

Homework-5

Group 4

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Problem 1

The file ToyotaCorolla.xlsx contains the data on used cars (Toyota Corolla) on sale during late summer of 2004 in The Netherlands. It has 1436 records containing details on 38 attributes, including Price, Age, Kilometers, HP, and other specifications. The goal is to predict the price of a used Toyota Corolla based on its specifications.

Data Preprocessing: Create dummy variables for the categorical predictors (Fuel Type and Color). Split the data into training (50%), validation (30%), and test (20%) datasets.

```
library(dplyr)

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##   filter, lag

## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union

library(readxl)
library(ggplot2)
library(caret)

## Loading required package: lattice

library(tree)
library(rpart)
library(rpart.plot)
library(magrittr)
library(reshape2)

ToyotaCorolla <- read_xlsx("ToyotaCorolla.xlsx", sheet = "data")

dummies <- dummyVars(~ Fuel_Type + Color, data = ToyotaCorolla, sep = ".")
dummies <- predict(dummies, ToyotaCorolla)

ToyotaCorolla <- cbind(
  select(ToyotaCorolla, -c("Fuel_Type", "Color")),
  as.data.frame(dummies)
)
```

```

set.seed(20)
split_sample <- sample(1:3,
  size = nrow(ToyotaCorolla),
  prob = c(0.50, 0.30, 0.20),
  replace = TRUE
)

train_data <- ToyotaCorolla[split_sample == 1, ]
validation_data <- ToyotaCorolla[split_sample == 2, ]
test_data <- ToyotaCorolla[split_sample == 3, ]

```

(a) Run a regression tree (RT) with the output variable Price and input variables

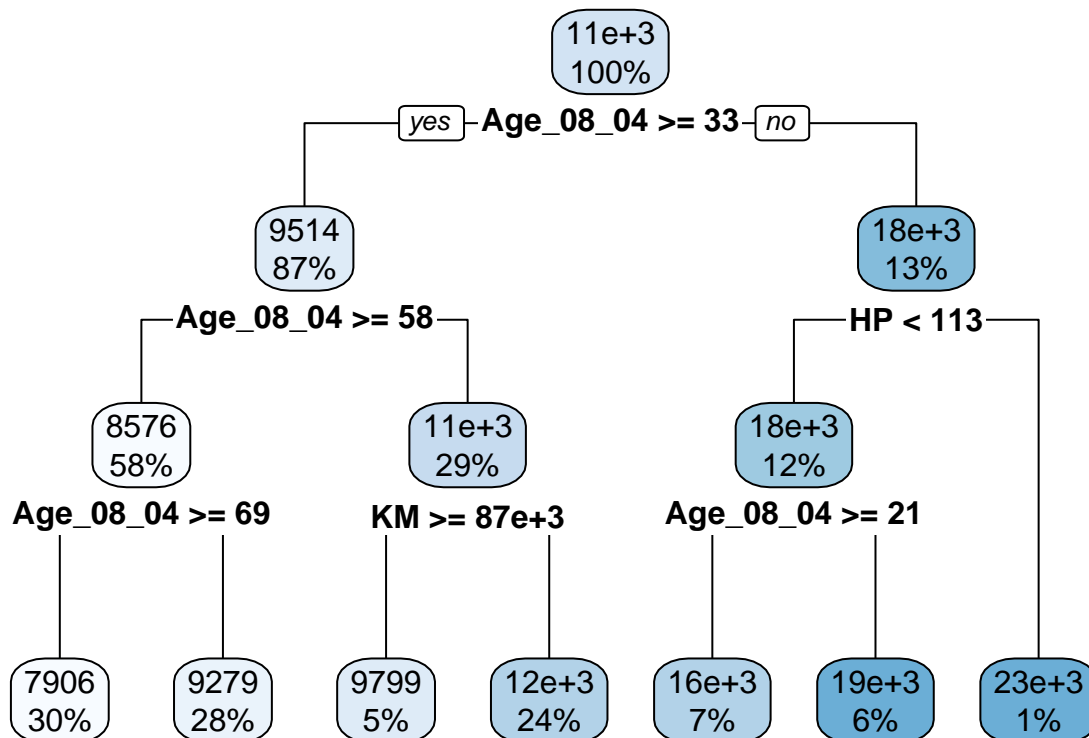
Age_08_04, KM, Fuel_Type, HP, Automatic, Doors, Quarterly_Tax, Mfg_Guarantee, Guarantee_Period, Airco, Automatic_Airco, CD Player, Powered_Windows, Sport_Model, and Tow_Bar.

```

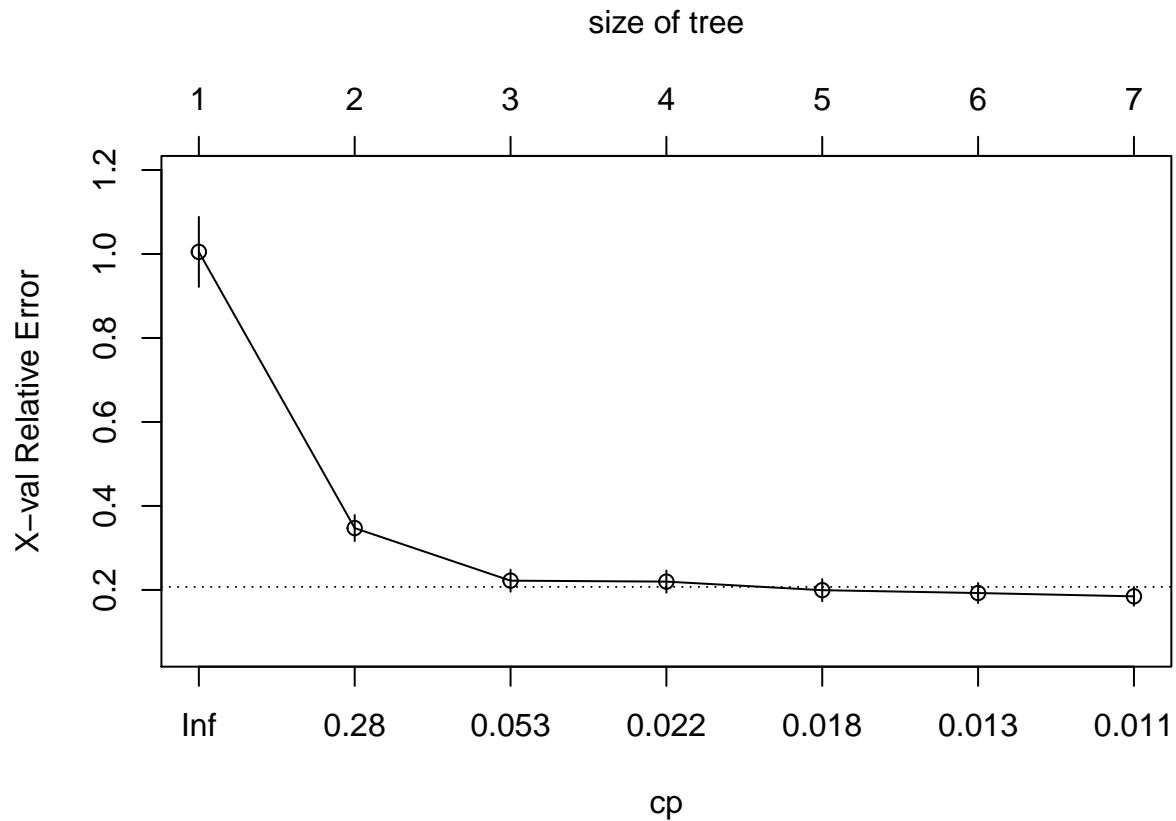
reg.tree <- rpart(Price ~ Age_08_04 + KM + Fuel_TypeCNG + Fuel_TypeDiesel +
  Fuel_TypePetrol + HP + Automatic + Doors +
  Quarterly_Tax + Mfr_Guarantee + Guarantee_Period + Airco + Automatic_airco + CD_Player + Powered_Windows +
  Tow_Bar,
data = train_data,
method = "anova"
)

rpart.plot(reg.tree)

