Class-3

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» Fourier series Co-efficients (Reals)

For a given periodic function of period T, we define Fourier co-efficients

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cdot\cos(n\omega_0 t)\,dt \quad , n = 0, 1, 2, 3, 4, \dots$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cdot\sin(n\omega_0 t)\,dt \quad , n = 1, 2, 3, 4, \dots$$

where $\omega_0 = \frac{2\pi}{T}$.

» Fourier Series (Reals)

Fourier series for a given Fourier co-efficients a_n and b_n is

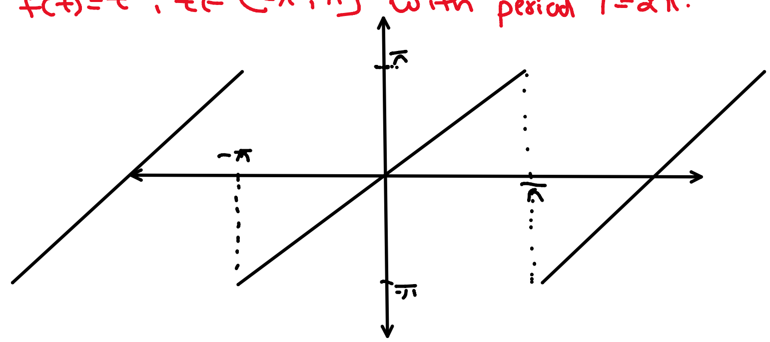
$$\frac{a_0}{2} + \sum_{n=}^{\infty} a_n\cdot\cos(n\omega_0 t) + b_n\cdot\sin(n\omega_0 t)$$

» Example: Find Fourier Co-efficients for $f(t)=t$, $t\in(-\pi,\pi]$ with period $T=2\pi$.

Clearly $\omega_0 = \frac{2\pi}{2\pi} = 1$

$n=0$, $a_0 = \frac{2}{2\pi}\int_{-\pi}^{\pi} t\cdot\cos(0)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} t\,dt = 0$

for $n\neq 0$, $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{\underset{\text{odd}}{t}\cdot\underset{\text{even}}{\cos(n\omega_0 t)}}_{\text{odd}}\,dt = 0$



$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} t\cdot\sin(n\omega_0 t)\,dt = \frac{2}{\pi}\int_{0}^{\pi} t\cdot\sin(n\omega_0 t)\,dt$$

$$= \frac{2}{\pi}\int_{0}^{\pi} t\cdot\sin(nt)\,dt$$

$$= \frac{2}{\pi}\frac{1}{n}\int_{0}^{\pi} nt\cdot\sin(nt)\,dt$$

$$= \frac{2}{n\pi}\{-t\cos(nt)+\sin(nt)\}_0^{\pi}$$

$$= \frac{2}{n\pi}\{-\pi(-1)^n\}$$

$$= \frac{2(-1)^{n+1}}{n}$$

Note: $\int nt\cdot\sin(nt)\,dt$

$$= -nt\cdot\frac{\cos(nt)}{n} - \int -\frac{\cos(nt)}{n}\cdot n\,dt$$

$$= -t\cos(nt) + \sin(nt) + C.$$

Remark: * From the above example Fourier series correspondint the function given is

$$\sum_{n=1}^{\infty} (-1)^{n+1}\cdot\frac{2}{n}$$

* Still we yet explain about convergence for Fourier series we get by a given periodic function.

Q) What is the relation b/w a periodic function and the corresponding Fourier series?

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Is it true that every periodic function generates convergent Fourier series?