COSMOLOGY UPDATE 1998

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For two decades the hot big-bang model has been referred to as the standard cosmology – and for good reason. For just as long cosmologists have known that there are fundamental questions that are not addressed by the standard cosmology and point to a grander theory. The best candidate for that grander theory is inflation + cold dark matter. It holds that the Universe is flat, that slowly moving elementary particles left over from the earliest moments provide the cosmic infrastructure, and that the primeval density inhomogeneities that seed all large-scale structure arose from quantum fluctuations. There is now prima facie evidence that supports two basic tenets of this paradigm, and an avalanche of high-quality cosmological observations will soon make this case stronger or will break it. Key questions remain to be answered; foremost among them are: identification and detection of the cold dark matter particles and elucidation of the mysterious dark-energy component. These are exciting times in cosmology!

1 1998, A Remarkable Year for Cosmology

The birth of the hot big-bang model dates back to the work of Gamow and his collaborators in the 1940s. The emergence of the hot big-bang cosmology began in 1964 with the discovery of the microwave background radiation. By the 1970s, the black-body character of the microwave background radiation had been established and the success of big-bang nucleosynthesis demonstrated, and the hot big-bang was being referred to as the standard cosmology. Today, it is universally accepted and provides an accounting of the Universe from a fraction of a second after the beginning, when the Universe was a hot, smooth soup of quarks and leptons to the present, some 14 Gyr later. Together with the standard model of particle physics and ideas about the unification of the forces, it provides a firm foundation for speculations about the earliest moments of creation.

The standard cosmology rests upon three strong observational pillars: the expansion of the Universe; the cosmic microwave background radiation (CBR); and the abundance pattern of the light elements, D, ^{3}He, ^{4}He, and ^{7}Li, produced seconds after the bang (see e.g., Peebles et al, 1991; or Turner & Tyson, 1999). In its success, it has raised new, more profound questions: the origin of the matter/antimatter asymmetry, the origin of the smoothness and flatness of the Universe, the nature and origin of the primeval density inhomogeneities that seeded all the structure in the Universe, the quantity and composition of the dark matter that holds the Universe together, and the nature of the big-bang event itself. This has motivated the search for a more expansive cosmological theory.

In the 1980s, born of the inner space/outer space connection, a new paradigm emerged, one deeply rooted in fundamental physics with the potential to extend our understanding of the Universe back to 10^{-32} sec and to address the fundamental questions posed, but not addressed by the hot big-bang model. That paradigm, known as inflation + cold dark matter, holds that most of the dark matter consists of slowly moving elementary particles (cold dark matter), that the Universe is flat and that the density perturbations that seeded all the structure seen today arose from quantum-mechanical fluctuations on scales of 10^{-23} cm or smaller. It took awhile for the observers and experimentalists to take this theory seriously enough to try to disprove it, and in the 1990s it began to be tested in a serious way.

This could prove to be a watershed year in cosmology, as important as 1964, when the CBR was discovered. The crucial new data include a precision measurement of the density of ordinary matter and of the total amount of matter, both derived from a measurement of the primeval deuterium abundance and the theory of BBN; fine-scale (down to 0.3°) measurements of the anisotropy of the CBR; and a measurement of the deceleration of the Universe based upon distance measurements of type Ia supernovae out to redshift of close to unity.

Together, these measurements, which are harbingers for the precision era of cosmology that is coming, provide the first plausible, complete accounting of the matter/energy density in the Universe and evidence that the primeval density perturbations arose from quantum fluctuations during inflation. In addition, there exists a large body of cosmological data – from measurements of large-scale structure to the evolution of galaxies and clusters – that supports the cold dark matter theory of structure formation.

The accounting of matter and energy goes like this (in units of the critical density for $H_0 = 65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$): light neutrinos, between 0.3% and 15%; stars and related material, between 0.3% and 0.6%; baryons (total), $5\% \pm 0.5\%$; matter (total), $40\% \pm 10\%$; and vacuum energy (or something similar), $80\% \pm 20\%$; within the uncertainties, a total equalling the critical density (see Fig. 1).

The recently measured primeval deuterium abundance (Burles & Tytler,

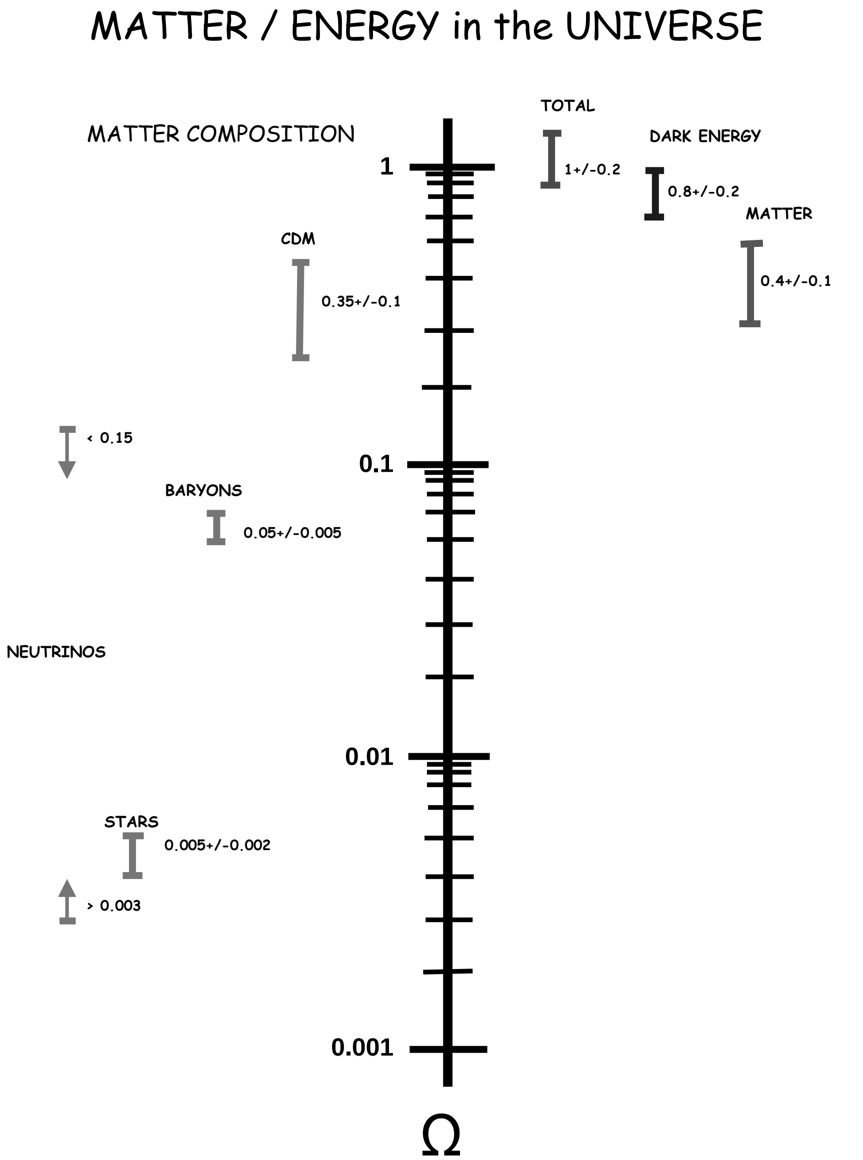

Figure 1: Summary of matter/energy in the Universe. The right side refers to an overall accounting of matter and energy; the left refers to the composition of the matter component. The upper limit to mass density contributed by neutrinos is based upon the failure of the hot dark matter model of structure formation (Dodelson et al, 1996; White, Frenk & Davis, 1983) and the lower limit follows from the evidence for neutrino oscillations (Fukuda et al, 1998). Here H_0 is taken to be $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

