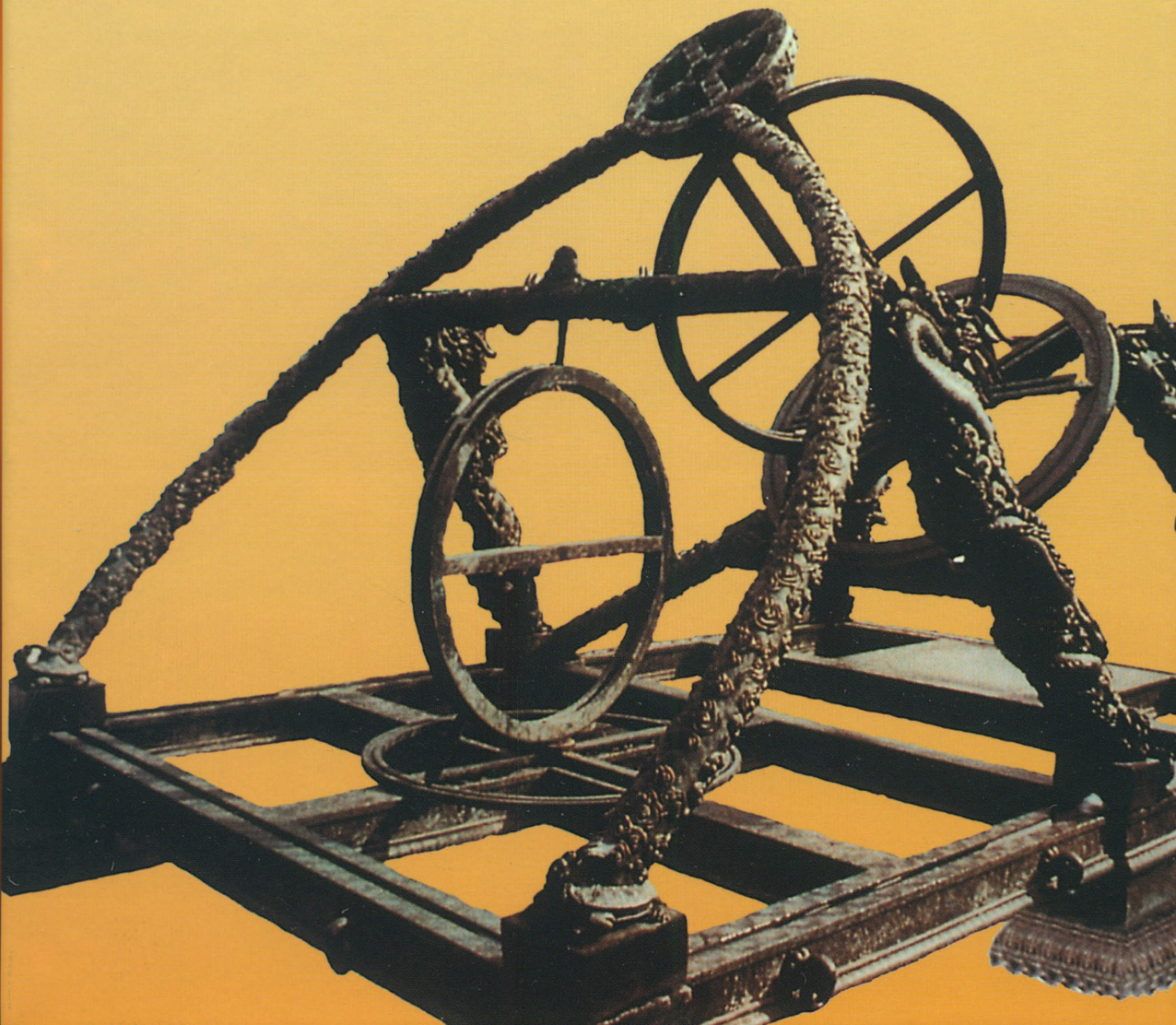
Early Chinese Work in Natural Science

A Re-examination of the Physics of Motion, Acoustics, Astronomy and Scientific Thoughts

Chen Cheng-Yih



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中華早期自然科學之再研討

程貞一著

石泉題

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Chen Cheng-Yih

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To Chén Hsū (Theresé) Hsiá-Chīng 程胥霞青
My mother who taught me by her deeds the meaning of *rén* 仁.

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Foreword

Language difficulties, cultural differences, and religious bias have long been formidable barriers facing many European missionary scholars and other sinologists in their earlier attempts to decipher, and evaluate objectively, what has been so richly documented in the Chinese literature and archaeological records for millennia. Recognition of early Chinese technological contributions to world civilization through such fine products of invention as porcelain wares, silk textile, pulp paper, the printing press, the magnetic compass, pyrotechnic rocket propellants (gunpowder), etc., required little literary translations. These could be the reasons why so many Western scholars have concluded that there was no indigenous science in the Chinese civilization, only philosophy in humanity as exemplified by Confucius' teaching, and technology.

The in-depth and extensive research by Joseph Needham and his distinguished co-workers, which resulted in the monumental book series published by the Cambridge University Press over the past four decades (1954–88), showed not only that mathematics and natural science did exist in China, but also that the ancient Chinese had excelled in algebra, digital algorism, astronomy, cosmology, cartography, and in many other areas over the long history of world civilization. In most cases, there was no question of indigeneity and originality. In some cases, questions were raised regarding the possibility of earlier Babylonian influence and transmission. One penetrating question asked by Needham and his co-workers was: Why did post-Renaissance modern science (or its equivalent) develop only in Europe, and in North America by extension, but not in China?[1] This, incidentally, was also the same puzzling question that has been asked by all Chinese scholars and intellectuals troubled by the apparent decline of Chinese civilization over the last few centuries, especially when measured in terms of science and technology, after such

1. Joseph Needham, *Science and Civilisation in China* (Cambridge University Press, Cambridge, 1959), vol. 3, p. 154.

a glorious early beginning and illustrious progress throughout the middle ages. In answering this and other questions, Needham and his co-workers suggested that fruitful union of mathematics with science and the rise of a mercantile-capitalism culture in post-Renaissance Europe were the two most important reasons. Successful union of mathematics and natural science, in turn, was attributed to the birth of the experimental-mathematical method as pioneered by Galileo. In contrast, mathematics and science had remained far apart in medieval European, as in Chinese, society.[2] Furthermore, Needham is also of the view that Chinese mathematics was application-oriented rather than abstract and analytical, and Dàoist philosophy, which led to a concept of an all-embracing Order in an organic universe with no room for Laws of Nature, had greatly inhibited the development of natural science in China.[3]

In retrospect, most scholars would agree that Galileo's experimental-mathematical verification of Copernicus' theory was the key for emancipating free thinking from medieval Europe, and subsequent development of Newtonian mechanics, electromagnetic theory, and quantum mechanics without the help of differential calculus or other suitable mathematical tools would have been extremely difficult, if not impossible. It was probably also true that a mercantile-capitalism culture was essential for sustaining the rapid development of modern science and technology in post-Renaissance Europe, as well as for providing a strong profit motive to launch Christopher Columbus' exploration of a westward short-cut sea route to India in 1492. Conversely, the absence of profit motive[4] was probably the reason for the discontinuation of Admiral Zhèng Hé's 鄭和 great maritime explorations (e.g. 37,000 soldiers and 62 ships in the first of seven expeditions) of the South China Sea, Indian Ocean, and East Africa, which preceded Columbus' explorations by almost a full century. The relative importance of the other factors, such as methodology and philosophy, which have been cited by Needham and his co-workers as having played decisive roles in the development of modern science, however, are not so obvious. Here scholarly opinions cannot be expected to be unanimous and interpretations of ancient Chinese philosophies may, indeed, differ. These are reflected in the comments of Professor Joseph Chen in the introductory and final sections of this book.

As stated in the Author's Note and Introduction, Professor Chen has

2. Joseph Needham, Ibid., vol. 3, p. 153.
3. Joseph Needham, Ibid., vol.3, p. 156.
4. Joseph Needham, Ibid., vol. 3, p. 557.

selected three topics of particular interest in this re-examination of natural science in the Chinese civilization. His choice of acoustics and astronomy as the main topics was stimulated by the findings of the recently completed international research programme on the –5th century Marquis Yǐ set-bells and the astronomical diagram of 28-*xiù* 宿 system discovered in 1978 in Húběi province, China, which helped to settle the question of originality of musical ratios and equatorial astronomy and the issues of true octave and chromatic scale generation raised by Needham and his co-workers. The third topic, physics of motion, was chosen to refute the erroneous notion of some leading historians of science that this topic was totally absent from Chinese physical thinking. By re-examining some of the ancient Chinese texts, Professor Chen found that the classical (non-relativistic) concept of space and time, the inertial properties of matter, Newton's First Law, indistinguishability of an isolated system at rest and an isolated system in constant rectilinear motion (Galilean transformation), etc., were, in fact, quite well understood by some of the early philosophers in the –5th to –1st century. Newton's Second Law was also qualitatively understood, though not mathematically formulated.

In his comments on philosophy, Professor Chen noted that the basic ideas about nature in the various philosophical schools of ancient China were remarkably similar in spite of their difference in humanistic teachings. The common view, in essence, was that celestial and terrestrial processes such as orderly placement and movement of heavenly bodies, birth and death of stars, flowing of water, etc., all developed spontaneously according to the 'way of nature' (i.e. the *dào* in Chinese philosophy). The 'way of nature' is not always self-manifesting and the cosmic order is not unchanging, so that all humans should learn the way of nature, respect the cosmic order and detect its changes. The perceptions of cosmic structure, origin and evolution, as well as suggested methods for learning the ways of nature and for detection of impending changes in cosmic order, were quite different in the various philosophical schools. Thus, the Mòhist concept of the spacetime continuum had led to later development of the Xuān-Yè 宣夜 theory [5] of an infinite universe with stars and other heavenly bodies floating in vacuum space taught by a school of Chinese cosmologists in the second century, which Needham and his co-workers had noted as being far more advanced in their comparative study of pre-Renaissance astronomy in different civilizations. The Dàoists had conceived a universe with chaotic beginning, which evolved incessantly and spontaneously into great *dào*,

5. Joseph Needham, Ibid., vol. 3, p. 219.

heaven, earth, and man, in that order. In the universe visualized by Chinese philosophical schools, a *Yīn-Yáng* 陰陽 principle and a force-field-like entity *qì* 氣 were also introduced to explain binary processes and interactions at a distance frequently observed in nature. To detect impending changes in cosmic order, the Confucians had developed the *Dà-Yǎn* 大衍 procedure for mathematical simulation of the binary production processes. In retrospect, the *Yīn-Yáng* principle can be flawed for its lack of vigour in specification and indiscretion in applications. The *Dà-Yǎn* procedure was good mathematics (precursor of the well-known Chinese Remainder Theorem and of the Binomial Theorem), but a useless tool for prediction of cosmic changes. Yet, these were extremely advanced physical concepts and daring intellectual exercises in their –4th century time frame. With these perceptive insights, Professor Chen shows that the cosmic view shared by all great Chinese philosophers, Dàoists included, was not only non-anthropocentric, but also non-organic. In fact, the basic idea that all cosmic processes developed spontaneously according to the way of nature, was exactly the same as that central to the philosophy of modern science. So, contrary to the earlier assessment of Needham and his co-workers, the net effect of ancient Chinese philosophy on the development of natural science was very positive rather than inhibitive. With these settled, we may now ask the remaining question: Was the rise of a mercantile-capitalism culture in the post-Renaissance Europe the only reason why modern science developed only in Europe, but not in China? Here, Professor Chen's answer seems to be that scientific thoughts and mathematics are, indeed, universal, but a sudden flourishing of science and technology in any particular civilization at a particular time requires fortuitous convergence of many favourable factors, so that the dynamics of social, political and intellectual interactions could have played important roles. This is certainly an interesting proposition which deserves further exploration.

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Author's Note

This work is based on the Sixth Annual East Asian History of Science Foundation Lecture, October, 1988, sponsored by the East Asian History of Science Foundation and the University of Hong Kong. In this written version, I have added the topic astronomy to the two topics, namely the physics of motion and acoustics, selected in the lecture for the re-examination of the early Chinese work in natural science and, in addition, expanded my lecture account of early scientific ideas and thoughts of nature, and their effect on the development of natural science. I am conscious of the fact that the three topics studied here do not necessarily represent adequately the nature of early Chinese work in natural science. But the physics of motion has been considered by leading historians of science as being totally absent from Chinese physical thinking. The accounts of early Chinese works on acoustic and astronomy in the pre-Qín period, on the other hand, contain serious mistakes.

After the presentation of each topic, comments and discussion are given in response to some of the characterizations of Chinese work made by leading sinologists and historians of science. Evaluations are also made to the claims that acoustics and astronomy in China have their roots in Babylonia. The study presented here, though limited only to three topics, serves to illustrate that the traditional accounts and characterizations of early Chinese work in natural science requires drastic modifications.

Some comment should be given with regard to the conventions used in this work. Chinese technical terms, titles, and quotations, on first appearance, are given in romanized form followed by Chinese characters and then English translations, enclosed in parentheses. Thereafter, they will generally be represented in their romanized form without English translations. The system of romanization used is the Hànyǔ pīnyīn 漢語拼音 system, but concatenation are not adopted for all compound Chinese characters. Concatenations are used for proper names such as the names of places. But for Chinese personal names, concatenations are not used to concatenate the first and second given names into one; they are

connected by a hyphen. Tonal marks are provided for all the romanized Chinese characters. For long quotations offset by indentation, the romanized form is omitted, only the Chinese characters and English translations are provided. The book titles in their Chinese character form are enclosed within double angle brackets 《 》, while chapter and article titles in their Chinese character form are enclosed within single brackets 〈 〉.

The method for indicating dates adopted in this work is the modified −/+ system in which the − sign corresponds to B.C. and the + sign corresponds to A.D. This system is used by Joseph Needham in his monumental series *Science and Civilisation in China*. The modified −/+ system does not contain a year 0 (as in the −/+ system used by astronomers). All the B.C. dates will have a − sign. The + sign is explicitly used only for the year A.D. 1 to 99. Any three- or four-digit year without a − sign is an A.D. year.

Acknowledgements

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Introduction

In the late 1940s, when I was a student in the Xú-Huì 徐滙 Middle High School in Shànghǎi taking high school mathematics, I was puzzled by the realization that none of the mathematical theorems in the book which we were studying came from Chinese civilization. One day, I finally asked my mathematics teacher, Mr Liú 劉, for the reason why. He was somewhat impatient with such an unexpected question, but he nevertheless replied with an authoritative tone:

> 我們唸的是西方數學. 如要唸中國數學定律,
> 你應唸《九章算術》.
> We are studying Western mathematics. If you want to study theorems in Chinese mathematics, you should read the *Jiǔ-Zhāng Suàn-Shù*.

That was the first time in my education that I had ever heard of the *Jiǔ-Zhāng Suàn-Shù* 《九章算術》 (*Mathematics in Nine Chapters*) for which I was very grateful. But there was something about this answer which was not quite satisfactory, and yet I could not, at that time, comprehend what it was.

As did many postwar students who went abroad, I decided to major in science, but I sustained an interest in the early Chinese work in mathematics. One day when I was working on a term paper for a sophomore course, the problem with my teacher's answer suddenly occurred to me. Mathematics and science should have no ethnicity or nationality! The theorem that summarizes into an analytic mathematic expression, the relation between vertical, horizontal and their hypotenuse, is the same whether it was proved by Pythagoras and called the Pythagoras (or Pythagorean) theorem in Greek civilization or proved by Shāng Gaō 商高 and called the *gōu-gǔ* 勾股 (width-length) theorem in Chinese civilization. Similarly, the triangular tabulation of binomial coefficients is the same whether it was discovered by Pascal and called the Pascal triangle in European civilization or discovered by Jiǎ Xiàn 賈憲 and called the *shì-suǒ* 釋鎖 ('key to unlocking') method in Chinese civilization. This is because mathematics is universal and cannot be

classified according to ethnicities or nationalities. Thus, it is misleading to speak of Western mathematics or Chinese mathematics. Ethnicity and nationality are certainly of great interest to the history of mathematics, but it seems natural to expect that textbooks on mathematics, as well as on science, should be free from the imposition of any particular civilization.

There is no doubt that in the Greek civilization, Pythagoras played an important role in the development of mathematics. But one must also acknowledge that long before the time of Pythagoras, the theorem that bears his name was already known to a number of early civilizations. Even the claim that it was the proof by Pythagoras (or Pythagoreans) that raised the theorem above the level of empirical knowledge has never been substantiated, since early accounts of Pythagoras' work are tangled in myths and without any description of the proof. At present, the earliest extant proof of the theorem known in the Greek civilization is found in the *Elements* by Euclid of the −3rd century and in the Chinese civilization is that of Shāng Gaō of the Zhōu 周 dynasty. The Shāng Gaō proof survived through its incorporation into the *Zhōu-Bì Suàn-Jīng* 《周髀算經》 (*Mathematics Classics of Zhōu Gnomons*) (see p.140 to p.143). Although it is well known that Pascal discovered the properties of binomial coefficients in the seventeenth century, this does not make the work of Jiǎ Xiàn on binomial coefficients in the eleventh century (see p. 212 to p.213) any less significant in the history of mathematics. His triangular tabulation of coefficients can be found in the thirteenth century book by Yáng Huī 楊輝.[1] One of the major contributions of modern historians of science, such as Joseph Needham and his collaborators, has been to help us realize that science and technology are rooted in a multicultural tradition.[2]

It was in the seventeenth century, under a new level of intellectual awareness in Europe, that the fusion of accumulated achievements from various civilizations began to take place and initiated a period of unprecedented progress in science. As a result, 'modern science' emerged. So influential was the 'modern scientific method' that it was soon adopted worldwide. We continue today, almost without exception, to approach science in this same general manner. Whether this loss of diversity is beneficial for the future progress of science remains to be seen. But the view, advocated by a number of scholars, that natural science did not exist before the development of the 'modern scientific

1. Yāng Huī 楊輝, *Xiáng-Jiě Jiǔ-Zhāng Suàn-Fǎ* 《詳解九章算法》(A *Detailed Analysis of the Mathematical Methods in the Nine Chapters*), 1261.
2. See the monumental series entitled *Science and Civilisation in China* (University Press, Cambridge, 1954–88) by Joseph Needham and his collaborators.

method', requires some comment. Such a view is not unrelated to the opinions held by some historians of science that there was no natural science, only technology, in ancient China.

It is certainly true that the development of the 'modern scientific method' and its integration into natural science were crucial for the emergence of 'modern science'. This does not, however, imply that before the development of this method, natural science did not exist. One must realize that natural science deals with the discovery and interpretation of the 'truths' of nature and the 'modern scientific method' is only a system of logical procedures devised by man for his inquiry of nature. Although methods are an integral part of natural science, they are not the products of inquiries into nature. The laws of nature are well-hidden and difficult to grasp. And it is well to keep in mind that no single pre-defined method can guarantee the success of a scientific inquiry of nature. Often, in fact, discoveries of laws of nature involve intangible influences from the individual, such as intuition and creativity, which defy rational analysis. Thus, methodology cannot be the only criterion for the evaluation of scientific accomplishments. It may also be worthwhile to note that the validity of the 'modern scientific method' does not preclude the possibility of other, yet unknown, methods of approaching natural science.

It has often been said that science progresses in leaps and stagnations. The leaps are triggered by revolutionary discoveries. Once the new possibilities offered by these discoveries are exploited, stagnation sets in until the next leap is triggered by further revolutionary discoveries. Although such a description provides, at most, the skeleton of the process without any consideration for the range of parameters which could influence its pace or even disrupt it, the description nevertheless brings into the picture the underlying cyclic nature of the process in the progress of science.

The stagnation of scientific progress in Chinese civilization in the last three centuries, compounded by the aggressions and exploitations by the then newly industrialized nations, as well as by the inability of the Chinese to respond to the changes properly and adequately, forced many Chinese intellectuals to undergo self-examination. In their haste to determine the cause for the absence of early Chinese participation in the development of 'modern science', almost every facet of Chinese civilization has, at one time or another, been blamed as the cause. Actions based on such hasty conclusions, though well intended, have done much to break down the very intellectual fibres that constitute Chinese civilization. Circumspect analyses and careful conclusions are necessary for prudent actions.

The fact that 'modern science' was first developed in Europe, and

not anywhere else, has led a number of scholars to conclude that there must be some conducive factors in Western civilization facilitating such a development. Considerations of why 'modern science' did not develop in China has also led a number of scholars to conclude that there must be some inhibitive factors in Chinese civilization preventing such a development. These types of reasoning are not very helpful to our understanding of the process for the development of science since they encourage the search and identification of isolated factors as the causes of the success or failure for the development of modern science. But the development of science is a dynamic process involving the interplay of many, often opposing, factors, and the dynamic compositions of these factors can change rapidly with time. This can best be appreciated if one compares Europe in the Dark Ages with Europe after the Renaissance.

Faced with the widespread ignorance and intellectual stagnation during the Dark Ages between the fifth and the tenth centuries, a pre-modern scholar, basing his thinking on the same type of reasoning, could equally well conclude that there must be some inhibitive factors in Western civilization preventing the progress of science. In fact, for any given civilization, one can always simultaneously prepare a list of factors which are favourable to the development of science and a list of factors which are unfavourable. The crucial factors cannot be identified purely from the intrinsic merits of the factors in such lists without considering the dynamics involved.

There are scholars who choose to identify the crucial factors with so-called 'crucial steps' by direct comparisons of historical paths. One must realize that the steps which make up a historical path are not necessarily logical or self-consistent. This implies that the historical path along which 'modern science' had developed cannot be the only path along which it could have developed. It is this non-uniqueness of historical paths that makes the identification of 'crucial steps' or the 'absence of crucial steps' somewhat dubious. One must be careful in distinguishing historical steps from logical steps when identifying the so-called 'crucial steps'.

An obvious requirement for an adequate evaluation of the status of a civilization in science is to have an objective account of the contributions of that civilization. This requirement is not at all as easy to provide as it appears to be. In comparison with the history of other activities, such as art, literature, society, war, etc., the history of science is a relatively new academic discipline which has emerged under the strong influence of European advances in 'modern science and technology'. It is, therefore, not unexpected that the majority of scholarly works in this field view the development of science within the lineage of Western civilization. Such a Eurocentric view tends to place

greater emphasis on early Greek contributions, neglecting contributions from other civilizations. It is not until recent decades that the contributions to science from other civilizations have begun to receive some recognition.

The recognition of Chinese work in science and technology as contributing an integral part in the development of science has signalled a significant change from the stereotyped view of science in ancient China generated by some of the early missionaries and traditional sinologists. In fact, the stereotyped view was so well rooted in Western literature on China that few later scholars found a need to consult original Chinese records for verification. It was the new source materials produced in recent decades by Needham and his collaborators, through their systematic research on science in China, that facilitated this change of perspective in the West. This change, together with the remarkable accumulation of artifacts discovered in recent decades by archaeological explorations of China's past, has promoted a burgeoning interest in the reexamination of early Chinese contributions to science and technology.

In this work, I re-examine the nature of natural science in early Chinese civilization (primarily in the pre-Qín 秦 period) for a few selected topics. The selection is based essentially on the recent lectures given in Hong Kong.[3] In addition to the topics of physics of motion and acoustics, I have added the topic of astronomy. The Chinese works in alchemy and medicine are better studied[4] than those, for example, in mechanics and acoustics. I, therefore, choose to first re-examine the topic of physics of motion[5] which is chosen not only because it is fundamental to natural science but also because of the important roles it later played in the development of 'modern science' through the works of Galileo (1564–1642)[6] and Newton (1642–1727).[7] The second topic to

3. The Sixth Annual East Asian History of Science Foundation Lecture, October, 1988, sponsored by the East Asian History of Science Foundation and the University of Hong Kong.
4. For a recent discussions on alchemy, see Ho Peng Yoke in *Li, Qi and Shu: An Introduction to Science and Civilization in China*. (Hong Kong University Press, 1985) pp.173–217; for medicine see Paul U. Unschuld in *Medicine in China: A History of Pharmaceutics* (University of California Press, 1986) and see Lu Gwei-Djen and Joseph Needham in *A History and Rationale of Accupuncture and Moxa* (The University Press, Cambridge 1980).
5. Based on a paper entitled 'Early Chinese Work on the Physics of Motion' presented at the Fourth International Conference on the History of Science in China held at the University of Sydney, Australia, May 16–21, 1986. A Chinese translation of this paper by Xú Huá-Kūn 徐華焜 and Wáng Jīn-Guāng 王錦光 is published in the *Kē xuéshǐ Yicóng* 《科學史譯叢》 (*Journal of History of Science in Translation*) No. 2 (31) 1–5, 1988.
6. See Galileo Galilei (1638).
7. See Issac Newton (1687).

be re-examined is acoustics[8] followed then by astronomy. Acoustics and astronomy were not only the major topics of early science but also enjoyed the most significant progresses. They both have the distinction of being the earliest fields in natural science to first establish a relationship with mathematics. Following these topics, I present a concise account of early scientific ideas and thoughts of nature, and their effect on the development of natural science.

One of the problems with analysing early science in any civilization lies with the interpretation of technical terms, because these terms have a tendency to acquire additional meaning as science progresses. Thus, through the use of technical terms, there is the possibility of reading modern meanings back into antiquity. This problem can be particularly serious for Western scientific traditions because the majority of 'modern' technical terms are derived directly from Greek terms and their meanings have been changed and enriched through the centuries. Although this is less a problem for Chinese scientific terms than for the Western terms, the same problem may arise in the translation of the Chinese terms. To minimize this problem, original texts, whenever possible, are reproduced for direct comparison.

It is also important to note that, in examining and discussing early works in science, relevant ancient materials are often organized and presented in current scientific notation and format. This is certainly convenient since current scientific notation and format are almost universally adopted and easily understandable to the readers. However, this very manner of organizing and presenting ancient materials is itself a modern input. Certain caution must be exercised in evaluating ancient materials presented in such a manner. Let us examine for the moment the topics in natural science considered here, namely physics of motion, acoustics, and astronomy. Although acoustics and astronomy were recognized, very early in Chinese civilization, as fields of learning under the name *lǜ* 律 (or *lǜ-xué* 律學, lǜology[9]) and *lì* 曆 (and/or *tiān-wén* 天文),

8. Based on papers, 'Bronze Dual-Tone Set-Bells and the Chromatic Scales' and 'The Significances of Marquis Yǐ Bronze Set-Bells in the History of Acoustics' presented, respectively, at the Fifth International Conference on the History of Science in China held at the University of California, San Diego, August 5–1, 1988, and at the International Culture and Science Exchange: Marquis Yǐ Set-Bells held at Wǔhàn, China, November 3–8, 1988.
9. The areas of early acoustics, that were of interest to the ancient Chinese known as *lǜ-xué* 律學, has been translated by some scholars as 'temperament' but the scope and interest of *lǜ-xué* are not well represented by those of the temperament. The term *lǜ-xué* means literally the study of *lǜ* 律. It would probably be better not to translate the term at all. If it is necessary, one probably may coin the term 'lǜology' to represent this branch of early science in Chinese civilization.

respectively, as evidenced by the extant writings devoted specifically to these topics, the physics of motion was certainly not identified nor was consciously pursued as a specific topic of learning in ancient times. Consequently, the organization of these materials in modern format has unavoidably introduced in them certain modern notions. For this reason, it is important for readers who are interested in these topics to consult the original texts for the context within which the materials originated. On the other hand, it should also be noted that materials without substance cannot be improved appreciably by simply organizing and presenting them in a different notation and format.

1 Early Chinese Work on the Physics of Motion

無動而不變, 無時而不移.
Zhuāng Zǐ · Qiū Shuǐ 《莊子 · 秋水》
(*The Book of Master Zhuāng*)

The direct physical experience of man in his immediate environment is predominantly mechanical. Ever since the dawn of civilization, humanity has been fascinated by the phenomenon of motion. They not only studied it but made ingenious use of it. But not until the phenomenon of motion was analyzed in terms of its general features and its relationship to force did the study of motion begin to take shape as a branch of physical science known as mechanics.

1.1 The Conceptualizations of Space and Time

To describe motion, it is necessary to have an understanding of space and time. The early Chinese conceptualization of space and time is best stated in the *Mò Jīng* 《墨經》 (*The Mòhist Canon*).[10] We have:

> 宇, 彌異所也.
> 久, 彌異時也.
> Space is that which extends to different positions.
> Time is that which extends to different moments.[11]

10. The *Mò-Jīng* which contains two canon sections and two sections on canon exposition was compiled in the early −4th century. The canon section contains a number of Mò Zǐ's own teaching of the −5th century. The book, as we now have it, also contains the work of later Mòhists down to the −3rd century.
11. The translations in this work are made in consultation with earlier translations whenever they are available. Every effort has been made to retain their original

The space and time as defined by these statements are the general continuum space and continuum time extending without bounds. These concepts are theoretical abstractions of bounded space and finite time intervals.

The Mòhists, Mò Jiā 墨家, were well aware of the fact that time cannot be perceived directly through our five senses. In the *Mò-Jīng*, it is stated:

> 知而不以五路, 說在久.
> There are things that we know but cannot be perceived by the five paths (i.e. our five senses). An example is time.

The Mòhists must have also deduced this concept of time from observations of changes. This is obvious from the word *jiǔ* 久 adopted to represent time, since it is a borrowed term which, according to the *Shuō-Wén Jiě-Zì* 《說文解字》 (*Analytical Dictionary of Characters*) of 121, means originally 'blocking' and, in the course of time, took on the connotation 'long duration' associated with the length of the blocking action. It was through such connections that the word was borrowed to represent the abstract concept of time.

In the *Mò-Jīng*, we find also discussions on the concept of a point in space and an instant in time. As a specific example of an instant in time, the Mòhists considered the instant of onset by defining the term *shǐ* 始:

> 始, 當時也.
> 始, 時或有久, 或無久, 始當無久.
> The onset-instant is an instantaneous moment in time.
> Time sometimes has duration and sometimes not,
> the onset-instant has no duration.

Here we see that the concept of an instant in time is visualized as an instantaneous moment (*dāng shí* 當時) with no duration.

Further insights on Mòhist interpretations of space and time can be found in the 'exposition' section of the *Mò-Jīng*. We have the following elucidation of the definitions of space and time:

> 宇, 東西家南北.
> 久, 古今旦莫.
> Space extends to the east, west, south and north [from] a *jiā* 家 (center).
> Time extends to the past, the present, the morning and the evening.

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meaning and writing style. For the convenience of the reader, the Chinese writings are reproduced with their English translations, with supplied interpretations given in square brackets [ ] and explanations given in parentheses. The reader is urged to also consult translations by Joseph Needham and his collaborators in the *Science and Civilisation in China*.

Here space is illustrated in terms of directions and time in terms of durations. It is significant that the concept of direction is introduced in the exposition of space. The character *jiā* 家, adopted to represent the centre from which the directions are specified, means literally 'home' and is defined only with respect to an individual and thus, its location can be arbitrarily chosen. This is indeed a remarkable anticipation of the concept of coordinates (see Fig. 1).

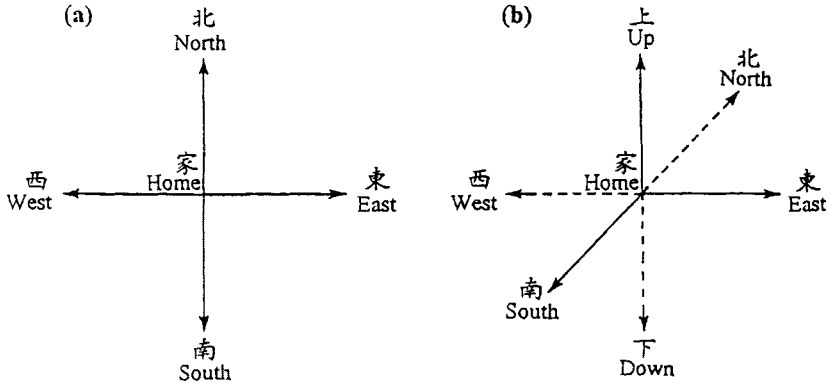

Figure 1. An illustration of space as described in (a) the 'exposition' section of the Mò-*Jīng* 《墨經》 and (b) the *Shī Zǐ* 《尸子》.

The Mòhist exposition of space and time is obviously not in the most general terms since the given four directions subtend only a two-dimensional space (see Fig. 1a). This limitation was soon removed by the work of Shī Jiǎo 尸佼 (fl. –4th century). We have from the *Shī Zǐ* 《尸子》 (*The Book of Master Shī*):

> 宇, 四方上下.
> 宙, 往古來今.
> Space extends to the four cardinal directions
> [as well as] the up and down directions.
> Time extends to the past and the coming of the present.

By the inclusion of the up and down directions in the exposition of space, Shī Jiǎo succeeded in describing a three-dimensional space consistent with the Mòhist definitions of space and time given above (see Fig. 1b).

The coinage of the character *zhòu* 宙 to replace *jiǔ* 久 for denoting the general concept of time in this context is of significance. It was in terms of *yǔ* 宇 and *zhòu* 宙 that the concept of the four-dimensional space-time continuum began to emerge early in Chinese civilization as the characters *yǔ* and *zhòu* were later joined together to form the compound word *yǔ-zhòu* 宇宙 to denote the concept of the universe (see p.156 to p.158).

1.2 Motion in Space

Having examined the early Chinese conceptualization of space and time, we are now in a position to consider their view of motion in space. In describing motion, the basic physical quantities that are of concern are the displacement in position and the duration in time from which other physical quantities of motion, such as velocity and acceleration, may be derived. We find in the *Mò-Jīng* a description of motion in terms of these basic quantities:

> 行, 脩以久, 說在先後.
> 行, 行者必先近而後遠.
> 遠近脩也, 先後久也.
> 民行脩必以久也.
> During motion, displacement is measured in terms of duration, for in motion there is 'earlier and later'.
> In motion, a moving object must first be at a near position and later at a distant position.
> The near and the distant constitute the displacement.
> The earlier and the later constitute the duration.
> The displacement of a person in motion must therefore be measured in terms of duration.

This description of displacement in terms of time duration is of interest since it appears to venture, in non-mathematical terms, toward the concept of 'velocity'. Although the character *xiū* 脩 is defined to be the change in position, it is not clear whether the directional property of displacement was appreciated and implied in the above definitions. Consequently, the term 'velocity' should be interpreted with caution. One thing, however, is certain; the concept of directions was used in the description of space (as mentioned on p.1 to p.3 and Fig. 1).

Mòhists viewed continuous change in location of an object as motion. This is discussed in the *Mò-Jīng*:

> 動, 域從也.
> 動, 偏際從, 若戶樞免瑟.
> Motion is change in location (*yù* 域).
> If the door-pivot is free from [the action of] the bolt, the open-edge [of the door] would yield to [swinging] motion.

It is of interest to note that the example of motion chosen in the 'exposition' section of the *Mò-Jīng* is the motion of a swinging door. This is not an example of pure translational motion, but an example of rotational motion of an extended body.

Motion is also a topic of quantitative study in mathematical treatises. In Chapter 6 of the *Jiǔ-Zhāng Suàn-Shù* 《九章算術》 (*Mathematics in Nine Chapters*), for example, there are a number of problems designed to study constant motion. Reproduced below as an illustration is Problem 16:

今有客馬, 日行三百里. 客去忘持衣, 日已三分之一,
主人乃覺. 持衣追及與之而還, 至家視日四分之三.
問主人馬不休日行幾何?
Given a host, a visitor, and a visitor's horse which travels at a speed of 300 *lǐ* 里 / day. The visitor forgot to bring his clothes which was not discovered by the host until one third of the day has gone by. The host chased and brought the clothes to the visitor, but when he returned home the day was three quarters gone. What would be the distance travelled by the host's horse in a day without resting?

Such problems are designed to study the interrelationships among speed, time, and distance in motions with constant speed. Both the method and the answer for problem 16 are given in the book; the distance travelled is found to be 780 *lǐ* 里.

In the early Chinese conceptualization of motion, the problem with reference was an important issue. This issue is best illustrated by a dialectical proposition found in the *Mò-Jīng*:

宇進無近說在敷.
偏宇不可偏舉宇也
進行者先敷近後敷遠.
Moving in space, there is nothing to indicate whether it is coming nearer [or going further away].
The explanation lies in the reference.
Space cannot, in general, be represented by certain selected districts;
[but only with respect to such districts can] a traveller first make reference as near and later make reference as far.

Contained in this proposition is the important physical insight of the relativeness of reference. The space-time continuum visualized by the early Chinese scholars is isotropic and homogeneous. The direction and position of a moving object in space are defined only with respect to a chosen reference.

The physical insights provided by the proposition undoubtedly helped the scholars of the Han period to first offer an explanation as to why the motion of the earth cannot be directly observed by an observer on earth. We have, from the 'Kǎo Líng Yào' 〈考靈曜〉 section of the *Shàng-Shū Wěi* 《尚書緯》 (*Treatise on the Book of Documents*) of the *c.*–1st century, the explanation:

地恒動不止而人不知.
譬如人在大舟中,
閉牖而坐, 舟行不覺也.
The earth is constantly in motion, without stop,
but one does not know about it.
Just as if one sat in a large boat with all the windows
[and doors] closed,
one would not sense the travelling of the boat.

In view of its time period, this is indeed a remarkable passage. Although the account provides no explanation as to how one knows that the earth is in motion, it certainly provides the first correct explanation as to why an observer on earth cannot directly observe the motion of the earth. The explanation is based on the realization that the state of motion and the state of rest are a matter of reference frames in which the observations of the object are made. This, in fact, is one of the theoretical foundations of Newtonian mechanics.

It should be noted that a two-dimensional rectangular coordinate system was certainly developed by the time of the Han period. This is evident from maps of the −2nd century unearthed from Han tombs. These maps are drawn on the scale of 1 *cùn* 寸 (Chinese inch) to 10 *lǐ* 里 (Chinese mile).[12] There is also evidence that, by the −4th century, a celestial equatorial coordinate system was developed to specify the positions of stars (see p.119 to p.125). There are, however, no extant records to indicate that coordinates were ever used to provide a quantitative description of an object in motion.

1.3 A Dynamical Consideration of Motion

In the study of motion, it is natural to ask questions such as why a moving object stops or what causes an object to move. In the past, inquiries of this type gradually led to the recognition of the role of force in motion. We have from the *Mò-Jīng*, an analysis on the cessation of motion:

> 止, 以久也.
> 止, 無久之不止, 當牛非馬, 若矢過楹.
> 有久之不止, 當馬非馬, 若人過梁.
> The cessation of motion (*zhǐ* 止) is due to blocking.
> [That the motion will] not stop in the absence of blocking
> is as [true as that] an ox is not a horse,
> like an arrow passing through between pillars.
> [That the motion would] not stop in the presence of blocking
> is like something being a horse and yet not a horse,
> like people passing over a bridge.

This is an interesting but difficult passage that requires careful analysis.

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12. See the *Gǔ Dì-Tú* 《古地圖》 (*The Ancient Maps*) and the accompanying book entitled *Gǔ Dì-Tú Lùn-Wén Jí* 《古地圖論文集》 (*A Collection of Research Papers on Ancient Maps*) (Wén-wù, 1977), p. 6.

Before discussing the physical content of this analysis, some remarks on the interpretation of the character *jiǔ* 久 are in order. Reproduced below is the entry for the character *jiǔ* 久 in the *Cí Hǎi* 《辭海》 (*Word Encyclopedia*):

【久】紀有切，音九，有韻。俗作乆。㊀永久也。易恆：『久而不已也。』㊁舊也。詳久要條。㊂待也。左傳昭二十四年：『寡君以爲盟主之故，是以久子。』㊃塞也。儀禮士喪禮：『羃用疏布久之。』注：『久讀爲灸，謂以蓋塞鬲口也。』按說文久字段注引士喪禮此文云：『久字不必改讀，蓋久本義訓從後𨸏之，引伸之則凡𨸏塞皆曰久。』

It is seen that this character has a number of meanings, namely (1) for a long time, forever; (2) old, worn out, formerly; (3) wait for, stop over; and (4) blocking. As mentioned earlier, the original meaning of *jiǔ* is 'blocking'. In the course of time, it began to be associated with the length of the blocking action. It was through such a connotation that this character gradually acquired its other meanings. In the context of the above analysis on cessation of motion, the character *jiǔ* 久 is used in its original meaning, 'blocking'.

The use of 'blocking' without the mention of force to explain the cessation of motion is somewhat ambiguous since it seems to suggest that the action of 'blocking', rather than the opposing force produced by the action, is responsible for the cessations. The analysis, nevertheless, provides valuable physical insights. After stating that the cessation of motion is caused by blocking, it attempts to differentiate the varying degrees in the influence of blocking on motion. It begins with the assertion that in the absence of blocking, the motion will not stop. It then points out that even in the presence of blocking, the motion may not necessarily stop. Hence, three cases are recognized here, namely (1)

with blocking, motion stops; (2) without blocking, motion never stops; and (3) with blocking, motion continues. These are cases related obviously to the strength of the blocking. Here, one sees the possibility of developing a quantitative understanding of the process had mathematization of physical concepts become available. Without mathematization, the Mòhists were only able to classify the third case as 'something being a horse and yet not a horse, like people passing over a bridge.' The implied comparison of the metaphor is the case of being in between. What the Mòhists did was to classify all possible cases involving situations in which the blocking is present but is not sufficient to stop the motion as in between cases.

The importance of this analysis, however, lies in its assertion that in the absence of blocking a moving object would not stop. This assertion is not at all self-evident. In fact, it is often stated in college physics textbooks[13] that 'before Galileo's time most philosophers thought that some influence or 'force' was needed to keep a body moving.' The physical insights of the Mòhist analysis were probably derived in part from their concern with the issue of reference as discussed earlier. There are also extant early records of observations of such physical phenomena of motion. For example, in the *Kǎo Gōng Jì* 《考工記》 (*The Artificer's Record*),[14] we have:

> 馬力既竭, 輈猶能一取焉.
> Although the pulling force of a horse has ceased,
> the cart shaft can still make a gain [in distance].

These types of observations undoubtedly were also helpful in realizing the existence of the physical property known later as 'inertia' related to the mass of the object.

The existence of such a physical property in rotational motion was later postulated by Zhāng Zài 張載 (1020–76) of the Northern Sòng 宋 in explaining the constant rotational motion of objects. He stated:

> 凡圓轉之物, 動必有機.
> 既謂之機, 則動非自外也.
> All objects engaged in rotational (and/or spinning) motion
> must have an (internal) 'mechanism' to sustain their motion.
> It is due to the existence of such a 'mechanism'
> that their motion is not caused by an outside agent.[15]

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13. See for example, D. Halliday and R. Resnick (1974), p. 61.
14. The *Kǎo Gōng Jì* was a Qí 齊 document compiled probably in the –5th century or earlier. It was incorporated in the *Zhōu Lǐ* 《周禮》 (*The Book of Zhōu Institutions*) in the –2nd century.
15. See p. 6b of vol. II of the *Zhāng-Zǐ Quán-Shū* 《張子全書》 (*The Complete Work of Master Zhāng*).

The existence of an internal 'mechanism' to sustain rotational motion advocated by Zhāng Zài can, of course, now be confirmed and the mechanism can be identified with 'rotational inertia' (or 'moment of inertia') which depends not only on the mass but also on the configuration of the object.

The role of force in motion is best summarized in the *Mò-Jīng*:

> 力, 形之所以奮也.
> 力, 重之謂, 下與重奮也.
> Force is that which causes the status (of things) to hasten.
> Weight is a force.
> It is the downwardness with weight that gives rise to the hastening.

Here force is visualized to be that which causes things to undergo hastened motion. The key word used to denote this phenomenon is the character *fèn* 奮 which means 'rush', 'hasten' or 'rouse'. According to the *Shuō-Wén Jiě-Zì*, *fèn* originally was a pictogram describing the process of a large bird taking off from a field. Thus, the coinage of this character to describe the phenomenon of changing states in motion is particularly revealing, since such taking-off processes necessitate acceleration to give rise to velocity changes. The Mòhist analysis not only recognizes weight as a force but also states its directional property. It further recognizes that the phenomenon *fèn* (i.e. the increase in 'velocity') during a fall is due to the downward force of weight.

This is indeed a remarkable physical analysis of the role of force in motion. It comes extremely close to the concept of acceleration. Without the aid of mathematics, a precise quantitative definition of acceleration is not possible. This undoubtedly impeded the Mòhists' ability, despite their remarkable physical insights, to analyse quantitatively the role of force in motion. As a result, their experiments with motion on inclined planes and with the motion of spheres did not yield a quantitative understanding of motion. And yet one must acknowledge that natural science had to wait more than 2,000 years before a quantitative understanding of the concept of acceleration was made possible through the work of Galileo and Newton in the seventeenth century.

Quantitative studies of motion with changing speeds are also found in mathematical treaties. Reproduced below is Problem 19 of Chapter 7 of the *Jiǔ-Zhāng Suàn-Shù* 《九章算術》:

> 今有良馬與駑馬發長安至齊, 齊去長安三千里.
> 良馬初日行一百九十三里, 日增十三里; 駑馬初
> 日行九十七里, 日減半里. 良馬先至齊, 復還迎駑馬.
> 問幾何日相逢及各行幾何?
> Given a good horse and an old horse to be dispatched from Chángān 長安 to Qí 齊, a distance of 3,000 *lǐ* 里. The good horse travels with an

initial speed of 193 *lǐ* 里 / day and an increase of 13 *lǐ* 里 / day; the old horse travels with an initial speed of 97 *lǐ* 里 / day and a decrease of 0.5 *lǐ* 里 / day. The good horse reached Qǐ 齊 first and then returned to meet the old horse. At which day will the two horses meet and by then what is the distances travelled by each horse?

The method used to solve this problem is the method of *yíng-bù-zú* 盈不足 a linear interpolation procedure devised primarily for interpolating apparent motion of planets (see p.159).

1.4 The Mechanization Movement

There are scholars who dismiss the significance of the Mòhist work as being an isolated case which had little, if any, impact on the general scientific development in Chinese civilization. This view is based not only on incorrect perceptions about the impact of the Mòhist work, but also on a faulty basis for evaluation since the significance of a scientific work should be measured in terms of its scientific merits. The conceptual framework formulated by Mòhists for the analysis of motion was revolutionary, though the revolutionary nature is less obvious now than it was then, more than 2,000 years ago. The activities of the Mòhist school actually spanned more than two centuries, from the middle of the −5th century to the second half of the −3rd century.

The work of Mòhists had a profound influence, not only on their contemporaries, but also on the subsequent development of science and technology in Chinese civilization. In the previous sections, I have mentioned the possibility of Mòhist influence on the work of Hàn and Sòng scholars on the physics of motion, including the brilliant explanation by Hàn scholars as to why the motion of the earth cannot be directly observed by an observer on earth. The influence of Mòhist work on the concept of time and space in the cosmological models is discussed on p.156 to p.165. In this section, I suggest that the Mòhist work on mechanics, in general, was responsible for initiating the 'mechanization movement' that began in China sometime in the −4th century.

It was certainly not a coincidence that a variety of mechanical contrivances began to appear in China parallel to the Mòhist formulation of a conceptual framework for the analysis of motion. We read, for example, of the invention of the *shuàn chē* 輲車, a mobile scaling-ladder with pulley and counterweight.[16] The prestige acquired from mechanical

16. See the second exposition section of the Mò Jīng. Needham has constructed, based on the description from Mò Jīng, an illustration of the counterweighted scaling-ladder, see *Science and Civilisation in China* (1962), vol. 4, sec. 26(c), p. 21.

inventions at that time is best illustrated by the attempts of the Dàoists (Dào Jiā 道家) to concoct the story of an ultimate machine. This is cast typically in the form of a Dàoist parable. We find in Chapter 5 of the *Liè Zǐ* 《列子》 (*The Book of Master Liè*):[17]

> King Mù of Zhōu 周穆公 made an inspection tour in the west, crossing the Kūnlún 崑崙 Mountains to the Yǎn-shān 弇山. While on his return journey, before reaching his central kingdom, a certain artificer by the name of Yǎn Shī 偃師 was presented to him. The king received him and asked him what he could do. He replied that he would do anything which the king commanded, but that he had a piece of work already finished which he would like to show him. 'Bring it with you tomorrow,' said the king, 'and we will look at it together.' So the next day, Yǎn Shī appeared again and was admitted into the presence of the king. 'Who is that man accompanying you?' asked the king. 'That, Sir,' replied Yǎn Shī, 'is my own handiwork. He can sing and he can act.' The king stared at the figure in astonishment. It walked with rapid strides, moving its head up and down, so that anyone would have taken it for a living human being. The artificer touched its chin, and it began singing, perfect in tone. He touched its hand, and it began posturing, keeping perfect time. It went through any number of movements that his fancy might happen to dictate. The king, looking on with Shèng Jī 盛姬 and his personal assistants, could hardly persuade himself that it was not real. As the performance was drawing to an end, the robot winked its eye and made advances to the concubines sitting by the king, whereupon the king became incensed and would have had Yǎn Shī executed on the spot had not the latter, in mortal fear, instantly taken the robot to pieces to let him see what it really was. And, indeed, it turned out to be only a construction of leather, wood, glue and lacquer, variously coloured white, black, red and blue. Examining it closely, the king found all the internal organs complete—liver, gall bladder, heart, lungs, spleen, kidneys, stomach and intestines; and over these, muscles, bones, and limbs with their joints, skin, teeth and hair, all artificial. Every part was fashioned with the utmost nicety and skill, and when it was put together again, the figure presented the same appearance as when first brought in. The king tested the effect of taking away the heart and found that the mouth could no longer speak; he took away the liver and the eyes could no longer see; he took away the kidneys and the legs lost their power of locomotion. The king was delighted. Drawing a deep breath, he exclaimed, 'Can it be that human skill is on a par with that of the "author of nature" (*zào huà zhě* 造化者)?' And forthwith, he gave an order for the spare chariot, in which he took home with him the artificer and his handiwork.[18]

17. The book attributed to Liè Yù-Kòu 列禦寇 (c. –5th century) was probably first compiled in the –4th century. The book was then edited by Liú Xiàng 劉向 in the fourth century with much Dàoist materials added.
18. This translation is from L. Giles (1912). I have made some minor changes here.

The motivation of the Dàoists in concocting a robot as the ultimate mechanical invention was to upstage the Mòhists who were enjoying prestige from their mechanical inventions. This motivation is revealed in their own words following the robot parable in Chapter 5:

> As for the cloud-scaling ladder (*yún tī* 雲梯) of Bān Shū 班輸 and the flying wooden kite (*mù yuān* 木鳶) of Mò Dí 墨翟, their masters probably thought that they had reached the limit of human achievement. But when Yǎn Shī's remarkable skill was brought to their knowledge by their pupils, Dōng Mén-Jiǎ 東門賈 and Qín Huá-Lí 禽滑釐, respectively, the two masters no longer dared to talk of their mechanical skill and hesitated often when they had the compass and the right-angled ruler in hand.

Of course, no robot was ever built at that time. The interest in the parable lies in the fact that it provides a measure of the impact of the 'mechanization movement' at that time.

In retrospect, it is not difficult to see that one of the difficulties with the early 'mechanization movement' was the inability to make use of mathematical knowledge in the conversion of mechanical knowledge into practical technology. Consequently, the movement, in comparison with that following the 'scientific revolution', was rather limited in scope and slow in rate. The early 'mechanization movement' had, however, a profound impact on Chinese civilization. The Chinese mechanical practice as seen in vehicles, projectiles and engines of various kinds was, if not often ahead, at least comparable to the European practice until the onset of the 'scientific revolution'.[19]

The 'mechanization movement' in China was not without resistance. We read in the *Zhuāng Zǐ* 《莊子》 (*The Book of Master Zhuāng*), of the late –4th century, the concern with the mechanical device for raising water, a counterbalanced bailing bucket known as the *gāo* 槔 (a type of swape). The setting of this Dàoist parable is between the Dàoists and the Confucians instead of the Mòhists. We have:

> Zǐ Gòng 子貢 had been touring in the south of Chǔ 楚 and was returning to Jìn 晉. As he passed Hànyīn 漢陰, he saw an old man working in a vegetable garden. Having dug his channels, he kept on going down into a well, carrying a large jar and returning with water for irrigation. This caused him much expenditure of strength for meagre results. Zǐ Gòng said to him, 'There is a mechanical contrivance (*xiè* 械) by means of which a hundred plots of ground may be irrigated in one day. Little effort will thus accomplish much. Would you, sir, not like to

19. See the volumes of documentation collected by Joseph Needham and his collaborators in *Science and Civilisation in China* (University Press, Cambridge, 1954–1988), in particular secs. 27–30.

> try it?' The farmer looked up at him and said, 'How does it work?' Zǐ Gòng said, 'It is a lever made of wood, heavy behind and light in front. It raises water quickly as if being drawn out and with such a quantity that it flows in a steady stream. Its name is gāo 槔 (swape).' The vegetable farmer suddenly changed his face and laughed, saying, 'I have heard about this device from my master. But he said, 'those who have cunning devices use cunning in their dealings' and 'those who use cunning in their dealings have cunning intentions'. Furthermore, with cunning intentions in one's heart one can no longer possess purity and simplicity and without purity and simplicity one cannot spiritually be at peace. Those who lose spiritual peace possess no longer the *dào* 道. It is not that I know not of the device; I would be ashamed to use such a device.'[20]

Despite the opposition of the Dàoists, mechanical knowledge was well integrated into the practical technology of the Chinese civilization.

1.5 Comments and Evaluations

The early Chinese work on motion has not been seriously studied among sinologists and historians of science. With very few exceptions, most scholars are of the impression that in Chinese civilization there was hardly any theoretical interest in motion. Even the basic concepts of space and time as given in the *Mò-Jīng* were misinterpreted. We have, for example, from the eminent sinologist, Marcel Granet, the following comments:

> For the ancient Chinese, time was not an abstract parameter, a succession of homogeneous moments, but was divided into concrete separate seasons and their subdivisions.
> Space was not abstractly uniform and extended in all directions, but was divided into the regions, south, north, east, west and centre.[21]

These comments certainly are contradictory with the definitions of space and time (see p.1 to p.3) given in the *Mò-Jīng*: 'Space is that which extends to different positions. Time is that which extends to different moments.' In the 'exposition' section of the *Mò-Jīng*, concepts of directions and durations are indeed introduced for the purpose of elucidating, respectively, the concepts of space and time. But these elucidations are appropriate and insightful and nowhere were suggestions made that 'time was not an abstract parameter' and 'space was not abstractly uniform and extended in all directions'.

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20. This translation is a modified version of earlier translations by J. Legge (1891) and by Lin Yü-T'ang 林語堂 (1948)
21. Marcel Granet (1934), pp. 88, 96.

The accounts of Chinese work on motion found among the work of sinologists consist usually of discussions of the *mìng-tí* 命題 on motion, devised by dialecticians such as Huì Shī 惠施 (c. −380 to c. −305) and Gōng Sūn Lóng 公孫龍 (c. −320 to c. −250) of the Warring States period of the Zhōu dynasty. Such *mìng-tí* are mostly found in the *Zhuāng Zǐ*. Let us re-examine the following *mìng-tí* on motion:

> 飛鳥之景未嘗動也.
> The shadow of a flying bird never was in motion.

> 鏃矢之疾而有不行不止之時.
> The rapid motion of a flying arrow consists of moments
> at which [the arrow] is not in motion and not at rest.

These *mìng-tí* are inevitably compared with the arrow paradox of Zeno recorded in the work of Aristotle:

> The arrow in flight is at rest.
> For if everything is at rest
> when it occupies a space equal to itself and
> what is in flight always occupies a space equal to itself.[22]

(where the explanation was probably provided by Aristotle himself). Such comparisons are usually made, however, only at a level of marvelling at their superficial similarities without examining their difference in content.

The analysis of an arrow in flight given in the arrow *mìng-tí*, unlike that of Zeno's paradox, is not fallacious. It describes the instantaneous aspects of the motion. We now know that, at any instance during its flight, the arrow would have simultaneously an instantaneous position and an instantaneous velocity. To the dialecticians who debated the *mìng-tí*, the fact that the arrow is located at a specific position at a given instant implies that, at that instant, it is not in motion. Yet, in the meantime, the arrow has a velocity so that, at that same instant, it is not at rest. As discussed earlier, an instant in time is defined by the Mòhists as an instantaneous moment with no duration. Thus, the arrow *mìng-tí* deals with the instantaneous aspects of the motion anticipating the concept of instantaneous position and velocity. This is probably one of the earliest adumbrations of the instantaneous behaviour of an object in flight without the help of a mathematical apparatus in taking the limit to infinitesimal time intervals.

The shadow *mìng-tí* may superficially appear to be similar to Zeno's arrow paradox since it considers that the shadow of a flying bird, like an arrow in flight, is not in motion. But there is a subtle difference between

22. Aristotle, Physics, 239b, 5–33. The translation is from J. Burnet (1908), p. 367.

the motion of an arrow in flight and the shadow of a flying bird. The difference lies in the dynamics of the motion. Unlike the flying arrow, the moving shadow can neither possess inertia nor have forces acting directly upon it. Notice that the *mìng-tí* specifically depicts the motion of the shadow, not the motion of the flying bird itself. Based on the dynamic analysis of motion advanced by the Mòhists, the dialecticians can successfully argue that a shadow cannot by itself engage in motion dynamically. It derives its apparent motion from the flying bird.

Needham was much better informed of early Chinese work on motion and included the work of Mòhists in his treatment of this topic. His interpretation of Mòhist work was, however, partly hindered by the changes and incorrect interpretations made by the Qīng 清 scholars. Consider, for example, the passage on dynamics discussed in the last section. Needham accepted the suggestion of Sūn Yí-Ràng 孫詒讓 made in 1894 and replaced the original character *yǔ* 與 (with or and) with the character *jǔ* 舉 (to raise) and translated the character *fèn* 奮 as 'motion'.[23] Thus, the passage becomes:

> 力, 形之所以奮也.
> 力, 重之謂, 下舉重奮也.
> Force is that which causes shaped things (i.e. solid objects) to move.
> Weight is a force.
> The fall of a thing, or the lifting of something else,
> is motion due to heaviness.

which has a very different meaning. Take the last sentence for example. Instead of stating that the phenomenon of *fèn* during a fall is due to the downward force of weight, it takes on the meaning that the falling and lifting of things are motions due to heaviness.

In a footnote of this passage, Needham did express his reservation in translating *fèn* as motion. He stated that 'if the Mòhist writer had not had a vague idea of acceleration at the back of his mind, he would have used obvious words such as *hsing* [*xíng* 行], *i* [*yí* 移] and *tung* [*dòng* 動].'[24] He also pointed out that 'the word *fèn* is of particular interest here, since it connotes rushing or accelerated movement, and originally meant the taking-off of a bird from the field in flight.' Despite his reservations on the interpretation of the character *fèn* 奮, Needham nevertheless accepted Sun's suggestion and misinterpreted the Mòhist analysis of the role of force in motion and their interpretation of falling motions.

Needham thus concluded that 'the study of motion (kinetics and cinematics) seems to have been, on the whole, conspicuously absent

23. Joseph Needham (1962), vol. 4, sec. 26(c), p. 19.
24. Ibid., footnote a.

from Chinese physical thinking'.[25] Although he pointed out that this absence of Chinese discussions on dynamics 'seems to have had no inhibitory effect at all upon practical technology in the eotechic phase' and acknowledged that 'so far as vehicles, projectiles, and engines of all kinds were concerned, Chinese mechanized practice was ahead of European, not retarded, down to the very time when the scholastics of the fourteenth century were preparing the way for Galileo, and even later.'[26] Needham's view of Chinese work on motion is essentially the same as those of earlier scholars, namely, that there was hardly any theoretical interest in motion.

But the conceptual framework formulated by early Chinese scholars for the interpretation and analyses of motion is highly theoretical. It provides a sound interpretation of space-time continuum, as well as points in space and time; it emphasizes the importance and the relative nature of reference in observing motion; it realizes that in the absence of a blocking force, an object in motion will not stop; it interprets the phenomenon of *fèn* (i.e. the hastened motion) of an object in motion as caused by force; and it correctly explains that the increase in velocity of an object in free fall is the result of the downward force of weight.

The early Chinese conceptualization of motion was free from the fallacious Aristotelian concept of 'natural place' according to which an object can exhibit either 'natural' motion in seeking its 'natural place' or 'violent' motion compelled by some external force. Nor did the early Chinese analysis of motion encounter the complication of the Aristotelian theory of 'antiperstasis' in which the motion of an object was visualized to be propelled by the air rushing into the 'horror vacui' created by the moving object.[27] The distinguishing feature of the Mòhists' work on motion lies in their ability to strip away the peripheral and secondary phenomena and to concentrate on the essential physical features of the motion.

This was probably related to the Mòhist methodology known as the *xiào* 效 (model-thinking):

> 效者, 為之法也.
> 所效者, 所以為之法也,
> 故中效則是也, 不中效則非也.
> 此效也
> That being modelled is considered to be the principles.
> A model so constructed should contain these principles.
> If the model succeeds in manifesting [these principles],
> then [the reasoning] is correct.

25. Ibid., p. 55.
26. Ibid., p. 59.
27. See for example, R. Dugas (1950); C.B. Boyer (1950).

> But, if the model fails in manifesting [these principles],
> then [the reasoning] is wrong.
> Such is [the method of] *xiào*.[28]

Indeed, the method of model-thinking discussed above is very advanced for its time and the logic in its arguments bears a strong resemblance to that of our contemporary scientific model. Needham has suggested that 'the general attitude of Chinese thinkers towards conceptual model-making might have induced in them by the structure of their language and this perhaps enabled them to attain a sophistication in differentiating those intellectual operations which can be carried on with models from those which cannot, only now being rediscovered and developed by modern philosophers of science.'[29]

Following the discussions of the method of *model-thinking*, we find, in the *Minor Illustration* section of the *Mò-Zǐ* 《墨子》 (*The Book of Master Mò*), discussions on reasoning by comparison (*pì* 辟 [譬]), paralleling (*móu* 侔), analogy (*yuán* 援) and extension (*tuī* 推). The passage for the reasoning by extension runs as follows:

> 推也者, 以其所不取之同於其所取者, 予之也.
> [Reasoning by] extension consists of affirming the similarities
> of those not yet considered with those already considered.

These methodologies had undoubtedly helped the Mòhists in their analysis of the physics of motion.

The conceptual framework developed by the early Chinese scholars for the analysis of motion is entirely compatible with our current conceptualizations. What was absent in the early Chinese work on motion was the means to express their physical concepts in modern mathematical terms. But such mathematization of physical concepts of motion did not begin until the time of Galileo and Newton in the seventeenth century.

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28. An excellent discussion on Hu Shi's interpretation versus Maspero's interpretation on *xiào* 效 and *tuī* 推 is given by Needham (1956) on pp. 182 to 184, vol. 2 of *Science and Civilization in China*. I agree with the interpretations of Needham in translating *xiào* as 'model-thinking' and in accepting Hu Shi's interpretation of *tuī* as 'induction'.
29. Joseph Needham (1956), vol. 2, p. 184.

2 Early Chinese Work on Acoustics

詩言志, 歌永言, 聲依永, 律和聲, 八音克諧.
Shùn Diǎn 《舜典》
(The Canon of Shùn)

We live in an environment filled with sound waves. Much of our acoustical experience comes naturally and spontaneously. The early students of acoustics certainly did not know that the audible sensation of sound is produced by mechanical wave motions in an elastic medium, and that sound waves are longitudinal waves with the disturbances in the medium moving in the same or opposite direction from that of the sound. Nor did they know about the physio-neurological nature of the human auditory system. They nevertheless made penetrating analyses based on their perception of sound. In the following subsections, I will discuss the early Chinese perception of sound, their understanding of the physical nature of sound, and their contributions to musical acoustics.

2.1 The Perception of Sound

Our perception of sound may be expressed in terms of three characteristics, namely: pitch, loudness and quality. Although these characteristics are subjective perceptions, they can be correlated to physical properties of sound waves. Modern science has revealed that pitch is related to the frequency, loudness is related to the amplitude, and quality is related to the harmonic structures of the sound wave. In Chinese classics, there are two characters for sound, namely *shēng* 聲 and *yīn* 音. Before they became synonyms and formed the generic compound *shēng-yīn* 聲音 for sound in general, they had a subtly different meaning.

An old definition of these two characters is preserved in the *Shǐ Jì* 《史記》 (*Historical Records*):

> 單出曰聲, 雜比曰音.
> A singly emerged sound is called *shēng*.
> An assorted comparative sound is called *yīn*.

This definition, though not precise from a modern acoustic point of view, points out unmistakably that there was a fine distinction between *shēng* and *yīn*. In early classic usages, *shēng* refers to sound in connection with its tonal relations such as the notes in the *wǔ-shēng* 五聲 (pentatonic scales), while *yīn* refers to sound in connection with its tone quality in harmonic relations such as the *bā-yīn* 八音 (eight tone qualities). I will interpret *shēng* and *yīn* according to the context in which they appear.

2.1.1 Pitch (Yīn-Lǜ 音律)

The Chinese term for pitch is *yīn-lǜ* 音律 or simply *lǜ* 律. The earliest extant mention of pitch is found in the 'Shùn Diǎn' 〈舜典〉 ('The Canon of Shùn') of the *Yú Shū* 《虞書》 (*The Book of Yú*) which survived through its incorporation in the *Shàng Shū* 《尚書》 (*The Book of Documents*). We have:

> 詩言志, 歌永言, 聲依永, 律和聲,…….
> Ode expresses thought in lyric,
> Song puts lyric in singing,
> Sound is produced to facilitate singing,
> Pitch is used to harmonize sound,...

The term *lǜ* 律, as it is described here, seems to suggest that certain standards for pitch might have already existed at the time of Shùn 舜.

This is consistent with the following descriptions given elsewhere in the *Yú Shū*. Among the descriptions of the activities and policies implemented by Shùn after taking over the responsibilities of the king of Yáo 堯, it is stated in the 'Shùn Diǎn':

> 同律度量衡.
> Standardize (*tóng* 同, literally 'equalize') *pitches* (*lǜ* 律),
> measures and weights (*dù liáng héng* 度量衡).

In another document of the *Yú Shū*, 'Yì Jì' 〈益稷〉, we find the earliest extant mention of six pitches, *liù lǜ* 六律, in connection with the pentatonic scale, *wǔ-shēng* and the eight tone qualities, *bā-yīn*. Although the names of the six pitches are not given in this document, it is significant that the six pitches should be mentioned together with the pentatonic scale and the eight tones at this early stage, indicating their importance in music making.

We now know that the perceived difference in pitch comes primarily from the change in frequency of the sound wave. Low pitches correspond to low frequencies and high pitches correspond to high frequencies. Language deals with the differences in pitch, varying greatly from culture to culture. The Romans, for example, described high pitch as *acutus* (*cutting* or *sharpened*) and low pitch as *gravis* (*heavy*). The Chinese, on the other hand, used the terms *qīng* 清 (*clear*) and *zhuó* 濁 (*muddy*) to describe the high and low pitches, respectively. The human auditory system is sensitive to a limited range of frequencies. The sonic range which varies with individuals is from about 25 to 18,000 Hz.

The early Chinese were aware, at least qualitatively, of the existence of a range of pitches for which the ear is sensitive. Indications of this can be found in a discourse that took place in −522 between the High King Jǐng of Zhōu and his minister Shàn Mù Gōng 單穆公 as recorded in the *Guó Yǔ* 《國語》 (*Discourse on the States*). In remonstrating the decision of the king to have the Wǔ-Yì 無射 bell melted down and converted into the Dà-lín 大林 (i.e., Lín-zhōng 林鐘) bell of lower pitch, Shàn Mù Gōng stated:

> 耳之察和也, 在清濁之間.
> 其察清濁也, 不過一人之所勝.
> 是故先王之制鐘也, 大不出鈞, 重不過石.
> 律度量衡於是乎生.
> The ear functions harmoniously within a certain range of high and low pitches.
> The determination of this pitch-range should not be left for individuals.
> For this reason bells constructed by our ancient kings never exceeded their corresponding size in *jūn* 鈞 and weight in *dàn* 石.
> This is where the specifications of measures and weights for pitches originated.[30]

The characters *jūn* and *dàn* in the above are units for size and weight, respectively. This statement leaves little doubt that the early Chinese were aware of the existence of an audible pitch-range. Bells were cast in accordance with specifications to preserve pitch standards.

According to music master Zhōu Jiū 州鳩, the acoustic adviser to the High King Jǐng 景, there were twelve pitches from antiquity preserved in bells. The names of these pitches are given in a discourse, that took place in −522, again in connection with the king's decision to convert the Wú-Yì bell to the Dà-lín bell of lower pitch. In this discourse, as recorded in the *Guó Yǔ*, Zhōu Jiū stated that the twelve pitches are obtained from the *liù-lǜ* 六律 by inserting six interstitial pitches called *liù-jiān* 六間. The names of these pitches are reproduced below:

30. The 'Zhōu Yǔ' 〈周語〉 chapter of the *Guó Yǔ* 《國語》, vol. 3.

Lǜ 律		*Jiān* 間[31]	
Huáng-zhōng	黃鐘	Dà-lǚ	大呂
Tài-cù	太蔟	Jiá-zhōng	夾鐘
Gū-xiǎn	姑洗	Zhòng-lǚ	仲呂
Ruí-bīn	蕤賓	Lín-zhōng	林鐘
Yí-zé	夷則	Nán-lǚ	南呂
Wú-yì	無射	Yìng-zhōng	應鐘

This set of twelve pitch-names can also be found in the *Yuè Lìng* 《月令》 (*Monthly Ordinances of Zhōu*) and in the *Zhōu Lǐ* 《周禮》 (*The Book of Zhōu Institutions*). In the *Zhōu Lǐ*, the six interstitial pitches are called *liù tóng* 六同 instead of *liù-jiān* and interpreted as the *yīn* 陰 pitches with the original *liù-lǜ* as the *yáng* 陽 pitches.

Thus, no later than the –6th century, the early Chinese certainly had in their possession the *shí-èr-lǜ* 十二律 (twelve pitches) which were used for regulating sound. The twelve pitches were obtained by subdividing the intervals in the *liù-lǜ* with the six interstitial pitches. An interesting question is how, in the first place, the original *liù-lǜ* were developed. Some light on this question is also provided by music master Zhōu Jiū. From the same discourse in which the above pitch names are given, we have the following statements:

> 律所以立均出度也,
> 古之神瞽考中聲而量之以制.
> 度律均鐘, 百官軌儀.
> 紀之以三, 平之以六, 成於十二, 天之道也.
> The reason that ranges and degrees can be established for pitches is due to Shēn Gǔ 神瞽 of antiquity who investigated and determined quantitatively the *zhōng-shēng* 中聲 (middle tone) as the reference.
> The degrees of the pitches and tuned bells are the standards observed by all officials.
> [Based on the *zhōng-shēng*] we first establish three pitches, then level them out evenly into six pitches, and finally bring them to completion in twelve pitches. This is the *dào* 道 of nature.[32]

This suggests that the *liù-lǜ* (six pitches) were derived from 'trichords' (three-note tone rows) (see p.41 to p.43). The pitch system was specified in reference to the *zhōng-shēng* 中聲 determined by Shén Gǔ.

In commenting on Zhōu Jiū's discussion on the establishment of the range and degree of pitch, Wéi Zhāo 韋昭, a third century commentary of the *Guó Yǔ*, stated:

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31. This indicates that, in the Zhōu pitch system, the three *lǚ* 呂 *pitches and three zhōng* 鐘 pitches listed here were later additions. The practice of using two characters to denote the names of pitches appeared rather early as indicated by the inscriptions found on the unearthed musical instruments such a the *biān-qìng* 編磬 (set stone-chimes) of the Shāng dynasty (see p.41 to p.43).
32. The 'Zhōu Yǔ' 〈周語〉 chapter of the *Guó Yǔ* 《國語》, vol. 3.

均者, 均鐘木, 長七尺, 有絲之, 以均鐘者.
The term *jūn* 均 referred to here is the *jūn-zhōng-mù* 均鐘木,
an instrument which is seven *chǐ* 尺 (feet) in length mounted with string (or strings) and is used for tuning bells.

This comment sheds light on the method of tuning in ancient times. The use of the term *jūn* 均 for 'tuner' is of interest. *Jūn* literally means 'even', 'equal' or 'in balance'. By interpreting the process of tuning as that of making pitches equal or even, one understands the use of the term *jūn* for tuning. The seven-foot tuner described by Wéi Zhāo is similar in nature to the monochord mentioned by Boethius (480–534) in Greek civilization.[33] There are, however, indications that the seven-foot tuner probably had more than one string (see p.95 and Fig. 26 below).

2.1.2 Loudness (Yīn Liàng 音量)

Loudness is a subjective measure of the auditory sensation produced by sound. The difference in loudness comes primarily from the change in the amplitude of the sound waves. For two sound waves of the same frequency, the one with the greater amplitude sounds louder. In Chinese, the louder sound is called *dà* 大 while the softer sound is called *xì* 細. The modern Chinese term for loudness in acoustics is *yīn-liàng* 音量.

Early Chinese musicians apparently took great care in their arrangement of musical instruments in an orchestra so that loud instruments, such as bells, do not drown out soft instruments, such as zithers. An early discussion of this principle for instrument arrangement is found in the *Guó Yǔ*:

聲應相保曰和, 細大不踰曰平.
細抑大陵, 不容於耳, 非和也.
聽聲越遠, 非平也.
Musical tones and their sympathetic sounds are said to be in harmony (*hé* 和), if they mutually support (*bǎo* 保) each other.
Soft and loud tones are said to be in balance (*píng* 平) if one does not overtake (*yú* 踰) the other.
If soft tones are suppressed by overtowering loudness so that the ear can no longer accommodate it, there is no harmony.
If sounds we hear are far apart in their loudness, there is no balance.

故樂器, 重者從細, 輕者從大.
Therefore, musical instruments are to be arranged with the heavy instruments (such as the bells) engaging in soft tones and the light instruments (such as the lute and zither) engaging in loud tones.[34]

33. See the *De Institutione Musica*, ed. G. Friedlein. bk. 1, secs. 10–11, pp. 197–8.
34. The 'Zhōu Yǔ' 〈周語〉 chapter of the *Guó Yǔ* 《國語》, vol. 3.

Here, the principle of balance and harmony in sound volume is applied to the arrangement of musical instruments when they are playing together.

