An Alternative to Compactification

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Abstract

Conventional wisdom states that Newton's force law implies only four non-compact dimensions. We demonstrate that this is not necessarily true in the presence of a non-factorizable background geometry. The specific example we study is a single 3-brane embedded in five dimensions. We show that even without a gap in the Kaluza-Klein spectrum, four-dimensional Newtonian and general relativistic gravity is reproduced to more than adequate precision.

1 Introduction

There exists "lore" that convinces us that we live in four non-compact dimensions. Certainly Standard Model matter cannot propagate a large distance in extra dimensions without conflict with observations. As has recently been emphasized, this can be avoided if the Standard Model is confined to a (3+1)-dimensional subspace, or "3-brane", in the higher dimensions [1-8]. However, this solution will not work for gravity, which necessarily propagates in all dimensions as it is the dynamics of spacetime itself. The experimental success of Newton's $1/r^2$ law and general relativity would therefore seem to imply precisely four non-compact dimensions. Additional dimensions would be acceptable, so long as they are compact and sufficiently small to be consistent with current gravitational tests. One further piece of lore is that if there are n extra compact dimensions, the Planck scale is related to the higher dimensional scale of gravity, M, through the relation $M_{Pl}^2 = M^{2+n}V_n$, where V_n is the extra-dimensional volume.

The point of this letter is to argue that none of the statements about gravity in the previous paragraph is necessarily true. The previous properties rely on a factorizable geometry, namely the metric of the four familiar dimensions is independent of coordinate in the extra dimensions. The story can change significantly when this assumption is dropped. Perhaps the most dramatic consequence is that we can live in 4+n non-compact dimensions, in perfect compatibility with experimental gravity. We will give an example with one additional dimension (n = 1). We will show that M_{Pl} is determined by the higher-dimensional curvature rather than the size of the extra dimension. This curvature is not in conflict with four-dimensional Poincare invariance. Earlier work on non-compact extra dimensions studied trapping of matter fields to be effectively four-dimensional [9] or studied finite-volume but topologically non-compact extra dimensions [10]. Here we carefully study the trapping of gravity itself to be effectively four-dimensional, where the extra-dimensional volume is infinite.

The reason the above statements can be true is that a curved background can support a "bound state" of the higher-dimensional graviton, which is localized in the extra dimensions. So although space is indeed infinite in extent, the graviton is confined to a small region within this space. The existence of a bound state can be understood as follows. Small gravitational fluctuations satisfy a wave equation of the form

$$\left(\partial_{\mu}\partial^{\mu} - \partial_{j}\partial^{j} + V(z_{j})\right)\hat{h}(x^{\mu}, z_{j}) = 0, \tag{1}$$

with a non-trivial "potential", V, arising from the curvature. The μ indices run from 0 to 3 whereas the j labels the additional dimensions. (We have dropped Lorentz indices on the fluctuations for simplicity.) General fluctuations can be written as superpositions of modes, $\hat{h} = e^{ip.x}\hat{\psi}(z)$, where $\hat{\psi}$ is an eigenmode of the following equation in the extra dimensional coordinates:

$$\left(-\partial_j \partial^j + V(z)\right)\hat{\psi}(z) = -m^2 \hat{\psi}(z), \tag{2}$$

and $p^2 = m^2$. This implements the Kaluza-Klein (KK) reduction of the higher-dimensional gravitational fluctuations in terms of four-dimensional KK states, with mass-squared, m^2 ,

given by the eigenvalues of Eq. (2). It is useful to note that Eq. (2) takes the form of an analog non-relativistic quantum mechanics problem. If there is a zero-mode (which is guaranteed if the background preserves four-dimensional Poincare-invariance) which is also a normalizable state in the spectrum of Eq. (2), it is the wave function associated with the four-dimensional graviton. This state is indeed a bound state and falls off rapidly away from the brane.

In addition there exists a tower of KK modes. If there were a gap, as is conventional in product space compactifications, one would reproduce four-dimensional gravity up to the scale determined by the gap. Instead, in our theory, there is a continuous KK spectrum with no gap. However, four-dimensional physics is extremely well approximated because the bound state mode reproduces conventional four-dimensional gravity, while the other KK modes give only a small correction, as we will demonstrate.