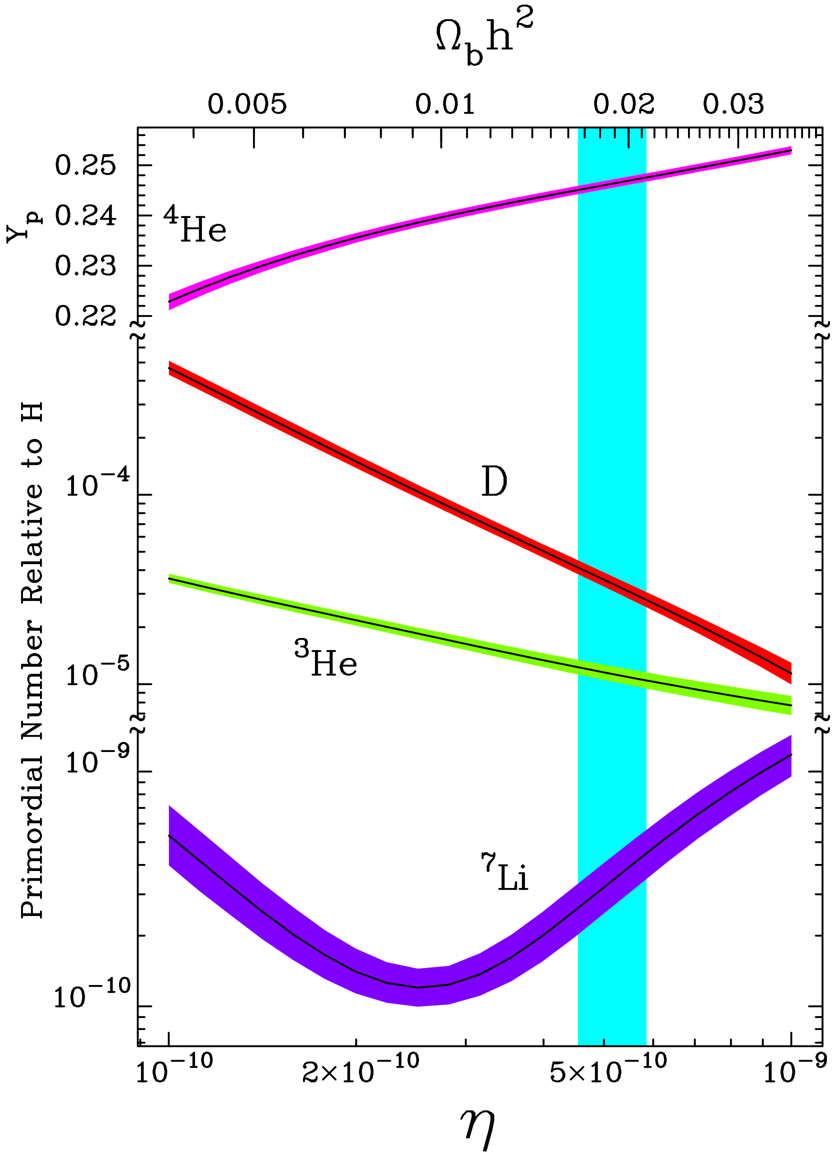

Figure 2: Predicted abundances of ^{4}He (mass fraction), D, ^{3}He, and ^{7}Li (number relative to hydrogen) as a function of the baryon density; widths of the curves indicate "2σ" theoretical uncertainty. The dark band highlights the determination of the baryon density based upon the recent measurement of the primordial abundance of deuterium (Burles & Tytler, 1998a,b), $\Omega_B h^2 = 0.019 \pm 0.0024$ (95% cl); the baryon density is related to the baryon-to-photon ratio, $\rho_B = 6.88\eta \times 10^{-22}\,\mathrm{g\,cm^{-3}}$ (from Burles et al, 1999).

1998a,b) and the theory of big-bang nucleosynthesis now accurately pin down the baryon density (Schramm & Turner, 1998; Burles et al, 1999), $\Omega_B = (0.019 \pm 0.0012)h^{-2} \simeq 0.05$ (for $h = 0.65$); see Fig. 2. Using the cluster baryon fraction, determined from x-ray measurements (Mohr et al, 1998; Evrard, 1996) and SZ measurements (Carlstrom, 1999), $f_B = M_{\rm baryon}/M_{\rm TOT} = (0.07 \pm 0.007)h^{-3/2}$, and assuming that clusters provide a fair sample of matter in the Universe, $\Omega_B/\Omega_M = f_B$, it follows that $\Omega_M = (0.3 \pm 0.05)h^{-1/2} \simeq 0.4 \pm 0.1$. Other direct measurements of the matter density are consistent with this (see e.g., Turner, 1999).

That $\Omega_M \gg \Omega_B$ is strong, almost incontrovertible, evidence for nonbaryonic dark matter; the leading particle candidates are axions, neutralinos and neutrinos. The recent evidence for neutrino oscillations, based upon atmospheric neutrino data presented by the SuperKamiokande Collaboration, indicates that neutrinos contribute at least as much mass as bright stars; the failure of the, top-down, hot dark matter scenario of structure formation restricts the contribution of neutrinos to be less than about 15% of the critical density (see e.g., Dodelson et al, 1996; White, Frenk & Davis, 1983). Because relic axions and neutralinos behave like cold dark matter (i.e., move slowly), they are the prime particle dark-matter candidates.

The position of the first acoustic peak in the angular power spectrum of temperature fluctuations of the CBR is a sensitive indicator of the curvature of the Universe: $l_{\rm peak} \simeq 200/\sqrt{\Omega_0}$, where $R^2_{\rm curv} = H_0^{-2}/|\Omega_0 - 1|$. CBR anisotropy measurements now span multipole number $l = 2$ to around $l = 1000$ (see Figs. 3 and 4); while the data do not yet speak definitively, it is clear that $\Omega_0 \sim 1$ is preferred. Several experiments (Python V, Viper, MAT and Boomerang) with new results around $l = 30 - 700$ should be reporting in soon. Ultimately, the MAP (launch in 2000) and Planck (launch in 2007) satellites will cover $l = 2$ to $l = 3000$ with precision limited essentially by sampling variance, and should determine Ω_0 to a precision of 1% or better.

The same angular power spectrum that indicates $\Omega_0 \sim 1$ also provides evidence that the primeval density perturbations are of the kind predicted by inflation. The inflation-produced Gaussian curvature fluctuations lead to an angular power spectrum with a series of well defined acoustic peaks. While the data at best define the first peak, they are good enough to exclude many models where the density perturbations are isocurvature (e.g., cosmic strings and textures): in these models the predicted spectrum is devoid of acoustic peaks (Allen et al, 1997; Pen et al, 1997).

The oldest approach to determining Ω_0 is by measuring the deceleration of the expansion. Sandage's deceleration parameter, $q_0 \equiv -(\ddot{R}/R)_0/H_0^2 = \frac{\Omega_0}{2}[1 + 3p/\rho]$, depends upon both Ω_0 and the equation of state, $p(\rho)$. Because

