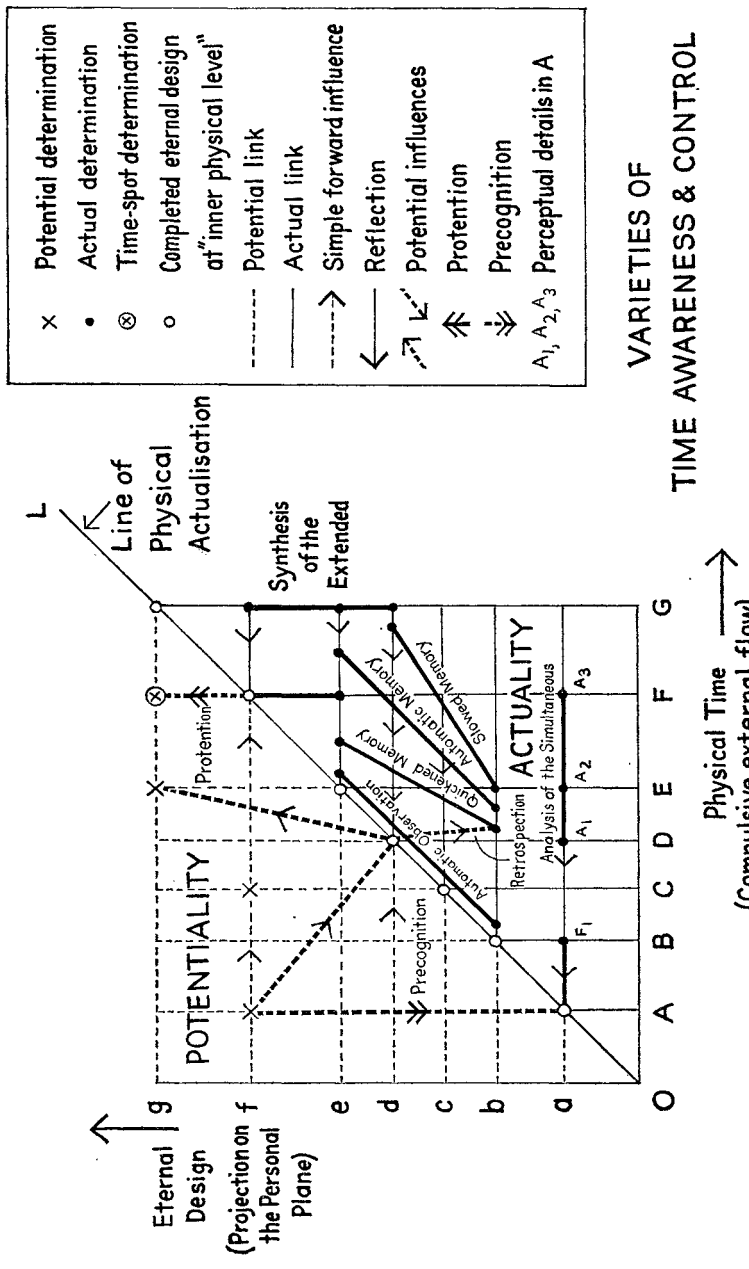
On some particular occasion it may indeed seem that two events 1/25th of a second apart are simultaneously present to us. But the meaning is either that we have mistaken a *synthesis of the extended* for a *physical simultaneity*, or we have been so confused by extraneous factors that we are unable to recognize the events adequately enough to distinguish between their times of occurrence in the 'eternal design'.

Varieties of Time Awareness and Control

Before turning to the consideration of varieties of time awareness and control with which we associate the word 'future', it may be helpful to clarify further, with the help of the accompanying representative diagram, the nature of the varieties so far discussed.

It should be noted first that a succession of events is shown on two scales: a horizontal one representing the 'compulsive external flow' of *physical time*, as it would be measured by some apparatus outside us; and a vertical one with the same intervals, representing the 'projection on to our personal plane' of the developing *eternal design*. Between, at an angle of 45 degrees, is the 'Line of Physical Actualization', *OL*, on which the same events are represented by small circles. To the left of this line we have a region of *potentiality* where the 'eternal design' is built up by means of influences of many kinds. To the right we have a region of *actuality* where acts of attention and recognition bring into play a reflection by which we come to know the eternal structures lying to their left in the diagram.

Thus in *Analysis of the Simultaneous*, applied to the event *a* as actualized physically, we discover perceptual details A_1, A_2, A_3 at successive moments of physical time, *D,E,F*. But all refer to the one 'eternal' event *a* as actualized on the line *OL*. In *Synthesis of the Extended*, on the other


Potential determination
Actual determination
Time-spot determination
Completed eternal design at "inner physical level"
Potential link
Actual link
Simple forward influence
Reflection
Potential influences
Protention
Precognition
A_1, A_2, A_3 Perceptual details in A
VARIETIES OF
TIME AWARENESS & CONTROL
L
Line of Physical Actualisation
Synthesis of the Extended
Eternal Design (Projection on the Personal Plane)
g
f
e
d
c
b
a
O
A
B
C
D
E
F
G
POTENTIALITY
ACTUALITY
Protention
Precognition
Automatic Observation
Quickened Memory
Automatic Memory
Slowed Memory
Retrospection
Analysis of the Simultaneous
F_1
A_1
A_2
A_3
Physical Time
(Compulsive external flow)

hand, the elements *d, e, f* of the eternal design, as actualized on the line *OL*, are comprehended by reflection at the physical instant *G*.

The lines marked 'automatic memory', 'quickened memory' and 'slowed memory' represent recurrence of the intelligible content of the events *B,C,D, E* at the same rate as that of their original physical occurrence, or more quickly, or more slowly, as the case may be. The recollections represented by such lines of intermediate gradient are usually very much covered over by imaginative additions, so that the *primordial* recollection is virtually lost.

The line marked 'retrospection' represents an unexplained passage back to event *b*, prompted by the actualization of event *d*.

The line marked 'automatic observation' is intended to suggest how a small lapse of time is required for recognition of each event. A more accurate representation might be a broken series of horizontal and vertical lines close to the line *OL* on its right.

Protention ('Time-spot Determination')[1]

The next variety of time awareness and control to be considered is again one which is scarcely recognized except by musicians—more precisely, by experienced musical performers; but references to it, generally somewhat obscure, may be found in writings on certain sports, for example, *Zen in the Art of Archery*, by Eugen Herrigel.[2]

To take a musical illustration, let us suppose that a performer wishes to play a musical phrase having a duration of about 5 seconds; or he may have already played most of a long phrase and now has to play the concluding 5 seconds of it. If his power of synthesizing the extended is well developed he will be able to hold the phrase as a whole in his mind while he is still playing the earlier part of it. Thus if he has played the first half of a 10-second phrase and now has to continue to the second half, the first half may be in his mind as a primordial recollection, while the second half (forming an intelligible whole with the first half) is in a certain way imaginative. But this 'imagination' is not an imagination which merely copies actualizations of the physical past in a fantasy-world of the present. Instead of looking to physical actualizations 'located' in the past, it looks to something which is *not yet* physically actualized and is therefore 'located in the future', as we may say. The use of the words 'located' and 'future' indicates that, without perhaps realizing it, we have to do here with the level of developing eternal design, where time (in a transcendental sense of the word) has space-like properties. And indeed, it is clear that if 'actualized' means 'known in an *act* of recognition', then the 'future' part of the melody, in the region of physical potentiality, is also actualized, but transcendentally.

[1] 'Protention' is Husserl's term.

[2] New York, Pantheon Books, 1953.

To avoid confusion of thought it seems to me very necessary to distinguish between two senses of the terms 'past' and 'future' by means of suitable epithets. On the one hand there is what I shall call the *physical* past and future, i.e. imaginations associated with measures of time before or after the measure of time which is being recorded *now*. On the other hand there is the primordially knowable 'past' and 'future' at the level of eternal design, the past being known by primordial recollection (called *retention* by Husserl) and the future by the act of *protention* which we are now considering. The 'past' and 'future' thus directly knowable may be described as *transcendental* or *formative*. The first of these terms emphasizes that the level at which they are known is totally different from that of physical awareness and logically prior to it. The latter term emphasizes that there is nevertheless a *mergence* of the one level in the other at the physical present, in such a way that the physical manifestation is recognized, and rationally comprehended and 'explained', by the 'causal structure' discoverable 'within' it at the transcendental level.

Since we are engaged upon a phenomenological analysis, we must take the utmost care to avoid lapsing into theories or thinking in terms of measures (which presuppose theories), and must keep our aim centred on the description of direct experience. Returning then to our analysis of musical performance, we find that over the course of years a performer can develop a certain most definite power of control, which is the *sine qua non* of genuine artistry in musical performance. It is a power by which an intelligibly creative but effortless *intention* is focused on some element in the *formative future* of a synthesized past-future conception such as a phrase in course of performance, the intention being to 'cause' its actualization at a corresponding physical time. That is to say, the focus and intention ensures that such element enters into the intelligible structure of that which appears physically in due course.

The famous pianoforte pedagogue, Tobias Matthay, introduced a term to suggest this *pin-pointing* of some detail of expression desired at some definite future instant and the conceptual *holding* of this (transcendentally future) instant in mind while physical time unfolds, till eventually the eternally conceived design actualizes physically at such a physical time as was precisely conceived in the region of potentiality. He spoke of the *time-spot*, and described at as the 'point of union between conception and execution'.

Although the focus may be on one particular note of a phrase, or one particular movement of the hand or finger, timed precisely at one instant of the phrase, it must be remembered that the power can be exerted only in a synthesis of the extended. Otherwise it would have no quality of *intelligible* completion of what is so far physically actualized only in part. Consequently the performer has a fairly clear conception

of the *whole* of what is to come, and he knows that if the one feature on which he particularly focuses comes into physical actualization as he desires, then the rest of what is to come cannot actualize itself very wrongly.

It is clear that without such a power of intelligent control of the future there would be no such thing as conscious control of artistic expression —only a blind hoping that one's physiological reactions will copy automatically certain requirements previously laid down in memory.

Precognition

'Time spot determination' is a kind of precognition which brings about what it precognizes. From our knowledge of the characteristics of this kind of precognition the following series of propositions concerning 'awareness of the future' may be set forth:

(1) We may be aware of a time-spot determination in eternal design before it is physically actualized.

(2) That determination is not complete because the cooperation of subjective and transcendental structures in the eternal design is required before physical actualization can occur.

(3) There are degrees of intention as regards time-spot determination, and subsidiary features may be actualized physically without any force of intention on them particularly.

(4) One time-spot determination, even a powerfully intended one, may be modified or cancelled out by another, or by circumstances over which we have no control.

(5) Awareness by us of a time-spot determination (in the future) affects us in the present.

(6) Time-spot determinations by one person will in general affect the physical actualizations for other people.

(7) Present physical actualization is the total fulfilment of a potentially existing eternal design developed by means of determinations from past, present and future, in various minds and inanimate objects whose connections transcend simple location in space and time.

Accordingly there seems to be no *a priori* reason why we should not sometimes, without force of intention, be aware of certain determinations of eternal design before they are physically actualized. The awareness might be of a general tendency only, or of a partly-imaginative sensory representation of such tendency, or of a very complete but still not quite final representation of nearly settled tendencies. In any case the representation must occur with the cooperation of subjective tendencies in the observing subject.

All these possibilities may be classified as types of precognition, and are shown in the diagram by a vertical line at the physical time when some representation of the determination in question, belonging to the

future part of an eternal structure, becomes transcendentally actualized in the observing subject.

It would be beyond the scope of this book to try to establish, from the voluminous evidence and published researches on the subject, that precognition actually occurs. All that I wish to show here is that precognition is not in any way incompatible with modern scientific theories, and that any antipathy which we may feel rests largely on belief in the *simple location* of events in a single time-scale, i.e. in a completely outdated seventeenth century world-view.

Thus 'the essential difficulty besetting all naïvely realistic accounts of precognition', according to H. A. C. Dobbs,[1] has two aspects: '(1) the *Epistemological* aspect: how can it be even logically possible to cognize something which does not yet exist? (2) the *Causal* aspect.' As regards (1), let us remove the ambiguity of wording, by putting the question in the following form: 'How can it be logically possible to cognize something of the *formative future* before it has been actualized at the *physical level*?' The answer is then clear. There is no logical difficulty, because what we cognize does already exist, being known by 'time-spot' awareness, in the developing parts of the eternal design. As regards (2), the concept of *cause* as a rigorous association of events of a certain kind, earlier in the physical time-scale, with events of another kind, later in the physical time-scale, falls away when we have *two* levels of time-awareness with various kinds of mergence of the one in the other. The impression of such 'cause', always preceding its effect, is due to our identifying intelligible quasi-temporal structure (capable of being explored backwards and forwards) with the onward urge of physical time—or perhaps due to our simply denying the former.

The greatest problem for the ordinary person, especially those who have had several experiences of precognition, is probably in accounting for the fact that other people, sometimes many thousands, may take part in the fulfilment, months or even years later, so that it seems none of these people had freedom of will to do otherwise.

If it be granted that precognitions of this type do occur (as much detailed and well-testified evidence asserts), the conclusion to be drawn in regard to 'free will' seems to be as follows. Individuals, by their character tendencies and in other ways, contribute to the building up of a progressive eternal design, and thus dominant tendencies to physical actualization come into being potentially. These the individual cannot then appreciably affect unless he is in a state of recollective release, so that he stands apart from those tendencies. The physical manifestation of events is to that extent determinate. But our mental attitude to them, and any consequent development of intelligence and character, as well as the will to achieve recollective release, lie right above the level of

[1] 'Time and E.S.P.', *Proc. S.P.R.*, 54, August 1965, p. 252.

physical actualization and are not at all determinate in the same sense. On the contrary, these are the things that help to determine the 'future' parts of the eternal design.

The obviously frequent occurrence of imaginative counterfeits of memory, precognition, analysis and synthesis indicates that the greater part of our diagram, which was presented as a diagram of varieties of time awareness and control as *fulfilled in physical actualization*, is to be duplicated in a diagram of imaginative awareness; also, that the two fields of actualization—physical and imaginative—may cooperate in a blended actualization. There are evidently features of the original diagram which cannot be duplicated imaginatively (true physical actualization, recollective release, etc.); but it is not necessary to look into this matter further.

We must finally recognize that the determinations in eternal design which will eventually be actualized in physical events *A*, *B*, *C*, *D*, . . . must be *intersubjective*, in such a way that the actualized time-intervals are recognizable mutually as referring to the same pairs of events, yet they are in general different in respect of observed magnitude for subjects differently located or in relative movement. Each of the observing subjects therefore requires a diagram with his own series of physical time-intervals, uncorrected for location and movement; and all the diagrams are connected somehow at the level of eternal design. Otherwise there would not be objective events which are identifiably the same for different subjects.

THE TRANSCENDENTAL ORIGIN OF PHYSICAL TIME-MEASURE

In the light of the foregoing discussions an attempt may be made to illuminate another difficult problem—how it can come about that there are *standards of time-duration* upon which all people can agree on all occasions, past, present and future. It must be remarked that when we have thrown over the Fallacy of Simple Location in space and time, we can no longer suppose that a certain structure of physical objects in a given locality actually goes through some cyclic process in a perfectly regular way, absolutely unaffected by other objects or by the minds that make use of it as a clock for the timing of observed events. On the contrary, everything that appears to an observer in space and time depends on his physical, physiological and mental state on the occasion in question, as well as on other physical and possibly personal influences. Universality does not lie immediately in observations which are dependent in these ways.

As we have seen in earlier discussions, universality is a conceptual

matter transcending local and subjective accidents, and must be put into nature approximatively by some operational procedure which Nature somehow can admit. How then does it come about that Nature admits the application of operational procedures such as the approximative establishing of congruence in space (ideal rigidity) and congruence in time (ideal clock-regularity)? It is no adequate phenomenological answer to say simply that we find that nature provides nearly rigid bodies and nearly accurate clocks, and that it is possible to impose 'corrections' consistently. For that would be again an attempt to explain the universal in terms of accidents.

As far as an individual subject is concerned, constancy of form is a conceptual judgement having its root in primordial recollection. That is to say, the form of one perception can be compared with the form of a similar perception at a later physical time by synthesizing the two as one synthesizes the various portions of a melody and compares the respective time-durations between pairs of notes. This comparison in respect of 'eternal design' is absolute—not to be assessed in terms of some more primordially valid comparison. For if we have judged a certain form observed on one occasion to be the same as the form observed on another occasion, nothing whatever can cause us (so long as we judge by primordial recollection) to think them different on another occasion. If we *did* think them different, we could not have primordially recollected them as they really are, timelessly.

But what if there are *two* observing subjects making the comparison between time-durations? Suppose that there is a pair of identifiable events A, B, and another pair C, D, and that for an observer O the time-interval from A to B is judged by primordial recollection to be the same as the time-interval from C to D. Then, in general, the corresponding time-intervals will not be the same for the second observer Q. The procedure in mathematical physics is to construct an abstract (unobservable) time-duration in each case by correcting for the velocities of the object and observer relative to the medium of sound, or for the velocity of light, and imposing far more elaborate corrections if need be for inequalities in the intervening medium—all these corrections being specially contrived so that we eventually arrive at an abstract measure which can reasonably be adopted by both observers.

Nothing of this technique is phenomenological. Nothing of it is self-evidently given to us. Nevertheless we can observe the bare fact that a formal consistency is obtainable by the use of rules which are universally applicable, without preference to particular observers. Since this formal consistency is found in the application of the rules to nature, in this sense it is found to exist in nature—or to express the conclusion less misleadingly, it is found in the 'background' or transcendental 'potentiality' of nature.

It appears therefore that the primordial origin of congruence in respect of space and time lies in the potentiality of nature, transcending the local and personal distinctions between space and time, and arrived at by cancelling out somehow the subjective components of the eternal design. But such 'cancelling out' is artificial and necessarily incomplete. What remains (assuming our cancelling out is reasonably correct) will be aspects of the eternal design which are insufficient to determine physical observations, even when we have attempted to correct for subjective conditions and produce an *ideally correct* measure for the observation. For the eternal design obviously cannot fall into two wholly unrelated parts, objective and subjective, without influence on each other.

The term 'subjective', in the foregoing discussion, refers to two broadly distinguishable kinds of observational conditions: (1) those associated with the locality, orientation and state of movement of the observer, as well as the conditions intervening between him and the object observed, all these characteristics being regarded simply as mathematical structures or measures; and (2) those associated with the 'mental state' of the observer and the qualities of his observations. The corrections devised by mathematical physicists are aimed at cancelling out conditions of type (1), while those of type (2) are ignored. Here we have a relic of the classical distinction between *primary* and *secondary* qualities, a distinction which is again artificial because primary qualities cannot be observed without secondary ones, and the conditions which alter secondary qualities also alter primary ones.

Nevertheless, the fact that congruence in respect of both space and time *can* be approximatively established, consistently for various observers, by the method of cancelling out conditions of type (1) as far as possible, enables us to draw a conclusion which may be expressed as follows. In the *abstract* space-time at which mathematical physics thus arrives, and which may be taken to correspond to some field of potentiality genuinely to be found at the level of eternal design, there is the possibility of structures which serve automatically to correlate the magnitudes of spatio-temporal intervals in different localities of the universe and at different epochs of physical time.

When we examine the phenomena of nature with a view to discovering where this possibility of primitive spatio-temporal congruence may be found operative, it seems that the structures concerned must be simply pure sinusoidal electromagnetic waves, regarded quantum-theoretically as probability structures in an abstract space-time which is not physical but represents a universally conceptual and intersubjectively permeating sphere of creativity.

From other structures which we may suppose produce such waves and are stabilized correspondingly we may suppose approximately rigid bodies are ultimately constituted, and finally the primitive congruences

are 'anchored' to approximately rigid structures of such a kind that the time congruence is able to manifest itself in approximate regularity of pulsation or cyclic motion.

It is to be observed that the potential existence of these structures, as intelligible elements in the substructure of nature, provides no argument whatever for determinism in human life. The 'mergence free' physics[1] which symbolizes them is only a very artificially truncated portion of the eternal design, limited to phenomena in the physical world, yet cut away from everything to do with *observation* and *voluntary control* of such phenomena as well as from everything else in life.

[1] Above, p. 65.

CHAPTER 20

INDIRECT KNOWABILITY OF INNER CONSTITUTION: THE INTERPRETATION OF QUANTUM THEORY

'This world-order, which is the same for all, none of the gods or men has made; but it was ever, is now, and ever shall be an ever-living Fire, kindling in measures, and being extinguished in measures.'

Heraclitus

'Wenn man den Schritt von der klassischen Physik zur Quantentheorie als endgültig betrachtet, wenn man also annimmt, dass die exacte Naturwissenschaft auch in Zukunft den Begriff der Wahrscheinlichkeit oder Möglichkeit, der "Potentia", in ihren Grundlagen enthalten wird, so rücken dadurch manche Probleme aus der Philosophie früherer Zeiten in ein neues Licht, und umgekehrt kann das Verständnis der Quantentheorie durch das Studium jener früheren Fragestellungen vertieft werden.'

W. Heisenberg, 'Die Plancksche Entdeckung und die Philosophischen Grundfragen der Atomlehre', *Mitteilungen aus der Max-Planck-Gesellschaft*, 3 (1958), 149

'There remains now the still unsolved task of carrying through a philosophy corresponding to the level of meditation of quantum mechanics.'

C. F. von Weizsäcker, *The World View of Physics*, London 1952, p. 135

What is 'indirect' knowledge? It might be said that we have indirect knowledge of the life of Julius Caesar, of another person's thoughts, of the other side of the moon as photographed automatically from a space-ship, of the structure of atoms as deduced from X-ray spectra, of the existence of seven roots of a given seventh degree algebraic equation not yet solved.

In all such cases it would perhaps be a more satisfactory mode of speech to say that we do not in fact have any knowledge of the person, thing, or conceptual structure in question. We do not know Julius Caesar; we only imagine what kind of happenings probably occurred in his life. We do not actually see the other side of the moon; we only see a photographic emulsion on which the light and dark shading is

reasonably taken as copying other shadings reasonably taken as caused by light reflected from the surface of the moon; and so with the other examples. In what sense, then, can we have 'indirect' knowledge of the inner constitution of nature? Do we only *imagine* the sort of existence or conditions which might reasonably be supposed to be constituted 'behind the scenes'? In that case, what is the kind of 'reason' to which we are appealing?

The view which I adopt in this chapter is that there is a certain way of knowing something of the inner constitution of nature and life in general—a way which ought not to be called 'direct', but is also not 'indirect' in the senses just considered. That is, such knowledge is neither imagined, plausibly postulated, inferred, nor deduced. An example, already given in another context, may clarify this assertion.

If we construct a rectangle 5×12 inches, we have knowledge that *if* we were then to rule straight lines across it one inch apart, making squares each of (approximately) one inch side, there would be 60 of them. That is, we know, without any actual construction, that the area would be approximately 60 square inches. I do not call this a *deduction*, because we do not consciously or unconsciously choose axioms and then proceed by deductive steps to the conclusion. On the contrary, by *self-evident apprehension* of the original spatial structure we see that such further constructions and their arithmetical results are implicit potentially in its Gestalt, by virtue of the congruently repetitive properties which constitute the Euclidean isometry group. Consequently, to the extent that an actual lamina is found to copy a lamina ideally 5×12 inches, to a corresponding extent must the area of the actual lamina be 60 square inches.

Pythagoras's Theorem and other more elaborate geometrical propositions are far more difficult to see as self-evident potentialities of the Euclidean isometry group. But it still seems reasonable to accept the general principle that, having arrived at a genuine insight into the truth of the theorem by discursive reasoning, we know more than merely how to repeat a series of deductive steps from chosen axioms. To believe that the geometrical property established is no more and no less than the particular set of axioms and deductive steps which we have used in order to arrive at an intuition of its truth, would be obvious psychologism.

It thus seems that we can to some extent come to appreciate by *direct intuition* (so to speak) that the result of a geometrical theorem is a property implicit in the constitution of the Euclidean isometry group. Our comprehension of it *now* is an absolute, timeless, comprehension, and not a comprehension spread over a time-succession of deductive steps. I suggest, therefore, that it is an appropriately suggestive mode of speech to say that our knowledge of the truth of a

theorem such as Pythagoras's Theorem (when we have come to appreciate its truth in the way indicated) is 'substantially direct', but is exceedingly hampered by the complexity and abstruseness of the potential relationships concerned.

'Measurement-Tools' and Physical Laws

When we pass to the consideration of Newtonian mechanics, the connection with a self-evident underlying constitution seems at first sight altogether wanting (it is true that in earlier centuries much of Newtonian mechanics was taken as self-evident; but no one would think so today). Newtonian *kinematics*, however, if limited to one inertial frame, is nothing but the local Euclidean geometry plus the 'linear' congruence-geometry of time, both being *ideal measurement-tools* which we know can be implemented in the actual measurement of phenomena. We know this ideal system of measurement by self-evident comprehension, although we do not know in the same self-evident way the extent to which the final result should be regarded as approximate because our implementation of the fundamental structures (unit distance, unit time-interval) is necessarily approximate.

If we now enlarge our ideal measurement-tool by means of further operational rules for the measuring of mass, force, etc., it is clear that much of this is also self-evident—not in the sense that nature *gives* it to us, but in the sense that nature *permits* the implementation of the conceptually self-evident features of our 'tool'. Thus if we begin with the ideal concepts of *force-field* (i.e. a vector field in ideal Euclidean space) and *mass-particles* capable of superposition, we can establish a *mass-congruence* and a definition of *dynamical force* in terms of ideal experiments as follows.

Let different masses, obtained by superposition, be taken at a given point. If acceleration is multiplied by a factor k, the mass shall be taken as also multiplied by k. The *static* character of the field (a pre-requisite in this experiment) is established by the consistency of the results when the experiment is very many times repeated. Masses at different points are compared by transportation, conservation of mass being likewise tested by consistency when the experiment is repeated in many different ways. If there is inconsistency, suitable corrections must be imposed so as to establish an *ideal* static field or *ideal* mass-conservation, as the case may be. *Forces* at different points are then defined as mass $\times$ acceleration, in units established in any convenient way. The possibility of all this is self-evident, as is also its applicability to nature in a series of successive approximations.

What cannot be seen by self-evidence is the justifiability of identifying concepts employed in various measurement-tools and so constructing a more comprehensive field of theoretical physics: for instance, iden-

tifying *dynamical* and *statical* measurements of force, *inertial* and *gravitational* measurements of mass, *electrostatic* and *electromagnetic* measurements of current, and so on. The identification, as in the case of the two kinds of force, may be extraordinarily persuasive, as if no other alternative faced us; but none the less it is scarcely 'self-evident'.

The conclusion seems to be that, in spite of the complexity and abstruse interrelatedness of most of the methods of measurement in physics, there may be a certain barely discernible core of constitution partly given in nature and partly devised by man. The difficulty is to sort out what is given, what is representatively devised, and what is a mere 'saving of appearances'.

In the previous chapters I have tried to show how in the space-time kinematics of the Theory of Relativity we can indeed sort out phenomenologically, with considerable success, what is given in nature and what is devised by man, while rejecting what merely 'saves the appearances'. As regards mechanics, it is clear from the complexity revealed in quantum theory that the comparatively simple formulae of Newtonian mechanics must be a vast simplification in which the primordially given constitution has passed completely out of sight. Our only hope of reaching by scientific means an understanding of those aspects of the constitution of nature from which mechanical and electrical laws originally stem is by trying to sort out phenomenologically the various strands of thinking and practice in *quantum theory*.

Difficult as this task undoubtedly is, I believe a careful survey of the experimental and theoretical methods may lead us to the conviction that a certain indisputable light has been thrown on the 'fine-structural' constitution of nature—not, of course, as it is in any particular manifestation to our physical senses, but as it is universally and transcendentally. For the purposes of this chapter, I call such a mode of knowledge 'indirect'—in contrast with the modes of knowledge discussed in the next chapter, which are to be called 'direct'.

THE INTERPRETATION OF QUANTUM THEORY

The mathematical formalism and procedures of quantum theory are the precise language of atomic and subatomic physics. To check the correctness of any conception which we may form in regard to 'particles', 'waves', interaction', and so on, in this sphere of knowledge, we must put aside 'popular' explanations and come back to the formalism and procedures to see what they imply.

But we must also allow for the fact that the formalism and procedures are devised so as to agree with experimental observations. The agreement is always open to improvement, especially when new experimental techniques are discovered. The formalism or its application may also be

open to criticism on logical grounds. Consequently the theory is continually being modified by the introduction of new mathematical techniques and the discarding of old ones. The change from the Old Quantum Theory to Quantum Mechanics about 1925 virtually created a new science. Quantum Field Theory, dating from 1934, and the rapid developments in Elementary Particle Theory during the last ten years, have also involved very substantial changes not merely in techniques as such but also in interpretative background.

Even the specialist, applying a mathematical formalism to an experimental investigation, needs an interpretative background to help him to think in physical terms about his mathematical procedures and to explain and justify them to other specialists. He makes use of what we may call the 'specialist language of words' (SL), as distinct from the bare mathematical language (ML) which consists of nothing but numbers and other mathematical symbols of general application. There is also the 'ordinary language of words' (OL). The specialist language of words inevitably joins company with this when (as happens continually in quantum theory) the course of research is novel or obscure, with 'snags' and logical defects making it doubtful to what extent the proposals can be regarded as having physical content.

In order to have a definite standpoint, as adequate as possible to the experimental situations concerned, I shall adopt here the 'language' of quantum field theory—communicated, of course, with the help of ordinary language. When it is necessary to discuss elementary quantum mechanics[1] this is to be regarded as a simpler introductory formalism which, by itself, is evidently very deficient. I do not think it is a valid argument against the adoption of the field-theoretic language to say that it also is deficient in certain fairly obvious ways. For the logical and practical superiority of quantum field theory over elementary quantum mechanics makes the deficiencies of the former seem superficial.

It should perhaps be remarked, before proceeding, that quantum-theoretic languages are not applicable to macroscopic objects such as a house or a human being. What could be meant, either mathematically or observationally, by 'the wave-function representing Einstein'?

THE UNLOCALIZABILITY OF 'PARTICLES' IN THE 'FREE FIELD'

Before considering general interpretative standpoints or any details of the formalism, it will be best to have plainly before us certain

[1] This term, currently used in discussions on the Philosophy of Science, here stands for (1) the formalism strictly established on the ground of a Fourier transformation between complementary sets of variables $r_1, r_2, \ldots t; p_1, p_2, \ldots$ H and an equation between H and other variables or their operational equivalents, and (2) the application of this formalism to problems which do not need the trial introduction of semi-classical or semi-field-theoretical techniques or any abandoning of the mathematical conditions of validity (see p. 321 below).

experimental evidence which runs counter to the older mechanistic views on space, time and matter. This will serve to focus attention on important features which must somehow be built into any scientific theory of radiation and elementary particles.

The Photoelectric Effect

The Photoelectric Effect has been very generally supposed to show that a monochromatic light-wave must be regarded as an ensemble of photons—i.e. localized particles or wave-packets, each carrying one quantum of energy and having an average frequency which is that of the light itself.

In figure 1, ultraviolet light of frequency ν falls on the metal plate P in a well-evacuated tube and immediately an electric current is found

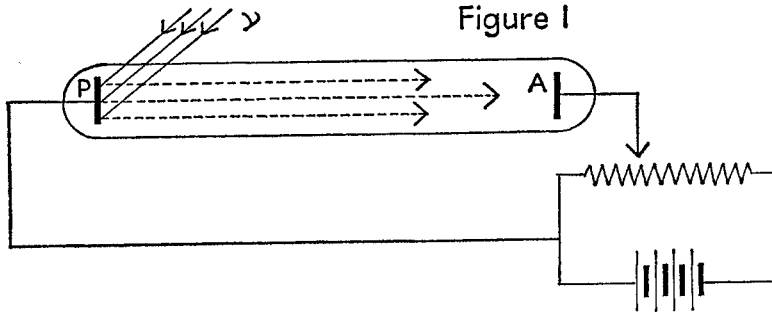


Figure I

to flow through a circuit joining A to P outside the tube. If A is kept at a positive potential the current is considerable, and this is taken to show that negative charges (electrons) are drawn from P to A inside the tube. A minimum frequency ν_0 of incident light, however, is necessary to release these electrons. If now (as shown in the figure) we reverse the battery connections and find the *negative* potential which is just necessary to turn back all the charges, we have a means of measuring their maximum kinetic energy $\frac{1}{2}mv^2$. This is found to be proportional to $\nu-\nu_0$, in fact $h(\nu-\nu_0)$, where h is Planck's constant, $6\cdot6 \times 10^{-27}$ erg-sec.; it does not at all depend on the *intensity* of the incident light. Even if the light is made many times brighter, still the maximum kinetic energy of the electrons remains precisely the same. The conclusion is that only energy $h\nu$ can be imparted to an electron in the plate P.