An obvious question on loudness is its role in the audibility of sound. Modern acoustics has shown that the question of audibility for the human ear depends not only on loudness, but also on the frequency of sound. Lǎo Zǐ 老子 made some interesting observations on the audibility of sound which seems to have some bearing on the threshold of hearing. We have from the *Dào-Dé Jīng* 《道德經》 (*Canon of the Virture of Dào*), the statement: '聽之不聞名曰希. That which is listened to but not heard is called *xī* 希.' The term *xī*, as it is defined here, obviously has something to do with sound, since in another Lǎo Zǐ's statement, he used the term *xī shēng* 希聲 which Needham translates as 'tenuous note'. Based on this definition for *xī*, Lǎo Zǐ apparently had in mind for *xī shēng*, a sound which cannot be heard but can still produce certain hearing sensations in the ear. Such a view of 'inaudible' sound is not unrelated to the modern concept of the threshold of hearing, which is the intensity of the smaller oscillation that can be perceived by the ear.

At the time of Lǎo Zǐ, concepts such as frequency and intensity had not yet been developed. Lǎo Zǐ could not have distinguished the audibility of *xī shēng* in relation to its frequency and intensity. But he probably was aware that most *xī shēng* are low-pitch sounds and that their audibility depends sensitively on loudness. Thus, when he said '大音希聲 Loud sound contains *xī shēng*' he probably was referring to the audibility of those 'inaudible' low-pitch sounds at a louder level. Being a Dàoist statement, one cannot exclude the possibility that the statement contains an implied metaphor. Even then, the acoustic content of the term *xī shēng*, which Lǎo Zǐ coined to express his metaphor, is still significant.

2.1.3 Quality (Yīn-Zhì 音質)

Tone quality, which is also known as timbre or tone colour (Klangfarbe), is the perception of sound in relation to the dynamic structure of the sound. It is this characteristic that permits one to recognize and distinguish sounds with the same pitch and loudness, but produced, for example, by different musical instruments. The Chinese term for tone quality is *yīn-sè* 音色 and in classic usage is simply *yīn* 音. The modern term for tone quality in acoustics is *yīn-zhì* 音質. Modern acoustics has shown that the quality of sound is determined by the dynamic blend of upper partials (harmonics or overtones). This can be easily demonstrated by the analysis of the wave form and harmonic spectrum. In ancient times, acousticians were not benefited by such knowledge; they based their analysis of tone quality on perceptions.

The early Chinese acousticians identified tone quality with sound-

producing materials and began to classify sound in accordance with such materials. For example, the sounds of pan-pipes (*pái-xiāo* 排簫) and a reed mouth-organ (*shēng* 笙) are identified to have the tone quality of bamboo,[35] and the sounds of a zither (*sè* 瑟) and a lute (*qín* 琴) are identified to have the tone quality of silk. With their interest in different tone qualities, the search for materials to produce rich sound began very early in Chinese civilization. This led to the identification of eight distinct tone qualities in association with sound-producing materials. These eight tone qualities are referred to simply as *bā-yīn* 八音, the 'eight tones'.

It is in the 'Shùn Diǎn' of the *Yú Shū* that we find the earliest mention of the 'eight tones' in a passage on mourning the death of King Yáo:

> 百姓如喪考妣. 三載, 四海遏密八音.
> Citizens mourned as if the death of their parents. For three years, the 'eight tones' (*bā-yīn* 八音) were silent within the four seas.

This passage reveals the possibility that the eight types of tone qualities were probably identified very early.

A discussion on the function of the *bā-yīn* is also found in a dialogue between King Shùn and his music master Kuí 夔 in the 'Shùn Diǎn'. A portion of this dialogue has already been quoted in the section on pitch (see p.20). With the statement on *bā-yīn*, we have:

> ……聲依永, 律和聲, 八音克諧
> ...Sound is produced to facilitate singing. Pitch is used to harmonize sound. 'Eight tones' are provided to enrich harmony.

Here we see the appreciation of harmonic subtleties offered by tone qualities. The term 'eight tones' is also mentioned in a subsequent passage in the *Yú Shū* in connection with the pentatonic scale and six pitches as noted earlier.

Though the term *bā-yīn* is also mentioned in the *Zuǒ Zhuàn* 《左傳》, it is in the *Zhōu Lǐ* 《周禮》 that an extant list of the *bā-yīn* is first found:

jīn	金	(metal)	*shí*	石	(stone)
tǔ	土	(clay)	*gé*	革	(skin)
sī	絲	(silk)	*mù*	木	(wood)
páo	匏	(gourd)	*zhú*	竹	(bamboo)

Each of these sound-producing materials represent a basic tone quality. These eight tone qualities mentioned in the *Zhōu Lǐ* are found in a delineation of the duties of musical officials and technicians in the Zhōu Institutions.

35. Similar to the practice of identifying the clarinet tone with mcingo wood.

In addition to being responsible for instituting music with the two sets of six pitches that harmonize *yīn* and *yáng*, the Grand Music Masters (*Dà-Shī* 大師) were also responsible for making sure that:

> 皆文之以五聲: 宮商角徵羽.
> 皆播之以八音: 金石土革絲木匏竹.
> All music is composed in pentatonic intonations: *gōng* 宮, *shāng* 商, *jué* 角, *zhǐ* 徵, and *yǔ* 羽.
> All music is performed in 'eight tones': *jīn*, *shí*, *tǔ*, *gé*, *sī*, *mù*, *páo*, and *zhú*.

In the Zhōu Institution, the selection of tone qualities for a performance was determined by the occasion through the use of an appropriate combination of musical instruments. Lists of musical instruments for different ritual occasions can be found in the *Yuè Jì* 《樂記》 (*Record of Ritual Music*).

There are scholars who consider the *bā-yīn* to be a classification of musical instruments. This is not unexpected since the classification of musical instruments is closely related to the classification of tone qualities. In 1936, Schaeffner commented that the *bā-yīn* was 'probably the oldest extant classification of musical instruments in any civilization.'[36] Needham and Robinson have compared the *bā-yīn* classification with the Graeco-Roman classification of musical instruments, namely wind instruments, stringed instruments and percussion instruments, and concluded that the Graeco-Roman classification was more scientific.[37] The point that needs to be emphasized here is that the *bā-yīn* classification was based on tone qualities and not on musical instruments, even though there is an intimate connection between the two.

It is certainly true that sounds produced, for example, by different percussion instruments have certain common features in tone quality which are, in general, different from sounds produced, say, by stringed instruments. But the Graeco-Roman classification would be equally unsatisfactory for the classification of different tone quality. Consider, for example, a bronze drum and a timpani with a skin drumhead. Though they both are percussion instruments, the sounds they produce do have certain basic differences in their tone qualities. The sound of a bronze drum has a metallic quality which is also found, for example, in the sound of a bronze bell. Thus, there is a difference in the classification between musical instruments and tone qualities. Other than the *Yuè Jì*

36. A. Schaeffiner (1936), p. 124.
37. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 142.

which makes no mention of the term *bā-yīn* and lists *jīn*, *shí*, *sī* and *zhú* among musical instruments, Chinese classic writings have been very specific about the term *bā-yīn*, the eight tone qualities, and the term *yuè-qì* 樂器, the musical instruments.

Perhaps it is worthwhile to note that in ancient China there was also the practice of using the general term *xián* 弦 for stringed instruments and the general term *dàng* 蕩 for wind instruments.[38] Although there was no general known term for all types of the percussion instruments, the term *gǔ* 鼓 was sometime also used to represent percussion instruments other than drums. It is important to note that even in modern acoustics, there does not yet exist a satisfactory system for classifying tone quality. The five-fold classification of ideophones, membramophones, chordophones, electrophones and aerophones is again a classification based on musical instruments. Each complex tone has its own unique characteristic harmonic spectrum and wave form.

The early Chinese were certainly aware of the fact that different musical instruments made of the same sound-producing material have further differences in tone quality. Their interest in such differences was undoubtedly one of the motivations for searching and inventing different musical instruments. In the *Shī Jīng* 《詩經》 (*Book of Odes*), a total of twenty-nine musical instruments are already mentioned. In the same section on the delineation of duties of musical officials and technicians in the *Zhōu Lǐ*, a lengthy discussion of different types of sound in relation to musical instruments is found in the entries for the musical official with the title Diǎn Tóng 典同. The principal responsibility of Diǎn Tóng was in musical instrumentation, including the construction and maintenance of musical instruments. The discussion of different types of sound is reproduced below:

> 高聲硱, 正聲緩, 下聲肆.
> The upper part produces a muffled (*gǔn* 硱) sound.
> The front part produces a gradual (*huǎn* 緩) sound.
> The lower part produces a vigorous (*sì* 肆) sound.
>
> 陂聲散, 險聲斂.
> The outward slanted part produces a scattered (*sàn* 散) sound.
> The inward inclined part produces a hoarded (*liǎn* 斂) sound.
>
> 達聲贏, 微聲韽, 回聲衍.
> An 'outreaching sound' accrues (*yíng* 贏).
> A 'minute sound' diminishes (*ān* 韽).
> An 'echo sound' reverberates (*yǎn* 衍).

38. Yáng Yìn-Liú 楊蔭瀏 (1981), p. 41.

侈聲筰, 合聲鬱.
An 'open sound' is unimpeded (*zuó* 筰).
An 'enclosed sound' is stifled (*yù* 鬱).

薄聲甄, 厚聲石.
The thin part produces a crispy (*zhēn* 甄) sound.
The thick part produces a solid (*shí* 石) sound.

It must be emphasized that the translation given here only serves as a suggestive interpretation. Specific definitions and explanations for many of the technical terms used here are no longer available. Despite this difficulty, it is apparent that the passage describes twelve different types of sound produced by or in relation to different parts in musical instruments.

A second century commentator, Zhèng Xuán 鄭玄 (127–200), commented that this is a classification of bell sounds. This view is partly supported by the following statement found in the *Kǎo Gōng Jì* 《考工記》 (*The Artificers' Record*) on bell-making:

鐘已厚則石, 已薄則播, 侈則筰, 弇則鬱.
Bells with thick walls produce heavy solid sound,
with thin walls produce scattered sound,
with open mouth produce unimpeded sound, and
with closed mouth produce stifled sound.

A number of other commentators disagreed with Zhèng Xuán, claiming that the classification of the twelve types of sound should apply to all instruments since the description of the sound is not found in the entries for the Bell Master (*Zhōng-Shī* 鐘師) but is found in the entries for the Diǎn Tóng, who is responsible for musical instrumentation in general.

I would like to call attention to the fact that right after the list of the twelve tones in the entries for the Diǎn Tóng, we also have the following statement:

凡為樂器, 以十有二律為之數度,
以十有二聲為之齊量.
Whenever making musical instruments, it is necessary to use the twelve pitches to determine their measurements, and to use the twelve sounds to maintain the [tone] qualities.

There seems no doubt that the twelve sounds apply to all musical instruments.

In addition to the tone quality of sound due to the sound-producing materials and the configuration of musical instruments, the ancient Chinese were also interested in the tone quality resulting from the different ways of playing musical instruments. Consider, for example, the playing of the ancient lute (*gǔ qín* 古琴), the only stringed musical instrument in use from antiquity which has no frets and markings for

nodes of vibration on the board. Any given note could be played with a variety of subtleties in touch. Thus, the tone quality of lute sound is entirely a question of touch. On lute playing, Needham and Robinson provided the following description:

> Recognition of individual harmonics 'floating sound' (*fàn yīn* 泛音), using the same string, was already well advanced in the time of Hsi Khang 嵇康 (223–62). In Europe on the contrary this came very late, not before the eighteenth century. Indeed, the technique of playing the *chin* 琴 mainly depends on exploiting the production of different timbres at the same pitch, and this was already developed to perfection by the later Sung (+12th century).[39]

According to the description given in the *Lǚ-Shì Chūn-Qiū* 《呂氏春秋》 (Master Lǚ's Spring and Autumn Annals) of −239 and *Huái-Nán Zǐ* 《淮南子》 (The Book of Prince Huai-Nan) of −120, the techniques of playing *sè* 瑟 (zither) and *qín* 琴 (half-tube zither) had already reached a high level of art by the −3rd century.

2.2 The Physical Nature of Sound

In addition to their perception of the subjective qualities of sound, the early Chinese had also discovered a number of physical attributes of sound. In this section, I will discuss some of the physical properties of sound recognized by the early Chinese based on records found in the Chinese classics.

2.2.1 The Sound of Thunder

As it is true in all early civilizations that nature's grand display of sound in thunderstorms commanded the attention of early observers of nature, the early Chinese observers were fascinated by the lightning and thunder. Statements about the sound of thunder are found in the *Yì Jīng* 《易經》 (*Book of Changes*): '動萬物者, 莫疾乎雷. Nothing is quicker than thunder in causing things to rumble.'[40] In the *Guo Yǔ*: '陰陽分布, 震雷出滯. The distribution of *yīn* and *yáng* produces shaking thunders through viscidity (*zhì* 滯).'[41] And in the *Gǔliáng Zhuàn* 《穀梁傳》 (*Master Gǔliáng's Commentary on the Spring-Autumn Annals*):

39. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 132.
40. The 'Shuō Guà' 〈說卦〉 section of the *Yì Jīng* 《易經》.
41. The 'Zhōu Yǔ' 〈周語〉 chapter of the *Guó Yǔ* 《國語》, vol. 1.

> 震, 雷也; 電, 霆也.
> Thunder is that produced by shaking (*zhèn* 震).
> Lightning is that produced by *electricus* (*diàn* 電).[42]

These statements provide early comments on the speed of thunder and descriptions on the physical causes of thunder and lightning. Thunder is visualized due to a shaking motion or viscidity resulting from the interaction of the *yīn* and *yáng*.

It is in the *Yuè Jì* that the concept of *qì* 氣 is introduced to explain the interaction between the *yīn* and *yáng* in the production of thunderstorms. We have:

> 地氣上齊. 天氣下降, 陰陽相摩.
> 天地相蕩, 鼓之雷霆, 奮之以風雨.
> The ascent of *qì* 氣 from the earth and the descent of *qì* from the *tiān* 天 give rise to the mutual friction between the *yīn* and *yáng* and the mutual undulation between the earth and heaven and result in the drumming of thunder and lightning and the rushing of wind and rain.

The concept of *qì*, like the concept of *yīn* and *yáng*, is an important scientific, as well as philosophical, concept in the classical periods of scientific development in Chinese civilization. *Qì* is visualized as the media for *yīn-yáng* interactions, as well as the basic constituent entity of all things in nature (see p.198 to p.200).

The early views on the thundering as described above were paraphrased in one way or another by a number of later writers. We have, for example, from the *Huái-Nán Zǐ* of −120, the statements:

> 陰陽相薄, 感而為雷, 激而為霆.
> The mutual closeness of the *yīn* and *yáng* induces thunder and stimulates lightning.
>
> 疾雷不及塞耳, 疾霆不暇掩目.
> Fast thunder allows not enough time to block one's hearing.
> Fast lightning allows no spare instant to block one's sight.

From these statements it is apparent that the early Chinese were also interested in the physical aspects of thunder and attempted to explain rationally the phenomena of lightning and thunder. Although concepts, *yīn*, *yáng*, and *qì*, based on the present criterion, were not precisely defined, the very use of abstract concepts for interpreting natural phenomena indicates a desire for an objective understanding of nature.

2.2.2 The Production and Propagation of Sound

Further information on early knowledge of the physical properties of sound are found among the work of musical masters and makers of

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42. The 'Yǐn Gōng' 〈隱公〉 section of the *Gǔliáng Zhuàn* 《穀梁傳》.

musical instruments. According to the *Kǎo Gōng Jì*, the early Chinese bell-makers certainly knew that sound is produced by vibrations:

> 薄厚之所震動, 清濁之所由出.
> The high pitch (*qīng* 清) and low pitch (*zhuó* 濁) come from the difference in vibrations (*zhèn-dòng* 震動) due to the thinness and thickness [of the bell walls].[43]

This statement indicates that the bell-makers recognized not only that sound is produced by vibration but also that there is a relationship between the thickness of the vibrating walls and the pitch of the sound produced by the vibration.

In Chinese classic usages, the character *jí* 疾 means rapid (or fast) and *shū* 舒 means slow. Thus, sound produced by rapid vibration is called *yīn jí* 音疾 (or *shēng jí* 聲疾) and by slow vibration *yīn shū* 音舒 (or *shēng shū* 聲舒). Such technical terms are found in the description of acoustics of bells and drums in the *Kǎo Gōng Jì*. It is important to appreciate the relation between this terminology and the terminology for pitch. The terms *qīng* 清 and *zhuó* 濁 for high and low pitches represent a perceptive description of sound in relation to the rate of vibration; while the terms *yīn jí* and *yīn shū* represent a physical description of sound in relation to the rate of vibration.

An explicit statement on the direct relation between the physical and perceptive descriptions of sound is found in the *Guǎn Zǐ* 《管子》 (*The Book of Master Guǎn*):

> 音疾以清, [音舒以濁].
> The sound of rapid vibration has a high pitch.
> [The sound of slow vibration has a low pitch.][44]

This shows clearly that, even though a quantitative description of frquency had not yet been developed, the early acousticians recognized not only that the difference in pitch comes from the diference in rate of vibration, but also the proportional relation that the faster is the vibration the higher is the pitch.

In addition to the production of sound by vibrations, another important physical question is how sound propagates from one place to another. In the *Kǎo Gōng Jì*, we read the following statement:

> 鐘大而短, 則其聲疾而短聞;
> 鐘小而長, 則其聲舒而遠聞.
> A large, but short bell produces sound with rapid vibration and a short audible distance.

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43. The 'Fú Shì Wéi Zhōng' 〈鳧氏為鐘〉 section of the *Kǎo Gōng Jì* 《考工記》.
44. The 'Dì Yuán' 〈地員〉 chapter (58) of the *Guǎn Zǐ*. In the present version, the second sentence '*yīn shū yǐ zhuó*' 音舒以濁 is missing.

> A small, but long bell produces sound with slow vibration and a large audible distance.[45]

This is a significant statement on the audibility of a bell's sound at a distance, because the two dimensions mentioned in the statement are determined by the diameter and length of the bells which are the primary parameters that relate, respectively, to the frequency, f, and the displacement amplitude, a, of the bell's vibrations for a given striking force. Modern acoustics tells us that the intensity, I, of a sound wave is directly proportional to the square of its frequency and its amplitude:

$$I \propto f^2 a^2$$

And in free, open space the intensity of a sound decreases according to the inverse square of the distance from the source. Thus, the audible distance of a sound is determined by the frequency and amplitude of the sound wave. This agrees essentially with the findings of early Chinese bell-makers as they discovered that the audible distance of a bell's sound depends on the interplay of the diameter and the length of the bell.

Such an experimental investigation on the dependence of the audible distance of a bell's sound on the dimensions of the bell was a significant step toward a scientific inquiry into the physical nature of sound propagation. The early Chinese probably looked to the propagation of a disturbance in water as a mental image for the propagation of sound. This is suggested by the hydraulic terms, *qīng* (clear) and *zhuó* (muddy), used to describe high and low pitches of sound. Thus, the analogy between the expanding pattern of ripples on water and the propagation of sound in the air probably also began early in Chinese civilization. However, no extant record with explicit mention of this mental connection between waves in water and in air is available earlier than the first century.

It is in a lengthy discussion by Wáng Chōng 王充 (+27–+97) against the superstitious belief that *tiān* 天 (heaven) can hear what man says, that we find the earliest extant record of such an analogy. The discussion is found in his book, *Lùn Héng* 《論衡》 (*Discourse Weighed in Balance*). At one point in the discussion, Wáng Chōng argues that physically it is impossible even for the voice of man to reach the height of *tiān*. The argument is presented in terms of an example that the voice of a cricket cannot reach the height of an upstairs terrace. He explains that the reason a man sitting in an upstairs terrace cannot hear the sound made

45. The 'Fú Shì Wéi Zhōng' 〈鳧氏為鐘〉 section of the *Kǎo Gōng Jì* 《考工記》.

by a cricket below is because the man is outside the range of the sound produced by the cricket, so that 'the sound disturbed air cannot reach [the ear of the man]'. He then makes use of the disturbance in water produced by a small fish as an analogy to illustrate that the range of propagation of the disturbance is limited. He estimates:

> 所振蕩者不過百步. 而一里之外
> 澹然澄靜, 灘之遠也.
> The region of the undulating movements, at most, do not exceed one hundred paces. At a distance one mile away, [the water] is still and transparently calm. This is because it is far away [from the disturbance caused by the fish].

This discussion provides a clear physical picture as to how Wáng Chōng visualized the transmission of sound in air.

2.2.3 The Phenomenon of Resonance

An important physical phenomenon of sound that was discovered early in Chinese civilization is resonance. A description of the demonstration of this phenomenon by Lǔ Jù 魯遽 of Western Zhōu is found in the *Zhuāng Zǐ* 《莊子》 (*The Book of Master Zhuāng*):

> 吾示子乎吾道, 於是乎為之調瑟.
> 廢一於堂, 廢一於室,
> 鼓宮宮動, 鼓角角動, 音律同矣.
> Let me show you my *dào* 道 in what is called the tuning of the *sè* 瑟 (zither).
> Place a [tuned] zither in a hall and another in a room.
> The striking of the *gōng* 宮 note of one zither causes the *gōng* note of the other zither to vibrate.
> The striking of the *jué* 角 note of one zither causes the *jué* note of the other zither to vibrate.
> This is because the notes are of the same pitch.[46]

This is a remarkable demonstration of the phenomenon of resonance between strings having the same natural frequencies. With air providing the coupling, when one is struck, sympathetic vibrations result in the other string.

The resonance phenomenon of Lǔ Jù certainly achieved wide recognition in ancient times. The phenomenon was discussed in a number of works. We have from the *Lǚ-Shì Chūn-Qiū* of –239:

> 聲比則應.
> 故鼓宮而宮應之, 鼓角而角動.

46. The 'Zá Piān' 〈雜篇〉 volume (8) of the *Zhuāng Zǐ* 《莊子》.

> Sounds which match in pitch (*bǐ* 比, literally 'comparison') can resonate (*yìng* 應, literally 'respond').
> This is the reason why when a *gōng* note is struck, other *gōng* notes would resonate and when a *jué* note is struck, other *jué* notes would vibrate.[47]

Here *shēng-bǐ zé-yìng* 聲比則應 is a concise statement on resonance for sound in general. It is expressed in terms of two key technical terms, *bǐ* 比 and *yìng* 應. The term *bǐ*, which means literally 'comparison', is coined to represent 'matching in pitch', a condition for resonance first pointed out by Lǔ Jù. The term *yìng*, which means literally 'respond', is coined to qualify the term *dòng* 動 by emphasizing the sympathetic aspects of the vibrations in resonance. These technical terms probably were not yet developed at the time of Lǔ Jù's work, but later became common usages in subsequent accounts of resonance in sound. An equally concise statement of sound resonance is also found in the 'Wén Yán' 〈文言〉 section of the *Yì Jīng*: '同聲相應 Identical sounds respond mutually.'

In the *Chūn-Qiū Fán-Lù* 《春秋繁露》 (*The Abundant Dews of the Spring-Autumn*) of –135 by Dǒng Zhòng-Shū 董仲舒, we have:

> 試調琴瑟而錯之, 鼓其宮則他宮應之,
> 鼓其商而他商應之. 五音比而自鳴,
> 非有神, 其數然也.
> Test the tuning of [stringed instruments such as] *qín* (half-tube zither) and *sè* (zither) against mistakes. The striking of the *gōng* note should cause the other *gōng* notes to resonate (*yìng*). The striking of the *shāng* note should cause the other *shāng* notes to resonate. When the pentatonic notes are matched in their pitches (*bǐ* 比), each can engage in resounding (*zì míng* 自鳴, literally 'autosound'). There is nothing supernatural about it. It all depends on their numerical specifications.[48]

Here we have an explicit description on the use of resonance phenomenon as a practical test for the tuning of stringed musical instruments.

Other than the resonance phenomenon involving fundamental frequencies described above, the Lǔ Jù's description, as recorded in the *Zhuāng Zǐ*, contains a description of yet another resonance phenomenon:

> 夫或改調一弦, 於五音無當也.
> 鼓之, 二十五弦皆動.
> Now, for example, let us change the tuning of one of the strings so that it is not in accordance with any of the pentatonic notes; the strike of this string would cause all the twenty five strings [of the zither] to vibrate.[49]

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47. The 'Zhào Lèi' 〈召類〉 chapter of vol. 20 of the *Lǚ-Shì Chūn-Qiū* 《呂氏春秋》.
48. Chapter 30 of the *Chūn-Qiū Fán Lù* 《春秋繁露》.
49. The 'Zá Piān' 〈雜篇〉 volume (8) of the *Zhuāng Zǐ* 《莊子》.

Apparently, this claim was not reproducible since later in the *Huái-Nán Zǐ*, we have the following comment:

> 故叩宮而宮應, 彈角而角動,
> 此同音之相應也.
> 其於五音無所比, 而二十五弦皆應,
> 此不傳之道也.
> The reason that the plucking of a *gōng* note causes other *gōng* notes to resonate, and the striking of a *jué* note causes other *jué* notes to vibrate, is because identical notes resonate mutually. However, for the case in which there is no match (*bǐ* 比) in pitch with any of the pentatonic notes and yet all twenty-five strings resonant, its *dào* did not come down to us.[50]

From this comment, it is apparent that experimentations with such a resonance had also been made. However, none of the associates of the Prince of Huái-Nán was able, using one string, to create sympathetic vibrations in all the strings of a zither.

2.2.4 The Acoustics of Bells

Bells, either as musical instruments or as instruments for the standardization of pitches, played an important role in the development of physical acoustics (see p.30 to p.33 and p.44 to p.47) in Chinese civilization. In this section, I will examine the acoustical properties of the Chinese set-bells. According to archaeological excavations, bells began to appear in China around the −3rd millennium.[51] They were first made of clay in the neolithic period, and no later than the early Shāng dynasty, they were cast in bronze. In the course of their development, they found their way into a variety of usages. The prominent bells of ancient China can, in general, be divided into the *lǐ-zhōng* 禮鐘 and the *yuè-zhōng* 樂鐘 categories. The *lǐ-zhōng* were used primarily for ceremonial and religious purposes, while the *yuè-zhōng* were used for musical performances. All the *yuè-zhōng* are constructed in sets (known as the *biān-zhōng* 編鐘, the set-bells) which are tuned harmonically so that their strike tones form musical intervals, chords or scales. Set-bells played an important role in bridging musical acoustics with physical acoustics in the Chinese civilization. Shown in Figure 2 are the excellent set-bell specimens discovered in the tomb of Marquis Yǐ of Zēng 曾侯乙 in 1978.[52] From the

50. The 'Qí Sú Xùn' 〈齊俗訓〉 volume (11) of the *Huái-Nán Zǐ* 《淮南子》.
51. A clapperless pottery bell of −3rd millennium was unearthed from a Lóngshān 龍山 cultural stratum located at Kèshěngzhuāng 客省莊 in Chánggān 長安.
52. See the Léigǔdūn 擂鼓墩 Archaeological Team of the No. 1 Tomb in Suíxiàn 隨縣 (1979).

date inscribed on a *bó* 鎛 bell in the tomb, a gift to Marquis Yǐ from King Xióng Zhāng of Chǔ 楚惠章王, it is determined that the tomb was sealed around the 56th year in the reign of King Huì of Chǔ 楚惠王 (–433). Thus the Marquis Yǐ set-bells were cast probably in the –5th century. Unlike the *lǐ-zhōng*, *yuè-zhōng* were all cast with an almond-shaped cross section (see Fig. 3), which was described by Shěn Guā 沈括 as the *hé-wǎ-shì* 合瓦式 (the form of two joined segments of curved roof tiles).[53] Such set-bells do not have a central axial symmetry, but only planar symmetries.

Figure 2. A photograph of the bronze set-bells (*biān-zhōng* 編鐘) of the –5th century unearthed in 1978 from the tomb of Marquis Yǐ located in the present Suí 隨 county, Húběi province. The largest bell weighs 203.6 kg with a height of 153.4 cm and the smallest weighs 2.4 kg with a height of 20.4 cm.

Modern acoustics has revealed that the vibrations of bells are very complex. One expects for a bell with a central axial symmetry (such as a temple bell in the *lǐ-zhōng* category), that the frequency depends on the thickness, t, of the bell walls and the diameter, d, of the circular cross section at the open end of the bell. Since the mass, m, of a bell and the thickness, t , of the bell walls are both proportional to its diameter, one therefore expects that the frequency of a bell is inversely proportional to the bell diameter,

$$f \propto d^{-\delta},$$

53. See the *Bǔ Bǐ-Tán* 《補筆談》 (*Addendum to the Meng-Xi Essays*) of the *Mèng-Xī Bǐ-Tán* 《夢溪筆談》 (*Qián-Dào* 乾道 edition).

with an exponent, δ, dependent upon the specific characteristics of the bell. For set-bells with almond-shaped cross sections, the situation is more complicated since the cross section of such a bell can no longer be defined by a single diameter. But in relation to the mass of the bell, the frequency should have a similar descending curve dependence. Such a dependence is found among the unearthed set-bells.[54]

All set-bells have two major planes of mirror symmetry. The plane containing the *xiǎn* 銑 (spine) lines (see Fig. 3) that join the two portions of a bell to form an almond-shaped cross section is the major symmetry

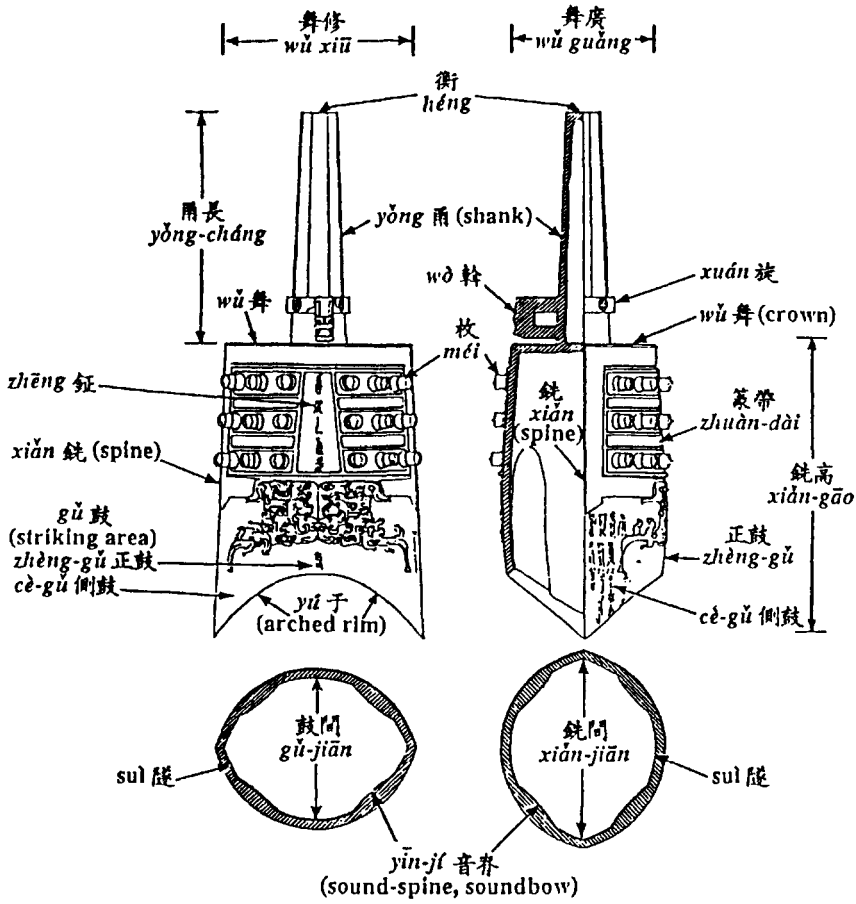

Figure 3. An illustration of the shape and the technical terms of a *yǒng-zhōng* 甬鐘, a type of ancient Chinese musical bell (*yuè-zhōng* 樂鐘), constructed with an almond-shaped cross-section in sets known as *biān-zhōng* 編鐘 (set-bells).

54. Lín Ruì 林瑞 et al. (1981), Figure 3.

plane of the bell. The second symmetry plane of the bell is that which bisects the major symmetry plane. Such a geometrical arrangement creates unequivalent strike zones for the bell, permitting the bell to be tuned to produce two distinct strike tones. The design also gives rise to discontinuous slopes at the points along the joining lines on the bell. Such discontinuities, together with the bell's geometrical shape, cause the sound of set-bells to decay faster than the sound of a bell with a central axial symmetry.

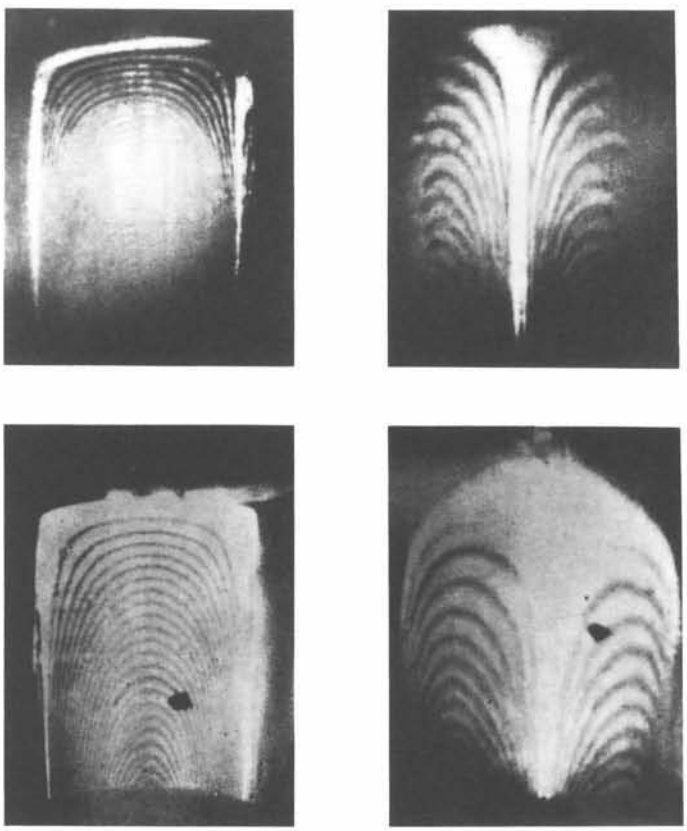

Figure 4. Holograms of the fundamental vibrational modes of the M-3-1 bell (upper figures) and the M-3-2 bell (lower figures) of the Marquis Yǐ set-bells excited by striking the *zhèng gǔ* 正鼓 strike point (left) and the *cè gǔ* 側鼓 strike point (right) as measured by Jiǎ et al.

These acoustic properties of almond-shaped bells were advantageously used by the ancient Chinese acousticians in their construction of set bells for musical performances. The discovery of the Marquis Yǐ set-bells (see Fig. 2) in 1978 revealed that the art of casting

set-bells had reached a very high level by the -5th century. The measured frequencies of the 64 Marquis Yǐ set-bells form a chromatic scale covering a range of 5½ octaves.[55] Each bell accounts for two notes in the scale. The two notes, usually a third apart, are produced by striking the bell at the *zhèng gǔ* 正鼓 and *cè gǔ* 側鼓 strike points. Holographic studies of the vibrational behaviour of such two-tone set-bells have also been carried out. As expected, two distinct families of vibrational modes can be excited by striking the bell at the two strike points. Reproduced in Figure 4 are laser holograms of the fundamental vibrational modes excited by striking the M-3-1 bell and the M-3-2 bell of the Marquis Yǐ set-bells at the *zhèng gǔ* and the *cè gǔ* strike points as measured by Jiǎ et al.[56] The lighter parts show the nodal lines and the darker parts show areas of vibrational amplitudes.

The use of the two-tone properties of set-bells for the construction of chromatic scales was indeed a remarkable invention. It required not only high precision foundry techniques but also advanced acoustical knowledge. Once a bell was cast, there would be little room for modifying the bell's pitches. Fine tunings were evidently made by sanding, as indicated by the sanding marks of different degrees found on the inner walls of the bells in the *suì* 隧 grooves (see Fig. 3). The foundry techniques and acoustic knowledge, as revealed by the Marquis Yǐ set-bells and the inscriptions on these bells, were more advanced than those given in the *Kǎo Gōng Jì*. It is of great interest to investigate to what extent was the practical knowledge in acoustics based on theoretical understanding.

2.3 Musical Acoustics

As mentioned earlier, living in an environment filled with sound waves, much of man's acoustical experience comes naturally and spontaneously. One of the early acoustical experiences that excited human aesthetical interest was the discovery of musical intervals. Certain individual tones, when played together (or in succession), sound pleasing to the human ear. The subtlety of this discovery lies in the fact that not all individual pleasing tones sound pleasing when played together. Such observations

55. See the Bronze Research Division of the Shànghǎi Museum (1982). The measurements were made by Pān Dǔ-Wǔ 潘篤武, Liú Guì-Xīng 劉貴興, of Fùdàn University and Mǎ Chéng-Yuán 馬承源 and Pān Jiàn-Míng 潘建明 of the Shànghǎi Museum.
56. Jiǎ Lǒng-Shēng 賈隴生, et al. (1981). The holograms for the M-3-2 bell are published by the Húběi Provincial Museum in a book report entitled *Zēng-Hóu-Yǐ Mù* 《曾侯乙墓》 *(The Tomb of Marquis Yǐ)* (1989), vol. 2, Plate 294.

led to the discovery of musical intervals, known as consonances. The extraordinary nature of this acoustical discovery lies in the fact that the discovery was based on aesthetical considerations rather than on physical criteria. This interaction with aesthetics permitted a remarkably early advancement in musical acoustics. In this section, I will discuss the early Chinese work in musical acoustics.

2.3.1 The Beginning Stage of Musical Acoustics

The onset of musical acoustics certainly began long before the invention of musical scales. We are, however, not in a position to learn much about this development. The most important information of early acoustic systems came primarily from ancient musical instruments discovered in archaeological excavations. In Chinese civilization, the earliest extant musical instruments can be traced to the *gǔ-shào* 骨哨 (bone whistle) of c. –6000 of the Neolithic Hémǔdù 河姆渡 culture and the *táo-xūn* 陶塤 (globular pottery flute) of c. –5000 of the Neolithic Yǎngsháo 仰韶 culture. One of the most significant archaeological discoveries of musical instruments of this early period is undoubtedly the *qī-kǒng gǔ-dí* 七孔骨笛, the seven-hole flute made from animal bones, unearthed in 1987 at Jiǎhú 賈湖, Wǔyáng 舞陽 county, Hénán province.[57] The second Wǔyáng cultural stratum, from which a total of sixteen such *gǔ-dí* (bone flutes) were unearthed, has been carbon-dated to the –6th millennium.[58] Among the sixteen *gǔ-dí* unearthed, the one with the assigned code M282 : 20 has no cracks and is well preserved and capable of producing tones. It has, in addition to the seven finger holes, two holes at its open-ends and a tiny hole just above the seventh finger-hole (see Fig. 5). The length of the *gǔ-dí* is 22.2 cm. It was from a few such rare unearthed musical instruments, which have preserved their ability in producing tones, that precious information on prehistorical achievements in musical scales was obtained.

Experimenting with its sound production indicates that the *gǔ-dí* is capable of producing a total of eight tones, with seven tones corresponding to the seven finger holes and an additional tone corresponding to the pipe tone with all the finger holes closed. Due to the additional small hole on top of the seventh finger hole, a variation in the tone of the 7th note can be produced. Measurement indicates that

57. See the Hénán Research Institute of Cultural Relics (1989).
58. The dates determined from the two samples taken from the second Wǔyáng 舞陽 cultural stratum are 7137±128 (7762±128, calibrated) and 7105±122 (7737±122, calibrated) years ago.

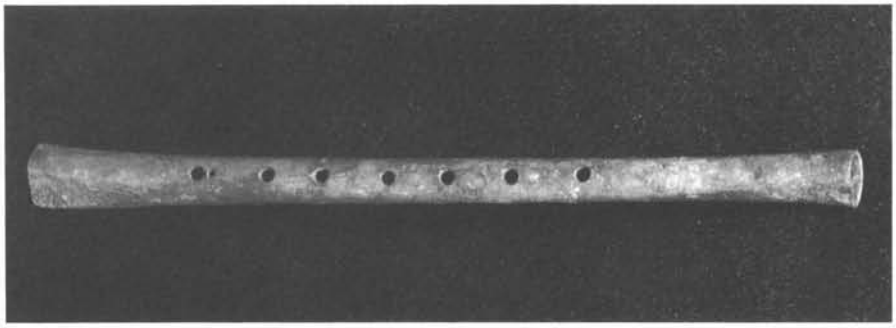

Figure 5. A photograph of a *qī-kǒng gǔ-dí* 七孔骨笛(M282 : 20), a seven-hole flute made of bone, of the –6th millennium unearthed in 1987 at Jiǎhú 賈湖, Wǔyáng 舞陽 county, Hénán province.

the interval between the 1st and the 7th notes is approximately an octave. Together with the pipe note, the eight notes of the *gǔ-dí* form an heptatonic scale.[59] This is indeed a remarkable achievement at this early stage of human civilization. The existence of a Neolithic heptatonic scale has a significant impact on the current view of the origins of musical acoustics in the Chinese civilization (see p.66 to p.78).

The number of unearthed musical instruments, which have preserved their ability to produce tones, is increased as one examines the musical instruments from the Shāng dynasty. The Shāng bronze bells and stone-chimes are usually found in sets known later as the *biān-zhōng* and *biān-qìng* and the majority of them are in sets of three. However, due to corrosion, the tones of the Shāng set-bells are less reliable than those of the stone-chimes. A set of three Shāng stone-chimes unearthed from a pit of the Yīn 殷 ruins in Ānyáng 安陽 is shown in Figure 6, together with the rubbings of pitch names engraved on the stones.[60] Measurement of the fundamental frequencies of these stone-chimes has been made by the Ethnic Music Research Institute of the Central Music Academy in Běijīng.[61] In Table 1, a comparison of the measurements with the results obtained from theoretical considerations is given.[62] The theoretical values are obtained by considering the frequency 948.6 Hz of the Yǒng-qǐ 永啟 pitch to be given as the fundamental and from which one obtains

59. See Huáng Xiáng-Péng 黃翔鵬 (1989).
60. From the *Shuāng-Jiàn-Yí Gǔ-Qì-Wù Tú-Lù* 《雙劍誃古器物圖錄》, the three *biān-qìng* 編磬 were a collection of Yú Xǐng-Wǔ 于省武 and are now preserved at the Palace Museum in Běijīng.
61. Lǐ Chún-Yí 李純一 (1981), p. 40.
62. A similar comparison was also given by Needham and Robinson, but their values were based on the incorrect interpretation of the Chinese twelve-pitch scale with the 'octave' taken to that of the 'spiral of fifths' (see p.83 to p.85).

the major second and the perfect fourth, frequencies 1067.2 Hz and 1264.8 Hz, by using factors 8/9 and 3/4, respectively. The interval arrangement in the stone-chimes fits well into the first three notes in the *shāng* 商 and *zhǐ* 徵 modes generated in accordance with the up-and-down procedure given in the *Guǎn Zǐ* (see p.48 to p.49).[63]

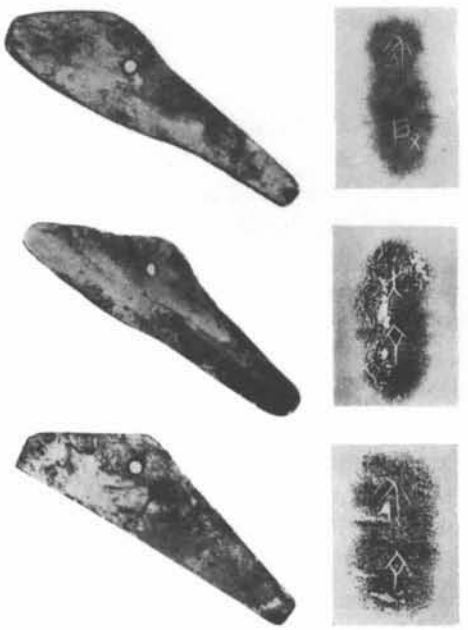

Figure 6. A set of three *biān-qìng* 編磬 (stone-chimes) unearthed from the Yīn 殷 ruins in Ānyáng 安陽 (now preserved in the Palace Museum in Běijīng) together with rubbings of the pitch names from the *biān-qìng* inscriptions.

Table 1. A Comparison of Measured Frequencies of a Set of Three Shāng Stone-Chimes with Frequencies Determined from Theoretical Considerations.

Pitch Name	Measured vib/sec	Theoretical vib/sec	Yǒng-qǐ 永啟 Mode-Key	
			Shāng Mode	*Zhǐ* Mode
Yǒng-qǐ 永啟	948.6	948.6 (given)	*shāng* 商 (re)	*zhǐ* 徵 (sol)
Yǒng-yú 永余	1046.5	1067.2	*jué* 角 (mi)	*yǔ* 羽 (la)
Yāo-yú 夭余	1278.7	1264.8	*zhǐ* 徵 (sol)	*gōng* 宫 (do)

63. It must be stressed that the up-and-down procedure was a later development, but the interest here lies in the appearance of the proper intervals in the specific sequence, among these Shāng stone-chimes.

The comparison indicates that the intervals between the pitches of the stone-chimes do indeed correspond to the major second and the perfect fourth:

Yǒng-qǐ 永啟—Yǒng-yú 永余 $$\frac{948.6}{1046.5} = \frac{8}{9} + 0.018$$

Yǒng-qǐ 永啟—Yāo-yú 夭余 $$\frac{948.6}{1278.7} = \frac{3}{4} + 0.018$$

The measured values, especially for the perfect fourth, are very close to the true values. The arrangement of the intervals fits well into the *shāng* 商 and the *zhǐ* 徵 modes. The remarkable agreement suggests an appreciation for the structure of the 'trichord' which is composed of a whole tone and a minor third. In fact, the role of 'trichords' in a pentatonic scale is similar to the role of 'tetrachords' in an heptatonic scale.[64]

As discussed earlier on p.20 to p.23, no later that the –6th century, the early Chinese had in their possession twelve pitches which were used for regulating sound. According to music master Zhōu Jiū, the acoustic adviser to the High King Jǐng, the twelve pitches, all designated by two-character names, were obtained from a set of six standard pitches by introducing six interstitial pitches to subdivide the pitch intervals. He also stated, as recorded in the *Guó Yǔ*, that development of the twelve pitches began originally with the middle tone determined by Shén Gǔ 神瞽 and from which 'we first establish three pitches, then level them out evenly into six pitches, and finally bring them to completion in twelve pitches.' It is of great interest to speculate that the three pitches mentioned by Zhōu Jiū correspond to a 'trichord' as exemplified by this set of three Shāng stone-chimes given in Figure 6 and Table 1. Also of significance, are the names of the three pitches which are all disyllabic (two-character) names.

By the Shāng dynasty, the musical instrument *táo-xūn* 陶塤 (globular pottery flute), first appeared in the Neolithic Yǎngsháo culture about –5000, had already developed into an important melodic instrument. Archaeological specimens of the unearthed *táo-xūn* reveal an interesting pattern of a gradual increase in the number of finger holes as the instrument advanced in time. Three *táo-xūn*, one large and two small, were unearthed from a Yīn tomb at Liúligé 琉璃閣 in Huīxiàn 輝縣, Hénán. Unfortunately, not only was one of the small *táo-xūn* badly broken, but the blow hole at the apex of the large one was also damaged.

64. Chén Chēng-Yīh 程貞一 (1984).

The undamaged small *táo-xūn* is, however, well preserved. A photograph of this *táo-xūn*, together with the damaged large *táo-xūn*, is reproduced in Figure 7. The small *táo-xūn* has, in addition to the blow hole, five asymmetrically placed finger holes. With the various playing finger configurations, the *táo-xūn* is capable of producing a set of eleven distinct tones. These tones, measured by the Ethnic Music Research Institute of the Central Music Academy in Běijīng, are shown in Figure 7.

It should be noted that these notes, given on the staff, are not the equal-tempered notes and their positions on the staff denote that the frequencies of these notes approximate those notes. The absolute deviation in frequency from the corresponding notes are not available since the measured frequencies of the eleven notes were not published with the above result. A significant finding from the scale of the *táo-xūn* is the appearance of the semitone intervals. In fact, seven of the twelve consecutive semitone intervals in an octave are found among the eleven notes of the *táo-xūn*. This seems to suggest that a twelve-note scale was probably already in the making during the Shāng dynasty, if not theoretically, at least empirically.

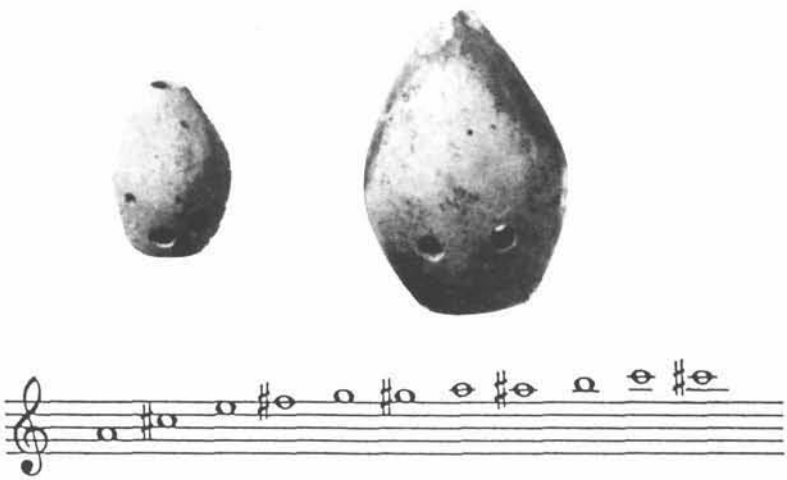

Figure 7. Two *táo-xūn* 陶塤 (globular pottery flutes) unearthed from a Yīn 殷 tomb at Liúligé 琉璃閣 in Huīxiàn 輝縣, Hénán. The small one (height 4.3 cm, maximum diameter 3.1 cm) has five finger holes, two at one side and three at the other side; the large one (height 7.3 cm, maximum diameter 5.1 cm) has a damaged blow hole.

2.3.2 Pitch Standardization and the *Xuán-Gōng* 旋宮 Principle

An important early musicological theory is the *xuán-gōng* 旋宮 principle. It states that each note of a scale can in turn act as the fundamental. From the *Lǐ-Jì* 《禮記》 (*Records of Rites*), we have the statement:

> 五聲, 六律. 十二管, 旋相為宮也.
> [Each element of] a five-tone scale, a six-pitch scale and a scale of twelve pipe-pitches acts, in turn, as the fundamental.

The *xuán-gōng* 旋宮 principle came from the realization that the intervals between notes in a scale are relative because their ratios are independent of the absolute value of the reference pitch (frequency). Thus, any one of these notes can be the fundamental with the lowest pitch in the octave.

An early Chinese practice to designate the degrees of tones in a pentatonic scale was to use a set of monosyllabic characters which, in ascending order, are: *gōng* 宮, *shāng* 商, *jué* 角, *zhǐ* 徵, *yǔ* 羽. In terms of these characters, intervals between notes can then be expressed explicitly. Since the ordering in the progression of the harmonics in an octave, known as the *diào* 調 (mode) of the scale, is specified only with respect to the fundamental, a *diào* (mode) became, in the course of time, identified with the character that designates the fundamental. The *xuán-gōng* principle states the fact that each of the five notes can act as the fundamental, giving rise to a total of five different *diào* in a pentatonic scale corresponding to the five possible permutations of the five cyclically ordered notes. Thus, the *shāng diào* 商調 means the scale is arranged with the Shāng note being the fundamental. By the −5th century and possibly much earlier, a theoretical method for calculating the harmonics in a specific mode of the scale was developed (see p.48 to p.51).

Parallel to the development of relative scales, efforts were also made to develop a scale in absolute pitches. By the −6th century, the early Chinese had in their possession twelve pitches which were used for regulating sound. Since each pitch can be used as a key to start the fundamental, the varieties in modes is further enhanced by the number of pitches. Such multiplicities in mode-keys were well recognized with the discovery of the *xuán-gōng* principle. A statement on such an appreciation can be found in the work of Mencius 孟子 (Mèng Kē 孟軻, c. −372 to −289):

> 既竭耳力焉繼之以六律正五音不可勝用也.
> To further develop the utmost of their hearing ability, they (sages) use the six pitches alternatively to key the pentatonic scale [resulting such a multiple of mode-keys] that one cannot exhaust their use.[65]

An explicit discussion of the multiplicities for pentatonic scales is found in *Huái-Nán Zǐ*:

65. See the 'Lí Lóu' 〈離婁〉 chapter (4) of the *Mèng Zǐ* 《孟子》.

一律而生五音, 十二律而為六十音.
One pitch generates a set of five notes.
Twelve pitches form a set of sixty notes.

With the names of the twelve pitches developed early in the Zhōu period (see p.21 to p.22), ancient Chinese musicians and acousticians had in their possession a unique system of nomenclature for mode-keys. Although recent archaeological discoveries indicate that there were a number of parallel systems for pitch-names in use among the various Zhōu states,[66] the principle of nomenclature for mode-keys, nevertheless, remained the same. Thus, the terms, Huáng-zhōng—*gōng* 黃鐘一宮 and Tài-cù—*jué* 太簇一角, for example, refer to the *gōng* mode in the Huáng-zhōng pitch (key), and the *jué* mode in the Tài-cù pitch (key), respectively.

The early interest in the standardization of pitch probably came from the practical need of playing different musical instruments together harmoniously for a greater variety in timbre. From the *Shī Jīng* 《詩經》, it is apparent that the practice of playing several musical instruments together began very early in Chinese civilization. In order to do so, musical instruments with unadjustable pitches must be properly tuned in advance. The fact that notes with almost the same measured frequencies are found among different unearthed Shāng instruments reveals that attempts were probably already made in the Shāng dynasty to standardize pitch.[67] An early mention of standardization of pitch is found in the 'Shùn Diǎn' of the *Shàng Shū* 《尚書》 (*The Book of Documents*) (see p.20). With the discovery of the Marquis Yǐ set-bells, we now have explicit information on the frequencies of a number of standard pitches. The inscriptions on the Top 2–6 bell of the Marquis Yǐ set-bells reveal that this bell's *shāng* note is the *gōng* of the Huáng-zhōng pitch. Thus, from the measured frequency of this note, one obtains for the Huáng-zhōng pitch at the –5th century, a frequency of 410.1 vibs/sec.

In 1986, a number of pitch-pipes were unearthed from a Chǔ 楚 tomb (M21) of the Warring States period located at the present Yǔtáishān 雨台山 in Jiānglíng 江陵, Húběi 湖北.[68] These unearthed pitch-pipes are made of nodeless bamboo (open ends) with different lengths and diameters. Though most of the pitch-pipes were broken, four of them still have readable inscriptions.[69] The fragments of extant inscriptions are reproduced below:

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66. Recent archaeological discoveries, such as the Marquis Yǐ set-bells, provide a unique opportunity to study parallel systems of nomenclature for note, pitches, and mode-keys that existed in the Zhōu period.
67. See Yáng Yìn-Liú 楊蔭瀏 (1981), p. 26 and Lǐ Chún-Yī 李純一 (1981), p. 48.
68. See the Húběi Provincial Museum (1988).
69. Tán Wéi-Sì 譚維四 (1988).

M21: 17-1 定新(新)鐘之宮為濁穆□.
坪皇角為定吝(文)王商□.

Assign (dìng 定) the fundamental (*gōng* 宮) of the Xīn-zhōng 新鐘 pitch to be the [...] of the lower Mù-[zhōng] 穆 [鐘] pitch...; The *jué* 角 of the Píng-huáng 坪皇 pitch is assigned to be the *shāng* 商 of the Wén-wáng 文 (吝) 王 pitch....

M21: 17-2 □姑洗(洗)之宮為濁吝(文)王羽(羽)為濁□.
The fundamental (*gōng* 宮) of the Gū-xiǎn 姑洗 pitch corresponds to the *yǔ* 羽 of the lower Wén-wáng 文(吝)王 pitch and the [...] of the lower....

M21: 17-3 □之宮為濁獸鐘羽(羽)□.
The fundamental of the [...] corresponds to the *yǔ* 羽 of the lower Shòu-zhōng 獸鐘 pitch... .

M21: 17-4 □為濁穆鐘□.
...corresponds to the [...] of the lower Mù-zhōng 穆鐘 pitch... .

These inscriptions provide not only the names of the pitches but also explicit statements on assigning pitches through the usage of the term *dìng* 定. All the pitch names: Xīn-zhōng 新(新)鐘, Mù-zhōng 穆鐘, Píng-huáng 坪皇, Wén-wáng 文王 (吝王, 吝王), Gū-xiǎn 姑洗, and Shòu-zhōng 獸鐘, found in the pitch-pipe inscriptions are identifiable with the pitch names inscribed on the unearthed Marquis Yǐ set-bells and set stone-chimes of the state of Zēng.

These unearthed pitch-pipes are the earliest extant specimens that are currently available. The dimensions of these pipes provided by their remnants are reproduced in Table 2 for the four pitch-pipes discussed above. It is seen that the diameters for two of the four pitch-pipes are still measurable. Unfortunately, the exact lengths of these two pitch-pipes are not available to shed further information on their actual frequencies for a direct comparison with the measured frequencies of the corresponding pitches in the Marquis Yǐ set-bells.

Table 2. Names and Measures of the Unearthed Chǔ 楚 Pitch-Pipes[a]

Assigned Specimen Code	Chu 楚 Pitch-Pipe's Pitch-Name	Pitch-Pipe Outer Diameter	Pitch-Pipe Inner Diameter	Remnant Pitch-Pipe Length
M21: 17-1	Xīn-zhōng 新鐘	0.9 cm	0.6 cm	10.9 cm
M21: 17-2	Gū-xiǎn 姑洗	1.0 cm	0.7 cm	13.3 cm
M21: 17-3	—	—	—	6.2 cm
M21: 17-4	—	—	—	5.9 cm

[a] Unearthed in 1986 from a Chǔ tomb (M21) of the Warring States period located at the present Yǔtáishān 雨台山 in Jiānglíng 江陵, Húběi 湖北.

2.3.3 Mathematical Generation of Pentatonic Scales

A mathematical procedure for the generation of musical scales has come down to us in the *Guǎn Zǐ*. The relevant passage runs as follows:

> 凡將起五音, 先主一而三之, 四開以合九九.
> 以是生黃鐘小素之首, 以成宮;
> 三分而益之以一, 為百有八, 為徵;
> 不無有三而去其乘, 適足以是生商;
> 有三分而復於其所, 以是生羽;
> 有三分而去其乘, 以是生角.
> As for the generation of the five tones [by a mathematical procedure], one begins first by taking the principal (*zhǔ* 主) to be unity and then multiplying it by the number 3. The fourth power of this number gives 9 × 9 (i.e. 81) which is the smallest leading number (*xiǎo shù zhī shǒu* 小數[素]之首) for the generation [of the scale] from [the tone] *gōng* 宮 in the Huáng-zhōng 黃鐘 pitch.
>
> By increasing the number by one-third of its value, one obtains the number 108 for [the tone] *zhǐ* 徵. By decreasing the number so obtained (i.e. 108) by nothing more than one-third of its value, one obtains [the number 72] which is sufficient to generate [the tone] *shāng* 商. By giving back to [the number 72] one-third of its value, one finds [the number 96] for the generation of [the tone] *yǔ* 羽. By decreasing the number so obtained (i.e. 96) by one third of its value, one finds [the number 64] for the generation of [the tone] *jué* 角.[70]

It is apparent that this passage describes a mathematical procedure for the generation of a set of five notes.

In current mathematical notation, the procedure can be summarized as follows:

$(1 \times 3)^4 = 9 \times 9 = 81$	for the note *gōng* 宮
$81\ (1 + \frac{1}{3}) = 108$	for the note *zhǐ* 徵
$108\ (1 - \frac{1}{3}) = 72$	for the note *shāng* 商
$72\ (1 + \frac{1}{3}) = 96$	for the note *yǔ* 羽
$96\ (1 - \frac{1}{3}) = 64$	for the note *jué* 角

Thus, the procedure involves the assignment of the number 81 to the note *gōng*, and then perform the appropriate operations 1 + 1/3 or 1 − 1/3 to generate numbers for other notes as illustrated in Figure 8. For convenience, the numbers, 1 + 1/3 and 1 − 1/3, are replaced in Figure 8,

70. See the 'Dì-Yuán' 〈地員〉 chapter (58) of the *Guǎn Zǐ* 《管子》.

by their equivalent numbers 4/3 and 2/3, respectively. Since the intervals between notes are given by the ratios of their corresponding numbers, the scale so generated is independent of the initial number 81 assigned to the note *gōng*. The passage explains that this number equals the fourth power of the number 3 and is the smallest leading number for the generation of the scale from note *gōng* in the Huáng-zhōng pitch. This seems to imply that number 81 is assigned to the note *gōng* for the purpose of avoiding non-integers because, in generating the rest of the four notes in the scale, the procedure requires the number to be divided by 3, four times, and the number 81 is the smallest integer that can be evenly divided by 3 four times.

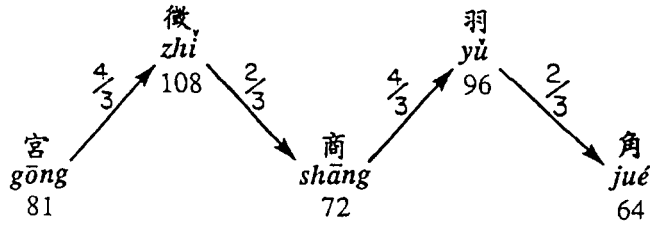

Figure 8. The procedure for generating the pentatonic scale in the *zhǐ* 徵 mode by the up-and-down generation procedure as described in the *Guǎn Zǐ*.

With the lowest note being the *zhǐ* note, the five notes so generated form a pentatonic scale in the *zhǐ* mode. A study of its intervallic structure reveals that the scale is composed of the following harmonic progression:

zhǐ	徵	1	fundamental
yǔ	羽	8/9	major second
gōng	宫	3/4	perfect fourth
shāng	商	2/3	perfect fifth
jué	角	16/27 (~ 3/5)	major sixth

Except for the major sixth, the ratios are exactly equal to those found in natural (or just) intonation.

2.3.4 The Up-and-Down Principle

As it is seen in Figure 8 above, these five notes are generated in the *Guǎn Zǐ* either by the multiplication of the factor 4/3 (1 + 1/3) or by the factor 2/3 (1 − 1/3). The principle behind the *Guǎn Zǐ* procedure is known as the up-and-down principle. An illustration of the principle in the current notation is shown in Figure 9. The down-generation with the ratio 2/3 generates a perfect fifth since it corresponds to decrease the

length of the vibrating string by 1/3 (i.e. 1 − 1/3).[71] The up-generation with the ratio 4/3 generates also a perfect fifth but an octave lower, since the operation to add 1/3 (i.e. 1 + 1/3) the length of the vibrating string is equivalent to shorten the length by 1/3 and then double the remaining length. In Figure 9, it is seen that the note Y_1, down-generated from note X, can also be generated by first up-generating the Y note then followed by lowering the note by an octave to obtain the Y_1 note. Notes Y and Y_1, being up-and-down generated from the same note X, are always an octave apart as clearly illustrated in Figure 9. Thus, the *Guǎn Zǐ* procedure is, in fact, a procedure that generates by the fifth (*wǔ-dù xiāng-shēng* 五度相生) and the up-and-down principle is a principle that regulates the octaval relation of the generated notes.

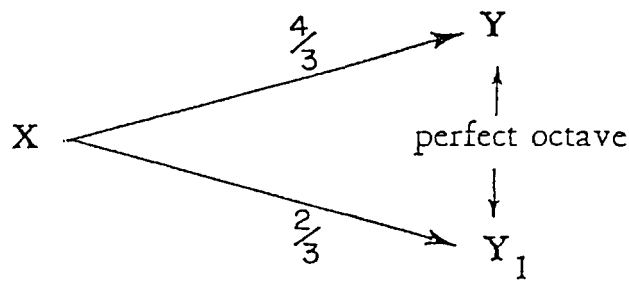

Figure 9. The up-and-down generation principle

From the above analysis, it is obvious that the *Guǎn Zǐ* procedure, also known in Chinese as the *sān-fēn sǔn-yì* 三分損益 method, is actually a general method which can be used to generate scales in different modes or with different number of notes. The explicit harmonic ratios used are the perfect fifth 2/3 and the perfect octave 1/2. This is acoustically just. The perfect fifth interval is used to generate all the notes, while the octave is used, when needed, to regulate the generated notes so that they all lie within an octave. Thus, the up-and-down principle insures that the notes generated by the fifth are lying within the compass of an octave.[72]

As an illustrative example, let us consider the generation of the pentatonic scale in the *shāng* mode. For the *shāng* mode, the lowest of the

71. It should be noted that the concept of frequency is a later development. The terms, up-generation and down-generation, refer to the increase and decrease in the length of a vibrating string. Hence, a down-generation actually generates a higher notes, and an up-generation a lower note.
72. Chén Chēng-Yīh 程貞一 (1987), pp. 168–70.

five-note scale within an octave should, of course, be a *shāng* note. If one follows the procedures given in the *Guǎn Zǐ*, to generate the *shāng* note from the *gōng* note, it requires two consecutive up-generations to obtain the lower *shāng* note. The second up-generation is required because the *shāng* note obtained by the down-generation from the *zhǐ* note would lie higher than the *gōng* note. Hence, it is necessary to lower the note an octave, which is accomplished by switching the procedure directly to the up-generation as shown in Figure 10.

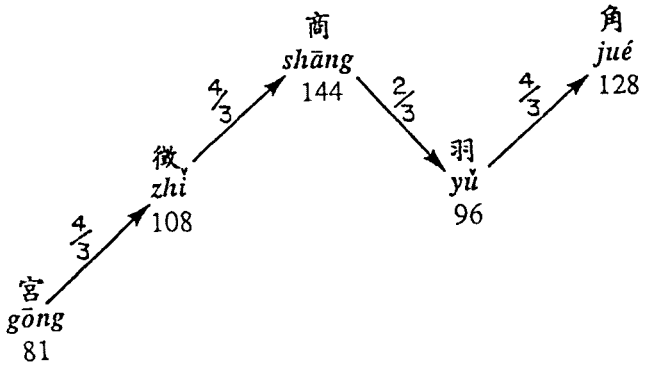

Figure 10. The procedure for generating the pentatonic scale in the *shāng* 商 mode by the up-and-down generation procedure as described in the *Guǎn Zǐ*.

The pentatonic scale in the *shāng* mode, so generated, is composed of the following harmonic progression:

shāng	商	1	fundamental
jué	角	8/9	major second
zhǐ	徵	3/4	perfect fourth
yǔ	羽	2/3	perfect fifth
gōng	宫	9/16 (~ 5/9)	minor seventh

Except for the minor seventh, the ratios are exactly as those found in natural (or just) intonation. The *shāng* mode differs from the *zhǐ* mode given in the *Guǎn Zǐ* by only one note; instead of the major sixth, the *shāng* mode has the minor seventh. It is straightforward to generate the pentatonic scale in the rest of the modes. This demonstrates the viability of the up-and-down principle and the general nature of the procedure. Based on the extant materials, it is evident that all the five pentatonic modes had been actively used in early Chinese music.

2.3.5 The Generation of the Chromatic Scales

Although the *Guǎn Zǐ* procedure given above uses the up-and-down

principle only to generate five notes within an octave for pentatonic scales, the principle can be used to generate, for example, chromatic scales. A research of the early Chinese work on generating chromatic scales reveals two methods that have came down to us, one from the *Lǚ-Shì Chūn-Qiū* based on the up-and-down principle and the other from the inscriptions of the Marquis Yǐ *niǔ* 鈕 set-bells based on the *jué – zēng* 角一曾 method. In this subsection, I examine both these methods.

The Lǚ-Shì Chūn-Qiū 《呂氏春秋》 Method

A description of the up-and-down procedure for the generation of chromatic scales is given in the *Lǚ-Shì Chūn-Qiū*. We have from Chapter 6 'Yīn Lǜ'〈音律〉 ('Tonal Pitch') the passage:

> 黃鐘生林鐘, 林鐘生太蔟, 太蔟生南呂, 南呂生姑洗,
> 姑洗生應鐘, 應鐘生蕤賓, 蕤賓生大呂, 大呂生夷則,
> 夷則生夾鐘, 夾鐘生無射, 無射生仲呂.
> 三分所生益之一分以上生,
> 三分所生去其一分以下生.
> 黃鐘, 大呂, 太蔟, 夾鐘, 姑洗, 仲呂, 蕤賓, 為上.
> 林鐘, 夷則, 南呂, 無射, 應鐘, 為下.
> The Huáng-zhōng 黃鐘 pitch generates the Lín-zhōng 林鐘 pitch.
> The Lín-zhōng pitch generates the Tài-cù 太蔟 pitch.
> The Tài-cù pitch generates the Nán-lǚ 南呂 pitch.
> The Nán-lǚ pitch generates the Gū-xiǎn 姑洗 pitch.
> The Gū-xiǎn pitch generates the Yīng-zhōng 應鐘 pitch.
> The Yīng-zhōng pitch generates the Ruí-bīn 蕤賓 pitch.
> The Ruí-bīn pitch generates the Dà-lǚ 大呂 pitch.
> The Dà-lǚ pitch generates the Yí-zé 夷則 pitch.
> The Yí-zé pitch generates the Jiá-zhōng 夾鐘 pitch.
> The Jiá-zhōng pitch generates the Wú-yì 無射 pitch.
> The Wú-yì pitch generates the Zhòng-lǚ 仲呂 pitch.
> The addition of 1/3 of its value gives rise to the up-generation.
> The subtraction of 1/3 of its value gives rise to the down-generation.
>
> Huáng-zhōng, Dà-lǚ, Tài-cù, Jiá-zhōng, Gū-xiǎn, Zhòng-lǚ,
> and Ruí-bīn are obtained by up-generation.
> Lín-zhōng, Yí-zé, Nán-lǚ, Wú-yì,
> and Yīng-zhōng are obtained by down-generation.

The procedure for generating the twelve pitches described by this passage is illustrated in Figure 11. It is seen that procedure is again made up of the up- and down-generations. In fact, the *Lǚ-Shì Chūn-Qiū* passage contains the explicit definitions for the up- and down-generations:

> 三分所生益之一分以上生.
> 三分所生去其一分以下生.

The addition of 1/3 of its value gives rise to the *shàng-shēng* 上生 (up generation).
The subtraction of 1/3 of its value gives rise to the *xià-shēng* 下生 (down generation).

This indicates that the *Lǚ-Shì Chūn-Qiū* procedure is, in fact, the *Guǎn Zǐ* procedure with a different application.

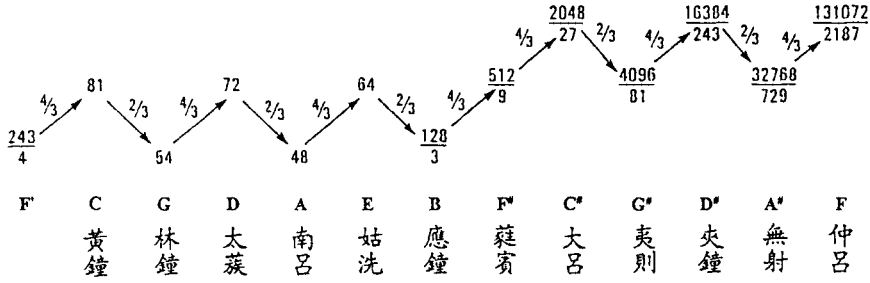

Figure 11. The generation of the chromatic scale by the use of the up-and-down principle as described in the *Lǚ-Shì Chūn-Qiū*.

For comparison, the number 81 from the *Guǎn Zǐ* procedure is again used as the starting number for the Huáng-zhōng pitch in Figure 11 to generate the rest of the pitches within the octave. The intervallic relations given by the ratios of these numbers indicate a chromatic structure. To make use of the established nomenclature for chromatic sequence: C, C#, D, D#, E, F, F#, G, G#, A, A#, B, C_1 for the purpose of discussion, the pitch Huáng-zhōng is arbitrarily identified with the note C. The intervals between adjacent pitches in the chromatic sequence can easily be calculated by the use of the numbers given in the Figure 11.

The calculated results reveal that the chromatic scale is made up of two types of semitones (see Fig. 12), namely the major and the minor semitones. One has, for example:

$$\frac{\text{仲呂 F}}{\text{姑洗 E}} = \frac{131072/2187}{64} = \frac{2048}{2187} \quad \text{(major)}$$

$$\frac{\text{蕤賓 F\#}}{\text{仲呂 F}} = \frac{512/9}{131072/2187} = \frac{243}{256} \quad \text{(minor)}$$

The combination of the major and minor semitones gives the major whole tone

$$\left(\frac{243}{256}\right)\left(\frac{2048}{2187}\right) = \left(\frac{3^5}{2^8}\right)\left(\frac{2^{11}}{3^7}\right) = \frac{8}{9}\,. \tag{1}$$

This relationship between the semitone and whole tone is the foundation of the *Lǚ-Shì Chūn-Qiū* chromatic scale. As seen from Figure 12, an

octave in such a chromatic scale is made up of seven minor semitones and five major semitones. They satisfy, as expected, the following numerical relation for an octave:

$$\left(\frac{243}{256}\right)^7 \left(\frac{2048}{2187}\right)^5 = \left(\frac{3^5}{2^8}\right)^7 \left(\frac{2^{11}}{3^7}\right)^5 = \frac{1}{2}\,. \tag{2}$$

With only two different semitones, these chromatic scales are rather flexible in accommodating mode shifts.

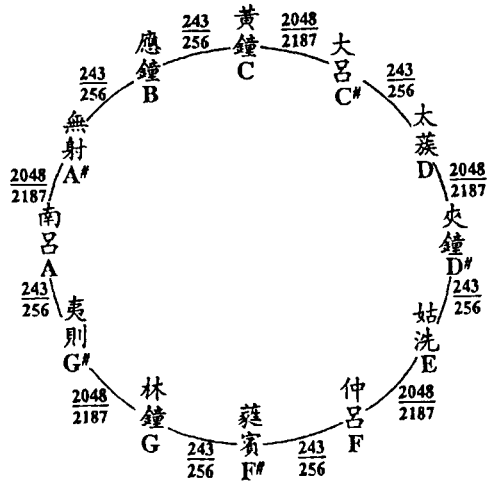

Figure 12. The semitonal structure of the octave circle of the chromatic scale generated by the up-and-down procedure described in the *Lǚ-Shì Chūn-Qiū*.

