Chapter 4 - Real Fluid Effects ($\nu \neq 0$)

Potential Flow Theory $\rightarrow$ Drag $= 0$.

Observed experiment (real fluid $\nu << 1$ but $\neq 0$) $\rightarrow$ Drag $\neq 0$.

In particular the **total** drag measured on a body is regarded as the sum of two components:
the **pressure** or **form** drag, and the **skin friction** or **viscous** drag.

Total Drag	=	**Pressure Drag**	+	**Skin Friction Drag**
Profile Drag		or Form Drag		or Viscous Drag
		$\underbrace{\text{Drag Force due to Pressure}}$		$\underbrace{\text{Drag Force due to Viscous Stresses}}$
		$\iint\limits_{S} p\hat{n}ds$		$\iint\limits_{S} \tau\hat{t}ds$

where $\hat{n}$ and $\hat{t}$ are the normal and tangential unit vectors on the body surface respectively.
The pressure and the viscous stresses on the body surface are p and τ respectively.

The form drag is evaluated by integrating the pressure along the surface of the body. For
bluff bodies that create large wakes the form drag is $\sim$ total drag.

The skin friction drag is evaluated by integrating the viscous stresses on and along the
body boundary. For streamlined bodies that do not create appreciable wakes, friction drag
is dominant.

4.1 Form Drag

4.1.1 Form Drag on a Bluff Body

Consider a sphere of diameter d:
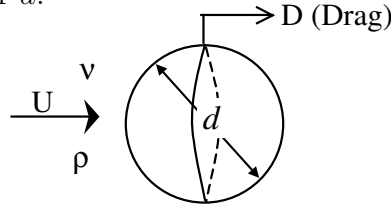
If no DBC apply then we have seen from Dimensional Analysis that the **drag coefficient** is a function of the Reynolds number only:

$$C_D = C_D(R_e)$$

The drag coefficient C_D is defined with respect to the body's projected area S:

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 S} = \frac{D}{\frac{1}{2}\rho U^2 \underbrace{\pi d^2/4}_{\text{Projected area}}}$$

The Reynolds number R_e is defined with respect to the body's diameter d:

$$R_e = \frac{Ud}{\nu}$$

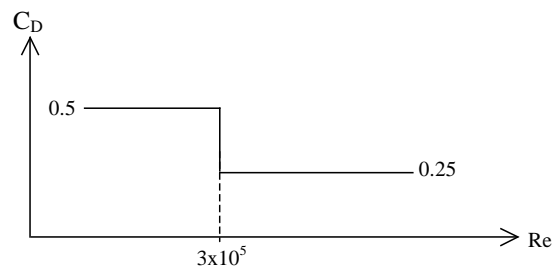The following graph shows the dependence of C_D on R_e as measured from numerous experiments on spheres.

Figure 1: Drag coefficient (C_D) for a sphere for $R_e > 10^2$.

i) **Bluff Body $\rightarrow$ Form Drag**

For a bluff body (examples: sphere, cylinder, flat plate, etc.) there is appreciable flow separation and a wake is formed downstream of the body. The pressure within the wake is significantly smaller than that upstream of the body. Therefore the integral of the pressure along the body boundary (= form drag) does not evaluate to zero as predicted by P-Flow.

In general, for bluff bodies $\boxed{\text{form drag} >> \text{friction drag}}$

Flow separation

Real Flow

Potential Flow
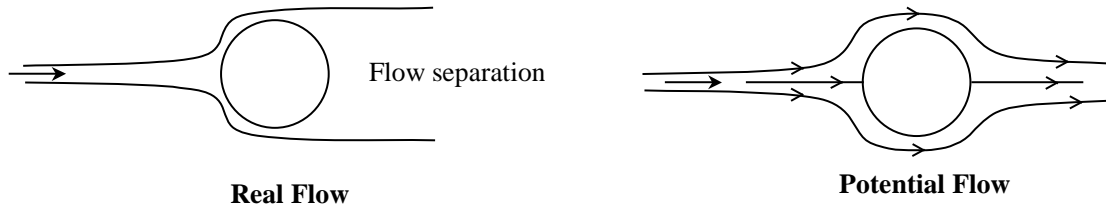
Assume: pressure in the wake $\sim p_\infty$ (pressure at infinity)

