

3.225 Electronic and Mechanical Properties of Materials

Test - Elasticity Friday July 19, 2002

- 16 1. A fibre composite is transversely isotropic and has the following elastic constants:

$$E_1 = E_2 = 5 \text{ GPa} \quad E_3 = 25 \text{ GPa} \quad v_{12} = -\frac{\epsilon_2}{\epsilon_1} = 0.25 \quad v_{13} = -\frac{\epsilon_3}{\epsilon_1} = 0.33 \quad G_{13} = 8 \text{ GPa}$$

The composite is subjected to the stress state:

$$\sigma_{ij} = \begin{bmatrix} 10 & 0 & 8 \\ 0 & 5 & 3 \\ 8 & 3 & 5 \end{bmatrix} \text{ MPa}$$

S_{ij} 10
 $S_{ij} \times \sigma_i$ 3
 $U = \frac{1}{2} \sigma_i \epsilon_i$ 3

Calculate the strain energy in the composite under this stress state.

- 9 2a) 3 Why are most crystalline materials linear elastic?
b) 3 Why are rubbers almost incompressible?
c) 3 Why does the Young's modulus of a crystalline material decrease with increasing temperature while the Young's modulus of a rubber increases with increasing temperature?

$$\textcircled{1} \quad \begin{aligned} E_1 = E_2 &= 5 \text{ GPa} & v_{12} &= 0.25 & G_{13} &= 8 \text{ GPa} \\ E_3 &= 25 \text{ GPa} & v_{13} &= 0.33 \end{aligned}$$

$$\sigma_{ij} = \begin{bmatrix} 10 & 0 & 8 \\ 0 & 5 & 3 \\ 8 & 3 & 5 \end{bmatrix} \text{ MPa} \quad u = ?$$

$$u = \frac{1}{2} [\sigma_1 \epsilon_1 + \sigma_2 \epsilon_2 + \sigma_3 \epsilon_3 + \sigma_4 \epsilon_4 + \sigma_5 \epsilon_5 + \sigma_6 \epsilon_6]$$

NEED COMPLIANCE MATRIX TO OBTAIN STRAINS

$$S_{11} = S_{22} = 1/s = 0.2 \text{ GPa}^{-1}$$

$$S_{33} = 1/b = 0.04 \text{ GPa}^{-1}$$

$$S_{44} = 1/G_{23} = 1/G_{13} = 1/g = 0.125 \text{ GPa}^{-1}$$

$$S_{55} = 1/G_{13} = 1/g = 0.125 \text{ GPa}^{-1}$$

$$\sigma_6 = 0 \Rightarrow \text{DON'T NEED } S_{66}$$

$$\text{APPLY } \sigma_1 \text{ ONLY : } v_{12} = -\frac{\epsilon_2}{\epsilon_1} = -\frac{S_{12} \sigma_1}{S_{11} \sigma_1} \quad S_{12} = -v_{12} S_{11} = -\frac{v_{12}}{E_1} = -\frac{0.25}{5}$$

$$S_{12} = -0.05$$

$$\text{APPLY } \sigma_1 \text{ ONLY } v_{13} = -\frac{\epsilon_3}{\epsilon_1} = -\frac{S_{13} \sigma_1}{S_{11} \sigma_1} \quad S_{13} = -v_{13} S_{11} = -\frac{v_{13}}{E_1} = -\frac{0.33}{5}$$

$$S_{13} = -0.0660$$

$$\nexists \quad S_{23} = S_{13} = -0.0660$$

APPLY σ_3 ONLY

②

$$\epsilon_1 = S_{11}\sigma_1 + S_{12}\sigma_2 + S_{13}\sigma_3 = (0.2)(0.010) + (-0.05)(0.005) + (-0.066)(0.005) \\ = 0.00142$$

$$\epsilon_2 = S_{12}\sigma_1 + S_{22}\sigma_2 + S_{23}\sigma_3 = (-0.05)(0.010) + (0.2)(0.005) + (-0.066)(0.005) \\ = 0.00017$$