```



```
plotcp(reg.tree)
```



```
print(reg.tree$variable.importance)
```

```
##      Age_08_04  Automatic_airco      KM      Quarterly_Tax
##      7716864493      2456255799      1812651349      837860826
##      HP Guarantee_Period      CD_Player      Fuel_TypeDiesel
##      811189489      336999867      219398681      34185858
##      Fuel_TypePetrol      Doors      Airco      Powered_Windows
##      34185858      31068708      27966026      18322568
##      Mfr_Guarantee      Sport_Model
##      17358223      3107805
```

(i) Which appear to be the three or four most important car specifications for predicting the car's price?

- The most important car specifications for predicting the car's price are Age_08_04, KM and Automatic_airco.

(ii) Compare the prediction errors of the training, validation, and test sets by

examining their RMS error and by plotting the three boxplots. What is happening with the training set predictions? How does the predictive performance of the test set compare to the other two? Why does this occur?

```
train_preds <- predict(reg.tree, train_data)
validation_preds <- predict(reg.tree, validation_data)
```

```

test_preds <- predict(reg.tree, test_data)

train_error <- RMSE(train_preds, train_data$Price)
validation_error <- RMSE(validation_preds, validation_data$Price)
test_error <- RMSE(test_preds, test_data$Price)

cat("Train Data RMSE", train_error, "\n")

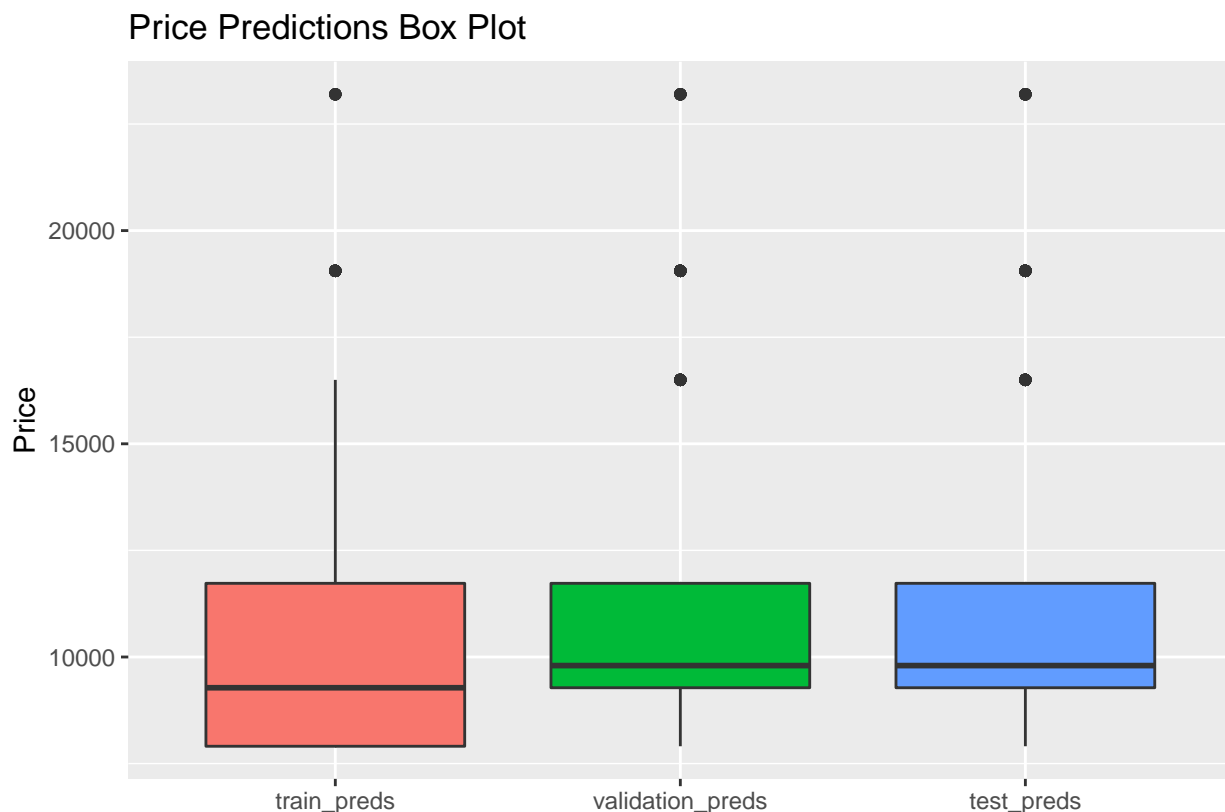
## Train Data RMSE 1345.008
cat("Validation Data RMSE", validation_error, "\n")

## Validation Data RMSE 1397.415
cat("Test Data RMSE", test_error, "\n")

## Test Data RMSE 1591.446

df <- melt(as.data.frame(cbind(train_preds, validation_preds, test_preds)))
ggplot(data = df, aes(x = variable, y = value, fill = variable)) +
  geom_boxplot() +
  theme(legend.position = "none") +
  ggtitle("Price Predictions Box Plot") +
  labs(x = "", y = "Price")

