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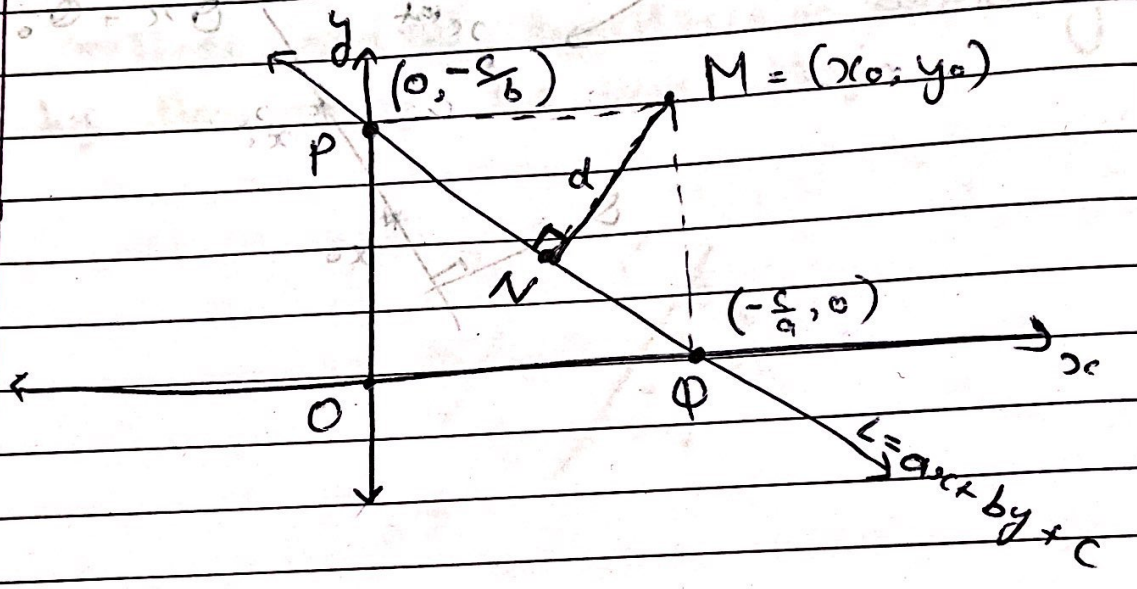
10/28/21 **In-Class Problem 10**

Derive the equation for distance from a point (x_0, y_0) to a line $ax+by+c=0$

$\Rightarrow$ Let visualize the line $L: ax+by+c=0$



Now we can see that the line L touches the y and x axes at points $P\left(0, -\frac{c}{b}\right)$ and $Q\left(-\frac{c}{a}, 0\right)$ respectively.

The point $M(x_0, y_0)$ is the one we need to calculate the distance of the line from.

The distance of M from line L is the shortest distance ~~of a point (here M)~~ of a line we can draw from M connecting L.

A shortest distance will always be a perpendicular drawn from M onto L.

∴ We need to calculate the ~~distance~~ length d of the perpendicular NM.

So, let us utilize the property of area of triangle:

$$\text{Area } \Delta PM\Phi = \frac{1}{2} \times \text{Perpendicular} \times \text{base}$$

$$= \frac{1}{2} \times MN \times P\Phi$$

$$\therefore MN = \frac{2 \times \Delta PM\Phi}{P\Phi} \rightarrow (1)$$

$$\text{Now, } P\Phi = \sqrt{\left(0-\left(\frac{-c}{a}\right)\right)^2 + \left(\frac{-c}{b}-0\right)^2}$$

$$= \sqrt{c^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}$$

$$= c\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

Substituting that in (1),

$$MN = \frac{2 \times \Delta PMQ}{c \times \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \rightarrow (2)$$

Now we also have the formula,

$$\text{area } \Delta PMQ = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

$$= \frac{1}{2} \left| 0(y_0 - 0) + x_0\left(0 - \left(\frac{-c}{b}\right)\right) - \frac{c}{a}\left(\frac{-c}{b} - y_0\right) \right|$$

$$= \frac{1}{2} \left| \frac{x_0 c}{b} + \frac{c^2}{ab} + \frac{c y_0}{a} \right|$$

Multiplying and dividing by ab,

$$= \frac{1}{2} \left| c\left(\frac{x_0 ab}{ab^2} + \frac{cab}{(ab)^2} + \frac{y_0 ab}{a^2 b} \right) \right|$$

$$= \frac{1}{2} \left| \frac{c a x_0}{ab} + \frac{c^2}{ab} + \frac{y_0 cb}{ab} \right|$$

$$= \frac{c}{2ab} \left| a x_0 + b y_0 + c \right| \rightarrow (3)$$

Substituting (3) in (2),

$$MN = \frac{2\cancel{x}\ \cancel{2} \times |ax_0 + by_0 + c|}{2ab \times \sqrt{\frac{a^2+b^2}{(ab)^2}}}$$

$$= \frac{|ax_0 + by_0 + c|}{\cancel{ab}} \Big/ \frac{\sqrt{a^2+b^2}}{\cancel{ab}}$$

$$\therefore d = MN = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}}$$

Solved.