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CS411 - AS3 PROBLEMS

A) (1,1) (4,7) Given Points
$u=0$ $u=1$

$u = 0.3$?

$x = x_0 + (x_1 - x_0) * u \Rightarrow 1 + (4-1) * 0.3 = 1.9$
$y = y_0 + (y_1 - y_0) * u \Rightarrow 1 + (7-1) * 0.3 = 2.8$

$\therefore$ Point $\rightarrow$ $(\underset{x}{1.9}, \underset{y}{2.8})$ //

B) Parametric continuity defines the curve as a function whereas geometric continuity defines the curve as a shape.

c) Given (0,0) (2,2) and tangents (1,1) (1,-1)

$P(0) = P_k = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$P(1) = P_{k+1} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$P'(0) = dP_k = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$P'(1) = dP_{k+1} = \begin{bmatrix} 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$$\therefore \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{M_H //} \underbrace{\begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}}_{P} //$$

D) (2, 2) (4, 2) tangent (1, 1) (1, −1)
$u=0$ $u=1$

$u = 0.5$? $\quad P(u) = [0.5^3 \;\; 0.5^2 \;\; 0.5 \;\; 1]\, M_H\, P$

$$X(0.5) = [0.5^3 \;\; 0.5^2 \;\; 0.5 \;\; 1]\, M_H \begin{bmatrix} 2 \\ 4 \\ 1 \\ 1 \end{bmatrix}$$

$$Y(0.5) = [0.5^3 \;\; 0.5^2 \;\; 0.5 \;\; 1]\, M_H \begin{bmatrix} 2 \\ 2 \\ 1 \\ -1 \end{bmatrix}$$

$$X(0.5) = [0.125 \;\; 0.25 \;\; 0.5 \;\; 1] \begin{bmatrix} -2 \\ 3 \\ 1 \\ 2 \end{bmatrix} \Rightarrow 3$$

$$Y(0.5) = [0.125 \;\; 0.25 \;\; 0.5 \;\; 1] \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow 2.25$$

$\therefore$ $X, Y(0.5) = (3, 2.25)$

E) $$b(u) = M_H^T u = \begin{bmatrix} 2u^3 - 3u^2 + 1 \\ -2u^3 + 3u^2 \\ u^3 - 2u^2 + u \\ u^3 - u^2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.125 \\ -0.125 \end{bmatrix}$$

$$X(0.5) = 0.5(2) + 0.5(4) + 1(0.125) - 1(0.125)$$
$$= 3 \,//$$

$$Y(0.5) = 0.5(2) + 0.5(2) + 0.125(1) - 0.125(-1)$$
$$= 2.25 \,//$$

$\therefore$ $(X, Y)(0.5) = (3, 2.25) \,//$

F) (1,1) (2,2) (4,2) (5,1) $u = 0.5$

$u=0$ $u=1$ tension $= 0.5$.

cardinal spline,

$$M_c = \begin{bmatrix} -0.5 & 1.5 & -1.5 & 0.5 \\ 1 & -2.5 & 2 & -0.5 \\ -0.5 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

xcoords $= [1, 2, 4, 5]$ $McX = M_c \times X$

ycoords $= [1, 2, 2, 1]$ $McY = M_c \times Y$

$McX = [-1, 1.5, 1.5, 2]$

$McY = [0, -0.5, 0.5, 2]$

$u_x = u_y = [0.125 \quad 0.25 \quad 0.5 \quad 1]$

$u_x \times McX = 3$

$u_y \times McY = 2.125$

$\therefore$ $(u=0.5)$ $(x,y) = (3, 2.125)$ //

G) $c_0(u) = -su^3 + 2su^2 - su$ $= -0.0625$

$c_1(u) = (2-s)u^3 + (s-3)u^2 + 1$ $= 0.5625$

$c_2(u) = (s-2)u^3 + (3-2s)u^2 + su$ $= 0.5625$

$c_3(u) = su^3 - su^2$ $= -0.0625$

$X(u) = c_0(u) P_{k-1} + c_1(u) P_k + c_2(u) P_{k+1} + c_3(u) P_{k+2}$

$x(u) = -0.0625\times1 + 0.5625\times2 + 0.5625\times4 - 0.0625\times5 = 3$ //
$y(u) = -0.0625\times1 + 0.5625\times2 + 0.5625\times2 - 0.0625\times1 = 2.125$ //

$\therefore\ (x, y)\ (0.5) = (3, 2.125)$ //

h) $(1,1)\ (2,2)\ (4,2)\ (5,1)$ $\quad u = 0.5.$
$u=1$ $\qquad\qquad\qquad\qquad$ $u=1$

$P(u) = u^T M_B P$

$$M_B = \cancel{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$u^T = \begin{bmatrix} 0.125 & 0.25 & 0.5 & 1 \end{bmatrix}$ $u = (0.5)$

$$P_x = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix} \qquad P_y = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

$M_B P_x = \cancel{[1, +4, 3, +12]}\ \ [2, +3, 3, 1]$
$M_B P_y = \cancel{[1, 1, +4, 1, 3, 0]}\ \ [0, -3, 3, 1]$

$u_T M_B P_x = \cancel{+12.625}\ 3$

$u_T M_B P_y = \cancel{\quad}\ 1.75$

$\therefore\ (x, y)_{(u=0.5)} = \cancel{(+12.625, -3.625)}$
$= (3, 1.75)$ //

i) $b_u = M_B^T u = \begin{bmatrix} (1-u)^3 \\ 3u(1-u)^2 \\ 3u^2(1-u) \\ u^3 \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.375 \\ 0.375 \\ 0.125 \end{bmatrix}$

$P_x = 0.125 \times \underline{1} + 0.375 \times \underline{2} + 0.375 \times \underline{4} + 0.125 \times \underline{5}$

$= 3$ //

$P_y = 0.125 \times \underline{1} + 0.375 \times \underline{2} + 0.375 \times \underline{2} + 0.125 \times \underline{1}$

$= 1.75$ //

$P(x, y)_{u=0.5} = (3, 1.75)$ //

j)

k) Bezier polynomials are ~~[illegible]~~ easier to work with because

1) The zeros of the polynomials are ~~[illegible]~~ at $u=0$ or $u=1$. Therefore, for each blending polynomial, without zeros in the interval, each polynomial must be smooth.

Representing Bezier curves in terms of its blending polynomials is a convex sum

(5)

So, the polynomials must lie in the convex hull of the 4 control points. ~~[illegible]~~ Though the polynomial doesn't interpolate all the control points, it cannot be far from them.

l) (1, 1) (2, 2) (4, 2) (5, 1)

knot vector [0, 1, 2, 3, 4, 5, 6]

$u = 2$.

m) ~~[illegible]~~ [illegible] + ~~[illegible]~~ [illegible]