```



We can see from the RMSE error that the model performs the best on the train_data but that is to be expected. It performs slightly worse on the validation data. It performs the worst on the test data.

We can see that the training set predictions are similar to the actual Price values of the train data with a

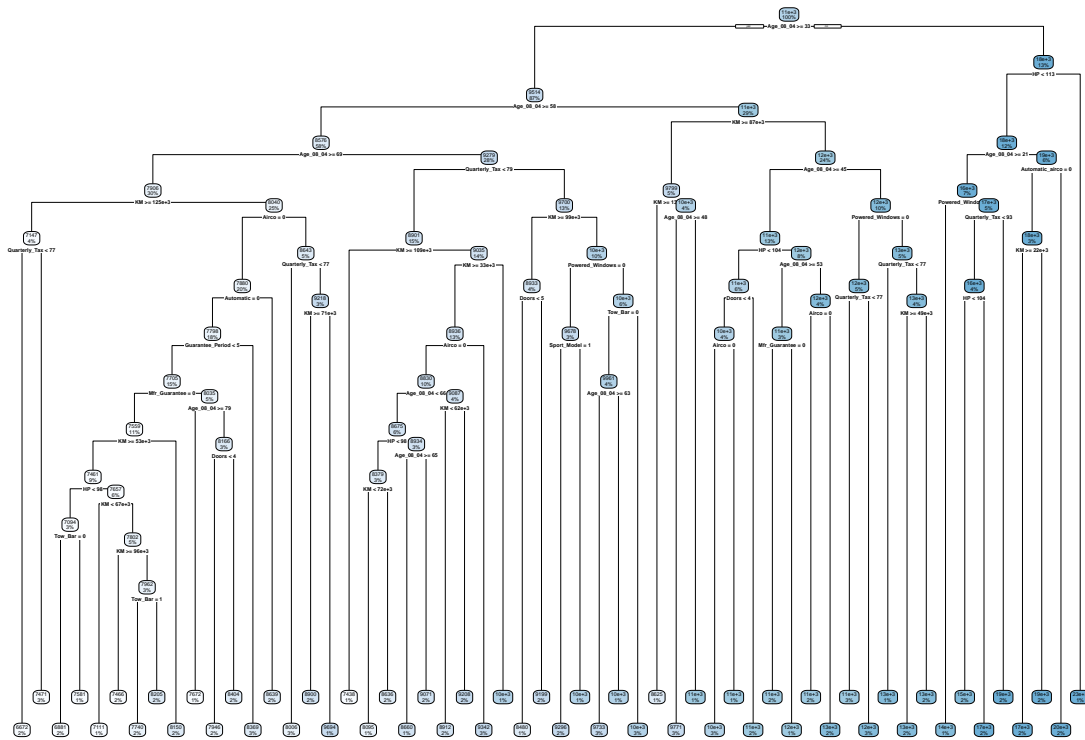
mean of approx \$10000. There are 2 samples which are outliers in all three of the sets.

(iv) If we used the full tree instead of the best pruned tree to score the validation set, how would this affect the predictive performance for the validation set? (Hint: Does the full tree use the validation data?)