In Table 3, the intervallic structure of the chromatic scale so generated is given in each of the twelve pitches (keys). For comparison, the intervallic structure of natural (just) intonation is also given in Table 3. It is seen that the *Lǚ-Shì Chūn-Qiū* chromatic scales contain different degrees of admixture of the natural intonation in its mode-keys.[73]

The Jué – Zēng 角 – 曾 Method

Another early method for generating chromatic scale is revealed by the Marquis Yǐ *niǔ* 鈕 set-bells in top-2 and top-3 groups (see Fig. 2). In this method, use is made of the up-and-down principle to generate four core notes and from these core notes the remaining eight notes in the chromatic scale are generated by the use of the *jué* 角 and the *zēng* 曾 intervals.[74] The method is best appreciated by examining the note names

73. See p.110 to p.112 below for an analysis of the relation to the natural intonation.
74. Chén Chēng-Yīh 程貞一 (1987), pp. 171–76. and Chén Chēng-Yīh (1994).

Table 3. A Comparison of the Intervallic Structures of Chromatic Scales in the Twelve Pitches (Keys) with the Natural (Just) Intonation.

Keys / Intervals	黃鐘	大呂	太蔟	夾鐘	姑洗	仲呂	蕤賓	林鐘	夷則	南呂	無射	應鐘	Natural (Just) Intonation
	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	
Fundamental	1	1	1	1	1	1	1	1	1	1	1	1	1
Minor second	$\frac{2048}{2187}$	$\frac{243}{256}$	$\frac{2048}{2187}$	$\frac{243}{256}$	$\frac{2048}{2187}$	$\frac{243}{256}$	$\frac{243}{256}$	$\frac{2048}{2187}$	$\frac{243}{256}$	$\frac{2048}{2187}$	$\frac{243}{256}$	$\frac{243}{256}$	$\frac{15}{16}$
Major second	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{59049}{65536}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{59049}{65536}$	$\frac{8}{9}$	$\frac{8}{9}$
Minor third	$\frac{16384}{19683}$	$\frac{27}{32}$	$\frac{16384}{19683}$	$\frac{27}{32}$	$\frac{27}{32}$	$\frac{27}{32}$	$\frac{27}{32}$	$\frac{16384}{19683}$	$\frac{27}{32}$	$\frac{27}{32}$	$\frac{27}{32}$	$\frac{27}{32}$	$\frac{5}{6}$
Major third	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{6561}{8192}$	$\frac{64}{81}$	$\frac{6561}{8192}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{6561}{8192}$	$\frac{64}{81}$	$\frac{6561}{8192}$	$\frac{64}{81}$	$\frac{4}{5}$
Perfect fourth	$\frac{131072}{177147}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
Augmented fourth	$\frac{512}{729}$	$\frac{729}{1024}$	$\frac{512}{729}$	$\frac{729}{1024}$	$\frac{512}{729}$	$\frac{729}{1024}$	$\frac{729}{1024}$	$\frac{512}{729}$	$\frac{729}{1024}$	$\frac{512}{729}$	$\frac{729}{1024}$	$\frac{512}{729}$	$\frac{32}{45}$
Perfect fifth	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{177147}{262144}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
Minor sixth	$\frac{4096}{6561}$	$\frac{81}{128}$	$\frac{4096}{6561}$	$\frac{81}{128}$	$\frac{81}{128}$	$\frac{81}{128}$	$\frac{81}{128}$	$\frac{4096}{6561}$	$\frac{81}{128}$	$\frac{4096}{6561}$	$\frac{81}{128}$	$\frac{81}{128}$	$\frac{5}{8}$
Major sixth	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{19683}{32768}$	$\frac{16}{27}$	$\frac{19683}{32768}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{19683}{32768}$	$\frac{16}{27}$	$\frac{3}{5}$
Minor seventh	$\frac{32768}{59049}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{32768}{59049}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{9}$
Major seventh	$\frac{128}{243}$	$\frac{128}{243}$	$\frac{128}{243}$	$\frac{2187}{4096}$	$\frac{128}{243}$	$\frac{2187}{4096}$	$\frac{128}{243}$	$\frac{128}{243}$	$\frac{2187}{4096}$	$\frac{128}{243}$	$\frac{2187}{4096}$	$\frac{128}{243}$	$\frac{8}{15}$
Octave	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

and their arrangement in relation to the chromatic scale (see Fig. 13) and the measured frequencies of these *niǔ* 鈕 bells (see Table 4).

Shown in Figure 13 is the layout of these *niǔ* bells. It is seen that the notes in accordance with the arrangement of the bells exhibit a chromatic structure in ascending order from left to the right and from bottom to the top, encompassing a little over two octaves.[75] For convenience, the lowest *gōng* note is arbitrarily identified with the letter note C to make use of the established nomenclature for chromatic sequence. It is seen that the bell notes fit together systematically to form the chromatic scale.

A clue for the reconstruction of the procedure used in generating the chromatic scale is found in the nomenclature adopted for note names. Among these *niǔ*-bell notes, there are four pentatonic notes, i.e. the *zhǐ*, *yǔ*, *gōng*, and *shāng* notes each associated with two additional notes bearing their names as follows:

zhǐ	徵	*zhǐ-jué*	徵角	*zhǐ-zēng*	徵曾
yǔ	羽	*yǔ-jué*	羽角	*yǔ-zēng*	羽曾
gōng	宫	*gōng-jué*	宫角	*gōng-zēng*	宫曾
shāng	商	*shāng-jué*	商角	*shāng-zēng*	商曾

This suggests that the four pentatonic notes function as the core from which the additional eight notes in a chromatic scale are generated by the use of the *jué* and *zēng* intervals. The notes generated using the *jué* intervals are labelled with the additional character *jué* 角, while the notes generated using the *zēng* intervals are labelled with the additional character *zēng* 曾. This pattern of generation of a chromatic scale from four core notes is illustrated (right diagram) in Figure 13 in conformity with the arrangement of the bells (left diagram).[76] It is seen how readily a chromatic scale can be generated using this scheme. This generation method is different from the up-and-down method since the fifth interval is used only to generate the *zhǐ*, *yǔ*, *gōng*, and *shāng* four core notes. The *zhǐ-jué*, *yǔ-jué*, *gōng-jué*, and *shāng-jué* notes are generated from the core notes with the *jué* interval and the *zhǐ-zēng*, *yǔ-zēng*, *gōng-zēng* and *shāng-zēng* notes are generated from the core notes with the *zēng* interval.

In order to examine this ancient method for generating chromatic scales, one must first know the actual ratios used for the *jué* and *zēng* intervals in the generation of the eight notes from the four core notes.

75. Based on this pattern, it is not difficult to see that the bell corresponding to the notes *gōng* 宫 and *zhǐ-zēng* 徵曾, next to the bell denoted by T-3-2, is, in fact, missing. The bell denoted by T-3-1, which was found in the position of the missing bell in the top-3 group, belongs to the top-2 group.

76. See Chén Chēng-Yīh 程貞一 (1987), p. 163, Figure 2 and p. 174, Figure 7.

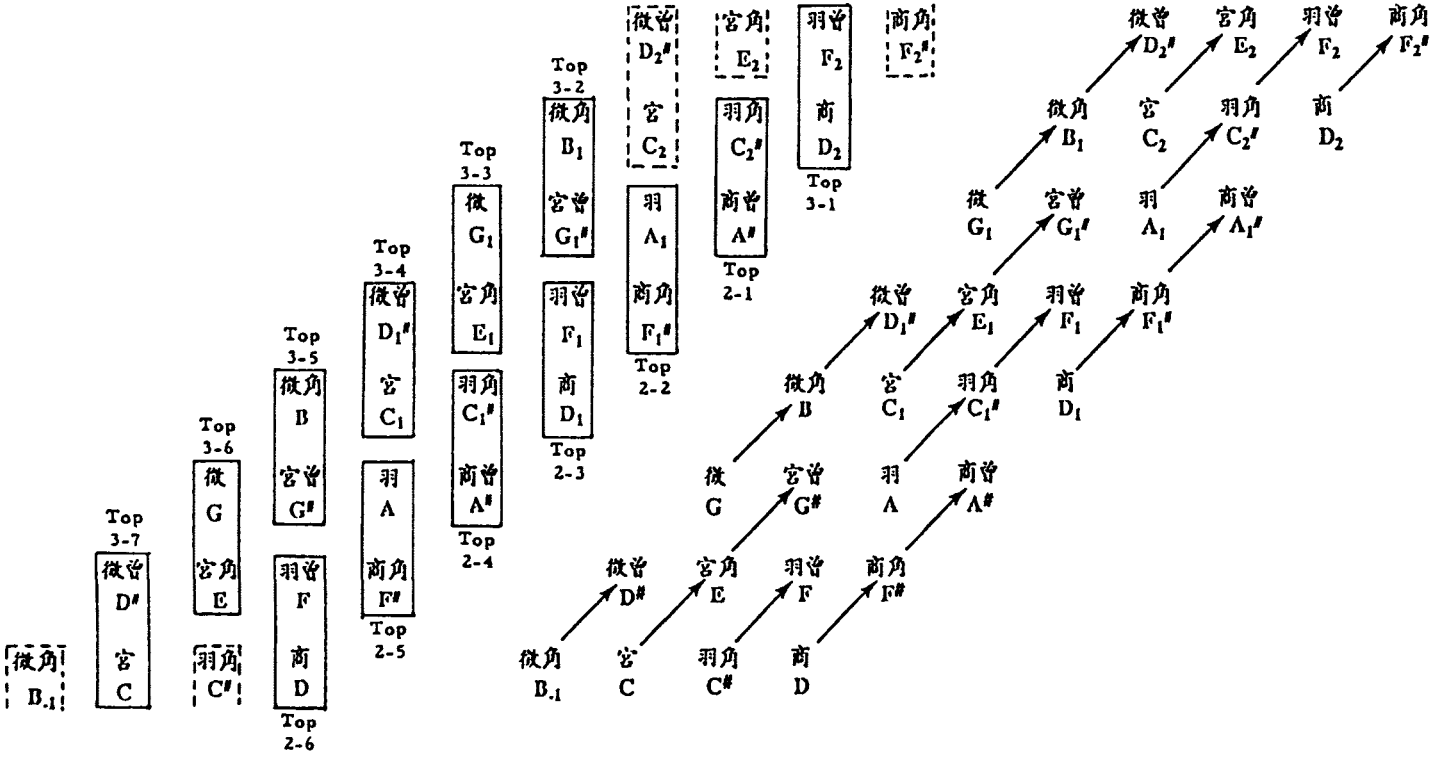

Figure 13. Left, the pattern of a chromatic scale composed of pairs of notes from the Marquis Yǐ *niǔ* bells (-5th century) in the top-2 and top-3 groups. The note-names are from the bell inscriptions and the lowest *gōng* 宫 note is arbitrarily identified with the C note to make use of the established chromatic sequence. Right, the pattern of note generation within the same arrangement between bells and bell notes as given in the left illustration. The notes are in ascending order from left to right and from bottom to top.

Table 4. Measured Frequencies of the *Zhèng-gǔ* 正鼓 and *Cè-gǔ* 側鼓 Notes of the Marquis Yǐ *Niǔ-Bells* in the T-2 and T-3 Groups

Chromatic Note Sequence	*Niǔ* 鈕 Bell Notes	1st Octave	2nd Octave	3rd Octave
gōng 宮 C	*Zhèng-gǔ* 正鼓 Note (T-3-7, T-3-4)	a 362.2 b 365.1 c 364.1	a 728.5 b 731.7 c 729.8	
yǔ-jué 羽角 C#	*Cè-gǔ* 側鼓 Note (T-2-4, T-2-1)		a 814.9 b 818.9 c 817.8	a 1668.9 b 1678.7 c 1673.8
shāng 商 D	*Zhèng-gǔ* 正鼓 Note (T-2-6, T-2-3, T-3-1)	a 407.7 b 410.1 c 409.0	a 818.7 b 822.7 c 820.6	a 1775.3 b 1781 c 1778.1
zhǐ-zēng 徵曾 D#	*Cè-gǔ* 側鼓 Note (T-3-7, T-3-4)	a 443.8 b 446.6 c 446.2	a 885.6 b 890 c 889.2	
gōng-jué 宮角 E	*Zhèng-gǔ* 正鼓 Note (T-3-6, T-3-3)	a 469.1 b 471.9 c 470.2	a 938.3 b 944 c 940.4	
yǔ-zēng 羽曾 F	*Cè-gǔ* 側鼓 Note (T-2-6, T-2-3, T-3-1)	a 497.3 b 500.5 c 501.2	a 979.3 b 998 c 996.6	a 2111.2 b 2117 c 2111.9
shāng-jué 商角 F#	*Zhèng-gǔ* 正鼓 Note (T-2-5, T-2-2)	a 541.1 b 543.5 c 542.0	a 1094.7 b 1101 c 1097.4	
zhǐ 徵 G	*Cè-gǔ* 側鼓 Note (T-3-6, T-3-3)	a 569.0 b 571.2 c 570.1	a 1134.6 b 1140 c 1138.8	
gōng-zēng 宮曾 G#	*Zhèng-gǔ* 正鼓 Note (T-3-5, T-3-2)	a 599.3 b 599.3 c 598.1	a 1237.3 b 1243 c 1236.5	
yǔ 羽 A	*Cè-gǔ* 側鼓 Note (T-2-5, T-2-2)	a 640.5 b 642.6 c 641.6	a 1300.4 b 1306.8 c 1305.6	
shāng-zēng 商曾 A#	*Zhèng-gǔ* 正鼓 Note (T-2-4, T-2-1)	a 681.3 b 683.5 c 681.0	a 1381.7 b 1390 c 1385.7	
zhǐ-jué 徵角 B	*Cè-gǔ* 側鼓 Note (T-3-5, T-3-2)	a 713.5 b 716.4 c 716.4	a 1442.0 b 1447 c 1446.7	

[a] Measurements carried out in 1978 by the Music Institute of the Běijīng Academy of Literature and Arts (see Wáng Xiāng, pp. 71–3);

[b] Measurements carried out in 1979 by the Bronze Research Division of the Shànghǎi Museum and the Physics Department of the Fùdǎn University (see the Bronze Research Division of the Shànghǎi Museum, pp. 90-2);

[c] Measurements carried out in 1980 by the Harbin University of Science and Technology (see Húběi Provincial Museum (3), vol. 1, pp. 110–5).

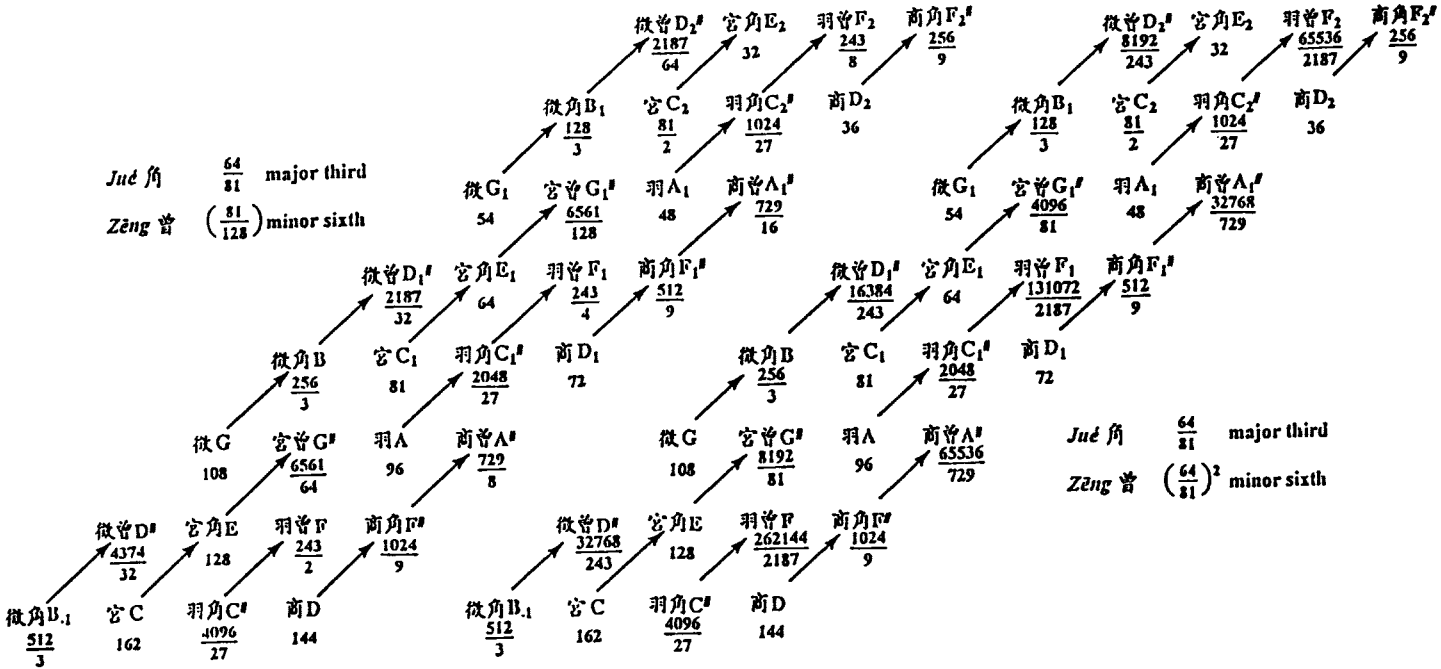

Figure 14. The generation of chromatic scales from the *zhǐ* 徵, *yǔ* 羽, *gōng* 宫, and *shāng* 商 four core notes by the use of the approximate major-third ratio 64/81 for the *jué* 角 interval and both the approximate major-sixth ratio 81/128 and $(64/81)^2$ for the *zēng* 曾 interval. The core notes are generated by the fifth 2/3 in accordance with the up-and-down principle.

Examining the chromatic sequence of the notes generated from the core notes together with the measured frequencies tabulated in Table 4, it is apparent that the *jué* and *zēng* intervals correspond to the major third and minor sixth, respectively. From the work of the *Guǎn Zǐ* and the *Lǚ-Shì Chūn-Qiū*, we have the approximations 64/81 or 6561/8192 for the major third 4/5, and 81/128 or $(64/81)^2$ for the minor sixth 5/8, leading to four sets of possible combinations for the two intervals:

Set 1. *jué* 角 interval $\cong \frac{64}{81}$ *zēng* 曾 interval $\cong \frac{81}{128}$

Set 2. *jué* 角 interval $\cong \frac{64}{81}$ *zēng* 曾 interval $\cong (\frac{64}{81})^2$

Set 3. *jué* 角 interval $\cong \frac{6561}{8192}$ *zēng* 曾 interval $\cong \frac{81}{128}$

Set 4. *jué* 角 interval $\cong \frac{6561}{8192}$ *zēng* 曾 interval $\cong (\frac{64}{81})^2$

Calculations based on the four sets of the *jué* and *zēng* intervals are carried out. Shown in Figure 14 are the computations for sets 1 and 2. The octave circles yield from the calculation are shown in Figure 15 for the four sets of intervals.

In these calculations, the numbers for the *zhǐ*, *yǔ*, *gōng*, and *shāng* core notes are taken from those generated by the fifth based on the up-and-down principle of the *Guǎn Zǐ*. As shown in a previous work, the chromatic scales generated by using the Set 2 ratios is identical to that generated in the *Lǚ-Shì Chūn-Qiū*.[77] It is seen, from the four octave circles given in Figure 15, that the chromatic scales generated from the rest of ratio sets, except for Set 4, are equivalent to that of the *Lǚ-Shì Chūn-Qiū* since their octave circles are also composed of five major (2048/2187) and seven minor (243/256) semitones (see Eq. (2) and Fig. 12). These octave circles are related to that of Set 2 by a rotation with respect to the perpendicular axis through the centre of the circles.

Two additional sets of combinations are constructed involving the exact ratios for the two intervals:

Set 5. *jué* 角 interval $= \frac{4}{5}$ *zēng* 曾 interval $= \frac{5}{8}$

Set 6. *jué* 角 interval $= \frac{4}{5}$ *zēng* 曾 interval $\cong (\frac{5}{8})^2$

77. Ibid., p. 183.

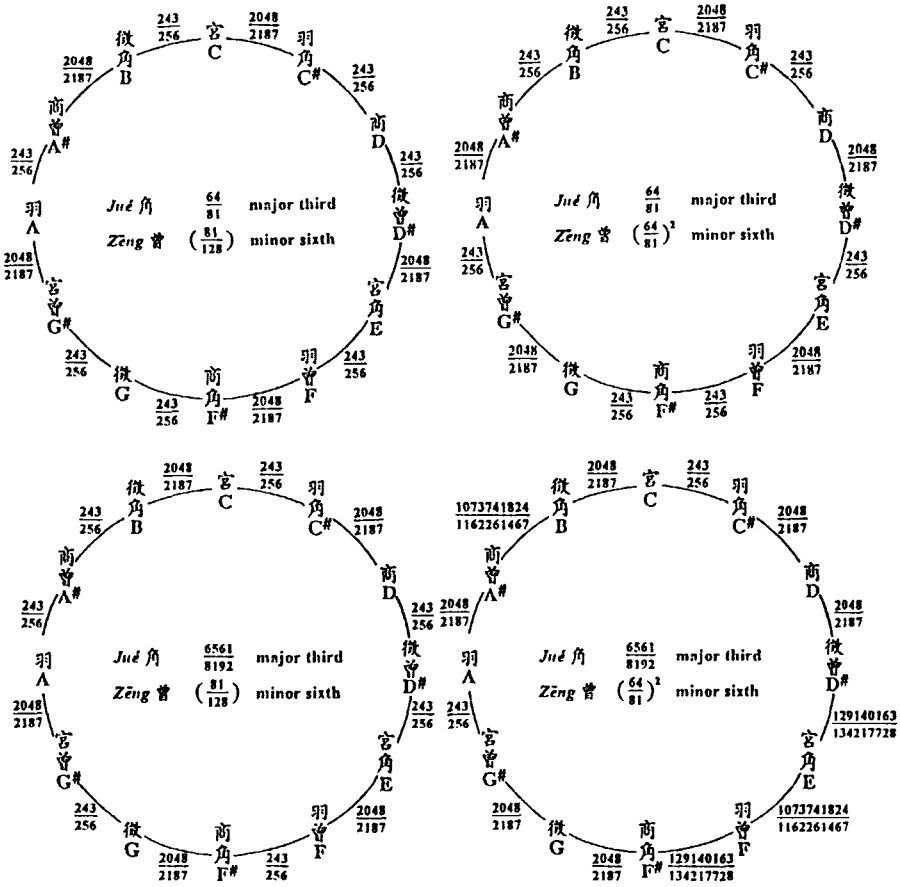

Figure 15. The octave circles of the chromatic scales generated from four core notes the *zhǐ* 徵, *yǔ* 羽, *gōng* 宫, and *shāng* 商 by using four combinations (Sets 1–4) of the approximate major third and minor sixth ratios for the *jué* 角 and *zēng* 曾 intervals.

In Set 5, the exact ratios for both the major third 4/5 and minor sixth 5/8 are used and in Set 6, only the ratio 4/5 for the major third is used. Calculations based on the two sets of the *jué* and *zēng* intervals are also carried out. Shown in Figure 16 are the calculations for sets 5 and 6.

The situation is different for Set 5 and Set 6 involving the exact ratios. The octave circles yielded from the calculation given in Figure 16 are shown in Figure 17. It is seen that these octave circles are composed not of two but of four different semitones—six (15/16), three (24/25), two (128/135) and one (25/27):

$$\left(\frac{15}{16}\right)^6 \left(\frac{24}{25}\right)^3 \left(\frac{128}{135}\right)^2 \left(\frac{25}{27}\right) = \frac{1}{2}. \tag{3}$$

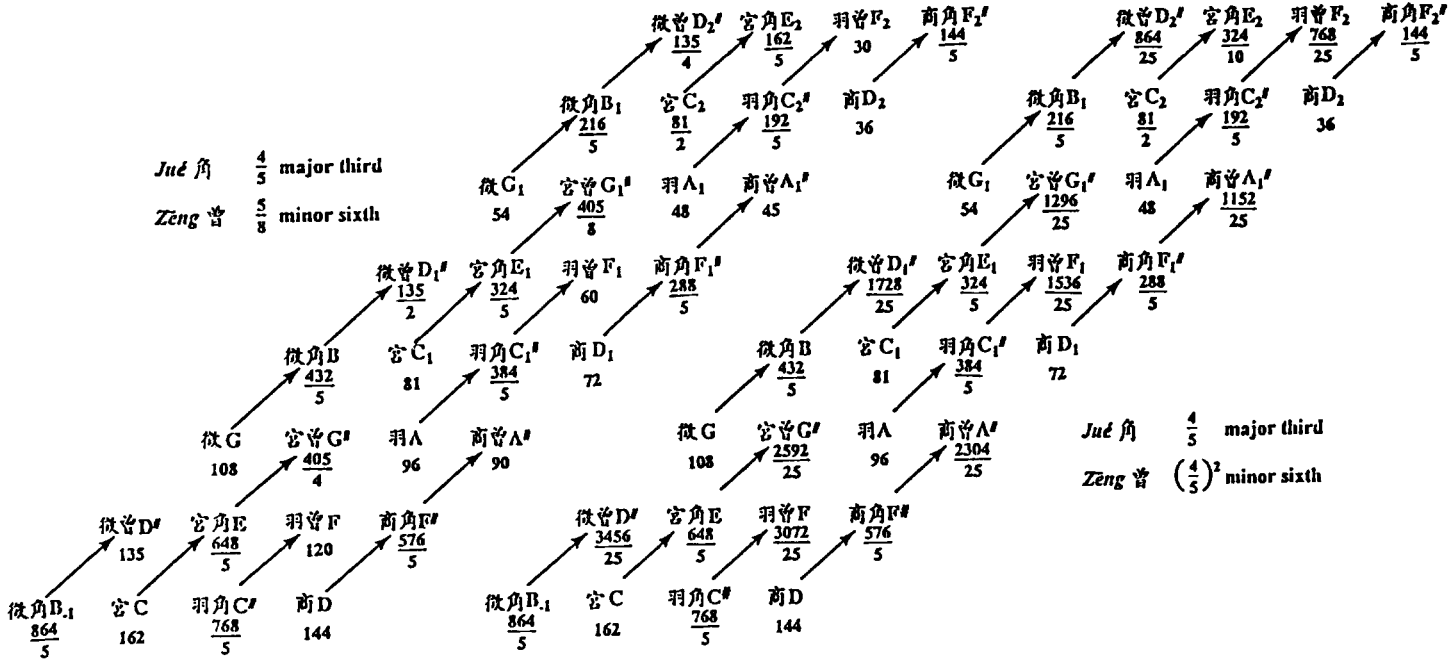

Figure 16. The generation of chromatic scales from the four core notes *zhǐ* 徵, *yǔ* 羽, *gōng* 宫, and *shāng* 商 by the use of the exact ratio 4/5 of major third for the *jué* 角 interval and both the exact 5/8 and approximate $(4/5)^2$ ratios of minor sixth for the *zēng* 曾 interval. The core notes are generated by the fifth 2/3 in accordance with the up-and-down principle.

This semitone-octave relationship is exactly that of a chromatic scale in the natural (just) intonation. With four different semitones, scales in natural intonation are less flexible to accommodate mode shifts than the *Lǚ-Shì Chūn-Qiū* scales which are composed of only two different semitones (see Fig. 11 and Eq. (2)).

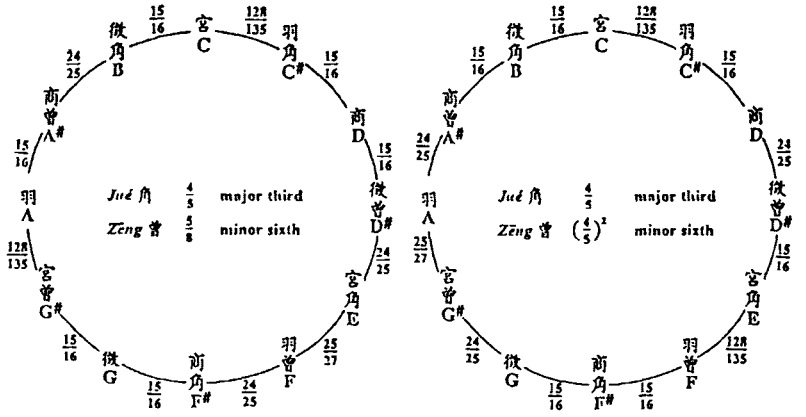

Figure 17. The octave circles of the chromatic scales generated from the four core notes *zhǐ* 徵, *yǔ* 羽, *gōng* 宫, and *shāng* 商 by using the exact ratio 4/5 of major third for the *jué* 角 interval and both the exact 5/8 and approximate $(4/5)^2$ ratios of minor sixth for the *zēng* 曾 interval.

The octaval relation given in Equation (3) implies that chromatic scales generated using either Set 5 or Set 6 combinations of the *jué* and *zēng* intervals are both in natural intonations. In Table 5, the harmonic intervals in the twelve modes are tabulated for the scales generated by the use of the major third 4/5 and minor sixth 5/8 ratios for the *jué* and *zēng* intervals, respectively (Set 5). It is seen that all the harmonic intervals in the *zhǐ* 徵 mode match exactly with that of the natural intonation. For the set of chromatic scales generated using the approximate minor sixth $(4/5)^2$ for the *zēng* interval as in Set 6, it is in the *zhǐ-jué* 徵角 mode that all the harmonic intervals match exactly with the natural intonation. This can be easily seen from the octave circles given in Figure 17 since they are related to each other by a rotation between the *zhǐ* 徵 and *zhǐ-jué* 徵角 notes with respect to the perpendicular axis through the centre of the circles.

2.4 Comments and Evaluations

In Chinese civilization, acoustics share with astronomy the distinction of being the first two branches of early studies of nature that received

Table 5. The Progression of Harmonic Intervals of the Chromatic Scales Generated by the Use of the Exact Ratios 4/5 and 5/8 of the Major Third and Minor Sixth for the *Jué* 角 and *Zēng* 曾 Intervals

Mode Key 律調 / Intervals	宫	羽角	商	徵曾	宫角	羽曾	商角	徵	宫曾	羽	商曾	徵角	Natural 纯律
	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	(Just) Intonation
Fundamental	1	1	1	1	1	1	1	1	1	1	1	1	1
Minor second	$\frac{128}{135}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{24}{25}$	$\frac{25}{27}$	$\frac{24}{25}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{128}{135}$	$\frac{15}{16}$	$\frac{24}{25}$	$\frac{15}{16}$	$\frac{15}{16}$
Major second	$\frac{8}{9}$	$\frac{225}{256}$	$\frac{9}{10}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{225}{256}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{9}{10}$	$\frac{8}{9}$	$\frac{8}{9}$
Minor third	$\frac{5}{6}$	$\frac{27}{32}$	$\frac{5}{6}$	$\frac{64}{75}$	$\frac{5}{6}$	$\frac{27}{32}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{64}{75}$	$\frac{27}{32}$	$\frac{64}{75}$	$\frac{5}{6}$	$\frac{5}{6}$
Major third	$\frac{4}{5}$	$\frac{25}{32}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{25}{32}$	$\frac{4}{5}$	$\frac{25}{32}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{25}{32}$	$\frac{4}{5}$
Perfect fourth	$\frac{20}{27}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{20}{27}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{512}{675}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
Augmented fourth	$\frac{32}{45}$	$\frac{45}{64}$	$\frac{45}{64}$	$\frac{32}{45}$	$\frac{25}{36}$	$\frac{18}{25}$	$\frac{45}{64}$	$\frac{32}{45}$	$\frac{32}{45}$	$\frac{45}{64}$	$\frac{18}{25}$	$\frac{25}{36}$	$\frac{32}{45}$
Perfect fifth	$\frac{2}{3}$	$\frac{675}{1024}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{27}{40}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{27}{40}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
Minor sixth	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{16}{25}$	$\frac{5}{8}$	$\frac{16}{25}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{16}{25}$	$\frac{5}{8}$	$\frac{16}{25}$	$\frac{5}{8}$	$\frac{5}{8}$
Major sixth	$\frac{16}{27}$	$\frac{75}{128}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{16}{27}$	$\frac{3}{5}$	$\frac{75}{128}$	$\frac{3}{5}$	$\frac{16}{27}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{75}{128}$	$\frac{3}{5}$
Minor seventh	$\frac{5}{9}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{128}{225}$	$\frac{5}{9}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{9}$	$\frac{128}{225}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{9}$	$\frac{5}{9}$
Major seventh	$\frac{8}{15}$	$\frac{135}{256}$	$\frac{8}{15}$	$\frac{8}{15}$	$\frac{25}{48}$	$\frac{27}{50}$	$\frac{25}{48}$	$\frac{8}{15}$	$\frac{8}{15}$	$\frac{135}{256}$	$\frac{8}{15}$	$\frac{25}{48}$	$\frac{8}{15}$
Octave	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

official recognition. Together with astronomy, acoustics occupied a unique position (such as 'Lǜ-Lì Zhì' 〈律曆志〉) in annals and official historical records throughout the ancient history of China. Literary records of work in acoustics go back as far as those in astronomy. This is not unexpected since acoustics, like astronomy, has the distinction of being one of the earliest fields in natural science to first establish a relationship with mathematics. However, historical studies of early Chinese work in acoustics were clouded by a number of issues and did not even achieve the same level of scholarship as studies of Chinese work in astronomy. Unfounded claims, incorrect dating, as well as misconceptions of Chinese work in acoustics, continue to propagate in literature. The stereotyping of Chinese music as monotonic, pentatonic and lacking of semitones and the true octave certainly did not help the situation.

In addition to these issues, there are also major omissions in the study of ancient Chinese work in acoustics. In commenting on the question 'What did the early Chinese thinkers believe sound to be?' that they raised, Needham and Robinson stated: 'Sound was regarded as but one form of an activity of which flavour and colour were other.'[78] This characterization of the ancient Chinese belief of sound reflects well the majority of views of sinologists and historians of science. While there were indeed attempts to subjectively correlate sound with flavour and colour, such attempts do not constitute the totality of the Chinese approach to sound nor represent the nature of the work of the ancient Chinese acousticians.

However, in comparison with the Greek approach to acoustics, Needham and Robinson have made the following remarks:

> In acoustics, as in so many branches of science, the Chinese approach was rather different from the European. Where ancient Greece was analytic, ancient China was correlative.[79]

Such a sweeping generalization requires caution. In fact, the early Chinese work in physical acoustics has not yet been adequately studied. As can be seen from p.19 to p.63, contrary to the above characterization, the analysis of sound by the ancient Chinese acousticians was analytical. It demonstrates an appreciation, not only of the abstraction of sound, but also of the physical attributes of sound.

It is beyond the scope of this work to comment on all the major issues that confront the study of acoustics in ancient China. In this section, I will, however, bring up a few of these issues for discussion.

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78. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 132.
79. Ibid., p. 128.

2.4.1 The Origins of Acoustics in Ancient China

One of the major issues that brought the early Chinese work in acoustics into the center of controversy was the origins of acoustics in ancient China. In this section, I will examine and comment in some detail the various hypotheses, claims, and issues that are confronting the views on the origins of acoustics in ancient China.

The Historical Background and Amiot's Dilemma

When Matteo Ricci (1552–1610) attended, for the first time in 1599, a ceremony at a Confucian temple in Nánjīng, 'classical music' in Europe as we know it today, was not yet in existence. In fact, musicians such as Antonio Vivaldi (1675–1741) and Johann Sebastian Bach (1685–1750) had not yet been born. The primary musical experience of Ricci was with the Gregorian Chant Church music. Ricci was astonished by the timber variety in the music that accompanied the ceremonies at the temple.[80] This contrast between the Chinese and European music, noticed by Ricci, primarily reflects cultural differences. It does not necessarily reflect a basic difference in acoustic knowledge. At the level of musical acoustics, there was important progress being made in China at that time. Fifteen years before the arrival of Ricci in China, an exact mathematical solution for the equal-ratio chromatic scale had already been worked out. Theoretical methods for approximating end-corrections in pitch-pipes were also discovered. These works were published in *Lǜ-Xué Xīn-Shuō* 《律學新說》 (*A New Account of Lǜology*) in 1584 by Zhū Zài-Yù 朱載堉.[81]

As cross-currents of science and technology between the East and West were accelerated by the increasing contact of Western missionaries in China, the question of origins of a number of scientific and technological developments became an issue. When Jean Joseph-Marie Amiot (1718–93) published his *Mémorie sur la Musique des Chinis tant anciens que modernes* in 1780, approximately two centuries after the arrival of Ricci in China, the difference between China and Europe in the rate of progress being made in science and technology, as well as in music, was startling. There was hardly any significant progress made in China, but in Europe the progress was accelerating exponentially. The

80. See *Fonti Ricciane, Storia dell'Introduzione del Cristianesimo in Cina*, 1597–1611 (Rome, 1949), ed. Pasquale M. d'Elia, vol. II, p. 70.
81. For a recent discussion of the publication date of the original work, see Dài Niàn-Zǔ 戴念祖 (1986), pp. 91–7. See also the discussion by Joseph Needham (4) (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), pp. 214–28.

movement of 'classic music', initiated by fundamental works, such as Bach's *Well-Tempered Clavichord*, had also reached one of its finest stages as Wolfgang Amadeus Mozart (1756–91) brought into the world his immortal musical compositions. Amiot's claim:

> *l'Heptachorde des Grecs anciens, la lyre de Pythagore, son inversion des tétrachordes diatoniques, et la formation de son grand système, sont autant de larcins faits aux Chinois du premier âge*

commanded little attention. Amiot was probably the only Jesuit who took the opposite view from the majority and advocated that the flow of acoustic knowledge was not from the west to the east, but the reverse.

The Claim of Chavannes

By the nineteenth century, China's prestige abroad had further declined, and music in Europe continued to make remarkable advances under the leadership of great composers, such as Ludwig van Beethoven (1770–1827) and Johannes Brahms (1833–97). It was not unexpected that, in the public's opinion, Chinese music should be stereotyped as monotonic, pentatonic and lacking semitones. What was unexpected was the appearance of such stereotyping in the scholarly work of influential sinologists, such as E. Chavannes. In his *Les Mémoires Historiques de Se-Ma Ts'ien*, Chavannes made the remark 'le caractère tapageur et monotone de leur musique est d'ailleurs bien connu'[82] about the Chinese and their music. He not only held the view that knowledge of the acoustic system in China came from Greece, but he also claimed that it was brought to China during Alexander's expedition. This would place the time of transmission at a period corresponding to the latter part of the Warring States period in Chinese civilization. In order to accept such a claim, one must ignore a number of known historical facts and yet, according to Needham and Robinson,[83] Chavannes' claims 'have been accepted for the last fifty years.'

A Needham-Robinson Hypothesis

An important work on the history of acoustics in ancient China is that of Needham and Robinson, of the present century, which appears in the *Science and Civilisation in China*. After reviewing the works of Amiot and Chavannes, they made the following comments:

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82. E. Chavannes (1989), *Les Mémoires Historiques de Se-Ma Ts'ien*, vol. 3, p. 642.
83. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 177.

> Chavannes' hypothesis must be dismissed not merely because the Chinese were tuning sets of twelve bells in the same century as that which is said to have seen the lifetime of Pythagoras, and in any case long before any possible influences of Alexander's expedition could have brought the Greek formula into the Chinese literature; but also because the Chinese gamut is in its structure essentially unlike the Pythagorean scale. Yet Amiot's notion that a transmission took place in the other direction at such an early date can no longer be taken seriously either.[84]

In the view of Needham and Robinson, the knowledge of harmonic ratios originated neither from ancient China nor from ancient Greece; instead, they postulated that it originated from ancient Babylon, and from there, the knowledge of harmonic ratios for musical intervals was transmitted westward to the Greek civilization and eastward to the Chinese civilization. The relevant passage is reproduced below:

> The simplest alternative hypothesis for which good reason can be found is that there radiated east and west from Babylon the germ of an acoustic discovery which was developed in one way by the Greeks and in another by the Chinese; namely, that the pitch of notes emitted by strings when plucked is in part determined by their length.[85]

Although they stressed that 'a Babylonian origin for these discoveries is hypothetical, of Babylonian music [they] know very little,' they nevertheless presented a number of arguments to support their hypothesis. It is worthwhile to further examine the arguments presented by Needham and Robinson.

Other than the epigraphical evidence of lyres found in the Sumerian and Babylonian tablets, there is little extant information on early Babylonian music and acoustic knowledge. Needham and Robinson support their view of a Babylonian origin by the arguments that there are evidences indicating that both Greek and Chinese traditions gave credit for the origin of the acoustic system to a foreign country, and that the acoustic knowledge was not properly understood by either the Greeks or the Chinese, thus supporting the view of a foreign transmission.

The Hypothesis of Babylon-to-Greece Westward Transmission

The hypothesis of the westward transmission of musical ratios from Babylon to Greece probably has certain historicities, since, long before the capture of Babylon by Alexander, the story of Pythagoras' eastward journey had already been noted by Greek authors such as Aristotle.[86]

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84. Ibid., pp. 126–228.
85. Ibid., p. 177.
86. Aristotle, *Metaphysics* I, 1.

Later authors, such as Iamblichus (c. third century), went so far as to say that knowledge of harmonic ratios was brought back to Greece by Pythagoras from his eastward travels.[87] Perhaps, more significant were the early Greek accounts of Pythagoras' discovery, since they contained what Needham and Robinson called 'misapplications of harmonic laws.' A summary of Needham and Robinson's account of Pythagoras' discovery is reproduced below:

> Pythagoras, passing by a forge, heard the hammers ringing out to form the intervals of the octave, fifth and fourth. After inspecting them he realised that this was due to the different weight of the heads of the hammers, which produced different notes according to their mass. He therefore made four similar weights the basis of his experiments, but no matter what he tested, strings by tension, vases by striking, flutes or monochords by measurement for length, he always found that the numbers 6, 8, 9, 12 formed the proportions of the consonances, 6 : 12 the octave, 8 : 12 the fifth, 9 : 12 the fourth.

Such a description of Pythagoras' discovery can be found in the works of Nicomachus of Gerasa (fl. 100), Iamblichus and Boethius.[88] And the claim that the consonances in the forge were produced by the proportionate weights of the heads of the hammers was indeed incorrect, indicating the lack of a correct understanding of musical ratios. Thus, the hypothesis of the westward transmission from Babylon to Greece, postulated by Needham and Robinson, is not incompatible with historical records found in Greek civilization.

The Hypothesis of Babylon-to-China Eastward Transmission

The hypothesis of the eastward transmission of musical ratios from Babylon to China requires, however, further substantiation. The source cited by Needham and Robinson for their claim that 'a Chinese tradition gave credit for the origin of the acoustic system to a foreign country' is the story told in the 'Gǔ Yuè' 〈古樂〉 ('Ancient Music') chapter of the *Lǚ-Shì Chūn-Qiū*:

> 昔黃帝令伶倫作為律, 伶倫自大夏之西,
> 乃之阮隃之陰, 取竹於嶰谿之谷,
> 以生空竅厚鈞者, 斷兩節間, 其長三寸九分.
> 而吹之, 以為黃鐘之宮, 吹曰[舍少],
> 次制十二筒, 以之阮隃之下, 聽鳳凰之鳴,
> 以別十二律, 其雄鳴為六, 雌鳴亦六,
> 以比黃鐘之宮適合.
> 故曰[黃鐘之宮, 律呂之本].

87. Iamblichus, *Introductio Nicomachi Arithmetic* (Tennulius' ed).
88. Nicomachus, *Encheiridion Harmonices* (Meibom's ed.), bk. I, p. 10; Boethius, *De Institutione Musica*, ed. G. Friedlein, bk. 1, secs. 10–11, pp. 197–8.

> In ancient times, Huáng-Dì 黄帝 ordered Líng Lún 伶倫 to establish pitches (*zuò wéi lǜ* 作為律). For this purpose, Líng Lún went from the west of Dà-Xià 大夏 to the northern slopes of Ruǎn-Yú 阮隃 (i.e. the northern range of the Kūn-Lún 崑崙 mountain) and obtained, from the valley of Xiè-Xī, 嶰谿, bamboo stems with uniform aperture (kōng-qiào 空竅) and thickness. By cutting one between the nodes to a length of 3.9 *cùn* 寸 and blowing it, he obtained the fundamental for the Huáng-zhōng pitch. After test-blowing, he said that this is satisfactory and proceeded with the construction of the twelve bamboo pitch-pipes. He then went further down to the foot of Ruǎn-Yú mountains, and listened to the singing of the *fèng huáng* 鳳凰 birds for the purpose of classifying the pitches of the twelve pitch-pipes. Six male bird sounds and six female bird sounds, compatible with the fundamental of the Huáng-zhōng pitch, were identified. It is, therefore, said that Huáng-zhōng fundamental is the root for generating the *lǜ* 律 and *lǚ* 呂 [two groups of] pitches.

Clearly, the story concerns the standardization of the Huáng-zhōng fundamental for the construction of an acoustic system with twelve pitches rather than the construction of the acoustic system itself. It is highly doubtful whether the story may be construed as evidence of the Chinese tradition giving credit for the origin of the acoustic system to a foreign country, since enshrined in this story is probably an ancient Chinese philosophical belief that there exists a universal pitch in harmony with nature. Efforts were made to standardize the Huáng-zhōng as the universal pitch and to use it as the fundamental to all pitches in harmony with nature. Thus, the story of Líng Lún's westward journey to the bamboo country in search of non-tapering bamboo stems was an effort to construct such a Huáng-zhōng pitch. Once the appropriate bamboo stems were found and the Huáng-zhōng pitch was established, a set of twelve pitch-pipes was then constructed based on the Huáng-zhōng fundamental. The description of Líng Lún's comparison of the sound of the pitch-pipes so constructed with the singing of the *fèng huáng* birds was an effort to reassure that harmony with nature was indeed preserved in the pitch-pipes constructed with the Huáng-zhōng fundamental, thus reconfirming the assertion that the Huáng-zhōng fundamental is the root for generating the *lǜ* 律 and *lǚ* 呂, two groups of pitches in nature.

The story of Líng Lún's journey was cast in the setting of Huáng-Dì, thus making it difficult to date the story and to determine the story's historicity. However, the description of Líng Lún's identification of the *lǜ* 律 and *lǚ* 呂 groups in the twelve pitch-pipes and his confirmation of the Huáng-zhōng fundamental as the root for the generation of the *lǜ* and *lǚ* pitch groups suggests that, at the time when the story was told, the practice of classifying pitches into the *lǜ* 律 and *lǚ* 呂 groups had already existed. Since the *shí-èr-lǜ* system was developed by subdividing the

intervals in the *liù-lǜ* 六律 (the *lǜ* 律 group) with the six interstitial pitches known as the *liù-jiān* 六間 (the *lǚ* 呂 group) (see p.21 and p.22), divisions of the twelve-pitch system into the *lǜ* 律 and *lǚ* 呂 groups could not appear before the development of the twelve-pitch system. This indicates that the knowledge of a twelve-pitch system had already known when the westward journey was made.

The reason given in the story for the journey to the bamboo country was to obtain non-tapering bamboo stems with uniform aperture and thickness. This is not only a legitimate reason but also a reason with scientific merits since, for the standardization of the Huáng-zhōng pitch and the construction of the pitch-pipes, uniformity in the bamboo's aperture is important. Based on these observations it is clear that the journey was made in search of the suitable bamboo stems for the standardization of pitch, and that the knowledge of pitch systems had probably already existed by the time of the journey. Furthermore, the westward journey was not to a foreign country but to Ruǎn-Yú 阮隃, a bamboo country in the northern range of Kūn Lún 昆崙 Mountain. Thus, the story of Líng Lún's westward journey, even disregarding the uncertainties on its dating and its historicity, cannot be taken as evidence of giving credit to a foreign country for the origin of the acoustic system in ancient China as claimed by Needham and Robinson.

Needham and Robinson have also cited two cases of misapplication of harmonic laws in Chinese work, one with the construction of stone-chimes and the other with the bronze formula for foundry, as evidence in support of their hypothesis of the eastward transmission of musical ratios from Babylon to China. Their claim of the misapplication of harmonic laws in the construction of stone-chimes was based on measurements made by Amiot of certain Sòng dynasty stone-chimes that he saw at the imperial court. These measurements, however, are not in agreement with any of the extant sets of stone-chimes that have been studied, including those unearthed in recent decades, such as the Marquis Yǐ stone-chimes of the –5th century and the set of three Shāng stone-chimes discussed on p.41 to p.43. In addition, Amiot also remarked that other stone-chimes of more recent times that he examined did not use these proportions.[89] Thus, it appears that the stone-chimes seen by Amiot at the imperial court were an exception, and cannot be used to represent the general construction of stone-chimes in ancient China.

The claim of misapplication of harmonic laws in the foundry technique is based on the passage in the *Kǎo Gōng Jì* as reproduced below:

89. See the appendix, 'Essay on the Sonorous Stone of Chia', in Amiot's *Mémoire sur la Musique des Chinis tant anciens que moderne*, 1780.

六分其金而錫居一, 謂之鐘鼎之齊.
五分其金而錫居一, 謂之斧斤之齊.
四分其金而錫居一, 謂之戈戟之齊.
叁分其金而錫居一, 謂之大刃之齊.
五分其金而錫居二, 謂之削殺矢之齊.
金錫半, 謂之鑒燧之齊.

'Six portions of *jīn* 金 with one portion of tin' is the bronze formula for bells.[90]
'Five portions of *jīn* with one portion of tin' is the bronze formula for axes.
'Four portions of *jīn* with one portion of tin' is the bronze formula for spears.
'Three portions of *jīn* with one portion of tin' is the bronze formula for swords.
'Five portions of *jīn* with two portions of tin' is the bronze formula for arrows.
'*Jīn* and tin each half' is the bronze formula for metal mirrors.

This set of formulas describes systematically the proportions of tin and copper metals that compose a series of bronze alloys appropriate for making the varieties of vessels and implements. Based on the interpretation of the character '*jīn*' 金, these formulas could yield two possible sets of values for the proportions of copper content. Needham and Robinson took the character '*jīn*' to mean bronze and obtained for the proportion of copper content the ratios 5/6, 4/5, 3/4, 2/3, 3/5, and 1/2.[91] These ratios correspond exactly to the intervals minor third, major third, perfect fourth, perfect fifth, major sixth, and octave in the natural intonation. The appearance of these musical ratios in the bronze formulas has been interpreted by Needham and Robinson to be a case of misapplication of harmonic laws by the ancient Chinese foundry workers. This is, however, a highly speculative interpretation.

One of the implications of this interpretation is that, when these formulas first appeared in China, the Babylonians should have already had in their possession the knowledge of natural (just) intonation. Based on Needham's estimation, 'the passage [from the *Kǎo Gōng Jì*] cannot be later than the −3rd century and may be a great deal earlier.'[92] However, there is no evidence for the development of natural intonation in Babylonia by the −3rd century. On the other hand, evidence for natural intonation has now been found in the unearthed Marquis Yǐ *niǔ* set-bells

90. Character *jì* 齊 is an earlier form of the character *jì* 劑, which means formula or prescription.
91. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 180.
92. Recent archaeological evidence indicates that these formulas appeared no later than the −5th century and probably a great deal earlier. See Wénrén Jūn 聞人軍 (1984).

of the –5th century as discussed above on p.54 to p.63. In addition, if these ratios in the bronze formulas were indeed adopted from harmonic laws as claimed by Needham and Robinson, a logical question to ask is why these ratios were not used to cast bronze bells with different pitches instead of casting different bronze implements which had no relation to sound production?

There are also strong possibilities that the interpretation of the character '*jīn*' as bronze may not be correct. Although character '*jīn*' is a general term for metal, in ancient times, copper with high purity was also called *jīn*. If the character '*jīn*' in the formulas are taken to mean copper metal, a different set of ratios 6/7, 5/6, 4/5, 3/4, 5/7, and 1/2 for copper content is then obtained and, in this case, not all the ratios have a match with intervals in the natural intonation. Recent studies, based on the unearthed set-bells of Marquis Yǐ, reveal that the tin content in the bronze of these bells of the –5th century is, on the average, 13.75%, indicating that the interpretation of *jīn* to mean copper is probably correct.[93] Hence, the bronze formula for bells as given in the *Kǎo Gōng Jì* for the content of copper and tin yields the ratios 6/7 and 1/7, respectively. This formula for bell foundry is the only one among the formulas listed in the *Kǎo Gōng Jì* which can be construed to have anything to do with sound and yet its copper-content ratio 6/7 does not correspond to any intervallic ratios of natural intonation. Thus, this makes the interpretation that the bronze formulas were the result of a misapplication of harmonic laws highly doubtful, independent of the question whether the Babylonians knew the appropriate rations of the natural intervals. Contrary to the claim of Needham and Robinson, the ratios in the bronze formulas of the *Kǎo Gōng Jì* have nothing to do with the harmonic laws.

Marquis Yǐ Set-Bells and McClain's Claim

The discovery of the two-tone set-bells of the Marquis Yǐ of Zēng had a major impact on the history of acoustics. These bells have revealed that by the –5th century, approximately 64 years after the death of Pythagoras (c. –570 to c. –497),[94] chromatic scales in natural intonation had already been developed (see p.54 to p.63). Chromatic scales, extending over a range of more than five octaves, are found encased in the Marquis Yǐ set-bells. This archaeological find not only dismisses outright the claim of Chavannes that the acoustic system in ancient

93. See Lǐ Zhòng-Dá 李仲達 et al. (1984).
94. Based on the date in the inscriptions on the *bó-zhōng* 鎛鐘, a gift from King Xióng Zhāng 酓章 of Chǔ, Marquis Yǐ died around –433.

China was brought to China from Greece during Alexander's expedition, but it also seriously challenges the traditional claim on the priority of Pythagoras' discovery of harmonic ratios of musical intervals.[95]

However, there are those scholars who view the evidence provided by the Marquis Yǐ set-bells as having further enhanced the Needham-Robinson hypothesis that acoustic systems in ancient China originated in Babylon. McClain, in support of such a view, provided the following argument:

> All of the tonal-calendrical associations in China and Greece can be traced to one 'Just' harmonic matrix implicit in the ancient Babylonian multiplication table and alluded to in the mythology of Enuma Elish, the Babylonian creation myth, at least fourteen centuries before the Zēng bells were cast. Only when the musical octave ratio 2 : 1 is expanded to 720 : 360—that is, to a cycle of 360 units—are there sufficient integers at the appropriate ratios to define a twelve-tone scale in a way which satisfied the ancient affection for perfect symmetry, a dominant notion in art, science, and political theory.[96]

This argument is based, however, on a subjective interpretation of the relation between the twelve-tone scale and the cycle of 360 units and on the claim that tonal-calendrical association in China and Greece both originated in Babylon. The association of acoustics with astronomy in ancient Greece and Babylon, as well as in China, has been discussed by Needham and Robinson. They have also interpreted that the appearance of such associations in the Chinese and Greek civilizations is a consequence of transmission from the Babylonian civilization. A relevant question here is to what extent can ideas, which are common to different civilizations, be viewed as more than a part of human nature but as evidence of transmission?

The Association of Acoustics with Astronomy

The intuitive idea that harmonic sound is in some way associated with astronomical phenomena must have been rather appealing and readily acceptable to early students of nature. Attempts to associate acoustics with astronomy can be found in a number of ancient civilizations. One well-known example of such an association in Greek civilization is the 'harmony of spheres' (or 'music of spheres'). In the Pythagorean scheme, the universe is powered by a central fire around which, moving in circles,

95. This traditional claim is also widely accepted in the scientific communities, see for example, *The Feynman Lectures on Physics* (Addison-Wesley Publishing, 1963), vol. I, pp. 501 to 50-4.
96. Ernest G. McClain (1985), 147–3.

are celestial bodies such as the sun, moon, earth and the invisible counter-earth, all assumed to be spheres. According to their distances from the centre, the whirling celestial bodies produce tones which are in harmony with each other. Despite the fact that no one has ever heard the harmony, this fanciful idea was taken up by a number of later scholars who invented various schemes to produce harmonic ratios for the tones of celestial spheres.[97]

As an example of Babylonian association of harmonic ratios with astronomical phenomena, Needham and Robinson quoted the determination of seasons told in the *Moralia* by Plutarch:

> The Chaldeans say the Spring stands to Autumn in the relation of a Fourth, to Winter in the relation of a Fifth, and to Summer in the relation of an Octave. But if Euripides makes a correct division of the year into four months of Summer, and of Winter a like number, of well-loved Autumn a pair, and of Spring a like number, the seasons change in the octave proportion.[98]

According to Plutarch, Babylonians considered the division of four seasons to be governed by harmonic ratios. Needham and Robinson comment that 'the numbers which give these proportions are in fact spring 6, autumn 8, winter 9, and summer 12, the numbers used by Pythagoras for the musical consonances.' From these numbers and the relation that the four seasons equal 12 months, they calculated the seasons in Babylon as spring 2.1 (i.e. 72/35) months, autumn 2.7 (i.e. 96/35) months, winter 3.1 (i.e. 108/35) months, and summer 4.1 (i.e. 144/35) months.

Examples of associating acoustics with astronomy are also found in the Chinese civilization. But no examples are found in which harmonic ratios are explicitly used in the determination of astronomical phenomena such as those of the Babylonian and Greek civilizations discussed above. Among the extant records, the only example that probably made use of numbers from acoustics for calculations in astronomical work, is found in the work of Chén Zǐ 陳子 as recorded in the *Zhōu-Bì Suàn-Jīng* 《周髀算經》 (*Mathematics Classics of Zhōu-Gnomons*). For the assumed diameter in which sunlight is visible on the earth, Chén Zǐ came up with the value 810,000 *lǐ* 里 (Chinese mile) (see p.133). It is possible that this diameter was obtained by adopting the number 81 from musical acoustics. Consequently, this work may be viewed as an example of associating acoustics with astronomy. However, a noticeable difference

97. See for example, Johannes Kepler (1571–1630), *Harmonices Mundi libri V*.
98. Plutarch, *Moralia*, 'Creation of Soul' (translated by John Philips, 1694), see also Joseph Needham (1962), vol. 4, sec, 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 181.

between this example and those identified in reference to Babylon and Greece is that the number adopted here is not the ratio of consonances, but a number used to generate the musical scale.

Examples of associating acoustics with astronomy in Chinese civilization are found primarily between the tonal system and the calendar. In the *Yuè Lìng*, we have the following list of association between month and pitch:

1st month of Spring	Tài-cù	太蔟
2nd month of Spring	Jiá-zhōng	夾鐘
3rd month of Spring	Gū-xiǎn	姑洗
1st month of Summer	Zhòng-lǚ	仲呂
2nd month of Summer	Ruí-bīn	蕤賓
3rd month of Summer	Lín-zhōng	林鐘
1st month of Autumn	Yí-zé	夷則
2nd month of Autumn	Nán-lǚ	南呂
3rd month of Autumn	Wú-yì	無射
1st month of Winter	Yìng-zhōng	應鐘
2nd month of Winter	Huáng-zhōng	黃鐘
3rd month of Winter	Dà-lǚ	大呂

The tonal-calendrical association exemplified by the *Yuè Lìng* is based on pattern recognition. There is no admixture or cross borrowing either of numerical calculation or physical principle between the two fields. The seasons and months in the calendar were determined based on astronomical observations while the tonal system is calculated from acoustic measurements. This type of association is fundamentally different from those in which numerical relations in acoustics are adopted for the determination of phenomena in astronomy as discussed above.

It is worthwhile to note that the acoustic and calendrical systems do have certain similarity in their mathematical properties. It is obvious that a year is subdivided into twelve lunations just as an octave is subdivided into twelve semitones. However, in nature, a year cannot be evenly divided into twelve exact lunations just as an octave cannot be evenly divided into twelve untempered semitones. A mathematical solution found by the early students of nature in ancient China was to make use of two different month-lengths to divide a year into twelve months and two different semitonal-intervals to divide an octave into twelve semitones.[99] This analogy in phenomena and similarity in solutions, were

99. As discussed from p.21 and p.22 above, the twelve pitches were obtained from the subdivision of the pitch intervals in the six standard pitches by six interstitial

much too obvious for early students of nature to ignore. This was probably one of the reasons why the potential association between acoustic and calendrical systems was fascinating even to learned scholars. Eventually, such an association was institutionalized in the Chinese civilization. Beginning with the Hàn dynasty many Chinese official historical records, with the noticeable exception of the *Shǐ Jì*, have presented the two fields together under the title, 'Lǜ-Lì Zhì' 〈律曆志〉.

There is no doubt that the practice of associating acoustics with astronomy was common in ancient Babylon, China, and Greece, but the actual implementation of this association in China was not similar to those in Babylon and Greece. The differences exemplified by the extant records cast significant doubt upon the conclusion that the association of acoustics with astronomy found in the three civilizations is the result of transmission.

A Single Origin versus Multiple Origins

An implicit assumption in the Needham-Robinson hypothesis, as well as in the claims of Amiot, Chavannes, and McClain, is that acoustics had a single origin. Is this a valid assumption? Certainly there is no logical reason to exclude *a priori* the possibility of more than one origin. The progress of human civilization has been rather repetitive and uncoordinated. Separate discoveries and independent progress toward common concepts and/or ideas are more the rule rather than the exception, especially in ancient times. The fact that the same harmonic ratios, 3 : 4, 2 : 3, and 1 : 2 are found among different civilizations does not constitute proof of a single origin for their discovery. The human ear has no ethnic or regional difference in its ability to identify consonances.[100] It is certainly

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pitches. We have from the *Guó-Yǔ* the description:

> [Based on the *zhōng-shēng* 中聲] we first establish three pitches, then level them out evenly into six pitches, and finally bring them to completion in twelve pitches. This is the *dào* 道 of nature.

Thus, it is clear that the idea of having twelve tones in one octave is not patterned after the twelve months in a year. Even if the twelve-semitone octave was a solution patterned after the twelve-month year, the fact that an octave can also be solved in the same pattern would still, in itself, be fascinating. The association between acoustic and calendrical system remained at the level of pattern association since there was no admixture or cross-borrowing either of numerical calculation or physical principle between the two fields.

100. In his description of Pythagoras' discovery of the consonances, Jacob Bronowski (1973) stated that 'Pythagoras had found that the chords which sound pleasing to the ear—the Western ear—correspond to exact divisions of the string by whole numbers' (p. 156). The qualifying phrase, 'the Western ear', used here seems to suggest otherwise.

conceivable that the same harmonic ratios could be discovered independently by different investigators from different civilizations.

Tacit assumption, without concrete evidence, of a single origin for the discovery of harmonic ratios would certainly affect one's interpretation of the meagre trace of early acoustics that has survived the course of time. Consider, for example, the works of Amiot and Chavannes on the origin of the acoustic system. It is evident that, with their individual supposition of a single origin, they had committed not only their choice within the Chinese and Greek civilizations for the development of the acoustic system, but also their belief in the existence of transmissions between the two civilizations at a very early stage in the development of musical acoustics. Such commitments certainly influenced their conclusions. Though Needham and Robinson expanded the number of civilizations involved, they again assume a single Babylonian origin. This is evident from their presentation, since no effort was made to justify Babylon as the only origin of acoustics. They tried instead to provide secondary evidence of foreign transmission to China and to Greece to support their view of a Babylonian origin. The incorrect claim of the bronze formulas of the *Kǎo Gōng Jì* as an example of the Chinese 'misapplications of harmonic laws', as well as the misinterpretation of the story of Líng Lún's westward journey as 'evidence' of a 'Chinese tradition gave credit for the origin of the acoustic system to a foreign country,' reflect the influence of their tacit belief in the existence of transmission, implicitly implied by their single-origin assumption. If one follows, for example, their argument that misapplication of harmonic laws is probably an indication of receiving the laws from a foreign source, a possible conclusion, based on the Babylonian use of musical ratios for the determination of seasons, would be that the harmonic laws in ancient Babylon were also of a foreign origin.

The recent archaeological discoveries of ancient Chinese musical instruments have greatly improved the available evidence of the developmental stage of musical acoustics in China. Of particular importance are the Neolithic seven-hole flutes of c. −5700 (see Fig. 5) discussed on p.40 and p.41 above, and the two-tone set-bells of Marquis Yǐ of the −5th century (see Figs. 2) discussed from p.35 to p.39 above. These discoveries indicate not only that musical acoustics began very early in Chinese civilization but also that acoustics in ancient China, in comparison with her contemporaries, was rather advanced. These discoveries further enhance the view that the acoustic system that existed in ancient China was most likely internally developed.

2.4.2 The Issues of the Octave and the Perfect Fourth

One of the points repeatedly emphasized by Needham and Robinson in their comparison of Chinese and Greek works in musical acoustics is that the Pythagorean acoustic system is basically different from the Chinese acoustic system. The primary difference lies in the proportions of the octave and the fourth. The claim of lacking the true octave and the perfect fourth in the Chinese musical scales was made long before the work of Needham and Robinson, but their work is a comprehensive and recent account based on detailed comparisons. As such, I will again make frequent references to their work.

The Issue of the True Octave

A common belief held by sinologists and historians of musical acoustics is that there is no true concept of the octave in Chinese musical theory. From the *Acoustics of Music* by Bartholomew, we have, for example, the passage:

> The octave is the only interval which is found in every scale, no matter in what country or what period of history it was used. Even in Chinese music, in which it is theoretically impure, it is probably produced fairly correctly in actual practice.[101]

From the work of Needham and Robinson, we have the passage:

> The Chinese gamut of pitches, on the other hand, requires only the simplest mathematics and does not use the octave as a starting point. Indeed, it does not even include a true octave at all.[102]

In this section, I examine the validity of such views and compare the Chinese acoustic system with the Pythagorean system.

The Chinese method for generating scales is given on p.48 and p.49 (pentatonic scales), p.49 to p.51 (the up-and-down principle), and p.51 to p.63 (chromatic scales) above. In all these cases, the true concept of the octave is not only present but is also essential for the implementation of the method. To demonstrate that the concept of the octave is not manifested from interpretation, and indeed is found explicitly in the original materials, I show in Figure 18 a superposition of the pentatonic scale given in the *Guǎn Zǐ* (Fig. 8) with the chromatic scale given in the *Lǚ-Shì Chūn-Qiū* (Fig. 11).[103] From Figure 18, it is seen that the '*zhǐ*' note

101. Wilmer T. Bartholomew, *Acoustics of Music* (Prentice Hall, New York, 1952), pp. 163–4.
102. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 172.
103. Chén Chēng-Yīh 程貞一 (1984).

and the 'G' note, with a ratio 108 : 54, are exactly an octave apart in the Lín-zhōng pitch. Similarly, the '*yǔ*' and the 'A' notes, with a ratio 96 : 48, are an octave apart in the Tài-cù pitch. This is, of course, expected since the up-and-down principle is based on the true concept of the octave as well illustrated in Figure 9 above.

The original procedure of the *Lǚ-Shì Chūn-Qiū* for generating chromatic scales provided yet another explicit example of the use of the

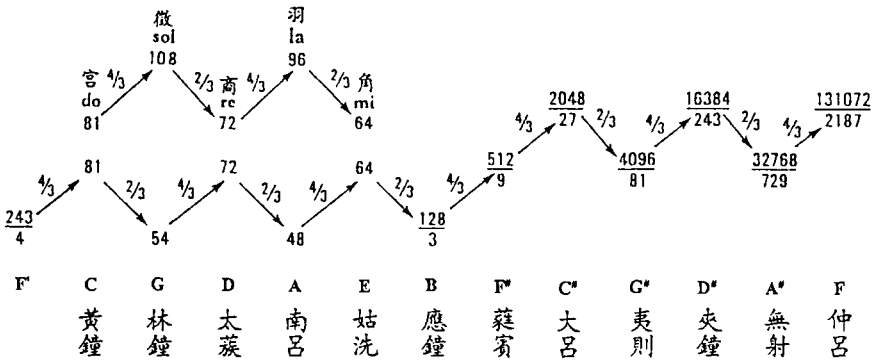

Figure 18. An illustration of the octave relationship between the pentatonic and the chromatic scales generated by the up-and-down principle.

true concept of the octave. After up-generating the Ruí-bīn (F#) pitch, the procedure up-generates the Dà-lǚ (C#) pitch (see Fig. 11); that is, instead of following an up-generation with a down-generation, the procedure calls for two consecutive up-generations. This deviation from the up-and-down pattern is required to keep the generated Dà-lǚ (C#) pitch within the octave range. To facilitate visualization, let us change the number 81 for the Huáng-zhōng fundamental to 1 so that all notes within the octave range must lie between 1 and 1/2. As illustrated in Figure 19, the number obtained by down-generating the Dà-lǚ (C#) pitch is 0.4682, lying outside of the octave range. However, by up-generating the Dà-lǚ (C#) pitch, a value of 0.9364 is then obtained. To keep the generated Dà-lǚ (C#) pitch lying within the octave range of the Huáng-zhōng fundamental, the *Lǚ-Shì Chūn-Qiū* procedure calls for an up-generation. As expected, the two values for the Dà-lǚ (C#) pitch are an octave apart, (0.4682/0.9264) = 1/2 . This demonstrates that the true concept of octave was not only well understood but also explicitly used both in practice and in theory by the early Chinese acousticians and musicians.

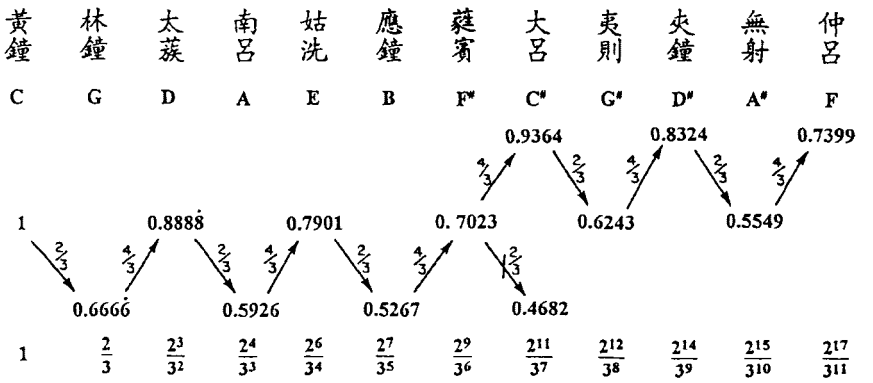

Figure 19. The generation of the chromatic scale with the perfect fifth interval based on the up-and-down principle as described in the *Guǎn Zǐ* and the *Lǚ-Shì Chūn-Qiū*.

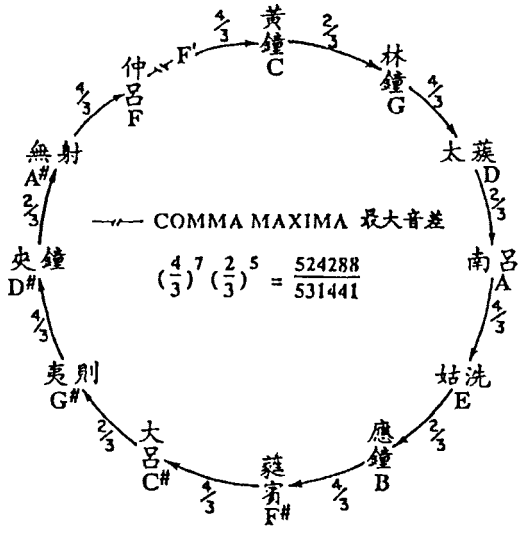

Figure 20. The 'spiral of fifths' obtained by recasting the up-and-down procedure for generating chromatic scale given in the *Lǚ-Shì Chūn-Qiū* (see Fig. 11).

The Issue of the 'Chinese Octave'

An important point to be clarified on the 'issue of octave' is the so called Chinese 'octave'. Needham and Robinson are of the view that the Chinese twelve-pitch system is a gamut of 'spiral of fifths' and the octave found in the system is not the true octave but an approximate one which they called the Chinese 'octave'. To examine their claims, let us first recast the up-and-down procedure for generating the twelve pitches

given in Figure 11 in the form of 'spiral of fifths' as shown in Figure 20. It is apparent that the last pitch generated in the procedure is the Zhòng-lǚ (F) pitch, not the initial (F') pitch that generates the Huáng-zhōng (C) pitch. The interval between the Zhòng-lǚ (F) pitch and the initial (F') pitch is given by seven up-generations and five down-generations. We have, therefore,

$$(\frac{4}{3})^7 (\frac{2}{3})^5 = (\frac{2^{19}}{3^{12}}) = \frac{524288}{531441}.$$

This demonstrates that the interval between pitches F' and F is the well-known comma maxima.

In their analysis of the Chinese up-and-down method, Needham and Robinson have correctly noted that the Chinese method makes use of the perfect fifth to generate the system of twelve pitches, but they are incorrect to conclude that the octave found in the Chinese method is the 'octave' in the 'spiral of fifths', 262144/531441, (i.e, ½ of the comma maxima). The relevant passage is reproduced below:

> ...the discrepancy between a Chinese 'octave' so produced and the true octave is apparent, the ratio of a true octave being 1 : 2, and of an 'octave' in the spiral of fifths 262,144 : 531,441. The ratio, therefore, between a true octave and a Chinese 'octave' is as 524,288 : 531,441. This is known as the comma maxima, or more frequently today on account of the mistaken association of the name of Pythagoras with the spiral of fifths, the Pythagorean comma.[104]

Here the claim is not that the true octave was unknown, but that the octave derived by the Chinese method of generating scales was not the true octave. Hence, they conclude:

> It should not be imagined that this gamut ever functioned as a scale and it is erroneous to refer to the 'Chinese chromatic scale'.[105]

This is also the reason why, in their work, the Chinese twelve pitches are called a gamut rather than a scale. The conclusions of Needham and Robinson on the function of the twelve pitches and on the Chinese 'octave' are, however, based on their incorrect interpretation of the generation procedure and the role of the comma maxima in the determination of the octave.

Although the text of the *Lǚ-Shì Chūn-Qiū* explicitly states that the Huáng-zhōng pitch is obtained by the up-generation (see p.52 to p.54),

104. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), pp. 213–214.
105. Ibid., p. 170.

the (F') pitch, from which the Huáng-zhōng (C) pitch is generated, is neither given a name nor selected among the twelve pitches that constitute the scale. An important point to make is that this unnamed (F') pitch is not identified with the Zhòng-lǚ (F) pitch, the twelfth note generated in the procedure. This implies that the Huáng-zhōng (C) pitch at an octave higher is not generated from the Zhòng-lǚ (F) pitch. Hence, the octave of the twelve-pitch system is not determined by the note generated from the Zhòng-lǚ (F) pitch.

