20.3.3.3.3.1 User Algorithm for SV Clock Correction. The polynomial defined in the following allows the user to determine the effective SV PRN code phase offset referenced to the phase center of the antennas (Δt_{sv}) with respect to GPS system time (t) at the time of data transmission. The coefficients transmitted in subframe 1 describe the offset apparent to the two-frequency user for the interval of time in which the parameters are transmitted. This estimated correction accounts for the deterministic SV clock error characteristics of bias, drift and aging, as well as for the SV implementation characteristics of group delay bias and mean differential group delay. Since these coefficients do not include corrections for relativistic effects, the user's equipment must determine the requisite relativistic correction. Accordingly, the offset given below includes a term to perform this function.

The user shall correct the time received from the SV with the equation (in seconds)

$$t = t_{sv} - \Delta t_{sv} \tag{1}$$

where

t = GPS system time (seconds),

t_{sv} = effective SV PRN code phase time at message transmission time (seconds),

Δt_{sv} = SV PRN code phase time offset (seconds).

The SV PRN code phase offset is given by

$$\Delta t_{sv} = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \Delta t_r \tag{2}$$

where

a_{f0}, a_{f1} and a_{f2} are the polynomial coefficients given in subframe 1, t_{oc} is the clock data reference time in seconds (reference paragraph 20.3.4.5), and Δt_r is the relativistic correction term (seconds) which is given by

$$\Delta t_r = Fe(A)^{1/2} \sin E_k.$$

The orbit parameters (e, A, E_k) used here are described in discussions of data contained in subframes 2 and 3, while F is a constant whose value is

$$F = \frac{-2\mu^{1/2}}{c^2} = -4.442807633\,(10)^{-10} \text{ sec/(meter)}^{1/2},$$

where

$$\mu = 3.986005 \times 10^{14} \frac{\text{meters}^3}{\text{second}^2} \quad = \quad \text{value of Earth's universal gravitational parameters}$$

$$c = 2.99792458 \times 10^{8} \frac{\text{meters}}{\text{second}} \quad = \quad \text{speed of light.}$$

Note that equations (1) and (2), as written, are coupled. While the coefficients a_{f0}, a_{f1} and a_{f2} are generated by using GPS time as indicated in equation (2), sensitivity of t_{sv} to t is negligible. This negligible sensitivity will allow the user to approximate t by t_{SV} in equation (2). The value of t must account for beginning or end of week crossovers. That is, if the quantity t - t_{oc} is greater than 302,400 seconds, subtract 604,800 seconds from t. If the quantity t - t_{oc} is less than -302,400 seconds, add 604,800 seconds to t.

The control segment will utilize the following alternative but equivalent expression for the relativistic effect when estimating the NAV parameters:

$$\Delta t_r = -\frac{2 \vec{R} \bullet \vec{V}}{c^2}$$

where

$\vec{R}$ is the instantaneous position vector of the SV,

$\vec{V}$ is the instantaneous velocity vector of the SV, and

c is the speed of light. (Reference paragraph 20.3.4.3).

It is immaterial whether the vectors $\vec{R}$ and $\vec{V}$ are expressed in earth-fixed, rotating coordinates or in earth-centered, inertial coordinates.

20.3.3.3.3.2 L1 - L2 Correction. The L1 and L2 correction term, T_{GD}, is initially calculated by the CS to account for the effect of SV group delay differential between L1 and L2 based on measurements made by the SV contractor during SV manufacture. The value of T_{GD} for each SV may be subsequently updated to reflect the actual on-orbit group delay differential. This correction term is only for the benefit of "single-frequency" (L1 or L2) users; it is necessitated by the fact that the SV clock offset estimates reflected in the a_{f0} clock correction coefficient (see paragraph 20.3.3.3.3.1) are based on the effective PRN code phase as apparent with two frequency ionospheric corrections. Thus, the user who utilizes the L1 frequency only shall modify the code phase offset in accordance with paragraph 20.3.3.3.3.1 with the equation

$$(\Delta t_{SV})_{L1} = \Delta t_{SV} - T_{GD}$$

where T_{GD} is provided to the user as subframe 1 data. For the user who utilizes L2 only, the code phase modification is given by

