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2.72 Elements of Mechanical Design

Spring 2009

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2.72

*Elements of
Mechanical Design*

Lecture 08: Flexures

Schedule and reading assignment

Quiz

- ❑ Today: Bearing layouts (mid-class)
- ❑ Thursday: Hale 6.1
- ❑ Soon: Bolted joint qualifying quiz

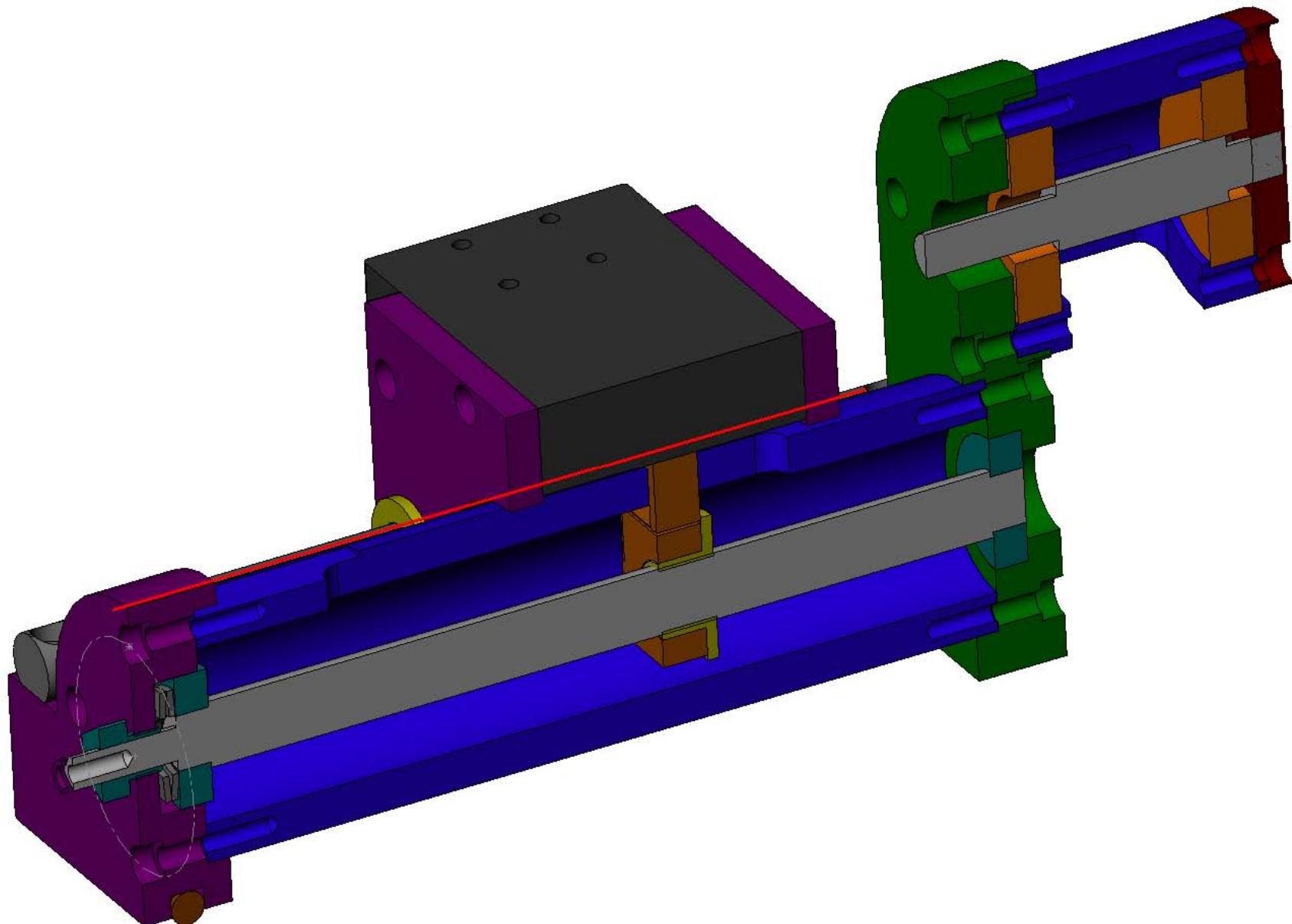
Topics

- ❑ Flexure constraints and bearings... Degrees of Freedom

Reading assignment

- ❑ Thursday:
 - *Layton Hale's thesis – Read 2.6, 2.7, 6.1, skim rest of Chapter 6*
 - *Chapter 7 is cool to look at*
- ❑ Tuesday:
 - *Read: 8.1, 8.3 – 8.5, 8.7, 8.9 – 8.11*
 - *Skim: 8.6, 8.8, 8.12*

Examples drawn from your lathe



Mechanisms: Compliant vs. rigid

Rigid mechanisms

- ❑ Sliding joints
- ❑ 100s of nm resolution
- ❑ Large range
- ❑ kg load capacity

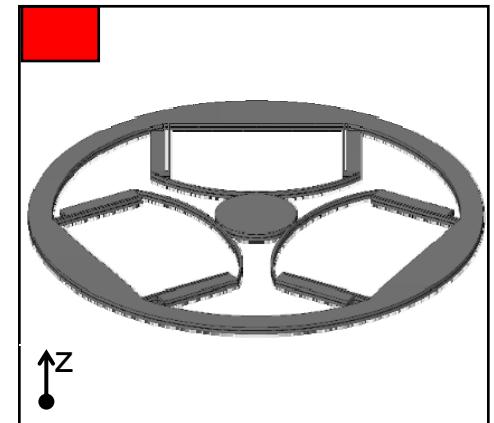
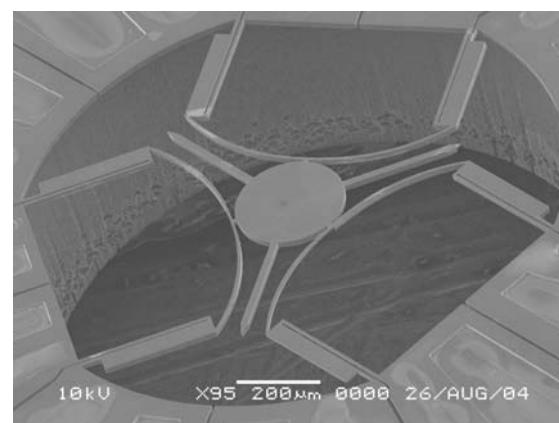
Images removed due to copyright restrictions. Please see

http://www.physikinstrumente.com/en/primages/pi_m850_tip_i4c_o_eps.jpg

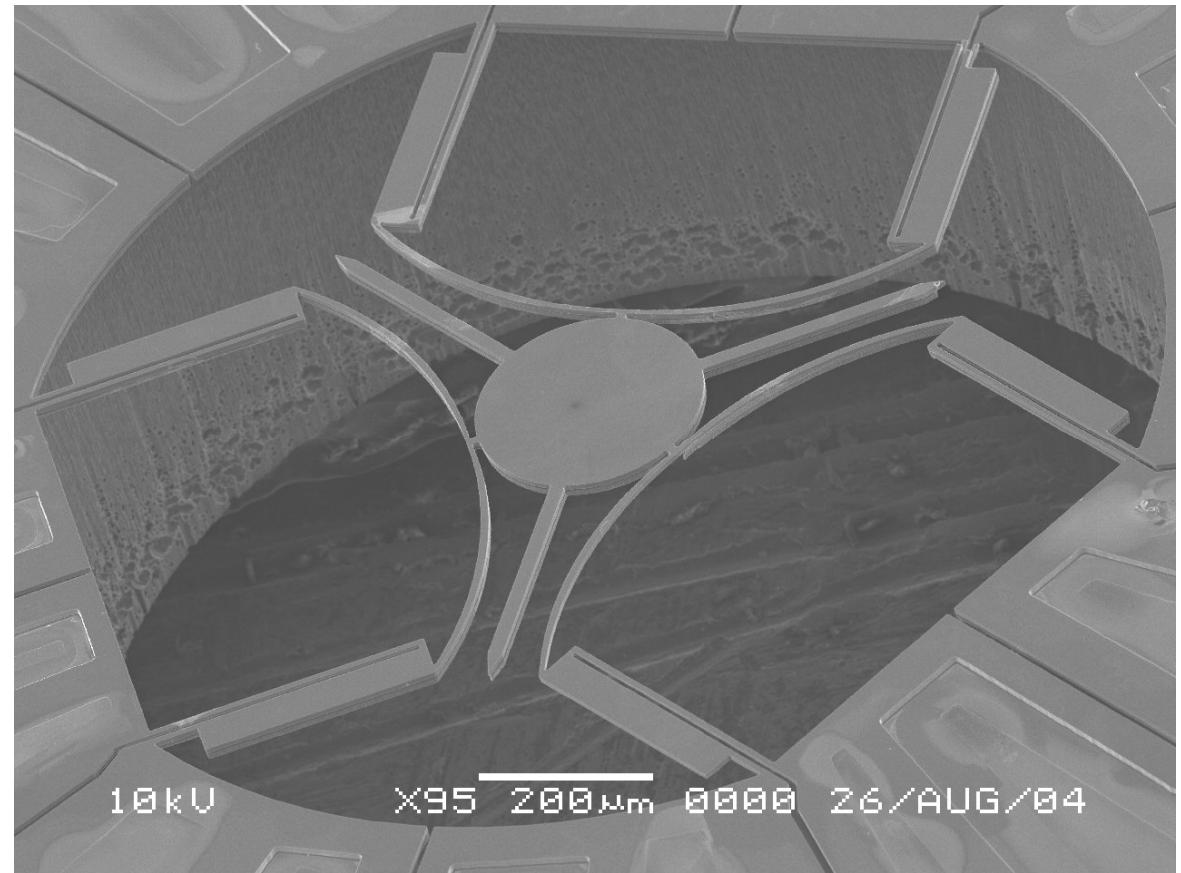
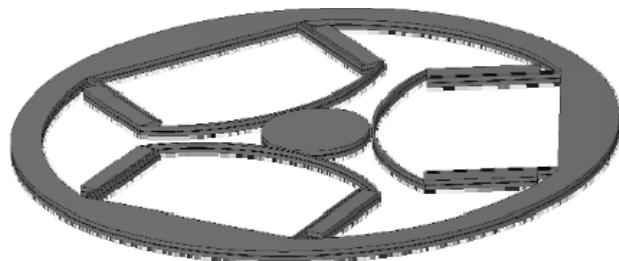
<http://www.hexamods.net/images/M850Ani160-1-slow.gif>

Compliant mechanisms

- ❑ Motion from member compliance
- ❑ Angstrom resolution
- ❑ Limited range
- ❑ Limited load capacity