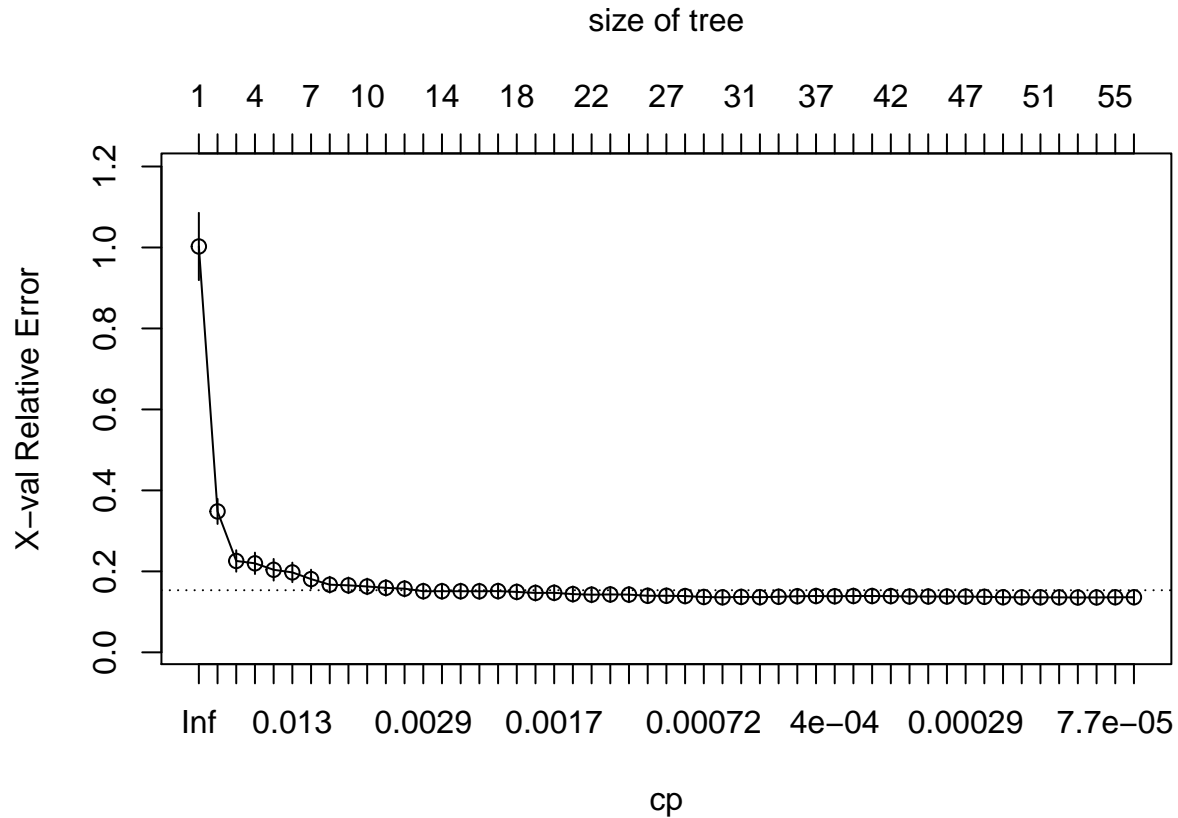
```
full.tree <- rpart(Price ~ Age_08_04 + KM + Fuel_TypeCNG + Fuel_TypeDiesel +
  Fuel_TypePetrol + HP + Automatic + Doors +
  Quarterly_Tax + Mfr_Guarantee + Guarantee_Period + Airco + Automatic_airco + CD_Player + Powered_Windows +
  Tow_Bar,
data = train_data,
method = "anova",
control = list(cp = 0)
)

rpart.plot(full.tree)
```

Warning: labs do not fit even at cex 0.15, there may be some overplotting



```
plotcp(full.tree)
```



```
train_preds <- predict(full.tree, train_data)
validation_preds <- predict(full.tree, validation_data)
test_preds <- predict(full.tree, test_data)

train_error <- RMSE(train_preds, train_data$Price)
validation_error <- RMSE(validation_preds, validation_data$Price)
test_error <- RMSE(test_preds, test_data$Price)

cat("Train Data RMSE", train_error, "\n")

## Train Data RMSE 971.6847
cat("Validation Data RMSE", validation_error, "\n")

## Validation Data RMSE 1202.67
cat("Test Data RMSE", test_error, "\n")

## Test Data RMSE 1497.919
```

Using the full tree, we significantly reduce the RMSE error on both the validation and the test sets. Although the full tree doesn't use the validation data, it has more decision boundaries which are correctly separating the validation data hence reducing its error.

(b) Let us see the effect of turning the price variable into a categorical variable. First, create a new variable that categorizes price into 20 bins of equal counts. Now

repartition the data keeping Binned Price instead of Price. Run a classification tree (CT) with the same set of input variables as in the RT, and with Binned Price as the output variable.

```
ToyotaCorolla$Price <- cut(ToyotaCorolla$Price, breaks = seq(4300, 32500, by = 1410))
```

```
set.seed(20)
```

```
split_sample <- sample(1:3,  
  size = nrow(ToyotaCorolla),  
  prob = c(0.50, 0.30, 0.20),  
  replace = TRUE  
)
```

```
train_data <- ToyotaCorolla[split_sample == 1, ]
```

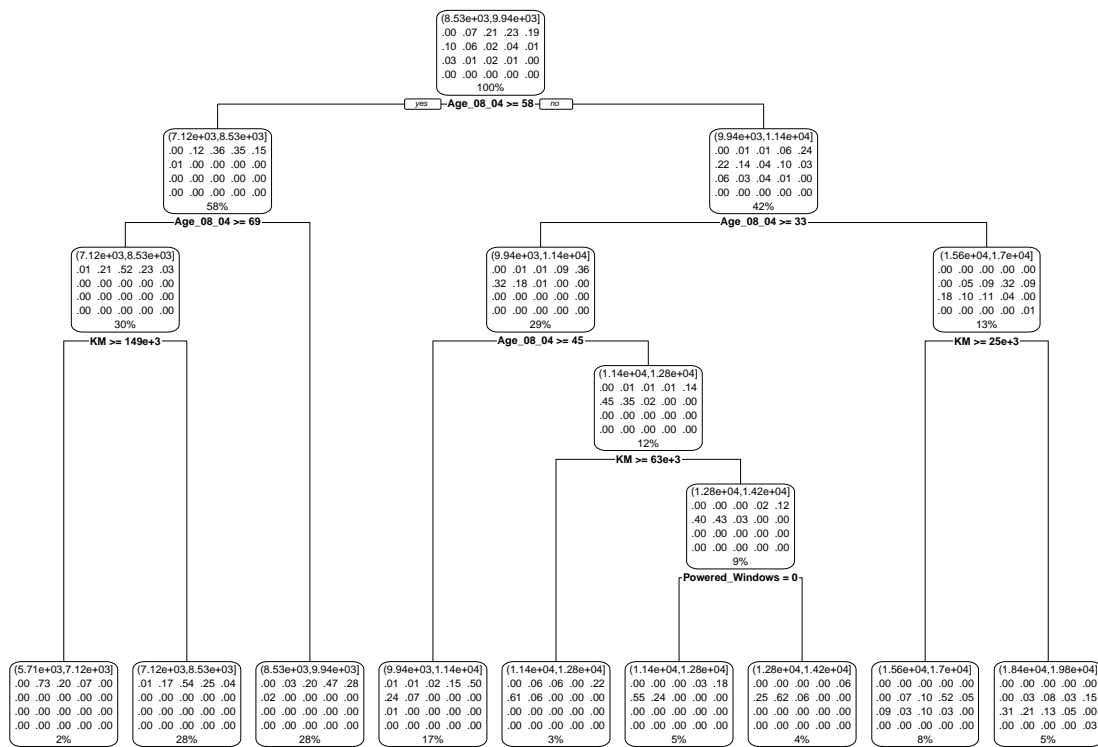
```
validation_data <- ToyotaCorolla[split_sample == 2, ]
```

```
test_data <- ToyotaCorolla[split_sample == 3, ]
```

