Assignment 5 - Hypothesis Testing

Math 363 - November, 2009

1. During the 1980s, the general consensus is that about 5% of the nation's children had autism. Some claimed that increases certain chemicals in the environment has led to an increase in autism.
   (a) Write an appropriate hypothesis test for this situation.
   (b) Give an appropriate test for this hypothesis, stating what are the necessary conditions for performing the test.
   (c) A recent study examined 384 children and found that 46 showed signs of autism. Perform a test of the hypothesis and state the p-value.
   (d) What are your conclusions? State how you use the p-value.

2. A company with a fleet of 150 cars found that the emission system of 7 our of the 22 cars tested failed to meet pollution guidelines.
   (a) Write a hypothesis to test if more than 20% of the entire fleet might be out of compliance.
   (b) Test the hypothesis based on the binomial distribution and report a p-value.
   (c) Is the test significant at the 10%, 5%, 1% level?

3. National data in the 1960s showed that about 44% of the adult population had never smoked.
   (a) State a null and alternative hypothesis to test that the fraction of the 1995 population of adults that had never smoked had increased.
   (b) A national random sample of 891 adults were interviewed and 463 stated that they had never smoked. Perform a z-test of the hypothesis and give an approriate p-value.
   (c) Create a 98% confidence interval for the proportion of adults who had never been smokers.
   (d) Give the value of the power function $\pi(p)$ for $p = 0.46, 0.48, 0.50, 0.52$ with the choice of $\alpha = 0.02$ and a "greater than" alternative hypothesis.
   (e) Compute the power function for these values if we increase the sample to 1600. Explain why these values increased.

4. One of the lenses in your supply is suspected to have a focal length f of 9.1cm rather than the 9cm claimed by the manufacturer.
   (a) Write an appropriate hypothesis test for this situation.
   (b) The focal length f is determined by using the thin lens formula,

$$\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}.$$

Here s_1 is the distance from the lens to the object and s_2 is the distance from the lens to the real image of the object. The distances s_1 and s_2 are each independently measured 25 times. The sample mean of the measurements is $\bar{S}_1 = 26.6$ centimeters and $\bar{S}_2 = 13.8$ centimeters, respectively. The standard deviation of the measurement is 0.1cm for s_1 and 0.5cm for s_2.

Give an estimate $\hat{f}$ based on these measurements and the thin lens formula.

(c) Use the delta method to give the standard deviation of $\hat{f}$.

(d) Use this to devise a z-test for the hypothesis and report a p-value for the test.

5. The body temperature in degrees Fahrenheit of 52 randomly chosen healthy adults is measured with the following summary of the data:

$$n = 52, \quad \bar{x} = 98.2846 \quad s = 0.6824.$$

(a) Are the necessary conditions for constructing a valid t-interval satisfied? Explain.

(b) Find a 98% confidence interval for the mean body temperature and explain its meaning.

(c) Give a two-side hypothesis test for a mean body temperature of 98.6^o Fahrenheit and use the information above to evaluate a test with significance level $\alpha = 0.02$.

(d) Find the power of the test at the parameter value $\mu = 98.2$ and indicate this value using the cutoff value for the test and drawing the sample distribution for the null and alternative hypothesis.

6. Drivers of cars calling for regular gas sometimes premium in the hopes that it will improve gas mileage. Here a rental car company takes 10 randomly chosen cars in its fleet and runs a tank of gas according to a coin toss, runs a tank of gas of each type.

Car #	1	2	3	4	5	6	7	8	9	10
Regular	16	20	21	22	23	22	27	25	27	28
Premium	19	22	24	24	25	25	26	26	28	32

(a) Write an appropriate hypothesis test for this situation and state the testing procedure appropriate to this circumstance.

(b) Compute the necessary summary statistics for the test in part (a).

(c) Perform the t-test and report the p-value.

(d) Compare your result to that of a two sample t-test.