If we compute from experimental data the energy which each atom affected by the wave may be expected to receive, assuming that the energy of the wave is distributed uniformly between them all, we find, with a moderate intensity of illumination, that the time required for any single atom to accumulate 1 quantum of energy from the wave

would be at least 50 seconds, and probably much more.[1] Precise measurement shows, however, that if there is a time-lag between arrival of the light at P and the starting of the photoelectric current, it is less than 3×10^{-9} seconds. It seems certain, therefore, that either (1) a single atom is able, virtually instantaneously, to 'condense' a quantum of energy from the continuous field passing over many millions of other atoms in its neighbourhood—in this case we are unable to see how the condensation is effected or how the choice of a single atom would be made; or (2) the wave has already broken up into quanta of energy in isolated wave-packets when it arrives at *P*.

In passing, it may be remarked that the formalism of quantum field theory admits, with qualifications and severe limitations, not only the concept of a 'wave-packet' but also that of an 'ingoing wave'. Wave-packets, in the case of the 'free field', can be formalized only by a technique which seems to be artificial and is hardly ever considered in the literature. It does not permit the allocation of a separate quantum of energy to each separate wave-packet, but the energy is possessed by the field as a whole. The 'ingoing wave' also cannot properly be said to 'condense' the energy, which is, on the contrary, transferred probabilistically from the field again regarded as a whole.

If we are only interested in explaining the photoelectric effect, it is of no concern which of the two possibilities just mentioned occurs; but until recently the first ('ingoing wave') was generally ruled out as absurd. As we shall see, however, in interference experiments it is the second possibility which has to be ruled out, and something like the first possibility must therefore be accepted as occurring.

Interference

In the classical two-slit experiment of Thomas Young (1802) a source *S*, which is restricted to a very small region, radiates monochromatic light towards a screen *A* in which there are two very narrow slits close together. Light from each of the slits then spreads out and reaches a further screen *B*. A pattern of bright and dark bands of light is observed on *B*, depending on the distance apart of the slits, the distance between *A* and *B*, and the wave-length of the light. (Figure 2 is drawn very much out of scale, since even when the distance between *A* and *B* is one metre, and the distance between the slits is 1 mm., the distance between the bands is only a fraction of a millimetre.) For accuracy of observation and measurement a travelling microscope can be set up in place of the screen B.

The explanation by classical wave-theory is simple and completely accurate. At a place on screen *B* where the phases are the same, the

[1] F. K. Richtmyer and E. H. Kennard, *Introduction to Modern Physics*, New York 1947, pp. 93 ff.

field intensities of the waves from the two slits are added, so that a bright band results; at a place where the phases are opposite, like the crest of one water-wave and the trough of another placed together, the fields annul each other and we have a dark band.

Now taking over the previous results for the photoelectric effect, let the source be of ultraviolet light and the screen *B* be of photosensitive material, so that in place of visible bands we have the emission of photoelectrons according to a statistical distribution in similar bands (but of course far closer together). Since each photoelectron is released

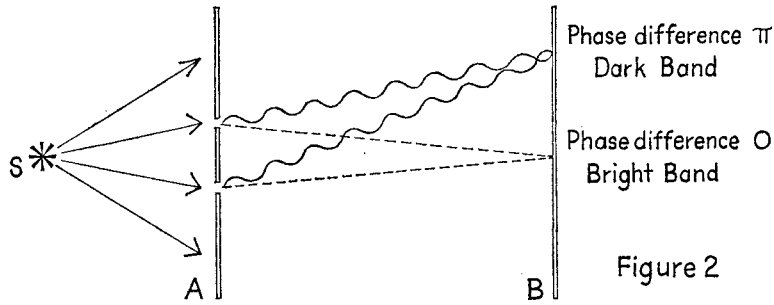


Figure 2

by one photon of energy, it is natural to ask through which slit any particular photon passes.

But the supposition that any particular photon goes through one of the slits necessitates admitting that the photon is somehow guided from the slit to the region of a bright band, in a way, therefore, which is influenced by the position of the slit through which it does *not* pass—a slit which therefore could have been filled in during the time of transit and so made non-existent.

Since a non-existent slit could not guide anything, we are led to conclude that either

(1) Two or more photons are always passing through both slits simultaneously, and the photoelectrons released on screen *B* are released by the absorption of one photon from the combined wave at the region of *B* in question. And this must be so, however much the intensity of the source *S* is reduced, provided the statistical distribution in bands remains unchanged.
Or
(2) It is illegitimate to allocate an individualized identity to photons, such that each photon must be conceived to go through one or other of the slits. On the contrary, unless transit through one of the slits is blocked, the wave proceeding from *A* to *B*, even when possessing only one photon of energy, must go through *both* slits at once.

However fantastic supposition (2) may seem, it has always been felt that supposition (1) is far more so, since it is impossible to conceive of

any reason why photons should be linked in pairs before they can get through the slits.

A possible escape from the dilemma may appear to lie in the *proviso* appended to supposition (1). If, when the intensity of light from the source S is reduced sufficiently, the usual statistical distribution in bands disappears and is replaced by a distribution that would result from the use of a single slit, this would be an indication that separate photons are now passing each through one slit without interference from the other.

The deciding factor thus seems to be whether the statistical distribution remains unchanged even when the intensity is reduced till, on the average, only one photon, or less, is in the apparatus at any time.

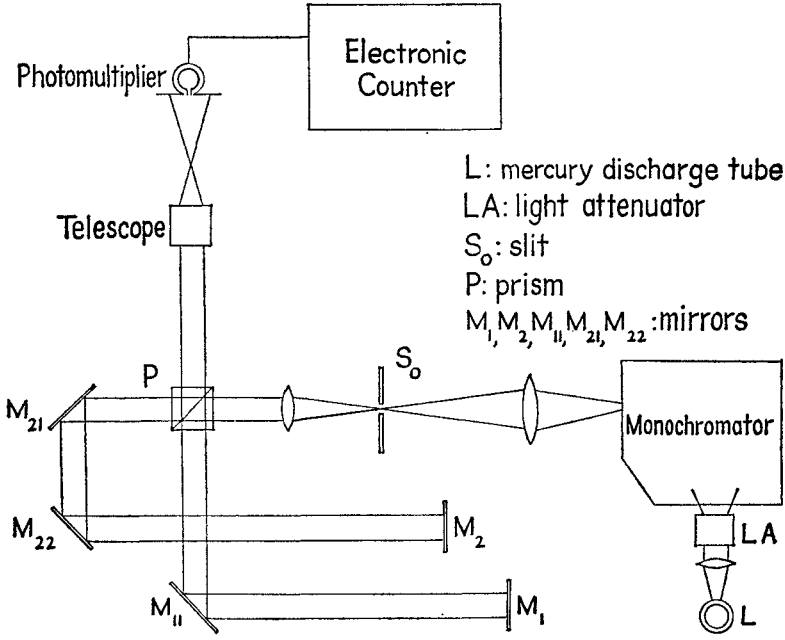
An experiment to decide this question was carried out in 1957 by L. Jánossy and Z. Náray, following on earlier experiments not perhaps altogether free from objection. The conclusion was absolutely definite that the statistical distribution remains unchanged even when less than one photon (as calculated classically) is in the apparatus at one time.

It seems clear, therefore, that we must accept that it is illegitimate to think of photons as individualized in such a way that they would have to go through one or other of the slits. The photons must not even be thought of as *in existence* till the absorption takes place. For if they were in existence, then in the case when *less* than one photon is in the apparatus at any time, we would have to accept that the supposedly existing energy accumulates itself (without passing out of the apparatus) till precisely one photon is available at the point where the next absorption is to take place.

The view thus forced on us can be expressed positively in the following way. Consistently with quantum field theory, *the free field represents a certain potentiality whereby a transfer of energy, momentum, etc., is actualized* (*with a certain probability according to the total circumstances*) *when there is another field capable of admitting just that transfer.* The probability, as we calculate it with remarkable success in innumerable cases, depends on the abstruse and complicated mathematical structure which we are led to impute to the fields.

Since this conclusion is exceedingly important, not only as establishing the correctness of quantum field theory in this respect, but also for its philosophical significance, it seems advisable to give further details concerning the 1957 experiment of Jánossy and Náray. The source used was a mercury discharge tube, the line $\lambda = 5461$ Å being selected by a monochromator, and interference was brought about (as in the Michelson–Morley experiment) through division of the beam into two by a semi-transparent mirror. Each beam travelled a total distance of 13·85 metres, being reflected on the way for convenience. A square area of about 2 mm side, in the region of interference, was explored

with a photomultiplier whose slit (0·2 mm wide) was moved in steps 0·25 mm across the pattern, the number of photons absorbed per second being obtained at each stage by an electronic counter. The placing of the bands (about 1·5 mm apart) was conspicuously the same for a low intensity (less than one photon in the apparatus at any time) as for an intensity 10,000 times as great.[1]



The diagram shows the arrangement of the apparatus in a simplified way. To avoid temperature and mechanical disturbances the whole was built in an underground tunnel cut in rock 30 metres under the surface of the earth. The interferometer was mounted directly on the concrete floor of the tunnel. After the apparatus had been left for a few days to rest it was operated by remote control from a different compartment.

OTHER OBSERVED PARTICLE-LIKE EFFECTS

The 'particle-like effects' which we call scintillations might be more accurately characterized as 'localized quantum-absorption effects, resulting in the emission of visible electromagnetic waves'. This

[1] L. Jánossy and Z. Náray, 'Investigations into Interference Phenomena at Extremely Low Light Intensities, by means of a large Michelson Interferometer' *Nuovo Cimento Suppl.*, **9**, 2 (1958).

phraseology has the advantage that it leaves open the question whether anything like a localized particle or wave-packet can be supposed to follow a trajectory to the place of absorption.

Among other types of apparatus or experiment by which localized quantum-absorption effects may be observed are the Geiger Counter, Crookes's Spinthariscope, and Michelson's Oil Drop Experiment. The Brownian movement of small particles visible in liquid or gas suspension exhibits effects of molecular bombardment which are capable of being dealt with quantitatively to some extent by classical methods.

In Crookes's Spinthariscope the number of scintillations produced per minute in the neighbourhood of a radioactive particle is counted and compared with the loss in mass of the particle in the same period of time, as estimated by other means. Good agreement is found with other experiments in which the mass or charge of 'particles' is measured. Must one then suppose that minute localized particles of mass m leave the radioactive particle and travel along definite trajectories to some place on the screen where the same mass m is absorbed? As in the experiments just considered, such a hypothesis could never be tested experimentally, and is quite unnecessary (if not absolutely ruled out) in quantum field theory. Mass-transfer occurs in a similar way to energy-transfer, mass and energy being both measures of the quanta of a field.

In contrast, the tracks produced in cloud chambers, bubble chambers or nuclear emulsions seem to be clear evidence of the existence of a particle-like structure travelling along the line of the tracks and producing visible effects by its successive interactions with other systems on the way. In quantum field theory it is possible to formalize an outgoing wave proceeding from a region of interaction. This wave, however, like those in the two-slit experiment, is not an observable actuality, but is associated with the *probability-density* that an observable phenomenon should occur in any chosen situation when interaction with systems located there leads to observation. Accordingly, in the case of these tracks, the view we should adopt is rather that a succession of interactions occurs with very high probability in an approximate straight line, without necessarily implying the transit of a minute wave-packet or other sharply localized object.

The clearest evidence for the localization of atomic systems comes perhaps from the electron microscope, which is able to show, for instance, the regular lines of molecules in a crystal. Quantum field theory can deal with a single localized system of two 'particles'. Whether or not a satisfactory formalism exists to localize more than one such system, or more complicated molecular systems, is a question which we must leave for specialists to decide.

A very remarkable experiment by Estermann, Frisch and Stern[1] provides direct verification of the momentum formulae for a wave-packet, $mv = h\nu$, the velocity v of the particles being obtained on the assumption that they pass individually between the teeth of a ratchet wheel revolving very rapidly. It may be thought far-fetched to account for this result by supposing that there is only a *field* expressing potentiality for absorption, and that this is cut by the ratchet teeth into sections holding a quantum or more of energy. On the other hand, any field-theoretical treatment of this problem would have to proceed on this latter basis. The additional positing of actual particles in transit would then be a definitively *ad hoc* hypothesis, ruled out in terms of Weyl's Principle.

FOUNDATIONS OF QUANTUM MECHANICS: THE WAVE FUNCTION

Quantum Theory might be described as the science which deals with measurements in terms of mathematical *operators* of a very general kind, these operators providing a range of possibilities for numerical results when they operate on a certain *field*. The field is a mathematical structure expressed in terms of variables such as x, y, z, t (or more than one set $x_1, y_1, z_1, x_2, y_2, z_2$, etc.) in an abstract space and time, and expectation values of these and other variables are ultimately identified with likely values for actual measurements made under corresponding conditions in the space-time of actual observations, using the same coordinate-symbols.

At the fountainhead of the theory is the accepted difference between the unobservable but precise mathematical 'world' and the visible and tangible world of actual but statistically uncertain observations made on particular occasions.

It is remarkable that both the experimental operations and the conditions in nature 'behind the scenes' can be described with the precision of a mathematical formalism.

The Classical Uncertainty Principle for Wave-Packets

It is convenient to begin by looking at some simple properties of wave-packets. The conclusions to be drawn first of all are of general application, wherever ranges of frequencies and wavelengths are compounded (or can be treated as compounded) in a single wave-field. They can be established both by mathematical analysis and by experiment—the latter, for instance, by observing the expanding circular ripples when a stone is dropped into a pond. There is a *phase-velocity* u of the waves constituting the packet, and a *group-velocity* v of the packet

[1] Richtmyer and Kennard, *op. cit.*, p. 258.

as a whole. If the packet is very small it spreads fairly rapidly, showing that it can be taken to consist of 'components' with a fair range of phase-velocities—some components will proceed more rapidly while others lag behind. On the other hand, if the packet is permitted to cover quite a large region of space the velocity of the ripples can be fixed very precisely.

Instead of phase-velocities, let us now deal with wave-numbers ν' (the number of wave-lengths per cm.). It is then easy to show that the approximate extent Δx of the packet and the approximate range $\Delta\nu'$ of wave-numbers of the components are connected by the relation $\Delta x.\Delta\nu' \gtrsim 1$. Also, if Δt is the approximate time for the packet to pass a fixed point in space and $\Delta\nu$ is the approximate range of frequencies of the components, then $\Delta t.\Delta\nu \gtrsim 1$. These are limitations placed on all measurement by means of light or any other kind of wave-field; and there is unquestioned evidence, of course, that elementary 'particles' such as the electron, and even composite particles, must be associated with wave-properties of this kind. Hence we may conclude that all measurement is subject to such limitations.

The Quantum Concepts of Energy and Momentum

In the photoelectric effect the access of kinetic energy $\frac{1}{2}mv^2$ of an electron released from the plate P was equated to the quantity $h\nu$ derived from the incident light. According to both experiment and theory the energy of a 'particle' in a free field, photon or otherwise, is $h\nu$, where ν is the average frequency of its wave-structure.[1] By giving this multiple of frequency the name of 'energy' we mean (1) that the whole or any part of it can be transformed into mass or kinetic energy of motion as calculated classically, and (2) the total energy in these interchanges is conserved. But there seems to be not the slightest indication of a commonsense explanation how such a transformation can take place or why there should be conservation of what is in effect a total of frequencies. In the present state of our knowledge we must simply accept that countless experimental results work out accurately if these assumptions are made.

Likewise, with the same kind of interpretation, both experiment and theory show that the 'momentum' mv of a free particle is $h\nu'$. And again there seems not the slightest indication of a commonsense explanation how the number of wave-lengths per centimetre can be transformed into the product of mass and velocity, or how, in interaction, the total of such wave-numbers is conserved.

Accepting these facts, however, we obtain at once the quantum-

[1] In quantum field theory a wave-packet can be constructed as a 'composite state vector' (see below), and the energy will be a weighted mean of the separate energies $h\nu$, according to the weights of the state-vectors combined.

mechanical uncertainty relations $\Delta E.\,\Delta t \gtrsim h$ and $\Delta x.\,\Delta p \gtrsim h$. These accordingly impose limitations of accuracy on the classical determination of energy and momentum of localized atomic or subatomic wave-structures along with measurements of position and time for any event in the course of its motion. Or rather, perhaps we should say that however accurate our classical measurements of energy, momentum, position and time may be, we cannot take them as accurate measures of corresponding quantum-theoretical concepts.

The Operator Formalism

Consider now any wave-like or vibrational effect, capable of being analysed into a set or range of 'components' with definite frequencies. In quantum theory the 'components' are called *stationary states*. In the case of a classical vibrating system, such as a stretched string, they are called *normal modes* of vibration, or *harmonics*. The mathematical technique of Fourier Transforms enables us to abstract the various contributory vibrations with their correct amplitudes. Moreover, further developments of the technique provide us with formulae for the average or expectation value of the frequency ν in the system at any time and with an estimate of the range of ν (standard deviation).

On account of the relation between energy and frequency in quantum mechanics, the expectation value of the energy comes to

$$E = \int \psi^* \left(i\hbar \frac{\partial}{\partial t}\right) \psi d\tau \tag{1}$$

taken over all the variables, ψ being the normalized wave-function. The measurement of energy thus corresponds to the operator $i\hbar\, \partial/\partial t$, which may be written simply as the symbol H.

Textbooks often introduce the various initial formulae of quantum mechanics in succession as postulates, regardless of mutual consistency. A more logical way of proceeding is to postulate simply a Fourier transformation between complementary sets of variables x, t and p, H (here x stands for all space variables, and p for all momentum variables), while each system is given by an operator relation between H and x, p and possibly t (the wave-equation). One then deduces easily all the operator equivalences $p = -i\hbar\, \partial/\partial x$, $x = i\hbar\, \partial/\partial p$, etc., the 'commutator' relations $xp-px = i\hbar$, etc., the uncertainty relations, the equation of motion $i\hbar\dot{F} = [F, H]$, and the classical Hamiltonian equations of motion for expectation values. With a relativistic wave equation, moreover, the relativistic formulae for the energy and momentum of a free particle (wave-packet) are deduced, as well as the quantum formulae.

The calculation of energy-levels in 'stationary states' (regularly

periodic motion, normal modes) in simple cases yields, with the help of postulates of emission, well-confirmed formulae for optical spectra and the effects of electric and magnetic fields on them.

There can be no question that here we have a far more accurate description of processes in nature than classical physics, obscured only by the strange connection between energy and frequency and between momentum and wave-number, and hampered by its restriction to comparatively simple atomic systems.

The closeness of the agreement indicates, perhaps, that the originating causal laws of movement and forces in nature are to be found in an ultimate wave-structure in fields and concentrations. The outward manifestations that we call momentum, energy and force are end-results in which the unobserved causal world has taken on, in our imagination, a sensory character in accordance with its effect on our sense of touch and bodily resistance.

QUANTUM FIELD THEORY

Among the unsatisfactory features of elementary quantum mechanics may be mentioned the following:

(1) The electromagnetic field, to which the classical uncertainty principle applies, should be quantized in respect of conjugate variables **E** and **H**, or **P** and **A**. But in elementary quantum mechanics these quantities are inserted into the wave equation as exact functions of the space and time variables.

(2) Only a very limited and makeshift treatment of interaction is possible, by the introduction of semi-classical methods and postulates not integral with the rest of the theory. Since all observation is the result of interaction, the theory ends by being unrealistic and inaccurate in the most vital aspects of its application.

Quantum field theory begins by taking the symbols ψ, which in elementary quantum mechanics were the functions directly describing the field, as standing for *operators* subject to quantization rules (i.e. commutator formulae). Operators of such a kind are introduced for each type of field—electromagnetic, fermion, pion, etc.—with which the problem is concerned.

As we have seen, the effect of this technique is to substitute expectation values and a range of possibilities for single precise values, and at the same time, under certain conditions, to substitute for a single state a discrete set of possible states. Thus a problem in elementary quantum mechanics concerning a single proton-electron system in an unquantized electromagnetic field now becomes a multiplicity of possible fermion states, superposed on an electromagnetic field which also has a multiplicity of possible quantum-states.

Field Operators and State Vectors

To discuss the working-out of these ideas, and their significance for a philosophy of nature, we must have recourse to the 'specialist language'. The *field*, as distinct from the operators ψ, A_μ, etc., is exhibited with the help of:

(1) A conveniently chosen Schrödinger representation consisting of a set of orthonormal functions of space and time and any other determining quantities such as 'particle-spin', 'isotopic spin'. These I shall call *basis functions*. They are strictly of the Schrödinger type (S) if each contains as a factor a certain term periodic in t, namely $e^{-iE_\alpha t/\hbar}$, i.e. each is a stationary state. If this or any other dependence on t is omitted or removed in some way, the representation is of the Heisenberg type (H). The basis functions are distinguished not only by their energy $E\alpha$ but also by certain other parameters (eigenvalues) forming a discrete or continuous set of real numbers.

(2) *Occupation vectors*[1] on which the field operators operate, and which specify merely how many quanta are associated with each of the basis functions, or, as we say, are in each 'state'. It is not considered necessary to exhibit a mathematical form for these 'vectors', other than by the use of a symbol such as $|n_1, n_2, \ldots\rangle$ obeying rules very similar to those obeyed by wave-functions in elementary quantum mechanics, but subject to modification when operated on by the field operators.

On analogy with the vibration of a stretched string, one can think of the basis functions, in the Schrödinger representation, as giving the spatial form and frequency of the possible harmonics, and the state vectors as being each a table of the respective amplitudes (or energies, rather) present on any occasion. But in quantum physics the energies, if we take units that make $\hbar = 1$, must be integral multiples of the respective frequencies. In the case of 'interaction' such as a momentary touch somewhere, a certain energy associated with one or more harmonics is destroyed, and part of it may be supplied to other harmonics. The creation and destruction of quanta in various states (loosely called 'particles'), which is one of the most striking features of quantum field theory, is to be thought of in a similar way.

The product of an occupation vector and the basis function or functions whose presence it indicates is called a *state vector*. It can also be of (S) or (H) type. Operation by the field operators on an occupation vector, and other necessary procedures, bring about a linear combination of one-quantum state vectors, or two-quantum state vectors, and so on, as the case may be. We may speak of these as *composite* state vectors.

[1] My term. Ordinarily the term 'state vector' is used for these as well as for the products and composite forms which I refer to below, more specifically, as 'state vectors'. Schiff uses the term 'wave functionals' (*Quantum Mechanics*, 1955, p. 354).

Characterization of the Field: The Hamiltonian

Wave equations are of a certain broad type, and the necessity that conditions of special relativity should be obeyed limits the possibilities very considerably. The ones required in quantum theory can all be derived from a single Lagrangian or Hamiltonian operator, and thus specify the possible mathematical forms of field operators which may be scalar (real or complex), spinor, or four-vector type, to mention the chief possibilities. These correspond, respectively to the pion, fermion and electromagnetic (photon) fields found in nature.

The mathematical form of the field operators is further limited by the quantization rules which we impose in order that there may be any integral number of quanta ('particles') in each state. The field operators can then be expressed in terms of the chosen basis functions, with multipliers which of course must be operators also. These individualized operators, one for each state, are the creation and destruction operators of the field. Generally four forms can be constructed, for the creation or destruction of a 'particle' or 'anti-particle' of the field in some particular state. The manner of creation or destruction is determined by the *interaction Hamiltonian*. But first we must consider what is meant by the 'free field'.

Wave equations may be 'homogeneous',[1] or they may contain a 'source function'. The difference is clearly seen in the case of the electromagnetic field, which may be created by charges, currents or magnetic fields, but then propagates itself into regions even astronomically far from their source. The field-formalism of the former type of wave equation is fairly straightforward, and is said to describe the 'free field'. The other type generally causes severe mathematical or logical difficulties.

Interaction

The most logical way of proceeding is to regard every source as a field, perhaps localized in a very small region, and to construct the Hamiltonian operator for the whole by adding together the separate Hamiltonian operators for the free fields and an 'interaction Hamiltonian' which combines the field operators of the separate fields in a trial expression of relativistic type, suggested by classical analogies and a survey of the very few possibilities (perhaps only one exists) for a suitable mathematical expression. Although this method requires, for the most part, to be worked out by methods of approximation (perturbation theory), and occasional adjustments have to be made to eliminate infinite quantities that arise in the calculations, the extraordinarily complete qualitative correspondence with experimental

[1] 'Homogeneous' in the sense that the field operator ψ appears in each of several added terms, after operators of various kinds.

data of elementary particle physics, and in a number of cases quantitative confirmation of almost unbelievable accuracy, must call forth intense admiration.

From the point of view of the discussions in this book, interest centres chiefly on the manner in which interaction, with its accompanying destruction and creation of one-quantum states, is supposed to take place. As the theory is presented in its most general form, the interaction need have no particular localization. There is merely a mingling of fields, coming into effect as the interaction is gradually 'switched on', and resulting in a changed mathematical form of the field when the interaction is 'switched off'. If at first there are two non-interacting fields, associated with different types of 'particle', and the states of these fields as regards momentum and other physically measurable quantities is prescribed, then the final form of the field gives us the probabilities for all the various possible states of the particles resulting—that is to say, the probability that if a detector of some kind were suitably placed so as to be able to record directly or indirectly the momentum or some other measure of one of the fields, a further interaction (e.g. a scintillation) would take place, yielding such measures. There is no means in the theory for formalizing this second interaction otherwise than as an expectation value.

For the sake of clarity we may consider the special case of a pion field of monoenergic plane waves, transformed, by interaction with another field in a finite region, into a spherically radiating beam of different frequency.[1] For basis functions one naturally uses spherical Bessel functions and harmonics (angular momentum representation) and an incoming one-quantum state vector becomes representable as a linear combination of one-quantum state vectors in the new representation. The interaction Hamiltonian, by reason of the fact that it contains the field operators of both fields, brings about, as it is gradually switched on, a correspondingly gradual and probabilistic destruction and creation of quantum states with different energies, various terms in the calculation surviving only if the total energy in any typical destruction-creation process is conserved. As a result there is gradually built up, probabilistically, an outgoing field of definite energy and velocity according to the angle of deflection.

The same kind of operation is represented in the formalism of a 'particle' undergoing gradual deflection in an electromagnetic field. One speaks, for instance, of 'electron self-energy', implying that when undergoing deflection in an electromagnetic field the electron-field undergoes a probabilistic destruction of quanta in various of its states and creation of quanta in other states of different energy.

[1] J. Hamilton, *The Theory of Elementary Particles*, Oxford 1959, pp. 33 ff., 216 ff., 459.

Localization, Sources

Experiments to verify the results of quantum field theory concerning 'elementary particles' always involve interaction in some localized region where changes in the field are supposed to take place, and further interactions where localized observations are made. The difficulty in formalizing these localizations seems to be one of the chief defects in quantum field theory.

A method which gave, more than twenty years ago, deductions of the law of Coulomb force between electric charges and Yukawa force between nucleons was to insert point-singularities as rigid sources in the wave equation.[1] The absence of field-structure in the sources, and their rigid placement, make this method logically objectionable in the extreme.

A wave-packet representing a localized particle in the free field may be formed by the linear combination of one-quantum states; or even several such particles moving independently might in principle be formed in the same way. These might be given a total of any number of quanta, from one upwards, possessed by the group as a whole if the same small range of energies is covered by all. It seems conceivable that a mathematical technique might be devised to cope with the formation of one such localized particle as a result of interaction with particular atomic systems.

Perhaps the best method of localization at present, though applicable only to very much simplified problems, is the use of a Schrödinger representation such as the Angular Momentum representation, which centres the field round a chosen fixed point.

This difficulty over localization seems to indicate, more than anything else, the need for some fundamental extension or revision of the theory, or even its replacement by a theory of much wider scope. Nevertheless, it seems impossible to believe that the general principles and significance of quantum field theory, which agrees with experience so remarkably, can be far wrong.

GENERAL TYPES OF INTERPRETATIVE STANDPOINT

Almost without exception, discussions of the philosophical significance of quantum theory have in the past proceeded on the basis of elementary quantum mechanics. This has, I feel, tended to make them unrealistic. I believe most of the subjects of controversy resolve themselves very easily when one deals with them in the language of quantum field theory.

Considering the matter very broadly, there are four attitudes one can

[1] G. Wentzel, *Quantum Theory of Fields*, translation Interscience, New York 1949, pp. 43 ff., 127 ff.

adopt to the formalism and confirmations of quantum theory. I shall describe them as the *Positivist* or P-view, the *Model-Motivated* or M-view (commonly, but misleadingly for philosophers, called 'phenomenological'), the *Simply-Locational* or L-view, and the *Analytic* or A-view, which is properly phenomenological. Naturally these are extremes, and many intermediate or shifting positions may be taken.

In the P-view (which could also be called the pragmatist view) the only consideration is that the formalism reasonably chosen for particular problems 'works well' to the extent that very accurate agreement with experiment is obtained in a large variety of cases. According to Jordan, for instance, any statement which does not 'fit completely into a system of pure description of observed results . . . must be eliminated and declared basically meaningless'.[1] Such statements are characterized dyslogistically as 'metaphysical'. In particular—and here proponents of the analytic view would largely agree—it is illegitimate to visualize particle trajectories in the space of the laboratory between the position where a certain system (such as a radioactive particle) is placed and the position where an effect imputed to it (such as a scintillation) is observed. The 'trajectory language' is used, but only as an aid to thinking and communication; it is not intended to be taken literally.

In comment on Jordan's statements, just quoted, I should like to say that in quantum theory only the actual experimental observations themselves fit completely into a system of pure description of observed results. Perhaps what Jordan means would be more accurately stated by saying that one should not try to visualize observable happenings corresponding to those procedures of the formalism which do not describe observable results. This is an entirely logical standpoint, provided that by 'observable happenings' one means 'observable in the ordinary way in the physical world'.

The L-view provides a complete contrast to the P-view. Here emphasis is laid on certain facts of seemingly obvious causal connection. For instance, beams of 'particles' or waves can be collimated, obstructed or deflected by obstacles at various points and in this way we can actually control *what happens between* the 'preparation' of a 'system' and observation of its distant effects, so that the happenings 'between' can be properly located in space and time (subject, as all measurements must be, to certain techniques of approximation). It would of course be agreed that a change in what happens 'between', such as the performing of a certain experiment there, would alter the observations finally expected and sometimes substitute something altogether different. But this is the very reason why the view that something definite 'happens between' is thought to be forced on us.

[1] P. Jordan, *Physics of the 20th century*, New York 1944, p. 46.

In this view, note is also taken of the fact that cloud-chamber or bubble-chamber tracks can be followed out and made to curve by the influence of electromagnetic fields. Moreover, if the apparatus contains a small crystal, the rows of molecules of which can be detected under the electron microscope, and we accept that these rows remain in the same 'place' (approximately) while an electron-beam strikes the crystal and is deflected, then it seems that in the event of the crystal being moved during the experiment we must accept that there is a like justification for depicting (approximate) trajectories for the molecules from one position to another.

The A-view agrees that under some circumstances the depiction of an approximate trajectory may have a kind of warrant. The aspect of the L-view which it cannot accept is that locations in space and time, as in an existing 'container' of space, are causally fundamental and that all other details of substructure should in consequence be similarly located. This is the Fallacy of Simple Location, as Whitehead called it. In opposition to this fallacy there is what is called in the Copenhagen Interpretation of quantum theory the principle of 'The Indivisibility of the Phenomenon'. But the A-view (and also the Copenhagen Interpretation) would go farther than this, and say that the full primitive structure of the causal background is unrepresentable in any one-level, quasi-materialistic, space-time way.

The A-view, as I conceive of it, is simply the endeavour to discover the presuppositions of the formalism and procedures of physics, and to decide by the self-evidence of rational intuition to what extent they are to be regarded as given in nature and to what extent conveniently devised by man.

The central feature of the A-view is the insight that, however accurately the existing formalisms of quantum theory may agree with experimental results, nature does not *give* us the formalism. On the contrary we devise the formalism, and elaborate and improve it from time to time, so as to make it a more and more comprehensive and consistent 'measurement tool' for what is given in nature. There must therefore be laws of nature to which we endeavour to fit a formalism, and these laws of nature cannot consist in the bare rules of the existing formalism. The formalism therefore *signifies* relationships given in nature which it is the purpose of analysis to uncover, along with other things which are not given in nature and which again analysis tries to distinguish. Some of the relationships given in nature may be unrecognized presuppositions of the positivist approach, and it would be claimed that by uncovering these the formalism itself might be made successively more comprehensible and mathematically as well as physically more satisfactory. This has happened many times in the past, as when Einstein (following Poincaré) uncovered the presupposi-

tions of time-measurement and reinterpreted the Lorentz Transformation accordingly.