pressure on the upstream boundary of the body $\sim p_s$ (stagnation pressure)
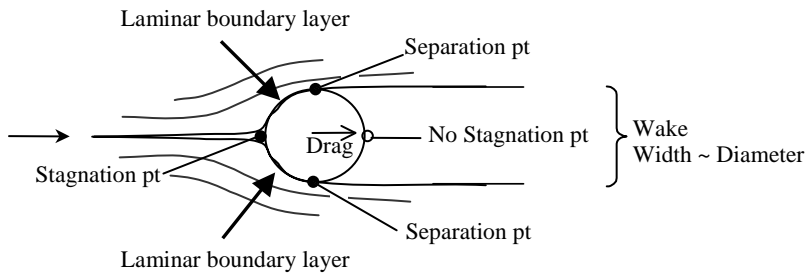
Then:

$$D = \left[\underbrace{C_D}_{\substack{\text{Friction Coefficient} \\ \text{to be determined}}} \underbrace{(\text{Projected/frontal area}}_{S})(p_s - p_\infty) \right] \underset{\substack{\uparrow \\ \text{Bernoulli}}}{=} C_D S\left(\tfrac{1}{2}\rho U^2\right)$$

ii) $C_D = C_D(R_e) \rightarrow$ **Regime Dependence**

In general, for typical bluff bodies such as spheres, it is found that $C_D = C_D(R_e)$ has a form similar to that shown in Figure (1). This means that C_D has a 'regime dependence' on R_e.
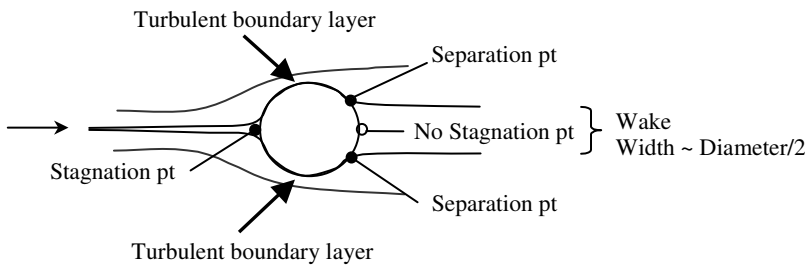
There are two main **regimes** of interest:

- **Laminar regime:** $\boxed{R_{e \text{ no separation}} < R_e < R_{e \text{ critical}} \cong \quad (3 \times 10^5) \atop \text{for a smooth sphere with smooth inflow}}$

Laminar boundary layer

Separation pt

Stagnation pt

Drag

No Stagnation pt

Separation pt

Laminar boundary layer

Wake
Width ~ Diameter

- Wide wake
- Early separation
- 'Large' $C_D = O(1)$

- **Turbulent regime:** $\boxed{R_e > R_{e \text{ critical}}}$

Turbulent boundary layer

Separation pt

Stagnation pt

No Stagnation pt

Separation pt

Turbulent boundary layer

Wake
Width ~ Diameter/2

- Narrow wake
- Delayed separation
- Smaller C_D

iii) **Cylinder** The drag coefficient for a cylinder is defined as: $\boxed{C_D = \dfrac{D/L}{\frac{1}{2}\rho U^2 d}}$
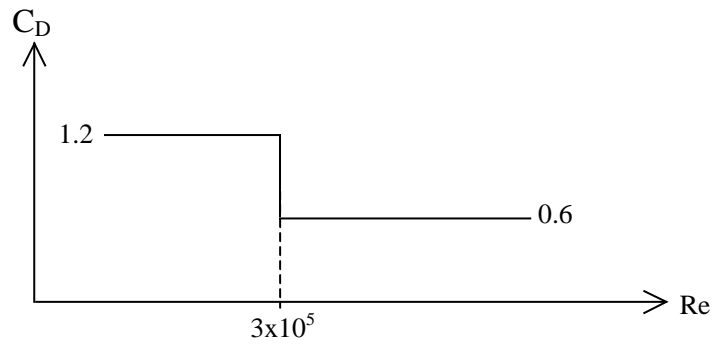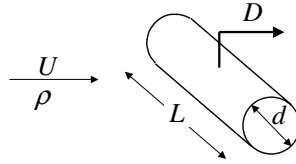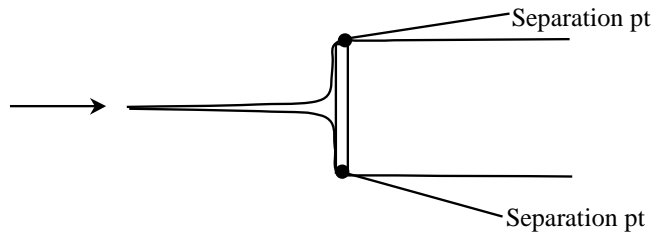