It is here that Needham and Robinson made both an incorrect interpretation of the procedure and an incorrect insertion into the Chinese twelve-pitch system, the so called Chinese 'octave'. The relevant passage is reproduced below:

> ...the notes of the gamut, twelve in all (to which a thirteenth, the octave, may be added by simply continuing the calculation an additional step),...[106]

The procedure used by the Needham and Robinson for adding the thirteenth note is shown in Figure 21 (the left circle). The added note, denoted by c', is produced from the Zhòng-lǚ (F) pitch by up-generation. It is obvious, from the comma maxima between the Zhòng-lǚ (F) pitch and the unnamed (F') pitch, that the interval between the Huáng zhōng (C) and the added thirteenth note (c') is not 1/2 but 262144/531441. However, the Chinese procedure certainly did not call for an additional step to generate the thirteenth note, nor did the procedure recognize this thirteenth note as the 'octave' note. As shown in Figure 9 (on p.50), the octave of a note is its up-and-down pair generated from the same source note. Thus, the octave of the Huáng-zhōng (C) pitch is not generated from the Zhòng-lǚ (F) pitch as assumed by Needham and Robinson, but generated from the same pitch that generates the Huáng-zhōng (C) pitch. As illustrated in the right circle of Figure 21, the octave so generated (denoted by c) lies exactly an interval of 1/2 higher than the Huáng-zhōng (C) pitch.

The term 'spiral' used by Needham and Robinson to describe the procedure is rather misleading.[107] The procedure given in the *Lǚ-Shì Chūn-Qiū* is used to generate only twelve pitches lying within the

106. Ibid., p. 174.

107. In the work of Jīng Fáng 京房 –77 to –37, he extended the up-and-down principle from 12 to 60 generations and obtained a microtonic scale with 60 microtonic notes. It is important to emphasize that Jīng Fáng's work not only appeared rather late, but also was not an attempt to use the up-and-down principle for generating either the octave or any notes outside of the octave; all the 60 microtonic notes are lying within the compass of an octave.

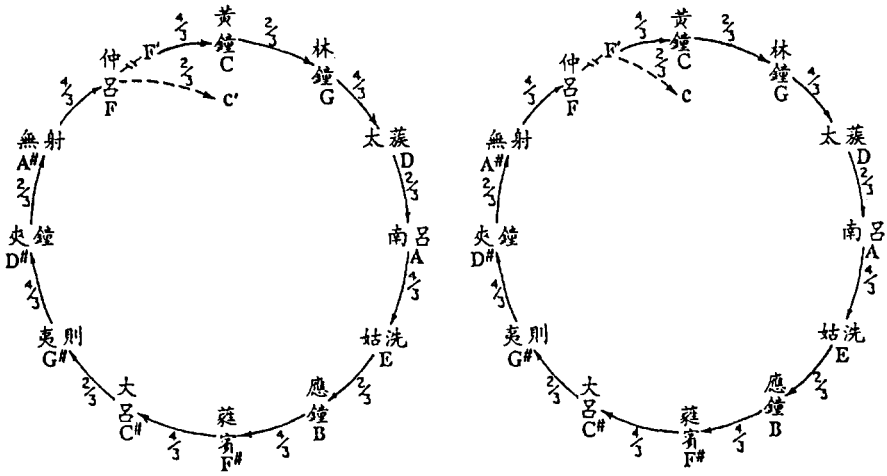

Figure 21. Left, the 'spiral of fifths' used by Needham and Robinson in their construction of a gamut of twelve notes plus an added thirteenth note (denoted by c') which they called the Chinese 'octave'. Right, the generation of an octave (denoted by c) in the up-and-down procedure cast in the form of 'spiral of fifths'.

compass of an octave (see Fig. 19).[108] Because the high and low notes generated from the same note is automatically an octave apart, it is possible for the procedure to select, at each step, the appropriate generation to keep the generated note within the desired octave. This is, in fact, the operational principle that governs the up-and-down procedure as discussed on p.49 to p.51. To examine the octaval structure of the twelve-pitch scale, I have included in Figure 22 an illustration of two 'spirals of fifths', an octave apart, linked through the up-and-down principle (dashed arrow lines). The pitches in the 'spiral of fifths' of a higher octave are denoted by the character *qīng* 清. It is seen from the illustration that one can easily move from one octave to another along the dashed arrow lines without going through the comma maxima *cut*.

This demonstrates that the true octave was not just merely known in practice but was also incorporated theoretically into the Chinese method for generating scales as shown in Figures 19 and 22 above and in Figure 12 and Equation (2) on p.54. The comma maxima never, in fact, entered

108. It is worth noting that, in the ancient Chinese practice, a scale need only be given in terms of the notes lying within an octave, since, with the octave note, the scale begins to repeat itself. Thus, an octave note is visualized simply as the fundamental lying an octave higher. To cite the pentatonic scale in the *jué* mode, for example, only the five notes, *jué*, *zhǐ*, *yǔ*, *gōng*, and *shāng*, inside the octave need to be specified and the octave *jué* note is implicitly understood.

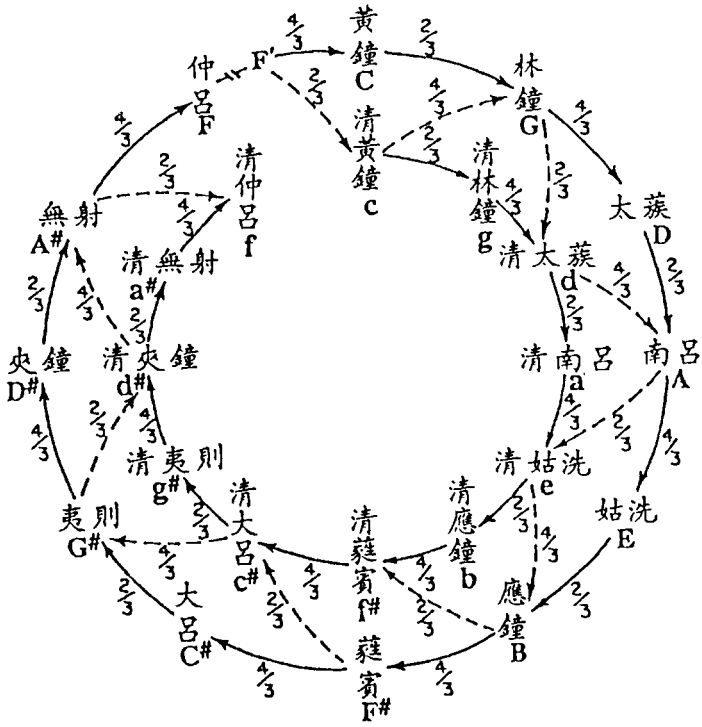

Figure 22. An illustration of bypassing comma maxima *cut* in the octaval relationship of the *Lǚ-Shì Chūn-Qiū* chromatic scale in the form of two linked 'spirals of fifths'.

the Chinese scales generated by the methods described either in the *Guǎn Zǐ* or in the *Lǚ-Shì Chūn-Qiū*. Consequently, the conclusion of Needham and Robinson on the Chinese 'octave' and the function of the Chinese twelve pitches must now be corrected.

The Issue of the Perfect Fourth

Having clarified the issue of Chinese 'octave', we are now in a position to examine the issue of the perfect fourth. Needham and Robinson have remarked:

> It will be seen that the Pythagorean scale and the Chinese gamut (the spiral of fifths) are not identical, either in the general manner of their construction or in the particular proportions of certain notes, i.e. the octave and the fourth.[109]

109. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 176.

Examining Table 3, it is seen that indeed the proportion of the fourth in the Huáng-zhōng—*gōng* mode-key of the Chinese scale is not 3 : 4, the perfect fourth, as in the Greek scale. The fourth in the Huáng-zhōng—gōng mode-key is sharper than the perfect fourth:

$$\frac{131072}{177147} = \frac{3}{4} - \frac{7153}{708588}.$$

This explains the observation of Needham and Robinson on the fourth.

However, the above difference between the fourth is found only for the scale in which the Huáng-zhōng pitch acts as the fundamental (see Tables 3 and 6). In all the other eleven mode-keys, the proportion of the fourth is exactly 3:4. It is well documented that, in ancient times, each of the twelve pitches had functioned as the fundamental and all of the twelve mode-keys were actively used. Thus, the appearance of a sharper fourth in one of the twelve keys cannot be regarded as a general feature of the Chinese scales.

The Pythagorean scale, referred to by Needham and Robinson, corresponds to our present 'major' mode in diatonic intonation in ascending note-ordering:

whole-whole-semi-whole-whole-whole-semi

which has been identified with the Greek 'Ionian' mode.[110] To compare with the Pythagorean scale, I listed, in Table 6, the heptatonic scales embedded in the *Lǚ-Shì Chūn-Qiū* scales found in Table 3. Table 6 reveal that out of the twelve mode-keys, the following seven have identical intervallic structure as the Pythagorean intonation:

Dà-lǚ	大呂	Tài-cù	太蔟	Gū-xiǎn	姑洗
Ruí-bīn	蕤賓	Lín-zhōng	林鐘	Nán-lǚ	南呂
Yìng-zhōng	應鐘				

This implies that the Pythagorean scale is also found in the Chinese system of scales. Based on the inscriptions of the Marquis Yǐ set-bells and stone-chimes of the −5th century, it is evident that among the seven mode-keys, the Tài-cù 太蔟, Gū-xiǎn 姑洗, Ruí-bīn 妥賓(蕤賓), and Yìng-zhōng 鄦鐘(應鐘) were also found in the state of Zēng.[111] This indicates that these mode-keys were in existence no later than the −5th century, contemporary to the Pythagorean period in the Geek civilization.

110. This note-ordering of the Pythagorean intonation has also been identified with the Greek Lydian mode, see Frederick V. Hunt, p. 16.
111. See the Húběi Provincial Museum (1), pp. 4–13.

Table 6. A Comparison of the Intervallic Structures of the Pythagorean Scales ('Ionian' Mode) with Heptatonic Scales Embedded in the Chromatic Scales (in Twelve Pitches, see Table 3) Generated in the *Lǚ-Shì Chūn-Qiū*.

Mode Key 律調 / Intervals	黃鐘	大呂	太蔟	夾鐘	姑洗	仲呂	蕤賓	林鐘	夷則	南呂	無射	應鐘	Greek 'Ionian' Intonation	Natural 純律 (Just) Intonation
	C	C#	D	D#	E	F	F#	G	G#	A	A#	B		
Fundamental	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Major second	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{59049}{65536}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{59049}{65536}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$
Major third	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{6561}{8192}$	$\frac{64}{81}$	$\frac{6561}{8192}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{6561}{8192}$	$\frac{64}{81}$	$\frac{6561}{8192}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{4}{5}$
Perfect fourth	$\frac{131072}{177147}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
Perfect fifth	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{177147}{262144}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
Major sixth	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{19683}{32768}$	$\frac{16}{27}$	$\frac{19683}{32768}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{19683}{32768}$	$\frac{16}{27}$	$\frac{16}{27}$	$\frac{3}{5}$
Major seventh	$\frac{128}{243}$	$\frac{128}{243}$	$\frac{128}{243}$	$\frac{2187}{4096}$	$\frac{128}{243}$	$\frac{2187}{4096}$	$\frac{128}{243}$	$\frac{128}{243}$	$\frac{2187}{4096}$	$\frac{128}{243}$	$\frac{2187}{4096}$	$\frac{128}{243}$	$\frac{128}{243}$	$\frac{8}{15}$
Octave	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

Instead of examining the Pythagorean scale embedded in the chromatic scales, one can also examine the heptatonic scales generated directly in accordance with the up-and-down principle. The heptatonic scales can simply be constructed from the first seven generated notes in Figure 11. The intervallic structure given by these seven notes is also shown in Figure 23. It is seen that the scales so generated are composed of five whole-tone intervals 8/9 and two semitone intervals 243/256, in agreement with the Greek note arrangement described in the passage ascribed to Philolaus (see p.103). The notes of these scales satisfy the following octaval relation:

$$\left(\frac{243}{256}\right)^2 \left(\frac{8}{9}\right)^5 = \left(\frac{3^5}{2^8}\right)^2 \left(\frac{2^3}{32}\right)^5 = \frac{1}{2}$$

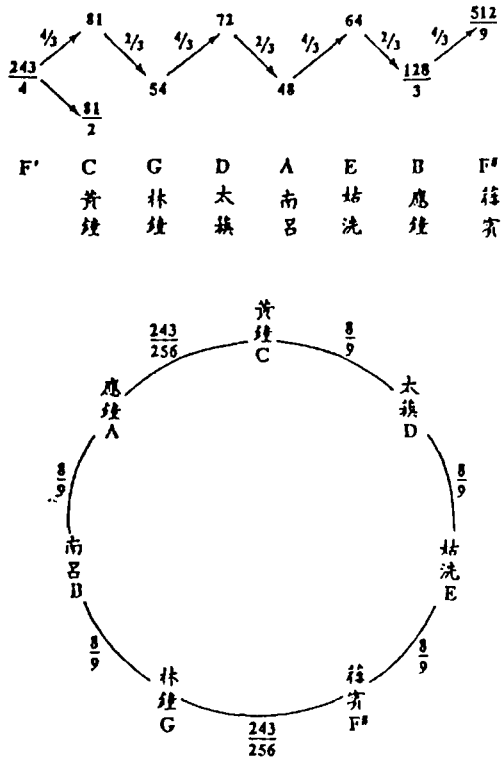

Figure 23. The heptatonic scales and their octave circle generated in accordance with the up-and-down principle.

In Table 7, the progression of harmonic intervals for each of the seven modes are tabulated. It is seen that intervals of the Lín-zhōng—*gōng* mode-key has the same intervals as the Greek scale in the 'Ionian' mode.

Table 7. A Comparison of the Greek Scale in the 'Ionian' mode with Heptatonic Scales Obtained in Accordance with the *sān-fēn sǔn-yì* 三分損益 Method Based on the Up-and-Down Principle

Mode Key 律調 / Intervals	黃鐘	太蔟	姑洗	蕤賓	林鐘	南呂	應鐘	Greek 'Ionian' Intonation
	C	D	E	F#	G	A	B	
Fundamental	1	1	1	1	1	1	1	1
Minor second				$\frac{243}{256}$			$\frac{243}{256}$	
Major second	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$
Minor third			$\frac{27}{32}$	$\frac{27}{32}$		$\frac{27}{32}$	$\frac{27}{32}$	
Major third	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{64}{81}$	$\frac{64}{81}$
Perfect fourth		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
Augmented fourth	$\frac{512}{729}$			$\frac{729}{1024}$				
Perfect fifth	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$		$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
Minor sixth			$\frac{81}{128}$	$\frac{81}{128}$			$\frac{81}{128}$	
Major sixth	$\frac{16}{27}$	$\frac{16}{27}$			$\frac{16}{27}$	$\frac{16}{27}$		$\frac{16}{27}$
Minor seventh		$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$		$\frac{9}{16}$	$\frac{9}{16}$	
Major seventh	$\frac{128}{243}$				$\frac{128}{243}$			$\frac{128}{243}$
Octave	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

It is evident from Table 6 (as well as Table 3) that almost the identical situation exists with the perfect fourth transpiring for the perfect fifth. One finds that the fifth in the Zhòng-lǚ (F) mode-key is flatter than the perfect fifth (see Table 6):

$$\frac{177147}{262144} = \frac{2}{3} + \frac{7153}{786432}.$$

This difference in the fifth is again found only in one mode-key, namely the Zhòng-lǚ (F) mode-key, but not in the rest of the eleven keys. Should one conclude from this observation that the Pythagorean scale and the Chinese scale are different in the proportion of the fifth? This, in fact, is the same question regarding to the fourth raised above. The answer for both questions should be negative, not only because neither the Zhòng-lǚ (F) nor the Huáng-zhōng (C) mode-key alone represents the Chinese scale system, but also because the Ionian mode (or the Lydian mode) alone does not represent the Greek scale system. The ancient Greeks were also fascinated by the changes in note-ordering and intervallic variations in musical scales and used to name different modes of their scales after their tribes. If one applies the *xuán-gōng* principle to the scale represented by the Ionian mode, one would obtain a set of scales with the same overall structure as those given in Table 7. But unfortunately the early Greek modal system is not well documented and its history is obscure.[112]

From the above comparison given in Table 6, it is apparent there are strong similarities between the Chinese scale system and the Greek scale system. With the exception of two mode-keys in which the proportions of the fourth and the fifth are not perfect, the majority of the mode-keys has the perfect fourth and perfect fifth. In fact, the perfect fourth and perfect fifth are also found in the earliest extant Chinese scale generated theoretically, namely the *zhǐ* mode pentatonic scale (see p.48 to p.49) recorded in the *Guǎn Zǐ*. Consequently, one can conclude that the perfect fourth and perfect fifth, just like the true octave, were not only known in practice but were also generated theoretically in the Chinese musical scales.

It should also be noted that the appearance of the sharper fourth in the Huáng-zhōng mode-key is not unique since by a different choice of the twelve pitches one could move the sharper fourth from the Huáng-zhōng—*gōng* mode-key to another mode-key without altering the overall structure of the Chinese scale system. As mentioned earlier, the twelve pitches selected in the *Lǚ-Shì Chūn-Qiū* did not include the unnamed (F') pitch. If, in the twelve selected pitches, one replaces the Zhòng-lǚ (F) pitch with the unnamed (F') pitch, a different permutation in the semitonal structure of the twelve-pitch scale would then be obtained.

112. Studies on intervallic variations with respect to modality in the early Greek scale system are not available. The Ionian mode compared in Tables 7 and 8, for example, was in fact not officially accepted until the sixteenth century even though the mode probably had long beem used in ancient Greece. For a recent work on the topic of Greek modality, see R.P. Winnington-Ingram (1968).

Shown in Figure 24 is a comparison of the semitonal structures for the two alternative selections. Although the permutations of semitones are different, the two selections, owing to the cyclic nature of the scale system, are in fact equivalent with respect to the overall structure of the system. This observation can be demonstrated as follows: if one takes the line through the F' pitch and the Yìng-zhōng (B) pitch as the symmetry axis of the octave circle (Fig. 24, left circle) and rotates the circle an angle of 180° about the axis, one obtains the semitonal structure of the original selected twelve pitches (Fig. 24, right circle). This indicates that the two selections are equivalent. But in the second selection, the sharper fourth would no longer be found in the Huáng-zhōng (C) mode-key; instead it would be found in the Lín-zhōng (G) mode-key. Thus, the appearance of the sharper fourth in the Huáng-zhōng mode-key is a consequence of the specific choice made in the *Lǚ-Shì Chūn-Qiū*.[113]

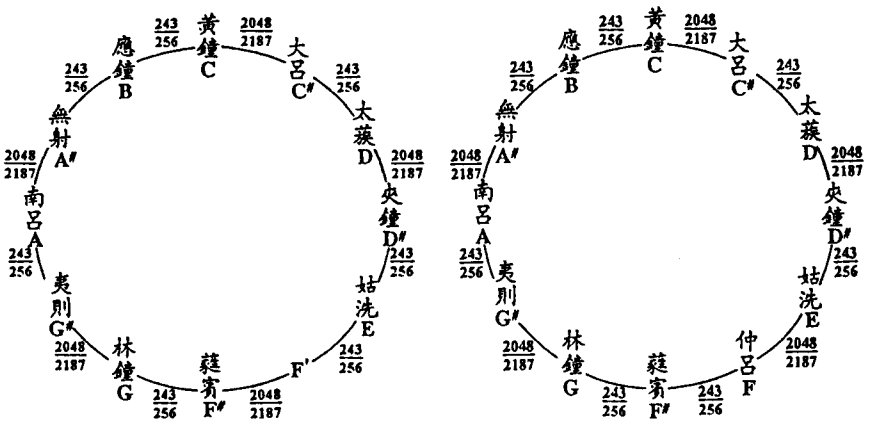

Figure 24. A comparison of the semitonal structures of the twelve-pitch scale given in the *Lǚ-Shì Chūn-Qiū* (the right circle) with that obtained by an alternative selection of the pitches (the left circle).

2.4.3 The Issue of a String versus a Pitch-Pipe

Having demonstrated that there is no concrete evidence in the claims that the knowledge of harmonic ratios was transmitted to China either

113. In view of the significant role of the Huáng-zhōng (C) pitch in Chinese musical acoustics, it is of interest to inquire whether the selection of the *Lǚ-Shì Chūn-Qiū* was made deliberately to depict the Huáng-zhōng (c) from the rest of the pitches. There is no doubt that the author of the passage on the up-and-down procedure was aware of the unnamed pitch (F') since it is stated in the passage explicity that the Huáng-zhōng (C) pitch was obtained by up-generation. Hence, he could equally make the alternative choice by including the unnamed pitch (F') in place of the Zhòng-lǚ (F) pitch.

from Greece or from Babylon and that these ratios were not only known but also generated theoretically in the Chinese musical scales, I would now like to discuss yet another issue confronting the harmonic ratios as to how and when the ancient Chinese musical acousticians discovered these ratios. Among those scholars who consider that the knowledge of ratios in ancient China was developed independently, there has been considerable debate as to whether the discovery was made on a stretched string or a pitch pipe.

There is no doubt that the discovery of musical intervals was made long before the actual development of musical scales. Consequently, it is certain that some basic knowledge of consonance was known to the early Chinese musicians long before they came up with the scheme of using an ordered set of monosyllabic characters (*gōng*, *shāng*, *jué*, *zhǐ*, *yǔ*), to designate the degrees of the five notes in a pentatonic scale. By the time when the up-and-down method for numerical generation of scales was developed, the Chinese acousticians must have had in their possession specific knowledge of harmonic ratios. As it is seen from p.48 to p.51, not only is the generation method constructed in terms of the precise ratios of the perfect fifth and octave, it also generates the explicit ratios for the intervals designated by the set of characters.

After a discussion on the development of this system of characters for designating the relations of notes in pentatonic scales, Needham and Robinson made the following conclusion:

> We may therefore conclude that by the –4th century a scale of five notes was without doubt used and the relations of the notes in this scale were designated by the terms *kung*, *shang*, *chio*, *chih*, and *yü*. We cannot say precisely what were the intervals between these five notes, however, without further information such as the relative lengths of five tuned strings of the same material at the same tension, or of five bamboo pipes of known dimensions identically blown. This precise information is not given in Chou texts.[114]

Apparently, Needham and Robinson have overlooked the calculation of these intervals worked out in the 'Dì-Yuán'〈地員〉('Elements of Earth') chapter of the *Guǎn Zǐ*. In fact, this reference is not even cited in their work on acoustics in ancient China. But, it is through this reference that a complete record on the precise information of the intervals between notes designated by these characters has come down to us.

Summarized in Figure 24 are the calculated ratios for the intervals designated by these characters, as worked out in the *Guǎn Zǐ* (see p.48 to

114. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 161.

p.51). From these ratios, it is seen how harmonic ratios were expressed in terms of these characters. Obviously, the major second 8/9 is given by the *gōng-shāng* 宫—商, *shāng-jué* 商—角, and *zhǐ-yǔ* 徵—羽 intervals. The perfect fourth, 3/4, is given by the following intervals:

shāng-zhǐ	商—徵	(8/9) (27/32) = 3/4
jué-yǔ	角—羽	(27/32) (8/9) = 3/4
zhǐ-gōng	徵—宫	(8/9) (27/32) = 3/4
yǔ-shāng	羽—商	(27/32) (8/9) = 3/4

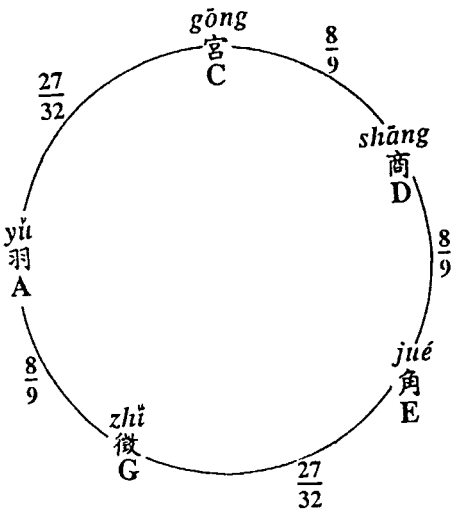

Figure 25. The intervallic structure of pentatonic scales designated by the set of monosyllabic characters: *gōng*, *shāng*, *jué*, *zhǐ*, *yǔ*, as calculated in the *Guǎn Zǐ* (see p.48 to p.49).

Similarly, the perfect fifth, 2/3, is given by the following intervals:

gōng-zhǐ	宫—徵	(8/9) (8/9) (27/32) = 2/3
shāng-yǔ	商—羽	(8/9) (27/32) (8/9) = 2/3
zhǐ-shāng	徵—商	(8/9) (27/32) (8/9) = 2/3
yǔ-jué	羽—角	(27/32) (8/9) (8/9) = 2/3

The octave, 1/2, is given by intervals between the same characters since from *gōng* to *gōng*, for example, one has (see the ratios given in Figure 25):

$$(\frac{8}{9})^3 (\frac{27}{32})^2 = (\frac{2^3}{3^2})^3 (\frac{3^3}{2^5})^2 = \frac{1}{2}.$$

Thus, one sees that, contrary to the comment of Needham and Robinson, the intervals designated by these characters have been precisely defined.

It is well known that the *Guǎn Zǐ* was compiled mainly in the –4th century by the academicians of the Jì-Xià 稷下 Academy. However, one must also recognize that this book was named after Guǎn Zhòng 管仲 (d. –645), not simply because he was an early statesman of the state of Qí 齊, but also because it contains much of the material coming from the Guǎn Zhòng periods. The calculation of the pentatonic scale, as recorded in the 'Dì-Yuán' chapter, was also from an early period. It is, therefore, of interest to further investigate the date when the system of characters for designating intervals was first used.

Although immediately following the above cited conclusion, Needham and Robinson have stated that 'some assistance may be looked for from archaeology', and they attempted to extract information on the intervals from unearthed Chinese musical instruments, their comparison of the measured frequencies of a set of three Shāng stone-chimes (see Table 1) with the theoretical results obtained from the up-and-down generation method[115] is, however, based on an incorrect interpretation of the octave (see p.81 to p.85). The measured frequencies actually reveal a structure of intervals which match well with the *shāng-jué-zhǐ* 商角徵 and the *zhǐ-yǔ-gōng* 徵羽宮 tricords (see Table 1) designated by a system of characters. As shown on p.43, the measured frequencies give a ratio, (8/9) + 0.018, which approximates the major second and a ratio, (3/4) + 0.008, which is in excellent agreement with the perfect fourth. This implies that, no later than the –11th century, the Shāng musicians had already in their grasp a remarkable knowledge of some of the basic intervals.

Since the work of Needham and Robinson, additional archaeological discoveries have shed much new light on the date of this system of characters. Among the new discoveries, the most significant is perhaps the Marquis Yǐ two-tone set-bells (see Fig. 2). The explicit appearance of these characters in the inscriptions on the bells for denoting the bells' intervals[116] suggests that they had already been developed by the –5th century. Literary evidence points to an even earlier date for the development of these characters. It should be noted that these characters were also mentioned, according to *Guó Yǔ*, in a discussion between the High King Jǐng and his acoustic adviser Zhōu Jiū that took place in –522 in connection with the King's decision to convert the Wú-Yì bell to the Dà-lín bell of a lower pitch.

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115. Ibid., p. 162.
116. See for example, Chén Chēng-Yīh 1987, Figs. 2 and 7, pp. 163 and 174.

Before frequency of vibration and its relation to sound was known quantitatively, early musical acousticians worked with the concept of pitch in terms of the length of the resonating agent that produced the pitch. There has been much debate among historians of science as to whether the early Chinese discovered the harmonic ratios on a stretched string. A number of scholars advocated the view that the early Chinese derived the harmonic ratios from the length of pitch pipes rather than from the length of strings and, to support their views, they cited the absence of artifacts and epigraphic evidence of pre-Qín Chinese string musical instruments as well as the frequent reference to pitch-pipes for pitch intervals as evidence. This is an important issue since it is now well known that the actual length of the resonating air column in the pitch-pipe is somewhat longer than the pipe length itself for the antinodal points of the air column lie a distance out from the open ends of the pipe.

It should be noted that among the earliest Zhōu texts, such as the *Shī Jīng* and the *Shàng Shū*, string musical instruments such as the *sè* 瑟 (zither) and the *qín* 琴 (half-tube zither) have frequently been mentioned.[117] Also, the *sè* and the *qín* are mentioned in the *Lún Yǔ* 《論語》 (Analects). Since the discovery of the musical instruments in the tomb of Marquis Yǐ in 1978, we now possess string instruments of both types dating back to the –5th century. In addition to the twelve 25-string *sè* and one 10-string *qín* unearthed from the tomb, there is also a 5-string tuning apparatus (see Fig. 26) probably related to the *jūn-zhōng-mù* 均鐘木 discussed by Wéi Zhāo 韋昭 (see p.22 to p.23).[118] These are advanced string instruments, suggesting that the development of string instruments in Chinese civilization probably began very early. This archaeological discovery is consistent with the early mention of these string instruments in the Zhōu texts. Hence, it removes any suspicion that the string musical instruments did not appear in ancient China until after harmonic ratios had already been discovered.

Intermingled in this issue of a string versus a pitch-pipe are two different but interrelated topics in musical acoustics which are the determination of harmonic ratios and the standardization of pitches. The frequent reference to pitch-pipes in Chinese texts is made in connection with the standardization of pitch and the preservation of the standardized pitches in their proper intervallic relations, and has nothing to do with the determination of the harmonic ratios. There is no doubt

117. In addition to the direct mention of *sè* and *qín*, the *Shàng Shū* also indirectly mentioned string instruments through the term *bā-yīn* 八音, the eight tones, which included the sound of *sī* 絲 (silk) as one of the eight basic tone qualities (see p.22 to p.26).

118. Chén Chēng-Yīh 程貞一 (1986).

Figure 26. The three types of string instruments unearthed in 1978 from the tomb of Marquis Yǐ. Top, a 5-sting apparatus (length 15cm, width 7.0-5.5cm). Middle, a 10-string *qín* 琴 (length 67cm, width 19cm). Bottom, a 25-string *sè* 瑟 (length 168cm, width 43.8cm).

that a stretched string permits a convenient way for an accurate determination of harmonic ratios, but it cannot be used to preserve a standardized pitch because of the difficulty, especially in ancient times, to keep the tension in the string constant for any substantial period. It is obvious that instruments, other than the string instruments, were needed for the standardization and preservation of a pitch. The instrument used by the early Chinese acousticians for this purpose was the pitch-pipes.

The use of pitch-pipes for the standardization and preservation of pitch was indeed a practical choice of scientific merit at that stage of development, not only because of the ease of their construction, but also because of their stability in sound production. Once the proper measurements of the pitch-pipe (length and diameter) are fixed, the pitch would not alter appreciably under normal conditions. In terms of a set of twelve pitch-pipes, a standardized scale of twelve notes known as the *shí-èr-lǜ* (twelve-pitch scale) was subsequently preserved. As the art of bell casting reached a stage of maturity, such a *shí-èr-lǜ* were also often encased in bronze set-bells.

In discussing the early Chinese work on pitch, Needham and Robinson asserted:

> Exact pitch probably did not become a dominating factor among them (the Chinese) till Babylonian influence made itself felt at the beginning of the –4th century.[119]

119. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 160.

Other than the improbable hypothesis of a Babylonian influence already discussed from p.66 to p.78, this view of Chinese work on pitch is also contrary to the evidence provided by the extant records and archaeological discoveries (see p.40 to p.47). What should be noted here is that a knowledge of the harmonic ratios alone is not sufficient to produce the exact pitch-intervals in the standardized *shí-èr-lǜ* in either the pitch-pipes or the set-bells.

A knowledge of harmonic ratios is certainly essential for the generation of the proper intervals in the *shí-èr-lǜ* but what the ancient Chinese acousticians needed was how to produce these intervals in the pitch-pipes and in the set-bells. This was not a simple task. Even with the knowledge of the proper intervals acting as a guide, the construction of the pitch-pipes and set-bells actually underwent a long period of *ad hoc* development before a certain systematical approximate was empirically developed. Although not much extant literary information is now available, an appreciation of such a development in set-bells can be found in the unearthed bells ranging from the simple Shāng set-bells of the −13th century to the sophisticated Marquis Yǐ set-bells of the −5th century. As for pitch-pipes, one must rely on literary information to trace their development since early pitch-pipes were made of perishable bamboo, and very few early specimens with measurable length and diameter have come down to us (see however p.46 and p.47).

Modern acoustics has revealed that although the actual length of the resonating air column in a pitch-pipe is longer than the pipe length itself due to the end effect, the difference in the two lengths is relatively small for pitch-pipes with small diameters. Hence, it is apparent that the intervals determined on a stretched string constitute a good initial approximation for the determination of the relative lengths of the pitch-pipes. The length so obtained for a specific interval can then be empirically altered to improve its precision. Unfortunately, explicit information on pitch-pipe length from the early periods is not available at present. The earliest extant record of the actual lengths of a set of twelve pitch-pipes for the *shí-èr-lǜ* is found in the *Shǐ-Jì* from c. −90.

Shown in Table 8 is a comparison of these pitch-pipe lengths with the lengths obtained from the intervals of the *shí-èr-lǜ* taken from the *Shǐ-Jì*. The calculated lengths are obtained by multiplying the corresponding ratios of the intervals with the length of the Huáng-zhōng pitch-pipe taken to be 8.1 *cùn* 寸 (Chinese inch). It is seen that the lengths of the pitch-pipes differ from those calculated. Examining the ratios of the *shí-èr-lǜ* intervals given in the *Shǐ Jì*, it is apparent that these ratios were generated by the up-and-down method [see the Huáng-zhōng

column of Table 3 on p.55.[120] The fact that the given pipe lengths differ from those obtained from the *shí-èr-lǜ* ratios indicates unambiguously that the acoustician who constructed these pitch-pipes certainly knew that the pitch-pipe length is not given by the length of the string of the corresponding notes and had in his possession certain empirical methods to make such a correction.

Modern acoustics reveal that the fundamental frequency f_n of an open-end pitch-pipe with a length l_n and a diameter d_n is given by the formula:

$$f_n = \frac{v}{2\,(l_n + 0.58\,d_n)} \tag{4}$$

where v is the speed of sound in air (v = 343 m/s in air at a temperature 20°C). Let the subscripts, n = 1, 2,...12, denote, in the ascending order, the twelve pitch-pipes, then the fundamental frequency f_1 of the Huáng-zhōng pitch-pipe with a length l_1 (8.1 *cùn*) and an unspecified diameter d_1 is given by the formula:

$$f_1 = \frac{v}{2\,(l_1 + 0.58\,d_1)}\,. \tag{5}$$

The structure of the *shí-èr-lǜ* given in the *Shǐ-Jì* requires the interval between the Dà-lǚ pitch and the Huáng-zhōng pitch to be a minor second with the ratio 2048/2187. Thus, the frequency f_2 of the Dà-lǚ pitch can be expressed in terms of the Huáng-zhōng pitch as follows:

$$f_2 = \frac{f_1}{\gamma_2} = \frac{v}{2\,\gamma_2\,(l_1 + 0.58\,d_1)}\,, \tag{6}$$

where γ_2 = 2048/2187. But in terms of its length l_2 and diameter d_2, the fundamental frequency of the Dà-lǚ pitch-pipe is also given by the formula:

$$f_2 = \frac{v}{2\,(l_2 + 0.58\,d_2)}\,.$$

Hence,

$$\frac{v}{2\,(l_2 + 0.58\,d_2)} = \frac{v}{2\,\gamma_2\,(l_1 + 0.58\,d_1)}\,.$$

120. The ratios given in the *Shǐ Jì* for the minor second, 1024/2187, minor third, 8192/19683, and perfect fourth, 65536/177147 are an octave higher than those of the rest of the nine intervals. But there is no question that all these ratios are generated by the up-and-down method (see Figs. 11 and 19 for the octaval structure).

Table 8. A Comparison of the Lengths of a Set of Twelve Pitch-Pipes with the Length Calculated Based on the Intervals of the *Shí-èr Lǜ* 十二律 Given in the *Shǐ-Jì* 《史記》 of c. –90.

Pitch Name		Pitch-Pipe Length *Shǐ-Jì* 《史記》, c. –90 (*cùn* 寸)		Length Correction (*cùn* 寸)	Calculated Length (*cùn* 寸) (uncorrected)	*Shí-èr-Lǜ* 十二律 Intervals[a] (in the Huáng-zhōng key) *Shǐ-Jì* 《史記》, c. –90		
Huáng-zhōng	黃鐘	長八寸十分一	8.1	—	8.1	1	*zǐ* 子	Fundamental
Dà-lǚ	大呂	長七寸五分三分一	7.533	–0.052	7.585	$\frac{2048}{2187}$	*wèi* 未	Minor second
Tài-cù	太簇	長七寸七分二	7.286	0.086	7.2	$\frac{8}{9}$	*yín* 寅	Major second
Jiá-zhōng	夾鐘	長六寸七分三分一[b]	6.733	–0.009	6.742	$\frac{16384}{19683}$	*yǒu* 酉	Minor third
Gū-xiǎn	姑洗	長六寸七分四	6.571	0.171	6.4	$\frac{64}{81}$	*chén* 辰	Major third
Zhòng-lǚ	仲呂	長五寸九分三分二	5.967	–0.027	5.993	$\frac{131072}{177147}$	*hài* 亥	Perfect fourth
Ruí-bīn	蕤賓	長五寸六分三分一	5.633	–0.056	5.689	$\frac{512}{729}$	*wǔ* 午	Augmented fourth
Lí-zhōng	林鐘	長五寸七分四	5.571	0.171	5.4	$\frac{2}{3}$	*chǒu* 丑	Perfect fifth
Yí-zé	夷則	長五寸四分三分二	5.167	0.110	5.057	$\frac{4096}{6561}$	*shēn* 申	Minor sixth
Nán-lǚ	南呂	長四寸十分八[c]	4.800	—	4.8	$\frac{16}{27}$	*mǎo* 卯	Major sixth
Wú-yì	無射	長四寸四分三分二	4.467	–0.028	4.495	$\frac{32768}{59049}$	*wù* 戌	Minor seventh
Yīng-zhōng	應鐘	長四寸二分三分二	4.267	—	4.267	$\frac{128}{243}$	*sì* 巳	Major seventh

[a] The intervals given in the *Shǐ-Jì* for the minor second, minor third and perfect fourth are, respectively, 1024/2187, 8192/19682, and 65536/177147, which are an octave higher than those listed in the above.

[b] This entry for the length of the Jiá-zhōng pitch-pipe before correction reads 長六寸一分三分一.

[c] This entry for the length of the Nán-lǚ pitch-pipe before correction reads 長四寸七分八.

This gives an explicit relation between the dimensions of the Dà-lǚ pitch-pipe and the dimensions of the Huáng-zhōng pitch-pipe:

$$l_2 = \gamma_2 l_1 + 0.58\ (\gamma_2 d_1 - d_2) \tag{7}$$

Following the same procedure, a general relation between the dimensions of the Huáng-zhōng pitch-pipe and any one of the twelve pitch-pipes is obtained:

$$l_n = \gamma_n l_1 + 0.58\ (\gamma_n d_1 - d_n) \tag{8}$$

where γ_n with n = 1, 2,..., 12 are the interval ratios for the *shí-èr-lǜ* (see Table 8).

From the above derivation and Equation (8), it is seen that the end-correction for the length of a pitch-pipe is given by the formula:

$$\Delta l_n = l_n - \gamma_n l_1 = 0.58\ (\gamma_n d_1 - d_n) \tag{9}$$

If the diameters of the twelve pitch-pipes are all equal, i.e., $d_1 = d_n$ for n = 1, 2,...,12, then the end-corrections reduce to the form,

$$\Delta l_n = -0.58\ (1 - \gamma_n)\ d_1.$$

Since $\gamma_n < 1$, then the end-corrections for the length of pitch-pipes should all be negative. But this is not the case as seen from Table 8. Hence, one may conclude that the twelve pitch-pipes referred to in the *Shǐ-Jì* did not all have the same diameter.

Based on the level of accuracy achieved for the intervals in the *shí-èr-lǜ* preserved in the unearthed set-bells, one would expect that the set of twelve pitch-pipes given in the *Shǐ-Jì* should produce the *shí-èr-lǜ* very accurately, for it is much easier to achieve a higher level of accuracy for pitch-pipes than for set-bells. If this is the case, then, the end-correction formula given by Equation (9) provides a window for us to look into the size of the diameters of the pitch-pipes given in the *Shǐ-Jì*, because for one to calculate the diameters of the twelve pitch-pipes all one needs is the knowledge of any one of the twelve diameters. By taking the diameters of the Dà-lǚ and the Huáng-zhōng pitch-pipes to be equal (i.e. $d_2 = d_1$), then with $\gamma_2 = 2048/2187$ and $\Delta l_2 = -0.052$ *cùn* (see Table 8), we have, from Equation (9), $d_2 = d_1 = 1.4$ *cùn*. This value for d_1 constitutes a good estimation of the diameter of the Huáng-zhōng pitch-pipe. In Table 9, the calculated diameters of the twelve pitch-pipes are tabulated for five different values of d_1, the diameter of the Huáng-zhōng pitch-pipe. It is seen that, for the case $d_1 = 1.4$ *cùn*, the diameters of the twelve pitch-pipes lie within a range of 1.4 *cùn* to 0.64 *cùn*. Roughly speaking, the diameters of pitch-pipes with the pipe lengths given in the *Shǐ-Jì* tend to decrease with the increase in the pitch of the pitch-pipes.

Table 9. A Study of the Diameters of a Set of Twelve Pitch-Pipes Calculated Based on the Length Corrections and the Intervals of the *Shí-èr Lǜ* 十二律 Given in the *Shí-Jì* 《史記》 of c. –90.

Pitch Name		Length (*cùn* 寸)	Length Correction (*cùn* 寸)	Diameter (*cùn* 寸)	Diameter (*cùn* 寸)	Diameter (*cùn* 寸)	Diameter (*cùn* 寸)	Diameter (*cùn* 寸)	*Shí-èr-Lǜ* 十二律 Intervals
Huáng-zhōng	黃鐘	8.1	—	1.00	1.20	1.40	1.60	1.80	Fundamental
Dà-lǚ	大呂	7.533	–0.052	1.03	1.21	1.40	1.59	1.78	Minor second
Tài-cù	太簇	7.286	0.086	0.74	0.92	1.10	1.27	1.45	Major second
Jiá-zhōng	夾鐘	6.733[a]	–0.009	0.85	1.01	1.18	1.35	1.51	Minor third
Gū-xiǎn	姑洗	6.571	0.171	0.50	0.65	0.81	0.97	1.13	Major third
Zhòng-lǚ	仲呂	5.967	–0.027	0.79	0.93	1.08	1.23	1.37	Perfect fourth
Ruí-bīn	蕤賓	5.633	–0.056	0.80	0.85	1.08	1.22	1.36	Augmented fourth
Lín-zhōng	林鐘	5.571	0.171	0.37	0.51	0.64	0.77	0.91	Perfect fifth
Yí-zé	夷則	5.167	0.110	0.43	0.56	0.68	0.81	0.93	Minor sixth
Nán-lǚ	南呂	4.800[b]	—	0.59	0.71	0.83	0.95	1.07	Major sixth
Wú-yì	無射	4.467	–0.028	0.60	0.71	0.83	0.94	1.05	Minor seventh
Yīng-zhōng	應鐘	4.267	—	0.53	0.63	0.74	0.84	0.95	Major seventh

[a] This entry for the length of the Jiá-zhōng pitch-pipe before correction reads 長六寸一分三分一, i.e. 6.133 *cùn*.
[b] This entry for the length of the Nán-lǚ pitch-pipe before correction reads 長四寸七分八.

There are historians of science who consider that the ancient Chinese pitch-pipes, for a given set of *shí-èr-lǜ*, were all made from pipes with the same diameter. Although this appears to be a reasonable assumption, in practice, it may not be the most convenient thing to do since, in ancient times, pitch-pipes were made of natural bamboo stems, not manufactured pipes. It would be more convenient to alter the length of the bamboo stems than to find twelve bamboo stems with exactly the same diameter.

An early description of the features of the bamboo stem desired for the construction of pitch-pipes is found in the *Lǚ-Shì Chūn-Qiū* which states that 'the bamboo stem should have an uniform (*jūn* 鈞) aperture (*kōng-qiào* 空竅) and thickness' as already quoted above from p.69 to p.70. There was no mention on the diameter of the aperture. The fact that ancient pitch-pipes were not necessarily made of bamboo stems with the same diameter is now confirmed by the remnants of pitch-pipes unearthed in 1986 from a Chǔ tomb (M21) of the Warring States period located at the present Yǔtáishān in 雨台山 Jiānglíng 江陵, Húběi (see p.46 to p.47). From these remnants, it is clear that the pitch-pipes were made of nodeless bamboo (open ends) with different lengths and diameters. Measurements of these remnants for four pitch-pipes reveal that the diameters for two of them are 0.6 cm and 0.7 cm (see Table 2, p.47).

2.4.4 Treatment of Semitones

In comparison with the Greek method, Needham and Robinson made the following remarks on semitones:

> The Greek used the interval of the tone as the basis of their scale structure, an octave being subdivided into a tone and two tetrachords, and a tetrachord being subdivided into two tones and a Pythagorean semitone or diesis. The Chinese did not become involved in the complications of the Greek semitones *apotomē* and *leimma*, but having advanced two steps from their fundamental note (the Huang-chung already referred to), went on to calculate a fourth note in the series by multiplying the length of the resonating agent of their third note by 2/3, which gave a length 16/27 that of the fundamental.[121]

The claim that 'the Chinese did not become involved in the complications of the Greek semitones' needs to be re-evaluated.

The musical scale now identified as the Pythagorean intonation, in the view of most historians of musical acoustics, is an inheritance from

121. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 173.

the harmonic ratios of simple numbers first discovered by Pythagoras. The earliest extant record on the elements of the musical scale is found in a passage ascribed to Philolaus (c. –5th century). A translation of the passage is quoted below:

> The content of the octave is the major fourth and the major fifth; the fifth is greater than the fourth by a whole tone; for from the highest string to the middle is the fourth, and from the middle to the lowest string is a fifth. Between the middle and third string is a tone. The major fourth has a ratio 4 : 3, the fifth 3 : 2, and the octave 2 : 1. Thus the octave consists of the five whole tones and two hemitones, the fifth consists of two tones and a hemitone.[122]

From this passage one can deduce that a whole tone interval is 8/9 and a semitone interval is 243/256. The arrangement of notes in the string instrument (a lyre) given in the passage is consistent with the conclusion that an octave is divided into five whole tones and two semitones. Thus, the octave satisfies the mathematical relation:

$$(\frac{8}{9})^5 (\frac{243}{256})^2 = (\frac{2^3}{3^2})^5 (\frac{3^5}{2^8})^2 = \frac{1}{2} . \tag{10}$$

It is clear from the passage that the octave in the Pythagorean scale is composed of whole tones and semitones. But the passage does not specify the note-ordering of the scales, because the sequence of the tones and semitones in both the upper and lower portions of the octave is not defined.

Examining the intervals between the consecutive tones in the pentatonic scale generated by the up-and-down procedure of the *Guăn Zĭ* (Fig. 8), it is seen the octave is divided into three whole tones (8/9) and two minor thirds (27/32). One has for the octave the relation:

$$(\frac{8}{9})^3 (\frac{27}{32})^2 = (\frac{2^3}{3^2})^3 (\frac{3^3}{2^5})^2 = \frac{1}{2} . \tag{11}$$

In comparison with the octave of the Pythagorean scale given by Equation (10), it is obvious that the pentatonic octave contains no semitones. In place of the two semitones, there are two minor third intervals. The fact that the minor third was not subdivided into a whole tone and a semitone in a pentatonic scale is a matter of preference and probably had nothing to do with the complication of semitones, since

122. Philolaus, frag. 6; (translation by Freeman), see *Die Fragmente der Vorsokratiker* by Hermann Diels, 1st ed., 1903; 5th ed., ed. W. Kranz (Wiedmann, Berlin, 1934–1937); for a complete translation of these fragments, see Kathleen Freeman, *Ancilla to the Pre-Socratic Philosophers* (Harvard University Press, 1948), p. 74.

both semitones and heptatonic scales had appeared very early in ancient China.[123] Semitones were certainly in full appreciation when the whole-tone intervals in the *liù-lǜ* were subdivided into semitones as the system of six pitches was enlarged into twelve pitches by the insertion of six interstitial pitches called *liù-jiān* (see p.21 and p.22).

By the time when the up-and-down principle was utilized to generate the twelve pitches theoretically, the treatment of semitones was very successful. From the analysis given from p.53 and p.54, it is seen that a whole tone is subdivided into two semitones, a major and a minor semitone as reproduced below from Equation (1):

$$\left(\frac{243}{256}\right)\left(\frac{2048}{2187}\right) = \left(\frac{3^5}{2^8}\right)\left(\frac{2^{11}}{3^7}\right) = \frac{8}{9} \qquad (1')$$

If all of the five whole tones in the Pythagorean scale (see Eq. 10) are subdivided into two such semitones, one would than obtain, for the octave, five major semitones and seven minor semitones:

$$\left(\frac{243}{256}\right)^7\left(\frac{2048}{2187}\right)^5 = \left(\frac{3^5}{2^8}\right)^7\left(\frac{2^{11}}{3^7}\right)^5 = \frac{1}{2} \qquad (12)$$

which is exactly the octave-semitone relation of the Chinese twelve-pitch scale given in Equation (2) and Figure 12 on p.54. This is the reason why, as seen from the comparison given in Table 6 on p.87, that the majority of the mode-keys are identical with the Pythagorean intonation. Thus, the claim that 'the Chinese did not become involved in the complications of the Greek semitones' is refuted.

2.4.5 An Evaluation of the Chinese Methods

In analyzing the Chinese up-and-down method of generating scales, Needham and Robinson made the following comparison:

> The mathematics of the Pythagorean scale, which are considerably more complicated than those of the Chinese gamut, and require a knowledge of means, were set forth by Plato, though for metaphysical rather than musical reasons; and a full description of its most extended form was given by Euclid.
>
> The Chinese gamut of pitches, on the other hand, requires only the simplest mathematics and does not use the octave as a starting-point.

123. One finds the terms *qī yīn* 七音 (heptatonic scale) and *shí-èr-lǜ* 十二律 (twelve-pitch scale) in Chinese classics such as the *Zuǒ Zhuàn* 《左傳》 and the *Gúo Yǔ* 《國語》. These scales probably appeared very early in the Chinese civilization as revealed by the ancient musical instruments unearthed in archaeological excavations (see p.40 and p.41).

> Indeed, it does not even include a true octave at all. The only mathematical operation needed is the multiplication of certain figures by 2/3 and 4/3 alternatively. [124]

In this subsection, the Chinese method is re-evaluated.

The Nature of the Up-and-Down Procedures

It has been shown on p.48 to p.54 that the underlying principle in the Chinese method of generating scales is the up-and-down principle. According to this principle, the notes of a scale can be generated by the use of a harmonic interval and the generated notes can be selectively regulated by the octave interval to lie within a given octave. Based on the two earliest extant records on this method found in the *Guǎn Zǐ* and the *Lǚ-Shì Chūn-Qiū*, the specific harmonic interval used for generating notes is the perfect fifth. It is instructive to further examine this choice and the principle upon which it is based.

Mathematically, it is known that the numbers which are capable of generating all the members in a group of twelve ordered elements are 1, 5, 7, and 11. Applied to the group of twelve notes in an octave, the generating numbers 1, 5, 7, and 11 correspond to generations using the minor second, perfect fourth, perfect fifth, and major seventh, respectively. This implies that, in principle, the perfect fifth is only but one of the four possible intervals, within this generation scheme, which are capable of generating the twelve notes in an octave.

The use of the minor second to generate the twelve notes (corresponding to the generating number 1) results in a stepwise generation in which the twelve notes are generated in ascending order from the fundamental to the minor second, then the major second, followed by the minor third, and so on. The use of the major seventh to generate the twelve notes (corresponding to the generating number 11) also results in a stepwise generation but in the reverse direction in which the twelve notes are generated in descending order from the fundamental to the major seventh, then the minor seventh, followed by the major sixth, and so on. Thus, it is apparent that these two ways of generating notes in an octave are interrelated. Although they are mathematically acceptable solutions, in practice the procedures were difficult to implement in ancient times since advanced knowledge of semitone intervals are required.

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124. Joseph Needham (1962), vol. 4, sec. 26(h) 'Sound (Acoustics)' (with Kenneth Robinson), p. 172.

The two remaining mathematical solutions require only the knowledge of the perfect fourth or the perfect fifth which were certainly known accurately in ancient times. In Figure 27, the patterns of these two alternative ways of generating twelve notes of an octave are illustrated. The sequence in generating the notes by the fifth (corresponding to the generating number 7), as illustrated in the figure on the left, is identical to that given in the *Lǚ-Shì Chūn-Qiū* (see p.52 to p.54). This is to be expected since the ancient Chinese used the fifth to generate their scales. The interest here lies in its relation to generation by the fourth (corresponding to the generating number 5). Can the Chinese up-and-down principle, applied here, generate the twelve notes by using the perfect fourth? From the patterns given in Figure 27, it is seen that generation by the fifth is inversely related to generation by the fourth.

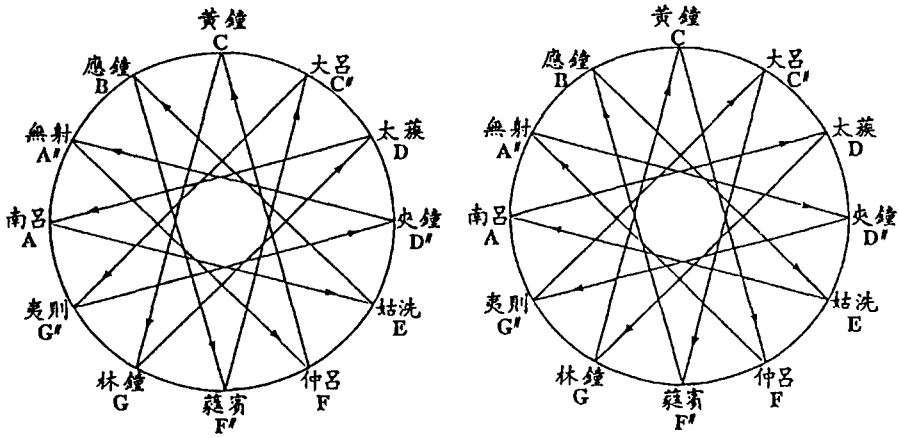

Figure 27. A comparison of the sequential patterns in the generation of twelve notes in an octave by the use of the perfect fourth intervals (right) and the perfect fifth interval (left).

To apply the up-and-down principle to the method of generation by the fourth, all one needs to do is to replace the factor 2/3 with 3/4 ,the perfect fourth interval, for the down-generation and to replace the factor 4/3 with 3/2 [i.e. (3/4)/(1/2)][125] for the up-generation. The twelve notes can then be generated by following the sequence of generation given in Figure 27, and by using the factor 3/2 for the up-generation and the factor 3/4 for the down-generation as shown in Figure 28, in accordance

125. This corresponds to the generation of a perfect fourth and then followed by the lowering the fourth by an octave.

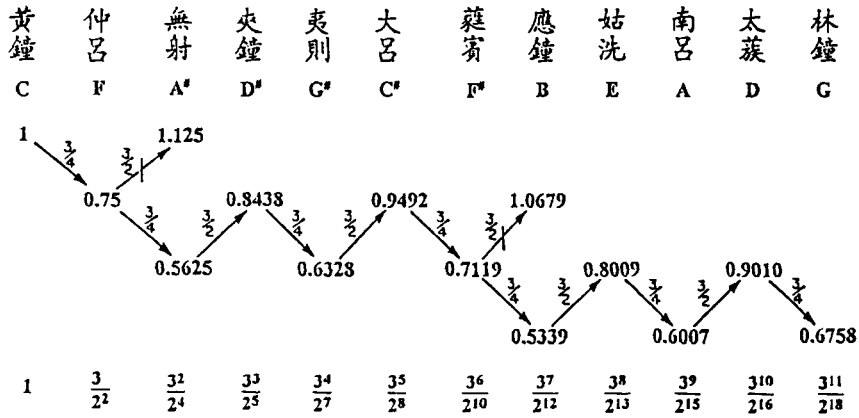

Figure 28. The generation of the chromatic scale with the perfect fourth intervals based on the up-and-down principle as described in the *Guǎn Zǐ* and the *Lǚ-Shì Chūn-Qiū*.

with the principle. The semitonal structure of the octave generated by the fourth is shown in Figure 29 (the right side) in comparison with that generated by the fifth (the left side). The only difference between the two semitonal structures is that the arrangement in semitone intervals between the Yìng-zhōng 應鐘 and the Huáng-zhōng 黄鐘 pitches and between the Huáng-zhōng and the Dà-lǚ 大呂 pitches are opposite to each other. Let the line through the Huáng-zhōng and the Ruí-bīn 蕤賓 notes be the symmetry axis, then a rotation of an angle of 180° about the axis of the right semitonal structure gives the left semitonal structure and vice versa. This demonstrates that the two semitonal structures are equivalent. Thus, the two scale systems generated by the perfect fourth and perfect fifth have, in fact, the same overall structures even though the names of pitch notes in the systems are arranged differently.

From this analysis, it is seen that the *shí-èr-lǜ* system derived by the ancient Chinese is a general solution. It is a mathematical accepted solution. Despite its simple appearance and requiring 'only the simplest mathematics', the *sān-fēn sǔn-yì* 三分損益 method based on the up-and-down principle is an analytical method capable of generating scales in a variety of number of notes. The up-and-down principle is based on the correct understanding of the concept of the octave and its relationship to the harmonics.

The *Lǚ-Shì Chūn-Qiū* chromatic scales so generated, being composed only two kinds of semitones, are quite flexible with respect to modulation in modes. The ancient Chinese musicians were interested in the modulation of pentatonic modes. We have, for example, from the *Zuǒ Zhuàn* and *Lǚ-Shì Chūn-Qiū*, the statements:

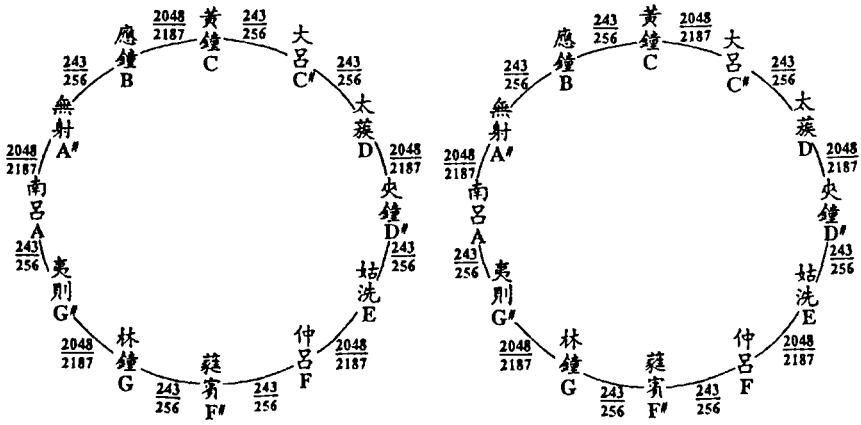

Figure 29. The semitonal structures of the octave of chromatic scales generated with the perfect fourth interval (right) and by the perfect fifth interval (left) based on the up-and-down principle as described in the *Guǎn Zǐ* and the *Lǚ-Shì Chūn-Qiū*.

> 七音, 六律, 以奉五聲.
> The seven-note scale and the six-pitch scale subordinate to the five-tone scale.
>
> 鑄十二鐘以和五音.
> The set of twelve bells is cast in order to harmonize five-note scales.

These statements indicate that the early interest in the variety of scales was motivated by the need in modulating pentatonic modes. Thus, the versatility of the *Lǚ-Shì Chūn-Qiū* scales was of importance.

Despite this versatility, the *Lǚ-Shì Chūn-Qiū* scales retained certain natural (just) intonation. From the comparison of Tables 3 and 7, it is apparent that the ratios that make up the *Lǚ-Shì Chūn-Qiū* scales are not at the same level of accuracy. They can be separated into two groups as shown in Table 10. Other than a few cases, the ratios in Group-I (major second, perfect fourth and fifth) are exact. The exceptions, as seen from Table 3, are the fourth in the Huáng-zhōng pitch (key), the fifth in the Zhòng-lǚ pitch (key), and the major second in both the Wú-yì and Zhòng-lǚ pitches (keys).

The ratios in Group-II of Table 10 are found only in approximations. The approximations listed in Column A are those found not only in Chinese civilization (see Table 3), but also among the scales of a number of other ancient civilizations. The approximations listed in Column B are not as commonly known, but in comparison with the natural intonation they are very close to be exact. In fact, the deviations from the natural intonation are often so small (less than one-tenth of one percent), it is

Table 10. An Analysis of the Intervals in the Chromatic Scales of the *Lǚ-Shì Chūn-Qiū* Generated in Accordance with the Up-and-Down Principle.

Group I			Group IIA	Group IIB
Fundamental	1	Minor second	$\frac{243}{256}$	$\frac{2048}{2187} = \frac{15}{16} - \frac{37}{34992}$
Major second	$\frac{8}{9}$	Minor third	$\frac{27}{32}$	$\frac{16384}{19683} = \frac{5}{6} - \frac{37}{39366}$
	$\frac{59049}{65536} = \frac{8}{9} - \frac{7153}{589824}$	Major third	$\frac{64}{81}$	$\frac{6561}{8192} = \frac{4}{5} + \frac{37}{40960}$
Perfect fourth	$\frac{3}{4}$	Augmented fourth	$\frac{512}{729}$	$\frac{729}{1024} = \frac{32}{45} + \frac{37}{46080}$
	$\frac{131072}{177147} = \frac{3}{4} - \frac{7153}{708588}$	Minor sixth	$\frac{81}{128}$	$\frac{4096}{6561} = \frac{5}{6} - \frac{37}{52488}$
Perfect fifth	$\frac{2}{3}$	Major sixth	$\frac{16}{27}$	$\frac{19683}{32768} = \frac{3}{5} + \frac{111}{163840}$
	$\frac{177147}{262144} = \frac{2}{3} + \frac{7153}{786432}$	Minor seventh	$\frac{9}{16}$	$\frac{32768}{59049} = \frac{5}{9} - \frac{37}{59049}$
Octave	$\frac{1}{2}$	Major seventh	$\frac{128}{243}$	$\frac{2187}{4096} = \frac{8}{15} + \frac{37}{61440}$

questionable whether they are significant in practice. Thus, embedded in the *Lǚ-Shì Chūn-Qiū* chromatic scales are pentatonic and heptatonic scales (such as those in the Jiá-zhōng—*zhǐ* and Lín-zhōng—*gōng* mode-keys) which are practically in natural (just) intonations.

The Procedure in the Marquis Yǐ Set-Bells

The method of using the fifths as the core notes for generating chromatic scales found in the Marquis Yǐ set-bells is designed specifically to allow input of additional harmonic intervals for generating the natural (just) intonation. The choice of the core notes and the input intervals are not at all arbitrary (Fig. 30). The success of the method in generating scales in the natural intonation was indeed a very important achievement in acoustics. In Figure 30, the generation of the four core notes by the fifths using the *Guǎn Zǐ* procedure (left octave circle) and the generation of the eight notes using the *jué* 角 and *zēng* 曾 intervals (right octave circle) are illustrated. For convenience, the ratios used in generating the four core notes and the eight additional notes are omitted in the illustrations.

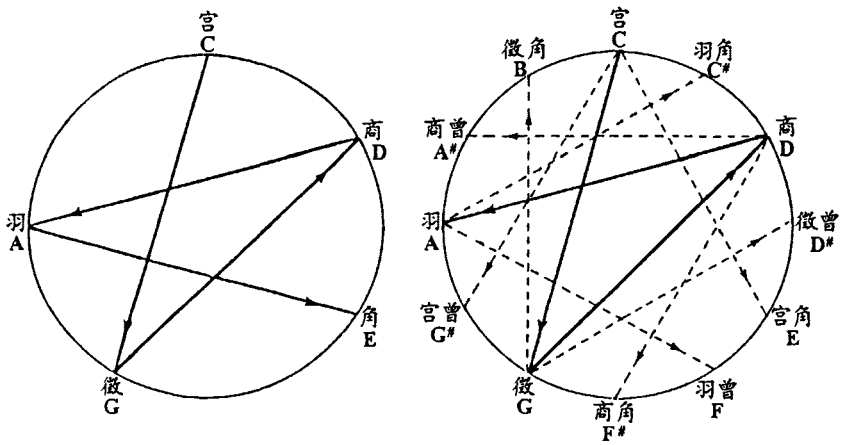

Figure 30. Left, the generation (solid lines) of the core notes in the octave circle. Right, the generation (dashed lines) of the additional eight notes from the core notes in the octave circle.

The uniqueness of the choice in the core notes and in the input intervals for generating the natural intonation is best appreciated by examining the right octave circle in Figure 30 after it is rotated as shown in Figure 31. It is certain that such a symmetry analysis was not directly available to the ancient Chinese acousticians. But it is through such a hidden symmetry illustrated in Figure 31 that makes the choice of the *jué* and *zēng* intervals favourable for the construction of the natural (just) intonation.

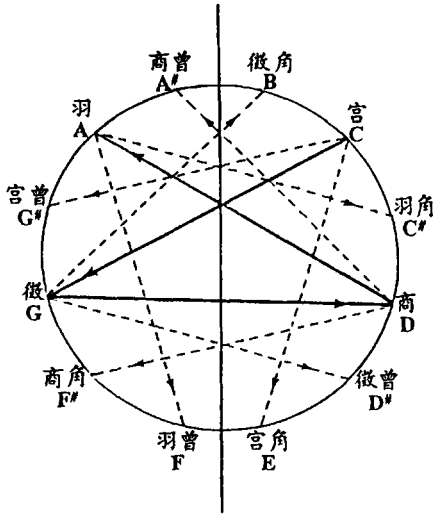

Figure 31. The hidden symmetry in the intervallic structure of the chromatic scale of the Marquis Yǐ *niǔ* bells. The solid lines denote the generation of the core notes and dashed lines denote the generation of the additional eight notes in the octave circle.

It is worthwhile to note that although natural intonation is aesthetically pleasing, the scale involves, however, four different semitones. As a result, the scales are less flexible to accommodate mode changes than the *Lǚ-Shì Chūn-Qiū* chromatic scales which are composed of only two different semitones. The competing interests in aesthetic tonality and modal versatility played an important role in the development of musical scales. History has revealed that the musicological need for mode modulation eventually led to tempering of scales. Early traces towards such a trend can already be found within the Marquis Yǐ set-bells. The intervallic structures of the Marquis Yǐ *yǒng* 甬 bells reveal attempts to modify the procedure for the generation of scales from the core notes of the fifths by using different input intervals such as the minor third. But it took more than two millennia before the equal-ratio chromatic scales composed only of one type of semitones was developed in the sixteenth century. A mathematical solution for the equal-ratio chromatic scale was provided by Zhū Zài-Yù 朱載堉 in 1584.[126] Before the development of the equal-ratio chromatic scale, the *Lǚ-Shì Chūn-Qiū*

126. See the *Lǜ-Xué Xīn-Shuō* 《律學新說》 (*A New Account of Lüology*), 1584. For a recent discussion of the work, see Dài Niàn-Zǔ 戴念祖 (1986), and also Joseph Needham and Kenneth Robinson, vol. 4, sec. 26(h), pp. 214–28.

chromatic scales were the most versatile scales in accommodating modulations and mode shifts. This again attests to the importance of the up-and-down principle and the validity of the *sān-fēn sǔn-yì* generation method. It should also be noted that despite the later dominance in the interest of versatile scales for mode shifts, the ability of theoretically generating chromatic scales in the natural (just) intonation as revealed by the Marquis Yǐ *niǔ* set-bells was also an important acoustic achievement at this early stage of development in musical acoustics.

3 Early Chinese Work in Astronomy

曆象日月星辰, 敬授人時.
Yáo Diǎn 《堯典》
(*The Canon of Yáo*)

In general, the Chinese works in astronomy are better studied than those in the physics of motion and in acoustics discussed above. However, for the early astronomical works, especially in the pre-Qín period, the accounts are often controversial and in some cases inadequate. Not only do there exist a number of issues concerning the origins of astronomy in China, but the accounts of the work of Chén Zǐ 陳子 contain serious mistakes. In this section, the early phase of Chinese work in astronomy is reexamined. The major written source of early Chinese work in astronomy is contained in the *Yáo Diǎn* 《堯典》 (*The Canon of Yáo*)[127] and the *Zhōu-Bì Suàn-Jīng* 《周髀算經》 (*Mathematics of Zhōu-Gnomons*). The discovery and decipherment of the *jiǎ-gǔ-wén* 甲骨文 (shell-bone inscriptions) during the end of the nineteenth century played an important role in restoring parts of the astronomical work in the Shāng dynasty (c. −17th century to −1111). Recent archaeological discoveries in China have provided further information on a number of unsettled questions about ancient Chinese astronomy.[128, 129]

127. There is no question that this document was based on certain oral tradition long after the time of Yáo. The crucial date is not the date when the document in the present form was compiled but the date when the astronomical knowledge contained in the document first emerged in history.
128. Xí Zé-Zōng 席澤宗 (1984).
129. Chén Chēng-Yīh 程貞一 (1986).

3.1 The Development of Calendrical Science

3.1.1 The Ancient Chinese Calendar

The earliest written account of a calendar is found in the *Yáo Diǎn*:

> 朞三百有六旬有六日, 以閏月定四時成歲.
> The *qī* 朞 (duration) of three hundred days plus six *xún* 旬 (ten-day periods) and six days, forms a year in which the four seasons are fixed by the use of the *rùn-yuè* 閏月 (intercalated month).

The account specifically mentions the seasons and the intercalated month, indicating that the ancient Chinese calendar was constructed in an attempt to keep in step with both the moon's phases and the seasons. In addition to the astronomical periods of a month (*yuè* 月) and a year (*suì* 歲), it also mentions an artificial ten-day period (*xún* 旬) similar in many ways to the present seven-day week.

The statement records that a seasonal year had 366 days but provides no information on the length of the month. It is, therefore, difficult to deduce how intercalation of the month was implemented. On the other hand, it is reasonable to expect that at a given stage of development, the length of the lunation should be determined, if not to a higher degree than, then at least to within the same degree of accuracy as, the length of a seasonal year, since the latter is much more difficult to measure than the former. By assuming that the same degree of accuracy was obtained, we obtain an upper bound of 29.6 days for the length of a lunation. The value favours the scheme of using a short month of 29 days and a long month of 30 days to keep in step with the moon's phase rather than the equal-month scheme.

The earliest extant records of the calendrical use of the 29-day short month and the 30-day long month are found in the *jiǎ-gǔ-wén* of the Yīn 殷 period of the Shāng dynasty in the −14th century. The shell-bone inscriptions also contain evidence that the intercalated month was not always inserted at the end of the year as the thirteenth month, and that the Yīn calendar occasionally contained a fourteen-month year.[130] A systematic investigation of the intercalation records reveals only two records of a fourteenth month among more than 150 records of a thirteenth month.[131] This observation seems to suggest that certain regularities in intercalation procedure had already been developed by the Yīn period.

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130. Dǒng Zuò-Bīn 董作賓 (1930).
131. Jao Tsung-I 饒宗頤 ed. (1990), vol. 2.

No explicit statements on the length of the lunation and tropical year were found among the shell-bone inscriptions. Dǒng (1930) deduced, from the information found in the inscriptions, a figure of 29.53 days for the lunation and 365.25 days for the tropical year, and suggested that the well-known *Sì-Fēn Lì* 四分曆 (Quarter-Remainder calendar) was handed down by the Shāng astronomers.[132] However, tacit assumptions were made in deducing these numbers and further evidence is needed before they can be accepted as those determined by the Shāng astronomers. Based on Dǒng's work, it is nevertheless apparent that the Shāng astronomers knew the length of a tropical year more accurately than the 366 days given in the *Yáo Diǎn*.[133] The shell-bone inscriptions also confirm that by the −14th century the Chinese calendar was a lunisolar one in which the moon's phase and seasonal changes were reconciled by expressing the twelve-month year in terms of combinations of short and long months and by the occasional use of an intercalated month. The Yīn calendar, revealed by the shell-bone inscriptions, is consistent with the account of the *Yáo* calendar given in the *Yáo Diǎn*.

By the Zhōu dynasty, a systematic procedure for intercalation was developed. The procedure consists of using seven intercalated months in every nineteen-year period (hereinafter 'the 7–19 intercalation procedure'). Detailed studies of the intercalation procedures, based on the chronicle and astronomical data systematically kept in the *Chūn-Qiū* 《春秋》 (*Spring and Autumn Annals*) between −722 and −476, were carried out by Wáng[134] and subsequently by Shinjo[135] and Yabuuchi.[136] The results of these studies are summarized in Table 11.[137] It is apparent that the 7–19 intercalation procedure was implemented systematically by the −6th century.

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132. Dǒng Zuò-Bīn 董作賓 (1945).
133. It is worth noting that the expression for 366 days is extremely archaic. Compare this expression with that for 547 days found in the shell-bone inscriptions of the −13th century:
三百有六旬有六日. Three hundreds plus six tens plus six days.
五百四旬七日 (𠄟 三 𠂤 十 日). Five hundreds, four tens, seven days.
Though the use of a particle between the units and the tens of a number was common in the early shell-bone inscriptions of the −14th century, the practice was nevertheless becoming rare in the later Shāng. On the other hand, the use of a particle between the tens and the hundreds is not found at all in the shell-bone inscriptions. This seems to indicate that the expression for 366 days was probably preserved from the pre-Shang period. This is also consistent with the work of Dǒng (Dǒng Zuò-Bīn 董作賓, 1945) that by the time of Yīn 殷, the length of a tropical year was known more accurately than to the nearest whole number of days.
134. Wáng Tāo 王韜, (1889).
135. Shinjō Shinzō 新城新藏 (1929).
136. Yabuuchi Kiyoshi 藪内清 (1969).
137. Chén Jiǔ-Jīn 陳久金 (1978).

Table 11. The Chinese '7–19' Intercalation Procedure

Period	#[a]	Period	Year with Intercalation							#[a]
−722 to −704	7	−589 to −571	3	6	8	11	13	16	19	7
−703 to −685	6	−570 to −552	3	5	8	11	14	17	19	7
−684 to −666	7	−551 to −533	3	5	8	11	13	16	19	7
−665 to −647	7	−532 to −514	3	6	8	11	13	16	19	7
−646 to −628	6	−513 to −495	3	6	8	11	13	16	19	7
−627 to −609	7	−494 to −476	3	5	8	11	13	16	19	7
−608 to −590	8									

[a]# denotes the number of intercalations.