$$(\Delta t_{SV})_{L2} = \Delta t_{SV} - \gamma T_{GD}$$

where, denoting the nominal center frequencies of L1 and L2 as f_{L1} and f_{L2} respectively,

$$\gamma = (f_{L1}/f_{L2})^2 = (1575.42/1227.6)^2 = (77/60)^2.$$

The value of T_{GD} is not equal to the mean SV group delay differential, but is equal to the delay differential multiplied by $1/(1-\gamma)$. That is,

$$T_{GD} = \frac{1}{1-\gamma}(t_{L1} - t_{L2})$$

where t_{Li} is the GPS time the i^{th} frequency signal is transmitted from the SV.

20.3.3.3.3.3 Ionospheric Correction. The two frequency (L1 and L2) user shall correct for the group delay due to ionospheric effects by applying the relationship:

$$PR = \frac{PR_2 - \gamma PR_1}{1 - \gamma}$$

where

PR	=	pseudorange corrected for ionospheric effects,
PR_i	=	pseudorange measured on the L-band channel indicated by the subscript.

and γ is as defined in paragraph 20.3.3.3.3.2. The clock correction coefficients are based on "two frequency" measurements and therefore account for the effects of mean differential delay in SV instrumentation.

20.3.3.3.3.4 Example Application of Correction Parameters. A typical system application of the correction parameters for a user receiver is shown in Figure 20-3. The ionospheric model referred to in Figure 20-3 is discussed in paragraph 20.3.3.5.2.5 in conjunction with the related data contained in page 18 of subframe 4. The $\frac{ERD}{c}$ term referred to in Figure 20-3 is discussed in paragraph 20.3.3.5.2.6 in conjunction with the related data contained in page 13 of subframe 4.

20.3.3.4 Subframes 2 and 3. The contents of words three through ten of subframes 2 and 3 are defined below, followed by material pertinent to the use of the data.