Figure 2: Drag coefficient (C_D) for a cylinder for $R_e > 10^2$

iv) **Bodies with Fixed Separation Points**

For bodies with fixed separation points, the drag coefficient is roughly constant, i.e., does not depend on R_e. For example, for a **flat plate or disc** $\boxed{C_D \approx 1.2}$
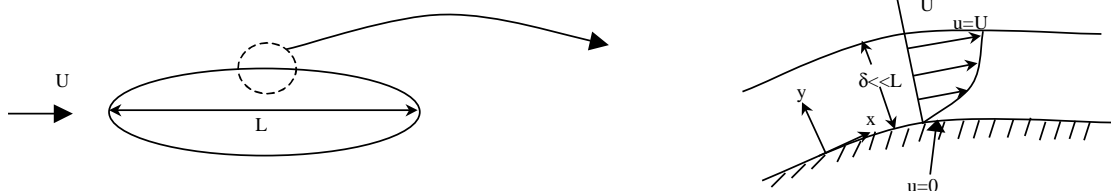
4.1.2 Boundary Layers

$$\frac{L}{UT}\left(\frac{\partial \vec{v}}{\partial t}\right)^* + \left(\vec{v}\cdot\nabla\vec{v}\right)^* = ... + \underbrace{\frac{\nu}{UL}}_{\frac{1}{Re_L}}\left(\nabla^2\vec{v}\right)^*$$

For most flows of interest to us $R_{e_L} >> 1$, i.e., viscosity can be ignored if U, L govern the problem, thus potential flow can be assumed. In the context of potential flow theory, drag = 0! Potential flow (no τ_{ij}) allows slip at boundary, but in reality, the no-slip condition applies on the boundaries. Otherwise, if $\nu \neq 0$ and a free-slip KBC is imposed then $\tau \sim \nu \frac{\partial u}{\partial y} \to \infty$ at the boundary.

Prandtl: There is a length scale δ (**boundary layer thickness** $\delta << L$) over which velocity goes from zero on the wall to the potential flow velocity U outside the boundary layer.

Estimate δ: Inside the **boundary layer**, viscous effects are of the same order as the inertial effects.

$$\nu\frac{\partial^2 U}{\partial y^2} \sim U\frac{\partial U}{\partial x} \to \nu\frac{U}{\delta^2} \sim \frac{U^2}{L} \to \frac{\nu}{UL} \sim \frac{\delta^2}{L^2} \to \frac{\delta}{L} \sim \sqrt{\frac{\nu}{UL}} = \frac{1}{\sqrt{R_{e_L}}} << 1 \text{ As } R_{e_L} \uparrow, \delta \downarrow$$

Generally: $R_{e_L} >> 1, \frac{\delta}{L} << 1$, thus potential flow is good **outside** a very thin boundary layer (i.e., provided no separation - a real fluid effect). For Reynolds number not $>> 1(R_e \sim O(1))$, then thick boundary layer ($\delta \sim O(L)$) and Prandtl's boundary layer idea not useful. If separation occurs, then boundary layer idea is not valid.

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4.1.3 Boundary Layers and Flow Separation

Boundary layers help understand flow separation.

Example for flow past a circle.
Outside the boundary layer P-Flow is valid. Let **capital** U denote the **potential flow** tangential velocity on the circle and let x denote the distance along the circle surface (i.e., $x =$ body coordinate).

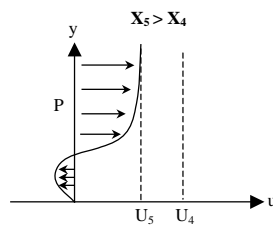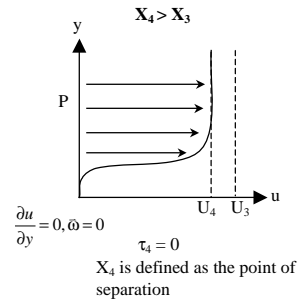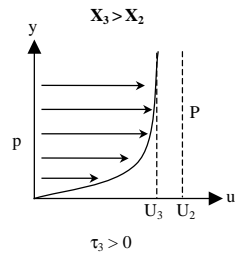From the steady inviscid x-momentum equation (steady Euler) along the body boundary ($y = 0$, $V = 0$), we obtain :
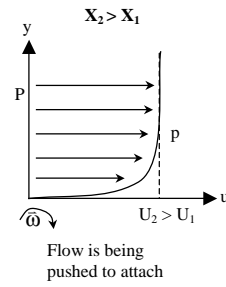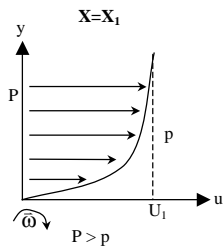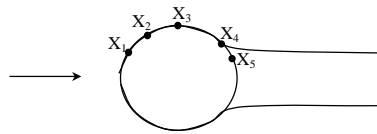
$$\boxed{U\frac{dU}{dx} = -\frac{1}{\rho}\frac{dp}{dx}} \tag{1}$$

Note 1: Equation (1) is used frequently in boundary layer theory.
Note 2: From Equation (1) $\rightarrow$ for flow past a flat plate $\frac{dp}{dx} = 0$ along the plate.

