Broken symmetries and mass formulae for vector mesons

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BROKEN SYMMETRIES AND MASS FORMULAE FOR VECTOR MESONS

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A B S T R A C T

The question of which kind of mass formulae [(mass)^2 vs. (mass)$^{-2}$] are correct for a multiplet of vector mesons is studied. It is shown that from a field theoretic point of view, one does not have a clear a priori preference for either kind of formulae.

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1. INTRODUCTION

One of the great successes of broken SU(3) symmetry has been the striking confirmation of the mass formulae, first obtained by Gell-Mann and Okubo [1].

For bosons, these formulae are generally taken as referring to the squares of the masses rather than the masses themselves [1]. In a study devoted to remedying the defects of the so-called "particle mixing" approximation, Coleman and Schnitzer [2] introduced an approximation for vector boson propagators which leads to mass formulae for vector boson multiplets in which the squares of the masses are replaced by the inverse squares. Recently higher symmetry schemes, in particular SU(4), have been studied [3] and both kinds of mass formulae have been used.

The purpose of this paper is to study the difference between these choices from a field theoretic point of view. We find that the validity of inverse mass squared sum rules in vector meson field theory does not necessarily imply that the inverse squares of the masses should be used, even if certain approximations to the propagators are accepted.

In Section 2, below, we review briefly the sum rules for vector meson fields. In Section 3 the arguments and assumptions which lead to inverse square mass formulae are presented from a somewhat different point of view than that of Ref. [2]. The difficulties encountered in this section are studied in more detail in Section 4, with the aid of a simple model of vector meson interactions. Section 5 contains a summary of our conclusions. Some calculational details are given in two appendices.

2. SUM RULES FOR VECTOR MESON FIELDS

Consider a set of N vector fields $\varphi_\mu^i(x)$, $i = 1, 2, \ldots N$. The vacuum expectation value

$$W_{\mu\nu}^i(x-y) = \langle 0 | \varphi_\mu^i(x) \, \varphi_\nu^i(y) | 0 \rangle \tag{2.1a}$$

may be written in the form

$$W_{\mu\nu}^i(x-y) = \int_0^\infty dm^2 \left[\rho_1^i(m^2) g_{\mu\nu} + \frac{\rho_2^i(m^2)}{m^2} \partial_\mu \partial_\nu \right] \Delta^{(+)}(x-y; m^2) \tag{2.1b}$$

The quantity $\Delta_{\mu\nu}^i(k)$ may be defined in terms of the spectral function ρ_1^i and ρ_2^i by

$$\Delta_{\mu\nu}^i(k) = D^i(k^2) g_{\mu\nu} - E^i(k^2) k_\mu k_\nu$$

where

$$D^i(k^2) = \int_0^\infty dm^2 \frac{\rho_1^i(m^2)}{k^2 - m^2 + i\varepsilon} , \qquad E^i(k^2) = \int_0^\infty \frac{dm^2}{m^2} \frac{\rho_2^i(m^2)}{k^2 - m^2 + i\varepsilon}$$

Except for $\mu = \nu = 0$, $\Delta^i_{\mu\nu}$ is just the Fourier transform of the vacuum expectation value of the time-ordered product of $\varphi^i_\mu(x)$ and $\varphi^i_\nu(y)$. In the absence of "particle mixing", i.e., with the various fields φ^i_μ corresponding to different sets of conserved quantum numbers, $\Delta^i_{\mu\nu}(k)$ may be regarded as the propagator of the vector meson "i", associated with the field $\varphi^i_\mu(x)$. For the sake of simplicity we shall assume no "mixing" of the fields occurs.

