

CS411 - Assignment 2

2A) Given 2D point (1,1), translate by (2,3)

Transformation Matrix:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+0+2 \\ 0+1+3 \\ 0+0+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \quad \text{(2DH)} //$$

2B) Given 2D point (1,1), transform by scale (2,2)

Transformation Matrix:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0+0 \\ 0+2+0 \\ 0+0+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \quad \text{(2DH)} //$$

2C) Given 2D point (1,1) → rotate by (2,2)

Transformation Matrix:

$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^\circ + (-\sin 45^\circ) + 0 \\ \sin 45^\circ + \cos 45^\circ + 0 \\ 0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix} \quad \text{(2DH)} //$$

2D) Given 2D point (1,1), find 2DH

$$2D \text{ point} = (1,1)$$

$$2DH \text{ point} = (1,1,1)$$

2E) Given 2DH (1,1,2) find 2D point

2DH point : (1, 1, 2)

2D point : $(\frac{1}{2}, \frac{1}{2})$

2F) Given 2DH (1,2,3) find // other 2DH point with same 2D point:

2DH : (1, 2, 3)

2DH other : $(\frac{1}{3}, \frac{2}{3}, \frac{3}{3}) = (0.33, 0.66, 1) //$

2G) Given 2DH point (1,2,3) find 2D point.

2DH : (1, 2, 3)

2D point : $(\frac{1}{3}, \frac{2}{3}) = (0.33, 0.66) //$

2H) what is 2DH (1,1,0)?

The point (1,1,0) in 2DH doesn't exist because $w=0$ it represents a line having the direction of (1,1).

2I) Given 2D point (2,5), rotate by 30° around origin.

2D point (2,5) \rightarrow 2DH (2,5,1)

Transformation Matrix:

$$\begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(\cos 30 - \sin 30) \\ 5(\sin 30 + \cos 30) \\ 0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} -0.679 \\ 5.3301 \\ 1 \end{bmatrix} //$$

2J) Given 2D point (2,5) rotate by 30° at point (1,2)

$$2D(2,5) \rightarrow 2DH(2,5,1)$$

$$\text{Transformation} = T(1,2) R(30^\circ) T(-1,-2)$$

Transformation Matrix:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0.134 \\ 1/2 & \sqrt{3}/2 & -2.232 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 1.134 \\ 1/2 & \sqrt{3}/2 & -0.232 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_M \times 2DH = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 1.134 \\ 1/2 & \sqrt{3}/2 & -0.232 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.366 \\ 5.098 \\ 1 \end{bmatrix}$$

$$2DH = (0.366, 5.098, 1)$$

$$2D \rightarrow (0.366, 5.098)$$

2K) Given 2D point (2,5) translate (3,4) rotate (45).

$$2DH(2,5,1) \\ \text{Transformation } T_M = R(45) T(3,4)$$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 4.949 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_M \times 2DH = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 4.949 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.8284 \\ 9.899 \\ 1 \end{bmatrix} \quad (3) //$$

$$2DH \rightarrow 2D = (-2.828, 9.899) //$$

2L) Given 2D (2, 5) rotate by 45° , then translate (3, 4)

$$2D \rightarrow 2DH(2, 5, 1)$$

Transformation $T_M = T(3, 4)R(45^\circ)$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 3 \\ 1/\sqrt{2} & 1/\sqrt{2} & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_M \times 2DH = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 3 \\ 1/\sqrt{2} & 1/\sqrt{2} & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.8786 \\ 8.9497 \\ 1 \end{bmatrix}$$

$$2DH \rightarrow 2D = (0.8786, 8.9497) //$$

2M) Convert (5, 6) world to camera. camera translated by (1, 2) rotate (45°)

$$2D \rightarrow 2DH(5, 6, 1)$$

Transform $T_M = R(45^\circ)T(1, 2)$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 2.121 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_M \times 2DH = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 2.121 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} \\ 9.898 \\ 1 \end{bmatrix} \rightarrow 2D(\sqrt{2}, 9.898) //$$

(4)

(5)

2N) Transformation Matrix = $T(T_x, T_y) R(\theta)$ //

2O) $W \rightarrow (1,1)(2,2)$ to $V_P(3,3)(4,5)$ //

$$W_2 V_P = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= T(V_{min}, V_{max}) S \left(\frac{U_{max} - U_{min}}{X_{max} - X_{min}}, \frac{V_{max} - V_{min}}{Y_{max} - Y_{min}} \right) T(-x_{min}, -y_{min})$$

$$W_2 V_P = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} //$$

2P) $LS \rightarrow (1,1)(2,2)$
 $P \rightarrow (1,1)$
 $W \rightarrow (0,0)(1,1)$

$$x_{min} = 0, \quad y_{min} = 0$$

$$x_{max} = 1, \quad y_{max} = 1$$

$$L = 0 \quad (x \geq x_{min})$$

$$R = 0 \quad (x \leq x_{max})$$

$$B = 0 \quad (y \geq y_{min})$$

$$T = 0 \quad (y \leq y_{max})$$

$$\left. \begin{array}{l} L=0 \\ R=0 \\ B=0 \\ T=0 \end{array} \right\} \text{Binary code}(1,1) = 0000 //$$

(5)

2Q)

Let Line segment start point = L_s
end point = L_E

find binary code for L_s & L_E

$(x < x_{min})$	$L = 1$	$L = 0$
$(x > x_{max})$	$R = 1$	$R = 0$
$(y < y_{min})$	$B = 1$	$B = 0$
$(y > y_{max})$	$T = 1$	$T = 0$

If $(binary(L_s) \neq 0)$ // inside
if $(binary(L_s) \neq 0 \text{ and } binary(L_E) \neq 0)$ // outside //

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