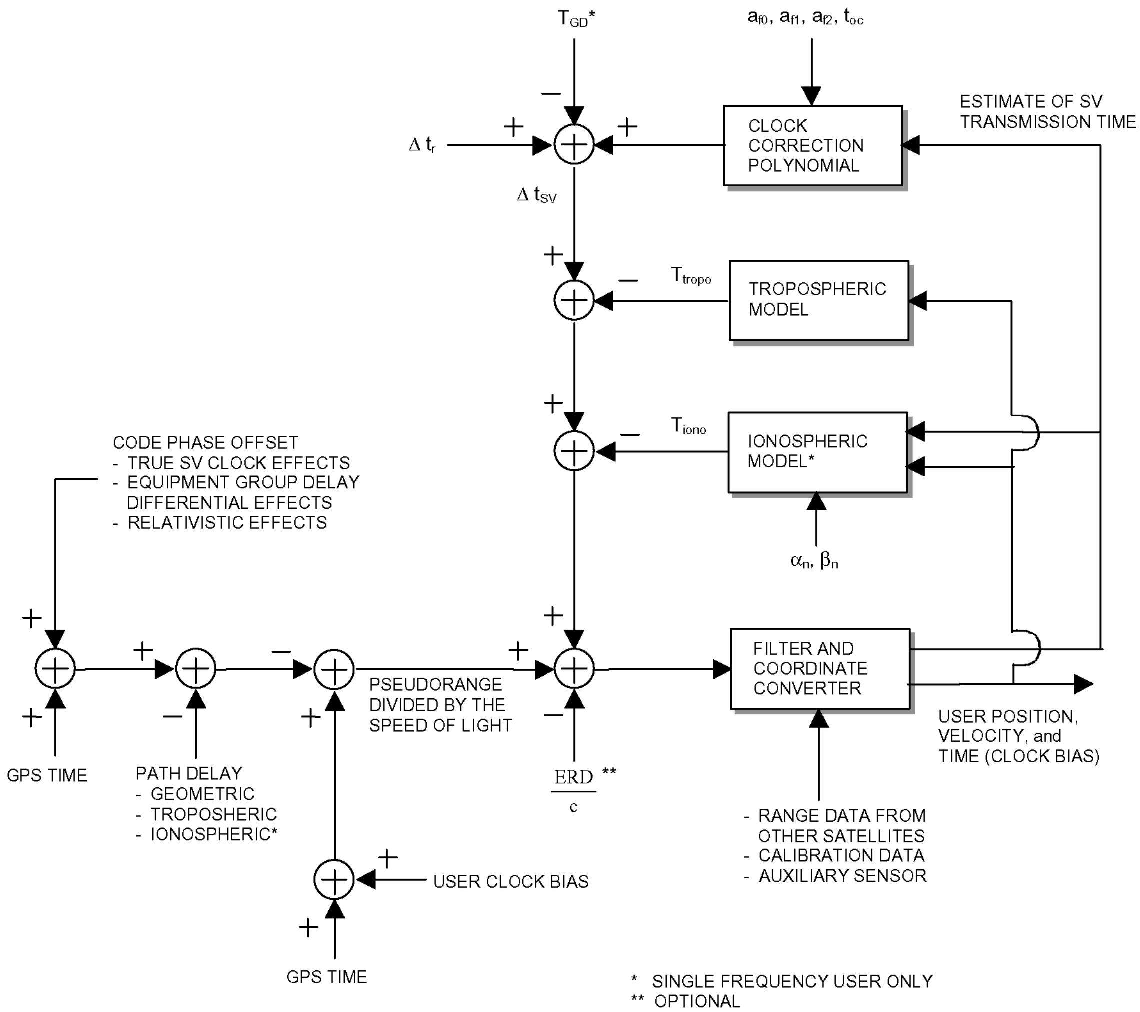

Figure 20-3. Sample Application of Correction Parameters

20.3.3.4.1 <u>Content of Subframes 2 and 3</u>. The third through tenth words of subframes 2 and 3 shall each contain six parity bits as their LSBs; in addition, two non-information bearing bits shall be provided as bits 23 and 24 of word ten of each subframe for parity computation purposes. Bits 288 through 292 of subframe 2 shall contain the Age of Data Offset (AODO) term for the navigation message correction table (NMCT) contained in subframe 4 (reference paragraph 20.3.3.5.1.12). The remaining 375 bits of those two subframes shall contain the ephemeris representation parameters of the transmitting SV.

The ephemeris parameters describe the orbit during the curve fit intervals described in section 20.3.4. Table 20-II gives the definition of the orbital parameters using terminology typical of Keplerian orbital parameters; it shall be noted, however, that the transmitted parameter values are such that they provide the best trajectory fit in Earth-Centered, Earth-Fixed (ECEF) coordinates for each specific fit interval. The user shall not interpret intermediate coordinate values as pertaining to any conventional coordinate system.

Table 20-II. Ephemeris Data Definitions	
M_0	Mean Anomaly at Reference Time
Δn	Mean Motion Difference From Computed Value
e	Eccentricity
$(A)^{1/2}$	Square Root of the Semi-Major Axis
$(OMEGA)_0$	Longitude of Ascending Node of Orbit Plane at Weekly Epoch
i_0	Inclination Angle at Reference Time
ω	Argument of Perigee
OMEGADOT	Rate of Right Ascension
IDOT	Rate of Inclination Angle
C_{uc}	Amplitude of the Cosine Harmonic Correction Term to the Argument of Latitude
C_{us}	Amplitude of the Sine Harmonic Correction Term to the Argument of Latitude
C_{rc}	Amplitude of the Cosine Harmonic Correction Term to the Orbit Radius
C_{rs}	Amplitude of the Sine Harmonic Correction Term to the Orbit Radius
C_{ic}	Amplitude of the Cosine Harmonic Correction Term to the Angle of Inclination
C_{is}	Amplitude of the Sine Harmonic Correction Term to the Angle of Inclination
t_{oe}	Reference Time Ephemeris (reference paragraph 20.3.4.5)
IODE	Issue of Data (Ephemeris)

The issue of ephemeris data (IODE) term shall provide the user with a convenient means for detecting any change in the ephemeris representation parameters. The IODE is provided in both subframes 2 and 3 for the purpose of comparison with the 8 LSBs of the IODC term in subframe 1. Whenever these three terms do not match, a data set cutover has occurred and new data must be collected. The timing of the IODE and constraints on the IODC and IODE are defined in paragraph 20.3.4.4.