If the field $\varphi^i_\mu(x)$ is transverse,

$$\partial_\mu \varphi^i_\mu(x) = 0$$

then necessarily

$$\rho^i_1(m^2) = \rho^i_2(m^2) \tag{2.2}$$

The equations of motion may be written in the form

$$\partial^\mu G^i_{\nu\mu} + (m^i_0)^2 \varphi^i_\nu = J^i_\nu$$

with $G^i_{\mu\nu}$ an antisymmetric tensor and m^i_0 the bare mass. The transversality of φ^i_ν is then equivalent to conservation of the current J^i_ν,

$$\partial^\nu J^i_\nu = 0$$

provided that m_o^i is non-vanishing. If φ_μ^i commutes with J_μ^i one can also prove the sum rule [4),5)]:

$$\int_0^\infty dm^2 \frac{\rho_2^i(m^2)}{m^2} = \frac{1}{(m_o^i)^2} \tag{2.3}$$

On defining a self-energy function $\Pi^i(k^2)$ by

$$\left[D^i(k^2)\right]^{-1} = k^2 - (m_o^i)^2 - \Pi^i(k^2) \tag{2.4}$$

the sum rule (2.3) and the equality (2.2) are seen to imply that

$$\Pi^i(0) = 0 \tag{2.5}$$

3. INVERSE SQUARE MASS FORMULAE

Assume now that the N vector mesons form a degenerate multiplet with common bare mass m_o, in the absence of any interaction. Take the Lagrangian $\mathcal{L}$ to have the form

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_B$$

where $\mathcal{L}_S$ admits SU(3) symmetry and $\mathcal{L}_B$ denotes a symmetry breaking interaction. Then we may write

$$\Pi^i(k^2) = \Pi_S(k^2) + \Pi_B^i(k^2) \tag{3.1}$$

where $\Pi_S(k^2)$ is the common value of the $\Pi^i(k^2)$ in the absence of $\mathcal{L}_B$ and $\Pi_B^i(k^2)$ includes all effects of $\mathcal{L}_B$. With $\mathcal{L}_B = 0$, the still common mass of the particles, m_S, is determined by solution of

$$m_S^2 - m_o^2 - \Pi_S(m_S^2) = 0$$

It follows that

$$\left[D^i(k^2)\right]^{-1} = k^2 - m_S^2 - \left[\Pi_S(k^2) - \Pi_S(m_S^2)\right] - \Pi_B^i(k^2) \tag{3.2}$$

If $\Pi_s(k^2)$ is slowly varying in an interval about $k^2 = m_s^2$ of the order of the mass splitting, we may write, for k^2 in this interval,

$$\Pi_s(k^2) - \Pi_s(m_s^2) \approx (k^2 - m_s^2)\, \Pi_s'(m_s^2) \tag{3.3}$$

where the prime denotes a derivative. From Eqs. (3.2) and (3.3) it follows that $D^i(k^2)$ has a pole at $k^2 = m_i^2$ where m_i^2 is the solution of

$$m_i^2 - m_s^2 \approx \xi^{-1}\, \Pi_B^i(m_i^2) \tag{3.4}$$

with

$$\xi = 1 - \Pi_s'(m_s^2)$$

Suppose now that $\mathcal{L}_s$ is such that the equality (2.2) and the sum rule (2.3) hold for all i. Then Eq. (2.3) holds with $\Pi^i \to \Pi_s$ and we have

$$\Pi_s(0) = 0 \tag{3.5}$$

If the sum rule (2.3) and equality (2.2) continue to hold when $\mathcal{L}_B \neq 0$, then we have also,

$$\Pi^i(0) = 0$$

so that, using Eqs. (3.1) and (3.5),

$$\Pi_B^i(0) = 0 \tag{3.6}$$

If it is a good approximation to write

$$\Pi_B^i(k^2) \approx \Pi_B^i(0) + k^2 \Pi_B^{i\,\prime}(0) \tag{3.7}$$

for $0 \lesssim k^2 \lesssim m_s^2$, and if Eq. (3.6) holds, Eq. (3.4) reduces to

$$m_i^{-2} \approx m_s^{-2} \left[1 - \xi^{-1} \Pi_B^{i\,\prime}(0) \right] \tag{3.8}$$

Now $\Pi_B^i(k^2)$ may be regarded as the matrix element

$$\Pi_B^i(k^2) = \langle i | \Pi_B^{op}(k^2) | i \rangle$$

of an operator $\Pi_B^{op}(k^2)$ in unitary spin space. If we also assume that Π_B^{op} transforms like the $I = 0$, $Y = 0$ component of an octet, e.g., this would be the case if $\mathcal{L}_B$ transformed in this way and only the terms of first order in $\mathcal{L}_B$ were important, Eq. (3.8) is equivalent to a G.M.O. formula for the multiplet under consideration, with inverse square masses replacing the squares of masses.