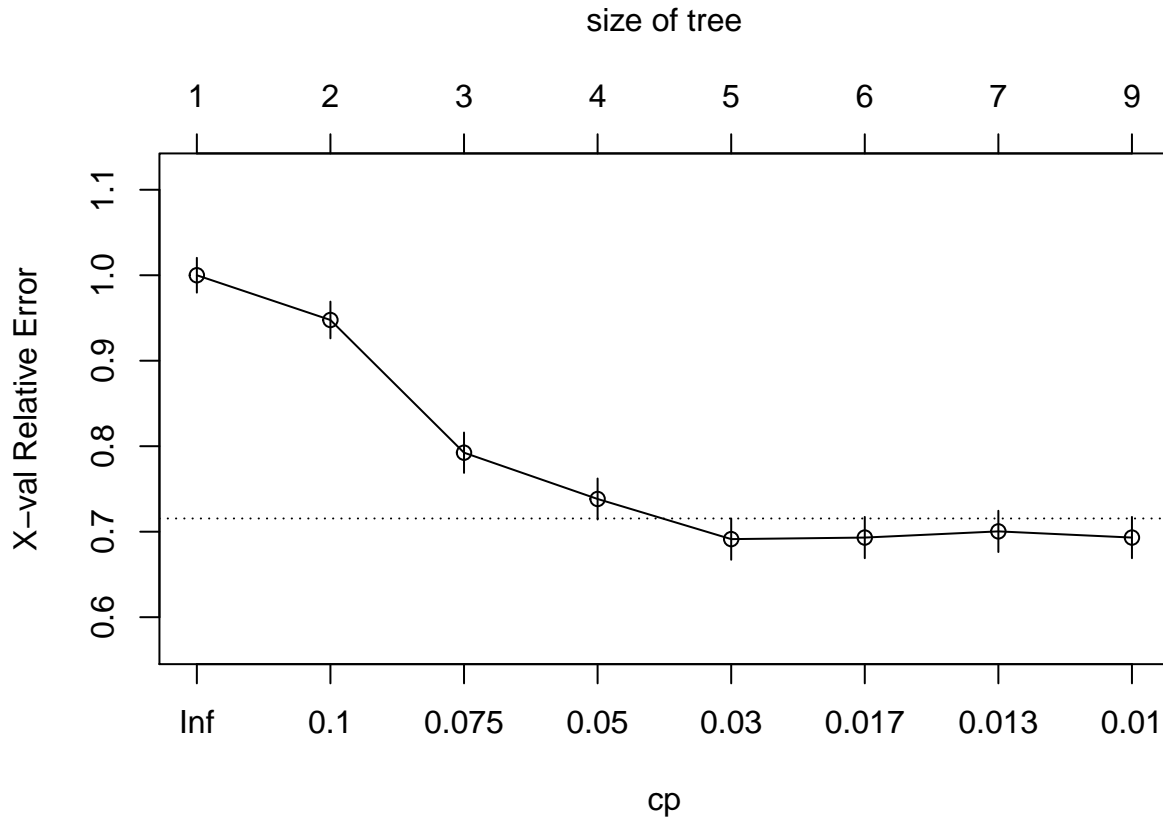
```
class.tree <- rpart(Price ~ Age_08_04 + KM + Fuel_TypeCNG + Fuel_TypeDiesel +  
  Fuel_TypePetrol + HP + Automatic + Doors +  
  Quarterly_Tax + Mfr_Guarantee + Guarantee_Period + Airco + Automatic_airco + CD_Player + Powered_Wind  
  Tow_Bar,  
  data = train_data  
)
```

```
rpart.plot(class.tree)
```

```
## Warning: All boxes will be white (the box.palette argument will be ignored) because  
## the number of classes in the response 20 is greater than length(box.palette) 6.  
## To silence this warning use box.palette=0 or trace=-1.
```



`plotcp(class.tree)`



```
print(class.tree$variable.importance)
```

```
##      Age_08_04      KM      Airco      Automatic_airco
##      127.3907916      53.2246381      19.4035608      18.5668915
##      CD_Player      Sport_Model      Quarterly_Tax      Mfr_Guarantee
##      18.2603264      15.7928942      7.3889731      6.7235932
##      Powered_Windows      HP      Fuel_TypePetrol      Guarantee_Period
##      6.5573410      3.8118783      1.1929470      0.9997784
##      Doors      Fuel_TypeDiesel      Fuel_TypeCNG      Automatic
##      0.8981243      0.7550886      0.3205369      0.1602685
```

(i) Compare the tree generated by the CT with the one generated by the RT. Are they different? (Look at structure, the top predictors, size of tree, etc.) Why?

- The top predictors of both the classification and regression trees remain more or less the same except of Quarterly_Tax which is only important in the regression tree.
- The classification tree is both structurally more complex as well bigger than the regression tree. This difference is created because in a regression tree, the response value is the mean of all the other values in the particular region however in the classification tree, the response is the majority class of the region.
- The difference in shape is so apparent because decision trees are sensitive to change in training data.

