12.005 Lecture Notes 1

Handouts - Course description & reserve list

- Motivation for core course

- Instructor background

- Student background

       Courses in physics and mathematics?

       EAPS area of focus?

- Textbooks

Mechanics: the study of the motion of matter and the forces that cause such motion.

Based on concepts of time, space, force, energy, matter.

Applications to point masses, solid bodies familiar from introductory physics.

Continuum mechanics - mechanics of parts of "bodies."

Continuum - define values of fields (e.g., density) as functions of position, i.e., at points.

Example : density $\rho(P) = \lim\limits_{V_n \to 0} \dfrac{M_n}{V_n}$

       $V_i \longrightarrow$ Sequence of volumes converging on P

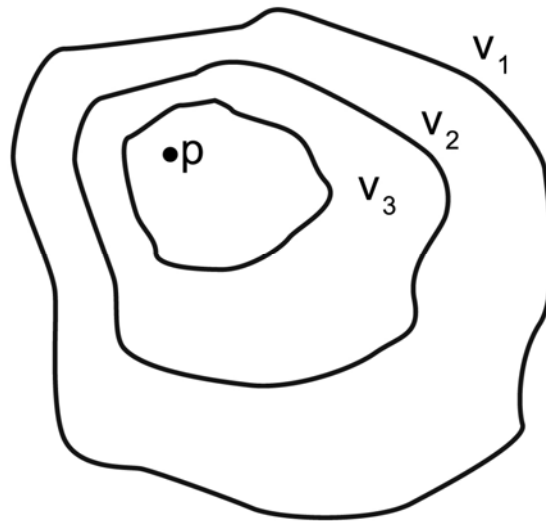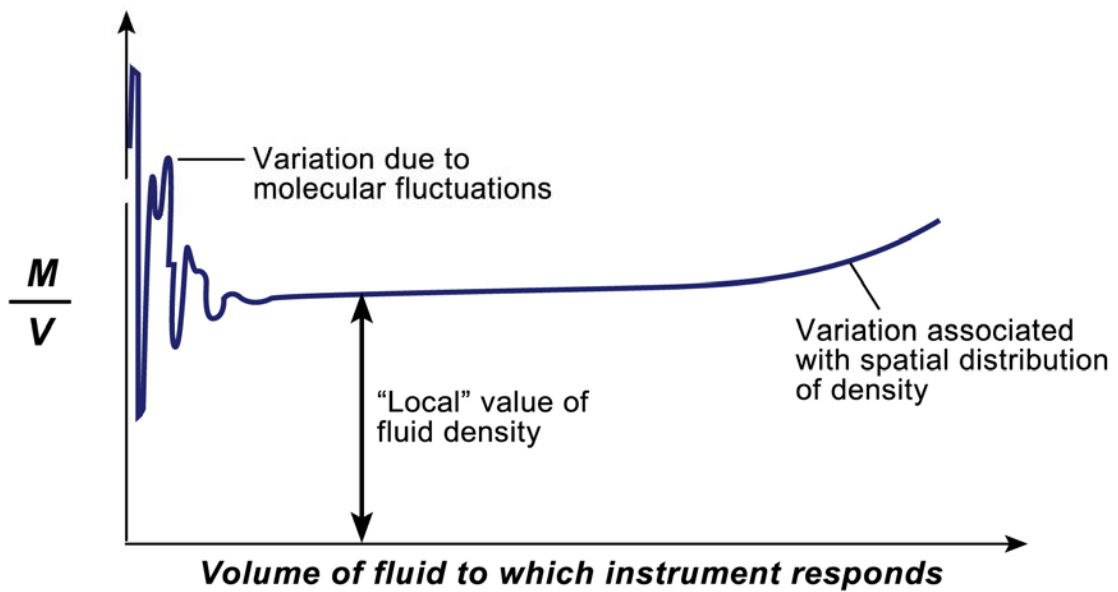 $M_i \longrightarrow$ Mass enclosed by V_i

Figure 1.1

Figure by MIT OCW.