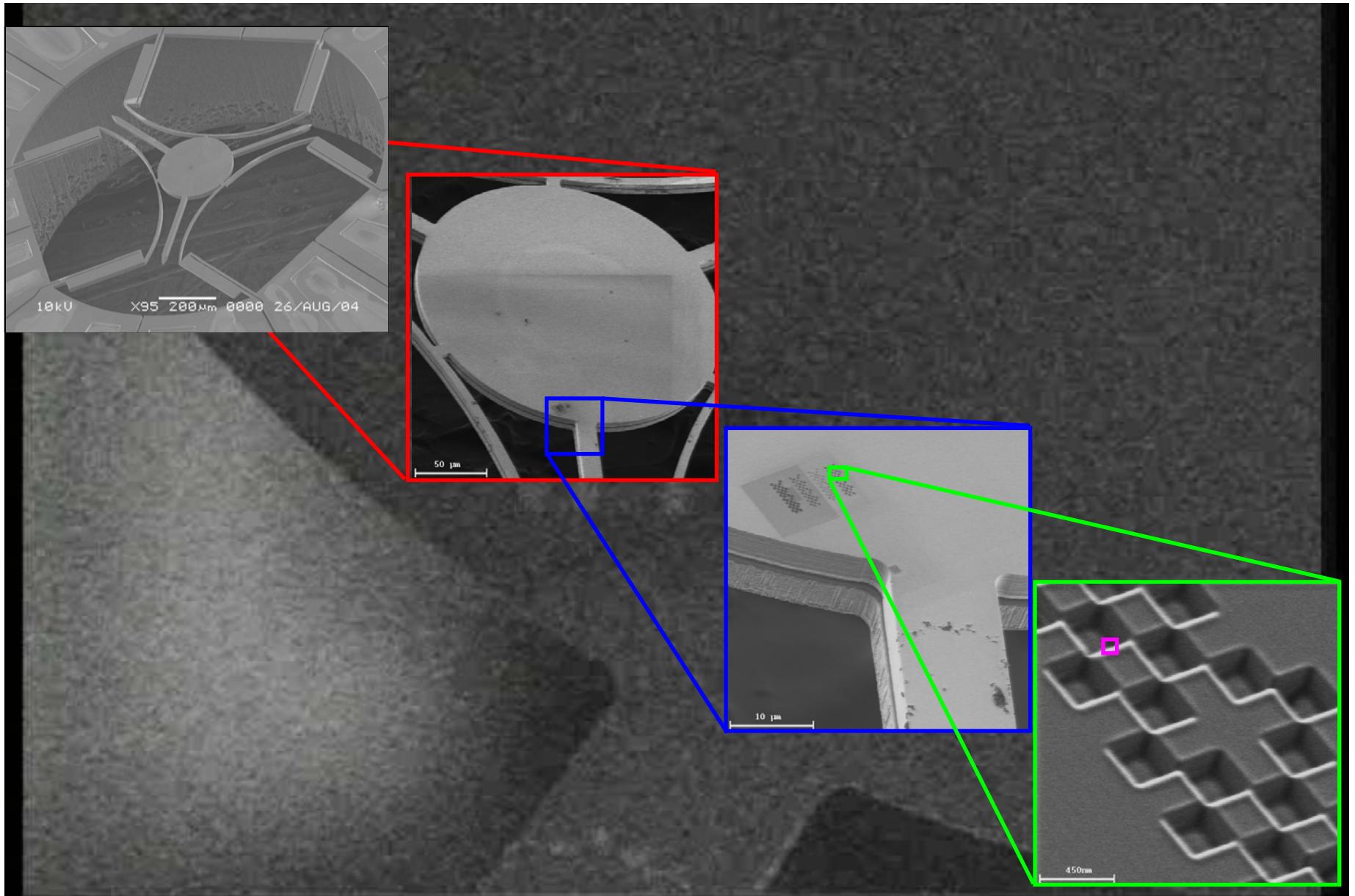


Micro-scale precision machines



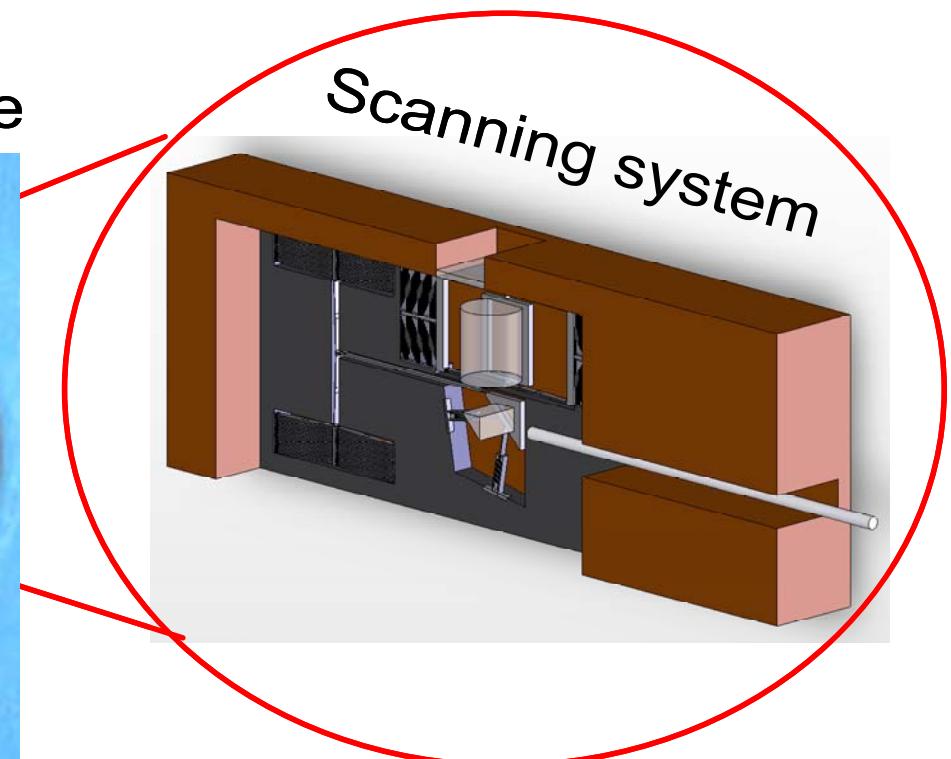
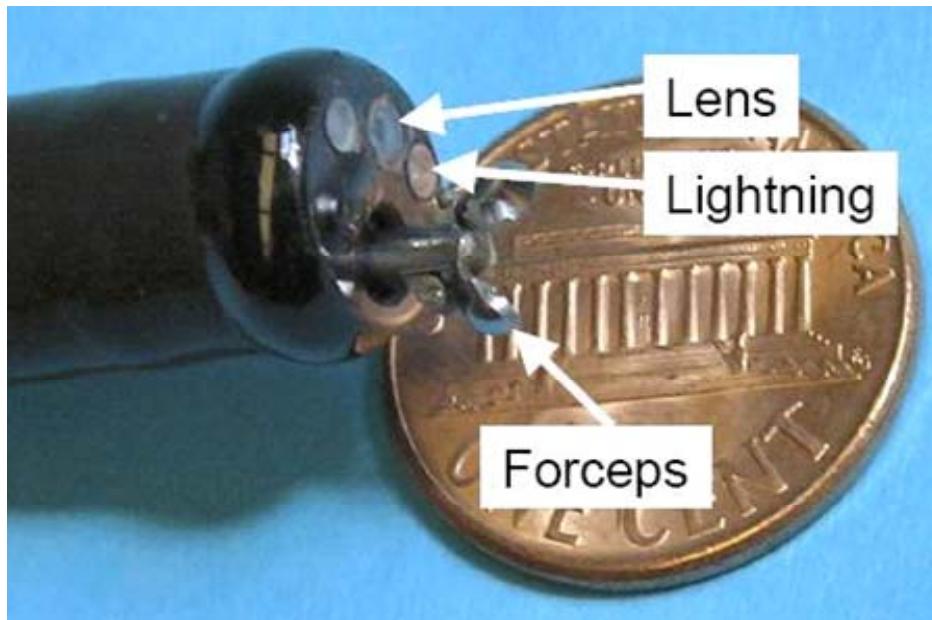
Static

SEM: Drs. Andras Vladar & Jason Gorman (NIST)
FIB: Dr. Konrad Jarush
(Hitachi)

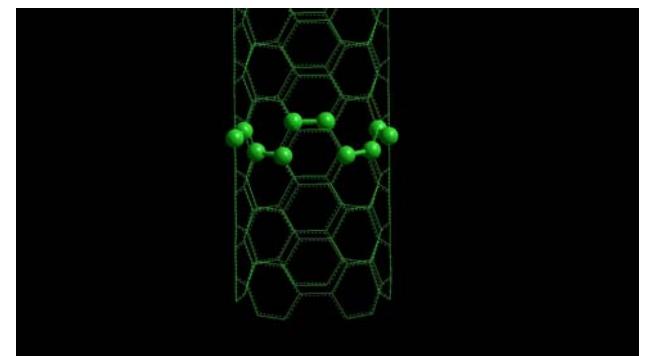
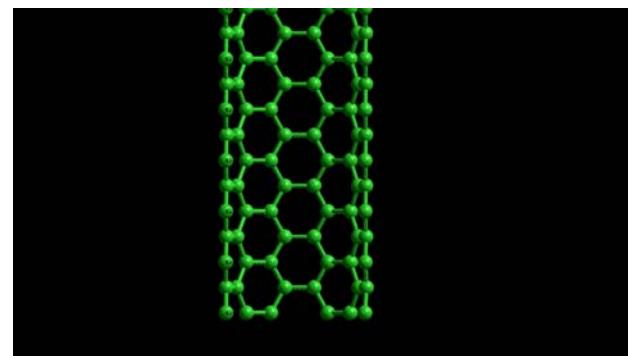
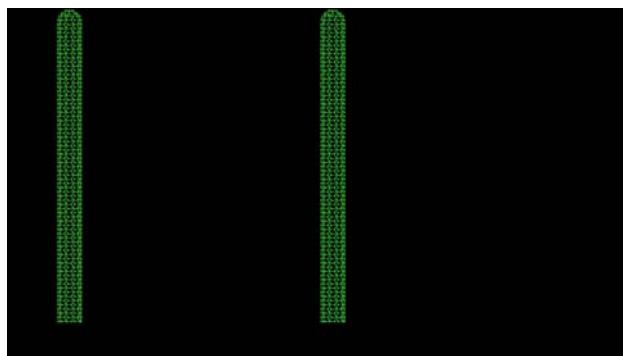
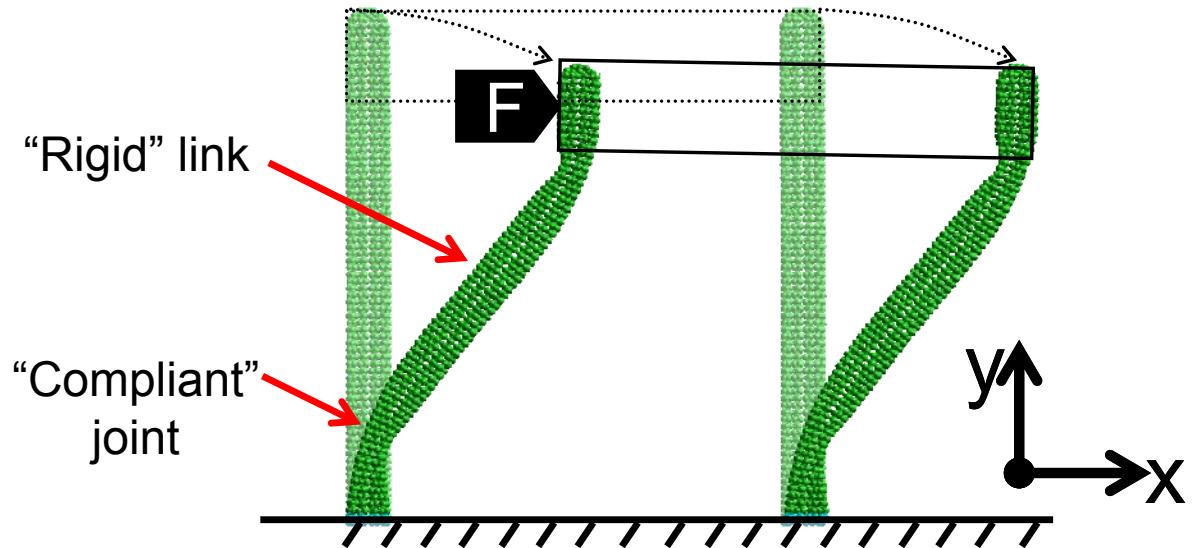
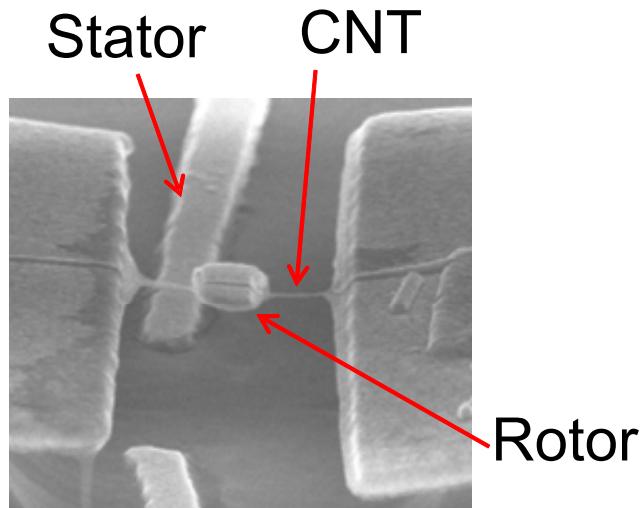


Meso-scale devices: Biomedical

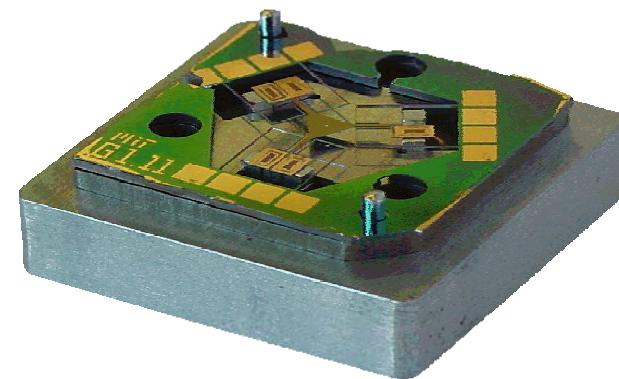
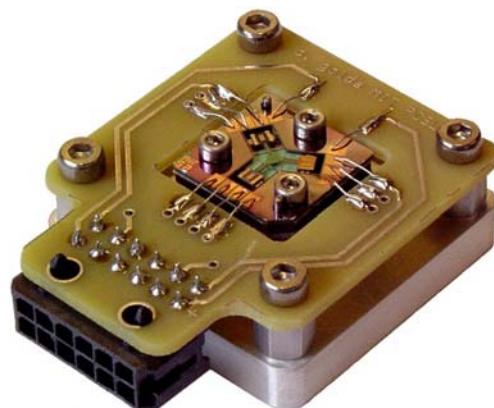
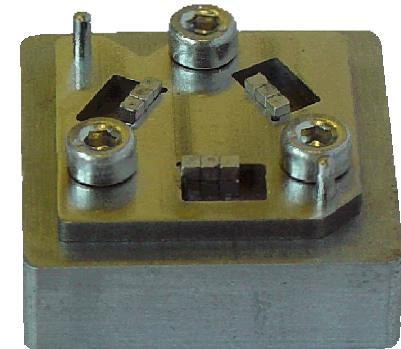
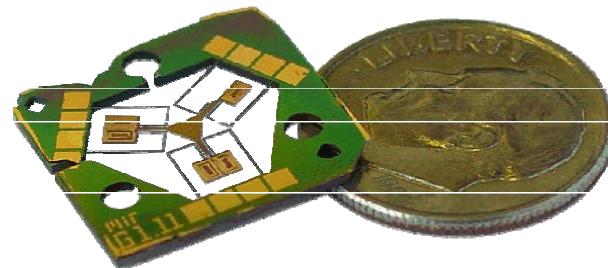
Two -photon endomicroscope



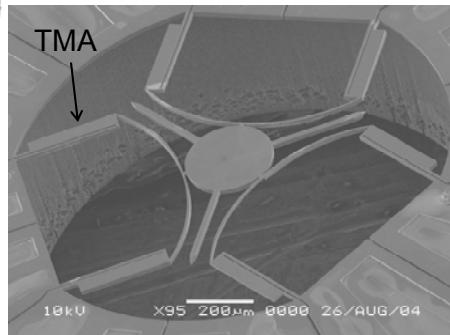
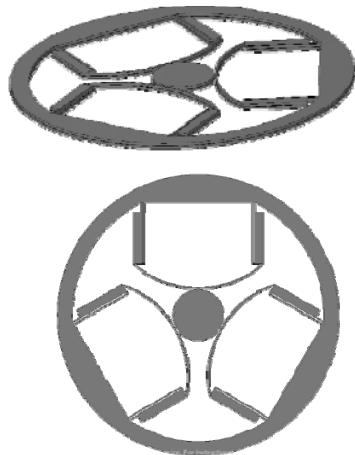
Nano-scale devices