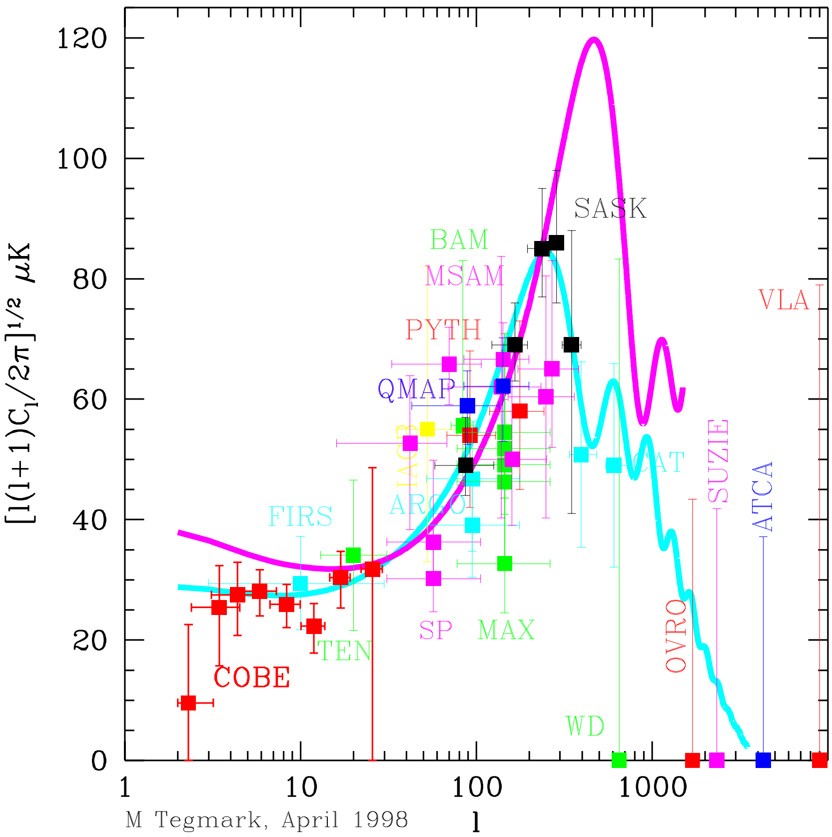

Figure 3: Summary of current CBR anisotropy measurements, where the temperature variation across the sky has been expanded in spherical harmonics, $\delta T(\theta, \phi) = \sum_i a_{lm} Y_{lm}$ and $C_l \equiv \langle |a_{lm}|^2 \rangle$. The curves illustrate CDM models with $\Omega_0 = 1$ (lighter) and $\Omega_0 = 0.3$ (darker). Note the preference of the data for a flat Universe (Figure courtesy of M. Tegmark).

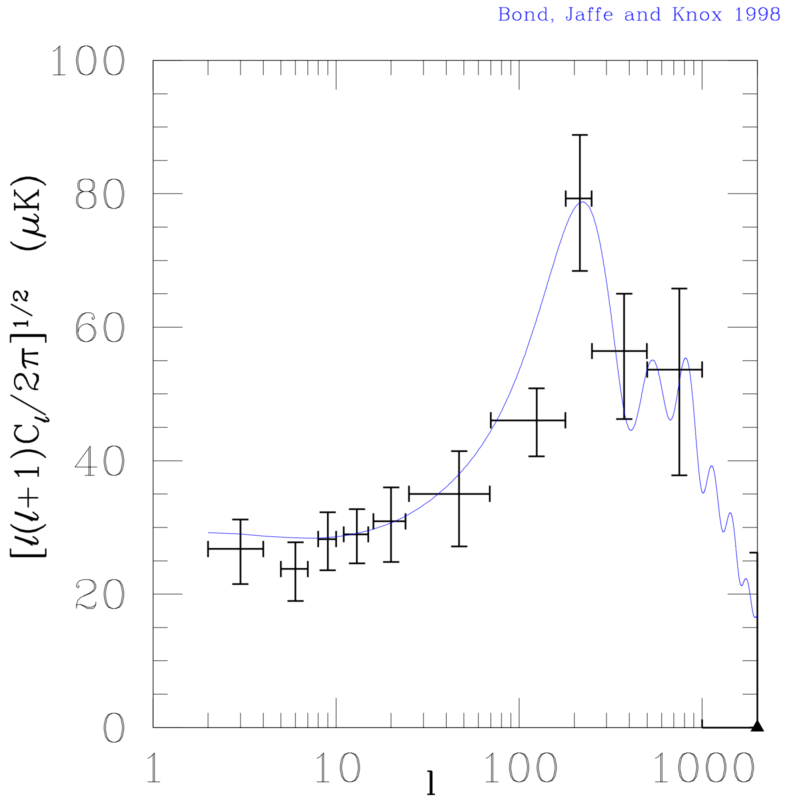

Figure 4: The same data as in the previous figure, but averaged and binned to reduce error bars and visual confusion. The theoretical curve is for the ΛCDM model with $H_0 = 65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_M = 0.4$ (Figure courtesy of L. Knox).

distant objects are seen at an earlier epoch, by measuring the (luminosity) distance to objects as a function of redshift the deceleration of the Universe can be determined. (If the Universe is slowing down, distant objects should be moving faster than predicted by Hubble's law, $v_0 = H_0 d$.) Accurate distance measurements to some fifty supernovae of type Ia (SNe Ia) carried out by two groups (Riess et al, 1998; Perlmutter et al, 1998) indicate that the Universe is speeding up, not slowing down (i.e., $q_0 < 0$). The simplest explanation is a cosmological constant, with $\Omega_\Lambda \sim 0.6$. This result makes the CBR determination of the total density ($\Omega_0 = 1$) and direct measures of the matter density ($\Omega_M \sim 0.4$) consistent: the "missing energy" exists in a smooth component that cannot clump and thus is not found in clusters of galaxies.

The concordance of the three measurements that bear on the quantity and composition of matter and energy in the Universe is illustrated in Fig. 5. The SN Ia results are sensitive to the acceleration (or deceleration) of the expansion and constrain the combination $\frac{4}{3}\Omega_M - \Omega_\Lambda$. (Note, $q_0 = \frac{1}{2}\Omega_M - \Omega_\Lambda$; $\frac{4}{3}\Omega_M - \Omega_\Lambda$ corresponds to the deceleration parameter at redshift $z \sim 0.4$, the median redshift of these samples). The (approximately) orthogonal combination, $\Omega_0 = \Omega_M + \Omega_\Lambda$ is constrained by CBR anisotropy. Together, they define a concordance region around $\Omega_0 \sim 1$, $\Omega_M \sim 1/3$, and $\Omega_\Lambda \sim 2/3$. The constraint to the matter density alone, $\Omega_M = 0.4 \pm 0.1$, provides a cross check, and it is consistent with these numbers. Cosmic concordance!

While the evidence for inflation + cold dark matter is not definitive and we should be cautious, 1998 could well mark a turning point in cosmology as important as 1964. Recall, after the discovery of the CBR it took a decade or more to firmly establish the cosmological origin of the CBR and the hot big-bang cosmology as the standard cosmology.

2 Inflation + Cold Dark Matter

Inflation has revolutionized the way cosmologists view the Universe and provides the current working hypothesis for extending the standard cosmology to much earlier times. It explains how a region of size much, much greater than our Hubble volume could have become smooth and flat without recourse to special initial conditions (Guth 1981), as well as the origin of the density inhomogeneities needed to seed structure (Hawking, 1982; Starobinsky, 1982; Guth & Pi, 1982; and Bardeen et al, 1983). Inflation is based upon well defined, albeit speculative physics – the semi-classical evolution of a weakly coupled scalar field – and this physics may well be connected to the unification of the particles and forces of Nature.