The Model-Motivated Approach

Lastly there is the M-view, which I characterize as a view which recognizes that in addition to the bare formalism and procedures a certain 'commonsense' conception of 'what is going on' is necessary if we are to decide reasonably how to apply the formalism and procedures in any particular experiment. In the M-view this 'commonsense' conception rests on the picturing of certain 'models' in space and time—that is, imagined structures and events thought of as in ordinary space and time, and considered to express some genuine truth about the circumstances, but recognized as in other respects (e.g. picturability in space and time) fundamentally false. It seems to me that this may be the commonest view among experimental and theoretical physicists.

The difference between the M-view and the A-view, so far as 'models' are concerned, is that the A-view is interested in models only to the extent that, in their use, some of the presuppositions of the theory and practice of atomic and subatomic observation and measurement may be discerned by genuine insight. The M-view, on the other hand, is coloured by the assumptions of the P-view. It accordingly regards models as heuristic aids to the construction of a more widely applicable formalism, and does not accept that use of a model can be seen to be warranted or unwarranted on any other ground than that of its bare utility.

The Born Interpretation

Sometimes one reads unguarded statements about, for example, 'the probability that a photon is in the volume $d\tau$'—whereas the fact is that no photon can be considered to be in any particular volume till an observation is made, and then, usually, the photon must be taken to be destroyed. It is an improvement to speak of 'the probability of finding the particle in the volume $d\tau$'. Yet even this form of words suggests that 'the particle' exists somewhere, although we are not quite sure where it is going to be detected. As the two-slit experiment shows, the 'particles' in the free field may not logically be supposed to exist anywhere except in the field as a whole.

From verbal expressions used it seems sometimes as if 'particles' in the field are being visualized as Euclidean points or small massy objects, distributed in a field which somehow expresses in a continuous way the statistical results of our measurements yet also conveys observable frequencies and momenta to us. Ideas of this kind seem

to indicate too narrow and mechanistic a view of the Born Statistical Interpretation.

According to the Born Interpretation, '$|\psi(\boldsymbol{r}, t)|^2 dx\, dy\, dz$ is the probability of finding a particle in the volume element $dx\, dy\, dz$ about the point $\boldsymbol{r}$ at the time t, when a number of precise position measurements are made on independent particles each of which is described by the one-particle wave-function $\psi(\boldsymbol{r}, t)$.'[1]

An obvious criticism of this is that it is impossible to say whether or not a 'particle' is described by $\psi(\boldsymbol{r},t)$. The objection may perhaps be better put by saying that the above quoted passage is not a correct description of what is done, and is in fact not operationally meaningful. The actual procedure in quantum theory is to fit a conjectural ψ to a given experimental situation, and to take the chosen ψ as sufficiently correct in so far as the agreement (from calculation of expectation values) is regarded as sufficiently accurate. To assess the agreement, statistical considerations are often needed, as in the two-slit experiment. But this is because the special feature of the predictions in that case is the variation of expectation values from point to point of the screen F. We must therefore observe scintillations in many regions in order to provide a possibility of verification. This kind of argument does not apply in the prediction of, say, the energy-levels of the Hydrogen spectrum. One observation of the spectrum provides confirmation.

The 'Subjective View'

What is sometimes called the 'subjective view' is the view that wave functions or state vectors do not stand for what is given in nature but express the extent of our knowledge, resulting from observations. Hence when a fresh observation is made, adding to our knowledge, the ψ imputed to the system or field will change.

In every branch of physics or other quantitative science, however, one frequently finds that a conjecture on which various deductions have been based has to be thrown over. We do not then conclude that there exists a certain formula synthesizing the results of our experiments, even when they are inadequate or incorrect, and that this formula must be 'tightened up' because of our newly acquired knowledge, until ultimately a 'pure state' of maximal knowledge is obtained. Scientific procedure is merely to start again with another conjecture, better adapted to what is given in nature.

It is true, however, that the state vector is always a kind of 'man-made average', since the conditions of the experiment can be neither measured nor formalized with all their infinity of detail. And we are at liberty to 'average' the conditions in any convenient way.

[1] Schiff, *op. cit.*, p. 22.

THE CAUSALITY PROBLEM

A problem in causality arises if we make certain assumptions: (1) that the mathematical forms of the fields and systems which we speculatively place in abstract space-time or other frames that may be necessary ('configuration space') constitute a *full and exact representation of existing physical entities or structures*; and (2) the inability of quantum theory to deal with emission, absorption and scattering otherwise than in terms of ranges of possibilities corresponds with an absolute limitation in nature, because of the all-comprehensive character and final truth of our formalism.

If these assumptions are accepted, it seems that there is in nature a causal hiatus between, on the one hand, systems and fields originating change or responding to it, and, on the other hand, the observations which are considered to result from these factors alone. To use Einstein's familiar expression, we have a 'dice-playing God'. Or, in clearer terms perhaps, we must abandon forever the hope of explaining how certain changes occur.

What are the alternatives? Obviously it can be pointed out that existing quantum theory is not in fact able to formalize, with perfect accuracy and final truth, everything that happens in nature. To mention one consideration only, the limitation of quantum theory to fairly simple experiments, with idealized boundary conditions contrived so as to make the mathematical problems soluble, forbids the supposition that it is an all-comprehensive tool for precisely predicting actual physical happenings. On the other hand one might hold that, in spite of limitations of this kind, the general character of quantum theory (bearing in mind its great successes) nevertheless does indicate a kind of causal hiatus in nature between a set of 'evolutionary' laws such as we have formulated mathematically and the full results of physical observation. In that case the hiatus is in the deductive linkage between the 'laws' contemplated and observations which we affirm have *no other explanatory substructure* than those laws.

Behind many of the discussions of this subject seems to lie the belief that we *ought* to be able to formalize, in a mathematical system of strict deduction, every possible event and experience in the universe.

Perhaps it is right to expect, in principle, such an explanation of the difference in positions taken by the various photon-absorptions in interference experiments (but this is forthcoming, to some extent at least, in the realization that quantum theory takes no cognisance of molecular differences in the screen, slits and intervening gas). On the other hand, to require that all mental and spiritual causation flowing into the act of observation and into the preparations for it and its

subsequent interpretation shall be replaceable by a step-by-step deductive formalism—this, surely, is to treat experience in the manner of Procrustes, lopping off the powers of the mind to control action by a synthesis of past, present and future, and forcing out the bare instants of physical observation and their spatio-temporal connections, in an endeavour to make them fill the eternal hierarchy of spirit. For further clarity over the 'causality problem' in quantum theory, it is necessary to consider the differences between the hypothetico-deductive and phenomenological methods, as discussed in Ch. 9 above (pp. 105 ff.). The discussion is continued below (pp. 334, 337 ff.).

'ESSENTIAL' AND 'SUPPLEMENTED' COPENHAGEN INTERPRETATIONS

By the 'Copenhagen Interpretation' is understood a certain general view, usually coupled with subsidiary and sometimes controversial beliefs, and associated particularly with the names of Bohr and Heisenberg. Following N. R. Hanson,[1] I think we may usefully distinguish an essential core of this hazily-defined and variable 'Copenhagen Interpretation'. It is, fairly clearly, the view that the techniques of quantum theory rest on or imply the *limited applicability of the classical concepts* of position, momentum, energy, etc. In particular, the quantum-theoretical concepts given these names receive particular quantitative values of classical significance *only when an observation is made*, and such quantitative values are not in general predictable by the formalism otherwise than statistically.

An important principle regarded as inherent in the limited classical applicability of the concepts of quantum theory is that of the *Indivisibility of the Phenomenon.* That is to say, we may not isolate part of the apparatus in our imagination and discuss the kind of observable process (e.g. movement of particles) which would be going on there if we were able to observe it. The only way in which we could observe something there would be to set up what is in effect a different experiment, also to be regarded indivisibly.

Along with this essential core of the Copenhagen Interpretation, Hanson couples the view that 'there exists at present no coherent, currently workable, and fully articulated conception of a microphysical theory which could do without this [quantum] postulate and this [duality] principle'. This seems to me a plain matter of fact, and need not be regarded as part of an 'interpretation'.

There are also various speculations which, from the point of view

[1] 'Three Cautions for the Copenhagen Interpretation's Critics', *Phil. of Science*, **26**, 326 f. (1959).

just explained, are redundant and non-evidential, yet have been put forward by some authors as an indispensable part of the Copenhagen Interpretation. The chief of these are as follows:

(1) The **Projection Postulate,** that when an observation a_k by operation α is made on a system, a_k being an eigenvalue of α, the system is left in the corresponding eigenstate ψ_k. By ψ_k one must understand the state vector of the entire interacting system, because this cannot be divided. But what is an 'observation'? If it is the emission of an electromagnetic wave (later received at the eye) as a result of absorption of a quantum from a field which interacts with an atomic system, the postulate tells us no more than that, if the emitted wave has a definite energy, momentum, or as the case may be, it is associated in the formalism with a state-vector identified by the same quantities as eigenvalues. The absorbed photon is of course destroyed. But then it has no separate eigenstate.[1]

The argument in the Einstein–Podolski–Rosen paradox[2] depends absolutely on a kind of projection postulate ('reduction of the wave-packet') to establish the 'incompleteness' of quantum mechanics. It goes wrong, as I see it, partly because it is based on an inadequate theory of interaction, and partly because it goes beyond the permissible form of projection postulate just stated.

It breaks up an interacting system into two sub-systems 1 and 2, and considers the effect of an observation α on 1, after interaction with 2 and when supposedly quite unrelated to 2, the observation α also being supposed to yield a measure a with certainty. Observations of 2 (corresponding to non-commuting observables β and γ) are envisaged as capable of being carried out (though not simultaneously) when 1 and 2 are quite unrelated. Assuming the projection postulate, suitably chosen observations of 1 will reduce 2 to eigenstates of β or γ. This means that the observation of β or γ will necessarily yield the eigenvalue b or c, as the case may be, with certainty, if one or the other observation were to be carried out. Hence, it is concluded, one can predict with certainty the measures b and c of non-commuting observables for system 2. Since quantum mechanics, in its ordinary applications, does not permit such determinations, Einstein, Podolski and Rosen declare that it does not give a complete description of 'physical reality'.

What the three authors have been demonstrating, however, is merely a contradiction, and thus an error in the assumptions or the argument somewhere. One error, at any rate, is obviously the assumption that observation of one system can tell us for certain what the results would

[1] The Projection Postulate is discussed in detail by H. Margenau, 'Measurements and Quantum States', Parts I and II, *Phil. of Science*, **30**, 1 f., 138 f. (1963).

[2] A. Einstein, B. Podolsky and N. Rosen, *Phys. Review*, **47**, 777 (1935).

be of various measurements of a totally unrelated system. That is, this form of the projection postulate ('reduction of the wave-packet') must be abandoned.

(2) **One-Level Duality,** i.e. the view that there is just one level of physical actuality, and no such thing as a *real background of potentialities* represented by mathematical structures in 'configuration spaces', and that there will *exist*, at that one level of actuality, either particles (Euclidean points?) or ideally continuous waves, according to the experimental set-up which we choose to arrange.

This has merely to be stated with some precision of language, I think, to be seen as self-contradictory and also incompatible with the essential features of the Copenhagen Interpretation.

(3) **Acausality,** i.e. the view that in nature (without regard to the limitations of the formalism) there is, to some extent, on the atomic or subatomic scale, *no reason whatever* why events should have occurred in one way rather than in another. From the analysis offered in this chapter and Chapter 9, I think it will be seen that this hypothesis is without experimental or theoretical backing.

Acausality in this sense, i.e. breakdown of the Principle of Sufficient Reason, must not be confused with *impossibility of exact prediction.* Such impossibility may be explained by many different circumstances, as for instance the fact that the power of exact prediction evidently requires exactitude of measurement both of preliminary conditions and of the 'verification', or the fact that exact prediction will involve an integration of exact knowledge of the whole universe past and present and of all natural laws concerned. To deny the possibility of this is very different, logically and philosophically, from denying the Principle of Sufficient Reason.

It may be that certain writers on these subjects will object to my analysis being called the 'Essential Copenhagen Interpretation', since they consider that the term 'Copenhagen Interpretation' has become firmly associated with the redundant hypotheses (2) and (3) which I have totally rejected. In that case, the difficulty being purely verbal, I would suggest that the analysis put forward here be called simply by another name. It could be described as an 'AP-interpretation', meaning: 'actualized, or adapted, potentiality.' This might perhaps make clear that in this view there is a creative realm of *potentiality* not physically observable as a whole, but actualized in physical observation when and according to the manner in which an observation is made; also, that in the construction of a 'measurement-tool' or mathematical system for the purpose of measuring and predicting phenomena on the atomic or subatomic scale, we are obliged to take cognisance of the fact that measurements and predictions cannot be exact, and that this results in our formalism having a smoothed-over

or statistical character, according to the type of experiment performed or observations made.

SOME UNORTHODOX OPINIONS

It would be out of place here to attempt a critique of all interpretations and expressed views not compatible with the one I am presenting here. But it may throw some of the points at issue into clearer light if I add a few comments on the ideas of Alfred Landé, as set forth with vigour and pertinacity in various publications, and also on those of David Bohm on the subject of 'hidden parameters'.

Landé describes his theory as a 'unitary particle' one.[1] No interpretation of quantum theory, however, can avoid mention of waves, unless all experiments involving frequency are tabooed. Landé takes his waves as probability ones, but imposes on them an actuality of particles which are apparently Euclidean points, without wave-structure of their own. The observed frequencies are somehow carried by the probability wave, though he tries to avoid this conclusion by a psychologistic definition of the wave function.

He states that a ψ-function is 'a well-ordered list of betting odds based on past statistical experience'.[2] We might perhaps be content to pass over the fact that the complex, many-level and componented ψ in quantum mechanics is not the same as the probability density for any measure of a given 'particle'—this latter being obtained by integrating over all the separate spaces for all the particles except the one concerned. It is more important to ask, 'Do the particles behave in any regular way *before we have collected our statistics?*' If so, then ψ describes a regularity in the background of nature, which is responsible for any regularities that may appear in our list.

Landé's chief attack is against 'dualism' or 'neodualism', by which he seems to mean the view that we should 'be content with two "pictures" or mental constructs, one of particles and one of waves, applicable to different experimental situations'.[3] In the view which I am presenting, however, the term 'particle' is never to be interpreted as standing for a frequency-less Euclidean point or core of substance confined to one space-time level and so 'picturable' in any usual sense of the term. In a strict sense it is an element in an unpicturable structure of potentialities, contributing to an indivisible whole; in a looser sense it is the (scarcely warranted) reification of the cause of scintillations, cloud-chamber tracks, etc. If we decide to use the term in this loose sense,

[1] *From Dualism to Unity in Quantum Physics*, Cambridge 1960, p. 102.

[2] *Ibid.*, p. 93. *Current Issues in the Philosophy of Science*, ed. H. Feigl and G. Maxwell, New York 1961, paper by A. Landé, p. 356.

[3] *Current Issues, etc.*, p. 351.

for convenience or heuristic value, we should not allow ourselves to forget that in doing so we have grossly oversimplified and misrepresented the mathematics which we have used to explain them.

Let us now consider how Landé accounts for the result of the two-slit experiment at its second stage. Assuming that the particles pass individually through one or the other slit, he must show how each one proceeds to the region of a bright band as determined by the position of the slit through which it does *not* pass. For this purpose he relies on the *ad hoc* explanation advanced by W. Duane in 1923[1] to explain electron diffraction by crystals.

A particle going through one slit is supposed to receive, from the screen in which the slit is made, one of a series of possible momenta p in a direction parallel to the screen and perpendicular to the slits. The reason is said to be because a screen with slits d apart is 'capable of changing its momenta' by quantized amounts $p = nh/d$ in the direction stated.[2] These amounts are postulated so that the position of the bands should come out correctly without appeal to waves of any kind. Obviously the formula must not be affected by the size, shape or mass of the screen, or even by the presence in it of other slits in front of which some object is held up so as to obstruct the particles coming through. One would like to ask what is supposed to happen when the slits are gradually shortened in length till they become small holes. But in any case this 'explanation' breaks down completely in the two-mirror and other types of interference experiment, for each of which a special *ad hoc* rule of quite a different kind would have to be devised.

These difficulties being supposedly overcome, Landé sets out to derive the formula $\psi(q,p) = \exp(2\pi iqp/\text{constant})$ for Schrödinger's amplitude function in the case of a single infinite plane wave. This formula, he says, is 'equivalent' to the formulae $E = h\nu$, $p = h\nu'$ for energy and momentum of a particle, and also to the quantum condition $qp - pq = i\hbar$.[3] Presumably, to deduce the quantum condition, Landé has taken p as standing for the operator $-i\hbar\,\partial/\partial q$, while to deduce the other two formulae he has taken it as standing for the eigenvalue of this operator when operating on the infinite plane wave (and similarly E is the eigenvalue of $i\hbar\,\partial/\partial t$). Apart from the mathematical incoherence, it has to be objected that there is no link-up of this symbolic algebra with energy and momentum in their experimental significances without the usual wave equation, yielding also the formulæ $p = mv$, $E = mc^2$ (in the relativistic case), where v is the group velocity of the wave-packet constructed from the components with a *range* of frequencies and wave-numbers.

[1] W. Duane, *Proc. Nat. Acad. Wash.*, **9**, 158 (1923).
[2] *Current Issues, etc.*, p. 69.
[3] *Ibid.*, p. 55.

Landé admits no such wave-packets or range of frequencies, etc. He takes his (mathematically inadmissible and infinitely extended) form of amplitude function as standing for the probability wave which is constructed from the 'list of betting odds' for a single point-particle. For its derivation he first of all assumes the projection postulate, though elsewhere he strongly rejects the 'reduction of the wave-packet'. The next step is to consider two operators with discrete spectra and to propose a 'transition probability' $P(A_i,B_j)$ from an eigenstate of one to an eigenstate of the other. To agree with accepted theory a complex quantity ψ is then associated with each P, making $P = |\psi|^2$. The exponential character of ψ is deduced from other considerations, and by making the index imaginary a periodic form results. It is not clear to me how Landé proposes to pass from this excessively simplified special case to the solutions (not exponential in qp) of wave equations in general, or how he can consistently accept the formalism of quantum mechanics yet refuse to apply it to the wave-equation of a free particle so as to deduce the possibility of a wave packet with all the properties which he accounts for otherwise.

Hidden Parameters

A few remarks may finally perhaps be ventured on the views of Bohm, that it should be possible to 'tighten up' quantum theory by the introduction of new 'parameters' or otherwise, so as to remove uncertainty either all at once or in successive stages. He writes:

'If we study the level of Brownian motion itself, we can expect to treat, in general, only the statistical regularities, but for a study of the precise details of the motion this level will not be complete. Similarly, one might suppose that in its present state of development, the quantum theory is also not complete enough to treat all the precise details of the motions of individual electrons, light-quanta, etc. To treat such details, we should have to go to some as yet unknown deeper level, which has the same relationship to the atomic level as the atomic level has to that of Brownian motion.'[1]

On this view it may be commented, in the first place, that all problems in quantum theory could in principle be framed so as to introduce new 'parameters', simply by expressing the experimental conditions mathematically in less simplified forms. The fact that our mathematical techniques are at present incapable of dealing with conditions more closely resembling what they seem to be in reality amounts to no serious objection; for this is equally the case in classical mechanics,

[1] *Causality and Chance in Modern Physics*, London 1957, p. 80.

which is none the less accepted as 'complete'. On the other hand it is clear that the present techniques of quantum theory provide no ground whatsoever for determination of precisely when absorption of a quantum from a wave-field will be made by an atomic system or precisely which of two or three systems will absorb it. Even if we accept, in general terms, that our mathematical techniques might be developed so that it becomes possible to formalize the experimental conditions in far greater detail, still it does not seem possible that the times and places of absorption would thereby be precisely determined. The theory does not remotely touch on the *manner* in which absorption takes place; it deals only with average *results* found in practice.

In the second place, many problems in quantum theory already introduce parameters or possibilities of measurement going beyond the general rule that the theory connects with experiment only through expectation values of operators and transition probabilities. Thus the form of a wave-packet, the relative intensities of the various components ('single particle states'), etc., escape this rule and yet are relevant to observation. The calculation of the length of wave-packets from observed line-breadth, it being assumed that the packets are all of approximately the same shape, might be one way in which we could make a measurement which escapes the general rule. Other estimates of the size of elementary particles, made on various grounds, are also estimates of the size of wave-packets or minute wave-fields, employing methods going beyond the regular techniques for connecting the theory with observation. As experimental research advances, still other methods may be discovered and built satisfactorily into the theory.

But now various other objections arise. The measurements which we might obtain in these other supposed ways would still not be in exact correspondence with numbers calculated by means of the (revised) formalism. For in so far as the formalism represents a genuine *givenness in nature*, and not *ad hoc* imaginations, it will represent an objectivity for all observers. In other words, it will represent a potentiality which comes into actuality, when any particular observation is made, according to the infinitely complex circumstances of that particular observation. Hence any formally predicted result would still be only an expectation.

Thus the supposed extension of the techniques of measurement and quantum theory would at best introduce a second-order expectation, treated more or less as classical irregularities of this kind are treated by the Theory of Errors—i.e. classified after the event, but not predicted.

Bohm indeed agrees that there will always be 'random fluctuations'. He writes:

'The determinations of any purely causal theory are always subject to

random disturbances, arising from chance fluctuations in entities, existing outside the context treated by the theory in question.[1]

It seems to me however that Bohm, while accepting 'random fluctuations' in principle, nevertheless wishes to do away with the uncertainty principle (conceiving it as unrelated to the inevitable randomness of atomic and subatomic measurement) and to reintroduce trajectories for classically-conceived 'particles'. Thus in an endeavour to explain the two-slit experiment, he has suggested that in addition to the wave, which goes through both slits, there are localized particles (*not* wave-packets) which are guided through one or other of the slits by existing but unformalized 'fluctuations' in the wave, and that neither particles nor wave exist without the other.[2]

This hypothesis offends against Weyl's Principle, i.e. it is definitively *ad hoc*. The trajectories of the supposed 'particles' through one or other of the slits could never be determined, and the attempt to do so would mean setting up a different experiment.

But Bohm himself runs against this hypothesis later,[3] when, in order to agree with certain verified applications of quantum field theory, he supposes that 'particles' are really 'particle-like concentrations of energy, charge, momentum, mass, etc. . . . always forming and dissolving . . . the particle does not move as a permanently existing entity'. To a certain extent, this latter view seems to give a true picture of what is being done in applying the quantum theory formalism.

On the other hand it must be remembered that these 'concentrations' connect with observation only in a statistical way, that is, they stand for potentialities and not for entities whose positions, momenta, etc., are precisely determinable in a container of space. Moreover, since the 'concentrations' are of (potential) wave-structure, it is evident that the uncertainty principle must apply to them. Such conclusions are not incompatible with the view that we simply do not know enough about the way in which absorption from a wave-field takes place, and that existing quantum theory is deficient in this and other respects.

But the chief point I am making here is that, however deep the analysis is taken, and however far the formalism is improved, measurement (and the formal predictions based on it) will always be *empirical*; and the first mention of 'prediction' in empirical science takes one out of the realm of exactitude. For, in actuality and in principle, the total conditions from which the prediction is thought to be made can only be sketched out in a simplified way.

Coupled with the impossibility of absolutely exact prediction is the impossibility of expressing the formal structures of quantum theory in terms of certainties of exact location in space and time, independent

[1] *Causality and Chance in Modern Physics*, p. 141.
[2] *Ibid.*, pp. 111 ff.
[3] *Ibid.*, p. 121.

of the various observational circumstances which may be set up in order to measure them.

As general conclusions I suggest therefore that (1) quantum theory may quite possibly be improved in various ways and enlarged in its techniques whereby correspondences with experiment can be obtained. But (2) it will remain true that the state vectors will stand for a *potentiality* which cannot be represented in terms of specific actualized structures at the one level of particular observations in a 'container' of space.

Heisenberg seems to have correctly summarized the position when he wrote that

'all the opponents of the Copenhagen Interpretation do agree on one point. It would, in their view, be desirable to return to the reality concept of classical physics or, to use a more general philosophic term, to the ontology of materialism. . . . This, however, is impossible, or at least not entirely possible.'[1]

[1] W. Heisenberg, *Niels Bohr and the Development of Physics*, London 1955.

CHAPTER 21

THE DIRECT KNOWABILITY OF TRANSCENDENTAL CONSTITUTION

> 'And when I speak of the other division of the intelligible, you will also understand me to speak of that knowledge which reason herself attains by the power of analysis [διαλεκτικὴ], using the hypotheses not as first principles, but only as hypotheses—that is to say, as steps and points of departure into a region which is above hypotheses, in order that she may soar beyond them to the first principle of the whole; and holding to this and then to that which depends on this, by successive steps she descends again without the aid of any sensible object, beginning and ending in ideas.'
> 'I understand you, he replied; not perfectly, for you seem to me to be describing a task far from easy.'
>
> Plato, *The Republic*, 511

> 'I.29. Thence one obtains recollective consciousness [*pratyakcetanā*] and the absence of obstacles.
> I.41. In one whose mind-disturbance is (thus) destroyed, (there is) an intimate concurrence of knower, knowing and known, resting on and imbued with that (condition of release).
> I.40. (In that state of release) his power (of contemplation extends) from the ultimate atomicity to the ultimate immensity.'
>
> Patanjali, *Yoga Sutras*

The common dualistic view, that besides the various kinds of awareness and activity of the individual subjective mind there is in nature an objective substratum which is *unknowable*, has a considerable amount of plausibility. It may seem obvious that a direct discernment of the details of atomic structure or of the constitution of distant galaxies must be absolutely beyond our powers. It may also be argued that since everything which we observe is an appearance according to the subjective occasion, no room is left for observation of an objective substructure, although this must exist to account for the continuing order of nature, affecting all people similarly.

But to accept the view that the substratum is unknowable would be to place at the root of all phenomenological analysis of nature an irremovable dualism, like a signboard instructing us: 'So far, and no further! What lies beyond is in the unknowable category.' How, then, can we know that anything is beyond?

A paradoxical situation can usually be resolved fairly quickly by exposing ambiguities of language and exhibiting each intended meaning

directly, that is, by clear phenomenological analysis. In the above contexts it is reasonable to take the word 'unknowable' as meaning, more precisely, 'not knowable by acquaintance'.[1] For it could be maintained, at least by Scientific Realists, that the substratum is certainly knowable 'by description' in terms of the formulations of mathematical physics. What we have to consider is whether such kind of knowledge is all that is available to us and all that it is reasonable to demand.

Knowability of the substratum by *acquaintance* seems to be definitely required by what Russell calls 'the fundamental principle in the analysis of propositions', namely:

> Every proposition which we can understand must be composed wholly of constituents with which we are acquainted.[2]

We can, of course, understand the propositions of mathematical physics in a certain way, without being acquainted with (i.e. directly perceiving) the elementary particles or wave-functions to which they may refer. This is because we are acquainted with the symbols and their accepted usage, and because some of the symbols stand for physical measures in a contemplated experiment. It does not follow, however, that we can understand the propositions of mathematical physics *as referring to something given in nature* other than the bare existence of certain measures. In fact, the claim that there is an unknowable substratum is precisely the claim that we can never come to understand the propositions in that way, but are limited to bare symbolic usages and the positivistic fitting of data to a formalism. In short, the order of nature has no inherent comprehensibility.

It is not necessary to consider further arguments for rejecting the theory of the unknowable substratum.[3] I propose in this chapter to answer the difficulty simply by continuing the phenomenological analysis into the regions where its power might be doubted, or otherwise by showing that many things postulated in mathematical physics *are* unknowable by acquaintance, for the reason that they do not occur in nature in the way supposed.

One important fact may with advantage be stressed in advance. In order to become acquainted with (or, as I shall say, have direct know-

[1] Cf. Bertrand Russell, *The Problems of Philosophy*, Ch. 3. In phenomenology, however, the 'intuitive knowledge of truths (of existence, relationship and circumstances)' is a variety of 'knowledge by acquaintance'. This is to be sharply contrasted with the 'derivative knowledge of truths' and 'knowledge (of things) by description', neither of which has phenomenological status. I shall describe the former types of knowledge as *direct* (or *ostensive*), and the latter as *indirect*.

[2] *Ibid.*, Home Univ. Library, p. 91.

[3] Other arguments have been outlined in Ch. 16, pp. 242 ff., and summary p. 280. The conceptual origins and historical development of the theory of the unknowable substratum are studied at length in Part IV.

ledge of) any of the numerous levels and types of transcendental constitution, we must ourselves be taken up to that level and must participate in that type of transcendental existence. Consequently, some aspects of constitution are easily known, while others require the possession or development of unusual faculties. But the evidence is sufficient, I believe, to confirm that every feature of constitution, of which we can justifiably assert that it is genuinely 'there' in the form supposed, must be, in principle, directly knowable.

The discussion of this difficult subject is necessarily very condensed and limited, since we are here verging on vast fields of research extending far beyond the scope of a single volume on space, time and nature.

TRANSCENDENCE 'IN'

Much evidence has already been given of the breakdown of the usual physical barriers in certain kinds of analysis. By the 'usual physical barriers' I mean fixation on the view that our minds are enclosed in the body or perhaps in some small region in the head, quite insulated from the minds of other people, and along with this, the adoption of an almost automatic 'simple location' of objects outside and the denial of connections between the objects and people 'outside' otherwise than by certain means which are also simply located in space and time.

The breaking of the barriers is achieved by *recollective release*,[1] which overcomes the onward urge of physical time and thus tends to reveal everything, including positions, distances, durations, and impressions of material substance, as *constituted in ideas*. That is to say, their givenness is primordially conceptual and open to phenomenological analysis. The following kinds of transcendental experience thus made possible have been described:

1. **Knowledge of Eternal Design** by primordial recollection, proceeding perhaps to analysis of the simultaneous, synthesis of the extended, or protention (described in Chapter 19). Particular and general kinds of knowledge of eternal design can be distinguished; for example, there may be protention of some particular time-spot, or there may be recognition of protention as a power of the mind which we can voluntarily exercise in a real relationship to outward physical experience of the future.
2. **Exercise of the Function of Spatial Identification,** so that our consciousness of position is neither exclusively 'here' nor exclusively 'there', but both positions are together knowable in a transcendental Gestalt of self-identifications. This was described in Chapter 10.
3. **Comprehension of Geometrical Gestalten** of objects, e.g. the straight line, in the spatial field of translations and rotations, as represented mathe-

[1] On 'release' in general, the power of 'primordial recollection', and the stabilized state of 'recollectedness' (recollective release), as conditions of phenomenological analysis and transcendental participation, see pp. 107–110, 295 ff.

matically by the Euclidean Isometry Group—this being the constitution of the function of spatial identification. In particular, we can perceive, along with the one central perspective of an object, the other *appresented* perspectives comprised in its three-dimensional Gestalt.[1]

4. **Empathy.** Direct awareness of communality with another mind (the term is also used of spatial identification with a particular solid object or living body, at rest or in movement).

Empathy was briefly considered in Chapter 10, and again, more fully, in Chapter 14. The views of Husserl and others are compared in Chapter 24 below. A few further remarks seem desirable here, referring more specifically to empathic knowledge of 'mental states', that is to say, of another person's feelings, motives, trends of character and thought, but not of his physical sensations directly.

Empathy, as a primordial power

It is obvious that we ordinarily come to a recognition of the feelings and viewpoint of another person by means of visual and audible appearances, particularly by the sight of expressions on the face and gestures of the body and the sound of words spoken. Such sensations are, for us, *clues* to the mind of the other person (i.e. to his existence as another real person and to his mental states), and I am maintaining here that they can bring us into a 'primordial' knowledge of his mind. In other words, they can bring us to some degree of face-to-face acquaintance with, or participation in, the other person's existence and mental states.

The alternative opinion is that we can only *infer* the state of the other person's mind from the clues, thus forming an imaginative supposition of what might be going on in the other person's mind; and that we are thus induced to *transfer* feelings or thoughts of our own, modelled on what we suppose are the feelings and thoughts of the other person, to the imagined locality of the other person's body.