7. In this problem, we will examine the sugar content of several national brands of cereals, here measured as a percentage of weight.

children	40.3	55.0	45.7	43.3	50.3	45.9	53.5	43.0	44.2	44.0					
	33.6	55.1	48.8	50.4	37.8	60.3	46.6	47.4	44.0						
adult	20.0	30.2	2.2	7.5	4.4	22.2	16.6	14.5	21.4	3.3	10.0	1.0	4.4	1.3	8.1
	6.6	7.8	10.6	10.6	16.2	14.5	4.1	15.8	4.1	2.4	3.5	8.5	4.7	18.4	

(a) Give a summary of these two data sets.

(b) Create side-by-side boxplots and interpret what you see.

(c) Use R to create a 95% confidence interval for the difference in mean sugar content and explain your result.

Assignment - Hypothesis Testing

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1. During the 1980s, The general consensus is that about 5% of the Nation's children had autism Some claimed that Increases certain chemicals in the environment has led to an Increase in Autism.

   a. write an appropriate hypothesis test for this situation

   b. Give an appropriate test for this hypothesis, stating what are the necessary conditions for performing the test

   c. A recent study examined 384 children and found that 46 showed signs of Autism Perform a test of hypothesis & state P-value.

   d. what are your conclusions ? state how to use P-value

Answer:- Step - 1 Null & Alternative hypothesis

Null Hypothesis : 5% of the Nation's children has Autism

$H_0 : P = 5\% = 0.05$

Alternative hypothesis : more than 5% of Nation's childen has Autism

$H_1 : P > 5\% > 0.05$

we will use one-tail Test because we will check only more than 5%

Step-2:- which Test should we use Z-test

Step-3:- find The value of Alpha

As not given in Problem we will assume 5% as α

2

Step-4 :- If z-critical < z-score

If p-value < significance value } Reject null hypothesis

Step-5 :- Data Gathering

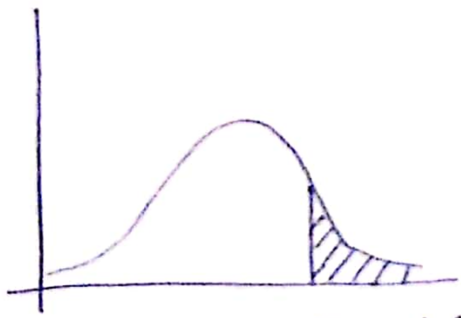
Step-6 :- Analyze Data

$p = 0.05 \quad n = 384 \quad q = 1-p = 0.95$

$$\hat{p} = \frac{46}{384} = 0.12$$

$$z\text{-score} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.12 - 0.05}{\sqrt{\frac{0.05 \times 0.95}{384}}}$$

$$= \frac{0.07}{0.011} = 6.29$$



Step-7 :- Statistical action

z-critical $= 1.65 \quad z$-score $= 6.29$

z-critical < z-score

we will reject the null hypothesis

Conclusion :- More than 5% of the nations children had Autism due to increases in certain chemical in the environment

Solution:- **Step-1:-** Hypothesis

Null hypothesis : 20% of the fleet out of compliance

$H_0 : P = 0.20$

Alternative hypothothesis :- more than 20% fleet out of compliance

$H_1 : P > 0.20$

It is a one tailed Test

Step-2:- we will perform Z-test

Step-3:- Significance level = 10% i.e $\alpha = 0.10$

Step-4:- Z-critical < Z-score → Reject null hypothesis

If P-value < Significance level → Reject null hypothesis

Step-5:- Collecting Data

Step-6:- Analysis of Data

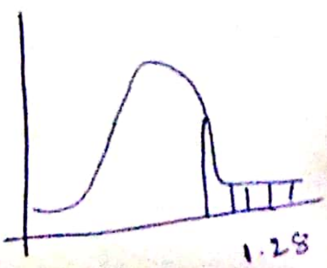
For Z Score $\quad Z\ \text{score} = \dfrac{\hat{P} - P}{\sqrt{\frac{Pq}{n}}} \qquad \hat{P} = 7/22 = 0.31$