It would be nice if there were a standard model of inflation, but there

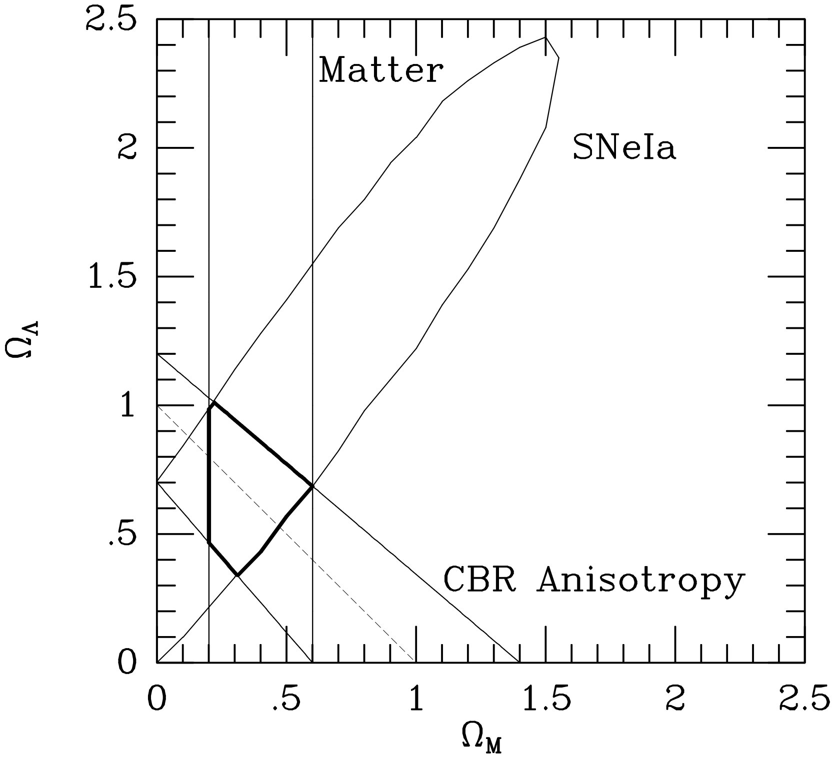

Figure 5: Two-σ constraints to Ω_M and Ω_Λ from CBR anisotropy, SNe Ia, and measurements of clustered matter. Lines of constant Ω_0 are diagonal, with a flat Universe shown by the broken line. The concordance region is shown in bold: $\Omega_M \sim 1/3$, $\Omega_\Lambda \sim 2/3$, and $\Omega_0 \sim 1$. (Particle physicists who rotate the figure by 90° will recognize the similarity to the convergence of the gauge coupling constants.)

isn't. What is important, is that almost all inflationary models make three very testable predictions: flat Universe, nearly scale-invariant spectrum of Gaussian density perturbations, and nearly scale-invariant spectrum of gravitational waves. These three predictions allow the inflationary paradigm to be decisively tested. While the gravitational waves are an extremely important and challenging test, I will not mention them again here (see e.g., Turner 1997a).

The tremendous expansion that occurs during inflation is key to its beneficial effects and robust predictions: A small, subhorizon-sized bit of the Universe can grow large enough to encompass the entire observable Universe and much more. Because all that we can see today was once so extraordinarily small, it began flat and smooth. This is unaffected by the expansion since then and so the Hubble radius today is much, much smaller than the curvature radius, implying $\Omega_0 = 1$. Lastly, the tremendous expansion stretches quantum fluctuations on truly microscopic scales ($\lesssim 10^{-23}\,$cm) to astrophysical scales ($\gg$ millions of light years).

The curvature perturbations created by inflation are characterized by two important features: 1) they are almost scale-invariant, which refers to the fluctuations in the gravitational potential being independent of scale – and not the density perturbations themselves; 2) because they arise from fluctuations in an essentially noninteracting quantum field, their statistical properties are that of a Gaussian random field.

Scale invariance specifies the shape of the spectrum of density perturbations. The normalization (overall amplitude) depends upon the specific inflationary model (i.e., scalar-field potential). Ignoring numerical factors for the moment, the overall amplitude is specified by the fluctuation in the gravitational potential, $\delta\phi \simeq (\delta\rho/\rho)_{\rm HOR} \sim V^{3/2}/m_{\rm PL}^3 V'$, which is also equal to the amplitude of density perturbations when they cross the horizon. To be consistent with the COBE measurement of CBR anisotropy on the 10° scale, $\delta\phi$ must be around 2×10^{-5}. Not only did COBE produce the first evidence for the existence of the density perturbations that seeded all structure (Smoot et al, 1992), but also, for a theory like inflation that predicts the shape of the spectrum of density perturbations, it fixed the amplitude of density perturbations on all scales. The COBE normalization began precision testing of inflation.

3 Cold Dark Matter – A Cosmological Necessity!

While we don't know what the cold dark matter consists of, there is overwhelming evidence that it must be there. (Generically, cold dark matter refers to particles that comprise the bulk of the matter density, move very slowly, interact feebly with ordinary matter (baryons) and are not comprised of ordinary matter.) The biggest surprise in cosmology that I can imagine is the nonexistence of cold dark matter. Here is a brief summary of the most compelling evidence for cold dark matter:

- For more than a decade there has been growing evidence that the total amount of matter is significantly greater than what baryons can account for. Today, the discrepancy is about a factor of eight: $\Omega_M = 0.4 \pm 0.1$ and $\Omega_B = (0.02 \pm 0.002)h^{-2} \simeq 0.05$. Unless BBN is grossly misleading us and/or determinations of the matter density are way off, most of the matter must be nonbaryonic. (The discovery of dark energy does nothing to change this fact; it explains the discrepancy between the matter density, $\Omega_M = 0.4$, inferred from matter that clusters, and the total density, $\Omega_0 = 1$, inferred from CBR anisotropy.)

- We now know that galaxies formed at redshifts of order 2 to 4 (see Fig. 6), that clusters formed at redshifts of 1 or less and that superclusters are forming today. That is, structure formed from the bottom up, as predicted if the nonbaryonic matter is cold. (Hot dark matter leads to a top-down sequence of structure formation; see, White, Frenk, and Davis, 1983.)

- The cold dark matter model of structure formation is consistent with an enormous body of data: CBR anisotropy, large-scale structure, abundance of clusters, the clustering of galaxies and clusters, the evolution of clusters and galaxies and their clustering, the structure of the Lyman-α forest, and a host of other data.

- The only plausible candidate for the bulk of the dark matter in the halo of own galaxy is cold dark matter particles. The last-hope baryonic candidate, dark stars or MACHOs, can account for only about half the mass of the halo and probably much less. This follows from the fact the microlensing rates toward the Magellanic Clouds, which are about 30% of that expected if the halo were comprised entirely of MACHOs, and the growing evidence that the Magellanic lenses are in the clouds themselves or other nonhalo components of the Galaxy.

The two leading particle candidates for cold dark matter are the axion and the neutralino. Both are well motivated by fundamental physics concerns and both have a predicted relic abundance that is comparable to the critical density. There are other "dark-horse" candidates including primordial black holes and superheavy relic particles, which should not be forgotten (Kolb, 1999). As far as cosmological infrastructure goes, they would be every bit as good as axions and neutralinos.

4 Neutrinos by the Numbers

Cosmic neutrinos are almost abundant as CBR photons: $n_{\nu\bar{\nu}} = \frac{3}{11}n_\gamma$ (per species) $\simeq 112\,\mathrm{cm}^{-3}$. Cosmologists are confident of their relic abundance (at least within the standard model of particle physics) because they were in thermal equilibrium until the Universe was a second old; thereafter, their weak interactions were too "weak" to keep them in thermal equilibrium and their temperature decreased as R^{-1}. Shortly after neutrinos "decoupled" electrons and positrons annihilated, raising the photon temperature slightly so that $T_\nu/T_\gamma = (4/11)^{1/3}$. Because the yields of big-bang nucleosynthesis are so sensitive to the phase-space distribution of neutrinos, the success of BBN is also a confirmation of the standard cosmic history of neutrinos.

The sensitivity of BBN to neutrinos allowed Steigman, Schramm and Gunn (1977) to use the yield of ^{4}He to constraint the number of light neutrino species. Their original limit, $N_\nu < 7$, bettered the laboratory limit at the time by almost a factor of 1000. A recent analysis (Burles et al, 1999) finds $N_\nu = 2.84 \pm 0.3$ (95% cl; see Fig. 7), not quite as good as the LEP determination of 3.07 ± 0.24, but still very impressive. Since we are convinced that there are just three standard neutrinos, both the LEP and BBN determinations are now used to search for the existence of new particles; in the case of BBN, light (mass less than about 1 MeV) species with sufficiently potent interactions to be present in significant numbers around the time of BBN. The current BBN limit, with the prior $N_\nu \geq 3$, is: $N_\nu < 3.2$ (95% cl).