Breakdown at small V (e.g., gas)

Effect of size of sensitive volume on the density measured by an instrument

Figure 1.2

Figure by MIT OCW.

Ignore fine structure. Assume:

    • **Continuity**: completely fills space (no pores or voids) and has properties describable by continuous functions. [Scale dependent. E.g., sand will be treated as a continuum]

    • **Homogeneity** - identical properties at all points   [Scale dependent.]

    • **Isotropy** - properties same in all directions. [Often not true. e.g., schist]

Behavior describable in terms of partial differential equations subject to boundary conditions. All functions will be "well behaved," except at a finite # of surfaces. Logical contradiction between mathematical process of taking limits & physical breakdown of continuum description at small scales ignored. (Assume - fields varying slowly enough that math defined before description falls apart.)

At core: Newton's 2nd Law: $F = ma$, or $F - ma = 0$.

    for this course, usually $a = 0$ => the governing equation is $F = 0$!

    • Applies not just to particles, entire bodies, but to regions within bodies.
    • Free-body diagram - cut open body (thought experiment), examine forces of interaction between surfaces.

Description of forces acting in the continuum - For taking limits, etc, forces/unit area make sense. Consider tractions **T**: Force/unit area on faces (vector quantities).

Units: SI - Pascal (Pa); 1 Pa = 1 N/m^2
    cgs: dyne/cm^2
    "geophysics": bar (1 atmosphere): 1 bar = 10^6 dynes/cm^2 = 0.1 MPa

A general description of the tractions acting on planes of arbitrary orientation requires:

    • **stress** - forces of interaction (2nd rank tensor)

        Newton's 2nd —> $\tau_{ij,j} + f_i = 0$.

    • **strain & rotation** - kinematics of motion & deformation (2nd rank tensors)

    • **rheology** (constitutive law) - relationship between stress & strain
        solids - elastic, plastic, brittle
        fluids - viscous, ideal
        other - e.g., Silly putty, paint,

We will quantify these quantities soon, but first a digression to a concrete example to bring things into focus. When I moved to southern CA ~ 1980, I was impressed by press coverage of a set of law suits pending against various governments (state & local). The question to be settled was who was responsible for an expensive housing development "sliding into the ocean." Briefly, the sequence of events was:

1) "Nature" deposited a terrace (mainly sand) beside the ocean.

2) The highway department made a road cut into the base of the sea cliff to put in a coastal highway.

3) Expensive houses were built on top of the terrace.

4) Lawns were watered lavishly.

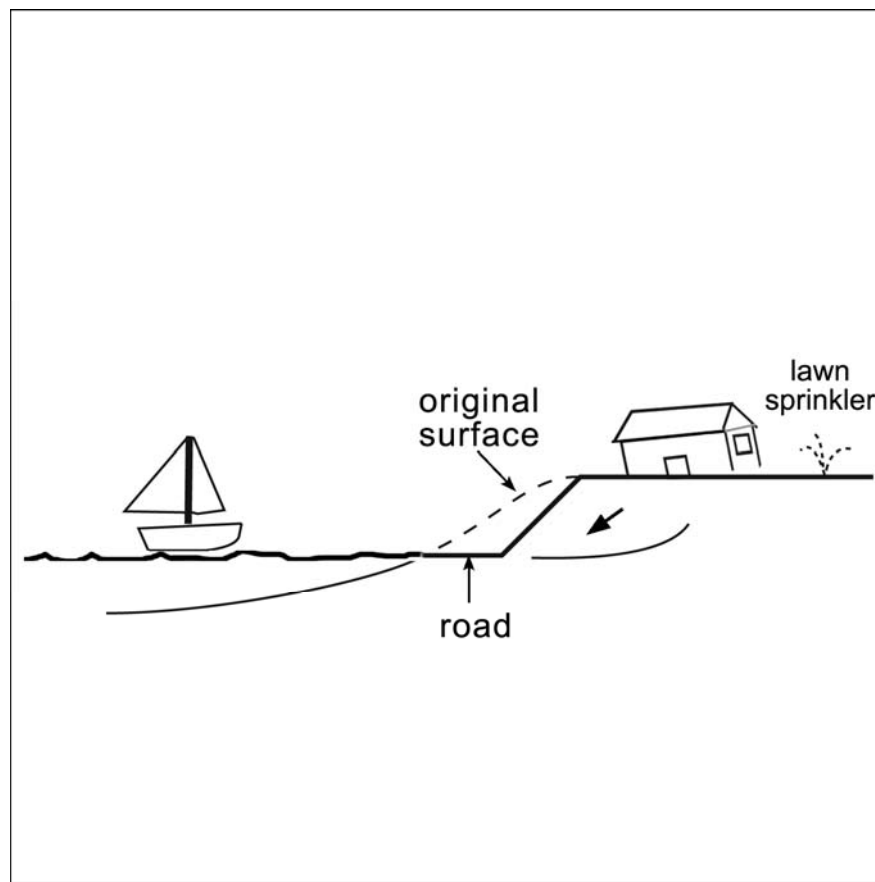5) Landslides started wrecking the expensive houses.