$$\epsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2 + S_{33}\sigma_3 = (-0.066)(0.010) + (-0.066)(0.005) + (0.04)(0.005) \\ = -0.00079$$

$$\epsilon_4 = S_{44}\sigma_4 = (0.125)(0.003) = .00038$$

$$\epsilon_5 = S_{55}\sigma_5 = (0.125)(0.001) = .00100$$

$$\epsilon_6 = 0$$

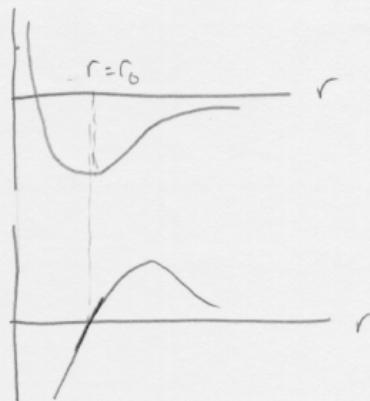
$$\text{Strain Energy} = \frac{1}{2} (\sigma_1\epsilon_1 + \sigma_2\epsilon_2 + \sigma_3\epsilon_3 + \sigma_4\epsilon_4 + \sigma_5\epsilon_5 + \overset{\circ}{\sigma_6}\epsilon_6) \\ = \frac{1}{2} [(10)(0.00142) + (5)(0.00017) + (5)(-0.00079) + (3)(0.00038) + (8)(0.001)] \\ = 0.01012 \text{ MPa} = 0.01012 \frac{\text{MN}}{\text{m}^2} \frac{\text{m}}{\text{m}} = 0.01012 \frac{\text{MJ}}{\text{m}^3} \\ = 10.1 \text{ kJ/m}^3$$

$$\#2 \text{ (a)} \quad E = \frac{1}{2} \left[\frac{d^2 U_a}{d\epsilon^2} - T \frac{d^2 S_a}{d\epsilon^2} \right]$$

\Rightarrow Crystalline materials - little change in entropy as strain $\Rightarrow \frac{dS_a}{d\epsilon} \rightarrow 0$

\Rightarrow energy separation curve U

\Rightarrow derivative of energy-separation $F = \frac{dU}{dr}$
 Curve at small strain
 \Rightarrow linear $\Rightarrow F \propto r$
 $\Rightarrow \sigma \propto \epsilon$



(3)

2(b) rubbers almost incompressible

\Rightarrow hydrostatic pressure - structure undergoes uniform volumetric compression

\Rightarrow relative position of atoms in the chains remains similar

\Rightarrow to deform rubber hydrostatically, have to deform C-C bonds

$$\Rightarrow K \propto \frac{d^2U_a}{d\epsilon^2}$$

\Rightarrow under tension or compression or shear, loading causes structure to become more ordered \Rightarrow large change in entropy without deforming C-C bonds $\Rightarrow E, G \propto \frac{d^2S_a}{d\epsilon^2} =$

\Rightarrow relatively easy to slide chains over one another compared to stretching or compressing C-C bonds

$$\Rightarrow K \gg E$$

$$\Rightarrow V = \frac{1}{2} - \frac{1}{6} \frac{E}{K} \stackrel{0}{\rightarrow} \Rightarrow \rightarrow \frac{1}{2} \Rightarrow \text{material is incompressible.}$$

2c) crystalline materials $\Rightarrow E \propto \frac{d^2U}{d\epsilon^2}$; the second derivative of U wrt r

decreases slightly as r increases. On heating r increases $\Rightarrow E \downarrow$.

rubbers \Rightarrow on heating, rubber has more thermal energy which increases the tendency of the rubber to go back to its random configuration

$$\Rightarrow E \uparrow$$

$$E \propto -T \frac{d^2S_a}{d\epsilon^2}; E = 2n_r kT; \text{ as } T \uparrow E \uparrow$$