By the Warring States period, the length of the synodic month (*shuò-wàng yuè* 朔望月) is determined to be $29\frac{499}{940}$ days and of the tropical year (*huí-guī nián* 回歸年) $365\frac{1}{4}$ days. With these numbers for the lunation (*shuò-shí* 朔實) and year (*suì-shí* 歲實), the '7–19' intercalation procedure yeilds the following relation in days:

$19 \times 365\frac{1}{4} = 235 \times 29\frac{499}{940} = 6939\frac{3}{4}$ days where $235 = [(19 \times 12) + 7]$ is the total number of months in 19 years with 7 intercalated months. This naturally led to the *Sì-Fēn Lì*. By this time, the basics of the intercalation procedure were well established.[138] It should be noted that the term *lì* 曆 in ancient usage means more than just a system for the specification and division of a year as implied by the term 'calendar'. It is worthwhile to examine the nature of calendrical science in ancient China.

3.1.2 The Nature of Calendrical Science in China

At a very early stage, calendrical science in China underwent different trends of development. One such trend placed an emphasis upon incorporating into the calendar additional astronomical phenomena such as lunar and solar eclipses and planetary motions. This led to the inclusion of planetary astronomy as an essential part of calendrical science. Thus, much of the Chinese work in planetary astronomy is found in calendrical treatises (see p.163 to p.164). Another important trend of development was to facilitate the civil functions of the calendar

138. See for example, the following remark by Mèng Kē 孟軻 (c. −372 to c. −289):
天之高也, 星辰之遠也, 苟求其故, 千歲之日至可坐而致也.
from the *Mèng Zǐ*《孟子》(*The Book of Master Mèng*) of the −3rd century.

by introducing counting cycles dissociated from the cycles of lunations and seasonal changes. These different developments contributed to the multifunctional characteristics of the later Chinese calendar.

The first known artificial period introduced into the ancient Chinese calendar was the ten-day *xún* 旬. The purpose was to subdivide the astronomical periods of a lunation and a seasonal year. Thus a three-*xún* was approximately a lunation and a 36-*xún* was approximately a tropical year. A ten-day period was a logical choice since Chinese numerals, whether in the ciphered grouping form found in the shell-bone inscriptions or in the positional counting rod form for computations, have always been decimal.

The names of the days in a *xún*, *jiǎ* 甲, *yǐ* 乙, *bǐng* 丙, *dīng* 丁, *wù* 戊, *jǐ* 己, *gēng* 庚, *xīn* 辛, *rén* 壬, and *guǐ* 癸 form a denary cyclic system known as the *gān* 干 system. There is evidence from the shell-bone inscriptions that by the time of the Shāng dynasty in the –2nd millennium, the denary day-count cycle had been incorporated into a sexagenary one. It is self-evident that the *gān-zhī* 干支 cyclic system used for the sexagenary day-count was derived from the *gān* system by combining its ten ordered characters with the twelve ordered characters, *zǐ* 子, *chǒu* 丑, *yín* 寅, mǎo 卯, *chén* 辰, *sì* 巳, *wǔ* 午, *wèi* 未, *shēn* 申, *yǒu* 酉, *xū* 戌, and *hài* 亥 of the duodenary *zhī* 支 system to give a total of sixty ordered combinations (see p.214), such as *jiǎ-zǐ* 甲子 or *yǐ-chǒu* 乙丑. Such combinations are among the most common characters for the day count in the shell-bone inscriptions.

It should be noted that the *gān-zhī* day count was introduced after the scheme of intercalation had already been invented. It was introduced, not to replace the day count within lunations and seasonal years, but to provide a reference day count common to all regions and periods. Thus, the statement 'the most ancient day count in Chinese culture did not depend on the sun and moon at all'[139] can be rather misleading. The *gān-zhī* day count was an important innovation of scientific merit in chronology.

It was the chronology of the *gān-zhī* day count that permitted us to deduce the length of a tropical year in the Spring-Autumn period (–771 to –477) of the Zhōu dynasty. In the *Zuǒ Zhuàn* 《左傳》 (*Master Zuǒqiū's Commentary on the Spring-Autumn Annals*), there are two dated records of the winter solstices: one is on the *xīn-hài* 辛亥 day in the first month of the fifth year of the reign of Xī Gōng 僖公 (i.e. –655), and the other is on the *jǐ-chǒu* 己丑 day in the second month of the twentieth year of the reign of the Zhāo Gōng 昭公 (i.e. –522). The number of days in the 133-year

139. Joseph Needham (3), (1959), vol. 3, p. 396.

period between the two winter solstices can be deduced from the *gān-zhī* day-count starting on the *xīn-hài* day and finishing on the *jǐ-chǒu* day. This yields a total of 48,578 days, giving a tropical year of $365\frac{33}{133}$ days.

The sexagenary *gān-zhī* system was later extended to the year count. The existing data permits the *gān-zhī* chronology to be traced precisely back to −841. Thus, unless there is an uncertainty as to a probable date greater than sixty years, the absolute date of events after −841 can be calculated with precision.

3.2 The System of *Xiù* 宿 and Positional Astronomy

3.2.1 The System of 28 *xiù*

The system of *xiù* 宿 originated from the use of star-groups to trace the apparent motion of the sun in relation to the seasons. According to the *Yáo Diǎn*, during the time of Yáo 堯 the seasons were determined by the meridian passage of four star-groups, known later as *xiù*. The vernal and autumnal equinoxes were determined by the culmination of the star-groups identified by the stars *Niǎo* 鳥 (α Hya) and *Xū* 虛 (β Aqr). The summer and winter solstices were determined by the culmination of the star-groups identified by the stars *Huǒ* 火 (Antares, α Sco) and *Mǎo* 昴 (η Tau). The relevant passage runs as follows:

> 日中星鳥, 以殷仲春.
> 日永星火, 以正仲夏.
> 宵中星虛, 以殷仲秋.
> 日短星昴, 以正仲冬.
> The day of medium length and [the culmination of] the star *Niǎo* 鳥 [serve to] adjust the middle of spring.
> The day of greatest length and [the culmination of] the star *Huǒ* 火 [serve to] fix the middle of summer.
> The night of medium length and [the culmination of] the star *Xū* 虛 [serve to] adjust the middle of autumn.
> The night of greatest length and [the culmination of] the star *Mǎo* 昴 [serve to] fix the middle of winter.

In the course of time, a total of 28 star-groups were identified as *xiù*. The character *xiù* 宿 literally means a temporary resting-place. With the mental picture of the 28 compartments identified with the 28 star-groups along the equatorial belt functioning as the resting-place for the apparent journey of the sun, the star-groups gradually became known as *xiù*, the 'equatorial compartments'. The familiar translation of *xiù* as 'lunar mansion' is rather misleading, since the equatorial compartments were

not only for the moon, but also for the sun and planets. From the early Chinese records, it is seen that the concept of *xiù* was developed primarily in relation to the apparent motion of the sun. Relationships to the moon and planets appeared gradually in the course of time.

By the time of the Western Zhōu, the *xiù* system had already been expanded into its final form, as all but four (nos. 7, 18, 23, and 26) of the 28 *xiù* (see Table 12) are found in the *Yuè Lìng* 《月令》 (*Monthly Ordinances of Zhōu*). An example entry is given below:

> 季秋之月, 日在房, 昏虛中, 旦柳中, 其日庚辛.
> In the third month of autumn, the sun is in the *Fáng* 房. *Xū* 虛 culminates at dusk. *Liǔ* 柳 culminates at dawn. It is the *gēng-xīn* 庚辛 day.

This illustrates the general way in which the system was applied. The *Yuè Lìng* was later incorporated into the *Lǐ Jì* 《禮記》 (*Record of Rites*) and the *Lǚ-Shì Chūn-Qiū*.

The specifics of the determinative stars, *jù-xīng* 距星, of the 28 *xiù* 宿 based on epoch +1900 are shown in Table 12.[140] Listed alongside each *xiù* is the name, magnitude, right ascension α, and declination δ of its determinative star lying on the hour circle. From the table, it can be seen that the *xiù*-determinative stars were not chosen on the basis of their magnitude; a number of them were of only fourth magnitude, and one (*Guǐ*, θ Cnc) was of sixth. This seems to suggest that they were chosen because of certain specific positional requirements. In fact, some of the determinative stars were quite far from the equator even after taking into consideration the precession of the equinoxes.

3.2.2 The Chinese Celestial Coordinates

The set of 28 *xiù* along the equatorial belt for tracking the position of the sun served conveniently as references to specify celestial positions in recording celestial objects. Examples of such usages are found as early as among the shell-bone inscriptions before the set of 28 *xiù* was even completed. Reproduced in Figure 32 is the oldest extant record of the sighting of a nova, dating from about −1300. The inscription reads:

> 七日己巳夕豈出新大星並火.
> On the seventh day of the month, a *jǐ-sì* 己巳 day, a great new star appeared at dusk in company with *Huǒ* 火 (Antares).

Here the position of the *xīn-xīng* 新星 (nova) is specified using the star *Huǒ*.

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140. Zhú Kě-Zhēn 竺可禎 (1944).

Table 12. Table of the 28 *Xiù* and Their Determinative Stars*

1		*Jiǎo* 角	α Virginis (1.2) 13 19 55 −10° 38' 22"	15		*Kuí* 奎	η Andromedae (4.2) 00 42 02 +23° 43' 23"
2		*Kàng* 亢	κ Virginis (4.3) 14 07 34 −09° 48' 30"	16		*Lóu* 婁	β Arietis (2.7) 01 49 07 +20° 19' 09"
3		*Dī* 氐	α^2 Librae (2.9) 14 45 21 −15° 37' 35"	17		*Wèi* 胃	41 Arietis (3.7) 02 44 06 +26° 50' 54"
4		*Fáng* 房	π Scorpii (3.0) 15 52 48 −25° 49' 35"	18		*Mǎo* 昴	η Tauri (3.0) 03 41 32 +23° 47' 45"
5		*Xīn* 心	σ Scorpii (3.1) 16 15 07 −25° 21' 10"	19		*Bì* 畢	ε Tauri (3.6) 04 22 47 +18° 57' 31"
6		*Wěi* 尾	μ^1 Scorpii (3.1) 16 45 06 −37° 52' 33"	20		*Zī* 觜	λ^1 Orionis (3.4) 05 29 38 +09° 52' 02"
7		*Jī* 箕	γ Sagittarii (3.1) 17 59 23 −30° 25' 31"	21		*Shēn* 參	ζ Orionis (1.9) 05 35 43 −01° 59' 44"
8		*Dǒu* 斗	ϕ Sagittarii (3.3) 18 39 25 −27° 05' 37"	22		*Jǐng* 井	μ Geminorum (3.2) 06 16 55 +22° 33' 54"
9		*Niú* 牛	β Capricorni (3.3) 20 15 24 −15° 05' 50"	23		*Guǐ* 鬼	θ Cancri (5.8) 08 25 54 +18° 25' 57"
10		*Nǚ* 女	ε Aquarii (3.6) 20 42 16 −09° 51' 43"	24		*Liǔ* 柳	δ Hydrae (4.2) 08 32 22 +06° 03' 09"
11		*Xū* 虛	β Aquarii (3.1) 21 26 18 −06° 00' 40"	25		*Xīng* 星	α Hydrae (2.1) 09 22 40 −08° 13' 30"
12		*Wēi* 危	α Aquarii (3.2) 22 00 39 −00° 48' 21"	26		*Zhāng* 張	μ Hydrae (3.9) 10 21 15 −16° 19' 33"
13		*Shì* 室	α Pegasi (2.6) 22 59 47 +14° 40' 02"	27		*Yì* 翼	α Crateris (4.2) 10 54 54 −17° 45' 59"
14		*Bì* 壁	γ Pegasi (2.9) 00 08 05 +14° 37' 39"	28		*Zhěn* 軫	γ Corvi (2.4) 12 10 40 −16° 59' 12"

* The entries—magnitude, right ascension α, and declination δ—for the *xiù*-determinative star lying on the hour circle are based on epoch +1900 (Zhú Kě Zhēn 竺可禎, 1944).

Such a usage of *xiù* star-groups led eventually to the development of a celestial coordinate system with the set of 28 *xiù* (each identified by a determinative star) along the celestial equator (*chì-dào* 赤道) as one of the reference circles. A description on the use of the *xiù* system for the celestial circle is given in the *Zhōu-Bì Suàn-Jīng* 《周髀算經》:[141]

141. See the *Zhōu-Bì Suàn-Jīng*, vol. 2, p. 9b.

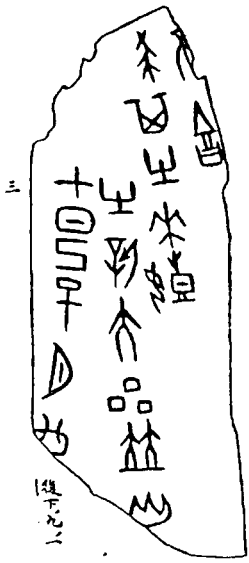

Figure 32. A record of the sighting of a nova inscribed on a bone dating from about −1300. It reads (the left two columns of characters): 'On the seventh day of the month, a *jǐ-sì* 己巳 day, a great star appeared at dusk in company with *Huǒ* 火 (Antares)'.

立二十八宿以周天曆度之法.
Set the 28 *xiù* as the method for specifying the degrees of the celestial circle.

It is perhaps worthwhile to note that the circumference along the celestial circles were divided into $365\frac{1}{4}°$ not 360°.

Recently, two round disks marked with 28-*xiù* were unearthed at Fùyáng 阜陽 in 1977 from the Rǔ-Yīn Hóu 汝陰侯 tomb of Western Hàn.[142] The 28-*xiù* disks are dated to −165. Engraved on the disks are the *xiù* names and the separation in degrees between the *xiù*-determinative stars on the equatorial circle. In Table 13, a comparison of these degrees from the unearthed *xiù* disks with those in *gǔ-dù* 古度 (anciently measured degrees) preserved in the *Kāi-Yuán Zhàn-Jīng* 《開元占經》 (*The Kāi-Yuán Reign Period Treatise on Astrology and Astronomy*) of 729 is given. It is seen that the two sets of values are in agreement. This implies that the selection of the determinative stars as reflected by the *xiù* disks of −165 was based on the ancient selection preserved in the *Kāi-Yuán Zhàn-Jīng*.

The celestial coordinate system developed by the astronomers of the Hún-Tiān 渾天 (Celestial Sphere) school made use of the equator-circle together with the hour-circle as the reference circles on the celestial

142. Ānhuī Provincial excavation team, Fùyáng District Museum, and Fùyáng County Cultural Bureau (1978), *Wén-Wù* 《文物》 (*Cultural Relics*) No. 8, pp. 18 and 19, figs. 8–10.

Table 13. Separation in Degrees between the *Xiù*-Determinative Stars

No.	*Xiù* 宿 Name	Degree Hàn 漢[a]	Degree Gǔ 古[b]	No.	*Xiù* 宿 Name	Degree Hàn 漢[a]	Degree Gǔ 古[b]
1	*Jiǎo* 角	?	12	15	*Kuí* 奎	14	12
2	*Kàng* 亢	11	[9]	16	*Lóu* 婁	15	15
3	*Dī* 氐	1?	17	17	*Weì* 胃	11	11
4	*Fáng* 房	7	7	18	*Mǎo* 昴	15	15
5	*Xīn* 心	11	12	19	*Bì* 畢	15	15
6	*Wěi* 尾	9	9	20	*Zī-Xī* 觜觿	6	6
7	*Jī* 箕	10	10	21	*Shēn* 參	9	[9]
8	*Nán-Dǒu* 南斗	22	22	22	*Dōng-Jǐng* 東井	26	29
9	*Qiān-Niú* 牽牛	9	9	23	*Yú-Guǐ* 輿鬼	5	5
10	*Wù-Nǚ* 婺女	10	10	24	*Liǔ* 柳	18	18
11	*Xū* 虛	14	14	25	*Qī-Xīng* 七星	12	13
12	*Wēi* 危	6	9	26	*Zhāng* 張	?	13
13	*Yíng-Shì* 營室	20	20	27	*Yì* 翼	?	13
14	*Dōng-Bì* 東壁	15	15	28	*Zhěn* 軫	?	16

a. *Hàn-dù* 漢度 from the 28-*xiù* 宿 round disk of −165 unearthed in 1987 from the tomb of Marquis Rǔ-Yīn 汝陰, see Anhui Provincial excavation team et al. (1978).

b. *Gǔ-dù* 古度 from the *Kāi-Yuán Zhàn-Jīng* 《開元占經》, Táng 唐.

sphere with the two extreme points identified as the north and south poles. Thus, the position of a celestial object is specified by the *qù-jí-dù* 去極度, the distance from the north pole in degrees, and the *rù-xiù-dù* 入宿度, the distance in degrees to the *xiù*-determinative star of the nearest *xiù* in the right ascension. Evidence for such a coordinate system in Chinese civilization can be traced to the −4th century works of Shí Shēn 石申 of the state of Wèi 魏 and Gān Dé 甘德 of the state of Qí 齊 on star listings.[143]

A comparison of the equatorial celestial coordinate system used by the ancient Chinese astronomers with that used universally in modern astronomy is given in Figure 33. In the Chinese system, the position of a celestial object, such as a star, is specified by the *qù-jí-dù* 去極度 p (north polar distance) and the *rù-xiù-dù* 入宿度 β (in-*xiù* distance) in the right

143. See the *Shí-Shì Xīng-Jīng* 《石氏星經》 (*The Star Manual of Master Shí*), much of the work is preserved in the form of quotations in later works especially the *Kāi-Yuán Zhàn-Jīng*.

ascension relative to the nearest *xiù*-determinatives (see Fig. 33a). On the other hand, in modern astronomy, the position is specified by the declination δ and the right ascension α with respect to the vernal equinox (see Fig. 33b). It is apparent from the comparison that the two celestial coordinate systems both use the hour-circle and the equator as their reference circles.

The fact that the north polar distance is simply the complement of declination and the distance from the determinative star of the nearest *xiù* is equivalent to right ascension seems to suggest that the two celestial coordinate systems were probably related in their development. It is well known that ancient Greek as well as medieval European astronomers used the ecliptic and the circle of the celestial longitude as their reference circles. Consequently, a star's position is expressed in terms of ecliptic longitude and latitude. Needham has commented that 'since the Renaissance, and especially since the work of Tycho Brahe, modern astronomy has made universal use of equatorial coordinates, using right ascension and declination.'[144] Thus, the equatorial system began to emerge as the major celestial coordinate system in the sixteenth century.

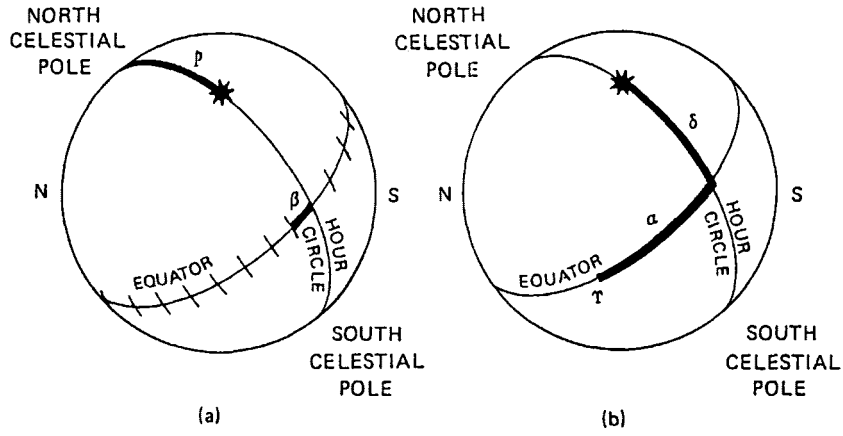

Figure 33. Illustrations of the celestial equatorial coordinate systems used in (a) ancient Chinese astronomy and (b) modern astronomy.

It should be noted that the ecliptic coordinates were also used by the Chinese astronomers. An early reference is again found in the −4th century work of the *Hún-Tiān* school. As recorded in the 'Lǜ Lì Zhì' 〈律曆志〉 of the *Hòu-Hàn Shū* 《後漢書》 (*History of the Later Hàn Dynasty*), the astronomer Jiǎ Kuí 賈逵 made in +92 the following observation in which

144. Joseph Needham, (1959), vol. 3, p. 266.

he also cited measurements from the *Shí-Shì Xīng-Jīng* 《石氏星經》 (*The Star Manual of Master Shí*):

> 石氏星經曰:「黃道規牽牛初直斗二十度,
> 去極百一十五度.」于赤道斗二十一度也.
> According to the *Shí-Shì Xīng-Jīng*, 'the winter solstice (i.e. *qiān-niú chū* 牽牛初)[145] on the huáng-dào 黃道 (ecliptic circle) is 20° from the determinative star of the *Dǒu* 斗 star-group, and the *qù-jí-dù* 去極度 is 115°' [However] the winter solstice is now 21° from the determinative star of the *Dǒu* star-group on the chì-dào 赤道 celestial equator.

The statement records the discovery of Jiǎ Kuí 賈逵 in +92 that the position of the winter solstice is not fixed but also changes. This was an important observation of the *suì-chā* 歲差 (precession of the equinoxes) phenomenon.[146] In addition, the interest here is that the statement indicates that the concept of ecliptic (*huáng-dào* 黃道) already appeared in the Shí Shēn's work of the −4th century. An explicit description of the *huáng-dào* is found in the work of Liú Xiàng 劉向 of the −1st century:

> 日、月循黃道, 南至牽牛, 北至東井.
> The sun and moon revolve along the ecliptic (*huáng-dào* 黃道), south to the *Qiān-Niú* 牽牛 star group and north to the *Dōng-Jǐng* 東井 star group.[147]

These passages imply that the concept of ecliptic was not only known to Shí Shēn in the −4th century, but was explicitly used in later observation of planetary motion. Thus, the adoption of the celestial equator for coordinates was a matter of choice. Although the equatorial coordinates were favoured by the Chinese astronomers, the ecliptic coordinates were certainly used. By the Eastern Hàn, the motions of the planets were, in fact, examined in terms of the ecliptic instead of the equator.

It is also of interest to note that the above statement from the *Shí-Shì-Xīng-Jīng* quoted by Jiǎ Kuí also provides information on the angle ϕ between the ecliptic and celestial equator (*huáng-chì jiāo-jiǎo* 黃赤交角). From the *qù-jí-dù* 去極度 115° given for the winter solstice, one obtains for this angle:

$$\phi = 115° - 91\frac{5}{16}° = 23\frac{11}{16}°$$

where the $91\frac{5}{16}°$ is the *qù-jí-dù* of the celestial equator obtained by

145. In the ancient work up to the Tài-Chū 太初 Calendar, the winter solstice was found at the *qiān-niú chū* 牽牛初. Hence, the term became identified with the winter solstice as it is used here in the *Shí-Shì Xīng-Jīng*.
146. The observation of positional change of winter solstice was probably already made in the Tài-Chū 太初 Calendar of −104.
147. See the 'Lǜ Lì Zhì' 〈律曆志〉 of the *Hòu-Hàn Shū* 《後漢書》.

dividing $365\frac{1}{4}^{\circ}$ by 4. The angle $23\frac{11}{16}^{\circ}$ obtained from the measurement of Shí Shēn in the −4th century, is the angle between the ecliptic and celestial equator. Converting to the 360° basis, one obtains 23°44'38". Based on current values, the angle in the −4th century should be around 23°20'52".

3.3 Chén Zǐ 陳子 and Mathematical Astronomy

Chén Zǐ's work came down to us as an integral part of the *Zhōu-Bì Suàn-Jīng* 《周髀算經》 (*Mathematics Classics of Zhōu-Gnomons*), a Chinese classic on mathematics and astronomy which was thought traditionally to be the work of Western Zhōu period. The book in its present form is in two volumes. The first volume includes the work of mathematician Shāng Gāo 商高 in the form of a dialogue with Zhōu Gōng 周公 (the Duke of Zhōu), and the work of astronomer Chén Zǐ in the form of a dialogue with his student Róng Fāng 榮方. The rest of the first volume, as well as the second volume, which are not in dialogue form, contain the work of Lǚ Bù-Wéi 呂不韋 and the astronomers of the Gài-Tiān 蓋天 school of the −3rd century.

The crucial data for the determination of the date of Chén Zǐ's work is not the date of completion of the book in the form that came down to us as implied by a number of scholars,[148] but rather the date of the content and the dialogue itself. There is consistent evidence indicating, not only that the two dialogues were written earlier and independently, but also that their contents had not been altered when they were incorporated together with the work of the Gài-Tiān school to form the *Zhōu-Bì Suàn-Jīng* in its present form.[149]

Other than the family name Chén 陳, very little is known about the personal life of Chén Zǐ. Based on the references made of the former high kings of Zhōu in the Chén Zǐ – Róng Fāng dialogue, it is evident that he must have lived much later than the −11th century. According to the records in the *Shǐ-Jì* the Róng 榮 family, beginning with Róng Bó 榮伯, served in the Zhōu court. The last known figure of the Róng family recorded in the *Shǐ-Jì* was Róng Yí-Gōng 榮夷公 who served as the Minister of King Lì 厲 of Zhōu, until shortly before −841. If Róng Fāng, who appeared in the dialogue with Chén Zǐ, was indeed a member of the Róng family of that period, this would place Chén Zǐ in the Western

148. See for example, Qián Bǎo-Cóng (1) 錢寶琮 (1929).
149. This observation is also supported by the technical term for the hypotenuse, *jìng* 徑, found in the Shāng Gāo—Zhōu Gōng dialogue. The term *jìng* 徑 was later changed to *xián* 弦.

Zhōu period in agreement with the traditional view of the work. There is also evidence indicating that Chén Zǐ's work was known to Zēng Zǐ 曾子 (Zēng Cān 曾參, c. −505 to c. −440) of the *Rú Jiā* 儒家.[150] This then gives a lower limit to the date of Chén Zǐ's work. Hence, Chén Zǐ must have lived in a period which is either earlier than or contemporary to that of Zēng Zǐ. A conservative estimate places Chén Zǐ's work in the −5th century or earlier.

3.3.1 The Chén Zǐ's Earth-Sun Model

It has often been mentioned that Chén Zǐ's cosmological view was that of the *Gài-Tiān* 蓋天 in which the *tiān* 天 (sky) is visualized as a half dome covering the *dì* 地 (earth) which is visualized to be in the form of a square.[151] Examining the entire text of the Chén Zǐ's work, no mention of a half-dome sky and a square earth is found. In fact, such a view is not entirely compatible with the views of Chén Zǐ. In Chén Zǐ's model, the sky is implicitly assumed to be an empty space in which the sun is free to orbit around the north celestial pole in a plane which is parallel to the plane of the assumed flat surface of the earth.

The passage which describes the relationship between the sun-shadow measurements and the apparent motion of the sun found in Chén Zǐ's dialogue is reproduced below:

> 夏至南萬六千里 ($\varkappa_0$), 冬至南十三萬五千里. 日中立竿測影, 此一者, 天道之數. 周髀長八尺 (h), 夏至之日, 晷一尺六寸 (λ_0). 髀者, 股也; 正晷者, 勾也. 正南千里 ($\varkappa_1 = \varkappa_0 - 1000$ 里) 勾一尺五寸 (λ_1), 正北千里 ($\varkappa_2 = \varkappa_0 + 1000$ 里) 勾一尺七寸 (λ_2).
>
> [The sun lies] at the summer solstice, 16,000 *lǐ* 里 (Chinese mile) [$\varkappa_0$] to the south and, at the winter solstice, 135,000 *lǐ* to the south. One of the locations (i.e. the summer solstice) was determined [anciently] by the shadow length of a vertical rod under the sun and is a constant of celestial nature. At the summer solstice, the sun shadow length of an 8 *chǐ* 尺 (Chinese foot) [h] Zhōu gnomon is measured to be 16 *cùn* 寸 (Chinese inch), [λ_0]. [Now if we] let the gnomon height be the *length* (*gǔ* 股) and the shadow length be the *width* (*gōu* 勾) [of the right-angled triangle formed by the gnomon-shadow system], then the *width* (i.e. the shadow length of the gnomon) would be 15 *cùn* [λ_1] for a gnomon 1,000 *lǐ* directly to the south [$\varkappa_1$], and the *width* would be 17 *cùn* [λ_2] for a gnomon 1,000 *lǐ* directly to the north [$\varkappa_2$].

The passage describes the sun-shadow measurements (λ_0, $\varkappa_0$) at an observational site and two additional sets of measurements, (λ_1, $\varkappa_1$) and

150. Chén Chēng-Yīh 程貞一 (1987), Appendix I, pp. 29–32.
151. See for example, Joseph Needham, (1959) vol. 3, pp. 210–314 and H. Chatley (1938).

(λ_2, $\varkappa_2$), at locations specified with respect to the observational site. Figure 34 illustrates the relationship of the measurements (λ_1, $\varkappa_1$) for a gnomon located 1,000 *lǐ* to the south and measurements (λ_2, $\varkappa_2$) for a gnomon located 1,000 *lǐ* to the north with respect to the measurements (λ_0, $\varkappa_0$) for a gnomon located at the reference site. These measurements are supposedly to be carried out 'simultaneously' at the summer solstice with the three gnomons having the identical height *h*.

It is seen from the illustration that these sets of measurements are interrelated through a set of three similar right-angled triangles. In current mathematical notation, we have:

$$\frac{H_0}{h} = \frac{\varkappa_1 + \lambda_1}{\lambda_1} = \frac{\varkappa_0 + \lambda_0}{\lambda_0} = \frac{\varkappa_2 + \lambda_2}{\lambda_2} \tag{13}$$

where H_0 represents the vertical height of the sun, *h* is the height of the

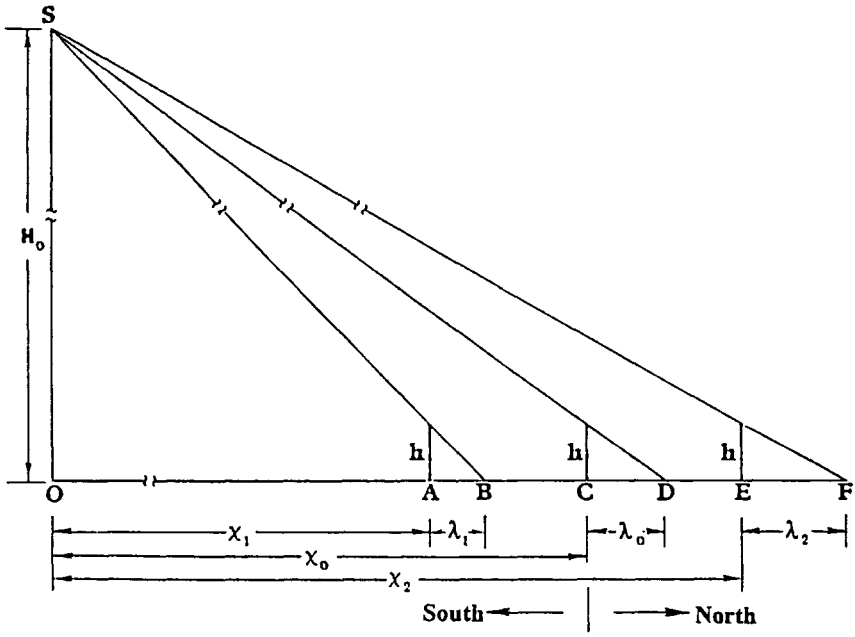

Figure 34. The similar right-angled triangles reconstructed based on the description of the sun-shadow measurements and the apparent motion of the sun given by Chén Zǐ.

Zhōu gnomon, $\varkappa_1$, $\varkappa_0$, and $\varkappa_2$ are the horizontal distances of the gnomons from the vertical position of the sun, and λ_1, λ_0, and λ_2 are the sun shadow lengths of the gnomons. From these similar right-angled triangles, Chén Zǐ derived the important formula:[152]

152. The derivation of Chén Zǐ's formula is examined from p.143 to p.146.

$$(\varkappa_2 - \varkappa_1) = \frac{\varkappa_0}{\lambda_0} (\lambda_2 - \lambda_1) \tag{14}$$

which provides an explicit relation between the change in shadow lengths and the change in the positions of the gnomons.

An important step in Chén Zǐ's work was to convert this mathematical relationship between the shadow lengths and the positions of the gnomons to a relationship between the shadow lengths and the positions of the sun. Here, Chén Zǐ asserted that the change in distance $(\varkappa_2 - \varkappa_1)$ between the gnomon's positions in the north-south directions corresponds exactly to the change in distance in the sun's positions, provided that at these positions the sun gives the same change in the shadow length $(\lambda_2 - \lambda_1)$ aligned in the north-south directions as observed at the same observational point on the earth. Based on this assertion, Chén Zǐ was able to determine the different positions of the sun by simply measuring the corresponding shadow length aligned in the north-south directions, using only one gnomon at a fixed observation point (see p.131 to p.133).

It is not apparent how Chén Zǐ arrived at this interpretation of correspondence between the sun's and gnomon's positions since no demonstration has come down to us. Under the assumption that the plane of the sun's orbit is parallel to the plane of the assumed flat surface of the earth, Chén Zǐ's interpretation of correspondence is indeed correct. This can be verified by examining the illustration given in Figure 35. The upper portion of Figure 35 illustrates the setup for the simultaneous shadow-length measurements at three gnomons located at different positions as the case given in Figure 34 for which Chén Zǐ's formula was first derived. In practice, such a measurement would, of course, be difficult to carry out in ancient times. What Chén Zǐ did was to substitute such simultaneous measurements by separate measurements carried out at different times for the corresponding positions of the sun but at a fixed observational point. The lower portion of Figure 35 illustrates the superposition of three such separate measurements of shadow length for three sun's positions carried out at a fixed gnomon position. In this case, the similar right-angled triangles exhibit the identical mathematical relations as those given by Equation (13). Thus Chén Zǐ's formula given by Equation (14) can again be derived. Consequently, the correspondence principle, as presumed by Chén Zǐ, is indeed valid within the earth-sun model that he adopted. This demonstrates Chén Zǐ's remarkable intuition and reasoning ability.

In Chén Zǐ's model, the geometrical form of the earth, other than the flat surface, is not explicitly specified. The sky is assumed to have no specific form and to allow the sun to move in it freely. The apparent

motion of the sun is to circle the north pole daily with a diameter which changes in synchronization with the seasonal cycle. Though the model is not complete and contains incorrect assumptions, it nevertheless is physical and permitted Chén Zǐ to use mathematics to analyse his measurements of the sun and to examine the sun's apparent motion.

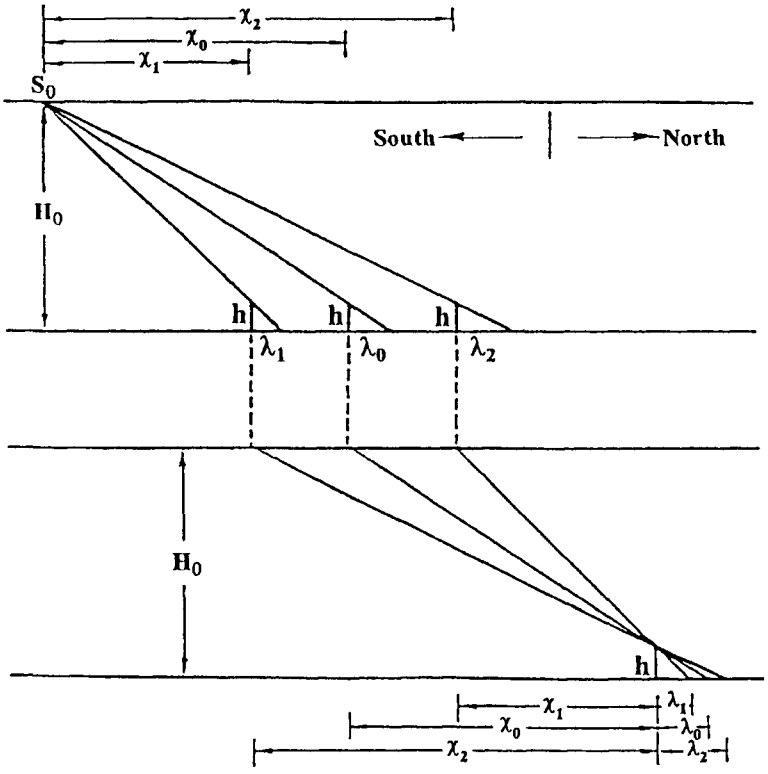

Figure 35. The correspondence principle in Chén Zǐ's model in which the plane of the sun's orbit is taken to be parallel to the plane of the assumed flat surface of the earth.

In Chén Zǐ's formula, the relation between the change in shadow lengths ($\lambda_2 - \lambda_1$) and the change in positions ($\varkappa_2 - \varkappa_1$) of the sun is expressed in terms of the reference set of measurements ($\varkappa_0$, λ_0). Returning now to Chén Zǐ's passage quoted above, we have $\varkappa_0 = 16{,}000$ *lǐ*, and $\lambda_0 = 16$ *cùn*. This gives Chén Zǐ's formula in its explicit form used for numerical calculations reported in the *Zhōu-Bì Suàn-Jīng*:

$$\varkappa_2 - \varkappa_1 \cong 1000\,(\lambda_2 - \lambda_1)\,\frac{\text{lǐ}}{\text{cùn}} \tag{15}$$

The formula states that each 1 *cùn* change in shadow length corresponds 1,000 *lǐ* change in the sun's position, a result which was adopted late by Gài-Tiān school in their model calculations.

The views of historians of science on the origin of this result given in Equation (15) can be categorized into two groups. One group considers it to be a result of direct measurements[153] and the other considers it to be a postulation of Chén Zǐ.[154] Here I would like to emphasize that this result was neither that of a direct postulation nor that based purely on direct measurements. It was a derived result based not only on the measurements but also on mathematical analyses within the validity of the earth-sun model visualized by Chén Zǐ. The specific numerical result given by Equation (15) depends critically on the values $\varkappa_0$ = 16,000 *lǐ* and λ_0 = 16 *cùn*. The crucial question lies in the origins of these values. In the passage quoted above, Chén Zǐ stated explicitly that they were measured values.

There is no question that 16 *cùn* was the sun shadow length of an 8-*chǐ* Zhōu gnomon measured at summer solstice.[155] However, the description for the determination of the value 16,000 *lǐ*, given in the passage, is rather vague. The passage states simply that: 'one of the locations (i.e. the summer solstice) was determined [anciently] by the shadow length of a vertical rod under the sun and is a constant of celestial nature (日中立竿測影, 此一者, 天道之數).' At the appropriate location, it is possible to determined the point at which the sun shadow length of a vertical pole is approximately zero at noon summer solstice. But in ancient times, it would be very difficult to measure the horizontal distance between this point and the observational point of Chén Zǐ, since it was a considerable distance separated by mountains and rivers. The passage provides no information as to how the distance from the location of zero shadow length at noon summer solstice to the observational site was determined. This seems to imply that 16,000 *lǐ* was an adopted number rather than a value determined by Chén Zǐ himself. Chén Zǐ adopted the number, perhaps on its authority, and made no attempt to assess its accuracy.

The lack of understanding of the origin of number 16,000 *lǐ* and the nature of the relation ('each 1 *cùn* change in shadow length corresponds 1,000 *lǐ* change in sun's position') has been a source of difficulties that hampered the work from being objectively evaluated. It is worthwhile to note that the value of $\varkappa_0$ in Chén Zǐ's model may be viewed as a scaling

153. The accuracy of this relation has long been questioned. Lǐ Chún-Fēng 李淳風 of the Táng dynasty, for example, had conducted several measurements and demonstrated the inaccuracy of this result.
154. See for example, Yoshio Mikami 三上義夫 (1911); Qián Bǎo-Cóng (2) 錢寶琮 (1958); H. Chatley (1938); and Shigeru Nakayama 中山茂 (1969), pp. 24–35.
155. For a recent assessment of the accuracy of this measurement, see Chén Chēng-Yīh 程貞一 and Xí Zé-Zōng 席澤宗 (1991), pp. 369–371.

parameter; it affects the numerical dimensions of the model but not the underlying theory that constitutes the model. This can be easily seen from the following alternative form of Chén Zǐ's formula:

$$\varkappa_2 - \varkappa_1 = \left(\frac{\lambda_2 - \lambda_1}{\lambda_0}\right) \varkappa_0 \tag{16}$$

Obviously from Equation (16), all the distances between different positions of the sun are proportional to $\varkappa_0$ in Chén Zǐ's model and the proportional constant can be determined from the measurement of the appropriate shadow lengths. This implies, of course, that the entire analysis made by Chén Zǐ can be carried out in terms of $\varkappa_0$ without altering any of his theory.

3.3.2 Analysis of Sun's Apparent Motion

With the understanding of the corresponding principle, Chén Zǐ was able to theoretically determine, from shadow-length measurements, the radius of the sun's apparent daily circles around the north pole and to analyse their seasonal shifts based on his model.

The Daily Circles of the Sun

To examine the seasonal shifts in the sun's daily circles, Chén Zǐ analysed the predicted change in the radius of the circles based on the shadow-length measurements. Figure 36 illustrates the relative vertical positions of the sun at the summer and winter solstices (S, W) and the vernal and autumnal equinoxes (V) with respect to the north celestial pole (N) as projected along the north-south line directions with respect to the observation site (Y). Although the site of observation used by Chén Zǐ is not explicitly specified in the text, it was probably located either at Yáng Chēng 陽城, the ancient Zhōu observatory first established by Zhōu Gōng 周公, or at Luò Yáng 洛陽, the Eastern Zhōu capital also established by Zhōu Gōng. Based on the observation that Chén Zǐ probably lived in the −5th century or earlier but much later than the −11th century, the observational site was probably at Luò Yáng.[156]

In order to determine the radius of the sun's apparent daily circle at the summer solstice, i.e. the distance NS in Figure 36, it is necessary to determine the horizontal distance of the observation site from the north

156. Based on Chén Zǐ's account, the shadow length of an 8-*chǐ* gnomon measured at the observational site should be 16 *cùn* at summer solstice. Accounts of the shadow-length measurement of an 8-*chǐ* gnomon at Yáng Chéng by the Zhōu official astronomers gave a length of 15 *cùn* at summer solstice.

pole, NY. To determine this distance, Chén Zǐ made further measurement of the *gōu* 勾 of an 8-*chǐ* gnomon for the north pole. We have the statement from the text of the dialogue:

> 今立表高八尺, 以望極, 其勾一丈三寸.
> Now let us set an 8-*chǐ* gnomon and observe the north pole. the *gōu* 勾 measures 1 *zhàng* 丈 3 *cùn* 寸.

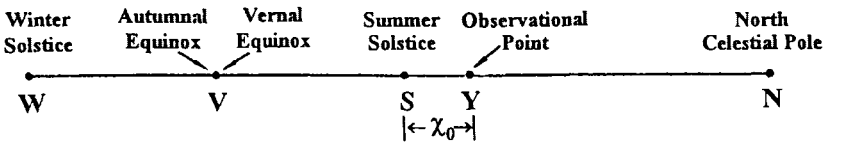

Figure 36. An illustration of the relative distance between the north celestial pole (N) and the sun at the summer and winter solstices (S, W) and at the vernal and autumnal equinoxes (V) along the north-south line directions projected on the earth with respect of the observation point (Y).

This implies that for the north pole, $\lambda_N = 103$ *cùn*. From Chén Zǐ's formula, Equation (16), we have:

$$NY = \varkappa_N - \varkappa_Y = \left(\frac{103 - 0}{16}\right) \varkappa_0 = 6.4375\, \varkappa_0 \tag{17}$$

Since the distance $YS = \varkappa_0$, it then follows that the apparent radius r_S of the sun's circle at the summer solstice is:

$$r_S = NS = NY + YS = 6.4375\, \varkappa_0 + \varkappa_0 = 7.4375\, \varkappa_0 \tag{18}$$

Similarly, from the measurement $\lambda_W = 135$ *cùn*, we have for the apparent radius r_W of the sun's circle at the winter solstice:

$$YW = \varkappa_W - \varkappa_Y = \left(\frac{135 - 0}{16}\right) \varkappa_0 = 8.4375\, \varkappa_0$$

$$r_W = NW = NY + YW = 6.4375\, \varkappa_0 + 8.4375\, \varkappa_0 = 14.875\varkappa_0 \tag{19}$$

The apparent radius r_V of the sun's circle at the vernal and autumnal equinoxes can then be obtained from those of the summer and winter solstices:

$$r_V = NV = \frac{NS + NW}{2} = 11.15625\varkappa_0 \tag{20}$$

This is the predicted radius of the sun's apparent daily circle by Chén Zǐ's model expressed in term of $\varkappa_0$.

The model predicts seasonal shift in the radius of the sun's daily circles, even though the values of the shift are incorrect. The predicted shift accounts for the sun's proper annual motion and is independent of the actual value used for $\varkappa_0$. The substitution of $\varkappa_0$ = 16,000 *lǐ* in Equations (18), (19) and (20) yields for the radius of the sun's daily circle the numbers given by Chén Zǐ, namely 119,000 *lǐ* at the summer solstice, 178,500 *lǐ* at the vernal and autumnal equinoxes, and 238,000 *lǐ* at the winter solstice.

The Day-and-Night Phenomenon

In Chén Zǐ's model, the earth is assumed to be fixed in space with a flat surface and the sun is orbiting in a plane parallel to the earth's surface. It would be difficult to incorporate features in the model to account for the phenomenon of day and night. Chén Zǐ sought the explanation from the light emitted by the sun. He visualized that the sun emits light in such a way similar to a spotlight; hence the phenomenon of day and night is a consequence of being inside or outside of the sunlight spread as the sun revolves around the north pole.

Chén Zǐ asserted that the diameter of the maximum area that the sun can cover in a complete revolution about the north pole is 810,000 *lǐ*.[157] Since, in Chén Zǐ's model, the sun's circle is largest at winter solstice, r_W = 238,000 *lǐ*, the radius r_1 of the sunlight spread on the surface of the earth is given by the following calculation:

$$r_1 = \left(\frac{810000}{2} - 238000\right) \text{ lǐ} = 167000 \text{ lǐ} \tag{21}$$

This is the spread radius of the sunlight given by Chén Zǐ.[158]In terms of the radius r_1, Chén Zǐ was able to account for certain features of the day and night phenomenon. The day begins, for example, at the observational site as soon as the sunlight spread reaches Y from the east and the night begins as soon as the sunlight spread leaves Y towards west as the sun revolves around the north pole. Since, in Chén Zǐ's model, the radius of the sun's daily circle changes from the smallest at the summer solstice to the largest at the winter solstice, the length of the day changes correspondingly from the longest at summer solstice to the shortest at winter solstice.

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157. For a discussion on the origin of this value see p.134 to p.136.
158. See the statement 日照四旁各十六萬七千里.

In addition to providing an explanation for the seasonal change in the length of day and night, Chén Zǐ's model also predicts the summer light and the winter darkness at the north pole. The relevant passage goes as follows:

> 春分之日夜分以至秋分之日夜分, 極下常有日光.
> 秋分之日夜分以至春分之日夜分, 極下常無日光.
> The polar region under the north celestial pole is always illuminated during the period from the vernal equinox to the autumn equinox, and always dark during (the other half of the year) from the autumn equinox to the vernal equinox.

Illustrative calculations of the sunlight spread in the north polar region are reported for a number of the sun's positions. The results for the summer and winter solstices are reproduced below:

> 夏至之日中與夜半, 日光九萬六千里過極相接.
> At the summer solstice between midday and midnight, the illuminated areas in the north polar region overlaps with a width of 96,000 *lǐ*.

> 冬至之日中與夜半, 日光不相及十四萬二千里, 不至極下七萬一千里.
> At the winter solstice between midday and midnight, the illuminated areas (in the north polar region) have a gap of 142,000 *lǐ*, i.e. 71,000 *lǐ* short from the north pole.

The areas of illumination and darkness in the north polar region as depicted in the above two passages are illustrated in Figure 37. In Chén Zǐ's model, the sun in its daily circle is at the circle's extreme north at midnight and extreme south at midday, with respect to the observational site.

One of the questions about the calculation is how did Chén Zǐ come up with the diameter of 81 *wàn-lǐ* 萬里 (810,000 *lǐ*) for the maximum area that the sun covers in a complete revolution about the north pole, since no clear explanation is found in the text. A passage that alludes to the possible source of the diameter runs as follows:

> 冬至、夏至觀律之數、聽鐘之音,
> 冬至晝、夏至夜差數及日光所還觀之,
> 四極徑八十一萬里.
> By examining the pitch numbers and listening to the bell tones for the winter and summer solstices, and by observing the differences in the sun's positions and areas of illumination between the day of the winter solstice and the night of the summer solstice, one obtains the 81 *wàn-lǐ* 萬里 (810,000 *lǐ*) for the diameter of the four extreme points.

Based on this passage, it appears that the number 81 could have been borrowed from acoustics since, as discussed from p.48 to p.49 above, the number was used in the *Guǎn Zǐ* for generating musical scales by

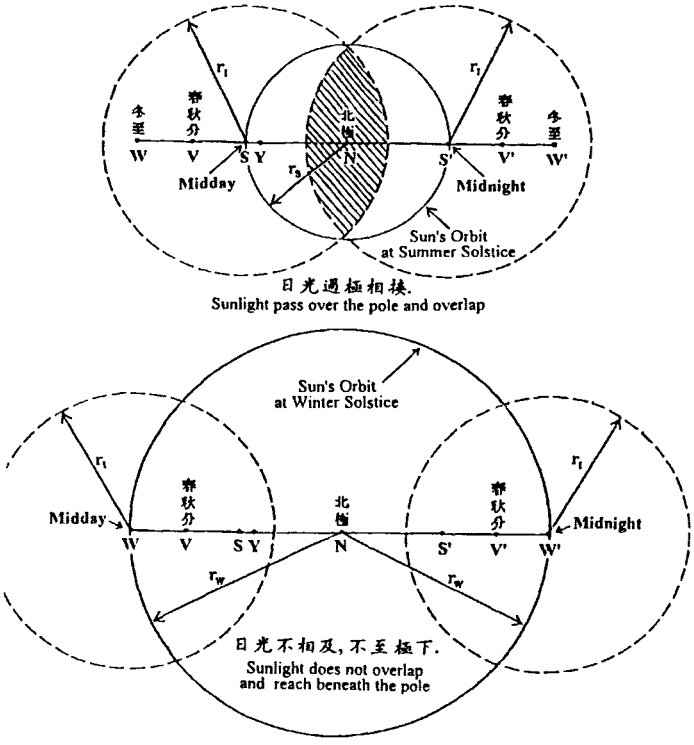

Figure 37. The distribution of sunlight spread from the extreme north and south points of the sun's circle at the summer and winter solstices according to Chén Zǐ's model.

assigning it to the *gōng* 宮 note in the Huáng-zhōng 黃鐘 pitch. However, the number was adopted in acoustics for the purpose of avoiding non-integers and it has no other significance because the scales so generated are independent of the number adopted.

In discussing the principles of methodology with Róng Fāng, Chén Zǐ made the following remarks:

> 同術相學, 同事相觀.
> One learns from similar and related methods,
> and observes similar and related phenomena.

Just as the distant range of a sound in which one can hear is determined by the propagation of the sound, Chén Zǐ assumed:

> 人望所見遠近, 宜如日光所照.
> The distant range in which one can see is defined by the illuminative range of the sunlight.

In analysing sunlight, Chén Zǐ apparently considered that the phenomena of sound and light are similar and made use of the knowledge from acoustics. Although the principle of 'observing similar and related phenomena and learning from similar and related methods' is a valid approach, the adoption of the number 81 from acoustics for the diameter of the maximum area covered by the sun was not based on the proper usage of the principle.

3.3.3 Determination of Sun's Diameter and Distance

Chén Zǐ also attempted to determine the sun's diameter and its distance. The passage that describes the measurement of the ratio between the sun's diameter and its distance is reproduced below:

> 日益表南, 晷日益長, 候勾六尺.
> 即取竹空, 徑一寸, 長八尺, 捕影而視之.
> 空正掩日, 而日應空之孔.
> 由此觀之, 率八十寸而得徑一寸.
> The further south is the sun to the gnomon, the longer the sun shadow would be. Let us wait until the shadow reached a length of 6 *chǐ* and then take an 8-*chǐ* sighting tube (*zhú-kōng* 竹空)[159] with a diameter of 1 *cùn* to look for the image of the sun. When the opening of the tube fits exactly to the visual rim of the sun, the measurement gives a ratio of 80 *cùn* in length to 1 *cùn* in diameter.

Apparently, the instrument used by Chén Zǐ for the measurement was a zhú-kōng 竹空 (sighting tube). An illustration of the measurement is given in Figure 38. In Chén Zǐ's model, the measured 80 to 1 ratio relates to the sun's diameter d_S with its distance R_S. Thus, we have

$$\frac{d_S}{R_S} = \frac{d}{t} = \frac{1}{80} \tag{22}$$

Once the vertical separation H_0 between the plane of the sun's apparent motion and the plane of the surface of the earth is determined in Chén Zǐ's model as illustrated in Figure 39, the distance between an observer

159. The name *zhú-kōng* is given explicity in the text of Chén Zǐ's work (see the quoted text). There are suggestions that the punctuation of the text should be such that the character '*kōng*' 空 is associated with the character '*jìng*' 徑 to give *kōng jìng* 空徑 as the following:
即取竹, 空徑一寸, 長八尺, 捕影而視之.
空正掩日, 而日應空之孔.
This choice of punctuation is not consistent with the usage of the character '*kōng*' 空 as it appeared in the last sentence quoted above, especially the phrase '*kōng zhī kǒng*' 空之孔. Consequently, I adopt the punctuation given in the text. The name *zhú-kōng* 竹空 is probably the earliest extant name of a sighting tube.

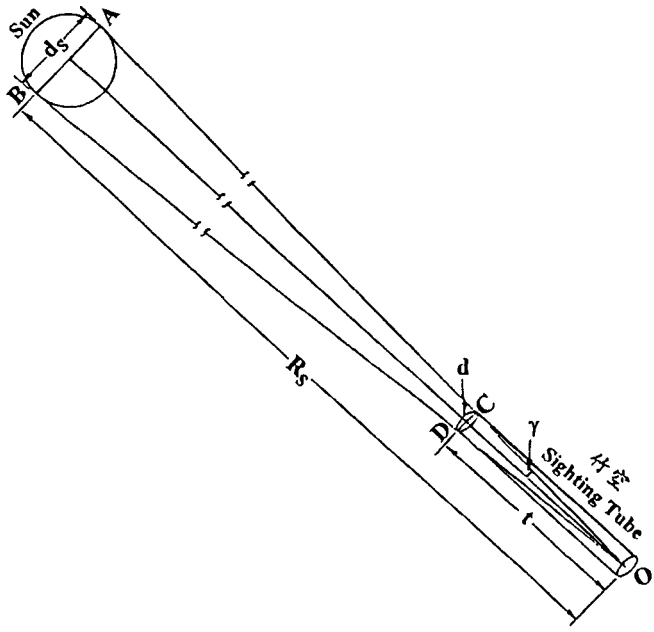

Figure 38. An illustration of the sighting-tube measurement of the sun's diameter as described by Chén Zǐ in the *Zhōu-Bì Suàn-Jīng*.

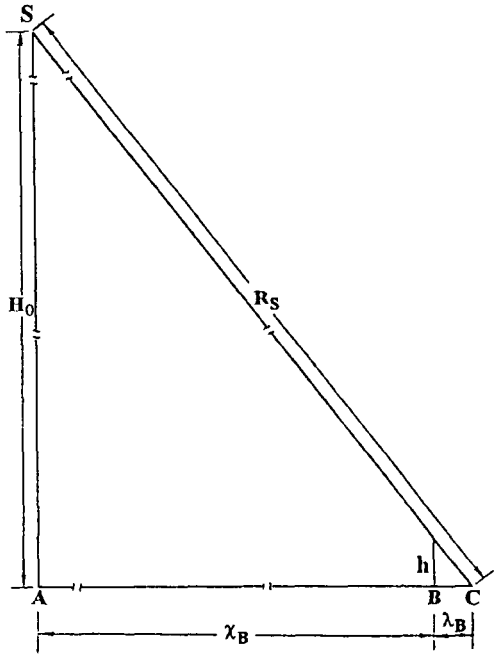

Figure 39. The vertical 'height' H_0 of sun and the assumed plane of the earth surface in Chén Zǐ's model.

and the sun any where along the line of north-south directions can easily be determined by the use of Chén Zǐ's formula and the *gōu-gǔ* 勾股 theorem. This is, however, not at all straightforward since the ancient formula for the height (See Eq. (38) on p.146):

$$H_0 = \left(\frac{\varkappa_B}{\lambda_B}\right) h + h$$

requires the value for the horizontal AB distance, $\varkappa_B$ (Figure 39).

Chén Zǐ made use of his formula to determine the required distance. He considered the case in which the sun was located far in the south such that the shadow length of 8-*chǐ* gnomon was 60 *cùn*. Utilizing the measurements at the summer and winter solstices, the horizontal AB distance, $\varkappa_B$, of the sun can be determined:

$$\varkappa_B = \left(\frac{\varkappa_W - \varkappa_S}{\lambda_W - \lambda_S}\right) \lambda_B = \left(\frac{8.4375\varkappa_0 - \varkappa_0}{135 - 16}\right) 60 = 3.75\ \varkappa_0. \tag{23}$$

The substitution of $\varkappa_B$ given by Equation (23) in the ancient height formula yields:

$$H_0 = \left(\frac{3.75\ \varkappa_0}{60}\right) 80 + 80 \cong 5\ \varkappa_0. \tag{24}$$

With the *gōu* 勾 given by the value of $\varkappa_B$ and the *gǔ* 股 given by the value of H_0, Chén Zǐ determined the distance to the sun by the use of the *gōu-gǔ* theorem:

$$R_S = SC = \sqrt{(\varkappa_B + \lambda_B)^2 + H_0^2} \cong 6.25\ \varkappa_0 \tag{25}$$

The diameter of the sun, d_S, is then obtained by substituting R_S from Equation (25) into Equation (22):

$$d_S = \frac{R_S}{80} = 0.078125\ \varkappa_0. \tag{26}$$

The substitution of $\varkappa_0$ = 16,000 *lǐ* yields d_S = 1250 *lǐ*, the numbers given by Chén Zǐ.

The flat surface assumption has a very limited range of validity. For the large dimension of distances considered by Chén Zǐ in dealing with the sun's measurements, it breaks down completely. Consequently, the values for the sun's diameter and its distance are incorrect. Although these derived values are grossly in error, the actual 'sighting-tube measurement obtained by Chén Zǐ was rather creditable. This can be seen by converting the measured 80 to 1 ratio into the angular diameter, θ, of the sun. From Figure 38, it is seen that the angular diameter $\theta = 2\gamma$ and

$$\gamma = \tan^{-1}\left(\frac{d/2}{t}\right) = \tan^{-1}\left(\frac{1/2}{80}\right) = \tan^{-1}(0.00625) = 21'31''$$

Hence,

$$\theta = 2\gamma = 2\ (21'31'') = 43'02''. \qquad (27)$$

In comparison with the actual value, 31'51", for the sun's average angular diameter, the value derived from Chén Zǐ's sighting-tube measurement is only 34% larger, an accuracy which was not surpassed until the work of Archimedes in the –3rd century.[160]

3.3.4 Chén Zǐ's Formula and the *Chóng-Chā* 重差 Method

Although Chén Zǐ's model has a limited range of validity, the analysis that led him to the combined use of mathematical formulas and measurements are correct within the physical model visualized by him. At the time of Chén Zǐ, such a combined approach was an important innovation. In addition, Chén Zǐ's formula was also responsible for the development of the *chóng-chā* 重差 method (see p.146 to p.149), a procedure that played an important role in the art of surveying in ancient China found in the work of Liú Huī 劉徽 and Zhào Shuǎng 趙爽 (Zhào Jūn-Qīng 趙君卿) of the third century. In this section, the mathematical contents of Chén Zǐ's work, in particular the derivation of Chén Zǐ's formula and its relation to the *chóng-chā* method, are examined in connection with the mathematical work of Shāng Gāo 商高 which, as mentioned earlier, is also incorporated in the *Zhōu-Bì Suàn-Jīng*.

It is perhaps worthwhile to note again that the current mathematical notation adopted here was not available to Shāng Gāo and Chén Zǐ and there is the danger of reading modern concepts into the ancient mathematical relations by expressing them in symbols and equations. Thus, in dealing with mathematical relations, such as Chén Zǐ's formula, within the frame-work of current mathematical notation, one must be careful to neither alter the original mathematical reasoning nor deviate from the original steps of derivation. This is possible because mathematical relations such as those used by Chén Zǐ, though expressed in statements, can equivalently be represented in symbols and equations. An algebraic equation, for example, can often adequately be represented by an open statement that specifies the pattern relation and permits the substitution of variables, regardless whether the pattern and variables are expressed in terms of words or symbols.

160. A. E. Shapiro (1975).

Consider, for example, the mathematical statement '勾股各自乘, 並而開方除之. Multiply the width and the length each by itself, and then take the square root of their summation' used by Chén Zǐ to determine the *jìng* 徑 (hypotenuse)[161] in the *gōu-gǔ* 勾股 (length-width) theorem. This is an open mathematical statement in which the variables are represented by technical terms applicable to general cases. If one replaces the mathematical operations in the statement such as *zì-chéng* 自乘 (square), *bìng* 並 (add), and *kāi-fāng-chú-zhī* 開方除之 (taking square root) by current symbols, the statement for the *jìng* is transformed into an equation:

$$jìng = \sqrt{gōu^2 + gǔ^2} \ (徑 = \sqrt{勾^2 + 股^2}) \tag{28}$$

In this form, not only the mathematical content expressed in Chén Zǐ's statement is preserved and readily understood, but the technical terms: *gōu* 勾 (width), *gǔ* 股 (length), and *jìng* 徑 (hypotenuse) are also readily identified with the appropriate variables in the corresponding theorem known in the West as the Pythagorean theorem.

In the present work, current mathematical notation is used only to facilitate the presentation since it is universally adopted and easily understandable. For the underlying logic in the derivation of the mathematical relations, I follow the original steps of reasoning preserved in text as closely as possible.

The Jī-Jǔ 積矩 *Method of Shāng Gāo* 商高

An important method devised by the early Chinese mathematicians for the derivation and proof of mathematical relations and theorems is the *jī-jǔ* 積矩 method described in the work of Shāng Gāo.[162] Shāng Gāo's work survived through its incorporation in the *Zhōu-Bì Suàn-Jīng* in the form of a dialogue with Zhōu Gōng 周公.

From the introductory dialogue, it is apparent that Shāng Gāo's work is included in the book to provide the mathematical basics for the astronomical work incorporated in the latter part of the book. The passage runs as follows:

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161. It should be noted that the technical term *jìng* for hypotenuse used in the work of Shāng Gāo and Chén Zǐ is an old term, and it was later replaced by the term *xián* 弦 in the later Zhōu and Qín period.

162. Very little is known about the personal life of Shāng Gāo except that he lived before Chén Zǐ. According to Zhào Shuǎng 趙爽, the third century commentator of the *Zhōu-Bì Suàn-Jīng*, Shāng Gāo lived in the Zhōu and was well known for his accomplishments in mathematics. The entry in the *Shāng Zhōu Zhì* 《商州志》 indicates that Shāng Gāo was bestowed, in the early Zhōu, with the title Zǐ Nán 子男 of Shāng 商.

> 昔者周公問於商高曰:
> 竊聞乎大夫善數也.
> 請問古者包犧立周曆度,
> 夫天不可階而升, 地不可將尺寸而度,
> 請問數從安出?
> Of old, Zhōu Gōng addressed Shāng Gāo, saying, 'I have heard that the Dà-Fū 大夫 Grand Master is versed in mathematics. May I venture to inquire how Bāo Xī 包犧 anciently established the degrees of the celestial circles? There are no steps by which one may ascend the sky, and the earth is not measurable with a foot-ruler. I should like to ask the origin of the methods.'

Shāng Gāo then replied:

> 數之法出於圓方.圓出於方, 方出於矩,
> 矩出於九九八十一.
> The mathematical method is derived from [the properties of] circles and squares. A circle is derived from a square and the square from a right-angled ruler (*jǔ* 矩). A right-angled ruler is derived from the [arithmetic principle of] $9 \times 9 = 81$.

In these few short sentences, Shāng Gāo was able to convey the important view that mathematical knowledge is interrelated and the method is derived from the combined use of mathematical knowledge.

Following this short explanation, Shāng Gāo introduced the *jī-jǔ* method through an explicit example in which the *gōu-gǔ* theorem is derived. The passage is reproduced below:

> 故折矩, 以為勾廣三, 股修四, 徑隅五.
> 即方其外, 半之一矩環而共盤.
> 得成三四五, 兩矩共長二十有五, 是謂'積矩'.
> Thus, let us cut a rectangle [diagonally]. [If we] suppose that the width (*gōu* 勾) is 3 [units] wide and the length (*gǔ* 股) is 4 [units] long, then the hypotenuse (*jìng* 徑) would be 5 [units] long. Now, after drawing a square on the outside [of the hypotenuse], circumscribe it by half-rectangles so as to form a (square) plate. Then the 3-4-5 relation [for the width, length, and hypotenuse] can be established, since the total difference (*gòng-cháng* 共長) between [the square plate] and the two rectangles is an area of 25 [units^2]. This method of proof is called *jī-jǔ* 積矩 (literally 'piling up the rectangles').

An illustration of the derivation of the *gōu-gǔ* theorem by the *jī-jǔ* method of Shāng Gāo is given in Figure 40 in three stages. Figure 40(a) illustrates the construction of the right-angled triangles (from rectangles) to be used later to form the square plate given in Figure 40(c). Figure 40(b) illustrates the construction of a square on the hypotenuse of such a right-angled triangle. Figure 40(c) illustrates the formation of a square plate by circumscribing the hypotenuse square of Figure 40(b) with the right-angled triangles constructed in Figure 40(a).

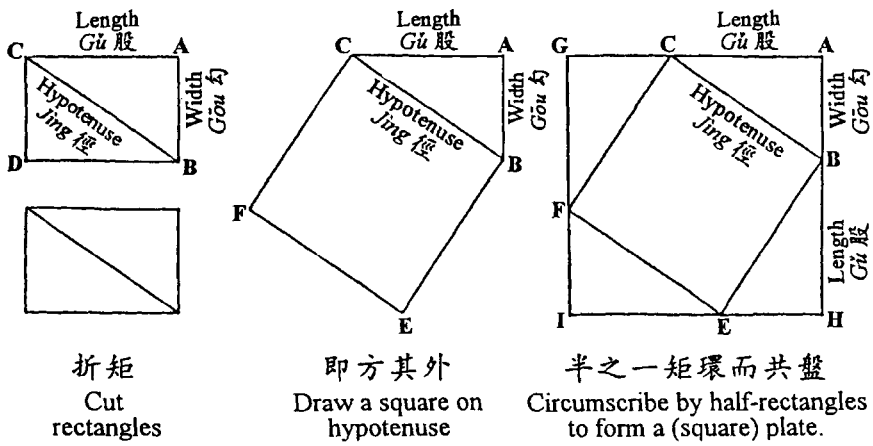

Figure 40. An illustration of the derivation of the *gōu-gǔ* 勾股 theorem by the *jī-jǔ* 積矩 method of Shāng Gāo 商高.

It is seen that the total area, $(gōu + gǔ)^2$, of the square AHIG is composed by construction of the area, $jìng^2$, of the centre square BEFC and the areas, $2(gōu \times gǔ)$, of the four corner right-angled triangles coming from two identical rectangles ABDC. Hence, the area of *jìng* square is given by the difference of the total area and the areas of the two rectangles:

$$jìng^2 = (gōu + gǔ)^2 - 2(gōu \times gǔ).$$

This yields the *gōu-gǔ* theorem:

$$jìng^2 = gōu^2 + gǔ^2 \ (徑^2 = 勾^2 + 股^2). \tag{29}$$

Hence, Shāng Gāo's description given above is a derivation of a general mathematical relation known later as the *gōu-gǔ* theorem.[163] The relation is given in technical terms, *gōu* 勾, *gǔ* 股 and *jìng* 徑, as well as in a specific example with numbers 3, 4 and 5. Though the derivation is not structured in an axiom deductive form, it is logically based and made use of both geometric and algebraic reasonings. The proof is appealing to one's intuition and has certain pedagogical merits.

The term *jī-jǔ* 積矩 literally means 'gathering or piling up rectangles'. As its name suggests, the *jī-jǔ* method makes use of the area of a *jǔ* 矩 as the basis for establishing relations. By analysing the relation between the

163. In the passage, Shāng Gāo refers this mathematical relation as the 3-4-5 relation. It was a common practice among ancient workers to name a general relationship by a well-known specific case. A well-known example in mathematics is to name the multiplication table as the *Jiǔ-Jiǔ biǎo* 九九表 (9 × 9 table).

total area and areas of its various parts, mathematical relations can be established, thereby formula and theorem are derived. The inner logic in the *jī-jǔ* method of proof is the axiom that the whole is the sum of its parts. This method of proof was gradually generalized from two-dimensional problems based on areas to three-dimensional problems based on volumes, known later as the *chū-rù xiāng-bǔ* 出入相補 method. Such a method of proof corresponds to the dissection proof found later in the Western work.[164]

Derivation of Chén Zǐ's Formula

Based on the *jī-jǔ* method of Shāng Gāo, there are two alternative ways that Chén Zǐ's formula might have been derived. By adopting the *gōu-gǔ* proportionality derived by the *jī-jǔ* method, Chén Zǐ could have easily derived the formula. Chén Zǐ's formula could also be derived by the direct use of the *jī-jǔ* method without using the *gōu-gǔ* proportionality.

(1) Derivation via the gōu-gǔ 勾股 proportionality

The *gōu-gǔ* proportionality refers to the proportional relations found among similar right-angled triangles. In discussing Chén Zǐ's formula on p.126 to p.128 above, such proportional relations are used [see Equation (13)] without proof. Since Chén Zǐ's formula could well be derived by the use of the proportional relations, the derivation is examined within the mathematical realm of Shāng Gāo and Chén Zǐ.

In Shāng Gāo's work, the *gōu-gǔ* proportional relations were derived by the *jī-jǔ* method. An illustration of the derivation is given in Figure 41. To establish the desired relationships in areas, the *jī-jǔ* method calls for the construction of the rectangle ADCB from the right-angled triangle ACB. By construction, each pair of the right-angled triangles at the opposite sides of the line AC are equal. Hence the area, $H_0\lambda$, of rectangle HDCF is equal to the area, $h(\varkappa + \lambda)$, of the rectangle EGCB. The equality of these two areas, $H_0\lambda = h(\varkappa + \lambda)$, gives the relation:

$$\frac{H_0}{h} = \frac{(\varkappa + \lambda)}{\lambda}. \tag{30}$$

Similarly, the equality of the areas HDGO and EOFB, $(H_0 - h)\lambda = h\varkappa$, gives the relation:

$$\frac{(H_0 - h)}{\varkappa} = \frac{h}{\lambda}. \tag{31}$$

164. See Chén Chēng-Yīh 程貞一 (1987) Appendix II, pp. 35–44.

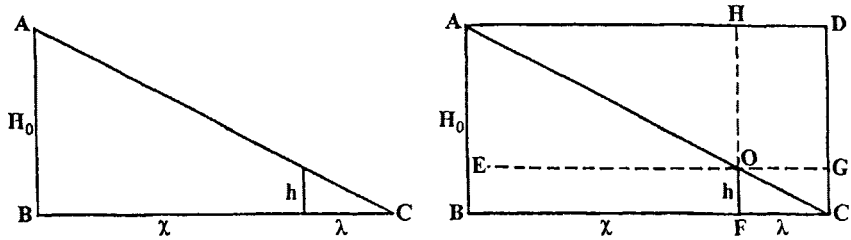

Figure 41. An illustration of the derivation of the *gōu-gǔ* 勾股 proportionality by the *jī-jǔ* 積矩 method of Shāng Gāo 商高.

The availability of the *gōu-gǔ* proportionality permitted the early use of right-angled ruler to determine heights, depths etc., as described by Shāng Gāo in the passage reproduced below:

> 平矩以正繩, 偃矩以望高,
> 覆矩以測深, 臥矩以知遠,
> A leveled right-angled ruler serves to align cords.
> A recumbent right-angled ruler serves to survey heights.
> A reversed right-angled ruler serves to measure depths.
> A flat right-angled ruler serves to determine distances.

In the work of Chén Zǐ, the *gōu-gǔ* proportionality is used on a number of occasions.

Considering the case that Chén Zǐ's formula was derived by the use of the *gōu-gǔ* proportionality, one would then have the proportional relations given by Equation (13) on p.127, obtained by repeatedly applying the *gōu-gǔ* proportionality of Equation (30) to the right-angled triangles in Figure 34. This implies that the south measurements ($\varkappa_1$, λ_1) and the north measurements ($\varkappa_2$, λ_2) are both related to the measurements ($\varkappa_0$, λ_0) at the observational site as follows:

$$\frac{\varkappa_1 + \lambda_1}{\lambda_1} = \frac{\varkappa_0 + \lambda_0}{\lambda_0}, \qquad \frac{\varkappa_2 + \lambda_2}{\lambda_2} = \frac{\varkappa_0 + \lambda_0}{\lambda_0}.$$

On reduction, these relations become $\varkappa_1 \lambda_0 = \varkappa_0 \lambda_1$ and $\varkappa_2 \lambda_0 = \varkappa_0 \lambda_2$; and their difference yields Chén Zǐ's formula [Equation (14)]. There are indications, however, that the formula was probably derived by the *jī-jǔ* method directly.

(2) Derivation by the Jī-Jǔ 積矩 method

To derive Chén Zǐ's formula by the *jī-jǔ* method directly, one must first gather the right-angled triangles in Figure 34 to form rectangles as shown in Figure 42. It is seen that rectangle STBO is constructed from right-

angled triangle SBO; rectangle SVDO from right-angled triangle SDO; and rectangle SYFO from right-angled triangle SFO.