The set-up for our theory is a single 3-brane with positive tension, embedded in a five-dimensional bulk spacetime. In order to carefully quantize the system, and treat the non-normalizable modes which will appear in the Kaluza-Klein reduction, we choose to first work in a finite volume by introducing another brane at a distance πr_c from the brane of interest, and taking the branes to be the boundaries of a finite fifth dimension. We will eventually take this second brane to infinity, thereby removing it from the physical set-up. Analogous domain walls were discussed in Ref. [11] and references therein. The action for our system is

$$S = S_{gravity} + S_{brane} + S_{brane'}$$

$$S_{gravity} = \int d^4x \int dy \sqrt{-G} \{-\Lambda + 2M^3R\}$$

$$S_{brane} = \int d^4x \sqrt{-g_{brane}} \{V_{brane} + \mathcal{L}_{brane}\},$$
(3)

where R is the five-dimensional Ricci scalar made out of the five-dimensional metric, G_{MN} , and Λ and V_{brane} are cosmological terms in the bulk and boundary respectively. The coupling to the branes and their fields and the related orbifold boundary conditions are described in Refs. [8](see also [4]). (The new coordinate y is $r_c\phi$ in the coordinates of Ref. [8].)

The solution to Einstein's equations was derived in Ref. [8] and is

$$ds^{2} = e^{-2k|y|} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + dy^{2}, \tag{4}$$

where $0 \le y \le \pi r_c$ is the extra-dimensional coordinate and r_c is essentially a compactification "radius". It can be identified as a slice of AdS_5 . The solution holds only when the boundary and bulk cosmological terms are related by

$$V_{brane} = -V_{brane'} = 24M^3k, \quad \Lambda = -24M^3k^2,$$
 (5)

which we assume from now on. We remind the reader that this condition amounts to setting the cosmological constant of the four-dimensional world to zero in this context, and we simply accept this necessary fine tuning without further explanation here. Notice that in the solution given in Eq. (4), we have reversed the labels for the "visible" and "hidden" branes relative to Ref. [8]. The solution for the background metric is the same, with the metric exponentially falling from one brane to the other. However, whereas in the solution to the hierarchy problem proposed in Ref [8] the massless graviton wavefunction is biggest on the hidden brane, in the scenario considered here it is critical that the graviton is "bound" to the visible brane.

We also review the derivation of the four-dimensional effective Planck scale, M_{Pl} as given in Ref. [8]. The four-dimensional graviton zero mode follows from the solution, Eq. (4), by replacing the Minkowski metric by a four-dimensional metric, $\overline{g}_{\mu\nu}(x)$. It is described by an effective action following from substitution into Eq. (3),

$$S_{eff} \supset \int d^4x \int_0^{\pi r_c} dy \ 2M^3 r_c e^{-2k|y|} \sqrt{\overline{g}} \ \overline{R}, \tag{6}$$

where \overline{R} denotes the four-dimensional Ricci scalar made out of $\overline{g}_{\mu\nu}(x)$, in contrast to the five-dimensional Ricci scalar, R, made out of $G_{MN}(x,y)$. Because the effective field is four-dimensional, we can explicitly perform the y integral to obtain a purely four-dimensional action. From this we derive

$$M_{Pl}^2 = 2M^3 \int_0^{\pi r_c} dy e^{-2k|y|} = \frac{M^3}{k} [1 - e^{-2kr_c\pi}]. \tag{7}$$

We see that there is a well-defined value for M_{Pl} , even in the $r_c \to \infty$ limit. This is a clue that one can get a sensible effective four-dimensional theory, with the usual Newtonian force law, even in the infinite radius limit, in sharp contrast to the product-space expectation that $M_{Pl}^2 = M^3 r_c \pi$.