(ii) Predict the price, using the RT and the CT, of a used Toyota Corolla with the specifications listed in Table below.

```

df <- data.frame(
  Age_08_04 = 77,
  KM = 117000,
  Fuel_TypePetrol = 1,
  Fuel_TypeDiesel = 0,
  Fuel_TypeCNG = 0,
  HP = 110,
  Automatic = 0,
  Doors = 5,
  Quarterly_Tax = 100,
  Mfr_Guarantee = 0,
  Guarantee_Period = 3,
  Airco = 1,
  Automatic_airco = 0,
  CD_Player = 0,
  Powered_Windows = 0,
  Sport_Model = 0,
  Tow_Bar = 1
)

df.pred.reg <- predict(reg.tree, df)
df.pred.class <- as.data.frame(predict(class.tree, df))
df.pred.class <- attributes(which.max(df.pred.class))$names

cat("Prediction of Regression Tree:", df.pred.reg, "\n")

## Prediction of Regression Tree: 7906.141
cat("Prediction of Classification Tree:", df.pred.class, "\n")

## Prediction of Classification Tree: (7.12e+03,8.53e+03]
rm(list = ls())

```

(iii) Compare the predictions in terms of the predictors that were used, the magnitude

of the difference between the two predictions, and the advantages and disadvantages of the two methods.

- The Regression Tree predicts the new data Price to be \$7906.141. While the classification tree predicts that the price will be in the category/bin/range \$7120-\$8530.
- The advantage of using regression trees is that we can obtain an exact numerical prediction of our data.
-

However classification trees can prove to be more useful in predicting numerical data in situations where we are more concerned with the range estimation of the data rather than its average value.

Problem 2

```
Banks <- read_xlsx("Banks.xlsx", sheet = "Bank Financial Condition")
```

```
fit.full <- glm(`Financial Condition` ~ `TotLns&Lses/Assets` + `TotExp/Assets`,
  data = Banks,
  family = "binomial"
)
```

```
summary(fit.full)
```

```
##
## Call:
## glm(formula = `Financial Condition` ~ `TotLns&Lses/Assets` +
##      `TotExp/Assets`, family = "binomial", data = Banks)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.64035  -0.35514   0.02079   0.53234   1.03373
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)      -14.188      6.122  -2.317  0.0205 *
## `TotLns&Lses/Assets`    9.173      6.864   1.336  0.1814
## `TotExp/Assets`      79.964     39.263   2.037  0.0417 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 27.726  on 19  degrees of freedom
## Residual deviance: 12.831  on 17  degrees of freedom
## AIC: 18.831
##
## Number of Fisher Scoring iterations: 6
```

```
exp(coef(fit.full))
```

```
##              (Intercept) `TotLns&Lses/Assets`      `TotExp/Assets`
##      6.893258e-07      9.635549e+03      5.344393e+34
```

(a) Write the estimated equation that associates the financial condition of a bank with its two predictors in three formats:

(i) The logit as a function of the predictors

Logit(Financial Condition = Strong) = $-14.188 + 9.172 * \text{TotLns\&Lses/Assets} + 79.964 * \text{TotExp/Assets}$
 Logit(Financial Condition = Weak) = $-14.188 + 9.172 * \text{TotLns\&Lses/Assets} + 79.964 * \text{TotExp/Assets}$

(ii) The odds as a function of the predictors

Odds(Financial Condition = Strong) = $e^{(-14.188 + 9.172 * \text{TotLns\&Lses/Assets} + 79.964 * \text{TotExp/Assets})}$
 Odds(Financial Condition = Weak) = $e^{(-14.188 + 9.172 * \text{TotLns\&Lses/Assets} + 79.964 * \text{TotExp/Assets})}$

(iii) The probability as a function of the predictors

Probability(Financial Condition = Weak) = $(e^{(-14.188 + 9.172 * \text{TotLns\&Lses/Assets} + 79.964 * \text{TotExp/Assets})}) / (1 + e^{(-14.188 + 9.172 * \text{TotLns\&Lses/Assets} + 79.964 * \text{TotExp/Assets})})$

$\text{Probability}(\text{Financial Condition} = \text{Strong}) = 1 / (1 + e^{(-14.188 + 9.172 * \text{TotLns\&Lses/Assets} + 79.964 * \text{TotExp/Assets})})$

(b) Consider a new bank whose total loans and leases/assets ratio = 0.6 and total

expenses/assets ratio = 0.11. From your logistic regression model, estimate the following four quantities for this bank: the logit, the odds, the probability of being financially weak, and the classification of the bank.

```
df <- data.frame(`TotLns&Lses/Assets` = 0.6, `TotExp/Assets` = 0.11)
colnames(df) <- c("TotLns&Lses/Assets", "TotExp/Assets")

logit <- -14.188 + 9.172 * df$`TotLns&Lses/Assets` + 79.964 * df$`TotExp/Assets`

odds <- exp(logit)

probability <- odds / (1 + odds)

bank_class <- predict(fit.full, df, type = "response")

cat("Logit:", logit, "\n")

## Logit: 0.11124
cat("Odds:", odds, "\n")

## Odds: 1.117663
cat("Probability of being Financially Weak:", probability, "\n")

## Probability of being Financially Weak: 0.5277814
cat(
  "Classification of the Bank:",
  ifelse(bank_class > 0.5, "Financially Weak", "Financially Strong"), "\n"
)

## Classification of the Bank: Financially Weak
rm(list = ls())
```

(c) The cutoff value of 0.5 is used in conjunction with the probability of being

financially weak. Compute the threshold that should be used if we want to make a classification based on the odds of being financially weak, and the threshold for the corresponding logit.

