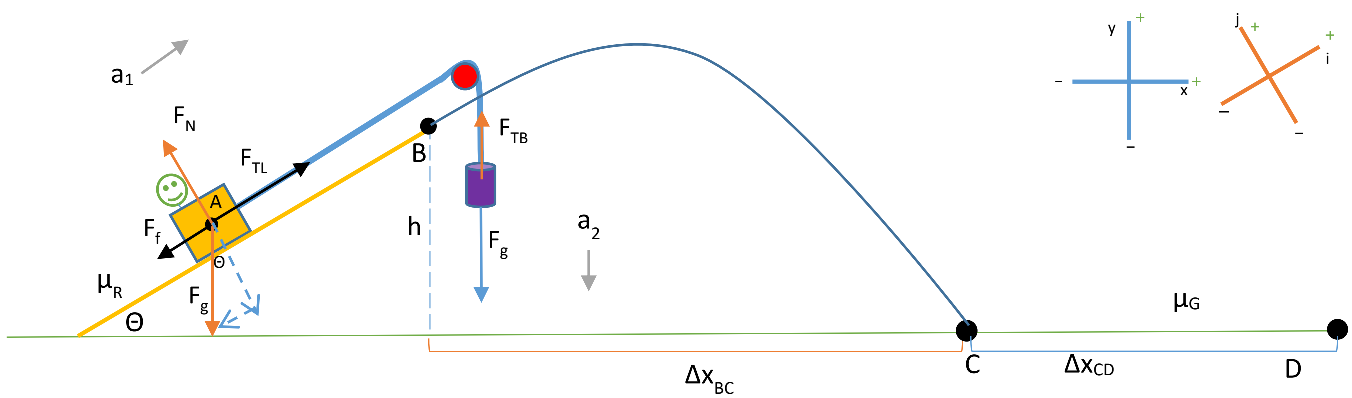
Über Problem

Manjusha Chava October 18, 2016 Section A

Problem Description

Leaping Larry decided to make a laborious launcher for his luxury luge using a pulley and ramp system (see diagram). His method was to attach one end of a massless stretch-less rope to a barrel of rocks and to hold the other end of the rope he placed the rope over a massless frictionless pulley, and then walked down the ramp as far down as possible to point A (where L = h). When he sat in the luge, he accelerated up the ramp to point B and then launched off the top at the same angle as the ramp (all while releasing the rope and avoiding the pulley). He flew through the air as a projectile to point C, transitioning <u>all of his speed</u> into the horizontal direction, and eventually slid to a stop at point D.

Diagram



Givens

Total mass of Larry and luge	27 (kg)
Total mass of barrel and rocks	42 (kg)
Ramp angle above the horizontal	35 (deg)
Coefficient of friction between luge and ramp	.24
Height of the ramp	7.7 (m)
Total horizontal distance from vertical base of ramp to final location	45 (m)
Coefficient of friction between luge and ground	???
Assumptions	$a_2 = -(a_1)$ $F_{TL} = F_{TB}$

Step 1:

Solve for the normal force of the luge in the perpendicular (j) direction.

$\underline{\sum F_j:}$

$$F_N - F_g(\cos\Theta) = m_L a_1$$

$$F_N - 27(9.8)(\cos 35) = m_L(0)$$

$$\underline{F_N = 216.75\ \text{N}}$$

Step 2:

Find the tension of the luge in the parallel (i) direction.

$\underline{\sum F_i:}$

$$F_{TL} - F_g(\sin\Theta) - F_f = m_L a_1$$

$$F_{TL} - m_L g(\sin\Theta) - \mu F_N = m_L a_1$$

$$\underline{F_{TL} = m_L g(\sin\Theta) - \mu F_N + m_L a_1}$$

Step 3:

Find the tension in the pulley.

$\underline{\sum F_y}$: $F_{TB} - F_g = m_B a_2$

$F_{TB} - m_B g = m_B a_2$

$\underline{F_{TB} = m_B g + m_B a_2}$

Step 4:

Solve for the acceleration of the luge at point B.