Thus, even if we accept Eq. (3.7) and the assumed transformation property of Π_B^{op}, the validity of inverse square mass formulae depends on the inequality

$$\Pi_B^i(0) \Big/ m_i^2 \, \Pi_B^{i\,\prime}(0) \ll 1 \tag{3.9}$$

Now, although $\Pi^{i}_{B}(0) \equiv 0$ if Eqs. (2.2) and (2.3) hold, both for $\mathcal{L}_B = 0$ and $\mathcal{L}_B \neq 0$, the equality (2.2) depends on φ_{μ}^{i} interacting with a conserved current. If we consider, for example, the octet K*, ρ and ω, this could be arranged for the ρ and ω fields by taking as the interaction current the conserved currents of isospin and hypercharge for the ρ and ω respectively. But, with $\mathcal{L}_B \neq 0$, there exists no conserved current with which the members of the K*, $\overline{K}$* doublets can interact. Thus, in general we cannot expect that $\Pi^{i}_{B}(0) = 0$ for all i and the question of the validity of the inverse square mass formulae becomes model dependent even whithin the framework of the assumption already made. We note that within this framework, if the ratio in (3.9) is $\gg 1$, we obtain the usual mass squared formula whereas if the ratio is ~ 1, no simple formula results.

4. A SIMPLE MODEL

We now consider a simple model of symmetry breaking and examine to what extent the inequality (3.9) is satisfied. We take (a sum on i is understood)

$$\mathcal{L}_S = -\frac{1}{4} G^i_{\mu\nu} G^{i\,\mu\nu} - \frac{m_0^2}{2} \varphi^i_\mu \varphi^{\mu\, i} - \frac{\mu_0^2}{2} \phi^i \phi^i - \frac{1}{2}\left[\partial_\mu \phi^i + \lambda(\varphi_\mu \times \phi)^i\right]\left[\partial^\mu \phi^i + \lambda(\varphi^\mu \times \phi)^i\right] \tag{4.1}$$

corresponding to a gauge theory of eight massive vector mesons [fields $\varphi_\mu^{\,i}(x)$] and eight scalar or pseudoscalar mesons [fields $\phi^i(x)$]. In Eq. (4.1),

$$G^i_{\mu\nu} = \partial_\mu \varphi^i_\nu - \partial_\nu \varphi^i_\mu + \lambda(\varphi_\mu \times \varphi_\nu)^i$$

with $\quad (\varphi_\mu \times \varphi_\nu)^i = f^{ijk} \varphi^j_\mu \varphi^k_\nu$

and the f^{ijk} are the structure constants of SU(3). This is the immediate generalization of gauge theories of isospin [SU(2)] to SU(3) [6]. For $\mathcal{L}_B$ we take

$$\mathcal{L}_B = -\delta\mu^2 \phi^i \left(F_8^{\,ij} + a D_8^{\,ij}\right)\phi^j \tag{4.2}$$

With the above choice of $\mathcal{L}_S$ we shall have $\Pi_S(0) = 0$ so that $\Pi^i_B(0) = \Pi^i(0)$. Rather than attempt to compute $\Pi^i(0)$ directly we note that from Eqs. (2.1a) and (2.1b) it follows readily that

$$\langle 0 | \left[\partial^\mu \varphi^i_\mu, \varphi^i_0(y)\right]_{x_0=y_0} | 0 \rangle = i\,\delta(\vec{x}-\vec{y}) \int_0^\infty \left(\rho^i_1 - \rho^i_2\right) dm^2 \tag{4.3}$$

and from Eqs. (2.3) and (2.4) that

$$\left[(m_0^i)^2 + \pi^i(0)\right]^{-1} - (m_0^i)^2 = \int_0^\infty (\rho_1^i - \rho_2^i)\, \frac{dm^2}{m^2}. \qquad (4.4)$$

