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In-Class Problem 5

Prove That:

$$\int_D \int_x \int_y (h_D(x)-y)^2 p(x,y) P_r(D) \, dx\, dy\, dD$$
$$= E_{x,D}\left[(h_D(x) - \bar{h}(x))^2 \right]$$
$$+ E_{x,y}\left[(\bar{y}(x) - y)^2\right] + E_x\left[(\bar{h}(x) - \bar{y}(x))^2\right]$$

$\Rightarrow$ Let's look at the LHS, it is nothing but the Expectation of the test error given $h_D(x) \sim P(D)$, $x, y \in P(x,y)$ [i.i.d]

$$= E_{x,y,D}\left[(h_D(x) - y)^2\right]$$

$$= E_{x,y,D}\left[\left((h_D(x) - \bar{h}(x)) + (\bar{h}(x) - y) \right)^2 \right]$$

adding and subtracting the expected classifier $\bar{h}(x)$

$$= E_{x,D}\left[(h_D(x) - \bar{h}(x))^2\right] + 2E_{x,y,D}\left[(h_D(x) - \bar{h}(x))(\bar{h}(x) - y)\right]$$
$$+ E_{x,y}\left[(\bar{h}(x) - y)^2\right]$$

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$$= E_{x,D}\left[(\bar{h}_D(x) - \bar{h}(x))^2\right] + 2E_{x,y}\left[E_D\left[h_D(x) - \bar{h}(x)\right](\bar{h}(x) - y)\right] + E_{x,y}\left[(\bar{h}(x) - y)^2\right]$$

$$= E_{x,D}\left[(\bar{h}_D(x) - \bar{h}(x))^2\right] + 2E_{x,y}\left[E_D\left[h_D(x)\right] - \bar{h}(x)\right](\bar{h}(x) - y)\right] + E_{x,y}\left[(\bar{h}(x) - y)^2\right]$$

$$= E_{x,D}\left[(h_D(x) - \bar{h}(x))^2\right] + E_{x,y}\left[(\bar{h}(x) - \bar{h}(x))(\bar{h}(x) - y)\right] + E_{x,y}\left[(\bar{h}(x) - y)^2\right]$$

$$= E_{x,D}\left[(h_D(x) - \bar{h}(x))^2\right] + 0 + E_{x,y}\left[(\bar{h}(x) - y)^2\right]$$

$$= E_{x,D}\left[(h_D(x) - \bar{h}(x))^2\right] + E_{x,y}\left[(\bar{h}(x) - \bar{y}(x) + (\bar{y}(x) - y))^2\right]$$

adding and subtracting expected label

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$$= E_{x,D}\left[(h_D(x) - \bar{h}(x))^2\right] + E_{x,y}\left[(\bar{y}(x) - y)^2\right]$$

$$+ 2E_{x,y}\left[(\bar{h}(x) - \bar{y}(x))(\bar{y}(x) - y)\right] + E_x\left[(\bar{h}(x) - \bar{y}(x))^2\right]$$

this term is also 0, similar breakdown to as the $E_{D}[h_D(x)]$ term.

noise

$$= E_{x,D}\left[(h_D(x) - \bar{h}(x))^2\right] + E_{x,y}\left[(\bar{y}(x) - y)^2\right]$$

Variance

$$+ E_x\left[(\bar{h}(x) - \bar{y}(x))^2\right]$$

$Bias^2$

LHS = RHS.

Hence Proved.