I reject this opinion, first because it does not seem to describe my own experience correctly, except perhaps in those cases where I am unable to establish what I call 'empathy' and substitute a guess at the other person's mental state; and secondly because it seems to me a clear instance of the Abacus Fallacy and the psychologistic confusion of the *order of approach* with the *order of reality*.[2]

When I associate with another person whom I know intimately, I am not busy planting my own feelings on to visible shapes and audible words. What comes to me 'from the other person' is very often surprisingly different from my own feelings and impressions, and I compare the two sets of feelings or impressions without thought of localization in bodies or transference of one set into another position.

[1] Above, pp. 130 ff., and below, p. 409.
[2] Above, p. 147.

In the order of approach, the clues come first, and our awareness of the other person's mental state seems derivative. But in the order of reality, feelings as such and in themselves are not localized in a container of space or in any physical body, but they acquire a derivative expression. For in recollective release the fixation on the physical body disappears and we know the feelings primordially in their own 'sphere', or at their own 'level' of creativity; and then, when recollective release is lost or abandoned, fixation on physical appearances is resumed, like the putting on of an outer garment.

Hence it seems to me clear that we can have a knowledge of other minds extending to a real participation (Einsfühlung[1]) in their mental states. What is necessary to reveal this is to suspend fixation on bodily appearances so that feelings and other impressions of personal life are presented out of space, in their essential ('primordial') nature. Reflection then discloses a distinction between what is 'ours' and what 'flows in' from another personal source. In this analysis associations with physical appearances are indeed strong support, but none the less outwardly imposed.

Theories of *inference* and *transference* presuppose the known existence of other people to whom (we infer) a transference of our emotions must be made. Hence to argue that this is our ground for believing in other people is to argue in a circle.

Just as there is nothing in physical science which runs against the possibility of varieties of time awareness and control, or against the exercise of the function of spatial identification, so there seems to me nothing which runs against the possibility of empathy. Physical science is the study of measurable representations in the container of space and in physical time, not taking into account anything that the human mind may do otherwise than by preparing the apparatus, carrying out the measurements, systematizing them, and making predictions which may be valid under the limited circumstances envisaged.

By its own hypotheses, therefore, the scope of physical science does not extend to the fields of knowledge which we are now considering. To suppose that these fields of knowledge are thus proved non-existent is a mere *non sequitur*, suggested perhaps by the Fallacy of the Conceptual Closed Shop.[2]

Correspondence: The Phenomenological 'As If'

Let us now consider again the perception of a three-dimensional Gestalt —for instance, that of a cube. Such perception is said to be 'transcendent in' the ordinary physical perception of the cube because the conscious exercise of recollective release from fixation on space (i.e. from 'simple

[1] *Wesen und Formen der Sympathie*, 3rd ed., Bonn 1931, section C. III.
[2] Above, p. 101.

location') is needed for us to become aware of it as constituted in a logically prior way, and not merely as an accompaniment, surface 'coloration', or by-product of the visible and tangible appearance presented to us in our ordinary state of physical observation. Yet we must admit that the three-dimensional Gestalt is *present* as a 'discoverable cause' of all the particulars of three-dimensional form ordinarily observed, being found by phenomenological analysis of the objective presentation as a whole.

This state of affairs can be more briefly described by saying that the three-dimensional Gestalt is constituted at a 'level of creativity' which is *immanent* in ordinary physical perception and is correspondingly represented in it.

The same kind of relationship is seen more easily perhaps in cases where the term 'transcendence' (of one level in another) seems too strong a term to use. Thus, returning for a moment to the case of emotional expression, it is to be remarked that we do not usually consider what expression may be on our face when we are irritated or feel sudden interest or disappointment. We live, so to speak, in an autonomous world of the emotions, and we have to adopt the viewpoint as of another person if we want to become aware of the sort of facial expression which we then have.

If a four-note chord is momentarily struck on the piano, the first impression we receive is usually one of its general tone-quality. But then, almost at once, we may proceed to analyse it, so far as we can adopt a condition of primordial recollection. The analysis may reveal to us the four constituent notes, and these, we feel, 'explain' its quality. To hear the four notes is merely to hear the chord at last clearly and comprehendingly. Yet the tone-quality of the chord is something additional, *corresponding* to the relations between the notes as well as to the notes individually, but not identical with them. The state of affairs is genuinely 'as if' the hearing of the four notes was *in* the quality, constituting and explaining it. In the same way, emotion is genuinely 'as if' *in* the facial expression of it, when this is looked at in its character as an expression of emotion. Yet again there is something additional, namely the visible and tangible aspects of the expression, necessary for the actualization of the emotion 'as if' at the level of physical observation.

Each additional aspect (facial form, tone-quality as such) is also open to phenomenological analysis, uncovering its 'explanation'. Hence we arrive at the view that what exists at one level of creativity (emotion) or in one limited field of creative possibilities (combinations of pure tones), manifests itself correspondingly at another level (visual observation, sense of tone-quality) by utilizing additional material in subsidiary fields; and this additional material has also its 'discoverable causes'.

The words 'as if', in these contexts, should not be interpreted as

meaning that the intelligible details 'explaining' the outer sensory effects are merely *imagined*—'not really there'. On the contrary, the outer sensory effect could not be recognizably there unless the 'inner' intelligible structure was potentially discernible there also.

Creative Influence

It could be said that what is immanent in an appearance *creatively flows into* it. This metaphorical mode of expression, however, seems more suited to express the relationship between the eternal design and the particular actualization concerned.

Thus *protention*, as in the 'time-spot determination' of some future note in a melody, which is being performed, would be confusingly described as 'immanent' in the physical perception of the note when actually played. The process of actualization will in general bring about slight but discernible differences between what was conceived in protention and what is discoverable by analysis of the actual physical sound. Only the latter is immanent in the physical sound; yet the protention has gone towards its determination.

Again, suppose that a strong emotion has gripped us, but in order not to show it we have exerted the muscles of the face in such a way as to prevent the normal form of manifestation. The emotion exists at its own level, and has creatively influenced the expression of the face; but to describe it as *immanent* in the expression is confusing.

It seems, from such examples, that we must envisage at the transcendental level of eternal design a continual build-up of connections having relation to past, present and future, and to 'objects' which may be manifested on occasion in various positions and with various appearances, as well as to individual 'minds' with different character dispositions, these 'minds' also tending to be manifested in certain bodily forms according to the occasion. The design is then in a certain way *fulfilled* by the actualization, in the physical present, of the creative influences bearing upon the equivalent time-spot from the 'eternal' past, present and future.

It is important not to fall into the error of supposing that each elementary particle or small region of a field (simply located) goes its way according to physical laws, and that the large-scale effects are thereby determined. Exactly the opposite is the case. Even in mathematical physics, the field equations determine nothing; but the instant the 'boundary conditions' are sufficiently decided, the field is determined accordingly at all points and times.[1]

So, if we take up a torch to flash a beam of light on a distant tree, the instant we switch it on, the light takes its course very largely as we wish,

[1] In Quantum Mechanics the determination is only of the wave-functions, at the level of potentiality.

and any waves or quanta which we may suppose are constituted 'behind the scenes' must behave accordingly. Likewise, if a tennis-player decides to hit the approaching ball in a certain way at a certain anticipated time, the dominant creative influence of his *protention* determines very largely (if he is expert) all the subsidiary motions and bodily changes.

These are illustrations of a principle which I shall call the Principle of **The Whole Before the Parts**. This is to the effect that if there is the creative determination of a whole, whether potentially or actually, the parts which are to be found in its intelligible constitution—even if they are supposedly of the order of elementary particles or quanta—must go with it, and so tend to 'fall into place' as the necessary 'as if' substructure of the macroscopic whole.

Theory of Actualization Further Extended

The Theory of Actualization outlined in Chapter 10 was presented as a theory for the 'Physical Grade' only. We have now reached a point where it is necessary to take into account the fact that our knowledge of physical objects is *intersubjective*; that is to say, there must be a constitution of nature which is common to different observing subjects. From this it is clear that the diagram (p. 146) of 'levels of objectivity' at the Physical Grade is very artificially limited. It suggests, indeed, that 'Reason', 'Actuality' and 'Matter' are all the exclusive concern of a single solipsistic subject occupied merely in physical observation. There should therefore be connections marked outside the diagram, representing the ways in which the solipsism is transcended, and the ways in which the various other activities of the subjective mind (besides physical observation) can be exercised and have creative influence.

Any diagrammatic representation of conceptual relations as complex as those with which we have to deal here is bound to seem a very inadequate makeshift. I venture, nevertheless, to make some suggestions for supplementing the former diagram, because hardly anything could be more absurdly inadequate than a world-view leaving out other people and the power of each individual to reflect, think, imagine, feel and will.

First, we must take our diagram of the 'Physical Grade' as representing, more precisely, some particular individual's physical awareness on some occasion, in all its actual and potential aspects. Then, when we show separate diagrams for different individuals observing the 'same' objects in the physical world, we ought also to show them in some way cohering or connected at the level of intelligible structure; for any communal objectivity, as we have seen, is intelligible and not sensory or subjective in the ordinary ways characteristic of physical life.

We may not suppose, however, that the intelligible structure discoverable in one individual's physical observation altogether coincides with that of another individual. The strictly communal potentialities in

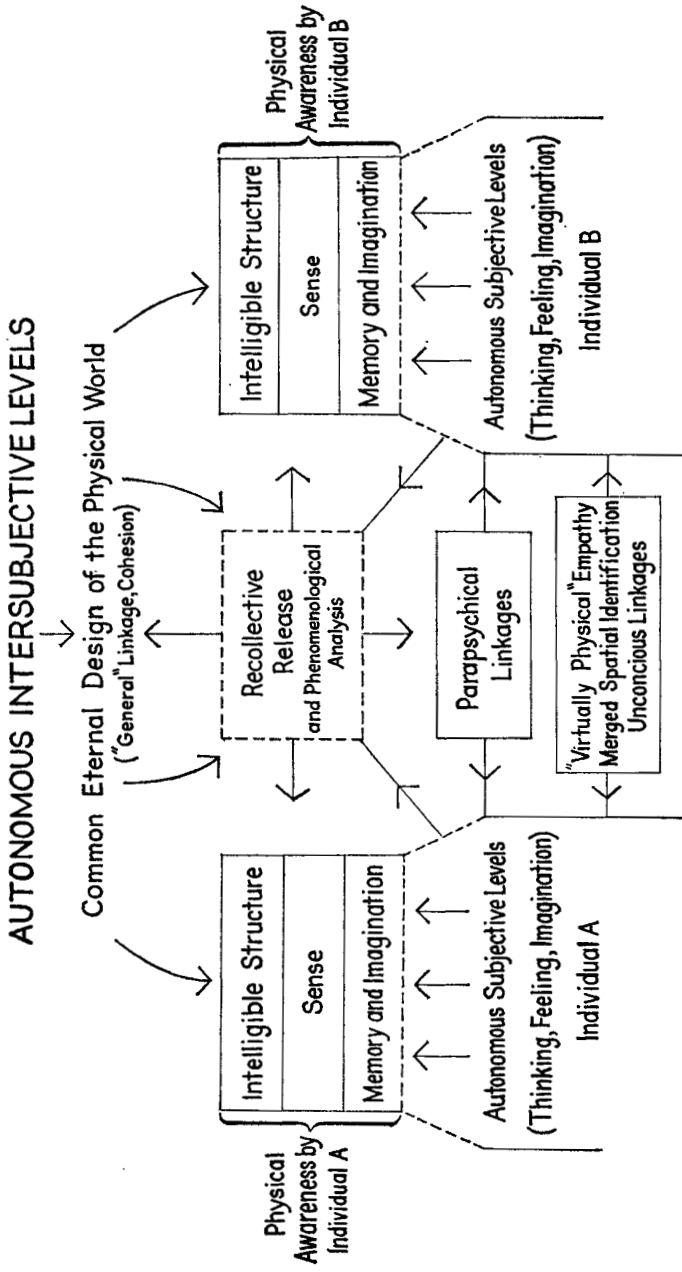


DIAGRAM OF TRANSCENDENCE
OF INDIVIDUAL PHYSICAL AWARENESS

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the eternal design cannot be identical with the intelligible structure of any particular physical observation by some individual. They must be something more primitive, which *corresponds* to that intelligible structure according to the individual's circumstances and mental state. At the top of the diagram for each individual, therefore, the common eternal design of the physical world is shown as at a superior level and creatively flowing into the intelligible structure of his physical experience.

The common eternal design of the physical world is also able to flow creatively into perceptions of spatial identification and empathy, precognitions, primordial recollections of physical experience, and (much overlaid by additions or counterfeits) memories. For all these have a communal origin. The two curved arrows descending more centrally from the common eternal design indicate the direction of this influence, namely towards conditions of detachment or release from physical awareness.

The various ways in which any individual can become detached or released from his strictly physical awareness should also be indicated somehow in the completed diagram. This occurs most evidently (1) when the conditions of physical experience and its subjective accompaniments are analysed phenomenologically, (2) when by a specially strong establishment of recollective release we become temporarily 'stabilized' at some non-physical level of experience, or (3) when we pass to conditions of thinking, imagination, and so on. Differences in the *degree* of release from the physical state, as between (1) and (2), are not represented in the diagram, but we must try to indicate how the various kinds of detachment or release come about.

The broken line at the side of the region marked 'Autonomous Subjective Levels' indicates the dissolution of sensory and other subjective fixations (especially fixation on the onward urge of time) so that recollective release becomes possible. If then the universality and intelligible coherence of what is contemplated is presented to us in a strongly apodeictic way, I shall describe it as *mystically* known, and I shall assume that the knowledge has then a stabilized character. Scholarly usage of the term *mystical* today, I believe, forbids our using it in respect of momentary insights, after which we return at once and wholly to a more normal state, without that apodeictic knowledge. If what is contemplated is presented to us in a less universal and apodeictic way, but still clearly transcending the ordinary conditions of physical space and time (as, for instance, a fairly vivid and unmistakable experience of empathy), I shall describe it as *parapsychically* known.

Phenomena of either a mystical or parapsychical kind may also, as we have seen, *merge* in physical experience, so that both physical and non-physical phenomena seem to be known at once. It is then impossible to draw sharp distinctions. A very strong experience of empathy (as, for

example, in intense mutual love) in which time seems to stop, might be suitably described as mystical, while the weak and elusive experience of empathy which we have in ordinary relations with other people with whom we are fairly well acquainted might be described as 'virtually' at the physical level.

In this last category (the 'virtually physical') it seems reasonable to include also the ordinary experience of three-dimensional spatial identification, which presents itself almost as an attribute of visual appearances. When the spatial identification is exceedingly vivid, however, I would classify it as parapsychical.

At the foot of the completed diagram the boundary line for each individual is shown broken, in order to indicate the ease with which creative influences which are suitably called 'subjective' break through to the 'memory and imagination' aspects of physical observation. Reciprocally, we may begin with the memories and imaginations which can be discovered implicit in physical sensation, and then 'locate' our consciousness in the connected spheres of thought, feeling, imaginations, etc. I call these spheres of awareness 'autonomous' because it is clear that we can remain for a considerable period in a state of deep thought, imagination, strong feelings, or dream, without being aware of the corresponding changes which are potentially observable in our physical bodies or in the physical world around.

It is clear that the primordial recollection which stops the urge of time and clears us of fixation also begins, as it were, at an autonomous subjective level. But the *established state* of recollective release and accompanying phenomenological analysis is not at any subjective level, being, on the contrary, in any region whatsoever which is to be phenomenologically explored. The rectangle marked 'Recollective Release,' with its broken boundary, is therefore to be conceived of as applicable to every region which the individual is able to reach for the purpose of analysis or stabilized participation.

Complicated though the diagram may seem, I should like to reiterate that it is only an attempt to indicate suggestively the results of phenomenological analysis of the various conditions, stabilized or otherwise, in which the limitations of ordinary physical awareness are transcended.

There seems to me again nothing in physical science which runs against accepting that intersubjective linkage can be represented to us in forms of awareness at various levels, mystical, parapsychical, or virtually physical. Existing and well-established physical laws have to do with the actualized physical forms teleologically determined by certain boundary conditions which are otherwise decided upon. They have nothing to say upon the possibility of linkages at a transcendental level manifesting themselves in corresponding forms at a non-physical level.

Stabilized Levels of Creativity: Transcendence 'Over'

It has already been remarked that emotion may develop itself in a 'world of the emotions', while the normal expression of emotion in the physical body is ignored or even inhibited. Dream experience provides a much clearer example of events in 'another world' proceeding autonomously; for not only do we have then a function of spatial identification, with a corresponding sense of sight and possibly also hearing, touch, etc., but the stabilization seems to be secured by or associated with the persistence of a characteristic state of the physical body, namely, sleep. The existence of a correspondence between manifestations on two levels does not prevent the two levels being each 'autonomous' in my sense of the term. What makes the dream state and the physical state autonomous is the fact that we can continue in either state for a considerable period, during which any corresponding manifestations in other states are observed *by other people*, not by us. On the other hand, the feelings and thought which come to us when in a state of ordinary physical activity are momentary merging impressions in physical experience, not to be described as 'autonomous'.

We now have to consider whether a stabilization such as occurs in a state of dream, and may also occur to some extent in a state of emotion or deep thought, can also occur in states which are more appropriately described as 'transcending' the physical state. Are we limited, in our knowledge of eternal design, to just those portions which we can reach by primordial recollection or protention in correspondence with some physical actuality of the past, present or future? Or can we to some extent become free to survey the constitution of nature and human life in all its aspects autonomously? We can deal with these questions in terms of reason, or in terms of evidence.

Taking the arguments of reason first, the general terms which I have introduced to describe the power by which we come to knowledge of the eternal design (whether by primordial recollection, protention, or any other way) is *recollective release*. By this power our bondage to the onward urge of physical time, and the strictness of the correspondence between events in the level of eternal design and those in the physical level, are broken. A synthesis of the extended at the level of eternal design can be surveyed without regard to how physical time may be progressing (we only notice the passage of physical time when we return to the physical level). In other words our survey of the level of eternal design is, in principle, autonomous, even if we do not have the requisite power at any given stage of our lives or in the given circumstances to carry the survey very far.

Another argument of reason arises from the fact that the dream state is autonomous. Experience in the dream state must have its intelligible constitution, even if we can discern nothing of it when carried along in it

unreflectingly (I assume that everything has its sufficient reason). There is therefore in the constitution of nature an intelligible provision for the coordinated and stabilized working of senses and other human faculties (all in analogous forms) at more than one autonomous level. Now what is intelligible manifests itself most clearly and controllably at the transcendental levels. Hence we can expect stabilization and autonomy to manifest itself clearly and controllably at the transcendental levels.

Considering the appeal to evidence, we find that *paraphysically* represented linkage, of numerous kinds recognized by present-day academic research on the subject, is commonly autonomous (and not merely mergent in physical phenomena), though usually for brief periods only. The chief categories are extra-sensory-perception (ESP), out-of-the-body experience (of the type which I have elsewhere called 'full separation'[1], telepathy, and precognition. The first has reference to transcendence of space in respect of certain presentations, thus providing a form of *extended conceptuality*.[2] In the second, there is full transcendence of fixation on the physical body and its 'simple location', along with marked stabilization in another spatially-represented 'world'. In the third, there is transcendence of 'enclosure' as regards personality structure. In the last, there is transcendence of the rigid identification of eternal design and physical time. To attempt to provide a satisfactory survey of the evidence would make far too extensive a digression.

There is special need here to consider the levels to which we are taken by the highest degrees of phenomenological analysis, which disclose the apodeictically intelligible features of constitution in the most intimate way. The precise manner in which these features may be manifested *at or near their own level* (when sufficiently stabilized) or *mergingly* 'as if' in the physical level, is clearly not a matter for conjecture. The evidence which has accumulated since earliest times, and is today being presented and studied in a careful scientific spirit, is of what is called *mystical* experience. The importance of such experience for the purposes of the present chapter lies in the fact that we have here a field of autonomous knowledge of transcendental constitution, precisely such as the difficulties exposed at the beginning of the chapter indicated must be available to us. Hence the paradoxes which burdened the problem of the substratum prove to be groundless.

Where there is a reluctance to accept the corroborative evidence on this kind of experience accumulated through the ages, I suspect that it may be due to two reasons. There may be too strong an unconscious assumption of 'simple location' in space and time, so that the existence of a *conceptual* constitution of all things is felt to be both unnecessary

[1] *The Mystical Life*, London 1961, Part II.
[2] Chapter 16 above, esp. p. 247.

and fantastic. And the second reason is that mystical experiences are thought to be always and wholly in the same category as subjective imaginations, with no 'reality' in relation to that of physical experience. It is not realized that the descriptions and explanations accepted by authorities on the subject (many of them trained as scientists) refer to manifestations of a *distinctly transcendental* kind—manifestations which intimately represent what is universal, intelligible, apodeictic, and logically prior to physical manifestations.

'Reality'

Much confusion in discussions of transcendental knowledge is bound up in the varying usage of the word 'real'. Thus it may be considered that

(1) Objects are 'real' if we can see or touch them with our physical senses.
(2) Objects and other phenomena are 'real' if we can measure them by means of physical apparatus.
(3) The unobservable entities of theoretical physics (atoms, electrons, quanta, etc.) are 'real' because the theory concerning them accounts so well for objective physical appearances.
(4) Objects of the physical function of spatial identification are 'real'.
(5) Presentations are 'real' if not imagined but 'given in nature'.

Although these usages are all clear when the context is understood, the usage which I have chiefly in mind in this book is quite different, since it recognizes degrees of reality. Thus I would say that nothing actual or potential is to be characterized as absolutely 'real' in such a way that all other existences are 'unreal'. But there are stabilized levels to which (when we know them directly) we respond with the spontaneous and reasonable judgement that they have each a certain degree or kind of 'reality' in relation to the other levels. The most transcendent levels are the ones with the highest degree of reality—this term 'highest' expressing our instinctive judgement over such matters.

An impression of unreality is also caused by a *mixture* of states. Any presentation which weakly merges in another level (as thought does in the physical level) seems unreal. But it would again go far beyond the scope of this book to investigate all the circumstances upon which our impression of 'reality' depends.

The Non-Phenomenological 'As If'

It is obviously unreasonable to demand that we should be able to duplicate the work of nature and show how to produce a total experimental or astronomical effect from an infinity of substructural details, past, present, and possibly future, personal or impersonal, involving both us and other people. All that we should expect to be able to do is

to analyse occasions which are offered to us, in the exact form in which they appear, and subsequently, by noting regularities (stemming from the intelligible relations in the eternal design), to make 'constructions' whose outcome is likely to be more or less as expected.

It is very important, in the analysis, not to confuse one occasion with another. The observation of a natural scene, for instance, is not the same as the observation of cloud-chamber tracks or interference bands, and there is no evident requirement that our observation of the natural scene shall be analysable into separate effects of particles or waves or anything of the kind. Are we to suppose that there are light-waves in physical actuality crossing and re-crossing from every atom in the panorama around us to every other atom in it?—that these electromagnetic fields, infinity upon infinity, are actually superposed at each point, and that each quantum of energy goes its way independently of all the others? If indeed we set up a closed box with an arrangement of slits and gratings inside, in order to isolate particular wavelengths (as we suppose), we should bring about a wavelike effect according to the construction of the apparatus. But although we then see or photograph the wavelike effect, there is no evidence that wavelike formations progress through the container of space so as to enter into the intelligible structure of the experience of seeing a distant tree, and thus to 'explain' it. To speak, in the same breath, of 'intelligible structure' and 'progress in the container of space' is at once to expose the illogicality of one's position. All that we know is that certain effects are visible 'as if' (non-phenomenologically) certain mathematical constructions and calculations produced them.

If we ask how this can come about, a possible answer seems to be that the mathematical constructions and calculations have a certain correspondence, probably very remote and oversimplified, with the general constitution of the eternal design, so far as physical manifestation is concerned.

SUMMARY

CHAPTER 19

The theory of *simple location* in time is a gross simplification which pictures events in an absolute line of succession, leaving nothing to be said about them except what concerns their measures and our psychological impressions at each instant or of duration, memory, etc. There is then no internal relation between one event and another, and consequently no internal justification of the order of nature. Phenomenology, on the other hand, explores every kind of awareness concerning time, conceptual or psychological. It uncovers varieties of time-awareness and control, and thus features of intelligible constitution, which the theory of simple location entirely ignores.

The power of *synthesis of the extended* enables a 'horizontal' time-spread to be comprehended as a whole, time-durations and tempi being realized as conceptual. There are then two kinds of correspondence with physical events, namely, an *identity of intelligible content* (capable of being explored backwards and forwards in its 'eternal design') and a *physiological correlation* with what is happening physically at the instant of any particular contemplation.

Primordial Recollection is the power of returning to the contemplation of the identity of intelligible content of some physical event or series of events, that is to say, a viewing of it not as a new image but as the original, found again in essence. By virtue of this power an *analysis of the simultaneous* is possible, without losing the details that were contemplated 'previously' (in the physiological correlation). 'Automatic observation of the present' is a brief reflection, primordial in type and extending over a short period of physical time (the 'specious present'), but occurring as a psychological reaction and not as controlled phenomenological analysis.

A diagram is provided, representing the relationship between these and other varieties of time-awareness and control, such as *protention* (a kind of future determination in the eternal design), various kinds of memory, and precognition.

Physical time-measurement, in respect of which all people can come to approximate agreement, must have a transcendental origin in ideal time-congruence, intimately bound up with ideal space-congruence. A potentiality in the constitution of nature, mathematically represented as pure sinusoidal wave-motions, may be presumed to actualize itself corporately in such a way that the primitive congruences are 'anchored' in very nearly rigid bodies and very nearly regular time-cycles.

CHAPTER 20

There is a certain way of knowing something of the inner constitution of nature and life which ought not to be called 'direct' but is also not entirely 'indirect', i.e. not merely plausibly imagined. The difficulty is to sort out

what is given, what is representatively devised, and what is a mere 'saving of appearances'. The light thrown by Quantum Theory on the fine-structural constitution of nature is called 'indirect' in this chapter, although our object is to discover, by analysis of the totality of results and effective representations, the general character of what is *given*.

Four approaches to Quantum Theory are found today: (1) *positivist*, or *pragmatic*, (2) *model-motivated*, (3) *simply-locational*, and (4) *analytic*, or strictly phenomenological in regard to both formalism and givenness in nature.

The 'subjective view' takes symbols ψ as constructed by rule in accordance with observations made, so that further observations may require a change of ψ. The alternative view is that experimentation on space-ensembles or time-ensembles, carried far enough, would determine, for the problem in question, a form of ψ which conveniently represents the substructure given in nature. It is this latter ψ which obeys the wave-equation.

Three outstanding problems are (A) precisely where and how *emission and absorption* occur; (B) *causality*—if the formalism corresponds exactly with everything given in nature, the hiatus between formalism and observation would imply contravention of Helmholtz's Principle, and a 'dice-playing God'; (C) the *individualization of particles*, either as wave-packets or as exchanges of energy. Can these be phenomenologically localized?

Quantum Theory has nothing to say on the first problem, since emission and absorption are treated statistically. On the question of causality, the answer seems to be simply that the formalism of Quantum Theory does *not* correspond *exactly* with *everything* given in nature.

The 'localized particle hypothesis' runs into insoluble contradictions when applied to the interpretation of interference experiments. Nevertheless other experiments show that 'bound' particles are sometimes phenomenologically represented to us as localized; or a represented localization of particles in transit may be convenient in some experiments and not obviously inadmissible.

The formalism of Quantum Theory makes allowance for evident limitations in physical experimentation with small-scale phenomena. The necessary use of wave structured radiation in measurement of such phenomena introduces an inevitable 'uncertainty' remarkably well represented in the formalism; the link up with classical measurements of energy and momentum (so that uncertainty attaches to these also) is not phenomenologically established, but is excellently confirmed by experiment. The formal mathematical wave-fields in terms of which the various correspondences with experiment appear are wholly unrepresentable as actualities on a single space-time map.

The formalism of elementary Quantum Mechanics, despite its large range of agreement with experiment, is open to both logical and experimental objections. Field Theory provides a formalism free of the most serious of these objections, but in its turn is open to other objections. Viewing quite generally these attempts to describe symbolically and formalize the results of small-scale experimentation, we seem to be impelled to the 'Essential Copenhagen Interpretation'. Classical concepts of position, time, energy, momentum, etc., are to be replaced by potentialities unrepresentable in a one-level space and time, and observational results are the manifestation

upon occasion of the whole indivisible background potentiality (transcending classical spatio-temporal concepts) according to the total conditions of the experiment.

CHAPTER 21

By the 'usual physical barriers' is meant fixation on 'enclosure' in the container of mind, with simple location of objects outside. This fixation is dissolved by *recollective release*, which discloses all givenness as primordially conceptual and open to phenomenological analysis. Kinds of 'transcendental experience' already dealt with (e.g. comprehension of Gestalten) are listed. The theory that *empathy* is a mere inference from appearances, followed by transference of our feelings to another location, is rejected; my feelings, and those of another person, are considered to be essentially prior (in the 'order of reality') to their localization in physical space, and essentially distinguishable one from the other without such localization.

The intelligible structure of the Gestalt is said to be 'transcendent in' ordinary physical perception because it is reached by some degree of recollective release from 'simple location'. Nevertheless, the physical appearances *correspond* to the intelligible structure *immanent* in them and so 'explaining' them. Simple immanence of this kind is to be distinguished from *creative influence*, as of the unfulfilled portion of the eternal design on the actualization of the whole now taking place (by *protention*).

The Theory of Actualization in Chapter 10 is further extended so as to include varieties of transcendence of individual physical awareness (simply located) and 'intersubjective linkages' of various kinds to which we are led by phenomenological analysis. Two degrees of apodeictic presentation of transcendental constitution are distinguished: *mystical* and *parapsychical*. An unrecognized or barely recognized degree of transcendence is called *virtually physical*.

It is possible to remain in stabilized (and autonomous) 'levels of creativity', of which the most familiar is dream. Mystical and parapsychical levels can also be stabilized. Comprehension of the transcendental constitution of nature does not require that we should be able to analyse fully, down to its supposed atomic or cosmical influences, some particular physical experience. The comprehension for which we can reasonably seek is of the general transcendental constitution at the higher stabilized levels and its creative nfluence on the physical level.

Arbitrary restrictions of the term 'real' to observed or unobservable physical entities makes for much confusion. Every stabilized level has a 'givenness' which will impress us as 'real' in its own way. The impression of 'unreality' of an experience is conveyed to us by many factors: the 'lowness' of the level in the hierarchy of creative influence or the 'order of reality', subjective lack of control and detachment, mergence (as of thought) in a stronger state.

PART IV

HISTORICAL CRITIQUE: THE RISE AND FALL OF SCIENTIFIC DUALISM

(Non-Mental Causal Substructure, plus Subjective Mind)

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CHAPTER 22

PRELIMINARY DIAGNOSIS AND TERMINOLOGY

'It required the crude and uncompromising virility of the *buccinatores novi temporis* to take the new science out of the worn framework of the traditional metaphysics which they inherited with it. If their next action was to try to reverse the position and to place metaphysics in a scientific framework, if they thought it possible to carry the methods and conclusions of triumphant science into subjects where they could bring only a new confusion based on another misunderstanding, doubtless that was the price that the dying remnant they had vanquished taught them to exact.'

A. C. Crombie, *Robert Grossteste and the Origins of Experimental Science*, 1100–1700, Oxford 1953, p. 3

'In spite of the central role played by this concept [of potentiality and becoming] in the thought of Plato, as indeed in the whole history of western philosophy up to the time of Descartes, and in spite of the fact that it thoroughly permeates all intelligible speech, potency is generally slurred over by present-day schools as an "Aristotelian" technicality. Change (i.e. the empirical world) then becomes unintelligible . . . This neglect of change, and its hierarchical or causal nature, makes it impossible to recognize that world-structure of becoming within which nature (φύσις) and man (νόμος) *both* exist at different ontological levels.'

John Wild, *Husserl's Critique of Psychologism* (from 'Philosophical Essays in Memory of Edmund Husserl', ed. Marvin Farber, Harvard U.P., 1940, p. 39

Duality and Dualism

Life is full of dualities. To every quality there is an opposite or complement—evil the opposite of good, objective materiality the opposite of mental life, effect the complement of cause, actuality the complement of potentiality. A *dualism*, on the other hand, arises when the two terms of a duality are given a special interpretation in which they assume the character of mutually exclusive principles or categories of existence, at work conjointly in the foundations of human experience and thought.