3.225 Electronic and Mechanical Properties of Materials
Mechanical Properties: Test 2
Friday July 26, 2002

- 1a) At a temperature of 50°C, an amorphous polystyrene ($T_g = 40^\circ\text{C}$) has a 10 minute relaxation modulus of 0.05 GPa. How long does it take to reach the same value of the relaxation modulus at a temperature of 60°C? (10 points)
Time-temperature shift constants:

T_0 (°K)	C_1	C_2
T_g	-17.44	51.6
$T_g + 50$	-8.86	101.6

(15)

- 1b) Spring-dashpot models can be used to represent the linear viscoelastic behaviour of polymers. The simplest such models are the Maxwell and Voigt models with the spring and dashpot in series and parallel, respectively. Describe the limitations of the Voigt spring-dashpot model. (5 points)

- 2a) An aluminum alloy ($\sigma_y = 170 \text{ MPa}$) component is subjected to the following stress state:

$$\sigma_{ij} = \begin{bmatrix} 10n & 3n & 2n \\ 3n & 5n & n \\ 2n & n & 10n \end{bmatrix} \text{ MPa}$$

(20)

What value of n is sufficient to initiate yield in the component? (5 points)

- 2b) Why is the intrinsic lattice resistance of covalently bonded solids much larger than that of metals? (5 points)
- 2c) Describe two methods of alloying metals to increase their yield strength. Develop equations describing the increase in yield strength. (10 points)

Avg $\frac{47}{60}$

#3 (15)

SD 5.7

#1 (10)

3a) Creep data are given in the attached tables. Calculate the steady state creep strain rates for 316 stainless steel with a grain size of $100\mu\text{m}$ at a stress of 150 MPa and a temperature of 1000°C for: diffusional flow and power law creep. The atomic volume for 316 SS is $1.21 \times 10^{-29} \text{ m}^3$. Note that Boltzmann's constant is $k = 1.38 \times 10^{-23} \text{ J}/\text{K}$ and the gas constant $R = 8.314 \text{ J}/\text{mole}/\text{K}$. (10 points)

3b) The two main mechanisms of diffusional flow are vacancy diffusion and grain boundary diffusion. Why is the activation energy for vacancy diffusion higher than that for grain boundary diffusion? (5 points)

4 A double edge notch specimen of PMMA is loaded in tension. The width, W of the specimen is 30mm, the thickness B is 5 mm and the edge notch length a is 3mm. The fracture toughness of the PMMA is $0.5 \text{ MPa m}^{1/2}$ and the yield strength is 75 MPa. The geometrical factor Y for the stress intensity factor calculation is 1.12 for a double edge notch specimen.

- 4 a) What stress is required to break the specimen? (4 points)
- 3 b) What is the plane strain plastic zone size for this material just prior to fracture? (3 points)
- 3 c) Does the test specimen meet the requirements of a plane strain fracture toughness test? (3 points)

(60 points total)