P$-$Flow solution on body$\big|_{y=0}$: $U\dfrac{dU}{dx} = -\dfrac{1}{\rho}\dfrac{dp}{dx}$
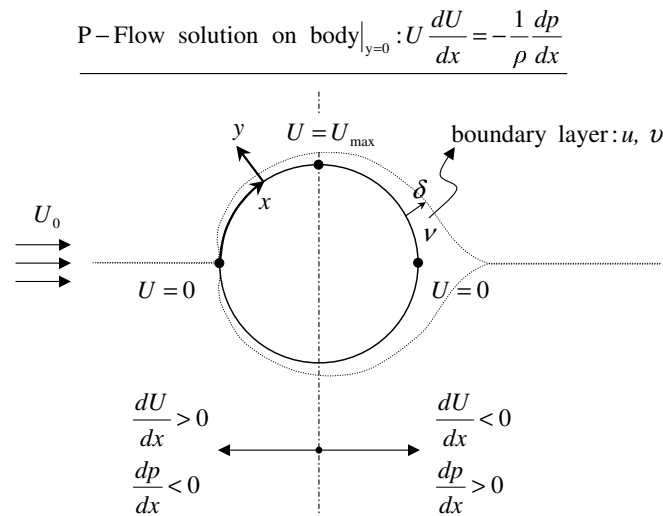
$\frac{dU}{dx} > 0$	Acceleration	$\frac{dU}{dx} < 0$	Deceleration
$\frac{dp}{dx} < 0$	'Favorable' pressure gradient	$\frac{dp}{dx} > 0$	'Adverse' pressure gradient

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X=X$_1$

y

P

p

u

U_1

ω

$P > p$

X$_2$ > X$_1$

y

P

p

u

$U_2 > U_1$

ω

Flow is being
pushed to attach

X$_3$ > X$_2$

y

P

p

u

U_3 U_2

$\tau_3 > 0$

X$_4$ > X$_3$

y

P

u

U_4 U_3

$\dfrac{\partial u}{\partial y} = 0, \bar{\omega} = 0$

$\tau_4 = 0$

X$_4$ is defined as the point of
separation

X$_5$ > X$_4$

y

P

u

U_5 U_4

Separated Flow $\tau_5 < 0$
Flow reversal

A better way to think about separation is in terms of diffusion of vorticity.

ω=0 outside B.L.

ω added to fluid

ω removed from fluid
by diffusion
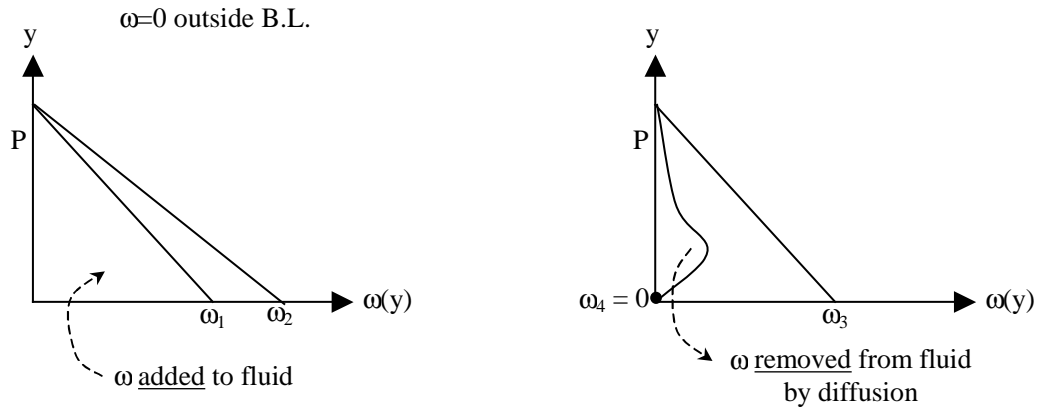
Think of vorticity as heat; $\omega(y)$ is equivalent to a temperature distribution. Note:

$$\frac{D\vec{V}}{Dt} = ... + \nu\nabla^2\vec{V} \quad \text{and} \quad \frac{D\vec{\omega}}{Dt} = ... + \nu\nabla^2\vec{\omega}$$