Clearly, there is no problem with taking the $r_c \to \infty$ limit of the background metric given above. This will remove the "regulator" brane from the set-up. However, we still need to determine whether the spectrum of general linearized tensor fluctuations $G_{\mu\nu} = e^{-2k|y|}\eta_{\mu\nu} + h_{\mu\nu}(x,y)$ is consistent with four-dimensional experimental gravity. This requires an understanding of all modes that appear in the assumed four-dimensional effective theory. We therefore perform a Kaluza-Klein reduction down to four-dimensions. To do this, we need to do a separation of variables; we write $h(x,y) = \psi(y)e^{ip\cdot x}$, where $p^2 = m^2$ and m^2 permits a solution to the linearized equation of motion for tensor fluctuations following from Eq. (3) expanded about Eq. (4):

$$\left[\frac{-m^2}{2} e^{2k|y|} - \frac{1}{2} \partial_y^2 - 2k\delta(y) + 2k^2 \right] \psi(y) = 0, \tag{8}$$

where the assumed orbifold boundary conditions tell us to consider only even functions of y. The effect of the regulator brane will be considered later; here it has been taken to infinity. The $\mu\nu$ indices are the same in all terms if we work in the gauge where $\partial^{\mu}h_{\mu\nu} = h^{\mu}_{\mu} = 0$, so they are omitted. Here m is the four-dimensional mass of the KK excitation.

It is more convenient to put the above equation into the form of an analog non-relativistic quantum mechanics problem by making a change of variables, $z \equiv sgn(y) \left(e^{k|y|} - 1\right)/k$,

 $\hat{\psi}(z) \equiv \psi(y)e^{k|y|/2}$, $\hat{h}(x,z) \equiv h(x,y)e^{k|y|/2}$. Eq. (8) then reads

$$\left[-\frac{1}{2}\partial_z^2 + V(z) \right] \hat{\psi}(z) = m^2 \hat{\psi}, \tag{9}$$

where

$$V(z) = \frac{15k^2}{8(k|z|+1)^2} - \frac{3k}{2}\delta(z).$$
 (10)

Much can be understood from the general shape of this analog non-relativistic potential.

First, the δ -function supports a single normalizable bound state mode; the remaining eigenstates are continuum modes. We have already discussed the role of the bound state mode as the massless graviton of the effective four-dimensional theory. With the explicit form of the KK "potential", we can also understand the properties of the continuum modes. First, since the potential falls off to zero as $|z| \to \infty$, there is no gap, and the continuum modes asymptote to plane waves. Furthermore, the amplitudes of the continuum modes is suppressed near the origin, due to the potential barrier near z=0. Finally, the continuum KK states have all possible $m^2>0$.

The precise continuum modes are given in terms of Bessel functions, and are a linear combination of $(|z|+1/k)^{1/2}Y_2(m(|z|+1/k))$ and $(|z|+1/k)^{1/2}J_2(m(|z|+1/k))$. The zero mode wavefunction follows (after changing variables) from Eq. (4), $\hat{\psi}_0(z) = k^{-1}(k|z|+1)^{-3/2}$. We can better understand the KK modes by studying the small and large argument limits of the Bessel functions. For small $\frac{m}{k}(|z|+1)$ we have

$$J_2(m(|z|+1/k)) \sim \frac{m^2(|z|+1/k)^2}{8}, \quad Y_2(m(|z|+1/k)) \sim -\frac{4}{\pi m^2(|z|+1/k)^2} - \frac{1}{\pi}$$
 (11)

Therefore to satisfy the boundary condition implied by the δ -function potential on the brane at z = 0, for small m (relevant at long distances) we must choose the linear combination,

$$\hat{\psi}_m \sim N_m(|z| + 1/k)^{1/2} \left[Y_2(m(|z| + 1/k)) + \frac{4k^2}{\pi m^2} J_2(m(|z| + 1/k)) \right]. \tag{12}$$

Here N_m is a normalization constant. For large mz,

$$\sqrt{z}J_2(mz) \sim \sqrt{\frac{2}{\pi m}}\cos(mz - \frac{5}{4}\pi), \quad \sqrt{z}Y_2(mz) \sim \sqrt{\frac{2}{\pi m}}\sin(mz - \frac{5}{4}\pi).$$
(13)

Let us now consider what happens when we reintroduce the regulator brane at $y_c \equiv \pi r_c$, that is $z_c \equiv (e^{k\pi r_c} - 1)/k$. It simply corresponds to a new boundary condition at z_c ,

$$\partial_z \hat{\psi}(z_c) = -\frac{3k}{2(kz_c + 1)} \hat{\psi}(z_c). \tag{14}$$

¹ Though the zero mode is not a Bessel function, it is the limit of $m^2(|z|+1)^{1/2}Y_2(m(|z|+1))$ when $m \to 0$.