TABLE 6.5

DIFFUSIONAL CREEP' PARAMETERS

MATERIAL	Q_v (kJ/mole)	D_{ov} (m^2/s)	η_B (kJ/mole)	δD_{ob} (m^3/s)
Titanium Carbide	740	4.4	543	2×10^{-9}
Zirconium Carbide	720	1×10^{-1}	468	5×10^{-12}
Tungsten	585	5.6×10^{-4}	385	3.3×10^{-13}
Silicon	496	9×10^{-1}	(300)	(1×10^{-15})
Uranium Dioxide	452	1.2×10^{-5}	293	2×10^{-15}
Tantalum	413	1.2×10^{-5}	(280)	(5.7×10^{-14})
Molybdenum	405	5×10^{-5}	(263)	(5.5×10^{-14})
Niobium	401	1.1×10^{-4}	(263)	(5.0×10^{-14})
Vanadium	308	3.6×10^{-5}	(209)	(5×10^{-14})
Chromium	306	2.8×10^{-5}	(192)	(5×10^{-15})
Cerium	287	8.0×10^{-4}	(172)	(1.0×10^{-17})
Niobium-Nichromes	285	1.6×10^{-4}	115	3.5×10^{-15}
Nickel	284	1.9×10^{-4}	115	3.5×10^{-15}
Stainless Steels	280	3.7×10^{-5}	167	2.0×10^{-13}
Y-Iron (Austenite)	270	1.8×10^{-5}	159	7.5×10^{-14}
Magnesium Oxide	261	2.5×10^{-10}	(230)	(1.3×10^{-15})
Low Alloy Steels	251	2.0×10^{-4}	174	1.1×10^{-12}
α-Iron, Mild Steel	251	2.0×10^{-4}	174	1.1×10^{-12}
Sodium Chloride	217	2.5×10^{-2}	155	6.2×10^{-10}
Lithium Fluoride	214	7.4×10^{-3}	-	-
Copper (and alloys)	197	2×10^{-5}	104	5×10^{-15}
Silver	185	4.4×10^{-5}	90	4.5×10^{-15}
Aluminum (and alloys)	142	1.7×10^{-4}	84	5.0×10^{-14}
Magnesium (and alloys)	135	1.0×10^{-4}	92	5.0×10^{-12}
Lead	109	1.4×10^{-4}	66	8.0×10^{-14}
Zinc	92	1.3×10^{-5}	61	1.3×10^{-14}
Ice	59	9.1×10^{-4}	(38)	(8.2×10^{-13})

TABLE 6.6

ARCHETYPAL DIFFUSION EQUATIONS

CLASS OF SOLID	LATTICE DIFFUSION COEFFICIENT m^2/s	BOUNDARY DIFFUSION COEFFICIENT m^2/s
Metals Carbides	$D = 0.2 \exp - \{24.0 \frac{T_K}{T}\}$	$\delta D_b = 7 \times 10^{-14} \exp - \{12.4 \frac{T_K}{T}\}$
Alkali Halides	$D = 1.5 \times 10^{-6} \exp - \{9.2 \frac{T_K}{T}\}$	$\delta D_b = 1 \times 10^{-11} \exp - \{15.1 \frac{T_K}{T}\}$
Alkali Metals	$D = 2.5 \times 10^{-5} \exp - \{14.7 \frac{T_K}{T}\}$	-
H.C.C. (2) Transition Metals	$D = 1.6 \times 10^{-4} \exp - \{17.8 \frac{T_K}{T}\}$	$\delta D_b = 1.4 \times 10^{-11} \exp - \{11.7 \frac{T_K}{T}\}$
H.C.C. Metals	$D = 4.9 \times 10^{-5} \exp - \{17.2 \frac{T_K}{T}\}$	$\delta D_b = 2.7 \times 10^{-14} \exp - \{10.8 \frac{T_K}{T}\}$
Hexagonal (Graphite)	$D = 2.4 \times 10^{-4} \exp - \{20.0 \frac{T_K}{T}\}$	-
Alkali Halides	$D = 1.7 \times 10^{-3} \exp - \{22.5 \frac{T_K}{T}\}$	-
Tetragonal (Ta)	$D = 3.2 \times 10^{-4} \exp - \{21.9 \frac{T_K}{T}\}$	-
Simple Oxides	$D = 3.3 \times 10^{-4} \exp - \{23.4 \frac{T_K}{T}\}$	$\delta D_b = 4.7 \times 10^{-15} \exp - \{14.7 \frac{T_K}{T}\}$
Silicates (Olivine)	$D = 3.0 \exp - \{21.3 \frac{T_K}{T}\}$	-
Metals Elements	$D = 5.5 \times 10^{-1} \exp - \{26.3 \frac{T_K}{T}\}$	$\delta D_b = 2.7 \times 10^{-14} \exp - \{11.3 \frac{T_K}{T}\}$
Ice	$D = 1 \times 10^{-1} \exp - \{26.3 \frac{T_K}{T}\}$	-
Diamond Cubic Elements	$D = 6.3 \times 10^{-2} \exp - \{23.9 \frac{T_K}{T}\}$	-