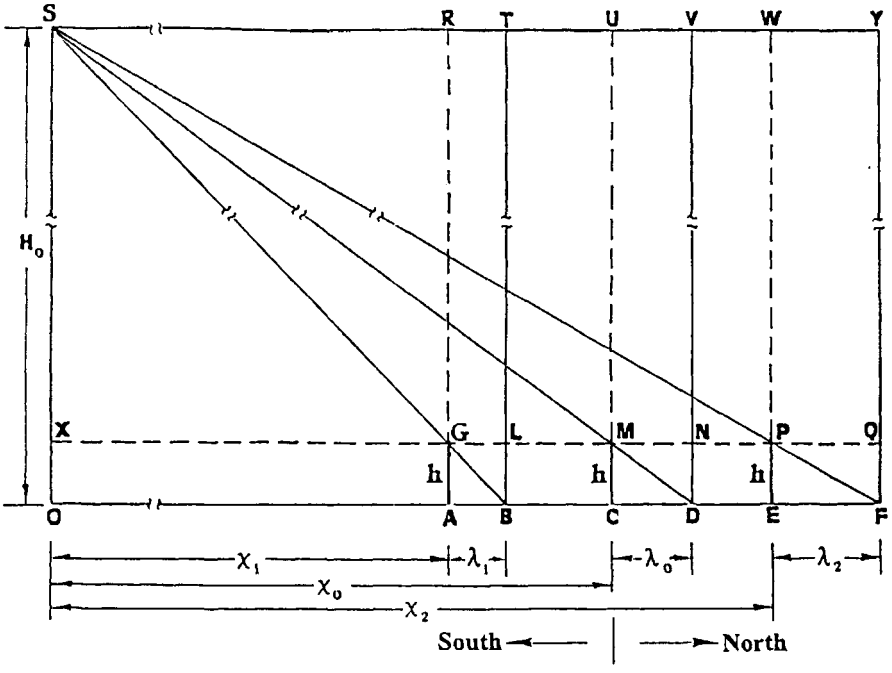

Figure 42. An illustration of the proof of Chén Zǐ's formula by the *jī-jǔ* method.

By construction, the rectangle SYFO is evenly divided into two parts by line SF, thus establishing the area relations ΔSPX = ΔSPW and ΔPFE = ΔPFQ for the right-angled triangles. It then follows that the areas of the two remaining rectangles at the opposite sides of line SF must be equal. Since the area of rectangle WYQP is $(H_0 - h)\ \lambda_2$ and the area of rectangle XPEO is $h\varkappa_2$, one obtains:

$$(H_0 - h)\ \lambda_2 = h\varkappa_2 \tag{34}$$

Similarly from rectangle STBO, one finds that the area, $(H_0 - h)\ \lambda_1$, of rectangle RTLG and the area, $h\varkappa_1$, of rectangle XGAO are equal:

$$(H_0 - h)\lambda_1 = h\varkappa_1 \tag{35}$$

From rectangle SVDO, one finds that the area, $(H_0 - h)\ \lambda_0$, of rectangle UVNM is equal to the area, $h\varkappa_0$, of XMCO:

$$(H_0 - h)\ \lambda_0 = h\ \varkappa_0 \tag{36}$$

One obtains, from the difference between Equation (34) and Equation (35):

$$(H_0 - h)\ (\lambda_2 - \lambda_1) = h\ (\varkappa_2 - \varkappa_1) \tag{37}$$

By combining Equation (36) with Equation (37), one obtains Chén Zǐ's formula

$$(\varkappa_2 - \varkappa_1) = \frac{\varkappa_0}{\lambda_0}\ (\lambda_2 - \lambda_1) \tag{14'}$$

This is primarily a geometric derivation of Chén Zǐ's formula based on the relations of areas established by the *jī-jǔ* method.

From the present perspective, it appears that the derivation based on the *gōu-gǔ* proportionality is simpler. However, Chén Zǐ's formula probably was derived by the direct application of the *jī-jǔ* method. This view is also supported by the work of the +3rd commentator, Zhào Shuǎng 趙爽, who provided, by the *jī-jǔ* method, a proof of the *chóng-chā* formula used by Chén Zǐ in connection with his attempt to determine the 'height' of the sun (see p.136 to p.139).

The Chóng-Chā 重差 Method

The *chóng-chā* method is a procedure that played an important role in the development of the art of surveying in ancient China. The method is closely related to Chén Zǐ's formula and was probably first used by Chén Zǐ in his attempt to determine the 'height' of the sun. Returning for the moment to Figure 39, the height H_0, according to the proportional relation of Equation (30), is given

$$H_0 = \left(\frac{\varkappa + \lambda}{\lambda}\right) h = \frac{\varkappa}{\lambda}\, h + h. \tag{38}$$

This is an ancient formula for measuring heights known to Shāng Gāo. However, for cases where the horizontal distance $\varkappa$ is not available, this expression for H_0 has no practical value for the determination of heights. It is for such cases that Chén Zǐ's formula provides an important solution to the problem known later as the *chóng-chā* 重差 method.

The classic Chinese work that provided a mathematical foundation for the determination of height, distance and depth in surveying is the *Hǎi-Dǎo Suàn-Jīng* 《海島算經》 (*Sea Island Mathematics Classic*) of 263.[165] In explaining the *chóng-chā* method, Liú Huī stated in the preface of his commentary:

165. The work was originally a part of Liú Huī's commentary of the *Jiǔ-Zhāng Suàn-Shù* 《九章算術》 (*Nine Chapters on Mathematics*) intended to be an extension of the *gōu-gǔ* chapter. Not until the early part of the Táng dynasty was it taken out of the commentary and printed as an independent book by Liú Huī 劉徽.

> 凡望極高測絕深而兼知其遠者, 必用重差.
> 勾股則必以重差為率, 故曰重差也.
> Whenever in determining the extreme height and depth [of an object] and do not know its actual horizontal distance [from the observation point], it is necessary to make use of the *chóng-chā* method. [In this method,] the ratio in the *gōu-gǔ* proportionality must be replaced by the *chóng-chā* ratio. This is why it is called the *chóng-chā* method.

The term *chóng-chā* 重差 means literally 'double differences'. Here it refers to the differences in two sets of measurements $(\lambda_1, \varkappa_1)$ and $(\lambda_2, \varkappa_2)$, namely $\varkappa_2 - \varkappa_1$ and $\lambda_2 - \lambda_1$. Therefore, Chén Zǐ's formula is a mathematical statement on the relations for the *chóng-chā*. From Equation (14), one obtains:

$$\frac{\varkappa}{\lambda} = \frac{\varkappa_2 - \varkappa_1}{\lambda_2 - \lambda_1} \tag{39}$$

It states that the ratio $\varkappa / \lambda$ from a single set of measurements is equal to the *chóng-chā* ratio, $(\varkappa_2 - \varkappa_1) / (\lambda_2 - \lambda_1)$, obtained from two sets of measurements.

Substitution of the *chóng-chā* ratio, $(\varkappa_2 - \varkappa_1) / (\lambda_2 - \lambda_1)$ given by Equation (39) for the $\varkappa/\lambda$ ratio in Equation (38) yields the *chóng-chā* height formula:

$$H_0 = \left(\frac{\varkappa_2 - \varkappa_1}{\lambda_2 - \lambda_1}\right) h + h \tag{40}$$

This implies that the height H_0 can be determined from measuring the differences in distances, $\varkappa_2 - \varkappa_1$, and in widths, $\lambda_2 - \lambda_1$, without the need to directly determine the distance $\varkappa$. The derivation of the *chóng-chā* height formula is consistent with Liú Huī's statement that in the *chóng-chā* method, the ratio in the *gōu-gǔ* proportionality must be replaced by the *chóng-chā* ratio. The relation with Chén Zǐ's formula revealed by the derivation indicate that the basic ideal of the *chóng-chā* method appeared first in Chén Zǐ's work on the measurement of the sun.[166]

The fact that the *chóng-chā* method was used by Chén Zǐ was certainly recognized by the +3rd commentator Zhào Shuǎng in the *Zhōu-Bì Suàn-Jīng*, for he provided a proof of the *chóng-chā* height formula using the *jī-jǔ* method. The relevant passage appeared in the 'Rì-Gāo-Tú Zhù' 〈日高圖註〉 ('A Commentary on the Diagram of Sun's Height') is reproduced below:[167]

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166. In his discussion of the *chóng-chā* method, Liú Huī also referred to the measurement of the sun as the early source of the method even though he made no explicit mention of Chén Zǐ or the *Zhōu-Bì Suàn-Jīng*.
167. The diagram of sun's height found in the present copy of the *Zhōu-Bì Suàn-Jīng* is not Zhào Shuǎng's illustration since it is not in agreement with his description given in the passage quoted here (see Fig. 35).

黃甲與黃乙其實正等.
以表高乘兩表相去為黃甲之實.
以影差為黃乙之廣, 而一所得,
則變得黃乙之袤.
上與日齊, 按圖當加表高.
今言八萬里者, 從表以上復加之.
青丙與青己其實亦等. 黃甲於青丙相連,
黃乙與青己相連, 其實亦等, 皆以影差為廣.

This passage is translated below in symbols and equations and is arranged in the sequence of Zhào Shuǎng's derivation.

The derivation is based on the *jī-jǔ* method. Following the description of Zhào Shuǎng, the right-angled triangles for the south measurements ($\varkappa_1$, λ_1) and the north measurements ($\varkappa_2$, λ_2) in Figure 34 are reconstructed into rectangles with the areas being labelled with the designated colour as shown in Figure 43. The Zhào Shuǎng proof can be examined in consultation with Figure 43 by following the derivation translated below:

Zhào Shuǎng's Derivation in Words	Zhào Shuǎng's Derivation in Symbols and Equations
黃甲與黃乙其實正等	yellow B area = yellow A area (i.e. □YQKZ = □PEAG).
以表高 h 乘兩表相去 ($\varkappa_2 - \varkappa_1$) 為黃甲之實.	$\because$ yellow A area = $h\ (\varkappa_2 - \varkappa_1)$
以影差 ($\lambda_2 - \lambda_1$) 為黃乙之廣, 而一所得, 則變得黃乙之袤.	WP = (黃乙之袤) yellow B area = $(\lambda_2 - \lambda_1)$ WP $= h\ (\varkappa_2 - \varkappa_1)$ $\therefore$ WP = 黃乙之袤 = $(\frac{\varkappa_2 - \varkappa_1}{\lambda_2 - \lambda_1})\ h$
上與日齊按圖當加表高.	H_0 = (黃乙之袤) + h $= (\frac{\varkappa_2 - \varkappa_1}{\lambda_2 - \lambda_1})\ h + h$
今言八萬里者, 從表上復加之	The value of 80,000 *lǐ* for H_0 is obtained after the gnomon height h is added.

It is seen that the derivation of the *chóng-chā* height formula given in the first portion of Zhào Shuǎng's passage is based on the proposition that the yellow B area (YQKZ) is equal to the yellow A area (PEAG). The proof of this proposition is given in the second portion of the passage and can be examined by following the derivation translated below:

Zhào Shuǎng's Derivation in Words	Zhào Shuǎng's Derivation in Symbols and Equations
青丙與青己其實亦相等.	∵ blue C area = blue F area (i.e. □GAOX = □TLGR).
黃甲與青丙相連, 黃乙與青己相連, 共實亦等.	yellow A area + blue C area = yellow B area + blue F area ∴ yellow B area = yellow A area (i.e. □YQKZ = □PEAG).
皆以影差 $(\lambda_2 - \lambda_1)$ 為廣.	yellow B area = $(H_0 - h)$ $(\lambda_2 - \lambda_1)$ yellow A area = h $(\lambda_2 - \lambda_1)$

In this proof, use is made of area relations relevant to the *gōu-gǔ* proportionality (see Fig. 41). By examining the rectangle TBOS with the diagonal line SB in Figure 43, one can readily verify that the rectangular area GAOX (the blue C area) equals the rectangular area TLGR (the blue F area) and similarly by examining the rectangle YFOS with the diagonal line SF, one can verify that the rectangular area PEOX (i.e. yellow A area + blue C area) equals the rectangular area YQPW (i.e. yellow B area + blue F area). This derivation of Zhào Shuǎng, given in his commentary of the *Zhōu-Bì Suàn-Jīng*, confers that the *chóng-chā* method was used in Chén Zǐ's work.

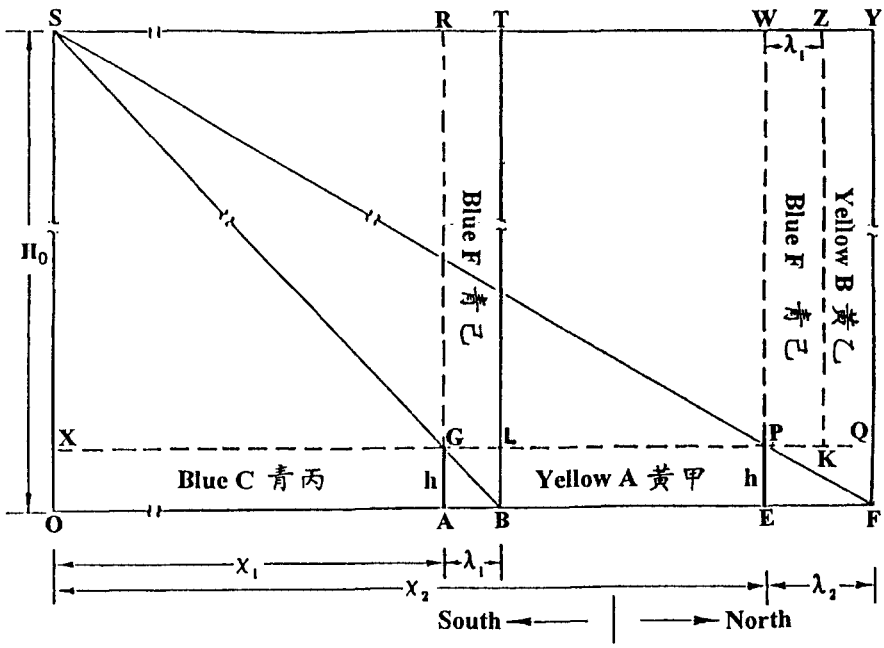

Figure 43. An illustration of Zhào Shuǎng's 趙爽 proof of the *chóng-chā* 重差 height formula by the *jǐ-jǔ* 積矩 method.

3.4 The Major Ancient Cosmological Ideas

In Chinese civilization, three major astronomical schools were identified by early astronomers. The best known reference is probably that of Cài Yōng 蔡邕 of the Eastern Hàn. The relevant passage is found in a memorial that he presented to the emperor in about 180:

> 言天體者有三家：一曰周髀，二曰宣夜，三曰渾天.
> 宣夜之學絕，無師法. 周髀術數具存，考驗天狀，
> 多所違失，故史官不用. 唯渾天者，近得其情，今史官所用候臺銅儀，則其法矣.
> Those who discuss the celestial system form three schools. The first is known as the Zhōu-Bì 周髀 (Zhou-Gnomon) school, the second is the Xuān-Yè 宣夜 (Infinite-Universe) school, and the third is the Hún-Tiān 渾天 (Celestial-Sphere) school. The teaching of the Xuān-Yè school has been interrupted and there is now no master in it. As for the Zhōu-Bì theory, though its methods and computations still remain, it proved incorrect and lacking in many ways when tested in explaining celestial structure. Therefore, the official astronomers do not use it. Only the Hún-Tiān theory approximates to the principles. The *tóng-yí* 銅儀 (bronze armillary) at the observatory now used by the official astronomers is constructed based on the Hún-Tiān method.

Cosmological and astronomical speculation probably began long before the establishment of any of the schools. In addition to the three schools mentioned by Cài Yōng, there was also the Gài-Tiān 蓋天 (Celestial Cover) school, but its relationship with the Zhōu-Bì school has caused much confusion in the literature. In this section, a brief account of these early schools is given.

3.4.1 The Zhōu-Bì 周髀 School

The Zhōu-Bì school was probably the first to reach the status of an astronomical school in Chinese civilization. The major astronomical tools of the school were the *bì* 髀 (gnomon) and *zhú-kōng* 竹空 (sighting tube). The representative work of the Zhōu-Bì school is that of the Chén Zǐ discussed from p.125 to p.139. As mentioned earlier, Chén Zǐ's work came down to us as an integral part of the *Zhōu-Bì Suàn-Jīng*, a book which was probably compiled by scholars of the Gài-Tiān school associated with Lǚ Bù-Wéi 呂不韋 of the −3rd century. Because of this association, Chén Zǐ's cosmological view has often been considered to be that of the Gài-Tiān in which the sky is visualized as a half dome covering the flat earth in the form of a square. This interpretation is incorrect not only because no mention of a half-dome sky and a square earth is found in the entire text of Chén Zǐ's work, but also because Chén Zǐ's work belongs to the Zhōu-Bì astronomical school.

Chén Zǐ himself gave credit of his work to the Zhōu-Bì school. The explicit dialogue runs as follows:

> 榮方曰:「周髀者何？」
> 陳子曰:「古時天子治周, 此數望之從周,
> 故曰周髀, 髀者表也.」
> Róng Fāng asked: 'What is Zhōu-Bì?' Chén Zǐ replied: 'It was that used by ancient high king to relate [the seasons] in Zhōu. The mathematical art originated from the Zhōu, and is, therefore, called *Zhōu-Bì*. *Bì* 髀 means gnomon.'

There is no doubt that the work of Chén Zǐ was a part of the grand Zhōu tradition.[168]

In fact, Chén Zǐ's work represents the best of the *Zhōu-Bì* astronomical school. The work contains sophisticated mathematical reasoning and credible measurements, but is based on an incorrect sun-earth model. The most critical of the model is the incorrect flat-earth assumption. Although in the Zhōu-Bì school the earth is assumed to be flat, its shape is not otherwise specified. It is, therefore, instructive to examine the source that later led to the square earth misinterpretation. The original source is probably the ambiguous term '*tiān yuán dì fāng*' 天圓地方 found in the last passage on mathematics presented in Shāng Gāo's dialogue:

> 環矩以為圓, 合矩以為方.
> 方屬地, 圓屬天, 天圓地方.
> 方數為典, 以方出圓.
> The revolution of a *jǔ* 矩 (right-angled ruler, functioning as a compass) produces circles. The union of *jǔ* produces squares (and rectangles). The [mathematical property of] squares pertains to earth, and that of circles pertains to sky, thus the term '*tiān yuán dì fāng*' (literally 'sky round and earth square'). Taking the mathematics of a square as the *diǎn* 典 (guide or rule), a circle can be derived from a square.[169]

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168. Based on Chén Zǐ's explanation of the term *Zhōu-Bì*, the character *Zhōu* 周 in the title of the book *Zhōu-Bì Suàn-Jīng* 《周髀算經》 refers to the Zhōu dynasty and the term *Zhōu-Bì* 周髀 should not be translated as 'the gnomon and the circular paths of heaven', see Needham, (1959), vol. 3, p. 19 and p. 210.

169. This 方數為典, 以方出圓 statement together with the earlier statement, 圓出於方, are probably the earliest statements on a method known later as the *gē-yuán shù* 割圓術 (circle-dissection method, or the method of exhaustion in the West). In the *Zhōu-Bì Suàn-Jīng*, this method is called *fāng yuán zhī fǎ* 方圓之法 (vol. 1, p. 55a). If one lets the square inscribed in a circle to approximate the circle, one would obtain a value of 2.828 for π, the ratio of the circumference to the diameter of the circle. However, if the sides of the inscribed square are doubled to form an eight-sided polygon, one would then obtain for π a value of 3.061. In the *Zhōu-Bì Suàn-Jīng*, the value for π is taken to be 3. For a recent account of the method, see Chén Chēng-Yīh 程貞一 (1987), pp. 19–24.

In interpretating the term '*tiān yuán dì fāng*', one should keep in mind that the presentation of the mathematics in Shāng Gāo's dialogue is cast in the form of responding to the questions how could the degrees of the celestial circles be established and what is the *dào* 道 in the use of the *jǔ* 矩 (right-angled ruler) since there are no steps by which one may ascend the sky, and the earth is not measurable with a foot-ruler? Thus, terms *yuán* and *fāng*, as they are used here, refer to their mathematical properties pertaining to the sky and the earth.

However, in later account of the Zhōu-Bì school, such as those in the *Jìn Shū* 《晉書》 (*History of the Jīn Dynasty*) of 635, the term is paraphrased as follows:

> 天圓如張蓋, 地方如棋局.
> The sky is round as an extended cover and the earth is square as a chess board.

The problem with such interpretations lies in the square earth, because the shape of the earth, other than being flat, is not specified in the Zhōu-Bì school.[170]

An early discussion indicating that the terms *yuán* and *fāng* in the context of '*tiān yuán dì fāng*' are not used to describe the geometrical forms of the sky and earth is found in the work of Zēng Cān 曾參 (−505 to −440), who was probably a comtemporary of Chén Zǐ.[171] After expressing his views in the *Zēng Zǐ* 《曾子》 (*The Book of Master Zēng*),[172] Zēng Cān argues:

> 如誠天圓而地方, 則四角之不揜也.
> If the sky were really spherical and the earth really square [in the geometrical sense], then the four corners of the earth would not be properly covered (*yǎn* 揜).

170. It should be noted that the idea of a square earth may very well have appeared in ancient speculations about the earth, but it is not part of the Zhōu-Bì school.
171. The period of Zēng Cān constitutes a lower limit to the uncertainty in the date of Chén Zǐ, who probably lived in a period earlier than that of Zēng Cān.
172. The *Zēng Zǐ* is a book put together by the followers of Zēng Cān on some of his original teachings. According to the 'Yì-Wén Zhì' 〈藝文志〉 section of the *Qián-Hàn Shū* 《前漢書》 (*History of the Former Hàn Dynasty*), the *Zēng Zǐ* has eighteen chapters, but subsequent mention of the book found in later works all stated that it has only ten chapters. During the Western Hàn, parts of the *Zēng Zǐ* were incorporated into the volumes on records of rites compiled by Zài the Elder, known as the *Dà-Dài Lǐ-Jì* 《大戴禮記》 (*Dà-Dài Record of Rites*). The book, as we have it now, has only ten chapters and was recovered from the *Dà-Dài Lǐ-Jì*. A commentary to the *Zēng Zǐ* was written in 1798 by Ruǎn Yuán 阮元.

Following this discussion of the sky-earth configuration, Zēng Cān further pointed out that terms *yuán* and *fāng* also had their technical meanings in the *yīn-yáng* 陰陽 interaction.[173]

In Chén Zǐ's model, the maximun area on the earth that could be covered by the sunlight is, in fact, a circle with a fixed radius. The centre of this circle is located directly below the north celestial pole. Beyond this area, the earth is not specified and nowhere in the model is the earth taken to be square. The fact that Chén Zǐ's model calculation is for the sun's motion based on shadow-length measurements and does not involve a specific shape of the sky provides a false impression that the work is adaptable to other sky-earth models. Examples of misapplication of the Chén Zǐ's formula are found in the Gài-Tiān and Hún-Tiān schools.

3.4.2 The Gài-Tiān 蓋天 School

The Gài-Tiān school was closely related to the Zhōu-Bì school and was probably associated with one of the most archaic of the ancient cosmological views. The earliest available Gài-Tiān model is that discussed by Shāng Gāo in the *Zhōu-Bì Suàn-Jīng*. The relevant passage runs as follows:

> 笠以寫天, 天青黑, 地黃赤, 天數之為笠也.
> 青黑為表, 丹黃為裏, 以象天地之位.
> Let a *lì* 笠 (conical sun-hat) describe the sky. With the blue and black representing the sky and the yellow and red the earth, the *lì* is constructed in accordance to the celstial numbers. To symbolize the sky-earth positions, the outer surface is made in blue and black and the inside red and yellow.[174]

In this model, the sky is visualized to be in the form of a *lì* 笠 and the earth is assumed flat (not explicitly stated).

Shāng Gāo's model described here was probably a model of the early

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173. The relevant passage runs as follows:
參嘗聞之夫子曰:「天道曰圓, 地道曰方, 方曰幽, 圓曰明.」
Cān 參 has heard in the past our master saying that 'The *dào* of sky is [of the quality of] the *sphere* (*yuán* 圓) and the *dào* of the earth is [of the quality of] the *square* (*fāng* 方). The *square* is said to be dim (*yōu* 幽) while the *sphere* bright (*míng* 明).'
The interaction of things classified as the *yōu* 幽 and *míng* 明 is discussed from p. 196 to p.197.

174. It is of interest to note that the color scheme of red and yellow is preserved in the names of the equator and the ecliptic. The equator is called *chì dào* 赤道 which means literally the 'red road'. The ecliptic is called *huáng dào* 黃道 which means literally the 'yellow road'.

phase of the Gài-Tiān Shuō 蓋天說 (celestial cover theory) since, by the time of Lǔ Bù-Wéi, sky and earth were visualized as two concentric domes. In this model, the sunlight is also assumed to have a fixed spread radius on the earth (see p.132 to p.136). A description of the moonlight from the school is given below:

> 日兆月月光乃出, 故成明月.
> The sun shines on the moon and generates the moonlight;
> thus the moon becomes bright.[175]

This is a clear statement on the identification of moonlight as the reflected light of the sun.

In the course of time, the term Gài-Tiān gradually became synonymous with the term Zhōu-Bì. By the fifth centry, Zǔ Gèng-Zǐ 祖暅子 replaced the Zhōu-Bì school by Gài-Tiān school in his discussion of the three major ancient cosmological ideas.[176] In the 'Tiān Wén Zhì' 〈天文志〉 section of the *Jìn Shū*, the Gài-Tiān Shuō is described as follows:

> 蓋天之說, 即周髀是也.
> 其本庖犧氏立周天曆度,
> 其所傳則周公受于殷商,
> 周人志之, 故曰周髀.
> The theory of Gài-Tiān is that of Zhōu-Bì. It originated from Bāo Xī's work in establishing the degrees of the celestial circles.[177] The propogation of this knowledge was that inherited by Zhōu Gōng from the people of Yīn Shāng. The Zhōu people studied and documented the theory and is, therefore, called Zhōu-Bì.

This account of the Gài-Tiān Shuō contains a number of inaccurate facts. Even though the Gài-Tiān view and the simple *lì* 笠 sky model mentioned by Shāng Gāo were very ancient and known to the Zhōu-Bì school , the major work of the Gài-Tiān model with sky and earth visualized as two concentric domes was done in the −3rd century near the end of the Zhōu dynasty, much later than the major work of the Zhōu-Bì school.

It is certainly true that use was made of Chén Zǐ's formula in the Gài-Tiān model calculations, but the adoption of this formula was not based on a correct understanding of Chén Zǐ's work. Chén Zǐ's formula was derived for two parallel planes assumed in Chén Zǐ's model but not valid for two concentric domes assumed in the Gài-Tiān model. In

175. See the *Zhōu-Bì Suàn-Jīng*, vol. 2, pp. 3a and3b.
176. See the *Tiān-Wèn Lù* 《天問錄》 (*Résumé of Astronomy*).
177. Traditionally, it is maintained that the idea of *zhōu-tiān lì-dù* 周天曆度 was established by Bāo Xī 包犧 (a cultural hero of the Neolithic period) or, at least, handed down from the period of Bāo Xī. There is no concrete evidence to support this view. It should, however, be noted that the translation of the term *zhōu-tiān* 周天 as celestial sphere cannot be correct since the concept of a celestial sphere was a later development.

addition, the upper plane in Chén Zǐ's model is visualized to be the plane of the sun's apparent motion and has nothing to do with the upper dome representing the sky in the Gài-Tiān model. In Chén Zǐ's model, the sky is not assumed to be in any specific form except that it allows the sun to travel freely. Thus the Gài-Tiān model is fundamentally different from Chén Zǐ's model. At most, the Gài-Tiān school, as represented by the last portion of the *Zhōu-Bì Suàn-Jīng*, could be considered a derivative of the Zhōu-Bì school.

3.4.3 The Hún-Tiān 渾天 School

The Hún-Tiān astronomical school also has its root in the ancient work on *zhōu-tiān lì-dù* 周天曆度 (establishing the degrees of the celestial circles) and was related to the Zhōu-Bì school. In the Hún-Tiān school, the sky is represented by a celestial sphere and its two extreme points are identified as the celestial north and south poles. The equatorial circle on the celestial sphere is marked with the 28 *xiù* 宿 (see p.118 to p.125). The equatorial circumference is divided into $365\frac{1}{4}^{\circ}$ with half of it above the earth and the other half below, so that only half of the 28 *xiù* are visible at one time. It was from such a usage of *xiù* star-groups that eventually led to the use of the *rù-xiù-dù* 入宿度 (in-*xiù* distance β, see Fig. 33a) relative to the nearest *xiù*-determinatives together with the *qù-jí-dù* 去極度 (north polar distance p, see Fig. 33a) to list the stars by Shí Shēn 石申 and Gān Dé 甘德 in the –4th century (see p.119 to p.125).

The earliest extant description of the celestial sphere is given by Shèn Dào 慎到 of the –4th century. The relevant statement, found in the *Shèn Zǐ*《慎子》(*The Book of Master Shen*), is reproduced below:

> 天體如彈丸, 其勢斜倚.
> The form of the sky is like a spherical bullet with an inclined leaning configuration [in rotation].

As a model representing the sky, the celestial sphere visualized by the Hún-Tiān school is a significant improvement over the half-dome of the Gài-Tiān school. This innovation led to the invention of a new astronomical instrument, *Hún-yí* 渾儀 (armillary sphere), which later became identified with the school.

During the Warring States period, as the Hún-Tiān school began to exercise greater influence, there were also a number of debates speculating on the sphericity of the earth.[178] But no concrete record of

178. See the *mìng-tí* 命題 of Huì Shī 惠施 recorded in the *Zhuāng Zǐ*《莊子》such as:
南方無窮而有窮.

the school's view of the earth is available prior to the first century. The best account is given by the well-known astronomer Zhāng Héng 張衡 (+78–139). From his 'Hún-Yí Zhù' 〈渾儀注〉 ('Commentary on the Armillary Sphere'), there is the following passage:

> 渾天如雞子, 天體圓如彈丸,
> 地如雞子中黃, 孤居于內.
> 天大而地小. 天表裏有水,
> 天之包地猶殼之裹黃.
> 天地各乘氣而立, 載水而浮.
> The celestial sphere is like a hen's egg in which the sky is round like a spherical bullet and the earth is like the yolk of the egg lying alone in the centre. The sky is large and the earth is small. The layer of sky contains water. The sky is wrapping around the earth just as the shell layer wraps around the yolk. The sky is supported by the *qì* 氣 and the earth floats on the water.

The sky-earth model of the Hún-Tiān school described here is indeed a remarkable improvement over those of the Zhōu-Bì and Gài-Tiān schools. This passage provides a clear description that the earth is spherical like the yolk of an egg. However, the interpretation still contains an apparent weakness in picturing the earth floating on the water.[179]

3.4.4 The Xuān-Yè 宣夜 School

The Xuān-Yè school has one of its roots in the notion that the world extends beyond the sky-earth system. Even in the Gài-Tiān school, the existence of something beyond the sky-earth system was discussed. The relevant passage from the *Zhōu-Bì Suàn-Jīng* runs as follows:

> 過此而往者, 未之或知,
> 或知者, 或疑其可知, 或疑其難知.
> What is extending beyond (the sky-earth system) is the unknown. One has doubts whether the unknown is comprehensible or too difficult to know.[180]

The south has at the same time a limit and no limit.
我知天下之中央, 燕之北、越之南是也.
I know the centre of the earth (*tiān-xià* 天下); it is north of the Yàn 燕 State and south of the Yuè 越 State.
which may be interpreted to mean that earth is spherical.

179. In the early Greek view, earth is heavier than waters so that waters surrounded the earth. According to Pierre Gassendi, a seventeenth-century French philosopher-astronomer, the earth in the world system of Ptolemy was visualized to be at the centre since it is the heaviest of the four 'elements' and surrounding it, in turn, were the waters, the air and a sphere of fire. (The sun, moon, five planets and stars were in still higher spheres.) See *The Cosmographical Glass: Renaissance Diagrams of the Universe* (The Huntington Library, 1977).
180. See the *Zhōu-Bì Suàn-Jīng*, vol. 1, p. 68b.

Apparently the Gài-Tiān school implicitly assumed the world extended beyond the sky-earth system.

By the Warring States period, the topic attracted intensive debate among philosophical schools from the point of view of time and space. As discussed from p.1 to p.3, space (*yǔ* 宇) is visualized, in the Mòhist school, to be that 'which extends to different positions' and time (*jiǔ* 久, later *zhòu* 宙) is visualized to be that 'which extends to different moments'. It was through the Mòhist conceptualization of the space and time that led to the brilliant synthesis of the concept of the space-time continuum and to the coining of the term *yǔ-zhòu* 宇宙 to denote the abstract concept of the universe. This conceptualization eventually led to the cosmological association of the concept of *yǔ-zhòu* with the the sky-earth system and the vastness beyond.

Cài Yōng 蔡邕 of the Eastern Hàn noted around 180 that 'the teaching of the Xuān-Yè school has been interrupted and there is now no master in it.' This statement probably did not refer to the specific teaching of the concept of an infinite universe, since this concept was accepted among the Eastern Hàn astronomers including the astronomers of the prevalent Hún-Tiān school. In fact, the well-known astronomer Zhāng Héng 張衡 of the Hún-Tiān school explicitly incorporated the idea of infinite universe in his Hún-Tiān model. In his *Líng Xiàn* 《靈憲》 (*Spiritual Constitution of the Universe*) of c. 118, Zhāng Héng identified the cosmos extending beyond the sky-earth system with the term *yǔ-zhòu*. The relevant passage is reproduced below:

> 過此而往者, 未之或知也.
> 未之或知者, 宇宙之謂也.
> 宇之表無極, 宙之端無窮.
> What is extending beyond (the sky-earth system) is the unknown. This extending through the unknown beyond is what is called the *yǔ-zhòu* 宇宙 (universe). There are no limits on the boundaries of space (*yǔ* 宇) and no bounds in the extremities of time (*zhòu* 宙).

Here the concept of an infinite universe is further affirmed by requiring no limits in space and no bounds in time.

A description of the Xuān-Yè teaching was recovered from the recollections of Qiè Yìn 郄蔭, a Hàn officer (*mì-shū láng* 秘書郎). The account is found in the 'Tiān Wén Zhì' 〈天文志〉 section of the *Jìn Shū* 《晉書》. The relevant passages run as follows:[181]

> 天了無質, 仰而瞻之, 高遠無極. 眼瞀精絕,
> 故蒼蒼然也. 譬之旁望遠道之黃山而皆青,
> 俯察千仞之深谷而窈黑, 夫青非真色, 而黑非有體也.

181. Similar passages are found in the 'Tiān Wén Zhì' 〈天文志〉 section of the *Suí Shū* 《隋書》, vol. 14, p. 5.

The sky is empty and void of substance (*wú-zhí* 無質). When one looks up at it, one can see that it is immensely high and far away, having no bounds (*wú-jí* 無極). It is the limitation of one's eyesight that causes the sky to appear as a vast blue. Just as if one looks horizontally at the yellow mountains at a great distance, and sees grey blue. Or as if one gazes down into a valley a thousand fathoms deep, one sees solid black. But the blue (one sees of the mountain) is not the true colour, nor does the black indicate substance.

日月眾星, 自然浮生虛空之中, 其行其止,
皆須氣焉. 是以七曜, 或逝或住, 或順或逆,
伏見無常, 進退不同, 由乎無所根繫, 故各異也.
The sun, the moon, and all the stars float naturally in empty space, and their motions all need *qì* 氣. Thus the seven luminaries (*qī-yào* 七曜)[182] sometimes appear and sometimes disappear, sometimes move forward and sometimes retrograde. Their presence does not have a common regularity and their motions are not the same. Since they are not interconnected, they each have a different behaviour.

故辰極常居其所, 而北斗不與眾星西沒也. 攝提填星皆東行, 日行一度, 月行十三度. 遲疾任情, 其無所繫著可知矣. 若綴附天體, 不得爾也.
[Among the celestial bodies] the pole star always keeps its place, and the *Běi-Dǒu* 北斗 (Great Bear) never sinks below the horizon in the west with the other stars. Those luminaries among the stars (i.e. the sun, the moon and planets) all move relatively eastwards (i.e. fall back), the sun making 1° a day and the moon 13°. Their speed depends on their individual natures, which show that they are not interconnected, for if they were fastened to the body of sky, this behaviour could not be manifested.

The views described in these passages are indeed very advanced for their time.

3.5 Comments and Evaluations

The ancient Chinese astronomers were often considered to be persistent and accurate observers of celestial phenomena but they had little, if any, theoretical and mathematical ability to analyse their observations. Maspero, for example, believed that astronomy in China depended upon the progress of the invention of astronomical instruments, and that this was because of a persistent backwardness in mathematics.[183] The early Chinese approach to astronomy has been widely characterized by

182. The term *qī-yào* 七曜 refers to the sun, moon, and the five planets: *Jīn* 金 (Venus), *Mù* 木 (Jupiter), *Shuǐ* 水 (Mercury), *Huǒ* 火 (Mars), and *Tǔ* 土 (Saturn).
183. H. Maspero (1950), 'L'Astronomie dans la Chine Ancienne; Histoire des Instruments et des Découvertes'. p. 15.

sinologists and historians of science as being non-theoretical and non-geometrical. In this section, I will examine the validity of such characterizations by presenting an analysis of Chinese views of the sky-earth system and the mathematical approach of Chén Zǐ discussed above.

Another issue of much debate is the origin of astronomy in China. A persistent view in the West is that astronomy in Chinese civilization has its roots in the Babylonian civilization. An investigation on the origins of astronomy in Chinese civilization is also carried out in this section.

3.5.1 An Analysis of the Chinese Cosmological Views

Among the astronomical schools discussed from p.150 to p.158, the Gài-Tiān 蓋天 school was the first one to be abandoned. By the time of the Eastern Han dynasty, the school essentially faded away.[184] Despite its wide coverage in Western literature,[185] the concentric two-dome model of the Gài-Tiān school had little influence on the Chinese view of the sky-earth system. This was due to the fact that before the school developed this model and reached its height through the work of astronomical groups associated with Lǚ Bù-Wéi 呂不韋 in the −3rd century, the Hún-Tiān 渾天 school was already well established through the work of astronomers such as Shí Shēn 石申 and Gān Dé 甘德 of the −4th century.[186] In fact, the curvature of the earth's surface introduced in the two-dome model was probably made in response to the views of the Hún-Tiān school. Other than a minor influence through the persistent use of the Chén Zǐ formula (p.165 to p.167), the Zhōu Bì 周髀 school also faded away in favour of the Hún-Tiān school.

In addition to the Hún-Tiān school, there was also the Xuān-Yè 宣夜 school (p.156 to p.158). The importance of this school has been an issue of debate. There is a consensus among sinologists that the Xuān-Yè school had minimal influence on the Chinese view of the sky-earth system. But this consensus was not supported by the available records. Needham was the first Western scholar to notice that the Xuān-Yè school pervaded Chinese thought to a greater extent than would appear at first sight. In commenting the Xuān-Yè school, Needham stated:

> These cosmological views are surely as enlightened as anything that ever came out of Hellas. The vision of infinite space, with celestial bodies at rare intervals floating in it, is far more advanced (and the

184. Except for a short and limited resurgence in the late fifth century by the astronomical group associated with Xiāo Yǎn 蕭衍 of Liáng 梁 during the Southern and Northern Dynasties.
185. Especially the reconstructed model by Herbert Chatley (1938).
186. See p.119 to p.125; p.155 to p.156.

> point is worth emphasizing) than the rigid Aristotelian-Ptolemaic conception of concentric crystalline spheres which fettered European thought for more than a thousand years. Though sinologists have tended to regard the Hsüan Yeh school as uninfluential, it pervaded Chinese thought to a greater extent than would at first sight appear.[187]

The comment that '[the Xuān-Yè school] pervaded Chinese thought to a greater extent than would at first sight appear' is indeed a very insightful observation.

Based on the recollection of the Hàn official, Qiè Yìn 郄蔭 (see p.157 to p.158), the major innovative views of the Xuān-Yè school are the following:

1. The sky is empty and void of substance (*wú-zhí* 無質) and is immensely high and far away, having no bounds (*wú-jí* 無極).
   天了無質, 高遠無極.
2. The sun, the moon, and all the stars float naturally in the empty space.
   日月眾星, 自然浮生虛空之中.
3. Their speed depends on their individual natures, which shows that they are not interconnected, for if they were fastened to the body of sky, this behaviour could not be manifested.
   遲疾任情, 其無所繫著可知矣. 若綴附天體, 不得爾也.

It can be shown that these views were mostly incorporated in the Hún-Tiān school. Shortly before the claim of Cài Yōng 蔡邕 in 180 that 'the teaching of the Xuān-Yè school has been interrupted and there is now no master in it', the concept of an infinite universe had already been incorporated in the Hún-Tiān school by Zhāng Héng 張衡 (see p.157). With this major view adopted, the Hún-Tiān school soon assimilated the Xuān-Yè school since the rest of the Xuān-Yè teachings were easily adapted into the Hún-Tiān teaching. Although the early astronomy in China had been dominated by the Zhōu-Bì 周髀 school, it gave way to the later schools during the Warring States. By the Qín-Hàn 秦漢 period, astronomy emerged with an observational tradition rooted in the Hún-Tiān 渾天 school and a theoretical tradition rooted in the Xuān-Yè 宣夜 school. The celestial sphere of the Hún-Tiān school has often been compared with that developed in the early Greek civilization. But the celestial sphere visualized by the ancient Chinese astronomers school was not subdivided into concentric crystalline spheres as in the Greek case. In the views of the early Greek astronomical schools, each of the concentric crystalline spheres represented a sky. Inside the sphere of stars

187. Joseph Needham (1959), vol. 3, p.220.

were the spheres associated with the sun, the moon and planets. It was the daily rotation of the crystalline spheres on a common axis powered by the Prime Mover that carried the sun, the moon, planets and stars from the east to the west.[188] To account the deviations from the uniform motion of a planet, an epicycle was introduced to allow the planet to move in the opposite sense on the epicycle as the centre of the epicycle moves along the *deferent* around the earth. Another method devised to account for the deviation from the uniform motion was to allow the planet to move around the earth in a circle whose centre was not the centre of the earth but a displaced point known as the *equant*.

In the views of the ancient Chinese astronomers school, there was only one sky represented by a celestial sphere and the stars rotate westward daily with the sky. The motions of celestial objects such as the sun, the moon and planets were visualized to be generated by *qì* 氣 on their own accord with their own characteristic paths (*guǐ-dào* 軌道 or simply *dào* 道) in space. These paths were assumed to be traceable by mathematical interpolations based on the measurements of the apparent orbital motions of the celestial objects around the earth. Hence in the Chinese sky-earth model, the motions of the sun, the moon and planets were represented by a network of their paths around the earth.[189]

It is apparent that the dynamics and mathematics of the Aristotelian-Ptolemaic celestial spheres and the Hún-Tiān celestial sphere were very different. In the Greek case, the apparent motions of the sun, the moon and planets were viewed as derived motions with their orbits arround the earth defined by the geometry of the crystalline spheres. Their paths were simulated by the introduction of the epicycles,

188. The primary source of ancient Greek work in astronomy is the *Almagest*, a treatise written by Claudius Ptolemy. Nothing is known about the personal life of Ptolemy except that he probably lived between 100 and 200. The book, as we have it now, originated from an Arabic translation in Baghdad of c. 827. The Arabic translation was probably based on the lost translation in Latin by Boethius. About 1230, the emperor Frederick II in Naples had it translated from Arabic to Latin. A summary appeared in print in 1496 in Venice, but the complete Latin translation was not published until 1515 by P. Liechtenstein.

189. By the –4th century, creditable measurements on planetary motion began to appear. One has, for example, from the *Kāi-Yuán Zhàn-Jīng* the following quotation from the work of Gān Dé's:

甘氏曰:「歲星凡十二歲而周, 皆三百七十日而夕入于西方, 三十日復晨出于東方.」

According to the Master Gan, the sidereal period of the *Suì-Xīng* 歲星 (i.e. *Mù-Xīng* 木星, Jupiter) is 12 years, it always disappears in the west in the evening after 370 days, and after 30 days reappears in the east in the morning.

The passage gives for the paths of revolution of the Jupiter, a sidereal period of 12 years (presently 11.862 years) and a synodic period of 400 days (presently, 398.884 days).

deferents and equants. In the Chinese case, the apparent motions of the sun, the moon and planets around the earth were treated as motions derived on its own accord. Their paths were the physical trajectories and assumed to be traceable by mathematical interpolations from observations. The majority of Chinese astronomers viewed the sky as being void of substance, allowing the sun, the moon and planets to move freely in it, and these celestial objects were not attached to the body of the sky. Such views persisted until the time of Matteo Ricci.

In enumerating the 'absurdities' of the Chinese, Ricci wrote in a letter in 1595 the following passage:[190]

> There is only one sky (and not ten skies). It is empty (and not solid). The stars move in the void (instead of being attached to the firmament).

In fact, the Míng-Qīng 明清 astronomers rejected the view introduced by the Jesuits that the skies are composed of solid substances. In commenting about the response of the Chinese astronomers to the European views, of the sky-earth system after the abolishment of crystalline spheres, Wylie states that 'Native scholars suppose that there is a close resemblance between the Xuān-Yè and the system introduced by Europeans.'[191]

The pictures of the sky-earth system and the universe visualized by the Chinese major astronomical schools were highly theoretical. However, the categorical characterization of the early Chinese approach to astronomy as being non-theoretical persists. Needham certainly has a better appreciation and understanding of the Chinese astronomical models, but his view, as expressed in the following passage, remains the same:

> Chinese astronomy is often reproached for its overwhelmingly observational bias, but the lack of theory was an inevitable result of the lack of deductive geometry. It might well be argued that the Greeks had had too much, since the apparent mathematical beauty of 'cycle on epicycle, orb on orb' came eventually to constitute a strait-jacket posing unnecessary difficulties for a Tycho, a Copernicus and a Galileo.[192]

This passage not only views that astronomy in ancient China lacked theory but also suggests that 'the lack of theory was an inevitable result of the lack of deductive geometry.' A pertinent question that needs to be addressed is what does the phrase 'lack of theory' mean? Does a physical

190. Cf. Joseph Needham (1959), vol. 3, p. 439. See Venturi (1911), vol. 2, pp. 175–85 and Bernard-Maître (1935), p. 48.
191. A.Wylie (1867), p. 86. Here it refers to the post-Jesuit Europeans.
192. Joseph Needham (1959), vol. 3, p. 220.

theory need to be expressed in a specific mathematical form to be considered a theory? Is it true that the physical theories of an astronomical model can only be expressed in terms of deductive geometry?

The lack of deductive geometry in Chinese astronomical modelling was further echoed, for example, in the following passage from Krupp:

> Like the Greeks, the Chinese also tried to understand the architecture of the universe and express the structure of cosmic order in cosmological theories that had to mimic the sky. Without deductive geometry, however, the Chinese never immersed themselves in the detailed modeling of planetary motion that led to the complexities of epicycles, deferents, and equants.[193]

The implication here is that, despite cosmological theories, without deductive geometry the Chinese were unable to immerse in detailed modelling of planetary motion. However, detailed modelling of planetary motion, can be carried out without the complexities of epicycles, deferents and equants.

In comparison with the Greek approach to astronomy, the Chinese approach was indeed less geometrically oriented but it was by no means non-geometrical or non-theoretical. The use of egg configuration to describe the sky-earth system discussed on p.156 is certainly geometrical. The treatment of the sun's motion in Chén Zǐ's work discussed from p.125 to p.149 is unquestionably geometrical. Though the geometry used in the Chinese work was not axiomatically deductive, it was certainly logically based. It provided proofs for theorems and derivations for formulas. Besides geometry, the Chinese also made simultaneous use of algebra. The point that needs to be emphasized is that the lack of Greek deductive geometry does not necessarily imply the lack of theory in astronomical models nor does it imply an inability to make a detailed modelling of planetary motion. The important difference between the early Chinese and Greek conceptions of the sky-earth system lies not in mathematics but in physics. The difference in mathematics reflects one's preference but the difference in physics reflects one's views of the sky and the motions of celestial objects. As mentioned earlier, the Chinese astronomers, unlike the Greek astronomers, viewed the universe to be infinite and the sky as being empty so that celestial objects could freely move in it. Thus, they viewed the apparent motions of the sun, the moon and planets as the motions derived on its own accord, not the motions derived by the rotation of the crystalline spheres. This physical difference is reflected in the models and mathematical treatments. In the Chinese

193. E.C. Krupp (1983), p. 333.

sky-earth model, the planetary motion was represented by a network of celestial paths around the earth, not by a network of concentric crystalline spheres as in the Greek sky-earth model. To examine the paths of the celestial objects, the Chinese astronomers preferred to trace their trajectories from observations by mathematical interpolations.

The concentric crystalline spheres of the Greek sky-earth model reflected well the Greek's geometrical view of the motions of the sun, the moon and planets. It permitted the Greek astronomers to invent epicycles, deferents and equants to analyse the deviations from the uniform rotation for each celestial object. On the other hand, the Chinese astronomers invented the *nèi-chā fǎ* 內插法 (method of interpolation) to analyse the accumulated planetary data. By this method, they were also able to trace the details of celestial motions and to account, for example, the change in the rate of motion and the eccentricity of the revolution. The early interpolation procedures involved the simple linear interpolations, known in mathematical work as the method of *yíng-bù zú* 盈不足.[194] It was used to interpolate short-distance ranges of the solar and lunar motions in the form of $F(x) = ax + b$ by the use of *tiǎo* 朓 (waning) and *nù* 朒 (waxing) information to determine the arbitrary constants a and b.

Interpolations based on higher order differences were gradually developed for the analysis of celestial motions. The development of the *zhāo-chā fǎ* 招差法 (method of finite differences) led to the use of the second-order interpolation in the form of $F(x) = ax^2 + bx + c$. It is in the construction of the *Huáng-Jí Lì* 皇極曆 of 604 by Liú Zhuó 劉焯 (544–610), one finds the use of the equal-interval second-differences interpolation method (*děng-jù èr-cì nèi-chā fǎ* 等距二次內插法) to compute, for example, the rate change in the motions of the sun, the moon and planets. The same quadratic interpolation was later used by Lǐ Chún-Fēng 李淳風 in the construction of the *Lín-Dé Lì* 麟德曆 of +664. The procedure was later modified by Zhāng Suì 張遂 (also known as Monk Yī Xíng 一行, +683–727) by allowing unequal intervals in interpolation. In constructing the *Dà-Yǎn Lì* 大衍曆 of +724, Zhāng Suì made use of the unequal-interval second-differences interpolation method (*bù-děng-jù èr-cì nèi-chā fǎ* 不等距二次內插法) for the calculation of the positions of the sun, the moon and planets. A better treatment of the sun's uneven motion is obtained by the use of the third-order interpolation first found in the work of Wáng Xún 王恂 (1235–81) and Guō Shǒu-Jìng 郭守敬 (1231–1316) for the construction of the *Shòu-Shí Lì* 授時曆 of 1280.[195] It

194. See chapter 7 of the *Jiǔ-Zhāng Suàn-Shù* 《九章算術》.
195. The cubic-interpolation formulas used by Wáng Xún 王恂 and Guō Shǒu-Jìng 郭守敬 are identical to the Newton-Sterling formulas of the eighteenth century.

is in the *Sì-Yuán Yù-Jiàn* 《四元玉鑑》 (*Precious Mirror of the Four Elements*) of 1303, Zhū Shì-Jié 朱世傑 first provided an interpolation procedure based on the method of finite differences (*zhāo-chā fǎ* 招差法) which can be generalized to an arbitrary high degree. Thus, the method of interpolation also constitutes a viable mathematical tool for examining planetary motion.

3.5.2 An Analysis of Chén Zǐ's Work

From the study given from p.125 to p.149 above, it is evident that, in many ways, Chén Zǐ's work was remarkably advanced for his time. In view of his outstanding achievements both in theoretical analysis and actual measurements at such an early stage of development in mathematical astronomy, an obvious question that needs to be addressed is: why has Chén Zǐ and his work not been adequately appreciated among historians of science?

The merit of Chén Zǐ's work has actually been a topic of considerable scholarly debate in the past. The debate rests primarily on two issues, one is on the formula, 'each 1 *cùn* change in shadow length corresponds 1,000 *lǐ* change (in the sun's position)', and the second on the question of whether the numbers that appeared in Chén Zǐ's work were measurements or fictitious numbers based on numerology. In this section, the work of Chén Zǐ is analysed within the context of these debates.

The Chén Zǐ 陳子 Formula and Its Application

The specific form of formula that was adopted by astronomers in the Gài-Tiān and Hún-Tiān schools for estimating the dimensions of the sun's motion is stated below:

> 周髀長八尺, 勾之損益寸千里.
> For an eight-chǐ Zhōu gnomon, each 1 *cùn* change in the *gōu* 勾 (sun shadow length) corresponds to 1,000 *lǐ* change [in the sun's position].[196]

which in current mathematical notation take the form given by Equation (15):

$$\varkappa_2 - \varkappa_1 \cong 1000\,(\lambda_2 - \lambda_1)\,\frac{li}{cun}$$

The major issue in the above formula has been whether it was a 'groundless postulation' or the result of direct measurements.[197]

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196. See the *Zhōu-Bì Suàn-Jīng*, vol. 1, p. 38b.
197. See for example, Yoshio Mikami 三上義夫 (1911); Qián Bǎo-Cóng (2) 錢寶琮 (1958); H. Chatley (1938); and Shigeru Nakayama 中山茂 (1969), pp. 24–35.

It is shown from p.125 to p.131 that the above formula is a special application of the general Chén Zǐ formula [Eq. (14)].

$$(\varkappa_2 - \varkappa_1) = \frac{\varkappa_0}{\lambda_0}(\lambda_2 - \lambda_1)$$

It is obtained by adopting the set of values $\varkappa_0$ = 16,000 *lǐ* and λ_0 = 16 *cùn*. The value 16 *cùn* for λ_0 is the sun shadow length of an 8-*chǐ* Zhōu gnomon at summer solstice measured at an observational site, probably Luò Yáng 洛陽. The value 16,000 *lǐ* for $\varkappa_0$ is the distance between the observational site and a point in the south at which the sun shadow length of a vertical pole is zero at noon summer solstice (see Fig. 36). This demonstrates that the specific form of Chén Zǐ's formula was not a 'groundless postulation', but a derived result based not only on the measurements but also on mathematical analyses.

Although Chén Zǐ's formula given by Equation (14) is mathematically exact within the validity of the earth-sun model visualized by Chén Zǐ, the special formula given by Equation (15) is limited not only to Chén Zǐ's model but also to the set of adopted values for ($\varkappa_0$, λ_0). As mentioned earlier, the value 16,000 *lǐ* for $\varkappa_0$ was probably adopted on its authority and no attempt was made by Chén Zǐ to assess its accuracy. It is the inaccuracy of this value that made this formula inaccurate even within Chén Zǐ's model.[198]

The formula 'each 1 *cùn* change in the sun shadow length corresponds to 1,000 *lǐ* change [in the sun's position]' was, however, such a convenient rule of thumb for estimating the dimensions of the sun's motion that it lingered among the ancient Chinese astronomers for a long period. It was used not only in the Gài-Tiān but also in the Hún-Tiān astronomical school. This is unfortunate since in the Hún-Tiān school the earth is visualized to be spherical and the sun is assumed to orbit around the earth. The adoption of this rule unwittingly reintroduced the errors associated with the flat-earth assumption in the estimation of the dimensions of the sun's motion. Though the accuracy of this rule was questioned by a number of astronomers, it was not totally abandoned until the work Zhāng Suì 張遂 and Nán Gōng 南宮 in the Táng dynasty around the eighth century.[199]

An early application of Chén Zǐ's formula in its general form was Chén Zǐ's attempt to determine the 'height' of the sun (see p.136 to

198. For a recent assessment of the accuracy of this measurement, see Chén Chēng-Yīh 程貞一 and Xí Zé-Zōng 席澤宗 (1991), pp. 369–71.
199. See also, for example, the work of Lǐ Chún-Fēng 李淳風 of the Táng dynasty who conducted several measurements and demonstrated the inaccuracy of this rule of thumb.

p.139). However, the work has been questioned by a number of sinologists. The debate is on the issue of whether the numbers which appeared in the work were fictitious numbers based not on measurements but on numerology. This issue arose from the observation that Chén Zǐ's method to determine the sun's diameter d_S was based on a right-angled triangle which satisfies a special case of the *gōu-gǔ* theorem (i.e. $6^2 + 8^2 = 10^2$).[200] This seems to suggest that the numbers used by Chén Zǐ were fictitious, chosen to fit into the 6-8-10 relation and not based on measurements.

In Chén Zǐ's model, the height H_0 is the vertical separation between the plane of the sun's apparent motion and the plane of the assumed flat surface of the earth. Thus, H_0 is a fixed value independent of the sun's position and the position of the observer. Once the value for H_0 is determined, Chén Zǐ had the freedom to choose any specific position of the sun to measure the ratio, d_S/R_S, and to determine the sun's diameter d_S, since, in principle, the value for d_S obtained from any position of the sun should be identical. This is certainly true for Chén Zǐ's model, if one ignores the practical problems encountered in observing and measuring celestial objects at astronomical distances. Obviously, at the time of Chén Zǐ, there was no such awareness of the observational problems.

Returning temporarily to see p.136 to p.139, it is seen that having determined $H_0 = 5\varkappa_0$ [Eq. (24)], Chén Zǐ made the deliberate choice to carry out the measurement for the ratio, d_S/R_S, when the shadow length of the gnomon was 60 *cùn* so that the sun was located at a horizontal distance $\varkappa_B = 3.75\varkappa_0$ [Eq. (23)] and consequently at a diagonal distance $R_S = 6.25\varkappa_0$ [Eq. (25)]. Taking $\varkappa_0 = 16{,}000$ *lǐ*, one has for $\varkappa_B$, H_0, and R_S the distances 60,000 *lǐ*, 80,000 *lǐ*, and 100,000 *lǐ*, respectively. These numbers satisfy the special 6-8-10 *gōu-gǔ* theorem. However, this is certainly an appropriate choice of the sun's position to illustrate the procedure for the determination of the sun's diameter [see Eq. (26)]. The ratio, $d_S/R_S = 1/80$, was indeed a value measured by Chén Zǐ. As shown on p.136 to p.139, the measurement is rather credible since the ratio yields the value 43'02" for the angular diameter, θ, of the sun (see Fig. 38), only 34% larger than the actual value, 31'51", for the average sun's angular diameter. Thus, the numbers in Chén Zǐ's calculation were not fictitious, and depended explicitly on observations and measurements.

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200. A number of scholars are of the opinion that Chén Zǐ made use of only certain special cases of the *gōu-gǔ* theorem. One has, for example, from Nakayama the following summary of the *gōu-gǔ* theorem in the *Zhōu-Bì Suàn-Jīng* 《周髀算經》: 'A right triangular theorem for the one case $3^2 + 4^2 = 5^2$. Although no demonstration is given, enthusiasm for the 'Pythagorian theorem' is apparent in the text. It was always expressed, however, as $6^2 + 8^2 = 10^2$.' See Shigeru Nakayama 中山茂 (1969), p. 25.

We should acknowledge that the inaccuracy of Chén Zǐ's predictions was due largely to his inadequate knowledge of the earth-sun system. It has nothing to do with the so called 'groundless postulations' or 'fictitious numerology'. In evaluating Chén Zǐ's work, one must be careful to distinguish his work from the later work that made improper use of his formula, since the formula was derived for a specific model centuries earlier.

Chén Zǐ and Aristarchus

Despite the limitations of his model, Chén Zǐ's analysis and measurements of the sun and its apparent motion was in many ways rather remarkable. In addition to his innovative theoretical approach, his measurements were very creditable. When he set up his pioneer sighting-tube measurements for the purpose of determining the ratio of the sun's diameter to its distance, he was certainly not aware of the observational problems involving the earth's atmosphere and astronomical distances of celestial bodies, let alone the concept of the sun's parallax. Nevertheless, Chén Zǐ's actual measurement of the ratio of the sun's diameter and distance was exceptionally accurate even by a standard much later than his own time (see p.136 to p.139). The only other available account of an early attempt to measure the sun, comparable in antiquity, is that of Aristarchus of Samos (c. −320 to −250) in Greek civilization.[201] A comparison of Chén Zǐ's work with that of Aristarchus is of interest not only on the difference in their models but also on the difference in their use of mathematics and measurements.

The work of Aristarchus is found in a treatise entitled 'On the Sizes and Distances of the Sun and Moon' survived through its incorporation in the fourth-century work of Pappus. Very little is known about the personal life of Aristarchus. However, he was probably a pupil of Strato and the first Greek astronomer to postulate that sun remains unmoved and the earth revolves about the sun.[202] Obviously Aristarchus had a much better view of the earth-sun system than Chén Zǐ.

Assuming that the triangle formed by the sun, the moon, and the earth at half-moon is a right-angled triangle as shown in Figure 44, Aristarchus took the elongation $\angle$MES of the moon from the sun to be 87°. Based on these numbers, he obtained the upper and lower bounds for the ratio of the earth-sun distance ES to the earth-moon distance EM:

201. Thomas L. Heath (1913); and O. Neugebauer (1975), part 2, pp. 634–43.
202. See the *Sand-Reckoner* of c. −216 by Archimedes.

$$18 < \frac{ES}{EM} < 20$$

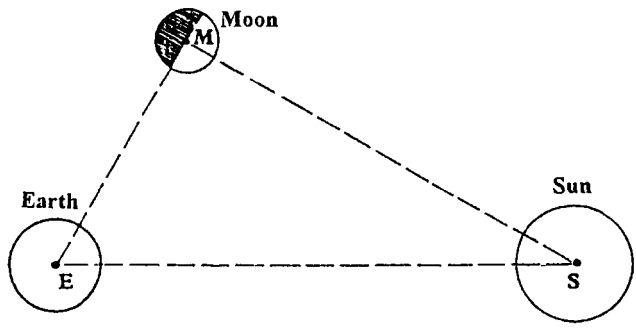


Figure 44. The right-angled triangle relationship among the sun, the moon, and the earth at half-moon as visualized by Aristarchus.

With these bounds for the distance ratio, Aristarchus further determined the bounds for the ratios of diameters of the sun, the moon and the earth by making use of the assumed geometric arrangement of a lunar eclipse as illustrated in Figure 45. The specific assumption made in the lunar eclipse is that the diameter of the earth's shadow IK is twice the diameter of the moon d_M. From these assumptions, Aristarchus derived for the ratio of the sun's diameter d_S to the earth's diameter d_E the following bounds:

$$\frac{19}{3} < \frac{d_S}{d_E} < \frac{43}{6}$$

This is the main result of the treatise and from which other bounds can be easily derived. It should be noted that in this work the sun and the moon are supposed to orbit around the earth. Thus, the geocentric hypothesis, not the heliocentric hypothesis, is adopted in the treatise.

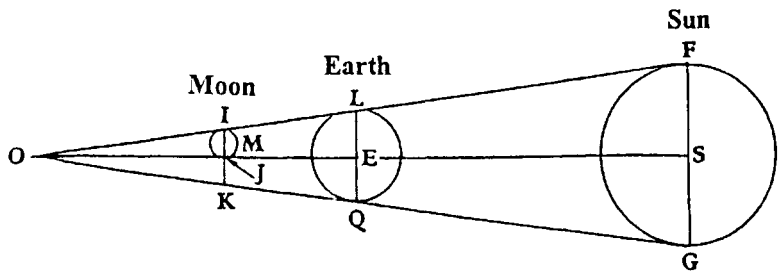


Figure 45. The geometrical relationship among the sun, the moon, and the earth at a lunar eclipse. Aristarchus assumed that the diameter of the earth's shadow IK at the moon is twice the moon's radius d_M = JI at the lunar eclipse. Diameter of the sun d_S = FG and the diameter of the earth d_E = LQ.

The bounds for the distance ratios and the size ratios predicted by Aritarchus do not, however, agree with nature. The ratio between the earth-sun and earth-moon distances should be 392, not bounded in between 18 and 20. The ratio between the sun's and the earth's diameters should be 109, not bounded in between 19/3 and 43/6. There are a number of possible reasons that contributed to the inaccuracy of these predictions. The numbers used in Aristarchus' calculation do not, however, reflect the empirical facts well. If these numbers were indeed based on measurements, they were rather inaccurate measurements.[203] Neugebauer commented that 'Aristarchus' treatise on the sizes and distances is a purely mathematical exercise which has as little to do with practical astronomy as Archimedes' *Sandreckoner*....' He also claims that $\angle MES = 87°$ is 'a purely fictitious number' and such numerical data were adopted for arithmetical convenience 'without consideration for observational facts.'[204]

In comparison with the work of Chén Zǐ presented from p.125 to p.149, Aristarchus' work is based on a more accurate physical earth-sun model than that of Chén Zǐ, but the work is more formal than Chén Zǐ's and adhered more to formulation than to empirical data. The mathematics used in the Aristarchus' work involves both arithmetic and geometry. Although less formal, Chén Zǐ's work made not only use of arithmetic and geometry but also algebra. Despite their difference in mathematics, both Aristarchus' work and Chén Zǐ's work well demonstrate the power of a mathematical approach to astronomical problems.

The inaccuracy in Chén Zǐ's work is due primarily to the incorrect earth-sun model, not so much to the measurements. In Chén Zǐ's work measurements played as an important role as the mathematical analysis. It was extremely difficult at the time of Chén Zǐ, no later than the −5th century and possibly much earlier, to demonstrate from observations and measurements that the earth is 'spherical'.[205] An earth-sun model similar to that of Aristarchus was also derived in China in the Hún-Tiān school (see p.155 to p.156) but much later than the time of Chén Zǐ. It is often

203. The apparent angular diameter of the moon given by Aristarchus in his treatise was 2° which is about four times larger than the actual value. Since the moon's angular diameter is relatively easy to measure, the value 2° was probably not the result of direct measurement.

204. See O. Neugebauer (1975), part 2, pp. 642–3.

205. It is perhaps relevant to note that the early acceptance of a spherical earth by the Greek astronomers was based primarily on religious and philosophical beliefs, and justifications based on physical observations and measurements were not supplied until much later. Pythagoras of the −6th century was probably among the first to view the earth as being spherical; but his contemporary, Anaximenes of Miletus, viewed the earth to be a flat disk.

easy for one enriched with current knowledge to be critical of past work, but the flat surface assumption made by Chén Zǐ should be judged within its proper historical context.

3.5.3 The Origins of Astronomy in China[206]

Despite our limited knowledge of early astronomy, objective evaluations of the relationship in astronomical works between the Babylonian and Chinese civilizations are possible. In this section, an attempt is presented in an effort to assess the claim of a Babylonian origin of astronomy in China.

Biot's Work and Needham's Claim

As mentioned earlier, the seasons during the time of Yáo were determined, according to the *Yáo Diǎn*, by the meridian passage of four *xiù* 宿 star-groups. The vernal and autumnal equinoxes were determined by the culmination of the star-groups identified by the stars *Niǎo* 鳥 (α Hya) and *Xū* 虛 (β Aqr). The summer and winter solstices were determined by the culmination of the star-groups identified by the stars *Huǒ* 火 (Antares, α Sco) and *Mǎo* 昴 (η Tau). Owing to the gradual westward movement of the equinoctial points along the ecliptic resulting from the change in direction of the earth's axis, the relationship between the seasons and the *xiù* described in the passage no longer holds.

By tracing backward the precession of the equinoxes, it is possible to estimate the date when the *xiù* season relationship recorded in the *Yáo Diǎn* could have been observed. Biot was the first scholar who made use of this astronomical fact and attempted to determine the date. Taking the hour of observation of culmination to be 6 p.m., Biot obtained a date of about −2400, in agreement with the traditional dating of the Yáo period.[207] Other than some uncertainties arising from the exact equatorial extension of the four *xiù* there appeared to be no escaping Biot's conclusion. However, by taking 7 p.m. as the hour of observation of culmination, Hashimoto showed that the date could be reduced to the

206. Based in parts on the paper entitled '*Yáo Diǎn* 《堯典》 and the Origins of Astronomy in China' by Chén Chēng-Yīh 程貞一 and Xí Zé-Zōng 席澤宗 presented at the Third International Conference on Archaeoastronomy, St. Andrew's University, Scotland (1990), and the paper entitled 'Dr. Needham and his View on the Origins of Equatorial Astronomy and Tonal System in China' by Chén Chēng-Yīh presented at the Symposium on Science and Civilization in China: A Celebration of the 90th Birthday of Dr. Joseph Needham, Shanghai (1990).

207. J.B. Biot (1862), p. 263.

–8th century.[208] Although there are no valid grounds for disregarding the traditional view that transit observations by Chinese astronomers were always made at 6 p.m., Hashimoto's calculation nevertheless highlights the sensitivity of the result to the precise hour of observation.[209]

Zhú approached the problem from a different perspective by examining the number of *xiù*-determinatives (stars identifying the equatorial extension of a given *xiù*, see p.118 to p.125) at different dates. His results are shown in Table 14. It is seen that the maximum number of *xiù*-determinatives occurred between –2300 and –4500, pointing again to a date earlier than –2300.[210]

Table 14. The *Xiù*-determinatives Used in the Chinese 28-*xiù* System within 10° of the Celestial Equator at Different Dates.

Date	#[a]	Names of the *Xiù*-determinatives
+1900	5	*Shēn* 參, *Xīng* 星, *Jiǎo*, 角, *Xū* 虛, *Wēi* 危
–230	8	*Shēn* 參, *Xīng* 星, *Yì* 翼, *Zhěn* 軫, *Kàng* 亢, *Dī* 氐, *Xū* 虛, *Wēi* 危
–2370	12	*Bì* 壁, *Kuí* 奎, *Lóu* 婁, *Bì* 畢, *Xīng* 星, *Zhāng* 張, *Yì* 翼, *Zhěn* 軫, *Fáng* 房, *Xū* 虛, *Wēi* 危, *Shì* 室
–3440	12	*Bì* 壁, *Kuí* 奎, *Jǐng* 井, *Xīng* 星, *Zhāng* 張, *Yì* 翼, *Zhěn* 軫, *Fáng* 房, *Xīn* 心, *Xū* 虛, *Wēi* 危, *Shì* 室
–4510	12	*Bì* 壁, *Kuí* 奎, *Jǐng* 井, *Liǔ* 柳, *Xīng* 星, *Zhāng* 張, *Yì* 翼, *Wěi* 尾, *Dǒu* 斗, *Xū* 虛, *Wēi* 危, *Shì* 室
–6650	10	*Bì* 壁, *Kuí* 奎, *Yì* 翼, *Wěi* 尾, *Jī* 箕, *Dǒu* 斗, *Niú* 牛, *Nǚ* 女, *Xū* 虛, *Wēi* 危
–8970	3	*Kuí* 奎, *Guǐ* 鬼, *Zhěn* 軫

[a] # denotes the number of *Xiù*-determinatives

It is worthwhile to recall that by the beginning of the present century, the study of ancient Chinese history, as evidenced by the *Shǐ-Jì* of the –1st century, had suffered severe setbacks. Many leading historians and sinologists had practically denied the existence of Chinese history prior to the Zhōu dynasty about the –11th century. Although the discovery of the shell-bone inscriptions of the Shāng dynasty and the subsequent identification of all but three of the thirty-one king's names

208. Masukichi Hashimoto 橋本増吉 (1928).
209. Zhào Zhuāng-Yú 趙莊愚 (1983) has further examined the effect due to different observation times at different possible sites and obtained a date between –2200 and –2000.
210. Zhú Kě-Zhēn 竺可禎 (1944).

in the inscriptions had restored much of the *Shǐ-Jì*'s account of the Shāng dynasty, a number of leading historians and archaeologists maintained the view that there was little development on Chinese soil prior to the –16th century and the pre-Shāng account of Chinese civilization given in the *Shǐ Jì* was not based on history.

Such a view on the antiquity of Chinese history undoubtedly affected the dating and the interpretation of the origin of the quadrantal *xiù* system, and clearly played a vital role in forming Needham's conclusion on the subject:

> In view of all that we know about ancient Chinese history, it seems very unlikely that the data in our text could refer to a time earlier than about –1500 at the most generous estimate, and therefore Hashimoto's conclusion is perhaps the most attractive. But the possibility remains open that the text is indeed the remnant of a very ancient observational tradition, not Chinese at all but the Babylonian.[211]

Here we see that, despite the consistency of two alternative calculations of Biot and Zhú based on the precession of the equinoxes, Needham favours Hashimoto's result and even postulates that the text itself is the remnant of a Babylonian observational tradition.

The Yáo Diǎn 《堯典》 *and the Date of Its Contents*

Our knowledge of prehistoric China has undergone dramatic changes in the last few decades as a result of archaeological explorations and discoveries. At the time of Yáo, there existed, for example, the Dàwénkǒu 大汶口 and the Lóngshān 龍山 cultures in the central and northeast plains, the Liángzhǔ 良渚 culture along the south-east coast and the Qíjiā 齊家 culture in the west. These were advanced Neolithic cultures.

Among the various Neolithic inscriptions found on pottery vessels and other artifacts that contain astronomical information, the most significant ones are probably those found on ceremonial pottery vessels of the Dàwénkǒu culture (c. –4300 to –1900) unearthed in 1959 at Língyánghé 陵陽河 in Jǔ 莒 county in the Shāndōng 山東 province as illustrated in Figure 46.[212] The Dàwénkǒu cultural stratum from which these pottery vessels were unearthed is C-14 dated to about –2500.

It is evident from Figure 46 that the two glyphs on the top vessels are pictograms of an axe and a hoe. The other two glyphs on the lower

211. Joseph Needham (1959), vol. 3, p. 246.
212. See the Shāndōng Provincial Cultural Relics Management and the Jìnán 濟南 Museum (1974).

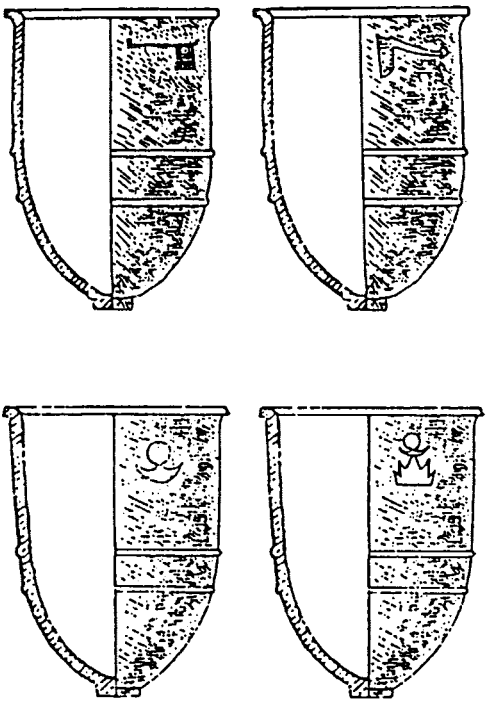


Figure 46. Illustrations of the four ceremonial pottery vessels with incised glyphs unearthed in 1959 from the Dàwénkǒu cultural stratum of c. −2500 at Língyánghé 陵陽河 in Jǔ 莒 county in Shāndōng 山東 province.

vessels are ideograms relating to the rising sun.[213] A logical supposition is that the four ceremonial pottery vessels bearing these inscriptions probably functioned within the context indicated by the meaning of these incised glyphs. Based on such an inference, it is seen that the context of these glyphs matches that recorded in the *Yáo Diǎn*.[214]

The discussion on astronomy in the *Yáo Diǎn* begins with the following passages:

> 乃命羲和, 欽若昊天.
> 曆象日月星辰, 敬授人時.
> [Yáo 堯] commanded Xī 羲 and Hé 和 to pay reverence to the grand celestial heavens, to delineate the regularities of the sun, the moon, stars, and constellations and to relate respectfully to people the seasons for observance.