$P = 0.20 \quad n = 22 \quad q = 1 - P = 0.80$

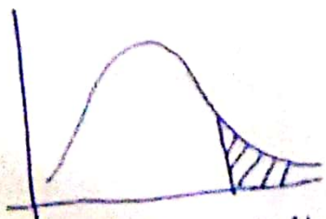
$$Z\text{-Score} = \frac{0.31 - 0.20}{\sqrt{\frac{0.20 \times 0.80}{22}}} = \frac{0.11}{\sqrt{0.0072}} = \frac{0.11}{0.085} = 1.18$$

By Z-table:-

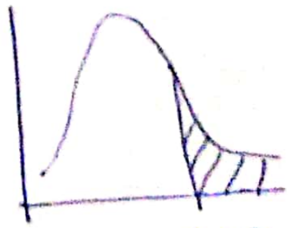
Z_c for 10% = 1.28   for 5% = 1.64   for 1% = 2.29



1.28



1.64



2.29

Z_c

4

At 10% Z - Critical = 1.28   Z score = 1.18

Z - Critical > Z - Score   P - value = 0.1002

1.28 > 1.18

We will accept the null hypothesis

At 5% Z - Critical = 1.64   P - value = 0.0505

1.64 > 1.18

We will accept the null hypothesis

At 1% Z - Critical = 2.29   P - value = ~~[illegible]~~

~~[illegible]~~

= 0.0110

2.29 > 1.18

We will accept the null hypothes

Q3 Solution:-

Null hypothesis : 44% of Adult Population Never Smoked

$H_0 : \mu = 0.44$

Alternative hypothesis:- more than 44% Never Smoked

$H_1 : \mu > 0.44$

* one Tailed Test - Right

Z-test will be used

Confidence level 98% hence $\alpha = 2\% = 0.02$

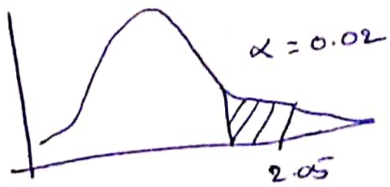
If Z-critical < Z-score we will reject null hypothesis

$p = 0.44 \quad n = 891 \quad \hat{p} = \frac{463}{891} = 0.519$

$q = 0.56$

$$Z\text{-Score} = \frac{0.519 - 0.44}{\sqrt{\frac{0.44 \times 0.56}{891}}} = 4.76$$