TABLE 6.4
POWER-LAW CREEP PARAMETERS

MATERIAL	Q_c kJ/mole °K	σ_0 MPa/m ²	n [-]
Titanium Carbide	740	.015	5.0
Zirconium Carbide	720	.052	5.0
Tungsten	585	0.41	4.7
Niobium	556	3.2	7.7
Silicon	496	.19	5.0
Uranium Dioxide	452	0.89	4.0
Tantalum	413	0.34	4.2
Hololybdenum	405	0.89	4.85
Nickel	401	0.11	4.4
Vanadium	308	0.62	5.0
Chromium	306	0.59	4.3
Ceramium	287	0.32	5.0
Nichromes	285	1.1	4.6
Nickel	284	0.5	4.6
316 Stainless Steels	270	33.5	7.9
304 Stainless Steel	270	11.3	7.5
γ -Iron (Austenite)	270	1.0	4.5
Magnesium Oxide	261	13.2	3.3
Low Alloy Steels	251	21.1	6.0
α -Iron, Mild Steel	251	1.92	6.9
Sodium Chloride	217	0.013	3.6
Lithium Fluoride	214	0.6	6.6
Copper	197	1.1	4.8
Silver	185	1.1	4.3
Aluminium	142	0.12	4.4
Magnesium	135	0.49	5.0
Lead	109	0.08	5.0
Zinc	92	1.24	4.5
Ice			

Test 2 3.225

$$1. (a) \log \alpha_r = \frac{C_1(T-T_0)}{C_2 + T - T_0} = \log \frac{t_T}{t_{T_0}}$$

$$T_g = 40^\circ C = T_0$$

$$T_1 = 50^\circ C, E_r(10\text{min}) = 0.05 \text{ GPa}$$

$$\Rightarrow T_g = T_0 \quad C_1 = -17.44 \quad C_2 = 51.6$$

$$\log \frac{t_{50^\circ}}{t_{40^\circ}} = \frac{-17.44(50-40)}{51.6 + 50 - 40} = -2.83$$

$$\therefore \frac{t_{50^\circ C}}{t_{40^\circ C}} = 10^{-2.83}$$

$$5 \quad t_{40^\circ C} = \frac{t_{50^\circ C}}{10^{-2.83}} = \frac{10\text{ min} \times 10^{+2.83}}{10^{-2.83}} = \underline{6780 \text{ min.}}$$

$$\log \frac{t_{60^\circ}}{t_{40^\circ C}} = \frac{-17.44(60-40)}{51.6 + 60 - 40} = -4.87$$

$$5 \quad \therefore \frac{t_{60^\circ C}}{t_{40^\circ C}} = 10^{-4.87}$$

$$t_{60^\circ C} = (10^{-4.87})(6780 \text{ min}) = 0.09 \text{ min.} \times \frac{60 \text{ sec}}{\text{min}} = \underline{5.5 \text{ sec.}}$$

(b) Voigt element limitations

- 5
- creep response - no initial elastic strain
 - no irrecoverable strain

- stress relaxation response - stress is constant with time, rather than decaying.

②

2(a)

$$\sigma_{ij} = \begin{bmatrix} 10n & 3n & 2n \\ 3n & 5n & n \\ 2n & n & 10n \end{bmatrix} \text{ MPa.} \quad \sigma_y = 170 \text{ MPa.}$$

n for yield?

$$\text{Van Mises: } \sigma_y = \sqrt{\frac{1}{2} [(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2] + 3\sigma_{12}^2 + 3\sigma_{23}^2 + 3\sigma_{13}^2}$$

$$= \sqrt{\frac{1}{2} [(25n^2 + 25n^2 + 0) + (3)(9n^2) + 3(4n^2) + 3n^2]}$$

$$= \sqrt{25n^2 + 27n^2 + 12n^2 + 3n^2}$$

$$= \sqrt{67} n$$

$$= 8.18 n.$$

S

$$n = \frac{170 \text{ MPa}}{8.18 \text{ MPa}} = 20.8.$$

(b) Intrinsic lattice resistance covalently bonded materials \rightarrow metals.

