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10/1/2021

In-Class Problem 7

⇒ θ_{MAP} for model where $y \sim N(\theta^T x, \sigma^2)$

and prior $\theta \sim \text{Laplace}\left(0, \frac{1}{\lambda}\right)$

$$P\left(\theta \mid Y, x, \sigma^2, \frac{1}{\lambda}\right) \propto \underbrace{P\left(Y \mid \theta, x, \sigma^2, \frac{1}{\lambda}\right)}_{\text{Likelihood}} \underbrace{P(\theta)}_{\text{Prior}}$$

$$\left(P(\theta \mid D) = P(D \mid \theta) \cdot P(\theta)\right).$$

taking log both sides.

$$\log \hat{\theta}_{map} = \log P(D \mid \theta) + \log(P(\theta)).$$

Here $P(D \mid \theta) = N(\theta^T x, \sigma^2) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-(y-\theta^T x)^2}{2\sigma^2}}$

$$P(\theta) = L\left(0, \frac{1}{\lambda}\right) = \frac{1}{2b} e^{-\lambda|\theta|}$$

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$$= \log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \sum_{i=1}^{m} \frac{(y - \theta^T x)^2}{2\sigma^2}$$

$$+ \log\left(\frac{1}{2b}\right) - \lambda|\theta|$$

(Getting rid of terms without θ)

$$\Rightarrow \theta_{MAP} = \underset{\theta}{\operatorname{argmax}} \left(\frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - \theta^T x_i)^2 + \lambda|\theta|_1 \right)$$

L1-Norm