Figure 1.3

Figure by MIT OCW.

We will examine the "continuum mechanics" of this situation & come up with some "expert opinion" about responsibility. We need to do two things: first, understand how sand responds to "internal forces," then understand how to describe these forces.

Assertion: Sand pile behavior, as a first approximation, is governed by **friction**. So are earthquake faults.
(reading: T & S, pp 351-353)

Consider an experiment to measure the frictional behavior of sand. One approach would be to place some sand between a block & a table, subjecting the block to a downward force F and a horizontal force V (in T&S notation). If the block has area A, there is a
normal traction $\sigma_n = F/A$
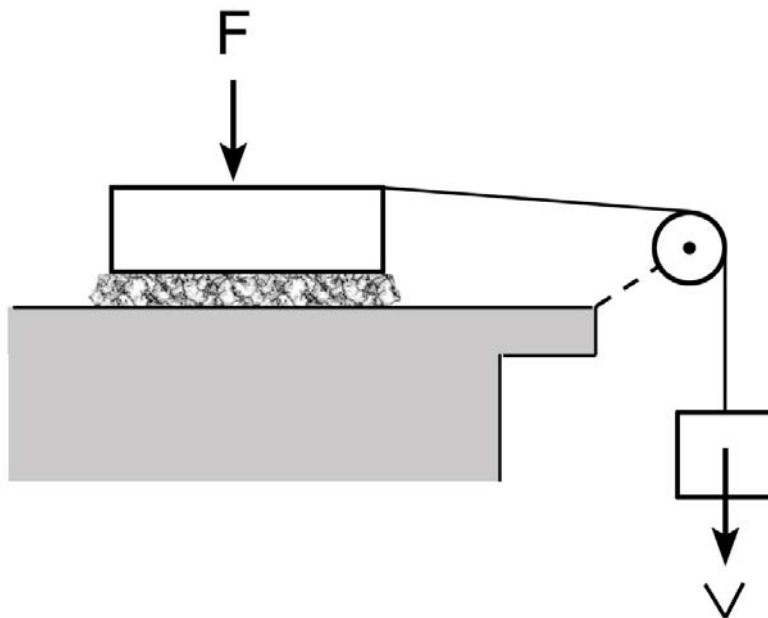and a **shear traction** $\tau = V/A$

Figure 1.4

Figure by MIT OCW.