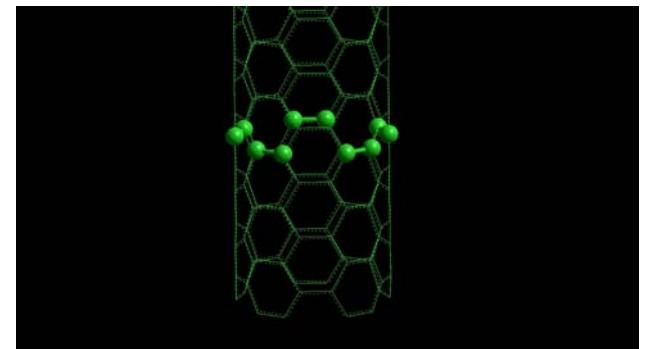
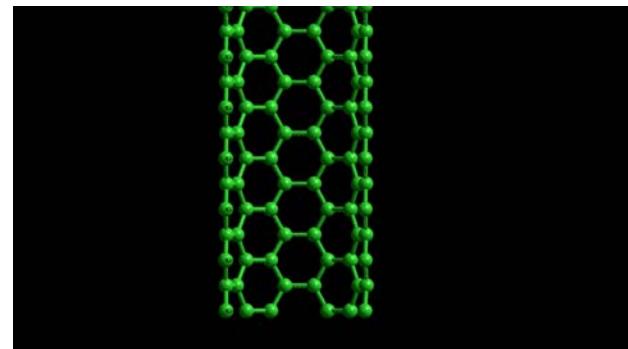
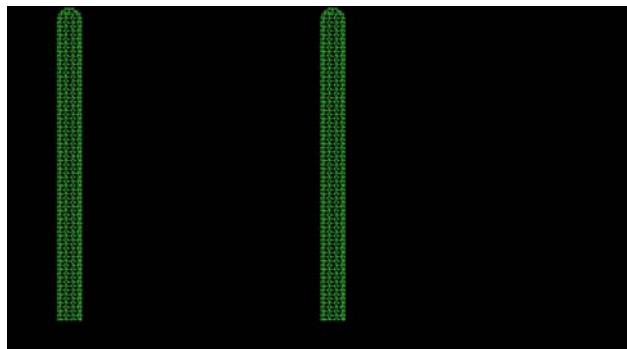
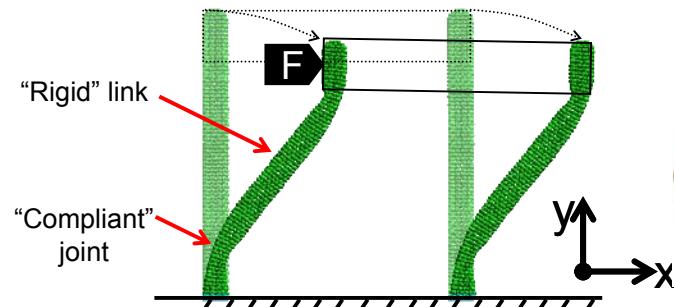
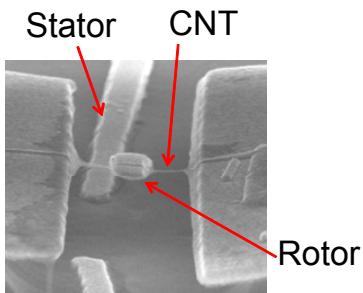
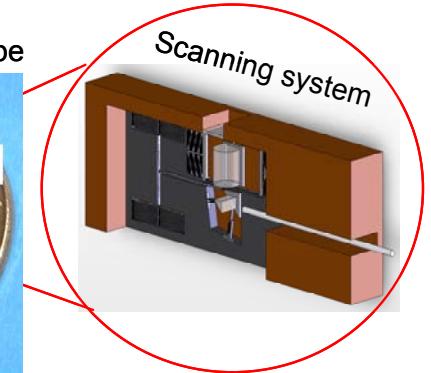
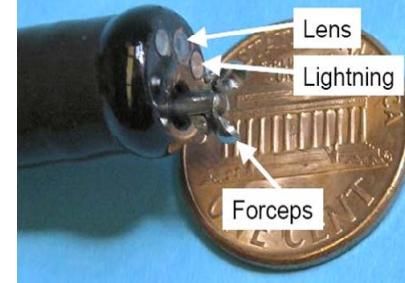
Meso-scale precision machines



Nano-scale devices



Two -photon endomicroscope



Dip pen nanolithography on DNA arrays

What is fundamentally different?

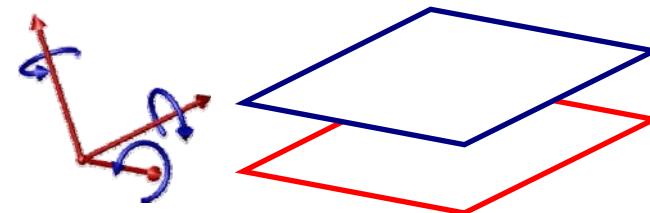
- Size → Physics → Fabrication
- Raw materials
- Surfaces vs. points or lines

Images removed due to copyright restrictions. Please see

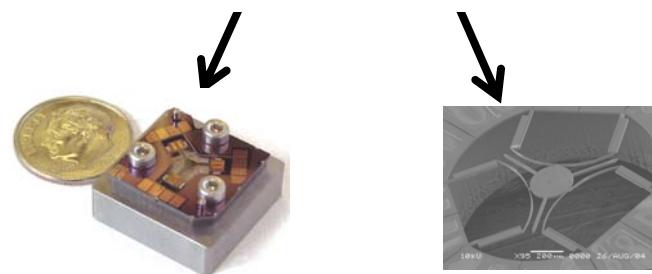
http://mcf.tamu.edu/images/DPN_process.png

http://www.nanoink.net/d/Nano%20-%20Part%201_Sm_Lo-Res_240x180.wmv

<http://images.iop.org/objects/nano/news/4/12/10/diagnal.jpg>



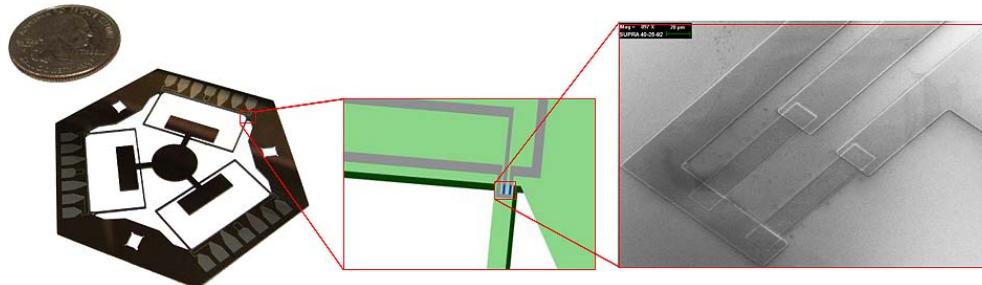
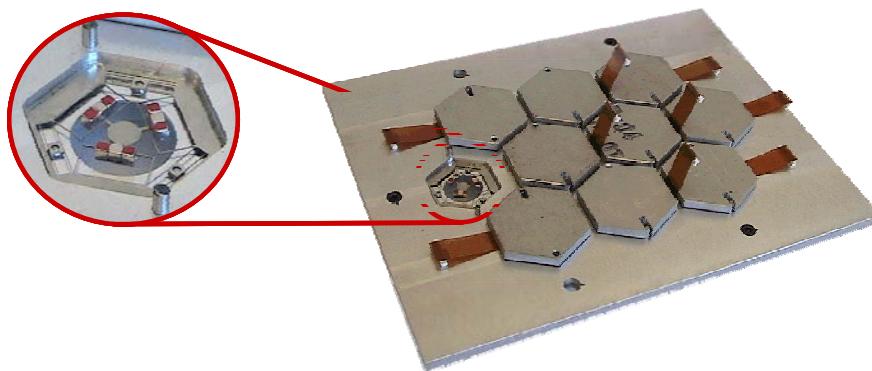
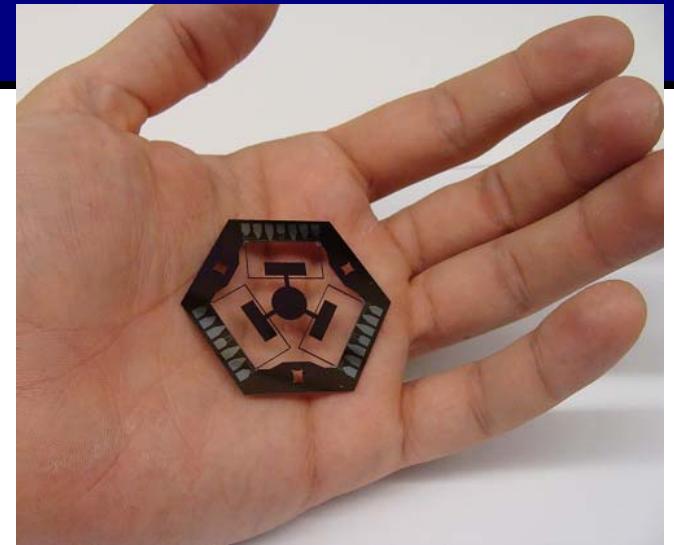
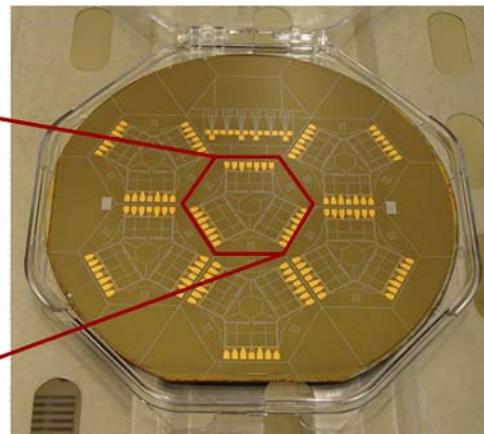
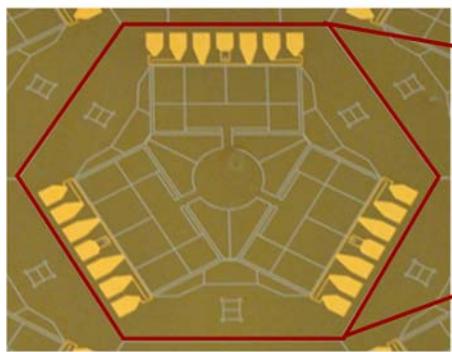
250 mm
Courtesy
PI



~20 mm

~1 mm

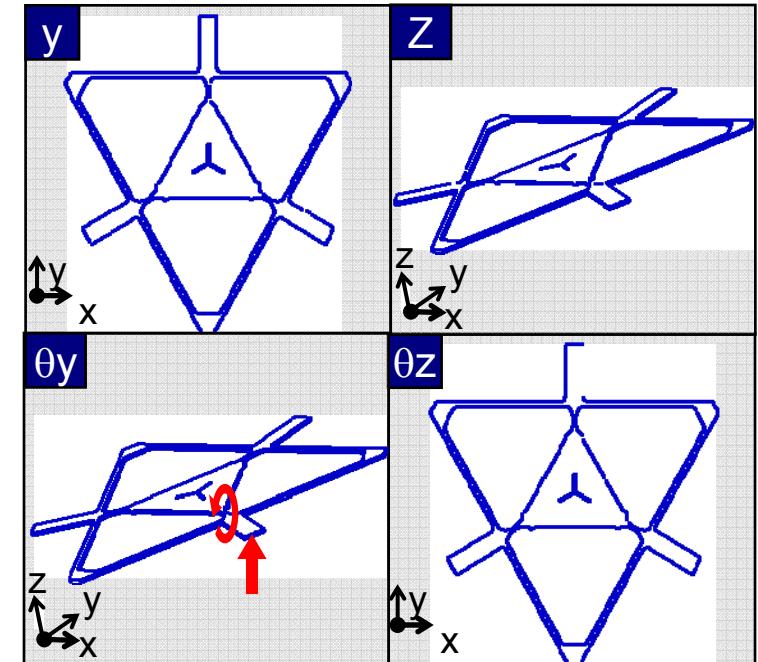
Nanomanufacturing



Advantages of flexures

Advantages

- Smooth, fine motion
- Linear/elastic operation in absence
- Failure modes are well understood
- Monolithic or assembled
- 2D nature lends to 2½D mfg.
- Miniaturization



Disadvantages

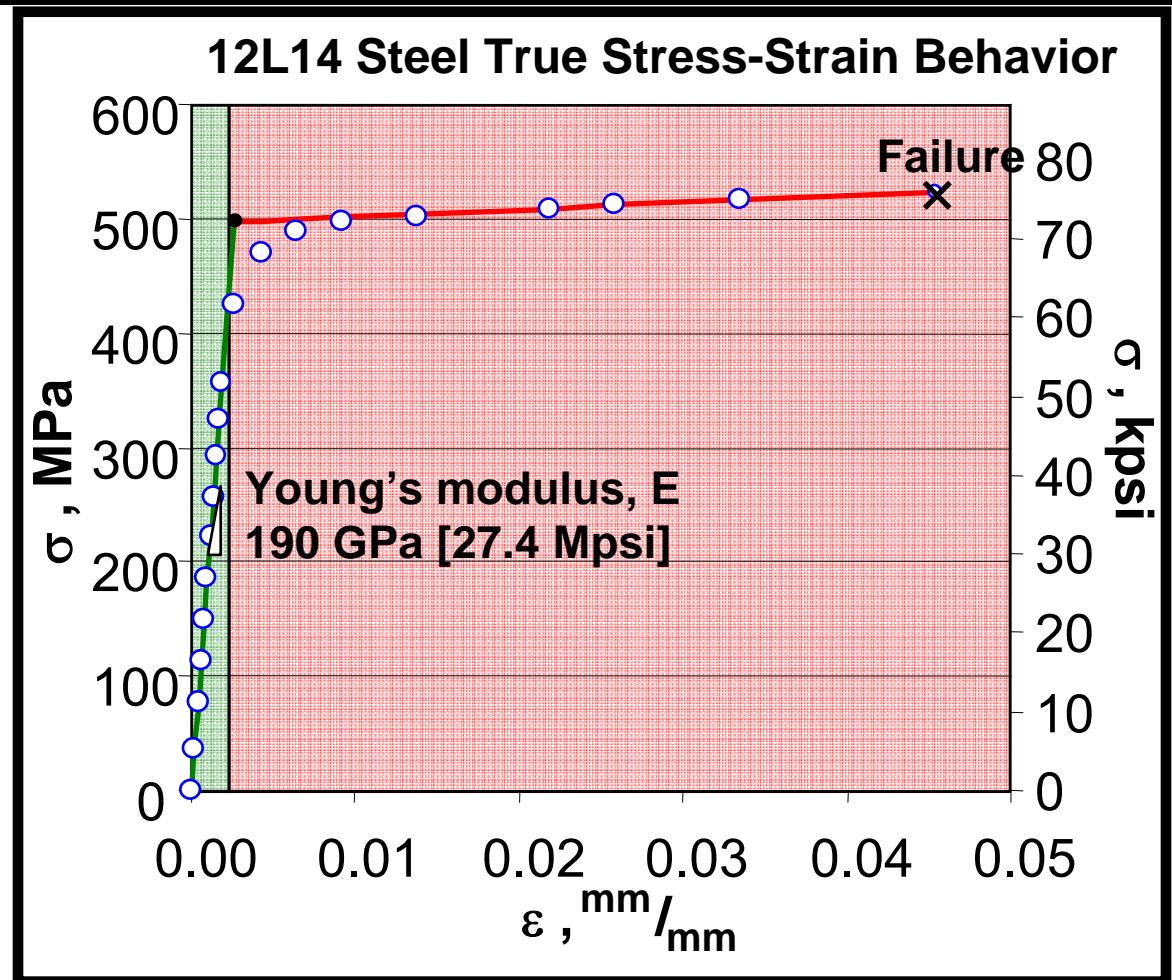
- Accuracy and repeatability sensitive to several variables
- Limited motion/stroke (usually a few to 10s % of device size)
- Instabilities such as axial or transverse buckling
- Dynamics
- Sensitivity to tolerance

