1. **Counting Solutions**

a)

1) $2x + 3y = 5$
$x + y = 2$

(multiply row 2 by -2 and add to row 1)

2) $2x - 2x + 3y - 2y = 5 - 4$
$x + y = 2$

(simplify)

3) $y = 1$
$x + y = 2$

(substitute y = 1 to row 2)

4) $y = 1$
$x = 2 - 1$

(find x)

5) $\mathbf{y = 1}$
$\mathbf{x = 1}$ **Unique solution**

b)

1) $x + y + z = 3$

$2x + 2y + 2z = 5$

(multiply row 1 by -2 and add to row 2)

2) $x + y + z = 3$

$2x - 2x + 2y - 2y + 2z - 2z = 5 - 6$

(simplify)

3) $x + y + z = 3$

$0 = -1$

There's **no solution** to this problem because no choice of x,y,z will make $0 = -1$ true.

In other words, $0 = -1$

$0x + 0y + 0z = -1$

No matter how we choose x,y,z, this equation is never true.

Thus, **no solution**.

c)

1) $\quad -y + 2z = 1$
$\quad 2x \quad + z = 2$

(find x and y)

2) $\quad y = 2z - 1$
$\quad 2x = 2 - z$

(divide row 2 by 2)

3) $\quad y = 2z - 1$
$\quad x = 1 - z/2$

There's an **infinite number of solutions** to this problem**.** This is because there's an infinite number of ways to choose z.
In other words, we can let z be any real number and we can find a corresponding x and y using the equations above, which will also be a solution. Thus, there's an infinite number of solutions to this problem.

Set of solutions: $(1 - t/2\ ,\ 2z - 1,\ t)$

d)

1) $x + 2y = 3$
$2x - y = 1$
$3x + y = 4$

(add row 2 to row 3)

2) $x + 2y = 3$
$2x - y = 1$
$5x = 5$

(multiply row 1 by -2 and add to row 2)

3) $x + 2y = 3$
$- y - 4y = 1 - 6$
$x = 1$

(simplify)

4) $x + 2y = 3$
$-5y = -5$
$x = 1$

5) $x + 2y = 3$

$y = 1$

$x = 1$

(plug in x and y to row 1)

6) $3 = 3$

$y = 1$

$x = 1$

7) **$x = 1$**

$y = 1$ **Unique solution**

e)

1) $x + 2y = 3$
$2x - y = 1$
$x - 3y = -5$

(row 3 minus row 1)

2) $x + 2y = 3$
$2x - y = 1$
$x - x - 3y - 2y = -5 - 3$

(multiply row 1 by -2 and add to row 2)

3) $x + 2y = 3$
$2x - 2x - y - 4y = 1 - 6$
$-5y = -8$

(simplify)

4) $x + 2y = 3$
$-5y = -5$
$-5y = -8$

(subtract row 2 from row 3)

5) $x + 2y = 3$

$y = 1$

$0 = -3$

This is the same situation as (b). There’s no values we can choose for x and y to make $0 = -3$ true. Thus, **no solution.**

2. Filtering out the Troll

a) $\begin{cases} m_1 = \cos(45^\circ)\vec{a} + \cos(-30^\circ)\vec{b} \\ m_2 = \sin(45^\circ)\vec{a} + \sin(-30^\circ)\vec{b} \end{cases}$

b) $\begin{cases} \frac{\sqrt{2}}{2}\vec{a} + \frac{\sqrt{3}}{2}\vec{b} = m_1 \\ \frac{\sqrt{2}}{2}\vec{a} - \frac{1}{2}\vec{b} = m_2 \end{cases}$

$$\begin{cases} \frac{\sqrt{3}}{2}\vec{b} + \frac{1}{2}\vec{b} = m_1 - m_2 \\ \frac{\sqrt{2}}{2}\vec{a} - \frac{1}{2}\vec{b} = m_2 \end{cases}$$

$$\begin{cases} \frac{\sqrt{3}+1}{2}\vec{b} = m_1 - m_2 \\ \frac{\sqrt{2}}{2}\vec{a} = m_2 + \frac{1}{2}\vec{b} \end{cases}$$

$$\begin{cases} \vec{b} = \frac{2}{\sqrt{3}+1}(m_1 - m_2) \\ \frac{\sqrt{2}}{2}\vec{a} = m_2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{\sqrt{3}+1}\right)(m_1 - m_2) \end{cases}$$

$$\vec{a} = \sqrt{2}\left[m_2 + \frac{1}{\sqrt{3}+1}(m_1 - m_2)\right]$$

$$\vec{a} = \sqrt{2}\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}m_2 - \frac{1}{\sqrt{3}+1}m_2 + \frac{1}{\sqrt{3}+1}m_1\right)$$

$$\vec{a} = \sqrt{2}\left(\frac{\sqrt{3}}{\sqrt{3}+1}m_2 + \frac{1}{\sqrt{3}+1}m_1\right)$$

$$\vec{a} = \boxed{\underbrace{\frac{\sqrt{2}}{\sqrt{3}+1}}_{u} m_1 + \underbrace{\frac{\sqrt{6}}{\sqrt{3}+1}}_{v} m_2}$$

c) All human beings are born free and equal in dignity and rights.

Optional: Taken from: Universal Declaration of Human Rights.

3. Homework Process and Study Group

Who else did you work with on this assignment?

I worked on this homework assignment alone.

How did you work on this assignment?

I first read Note 0 and Note 1, then I went through the lab presentation to learn about jupyter notebook. Then, I was able to understand the problems and solve them.