Uber Rocket

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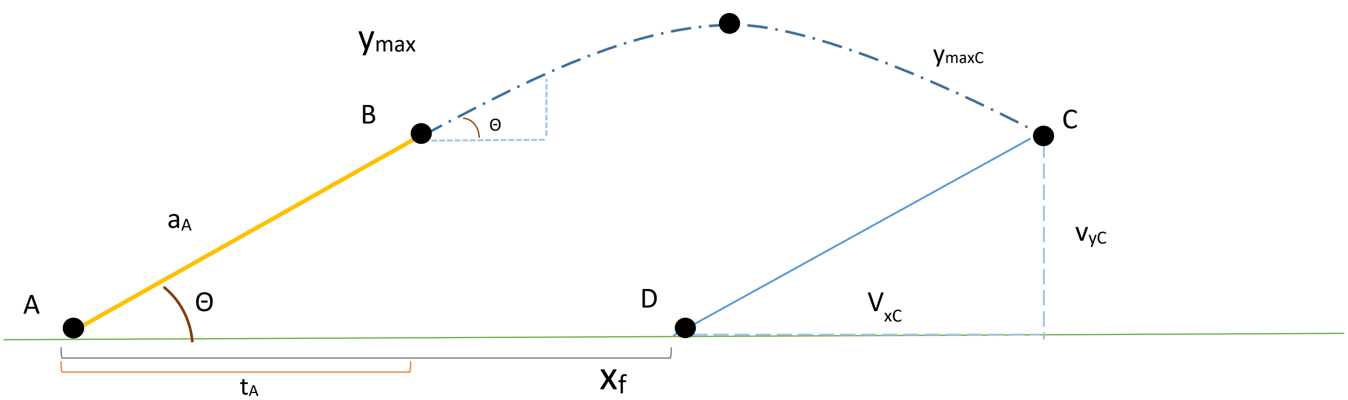
Problem Description

One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at a specified angle above the East horizontal. The rocket engine is designed to burn for specified time while producing a constant net acceleration for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens the rocket instantly changes speed and descends at a constant vertical speed. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind. Assume the wind affects the rocket only during the parachute stage.

Diagram



Givens

Launch angle	37 (deg)
Engine burn time	7.9 (sec)
Net acceleration of rocket while engine burns	7.6 (m/s^2)
Vertical distance rocket falls from max height before parachute opens	66 (m)
Rocket with parachute constant vertical speed	6 (m/s)
Wind and rocket with parachute constant horizontal speed	14 (m/s)

Step 1:

Calculate the initial velocities in both the x and y directions using the given angle, net acceleration of the rocket, and time.

<u>X-dir:</u> $v_{xA} = a_{xA}\Delta t + v_i \rightarrow v_{xA} = (7.6\cos 37)(7.9) + 0$

$\underline{V_{xA} = 47.9498\ m/s}$

<u>Y-dir:</u> $v_{yA} = a_{yi}\Delta t + v_i \rightarrow v_{yA} = (7.6\sin 37)(7.9) + 0$

$\underline{v_{yA} = 36.1329\ m/s}$

Step 2:

Find the height of y_a and the length of x_a using equation 1.

<u>y_A:</u> $y_A = \frac{1}{2}(v_f + v_i)(\Delta t) \rightarrow y_A = \frac{1}{2}(36.1329 + 0)(7.9)$

$\underline{y_A = 142.73\ m}$

<u>x_A:</u> $x_A = \frac{1}{2}(v_f + v_i)(\Delta t) \rightarrow x_A = \frac{1}{2}(47.9498 + 0)(7.9)$

$\underline{x_A = 189.402\ m}$

Step 3:

Find the maximum height of the rocket's projectile by taking the time at max height and substituting it in equation 3.

$\underline{y_{max}}$: $y_{max} = \frac{1}{2}at^2 + v_{yA}t + y_A$

$y_{max} = \frac{1}{2}(-9.8)t^2 + (36.1329)t + 142.73$

$t_{max}: \frac{-b}{2a} \rightarrow \frac{-36.1329}{-9.8} \rightarrow t_{max}: 3.68776\ s$

$y_{max} = -4.9(3.68776)^2 + 36.1329(3.68776) + 142.73$

$\underline{y_{max} = 209.395\ m}$

Step 4:

Find the height of point C using the max height and the distance the rocket falls from max height before parachute opens.

$\underline{y_C}$: $y_C = y_{max} - y_{maxC} \rightarrow y_C = 209.395 - 66$

$\underline{y_C = 143.395\ m}$

Step 5:

Find the time it takes the rocket to get from y_A to y_C by using equation 3 and v_{yA}.

$\underline{t_{AC}}$: $y_C = \frac{1}{2}at^2 + v_{yA}t + y_A$

$143.395 = \frac{1}{2}(-9.8)t^2 + (36.1329)t + 142.73$

$0 = -4.9t^2 + 36.1329t - 0.665$

$t_{AC} \underline{= 7.3569\ s}$ or $\cancel{0.018447\ s}$

Step 6:

Use equation 3 to find the distance between point A and C using v_{xA}, t_{ac}, and x_a.

$\underline{x_{AC}}$: $x_{ac} = \frac{1}{2}a\,t_{AC}^{\ 2} + v_{xA}t_{AC} + x_A$

$x_{ac} = \frac{1}{2}(0)(7.3569)^2 + (47.9498)(7.3569) + 189.402$

$x_{ac} = 0 + 352.762 + 189.402$

$\underline{x_{ac} = 542.164\ m}$

Step 7:

Calculate the time it will take for the rocket to reach a vertical y drop to 0 m from y_C.

t_f: $y_f = \frac{1}{2}a\,t^2 + v_{yC}t + y_C$

$0 = \frac{1}{2}(0)\,t^2 + (-6)t + 143.395$

$6t = 143.395$

$\underline{t = 23.8992\ s}$

Step 8:

Calculate the final displacement by using the distance from point A to C, v_{xC} and t_f in equation 3.

x_f: $x_f = \frac{1}{2}a_xt^2 + v_{xC}t + x_{AC}$

$x_f = \frac{1}{2}(0)(23.8992)^2 + (-14)(23.8992) + 542.164$

$x_f = -341.588 + 542.164$

$x_f = 200.576\ m$

$$\boxed{x_f = 200.6\ m}$$

The rocket will travel 200.6 m East of the initial point of launch.