It is easy to check that our zero-mode satisfies this new boundary condition. However, this condition does restrict the allowed continuum modes and quantizes the allowed values of m. For large z_c they are all in the plane-wave asymptotic regime of Eq. (13) when they satisfy the new condition. Therefore their masses are approximately quantized in units of $1/z_c$. Furthermore their normalization constants are predominantly those of plane waves, in particular, $N_m \sim \pi m^{5/2}/(4k^2\sqrt{z_c})$.

Having obtained the large but finite r_c asymptotics we can determine the proper measure for sums over the continuum states in the $r_c \to \infty$ limit. Because these asymptotics were dominated by plane wave behavior, this measure is simply dm after dropping the $1/\sqrt{z_c}$ factor in N_m to go a continuum normalization. We have also demonstrated the claim made in Ref. [8], that when z_c is kept large but finite, these KK states are quantized in units of $1/z_c$, which in Ref. [8] corresponded to the TeV scale. Also note that the normalized KK wavefunctions at the brane at z_c are all of order $1/\sqrt{z_c}$ since they are all plane waves at a maximum or minimum according to Eq. (14), which is kz_c times larger than $\hat{\psi}_0(z_c)$. This proves the claim of Ref. [8] that the KK states couple 10^{15} more strongly to matter on the brane at z_c than does the massless graviton.

Clearly, the $r_c \to \infty$ limit gives rise to a theory with a semi-infinite extra dimension. However, we were initially interested in studying a theory without the orbifold boundary condition, that is a theory with infinite extent in both the positive and negative z direction. It is trivial to extend the set-up we have studied to a fully infinite extra dimension by allowing even and odd functions of z rather than the restriction to purely even functions demanded by the orbifold conditions. From now on we will consider this to be the case. However, we will make use of the density of states we have found by the study of the finite volume situation.

Having found the KK spectrum of the effective four-dimensional theory, we can now compute the non-relativistic gravitational potential between two particles of mass m_1 and m_2 on our brane at z = 0, that is the static potential generated by exchange of the zero-mode and continuum Kaluza-Klein mode propagators. It is

$$V(r) \sim G_N \frac{m_1 m_2}{r} + \int_0^\infty dm \frac{G_N}{k} \frac{m_1 m_2 e^{-mr}}{r} \frac{m}{k}.$$
 (15)

Note there is a Yukawa exponential suppression in the massive Green's functions for m > 1/r, and the extra power of m/k arises from the suppression of continuum wavefunctions at z = 0 following from Eq. (11), discussed above. The coupling G_N/k in the second term is nothing but the fundamental coupling of gravity, $1/M^3$, by Eq. (7). Therefore, the potential behaves as

$$V(r) = G_N \frac{m_1 m_2}{r} \left(1 + \frac{1}{r^2 k^2} \right) \tag{16}$$

This is why our theory produces an effective four-dimensional theory of gravity. The leading term due to the bound state mode is the usual Newtonian potential; the KK modes generate an extremely suppressed correction term, for k taking the expected value of order the fundamental Planck scale and r of the size tested with gravity. Furthermore, since our propagators

are relativistic in general, going beyond the non-relativistic approximation we find all the proper relativistic corrections, again with negligible corrections from the continuum modes.

From the small m limit of the continuum wave functions, we also learn that the production of the the continuum gravitational modes from the brane at z=0 is suppressed by (dm/k)(m/k) due to the continuum wavefunction suppression there. This is very important, because it means the amplitude to produce the continuum modes in low-energy processes on the brane is extremely small, far smaller than gravitational strength. Were this not the case, we would be in danger of continuously losing energy to the additional dimension. Because of this suppression factor, the probability of producing KK modes is suppressed by $(p/k)^2$ relative to the zero mode, where p is the momentum of a process. For k of order the Planck scale, this is extraordinarily small for any process we presently observe, or are ever likely to observe.

