CS-411 – Assignment 3 (5%)

Curve and Surface Interpolation

Due by: October 19, 2017

The purpose of this assignment is to practice working with spline curves. Answer the following questions and implement the necessary code. Submit the assignment using the submission instructions of the first assignment. A skeleton program will be provided. You are not required to use the skeleton program and may modify it as needed.

1. Answer the following questions by writing the relevant computations and using a small JavaScript program to perform the computations:

   (a) Let $(1,1)(4,7)$ be two points. Assume an interpolation parameter $u = 0$ at $(1,1)$ and $u = 1$ at $(4,7)$. Find the coordinates of the 2D point between them using linear interpolation with an interpolation parameter of $u = 0.3$.

   (b) Explain the difference between parametric and geometric continuity. Explain the advantages and challenges that are associated with piecewise interpolation.

   (c) Given the points (0,0) (2,2) and corresponding tangents (1,1) (1,-1) respectively, write the 4 constraint equations that are used to compute the Hermite interpolation coefficients for the x coordinate of the interpolation curve between the points.

   (d) Given the 2D control points $(2,2)(4,2)$ with a tangent of $(1,1)$ at the point $(2,2)$ and a tangent of $(1,-1)$ at the point $(4,2)$, compute the coordinate of the point at parameter $u = 0.5$ using Hermite splines. Assume a parameter of $u = 0$ at $(2,2)$ and of $u = 1$ at $(4,2)$. Use the matrix form for the computations.

   (e) Repeat the previous question but this time using the blending function form of the computations.

   (f) Given a set 2D control points $(1,1)(2,2)(4,2)(5,1)$ with parameter $u = 0$ at $(2,2)$ and parameter $u = 1$ at $(4,2)$ find the coordinate of the point at $u = 0.5$ when using Cardinal splines. Use the matrix form for the computations. Assume a tension parameter of 0.5.

   (g) Repeat the previous question but this time using the blending function form.

   (h) Given a set 2D control points $(1,1)(2,2)(4,2)(5,1)$ with parameter $u = 0$ at $(1,1)$ and parameter $u = 1$ at $(5,1)$ find the coordinate of the point at $u = 0.5$ when using Cubic Bezier curves. Use the matrix form for the computations.

   (i) Repeat the previous question but this time using the blending function form.

   (j) Assuming that we want to add another cubic Bezier curve segment that will connect to the cubic Bezier curve segment in the previous question smoothly (with C1 continuity), compute the coordinates of the first control point in the second curve segment.

   (k) Explain the advantage of the Bezier curve blending functions (Bernstein polynomials).

   (l) Given a set 2D control points $(1,1)(2,2)(4,2)(5,1)$ and a knot vector $[0,1,2,3,4,5,6]$ find the coordinate of the point at $u = 2$ when using uniform quadratic B-splines.

   (m) Given the uniform knot vector [012345] for interpolating between 4 points using B-splines of degree $d = 2$, write the 4 B-spline blending functions $B_0^2(u), B_1^2(u), B_2^2(u), B_3^2(u)$.

2. Write a WebGL program for drawing a spline curve based on user input:

(a) Let the user click the canvas at arbitrary locations. Store and display the clicked locations as points.

(b) Upon receiving 4 points generate a Cardinal spline between the middle 2.

(c) Thereafter, upon receiving any additional points continue to add additional curve segments while guaranteeing C1 continuity between the added curve segments. Each added point should result in an added curve segment.

(d) Add two buttons for increasing/decreasing the tension parameter, and update the curve upon changing this parameter.

(e) Add the ability to move an object along the spline path you generated:

   i. Add a button to toggle animating an object moving along the spline curve path.

   ii. Add two buttons for increasing/decreasing the speed of the object.