Admonton's law (to summarize a lot of experiments) states that the shear force needed to cause sliding is related to the normal force pressing the block down as
$$V = f_s F$$
or, in terms of **tractions**
$$\tau = f_s \sigma_n$$

Digression: One very sensitive way of measuring f_s (developed at MIT) is to use an inclined plane & vary the inclination angle θ. Suppose this is done & loading is carried out by varying the # of bricks squeezing the sand. How will θ depend on the # of bricks?
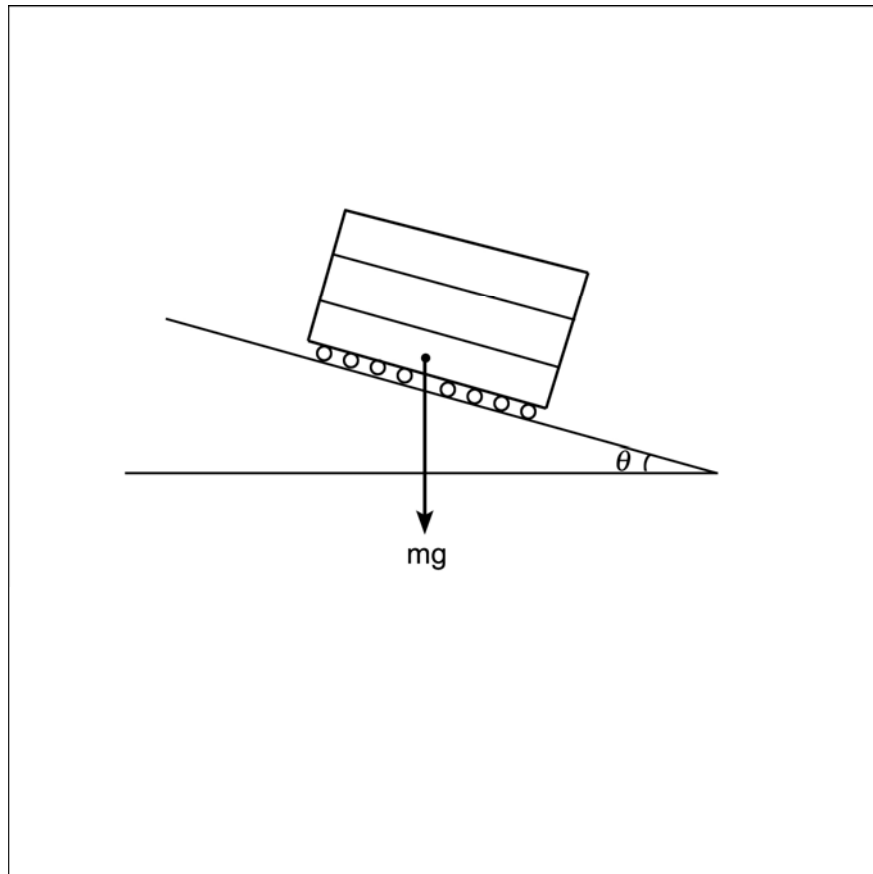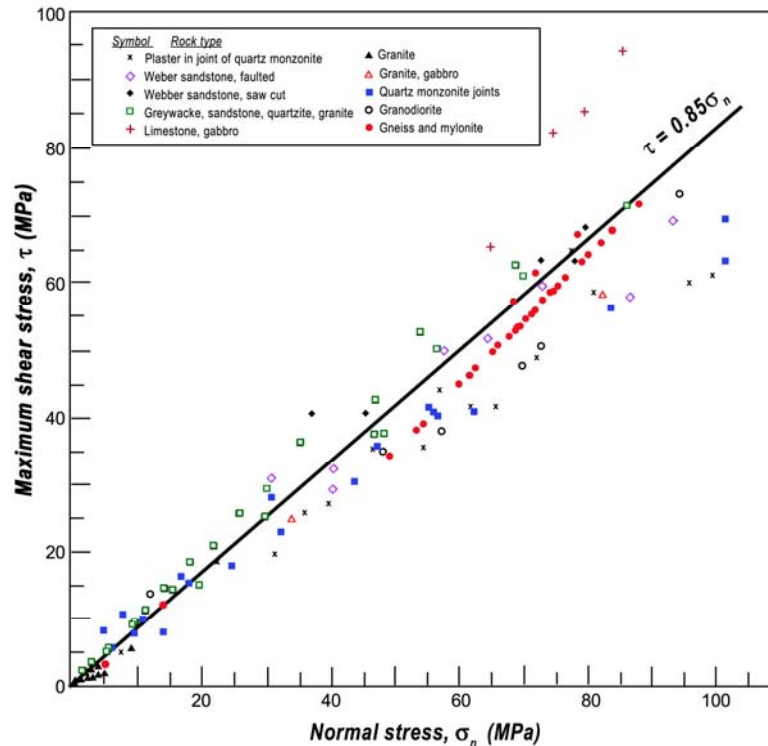