In this chapter we are concerned with two closely related dualities, namely, those of *cause* and *effect*, and of *object* and *subject*. A special philosophical view tends to arise on these topics when we reflect on the fact that colours, visible or tangible shapes, and all other phenomenal

parts of our experience of physical objects have the character of being subjective—peculiar to the person observing them. What remains when this phenomenal part is abstracted? A radical phenomenalist will say that nothing remains. Others will feel that there must be something to which natural law applies, something which has a continuing objectivity for various observers and passes therefore beyond what is subjective and private.

Scientific Dualism

In that case a simple solution of the problem is to say that what remains is an *objective substance* called 'matter', functioning as *cause*, while colours, shapes, smells, etc., of objects which we mentally perceive are *subjective attributes* associated with *effects* of matter. The subjective attributes or effects, being variable from person to person and liable to illusions of sense and misinterpretation, are regarded as a somewhat uncertain superstructure on the solid basis of 'matter' and its laws.

Possibly everyone, if questioned, would agree that this general view of the world is the indispensable background to ordinary commonsense management of human affairs. If, however, it is believed to be an accurate and exhaustive reduction of life and all that it means to the operation of two such mutually exclusive principles, then we have Scientific Dualism in its most primitive form. I shall describe it as 'simple matter-mind dualism'.

More carefully worded, this becomes Scientific Realism—probably the commonest view among scientists since the time of Descartes and Newton. We are scientific realists if we believe that the basic objects and motions spoken of in physical theories (particles, fields, vortices, spins, etc.), allowing for possible minor obscurities, have a 'real physical existence', very much as we imagine them to be; and moreover, they form a *non-mental substratum* which acts as cause of everything which happens or of which we can be aware, at least so far as the realm of 'nature' is concerned.

I take the term 'Scientific Realism' as implying no particular view concerning mental activities or their relationship to the substratum. Only the bare occurrence of a mental life of sensation, thought, intuition, feeling, will, etc., is implied, in some kind of correspondence with processes in the substratum. The essential feature is that the substratum is pictured as wholly outside the sphere of mental life of sensation, thought, etc., as the oceans and their movements are 'outside' the land-masses and affect them only by external contact.

Scientific realists therefore almost inevitably hold that, while the 'real physical entities' or structures *exist* in a substratum, they are not ostensively knowable, and their existence and nature have to be inferred in some way. This is epistemological **Agnosticism.**

The view that the reality of physical entities is guaranteed for us by the coherence and general verifiability of the theories concerning them is sometimes called **Physicalism.**

A more cautious view is merely that we are justified in proceeding as if the entities spoken of in physics have a real existence, so long as they provide the 'best explanation of the world'. This view can be described as epistemological **Pragmatism.**

The term **Materialism** seems usually to signify a version of Scientific Realism in which the 'real physical existences' are pictured as everywhere actualized in a 'container' of space, and these completely non-mental spatio-temporal existences are taken as deterministically causing everything mental.

The **Dialectical Materialism** of Marx and Engels criticized earlier materialistic views as 'mechanistic'. The term 'dialectical' is intended to suggest development by historical and social conflict. Social practice alone provides the test of 'reality' or 'truth', which in this way becomes knowable but incompletely so. Here, then, we seem to have a kind of sociological Physicalism, the 'entities' guaranteed being 'social-economic formations', 'natural resources', 'things for us', 'human nature', and the like.

In psychology, **Behaviourism** is similarly characterized by an 'objectivist method'. But in this case a continuing and recognizable 'reality' of objects is identified with a set of externalized observations by the subject. Attention being thus focused on presumed 'objective phenomena'—namely, *stimuli* and *responses*—the problem of the external world and the nature of physical law does not come into view. In the behaviourist's program we can detect something very like the Abacus Fallacy. The 'counters' of supposed external and continuing objects are manipulated in observational techniques and language without realizing that the manipulations, and the concepts from which they proceed, stem from a metaphysics of subject-object duality and a container-view of the external world. From this quasi-monistic, but latently-dualistic, standpoint, **Neo-Behaviourism** leans in the direction of Physicalism. A provisional guarantee is sought for 'motivation' theories, 'intervening variables', etc., assuming the priority of the object-manipulations of classical physics.

Phenomenalism

There are some fairly obvious objections to Scientific Realism and its related dualistic beliefs. If what is 'behind the scenes' is unknowable, how could the names allotted by us to entities supposedly existing there —'matter', 'particles', etc.—be other than mere words, unintelligible except as counters in a language-game? Does it mean anything clear and definite to say that there is something 'outside' all mental activities—

that something is 'given' in a manner which is, by its very character and constitution, *absolutely inexpressible* in terms of mental activities of any kind?

How can the existence of entities in an unknowable background be logically guaranteed by non-background observations? Is not the claim to possess such a guarantee a category mistake, as would be the claim that physical life logically guarantees a life after death? Is the claim any better if, instead of a guarantee, we assert merely the justifiability of a probable inference?

On the other hand, what does it mean to say that the reality 'behind the scenes' is 'incompletely' knowable? If we cannot know everything that is necessary to account for the relations between manifestations at one time and another and between one person or nation and another, then part of what is necessary remains unknowable and subject to the objections just made.

In answer to these objections, Phenomenalism asserts that there are not in fact *two* levels of nature, the visible and the invisible, but one. 'Physical objects', 'particles', etc., are defined in terms of physical observations, with the help of 'logical constructions' and 'conditional statements'. Any physical object is the 'permanent possibility' of observations of a stated kind being made *if* a normal observer were correspondingly circumstanced, the correspondence being according to certain formulae and rules of procedure as expressed in theoretical physics. Atomic structures, wave-functions, etc., are similarly explained as 'constructions' we have made for the purpose of unifying observational results. By such linguistic or symbolical devices, it is claimed, Phenomenalism reduces physical objects and entities of all kinds to the level of sensation (actual or possible).

A very plausible statement of this linguistic view occurs in Professor A. J. Ayer's early book, *Language, Truth and Logic*:

'Logical analysis shows that what makes these "appearances" the "appearances of" the same thing is not their relationship to an entity other than themselves, but their relationship to one another. The metaphysician fails to see this because he is misled by a superficial grammatical feature of his language namely, the fact that adjectives qualify nouns, and nouns so qualified stand for things.'[1]

But Phenomenalism in its turn is open to serious objections. The 'permanent possibility' *in nature*, needed to account for the similar appearances of an object at different times and to different people is not the same as the *verbal phrase* 'permanent possibility' or any equivalent symbolical expression. Any verbal phrase or formula is indeed a set of

[1] London 1936, p. 33.

here-and-now physical sensations. But the 'permanent possibility' *in nature* is something quite different. In fact it gets its peculiar character precisely because it is *not* reducible to particular sensations, but involves a coherence transcending particular observations in physical space and time. To suppose that the problem of physical objects is solved by shelving analysis of this transcending potentiality and devoting oneself to verbal forms seems a clear instance of the Abacus Fallacy.

Psychologism shows also in Ayer's explanation just quoted. To remove the ambiguity in the word 'makes' and elsewhere, he should have expressed himself somewhat as follows: 'Logical analysis shows that the way in which *we usually come to recognize* "appearances" as "appearances of" the same thing is by observing their resemblances to one another'. We then have to ask, 'What is the origin, in nature, of these resemblances?' Obviously they do not exist in nature merely because we recognize them on various occasions. Thus the dualistic difficulty is not disposed of, but is merely covered up by the ambiguous language.

Even if we provisionally admit that certain verbal forms are an adequate substitute for what is given in nature, there is the objection that any verbal definition of *physical* objects in terms of observation by *physical* instruments and persons in a normal *physical* state involves a vicious circle. Hence, in his later writings, Ayer concludes that 'the phenomenalist's programme cannot be carried through'.[1]

Hierarchism: Scientific, Phenomenological, Real

In the face of this impasse it behoves us to go back and examine the presuppositions of Scientific Dualism. In particular we may question whether

(1) the activities of 'mind' are in fact *privately* subjective, as in a water-tight enclosure, with other things 'outside';
and
(2) the 'objectivity' of those things 'outside' can be known only by inference from particular here-and-now sensations.

These are the presuppositions which seem to have led to absurdities. Let us therefore drop them, and admit, on the contrary, that 'objectivity' (actual or potential) may be ostensively knowable by certain mental powers distinct from ordinary here-and-now sensation. Then it is clear that a view of the external world results, quite different from that of Scientific Realism (with its non-mental substratum) or of Phenomenalism (with its 'home-made' logical constructions).

Discarding scientific dualisms does not in any way invalidate the

[1] 'Perception.' *British Philosophy in the Mid-Century*, ed. C. A. Mace, London 1957, p. 235.

accepted formalisms of modern physics, or the results of researches in biology and psychology. But it may do away with any supposed irreconcilability of the various fields of scientific endeavour with one another. Clarification of concepts in one field may then assist clarification of the concepts in quite other fields, and thus further stimulate research.

Gestalt Psychology, and Quantum Physics, for instance, alike point to a Theory of Actualization such as is outlined in Chapter 10. For universality of application the analysis must be taken far enough to reveal something of the mutual operation of levels of intelligibility, sense and imagination, and the internal structural relations of 'causality' by virtue of which one manifestation is analytically 'within' another and is thus felt to 'explain' it.

The philosophical view that results then is not a dualism, with causation 'upward' from a non-mental substratum to a mental superstructure, but an elaborate 'downward' *hierarchism*, concerning which, however, we can still adopt various standpoints.

First, we may accept a 'scientific' hierarchism, holding that (1) it is supported by the evidence of the experimental sciences, interpreted in the light of 'pure' commonsense, and (2) it constitutes an organized rational system, able to correlate and explain satisfactorily the findings in diverse spheres of human experience.

Second, we may accept a 'phenomenological' hierarchism, holding that phenomenological analysis (in its ordinary grades) directly discloses that a hierarchy of potentialities, passing on occasion into actuality, is implicit in the essential constitution of ordinary experience.

Third, we may hold that, besides this *epistemological* structure of experience, a deeper phenomenological analysis discloses an *ontological* structure of potentialities, which may also pass into actuality for us if the power of the analysis can take us into the corresponding transcendental level of *life*. We would then accept a 'real' hierarchism.

On account of the novelty of these ideas in a world still dominated by dualistic assumptions, it seems advisable to offer some further remarks on potentiality and phenomenological analysis (partly going over ground already covered in previous chapters) before turning to the historical survey.

Potentiality Characterized

In Hierarchism there is a twofold distinction between 'modes of creativity' making up 'life'—*not* a dualism, however, because the difference depends on viewpoint, which varies as the attention shades preponderatingly from one mode to the other, and converts details of the one mode into details of the other, without altering their intelligible character.

This crucially important distinction, oversimplified in the everyday subject-object duality and likewise in Scientific Dualism, can be set out in suggestive language as follows (noting that this is an attempt to convey varieties of *direct experience* which are outside the ordinary expressive uses of language):

(1) The Individual and Actual Mode is awareness in an *act of recognition* by an individual person. It can be either an awareness of particular *existences*, with perceptual details which are 'intelligible' through being constituted of timeless *essences* (i.e. recognizable 'ideas' or 'universals' of thought and cognition). Or it can be awareness of essences simply, either in thought or clearly and precisely at a transcendental level. Our awareness may be of innumerable kinds and degrees between the extremes of (*a*) unreflecting and confused subjective imaginations, and (*b*) totally detached and passive apprehension of what is *given*, clearly and precisely manifesting this in itself by participation.

The individual and actual life of direct awareness is subject to another important duality, namely, that of the *physical* and the *non-physical*, between which there may be various complex kinds of correspondence as well as varieties of mergence one in the other.

(2) The Common and Potential Mode constitutes the entire intelligible universe of potential existences, in some region of which the individual is circumstanced so that certain existences are actualized for him according to the occasion. Apart from what is actualized on any occasion, this universe has the character that it can rightly be considered to be *continuously* and *unconsciously* at work in each individual according as the potential existences concerned are (still speaking suggestively) closely accessible or distant, strongly operative or relatively inoperative. It is also to be considered as organized in various levels, according to a universal Reason, one level causally 'within' or 'above' another, and all directly knowable to those who are suitably situated (with respect to the 'intelligible universe') and have the necessary powers of discernment.

There are some apparent paradoxes in this attempted description of potentiality. How can we logically state that something is 'unconscious' in us and yet 'knowable'? On what ground can we say that there was something 'out of range' for us, which is *the same* as something which is now 'near'? How can what is 'entire' be knowable otherwise than by the discursive attention to various aspects of it? In which case, what does it mean to say that these aspects successively observed make a 'universe'? Possibly it is such difficulties which have led empiricists imbued with Descartes's ideal of 'clear and distinct' perception to reject or try to do without the concept of potentiality. But such a policy does not solve the problem of the communal nature-given continuity of objects, which is recognized by everyone.

By facing up to the paradoxes, we see that they all result from a limited, 'fixated' or irresistibly 'flowing', concept of time, in terms of which the past is irretrievably *gone* and observations at different times cannot be directly compared. The paradoxes fall away when we think in terms of a *released* or *transcendental* level of time, at which durations are *ideal qualities of coexistence* in a certain developmental order. At this level the 'contents' of time constitute a developing *pattern* rather than a *stream*; and a pattern can be studied all at once (from another level) or backwards and forwards, without its character altering.[1]

A certain familiar type of experience provides a simple illustration. We may be reading a book or conversing with other people, and then, suddenly realizing that a clock has struck, we find we can (in a moment of time) count the number of strokes. The strokes do not then occur for us in quick succession—six or seven in a quarter of a second, whereas in physical time they were each two seconds from the next; but the impression of two seconds between the strokes remains, even though we now perceive the whole series in a quarter of a second. Moreover, when we hear those six or seven strokes, the impression is exactly as if we were hearing them for the first time.

Transcendence

This power of passing 'out of time' (i.e. physical 'fixated' time) or backwards and forwards in time, and realizing the various perceptible parts as parts of a whole, extended in time or simultaneous, is exercised in a more controlled way by experienced musicians when they recall in a moment the durational-structure of melodies lasting for perhaps as much as ten seconds in physical time. A similar kind of *non-flowing* contemplation is exercised by all of us in comprehending the meaning and grammatical construction of a long sentence heard spoken to us by another person, especially if the verb comes at the very end.

The grasp of the total configuration of a physical object in three dimensions (a cube, for example), after a short visual or tactual exploration of it, requires a similar transcendence of time. Obviously the configuration as a whole is different from the separate spatio-temporal details discerned, and yet it comprehends them in a certain intelligible way.

Husserl[2] describes such transcendence of particular 'acts' at particular times as an 'insight into "real experience" ', which

> operates as a reflexion, and it has this remarkable peculiarity that that which is thus apprehended through perception is, in principle, characterized as something which . . . already was before this gaze was directed to it.

[1] A fuller discussion of time-transcendence appears in Chapter 19.
[2] See below, p. 407.

Thus the transcendental view shows us 'real experience'. The fixated and flowing time-presentation gives us indeed a 'full-blooded' presentation of objects; but it is one which is thereby almost deprived of its intelligible quality and made to seem mindless. In order to comprehend our physical experience we are obliged, though perhaps only momentarily, to resort to the transcendental level, i.e. to the 'intelligible universe' of potentialities.

Intelligible details of potentiality, like those of actuality, are discerned by an 'act' of recognition. But *this* 'act', if properly 'seen' in release, has the character of being universal, not limited to one person, or otherwise transcendental. The conclusion remains that there are two *modes* of creativity (types of experience, or however else we wish to name them), known in different ways, and thus two kinds of recognizing 'act'.

Previous chapters of this book have included examples of potentiality directly discerned in various circumstances; for instance, the discovery of *perceptual details* in a *total encounter*, only one such detail being focal at any moment;[1] the 'supraperspectival analysis' which discerns the implication of various non-actual perspectives in the one actual perspective of an object;[2] and the participation by the 'function of spatial identification'[3] or 'empathy'[4] in objects and people who, at the fixated level of space, are seemingly apart from us.

Consideration of the manner in which powers of phenomenological analysis are developed, so as to disclose the ontological constitution of potentialities in the 'intelligible universe' at the higher levels is omitted here, since the subject has been studied at some length in Chapter 8.

[1] Above, pp. 48 ff. [2] Pp. 131 ff. [3] Pp. 135 ff.
[4] Pp. 215 ff., 344 ff.

CHAPTER 23

FROM GALILEO TO KANT

'The cards are stacked by nature to produce dualistic doctrines.'

Charles Hartshorne, 'The Social Structure of Immediacy.'
(from *Philosophical Essays in Memory of Edmund Husserl*, ed. Marvin Farber, Harvard U.P., 1940, p. 227)

In western civilization the years AD 1600–1900 mark off what may perhaps be called the Age of Scientific Dualism. There took root then for the first time in the history of the world, it seems, a certain oversimplified world-view, which we can now appreciate was necessary for the development of the natural sciences, but which was fatal for any truly integral view of human life and experience. Hence arose numerous pseudo-problems, the study of which has no doubt helped in the end to clear the air.

Scientific Dualism, in some form or other, is generally assumed to be the orthodox view of scientists. I do not intend to suggest that it is now in fact generally rejected, because its foundations are seen to be overthrown by recent advances. My suggestion is, rather, that a readiness to recognize the severe limitations of this oversimplified world-view and to accept instead the concepts of *universal mind* and *hierarchic causation* is rapidly permeating the educated world, largely as the result of progress in the sciences themselves. I believe also that a growing body of influential scientists, who have thoroughly considered the foundations of their work and thinking, would agree that a reasonable attitude to the physical, biological and other natural sciences does in fact require a drastic overhaul of the world-view long accepted, almost without question, in these sciences.

The ancient religious philosophies founded on mystical experience—of the Upanishads, Buddhism, Plato, Neo-Platonism, and Christianity up to AD 1300—uniformly taught the knowability of an immense background structure, a subconscious potentiality, a hierarchy of creative powers, a divine Law and Reason (*logos*, *ṛta*), coming to manifestation on the physical or some other level according to the

mind of the beholder, who thus participates in those creative acts. Being a participator, the beholder was also a contributor, both at the outermost level and within, the human spirit being like a focus of powers in the ocean of the universal Spirit. All was thus the outpouring of a Spirit, neither matter nor mind in the limited modern sense of the terms, and deriving from a transcendent well-spring of Unity and Power.

By the 'trick' (*māyā*) of creation, the individual knower, patterned on the transcendent I AM, might be caused to *project* and *enclose*, making the real appearance of a 'world' and a 'descent' of the individual into 'matter'. Thus came about a deceptive dualism of object and subject, matter and thinking. Release was by denying *grasping* and *fixation*, passing beyond the sphere of thinking, and thus obtaining the freedom of knowledge of the Spirit at all levels.

Galileo: Truth in the Measured World

Associated with popular forms of this comprehensive mystical philosophy there grew up unfortunately, in the Dark and Middle Ages, the tendency to regard the investigation of the processes of nature by material means as an indulgence in illicit occult practices—an opening of the door to belief in powers of matter independent of spirit. To save the priority of spirit, authorities in religion tended to rely on experimentally untestable dogmas, thus blocking progress. Mere conservatism in thinking, a common phenomenon in all ages, also fought to preserve the traditional ideas about the world. A final reason for the long delay in the development of the exact sciences was the almost unbelievably primitive state of mathematics in Europe before the Renaissance, even in the simple skills of arithmetic.

By about AD 1600 the inevitable reaction against the traditional ways had reached its height, in GALILEO (1564–1642) and his contemporaries. In the scientific method of Galileo we see the clear lines of a new dualism, or rather a *trinalism*—'inner life', objective perceptions, causal substructure—with the chief division between the 'inner life' and what is 'external'. On the one hand there are all those subjective influences which the scientist rightly disregards or removes. On the other hand there is the scientist's 'onlooker mind', untouched by subjective distortions or preconceived ideas. This is concerned with measures and natural laws for occurrences in the 'container' of space, with systematizations and the testing of theories, and is ideally conceived of as recording accurately the attributes and behaviour of nature. It was only on such a ground, distinguishing the physical from the non-physical with the help of mathematics, that a securely based and progressive science of nature could arise.

Descartes: Extension and Thought

If, however, the chief division is made between sensation on the one hand and the causal substructure on the other hand, as was the intention of Galileo's distinction between *primary* qualities (of position, motion, etc.) and *secondary* qualities (of sense), we obtain a different dualism, which became the special concern of philosophers. Thus 'Galileo and the physicists', it has been said, 'peeled off' the phenomenal properties 'from material objects without caring much what became of them. Descartes salvaged the discarded peel'.[1] The work of DESCARTES (1598–1650) marks a turning point in the history of both science and philosophy: first, for its clear-cut emphasis on the concept of physical objectivity as consisting in mathematical or mechanical properties and laws, distinct from all appearances; secondly, for its thoroughgoing application of the empirical method—dependence on direct experience and analysis, as against untested dogma—both to the things of the outer world and to the 'inner world' of individual consciousness.

In Descartes's theory of perception, however, there seem to be two incompatible strands. The first, arising from his view of *res extensa* as a 'substance' apart from the mind, led to his Representative Theory of Perception, in which *organic impressions* in the brain stimulate an *innate idea* of the 'thing'. This almost inevitably suggests an *agnostic* scientific dualism, in which the thing itself remains unknown. But Descartes linked this view with the view that these innate ideas provide an adequate notion of objects 'directly implanted in the mind by God, and altogether independent of adventitious ideas of sense'.[2] This second strand led to the theories of MALEBRANCHE (1638–1715), for whom 'all things are seen in God', and of SPINOZA (1632–77), for whom extension and thought are parallel attributes of the one Substance. The other strand led to the psychologistic empiricism of JOHN LOCKE (1632–1704) and his successors.

Locke: The Mind and its Contents

The intention of the theory of 'clear and distinct ideas' existing 'innately in the mind' was to account for *necessity*, *universality*, and *objectivity* (the third term in the physical trinalism). Such 'ideas' were elements in the *order of reality*, apart from consideration of the *order of approach*.[3] They were implicit in human nature and creation, regardless of the time

[1] C. A. Mace, 'Some Trends in the Philosophy of Mind', *British Philosophy in the Mid-Century*, Ed. Mace, London 1957, p. 103.

[2] A. Boyce Gibson, *The Philosophy of Descartes*, London 1932, pp. 219, 229. Compare, 'All that is comprehended in the object of speculative geometry', I can be assured, through God, 'really exists external to me' (Med. VI).

[3] Boyce Gibson, *op. cit.*, pp. 251, 320 (see also the index).

and manner of our becoming critically aware of them. Locke, however, was impressed with the vagueness and inconsistency of the claim for 'innate ideas' and consequently for knowledge of the corresponding *order of reality*. 'If men have those innate truths originally, and before the use of reason, it is in effect to say, that men know, and know them not, at the same time' (Essay, 1.2.9).[1]

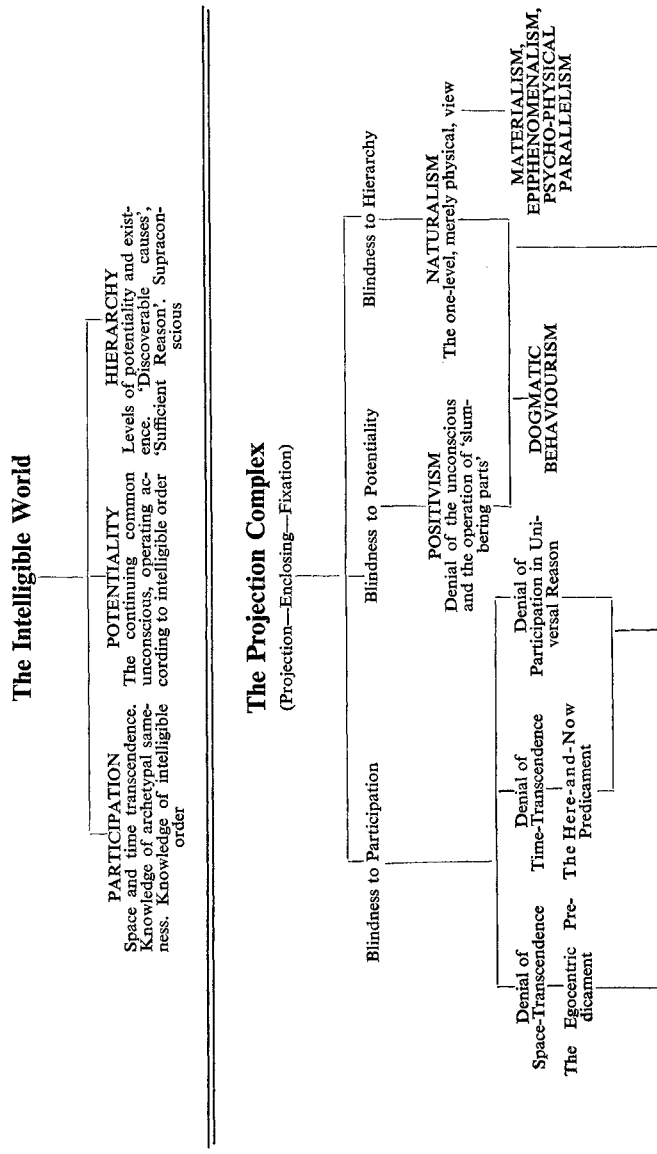
Having rejected the theory of innate ideas, Locke was obliged to find another way of explaining our knowledge of external things, as distinct from knowledge of their appearances. Corresponding to the two trends in the philosophy of Descartes there are two trends in Locke's theory of external objects. But one constitutes his chief 'platform', while the other creeps in almost inadvertently in various rather obscure expressions.

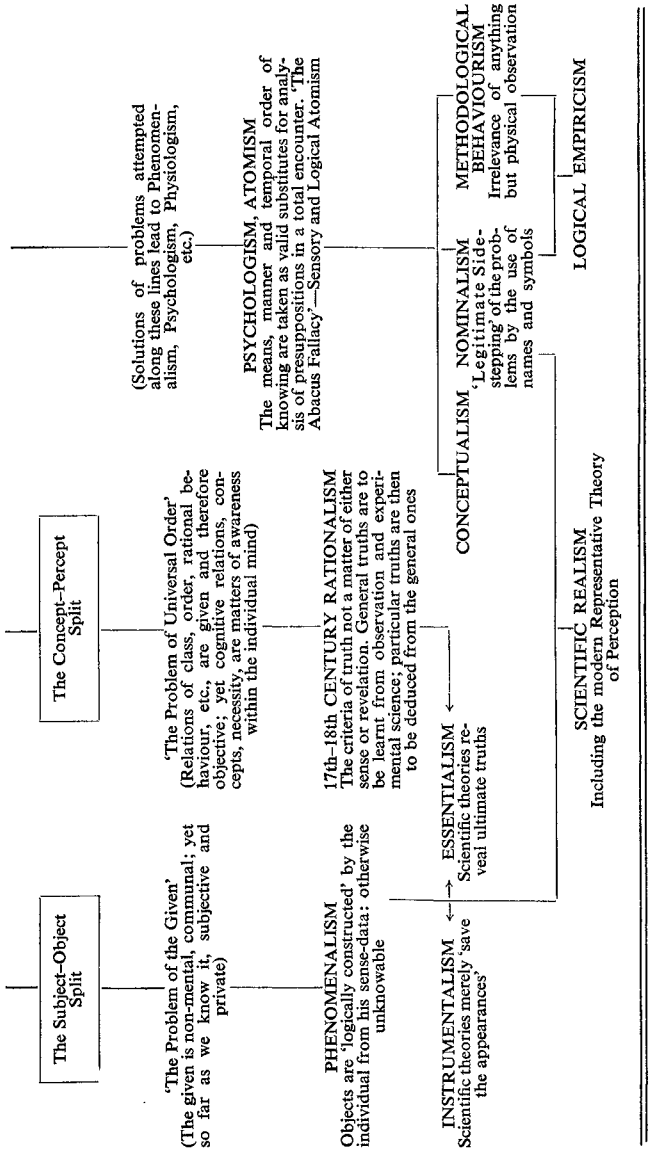
On the one hand he speaks as though the understanding consists of atomic items collected from sense experiences of the past: 'The understanding seems . . . not to have the least glimmering of any idea which it doth not receive from' sensation or reflection (2.1.5). Hence 'the mind knows not things immediately, but only by the intervention of the ideas it has of them' (4.4.3). From this trend of thought results the theory of the *unknowable substratum* of things. Indeed, it seemed certain (and perhaps still seems so to most people) that human nature cannot know 'directly' the corpuscular structure of nerve-currents, heat and other physical phenomena.[2]

On the other hand, according to Locke, in our knowledge of external things there is a 'blend' and 'conformity' which is in some way directly given, as in a *total encounter*: 'The qualities that affect our senses are, in themselves, so united and blended that there is no separation, no distance between them; yet it is plain that the ideas they produce in the mind enter by the senses simple and unmixed' (2.2.1). 'Simple ideas', he says, 'carry with them all the conformity with the reality of things which is intended or which our state requires.' More explicitly, this conformity appears to consist in a peculiar kind of *resemblance*. The ideas of primary qualities, 'extension, figure and motion', have 'patterns' which 'do really exist in the bodies themselves' (2.8.13).

[1] The way of escape from this dilemma, by appeal to an unconscious but knowable potentiality, was blocked by Locke's form of empiricism.

[2] The argument that we cannot know 'directly' the corpuscular structure of physical phenomena, and *therefore* the substructure of nature is unknowable, overlooks (1) that the 'corpuscular structure' has not been shown by physics or biology to be the essential substructure of our experience; it is merely an abstract form which we impute to certain physical effects which seem to *accompany* the experience; and (2) that the use of the term 'indirect' in regard to any means of knowledge presupposes the possibility of a 'direct' means of knowledge. For further discussion of this subject see Ch. 21, esp. pp. 354 ff.


The Intelligible World
PARTICIPATION
Space and time transcendence. Knowledge of archetypal sameness. Knowledge of 'intelligible order
POTENTIALITY
The continuing common unconscious, operating according to intelligible order
HIERARCHY
Levels of potentiality and existence. 'Discoverable causes', 'Sufficient Reason'. Supraconscious
The Projection Complex
(Projection—Enclosing—Fixation)
Blindness to Participation
Blindness to Potentiality
Blindness to Hierarchy
Denial of Space-Transcendence
The Egocentric Predicament
Denial of Time-Transcendence
The Here-and-Now Predicament
Denial of Participation in Universal Reason
POSITIVISM
Denial of the unconscious and the operation of 'slumbering parts'
NATURALISM
The one-level, merely physical, view
DOGMATIC BEHAVIOURISM
MATERIALISM, EPIPHENOMENALISM, PSYCHO-PHYSICAL PARALLELISM


The Subject–Object Split
The Concept–Percept Split
'The Problem of the Given'
(The given is non-mental, communal; yet so far as we know it, subjective and private)
'The Problem of Universal Order'
(Relations of class, order, rational behaviour, etc., are given and therefore objective; yet cognitive relations, concepts, necessity, are matters of awareness within the individual mind)
(Solutions of problems attempted along these lines lead to Phenomenalism, Psychologism, Physiologism, etc.)
PHENOMENALISM
Objects are 'logically constructed' by the individual from his sense-data: otherwise unknowable
17th–18th CENTURY RATIONALISM
The criteria of truth not a matter of either sense or revelation. General truths are to be learnt from observation and experimental science; particular truths are then to be deduced from the general ones
PSYCHOLOGISM, ATOMISM
The means, manner and temporal order of knowing are taken as valid substitutes for analysis of presuppositions in a total encounter. 'The Abacus Fallacy'—Sensory and Logical Atomism
INSTRUMENTALISM
Scientific theories merely 'save the appearances'
ESSENTIALISM
Scientific theories reveal ultimate truths
CONCEPTUALISM
NOMINALISM
'Legitimate Sidestepping' of the problems by the use of names and symbols
METHODOLOGICAL BEHAVIOURISM
Irrelevance of anything but physical observation
SCIENTIFIC REALISM
Including the modern Representative Theory of Perception
LOGICAL EMPIRICISM

'Growing Pains': A Depth-Diagnosis

If the Renaissance 'ushered in the birthday of the modern world', as Macneile Dixon has said,[1] then the coming of age may perhaps be placed near the end of the seventeenth century. By that time the rapid development of mathematics and experimental science after Galileo had led to the epoch-making work of NEWTON (1642–1727) in bringing 'celestial' phenomena under the rule of the same laws as terrestrial ones, to the experimental researches of BOYLE (1627–1691) in physics and chemistry and of HOOKE (1635–1703) in elasticity and theory of heat, and to the earlier developments of the wave-theory of light by HUYGENS (1629–1695). A new 'natural philosophy' of the universe, 'mechanical' and 'corpuscular', had arisen. This coming of age, however, brought with it a variety of 'growing pains', and the attempts to cure one kind of trouble led to other kinds, resulting here and there in what might almost be described as philosophical paralysis, long protracted.