Computation of the commutator in Eq. (4.3) with the help of the equations of motion yields (see Appendix I)

$$\int (\rho_1^i - \rho_2^i)\, dm^2 = -\frac{\delta\mu^2}{m_0^4}\, f^{ijk}\left(f^{8k\alpha} + a\, d^{8k\alpha}\right) \times f^{ij\alpha} \langle 0 | (\phi^j(0))^2 - (\phi^\alpha(0))^2 | 0 \rangle \qquad (4.5)$$

Although the vacuum expectation value is in general infinite, the last factor in Eq. (4.5) may well be finite since, in the absence of $\mathcal{L}_B$, $\langle 0|(\phi^j)^2|0\rangle$ is independent of j. Formally therefore, this last factor in Eq. (4.5) is of order $\delta\mu^2$, so that from Eq. (4.4) it may be inferred that $\pi_B^i(0)$ is of order $(\delta\mu^2)^2$ in the symmetry breaking, i.e., second order. When the symmetry breaking transforms like an eighth member of an octet and is treated to first order it is generally true that $\pi^i(0) = 0$, as

$$\pi^i(0) = \alpha + \beta\, d^{8ii} \qquad (4.6)$$

where α and β are constants, analogously to the Gell-Mann Okubo mass formula. Then $\pi_\rho(0) = \pi_\omega(0) = 0$ implies that $\pi_{K^*}(0) = 0$ to first order in the symmetry breaking. On the other hand, $\pi_B^{i'}(0)$ may be expected to be of order $\delta\mu^2$.

Unfortunately this does not imply that Eq. (3.9) is satisfied unless we restrict ourselves to <u>formal</u> first order perturbation theory, that is to "sufficiently small" $\delta\mu^2$. But to order $\delta\mu^2$ an inverse mass square formula is equivalent to the corresponding mass square formula so that no light is shed on the relative validity of these two choices.

We should emphasize that the problem of justifying any kind of mass formula, i.e., the absence of the (27) representation in $\Pi^{op}_B(k^2)$ when working to all orders in $\mathcal{L}_B$ is exactly the same as in the usual approach.

Another model we have considered is an octet of massive vector gauge particles interacting with a neutral vector meson coupled to the strangeness current. The coupling constant is such as to give a medium strong interaction: $g^2/4 \sim 0.1$. This is Ne'eman's "fifth interaction" [7]. In this model $\Pi^i_B(0)$ is again of second order in g but so are the mass differences and even less can be said than in the first model (see Appendix II).

5. CONCLUDING REMARKS

Our conclusion is that from a conventional field theoretic point of view it seems unlikely that a model-independent statement can be made regarding the relative validity of $(\mathrm{mass})^2$ versus $(\mathrm{mass})^{-2}$ types of formulae. Even useful model-dependent statements seem difficult to come by. The basic reason is that not all the particles in a vector multiplet can be expected to interact with conserved currents when the symmetry breaking interaction is turned on. But current conservation holds if and only if

$$\rho_1^i(m^2) \equiv \rho_2^i(m^2) \tag{5.1}$$

If the sum rule (2.3) holds one has $\Pi^i(0) = 0$ if and only if

$$\int_0^\infty \frac{\rho_1^i(m^2) - \rho_2^i(m^2)}{m^2}\, dm^2 = 0 \tag{5.2}$$

It seems very improbable that (5.2) can hold without (5.1), i.e., without current conservation. Although perhaps not impossible, the chances that $\Pi^i(0) = 0$ without both (2.3) and (5.1) holding seem remote. The possibility that there exist models for which $\Pi_B^i(0) \neq 0$ but the inequality (3.9) holds and for which Eq. (3.7) is a good approximation nevertheless appears to deserve further exploration.

