MAYANK BANSAL
A20392482

CS-411 - ASSIGNMENT 6

(PART I)

(A) Forward Texture mapping involves mapping the texture coordinates to the image coordinates where the texture is the source and image is the target.

Inverse mapping involves scanning the image and finding out what the texture should give us.

Forward Mapping $\rightarrow$ $P_i' = MP_i$

Inverse Mapping $\rightarrow$ $P_i = M^{-1}P_i'$

Forward mapping has the issue of holes and overlap, therefore inverse mapping is the preferred method of mapping because it avoids these issues.

(B) $v_1 = (1,2)$ $\quad v_2 = (4,2)$ $\quad v_3 = (4,6)$
$v_1' = (1,2)$ $\quad v_2' = (1,4)$ $\quad v_3' = (4,2)$

Forward Map $\rightarrow$ $P_i' = MP_i$

Inverse Map $\rightarrow$ $P_i = M^{-1}P_i'$

where $P_i' = \begin{bmatrix} p_1' & p_2' & p_3' \end{bmatrix}$ $\quad P_i = \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix}$

$$P_i' = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 2 & 6 \\ 1 & 1 & 1 \end{bmatrix} \qquad P_i = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore\ M = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 4 \\ 2 & 2 & 6 \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

$$M^{-1} = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 2 & 6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

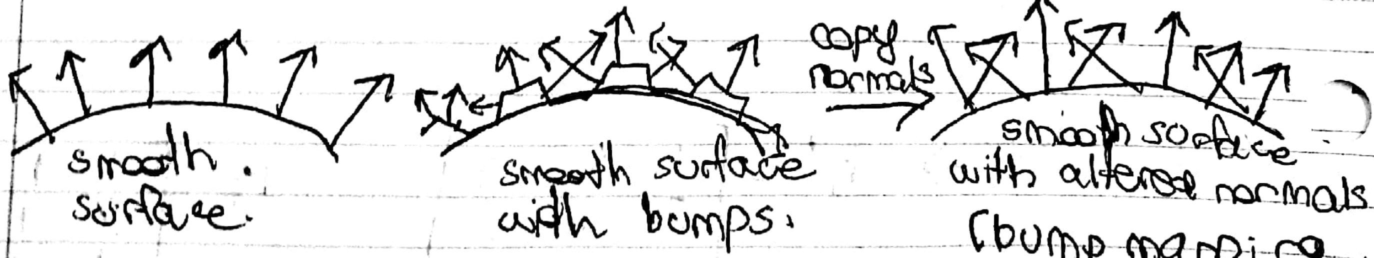
$$\therefore\ M = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1/3 & 0 & 4/3 \\ 1/3 & -1/4 & 1/6 \\ 0 & 1/4 & -1/2 \end{bmatrix} = \begin{bmatrix} 0 & 3/4 & -1/2 \\ 2/3 & -1/2 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} //$$

$$M^{-1} = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 2 & 6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1/3 & -1/2 & 7/3 \\ 0 & 1/2 & -1 \\ 1/3 & 0 & -1/3 \end{bmatrix} = \begin{bmatrix} 1 & 3/2 & -3 \\ 4/3 & 0 & 2/3 \\ 0 & 0 & 1 \end{bmatrix} //$$

(c) Texture mapping creates flat textures on a surface. If we use a texture with an image of bumps the lighting will not change and would look less realistic.

Mapping using bump mapping will change the lighting on a smooth surface when veiwed differently. we add bumps to a smooth surface. to allow for different lighting, therefore increasing realism.

we create the illusion of bumps as follows.



We change the normals of the smooth surface so that it looks like it has bumps but it's profile is smooth.

(D) Given a bump image $b(x,y)$ and $n=(n_x, n_y, n_z)$ we can compute the bump image using the following equations:

$N_x = n_x - b_u$

$N_y = n_y - b_v$

$N_z = n_z$

modified normal

original normal

where b_u, b_v are the u & v derivatives of the bump map.

$b_u(x,y) \simeq b(x+1,y) - b(x,y)$

$b_v(x,y) \simeq b(x,y+1) - b(x,y)$

//

(E) we need to add $-b_u$ to the x-coordinate

So, $b_u \simeq b(x+1, y) - b(x,y)$ where $x=4, y=5$

$\therefore b_u = b(5,5) - b(4,5) = 25 - 20 = 5.$

$\therefore$ the displacement added to x-coord is -5 //.

(F) Given $(R,G,B) = (0.2, 0.3, 0.9)$

$x = 0.2 \times 2 - 1 = -0.6$

$y = 0.3 \times 2 - 1 = -0.4$

$z = 0.9 \times 2 - 1 = 0.8$.

(G) Given $P_0 = [1,2,1]$ $P_1 = [4,2,2]$ $P_2 = [4,6,3]$

$(u_0, v_0) = [1,2]$ $(u_1, v_1) = [1,4]$ $(u_2, v_2) = [4,2]$

Compute N T & B.

$$[T \;\; B] = \frac{1}{\Delta u_1 \Delta v_2 - \Delta u_2 \Delta v_1} [E_1 \;\; E_2] \begin{bmatrix} \Delta v_2 & -\Delta u_2 \\ -\Delta v_1 & \Delta u_1 \end{bmatrix}$$

$\Delta u_1 = u_1 - u_0 = 0$ $\qquad$ $\Delta u_2 = u_2 - u_0 = 3$

$\Delta v_1 = v_1 - v_0 = 2.$ $\qquad$ $\Delta v_2 = v_2 - v_0 = 0$

$E_1 = P_1 - P_0 = (3, 0, 1)$ $\qquad$ $E_2 = P_2 - P_0 = (3, 4, 2)$

$$[T \;\; B] = \frac{1}{0 \times 0 - 3 \times 2} \begin{bmatrix} 3 & 3 \\ 0 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & -3 \\ -2 & 0 \end{bmatrix}$$

$$= \frac{1}{-6} \begin{bmatrix} -6 & -9 \\ -8 & 0 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -3/2 \\ -4/3 & 0 \\ -2/3 & -1/2 \end{bmatrix}$$

$T = (-1, -4/3, -2/3)$ $\qquad$ $B = (-3/2, 0, -1/2)$

$$N = T \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4/3 & -2/3 \\ -3/2 & 0 & -1/2 \end{vmatrix} \Rightarrow \hat{i}(2/3) - \hat{j}\left(\frac{-1}{2}\right) + \hat{k}(2)$$

$$\Rightarrow N = (2/3, 1/2, 2)$$

h) $M = \begin{bmatrix} T \\ B \\ N \end{bmatrix} = \begin{bmatrix} -1 & -4/3 & -2/3 \\ -3/2 & 0 & -1/2 \\ 2/3 & 1/2 & 2 \end{bmatrix}$

i) We use $M = \begin{bmatrix} T \\ B \\ N \end{bmatrix}$ to create a ~~tangent space~~ transformation of light direction vector L to normal map coord system.