Before glancing at the various conflicting views concerning the external world which came to the fore in the following 250 years we ought to make some attempt, like a good physician, to diagnose the root cause of the troubles. It is easy enough to say that nearly all these views were varieties of scientific dualism or attempts to substitute something less objectionable while retaining its chief presuppositions. What is wanted now, however, is a *depth-diagnosis*, penetrating to the inmost stronghold of dualistic belief.

What then is this 'inmost stronghold', or root cause of the troubles? Why are 'the cards . . . stacked by nature to produce dualistic doctrines'? The cause, I suggest, is that, in the endeavour to form a serviceable world-view as early in our lives as possible, we accept a certain persuasive entanglement of superficial impressions and half-truths, which we might say is offered by nature precisely because of its serviceability in ordinary life.

I propose to call this 'entanglement' the Projection Complex. The inserted chart shows its chief characteristics and also the lines of descent by which, I suggest, it finds its outcome in various well-recognized tendencies in philosophical or scientific thinking, plausibly supporting each other so that nothing can be put right piecemeal. By this means I hope the nature of these tendencies and their exemplification in post-Renaissance philosophies particularly may become clearer.

In the view I am presenting, the Projection Complex itself stems from ignorance of the 'intelligible world' as a hierarchially active reality in which we participate, this ignorance expressing itself in the tendency to *project*, *enclose* and *grasp* objects, i.e. totalities of manifestation in which sensations and spatio-temporal impressions, as well as details

[1] *The Human Situation*, London 1939, p. 33.

of substructure, can be clearly or faintly recognized. By such 'grasping'[1] the objects lose their conceptual character. Conceptuality then seems to reside solely in particular acts of the subjective mind. Mind and matter fall into their neatly-cut spheres of operation. And immediately, for philosophers, a number of insoluble problems arise.

There is the problem of *other people* (who are 'external' but possess a conceptual life as we do). There is the problem of the *past and future* (which we know only as present images). There is the problem of the *objectively given* (which the individual mind apprehends as private and subjective, yet is communal and supposedly non-mental). And there is the problem of *universal order* (which is given in nature yet conceptual in character).

Mostly the solutions offered consist in attempts to replace the *givenness of participation* (which is not admitted) by devices of the individual mind—names, symbols, language uses, descriptions of psychological reactions or the progressive means of learning, inferences and deductions. The difficulties, it is implied, are thus 'legitimately side-stepped', so that further analysis of the problems is unnecessary.

It is fairly easy, I think, to see why the Projection Complex should have suddenly taken such deep hold of the spirit of the times and remained deep-rooted, to a large extent, ever since. The power of *primordial recollection*, by which we are 'released' from the onward urge of time and come to a sense of participation, without 'fixation', 'enclosing', and consequent reduction of nature to one non-mental level, is very shadowy at first. It dies away if too strong an interest is taken in either outward activities or theoretical studies.

Every sane person retains, involuntarily, a power of primordial recollection sufficient for some participation in the lives of other people, so that a real human companionship and communication of ideas is possible (not, of course, without suitable expressive means). But social necessities do not likewise compel the development of any primordial recollection in respect of *sensory* impressions, in a manner which would clearly reveal space and the objects around as 'structures of ideas' in which we directly participate. The presence and operation of objects is strongly felt, but the universality flowing in from above, and the sense of space-transcendence, are blocked off. This will be specially so if, as science wisely requires in its investigations, there is an insistence on 'clear and distinct ideas' of what is happening physically, and if at the same time there is antipathy to ancient teachings on

[1] In the Buddhist twelvefold causal chain, leading to 'corruption and (spiritual) death', this 'grasping' is *upadāna*. See above, p. 109. Also *The Mystical Life*, pp. 242, 189.

participation because of prevalent dogmatism and speculative extravagance or vagueness.

Moreover, there was evidently a real need, about the time of Galileo and Descartes, for a demonstration of the (surface) independence of 'natural philosophy' from metaphysics and theology, so that progress might not be hampered by the mistaken application of analogies and principles proper to quite another sphere of experience.

A crucial point of discussion among the early scientists and their critics was the status of 'truths of nature' and the manner in which such truths were to be discovered. Nearly all earlier metaphysical and theological speculation having been rejected as non-evidential and irrelevant, and mystical insight having been passed over either from ignorance or because of its assumed irrelevance, there remained only the use of plausible analogies, to suggest some general hypothesis, with or without quantitative confirmations in experiment.

The danger then arises, either that the plausible analogy and 'models' constructed accordingly are taken as the plain and indisputable description of essential truth (an error into which Galileo fell), or that the deduction, from these analogies, of certain merely qualitative explanations of physical phenomena is too hastily taken as adequate confirmation of them (this was the error of continental rationalism in its support of Descartes's purely descriptive vortex theory of physical phenomena, based on an *a priori* rejection of the possibility of a vacuum). Some of the rationalists then, throwing stones from their glass house, levelled against Newton's theory of gravitational attraction the charge that it postulated an 'occult entity'—though this was a theory which indeed rested on quantitative confirmation.[1]

Newton: Natural Law, and the Deity

Newton's expressed views on the philosophy of nature, apart from mathematics and the details of experimental observation, are not extensive, but nevertheless demand attention. They embody several tendencies of thought, not always clear or mutually compatible. Mathematics and experimental investigation were to be regarded largely as *instruments*, the propositions of a theory being taken as 'accurately or very nearly true . . . till such time as other phenomena occur, by which they may be made more accurate, or liable to exceptions'.[2] Nevertheless he thought that 'natural philosophy', when successively

[1] For instance, Fontenelle, Leibniz, and Euler in his earliest writings. See F. Copleston, *A History of Philosophy*, London, VI. p. 8; Mary B. Hesse, *Forces and Fields*, London 1961, pp. 162, 170, 190.

[2] *Principia Mathematica*, trs. Cajori, Univ. of California Press, 1934, p. 400.

refined, could lead to a genuine knowledge of causes, even of 'the very first cause', superior to what is 'mechanical'.[1] He obviously had no doubt of the essential truth of the Copernican system and the concepts of force and momentum which he presented. Even his description of space as the 'sensorium' of God indicates possibly a view that our intuitions of space have an absoluteness which makes them in some way intuitions of ultimate reality.[2]

Although Newton's work killed, once for all, Aristotle's theory that terrestrial and celestial bodies obeyed essentially different laws, and although he strongly opposed the admission of 'the occult qualities of the Aristotelians', he retained some relics of Aristotelian or medieval apriorism. The concept of a 'Centre of the Universe' conflicted with the 'Newtonian relativity' expressed in the laws of motion; and acceptance of the Principle of Continuity, that 'matter cannot act where it is not', led him to believe, with Descartes, in the impossibility of the vacuum, and thus in the existence of a pervading aether.

Whether the aether was for Newton 'material' or 'immaterial' remains possibly in some doubt, since no clear distinction between these terms appears. The aether is 'a most subtle spirit', which is however 'propagated along the nerves'.[3] Generally, it seems, the dominance of the new mechanical and corpuscular philosophy leads Newton to reify spiritual things.

The reification appears most forcibly in the container view of space, which, for Newton (incongruously with his view of it as the 'sensorium of God') is a certain framework, namely Euclidean inertial frames, which are in some sense in *continuing physical actualization.* The objects and 'solid, massy particles'[4] placed in it thus take on the character of formations in a substance quite other than mind or spirit.

Rationalism and Newtonian Mechanics

The eighteenth-century rationalists who were not also atheists or agnostics held that God was to be inferred from experimental phenomena and laws taken as true *a priori*, not instruments. The essential truth of these laws was not to be doubted. Such views were held by Newton's immediate associates and successors, in spite of the growing empirical spirit in England, the laws in question being for them Newton's laws of motion and gravitation. On the Continent, however, the vortex theory of physical phenomena as developed by Descartes on the *a priori* ground of the supposed impossibility of a void held sway

[1] *Opticks*, London 1931, p. 369 (Query 28).
[2] *Ibid.*, p. 403 (Query 31), p. 370 (Query 28).
[3] *Principia*, General Scholium, p. 547.
[4] *Opticks*, p. 400 (Query 31).

for another generation or two, subject only to slight improvements by Huygens and others.[1]

Considerable stir was created in 1738 when MAUPERTUIS (1698–1759) an enthusiastic French supporter of Newton, confirmed the flattening of the figure of the earth near the poles, contrary to the vortex theory. His famous Principle of Least Action, though resting on the confused dynamical ideas of the time,[2] is important, not only for its stimulus to Euler and Hamilton in the development of variation methods in mathematical physics, but also for the significance which Maupertuis rightly claimed for it in the philosophy of nature. It expresses the truth (lacking in a mere statement of Newton's laws) that integral and teleological determinations are required in problems of motion if a solution is to be possible. Differential equations taken to express laws of nature yield nothing without boundary conditions and integration; they are to be interpreted as the results of some more primitive condition implying Universal Cohesion and the changing of the totality of events because of specific boundary-determinations. Hence Maupertuis regarded his Principle as showing that the system of nature is the work of an all-wise Creator and the manifestation of a world-soul. Thus he was led to reject the sharp distinction between extension and thought and to place conceptuality in the universe and not in the individual only. But in speaking of matter as having some kind of 'intelligence' or 'perception' he seems to have cancelled out the mystical character of his other views in people's minds and given the impression of teaching a kind of materialism.[3]

DIDEROT (1713–84), chief editor of the famous Encyclopédie of 1751–80, was one of those who regarded Maupertuis as a materialist. He himself tried to combine a strong leaning to materialism with the rejection of mathematics in the study of nature, a positivistic phenomenology, and a Leibnizian panpsychism. The views of D'ALEMBERT (1717–85), editor of the Encyclopédie jointly with Diderot, and one of the leading mathematicians of the time, were altogether more consistent and authoritative. In his attitude to the natural sciences he anticipated the twentieth-century positivistic 'Unity of Science' movement. 'Metaphysics must either become the science of facts or remain the field of illusions.'[4]

The classical development of Newtonian mechanics was completed

[1] For a short account of the vortex theory, see Mary B. Hesse, *Forces and Fields*, *op. cit.*, pp. 102–12.

[2] A general proof was first obtained in 1760, by Lagrange (E. T. Whittaker, *Analytical Dynamics*, Cambridge 1927, p. 248). For a thorough discussion of the controversial charge that Leibniz had anticipated Maupertuis, see M. Gueroult, *Dynamique et Métaphysique Leibniziennes*, Paris 1934, pp. 215–35.

[3] Copleston, *op. cit.*, VI, pp. 17 ff.

[4] *Ibid.*, p. 46.

by the great figures of the French school of mathematical physicists, especially LAGRANGE (1736–1813) and LAPLACE (1749–1827), who overcame the last serious objections to Newtonian gravitational theory in researches dated 1773 and 1787.[1] Newton's laws were then accepted as absolute truths of nature, the study of nature was reduced to a matter of coordinates and equations of motion, with formulae reasonably synthesizing the measures of other observed effects. Philosophical questions did not arise. When asked about God, Laplace replied that he had 'no need for that hypothesis'.

Leibniz: The Universal Harmony

We must now return to consider some parallel developments, beginning with Newton's great contemporary, LEIBNIZ (1646–1716), 'father of German philosophy'.

In the profound and wide-ranging philosophy of Leibniz there are three strands to be discerned. First, but of little importance in our present survey, is his rationalistic physics, which Bertrand Russell describes as 'a mass of confusions',[2] in spite of some valuable insights. Along with this we may perhaps include his 'invention' of the calculus —more correctly speaking, he gave the first accurate analysis of what was involved in the 'theory of indivisibles' and invented the symbolism of derivatives and integrals which has enabled the subject to be rapidly synthesized and developed in all its branches and applications. The second strand of thinking is seen in his prolonged efforts to construct a logical calculus or ideal philosophical language. In this we may see a leaning to logical atomism as a practical device.

The dominant theme and core of Leibniz's mature philosophy, however, is his mystico-rational metaphysics, the fundamentals of which seem discerned by genuine insight while the working-out and presentation is mostly by *a priori* or analogical argument. His system comprises (as a complete mysticism must) the philosophy of nature and the philosophy of its background in one coherent scheme of creation. Here too, unfortunately, undue apriorism causes trouble, which should not blind us however to the brilliance of his insights and their special relevance to problems of twentieth-century philosophy of science.

Everywhere we find emphasis on the rational universality of the creation, and the error of trying to divide it into parts held together by external connections. 'There is in the Divine mind an intelligible world,

[1] E. T. Whittaker, *A History of the Theories of Aether and Electricity; The Modern Theories*, London 1953, pp. 144 ff.

[2] *A Critical Exposition of the Philosophy of Leibniz*, Cambridge 1900, p. 91; also pp. 85, 89.

which I am wont to call the region of ideas.'[1] There is also a 'perfect harmony between the physical realm of nature and the moral realm of grace', so that 'things lead to grace by the very ways of nature' (M.87, 88). 'The totality of all spirits must compose the City of God', while this physical world is the expression of a Platonic world of real potentialities, of 'a reality in essences or possibilities, or rather in eternal truths' (M.85, 44).

The problem of *individuals* is solved by Leibniz with the help of a terminology which is partly traditional in mysticism and partly his own. Individuals (Monads) are to be thought of primarily as like the spark of consciousness in the human soul, or individualizations of the great Archetype of all: 'God alone is the primary unity or original simple substance, of which all created or derivative Monads are products and have their birth, so to speak, through continual fulgurations of the Divinity . . . limited by the receptivity of the created being, of whose essence it is to have limits' (M.47). Consequently 'souls in general are living mirrors or images of the universe of created things', and rational minds are also 'images of the Deity . . . capable of knowing the system of the universe' (M.83). From the principles of universality and continuity, everything that can be properly called an individual must be similar, like a point of spiritual perception in potentiality, modelled on the supreme Monad, and expressing 'the whole universe in its own way' (R.263). The primitive created units are called 'bare monads' (which we might perhaps liken to the substructure of elementary particles, but superior to physical space and time) and from these there is the formation of 'semi-mental' units 'by aggregation' (R.248, 271), in which aggregates there is 'domination and subordination' according to 'the degrees of their perfections' (R.141, M.70).

There is a (supra-physical) 'connexion or adaptation of all created things to each and of each to all', so that each simple substance is 'a perpetual mirror of the universe' (M.56). Each Monad is thus a 'perspective' of the universe, according to its 'special point of view' (M.56). Its 'passing condition', reflecting the universe and (like a thought) representing 'a multiplicity in the unit', is called a Perception (M.14). 'This representation can be distinct only as regards a small part of things', but is affected by 'a great multitude of little perceptions' which do not come distinctly into consciousness' (M.60, 21).

Sense awareness is described as *confused* perception, because we 'cannot separately enumerate the marks which are sufficient for distinguishing the thing from others, although the thing really has

[1] *Epistola ad Hanschium*, E. 445b. Robert Latta, *The Monadology*, Oxford 1898, p. 241. In the following quotations numbers following L are page-numbers in Latta; following M, are paragraphs in the *Monadology*; following R, are pages in Russell's book (note 2, p. 381 above). See also L. 343.

such marks and essential elements, into which its notion may be analysed' (L.49). This is not 'intellectualizing appearances', as Kant complained. For Leibniz's analysis is of perceptual details revealing to us the intelligible character of an object, which exists on the ground of *materia prima*, the principle of individuation and objectivity (R.268).

In his theory of space and the external world, Leibniz taught emphatically that materiality and extension are appearances only, but 'well-founded' ones (L.101, R.272). Spatial relations can be spoken of only when and where bodies are manifested: 'There is no space where there is no matter' (L.104).[1] Space and time are 'orders or arrangements of co-existing and successive things or phenomena' (L.101, R.251). They are the 'form of phenomena', not in Kant's sense, as subjective promptings by an unknowable thing-in-itself, but are real appearances or specializations integral with the total 'appearance to God' (R.253). Nevertheless 'there are no divisions' in space (measurements?) 'but such as are made by the mind' (R.245).

Leibniz's view of science was distinctly more *instrumental* than Newton's (M.46). He accepted Boyle's analogy of scientific hypotheses to the key to a cipher, which is justified by its results.[2] But he insisted continually on the need for analysis of presuppositions and the application of certain criteriological principles. The Principle of Sufficient Reason asserts, in effect, that the higher intelligible presuppositions which make any thought or thing what it is for us are capable of being discovered, and must be discovered if we are not to fall into error. The Principle of the Identity of Indiscernibles amounts to a looser form of what has been referred to above (Ch.8) as Weyl's Principle. Unfortunately Leibniz also accepted, as did Descartes, the Aristotelian Principle of Continuity, and accordingly rejected Newton's 'action at a distance', with or without an aether (which he normally accepted, but in this connection, regarded as 'occult').[3] Another reason for rejecting action at a distance (which his own monadology implied) may have been the feeling that an interaction at the physical level, in the container of space, was supposed, instead of a total adaptation at the transcendental level.

Although Leibniz's system conforms remarkably, if details concerning monads and 'reflection' are not asked for, with twentieth-century theories in physical science and psychology, in his day it was almost inevitably regarded as speculative metaphysics without any evident connection with physics, these being taken as two independent spheres of human thought.

[1] Leibniz is probably thinking here of *materia prima*, the principle of objectivity. His point is that space is an outward manifestation and not a substantive container. See R. 76, 144, 268.

[2] Hesse, *op. cit.*, p. 116.

[3] *Ibid.*, p. 162.

Berkeley: Perception, and the Inferred Greater Mind

Opposition to the rapid advance of naturalism came also from a quite unrelated quarter. BERKELEY (1685–1753), whose first writings were on mathematics, gave vivid and powerful expression in his philosophy to two connected aims: to counter materialism; and to 'put across' a principle, Existence is *percipi*, or *percipere*,[1] resembling that of Manifestation upon Occasion, and of course implying a potential sphere of 'spirit' or universal order. As part of his programme he put forward an extreme instrumentalism (denying the validity of Newtonian mechanics and gravitational theory as a description of essential truth). Locke's *unknowable substratum*, which he took as another name for mindless 'matter', came in for rejection on the ground of its supposed absolute unknowability. His strongly empirical turn of mind, however, induced in him also a marked psychologism, even nominalism, so that some rather obvious inconsistencies arise.

A natural starting-point to take in the consideration of Berkeley's philosophy is his demonstration that primary and secondary qualities are *both* phenomenal; that is, the actual space-picture of primary qualities—distance, size, motion, etc.—is dependent on the individual mind, in the same way as colours are. Physical theories are to a large extent arbitrary, and we must 'distinguish mathematical hypotheses from the natures of things' (DM.66).[2] On Locke's doctrine of resemblance, he argued discerningly that 'as these supposed originals [of our perceptions] are in themselves unknown, it is impossible to know how far our ideas resemble them, or whether they resemble them at all' (D.465).

Having shown that the claimed objectivity of primary qualities is a delusion, Berkeley has to explain what, in his view, constitutes the objectivity of 'real things' (P.34). His explanation depends on the division of 'the object of human knowledge and subject of discourse' into *ideas* (which we perceive, being 'imprinted on the senses' or else 'excited at pleasure'), along with *spirits* and *relations*, of which we have a 'notion' (P.89). By *ideas of sense* he means nothing like Locke's

[1] *Commonplace Book*, Vol. I of *The Works of George Berkeley*, ed. A. C. Fraser, p. 10. The later form substitutes '*esse*' for 'existence', while still translating it as 'existence' (*ibid.*, p. 259). If 'existence' means outward manifestation to some particular person on some particular occasion, the principle is the true and exceedingly important one of Manifestation Upon Occasion. Unfortunately, the term *esse* suggests (and Berkeley's language encourages this suggestion) that the whole continuing potentiality of the object 'in God' is signified, and that this therefore ceases when no individual mind is observing it. It seems that Berkeley was very much confused over the two senses of 'existence'.

[2] Abbreviations are, for *De Motu*, DM. with paragraph numbers; *Three Dialogues*, D. with page numbers in Fraser, Vol. I; *Principles of Human Knowledge*, P. with paragraph numbers; *Commonplace Book*, C. with page numbers in Fraser, Vol. I.

'object of the human understanding', but evidently has in mind something like a *total encounter* and manifestation upon occasion: 'Bodies, etc., do exist even when not perceived—they being powers in the active being' (C.61). There is an *ectypal* state of things 'created in time' from the archetypal (D.475). This state is 'relative'; for 'when things before imperceptible to creatures are, by a decree of God, [made] perceptible to them, then they are said to begin a relative existence, with respect to created minds' (D.472).

We have no right to speak of ideas of sense as 'fictions', since they possess that *real being* which we suitably call *reality*. Nevertheless such ideas are 'in the mind', or in a universal Mind (P.33–36). Berkeley, in fact, must admit a substratum of 'external' objects to explain their continuity and perceivability on various occasions by various people. But having shown that externality has a subjective character (being an 'idea'), and having rejected the substratum of 'matter' as not ostensively knowable and not providing any principle of 'action', he argues that the substratum is proved to be universal Spirit, of which we have a 'notion' on analogy with our individual spirit. He also touches on a hierarchic theory of *degrees of reality*, that is, ideas excited at pleasure, ideas of sense, and Spirit (D.472–4, P.36).

Berkeley's psychologism shows in the view that from the 'admirable connexion' of our ideas of sense 'we learn from experience' the laws of nature, which 'speak themselves the effects of a Mind more powerful and wise than human spirits' (P.30, 46). These 'settled laws' are not to be understood as *necessary* (P.31), like the laws of mathematics. The universal and necessary are always presented as subjective, and the universally true theorems of geometry are the result of private psychological acts of grouping, abstraction and representation (P.126; intro.15). Organized bodies and natural phenomena are 'clusters' or 'collections of ideas' (P.1). Even the notion of Spirit seems to be *inferred* as was the unknowable substratum called 'matter', or perhaps found by 'reflection' (D.450): 'Such is the nature of Spirit, as that which acts, that it cannot be of itself perceived, but only by the effects it produceth' (P.27). Indeed, 'we cannot know the existence of other spirits otherwise than by their operations . . . excited in us' (P.144–5, D.448).

Hume: The Empiricist of Human Nature

By thus asserting the unknowability of spirit Berkeley cut the ground from under his own feet, leaving reason to affirm the substratum of spirit while denying that of matter. Hence DAVID HUME (1711–1776), whose penetrating psychological analyses gave us the first thoroughgoing psychologistic empiricism, was easily able to undermine Berkeley's whole position. 'This theory', he says (Berkeley having nowhere

suggested that the intelligible world may be known by direct observation or genuine intuition)—'this theory of the universal energy and operation of the Supreme Being is too bold ever to carry conviction with it to a man sufficiently apprized of the weakness of human reason.'[1]

It is necessary to be clear about the epistemology on the ground of which Hume's conclusions are formed. 'Perceptions of the mind' are classified as either *impressions* or *ideas*. 'By the term *impression*', he says, 'I mean all our more lively perceptions, when we hear, or see, or feel, or love, or desire, or will. And impressions are distinguished from ideas, which are the less lively perceptions, of which we are conscious when we reflect on any of those sensations or movements above mentioned' (E.2). The term 'ideas' seems to cover all mental images and combinations of mental images, each detail of which is derived from a corresponding impression: 'All ideas, especially abstract ones, are naturally faint and obscure. When we entertain, therefore, any suspicion that a philosophical term is employed without any meaning or idea (as is but too frequent), we need but enquire, from what impression is that supposed idea derived?' (E.2). Ideas and impressions seem to differ, therefore, not in character, but only in vividness; and no other kind of awareness is admitted, unless it is of 'connexions' between ideas.

'There appear', he says, 'to be only three principles of connexion among ideas, namely, *Resemblance*, *Contiguity* in time or place, and *Cause* or *Effect*' (E.3). These connections are apparent or subjective. For 'all our distinct perceptions are distinct existences, and the mind never perceives any real connexion among distinct existences. . . . We only *feel* a connexion or determination of the thought, to pass from one object to another' (T:1, App.). Even 'the reference of an idea to an object' is 'an extraneous denomination, of which in itself it bears no mark or character' (T:1.1.7). 'Our senses . . . convey to us nothing but a single perception, and never give us the least intimation of any thing beyond' (T:1.4.2). Thus relations are purely subjective, being 'complex ideas' which 'generally arise from some principle of union among our simple ideas' (T.1.1.4); but how the 'principle of union' comes into being, or how it can be *given* in 'objects' is plainly outside the scope of this subjectivist and predominantly atomistic psychology.

Hume, one may say, was correct in demanding the ultimate reduction of everything of which we speak to *ostensive* knowledge, i.e. to formulations of what is directly 'read off' in experience and insight. But too exclusive a focusing on the atomic states of the individual mind and 'human nature' blacked out the recognition that participation, with

[1] *Enquiry concerning Human Understanding* (referred to below as E), 7.1. *The Treatise of Human Nature* is referred to below as T.

transcendence of physical space and time, could also be ostensive. Hence the existence of the external world becomes for him an insoluble problem. His approach to the problem is, however, illuminating.

He begins by admitting (T:1.4.2) that it is 'in vain to ask' *whether* 'there be body or not'. The fact that there are bodies is one which 'we must take for granted in all our reasonings'. But as to what causes 'induce us to believe in the [continued] existence of body', he concludes that an imaginative 'propension to bestow an identity on our resembling perceptions produces the fiction of a continued existence'. He then candidly admits that he 'cannot conceive how such trivial qualities of the fancy, conducted by such false suppositions, can ever lead to any solid and rational system'. In the surprising assertion, however, that 'carelessness and inattention alone can afford us any remedy', we may perhaps see a reluctant admission of the fact that by removing fixation on individual and subjective impressions and ideas (or placing a cloud of unknowing over them) we may come to that sense of participation which reveals the object in a *total encounter*.[1]

Kant: Rational Subject, Working Materials, and Unknowable Object

Hume, as we have seen, realized that Berkeley's theory of the 'Greater Mind' and 'spiritual potentialism' was incompatible with his psychologism. He therefore discarded the former, for which he saw no evidence. As a result, the continuing objectivity of things could not be reasonably explained: it had to be accepted as consisting merely in a subjective 'habit' or 'imaginative propension'. IMMANUEL KANT (1724–1804) was deeply impressed by the fact that this line of argument made objectivity, causality, the necessity of relations in space and time, and faith in God, unaccountable. He did not, however, re-admit 'spirit', a 'greater Mind', or the 'intelligible world' of Plato and Leibniz. Some other way of relating objectivity to reason had to be found, and this he called a 'transcendental doctrine of elements and method'.

The intention of this new doctrine was to substitute for certain 'dogmatic' views, which he regarded as unsupported by reason or evidence, other views which he considered to be according to reason and denominated 'critical'. In this way a sure foundation was to be provided for all science and philosophy. Here our discussion must be restricted almost entirely to the theory of the external world presented in the *Critique of Pure Reason*, 1781.

In Kant's thinking there are four distinct strands, which he endeavours to reconcile with the help of new insights into the working of the mind

[1] For a thorough discussion of this subject, see H. H. Price, *Hume's Theory of the External World*, esp. pp. 72 ff. (on the relation of Gestalt-theory to Hume), pp. 188–92.

and reason. To begin with, he plainly takes for granted a causal theory of nature broadly along Cartesian or Newtonian lines.[1] That is to say, there is a communal world of causal objects 'outside me':

'Outer sense is already in itself a relaxation of observation to something actual outside me' (Intro. xl, note).[2]
'Observation takes place only in so far as the object is given to us' (33).

He also refers to 'the subjective condition under which we can have outer observation, namely, liability to be affected by objects' (42). It is clear that such 'outer objects' must be communally-acting ones; in Husserl's terminology, they must be *intersubjective*. But this essential feature of physical objects has passed into the background of Kant's thinking. In fact, according to his strict 'transcendental doctrine' (as we shall see), such communal objectivity is 'unknowable'.

The implying of a communal objectivity becomes particularly evident when Kant speaks of *necessity* in connection with natural laws; for instance:

'Were I to posit the antecedent and the event were not to follow necessarily thereupon, I should have to regard the succession as merely subjective' (247).

Thus the 'merely subjective' is contrasted with 'necessity of causal connection'. Kant's answer to the crucially important question, how a (conceptual) 'necessity' can appear in (communal) nature, must be deferred till we are ready to consider his view of 'cause', Hume's discussion of which had particularly troubled him.

Hume's subjective idealism is in fact the second strand in Kant's thinking, and, unlike the first, is continually emphasized in the most explicit terms. It derives from Berkeley's demonstration that everything of sense, including relations in space and time, is an 'appearance' peculiar to the observing subject: '(Appearances) are nothing but sensible presentations, which, as such and in themselves, must not be taken as objects [Gegenstände] capable of existing outside our power of presentation' (A104).

In contrast with objects 'outside our power of presentation', Kant

[1] The unacknowledged presuppositions of Kant's *Critique of Pure Reason* are discussed at length by Husserl in *Die Krisis der Europäischen Wissenschaften*, § 28 and Beilage XV (The Hague, 1954).

[2] References by number simply are to pages in the translation by Norman Kemp Smith of the *Critique of Pure Reason*, London 1933. The letter A preceding indicates the first edition. I have, however, rendered *Anschauung* throughout as 'observation'. The translation 'intuition' usually adopted in philosophical writings seems to me quite inappropriate and confusing in all the passages quoted here.

introduces the concept of an *empirical object* (Objekt), which is directly observed, and identified by the unity of its successive presentations:

'Appearance, in contradistinction to the presentations of apprehension, can be presented as an object [Objekt] distinct from them only if it stands under a rule which distinguishes it from every other apprehension and necessitates some one particular mode of connexion of the manifold. The object is *that* in the appearance which contains the condition of this necessary rule of connexion' (236).

Since the 'synthetic unity of the manifold' which constitutes the object is open to *analysis* by the observing subject, Kant concludes that the *synthesis* is actually provided by the mind of the subject (130): 'We cannot present to ourselves anything as combined in the object which we have not ourselves previously combined. . . . Of all presentations *combination* is the only one which cannot be given through objects'. When the mind thus brings about (as in formless 'working materials') a 'synthetic unity of all appearances as regards their relation in time' (220), the observation 'becomes an object [Objekt] for me' (138).

It is to be noted that in the above quotation (236) Kant has again, by implication, used *necessity* of connection to characterize a communal objectivity in nature. But explicitly he insists that this necessity of connection is supplied by the individual mind, since only a mind can provide rational synthesis, and also perhaps because the observable results plainly vary with the condition of the particular subject. The opposing Leibnizian view, that a *communal* source of *concepts* of form (intelligible world) and subjective conditions cooperate in any presentation is ruled out for Kant, as for Hume, by the placing of concepts in the mind of the *individual* subject only and of communality in the (subconsciously presupposed) Newtonian space-container: 'The possibility of the category of *community*', he says, 'is only to be determined . . . through outer observation in space' (292).

Kant considers, further, that any 'appearance' implies the existence of a 'substance'—of the conservation of whose 'quantum' he offers a 'proof' (224)—and also a 'concept of reflexion' (333) or general formless principle of determinability called 'matter' (in the scholastic sense). In regard to both of these (although 'matter' is a 'concept'), a general kind of community or givenness in nature seems to be assumed:

'Appearances contain, in addition to observation, the matter for some (empirical) object in general' (207).

'The transcendental idealist [as Kant calls himself] allows to matter, as appearance, a reality which does not permit of being inferred, but is immediately perceived' (A371).

In regard to the term 'reality', Kant distinguishes between *empirical* or *subjective reality* (i.e. an appearance of reality, 520, A375, Wirklichkeit) and *objective reality* (Realität) which 'cannot contain any determinate observation, and therefore refers only to that unity which must be met with in any manifold of knowledge which stands in relation to an object' (A109). Objective reality, it seems, is the mere givenness of an empirical object in a recognizable succession of presentations, i.e. total encounter:

'That an object be given (if this expression be taken, not as referring to some merely mediate process, but as signifying immediate presentation in observation), means simply that the presentation through which the object is thought relates to actual or possible experience. . . . The *possibility of experience* is, then, what gives objective reality to all our *a priori* modes of knowledge' (195).

This is one of the very few passages where Kant seems to be distantly referring to the 'potentiality' which an object has to exhibit various specific presentations while remaining the *same* object for us and other people. But he has explicitly denied any *definite abstract form* of potentiality, manifesting itself conjointly with subjective conditions, as is required in modern scientific theories. The reason for this is that he believes, again in accord with Humean subjectivism, that such 'mediate process' could never be observed and is therefore mythical. His 'possibility of experience' is accordingly a simple, unanalysable conviction which guarantees objectivity by virtue of its own rational nature.