We have shown that a scenario with an infinite fifth dimension in the presence of a brane can generate a theory of gravity which mimics purely four-dimensional gravity, both with respect to the classical gravitational potential and with respect to gravitational radiation. It is also important to verify that the gravitational self-couplings are not significantly modified. For gravitions which couple to matter fields with gravitational strength, these have been tested at the 10^{-3} level of precision (see Ref. [13] for a review). Because the KK modes have p/k-suppressed coupling to matter on the brane relative to the zero mode, tests in which the KK modes ultimately couple to matter on the brane (ie detectors) are insensitive to the existence of the many new gravitational modes. It is only the zero mode which contributes at any measurable level. The zero mode exchanges and self-couplings are just those of a four-dimensional general relativistic dynamics described by Eq. (6).

However, it is important to also verify that the energy loss induced by gravitational self-interactions is also insignificant, that is the coupling of the zero mode to KK modes which do not ulimately couple back to matter on the brane. We will now demonstrate that this is also small. However, to do so requires an understanding of the limitations of the perturbative approach to the graviton fluctuations that we have implicitly assumed.

We will now show that for any finite energy, the graviton self-coupling gets large at an energy-dependent value of the coordinate z. Subsequently, we will demonstrate that fluctuations originating on the brane in low-energy processes have only a small probability to get to this large z. Graviton emission and the associated missing energy can be bounded within the framework of the low-energy effective theory, and we will argue that it is small.

We first explore the limitation of the linearized KK spectrum calculation we have performed. The problem can best be understood by considering the graviton couplings in position space in the extra dimension. We have solved for the KK modes in a linear expansion about the classical gravitional background. The linear approximation is well justified in the vicinity of the brane, but higher order terms in the perturbation are significant far from the brane.

The leading term giving the graviton coupling is

$$S \supset \int d^4x \int dz \sqrt{-G}R \supset \int d^4x \int dz z^{3/2} \frac{\hat{h}^{\mu\nu}(x,z)\partial_{\mu}\hat{h}_{\alpha\beta}(x,z)\partial_{\nu}\hat{h}^{\alpha\beta}(x,z)}{M_{Pl}}.$$
 (17)

In this language, it is clear that there is a strong coupling at large z. The source of the problem is that the coupling of the gravitons diverges as one approaches the AdS horizon. This is related to the standard blue-shift near the horizon of AdS space. This can be seen by examining the large-z behavior of our continuum KK modes from Eq. (13), $\hat{\psi}(z) \sim e^{imz/k}$. No matter how soft such a mode is when produced near the 3-brane (small m), it is blue-shifted into a hard mode as measured in the background five-dimensional metric at large z, $G_{MN} \sim \eta_{MN}/z^2$, $M, N = \mu, z$. Therefore at large z it can have strong gravitational couplings to other modes.

In fact, this strong coupling in our effective field theory description is probably essential if one is to obtain a consistent four-dimensional theory. This is because our theory respects four-dimensional general covariance and therefore each graviton mode in the four-dimensional effective theory must be coupled consistently. However, we know the wave function overlap between the zero mode and the continuum modes is very small. In position space in the five-dimensional theory, this must be compensated by a large coupling of the linearized modes at large z to reproduce the correct four-dimensional result.

The question remains whether this strong-coupling regime is problematic. We presume that if our set-up is embedded in fundamental string theory, then the strong coupling interactions at large z has an alternative description not encompassed by our perturbative approach. Essentially all the physics relevant to the four-dimensional world occurs on or near the brane. If strong interactions were a very frequent consequence of low-energy processes on the 3-brane however, we would lose all predictivity within the effective field theory framework of this letter. Fortunately this is not the case, as we show by a simple estimate. By Eq. (17), five-dimensional gravitational fluctuations have couplings of order Energy. $z^{3/2}/M_{Pl}$. We presently only perform experiments at energies of order a TeV or less. In order for gravitational fluctuations emerging from such processes to be coupled more strongly than the percent level for example, they must escape out to $z > 10^9$. Now, as we saw, there is negligible probability for low-energy brane processes to directly create continuum KK modes, they will almost exclusively result in massless gravitons (possibly offshell). So our problem reduces to finding the probability for these massless gravitons to be at $z > 10^9$. Consulting the massless graviton's wavefunction, this probability is 10^{-18} ! Thus, while low-energy brane processes can lead to strong gravitational interactions at large z, not captured by weakly coupled effective field theory, they almost never do. With almost unit probability, low-energy brane processes interact with the massless graviton mode according to an effective four-dimensional general relativity, without exciting continuum KK modes.