Any change in the subframe 2 and 3 data will be accomplished with a simultaneous change in both IODE words. The CS shall assure that the t_{oe} value, for at least the first data set transmitted by an SV after an upload, is different from that transmitted prior to the cutover.

A "fit interval" flag is provided in subframe 2 to indicate whether the ephemerides are based on a four-hour fit interval or a fit interval greater than four hours (reference paragraph 20.3.3.4.3.1).

The AODO word is provided in subframe 2 to enable the user to determine the validity time for the NMCT data provided in subframe 4 of the transmitting SV. The related algorithm is given in paragraph 20.3.3.4.4.

20.3.3.4.2 Subframe 2 and 3 Parameter Characteristics. For each ephemeris parameter contained in subframes 2 and 3, the number of bits, the scale factor of the LSB (which shall be the last bit received), the range, and the units shall be as specified in Table 20-III.

The AODO word (which is not an ephemeris parameter) is a five-bit unsigned term with an LSB scale factor of 900, a range from 0 to 31, and units of seconds.

Table 20-III. Ephemeris Parameters

Parameter	No. of Bits**	Scale Factor (LSB)	Effective Range***	Units
IODE	8			(see text)
C_{rs}	16*	2^{-5}		meters
Δn	16*	2^{-43}		semi-circles/sec
M_0	32*	2^{-31}		semi-circles
C_{uc}	16*	2^{-29}		radians
e	32	2^{-33}	0.03	dimensionless
C_{us}	16*	2^{-29}		radians
$(A)^{1/2}$	32	2^{-19}		$\text{meters}^{1/2}$
t_{oe}	16	2^{4}	604,784	seconds
C_{ic}	16*	2^{-29}		radians
$(OMEGA)_0$	32*	2^{-31}		semi-circles
C_{is}	16*	2^{-29}		radians
i_0	32*	2^{-31}		semi-circles
C_{rc}	16*	2^{-5}		meters
ω	32*	2^{-31}		semi-circles
OMEGADOT	24*	2^{-43}		semi-circles/sec
IDOT	14*	2^{-43}		semi-circles/sec

* Parameters so indicated shall be two's complement, with the sign bit (+ or -) occupying the MSB;
** See Figure 20-1 for complete bit allocation in subframe;
*** Unless otherwise indicated in this column, effective range is the maximum range attainable with indicated bit allocation and scale factor.

20.3.3.4.3 User Algorithm for Ephemeris Determination. The user shall compute the ECEF coordinates of position for the phase center of the SVs' antennas utilizing a variation of the equations shown in Table 20-IV. Subframes 2 and 3 parameters are Keplerian in appearance; the values of these parameters, however, are produced by the CS via a least squares curve fit of the predicted ephemeris of the phase center of the SVs' antennas (time-position quadruples; t, x, y, z expressed in ECEF coordinates). Particulars concerning the periods of the curve fit, the resultant accuracy, and the applicable coordinate system are given in the following subparagraphs.

20.3.3.4.3.1 Curve Fit Intervals. Bit 17 in word 10 of subframe 2 is a "fit interval" flag which indicates the curve-fit interval used by the CS in determining the ephemeris parameters, as follows:

0 = 4 hours,
1 = greater than 4 hours.

The relationship of the curve-fit interval to transmission time and the timing of the curve-fit intervals is covered in section 20.3.4.