Elastomechanics (σ & ε) relationship

Elastic
 $\sigma = \varepsilon \cdot E$

Plastic

Material	σ_y/E
Titanium V	1.00
Aluminum 7075	0.70
Stainless 316	0.09
Invar - Annealed	0.19



Important material properties

Nominal values

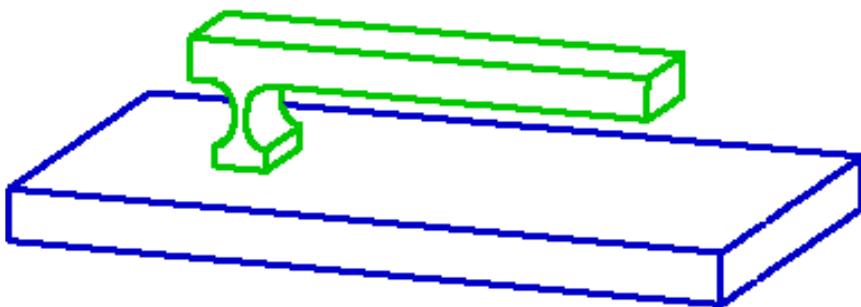
- Modulus
- Yield stress
- Coefficient of thermal expansion
- Thermal diffusivity
- Density

Material property ratios

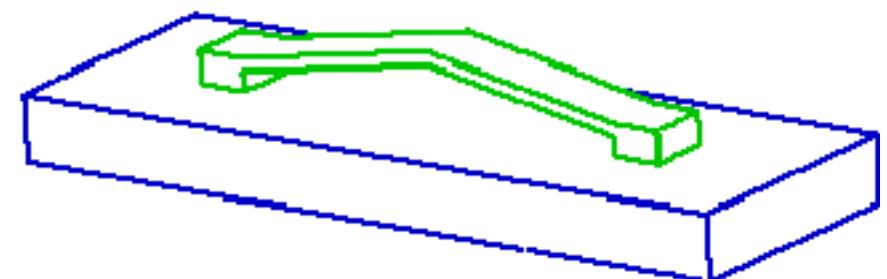
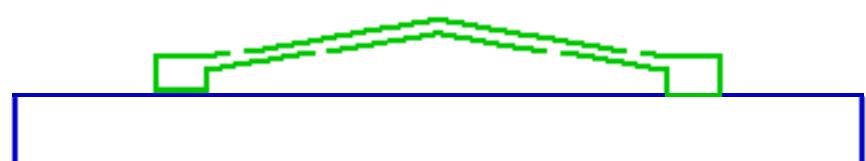
Material	Normalized Values			
	σ_y/E	$\alpha_{\text{diff}}/\alpha_{\text{CTE}}$	E/ρ	Cost
Titanium V	1.00	0.14	0.92	3.77
Aluminum 7075	0.70	1.00	1.00	1.00
Stainless 316	0.09	0.13	0.94	3.50
Invar - Annealed	0.19	0.87	0.70	5.21

Modules

Lever

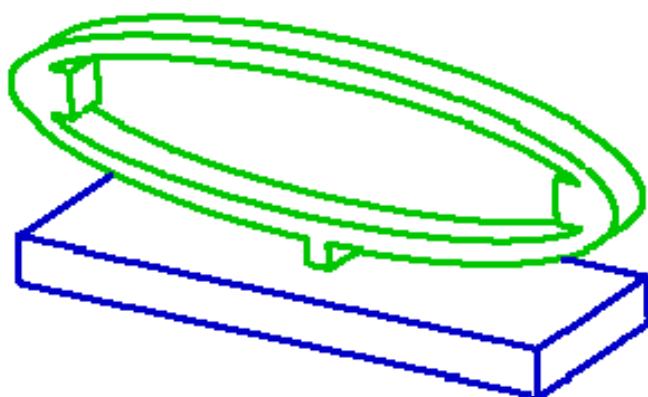
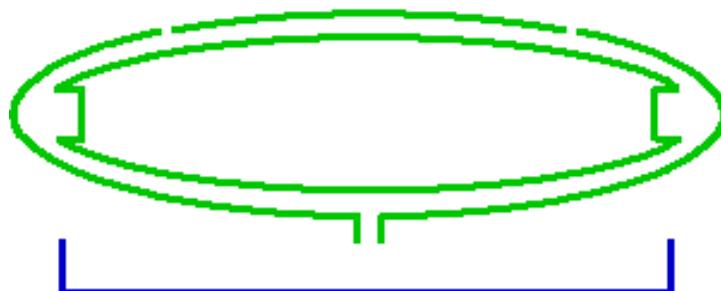


Chevron

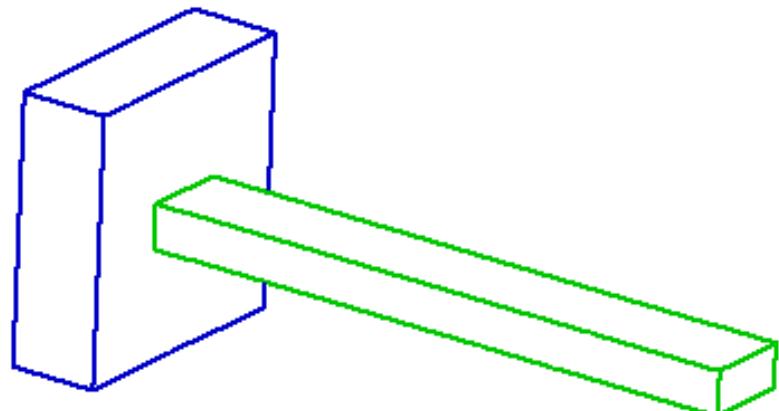
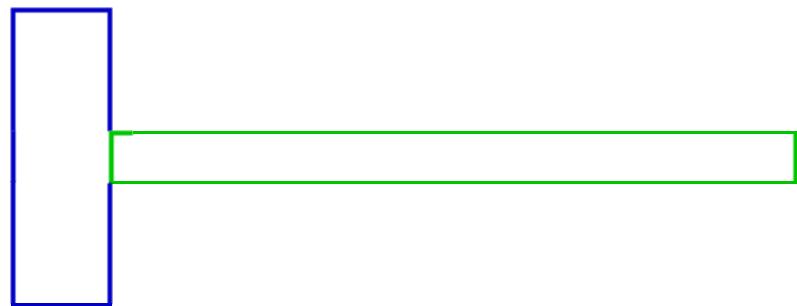


Modules cont.

Ellipse

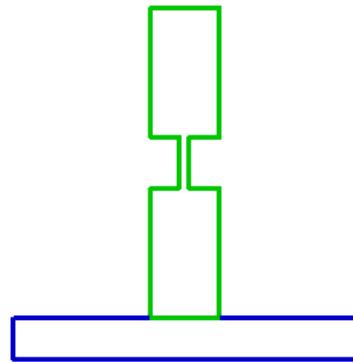


Cantilever/flexure blade

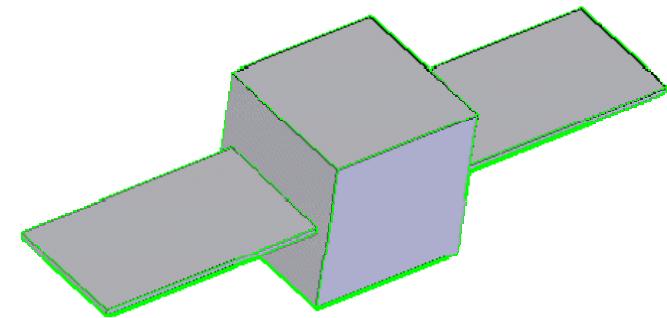


Modules cont.

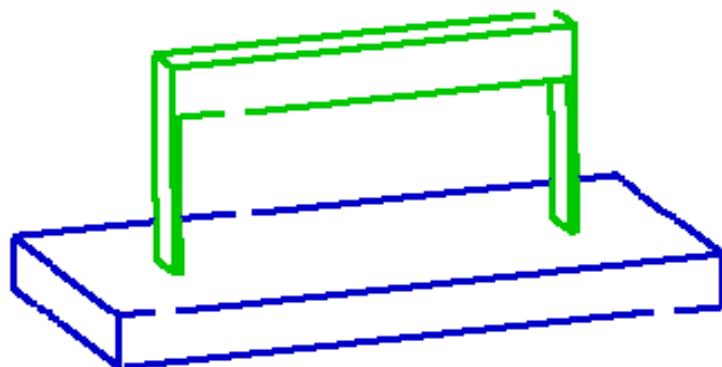
Flexure hinge



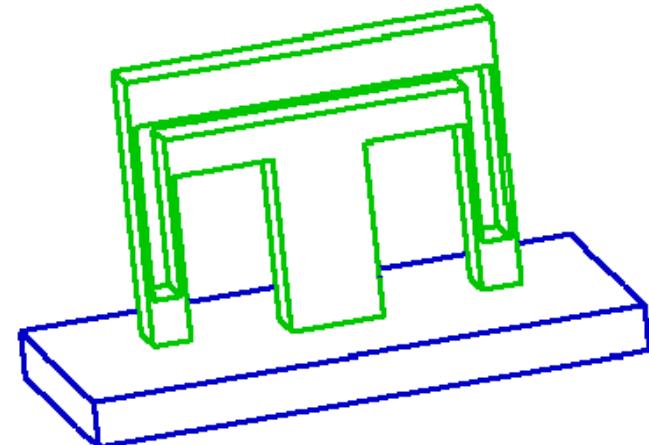
Torsion



Parallel four bar

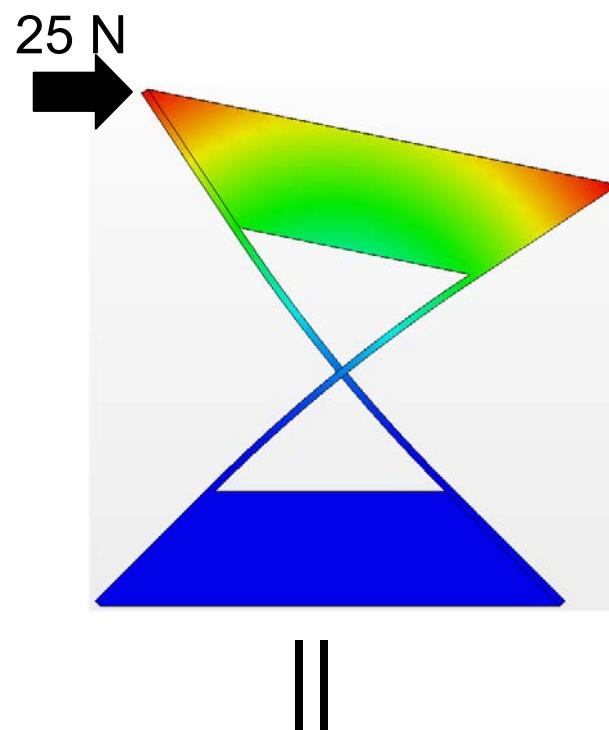
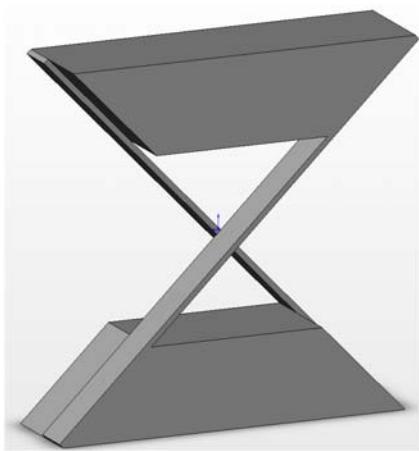


Double parallel four bar

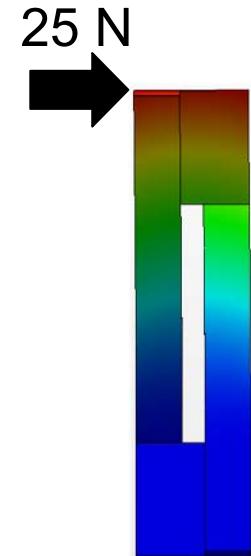
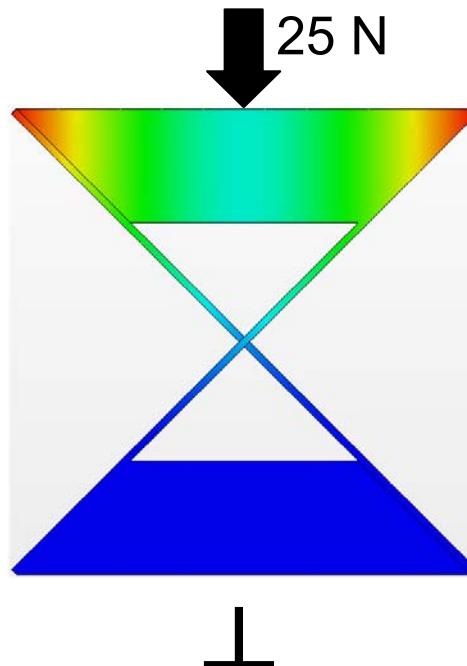