To conclude, we have found that we can consistently exist with an infinite fifth dimension, without violating known tests of gravity. The scenario consists of a single 3-brane, (a piece of) AdS_5 in the bulk, and an appropriately tuned tension on the brane. The need for this delicate adjustment is the equivalent of the the cosmological constant problem in this context, and is taken as a given and not solved.

In this framework, we have found that an inevitable consequence is a bound state graviton mode, whose shape is determined by the brane tension and bulk cosmological constant. There are no very large or small numbers assumed for the different gravitational mass scales in the problem, so the four dimensional Planck scale is comparable to the fundamental mass scale of the higher dimensional theory. In addition to the bound state mode, there is a continuum of Kaluza-Klein modes. These have very weak coupling to low-energy states on the brane, but are essential to the consistency of the full theory of gravity and would couple strongly to Planck-energy brane processes.

Notice that one interpretation of our result is as a solution to the moduli problem, for the particular modulus determining the distance between two branes. It says that the usual disasterous possibility, namely that the modulus runs away to infinity, is perfectly acceptable. Furthermore, in the $r_c \to \infty$ limit, the modulus is not coupled to matter on the brane, and the need for a modulus mass is eliminated. In fact, it is interesting to speculate that the problem associated with geometric moduli can be eliminated, because there is no compactification at all, and gravity is bound by a mechanism akin to that suggested in this letter. This would be a worthwhile problem to study within the context of fundamental string theory.

Because our effective theory clearly breaks down before $r_c \to \infty$ for (rare) processes involving Kaluza-Klein modes, it might be thought that this theory is in some sense compactified. It should be noted that this theory is clearly very different from truly compactified theories. First, the extra-dimensional measure is straighforwardly dy in the coordinate y, which takes values on the entire real line. The low dimensional Planck scale and all physical parameters of the effective four-dimensional theory are independent of r_c , so long as it is much greater than 1/k. Furthermore, at sufficiently low energies, the theory breaks down further and further from the origin, since the blue shifted energy becomes large further away. From these perspectives, the theory provides a well defined alternative to geometric compactification. However, it is intriguing to speculate that there exists a dual description of this theory in terms of a cut-off conformal field theory in four-dimensions, akin to the duals discussed in [12].

Many interesting questions remain to be addressed. Given a valid alternative to conventional four-dimensional gravity, it is important to also consider the astrophysical and cosmological implications. This different scenario might even provide a new perspective for solving uresolved issues in quantum gravity and cosmology.

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References

- [1] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B125 (1983) 136.
- [2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263, hep-ph/9803315;
 I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257, hep-ph/9804398.
- [3] G. Shiu and S. H. H. Tye, Phys. Rev. D58 (1998) 106007, hep-th/9805157.
- [4] R. Sundrum, Phys. Rev. D59 (1999) 085009, hep-ph/9805471.
- [5] Z. Kakushadze and S. H. H. Tye, Brane World, hep-th/9809147.
- [6] S. Nussinov and R. Shrock, Phys. Rev. D59 (1999) 105002, hep-ph/9811323.
- [7] A. Donini and S. Rigolin, Anisotropic Type I String Compactification, Winding Modes and Large Extra Dimensions, hep-ph/9901443.
- [8] L. Randall and R. Sundrum, A Large Mass Hierarchy from a Small Extra Dimension, hep-ph/9905221.
- [9] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B125 (1983) 139; M. Visser, Phys. Lett. B159 (1985) 22; M. Gogberashvili, Four Dimensionality in Non-compact Kaluza Klein Model, hep-ph/9904383.
- [10] M. Gell-Mann and B. Zwiebach, Nucl. Phys. B260 (1985) 56; H. Nicolai and C. Wetterich, Phys. Lett. B150 (1985) 347.
- [11] M. Cvetic and H. H. Soleng, Phys. Rep. 282 (1997) 159, hep-th/9604090.
- J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200; O. Aharony, S.
 S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Large N Field Theories, String Theory and Gravity, hep-th/9905111.
- [13] T. Damour, Gravitation and Experiment, gr-qc/9711061.