The odds of being financially weak are 1.11763 and the corresponding logit is 0.11124. So it is sufficient to say that the threshold for being classified as financially weak should be 0.5

(d) Interpret the estimated coefficient for the total loans & leases to total assets ratio (TotLns&Lses/Assets) in terms of the odds of being financially weak.

The coefficient for the TotLns&Lses/Assets ratio is 9.172 which means increasing the TotLns&Lses/Assets ratio by 1 increases the log odds of belonging to class “Financially Weak” by 9.172 (or the odds of belonging to class “Financially Weak” by $\exp(9.172)$)

(e) When a bank that is in poor financial condition is misclassified as financially

strong, the misclassification cost is much higher than when a financially strong bank is misclassified as weak. To minimize the expected cost of misclassification, should the cutoff value for classification (which is currently at 0.5) be increased or decreased?

We should decrease the cutoff to something lower than 0.5 so that the misclassification rate decreases.

Problem 3

A management consultant is studying the roles played by experience and training in a system administrator's ability to complete a set of tasks in a specified amount of time. In particular, she is interested in discriminating between administrators who are able to complete given tasks within a specified time and those who are not. Data are collected on the performance of 75 randomly selected administrators. They are stored in the file System Administrators.xlsx.

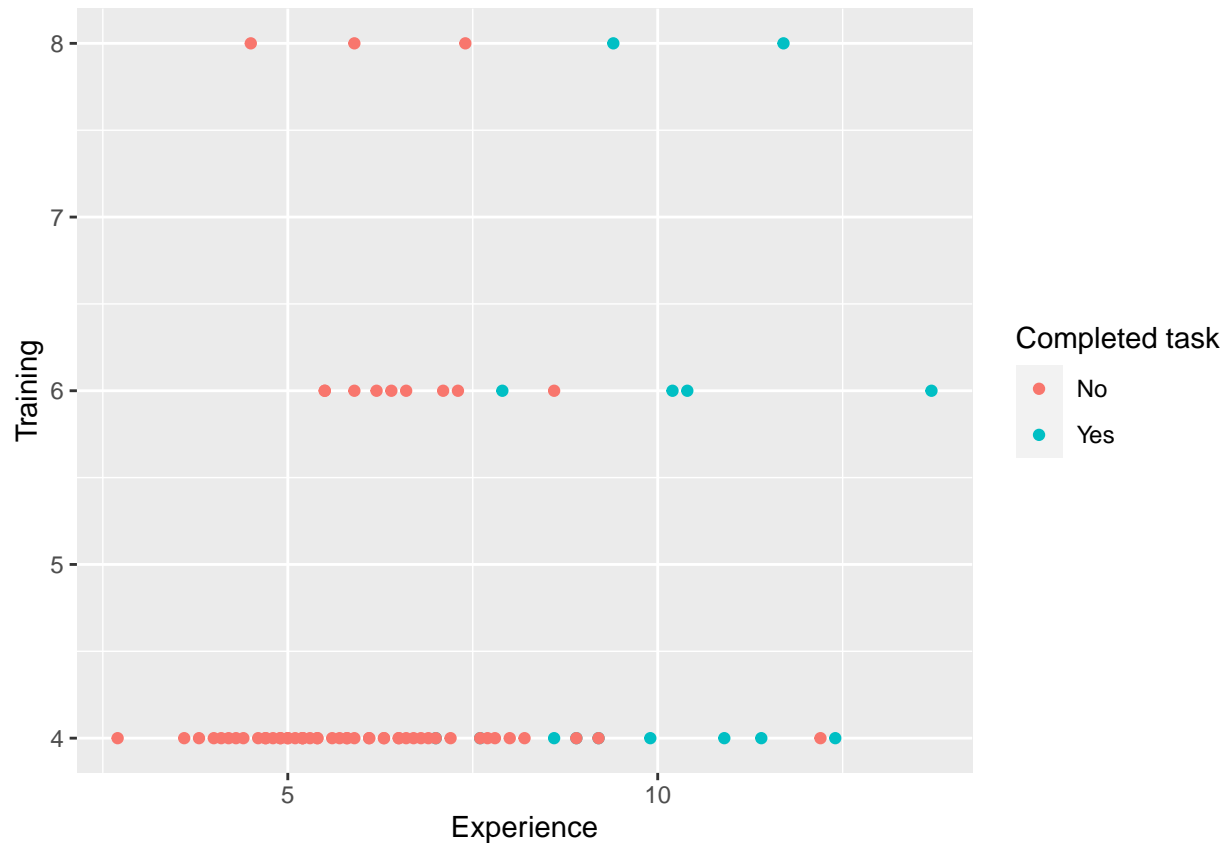
The variable Experience (X1) measures months of full-time system administrator experience, while Training (X2) measures the number of relevant training credits. The dependent variable Completed (Y) is either Yes or No, according to whether or not the administrator completed the tasks

```
System_Admin <- read_xlsx("System Administrators.xlsx", sheet = "data")
```

(a) Create a scatterplot of Experience versus Training using color or symbol to

differentiate programmers who complete the task from those who did not complete it. Which predictor(s) appear(s) potentially useful for classifying task completion?

```
System_Admin %>% ggplot(mapping = aes(
  x = Experience,
  y = Training,
  colour = `Completed task`
)) +
  geom_point()
```



The predictor Experience appears potentially useful for classifying task completion as the scatter plot clearly indicates that the completion of a task is associated with increasing experience.