Table 20-IV. Elements of Coordinate Systems (sheet 1 of 3)	
$\mu = 3.986005 \times 10^{14}$ meters^3/sec^2	WGS 84 value of the earth's universal gravitational parameter for GPS user
$\dot{\Omega}_e = 7.2921151467 \times 10^{-5}$ rad/sec	WGS 84 value of the earth's rotation rate
$A = \left(\sqrt{A}\right)^2$	Semi-major axis
$n_0 = \sqrt{\dfrac{\mu}{A^3}}$	Computed mean motion (rad/sec)
$t_k = t - t_{oe}$*	Time from ephemeris reference epoch
$n = n_0 + \Delta n$	Corrected mean motion
$M_k = M_0 + nt_k$	Mean anomaly
* t is GPS system time at time of transmission, i.e., GPS time corrected for transit time (range/speed of light). Furthermore, t_k shall be the actual total time difference between the time t and the epoch time t_{oe}, and must account for beginning or end of week crossovers. That is, if t_k is greater than 302,400 seconds, subtract 604,800 seconds from t_k. If t_k is less than -302,400 seconds, add 604,800 seconds to t_k.	

Table 20-IV. Elements of Coordinate Systems (sheet 2 of 3)	
$M_k = E_k - e \sin E_k$	Kepler's Equation for Eccentric Anomaly (may be solved by iteration)(radians)
$\nu_k = \tan^{-1}\left\{\dfrac{\sin\nu_k}{\cos\nu_k}\right\}$ $= \tan^{-1}\left\{\dfrac{\sqrt{1-e^2}\sin E_k/(1-e\cos E_k)}{(\cos E_k - e)/(1-e\cos E_k)}\right\}$	True Anomaly
$E_k = \cos^{-1}\left\{\dfrac{e+\cos\nu_k}{1+e\cos\nu_k}\right\}$	Eccentric Anomaly
$\Phi_k = \nu_k + \omega$	Argument of Latitude
$\delta u_k = c_{us}\sin 2\Phi_k + c_{uc}\cos 2\Phi_k$ — Argument of Latitude Correction	Second Harmonic Perturbations
$\delta r_k = c_{rs}\sin 2\Phi_k + c_{rc}\cos 2\Phi_k$ — Radius Correction	Second Harmonic Perturbations
$\delta i_k = c_{is}\sin 2\Phi_k + c_{ic}\cos 2\Phi_k$ — Inclination Correction	Second Harmonic Perturbations
$u_k = \Phi_k + \delta u_k$	Corrected Argument of Latitude
$r_k = A(1 - e\cos E_k) + \delta r_k$	Corrected Radius
$i_k = i_0 + \delta i_k + (\mathrm{IDOT})\, t_k$	Corrected Inclination

Table 20-IV. Elements of Coordinate Systems (sheet 3 of 3)	
$\left.\begin{array}{l} x_k' = r_k \cos u_k \\ y_k' = r_k \sin u_k \end{array}\right\}$	Positions in orbital plane.
$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)\, t_k - \dot{\Omega}_e\, t_{oe}$	Corrected longitude of ascending node.
$\left.\begin{array}{l} x_k = x_k' \cos\Omega_k - y_k' \cos i_k \sin\Omega_k \\ y_k = x_k' \sin\Omega_k + y_k' \cos i_k \cos\Omega_k \\ z_k = y_k' \sin i_k \end{array}\right\}$	Earth-fixed coordinates.

20.3.3.4.3.2 Parameter Sensitivity. The sensitivity of the SV's antenna phase center position to small perturbations in most ephemeris parameters is extreme. The sensitivity of position to the parameters $(A)^{1/2}$, C_{rc} and C_{rs} is about one meter/meter. The sensitivity of position to the angular parameters is on the order of 10^8 meters/semicircle, and to the angular rate parameters is on the order of 10^{12} meters/semicircle/second. Because of this extreme sensitivity to angular perturbations, the value of π used in the curve fit is given here. π is a mathematical constant, the ratio of a circle's circumference to its diameter. Here π is taken as

$$\pi = 3.1415926535898.$$

20.3.3.4.3.3 Coordinate Systems.

20.3.3.4.3.3.1 ECEF Coordinate System. The equations given in Table 20-IV provide the SV's antenna phase center position in the WGS 84 ECEF coordinate system defined as follows:

Origin*	=	Earth's center of mass
Z-Axis**	=	The direction of the IERS (International Earth Rotation Service) Reference Pole (IRP)
X-Axis	=	Intersection of the IERS Reference Meridian (IRM) and the plane passing through the origin and normal to the Z-axis
Y-Axis	=	Completes a right-handed, Earth-Centered, Earth-Fixed orthogonal coordinate system

* Geometric center of the WGS 84 Ellipsoid

** Rotational axis of the WGS 84 Ellipsoid

20.3.3.4.3.3.2 Earth-Centered, Inertial (ECI) Coordinate System. In an ECI coordinate system, GPS signals propagate in straight lines at the constant speed c* (reference paragraph 20.3.4.3). A stable ECI coordinate system of convenience may be defined as being coincident with the ECEF coordinate system at a given time t_0. The x, y, z coordinates in the ECEF coordinate system at some other time t can be transformed to the x′, y′, z′ coordinates in the selected ECI coordinate system of convenience by the simple** rotation:

$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = x\sin(\theta) + y\cos(\theta)$$
$$z' = z$$

where

$$\theta = \dot{\Omega}_e (t - t_0)$$

* The propagation speed c is constant only in a vacuum. The gravitational potential also has a small effect on the propagation speed, but may be neglected by most users.

** Neglecting effects due to polar motion, nutation, and precession which may be neglected by most users for small values of $(t - t_0)$.

20.3.3.4.3.4 Geometric Range. The user shall account for the geometric range (D) from satellite to receiver in an ECI coordinate system. D may be expressed as,

$$D = |\vec{r}(t_R) - \vec{R}(t_T)|$$

where

t_T and t_R are the GPS system times of transmission and reception, respectively,

and where,

$\vec{R}(t_T)$ = position vector of the GPS satellite in the selected ECI coordinate system at time t_T,

$\vec{r}(t_R)$ = position vector of the receiver in the selected ECI coordinate system at time t_R.

20.3.3.4.4 NMCT Validity Time. Users desiring to take advantage of the NMCT data provided in page 13 of subframe 4 shall first examine the AODO term currently provided in subframe 2 of the NAV data from the transmitting SV. If the AODO term is 27900 seconds (i.e., binary 11111), then the NMCT currently available from the transmitting SV is invalid and shall not be used. If the AODO term is less than 27900 seconds, then the user shall compute the validity time for that NMCT (t_{nmct}) using the ephemeris t_{oe} parameter and the AODO term from the current subframe 2 as follows:

$$\text{OFFSET} = t_{oe} \text{ [Modulo 7200]}$$

$$\text{if OFFSET} = 0\text{, then } t_{nmct} = t_{oe} - \text{AODO}$$
$$\text{if OFFSET} > 0\text{, then } t_{nmct} = t_{oe} - \text{OFFSET} + 7200 - \text{AODO}$$

Note that the foregoing computation of t_{nmct} must account for any beginning or end of week crossovers; for example,

$$\text{if } t^* - t_{nmct} > 302{,}400 \text{ then } t_{nmct} = t_{nmct} + 604{,}800$$
$$\text{if } t^* - t_{nmct} < -302{,}400 \text{ then } t_{nmct} = t_{nmct} - 604{,}800$$

* t is GPS system time at time of transmission.

Users are advised that different SVs will transmit NMCTs with different t_{nmct} and that the best performance will generally be obtained by applying data from the NMCT with the latest (largest) t_{nmct}. As a result, users should compute and examine the t_{nmct} values for all visible and available SVs in order to find and use the NMCT with the latest t_{nmct}. If the same latest (largest) t_{nmct} is provided by two or more visible and available SVs, then the NMCT from any SV with the latest t_{nmct} may be selected and used; however, the estimated range deviation (ERD) value provided by the selected NMCT for the other SVs with the same t_{nmct} shall be set to zero if those SVs are used in the positioning solution. It should be noted that the intended positioning solution accuracy improvement will not be obtained if the data from two different NMCTs are applied simultaneously or if the data from a given NMCT is applied to just a subset of the SVs used in the positioning solution (i.e., mixed mode operation results in potentially degraded solution accuracy).

It should be noted that the NMCT information shall be supported by the Block IIR SV only when operating in the IIA like mode of operation including the Autonav Test mode.