Module cont.: Cross flexure pivot

Deformation scale 1 : 1



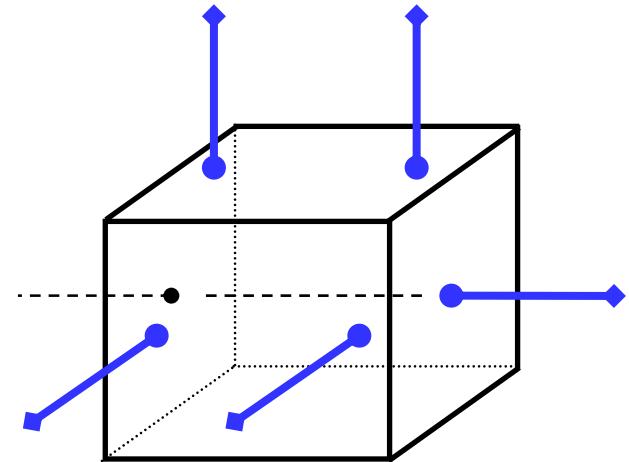
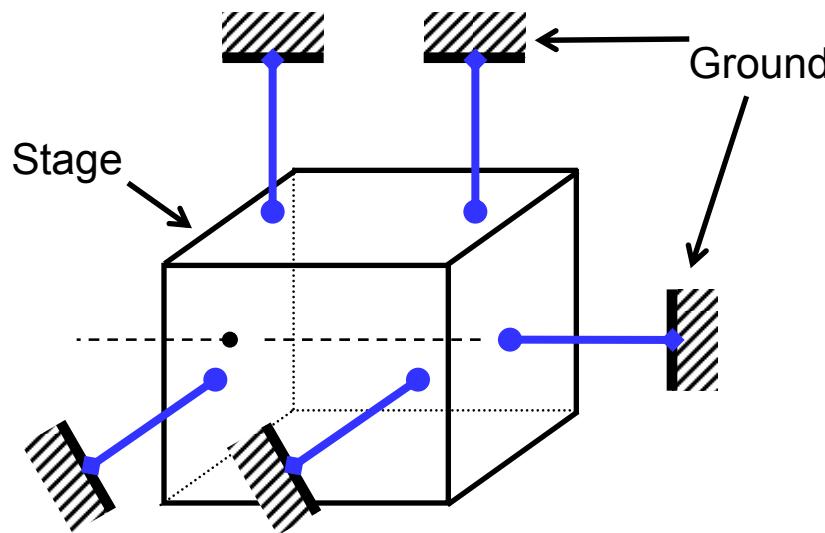
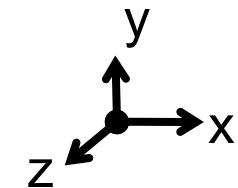
75 mm



Review of constraint fundamentals

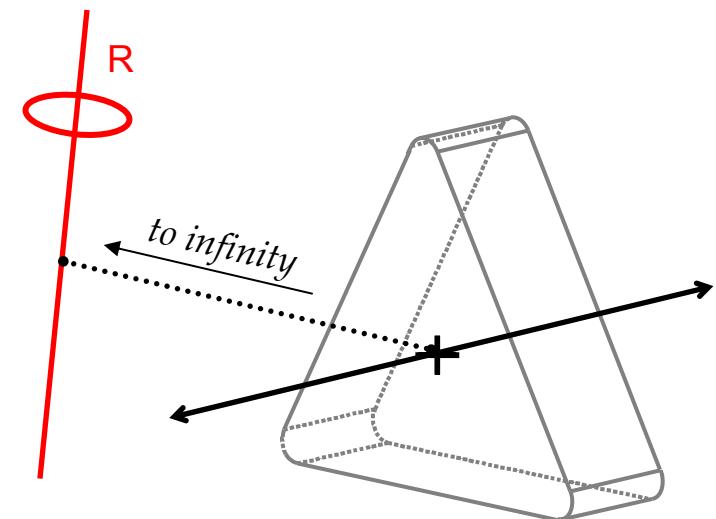
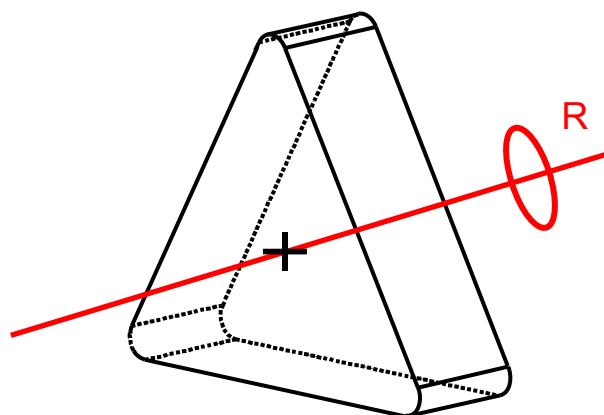
Rigid bodies have 6 DOF

- Constraints have lines of action
- $C = \# \text{ of linearly independent constraints}$
- $\text{DOF} = 6 - C \quad \rightarrow \quad F = 6 - C$



DOF in constraint-based design

A linear displacement may be visualized as a rotation about a point which is “far” away

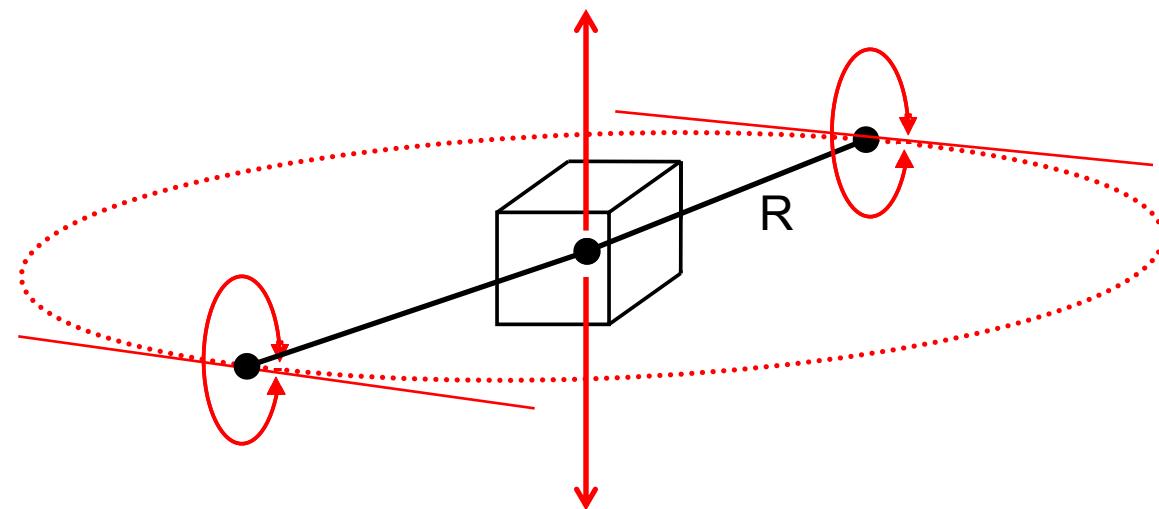


Two principles of projective geometry

Projective geometry comes in useful here

- Parallel lines intersect at infinity
- Translation represented by a rotation line at a hope of “infinite radius”

Image courtesy of John Hopkins
MIT MS Thesis



Constraint fundamentals

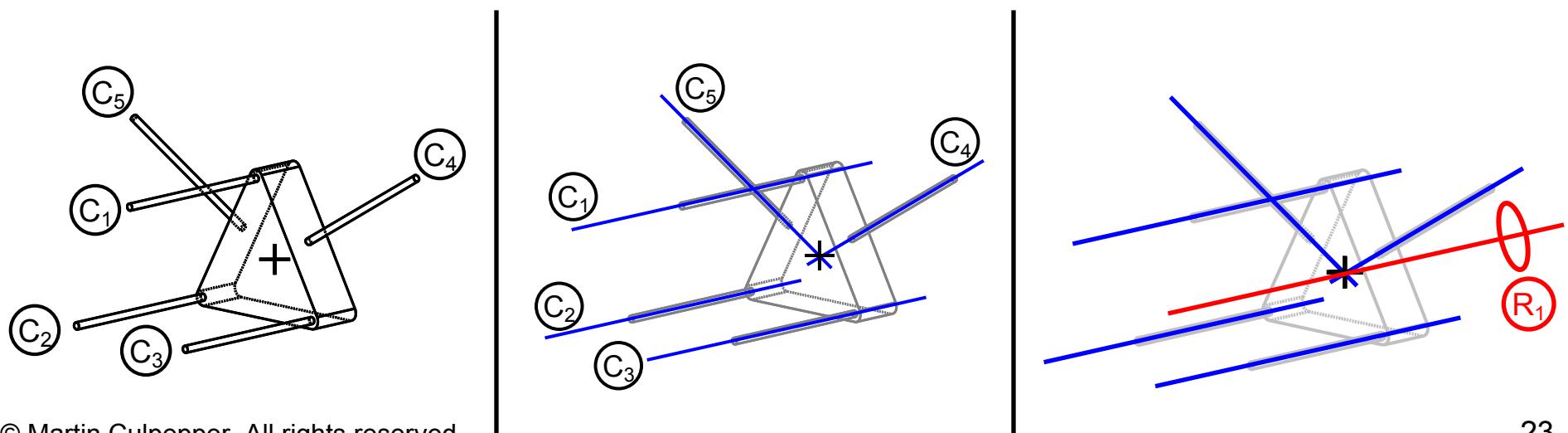
Blanding's RULE OF COMPLIMENTARY PATTERNS

- Each permissible **Freedom (F)** is a rotation about a line and each permissible freedom rotation line must intersect each **Constraint (C)**

Remember these principles of projective geometry

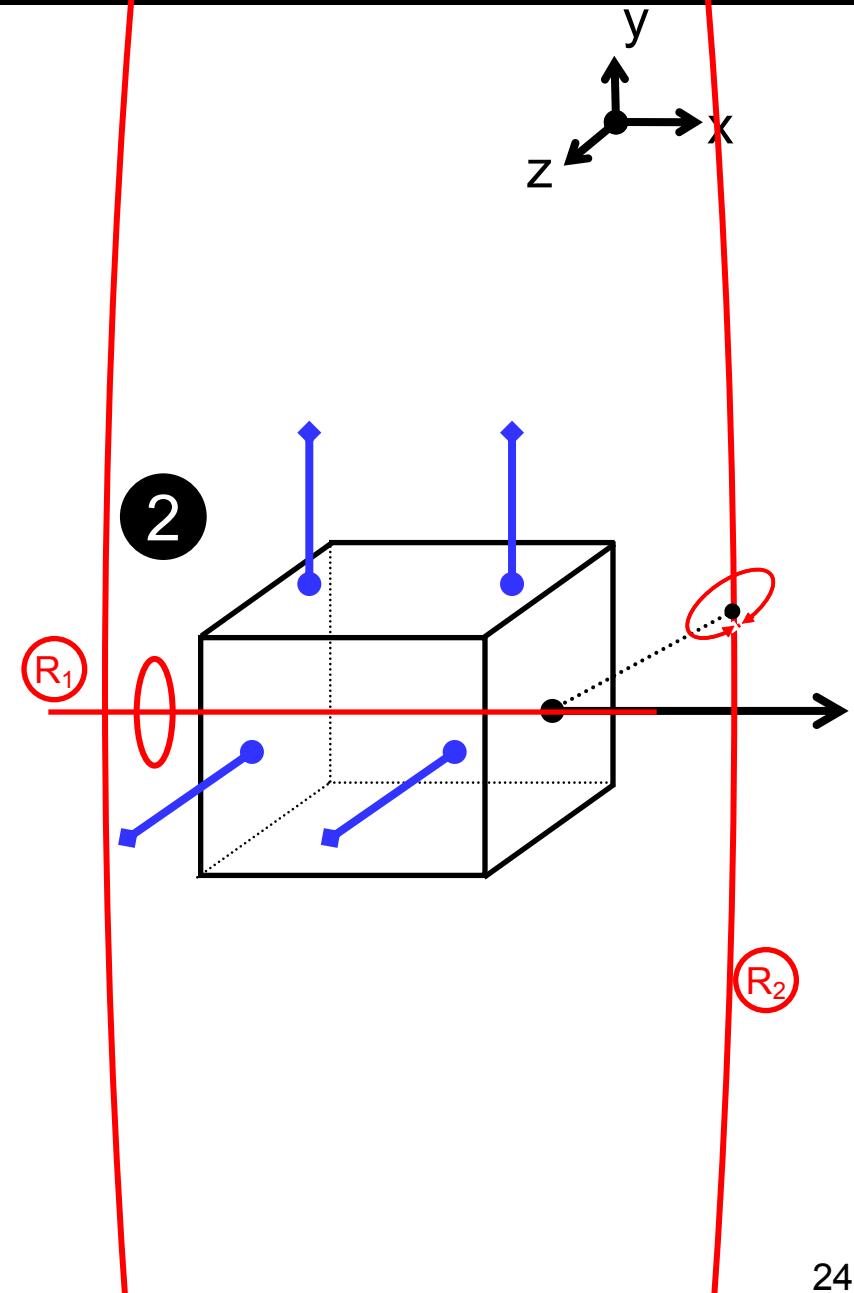
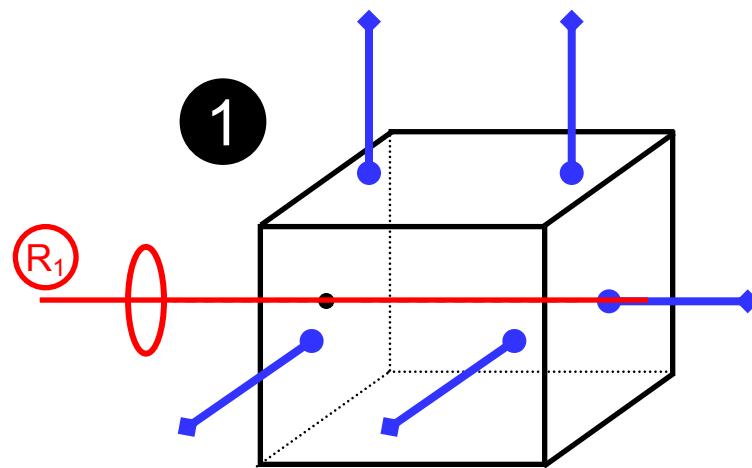
- Parallel lines intersect at infinity
- Translation represented by a rotation line at a hope of “infinite radius

$$R = 6 - C = 6 - 5 = 1 \dots \text{ so where is it?}$$



Examples

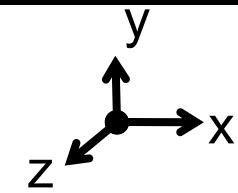
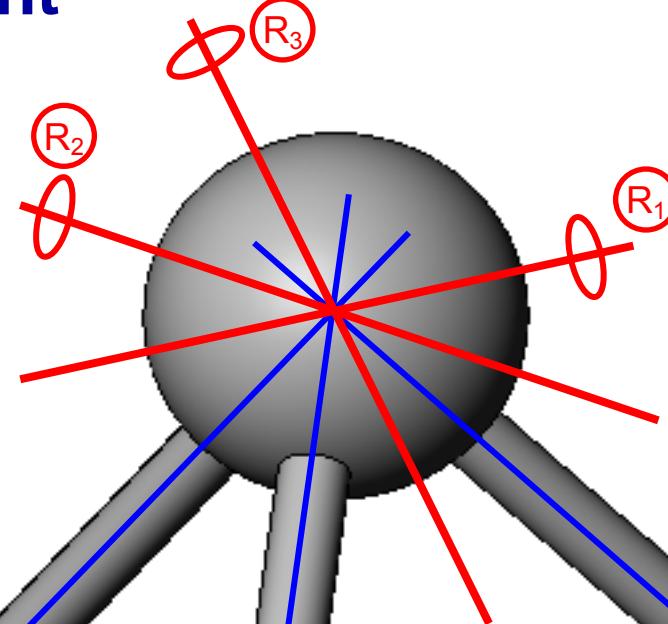
There will be a quiz on this NC



