9/14/21 In Class problem 1:

Prove that for a square matrix A:

A. $A + A^T$ is symmetric.

Let us denote $A + A^T$ as a new matrix $B \in R^{n \times n}$

i.e $B = A + A^T$

now, let us take a transpose of B,

$$B^T = (A + A^T)^T$$

recalling the property of transpose $(X+Y)^T = X^T + Y^T$, we can rewrite B^T as:

$$B^T = A^T + A$$

$$\Rightarrow B^T = B$$

since $B^T = B$, we can say B or $(A + A^T)$ is symmetric

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B. $A - A^T$ is anti-symmetric.

Let $B \in R^{n\times n}$ denote $A - A^T$

i.e $B = A - A^T$

taking a transpose of B,

$$B^T = (A - A^T)^T$$

again utilizing the same property,

$$B^T = A^T - A$$

$$\Rightarrow B^T = -(A - A^T)$$

$$\therefore B^T = -B \quad \text{or} \quad B = -B^T$$

Since $B = -B^T$, we can say that B or $(A - A^T)$ is anti-symmetric.

c. $A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$

We will prove that we can write

$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$, i.e A can

be written as a sum of symmetric and anti-symmetric matrices by proving that LHS = RHS.

Let us take transpose on both sides.

i.e $A^T = \left(\frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)\right)^T$

$\therefore A^T = \left(\frac{1}{2}(A+A^T)\right)^T + \left(\frac{1}{2}(A-A^T)\right)^T$

$\therefore A^T = \frac{1}{2}(A+A^T)^T + \frac{1}{2}(A-A^T)^T \rightarrow (1)$

Now, $A+A^T$ is symmetric,

$\therefore (A+A^T)^T = A+A^T \rightarrow (2)$

also $A-A^T$ is anti-symmetric

$\therefore (A-A^T)^T = -(A-A^T) \rightarrow (3)$

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using (2) and (3) in (1),

$$A^T = \frac{1}{2}(A + A^T) - \frac{1}{2}(A - A^T)$$

$$\therefore A^T = \frac{A}{2} + \frac{A^T}{2} - \frac{A}{2} + \frac{A^T}{2}$$

$$\therefore A^T = \frac{2A^T}{2}$$

$$\therefore A^T = A^T$$

LHS = RHS, hence proved.