Neutrinos were the first candidate for nonbaryonic dark matter (motivated by since refuted evidence for a 30 eV electron neutrino mass in 1978), and this led to the hot dark matter theory of structure formation. While many of its features are qualitatively correct, e.g., the existence of voids, walls and sheets, it predicted "top down" formation of structure (White, Frenk & Davis, 1983) and it is now very clear that structure formed from the "bottom up."

Because they are known to exist and are so abundant, neutrinos may well make up a significant part of the mass budget and be an interesting cosmic spice. Here are the numbers:

$$\begin{aligned}
\Omega_\nu &= \frac{m_\nu}{90h^2\,\mathrm{eV}} \simeq \frac{m_\nu}{40\,\mathrm{eV}} \\
\Omega_\nu/\Omega_B &= \frac{m_\nu}{1.7\,\mathrm{eV}} \\
\Omega_\nu/\Omega_* &= \frac{m_\nu}{0.3h\,\mathrm{eV}} \simeq \frac{m_\nu}{0.2\,\mathrm{eV}}
\end{aligned}$$

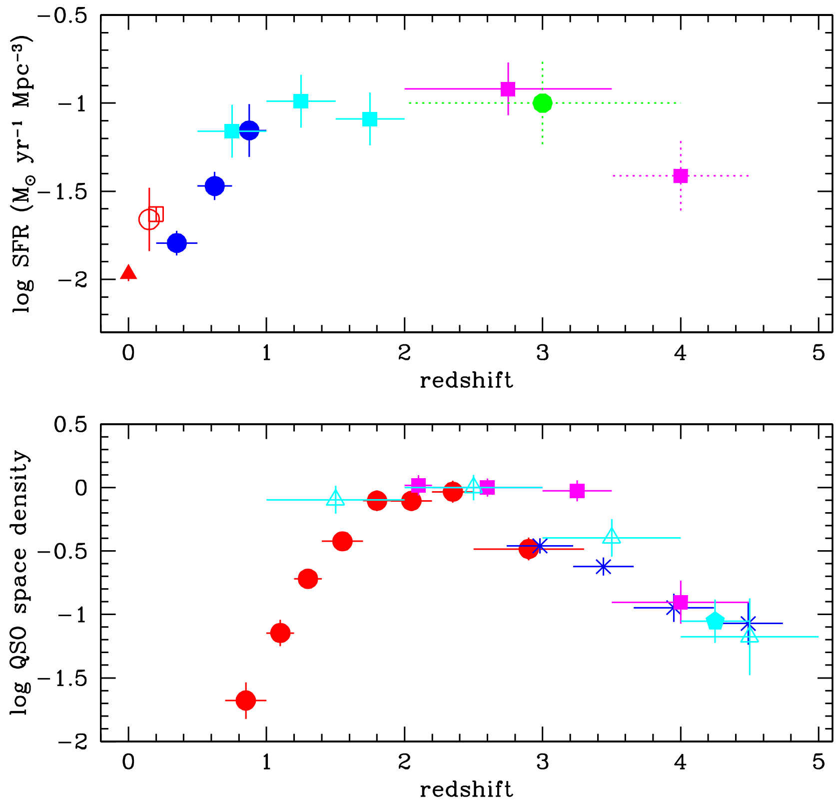

Figure 6: The history of galaxy formation in the Universe, as registered by star formation. The top panel shows the star formation rate vs redshift for galaxies, and the bottom for QSOs (which are hosted by galaxies). As can be seen in both panels, the epoch of galaxy formation occurs at redshifts of a few, as predicted by CDM. The points have been corrected for dust and the relative space density of QSOs (from Madau, 1999)

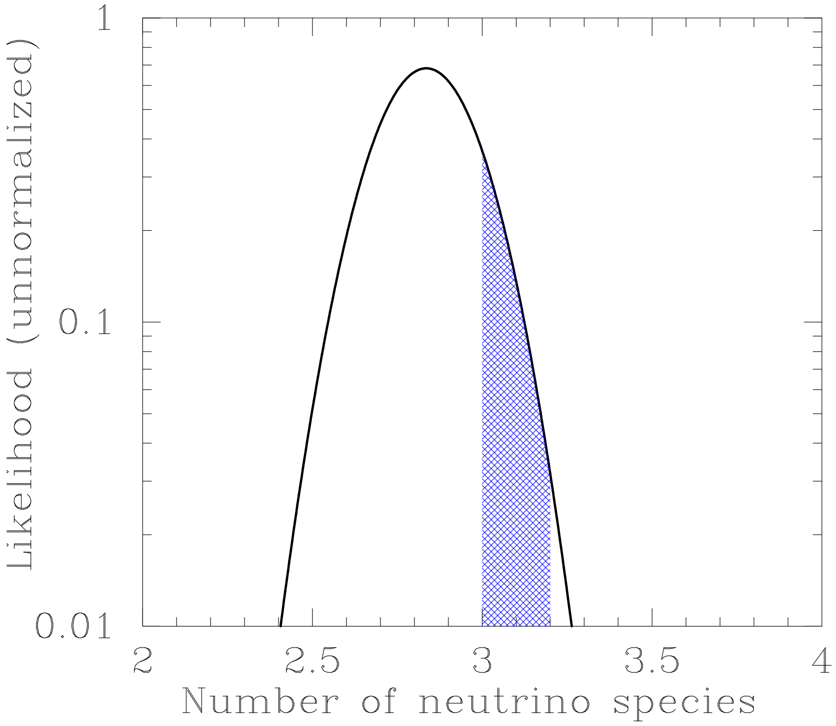

Figure 7: Likelihood function for big-bang production of ^{4}He as a function of N_ν. The BBN yield of ^{4}He (mass fraction Y_P) increases with increasing N_ν; for this analysis the deuterium-determined baryon density was assumed and $Y_P = 0.244 \pm 0.002$. The 95% confidence region is $N_\nu = 2.84 \pm 0.3$; the 95% cl upper limit (with the prior $N_\nu \geq 3$) is $N_\nu < 3.2$; shaded area indicates 95% confidence region for $N_\nu > 3$ prior (Figure courtesy of Scott Burles).

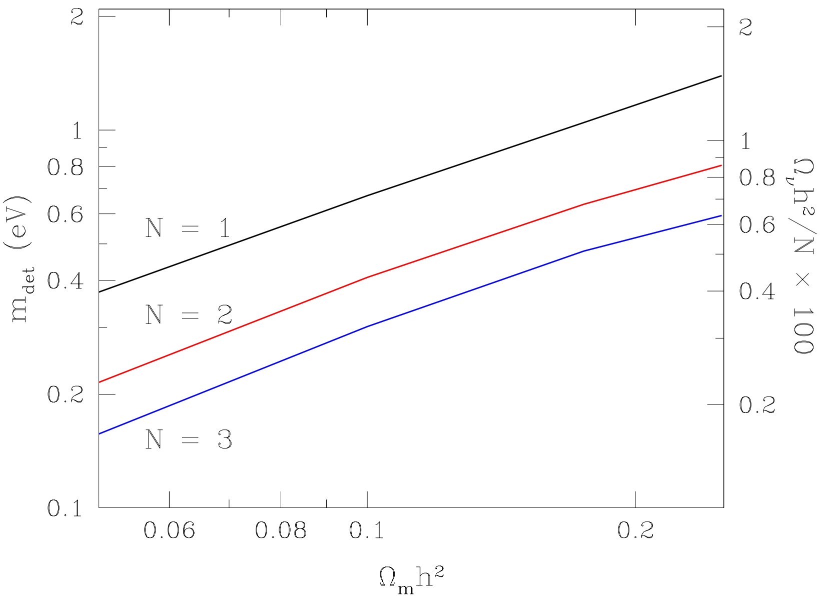

Figure 8: Projected "$2-\sigma$" neutrino-mass detection limit using Sloan Digital Sky Survey large-scale structure measurements. N is the number of neutrino species with the mass indicated and $\Omega_M h^2 \sim 0.15$ is the total mass density (from Hu et al, 1998).