213. See Yú Xǐng-Wǔ 于省吾 (1973); Táng Lán 唐蘭 (1975); and Chén Chēng-Yīh 程貞一 (1986).
214. Shào Wàng-Píng 邵望平 (1978).

The text then continues with discussions of duties and functions of the astronomy officials sent to different locations in the four directions. The following passage deals with the astronomy officials sent to the east:

> 分命羲仲, 宅嵎夷曰暘谷.
> 寅賓出日, 平秩東作.
> [Yáo 堯] commanded Xī-Zhòng 羲仲 to reside at the mountain side in the Yáng-Gǔ 陽谷 by the Yí 夷 tribe. There he was to enact the welcome reverence to the rising sun and to regulate the work of the east.

From these passages, we see that the description of the duties and functions of the Yáo astronomy officials sent to the east matches the possible functions of the ceremonial vessels as indicated by the meaning of the inscriptions found on them. Thus, the two pieces of pottery with ideograms and (Fig. 46, bottom) were probably the ceremonial vessels used by the Yáo astronomy officials to pay welcome reverence to the rising sun, while the two pieces with pictograms of an axe and a hoe (Fig. 46, top) were probably the ceremonial vessels used to give thanks for the harvest. All of this seems to suggest that the account of official activities of the Yáo astronomy officials recorded in the *Yáo Diǎn* was probably based on historical facts that occurred in the c.′–25th century. This archaeological discovery provides important evidence on the date of the *Yáo Diǎn*'s contents which is independent of, but consistent with, the date calculated based on the precession of the equinoxes.[215] Such a discovery has further strengthened the traditional date for the contents of the *Shàng Shū* and the reliability of the astronomical account given in the *Yáo Diǎn*. Needham's conclusion must, therefore, be re-evaluated based on new archaeological information.

The System of Xiù 宿 and Babylonian Planisphere

The claim of a Babylonian influence on the Chinese *xiù* system is based primarily on the interpretation of the equatorial significance of the Babylonian planisphere. A comparative study of the Babylonian planisphere is presented here.

It should be noted that our knowledge about ancient Babylonian astronomy of this early period is also very limited, coming primarily from archaeological discoveries. Two Babylonian tablets found in the British Museum (K250 and K8067), which contain the star and planet names *Elam*, *Akkad*, and *Amurru*, were probably among those coming from the old Babylonian period.[216] These stars and planets were also found in

215. Chén, Chēng-Yīh 程貞一 (1981).
216. See E.A. Wallis Budge, ed. (1912), vol. 33; and E.F. Weidner (1915).

association with *Ea* (*Enūma*), *Anu*, and *Enlil*, the three gods of *Elam*, *Aklad*, and *Amurru* as shown in Table 15. This list was reproduced by Schroeder from a Babylonian tablet (no. 218) of c. −1100 unearthed at Assur and preserved at the Berlin Museum.[217]

Table 15. Stars and Planets of *Ea*, *Anu*, and *Enlil*

Month	Stars of *Ea*	Stars of *Anu*	Stars of *Enlil*
1. Nisannu	IKU	DIL.BAT	APIN
2. Aiaru	MUL.MUL	SHU.GI	*A-nu-ni-tum*
3. Simanu	SIBA.ZI.AN.AN	UR.GU.LA	MUSH
4. Duzu	KAK.SI. DI.	MASH.TAB.BA	SHUL.PA.E
5. Abu	BAN	MASH.TAB.BA. GAL. GAL	MAR.GID.DA
6. Ululu	*ka-li-tum*	UGA	SHU.PA
7. Tashritu	NIN.MAH	*zi-ba-ni-tum*	EN.TE.NA. MASH.LUM
8. Arahsamna	UR.IDIM	GIR.TAB	LUGAL
9. Kislimu	*sal-bat-a-nu*	UD.KA.DUH.A	UZA
10. Tebetu	GU. LA	*al-lu-ut-tum*	A^{mushen}[a]
11. Shabutu	NU.MUSH.DA	SHIM.MAH	DA.MU
12. Addaru	KUA	[b]*Marduk*	KA.A

[a] The word *mushen* means 'bird', and A^{mushen} (or more precisely A_2^{mushen}) means 'eagle'.

Neugebauer has pointed out that 'deification of the Sun, Moon or Venus cannot be called astronomy' and 'the denomination of conspicuous stars or constellations does not constitute an astronomical science'.[218] Our interest in the stars and planets listed in Table 15 lies in the fact the they were associated with months and incorporated in the planisphere. Fragments of such planispheres (see Fig. 47) are found in tablets belonging to the '*Enma, Anu, Enlil*' series.[219] This series contained at least 70 numbered tablets with a total of about 7,000 astrological omens. The canonization of this enormous mass of omens is said to have extended over several centuries and reached its final form perhaps around −1000. Thus, the appearance of the planispheres in Babylonia is

217. O. Schroeder (1920), p. 119.
218. O. Neugebauer, (1957), p. 99.
219. E.A. Wallis Budge, ed. (1912), vol. 33, p. 6, pls. 11 and 12.

placed somewhere around −1200, contemporary with the Yīn 殷 period (c. −1385 to −1112) of the Shāng dynasty.

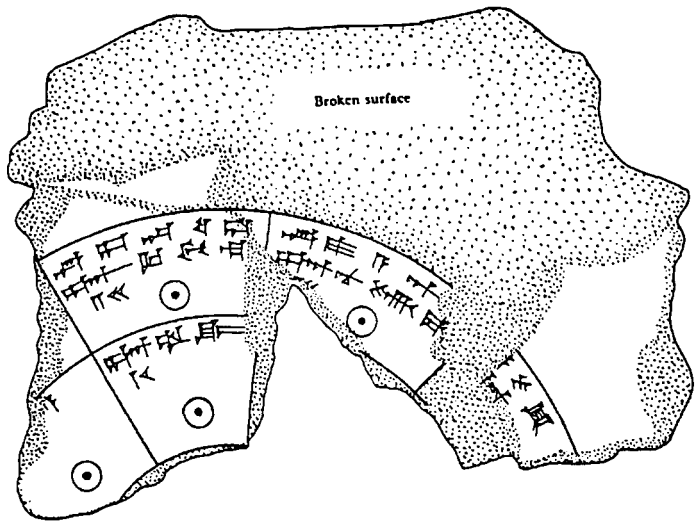


Figure 47. Fragment of a Babylonian planisphere (c. −1200).

The planisphere consists of a diagram of three concentric circles divided into a total of thirty-six sections by twelve radii. These sections contain the names of stars, constellations and planets, together with simple numbers in arithmetic progression. Unfortunately, only fragments of such planispheres have so far been discovered. Pinches (1900) first attempted to restore the planisphere and copied a number of fragments found in the collections stored in the British Museum.[220] The copied materials enabled Schott to publish the reconstructed diagram of the planisphere reproduced in Figure 48.[221] It can be seen that the planisphere consists of three 'roads', each marked with twelve stars corresponding to the months. The stars of the central road correspond to the stars of *Anu*; those of the outer road to the stars of Ea; and those of the inner road to the stars of *Enlil*. This arrangement is certainly consistent with the story told in the *Enma-Elish*, a Babylonian creation myth.

A common interpretation of the Babylonian planisphere is that it represented a division of the sky and the central road (*Anu*) of the

220. T.G. Pinches (1900); and A. Sacks (1955).
221. A. Schott (1934).

planispheres corresponds to the equatorial belt. The three-road divisions are compared with the combination of the *zhōng-guān* 中官 and *wài-guān* 外官 with the 28 *xiù* in the Chinese system.[222] In this comparison, the *zhōng-guān* corresponds to the region of the inner road (*Enlil*), the *wài-guān* to the region of the outer road (*Enma*), and the 28 *xiù* to the region of the central road (*Anu*). Such a comparison should not be viewed as an indication of a Babylonian influence since the Chinese division of the sky for celestial cartography was a logical extension of the equatorial *xiù* system, not modelled after the Babylonian three-road divisions.

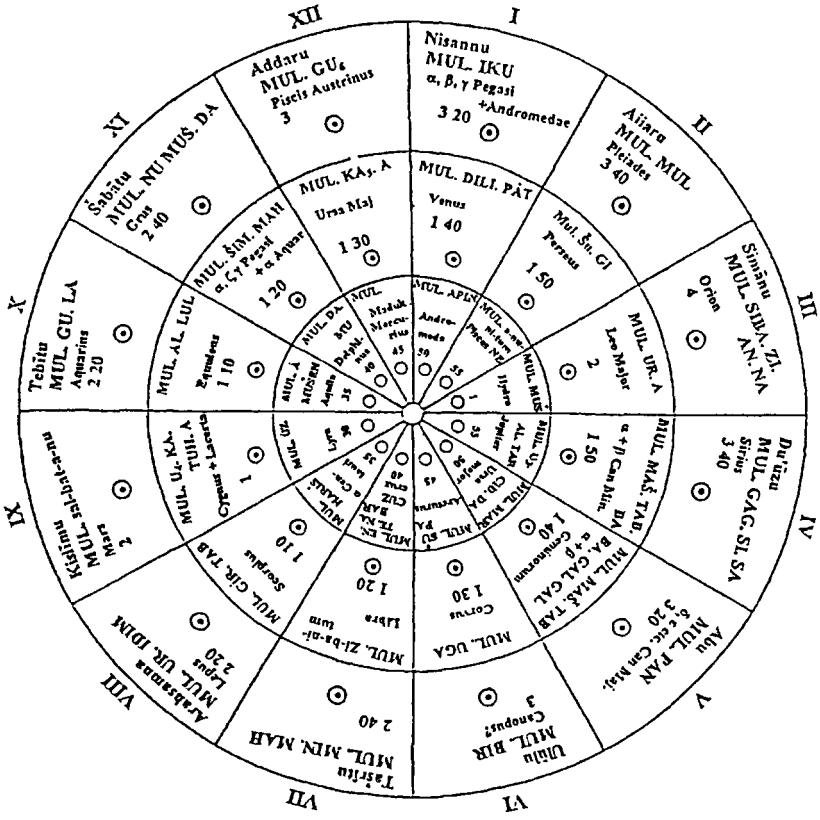


Figure 48. The reconstructed diagram of the Babylonian planisphere by Schott (1934).

222. For an account of the *zhōng-guān* and *wàn-guān* in English, see Needham (1959), vol. 3, pp. 263–5.

It is important in the comparison to examine the validity in the identification of the central road (*Anu*) of the Babylonian planisphere with the Chinese equatorial *xiù* system along the equatorial belt, since it assumes that the twelve stars in the central road mark the twelve months according to the times of their helical rising. However, if one examines these stars, one realizes that not all of them are helical stars. They even included a planet. The first month is marked by the planet Venus (*DIL. BAT*).[223] This is, of course, incorrect because planets cannot be fixed to any particular month. Evidently, the twelve stars of *Anu* did not form an equatorial system comparable with the quadrantal *xiù* system of the *Yáo Diǎn*. Even if one disregards the inscribed ceremonial pottery vessels and Biot's dating discussed above, and accepts the date of –800 for the observation of the quadrantal *xiù* system recorded in the *Yáo Diǎn*, it would still be difficult to demonstrate a Babylonian influence on the development of the Chinese equatorial *xiù* system.

The Origins of the Chinese Equatorial System

The polar and equatorial character of the Chinese system of 28 *xiù* ('equatorial compartments') has been a topic of much discussion. Needham has commented that 'many European scholars have found it almost impossible to believe that a fully equatorial system of astronomy could have grown up without passing through an ecliptic (zodiacal) phase, yet that undoubtedly happened.'[224] However, based on his claim that 'the (*Yáo Diǎn*) text is the remnant of a very ancient observational tradition, not Chinese at all but Babylonian',[225] Needham hastens to point out that 'it happened in Babylonia first.'[226]

However, Needham's claim that the *Yáo Diǎn* text is the remnant of a very ancient Babylonian observational tradition is primarily a speculation, not supported by evidence. In the last section, it is shown that there is no validity in the claim of a Babylonian influence on the development of the Chinese equatorial *xiù* system. The *Qī-héng-tú* 七衡圖 of the *Zhōu-Bì Suàn-Jīng* offered by Needham as evidence of traces of Babylonian influence[227] is also invalid since the diagram was not an attempt to divide the sky and has nothing in common with the Babylonian planisphere. The *Qī-héng-tú* is a diagram consisting of the

223. According to the Stars of *Anu* in Table 13, the twelfth month is marked by the planet Jupiter (*Marduk*).
224. Joseph Needham (1959), vol. 3, pp. 231, 252–8.
225. Joseph Needham (1959), vol. 3, p. 246.
226. Joseph Needham vol. 3, p. 231, footnote g, see also O. Neugebauer (2), (1957).
227. Joseph Needham (1959), vol. 3, pp. 256–7.

celestial north pole with seven concentric circles describing the change in the sun's apparent orbit with seasons (see p.131 to p.133), calculated from sun-shadow measurements by the use of Chén Zī's formula [Eq. (14)] under the parallel-plane sun-earth assumptions (see p.128).

The equatorial 28 *xiù* system, as we know it now, is divided into four groups known as the *sì gōng* 四宫 (the four palaces) or the *sì xiàng* 四象 (the four images), consisting of the Qīng-Lóng 青龍 (the Blue Dragon) in the east, the Zhū-Niǎo 朱鳥 (the Vermilion Bird) in the south, the Bái-Hǔ 白虎 (the White Tiger) in the west, and the Xuán-Wǔ 玄武 (the Black Snake-Tortoise) in the north. Each group contains seven *xiù*. Opinions differ considerably among scholars on the dates when the four divisions and the 28 *xiù* were first completed. Most scholars are of the view that the system was completed around the −3rd century as the earliest complete list of names of the 28 *xiù* is found in the *Lǚ-Shì Chūn-Qiū* of −239.[228] This would place the date later than the star-map work of Shí Shēn 石申 and Gān Dé 甘德 of the −4th century (p.121 to p.122).

Two centuries of controversy about the antiquity of the fully developed system of *xiù* have been set to rest by the discovery of an astronomical diagram of the −5th century with names of the 28 *xiù* written clockwise in an oval ring encircling the character *Dǒu* 斗, Ursa Major (see Fig. 49). This diagram, painted in red with a black background on the cover of a wooden chest (length 82.8 cm, width 47 cm, and height 44.8 cm),[229] was unearthed from the tomb of Marquis Yǐ, the same tomb where the bronze set-bells discussed above were unearthed.

All the characters for the names of *xiù* in the diagram of Marquis Yǐ are written in seal form.[230] In Figure 50, they are identified with the present printed form.[231] By comparing these with Table 12, it is seen that, other than for a few cases, the names of the *xiù* in the two sets are identical, except perhaps for homophone or stroke variations. The names provided by the astronomical diagram are invaluable in the study of the origins of the *xiù* names. A number of such studies are now available in the Chinese literature.

It should also be noted that from comparing the names of the stars given in the Babylonian planisphere (Fig. 48) with those in the Chinese 28 *xiù* (see Table 12 and Figs. 49 and 50), no significant relationship can

228. Among the extant Chinese classics, it is in the *Huái-Nán Zǐ* 《淮南子》 (*The Book of the Prince of Huái-Nán*) of −120 that both the four divisions and the complete list of 28 *xiù* are found.
229. See the Léigǔdūn Archaeological Team of the First Tomb in Suíxiàn (1979), pl. 5.
230. Wáng Jiàn-Mín 王健民 et al. (1979).
231. See Wáng Jiàn-Mín 王健民 et al. (1979); and Qiú Xī-Guī 裘錫圭 (1979).

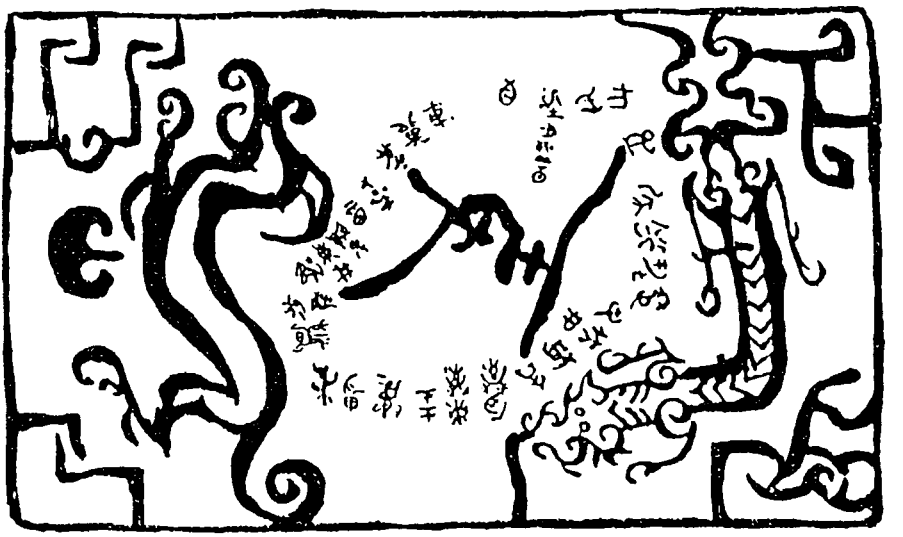


Figure 49. A copy of the astronomical diagram with names of the 28 *xiù* written clockwise in an oval ring encircling the character *Dǒu* 斗, Ursa Major unearthed in 1978 in Húběi from the tomb of Marquis Yǐ of Zēng dated to the –5th century.

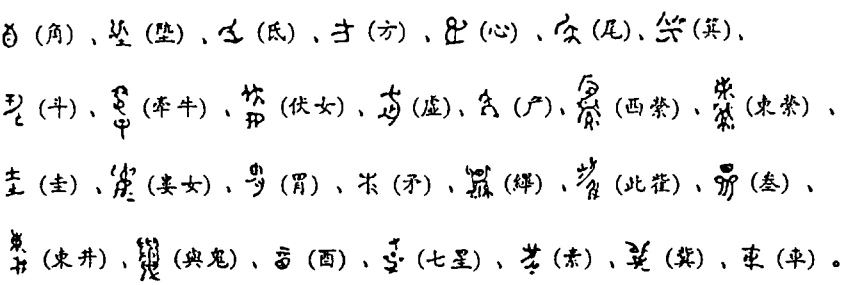


Figure 50. The identification of the names of the 28 *xiù* in the seal form in the –5th century astronomical diagram with those in the present form.

be found between the star nomenclatures of the two civilizations. Though in the past, a few star-names have been mentioned for possible identification, in each case the connection in meaning is vague and obscure.[232]

232. G. Thibaut, (1894).

The astronomical diagram of the −5th century also provides valuable information on the development of the star-group divisions. In the diagram, a dragon and a tiger are drawn on the opposite sides of the ring of the 28 *xiù*. An obvious interpretation is that the dragon corresponds to the Qīng-Lóng 青龍 palace in the east and the tiger corresponds to the Bái-Hǔ 白虎 palace in the west. This, however, leaves open the question about the remaining two palaces of the *sì gōng*, namely the Zhū-Niǎo in the south and the Xuán-Wǔ 玄武 in the north. Huáng et al. suggest that the two remaining palaces can also be accounted for if one includes the displays on the front and back sides of the wooden chest together with the astronomical diagram displayed on its cover (Fig. 51).[233]

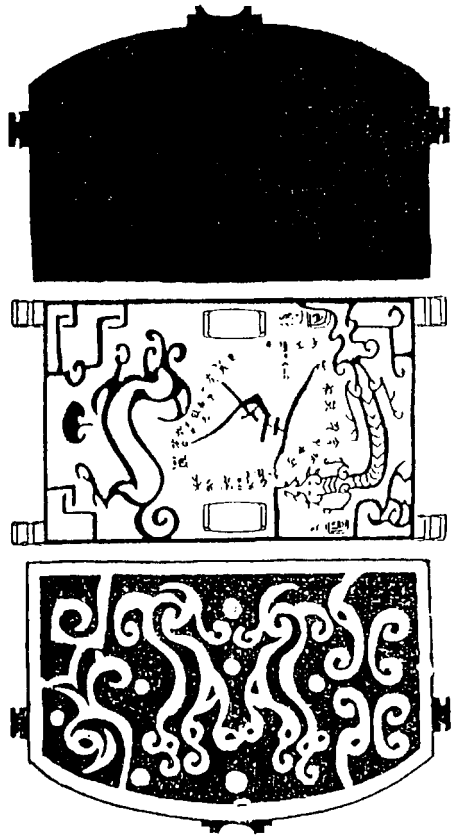


Figure 51. Designs from the front and back of a wooden chest unearthed in 1978 in Húběi from the tomb of Marquis Yǐ of Zēng, dated to the −5th century, displayed together with the astronomical diagram from its cover.

233. Huáng Jiàn-Zhōng 黄健中 et al. (1982).

By interpreting the design on the front side as birds and the total blackness on the back side as an indication of invisibility, they concluded that the Vermilion Bird palace and the Black Snake-Tortoise palace were indeed included in the overall presentation of the astronomical system on the wooden chest. To support their interpretation of total blackness as invisibility, Huáng et al. pointed out that the date 'the third day, *jiǎ yín* 甲寅' is entered below the second *xiù* (namely *Kàng* 亢) in the astronomical diagram.[234] The date, which was probably entered as a record of Marquis Yǐ's death, has been identified as the third day of the fifth month, corresponding approximately to the season of the Spring Equinox.[235] Consequently, the seven *xiù* in the north were invisible below the horizon at dusk.

A modified Babylonian system of 36 stars dating to c. −700 is found in the texts of the *Mul Apin* tablets series.[236] Except for eight bright stars somewhat far from the equator, the 28 stars are within an equatorial belt about 30° in width. This system, consisting only of fixed stars, is more advanced than the planisphere. By this time, the Chinese *xiù* system had also been expanded into its final form, as indicated by records found in the *Yuè Lìng* 《月令》 (*Monthly Ordinances of Zhōu*) of about the −7th century. All but four of the 28 *xiù* (nos. 7, 18, 23 and 26) are mentioned in the *Yuè Lìng*. Comparing the 36-star system of the Mul Apin with 28-*xiù* system, we still find no evidence of transmission in either direction.

Recent archaeological discoveries suggest that the system of *xiù* probably had a long history of development. In a tomb excavated in 1987 at Púyáng 濮陽 in the province of Hénán, images of a dragon and a tiger, made of shells, were found along the sides of the body of a male in the Yǎngsháo cultural stratum dating to about −5000 (Fig. 52).[237] The orientation of the skeleton is such that its skull points to the south. The interest in this discovery lies in the comparison shown in Figure 53. Taking the alignment of the body to the south together with the astronomical diagram unearthed from the tomb of Marquis Yǐ (Fig. 48), one sees that in both cases the dragon lies in the east and the tiger in the west. This seems to suggest that the astronomical significance attached to the images of the dragon and tiger seen in the astronomical diagram of the Marquis Yǐ might have its roots in ancient mythology. The relationship was probably established by ancient skywatchers, perceiving

234. Huáng Jiàn-Zhōng 黄建中 et al. (1979).
235. Wáng Jiàn-Mín 王健民 et al. (1979).
236. E.F. Weidner (1924).
237. Púyáng 濮陽 Cultural Relics Management and Púyáng Museum (1988), pl. 1. 1.

the configurations of the stars in the east to have the image of a dragon and those in the west to have the image of a tiger. Such an origin could also account for the fact that the *xiù* are not equally spaced and that some of the determinative stars were quite far from the equator.

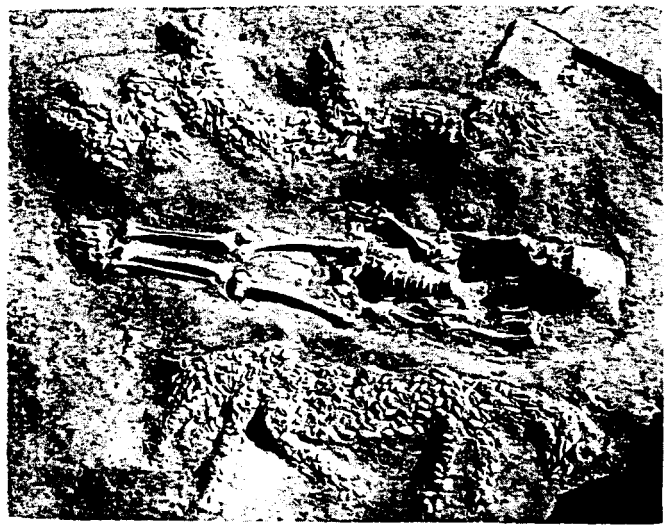


Figure 52. Images of a dragon and a tiger along the sides of a human body excavated in 1987 from the Yǎngsháo 仰韶 cultural stratum (c. –5000) at Púyáng 濮陽 in Hénán.

Figure 53. A comparison of the arrangement of the images of a dragon and a tiger from a tomb of the Yǎngsháo culture (c. –5000) with that of an astronomical diagram from the tomb of Marquis Yǐ (–433).

In this connection, two further mythical animals of antiquity are worth examining. Shown in Figure 54 is a drawing of a mythical animal

of the Yǎngsháo culture in the Gānsù 甘肃 tradition (c. −3000) unearthed in 1958 at Wǔshān 武山. This snake-like animal might be related to what later came to be known as the Xuán-Wǔ 玄武, the name for the star-group division in the north, since Xuán-Wǔ has been identified not only with the tortoise but also with the snake of antiquity.

Figure 54. A snake-like mythical animal painted on a flat-bottomed amphora of the late Yǎngsháo culture (c. −3000) unearthed in 1958 at Wǔshān in Gānsù province.

The other mythical animal worthy of note is the bird of the Hémǔdù culture of the Yángzǐ valley in the south. The astronomical significance of the Hémǔdù bird is revealed by an ivory engraving unearthed in 1977 from the third Hémǔdù cultural stratum (c. −4500) near Yúyáo 餘姚 in Zhèjiāng province. A copy of the engraving is shown in Figure 55.[238] In addition to the mythical birds, it consists of six stars with the sun in the middle. It seems to illustrate the presence of the sun in a star-group identified with the mythical bird. This interpretation raises the interesting question of whether the engraving is an early description of the *Niǎo Xīng* 鳥星, the 'Bird Star', mentioned in the *Yáo Diǎn* as well as in the shell-bone inscriptions (*jiǎ-gǔ-wén* 甲骨文) of the −14th century (see Fig. 56).[239] This seems to suggest that use of the Bird star to predict

238. Hémǔdù 河姆渡 Archaeological Excavation Team (1980), pl. 3, fig. 7 (T2263:79).
239. Dǒng Zuò-Bīn 董作賓 (1945), pt. 2, ch. 3, p. 1b.

the arrival of spring season probably had a very early beginning. The *Niǎo Xīng* has been identified with the 25th *xiù* (α Hya) central to the Zhū Niǎo star-group division in the south.

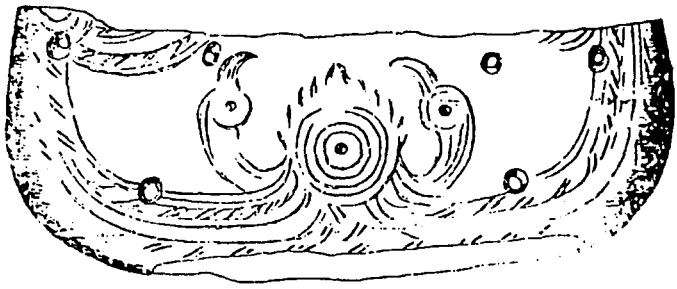


Figure 55. A copy of an ivory engraving unearthed in 1977 from the third Hémǔdù 河姆渡 cultural stratum (c. −4500) near Yúyáo 餘姚 in Zhèjiāng province.

Figure 56. A shell-bone inscription which mentions the 'Bird Star' (*Niǎo Xīng* 鳥星, the left two lines of the inscriptions). The inscription is from the −14th century.

It is perhaps worthwhile to note that, even by −700, concepts such as the celestial equatorial and ecliptic systems were probably not well developed. They were certainly not as clearly defined as they are in modern astronomy.

In the Babylonian system, the selection of the central-road stars along a 30° equatorial belt can be considered an intermediate step towards either an equatorial or an ecliptic system. Thus, the fact that the Seleucid Babylonian cuneiform texts of the −3rd and −2nd century give great prominence to the zodiac and made exclusive use of ecliptic

coordinates, should not be surprising. This can be attributed to parallel, independent developments in science.

The Origins of Chinese Calendrical Science

Much has been written over the last century about the Babylonian influence on Chinese calendrical science.[240] The sexagenary *gān-zhī* 干支 day-count system commonly found in the Chinese calendars has often been cited. It is well known that the Mesopotamian numeral system was sexagesimal. There can be little doubt that the sexagesimal fractions of the Greeks and Alexandrians, as well as the division of the circle into 360° were derived from it. However, the claim that the *gān-zhī* system was also derived from the Babylonian system is based on speculation.

Needham has pointed out that 'the number of degrees of the old Chinese circle was $365\frac{1}{4}$, not 360' and that 'sexagesimal fractions never played any part in Chinese calculations.'[241] In fact, the *gān-zhī* system is fundamentally different from the Babylonian sexagesimal system, not only in function, but also in concept.[242] Unlike the latter, the *gān-zhī* system is not a system of cardinal numerals but an ordered cyclic system. In addition, the *gān-zhī* system was not derived from any numeral principle with base 60, but from two separate cyclic systems: the denary *gān* 十 system and the duodenary *zhī* 支 system (see p.213 to p.214).

From the names of the months from the Old Babylonian Reign (−19th to −16th century) and their arrangement on Babylonian clay tablets of c. −1100 unearthed at Assur,[243] we learned that the vernal equinox, in the Babylonian calendar, was fixed in the first month (the month of *Nisannu*) and the month was always taken to begin with the 'new moon', the first appearance of the crescent moon after sunset.

There is evidence indicating that the Old Babylonian calendar also made use of an intercalation procedure to keep the lunations in step with the seasonal year. In this respect, there are indeed similarities between the Babylonian and Chinese calendar. The intercalation procedure in Chinese civilization was first discussed in the *Yáo Diǎn* and probably based on an oral tradition from the Yáo period (c. −2400), and records of intercalation are found in the shell-bone inscriptions of the Shāng dynasty (−17th to −12th century). In the Babylonian civilization, records

240. See for example, J. Edkins (1885); H. Oldenberg (1909); and C. Bezold (1919).
241. Joseph Needham, (1959), vol. 3, p. 82.
242. Chén, Chēng-Yīh 程貞一 (1980).
243. B.L. van der Waerden (1974), pp. 47 and 64.

of intercalation are also found very early in the cuneiform tablets of the Hammerabi period (−1728 to −1686).

According to a document found in the texts of a Babylonian cuneiform tablet, Hammerabi (c. −1728 to −1686) made the following order:

> Since the year is not good, the next month must be noted as a second Ululu [the sixth month]. Instead of delivering the tithes to Babylon on the 25th of Tishritu [the seventh month], have them delivered on the 25th of Ululu II.[244]

This indicates that intercalation was certainly used by the time of Hammerabi. The implementation of intercalations appears, however, to have been carried out in an *ad hoc*, rather than a systematic manner. The short- and long-month scheme was probably used. No information has come down to us on the length of the seasonal year before the Assyrian period. To judge by the *Yáo Diǎn* and the shell-bone inscriptions, there was no significant difference between the pace of calendar development in Babylonia and China.

On the basis of systematic investigations of available Babylonian tablets containing records on intercalated months in chronological order,[245] van der Waerden has shown that between −528 and −503 the intercalation procedure consisted of the insertion of three intercalated months in every eight-year period (hereinafter 'the 3–8 intercalation procedure') as shown in Table 16.

Table 16. The Babylonian '3–8' Intercalation Procedure.

Period	Year with Intercalation[a, b]			#[c]
−528 to −521	2U	4A	7A	3
−520 to −513	2U	4A	7A	3
−512 to −505	2U	4A	7A	3
−504	2U			

[a] U indicates that an intercalated month was inserted in the middle of the year.
[b] A indicates that an intercalated month was inserted in the end of the year.
[c] # denotes the number of intercalations.

The procedure was further improved by the use of seven intercalated months in every nineteen-year period (hereinafter 'the 7–19

244. F.X. Kugler (1909), p. 253, see also A. Pannekoek (1961), p. 31.
245. R.A. Parker and W.A. Dubberstein (1956).

intercalation procedure') at the turn of the −5th century, as shown in Table 17.[246] From this table, it is seen that by the −5th century the Babylonians had indeed mastered the 7−19 intercalation procedure by inserting an intercalated month on the third, sixth, eighth, eleventh, fourteenth, seventeenth, and nineteenth year in each nineteen-year period. According to the extant tablets of the last Babylonian period, the procedure was still in use in the first century.

Table 17. The Babylonian '7−9' Intercalation Procedure

Period	Year with Intercalation[a, b, c]								# [d]
−500 to −482	1A	3A	6A	(8A)	11A	14A	17U	19A	8
−481 to −463		(3A)	(6A)	8A	11A	14A	17U	19A	7
−462 to −444		3A	(6A)	8A	11A	14A	17U	19A	7
−443 to −425		3A	(6A)	8A	11A	14A	17U	19A	7
−424 to −406		3A	(6A)	8A	11A	14A	17U	19A	7
−405 to −387		3A	6A	8A	11A	(14A)	17U	19A	7

[a] U indicates that an intercalated month was inserted in the middle of the year.
[b] A indicates that an intercalated month was inserted in the end of the year.
[c] ( ) indicates that no record has yet been found among the tablets.
[d] # denotes the number of intercalations.

Similar studies of Chinese intercalation procedures, based on the chronicle and astronomical data systematically kept in the *Chūn-Qiū* 《春秋》 (*Spring and Autumn Annals*) between −722 and −476, are discussed above (see p.114 to p.116) and the results of these studies are summarized in Table 11. It is seen that the 7−19 intercalation procedure was also developed in the Chinese civilization and was implemented systematically by the −6th century. By comparing Table 11 with Tables 16 and 17, it is also apparent that both civilizations underwent a period of uncertainty before the 7−19 intercalation procedure was worked out satisfactorily and implemented systematically. Even then, there were still differences in the pattern of spacing the intercalated months. Thus, we fail to see any Babylonian influence on the Chinese calendar.

Indeed, if a case is to be made for transmissions during this period, it is probably in the reverse direction, for the changeover from the 3−8 intercalation procedure to the 7−19 intercalation took place in the Babylonian calendar in a relatively short period between −504 and −482,

246. B.L. van der Waerden (1974), p. 103.

and during this period, the Chinese astronomers had already systematically implemented the 7–19 intercalation procedure for almost a century. Although intercalation is a man-made procedure, it is, however, a straightforward procedure. One cannot *a priori* eliminate the possibility of a separate invention and independent origins of the procedure. Thus, in the absence of direct evidence of transmission, I prefer to work on the premise that I am dealing with parallel developments. Further studies are apparently required before a conclusive statement can be made.

4 Early Scientific Ideas and Thoughts of Nature in Ancient China

天地與我並生, 而萬物與我為一.
Zhuāng Zǐ · Qí Wù 《莊子 · 齊物》
(*The Book of Master Zhuāng*)

Much has been said about the influence of the various philosophical schools on the development of science in ancient China. It is beyond the scope of this work to examine individually the influence of the various philosophical schools. Although the major schools differ greatly in their philosophical views of social and ethical values and in their approaches to the studies of natural phenomena, their basic scientific ideas and thoughts of nature at this early stage were remarkably similar.

A common interest of the early students of nature was the phenomena of changes in nature. One has, for example, from the *Lún Yǔ* 《論語》 (*Analects*) of the Rú Jiā 儒家 the statement:

> 天何言哉? 四時行焉, 百物生焉, 天何言哉?
> What does the *tiān* 天 say?
> The progression of four seasons!
> The generation of all things!
> What does the *tiān* 天 say?

Here the term *tiān* 天 denotes the totality of nature.[247] In the view of Confucius, the intrinsic value of human relations is to be patterned in

247. The term *tiān* has often been translated as 'Heaven' but in the Confucian thought system, *tiān* has no religious connotations, even though it was occasionally used for moral support. The character *tiān* in the *Lún Yǔ* 《論語》 has several basic meanings, including of course the meaning 'heaven', depending on the context in which it is used. In the context of this quotation, *tiān* denotes the totality of nature.

congruence with the cosmic order. It is, therefore, of importance to observe the 'orders' of nature. His interest in the cosmic order eventually led him to the study of the patterns of changes in nature. According to the *Lún Yǔ*, Confucius, in his later life, devoted much of his studies to *yì* 易, the changes. Indeed, the *Yì Jīng* 《易經》 (*Book of Changes*), with the addition of the appendices by the Rú Jiā 儒家, contains much of the ancient studies on the patterns of changes in nature and humanity.

From the Dàoist writings, we have from the *Dào-Dé Jīng* 《道德經》 (*Canon of the Virtue of Dào*), for example, the statements:

> 道可道, 非常道; 名可名, 非常名.
> A *dào* 道 that could be documented is not the everlasting *dào*.
> A *míng* 名 that could be termed míng is not the everlasting míng.

and from the *Zhuāng Zǐ* 《莊子》 (*The Book of Master Zhuāng*):

> 論萬物之理也, 物之生也, 若驟若馳.
> 無動而不變, 無時而不移.
> 何為乎, 何不為乎, 夫因將自化.
> Examining the principles of all things, [one notices that] things forever generate suddenly (*zhòu* 驟) and rapidly (*chí* 馳).
> There is no move without change and no moment without movement.
> Are these changes made intentionally? Or are they not?
> Assuredly, they are, as if, spontaneous changes.

These statements indicate that Dàoists also paid special interest to the phenomena of changes in nature and viewed these changes with different philosophical implications.

It is in the Mòhist work that we find studies of *yì* that went beyond just the patterns and general impressions of changes. They differentiated the physical changes from the type of changes resulting from changes in substances and termed the latter as *huà* 化. A definition of *huà* 化 given in the *Mò Jīng* 《墨經》 (*The Mòhist Canon*) is reproduced below:

> 化, 徵易也.
> *Huà* 化 [is the process] in which the basic characteristics (*zhēng* 徵) [of the entity] are changed (*yì* 易).

Through such a differentiation, the term *huà* gradually came to represent, in natural science, those changes that would now be classified as biological and chemical changes, even though, at that time, such types of changes had not yet been clearly identified and specifically defined. In the 'Exposition' section of the *Mò Jīng*, the example given as a type of *huà* change is the biological change of metamorphoses. The modern term for chemistry, *huà-xué* 化學, is obviously derived from the term *huà* for it means literally the study of *huà*.

The sample quotations given above serve to indicate that there were, among the various leading schools, a considerable interest in the

phenomena of changes in nature. It was through the study of such phenomena that most of the early scientific ideas and thoughts on nature were synthesized. In this section, I shall examine some of the early scientific ideas such as the *yīn-yáng* 陰陽 principle, the concept of *qì* 氣, and the *wǔ-xíng* 五行 classifications and comment on their influence on the development of natural science in Chinese civilization.

4.1 Some Basic Scientific Ideas

4.1.1 The Patterns of Changes in Nature

The constant changes in nature are phenomena early inhabitants must learn to recognize and to adopt. A description of their effort in observing and studying these phenomena is given in the *Yì Jīng* 《易經》:

> 仰以觀於天文, 俯以察於地理.
> Looking up one observes the *wén* 文 (phenomena) of *tiān* 天 (heaven);
> Looking down one examines the *lǐ* 理 (patterns) of *dì* 地 (earth).

It was through such studies of the *wén lǐ* 文理 of nature that early observers became acquainted with certain basic patterns in nature that precipitated their interests. Among them, the most prevailing patterns are the binary and cyclic patterns commonly found among processes in nature such as those in astronomy and biology. In the course of time, binary and cyclic processes gradually emerged as the fundamental processes in nature as abstract significance began to be associated with their patterns. It was intuitively desirable for the early students of nature to analyse various phenomena of changes in terms of these fundamental processes and their symmetry patterns.[248] Such an approach certainly was of interest, not only to natural philosophers, but also to a number of early scientists.

The Cyclic Processes

Obvious examples of cyclic processes in nature are, of course, the astronomical cycles, such as seasonal changes. Their regularity was vital to the early inhabitants of the earth. In the course of time, the regularity of such astronomical processes projected a sense of dependability and continuity. Thus, a cyclic pattern in a process was viewed as an indication that the process is capable of sustaining itself with renewed cycles at a fixed periodicity. In the course of time, concepts such as

248. Joseph Chēng-Yīh Chén 程貞一 (1989), pp. 68–71.

permanence and inexhaustibility began to be associated with idealized cyclic processes. Such abstractions of cyclic patterns occurred very early in Chinese civilization and became an important feature in the interpretation of the phenomena of changes in nature.

An early usage of cyclic patterns was in the construction of artificial cycles. Due to daily interaction with astronomical cycles, it is not surprising to find that artificial cyclical systems first appeared in the work of calendrical science. The artificial cycles introduced in the Shāng calendar, as revealed by the unearthed shell-bone inscriptions (jiǎ-qǔ-wén 甲骨文) dating back to a period from the −14th to the −11th century, were the ten-day *xún* 旬 and the sexagenary *gān-zhī* 干支 system (see p.116 to p.118). The cycles were introduced to facilitate the civil functions of the calendar by providing a reference day count common to all regions and periods, detached both from lunations and from seasonal changes. This was an important innovation of scientific merit in chronology.

It was probably the discovery of the cyclic behaviour of harmonic progressions in acoustic systems that stimulated additional interest in cyclic patterns. Recall that acoustics and astronomy were the two branches of natural science to first establish precise relations with mathematics, as discussed on p.76 and p.77, it was probably the discovery of similar cyclic behaviour in acoustical and astronomical systems that enhanced the early Chinese practice of associating acoustics with astronomy. Not only was special significance attributed to the similarities found in the cyclical behaviours of the two systems, acoustics and calendrical science, both enjoyed official recognition and were often presented together in historical records under the title 'Lǜ-Lì Zhì' 〈律曆志〉 in Chinese historical records.

In addition to interpreting physical phenomena, cyclic patterns were also introduced to interpret biological phenomena. We have, for example, from the *Zhuāng Zǐ* the following statements:

> 萬物皆出於幾, 皆入於幾.
> All things arise from *jǐ* 幾 (germs) and go back to *jǐ*.
>
> 萬物皆種也, 以不同形相禪.
> 始卒若環, 莫得其倫, 是謂天均.
> All things are species which can manifest, in different forms, into each other, *xiāng shàn* 相禪. Their beginnings and endings form a cyclic process, but their proper orders are difficult to ascertain. This is the rhythm of nature.
>
> 日方中方睨, 物方生方死.
> The sun at noon is the sun declining;
> the creature at birth is the creature dying.

In the last statement, we see that parallelism is drawn on both the cyclic

pattern and the relative time between astronomical and biological phenomena.

Applications of the cyclic pattern are also found within the *yīn-yáng* principle and the *wǔ-xíng* classifications. These applications are discussed on p.196 to p.197 and p.200 to p.202 respectively.

The Binary Processes

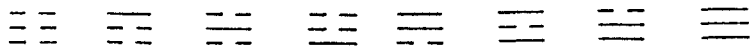
Another pattern found among the phenomena of changes in nature that acquired abstract significance to the early students of nature is the binary pattern. Very early in Chinese civilization, processes with a binary pattern were identified with those associated with *shēng* 生 (literally 'birth') as in generation and production, for it takes basically two entities to interact and to produce. The technical terms adopted to designate the two entities in such systems are the *yīn* 陰 and *yáng* 陽 characters. In the course of time, the binary *yīn-yáng* pattern became associated with concepts such as productivity and creativity.

The terms *yīn* and *yáng* have often been translated simply as 'female' and 'male'. However, the Chinese terms for designating genders are as follows:

Female	Male	Specifications
nǚ 女	*nán* 男	for human beings only
pìn 牝	*mǔ* 牡	for animals
cí 雌	*xióng* 雄	for birds
mǔ 母	*gōng* 公	for general use
yīn 陰	*yáng* 陽	for philosophical and scientific use

It is seen that the terms *yīn* and *yáng* are not used to specify gender in the biological sense.[249] They designate the two constituent elements that manifest the binary pattern in an abstract sense.

An early symbolic system constructed in terms of the binary pattern is the *guà* 卦 system in which the *yīn* and *yáng* elements are represented by two *yáo* 爻 symbols, known as the *yīn-yáo* 陰爻 — — and *yáng-yáo* 陽爻 —— . Taking n *yáos* at a time, the total number of combinations, allowing all possible permutations, is 2^n. We have, for example, by taking three *yáos* at a time, the trigram *guàs* 卦,



249. The term *yáng-mǎ* 陽馬 in mathematics, for example, represents a pyramid with a rectangular base and one lateral edge perpendicular to the base. The translation of *yáng-mǎ* as 'male horse' in a number of works by historians of mathematics is incorrect.

Each of the trigrams in the *guà* system symbolically represents a possible generation in a random three-stage binary process. I will return to the topic of *guà* on p.205 to p.213.

4.1.2 The Yīn-Yáng 陰陽 Principle

The *yīn-yáng* principle was an important scientific principle in Chinese civilization. It is based on concepts derived from binary processes. As described in the *Yì Jīng*:

> 一陰一陽之謂道.
> One *yīn* and one *yáng* constitute the *dào*.

In this *yīn-yáng* binary *dào*, 'what is *yīn*' and 'what is *yáng*' are not explicitly defined. They are identified through pattern recognition. Any two entities in a process exhibiting 'binary relationships' can be analyzed in terms of the principle. It is in the interpretation of the *yīn-yáng* binary relations that differentiates the significance of the principle and its applications.

To sustain the process, the interaction between the *yīn* and the *yáng* elements is often viewed to exhibit a cyclic pattern. Such a view is best stated in the following statements:

> 陽極反陰, 陰極反陽.
> As the *yáng* attains its maximum, it returns towards the *yīn*;
> As the *yīn* attains its maximum, it returns toward the *yáng*.[250]

> 陽還終始, 陰極反陽.
> The *yáng* moves cyclically between the end and the beginning, as the *yīn* attains its maximum and returns towards the *yáng*.[251]

Here, we see that, through the *yīn-yáng* interactions, the cyclic pattern is also incorporated into the principle. Such a synthesis makes the *yīn-yáng* principle accountable to both the binary and cyclic patterns, considered to be the most significant patterns in nature by the early Chinese students of nature.

It is important to note that in the synthesis the *yīn* and *yāng* cycles are not taken to be in phase with each other but out of phase by half a cycle. Thus, the *yīn* and *yāng* elements are in opposite configuration and their interactions are often described in terms of associated concepts such as balance and harmony. We have, for example:

> 陰淫寒疾, 陽淫熱疾.
> When *yīn* is in excess, one gets sick due to cold;
> When *yáng* is in excess, one gets sick due to hot.[252]

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250. See *Lùn Héng*《論衡》(*Discourse Weighed in Balance*), first century.
251. See *Guǐ Gǔ Zǐ*《鬼谷子》(*Book of the Devil Valley Master*).
252. See *Zuǒ Zhuàn*《左傳》(*Master Zuǒqiū's Commentary on the Spring-Autumn Annals*).

> 天地之和, 陰陽之調也.
> The harmony between the *tiān* and the earth is the result of balance between the *yīn* and the *yáng*.[253]

Applications based on considerations of balance and harmony are frequently found among works, for example, in alchemy, biology and medicine.

Such pattern-like descriptions of *yīn-yáng* interactions, as illustrated by the above examples, provide, however, no explicit information as to how the *yīn* and *yáng* elements interact. We need to investigate the early views on such interactions. Modern science has revealed that there are a number of binary systems in nature. Consider, for example, the system of electricity; the *yīn* and *yáng* elements would correspond to the negative and positive charges. In this case, the interaction between the *yīn* and the *yáng* elements can be expressed explicitly in terms of the properties of the charges and of their environment. Such a modern approach to interactions was, of course, not available in ancient times. In fact, at this early stage, no quantitative description of interaction was available at all. Even so, it is not difficult to appreciate the scientific merit of some of the mechanisms proposed for *yīn-yáng* interactions at this early stage.

A mechanism of particular importance visualizes the *yīn* and *yáng* elements to interact through an entity known as *qì* 氣. One of the best descriptions of such a type of *yīn-yáng* interactions is found in the *Zēng Zǐ* 《曾子》 (*The Book of Master Zēng*):

> 明者, 吐氣也, 是故外景.
> 幽者, 含氣也, 是故內景.
> 故火日外景而金水內景.
> 吐氣者施, 而含氣者化.
> 是以陽施而陰化也.
> The bright ones emit (*tǔ* 吐) *qì* 氣 and are, therefore, called *wài jǐng* 外景.
> The dim ones absorb (*hán* 含) *qì* and are, therefore, called *nèi jǐng* 內景.
> Hence, fire and sun are *wài jǐng* and metal and water are *nèi jǐng*.
> The one that emits is active (*shī* 施);
> and the one that absorbs is reactive (*huà* 化).
> This is the principle of *yáng* active and *yīn* reactive.[254]

This is an extraordinary description of interactions between *yīn* and *yáng* elements. It appears that the description is based on the concept of 'interaction at a distance'. From certain perspectives, the proposed mechanism of interaction through *qì* is remarkably modern. This is particularly true if one identifies *qì* with the modern concept of energy. Then, what is *qì*?

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253. See the 'Dà Yuè' 〈大樂〉 chapter of vol. 20 of the *Lǚ-Shì Chūn-Qiū* 《呂氏春秋》.
254. The work was incorporated in the *Zhōu Lǐ* 《周禮》 (*The Book of Zhōu Institutions*).

4.1.3 The Concept of Qì 氣

The character *qì* 氣 literally means vapour, or breath. In the course of time, it took on a certain proto-scientific significance and found its way into all branches of science. In the *yīn-yáng* principle, *qì* appears as the medium for the interaction between the *yīn* and the *yáng* elements. There are a number of scholars who translate *qì* as 'energy', 'field' or 'matter-energy'. Here we encounter the difficulties in translating an ancient concept discussed earlier in the introduction. The concepts of energy and field, as we understand them now in science, were certainly not developed in ancient time, let alone the matter-energy relations. However, as discussed below, the ancient Chinese considered *qì* to be the basic constituent of all things in nature and visualized it in many ways suggestive to the idea of 'energy' and 'field'.

The situation with the Chinese technical terms here is somewhat different from the Greek technical terms. Consider, for example, the Greek term 'atom', a concept advocated by Democritus as the basic constituent of things in nature. The concept of 'atom' has acquired additional meaning in modern science and is basically different from the early Greek concept of the atom. And yet, the term 'atom' is readily used for both. In fact, the connection between the Greek 'atom' with the modern 'atom' in concept is not much different from the connection in concept between the term *qì* and the modern terms 'energy' and 'field'. However, the term *qì* remains separately identifiable from the terms for 'energy' and 'field'. The problem only arises in the translation of the term *qì*. It is probably appropriate to use the term as it is and to not translate it.

Archaeological excavations revealed that using steam to cook began extremely early in Chinese civilization as evidenced by the unearthed Neolithic pottery cooking vessels known as *yǎn* 甗 (steamer).[255] Consequently, a certain appreciation of the dynamics of water-steam conversion must also have begun very early. In the course of time, it is recognized that, associated with the water-steam conversion, there is the process of transferring energy from fire to stream. In fact, an ancient form of writing the character *qì* 氣 is 氣, with the explicit appearance of the *huǒ* 火 (fire) radical. This early interest in water-steam conversion was probably one of the reasons that the term *qì* became associated with the

255. A *yǎn* 甗 is composed of two vessels, a *zèng* 甑 with a perforated floor surmounted on a *guàn* 罐 (pot) or a *lī* 甗 (caldron or pot with a tripod base) and a top cover. The earliest pottery *yǎn* (steamer) was unearthed in Bànpō 半坡, Xiàn 西安 from a Yǎngsháo 仰韶 cultural stratum, carbon dated to the c. −5th millennium.

phenomena of dynamic interactions and eventually led to the idea of visualizing *qì* as the media for *yīn-yáng* interactions.

Along another line of development, *qì* also acquired, in the course of time, the abstract significance of being 'vital' and began to be associated with the concept of *jīng* 精 (essence). Applications with interpretations along such a line are commonly found among works such as those in alchemy, biology and medicine. In the work of Zhuāng Zhōu 莊周, the phenomena of life and death are visualized as the conglomeration and dispersion of the *qì*. We have from *Zhuāng Zǐ* the passage:

> 生也死之徒, 死也生之始, 孰知其紀?
> 人之生, 氣之聚也.
> 聚則為生, 散則為死.
> 若死生為徒, 吾又何患?
> Life is the follower of death, and death is the predecessor of life;
> but who knows their order in the cycle?
> Man's life is due to the conglomeration of *qì*.
> The conglomeration of *qì* gives rise to birth,
> and dispersion of *qì* results in death.
> Since death and life thus attend upon each,
> why should I worry?

This passage also implies that life and death are parts of a cyclic process in nature.

It is in the work of Xún Qīng 荀卿 (−313 to −230) that we find the important discussion that *qì* is the basic constituent of all things. We have from the *Xún Zǐ* 《荀子》 (*The Book of Master Xún*) of −240:

> 水火有氣而無生, 草木有生而無知, 禽獸有知而無義.
> 人有氣, 有生, 有知, 亦且有義, 故最為天下貴也.
> Water and fire have *qì* but not life.
> Plants and trees have life but not perception.
> Birds and animals have perception but not the sense of *yì* 義.[256]
> Man has *qì*, life, and perception, and in addition, the sense of *yì*.
> Therefore, he is the noblest being on earth.

Here *qì* is visualized as the basic entity of all things in nature, whether they have life or not. This view of *qì* is of great scientific significance.

Needham has commented that Xún Qīng's classification is remarkably similar to the 'ladder of souls' of Aristotle.[257] But the concept of soul, or psyche (ψχη), has little in common with the concept of *qì*. In the 'ladder of souls', soul is used to differentiate living from non-living substance. Unlike the soul, the entity *qì* in the Xún Qīng's classification, is common to all things, including both living and non-living things.

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256. The character *yì* 義 has often been translated as 'righteousness', but it contains no connotation with the meaning of 'self-righteousness'.
257. Joseph Needham (1956), vol. 2, pp. 21–3.

Analysing this passage of Xún Qīng, together with the role of *qì* in *yīn-yáng* interactions discussed above, it is apparent, in light of modern science, that the entity *qì* could be interpreted to be related in concept with energy, field and matter-energy. However, *qì*, as it is given in the Chinese texts, is not identified by its intrinsic properties, but by its general relationship with others. One must be careful in translating *qì* as energy, field or matter-energy. *Qì*, as identified within the context given in the early Chinese texts, was indeed an important concept of great scientific merit.

Early applications of the concept of *qì* within the context of the *yīn-yáng* principle are discussed on p.205 and p.214 to p.217.

4.1.4 The *Wǔ-Xíng* 五行 Process-Classification

The *wǔ-xíng* 五行 classification of process was an early scientific idea that had an important impact, not only in early science, such as alchemy and medicine, but also in a number of pseudo-sciences, such as astrology and divination. It was also one of the earliest scientific systems that found its way into the Chinese socio-political arena as it was adopted to interpret the process of the rise and fall of the ruling house as being cyclic in nature, governed by the elements of the *wǔ-xíng* system.

The earliest extant record of the *wǔ-xíng* system is found in the 'Hóng-Fàn' 〈洪範〉 chapter of *Shàng Shū* 《尚書》 (*The Book of Documents*):

> 五行: 一曰水, 二曰火, 三曰木, 四曰金, 五曰土.
> 水曰潤下, 火曰炎上, 木曰曲直, 金曰從革, 土爰稼穡.
> *Wǔ-xíng*: 1. water, 2. fire, 3. wood, 4. metal, 5. earth.
> Water [is of the quality] that is soaking and descending.
> Fire [is of the quality] that is blazing and rising.
> Wood [is of the quality] that allows curving and aligning.
> Metal [is of the quality] that allows moulding and solidifying.
> Earth [is of the quality] that allows farming and harvest.

From this passage, it is apparent that the *wǔ-xíng*, as given in the 'Hóng-Fàn', is a list of five categories of qualities in nature, expressed in terms of five representative materials which are important and familiar to man. Thus, soaking and descending are qualities identified with water, while blazing and rising are qualities identified with fire. Similarly, qualities that allow curving and aligning are identified with wood, those that allow moulding and solidifying are identified with metal, and, finally those that allow farming and harvest are identified with earth.

It is important to note that these qualities, identified with the entities in the *wǔ-xíng* system, could only be manifested when the entities were functioning in their corresponding processes. This is to say that these qualities are as much the properties of the processes as those of the

entities. It is not difficult to see that the qualities of soaking and descending, identified with water, are also the properties of flow processes involving liquids and fluids, and the qualities of blazing and rising, identified with fire, are also the properties of combustion processes involving flaming and heat. Similarly, the qualities of wood that permit curving and aligning are the properties materialized in construction processes involving cutting and shaping, the qualities of the metal that permit moulding and solidifying are the properties transpired in metallurgical processes involving fabrication and congelation, and the qualities of the earth that permit farming and harvest are the properties manifested in agricultural processes involving planting and reaping.

Thus, *wǔ-xíng* should not be viewed as a system of five fundamental elements of matter.[258] It was an early attempt to classify the processes in nature. Unlike the binary and cyclic classifications discussed earlier, this classification is based, not on patterns, but on the properties of the processes. This was a scientific development that had an important impact on the progress of a number of early sciences in Chinese civilization. The five categories of processes identified in the *wǔ-xíng* system, namely the flow process, combustion process, construction process, metallurgical process, and agricultural process, are those which were of immense importance to the life of society in ancient times. The system was convenient and practical, for it makes use of familiar materials to identify and represent the processes.

Following the discussion of the *wǔ-xíng* system quoted above, the passage in the 'Hóng-Fàn' chapter of the *Shàng Shū* continues with a discussion on the association of the *wǔ-xíng* processes with descriptions normally used in describing the sensation of taste:

> 潤下作鹹, 炎上作苦, 曲直作酸, 從革作辛, 稼穡作甘.
> That which is soaking and descending causes saltiness.
> That which is blazing and uprising causes bitterness.
> That which allows curving and aligning gives sourness.
> That which allows moulding and solidifying gives acridness.
> That which allows farming and harvest gives sweetness.

It is not clear to what extent these associations with tastes were determined from chemical properties, but they are apparently subjective interpretations. It is, however, to these associations that one traces the beginnings of a branch of pseudo-science known as the *wǔ-xíng xué* 五行學 (the studies of *wǔ-xíng*) that flourished in the Hàn 漢 dynasty.

258. It has long been pointed out that the translation of the character *xíng* 行 which literally means 'walk', 'go', or 'motion' as 'element' has never been satisfactory.

In the *wǔ-xíng xué*, the elements of the *wǔ-xíng* system were used to correlate and classify a multitude of phenomena, many of which had little, if any, connection to the original classification. The elements of the *wǔ-xíng* system were also assumed to have certain mutual orders in such a way that they exhibited a cyclic pattern. At the height of such pseudo-scientific syntheses during the Hàn dynasty, the *wǔ-xíng* system was symbolically correlated with everything in the universe which could possibly be classified in fives.

Among the many applications of the *wǔ-xíng* system is the work of Zōu Yǎn 騶衍 on correlating the rise and fall of a ruling state with the elements in the *wǔ-xíng*. It was supposed that a ruling state derives its authority from the *dé* 德 (virtue), manifested from the corresponding element of the *wǔ-xíng*, in accordance with *wǔ-dé xiāng-shèng* 五德相勝, the principle of mutual conquest. We have:

> 五德之次, 從所不勝.
> 故虞土, 夏木, 殷金, 周火.
>
> The order of the *wǔ dé* 五德 (five virtues) is of such that each of them is followed by the one it cannot conquer.
> The dynasty of Yú [Shùn] 虞 [舜] [ruled by the virtue of] earth,
> the Xià 夏 dynasty [ruled by the virtue of] wood,
> the Shāng 商 dynasty [ruled by the virtue of] metal,
> and the Zhōu 周 dynasty [ruled by the virtue of] fire.

Here we see that the *wǔ-xíng* system is adopted to explain a socio-political phenomenon which has no relation at all to the original functions of the *wǔ-xíng* system. Even if the behaviour of the rise and fall of the ruling state was indeed cyclic in nature, Zōu Yǎn's interpretation and explanation would still be groundless rhetoric since there was no relation whatsoever between the *wǔ-xíng* and the socio-political phenomena of the rise and fall of a ruling state.

It is apparent that Zōu Yǎn made free use of symbolic correlation and intuitive speculation in his work. Sīmǎ Qiān 司馬遷 was very critical of Zōu Yǎn's work in his *Shǐ Jì* 《史記》 (*Historical Records*). The thought system of Zōu Yǎn was very different from that of the Mòhists. The mutual conquest principle postulated by Zōu Yǎn was criticized by the Mòhists:

> 五行毋常勝, 說在宜.
> The *wǔ-xíng* do not perpetually overcome one another.
> The reason is given under properness.

It is significant that the criticism was based on the lack of proper ground for Zōu Yǎn's claim.

4.2 Abstraction of the Processes of Nature

The basic scientific ideas and principles discussed above were not the products of any particular philosophical school. In fact they were, developed, in one early form or another, long before the coming of the philosophical schools. With the emergence of the philosophical schools, these basic scientific ideas were synthesized and utilized for the interpretation of the multitude of phenomena in nature, as well as in society and human behaviour. It was in such syntheses and utilizations of the basic ideas that separated the different philosophical schools. In this section, I shall examine how these early scientific ideas were used in the interpretation of the processes of nature, in particular, the production processes. This was a topic of immense interest to both early scientists and philosophers.

4.2.1 The Beginning of the Universe

The concept of the universe in Chinese civilization originated from the abstract concepts of space and time and is reflected by the term *yǔ-zhòu* 宇宙, set forth in the −4th century through the work of Mòhists and Logicians.[259] As discussed from p.1 to p.3, the term *yǔ-zhòu* literally represents a form of space-time continuum without bounds and limits. It was in this context that the concept was incorporated into the Xuān-Yè 宣夜 cosmological school (see p.156 to p.158) and later in the Hún-Tiān 渾天 cosmological school (see p.155 to p.156). This space-time view of the universe is significant not only because it is naturalistic but also because it is remarkably compatible with the modern concept of the universe.

A metaphysical view of the universe in its very beginning, as given in the *Dào-Dé Jīng* 《道德經》, is reproduced below:

> 有物混成, 先天地生;
> 寂兮寥兮, 獨立不改.
> 周行而不殆, 可以為天下母.
> 吾不知其名, 強字之曰道, 強為之名曰大.
> 大曰逝, 逝曰遠, 遠曰反.
> 故道大, 天大, 地大, 人亦大.
> 人法地, 地法天, 天法道, 道法自然.
> [In the beginning], before the existence of heaven and earth,
> there was complete chaos (*hùn* 混).
> Silent! Empty!
> Sufficient unto itself without change!

259. See for example, the *Shī Zǐ* 《尸子》 (*The Book of Master Shī*) and the *Mò Jīng* 《墨經》 (*The Mòhist Canon*).

> Revolving incessantly, never exhausted.
> Well might it be the 'mother' of all things under heaven.
> I know not its name, and *dào* is the courtesy-name given.
> If I were forced to classify it, I should call it great.
> Great is said to be that which is expanding and far-reaching,
> and yet capable of regenerating itself.
> Therefore, there are *dào* the great,
> heaven the great, earth the great, and man the great.
> Men are conditioned by earth, earth by heaven, heaven by *dào*,
> and *dào* by *zì-rán* 自然 (the spontaneity of itself, i.e. nature).[260]

In this description, the universe, before the existence of the earth and heaven, is visualized as being in a state of complete chaos (*hùn*). This chaotic state is viewed as a natural state without supernatural elements, again a confirmation of scientific naturalism. It was from this chaotic state that the universe entered a state of order in which all things were produced in accordance with the spontaneous *dào*, the 'mother' of all things. The *dào* was visualized as possessing abstract cyclic properties so that it had inexhaustible energy and the ability to regenerate repeatedly.

The ontological question on the existence of such states is discussed in a highly abstract form. We have, from the *Dào-Dé Jīng*, the statements for the term *wú* 無 (non-existence, or nothingness) and term *yǒu* 有 (existence, or actuality):

> 無, 名天地之始;
> 有, 名萬物之母.
> The term *wú* 無 (non-existence) designates 'the beginning of heaven and earth'.
> The term *yǒu* 有 (existence) designates 'the mother of all things'.

From these designations, it appears that the state of chaos, before the existence of heaven and earth, is perceived as *wú*, the state of non-existence, and the state of production of things, in accordance with the *dào*, is perceived as *yǒu*, the state of existence. In terms of the *wú* and *yǒu* states, the production of all things in nature is restated in the *Dào-Dé-Jīng* as below:

> 天下萬物生於有. 有生於無.
> All things under the heaven are produced from *yǒu* (existence), and *yǒu* (existence) is produced from *wú* (non-existence).

Influenced by the nature of cyclic processes, Dàoists assumed the state of non-existence and the state of existence are also cyclic:

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260. In this quotation, I replaced the character *wáng* 王 with the character *rén* 人 based on the content of the sentence and the sentence that followed it. The suggestion, that *wáng* is a misprint of *rén*, has been made by a number of scholars of Chinese classics. For a discussion, see for example, Yán Líng-Fēng 嚴靈峰 (1968), pp. 23–5.

有, 無相生.
Yǒu and *wú* are capable of mutual generation.

The cyclic relationship between the *yǎo* and *wǔ* states proposed by the Dàoists was not accepted by the Mòhists. In the Mò Jīng, we have the following discussions:

無不必待有, 說在有無.
Non-existence is not necessarily dependent on the existence.
The reason is given under the existence of the non-existence.

可無也, 有之而不可去, 說在嘗然.
There can be nonexistence.
Once [nonexistence] existed, it cannot be done away with.
The reason is given under former existence.

From these statements, it is seen that Mòhists considered that the state of non-existence could exist independent of the state of existence. Once it exists, it has consequences.

4.2.2 The *Dào-Shēng-Yī* 道生一 Model

Based on extant documents, a number of models were constructed to interpret the cosmogonical question of how things in the universe were produced in accordance with *dào* in the state of *yǒu* 有. From the *Dào-Dé Jīng*, we have the *dào-shēng-yī* 道生一 model of the Dàoists:

道生一, 一生二, 二生三, 三生萬物.
萬物負陰而抱陽, 冲氣以為和.
The *dào* produced one, the one produced two,
the two produced three, and the three produced all things.
All things backed by the *yīn* and embraced by the *yáng*
and saturated with *qì* in a state of *hé* 和 (harmony).

Here, one sees that the model is based on the *yīn-yáng* principle. Things are produced by the *yīn-yáng* interaction saturated with *qì*. The production pattern described in the *dào-shēng-yī* model projects a relation of linear progression.

4.2.3 The *Tài-Jí* 太極 Model

In view of the significance attached to the binary pattern, it is not at all surprising that one should find a production model that projects a binary progression. The *tài-jí* 太極 model, found in the *Yì Jīng* 《易經》 (*Book of Changes*), projects such a progression:

是故易有太極, 是生兩儀,
兩儀生四象, 四象生八卦.
Therefore, in the *yì* 易, there is the *tài-jí*, which produces *liǎng yí* 兩儀 (two entities) (i.e. the *yīn* and *yáng*).

> These two entities produce *sì xiàng* 四象 (four symbols), and the four symbols produce the *bā guà* 八卦 (eight trigrams).[261]

It is apparent that the pattern of progression in this model is in the form of 1 to 2, 2 to 4, and 4 to 8, which projects a binary pattern. The *tài-jí* model appears in the 'Xì-Cí Zhuàn' 〈繫辭傳〉, an appendix of the Yì *Jīng* considered to be the work of the Rú Jiā 儒家.[262]

The term *yì* 易, in the *tài-jí* model, refers to processes of changes responsible for the production of all things in nature. We have from the *Yì Jīng* the statements:

> 易, 無思也, 無為也, 寂然不動,
> 感而遂通天下之故.
> Yì, without thought and action,
> communicates (*suì tōng* 遂通) through mutual influence (*gǎn* 感),
> in silence without motion,
> the causes of all things under heaven.
>
> 天地感而萬物化生.
> The mutual influence between heaven and earth gives rise
> to the transformation and production of all things.

Here the interaction involved is again visualized to be spontaneous and capable of taking place at a distance.

On a closer examination of the *tài-jí* model, one sees that they are actually open statements. The only thing that is explicit in the model is the binary expansion. This is apparent because the elements, at all stages of the expansion, namely the *liǎng yí*, the *sì xiáng*, and the *bā guà*, as well as the *tài-jí* itself, are not explicitly defined. In fact, the *tài-jí* model, as it is given in the *Yì Jīng*, may be viewed as an algebraic statement in the form of a 2^n binary expansion, for n = 0 to 3. This is perhaps best illustrated in terms of the symbols that became, in the course of time, identified with these elements, as shown in Figure 57. The algebraic nature of the binary expansion is evident since the elements at each stage of the expansion are open elements and may be variously substituted in accordance with the process under consideration.

Evidently, the *tài-jí* model, just like the *dào-shēng-yī* model, is also based on the *yīn-yáng* principle. But incorporated in the *tài-jí* model, in addition, there is the mathematics of the binary expansion. Though the *tài-jí* model did not succeed in explaining the production processes in

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261. There are several translations of this passage available in the literature. See for example, Richard Wilhelm, *I Ging; Das Buch der Wandlungen* (Diederichs, Jean, 1924), Eng. tr. C.F. Baynes (Bollingen-Panthenon, New York, 1950).
262. It is of interest to note that the term *yì* 易 and the *Yì Jīng* 《易經》 (Book of Changes) are not at all mentioned in the *Dào-Dé Jīng* 《道德經》 (*Canon of the Virtue of Dào*).

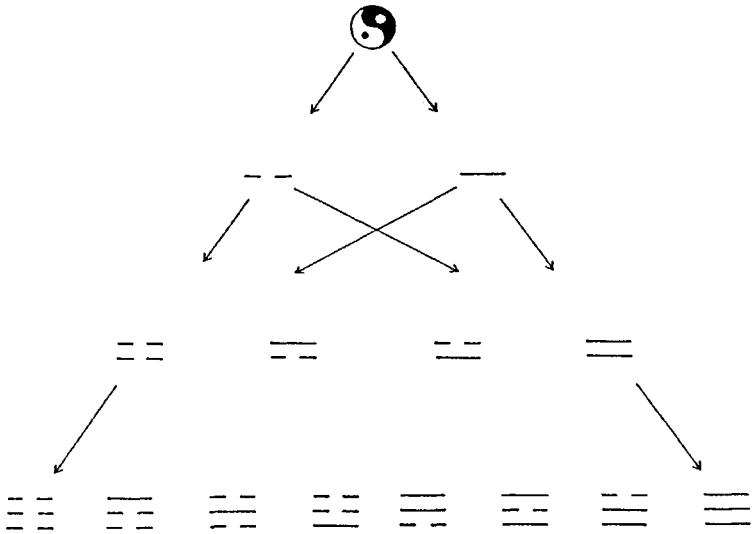

Figure 57. The binary pattern of production processes in the *tài-jí* model for the processes of nature.

nature, the model was, nevertheless, a remarkable synthesis.[263] Its use of binary symmetry and mathematical relations at this early stage of conceptualization of nature is still of great interest to the history of science. One must acknowledge that the production processes in nature, unlike topics such as acoustics and astronomy, is a much more comprehensive and heterogeneous topic. Even in modern science, there is not yet a unified theory capable of accounting for the production of different things in nature.

4.2.4 The *Dà-Yǎn* 大衍 Procedure

An early philosophical view in Chinese civilization is that man and nature coexist in harmony and every individual is ultimately his own source of authority. Accepting that processes in nature occur spontaneously and are ruled by chance, the early Chinese were intensely interested in predicting and anticipating events of nature as well as the outcome of man's action. An ancient practice to predict such events was *bǔ guà* 卜卦, a type of divinatory procedure used as an intuitive way to consult nature. The problem with *bǔ guà* lies, of course, in the fact that the probability predicted by the procedure has no relation to the

263. It is of interest to note that a zygote (fertilized egg cell) undergoes a binary expansion process in cleavage.

probability for the occurrence of the event. A seemingly rational argument, in ancient times, was that if the *tài-jí* model contains the truth for the production processes in nature, then, a *bǔ guà* procedure for consulting nature should simulate the model as closely as possible. The *dà-yǎn* 大衍 procedure given in the *Yì Jīng* is just such a *bǔ guà* procedure devised to simulate the *tài-jí* model as one consults the nature.

After the divinatory rhetorics are removed, the *dà-yǎn* procedure may be stated as follows:

1. Divide the total of 49 sticks into two portions arbitrarily and then remove one stick from one of the two portions
2. Count the sticks in each portion by fours, and then remove the remainders. (For the portion by fours evenly, take the remainder to be four.)
3. Take the remaining sticks (i.e., the total number of sticks minus those removed) to be the new total.
4. Repeat the procedure two more times.[264]

In current algebraic notation, the *dà-yǎn* procedure may be written in terms of the following equations:[265, 266]

$$x = 4n_1 + R_1 + 1, \qquad 4 \geq R_1 > 0 \tag{13}$$
$$y = 4n_2 + R_2, \qquad 4 \geq R_2 > 0 \tag{14}$$

where n_1 and n_2 are positive integers, R_1 and R_2 are the remainders to be removed, and the sum, $x + y$, is the total number subject to the procedure.

Beginning with the number 49, we have $x + y = 49$. Equations (13) and (14) then reduce to

$$4n + R = 48, \qquad 8 \geq R > 0 \tag{15}$$

where $n \equiv n_1 + n_2$ and $R \equiv R_1 + R_2$. Equation (15) yields two solutions:

$$n = 10, \qquad R = 8;$$
$$n = 11, \qquad R = 4.$$

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264. The *dà-yǎn* procedure is given in the 'Xì-Cí Zhuàn'〈繫辭傳〉 of the *Yì Jīng*. For the reader's convenience, the original Chinese passage is reproduced below:
大衍之數五十, 其用四十有九.
分而為二, 以象兩; 掛一, 以象三;
揲之以四, 以象四時, 歸奇於扐, 以象閏.
五歲再閏. 故再扐而後掛.
For a discussion of the procedure, see Shih-Chuan Chen 程石泉, 'How to Form a Hexagram and Consult the *I Ching*,' *Journal of the American Oriental Society* 92, 237 (1972).
265. Chén Chēng-Yīh 程貞一 (1980).
266. Luó Jiàn Jīn 羅見今 (1987), pp. 89–102.