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APPENDIX I

From the Lagrangian

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_B$$

given by (4.1) and (4.2) we obtain the field equations for the vector mesons,

$$\partial^\mu G^i_{\nu\mu} = -m_0^2 \varphi^i_\nu + J^i_\nu$$

$$J^i_\nu = \lambda \left[(\varphi^\mu \times G_{\mu\nu})^i + (\phi \times (\partial_\nu \phi + \lambda \varphi_\nu \times \phi))^i \right] \quad \text{(I.a)}$$

The commutation relations are

$$\left[\varphi^i_k(x), G^j_{0l}(y) \right]_{x_0 = y_0} = i \delta^{ij} \delta_{kl} \delta(\vec{x} - \vec{y})$$

$$\left[\phi^i(x), \tau^j_0(y) \right]_{x_0 = y_0} = i \delta^{ij} \delta(\vec{x} - \vec{y}),$$

where

$$\tau^j_\mu(y) = \partial_\mu \phi^j(y) + \lambda (\varphi_\mu \times \phi)^j$$

Since $G^i_{\mu\nu}$ is antisymmetric,

$$m_0^2 \partial^\nu \varphi^i_\nu = \partial^\nu J^i_\nu$$

Using the definition of J_ν^i , Eq. (I.a) and the equations of motion of the scalar meson field, which are

$$\partial^\nu \tau_\nu^i + \mu_0^2 \phi^i + \delta\mu^2 \left(f^{8ij} + a\, d^{8ij} \right) \phi^j = -\lambda \left(\varphi^\mu \times \tau_\mu \right)^i$$

we obtain

$$\partial^\nu J_\nu^i = \delta\mu^2 f^{ijk} \phi^j \left[f^{8k\alpha} + a\, d^{8k\alpha} \right] \phi^\alpha$$

By Eq. (I.a)

$$\varphi_0^n = \frac{\sum_{l=1}^{3} \partial^l G_{0l}^n + J_0^n}{m_0^2}$$

so that

$$\left[\partial^\mu \varphi_\mu^i(x) , \varphi_0^n(y) \right]_{x_0=y_0}$$

$$= -\delta\mu^2 \left[f^{ijk} \frac{\left(f^{8k\alpha} + a\, d^{8k\alpha} \right)}{m_0^2} \phi^j \phi^\alpha , \frac{(\phi \times \tau_0)^n}{m_0^2} \right]_{x_0=y_0}$$

$$= -\frac{\delta\mu^2}{m_0^4} f^{ijk} \left(f^{8k\alpha} + a\, d^{8k\alpha} \right) f^{nst} \left[\phi^j \phi^\alpha , \phi^s \tau_0^t \right]_{x_0=y_0}$$

$$= -\frac{i\,\delta\mu^2}{m_0^4} \delta(\vec{x}-\vec{y}) f^{ijk} \left(f^{8k\alpha} + a\, d^{8k\alpha} \right) f^{nst} \left(\phi^j \phi^s \delta^{\alpha t} + \phi^\alpha \phi^s \delta^{jt} \right)$$

Using $\langle 0|\phi^j\phi^s|0\rangle = \delta^{js}\langle 0|(\phi^j)^2|0\rangle$ in the absence of mixing and the anti-symmetry of f^{rst}, we obtain, for $r = i$,

$$\langle 0|\left[\partial^\mu \varphi_\mu^i(x), \varphi_0^i(y)\right]_{x_0=y_0}|0\rangle$$

$$= -i\,\delta(\vec{x}-\vec{y})\frac{\delta\mu^2}{m_0^4}\, f^{ijk}\left(f^{8k\alpha}+a d^{8k\alpha}\right) f^{ij\alpha}$$

$$\times\ \langle 0|(\phi^j)^2-(\phi^\alpha)^2|0\rangle$$

APPENDIX II

The correct form for the Lagrangian is actually

$$\mathcal{L} = -\frac{1}{4}\left(G^{\mu\nu\, i}\, G_{\mu\nu}^{i} \right) + \frac{G_{\mu\nu}^{i}}{2}\left(\partial^{\mu}\varphi^{\nu} - \partial^{\nu}\varphi^{\mu} \right)^{i}$$

$$+ \lambda\, G_{\mu\nu}^{i} \left(\varphi^{\mu} \times \varphi^{\nu} \right)^{i} + \frac{m_0^2}{2}\, \varphi_{\mu}\varphi^{\mu}$$

$$+ g\, G_{\mu\nu}^{i} \left(\chi^{\nu} \times \varphi^{\mu} - \chi^{\mu} \times \varphi^{\nu} \right)^{i} - \frac{1}{4}\, Q_{\mu\nu} Q^{\mu\nu}$$