The SuperKamiokande data, which indicate a neutrino mass-squared difference of around $10^{-2}\,\mathrm{eV}^2$, put the neutrino contribution to the cosmic mass budget at an amount at least comparable to that of bright stars. WOW! If the $0.1\,\mathrm{eV}$ mass corresponds to the lightest neutrino species or if the mass difference squared arises from nearly degenerate neutrino species, the total could even greater – $\sum_i m_{\nu_i} \simeq 1.7\,\mathrm{eV}$ would make neutrinos as important as baryons.

A neutrino mass of a few tenths of an eV is already very interesting from the point of view of large-scale structure formation. Hu et al (1998) have shown a neutrino species of this mass can have a potentially detectable influence on large-scale structure, one which could well be detectable with Sloan Digital Sky Survey data. (When CBR anisotropy data are folded in as well, the detection mass-limit might well be even lower.) At the other extreme, the effect on the formation of large-scale structure is so profound (and bad), that $\Omega_\nu > 0.15$ ($m_\nu \sim 4\,\mathrm{eV}$) can already be ruled out (Dodelson et al, 1996).

Finally, in the context of physics beyond the standard model, cosmology still has much to tell us about neutrinos. The properties of massive, decaying neutrinos can be severely constrained by BBN (Dodelson et al, 1994) and the CBR (Lopez et al, 1998). Further, a massive tau neutrino that decays a few seconds after the bang or later and produces relativistic particles, can change the balance of matter and radiation, leading to an interesting variant of cold dark matter called τCDM (see below, and Dodelson et al, 1996).

5 Inflation + CDM in the Era of Precision Cosmology

As we look forward to the abundance (avalanche!) of high-quality observations that will test inflation + CDM, we have to make sure the predictions of the theory match the precision of the data. In so doing, CDM + inflation becomes a ten (or more) parameter theory. For astrophysicists, and especially cosmologists, this is daunting, as it may seem that a ten-parameter theory can be made to fit any set of observations. This is not the case when one has the quality and quantity of data that will be coming. The standard model of particle physics offers an excellent example: it is a nineteen-parameter theory and because of the high-quality of data from experiments at Fermilab's Tevatron, SLAC's SLC, CERN's LEP and other facilities it has been rigorously tested and the parameters measured to a precision of better than 1% in some cases. My worry as an inflationist is not that many different sets of parameters will fit the upcoming data, but rather that no set will!

In fact, the ten parameters of CDM + inflation are an opportunity rather than a curse: Because the parameters depend upon the underlying inflationary model and fundamental aspects of the Universe, we have the very real possibility of learning much about the Universe and inflation. The ten parameters can be organized into two groups: cosmological and dark-matter (Dodelson et al, 1996).

Cosmological Parameters

1. h, the Hubble constant in units of $100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

2. $\Omega_B h^2$, the baryon density. BBN implies: $\Omega_B h^2 = 0.019 \pm 0.0024$ (95% cl).

3. n, the power-law index of the scalar density perturbations. CBR measurements indicate $n = 1.1 \pm 0.2$; $n = 1$ corresponds to scale-invariant density perturbations. Most models predict $n \approx 0.90 - 0.98$; the range of predictions runs from 0.7 to 1.2 (Lyth & Riotto, 1996).

4. $dn/d\ln k$, "running" of the scalar index with comoving scale (k = wavenumber). Most models predict a value of $\mathcal{O}(\pm 10^{-3})$ or smaller (Kosowsky & Turner, 1995).

5. S, the overall amplitude squared of density perturbations, quantified by their contribution to the variance of the CBR quadrupole anisotropy.

6. T, the overall amplitude squared of gravity waves, quantified by their contribution to the variance of the CBR quadrupole anisotropy. Note, the COBE normalization determines $T + S$.

7. n_T, the power-law index of the gravity wave spectrum. Scale-invariance corresponds to $n_T = 0$; for inflation, n_T is given by $-\frac{1}{7}\frac{T}{S}$.

Dark-matter Parameters

1. Ω_ν, the fraction of critical density in neutrinos. While the hot dark matter theory of structure formation is not viable, it is possible that a small fraction ($\Omega_\nu < 0.15$) of the matter density exists in the form of neutrinos.

2. Ω_X, the fraction of critical density in a smooth component of unknown composition and negative pressure ($w_X \lesssim -0.3$). The SN Ia results and CBR anisotropy provide strong evidence for such a component, with the simplest example being a cosmological constant ($w_X = -1$).

3. g_*, the quantity that counts the number of ultra-relativistic degrees of freedom around the time of matter-radiation equality. In the standard cosmology/standard model of particle physics $g_* = 3.3626$ (photons in the CBR + 3 massless neutrino species). The amount of radiation controls when the Universe became matter dominated and thus affects the present spectrum of matter inhomogeneity.

As mentioned, the parameters involving density and gravity-wave perturbations depend directly upon the inflationary potential. In particular, they can be expressed in terms of the potential and its first three derivatives:

$$
\begin{aligned}
S \equiv \frac{5\langle |a_{2m}|^2\rangle}{4\pi} &\simeq 2.2\,\frac{V_*/m_{\rm Pl}^4}{(m_{\rm Pl}V_*'/V_*)^2} \\
n - 1 = -\frac{1}{8\pi}\left(\frac{m_{\rm Pl}V_*'}{V_*}\right)^2 &+ \frac{m_{\rm Pl}}{4\pi}\left(\frac{m_{\rm Pl}V_*'}{V_*}\right)'
\end{aligned}
$$

$$
\begin{aligned}
\frac{dn}{d\ln k} &= -\frac{1}{32\pi^2}\left(\frac{m_{\rm Pl}{}^3 V_*'''}{V_*}\right)\left(\frac{m_{\rm Pl}V_*'}{V_*}\right) \\
&\qquad +\frac{1}{8\pi^2}\left(\frac{m_{\rm Pl}{}^2 V_*''}{V_*}\right)\left(\frac{m_{\rm Pl}V_*'}{V_*}\right)^2 \\
&\quad -\frac{3}{32\pi^2}\left(m_{\rm Pl}\frac{V_*'}{V_*}\right)^4 \\
T &\equiv \frac{5\langle|a_{2m}|^2\rangle}{4\pi} = 0.61(V_*/m_{\rm Pl}^4) \\
n_T &= -\frac{1}{8\pi}\left(\frac{m_{\rm Pl}V_*'}{V_*}\right)^2
\end{aligned}
$$

where $V(\phi)$ is the inflationary potential, prime denotes $d/d\phi$, and V_* is the value of the scalar potential when the present horizon scale crossed outside the horizon during inflation.

If one can measure S, T, and $(n-1)$, one can recover the value of the potential and its first two derivatives (see e.g., Turner 1993; Lidsey et al, 1997)

$$V_* = 1.65T\, m_{\rm Pl}{}^4, \tag{1}$$

$$V_*' = \pm\sqrt{\frac{8\pi}{7}\frac{T}{S}}\, V_*/m_{\rm Pl}, \tag{2}$$

$$V_*'' = 4\pi\left[(n-1)+\frac{3}{7}\frac{T}{S}\right] V_*/m_{\rm Pl}{}^2, \tag{3}$$

where the sign of V' is indeterminate (under the redefinition $\phi \leftrightarrow -\phi$ the sign changes). If, in addition, the gravity-wave spectral index can also be measured the consistency relation, $T/S = -7n_T$, can be used to test inflation.

Bunn & White (1997) have used the COBE four-year dataset to determine S as a function of T/S and $n-1$; they find

$$
\begin{aligned}
\frac{V_*/m_{\rm Pl}^4}{(m_{\rm Pl}V_*'/V_*)^2} = \frac{S}{2.2} &= (1.7\pm 0.2)\times 10^{-11} \\
&\quad \times\frac{\exp[-2.02(n-1)]}{\sqrt{1+\frac{2}{3}\frac{T}{S}}}
\end{aligned}
\tag{4}
$$

From which it follows that

$$V_* < 6\times 10^{-11} m_{\rm Pl}{}^4, \tag{5}$$

equivalently, $V_*^{1/4} < 3.4\times 10^{16}\,$GeV. This indicates that inflation must involve energies much smaller than the Planck scale. (To be more precise, inflation could have begun at a much higher energy scale, but the portion of inflation relevant for us, i.e., the last 60 or so e-folds, occurred at an energy scale much smaller than the Planck energy.)