$Z_c = 2.05$



Z-critical < Z-Score

2.05 < 4.76

Conclusion:- we will reject null hypothesis

more than 44% of adult population never smoked

Z- critical = 2.05

For P = 0.46 Z - Score = 3.554

Z - critical < Z - Score

Reject Null hypothesis

For P = 0.48 Z. Score = 2.349

Reject Null hypothesis

For P = 0.50 Z. Score = 1.144

Z - Score < Z - critical

we will accept Null hypothesis

For P = 0.52 Z. Score = - 0.060

we will accept Null hypothesis

e. If Sample is 1600

Z - Score = - 12.17

we will accept the Null hypothesis

Q4 - Answer:-

Step - 1:- Hypothesis

Null hypothesis :- focal length of lenses is 9 cm

$H_0 : \mu = 9$ cm

Alternative hypothesis :- focal length of lenses is 9.1 cm

$H_1 = \mu > 9$ cm

Step - 2 :- Determine the test   Z - test

Step - 3:- Significance level

Let's assume significance level as 1 %

$\alpha = 0.01$

It is a one-tailed - Right tailed test

Step - 4:- Decision Rule :-

If z - critical < z - score   Reject Null hypothesis

Step - 5:- Collecting Data

Step - 6:- Analysis of Data

$\bar{S_1} = 26.6$ cm $\bar{S_2} = 13.8$ cm

$\sigma_1 = 0.1$ cm $\sigma_2 = 0.5$ cm

$n_1 = 25$ $n_2 = 25$

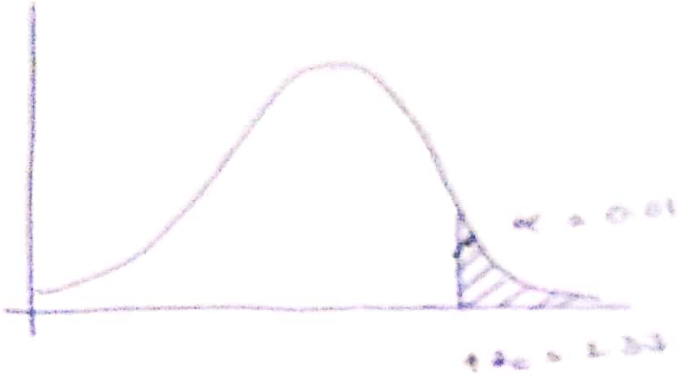
Z - score for Two Independent Samples

$$\text{Z- Score} = \frac{\bar{S_1} - \bar{S_2}}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}} = \frac{26.6 - 13.8}{\sqrt{\frac{(0.1)^2}{25} + \frac{(0.5)^2}{25}}}$$

$$= \frac{12.8}{0.102} = 125.5$$

Using z - table

z - critical = 2.33



z - critical < z - score

2.33 < 12.55

we will reject the null hypothesis

P value = 0.0099

P - value < significant value

0.0099 < 0.01

we will reject null hypothesis

Conclusion :- Focal length of the lenses = 9.1 cm

Scanned with CamScanner

Q 5 Answer:-

Step -1 :- Hypothesis

Null hypothesis:- Mean body temperature is 98.6

$$H_0 : \mu = 98.6$$

Alternative hypothesis:- Mean body temparature is not 98.6

$$H_A : \mu \neq 98.6$$

It is a Two Tail Test we have to check left & Right Tail for the Test

$$\frac{\alpha}{2} = \frac{0.02}{2} = 0.01$$

Step-2:- Determine The test

We will perform t - test

Step-3:- Significance level

as Given in problem $\alpha = 0.02$

Step-4:- Decision Rule

For critical value

t - critical < t - score

we will reject the Null hypothesis

Step-5:- Collecting Data

$n = 52 \quad \bar{x} = 98.2846 \quad S = 0.6824$

$\mu = 98.6 \quad D_f = n-1 = 52-1 = 51$

Step-6:- Analysis of Data

$$t\text{-Score} = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

$$= \frac{98.2846 - 98.6}{\frac{0.6824}{\sqrt{52}}} = \frac{-0.32}{0.0945}$$

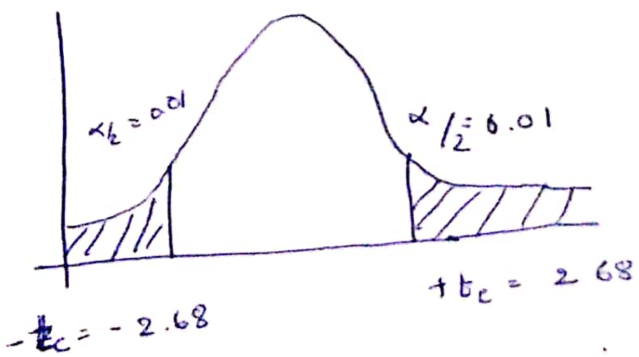
$$= -3.386$$

Using t-table

t-critical $= \pm 2.688$

using t-score

p-value is 0.001372



Step-7:-

Step-1:- Q-6 Answer

Hypothesis:-

Null hypothesis = No Difference between Premium & Regular gas

$H_0 = \mu_A = \mu_B$

Alternative Hypothesis:- mileage is not same with regular gas & premium gas

$H_1 = \mu_A \neq \mu_B$

It is a two tailed test we will check two tails.