$$+ \frac{Q_{\mu\nu}}{2}\left(\partial^{\mu}\chi^{\nu} - \partial^{\nu}\chi^{\mu} \right) + \frac{M^2}{2}\, \chi_{\mu}\chi^{\mu}$$

where χ_{μ} is represented as a vector in F spin space whose only non-vanishing component is the eighth one. By setting

$$\frac{\delta \mathcal{L}}{\delta G_{\mu\nu}^{i}} = 0$$

we find

$$G_{\mu\nu}^{i} = \partial_{\mu}\varphi_{\nu}^{i} - \partial_{\nu}\varphi_{\mu}^{i} + \lambda \left(\varphi_{\mu} \times \varphi_{\nu} \right)^{i} + g \left(\chi_{\nu} \times \varphi_{\mu} - \chi_{\mu} \times \varphi_{\nu} \right)^{i}$$

The field equation for φ_{μ}^{i} is

$$\partial^{\mu} G_{\mu\nu}^{i} = -\lambda \left(\varphi^{\mu} \times G_{\mu\nu} \right)^{i} + m_0^2\, \varphi_{\nu}^{i} + g \left(\chi^{\mu} \times G_{\mu\nu} \right)^{i} \qquad \text{(II.a)}$$

The commutation relations are

$$\left[\varphi_k^i(x)\,,\,G_{0\ell}^j(y)\right]_{x_0=y_0} = i\,\delta^{ij}\,\delta_{k\ell}\,\delta(\vec{x}-\vec{y})$$

$$\left[\chi_k(x)\,,\,Q_{0\ell}(y)\right]_{x_0=y_0} = i\,\delta_{k\ell}\,\delta(\vec{x}-\vec{y})$$

The current J_ν^i is defined by (II.a) as

$$J_\nu^i = \lambda\left(\varphi^\mu \times G_{\mu\nu}\right)^i - g\left(\chi^\mu \times G_{\mu\nu}\right)^i$$

Using the field equations of the χ_μ field

$$\partial^\mu Q_{\mu\nu} = M^2\chi_\nu - g\left(\varphi^\mu \times G_{\mu\nu}\right)^\delta$$

we find

$$-m_0^2\,\partial^\nu\varphi_\nu = \partial^\nu J_\nu^i = g\Big[\tfrac{1}{2}\left(Q^{\mu\nu} \times G_{\mu\nu}\right)^i$$

$$+\left(\chi^\mu \times \left(\lambda\varphi^\nu \times G_{\nu\mu} - m_0^2\varphi_\mu - g\,\chi^\nu \times G_{\nu\mu}\right)\right)^i$$

$$-\lambda\left(\tfrac{1}{2}\left(\chi^\nu \times \varphi^\mu\right) \times G_{\mu\nu} - \varphi^\mu \times \left(\chi^\nu \times G_{\nu\mu}\right)\right)^i\Big]$$

The vacuum expectation value of the commutator

$$\langle 0 | \left[\partial^\nu \varphi_\nu^i(x) \, , \varphi_0^i(y) \right]_{x_0 = y_0} | 0 \rangle$$

is a very long and complicated expression involving terms like

$$\frac{i g^2}{m_0^4} \left(f^{i8k} \right)^2 \langle 0 | \left(G_{0\ell}^k \right)^2 | 0 \rangle \, \delta(\vec{x} - \vec{y})$$

$$\frac{i g^2}{m_0^4} \left(f^{i8k} \right)^2 \langle 0 | \left(\chi_\ell \right)^2 | 0 \rangle \, \delta(\vec{x} - \vec{y})$$

$$\frac{i g}{m_0^4} \left(f^{ijk} f^{i8k} \right) \langle 0 | \chi_\ell \varphi_\ell^j | 0 \rangle \, \delta(\vec{x} - \vec{y})$$

all of which are of order g^2, the last one being so because the matrix element $\langle 0 | \chi_\ell \varphi_\ell^j | 0 \rangle$ is itself of order g.

We note that these all vanish for $i = 1, 2, 3, 8$ as $f^{i8k} = 0$ for $i = 1, 2, 3, 8$.

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