Finally, it should be noted that the 'tensor tilt,' deviation of n_T from 0, and the 'scalar tilt,' deviation of $n-1$ from zero, are not in general equal; they differ by the rate of change of the steepness. The tensor tilt and the ratio T/S are related: $n_T = -\frac{1}{7}\frac{T}{S}$, which provides a consistency test of inflation.

5.1 Present status of Inflation + CDM

A useful way to organize the different CDM models is by their dark-matter content; within each CDM family, the cosmological parameters vary. One classification is (Dodelson et al, 1996):

1. sCDM (for simple): Only CDM and baryons; no additional radiation ($g_* = 3.36$). The original standard CDM is a member of this family ($h = 0.50$, $n = 1.00$, $\Omega_B = 0.05$), but is now ruled out (see Fig. 9).

2. τCDM: This model has extra radiation, e.g., produced by the decay of an unstable massive tau neutrino (hence the name); here we take $g_* = 7.45$.

3. νCDM (for neutrinos): This model has a dash of hot dark matter; here we take $\Omega_\nu = 0.2$ (about 5 eV worth of neutrinos).

4. ΛCDM (for cosmological constant): This model has a smooth component in the form of a cosmological constant; here we take $\Omega_\Lambda = 0.6$.

Figure 9 summarizes the viability of these different CDM models, based upon CBR measurements and current determinations of the present power spectrum of inhomogeneity derived from redshift surveys (see Fig. 10). sCDM is only viable for low values of the Hubble constant (less than $55\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$) and/or significant tilt (deviation from scale invariance); the region of viability for τCDM is similar to sCDM, but shifted to larger values of the Hubble constant (as large as $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$). νCDM has an island of viability around $H_0 \sim 60\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $n \sim 0.95$. ΛCDM can tolerate the largest values of the Hubble constant.

5.2 The best fit Universe!

Considering other relevant data too – e.g., age of the Universe, determinations of Ω_M, measurements of the Hubble constant, and limits to Ω_Λ – ΛCDM emerges as the 'best-fit CDM model' (Krauss & Turner, 1995; Ostriker &

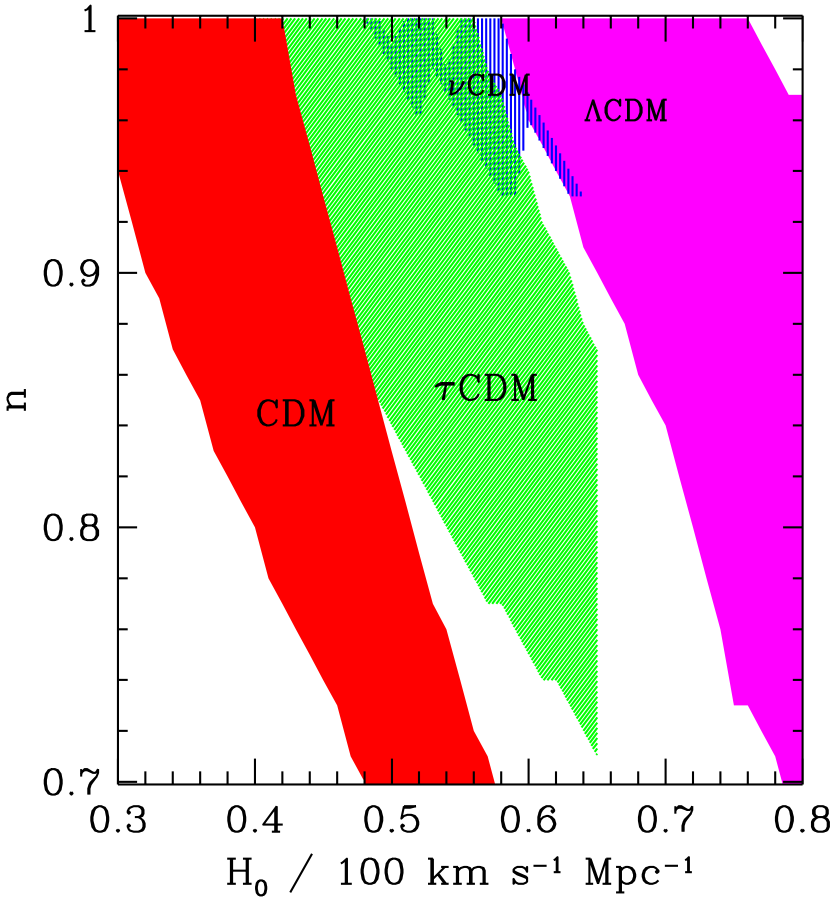

Figure 9: Summary of viable CDM models, based upon CBR anisotropy and determinations of the present power spectrum of inhomogeneity (Dodelson et al, 1996).

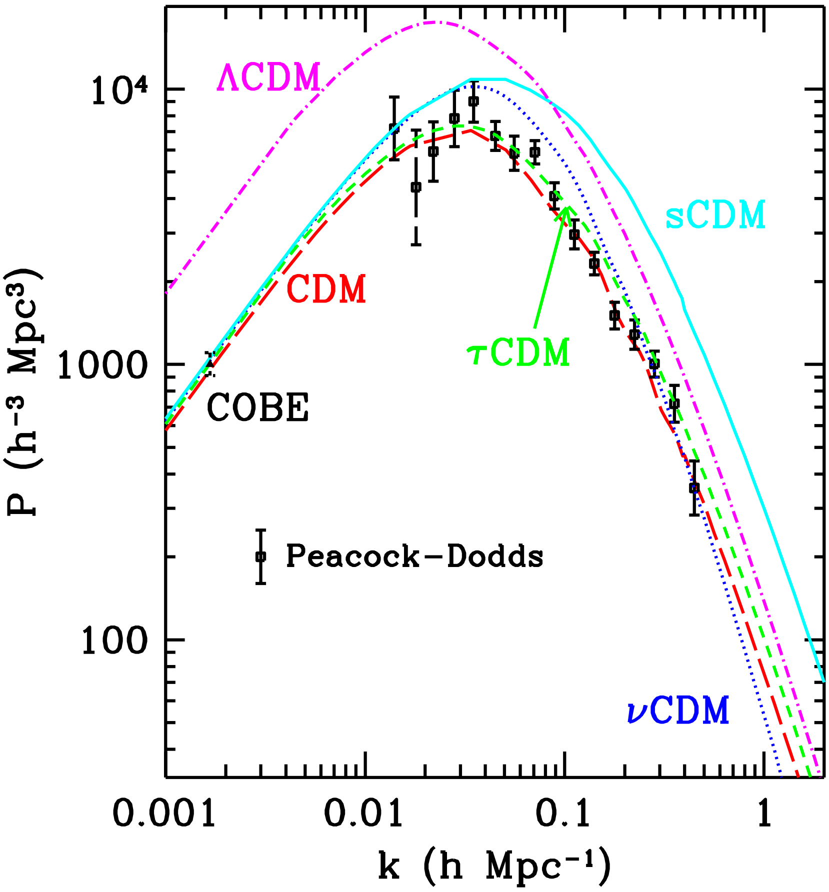

Figure 10: The power spectrum of fluctuations today, as traced by bright galaxies (light), as derived from redshift surveys assuming light traces mass (Peacock and Dodds, 1994). The curves correspond to the predictions of various cold dark matter models. The relationship between the power spectrum and CMB anisotropy in a ΛCDM model is different, and in fact, the ΛCDM model shown is COBE normalized.