Step-2:- Test

we will Perform T-test

Step-3:- Significance level

we will take default value 5%

$\alpha = 0.05$

Since it is a two tail test $\alpha/2 = 0.025$

Step-4:- Decision Rule

If T critical < t-test
If P value < Significance values
} Reject Null hypothesis

Step-5:- Data Collection

Step-6:- Analysis of Data

For Regular

$\bar{x}_1 = 23.1$

$n_1 = 10$

$s_1 = 3.72$

For Premium

$\bar{x}_2 = 25.1$

$n_2 = 10$

$s_2 = 3.44$

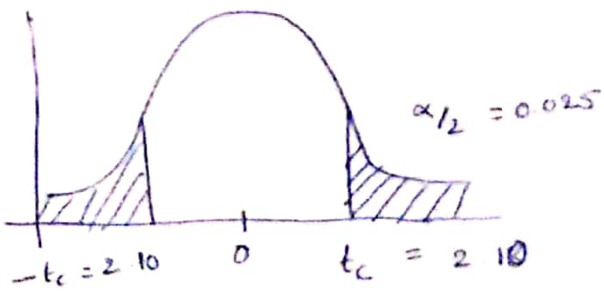
$$D_f = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}} = \frac{6.59}{0.212 + 0.155}$$

$$= \frac{6.59}{0.367} = 18.3$$

$$T\text{-test} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{23.1 - 25.1}{\sqrt{1.38 + 1.183}}$$

$$= \frac{2}{1.6} = 1.25$$

Using t-table t-critical = 2.10



$P = 0.24$

Step-7:- Statistical action t-critical > tscore

Using t-critical 2.10 > 2.24

Using P-value 0.22 > 0.05

we will accept the null hypothesis

There is no difference in milege of regular & Premium gas

Q.-7 Answer:-

Step-1 Hypothesis testing

Null hypothesis:- Sugar Content of brand of cerels for children and adult are same $H_0 = \mu_A = \mu_B$

Alternative hypothesis:- sugar content of brand of cerelas for children & adult are not same

$$H_1 = \mu_A \neq \mu_B$$

Step-2:- Determine the test

we will perfor t-test

step-3:- Significance level:- given as 95% as Confidence level $\alpha = 5\%$

It is a Two tailed test $\frac{\alpha}{2} = 0.025$

Step-4:- Decision Rule

For critical value

t-critical < t-test

For P-values

P value < Significance level

→ Reject Null hypothesis

step-5:- Collect data (Problem)

Step-6:- Data Analysis

It is a T-test for Two Sample random Variable

For sample of children

$\bar{x}_1 = \mu_A = 46.8$

$n_1 = 19$

$\bar{s}_1 = 6.41$

For sample of adult

$\bar{x}_2 = \mu_B = 10.16$

$n_2 = 29$

$\bar{s}_2 = 7.47$

$$d_f = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\left[\left(\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1}\right) + \left(\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}\right)\right]} = \frac{(2.16 + 1.92)^2}{\frac{4.67}{18} + \frac{3.70}{28}}$$

$$= \frac{16.64}{6.259 + 0.132} = 42.54$$

$$T\text{-test} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{46.8 - 10.16}{\sqrt{\frac{(6.41)^2}{19} + \frac{(7.47)^2}{29}}} = \cancel{\frac{\quad}{2.02}} =$$

$$= \frac{36.63}{2.02} = 18.13$$

using t-table $t_c = 2.02$

p value $= 0.0000$

Step-7: Statistical Decision

For critical value t-critical $<$ t-test score

$2.02 < 18.13$

For P-value $p <$ significance value

$0.0000 < 0.05$

Reject Null hypothesis

Conclusion:- The sugar content in different brands of cerelas for children & adult are not same.