$F_{TL} = F_{TB}$

$m_L g(\sin\Theta) - \mu F_N + m_L a_1 = m_B g + m_B a_2$

$m_L a_1 - m_B(-a_1) = m_B g - m_L g(\sin\Theta) - \mu F_N$

$a_1(27+42) = 42(9.8) - 27(9.8)(\sin 35) - 216.748(.24)$

$69a_1 = 207.81$

$\underline{a_1 = 3.0118\ m/s^2}$

Step 5:

Find the velocity of the luge at point B.

$\underline{v_B}$: $v_B^2 = v_i^2 + 2(a_1)(\Delta y)$

$v_B^2 = 0^2 + 2(3.0118)(7.7)$

$v_B^2 = 46.3814$

$\underline{v_B = 6.8104\ m/s}$

Step 6:

Find the time it takes for the luge to get from point B to point C.

$\underline{\Delta y_{BC}}$: $\Delta y_{BC} = \frac{1}{2} at^2 + v_{yB}t + y_i$

$0 = \frac{1}{2}(-9.8)t^2 + 6.8104(\sin\Theta)t + 7.7$

$0 = -4.9t^2 + 3.90628t + 7.7$

$\underline{t = 1.714s}$ $\cancel{t = -0.9168s}$

Step 7:

Use the time to find the horizontal distance between point B and C.

$\underline{\Delta x_{BC}}$: $\Delta x_{BC} = \frac{1}{2} at^2 + v_{xB}t + x_i$

$\Delta x = \frac{1}{2}(0)(1.714)^2 + 6.8104(\cos\Theta)(1.714) + 0$

$\Delta x_{BC} = 5.5787(1.714)$

$\Delta x_{BC} = 9.562m$

Step 8:

Find the horizontal distance between point C and point D.

$\underline{\Delta x_{CD}}$: $\Delta x_{BD} - \Delta x_{BC} = \Delta x_{CD}$

$45m - 9.562m = \Delta x_{CD}$

$\underline{\Delta x_{CD} = 35.438m}$

Step 9:

Find the velocity at point C in the x direction.

$\underline{v_{Cx}}$: $v_{Cx}^2 = v_{Bx}^2 + 2(a_x)(\Delta x)$

$v_{Cx}^2 = 5.5787^2 + 2(0)(9.5621)$

$v_{Cx}^2 = 5.5787^2$

$\underline{v_{Cx} = 5.5787\ m/s}$

Step 10:

Find the velocity at point C in the y direction.

$\underline{v_{Cy}}$: $v_{Cy}^2 = v_{By}^2 + 2(a_y)(\Delta y)$

$v_{Cy}^2 = 3.9063^2 + 2(9.8)(7.7)$

$v_{Cy}^2 = 166.18$

$\underline{v_{Cy} = 12.891\ m/s}$

Step 11:

Find the overall velocity at point C.

$\underline{v_C}$: $v_C^2 = v_{Cx}^2 + v_{Cy}^2$

$v_C^2 = (5.5787^2 + 12.891^2)$

$v_C^2 = 197.301$

$\underline{v_C = 14.046\ m/s}$

Step 12:

Find the acceleration at point C.

$\underline{v_C:}$ $v_D^2 = v_C^2 + 2(a_C)(\Delta x_{CD})$

$0^2 = 14.046^2 + 2(a_C)(35.438)$

$-197.301 = 70.876\, a_C$

$a_C = \underline{-2.7838 \text{ m/s}}$

Step 13:

Find the normal force when the luge is at point C.

$\underline{\sum F_y:}$ $F_N - F_g = m_L a_y$

$F_N - m_L g = m_L(0)$

$\underline{F_N = m_L g}$

Step 14:

Find the coefficient of friction between the luge and the ground.

$\underline{\sum F_x:}$ $F_f = m_L a_C$

$\mu_G F_N = m_L a_C$

$\mu_G \cancel{m_L} g = \cancel{m_L} a_C$

$\mu_G g = a_C$

$$\mu_G = \frac{a_C}{g}$$

$$\mu_G = \frac{-2.7838}{-9.8}$$

$\mu_G = 0.2841$

$$\boxed{\mu_G = 0.2841}$$

The coefficient of friction between the luge and the ground is 0.2841.