Steinhardt, 1995; Liddle et al, 1996; Turner, 1997b); see Fig. 11. Moreover, its 'smoking-gun signature,' accelerated expansion, has apparently been confirmed (Riess et al, 1998; Perlmutter et al, 1998) and it provides an excellent fit to the current CBR anisotropy data (see Fig. 4).

Despite my general enthusiasm, I would caution that it is premature to conclude that ΛCDM is anything but the model to take aim at. Further, it should be noted that the SN Ia data do not yet discriminate between a cosmological constant and something else with large, negative pressure (e.g., rolling scalar field or frustrated topological defects).

6 Checklist for the Next Decade

As I have been careful to stress the basic tenets of inflation + CDM have not yet been confirmed definitively. However, a flood of high-quality cosmological data is coming, and could make the case in the next decade. Here are some of the important aspects of inflation + CDM that will be addressed by these data:

- Map of the Universe at 300,000 yrs. COBE mapped the CMB with an angular resolution of around 10°; two new satellite missions, NASA's MAP (launch 2000) and ESA's Planck Surveyor (launch 2007), will map the CMB with 100 times better resolution (0.1°). From these maps of the Universe as it existed at a simpler time, long before the first stars and galaxies, will come a gold mine of information: Among other things, a definitive measurement of Ω_0; a determination of the Hubble constant to a precision of better than 5%; a characterization of the primeval lumpiness; and possible detection of the relic gravity waves from inflation. The precision maps of the CMB that will be made are crucial to establishing inflation + cold dark matter.

- Map of the Universe today. Our knowledge of the structure of the Universe is based upon maps constructed from the positions of some 30,000 galaxies in our own backyard. The Sloan Digital Sky Survey will produce a map of a representative portion of the Universe, based upon the positions of a million galaxies. The Anglo-Australian 2-degree Field survey will determine the position of several hundred thousand galaxies. These surveys will define precisely the large-scale structure that exists today, answering questions such as, "What are the largest structures that exist?" Used together with the CMB maps, this will definitively test the CDM theory of structure formation, and much more.

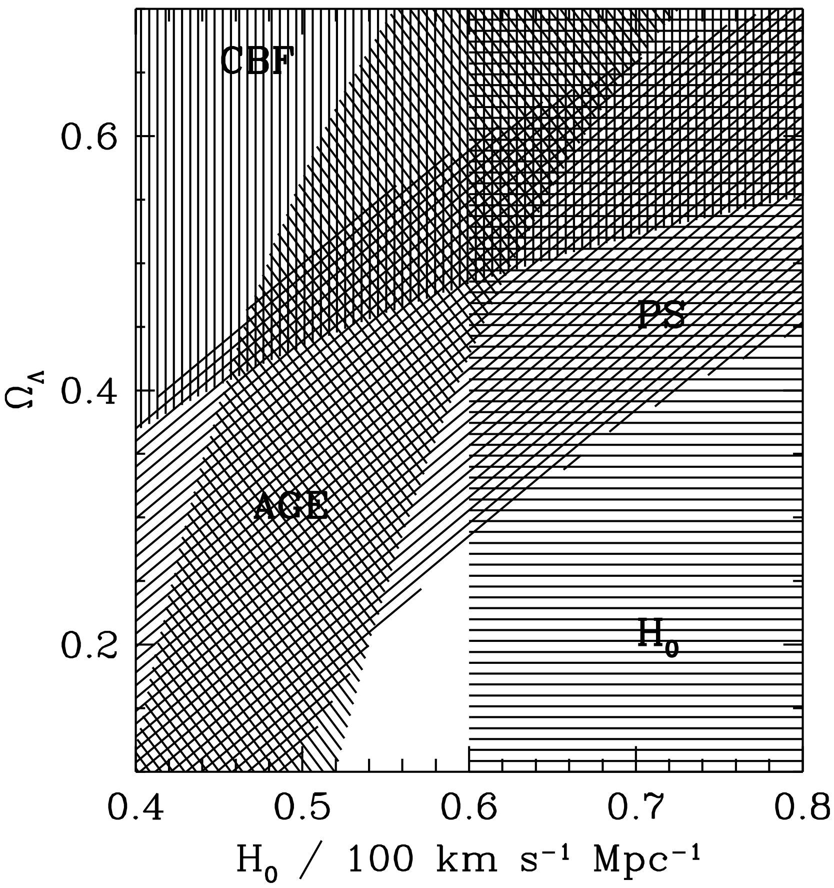

Figure 11: Constraints used to determine the best-fit CDM model: PS = large-scale structure + CBR anisotropy; AGE = age of the Universe; CBF = cluster-baryon fraction; and H_0= Hubble constant measurements. The best-fit model, indicated by the darkest region, has $h \simeq 0.60 - 0.65$ and $\Omega_\Lambda \simeq 0.55 - 0.65$.

- Present expansion rate H_0. Direct measurements of the expansion rate using standard candles, gravitational time delay, SZ imaging and the CMB maps will pin down the elusive Hubble constant once and for all. It is the fundamental parameter that sets the size – in time and space – of the observable Universe. Its value is critical to testing the self consistency of CDM.

- Cold dark matter. A key element of theory is the cold dark matter particles that hold the Universe together; until we actually detect cold dark matter particles, it will be difficult to argue that cosmology is solved. Experiments designed to detect the dark matter that holds are own galaxy together are now operating with sufficient sensitivity to detect both neutralinos and axions (see e.g., Sadoulet, 1999; or van Bibber et al, 1998). In addition, experiments at particle accelerators (Fermilab and CERN) will be hunting for the neutralino and its other supersymmetric cousins.

- Nature of the dark energy. If the Universe is indeed accelerating, then most of the critical density exists in the form of dark energy. This component is poorly understood. Vacuum energy is only the simplest possibly for the smooth dark component; there are other possibilities: frustrated topological defects (Vilenkin, 1984; Pen & Spergel, 1998) or a rolling scalar field (see e.g., Ratra & Peebles, 1998; Frieman et al, 1995; Coble et al, 1997; Caldwell et al, 1998; Turner & White, 1997). Independent evidence for the existence of this dark energy, e.g., by CMB anisotropy, the SDSS and 2dF surveys, or gravitational lensing, is crucial for verifying the accounting of matter and energy in the Universe I have advocated. Additional measurements of SNe Ia could help shed light on the precise nature of the dark energy. The dark energy problem is not only of great importance for cosmology, but for fundamental physics as well. Whether it is vacuum energy or quintessence, it is a puzzle for fundamental physics and possibly a clue about the unification of the forces and particles.

7 New Questions; Some Surprises?

Will cosmologists look back on 1998 as a year that rivals 1964 in importance? I think it is quite possible. In any case, the flood of data that is coming will make the next twenty years in cosmology very exciting. It could be that my younger theoretical colleagues will get their wish − inflation + cold dark matter is falsified and it's back to the drawing board. Or, it may be that it is roughly correct, but the real story is richer and even more interesting. This happened in particle physics. The quark model of the 1960s was based upon an approximate global $SU(3)$ flavor symmetry, which shed no light on the dynamics of how quarks are held together. The standard model of particle physics that emerged and which provides a fundamental description of physics at energies less than a few hundred GeV, is based upon the $SU(3)$ color gauge theory of quarks and gluons (QCD) and the $SU(2) \otimes U(1)$ gauge theory of the electroweak interactions. The difference between global and local $SU(3)$ symmetry was profound.

Even if inflation + cold dark matter does pass the series of stringent tests that will confront it in the next decade, there will be questions to address and issues to work out. Exactly how does inflation work and fit into the scheme of the unification of the forces and particles? Does the quantum gravity era of cosmology, which occurs before inflation, leave a detectable imprint on the Universe? What is the topology of the Universe and are there additional spatial dimensions? Precisely how did the excess of matter over antimatter develop? What happened before inflation? What does inflation + CDM teach us about the unification of the forces and particles of Nature? Last, but certainly not least, we must detect and identify the cold dark matter particles.

We live in exciting times!

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