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In-Class Problem 11

For hard margin SVM, prove that

$$b^* = -\frac{1}{2}\left[\min_{i:y_i=1} w^{*T}x_i + \max_{i:y_i=-1} w^{*T}x_i\right]$$

$\Rightarrow$

Our primal problem is,

$$\begin{cases} \min \frac{1}{2}\|w\|_2^2 \\ \text{s.t: } y_i(w^T x_i + b) \geq 1 \end{cases}$$

Let's assume we solved for this and calculated w^* and b^*,

then the closest datapoints to the decision boundary will satisfy,

$$\min_{i:y_i=1}(w^{*T}x_i) + b^* = 1 \quad [\text{for } y_i = 1] \rightarrow (1)$$

and

$$\max_{i:y_i=-1}(w^{*T}x_i) + b^* = -1 \quad [\text{for } y_i = -1] \rightarrow (2)$$

adding 1 and 2,

$$\max_{i:y_i=-1}(w_*^T x_i) + \min_{i:y_i=1}(w^{*T} x_i) + 2b = 0$$

$\therefore \cancel{b = -\frac{1}{2}\left[\max_{i:y_i=1} w^{*T} x_i\right.} +$

$$\therefore \boxed{b = -\frac{1}{2}\left[\min_{i:y_i=1} w^{*T} x_i + \max_{i:y_i=-1} w^{*T} x_i\right]}$$

Hence Proved