Flexure bearing systems

Spherical ball joint

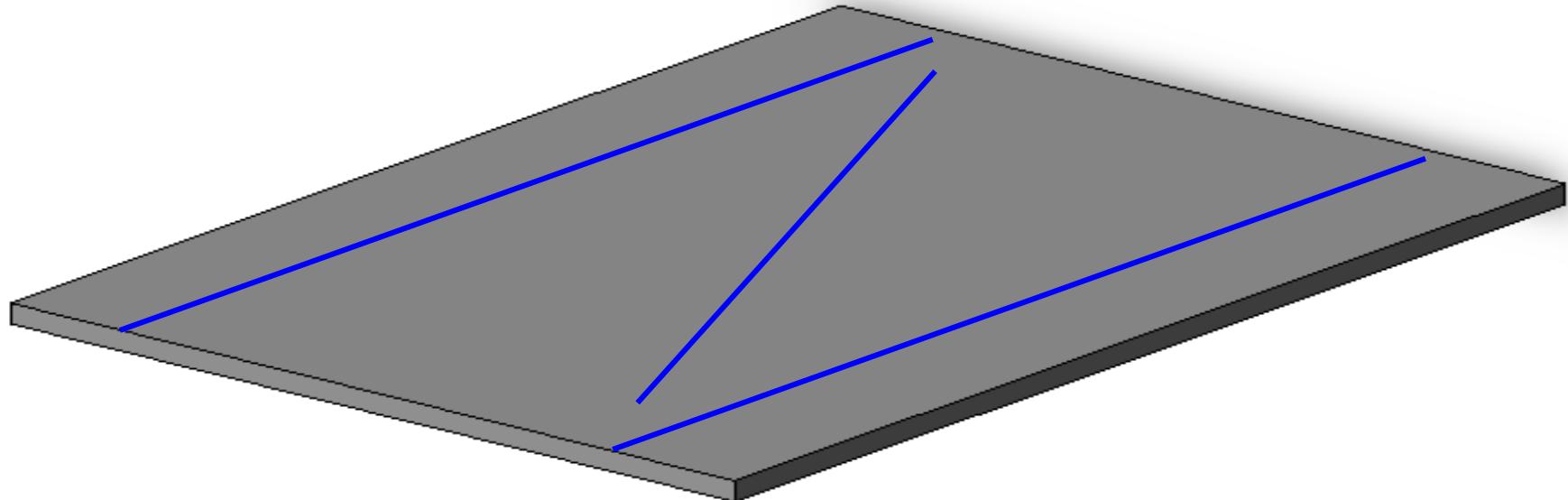
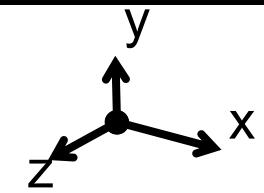
$$6 - C = F$$



Flexure bearing systems

Blade flexure

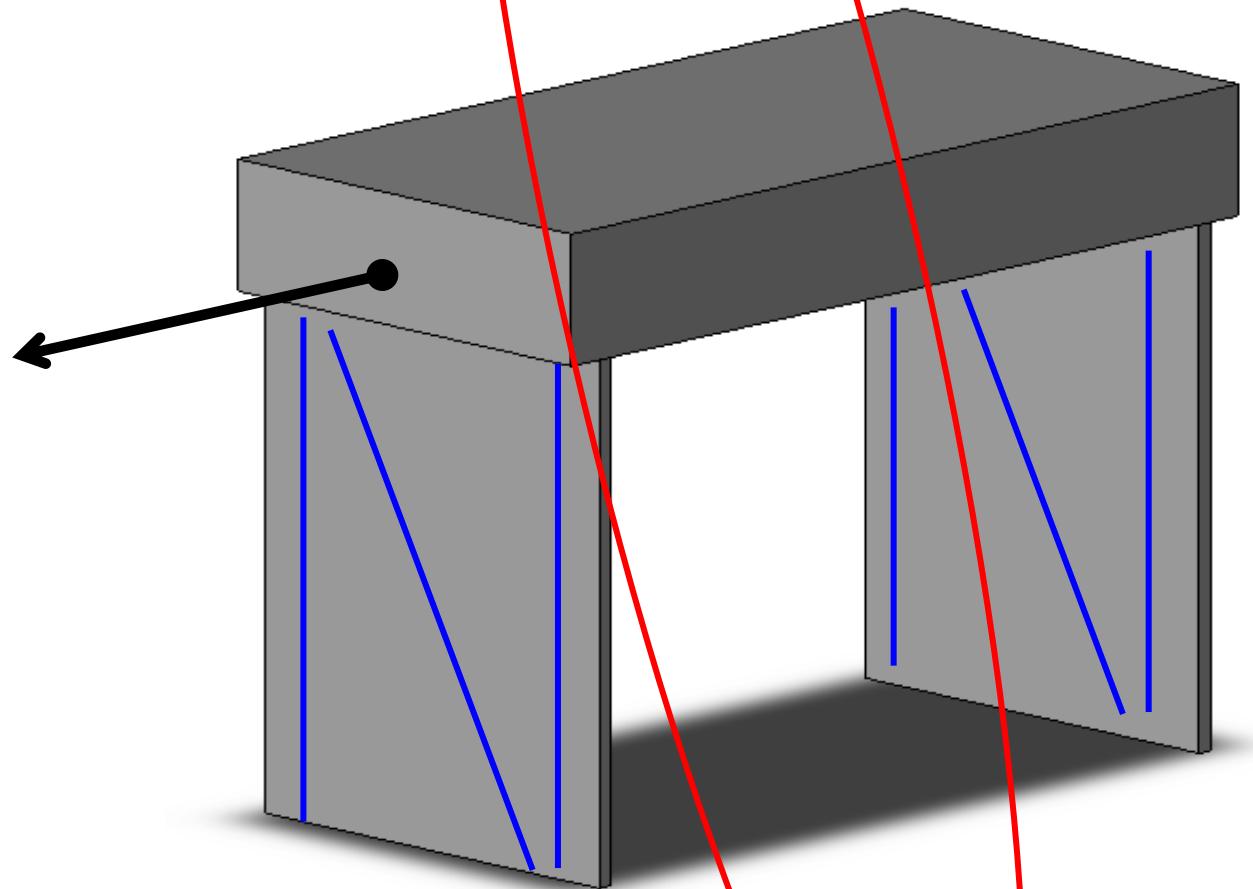
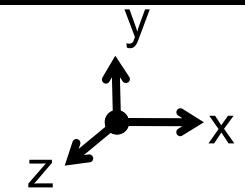
$$6 - C = F$$



Flexure bearing systems

Parallel guiding mechanism

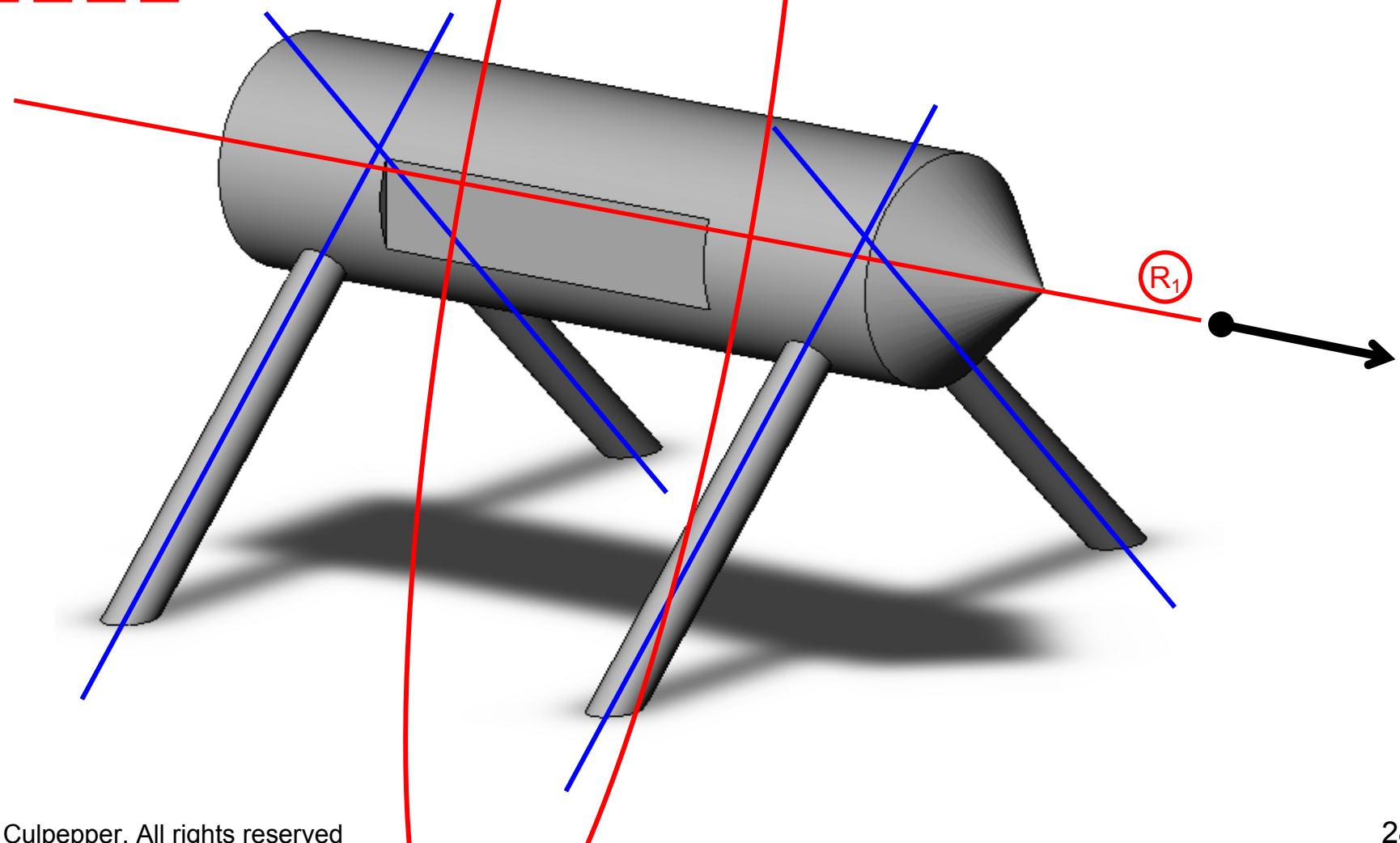
$$6 - C = F$$



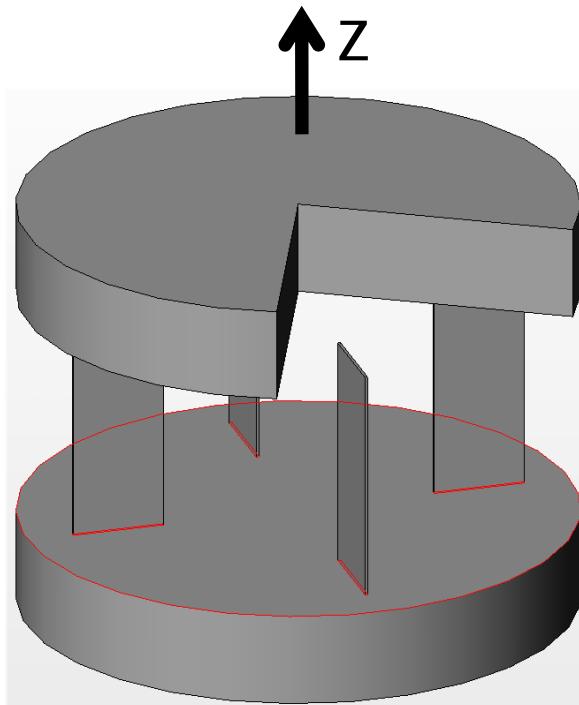
Flexure bearing systems

Doodle hopper...

$$6 - C = F$$



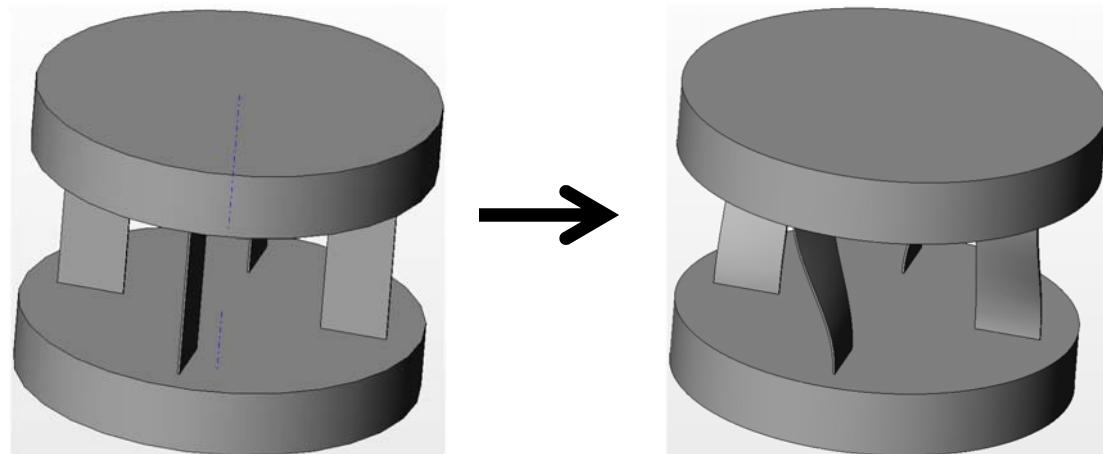
Parallel addition rules



What is parallel ? Elements are not in the same load path. Loads are split between the elements