(b) Run a logistic regression model with both predictors using the entire dataset as

training data. Among those who complete the task, what is the percentage of programmers who are incorrectly classified as failing to complete the task?

```
System_Admin$Completed_Task <- factor(System_Admin$`Completed task`,
  levels = c("No", "Yes"),
  labels = c(0, 1)
)

log_reg <- glm(Completed_Task ~ Experience + Training,
  family = binomial(),
  data = System_Admin
)

summary(log_reg)

##
## Call:
## glm(formula = Completed_Task ~ Experience + Training, family = binomial(),
##      data = System_Admin)
##
## Deviance Residuals:
```

```
##      Min      1Q      Median      3Q      Max
## -2.65306 -0.34959 -0.17479 -0.08196  2.21813
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -10.9813      2.8919  -3.797 0.000146 ***
## Experience    1.1269      0.2909   3.874 0.000107 ***
## Training      0.1805      0.3386   0.533 0.593970
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 75.060  on 74  degrees of freedom
## Residual deviance: 35.713  on 72  degrees of freedom
## AIC: 41.713
##
## Number of Fisher Scoring iterations: 6
```

```
log_red.pred <- predict(log_reg, newdata = System_Admin, type = "response")
```

```
log_red.pred
```

```
##           1           2           3           4           5           6
## 0.8833227256 0.7104041841 0.8607858148 0.9960915732 0.7419100430 0.9762161045
##           7           8           9          10          11          12
## 0.2698280096 0.4428550070 0.8315210600 0.9300686477 0.3617763914 0.5270974719
##          13          14          15          16          17          18
## 0.9746144299 0.1551727239 0.0854303709 0.0086861019 0.1304438403 0.0097121895
##          19          20          21          22          23          24
## 0.0077675696 0.0049629856 0.0039654656 0.0407145324 0.0069454891 0.0121376919
##          25          26          27          28          29          30
## 0.4428550070 0.0241239684 0.0151596615 0.0211288174 0.1705265274 0.0007337889
##          31          32          33          34          35          36
## 0.0624542525 0.0515988589 0.0786736805 0.0020206422 0.0097121895 0.0151596615
##          37          38          39          40          41          42
## 0.1582037909 0.1551727239 0.4485221325 0.9703814703 0.0235897833 0.0327678779
##          43          44          45          46          47          48
## 0.0241239684 0.0035443433 0.0113621331 0.1870631514 0.0854303709 0.0044364010
##          49          50          51          52          53          54
## 0.0327678779 0.0086861019 0.0770269246 0.0121376919 0.0693873778 0.0235897833
##          55          56          57          58          59          60
## 0.0062098690 0.1047650718 0.0121376919 0.0108581610 0.5270974719 0.0069454891
##          61          62          63          64          65          66
## 0.0373499891 0.2237715913 0.0031678009 0.0407145324 0.0561720643 0.0504877634
##          67          68          69          70          71          72
## 0.2318415026 0.0504877634 0.2653323995 0.0263296786 0.0189196130 0.0527330485
##          73          74          75
## 0.0638109991 0.0025301811 0.0135659355
```

As it can be seen the records 7,8,11,14,15 have been misclassified by the model among those who complete the task

the percentage of programmers who are incorrectly classified as failing to complete the task is 33.33%

(c) To decrease the percentage in part (b), should the cutoff probability be increased

or decreased?

clearly the cutoff probability needs to be decreased for the percentage of programmers who are incorrectly classified as failing to complete the task, to decrease.

(d) How much experience must be accumulated by a programmer with 4 years of

training before his or her estimated probability of completing the task exceeds 50%?

```
sample1 <- System_Admin[which(System_Admin$Training == 4), ]
sample1 <- sample1[order(-sample1$Experience), ]

log_reg1 <- glm(Completed_Task ~ Experience + Training,
  family = binomial(),
  data = sample1
)

summary(log_reg1)

##
## Call:
## glm(formula = Completed_Task ~ Experience + Training, family = binomial(),
##      data = sample1)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.3578  -0.3712  -0.2040  -0.1279   2.1349
##
## Coefficients: (1 not defined because of singularities)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -8.7486     2.3609  -3.706 0.000211 ***
## Experience     0.9397     0.2871   3.274 0.001062 **
## Training      NA           NA      NA      NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 49.723  on 56  degrees of freedom
## Residual deviance: 28.347  on 55  degrees of freedom
## AIC: 32.347
##
## Number of Fisher Scoring iterations: 6

log_red.pred1 <- predict(log_reg1, newdata = sample1, type = "response")

## Warning in predict.lm(object, newdata, se.fit, scale = 1, type = if (type == :
## prediction from a rank-deficient fit may be misleading

log_red.pred1

##           1           2           3           4           5           6
## 0.948022808 0.937943002 0.876952812 0.816684047 0.635144887 0.474165779
```



```
##           7           8           9           10           11           12
## 0.474165779 0.404840692 0.404840692 0.339114685 0.260548960 0.225996839
##           13           14           15           16           17           18
## 0.194819560 0.180500324 0.167015243 0.167015243 0.121020079 0.102408694
##           19           20           21           22           23           24
## 0.102408694 0.094087965 0.086378237 0.079244995 0.072653982 0.066571526
##           25           26           27           28           29           30
## 0.066571526 0.055802060 0.055802060 0.046687660 0.046687660 0.039000465
##           31           32           33           34           35           36
## 0.035627178 0.035627178 0.032535780 0.029704363 0.024740922 0.024740922
##           37           38           39           40           41           42
## 0.022572024 0.020589247 0.020589247 0.020589247 0.018777295 0.017122017
##           43           44           45           46           47           48
## 0.017122017 0.015610334 0.015610334 0.014230185 0.012970450 0.012970450
##           49           50           51           52           53           54
## 0.011820896 0.009815458 0.008942994 0.008147442 0.007422131 0.006760949
##           55           56           57
## 0.005609068 0.004652517 0.002002403
```

```
sample1[6, 1]
```

```
## Registered S3 method overwritten by 'cli':
##   method      from
##   print.tree tree

## # A tibble: 1 x 1
##   Experience
##   <dbl>
## 1      9.2
```

```
rm(list = ls())
```

It can be observed that 9.2 years of experience must be accumulated by a programmer with 4 years of training before his or her estimated probability of completing the task exceeds 50%