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CS411 - AS4 Problems

1. Rotate by 45° about axis given by $(0,1,0)$

$(0,1,0) \rightarrow$ y-axis.

$$R_y(\theta) = \cancel{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} //$$

$\theta = 45^\circ$ //

$$\begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} //$$

2. Rotate vector $(1,1,0)$ to be aligned with $(0,1,0)$ (y-axis) in homogeneous co-ordinates.

$(1,1,0) \rightarrow (0,1,0)$

we can rotate about z by 45°

$$R_z(\theta) = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. $R_{\hat{u}}(\theta) = I + \sin\theta \, U + (1 - \cos\theta) U^2$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} + \begin{bmatrix} -(\sqrt{2}-1)/\sqrt{2} & (\sqrt{2}-1)/\sqrt{2} & 0 \\ (\sqrt{2}-1)/\sqrt{2} & -(\sqrt{2}-1)/\sqrt{2} & 0 \\ 0 & 0 & -2+\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & (\sqrt{2}-1)/\sqrt{2} & 0 \\ (\sqrt{2}-1)/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & -2+\sqrt{2} \end{bmatrix} = R_y^T(\theta) R_z(\theta) R_y(\theta) //$$

1.4. $p = (1, 2, 3)$ $\quad u = (1, 0, 1)$
$v = (-1, -2, -3)$

Compute transformation matrix:

For direction of flight,

Rotation about $\rightarrow \left[\frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}} \right]$

$$R_1 = \cancel{\begin{bmatrix} -1/\sqrt{14} & -2/\sqrt{14} & -3/\sqrt{14} & 0 \\ 2/\sqrt{14} & -1/\sqrt{14} & -3/\sqrt{14} & 0 \\ 0 & 0 & -3/\sqrt{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \rightarrow \begin{bmatrix} -1/\sqrt{14} & -2/\sqrt{14} & -3/\sqrt{14} & 0 \\ 2/\sqrt{14} & -1/(\sqrt{14}) & -3/\sqrt{14} & 0 \\ -3/\sqrt{14} & -2/\sqrt{14} & 1/\sqrt{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation for the up vector will be a rotation about y-axis by 45°.

$$R_y = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translate by $(1, 2, 3)$

$$T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = T R_y R_1 = \cancel{\begin{bmatrix} \frac{-1}{2\sqrt{7}} & 0 & \frac{-1}{2\sqrt{7}} & 1 \\ 0 & -\sqrt{2}/\sqrt{7} & 0 & 2 \\ -3/2\sqrt{7} & 0 & -3/2\sqrt{7} & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} -0.75 & -0.8 & -0.37 & 1 \\ -0.53 & -0.26 & -0.53 & 2 \\ -0.37 & -0.94 & 0.75 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.5 $r_1 = [1,2,3,3]$ $r_2 = [2,3,4,4]$ $= \begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 3 & 4 & 4 \\ 3 & 4 & 4 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$r_3 = [3,4,4,5]$ $r_4 = [0,0,0,1]$

$v = (0,0,1)$ compute transform on v by M.

$$(M^{-1})^T = \begin{bmatrix} -4 & 4 & -1 & 0 \\ 4 & -5 & 2 & 0 \\ -1 & 2 & -1 & 0 \\ 1 & -2 & 0 & 1 \end{bmatrix} //.$$

Transformed v in 3DH $= (M^{-1})^T v =$

$$\begin{bmatrix} -4 & 4 & -1 & 0 \\ 4 & -5 & 2 & 0 \\ -1 & 2 & -1 & 0 \\ 1 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

$[-1, 2, -1, 1] \rightarrow$ 3D $(-1, 2, -1)$ //.

1.6 If we want to preserve the z-coordinate. we can use the orthographic projection matrix I which leaves the point at $(1,2,3)$//.

1.7. $p = (1,2,3)$ $v = (0,-1,1)$

Align v with z axis...

$$\begin{bmatrix} 1 & 0 & \cot\alpha\cos\phi & 0 \\ 0 & 1 & \cot\alpha\sin\phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

solving,

$\cot\alpha\cos\phi = 0$ — (1)

$-1 + \cot\alpha\sin\phi = 0$ — (2)

$\therefore \phi = \cos^{-1} 0 \Rightarrow \phi = 90$ //

substituting,

$-1 + \cot\alpha \sin 90 = 0$

$\Rightarrow \cot\alpha = 1 \Rightarrow \alpha = \cot^{-1}(1)$

$\alpha = 45^\circ$ //

$\therefore$ The point (3DH) $= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \\ 1 \end{bmatrix}$

In 3D $\Rightarrow (1, 5, 3)$ //

1.8 $p = (1, 2, 3)$ be 3D point. $f = 10$ //.

$$p' = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 3 \end{bmatrix}$$

3DH $\rightarrow$ 3D $\rightarrow (3.33, 6.66, 10)$ //