Figure 1.5

Figure by MIT OCW.

Another digression: An amazing result from rock mechanics, "Byerlee's law," is that f_s is essentially independent of rock type!

Maximum shear stress to initiate sliding as a function of normal stress for a variety of rock types.
The linear fit defines a maximum coefficient of static friction max t_a equal to 0.85.
Data from J.D. Byerlee, **Friction of Rocks,**
in ***Experimental Studies of Rock Friction with Application to Earthquake Prediction***
ed. J.F. Evernden, pp-55-77, U.S. Geological Survey, Menlo Park, Calif., 1977

Figure 1.6

Figure by MIT OCW.

Now, back to the sand pile. Admonton's law tells us that any time the shear traction on a plane reaches a value of $f_s \sigma_n$, the sand pile will fail. The complication is that we no longer have a brick & a bench to work with — what we need is a mathematical description of normal and shear tractions on arbitrary planes in a continuum. We start out with a special description of the tractions on a surface oriented with the coordinate system, then will go on to generalize.

It is useful to break tractions into their component parts (normal & shear):

Define σ_{ij} as force/unit area acting on face with normal n_i in direction n_j.

Consider now a small parallelepiped with faces in the co-ordinate planes and dimensions dx_1, dx_2, and dx_3. The tractions on the three faces can be resolved into their Cartesian components, one normal and two tangential to the face on which the traction acts:

$$T_{(1)i} = (\sigma_{11}, \sigma_{12}, \sigma_{13})$$
$$T_{(2)i} = (\sigma_{21}, \sigma_{22}, \sigma_{23})$$
$$T_{(3)i} = (\sigma_{31}, \sigma_{32}, \sigma_{33})$$

where $T_{(1)i}$ denotes the traction on the face normal to the axis x1, and so on. The nine components of the tractions form a 3×3 matrix

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

where for each component the first subscript denotes the co-ordinate axis to which the surface is normal, and the second subscript denotes the direction in which the component acts. For example, σ_{23} denotes the component acting on the face normal to x_2 in the direction of x_3.
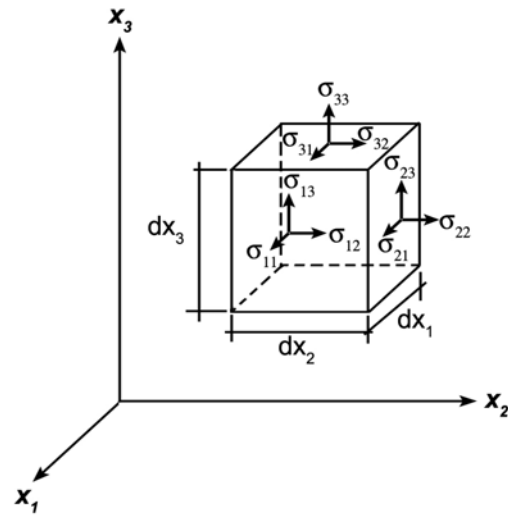
Figure 1.7

Figure by MIT OCW.

Case 1: Water at rest: What are the relations among components?

Case 2: A cube at the corner of the table: What are the relations among components?