- dislocations break bonds one at a time, yield related to $Q_b = \frac{2\text{Ha}}{z}$
- binding energy = Σ contributions from all atoms; similar for both coval. + metal. ($z \approx 12$)
- coval. solids - only nearest neighbors contribute ($z \approx 12$)
- metals - free e^- ; many layers of neighbors contribute (large z)
- Q_b for metals $\ll Q_b$ for ceramics
- intrinsic lattice resistance found by equating work done by disloc. in breaking 1 bond with $Q_b \Rightarrow T^\circ \propto Q_b \propto \frac{2\text{Ha}}{z}$

(c) alloying techniques

2x5

copy lecture notes

(3)

#3a

316 stainless

$$Q_c = 270 \text{ kJ/mole}$$

$$d = 100 \mu\text{m}$$

$$\sigma_0 = 33.5 \text{ MPa}$$

$$\sigma = 150 \text{ MPa}$$

$$n = 7.9$$

$$T = 1000^\circ \text{C}$$

$$= 1273^\circ \text{K}$$

$$R = 8.314 \text{ J/mole/K}$$

Power law creep: $\dot{\epsilon} = \dot{\epsilon}_0 \left(\frac{\sigma}{\sigma_0} \right)^n = A \exp(-Q_{act}) \left(\frac{\sigma}{\sigma_0} \right)^n$

$$= (1/\text{sec}) \exp\left(-\frac{270000 \text{ J/mole}}{(8.314)(1273^\circ \text{K}) \text{ J/mole/K}}\right) \left(\frac{150}{33.5}\right)^{7.9}$$

$$= (8.33 \times 10^{-12}) (1.39 \times 10^{-5})$$

$$= 1.15 \times 10^{-16} / \text{sec}$$

Diff flow

$$\dot{\epsilon} = \frac{2\sigma d^2}{kT d^2} (D_v + \frac{2\delta}{d} D_b)$$

$$D_v = D_{ov} \exp - \frac{Q_v}{RT}$$

$$= 3.7 \times 10^{-5} \exp\left(-\frac{284000}{(8.314)(273)}\right)$$

$$= 1.20 \times 10^{-16} \text{ m}^2/\text{s}$$

$$\delta D_b = 2.0 \times 10^{-13} \exp\left(-\frac{167000}{(8.314)(1273)}\right)$$

$$= 2.81 \times 10^{-20} \text{ m}^3/\text{sec}$$

$$\dot{\epsilon} = \frac{(2)(150 \times 10^6 \text{ N})}{(1.38 \times 10^{-23} \text{ Nm})} \frac{(1.21 \times 10^{-29} \text{ m}^3) \text{ K}}{(1273^\circ \text{K}) (0.0001 \text{ m})^2 \text{ s}} \left[1.2 \times 10^{-16} + \frac{2 \cdot 2.81 \times 10^{-20}}{0.0001} \right]$$

$$= (2.07 \times 10^7) (6.82 \times 10^{-16}) = 1.41 \times 10^{-8} / \text{sec}$$

$$5.62 \times 10^{-16}$$

④

- 3b) • Q is the activation energy for atom to jump between sites in the lattice

- Vacancy diffusion \Rightarrow need energy to form vacanc, U_f

$$\Rightarrow Q_v = Q + U_f$$

5

- grain boundary diffusion \Rightarrow gb - more open structure
- easier for diffusion to occur
- $Q_b \sim 0.6 Q$

$$\therefore Q_b < Q_v$$