We now turn to the third strand in Kant's thinking, namely, his intention to show how the logical character of physical laws, and especially the observed truth of geometrical theorems, can come about, in view of the difficulty that (following Hume) all logical concepts and operations are powers of the individual subjective mind. In short, how does 'human reason' get into nature? We have already considered Kant's view that the 'synthetic unity of appearances as regards their relation in time', which results in an observation becoming 'an object for me', is the work of the *observing mind*, while only the 'matter' of appearance is 'given to us *a posteriori*' (34). Consistently with this, Kant makes all observable characteristics of space and time similarly the work of the individual mind:

'By means of outer sense, a property of our mind, we present to ourselves objects as outside us, and all without exception in space' (37).

'Space does not present any property of things in themselves, nor does it present them in their relation to one another' (42).

'What we call outer objects are nothing but mere presentations of our sensibility, the form of which is space' (45).

'We can extract clear concepts of [space and time] from experience, only because we have put them into experience' (241).

In comment, it may be remarked that there are three ways in which we might be said to 'put space and time into experience'. By operational definitions, and in so far as the givenness of nature permits, we can create structures more and more closely approximating to the exact concepts of geometry, i.e. of abstract or ideal space and time (as explained in Chapter 6, above). This is obviously not what Kant has in mind, since he admits only one kind of 'space', and this is one in which relationships are already exact (in fact, Euclidean). Secondly, the meaning could be as in modern scientific theories, namely, that space-time appearances result from the cooperation of our subjective mind and bodily conditions with abstract but definite objective conditions. This interpretation is ruled out by quotation (42) above and numerous others.

Thirdly (and this seems to be what Kant explicitly intends) we can suppose that the form of appearances is wholly due to the individual subjective mind. What then if we picture various individual minds observing 'the same' phenomena in the communal world of nature? How would Kant explain the recognizable correspondence between the various appearances to the various observers, so that they agree that they are recognizing 'the same' thing. In what does this observed correspondence originate?

Kant shows some tendency to vacillation or obscurity over this point. In one place he explains that 'the form is *given* by itself' (my italics) and that matter 'presupposes a formal observation (time and space) as antecedently given' (324). Here, it seems, we have something like the Plato–Leibniz view that *form* is the communal (given) creative power of intelligence ('intelligible world'), while *matter* is the necessarily associated ground of actualization. (Kant, incidentally, has passed over Leibniz's teaching that each individual expresses 'the whole universe in its own way', and imputes to Leibniz the opposite view that space and time are 'determinations of things-in-themselves'.) The Plato-Leibniz view is also dimly suggested by Kant's description of form as *a priori* and of matter as a *posteriori* (34).

Kant's general procedure, however, in dealing with this and other questions involving *real communal existence*, runs along quite different lines, and bases itself on his 'transcendental' doctrine of *Schematism*

(176 f.). This is the king-pin of his great plan to reconcile the 'intellectual' with the 'sensible'.

The categories, or 'pure concepts of the understanding'—*substance*, *cause*, *reality*, *community*, etc.—are in themselves abstract and indeterminate until applied to appearances. But when an 'object is given for them', the concepts are no longer empty, but each has a *schema*, or phenomenal counterpart: 'The schemata of the pure concepts of understanding are thus the true and sole conditions under which these concepts obtain relation to objects [Objekte] and so possess significance' (185).

Thus, in conformity with his identifying of 'necessity' and 'givenness', Kant states that 'the schema of necessity is existence of an object at all times' (184). The meaning seems to be that we have only to apprehend *necessity* in a sequence of presentations to know that a corresponding *object* exists at all those times. This is our guarantee of continuing objectivity, implicit in human reason.

In the same way, actual causality is guaranteed by the observing of a 'rule of succession':

'The schema of cause, and of the causality of a thing in general, is the real upon which, whenever posited, something else always follows. It consists, therefore, in the succession of the manifold, in so far as that succession is subject to a rule.'

The problem of the communality of givenness of objects—what has been described above as the 'recognizable correspondence between the various appearances to various observers, so that they agree that they are recognizing the *same* thing'—is solved by introducing a schema of *mutual interaction*, which similarly guarantees communality:

The schema of community or reciprocity, the reciprocal causality of substances in respect of their accidents, is the coexistence, according to a universal rule, of the determinations of the one substance with those of the other.'

Reviewing the doctrine of Schematism as a whole, it appears that if the term 'causality' (for example) means no more and no less than 'the observing of what we judge to be a rule of succession of things', and if the term 'community' means no more and no less than 'such causality of interaction of one thing on another', then Kant's theory is tautologous (while it presupposes that the terms 'rule of succession' and 'interaction' are operationally precise). We can accept it as correct, on that understanding. But in the meantime the questions of primary importance have escaped attention—When, we may ask, is an impres-

sion of physical givenness veridical, and when deceptive, as with optical illusions, or hallucinations? When, if ever, does a rule of association between events of two classes *X* and *Y* become absolute? How does it come about that the various subjective presentations of a physically given occurrence to various people are accountable in terms of a mathematical space and time which is *not* itself the form of a presentation, but is abstract and common to all? To what extent is this abstract space and time given, and to what extent is it subjectively (but by some communal agreement) contrived? What stand should we take in regard to elements of substructure (such as harmonics in a sound heard) which *explain* the observed effect, but are not normally themselves observed? How can we explain the continued action of physical laws when no one is observing the phenomena? What are the possible 'means of interaction', in cases where no material medium can be discovered 'between'?

It seems that in the doctrine of Schematism we have an instance of what may be called the Fallacy of the Linguistic Blanket—the belief that we have only to attach a name or lay down a rule of linguistic usage (agreeing with one aspect of customary usage), and immediately the questions which were previously troubling us are either solved or shown to be senseless.

Elsewhere Kant offers some general views on the subject of natural law:

'By nature, in the empirical sense, we understand the connexion of appearances as regards their existence according to necessary rules, that is, according to laws. . . . Empirical laws can exist and be discovered only through ordinary practical experience [Erfahrung]' (263).

On the other hand, 'the order and regularity in the appearances, which we entitle *nature*, we ourselves introduce' (A125).

The first of these passages suggests *positivism*, or the 'reading-off' theory of nature. The second suggests *phenomenalism*, or the 'putting-in' theory of nature. In either case the attention is to avoid having to admit *potentiality*, i.e. givenness of relationship transcending the time and space of 'ordinary practical experiencing' (Erfahrung). Kant was perhaps induced to adopt these views by the belief, common at his time, that Newton's laws were absolute, and in fact, self-evident.

The fourth strand in Kant's thinking has already been much in evidence in the previous discussion. It is the striving to delimit the field of philosophical inquiry so as to rule out the views which he regarded as 'dogmatic' or superstitious.

Generally speaking, Kant adheres to a dualism of two types of experience (Erlebnis) in the broad sense, namely: (A) *Erfahrung*, i.e. 'ordinary practical experiencing' in this world, yielding 'observation' of 'appearances', either *outer* (of objects around us) or *inner* ('of ourselves and our inner state', 49); and (B) *Verstand*, i.e. 'understanding', taken to mean the exercise of discursive and directed thinking, or the forming of instinctive impressions such as 'necessity' or 'causality'. His keen philosophical insight showed him, however, that this analysis could not be complete without something transcendently *real* or more ultimately *given*, to implement or justify the instinctive interpretations which seem self-evident when we apply our understanding to the world, or 'think observations through the understanding' (as Kant describes the process).

Thus even beyond the schema, or phenomenal counterpart, of 'possibility' (i.e. possibility of appearances) must be a formless thing-in-itself or *transcendental object* which, by reason of its not entering into experience *A* or *B*, is unknowable:

'The true correlate of sensibility, the thing in itself, is not known, and cannot be known through these presentations: and in experience [Erfahrung] no question is ever asked of it' (45).

'If . . . we should attempt to apply the categories to objects [Gegenstände] which are not viewed as appearances, we should have to postulate an observation other than the sensible . . . such a type of observation forms no part whatsoever of our faculty of knowledge' (308).

Similarly, Kant felt called upon to deny outright all possibility of empathic participation, intersubjectivity, telepathy and precognition:

' . . . a special ultimate power of observing the future in advance [voraus anzuschauen] (and not merely inferring it), or lastly a power of standing in community of thought with other men, however distant they may be—are concepts the possibility of which is altogether groundless, as they cannot be based on experience [Erfahrung] and its known laws' (270).

The entire sphere of experience [Erlebnis], which Kant seems to deny, may be called experience *C*. Various later philosophers have admitted kinds of direct acquaintance going beyond 'ordinary practical experiencing and understanding', and have described them as *transcendental* (using the term therefore in a sense quite different from that of Kant). For philosophers who follow the Platonic tradition, believing

this to be experientially based, as well as for the great majority of eastern philosophers, believing in the efficacy of 'spiritual exercises', there is indeed a third sphere of 'experience'. Their philosophy is distinguished by the very fact that it looks beyond experience *A* and *B* to a sphere of higher reality, causally immanent in what is 'lower'.

If experience *C* is to be put forward as the centrally creative and causal category of knowledge, it is important that it should be characterized as clearly and effectively as is practicable in ordinary language. This we may attempt to do in terms of the 'depth diagnosis' outlined above[1] and the previous remarks on phenomenological and mystical analysis.[2]

Experience *C*, then, we may perhaps venture to say, is intellectual or perceptual knowledge which transcends in a certain clear and unmistakable way the onward urge of time, the rigid apartness of spatial objects, and the apparent isolation of the individual mind in its state of fixation on bodily impressions. It is a state of *release*. Fixation being overcome, the mind opens out into universality.

It is experience attained by a peculiarly enhanced power of reflection, beginning with 'non-attachment', 'recollection' and 'stoppage'[3] (of the current of time), and proceeding to 'essential insight' and a transcendent comprehension, by which details of intelligible structure in any experience can be held changelessly in view, even if (physically regarded) they are spread over a duration of time.

In its higher reaches it is characterized by an unmistakable kind of *unity*, and becomes, in the words of Leibniz, 'the knowledge of necessary and eternal truths . . . raising us to the knowledge of ourselves and of God'.[4]

Kant seems to recognize one variety of such experience, transcending time, in the 'pure synthesis of presentations' (104). Even so, the synthesis is made 'by means of the imagination'—a phrase which suggests only a shadowing forth of experience *C* in the privately subjective thinking of experience *B*. There are other kinds of knowledge or functions of the understanding (experience *B*) which have reference to what lies outside experience *A* and *B*, kinds which provide 'a touchstone of the value or lack of value' of experience *B* (26), and these Kant calls 'transcendental'. But such functions or knowledges are 'misemployed' if an attempt is made to apply them 'beyond the limits' of experience *A* and *B* (352).

[1] Page 376.

[2] Pages 107 ff.

[3] Further on 'stoppage' (*epoché*, phenomenological suspension) and 'primordial recollection', see the section on Husserl in Chapter 24 below, pp. 404 ff. Also pp. 107 ff., 295 ff.

[4] M. 29. See also M. 43, L. 343, L. 57.

To those who accept the primary 'reality' and explanatory force of experience *C*, Kant's original error may appear to lie in his endeavour to implant a subjective conceptuality onto the simple mechanistic features of Newtonian science. This endeavour necessarily fails, because scientific explanation shows that objectivity, in some abstract mathematical sense of the term, must cooperate with subjectivity in any experience. *Both* are conceptual, in structure and function. That is to say, when any detail of the structure or function of objectivity becomes known to us, whether in its form of potentiality (supraperspectival) or of actuality (as a particular perspective for us), it is evidently conceptual; so also, when unknown by us but conceived of as knowable by other people or in general, it is also suitably denominated conceptual. Hence an *intersubjective* conceptuality is demanded in scientific explanation; and empirical objects are for us the conjoint end-result of potentialities not themselves explicable in terms of simple locations in a three-dimensional container of space and a one-dimensional stream of time.

CHAPTER 24

DIVIDED STREAMS: THE RETURN TO POTENTIALITY

> 'When you are criticizing the philosophy of an epoch, do not chiefly direct your attention to those intellectual positions which its exponents feel it necessary to defend. There will be some fundamental assumptions which adherents of all the various systems within the epoch unconsciously presuppose.'
>
> A. N. Whitehead, *Science and the Modern World,* Cambridge 1925, p. 71

It was perhaps the boldness of Kant's attempt at an integral view of man and nature by extension of subjective analysis from the psychological to the rational that provided the greatest stimulus to later generations of philosophers. The inconsistency between the various strands of his thinking, however, had to be overcome. So long as the method of subjective analysis remained dominant (rather than the positivistic leanings) it was inevitable that the relics of Cartesian dualism and Newtonian absolutism should drop out, resulting in views resembling a Platonic idealism in which communality is conceptual and thus 'transcendentally' knowable.

We can see this tendency even in Kant's own later work, although he still endeavours to adhere as far as possible to the self-imposed limitations of his 'critical' method. He is persuaded that 'although the moral law is to be found in man's own nature, yet it . . . points beyond the individual limitations of man, [and] causes him to regard himself as a citizen of a great kingdom'.[1] In the Critique of Judgement (1790 things in themselves become 'noumenal realities' constituting a 'supersensible substrate' which is within as well as outside us. 'As a purpose of nature' the thing 'exists *by means* of all the other parts but is also regarded as existing *for the sake* of the others and of the whole' (286, 291). Here we have a real universal cohesion transcending physical space and time. Nevertheless, it does not seem that Kant admitted any possible *intelligible knowledge* of any structure of this universal order. We could have no more than a guarantee of it through the convictions of the moral law arising in us.

[1] H. Höffding, *A History of Modern Philosophy*, Vol. II, trs. B. E. Meyer, London 1900, p. 71.

FICHTE (1762–1814), who claimed to be developing Kant's ethical ideas, wholly abandoned his agnostic dualism and presented instead a system which is a remarkable western formulation of eastern mystical teachings (of which he seems to have had no direct knowledge). He distinguished the finite or empirical ego, non-egos, and the pure or infinite ego (*Atman*, Very Self, Universal Spirit). The pure ego is at the same time the 'innermost active essence of our being' and 'the ground of the world of objects or of limitations in which our finite ego is enclosed'. Moreover, the 'primitive activity' of the pure ego can be discovered by 'a kind of higher intuition, an intellectual intuition', to which we attain by a reconciliation or transcendence of opposites, in the 'antithetical method'. It is then seen that the world of our finite being comes into existence for us by a certain real projection: 'We project time, space . . . just as we project the sense-qualities.' The world then provides a ground for man's fulfilment in labour and struggle, 'the occasion for realizing the end of his existence', and 'the material of his duty'. Supplemented by an adequate theory of physical measurement and scientific method, Fichte's system would have provided, for the first time since Leibniz, a coherent philosophy of nature and its origination.[1]

In HEGEL (1770–1831), last of the German 'speculative idealists', the philosophy of nature unfortunately receives arbitrary and inadequate treatment.[2] But his leading principle, that 'the real is the rational, and the rational the real', expresses a valid and fruitful standpoint for such a philosophy.

Concurrently with this idealistic movement of thought we may notice a resumption of trends to materialism. This may be imputed perhaps to the effect of valuable scientific advances, as in the work of LAVOISIER (1743–94) in founding chemistry as an exact science, and the extension of the law of conservation of energy, by ROBERT MAYER (1814–78), about 1840, to all natural phenomena. This was somewhat confusingly understood by both Mayer and HELMHOLTZ (1821–94) as necessitating a single kind of *force* at the basis of all. The strict experimental justification, by the measuring of mechanical equivalents for heat and electricity, was provided by JOULE (1818–99). Along with these names it is suitable to mention that of DARWIN (1809–82), whose *Origin of Species* appeared in 1859 and was immediately taken as favouring materialism. Today, because of its bringing biology into the scheme of a rational universe, it can be considered rather as favouring the reverse doctrine, namely, the equivalence of rationality and universality of spirit.

Another line of scientific development, primarily mathematical, led

[1] *A History of Modern Philosophy*, pp. 154 ff. The systems of Leibniz and Fichte are compared in Latta, *op. cit.*, pp. 178 ff.

[2] Höffding, *op. cit.*, p. 183.

to an immense broadening of views on geometry and physics generally. The impossibility of entirely separating physical theories from a philosophy determining choices of various kinds was beginning to be felt. GAUSS (1777–1855), RIEMANN (1826–66) and Helmholtz, however, all inclined strongly to a *physical* view of geometry. The results of the mechanical operations of measurement were thought to be determined solely by the nature of the physical world or (in the case of Riemann) by its contents. Here we have still a 'container' theory of space, but the ideally exact properties of the container were conceived to be not necessarily Euclidean. Considerable confusion is evident in the writings of the time between geodesics in a formal geometry, light-rays observed physically, the original intuitions of space, and more or less arbitrary systems constructed out of these intuitions along with other concepts. This confusion seems still very evident today, when 'non-Euclidean' geometries are discussed.

In his 'Theory of Errors' Gauss threw light on the problem of the experimental inexactness of conceptually exact physical laws (as we suppose them to be). The deviations are classified as *systematic* (due to some definite physical cause) or *random* (due to a multitude of psychological and other unpredictable causes, weighing equally in all directions); and by taking a large 'sample' of readings, the 'most probable value' could be found and also a measure of the 'standard deviation' or of the 'probable error'. To deterministic materialists, all the causes must be considered exact in their operation, whether knowable or not. The impossibility of putting an exact and universal concept operationally into the materials of the physical world, and the consequent unbridgable difference between the *a priori* and the empirical (as between the universal and the particular) could hardly have attracted the notice of scientists of the time.

In various quarters there was a renewed sense of the inevitable entanglement of subjective or psychological ideas with physical ones. J. S. MILL (1806–73) carried psychologistic empiricism almost to the limit in his determination to root out *a priori* principles. He objected to the mention of unconscious presuppositions, as if a presupposition were a species of non-evidential dogmatism, and even took the view that the axioms of mathematics were derived from experience. In his definition of matter as a 'permanent possibility of sensation', however, he made a positive contribution to the solution of Hume's problem of the external world, focusing attention on the need for some background *potentiality*.[1] In so far as the 'permanent possibility' is held to be a subjective construction of the individual mind, or an imagination, or a mere way of formally uniting the actual observations by a conditional statement,

[1] *Examination of Sir William Hamilton's Philosophy, London* 1865, Chapters XI, XII.

the solution (?) would seem to go no farther than Hume. But in this problem Mill himself went clearly beyond phenomenalism of any kind. He asserted that 'the Permanent Possibilities should be classed by us as existences generically distinct from our sensations, but of which our sensations are the effect'. The sensations are thus separate manifestations of a continuing real potentiality; 'there is a bond of some sort among all the parts of the series', and this bond or 'uniting principle' is determined according to laws.[1] Mill does not discuss the question whether the bond is in a common unconscious or is something abstract and categorically unknowable.

The vitally important concept of psychological potentiality, involving some kind of transcendence of space and time and the individual mind, became clearer and more widely accepted through the writings of FECHNER (1801–87) and EDUARD VON HARTMANN (1842–1906). Fechner was the founder of psychophysics, and put forward a remarkable panpsychism, which exercised a strong influence on WILLIAM JAMES (1842–1910). Hartmann's famous book, *Philosophy of the Unconscious* (1869), was enormously influential, passing through ten editions in twenty years. He aimed 'to show that the scientific mode of explanation is insufficient', and that 'we must assume a spiritual principle at work'. This is the Unconscious—the true productive power in the actions of men, in world-history, and all evolution, the power whose universality makes the cooperation of individual beings possible, and which also expresses itself teleologically through the minutest structures of outward nature.[2]

The Ocean in Sight

Passing over the strong tendency to Kantian phenomenalism or materialism which became prevalent again in the last quarter of the century, we must take up again the line of mathematical development beginning with Riemann. The work of Riemann on the geometry of surfaces was followed by the systematic development of the Tensor Calculus by RICCI (1853–1925) and others, and by a further generalization in the theory of Continuous Groups of Transformations by SOPHUS LIE (1842–99). To the physicist it then became impossible to view geometry either as a set of axioms or as something merely 'given in nature'. Metrical geometries were at last seen as logically coherent systems of measurement, only some of which could be applied to the world for the consistent formulation of laws (of nature). It remained for HENRI POINCARÉ (1854–1912) to state explicitly the principle of the mathematical arbitrariness of the choice of geometry and the principle

[1] *Examination of Sir William Hamilton's Philosophy*, Ch. XI, p. 203. Notes to James Mill's *Analysis* ii (quoted Höffding, *op. cit.*, pp. 415 ff.).

[2] Höffding, *op. cit.*, pp. 533 ff.

of the Non-Absoluteness (Relativity) of Space, i.e. the non-operational character of the supposed ether, because of the group-structure involved in measurement. Views of this kind were actually helped forward by the positivism of ERNST MACH (1838–1916), who accepted nothing but actual measures and ways of systematizing them.

Poincaré's conclusions had been largely induced by certain famous experiments concerning light (Fizeau, Michelson-Morley), in which severe conceptual difficulties had arisen if one tried to account for them on the classical epistemology. The epoch-making step taken by Poincaré and, more explicitly, by ALBERT EINSTEIN (1879–1956) in 1905 was in effect the proposal to abandon the Galilean Transformation, or container theory of the second kind, and to adopt instead the Lorentz Transformation as a true description of the measured character of the world in a 'geometry' (i.e. Lie Group) of *space and time* together. The chief argument for this view was the lack of an operational definition for simultaneity.

Einstein's view of geometry remained predominantly *physical*—the geometry is read of, so to speak, by putting operational definitions into effect. Even in the later theory, of General Relativity, the geometry of the world was considered to be determined by the mass distribution in it, and this became the accepted interpretation. The container has become a set of differently shaped and time-ordered containers, but otherwise the dualism of object and onlooker remains much as before, and the universe is still imagined as a non-mental structure proceeding deterministically like a machine. To the end, Einstein continued to reify 'space' and call it (a new sort of) ether.

While recognizing with admiration the brilliance of Einstein's work in mathematical physics, it has to be admitted, I think, that the 'physical' view of geometry to which he gave the prestige of his name has had unfortunate results. In particular, it has fostered the idea that theories are built on measurements (read off with the help of physical apparatus), instead of the very measurements themselves presupposing theories. This positivistic view, foreign to his expressed opinions, goes along with oblivion of the fact that a geometry must be *put into the world* approximatively, and therefore the non-existence of precisely rigid bodies and the impossibility of manipulating even approximately rigid bodies outside a local region are alike irrelevant to the system of space and time measurement which we 'project' into the world.

The putting of geometry into the world being understood, Kant's dictum that we 'posit' things in themselves outside us can be justified, not only in a metaphysical sense but also in a physical sense—the 'things' being carriers of physical measurement. But the thing in itself is not unknowable. It is a *Gestalt potentiality*, recognized in a 'total encounter', and susceptible to measurement in countless ways.

Whitehead: 'Concrete Facts of Relatedness'

Group-objectivity (in a set of abstract spaces) was not at first seen by many physicists to have any bearing on the philosophical dictum that 'the real is the rational', or to lead therefore to the conclusion that 'the real' forms a whole, united by relations mental or spiritual in character and not mindlessly mechanistic.

Among those who early perceived the philosophical implications of the changed scientific outlook on space and time, first place is taken by A. N. WHITEHEAD (1861–1947), who showed clear recognition, in his Direct Realism, of the principle of the Total Encounter, and in his *panpsychism*, of the principle of Universal Space-Time Cohesion. Denial of the possibility of a real encounter with objects is characterized as the 'fallacy of vacuous actuality'.

This Direct Realism of Whitehead, however, led him to attack strongly the 'bifurcation of nature' into two fields of knowledge, namely, perception and background constructs. In *The Concept of Nature* (1926, abbreviated to CN) the causal substructure of science is included under the category of perceptive details: 'There is but one nature, namely the nature which is before us in perceptual knowledge', and 'the molecule is in the event in the same sense as the grin is really on the cat's face' (p. 40). In *Process and Reality* (1929, PR) empirical foundations seem to have vanished in a cloud of generalities, but the same broad view remains. The omnibus term, 'actual entity' is introduced to signify a Cartesian *substance* (pp. 25, 77) which has 'concrete facts of relatedness' (prehensions) with other 'actual entities', and may or may not be apprehended under the modes of 'presentational immediacy' or 'causal efficacy'. One actual entity has a 'potentiality' to contribute to another by prehensions. The 'extensive continuum' is the space-time presentation of actual entities to consciousness. (pp. 84 ff.).

All this amounts fairly clearly to a *one-level actuality* view of both space-time and its 'contents', perceived or unperceived. 'There is no going behind "actual entities" to find anything more real' (PR, p. 24). Even God is a 'primordial actual entity' (pp. 121 ff.).[1] The space-time form of this actuality is asserted, on *a priori* grounds, to be 'flat' (i.e. the transformations belong to the Poincaré group), with Maxwell's Electromagnetic laws also taken as true *a priori*.[2] The measures involved (time-instants, points, etc.) are 'extensive abstractions', also taken as provided in 'the deliverance of sense-awareness' (CN, p. 171).

[1] It is to be specially noted that, for Whitehead, 'objects' are (roughly) Lockian *ideas* (PR, pp. 71 ff.) and possess a 'real potentiality' to pass into actual entities (p. 123). Whitehead's reference to an 'object' as a 'potentiality for being a component in feeling' (*ibid.*) has therefore no connection at all with what I have called a *potential group-objectivity* obtaining definite manifestation only on the occasion of experience by an individual mind.

[2] PNK, p. 115; *The Principle of Relativity* (Cambridge 1922), pp. 29 f., 79 f.

The generalizations of *Process and Reality* transcend the subject-object dualism, which Whitehead overcame in *The Concept of Nature* by another piece of idealism. This we may perhaps call the 'myth of the non-subjective knower', for whom no subjective 'corrections' have to be imposed because no 'psychic additions' exist (CN, p. 185). 'Subjective' variations such as colours are to be accounted for, it would seem, by their obeying laws of nature not yet discovered by systematization. The inevitable conclusion is epiphenomenalism (a form of Scientific Dualism). Whitehead escapes this dilemma by accepting that 'data as they occur in the scientific laws do not relate all the entities which present themselves in our perception of nature' (p. 29 f.). A real 'spiritual world' may emanate into the fragmentary 'world of our common experience'. But these matters, he says, are outside 'the concept of nature' (pp. 46 ff.).

Bertrand Russell: 'The Art of Rational Conjecture'[1]

The influence of BERTRAND RUSSELL (1872–) has been greatest in stimulating a return of philosophy to the methods of analysis—though 'analysis' for him has been discursive reason and symbolic reduction rather than intuition. His early emphasis on logical 'atomism' and manipulation amounted to a sharpening of tools, which unfortunately induced many later philosophers to attend too exclusively to symbolic formulations or common language usage, so cutting themselves off from the awareness of transcendental issues and the totality of an encounter.

Russell's epistemology of the 1920's was a phenomenalism in which mind and matter were 'logical constructions' out of the 'neutral stuff' of sensation and images. But by 1927 (in *The Analysis of Matter*) he had come to reject this in favour of a Scientific Realism in which physical objects (as distinct from appearances) were neither knowable by supraperspectival analysis in a total encounter, nor categorically unknowable except by 'construction' from percepts (as in phenomenalism), but could have a kind of existence guaranteed for us by speculation on the presumed causal 'connection of "matter" with perception' (*op. cit.*, p. 215). The potentially 'mental' or spiritual character of objects in themselves is decisively rejected (pp. 199, 215, 387 f.). It is not clear how Russell reconciles his purely non-mental objects with the complex group-objectivity given in nature, according to the Theory of Relativity. He does not deny the possibility of 'unconscious perceptions' rationally unifying nature, but he maintains that they are 'useless as a basis for physics' (p. 200). His standpoint therefore remains the dualism of one physical world, non-mental and in effect unknowable, as against impressions in the human mind.

[1] This is the sub-title of a pamphlet by Russell, published in 1942, on *How to Become a Philosopher*.

Mention should perhaps be made at this point of Logical Positivism, the movement of thought directly or indirectly inspired by Russell's work and energetically campaigned for by the VIENNA CIRCLE (Schlick, Carnap, Neurath, and others). The central doctrine of Logical Positivism was the Verifiability Theory of Meaning, which no doubt helped greatly in inducing greater clarity in scientific and philosophical modes of expression and reasoning. In so far as the verifiability principle demanded operational definitions in the *broad* sense (Ch. 8 above) for every term used in science or philosophy, little objection could be made. But by limiting the verification to actual physical operations and the whole content of propositions to the one-level interpretation of atomic terms, the prevailing view amounted to the dogmatic application of a one-level naturalism of the crudest kind. It was not therefore long before the demands were drastically relaxed; but even in the 'Logical Empiricism' of today we can still see, perhaps, the dominance of a Kantian philosophy, arbitrarily limiting analysis to what has been described above as experience *A* or *B*.

Husserl: Encounter, Essence and Constitution

Concurrently with the increasing dominance of logico-empirical tendencies in philosophy during the first half of this century we find, in the *phenomenology* of EDMUND HUSSERL (1859–1938), the development of a widely influential philosophy which breaks entirely new ground, though having the same broad aim as that of logical empiricism—namely, to cut at the root of philosophical and scientific problems by clear analysis. Present-day attitudes to Husserl vary between the view sometimes expressed on the continent of Europe, that he is the most significant figure in twentieth-century philosophy, to the complete ignoring of his work by the majority of logical empiricists in England and the United States.

Husserl's phenomenology may be described as an analytical discipline which abandons the 'apparatus of inference and proof'[1] and brings into play a 'unique form of consciousness' (I.31)[2] for the disclosure of 'ultimate foundations and distinctions' (K.27) in what is given to us directly. It is completely opposed to psychologism, which tries to

[1] Marvin Farber, *A Presuppositionless Philosophy*, 'Philosophical Essays in Memory of Edmund Husserl', ed. Farber, Harvard U.P., 1940, p. 57 (this publication will be referred to as E).

[2] References to Husserl's works are abbreviated as follows: I, II are first and second volumes of *Ideen*, with section numbers, except where the page of II in the Nijhoff edition, 1952, is indicated by a p.; K. is *Die Krisis der Wissenschaften*; CM. *Cartesian Meditations*; TC. *The Phenomenology of Internal Time-Consciousness*, trs. James S. Churchill, Nijhoff, 1964. Boyce Gibson's translation is used for *Ideen* I. Translations from *Ideen* II, *Krisis*, and the concluding essay in E. (by Husserl himself) are my own.

explain matters of universal reason in terms of contingent psychological responses of individuals, and also to 'physicalistic objectivism' (K.6), which conceals the immediate 'seeing' (I.19) in a 'cloak of ideas', presuppositions, symbols, theories, inferences and approximate confirmations. But the phenomenologist does not turn away from the naturalistic 'theses'. On the contrary, he is enabled to 'step up' to them and 'transvalue' them (I.31) in terms of what is 'primordially given'.

Being thus unswayed by theories and presuppositions, while leaving the thesis essentially unaltered, the perceiving consciousness directs itself to analysis and comprehension in 'full freedom' (I.31). It focuses on and 'lives in'[1] the 'intentional act' of spontaneous recognition.[2] 'By virtue of the implications of the idea of intentionality, phenomenology is a philosophy of *encounter*.'[3] The encounter is at first with a *totality* comprising 'object-giving 'of all kinds—cognitive, cogitative, empirical. We consider conscious experiences 'in the concrete fullness and entirety with which they figure in their concrete context' (I.34). Then, separately, 'every cogitatio can become the object of a so-called inner perception, and eventually the object of a reflexive valuation' (I.38). This 'immediate SEEING (*νοεῖν*) is the ultimate source of justification of all rational statements . . . so far as its object-giving is primordial' (I.19). The exploration of the totality is not by 'busying' ourselves psychologically with it, but in a state of 'full freedom' and 'spontaneous' detachment (I.31, 23).

The vital core of Husserl's phenomenology is the description of this special detachment and mode of intelligible analysis in a state of 'alienation from regular life'.[4] Here, though without explicit reference, Husserl treads the same ground as eastern mystical[5] philosophies, in their account of the methods and stages of transcendental insight and the path towards 'the ultimate goal of all possible knowledge, the *sapientia universalis*'.[6] He provides, however, a more detailed logical analysis of rational and empirical observation than appears in eastern philosophies, whose chief interest is most often ethical and practical.

'The phenomenological *epoché*', he says, 'will turn out to be the neces-

[1] Helmut Kuhn, 'The Concept of Horizon', E. p. 109.