Add constraints so where there is a common DOF, then have mechanism DOF

Example: For instance, there are no conflicts in displacement to θz



Adapted from Layton Hale's Ph.D. Thesis (MIT)

Series addition rules

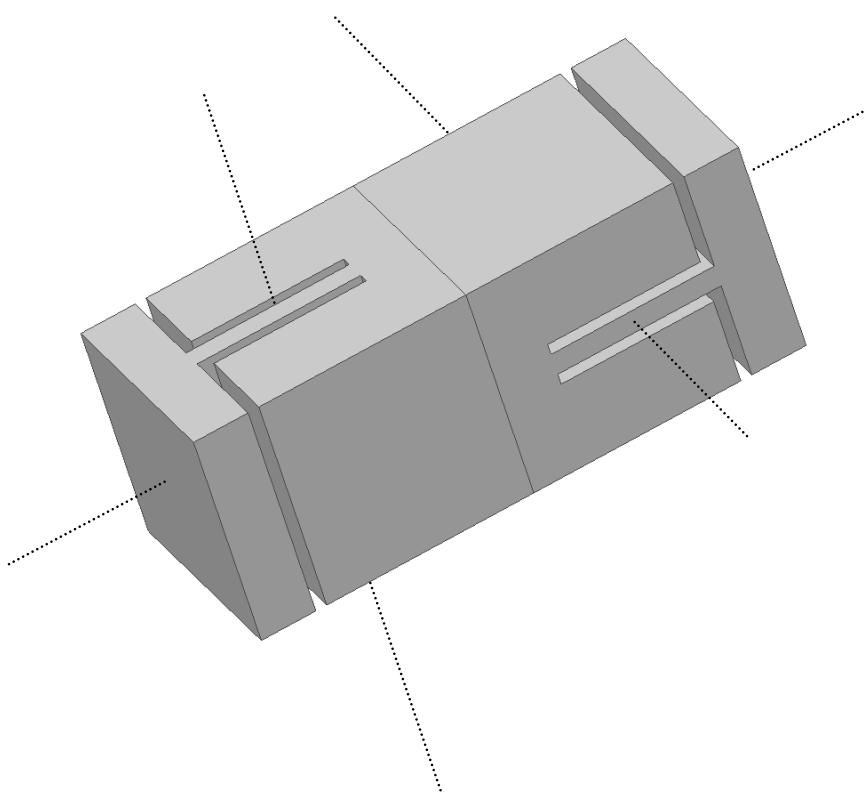
What is series?

- Differentiate series by load path
- Shared load path = series

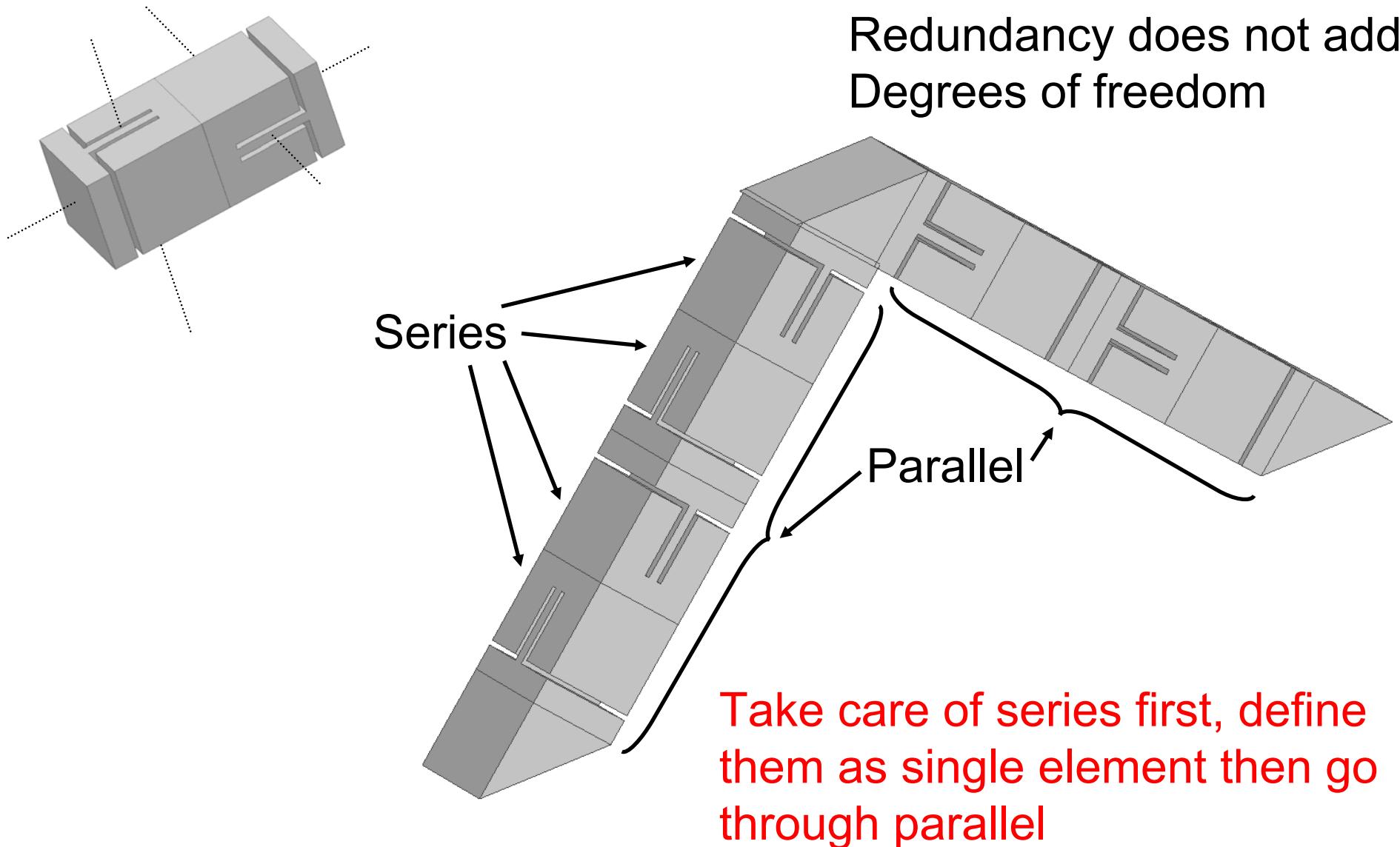
Series: Add DOF

Find common constraints

Follow the serial chain



Parallel and series systems



Accuracy

The accuracy of most flexures is sensitive to:

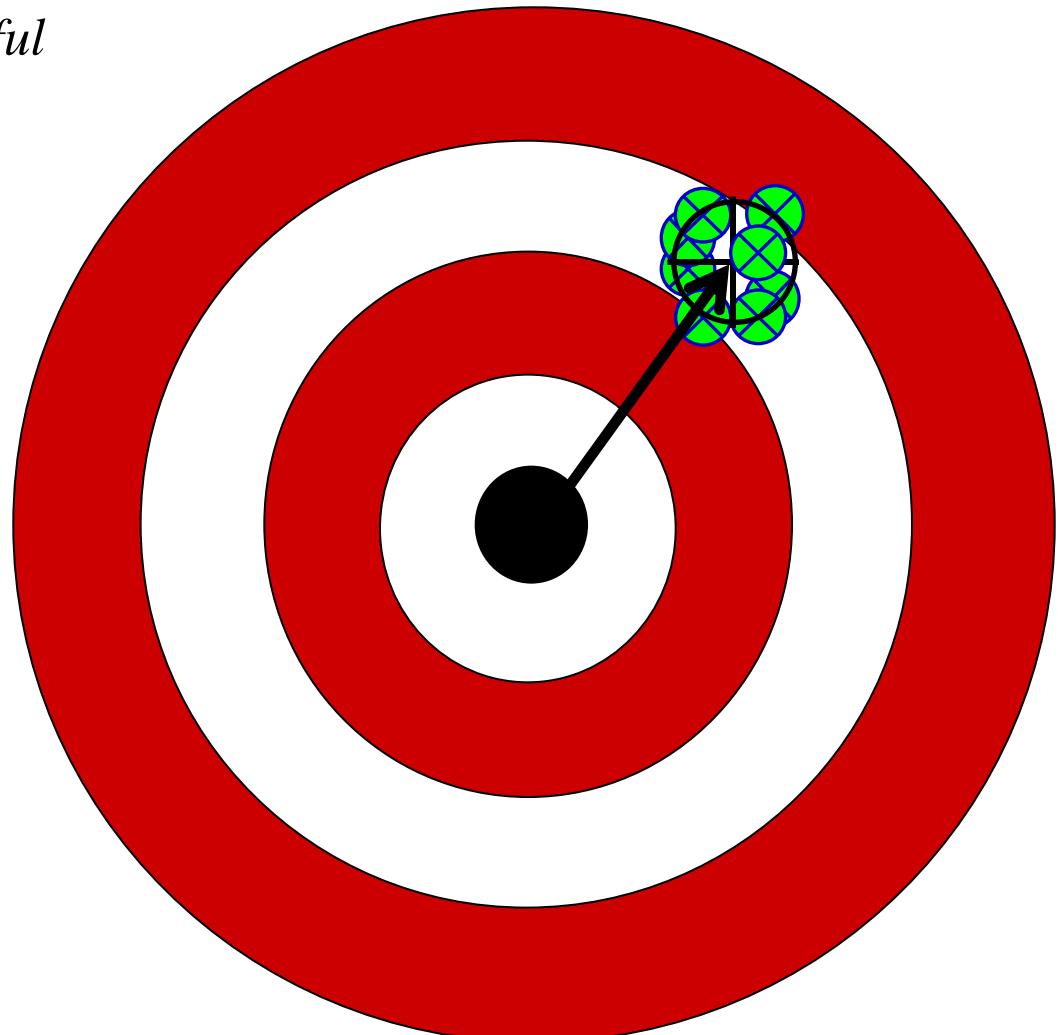
- 1. Small variations in dimensions, e.g. $\delta_{\text{thickness}}$
- 2. Young's Modulus (E)
- 3. Time variable errors
 - *Creep*
 - *Stress relaxation*
 - *Thermal*
 - *Dynamic/vibration*



Repeatability

Flexures can exhibit Angstrom-level repeatability if:

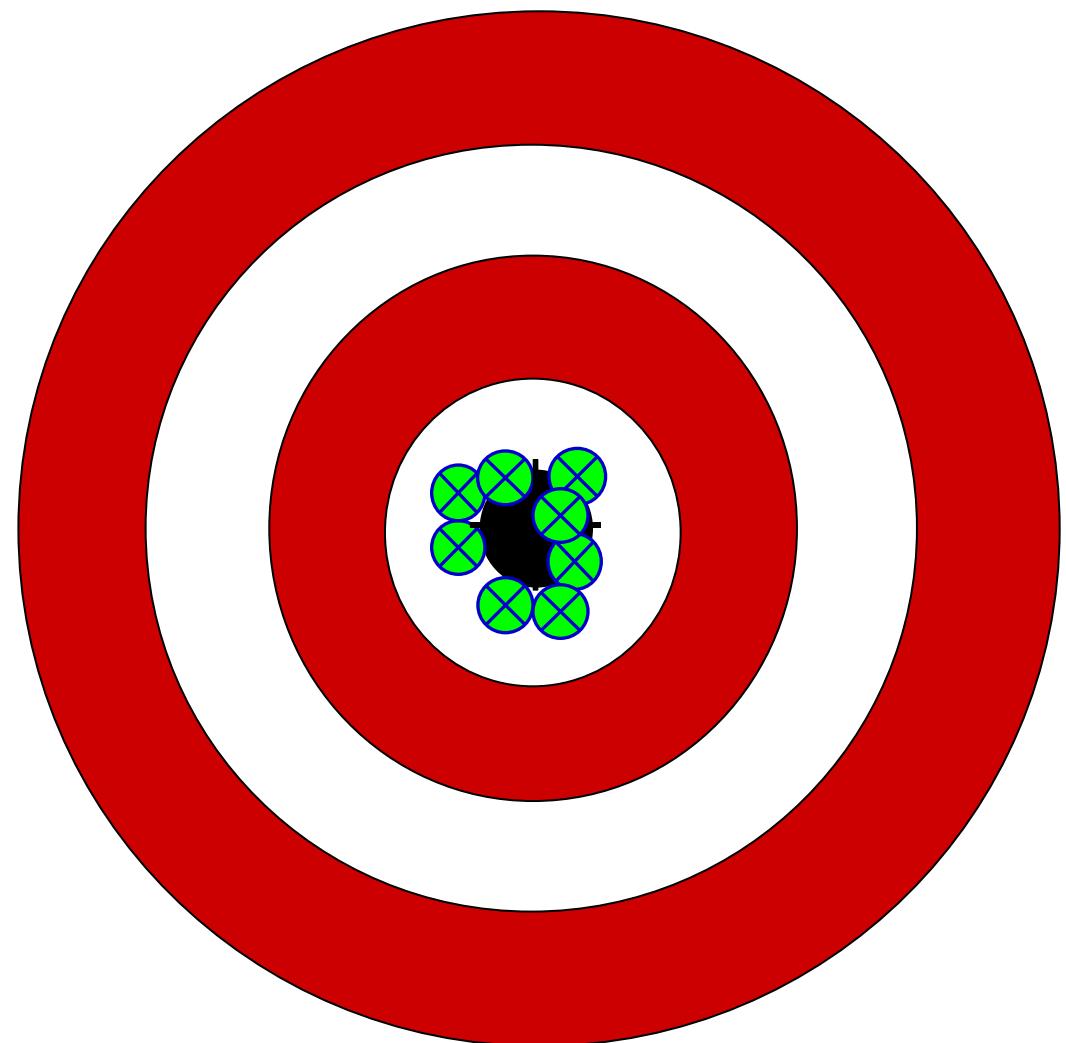
- Low material hysteresis
 - *Single crystal materials useful*
- No dislocation motion
 - $\sigma \ll \sigma_{yield}$
- Load is repeatable
 - *Magnitude*
 - *Direction*
- Assembly is correct
 - *No micro-slip*
 - *No friction in assembly*
 - *No yield during assembly*