This gives two possible paths, the path given by the first solution with the number 40, and the path given by the second solution with the number 44. The procedure is then repeated, with one of the two numbers, to generate the next possible paths. From number 40, one obtains the paths with numbers 32 and 36, and from number 44, one obtains the paths with numbers 36 and 40, as illustrated in Figure 58. It is significant that the *dà-yǎn* procedure allows a random generation of the paths, and the paths, so generated, simulate the 2^n binary expansion (see Fig. 58). This simulation of the 2^n binary expansion was probably intentional.

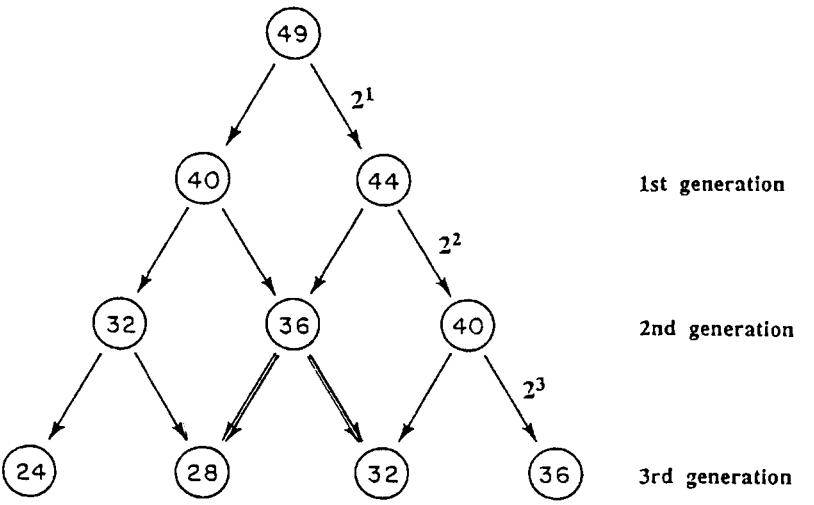

Figure 58. The 2^n binary expansion pattern generated by the *dà-yǎn* procedure for consulting the *tài-jí* model.

A question that is of interest is whether the inventor of the *dà-yǎn* procedure was aware of the mathematical content of his procedure. There is no question that the procedure was devised with the desire to simulate the binary expansion of the *tài-jí* model. But this can be accomplished, within the scheme of the procedure, by simply counting the sticks by fours without dividing the sticks into two portions. However, the presentation of the procedure was purposely made intriguing by allowing the division of the total number of sticks arbitrarily into two portions. To foresee the possibility of such a division necessitated an understanding of the mathematical content of the procedure.

There is no question that such a consultation for the prediction of events has no scientific basis, and the *dà-yǎn* procedure belongs to the pseudo-science of divination. The fact that *bǔ guà* provides no prediction

was certainly also realized by many of the early scholars. But the practice was, nevertheless, accepted for its historical and cultural significance. This was probably best stated by Xún Qīng 荀卿. We have from the *Xún Zǐ*《荀子》:

> 卜筮然後絕大事, 非以為得求也, 以文之也.
> To make important decisions after a divination is not because it provides the required prediction but because it is a cultural expression.

This realistic view expressed by Xún Qīng was shared by many of his contemporary as well as later scholars.

4.3 Comments and Evaluations

The brief account of the early scientific ideas and thoughts of nature, as presented above, is intended only as an illustration of the aspects of early Chinese work in science, which was of a more general nature than those dealing with specific topics, such as astronomy or acoustics. The interest here is in the influence of these early scientific ideas on the subsequent development of natural science in ancient China. In this section, I shall first discuss the influence of the scientific ideas discussed above on the progress of natural science for a few specific topics. Following this discussion, I shall then examine the influence of the *Yì Jīng* on the progress of natural science in ancient China.

4.3.1 The Mathematization of the Binary and Cyclic Patterns

The recognition of the cyclic and binary patterns for processes in nature was certainly an important first step toward an interpretation of the phenomena of changes in nature. It provided not only a physical basis for the analysis of the processes but also certain rudimentary mathematical properties for specifying the processes. It is of interest to elaborate on some of the progress resulting from the study and application of these patterns.

The Binary Permutations and Binomials

The study of binary patterns led to the discovery of a combinatorial principle for a binary system[267] which, in modern notation, may be stated as the following:

> For a binary system, the total number of combinations of n binary elements, allowing all possible selections and permutations, is 2^n.

267. Chén Chēng-Yīh 程貞一 (1980).

This principle was known, if not in theory at least in practice, since correct combinations of permutations were worked out for the n = 1 to the n = 6 *guà* 卦 system, corresponding to the 2-*guà* (monogram) to the 64-*guà* (hexagram) cases. As already discussed above, parts of the 2^n numbers were explicitly incorporated in the *tài-jí* model. This is probably one of the earliest known principles in combinatory analysis from an ancient civilization.

In 1703, Leibniz pointed out that the binary number system is contained in the Chinese binary *guà* system.[268] This is apparent if one takes the *yīn-yáo* 陰爻 — — symbol to be that for the numeral 0, and the *yáng-yáos* 陽爻 —— symbol to be that for the numeral 1, as illustrated for both the trigram and hexagram *guà* in the following examples:

䷀ $(1 \times 2^2) + (1 \times 2^1) + 0 = 6$

䷅ $(1 \times 2^2) + (0 \times 2^1) + 1 = 5$

䷄ $(1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + 1 = 58$

䷩ $(1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + 1 = 37$

Thus, all the *guà* symbols can indeed be interpreted as binary numerals in the scale of 2. This is not at all surprising since binary numerals are a manifestation of the general binary concept and the *guà* system is an abstract representation of the combinatorial properties of the binary concept.

In view of Leibniz's own statement, 'qui se sert des seuls caractères 0 et 1, avec des remarques sur son utilité, et sur ce qu'elle donne les sens anciennes figures chinoises de Fohy,' there were many debates among historians of science on the question of whether Leibniz's work on binary numerals was inspired or influenced in any way by the Chinese system of *guà*.[269] It is known that the binary numerals were first discussed by Leibniz in 1679, but a full account of the numerals was not published until 1703, after a long period of correspondence (beginning no later than 1697)[270] with the Jesuit Joachim Bouvet on the *Yì Jīng*.

The fact that the binary numerical principle is a part of the general binary concept represented by the *guà* does not necessarily imply that the early Chinese knew about the numerical properties of the *guà*. On the other hand, even if the binary numerical principle in the *guà* was not recognized by the early Chinese, it would not prevent a reader of the *Yì*

268. G.W. Leibniz (1703).
269. For a discussion of these debates, see Joseph Needham (1956), vol. 2, pp 340–5.
270. The *Yì Jīng* was available in Europe long before 1679 (see for example, the work of Nicolaus Trigault in 1626), but there is no known record indicating that Leibniz knew the book before 1697.

Jīng from realizing, or even gain further insights of, the binary numerals through the *guà* systems. The point that is of interest to us here is not so much the debate itself, but the observation that the *guà* system is an abstract representation of a correct binary combinatorial principle and the appreciation of its scientific merits depending on the user and in the context in which it is used. This is contrary to the claims of many sinologists and historians of science that the Chinese *guà* system was in itself inhibitive to the scientific development in ancient China. In fact, the *guà* system was of great importance, for example, to the early progress of mathematics in Chinese civilization.

An important contribution of the *guà* 卦 system to mathematics is the role it played in the discovery of the binomial coefficients.[271] Through the analysis of the structure of *guàs*, the ancient Chinese discovered certain distinct number patterns in the classifications of the *guàs* in term of combinations of the *yīn-yáng yáos*. For the trigram *guàs*, one has the following classifications:

☰ ☰	1	three *yīn-yáos*
☱ ☳ ☲	3	two *yīn-yáos* and one *yáng-yáo*
☴ ☶ ☵	3	one *yīn-yáo* and two *yáng-yáos*
☷	1	three *yáng-yáos*

For the hexagram *guàs*, one has, omitting the diagrams, the following classifications:

1	six *yīn-yáos*
6	five *yīn-yáos* and one *yáng-yáo*
15	four *yīn-yáos* and two *yáng-yáos*
20	three *yīn-yáos* and three *yáng-yáos*
15	two *yīn-yáos* and four *yáng-yáos*
6	one *yīn-yáo* and five *yáng-yáos*
1	six *yáng-yáos*

Such classifications of *guàs* in terms of *yáos* are found in the *Guà-Biàn Tú* 卦變圖 among various works on the *Yì Jīng*.

A significant progress was made by Jiǎ Xiàn 賈憲 (fl. c. 1010–25) of the Northern Sòng 宋 who developed the *zēng-chéng* 增乘 method for root-extraction by using the binomial coefficients. Jiǎ Xiàn's tabulation of these binomial coefficients is reproduced in Figure 59 from Yáng Huī's 楊輝 *Xiáng-Jiě Jiǔ-Zhāng Suàn-Fǎ* 《詳解九章算法》 (*A Detailed Analysis of the Mathematical Methods in the Nine Chapters*) of 1261.[272] The work of Jiǎ

271. Chén Chēng-Yīh 程貞一 (1980).
272. According to Yáng Huī (fl. the thirteenth century) in his *Xiáng-Jiě Jiǔ-Zhāng Suàn-Fǎ* of 1261, the triangular tabulation of the binomial coefficients came from Jiǎ Xiàn in the *Shì-Suǒ Suàn-Shū* 《釋鎖算書》 (*Mathematics Book on the Method of Unlocking by Binomial Coefficients*), a lost book probably by Jiǎ Xiàn.

Xiàn was of major importance in numerical equations since the work broke free from the traditional geometrical interpretation of root-extraction and set the stage for the development of numerical equations of higher degrees by the thirteenth-century mathematicians such as Qín Jiǔ-Sháo 秦九韶 and Yáng Huī.

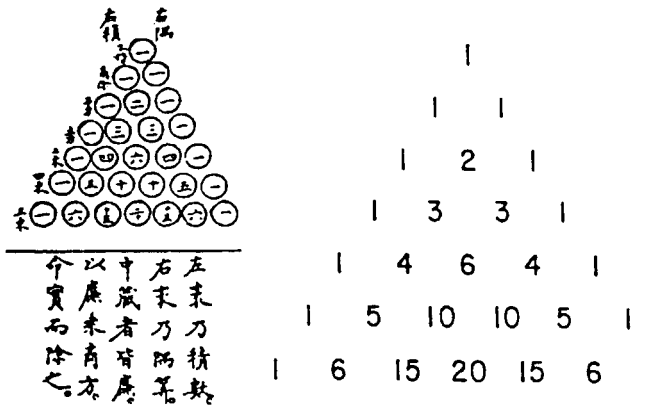

Figure 59. The triangular tabulation of binomial coefficients reproduced from Yáng Huī's 楊輝 *Xiáng-Jiě Jiǔ-Zhāng Suàn-Fǎ* of 1261, copied from ch. 16344 of the *Yǒng Lè Dà-Diǎn* 《永樂大典》 of 1407. In the text, it is noted that this tabulation (left) came from the *Shì-Suǒ Suàn-Shū* 《釋鎖算書》 and Jiǎ Xiàn 賈憲 made use of the method.

The Sexagenary Cycles and Combinatory

The study of cyclic patterns led one naturally to the concept of periodicity and continuity, as well as to associated concepts such as oscillation and balance. As mentioned earlier (p.116 to p.118), a major artificial cyclic system, invented for use in calendars, is the *gān-zhī* 干支 system with a periodicity of 60, first appeared in the Shāng 商 calendars for a day count. The *gān-zhī* system was constructed from the combination of the denary *gān* 干 system with the duodenary *zhī* 支 system. The construction of the *gān-zhī* system in itself is of interest to combinatory analysis.

If we denote the denary *gān* system and the duodenary *zhī* system by [A] and [B] and their elements by a_i, and b_i, respectively,

$$[A] = [\, a_1, a_2, \ldots, a_{10}], \qquad [B] = [\, b_1, b_2, \ldots, b_{12}],$$

then, the sexagenary *gān-zhī* system is defined by the combination of [A] and [B] as follows:

$$[AB] = [a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7, a_8b_8, a_9b_9, a_{10}b_{10}, a_1b_{11}\ a_2b_{12}, a_3b_1, a_4b_2, a_5b_3, a_6b_4, a_7b_5, a_8b_6, a_9b_7, a_{10}b_8, a_1b_9, a_2b_{10}, a_3b_{11}, a_4b_{12}, a_5b_1, a_6b_2, a_7b_3, a_8b_4, a_9b_5, a_{10}b_6, a_1b_7, a_2b_8, a_3b_9, a_4b_{10}, a_5b_{11}, a_6b_{12}, a_7b_1, a_8b_2, a_9b_3, a_{10}b_4, a_1b_5, a_2b_6, a_3b_7, a_4b_8, a_5b_9, a_6b_{10}, a_7b_{11}, a_8b_{12}, a_9b_1, a_{10}b_2, a_1b_3, a_2b_4, a_3b_5, a_4b_6, a_5b_7, a_6b_8, a_7b_9, a_8b_{10}, a_9b_{11}, a_{10}b_{12}]$$

This demonstrates that the system was derived in accordance with a predetermined combinatorial procedure. The *gān-zhī* system is probably the earliest extant example of a cyclic system derived by a combinatorial procedure.

4.3.2 The *Yīn-Yáng* 陰陽 Principle and the Concept of *Qì* 氣

The *yīn-yáng* principle is basically a symmetry principle, formulated specifically for the interpretation of binary processes. As it stands, it is a very general principle, permitting a range of interpretations and applications. In retrospect, one can easily see that the difficulties encountered by the principle was not in the basic concepts of principle but in the identification and interpretation of the *yīn-yáng* entities that constitute the binary system. It was the lack of a concrete criterion for identifying the *yīn-yáng* entities that caused the failure of the principle in some of its applications. This is partly a consequence of the fact that the principle is specified in terms of pattern relations of the entities rather than the intrinsic properties of the entities.

In analysing the various applications of the *yīn-yáng* principle, it is not difficult to see that, in many applications, the *yīn-yáng* entities were identified subjectively. On p.22, we have already encountered such an application in acoustics. In this case, the twelve pitches were subjectively identified as composed of six *yīn* pitches and six *yáng* pitches, known later as the *liù lǜ* 六律 and *liù lǚ* 六呂, respectively.[273] This lack of a concrete criterion for identifying the *yīn-yáng* entities certainly contributed to the impression that the *yīn-yáng* principle is an *ad hoc* principle and was inhibitive to the development of science in Chinese civilization. Such a view is based, however, not on historical facts, but on comparisons of the principle with modern 'progress' in science.

It should be noted, first of all, that modern 'progress' was not in existence before the seventeenth century. The history of science has revealed that science and technology enjoyed a long period of intermittent progress in Chinese civilization before the start of the

273. This identification which permits one to visualize the chromatic scale as composed of two hexatonic scales could be significant in musicology.

scientific revolution in Europe. Since the *yīn-yáng* principle was the dominant scientific principle during this period, one must, therefore, conclude that the progress of science in ancient China was, in fact, made under the explicit influence of the *yīn-yáng* principle. Viewed from its proper time frame, the *yīn-yáng* principle was indeed very advanced for its time. It was probably one of the earliest principles that incorporated into its interaction mechanism the properties of the symmetry of the system. The scientific merit of the principle depends on the context in which it is applied. This is perhaps best appreciated by explicit examples.

Let us examine how the natural phenomena of lightning was interpreted in terms of the *yīn-yáng* principle by the early Chinese. As mentioned in connection with the interpretation of thunder on p.29 and p.30, lightning is viewed to be that produced by *diàn* 電 (*electricus*) and thunder by *zhèn* 震 (shaking).[274] This interpretation is consistent with the term *shǎn-diàn* 閃電, for it visualizes lightning as the flash (*shǎn* 閃) of light given by *diàn*.[275] The identification of *diàn*, as the entity that gives rise to lightning, is certainly an important initial step toward an understanding of the phenomenon, but without a direct specification of the *diàn* itself, the identification is no more than a rhetoric. The question that one needs to specifically address is: what is *diàn*?

From the *Zhuāng Zǐ*, one has the following statement:

> 陰陽分爭故為電.
> The mutual contest (*zhēng* 爭) of the *yīn* and *yán* gives rise to the *diàn* (*electricus*).

Here, *diàn* is visualized as the entity produced by the mutual contest action in the *yīn* and *yáng* interactions. Such an interpretation, paraphrased in a number of different ways, is repeatedly found in Chinese works (see p.29 and p.30). From these accounts, the basic view of the early Chinese on the phenomenon of lightning can be summarized as follows: lightning comes from the *diàn* and *diàn* is produced through the *yīn-yáng* interactions.

It is from the *Yuè Jì* 《樂記》 (*Record of Ritual Music*), quoted earlier

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274. The terms *zhèn* 震 and *diàn* 電, as well as terms *léi* 雷 and *tíng* 霆, are also found in the *Shī Jīng* 《詩經》 (*Book of Odes*) as, for example, in 曄曄震電, 不寧不令, and 喤喤煒煒, 如霆如雷, respectively, but the relations between *léi* and *zhèn* and between *tíng* and *diàn* are not given.

275. Contrary to the general impression that *shǎn-diàn* 閃電 is a modern term for lightning, the term *shǎn-diàn* appeared very early in Chinese classics. It is found, for example, in the 'Zhǎng-Sūn-Shèng Zhuàn' 〈長孫晟傳〉 of the *Suí Shū* 《隋書》 (*The History of the Suí Dynasty* [581 to 617]), (636 to 656), as in the statement 聞其弓聲, 謂為霹靂, 見其走馬, 稱為閃電.

(p.30), that one finds the concept of *qì* being explicitly mentioned as the intermediate of the *yīn-yáng* interactions. This description is of particular interest, not only because it explicitly mentions *qì*, but also because it introduces into the *yīn-yáng* production mechanism for *diàn*, the action of *mó* 摩 (*yīn yáng xiāng mó* 陰陽相摩) which means literally 'rubbing'. It is of interest to note that the action of *mó* is a type of contact interaction which is basically different from the interaction at a distance involving the *qì*. Here, we see the emergence of further details on the production process for *diàn* in the *yīn-yáng* interactions.

With the development of the modern sciences of electricity and magnetism, the character *diàn* is now identified with 'electricity' just as the character *cí* 磁, from the name *cí-shí* 磁石 (magnetite), is identified with 'magnetism'. These identifications led naturally to the association of the *yīn* and *yáng* counterparts, with the positive and negative charges in the electricity and with the north and south poles in the magnetism. In addition, it also led to the association of the concept of *qí* in the two cases with the concept of an electric field and a magnetic field, respectively. Along with these identifications, one is then led to the following nomenclatures for the sciences of electricity and magnetism:[276]

yīn diàn-hè	陰電荷	negative charge
yáng diàn-hè	陽電荷	positive charge
yīn diàn-zǐ	陰電子	electron
yáng diàn-zǐ	陽電子	position
diàn-qì chǎng	電氣場	electric field
cí-qì chǎng	磁氣場	magnetic field
diàn-qì xué	電氣學	electricity
cí-qì xué	磁氣學	magnetism

Not until 'modern revolutions' against so called '*yīn-yáng* superstitions' in China did the terms *yīn*, *yáng*, and *qì* begin to disappear in these scientific terms. These same scientific terms, nevertheless, are still used in Japan.

In the above discussion, I used the adverb 'naturally' to qualify the identification of the various concepts and terms between *diàn* 電 and electricity and between *cí* 磁 and magnetism. This should not be interpreted as to imply that the modern sciences of electricity and magnetism were known to ancient Chinese. The concepts of electric and magnetic fields, the nature of electric and magnetic interactions, and the

276. The magnetic compass, an early Chinese invention, played an important role in first revealing a connection between electricity and magnetism, which led eventually to the development of electromagnetism by scientists such as Michael Faraday (1791–67) and James Clerk Maxwell (1831–1879). The connection was first discovered by Hans Christian Oersted in 1820, as he accidentally noticed that an electric current in a wire can affect a magnetic compass needle.

discoveries of electron and positron are all modern developments. They had nothing to do, whatsoever, with the *yīn-yáng* principle. But what is relevant here is that the pattern relations of *yīn* and *yáng*, as well as of *qì*, used to describe the entities and their interactions in the *diàn* and *cí* though vague, fit into the general patterns found in electricity and magnetism. This implies that views coming from the *yīn-yáng* principle, however incomplete and obscure, are not incompatible with the modern views, and may be directly improved. Had the modern science of electricity and magnetism been first developed in China, this terminology would naturally be adopted. The *yīn-yáng* principle cannot be considered as being inhibitive to the development of science in Chinese civilization. In fact, the principle provided an extremely favourable beginning towards the studies of nature. It is the failure of the later workers to improve the principle by providing further criteria for the identification of the *yīn-yáng* entities and by further investigating the nature of the *yīn-yáng* interactions that failed the principle.

4.3.3 The *Yì Jīng* 《易經》 and Natural Science

In the investigation of interfacing between philosophy and science in Chinese civilization, the *Yì Jīng* is often taken as the major work for analysing such a relation. In this section, I shall consider the effect of the *Yì Jīng* on the development of science in ancient China. Before doing this, it is useful to first present a very brief outline of the book.

The *Yì Jīng*, in its present form, consists of the 'Běn Jīng' 〈本經〉 ('Canonical Text') and 'Shí Yì' 〈十翼〉 ('Ten Wings') sections as listed below:

'Běn Jīng' 〈本經〉 ('Canonical Text')
- *guà* 卦 *tuàn* 彖.
- Yáo Ci 爻辭 (Appended Judgements)

'Shí Yì' 〈十翼〉 ('Ten Wings')
- 'Tuàn Zhuàn' (I, II) 〈彖傳〉(上, 下).
- 'Xiàng Zhuàn' (I, II) 〈象傳〉(上, 下).
- 'Xì-Cí Zhuàn' (I, II) 〈繫辭傳〉(上, 下).
- 'Wén Yán' 〈文言〉.
- 'Shuō Guà' 〈說卦〉.
- 'Xù Guà' 〈序卦〉.
- 'Zá Guà' 〈雜卦〉.

The 'Canonical Text', probably the oldest component of the book, is primarily a text of divination, containing a collection of omens and their attributed significances. It is in the 'Ten Wings', appended at different times, that philosophical interpretations of patterns of changes in nature are found. In these 'Ten Wings', there is the theoretical *tài-jí* model devised

for the interpretation of production processes in nature (see p.205 to p.207) and the philosophical conceptualizations of nature and humanity.

A Commentary on the Views of the Yì Jīng

There are more comments written on the effect of the *Yì Jīng* in the development of scientific thought in ancient China than actual studies of scientific thought systems in ancient China. It is beyond the scope of this book to provide an in-depth analysis of these comments. I will, however, examine a few selected comments from some of the well-known scholars.

From the *Science and Civilisation in China* series by Needham and his collaborators, one finds the following comments:

> I fear that we shall have to say that while the five-element and two-force theories were favourable rather inimical to the development of scientific thought in China, the elaborated symbolic system of the *Book of Changes* was almost from the start a mischievous handicap. It tempted those who were interested in nature to rest in explanation which were no explanations at all. The *Book of Changes* was a system for *pigeon-holing* novelty and then doing nothing more about it. Its universal system of symbolism constituted a stupendous *filing-system*. It led to stylization of concepts almost analogous to the stylization which have in some ages occurred in art forms, and which finally prevented painters from looking at Nature at all.[277]

Needham is certainly correct to visualize the system of sixty-four *guà* used for categorizing natural phenomena, as a stupendous cosmic filing-system.

Needham's view that 'it [the filing-system] tempted those who were interested in Nature to rest in explanations which were no explanations at all' appears, however, in direct contradiction with the views offered, for example, by Mote who observed that:

> *I Ching* has remained a bottomless well from which each age of Chinese thought has draw provocative insight.[278]

and by Masson-Oursel who visualized the *guà* system of categorizing natural phenomena to be a dictionary:

> It supposes a kind of translation of all natural phenomena into a mathematical language by means of a set of graphic symbols, germs of what Leibniz would have called a 'universal character'; thus constituting a dictionary permitting men to read Nature like an openbook, whether with intellectual or practical aims in view.[279]

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277. Joseph Needham (1956), vol. 2, p. 336.
278. F.W. Mote (1971), p. 15.
279. P. Masson-Oursel (1912, 1916).

The contradictions among these different views arise partly from the explanations of natural phenomena that one expected from the *Yì Jīng*. But the *Yì Jīng* is much more a philosophical book than it is a scientific book. It offers, in fact, no specific scientific explanations of natural phenomena but a systematic view on how natural phenomena are interrelated in terms of their patterns. It is the pattern symmetries, inter-pattern harmonies, and inter-process resonances, manifested by the systematic view of pattern relations, as well as the system of symbolism presented in combinatorial and algebraic mathematical forms, that have exerted the most sustaining, pervasive and stimulating influence on Chinese thought. From this respect, the views of Mote and Masson-Oursel are probably more compatible with the role played by the book in the development of scientific thought. What the *Yì Jīng* has offered is not specific explanations but sparks of ideas for thought.

We now come to the famous passage from the eminent sinologist James Legge, who, speaking apparently from personal experience, wrote:

> Chinese scholars and gentleman who have got some acquaintance with western science are fond of saying that all the truths of electricity, light, heat, and other branches of European physics are in the eight trigrams. When asked how then they and their countrymen have been and are ignorant of those truths, they say that they have to learn them first from western books, and then, looking into the *I*, they see that they were all known to Confucius more than two thousand years ago. The vain assumption thus manifested is childish, and until the Chinese drop their hallucination about the *I* as containing all things that have ever been dreamt of in all philosophies, it will prove a stumbling-block to them, and keep them from entering upon the true path of science.[280]

Needham, after quoting this passage, commented that:

> These words were written nearly a century ago, but now the pendulum has swung far in the opposite direction; indeed, the history of science in Asia suffers greatly from the fact that extremely few Chinese scientists have any time to spare for examination of what they regard as the follies of their own medieval ages.[281]

These two observations reported a century apart by Legge and Needham demonstrate how the perception of the *Yì Jīng* could change. This is expected since the intellectual and scientific background of the readers could vary greatly and, in addition, their views could be conditioned and swayed by a number of factors. It is, therefore, difficult for one to draw conclusions on the *Yì Jīng*, such as Legge did, based on the reactions of a few scholars that one had encountered.

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280. J. Legge (1899).
281. Joseph Needham (1956), vol. 2, p. 336.

On the other hand, the influence of a book does depend on its readers, for only through its readers could a book exercise its influence. To draw conclusions on the effect of the *Yì Jīng*, one should carry out, in addition to the evaluation of the book's merits, case studies of the influence so that the book's effects could be explicitly documented in their case histories. A case study of this nature will be presented below, but, before its presentation, I will first discuss the merit of the *tài-jí* model.

The Tài-Jí 太極 Model and Its Interpretations

One of the unique features of the *Yì Jīng* that was of interest to both philosophers and scientists is the theoretical *tài-jí* model. A presentation of the model is given on p.205 to p.207, where a discussion of its mathematical and scientific contents is presented. Here, I examine the model based on the interpretations from the point of view of the West. The *Yì Jīng* was transmitted to Europe shortly after the arrival of the Jesuits in China. The Latin translation of the *Wǔ Jīng* 《五經》, which includes the *Yì Jīng*, first appeared in Europe in 1626, in the work of Nicolaus Trigault. Discussions of the *Zhōu Yì* 《周易》 and 64 *guà* can also be found in the *Confudius Sinarum philosophus* by Henricus Coulpet published in 1687.

Matteo Ricci, before he arrived in Beijing in 1601, devoted much of his time in China to the study of Chinese classics in order to familiarize himself with Chinese culture. His Latin translation of the *Sì Shū* 《四書》 was completed and sent back to Europe in 1593. In his studies of the Chinese classics, Ricci interpreted the *tài-jí* as the 'material prima of the Scholastics'.[282] Based on this interpretation, John Rodriguez, later in 1631, made the following comments on the *tài-jí* model:

> According to the Occidental interpretation, the Great Ultimate (i.e. the *tài-jí*) is something material, is matter without intelligence and without consciousness. Unless there is the infinite, omnipotent, wise, and intelligent factor, how could it ever produce things?[283]

This comment is based essentially on the premise that, without supernatural power, it is not possible to create things. The fact that the

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282. See Fonti Ricciane, *Storia dell'Introduzione del Cristianesimo in China*, pt. II (Rome, 1949), p. 297, note b sqq. (ed. Pasquale M. d'Elia)
283. These comments are from a letter written by John Rodriquez in 1631. The letter is still preserved at the library of the University of Seoul and has been published in the *Shigaku Zasshi* 《史學雜誌》 (*The Historical Journal of Japan*), vol. XLIV. The translation in English was made by Rufus Suter and Matthew Sciascia, see Pasquale M. d'Elia, *Galileo in China* (1960), p. 43.

tài-jí model does not contain elements of creationism was a source of difficulty for Rodriguez. But the *tài-jí* model deals with the production of things from the *tài-jí*, and made no comment on the origins of *tài-jí* itself. Consequently, the basic view that things are produced spontaneously in nature according to certain basic principles is independent of the origins of the *tài-jí*. This view is consistent with that of modern science. Taking the big bang model of universe as an example, the basic view is that immediately after the big bang, the universe grew spontaneously governed by certain basic principles. The model tells us that the average density of matter is decreasing as the universe grown older, but it does not tell us anything about the original core of matter from which the big bang began.

The merit of the *tài-jí* model lies in its basic assumption that production of things in nature are spontaneous and can be interpreted in terms of certain basic principles. This is different from the views of the early creationists. At the time when the *Yì Jīng* found its way into Europe, the scientific revolution was in its beginning stage. It was the period that one saw the publication of the *Sidereus nuncius* (*Starry Messenger*) in 1610 by Galileo based on observations that he made with his redesigned telescope. And it is also the period that one saw the movement of scientific naturalism and the struggle to be free from the Church's view of science.[284] One can, therefore, appreciate how a production model based on the claim that all things are produced spontaneously without supernatural power would be received by the West.

In explaining the possible reason why Leibniz, in April 1701, sent a table of his binary numerals to Bouvet in China, with whom he had a long period of correspondence on the *Yì Jīng*, Needham offered the following explanation:

> He had done this because he attached religious and mystical significance to the binary arithmetic. 'All combinations arise from unity and nothing, which is like saying that God made everything from nothing, and that there were only two first principles, God and nothing.' Leibniz hoped that the Chinese might be induced to accept Christianity by such quasi-mathematical demonstrations.[285]

This explanation is of interest for it revealed how readily an idea can be recast into a different context.

Leibniz's interpretation of the binary production, in terms of nothing

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284. It is relevant to note that, at this junction, neither Galileo's *Discourses Concerning Two New Sciences* (1638) nor Newton's *Philosophiae Naturalis Principia Mathematica* (1687) were in existence.

285. Joseph Needham, *Science and Civilisation in China* (The University Press, Cambridge, 1956), vol. 2, p. 341.

and unity, can be visualized as a combination of the concept of *yīn* and *yáng*, with the concept of *wú* 無 (nonexistence) and *yǒu* 有 (existence), for the production processes in nature. By the identification of 'God' with the *yáng* symbol —— for unity as the state of *yǒu*, and 'nothing' with the *yīn* symbol — — for zero as the state of *wú*, Leibniz was able to recast the production model in naturalism into a production model in creationism. But, in addition to the binary mathematics in the symbols, the *tài-jí* model is based on the concept that the production processes are spontaneous, governed by binary dynamics of the *yīn-yáng* principle, requiring no infinite, omnipotent, wise or intelligent factor. It is these latter concepts, not included in the Leibniz's consideration, that were, in fact, the strength of the *tài-jí* model, and it is with these features that the Chinese thought system are identified. As noted by Jung[286] and by Mote,[287] the ancient Chinese minds with their unique conceptualizations have led to the development of cosmology and cosmogony which appear closer to the explanations offered by modern physics than those we find in myths and religious systems.

Qīn Jiǔ-Sháo 秦九韶 *and the Yì Jīng*

One way to examine the impact of the *Yì Jīng* on the development of science in China is to carry out specific case studies. The thirteenth-century mathematician, Qīn Jiǔ-Sháo 秦九韶 (1202–61), constituted a convenient case study for this purpose because of the availability, not only of his own work on the *Yì Jīng*, but also of his own comments on this topic.[288]

From the preface of his book entitled *Shù Shū* 《數書》 (*Mathematics Treatise*)[289] published in 1247, we have the following passage:

> 周教六藝, 數實成之,
> 學士大夫, 所從來尚矣.
> 其用本太虛生一, 而周流無窮.
> 大則可以通神明, 順性命.
> 小則可以經世務, 類萬物.
> 詎容以淺近窺哉!

286. See the forward by Carl Jung in Richard Wilhelm's *I Ging; Das Buch der Wandlungen* (Diederichs, Jean, 1924), Eng. tr. C.F. Baynes (Bollingen-Panthenon, New York, 1950).
287. F.W. Mote, *Intellectual Foundations of China* (Alfred A. Knopf, New York, 1971), p. 19.
288. Chēng-Yīh Chén 程貞一, 'The *Yì Jīng* and Qín Jiǔ-Sháo, A Case Study', Paper presented at the 'Fifth International Conference on the History of Science in China', held at the University of California, San Diego, August 5–10, 1988.
289. The book become known later as *Shù-Shū Jiǔ-Zhāng* 《數書九章》 (*Mathematics Treatise in Nine Sections*).

> The teaching of the six arts of the Zhōu [dynasty] can be considered as being complete due largely to the inclusion of mathematics. Mathematics has long been regarded highly by scholars and high officials. The function of mathematics originated from the *tài-xū-shēng-yī* 太虛生一 (i.e., the unity generated by *tài-xū* 太虛) and found unlimited applications. To its full extent, mathematics can be used to comprehend the miraculous universe and to understand the process of life. To a less extent, mathematics can be used to manage business and to analyse various things. How can one bear to let mathematics be viewed as plain and shallow?

From this passage, it is clear that Qín Jiǔ-Sháo, just like many other mathematicians of his period, as well as of earlier periods such as Liú Huī 劉徽, the well-known commentator of the *Jiǔ-Zhāng Suàn-Shù* 《九章算術》 (*Mathematics in Nine Chapters*), believed that the first principle of mathematics originated from *tài-xū* 太虛, an alterative name for *tài-jí*. This view was undoubtedly influenced by the *Yì Jīng* and the belief that the system contains the ultimate germs of truth in nature.

The passage also shows that Qín Jiǔ-Sháo not only placed great importance on mathematics, but he also had a very broad view of mathematics. His view that mathematics, in addition to managing business and to analysing various things, can be used to comprehend the miraculous universe and to understand the process of life was probably also influenced by the *Yì Jīng*. This broad view led him to the important supposition that:

> 數與道非二本也.
> Mathematics and *dào* are not from two different bases.

This is a premise of great scientific merit, very advanced for his time. This general view of the relations between mathematics and *dào* was probably also a consequence of the *Yì Jīng*'s influence.

From the work of Qín Jiǔ-Sháo, it is apparent that Qín actually devoted much of his research to the mathematics of the *Yì Jīng*. Because of his advanced view on the relation between mathematics and *dào*, he was able to analyse *dào* from a pure mathematical point of view. He pointed out that.

> 獨大衍法不載九章, 未有能推之者.
> 曆家演法頗用之, 以為方程者誤也.
> [Among the various mathematics,] only the rules of *dà-yǎn* is not contained in the *Jiǔ-Zhāng* [i.e. *Jiǔ-Zhāng Suàn-Shù* 《九章算術》], no one has yet been able to derive it. Calendar-makers, in carrying out their computations, had made considerable use of it. Those who considered it as belonging to *fāng-chéng* 方程 (tabulation of equations) were definitely wrong.

The *dà-yǎn* 大衍 rules referred to here are what is known in modern mathematics as the indeterminate analysis of the first degree. Libbrect[290] recently made a comparative study of the *dà-yǎn* rules with other methods, both ancient and modern, in particular the methods of Euler (1742) and Gauss (1801), and concluded that Sarton did not exaggerate when he called Qín Jiǔ-Sháo 'one of the greatest mathematicians of his race, of his time and indeed of all times.'[291]

At the end of the preface, Qín Jiǔ-Sháo provided a description of the topic contents of each of the chapters in his *Shù Shū*. From the description of the content of the first chapter, we have:

> 聖有大衍, 微寓于易, 其餘取策, 羣數皆捐.
> 衍而究之, 探隱知原, 數術之傳, 以實為體.
> 其書九章, 惟茲弗紀, 蹙家雖用, 用而不知.
> 小試經世, 姑推所為, 述大衍第一.
> The sage had the *dà-yǎn* method but it is integrated in the *Yì* 易 [i.e. Yì Jīng]. Counting the sticks using the remainder procedure, a set (group) of numbers may be generated. I followed this *yǎn* 衍 procedure and examined its hidden subtleties in order to search for its principle. Mathematics should be presented in terms of its essence (the basic principle). In the *Jǐu-Zhāng Suàn-Shù*, the method is not described. Although the method was used by calendar-makers, they did not grasp its principle. I have examined the method and derived a certain understanding of it. The *dà-yǎn* method is presented in the first chapter.

This description provides further insight into the relations between the *dà-yǎn* rules (or method) derived by Qín Jiǔ-Sháo and the *dà-yǎn* procedure of the *Yì Jīng*, since Qín stated explicitly that he actually experimented with the sticks in his investigation of the *dà-yǎn* procedure.

As shown on p.208, the *dà-yǎn* procedure can indeed be expressed in terms of indeterminate equations, Equations (13) to (15), with certain constraints. The constraints are introduced to reduce the solutions of the indeterminate equations to a fixed number 2. Thus, the *dà-yǎn* rules derived by Qín Jiǔ-Sháo may very well be inspired by the old *dà-yǎn* divinatory procedure devised to simulate the *tài-jí* model in an attempt to consult nature.[292]

In a summary of the status of his achievement in mathematics, Qín Jiǔ-Sháo stated:

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290. Ulrich Libbrecht (1973), pp. 370–81.
291. G. Sarton (1947), vol. III, p. 626.
292. It is worth noting that a method, without being explicitly contained in the *Yì Jīng*, can actually be inspired or even derived from the *Yì Jīng*. Thus, the denial of the probable *Yì Jīng* influence on Qin's *dà-yǎn* rules, based only on the observation that the *dà-yǎn* rules are not explicitly contained in the *Yì Jīng*, is not necessarily valid.

> 信知夫物莫不有數, 乃肆意其間.
> 旁諏方能, 探索杳渺, 贏若有得焉.
> 所謂通神明, 順性命, 固膚末於見.
> I deeply believe that nothing does not require mathematics. I devote my thought to it. By learning from those who are gifted and by pondering and exploring the intricate details, it appears that I have attained some crude understanding. Regarding the comprehension of the miraculous universe and the understanding of the process of life, my view is still shallow without depth.

Here Qín Jiŭ-Sháo stated explicitly that as for the comprehension of the miraculous universe and to the understanding of the process of life, he made no progress. This indicates that even though he believed that mathematics is capable of helping men to comprehend the miraculous universe and the process of life, he did not accept the explanation of the *Yì Jīng*. Thus, the present case study attests to the fact that the *Yì Jīng* had a positive influence on Qín Jiŭ-Sháo, and, through Qín's work, a positive influence on the progress of science in Chinese civilization.

In fact, the thirteenth-century work of Qín Jiŭ-Sháo would constitute the second major known progress in mathematics coming from the influence of the *Yì Jīng*, the first being the eleventh-century work of Jiă Xiàn who developed the *zēng-chéng* 增乘 method for root-extraction, by using the binomial coefficients derived from the *guà* system (see p.212 to p.213 and Fig. 59).

Epilogue

The re-examination of early Chinese works in natural science presented here, though limited only to three specific topics (the physics of motion, acoustics and astronomy) and a brief account of early scientific ideas and thoughts of nature, serves to illustrate that the traditional characterization of early Chinese work in natural science requires substantial modification. The common claim that the physics of motion as being 'totally absent from Chinese physical thinking' is in direct conflict with the extant Chinese records. Not only the physics of motion was not absent in Chinese physical thinking but the Chinese approach to motion was highly theoretical and, in many ways, is compatible with the 'modern' approach. The common view that the Chinese work on acoustic as being 'highly empirical and non-analytic' is not supported either by the extant Chinese records or by recent archaeological discoveries. In fact, the ancient Chinese not only had a highly developed physical acoustics but also provided, in musical acoustics, the only known analytic method from antiquity which is capable of generating musical scales in the pentatonic, heptatonic and chromatic intonations. And in addition, it is highly probable that Chinese acousticians had already developed, in the –5th century, an analytic method of generating chromatic scales in the natural (just) intonation. The study of early Chinese works on astronomy also reveals that the common characterization of Chinese works on astronomy as 'lacking theory' is not valid (see p.159 to p.165). In many ways, the physical theories in the Chinese cosmological views are quite compatible with the 'modern' views.

Thus, the re-examination of the early Chinese works in natural science presented here serves to illustrate that the problem with the lack of early Chinese participation in the development of modern science and technology is not a consequence of the lack of early scientific traditions in ancient China as commonly assumed. The ancient Chinese had remarkable early achievements not only in mathematics but also in science. The early scientific ideas and thoughts of nature shared by the Chinese philosophical schools were also conducive rather than inhibitive

to the development of science. The decline in science and technology in China is not due to the failure of their culture but due to the failure of the post-Sòng 宋 Chinese to overcome setbacks caused by social, political and world events and to build upon the sound foundation laid down by their ancestors.

It is certainly true that science and technology in China were on the decline soon after the death of the early Yuán scholars such as Wáng Xún 王恂 (1235–81), Guō Shǒu-Jìng 郭守敬 (1231–1361), and Zhū Shì-Jié 朱世傑 (fl. 1268–1303).[293] This was a consequence caused by the long costly war between the Mongols and the Sòng and by the gross neglect of the successors to Yuán Shì-Zǔ 元世祖. Following this decline, science and technology in the Ming dynasty entered a long period of stagnation and it was at this juncture in the late sixteenth century that cross-currents between the East and West were brought on by the appearance of the Jesuits in China. At that time, even with the decline and stagnation in China, the difference between the East and West in the level of science and technological achievements was not significant. When Matteo Ricci first entered Běijīng in 1601, for example, Galileo's revolutionary work *Sidereus Nuncius* (*Starry Messenger*), which brought Europe to a new level of intellectual awareness, was not yet in existence. A pertinent question that one needs to investigate is what had happened in China during the crucial century that followed this initial encounter with the West? One needs to know what had prevented China from responding to the rapid changes in science and technology that took place in Europe in this crucial century? It is beyond the scope here to address these questions in depth, but a brief review of the subsequent events that occurred in the cross-currents in astronomy during this crucial century is instructive since astronomy was one of the major early fields that sparked the 'scientific revolution' and was also one of the major fields involved in the exchanges between the East and West.

When historians talk about cross-currents in astronomy between the East and West in the period that began in the late sixteenth century, the main topics are usually the heliocentricity of our world system and the Galileo telescope. Both these topics are from the West and their introduction in China was of great importance. Besides these two topics, there are two additional topics of equal significance, namely the empty infinite space-time world-view and the equatorial coordinate and

293. It has often been said that early science in China reached its heights in the Sòng Yuán 宋元 periods, but the Yuán dynasty produced no outstanding scholars in science based on its own tradition. The early Yuán scholars were essentially products of the late Song, based on the Sòng tradition.

mounting system. Both these two topics were well developed in ancient China long before the sixteenth century and their history and relationship with cross-currents have not been well studied. However, the adoption of the equatorial system and the abandonment of the solid-sphere world-view in favour of the empty infinite space-time world-view were of fundamental and crucial importance to the development of 'modern' astronomy in Europe. A significant question here is why, with the introduction of these new concepts and innovations, astronomy progress rapidly in Europe but not in China?

Much has been written about the manner in which the concept of heliocentricity was introduced in China by the Jesuits. There are even historians who consider that the deliberate hiding of this new concept from the Chinese by the missionaries was responsible for the setback in astronomical reform in China. I do not share such a view since the delay in the concept of heliocentricity was not as crucial as a number of other factors that affected the astronomical reform during this period.[294] Unlike in Europe, where the heliocentricity view was one of the major driving forces for astronomical reform, the situation in China was basically different since the Chinese view of geocentricity of the world system was not as interwoven with religious implications as in the West. There was much less pressure to reject concepts other than geocentricity in China than in the West. In practice, the heliocentric view of taking the sun to be at rest, despite its conceptual importance, is in fact mathematically equivalent to the geocentric view of taking the earth to be at rest, since it is a matter of coordinate transformation. In fact, once the full concept of heliocentricity appeared in China in the early nineteenth century, its acceptance was rather swift.[295]

The introduction of the Galileo telescope in China marked the beginning of the European influence on observational astronomy in China. The first reference to the telescope in Chinese literature is in the *Tiān Wèn Lüè* 《天問略》 (*Explicatio Sphaerae Coelestis*) of 1615 by Emanuel Diaz (Yáng Mǎ-Nuò 陽瑪諾).[296] In this reference, a mention was made of a *qiǎo qì* 巧器 (clever instrument) recently constructed by a famous Western scholar in astronomical science who was unsatisfied with the capability of one's eyes. With this instrument, a one-foot object at a

294. It is obvious that China could have been spared the Tychonic theory of the world view if the heliocentric theory of Copernicus was not held back by the missionaries during the early periods of contact with the West. But the delay in the concept of heliocentricity was not a major cause for the setback of astronomical reform in China.

295. One can, of course, cite some scholars in China, just as in the West, who oppose the heliocentric view of the world system.

296. See the appendix of *Tiān Wèn Lüè*.

distance of 60 mile can be seen as clear as if it were in front of one's eyes. The reference goes on describing a number of the observational discoveries made with the aid of this instrument, but the optical design of the instrument was not mentioned. It is in the *Yuǎn-Jìng Shuō* 《遠鏡說》 (*An Exposition of Telescope*) of 1626 by J. Adam Schall von Bell (Tāng Ruò-Wàng 湯若望, 1591–1666) that one finds descriptions of a telescope and its optical design.[297]

The response to the telescope and the acceptance of the discoveries of Galileo by the Míng 明 astronomical communities were very decisive and swift as evidenced by the speed and depth of the astronomical reform carried out under the leaderships of Xú Guāng-Qǐ 徐光啟 (1562–1633) and later Lǐ Tiān-Jìng 李天徑 between 1629 and 1635.[298] The construction of such a telescope was included in their astronomical reform. A formal proposal for the construction of three such telescopes was submitted to the Emperor Yì Zōng 毅宗, and support was granted in 1632. Despite Xú's untimely death in the following year (1633), a telescope was constructed in 1634 under Lǐ. Hence, within seven years after the first appearance of a description on the optical design of a Galileo telescope in China, the Míng astronomers were able to construct their own telescopes for astronomical observations. In view of the long observational tradition in astronomy maintained throughout the centuries in Chinese civilization, such an enthusiastic response to a new astronomical instrument was not unexpected.

From the *Chóng-Zhēn Lì-Shū* 《崇禎曆書》 (*Chóng-Zhēn Treatise on Astronomical Science*) (137 volumes) compiled by the Míng reformers between 1629 and 1634, it is evident that the Míng reform, to a certain extent, was a one-sided adoption of the views of the West without an appropriate evaluation of their merit and relationship to the astronomical views and methods in China.[299] The reform was nevertheless timely and important. Ever since the death of Guō Shǒu-Jìng 郭守敬 (1231–1316) in the Yuán dynasty, astronomy in China had entered into a long period of stagnation. Thus, the new methods brought to China by the missionaries were important stimuli for astronomical reform.

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297. This book completed in 1626 with the assistance of his Chinese colleague Lǐ Zǔ-Bái 李祖白 was published in 1630 and later incorporated (under a changed title) in the *Xī-Yáng Xīn-Fǎ Lì-Shū* 《西洋新法曆書》 (*Treatise on Calendrical Science according to the New Methods from the West*) of 1645. There are claims that this work of J. Adam Shall von Bell was based probably on the book *Telescopium* (Francoforte, 1618) by Girolamo Sirturi.
298. For a recent account of this astronomical reform, see Hashimoto Keizo 橋本敬造 (1988).
299. For oppositions to the one-sided adoption, see for example, the criticism of the solidsphere world-view by Kē Zhòng-Jiǒng 柯仲炯 in his *Xuān-Yè Jīng* 《宣夜經》 of 1628.

In retrospect, the adoption of the telescope by the Míng astronomers was indeed remarkably fast. This expeditious adoption placed China in a position capable of competing favourably with the West. This situation could be meaningful, since after the Chinese acquired the knowledge to construct their own telescope in 1633, no significant progress on telescopes was made in Europe for more than a quarter of a century. Other than Kepler's suggestion of using a convex lens for the eye-piece to broaden the range, the refracting telescope remained unchanged from the original Galileo form. Not until after the appearance of Huygens' design of a micrometer in 1659, did a potentially viable improvement of the telescope as a measuring instrument became feasible. The invention of a reflecting telescope by Newton was not made until 1668.

However, the reform initiated by Xú Guāng-Qǐ was abruptly interrupted by the destruction and turmoil resulting from the change of rulers. As the disruption caused by the overthrow of the Míng 明 dynasty by the Manchus gradually subsided, astronomy emerged in the Qīng 清 dynasty under the direct control of missionaries. Soon after the Manchus entered Běijīng in 1644, Schall submitted to the newly established Qīng government a shortened version of the *Chóng-Zhēn Lì-Shū* 《崇禎曆書》 under the new title *Xī-Yáng Xīn-Fǎ Lì-Shū* 《西洋新法曆書》 (*Treatise on Astronomical Science According to the New Western Methods*) (103 volumes) and stated:

> 臣創立新法, 規制儀象, 以測諸曜視行,...
> 臣閱曆寒暑, 晝夜審視, 著為新曆百余卷.
> Your servant established new methods and constructed accordingly instruments for the observation of the motions of planets and stars.... Your servant examined and observed, day and night, the calendrical changes of seasons and authored more than one hundred volumes on new calendrical science.

The work was approved by the Qing government and the Shí Xiàn 時憲 Calendar was then officially adopted. Within six months after the Manchus entered Běijīng, Schall was appointed to the position of director of the new Qīng Imperial Bureau of astronomy.

Schall's control of the bureau not only created strong conflicts with the bureaucratic intellectuals in the Qīng government but also was a setback to the revival of astronomical reform. For the next 20 years, Schall made no effort to upgrade astronomy in China beyond the level of the *Xī-Yáng Xīn-Fǎ Lì-Shū*. Instead, he promoted the outdated Tychonic method as the *xī-fǎ* 西法 (Western method), a science which he supposed could only be produced in a Christendom.[300] In his effort to promote the

300. Needham (1959) has commented (vol. 3, p. 449): 'The implicit logic was that only Christendom could have produced (the new science). Every correct eclipse prediction was thus an indirect demonstration of the truth of Christian theology'.

xī-fǎ, he would not create, in the Imperial Bureau of Astronomy, a division with title *Xī Kē* 西科 (Western Division) since it would place, in his view, the *xī-fǎ* on the same level as the Muslim Division (*Huí-Huí Kē* 回回科) already existing in the bureau.

Not until 1664, was the bureaucratic intellectuals led by Yáng Guāng-Xiān 楊光先 finally successful in bringing charges against Schall. Among the various charges were using the church for political purposes and plagiarism. To submit a shortened version of the *Chóng-Zhēn Lì-Shū* as his own was indeed an act of plagiarism, since the bulk of the work and measurements was done by the Míng astronomical reformers even though portions of the book, based on methods from the West, was translated and carried out with Schall's assistance. Schall was found guilty and removed from the directorship. By this time, Emperor Kāng Xī 康熙 (who succeeded the throne in 1662) was only ten years old and was not in charge of the affairs himself. In 1665, the following year, the directorship of the Bureau of Astronomy was given to Yáng Guāng-Xiān, who had little knowledge about astronomy and relied on Wú Míng-Xuǎn 吳明烜 of the Muslim Division of the Imperial Bureau of Astronomy for information on calendrical science.

Astronomical reform was not revived until 1669, one year after Kāng Xī assumed the emperor's duty himself in 1668 at the age of 15. The Yáng – Schall conflict brought to focus the ulterior motive of the missionaries for bringing Western methods to China. Kāng Xī certainly learned of this disclosure and decided to participate in the decision making. Soon after Kāng Xī assumed the emperor's duty, Schall's assistant, Ferdinand Verbiest (Nán Huái-Rén 南懷仁, 1623–88) who taught the teenage Emperor Kāng Xī mathematics and astronomy of the West, again brought the charge that the calendar issued by Yáng Guāng-Xiān was inaccurate. Kāng Xī decided to end the long standing rivalry by calling Yáng and Verbiest together to test the accuracy of their respective calendars. Little did the young Emperor know how meaningless the test was in relation to astronomy. As a result of the test, which showed that the Huí Huí 回回 calendar was less accurate than the Shí Xiàn calendar, Kāng Xī removed Yáng from the directorship and placed Verbiest in charge of calendrical reform. In retrospect, it was indeed very unfortunate that the able astronomers such as Wáng Xī-Chǎn 王錫闡 (1628–82) were not consulted.[301]

The Qīng astronomical reform, ordered by the young Emperor Kāng Xī, was basically different from that originally envisioned by the Míng

301. Wáng Xī-Chǎn 王錫闡 was critical of Schall's incorrect interpretations of Chinese work and also the inconsistencies in the methods and views of the West.

reformers. Much of the new characteristics reflected the policies and the direct involvement of the emperor himself. The most elaborate attempt in reforming observational astronomy in the Qīng dynasty was the refitting of the Imperial (Běijīng) Observatory. This important project was carried out under the directorship of Verbiest, commissioned by Emperor Kāng Xī in 1669. After six years of construction and a large budget, the observatory was finally re-equipped in 1674 with the following six astronomical instruments:[302]

1. Ecliptic armillary sphere, *huáng-dào jīng-wěi-yí* 黃道經緯儀.
2. Equatorial armillary sphere, *chì-dào jīng-wěi-yí* 赤道經緯儀.
3. Celestial globe, *tiān-tí-yí* 天體儀.
4. Horizon circle, *dì-píng jīng-yí* 地平經儀.
5. Quadrant, *dì-píng wěi-yí* 地平緯儀 (or *xiàng-xiàn-yí* 象限儀).
6. Sextant, *jì-xiàn-yí* 紀限儀.

To the six instruments, two additional instruments were later added.[303]

7. Quadrant altazimuth, *dì-píng jīng-wěi-yí* 地平經緯儀.
8. Elaborate equatorial armillary sphere, *jī-héng fǔ-chén-yí* 璣衡撫辰儀.

The quadrant altazimuth (7) was constructed by Bernard Kilian Stumpf (Jì Lǐ-Ān 紀理安) between 1713 and 1715 and the elaborate equatorial armillary sphere (8) was constructed in 1744 under the directorship of Ignatius Kögler (Dài Jìn-Xián 戴進賢) assisted by Augustin von Hallerstein (Liú Sōng-Líng 劉松齡) and Anton Gogeisl (Bào Yǒu-Guān 鮑友管), perhaps also by Antoine Gaubil (Sòng Jūn-Róng 宋君榮 and de la Charme (Sūn Zhāng 孫璋).

In retrospect, one cannot but be puzzled by the questionable selections and designs of the instruments used in equipping the Imperial Observatory. All the instruments were essentially traditional instruments relying on naked-eye observations. Except for the sextant, all the instruments were previously known in China and the constructional improvements found in these instruments were all of minor nature. It is also curious why the quadrant was not constructed in combination with the horizon circle. An obvious deficiency of the newly constructed instruments was the absence of optical aids, since none was equipped with an optical device and the telescope was totally absent. In view of

302. See for example, Joseph Needham (1959), vol.3, pp. 451–2.
303. According to a photograph of the observatory, there was a smaller celestial globe, *hún xiàng* 渾象, which later disappeared after 1920. This smaller celestial globe has not been positively identified. It could be the globe of Guō Shǒu-Jìng.

the early interest in the telescope and the achievement made by the Míng scholars in the construction of a telescope soon after its introduction in China, this was indeed unexpected. In comparison with the astronomical instruments developed in Europe during this period (from the seventeenth to eigteenth centuries), the instruments at the Imperial Observatory constructed under the direction of Verbiest and his subsequent successors were already substandard.

After completing his instruments in 1673, Verbiest also wrote the *Líng-Tái Yí-Xiàng Zhì* 《靈臺儀象志》 (*Records of the Observatory Instruments*) to describe the principle and use of these instruments.[304] The book also includes, at the end, a table of star measurements supposedly obtained by the newly constructed instruments. However, in comparison with the measurements listed in the *Xī-Yáng Xīn-Fǎ Lì-Shū*, one finds that the new values for the star latitudes are identical with those listed in the latter book and the values for the star longitudes are all different from those listed by the same value 37'.[305] Since the measurements listed in the *Xī-Yáng Xīn-Fǎ Lì-Shū* were actually made by the Míng astronomers in 1629, the value 37' in longitudes, no doubt, was introduced by Verbiest to account for the precession of the equinoxes. This implies that the measurements listed in the Verbiest's book were not new measurements, but based on those of the Míng astronomers first published in 1634 in the *Chóng-Zhēn Lì-Shū*. This is indeed incomprehensible! One of the primary purposes for the refitting of the Imperial Observatory was in fact to determine more accurately the positions of the stars.

In explaining the absence of the telescope, it has recently been pointed out that 'at that time the telescope was not yet suited to determine precisely the positions of the stars, which was the need of the Qīng government for improving a calendar.'[306] The assertion that telescopes at the time were not yet suited for precisely determining the positions of the stars was probably correct based on the telescopes known to Verbiest. However, the situation with telescopes in Europe was rather different from the time when Verbiest left Europe in 1658. By the time the Imperial Observatory was commissioned for refitting in 1669, improvements in telescopes were already made in Europe for large-angle measurements by the use of cross-threads (*miáo-zhǔn chā-sī* 瞄準叉絲) and

304. This work was later incorporated into the *Yù-Dìng Yí-Xiàng Kǎo-Chéng* 《御定儀象考成》 (*Complete Studies of Astronomical Instruments*) edited by Ignatius Kögler and Augustin von Hallerstein in 1744.
305. See Research Group of Chinese History of Astronomy (1981), p. 231 and footnote 1. The number of stars listed in Verbiest's *Líng-Tái Yí-Xiàng Zhì* is 1366, 14 more than that listed in the *Xī-Yáng Xīn-Fǎ Lì-Shū*, but this was due to misprint.
306. See Xí Zé-Zōng 席澤宗 (in press) 'Why F. Verbiest Did Not Make a Telescope'.

for small-angle measurements by the invention of the screw micrometer (*xuán cè-wēi-qì* 旋測微器). Through the use of the cross-threads, one can accurately measure the direction one sees the star by bringing the focal image of the star to the exact intersection of the cross-threads. With the direction correctly determined, the positions of the stars can then be better measured from its projection on the celestial sphere. By the use of a screw to control two parallel threads in a Huygens' micrometer, higher accuracy can also be achieved for short angular-distance measurements, such as angular diameters of planets. In addition, improvement in mounting telescopes was also being pursued.[307]

With the telescope totally absent from the instruments at the Imperial Observatory, the Qīng astronomers were not in a position to compete favourably with their counterparts in Europe in observational astronomy. In fact, three years before Emperor Kāng Xī even commissioned Verbiest to re-equip the astronomical instruments at the Imperial Observatory, European astronomers, such as Pichard (1620–89) and Flamstead (1646–1719), had already independently carried out systematic measurements of the angle diameter of planets with telescopes equipped with a screw micrometer. There were certainly many debates in Europe on the accuracy of measurements obtained using telescopes with respect to other devices relying on the naked eye. But it was through such telescopic measurements and debates that eventually led to a better understanding in Europe of the effects of atmospheric refraction and aberration of light in astronomical measurements. The failure to keep up with the progress in observational astronomy, in my opinion, was one of the crucial factors that affected the progress of astronomical reform in China.

It should be noted that the significance of a telescope lies not just as a measuring instrument. In fact, telescopes made most of its early contributions to astronomy as an observational instrument rather than a measuring instrument. This is best appreciated by examining the discoveries made by Galileo through the use of his improved telescope. Given below is Galileo's summary of his observations:[308]

1. The moon's surface is not smooth and has, just as the earth's surface, mountains, rivers, lakes and oceans.[309]

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307. The equatorial telescope mounting in Europe reached its final stage in the work of James Short (1732 to 1768). The invention of the equatorial mounting can be traced to Guō Shǒu-Jìng of the thirteenth century in China as evidenced by his *jiǎn-yí* 簡儀 (simplified instrument). In Europe, equatorial mounting first appeared in the instruments of Tycho Brahe (see the greater equatorial armillary of 1585).
308. See Galilei Galileo, *Sidereus Nuncius* (1610).
309. The interpretation that there are rivers, lakes and oceans on the surface of the moon based on his telescope observations was, of course, not substantiated.

2. The surface of the sun has black spots.
3. Planet Jupiter has four moons.
4. Planet Venus has phase variations.
5. The Milky Way is composed of many fixed stars.
6. The number of fixed stars observable increases with the magnifying power of the telescope, and the rate of size increase for stars is much smaller than for the sun and moon.

Obviously, none of these observations provided by the telescope was based on precision measurements, nevertheless they were of great importance. The impact of these observations was particularly significant in the West due to the contrary nature of the observations, not only with respect to the traditional view of the world system but also with respect to certain philosophical and religious beliefs. These added dimensions in philosophy and religion greatly enhanced the impact of Galileo's observations.

It is worth noting that with the observations that the number of fixed stars observable increase with the telescope's power, and the rate of size increase for stars is much smaller than for the sun and moon, Galileo had also observed contradictions to the solid-sphere world-view. However, he failed to recognize the observation as an indication that stars are not attached on a crystalline sphere with a fixed ratio. Consequently, he did not initiate the process to abandon the solid-sphere world-view. Even with this failed interpretation, it illustrated the potential power of a telescope as an observational instrument.

It is evident that the telescope had been crucial to the development of the new astronomy. After a great effort to acquire the knowledge to construct their own telescope in 1633 from the missionaries by the Míng astronomers, it is ironic only to find, in the Qīng dynasty, a total absence of telescopes among the astronomical instruments in the most important and influential observatory in China. This absence of telescopes at the Imperial Observatory was detrimental to the Chinese astronomers. It not only handicapped them from competing with their counterparts in Europe, but also denied them the opportunities to develop experience and intuition by working directly with telescopes. In retrospect, it is evident that one of the driving forces in 'modern' observational astronomy has been the intuitive notion that optical devices can be further refined to provide better resolution and higher accuracy for celestial observations and measurements.

An obvious question to ask is why the Chinese did not benefit from the knowledge of telescope that they acquired from the missionaries so early in their astronomical reform? One may argue that should the astronomical reform, as envisioned by the Míng astronomers, be allowed to continue in the Qīng dynasty, astronomy in China would probably

have kept better pace with the new progresses made in Europe. Such a hypothetical argument does not, however, provide any insights to the question of what caused the reform to change from that envisioned by the Míng astronomers. Aside from the disruptions caused by the Lǐ Zì-Chéng 李自成 revolt and by the Manchu conquest which evidently were beyond the control of the astronomical community, one needs to investigate the reasons why after these disruptions subsided, astronomical reform did not revive with the same vigour as exhibited by the Míng astronomical reform?

Obviously, the problem was not due to the delay in the appearance of the concept of heliocentricity in China. In fact, the disruption in the Imperial Bureau of Astronomy and the selection of astronomical instruments for refitting the Imperial Observatory had more of a negative impact than the delay in the concept of heliocentricity. From the perspective of vision and innovation, Verbiest's re-equipment (1669–74) of the Qīng Imperial Observatory commissioned by Emperor Kāng Xī 康熙 was indeed inferior to Guō Shǒu-Jìng's 郭守敬 re-equipment (1276–79) of the Yuán Imperial Observatory commissioned by Yuán Shì-Zǔ 元世祖. In the work of Guō Shǒu-Jìng, the observatory instruments were not only state of the art at that time, but were further enhanced with the addition of a new instrument, *jiǎn-yí* 簡儀 (simplified instrument, see Fig. 60), that he invented. In the work of Verbiest, on the other hand, not only were the observatory instruments not upgraded to the level of the contemporary European instruments, they were deprived of a telescope, one of the most important new instruments.

Though Verbiest had undoubtedly been exposed to certain education in science while studying at the Leuven University, and probably was also instructed in astronomy by the church in preparation for his visit to China, he was not, after all, an astronomer nor an instrumentation specialist. After arriving in Běijīng, he served as an assistant to Schall until Schall's death in 1666, three years before he was commissioned to refit the imperial observatory. Apparently he was not in close contact with new European developments in astronomical instruments. The work of Verbiest reflected well his ability as an instrumental astronomer. The two additional instruments added by later missionaries did not improve the situation. In fact, no telescope was later added to the observatory. This prolonged absence of telescopes at the observatory was detrimental to the progress of observational astronomy.

In addition to the instruments constructed by the missionaries, the relationships between the missionaries and the Qīng astronomical community were also not conducive for rapid development of astronomy. It should be noted that the missionaries gained the official position of the Director of the Imperial Bureau of Astronomy, based not on their own

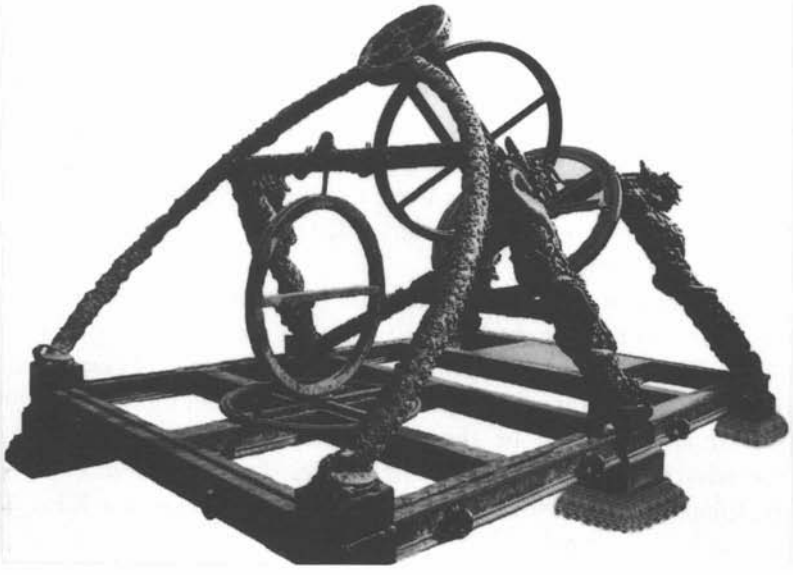

Figure 60. A 1437 replica of the original *jiǎn-yí* 簡儀 devised by Guō Shǒu-Jīng 郭守敬 in 1276, the earliest known astronomical instrument with an equatorial mounting. The original *jiǎn-jí* was melted down in 1715 by Jesuit Bernard Kilian Stumpf (Jì Lǐ-Ān 紀理安) for the purpose of making a quadrant. The replicate is now preserved in the Zǐjīnshān 紫金山 Observatory, Nánjīng.

achievements in astronomy nor on their experience in the field of astronomy, but on the general impression that they were informed with the new methods of science from Europe. Their blind assurance of Western superiority was constantly a source of difficulty. Consider, for example, Bernard Stumpf who in 1715 contributed the quadrant altazimuth to the Imperial Observatory. Even though he attained the position of director of the Imperial Bureau of Astronomy, he was at best a mediocre astronomer. In the *Cāo-Màn Zhī-Yán* 《操縵卮言》, Méi-Gǔ-Chéng 梅瑴成 (1681–1763) stated:

> 康熙五十四年, 西洋人紀理安欲炫其能而滅棄古法.
> 復奏制象限儀, 遂將台下所遺元明舊器作廢銅充用.
> 僅存明仿元制渾儀簡儀天體二儀而已.
> In the fifty-fourth year of Kāng Xī's reign (1715), Westerner Stumpf, intending to show off his ability and to eliminate ancient methods, proposed to construct a bronze quadrant. He destroyed the Yuán and Míng instruments stored below the observatory by melting them down as used bronze (to make a quadrant). Now only Míng copies of a simplified armillary-sphere (jiǎn-yí 簡儀) and a celestial globe (tiān-tí 天體) are left.

Ironically, the new instrument (a quadrant altazimuth) constructed by Stumpf was actually an attempt to correct a mistake in design made by Verbiest. It is an instrument obtained by simply combining a quadrant with a horizon circle. The quadrant altazimuth constructed by Stumpf was not only clumsy in design, but was also less valuable than some of the instruments that he had destroyed, in particular, the original *jiǎn-yí* (see Fig. 60), designed and constructed in the thirteenth century by Guō Shǒu-Jīng in his re-equipment of the Yuán 元 Imperial Observatory. The superiority attitude of the missionaries was not just a source of friction but was also responsible for the misrepresentation of the status of astronomy in China at the time.

Due to the fact that when astronomy reemerged in 1644 in the Qīng dynasty, Schall was the director of the Imperial Bureal of Astronomy and when the astronomical reform reemerged in 1669, Verbiest was in charge of refitting the Imperial Observatory, it is therefore natural to question whether the missionaries were responsible for the setback of the Qīng astronomical reform. As missionaries, they were of course under no obligation to bring the methods of the West to China. However, once they manipulated themselves into the Imperial Bureau of Astronomy and accepted positions in the bureau, they were responsible for their performance in the bureau. Thus, despite the goodwill in bringing the optical design of a telescope to China in his early days, Schall should be held responsible for the setback in the astronomical bureau in the twenty years when he was the director. For the same reason, Verbiest should also be held responsible for the inferior instruments constructed under his direction and for his decision to not include a telescope in refitting the observatory. On the other hand, one must also realize though the success of the bureau and of refitting project depended critically on the vision and ability of the individual in charge, the selection of the individual in charge is also of crucial importance. Thus, in my opinion, the important question to ask is not just whether Schall and Verbiest were responsible but also why they were selected for the task in the first place.[310] Why was the directorship of the Imperial Bureau of Astronomy placed in the hands of indivuduals such as Schall, Verbiest and Stumpf, as well as Yáng Guāng-Xiān, who were not competent in astronomy?

One may argue that events in China were such that the missionaries were able to connive their way into the directorship of the Bureau of Astronomy. This was perhaps the case with Schall,[311] but after the

310. See Chén Chēng-Yīh 程貞一 (1990) in Chinese.

311. It should be noted that the ability of translating European astronomical materials into Chinese language with Chinese assistants can hardly qualify one as an astronomer.

ulterior motive of the missionaries had been brought into focus in 1664 by the Yáng – Schall conflict, why was the practice of using missionaries continued until 1826 in the Dào-Guāng 道光 reign? Why then was Yáng appointed to the directorship? The answer had obviously to do with the Qīng authorities. Among the Qīng authorities, the person who should bear a substantial portion of the responsibility for the failure in selecting competent leaders in astronomical reform and the Imperial Bureau of Astronomy was Emperor Kāng Xī himself. In retrospect, this may appear ironical since Emperor Kāng Xī was not only a major driving force for the Qīng astronomical reform but he was also a capable ruler with a strong intellectual curiosity. In comparison with his contemporary rulers, for example, King Louis XIV (1641–1715) of France, Kāng Xī certainly showed a more personal interest in science and technology. Then, why should Kāng Xī be responsible? After all the selection of Verbiest for refitting the observatory was made when he was only sixteen years old.

One must realize, however, that the Kāng Xī reign (1662–1722) lasted more than half of a century and it was during this half of century that the gap in astronomy as well as in science between China and Europe was greatly widened. It was during his reign that one witnessed the development of calculus[312] and the Newtonian mechanics in Europe.[313] In addition, Kāng Xī was also known as a ruler who demanded to make major decisions himself. After the early mistakes, Kāng Xī certainly had adequate time and opportunities to reverse the situation but he did not. Not until near the end of his reign in 1714 did Kāng Xī order an upgrade of the *Xī-Yáng Xīn-Fǎ Lì-Shū* which after all was compiled by the Míng astronomical reformers between 1629 and 1634, almost a century earlier. The result was the book *Lì-Xiàng Kǎo-Chéng* 《曆象考成》 (*Compendium of Astronomical Science*) completed in 1722. Though this work was an improvement over the *Xī-Yáng Xīn-Fǎ Lì-Shū*, in comparison with the level of the European treatise on astronomy, it was rather substandard. In this attempt, astronomers outside the bureau were also invited to participate but the effort was rather late and limited.

The fallacy of Kāng Xī was not in the appreciation of science and technology, but in relating this appreciation at a national level. He, after all, was the ruler of a nation facing rapid changes around the world. He should have had the vision and moral duty to keep the nation contemporary with the emerging European nations. With his interest in science and technology, Kāng Xī was in a favourable position to

312. For a recent discussion on the development of the concept of limit see Chén Chēng-Yīh 程貞一 (1987), pp. 3–52.
313. See the *Philosophiae Naturalis Principia Mathematica* (1687) by Issac Newton.

accomplish at a national level, since in the second half of the seventeenth century, not only China had wealth and strength but also the gap in science and technology between China and Europe was still not yet very significant. Instead, Kāng Xī kept his interest and activity in science and technology within a small, controlled inner circle. He seems to have derived personal satisfaction in learning of new advances in science. But at a national level, in fact, he implemented no consequential policies and allocated no significant budget for the promotion and support of science and technology. No topics in science and technology were, for example, incorporated in the imperial examination. He employed missionaries frequently as a quick source of new scientific and technological knowledge from Europe but made no effort to acquire the knowledge directly. With all his interest in the new methods from the West, Kāng Xī had never sent off any team of Chinese scholars to Europe for facts finding and keeping up with European development in science and technology. In view of his interest in science and technology, and his intellectual capacity to appreciate their importance to the nation, it makes his lack of a concrete policy to promote science and technology at a national level even more inexcusable as a ruler of this critical period in the history of science.

From this brief examination of the events in the field of astronomy during this early period of crosscurrents with the West, it is apparent that the absence of the early Chinese participation in the development of 'modern' science is a very complex issue. One cannot exclude the dynamics of the socio-economical and political influence and seek answers simply in terms of the dubious concept of 'inhibitive factors' in the Chinese culture.

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