[2] Husserl particularly emphasizes that the 'intentional act' is 'spontaneous'. Hence the English word 'intention' (implying a deliberate exertion of will) seems misleading, and I usually speak instead of the 'act of recognition'—a phrase which seems to express exactly the required trinalism of subject, act, object.

[3] W. A. Luijpen, *Existential Phenomenology*, Duquesne U.P., Pittsburgh, 1960, p. 92.

[4] Fritz Kaufmann, 'Art and Phenomenology', E., p. 191.

[5] See Chapter 8 above, pp. 96 ff. Like most writers in the German philosophical tradition, Husserl's use of the term *mystisch* is purely dyslogistic. He takes it to imply 'theoretically invented feelings' which are taken to be genuine rational evidence (I.21), or perverse 'Platonic hypostatization', or reification of relations and qualities (TC. p. 24).

[6] Jacob Klein, 'Phenomenology and Science', E., p. 145.

sary operation which renders "pure" consciousness accessible to us, and subsequently the whole phenomenological region' (I.33). *Epoché* means 'stoppage' or 'suspension'. We are immediately reminded of the *upekkha* ('over-looking', utter non-involvement) and conquest of 'becoming' (*bhava*) in Theravada Buddhism, and the *vairāgya* (non-attachment), *samādhi* (contemplation 'illuminated by nothing but the meaning'), and 'checking' and 'one-pointed' conditions described in Patanjali's Yoga-Sutras.[1] Moreover, just as Patanjali says that in such states perception 'yields absolute truth' (*ṛta*), so Husserl insists that the phenomenological suspension is 'compatible with the unshakable because self-evidencing conviction of Truth' (I.31). The 'Principle of all principles' is that 'every primordial object-giving observation is a source of authority for knowledge' (I.24). This, of course, could not be so, if the condition were not a 'unique form of consciousness' in 'alienation from regular life'. Husserl calls it the condition of 'essential insight', and the objects of knowledge 'seen' then are *essences*.

In further clarification of this 'pure observation' (I.21) Husserl gives frequent consideration to the concepts of focus and periphery (Horizont), and also, in close relation with these, the concept of *potentiality*. As regards focus and periphery, he explains that

'what is actually perceived, and what is more or less clearly co-present and determinate (to some extent at least), is partly pervaded, partly girt about, with a *dimly apprehended depth or fringe* [Horizont] *of indeterminate reality*. I can pierce it with rays from the illuminating focus of attention with varying success' (I.27).

To bring essences to light we must 'bring to the normal distance . . . what floats before us shifting and more or less far removed' (I.67). 'The focal is girt about with a zone of the marginal; the stream of experience can never consist wholly of focal actualities' (I.35). Moreover:

'As it is with the world in its ordered being as spatial present, so likewise is it with the world in respect of its *ordered being in the succession of time*. This world now present to me has its temporal horizon, its intimately alive and its unalive past and future . . . . I can turn temporally forwards or backwards' (I.35).

If it be asked how the past and the future can be anything but imaginative suppositions of the present, Husserl has an answer again in accord with eastern mystical philosophies, but expressed in a way appropriate to investigation of western concepts of potentiality and the subconscious. Besides the 'secondary remembrance' which is ordinary

[1] Yoga-Sutras I.12, III.51; III.3; III.9, 12.

memory, there is a 'primary remembrance' or *retention*,[1] which consists in a *transcendence of the current of ordinary time*—a going back to what is still existing, rather than a reproduction of something which is now gone:

'This insight [into 'real experience'] operates as a reflexion, and it has this remarkable peculiarity that that which is thus apprehended through perception is, in principle, characterized as something which . . . already was before this gaze was directed to it' (I.45).

'With respect to intentional experiences . . . when unreflected on they are already there as 'background' and available for perception' (*ibid.*).[2]

Besides this potentiality in respect of time, Husserl distinguishes several varieties and levels of potentiality [Inaktualität, Möglichkeiten] in respect of space or transcendent constitution in general. He describes the change by which consciousness of some particular object or detail of it passes into subconsciousness or background consciousness as

'that remarkable modification which transfers consciousness in the *mode of actual orientation* [Zuwendung] to consciousness in the mode of non-actuality [Inaktualität] and conversely. At one time the experience is, so to speak, 'explicitly' aware of its objective content, at the other implicitly and merely *potentially*' (I.35).

The reference here is particularly to the various perspectives (Abschattungen) of a physical object, implying one another in a rational whole, while only one is actual at any instant. 'The visual image presenting one aspect of the thing is charged with potentialities. . . . By its very nature the initial aspect is one among countless others implied in the perception.'[3] The peculiar kind of 'implication' or involvement of non-actual perspectives in any actual orientation is called by Husserl Appräsenz (appresentation), in contrast with the Urpräsenz (original or primitive presentation) of the aspect which is actual for us at the time in question (II.44 ff. CM.5).

Potentiality in general, regarded as the structural and creative background of all ordinary experience, private or communal, is called by Husserl 'Constitution', and he investigates it at various 'levels'.[4] Husserl may be ranged with Plato among the very few philosophers who have

[1] TC.50, 57. See also Chapter 19 above.

[2] Compare Yoga-Sutras III.12: 'Then (in *samādhi*) the latent and the emergent are equally open to intellectual apprehension [*pratyaya*]'.

[3] I.41. Kuhn, *op. cit.*, pp. 111 ff. Also Chapter 10 above, pp. 130 ff.

[4] See, for instance, II, Beilage 1.

wrestled with the problems of potentiality, in this sense of the term. His very valuable contributions are occasionally vitiated by obscurities of terminology and lapses from strict phenomenological method, as some authors have commented. But it appears possible to overcome these troubles by a slight adjustment of view and terminology, in accord with later scientific and phenomenological researches.

In any discussion of this subject we must first of all consider the means of knowledge by which Husserl asserts that constitution and constitutive elements may be known, and how those means contrast with the 'act' by which a present 'actuality' is known.

'Intention' includes subject, act (of 'positing' or recognition), and object. Husserl maintains that the existence of an object of recognition (*noema*), distinct from the recognition itself, is a matter of direct observation that may become apodeictic (self-evidently certain) in the *epoché;* and this is so, whether the object is private or communal, inner or outer. This conclusion corresponds with the view expressed in previous chapters of the present book,[1] that for anything to be recognized, there must be a recognizable *encounter* with 'that' which can be recognized as remaining always 'the same' (in non-flowing time) while (in the current of outer time) we are analysing its intelligible structure by 'primary remembrance'.

To kinds of act specially fundamental or primitive in the process of gaining knowledge, Husserl attaches the epithets *originär*, *ursprünglich* (original, originary, primordial). He describes them in many ways, and not always quite consistently. A *primordial act* is 'immediate seeing', analogous to sensory perception, not imagination (I.23), direct, free of inference and proof (I.31). It is apodeictic, providing 'the ultimate source of justification for all rational statements' (I.19). It is private—'relative to an individual subject' (II. p. 308).

The condition of a primordial *judgement* is that it should be 'spontaneous' (TC. 186, I.23). Memory is primordial as a present observation, but not primordial in content. Primary remembrance (primordial recollection, in my terminology) is doubly primordial, related to two time levels, each directly observable (TC.186). 'The original presence [Urpräsenz] of an object . . . consists for the Ego in the possibility of bringing the object, in continuous primordial [ursprünglichen] perceptions, to original presence in respect of each of its properties, while, in this continuum of perceptions, the object is constantly [beständig] known in original presence' (II.44). The 'thing itself' (or 'transcendent object'; see below) is thus primordially known, while its properties and relations 'pose in a certain sense as mere differentiations' (I.10).

In order to define the term *primordial* more precisely I suggest that we take *primordial recollection* as a known condition of reflective freedom

[1] Pages 50, 211 ff., 292 ff.

from the current of time, and hence of 'pure objectivity' of awareness. Then we may define the term *primordial* more generally, in connection with observations and judgements, as signifying 'studied or exercised in an encounter, by primordial recollection', as opposed to being discursively inferred, imaginatively imposed or adulterated, or adopted merely because 'it works' or because of some habitual presupposition. (The claim that what is primordial is also 'private' is then an arbitrary postulate, and should be dropped. The claim that it is 'continuous' may perhaps be admitted as a valid description of primordial recollection.)

Accepting also that *potentiality* is the 'structural and creative background of ordinary experience, private or communal' (as suggested above), we have to ask whether, according to Husserl, it can be known in a primordial way.

Husserl's position seems to be that those kinds of potentiality which consist in impressions of unity or (what is perhaps the same thing) 'continuity' of possibilities in time or space are either primordially observable or observable by appresentation or a power analogous to it, while *other* kinds of potentiality 'inductively indicate themselves' or else are indicated by 'comprehension' of the possibilities.

Thus he explains that besides the 'manifold system of perspective variations' we have to recognize a 'thing' or 'transcendent object', which is 'the intentional unity—that which we are conscious of as one and self-identical within the continuously ordered flow of perceptual patterns as they pass off the one into the other' (I.41).

While the unity of the various perspectives and successive appearances of what is 'the same' object to us transcends all particular subjective aspects, it nevertheless has a givenness of which we can be directly and certainly aware, if the degree of clearness is not 'counterfeit' (I.68): 'The spatial thing which we see is, despite all its transcendence, perceived; we are consciously aware of it as given in its embodied form. We are not given an image or sign *in its place*' (I.43; see also I.52).

Where the potentiality in question is more remote than it is in the perception of physical objects (for example, in the thinking of other people), it sometimes seems as if Husserl's 'inductive indication' becomes a phenomenalistic 'logical construction' from actualities normally observed. In principle, however, a phenomenologist must admit the direct knowability of everything genuinely posited in the 'structural and creative background' of experience. In so far as a 'logical construction' is *not* knowable directly (i.e. primordially), it cannot be genuinely descriptive of what is *given*.

Intersubjectivity

Besides the 'intentional constitution' of things for the *individual* subject there is an 'intersubjective constitution' linking all individual points of

view, and we have to consider whether this also is open to observation. Among the various 'levels of constitutional investigation' (II. p. 307) two chief levels have first of all to be distinguished. There is the intersubjective constitution of 'objective things'; and there is the intersubjective constitution of the 'normal ego-community', the individuals of which observe 'the same' things in their various sensory ways and understand each other's point of view (II.43 f., p. 307).

Husserl's exposition of intersubjectivity in *Ideen* II is developed, by a somewhat complex analogical argument, from a wider use of the term *appresentation*, and with the help of the further concepts of *compresence* and *empathy* [Einfühlung].

An appresentation, in the wide sense of the term, is any observation that 'points back to primordial presentation [Urpräsenz]', i.e., to an encountered whole of any kind, or any aspect of it, but not necessarily to more than a few of its properties or details of possible appearance. Objects of perception which, when primordially present to one subject, can also 'ideally' be present primordially to other subjects, form a 'sphere of common primordial presence' for all subjects in communication with each other. In this way *Nature* is constituted. On the other hand there are objects of perception which are merely subjective, that is, they can be given primordially to only one subject (II.44). 'I experience the *bodies* of other people outside me as primordial presences like other "things", but the inwardness of the *mental* [Seelischen] through appresentation' (II.45).

As far as the 'sphere of common primordial presence' of *physical objects* is concerned, Husserl is evidently asserting that the *same* transcendental objects are knowable primordially by the various people who observe them (the *properties* of these objects seem to differ, but the *unity* which is encountered is the same). His view would be, perhaps, that there would be no 'common ground' or real 'communicability' without a 'sameness' of this kind. To account for the sameness in terms of other causes would be a piece of non-phenomenological theorizing, and would, moreover, lead to an infinite regress.

The position is not so clear in regard to other people's *mental states*. And we must ask whether the *appresentative* knowledge of other people's mental states is for Husserl a *real* participation (though not called primordial); or whether, on the contrary, it is privately conceived by us, following on a kind of inference. The discussion in *Ideen* II proceeds as follows.

In respect of inner and outer perceptions of an *individual* subject (for example, bodily impressions in the fields of the various senses), there is a 'system of appresentations which has its primordial establishment [Vorlage] in primordial connections of orderly coexistence, in such a way that the connected parts and series of parts are not only together

in their "compresence", but refer to one another'. Thus, if I feel an object with the hand and at the same time examine it visually, all the impressions I receive 'belong together' in compresence. 'Compresence then goes over into empathy. The touching hand of another person, which I see, appresents to me the solipsistic view of this hand [by its owner] and then everything which must belong to it in compresence.' This gives, however, only a 'beginning of comprehension of the other person's mental life'; and then various 'appresented interpretations' begin to operate all the time, making use of a 'system of signs' which 'express' the mental events of the other person. 'This manifold bodily expression appresents the mental being [seelisches Dasein] of the other person, who thus becomes [for me] a doubly-unified objectivity—humanity.' Thus 'bits of connectedness' are primordially given me, and 'the rest is then empirical extension, and transference in conceptual experience' (II.46).

From such explanations it might seem that Husserl does not admit any real participation in the mental lives of other people (as, for instance, his former disciple Max Scheler does,[1] or even a real participation in 'distant' objects by the function of spatial-identification (as do Lipps, Scheler, Merleau-Ponty, and others)[2] other than a mere perception of the *same unity*. This seems to be the view of Edith Stein, at the time of writing *Zum Problem der Einfühlung*.[3] She does not recognize the function of spatial-identification (*op. cit.*, p. 16), and regards empathy as a mode of knowledge by 'transference' (*ibid.*, p. 60). On the other hand it can be urged that a postulated 'transference' of 'my' self to the situation of another person, without the ability to participate simply and absolutely in the other person's mental life, contravenes altogether the phenomenological method, by a lapse into psychologism —mistaking the means for the constitutive 'order of reality' presupposed. How can I 'transfer' myself, if *that* to which I must transfer myself has not for me a knowable givenness logically prior to the act of transference?

This stricter phenomenological view of intersubjectivity is far more evidently adopted in Husserl's later writings, notably the fifth *Cartesian Meditation* and the essay on *Der Räumlichkeit der Natur*. Here he seems to outline a 'real hierarchism'[4] in which potentiality may become a directly knowable reality at a higher level. There is indeed an apparent tendency to phenomenalism, which explains all common objectivities as a kind of construction from the one physical (material and mental)

[1] See above, pp. 216 ff.

[2] Above, pp. 134 ff.

[3] English translation by W. Stein, 'On the Problem of Empathy', The Hague, Nijhoff, 1964.

[4] Above, pp. 365 ff.

level of 'reality'. But this seems to result from the desire to stress *both* the contingent contributions of the individual subject *and* the absolute intersubjectivity to which we are thereby led.

Thus although, at first, experience in the world occurs 'in the frame of real possibilities inductively indicating themselves', he sees these possibilities 'penetrating harmoniously and synthetically into the periphery and comprehending a portion of the World-field which is *really observable* [wirklich anschaulich] and which presents itself as attested Being [Sein]' (E.310, my italics). Likewise, although he speaks of the 'openness [Offenheit] of the world' (potentiality?) in terms of Kantian phenomenalism as a 'conceptually-posited infinity [gedanklich gesetzten Unendlichkeit]', it is 'implicitly an *already-formed* peripherality'.

Again, he explains (psychologistically) that the 'world-apperception' proceeds by an historical construction [Fortbildung] aimed at 'unity of world-view'. And 'because of experience-synthesis in unanimity [Einstimmigkeit] the kernel of actual experience counts as known reality'. Nevertheless, this outer kernel 'becomes the kernel of that which is indicated [vorgezeichnet] through it as the playground of possibilities' (E.310). Being the essential background of all experience, 'what pertains to constitution is, and alone is, absolute and ultimate necessity' (E.325).

If we suitably limit the denotation of the term 'primordial', if we ignore the possible traces of Kantian schematism and phenomenalism in Husserl's work, and admit the function of spatial-identification and empathy likewise as direct participation in transcendence of outer space (as he has admitted transcendence of outer time in 'primary remembrance' and the theory of 'perspectives'), there results a 'real hierarchism' whose general adoption would indeed resolve the 'Crisis of the Sciences' which he saw as a result of their idealized alienation from the *life-world* as a whole.

Quantum Theory and Physical Indeterminacy

Dualistic thought, which would explain all experience and events in terms of two levels of actuality—a level of mental life, and a level of non-mental causality—has run up against a big obstacle in Quantum Mechanics and Quantum Field-Theory. In these subjects the dualistic world-conception produces a hiatus which can be filled only by the irrational postulate of a 'dice-playing God' (to use Einstein's suggestive phrase), or, to speak more strictly, a simple rejection of the Law of Sufficient Reason for isolated experience, while postulating it for the same experiences in an ensemble.

These difficulties did not appear in the Old Quantum Theory, which was originated in 1900 by MAX PLANCK (1858–1947). The formalistic

inadequacies of this were rectified in the Matrix Mechanics of HEISENBERG (1901–) and the Wave Mechanics of SCHRÖDINGER (1887–), developed in the mid 1920's and subsequently shown to be equivalent, about the very time when Russell was engaged in writing *The Analysis of Matter*. Now, for the first time in physics, it was found necessary to build into the formalism certain procedures which express the impossibility of an exact correspondence between the empirical and the conceptually exact.

Subsequent developments, in 'elementary' Quantum Mechanics and the Quantum Theory of Fields, and intensive experimental studies of nuclear structure and elementary particles, have shown, in spite of many obscurities and outstanding problems, that a vast range of physical phenomena can be accounted for with almost miraculous precision by these formalisms, while probably no more than a handful of physicists today are prepared to admit that the hiatus expressed in the formalism could ever be filled without destroying the innumerable and astonishingly detailed correspondences with experiment.

It has become clear, in fact, quite apart from the formalism of quantum theory, that by the very nature of measurement there cannot be an exact correspondence between the abstract (but really operative) substructure and macroscopic measures. The concept of an exact measure, in the observable physical world, of everything involved in nature at the atomic level is operationally meaningless. For the act of measurement means the use of measuring instruments or 'materials' of finite size (slits, light waves, etc.); and these contribute to the thing measured, making a vicious circle. The circle cannot be overcome by a limiting process as parts of the apparatus are made smaller in order to isolate the positions of the smallest physical elements. For then the effect on those elements is more concentrated. And it still remains necessary to measure the *other* parts of the apparatus exactly (by new experiments) at the very time when the measurement in question is to be carried out, and to do this without disturbing the whole. Otherwise the problem to which the calculations and measurement refers is not exactly specified. But such additional exact measurements, infinite in number, are clearly impossible, and indeed would be impossible (without disturbing the apparatus) even if only one or two in number.

Any observation has therefore to be regarded as the macroscopic actualization, in accordance with the total experimental set-up, of a substructure not itself measurable but carrying the potentiality of a variety of measures. The multitudinous confirmations of quantum theory are derived from hypotheses which seem quite incompatible with that of one distinctly actualized 'physical space' and an onlooker mind.

Quantum theory thus builds reasonably into physics a recognition of

the unbridgeable difference between the *a priori* exactness of conceptual systems and the empirical character of any actual measurement. It provides exact laws in the substructure, but also a degree of unpredictability of outcome when a particular measurement is made. This 'unpredictability', moreover, is not incompatible with the Law of Sufficient Reason, unless one adopts a very narrow and absurd interpretation of this 'law', to the effect that a formalism which must perforce formalize *simplified* conditions for the experiment as a whole should nevertheless include the possibility of exactly formalizing every detail of the observational results found on any particular occasion.

It thus appears that the advances of science itself have ruled out the 'one actual space only' theory of physical causality, the 'reading-off' theory of measurement, the 'exact actualization everywhere' theory of experience, and accordingly also the 'non-mental fields and particles' theory of reality, replacing the last-named by a theory of *operations of reason* in a certain conceptual sphere, not in a single space-time frame or set of such frames, but as an intersubjective constitution 'behind the scenes'.

The formalism of quantum theory does not require the *knowability* of the intersubjective constitution, or any part of it, on any given occasion. What it enables us to do is to posit a type of mathematical structure as corresponding with certain features of comparatively simple types of experiment (proceeding without human influence, although set up by it)—with the expectation that measurements of certain other observable features in that type of experiment will agree, within reasonable limits, with what the formalism predicts.

It is maintained in this book, on the other hand, that a direct knowability, i.e. a phenomenological analysis, of selected regions of the intersubjective constitution of the universe of nature and humanity is in principle possible. This means that according to reason, the assumption of unknowability leads to a *reductio ad absurdum*; and secondly, that one can produce evidence of particular knowability, reasonably indicating the possibility of knowability in all other cases, given the requisite conditions.

Various *reductio ad absurdum* arguments could be offered—for instance, if something is postulated as knowable by inference but never directly, the inference could never be verified. It must remain for us, for all time, an *imagined* possibility, or a form of words and symbols which we have filled out with an imagined meaning. Where, then, is the intersubjective *reality* supposed to exist in nature and humanity?

As to the condition of mind and other circumstances necessary for us to carry out the phenomenological analysis in a specific field, and on a

specific occasion, the immense untapped resources of the mind for exploration of the higher levels of constitution have been indicated in various sections of the book, and instances of the more readily developed kinds of phenomenological analysis—recognition of the Gestalt of objects in space, or of the feeling and thought of another person face-to-face in human companionship—are obvious to everyone. Two courses are open to us: to say that the power of analysis reaches so far and no further, and to cease all effort or hope to develop them; or to watch insight gradually developing in us because, instead of rigidly fencing off mind from matter, we seek always for participation and the Higher Reality in a hierarchical universe.

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CONSPECTUS OF PRINCIPLES AND FALLACIES

FALLACIES

1. ABACUS: Supposing that some more or less mechanical procedure can be the ultimate source of knowledge; or that because we arrive at comprehension of *a priori* truths by empirical procedures, those truths are *nothing but* those empirical procedures. p. 41
2. CHICKEN-COUNTING: Anticipating a completeness of analysis and definition which is not attained and may be actually unattainable. p. 39
3. CONCEPTUAL CLOSED SHOP: Settling on a clear usage of certain names or symbols (with the aim to manipulate them like 'counters'); then, when we are presented with other concepts which appear still vague, trying to reduce them to relations between the existing 'counters', or else declaring them meaningless. p. 101
4. LINGUISTIC BLANKET: The belief that we have only to attach a name or lay down a rule of linguistic usage, and immediately the questions which were previously troubling us are either solved or shown to be senseless. p. 393
5. LIVING SKELETON: Accepting an explanation, belief or technique, a certain framework of which seems clear and workable, without realizing that an attempt to carry it out in detail would reveal contradictions or other inadequacies to the situation. p. 99
6. MERELY INTENSIVE DEFINITION: Thinking that a definition in terms of desired effects or properties is sufficient to establish existence without a corresponding 'existence theorem' or the phenomenological exhibiting of techniques by which the thing concerned can be found. p. 251
7. 'PHYSICAL' GEOMETRY: Assuming that an *a priori* exact and universally conceptual geometry can be 'read off' by examining the tabulated results of empirical observations by 'tools' whose manner of prefabrication is irrelevant. p. 169
8. PSYCHOLOGISM: Mistaking an analysis of psychological responses for an analysis of the strictly intelligible structure which prompts them. Mistaking the order of approach for the order of reality. [For a common special case, see the Abacus Fallacy] p. 107
9. SIMPLE LOCATION: Thinking of each event as completely characterizable, without reference to other events, merely by stating its x, y, z, t location in the (supposed) 'container' of space-time. p. 289

PRINCIPLES

1. CO-MOVING REPRESENTATION: Each observer of the class chosen for the purpose of making a universal representation (i.e. of the 'observer-gas') should preferably be represented to himself as at rest. p. 275
2. CONFORMALITY: [A particular application of the Principles of Prior Validity, 22, 23, to the correction for the gravitational effect of a mass-point.] Since *locally* a conformal representation has prior validity over non-conformal ones, the same applies in the universal representation. p. 277

3. CONCEPTUALITY OF DURATIONS AND TEMPI: Durations and tempi in a succession of events (as of notes in a musical phrase) have essential intelligible character, and are therefore capable of being held unchangingly in the mind, analysed phenomenologically, reproduced imaginatively, or primordially recollected—in a way analogous to what is possible with *spatial* intervals and gradients. p. 294
4. CONSISTENT CAUSALITY: Validity of representation is dependent on (1) the fact that the measuring system used must be capable of consistent application and interpretation, (2) the fact that voluntary action and choices by different observers cannot consistently be allowed to affect the measuring system and associated system of representation, and (3) topological considerations. p. 266
5. DEGREES OF PHENOMENOLOGICAL ANALYSIS: The discoverable causes in any object of attention, serving to 'explain' it, can be found and contemplated only by establishing a degree of phenomenological suspension and insight appropriate to the particular discoverable cause in question. p. 252
6. DISCOVERABLE CAUSES: In any phenomenon there is the potential presence of elements which serve to 'explain it'; these elements can be disclosed by phenomenological analysis, and sometimes with the help of material 'prompting extensional devices'. p. 252
7. EPISTEMOLOGICAL AVERAGE: In order to obtain a universal representation, the values of the parameters k, R_0 (and any others that might be included in the associated measurement system) are to be found by a process of fitting to available experimental data. A purely pragmatic principle; the stricter principles determining the most valid technique of fitting are not declared. p. 277
8. GESTALT RECOGNITION: A theory or interpretation which passes over the first need for wholes to be recognized, and postulates at once the existence of certain parts (perceptual details) or distinctions in the whole, is inadmissible. p. 265
9. HELMHOLTZ'S PRINCIPLE: A difference in the perceptions offering themselves to us is always founded on a difference in the real conditions. p. 114
10. IMAGINATIVE LOCALIZATION: When objects are known by an imaginative spatial identification their perceptual details and discoverable causes acquire a localization in the same place as the object itself, such localization having a quality which is partly imaginative and partly given. p. 141
11. IMPLICIT FINE-STRUCTURE: When macroscopic structures come into actualization for us, there is in principle discoverable, in the cognitive nature of each part and of the whole, a *fine-structure* the finding of which provides the 'explanation' of the macroscopic phenomena. [A special case of Principle 6 above.] p. 166
12. INDIVISIBILITY OF THE PHENOMENON (in Quantum Theory): Neither a wave nor a particle can be said to exist till an experiment is set up and considered as a whole; then observation may result in a phenomenon of one or the other kind. pp. 212, 332

13. INTERMEDIATE TRANSFORMATIONS: In the actualization of any object to any subject, a selection or adaptation of the various space-time fields of subject, chosen object, and intermediate objects, occurs, with an integration or synthesis, the final result of which is an actualization for the subject according to the total circumstances on each occasion of observation. p. 208
14. MEASUREMENT-TOOL SPACE-INVARIANCE: A representation which is constructed by certain general rules of procedure *independent of the choice of observer* from among the class of observers chosen for the purpose of making a universal representation (the 'observer-gas'), and independent also of the choice of any axes of reference which may be used by him, has prior validity over a representation in which this is not the case. p. 274
15. MEASUREMENT-TOOL TIME-INVARIANCE: A representation which is constructed by certain general rules of procedure (using a 'measurement-tool' with specification of a certain metric) and having the property that an alteration of the cosmical time-variable, whether by mistake or by passing to the future or past, has no effect on the representation of the 'observer-gas' basic to the measurement-tool, other than to alter its scale, has prior validity over one in which the representation of the 'observer-gas' suffers other deformations. This Principle is subject to a possible qualification, mentioned on p. 277. pp. 232, 275
16. OBSERVER-GAS AND NEBULAR LATTICE COINCIDENCE: If observations clearly indicate a distribution of large masses in the universe, such as would justify the addition of an idealized 'nebular lattice' to the 'material' of our measurement-tool, then prior validity will attach to the representation and measurement-tool in which the nebulae of the idealized lattice coincide with observers of the 'observer-gas' (i.e. the class of observers chosen for the purpose of making a universal representation). p. 275
17. ONTOLOGICAL ABSTRACTION: We can in principle abstract an objective constitution from the total perceived effects of an object which is identified by various persons. [Another name for Principle 31.] p. 208
18. OPERATIONAL IMPLEMENTATION:
    i. All terms and 'inner' operations of any theory must have been analysed phenomenologically, and the signs or words indicating them must be used consistently with that analysis;
    ii. In the case of a physical theory, some terms or 'inner' operations of the formalism must be implemented in 'outer' operations;
    iii. Reasonable correspondence of a physical theory with givenness in nature is further subject to the principles of Weyl and Helmholtz. p. 111
19. PERCEIVED CONCEPTUALITY: Perceptual details have two aspects, (1) a sensory aspect, or aspect of *pure existence*, as if void of intelligible quality, and (2) a conceptual aspect, or aspect of *pure essence*, as if void of sensory quality. p. 132
20. (PERSPECTIVAL) MANIFESTATION UPON OCCASION: No space-time perspective is actual for an observer till his personal standpoint is brought into relation with the object-Gestalt on some total occasion of observation. [A simpler expression of Principle 13.] p. 211

21. POTENTIAL GROUP-OBJECTIVITY: (a name for Principles 29, 28 and 13 conjointly). p. 208
22. PRIOR VALIDITY (I): If, among various possible measures which can be attached to a given measurable condition, each obtained by the consistent operational implementation of a conceptual system, there is one which is presupposed in the calculation of the others, that measure shall be said to have *prior operational validity* in relation to the others. p. 268
23. PRIOR VALIDITY (II): If, among various possible representations of the state of affairs, there is one which is required as an initial stage in the operational construction of the others, that representation shall be said to have *prior representational validity* in relation to the others. p. 269
24. PROMPTING EXTENSIONAL DEVICES: Means are provided in nature, or transcendentally, enabling us to obtain, in a way that is essentially direct but variable according to circumstances, veridical impressions of objects which, in the usual but 'artificial' world-view of physics, are held to be excessively far away or excessively small. p. 248
25. REAL LOCALIZATION: When objects are known by real spatial identification, their perceptual details and discoverable causes are presented as localized in the dame place as the object itself, such localization having its special quality of inescapable *givenness*. p. 141
26. RIEMANNIAN IMPLEMENTATION: A universal measurement-tool, having to be consistent with the local implementability of a flat space-time geometry, must be such that a differential invariant $g_{ij}dx^1dx^j$ in the coordinate increments is an invariant (ds^2), operationally implemented in terms of the Postulate of Proper Time and the Geodesic Postulate. pp. 274, 223
27. SUBJECT-OBJECT TRANSCENDENCE: There are perceptional conditions in which what is observed is neither 'objective' nor 'subjective', in the usual sense of these terms, but is constituted in a 'subject-object middle ground' (because simple location in the 'container' of space is transcended). p. 209
28. TOTAL ENCOUNTER: An object is recognized as a totality, and we can then proceed to discover perceptual details, each of which becomes a *focal actuality* as we bring it to the forefront of our attention. p. 212
29. TOTAL SUBJECT GESTALT: Each observing subject 'carries', or is 'anchored' in, a *proper* conceptual space-time field, with the potentiality of perceptibly passing to any of an infinity of other space-time fields in which there are correspondingly varied relations between subject and particular objects. p. 207
30. TOTAL OBJECT-GESTALT: Each object 'carries', or is 'anchored' in, a *proper* conceptual space-time field, with the potentiality of existing in an infinity of other space-time fields with correspondingly varied configurations. p. 207
31. TRANSCENDENTALLY OPERATIVE OBJECTIVITY: We must acknowledge, and can in principle find, a *givenness* common to or operative in the various perceived effects of an object identified and observed by various persons. p. 209
32. UNIFORMITY: (A principle with this name is sometimes postulated in

order to justify accepting an *absoluteness* of regularity, when no phenomenological justification has been found for it. Such Principle is not accepted here. Without phenomenological justification, impressions of uniformity must be considered always subject to empirical disturbance.) p. 187

33. UNIVERSAL COHESION: There is a real intelligible relationship of every object-Gestalt to every other object-Gestalt, prior to their conjoint manifestation in space and time for any observer on any occasion of observation. p. 210
34. WEYL'S PRINCIPLE: We may not assert the 'givenness in nature' of two different *entities* or *theoretical concepts* or *conditions* if, in terms of the formalism and geometry assumed and the class of experimental data admitted, such entities, concepts or conditions manifest themselves in identical ways. p. 112
35. WEYL'S PRINCIPLE (BROADENED VERSION): We may not assert the 'givenness in nature' of a certain standard condition, as distinct from other conditions of the same class, if, in terms of the formalism and geometry assumed and the class of experimental data admitted, this standard condition cannot be defined by the internal properties, given in nature, of that condition. p. 268
36. WHOLE BEFORE THE PARTS (or TELEOLOGICAL PRINCIPLE): Wherever there is the creative determination of a whole, actually or potentially, there are parts and constituents of substructure which may subsequently be discovered and will then be seen to have 'fallen into place' in the intelligible constitution of the whole, even though they were not individually intended or originally discerned. p. 348

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