Accuracy and repeatability

Difficult to obtain without calibration or adjustment

- Geometry
- Materials
- Loading
- Assembly/integration
- Environmental

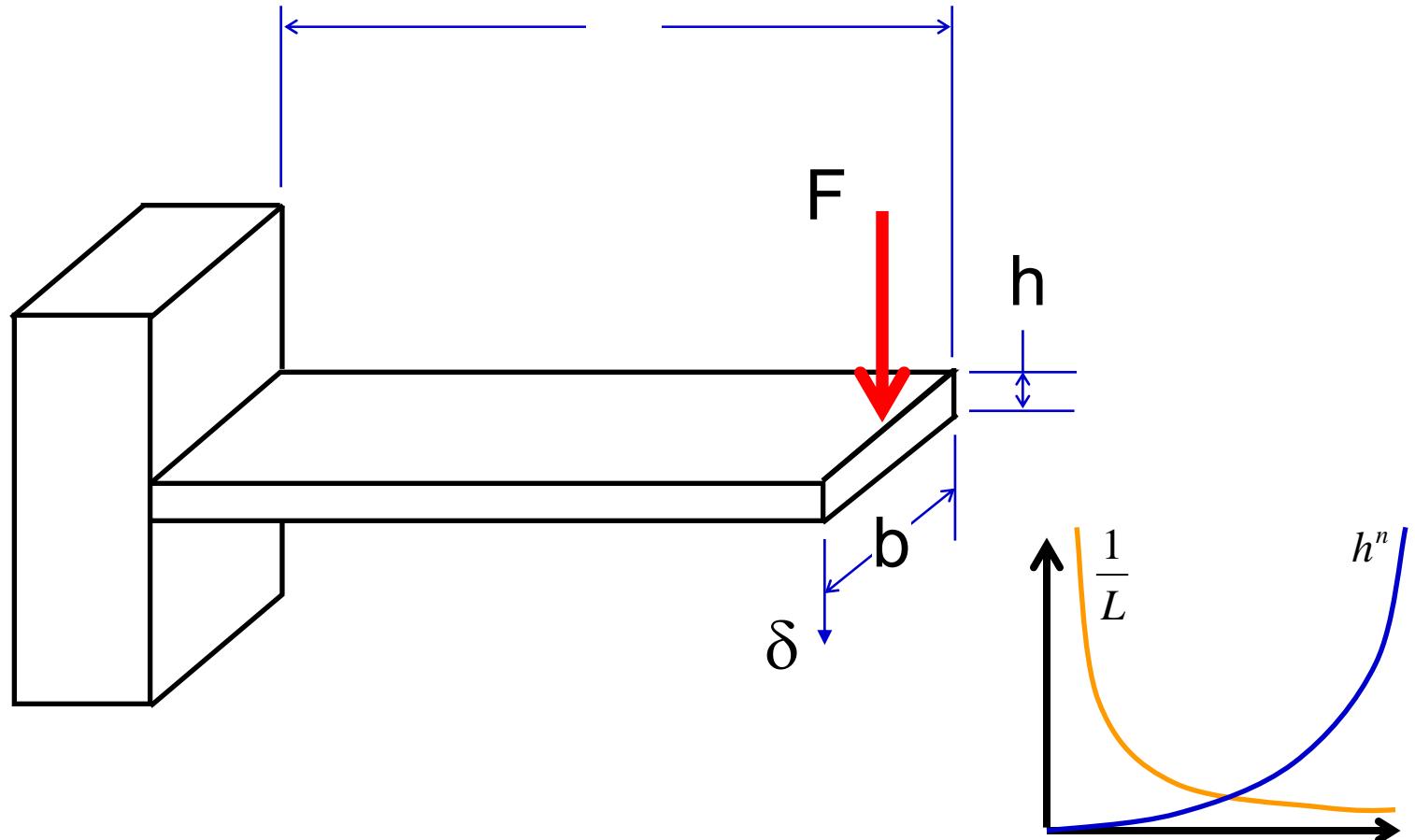


Links between kinematics and elasticity

Cantilever

$$\delta = \frac{F \cdot L^3}{3 \cdot E \cdot I}$$

$$I = \frac{1}{12} \cdot b \cdot h^3$$



$$F = \left(\frac{E \cdot b}{4} \cdot \left[\frac{h}{L} \right]^3 \right) \cdot \delta$$

$$k = \frac{dF}{d\delta} = \frac{d}{d\delta} \left\{ \frac{E \cdot b}{4} \cdot \left[\frac{h}{L} \right]^3 \cdot \delta \right\} \rightarrow \frac{E \cdot b}{4} \cdot \left[\frac{h}{L} \right]^3$$

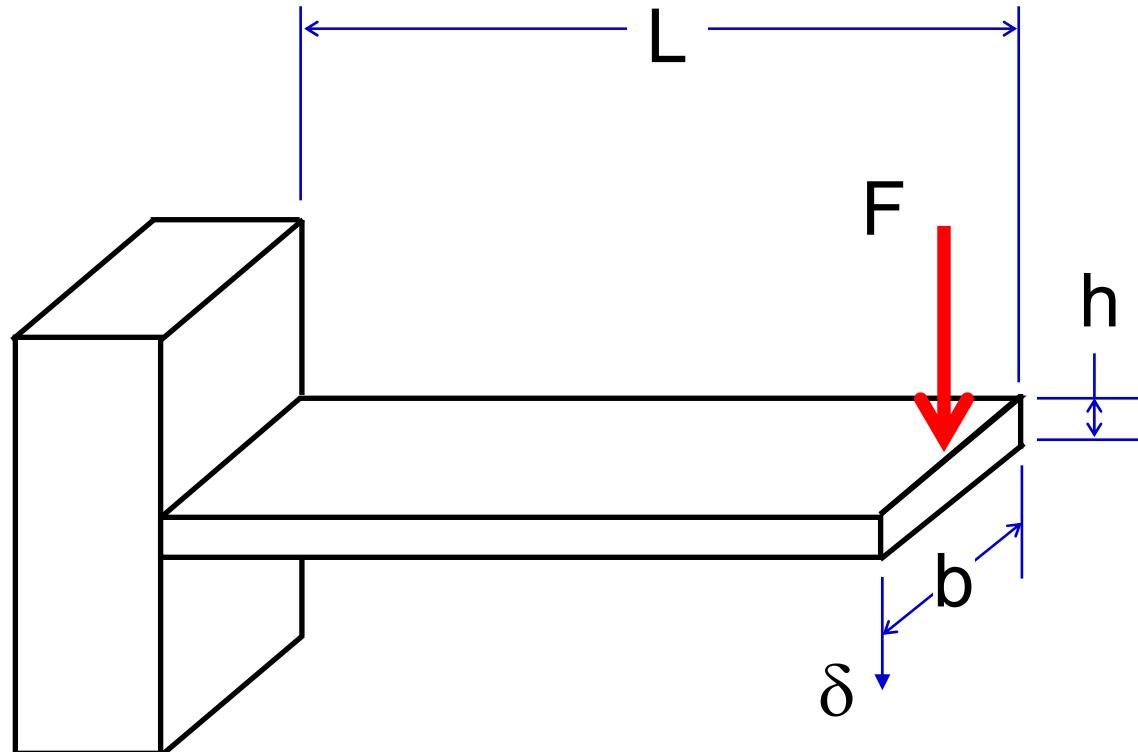
Links between kinematics and elasticity

Cantilever

$$\Delta L = 0.05 \cdot L$$

$$\Delta h = 0.05 \cdot h$$

$$\Delta b = 0.05 \cdot b$$



$$k + \Delta k = \frac{E \cdot (b + \Delta b)}{4} \cdot \left[\frac{h + \Delta h}{L - \Delta L} \right]^3 \rightarrow \frac{E \cdot b}{4} \cdot \left[\frac{h}{L} \right]^3 \cdot \left(1.05 \cdot \left[\frac{1.05}{0.95} \right]^3 - 1 \right) = k \cdot (1 + 0.42)$$

$$\Delta k = 0.42 \cdot k$$

Fabrication processes: EDM

EDM positives

- ❑ Accuracy (micrometers)
- ❑ 3D
- ❑ Surface finish (sub-micrometers)

Image removed due to copyright restrictions. Please see

http://www.physikinstrumente.com/en/about/images/pi_WIREEDMC_i4c_K50_eps.jpg

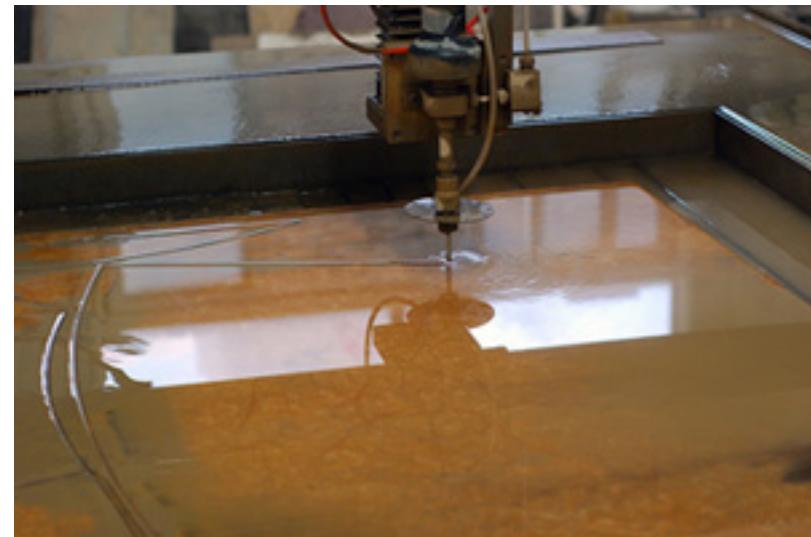
EDM drawbacks

- ❑ Time (mm/minute)
- ❑ Cost

Fabrication processes: Waterjet

Waterjet positives

- Low force
- Many materials including brittle materials and heat sensitive materials
- Rapid (inches/min)



Images courtesy of [xiaming](#) on Flickr.

Waterjet drawbacks

- Thickness limitations
- Kerf limitations
- Draft limitations
- Accuracy ~ 125 micrometers

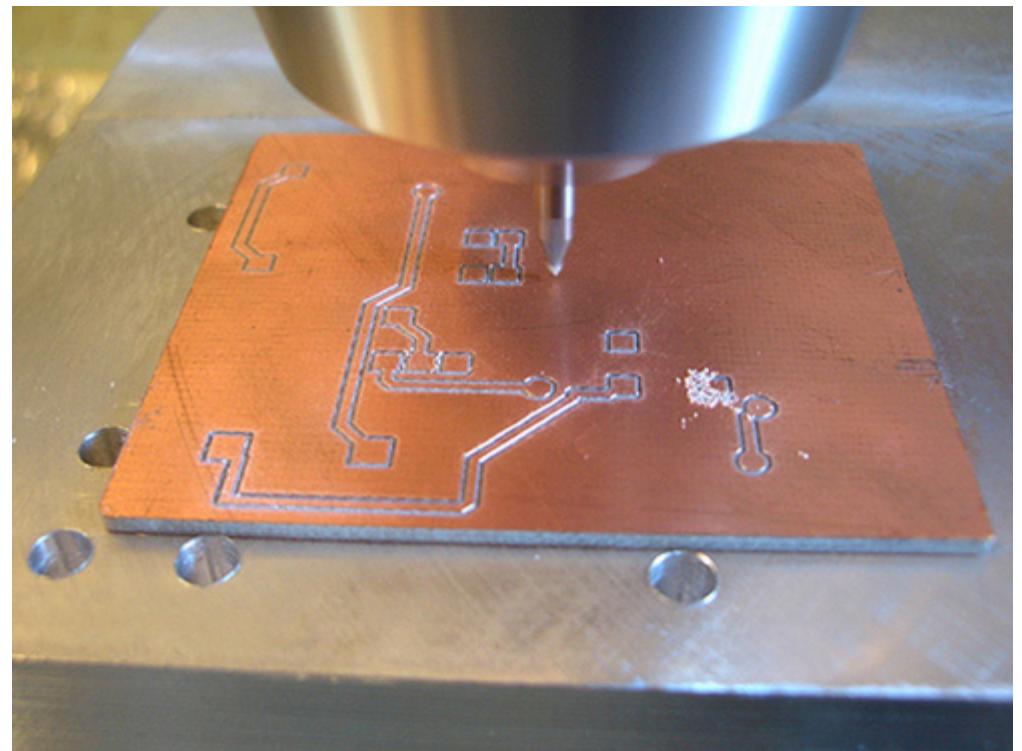


Images courtesy of [lansoper](#) on Flickr.

Fabrication processes: Milling/cutting

Milling/cutting positives

- Flexibility
- Any material
- Nearly any shape



Milling/cutting drawbacks

- Fixturing
- Compliance of parts
- Work hardening
- Surface damage

Image courtesy of [jiskar](#) on Flickr.

Please see any other image of milling, such as

http://students.washington.edu/dennyt/fsae/cnc/wc_fixtplate.jpg

Fabrication processes: Etching

Etching positives

- 2½ D topologies/shapes
- Monolithic
- Micron-level features

Images removed due to copyright restrictions. Please see:

http://www.ee.ucla.edu/~dejan/ee115c/ucla-graphics/IBM_metal_stack.jpg
http://www.stsystems.com/uploaded_files/1101/images/scallops.jpg

Milanovic, Veljko, et al. "Deep Reactive Ion Etching for Lateral Field Emission Devices." IEEE Electronic Device Letters 21 (June 2000): 271-273.

Milanovic, Veljko, et al. "Micromachining Technology for Lateral Field Emission Devices." IEEE Transactions on Electron Devices 48 (January 2001): 166-173.

Please see 371762. "How Microprocessor Work." February 14, 2009. YouTube. Accessed October 28, 2009.
http://www.youtube.com/watch?v=loMz_l_Fpx4

Assembly

Stress and energy

- ❑ Proper thickness of clamps and clamping load distribution
- ❑ Spring washer provide force source

Fusing

- ❑ Clamps members should “yield” before flexure
- ❑ Spring washer provide force source

Surface conformity

- ❑ Micro-slip is a major cause of hysteresis
- ❑ Deburring and potting/bonding

Images removed due to copyright restrictions.
Please see Fig. 8.5 and 8.6 in Smith, Stuart.
Flexures: Elements of Elastic Mechanisms.
Amsterdam, Holland: Gordon & Breach, 2000.

Misalignment = systematic errors