1. Mechanical Eigenvalues and Eigenvectors

a) $A\vec{x} = \lambda\vec{x}$

$(A - \lambda I)\vec{x} = \vec{0}$

$\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (5-\lambda)(2-\lambda) = 0$ $\quad \boxed{\lambda = 5, 2}$

for $\underline{\underline{\lambda = 5}}$: $A = \begin{bmatrix} 0 & 0 \\ 0 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

eigenspace $= \underline{\text{span}\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}}$

for $\underline{\underline{\lambda = 2}}$: $\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

eigenspace $= \underline{\text{span}\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}}$

b) $A\vec{x} = \lambda\vec{x} \qquad A\vec{x} - \lambda\vec{x} = \vec{0}$

$\det(A - \lambda I) = \begin{vmatrix} 22-\lambda & 6 \\ 6 & 13-\lambda \end{vmatrix} = (22-\lambda)(13-\lambda) - 36 = 286 - 35\lambda + \lambda^2 - 36$

$= 250 - 35\lambda + \lambda^2$

$= (\lambda - 10)(\lambda - 25)$

$\boxed{\lambda = 10, 25}$

for $\underline{\underline{\lambda = 10}}$: $\begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix} \sim \begin{bmatrix} 12 & 6 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix}$

$x_1 + \frac{1}{2}x_2 = 0$

$x_1 = -\frac{1}{2}x_2$

eigenspace $= \underline{\text{span}\left\{\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}\right\}}$

for $\underline{\underline{\lambda = 25}}$: $\begin{bmatrix} -3 & 6 \\ 6 & -12 \end{bmatrix} \sim \begin{bmatrix} -3 & 6 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

$x_1 - 2x_2 = 0$

$x_1 = 2x_2$

eigenspace $= \underline{\text{span}\left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right\}}$

c) $A\vec{x} = \lambda\vec{x}$

$A\vec{x} - \lambda\vec{x} = \vec{0}$

$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda) - 4 = \cancel{4} - 5\lambda + \lambda^2 - \cancel{4}$

$= \lambda^2 - 5\lambda$

$\boxed{\lambda = 0, 5}$

for $\underline{\lambda = 0}$: $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

eigenspace = $\underline{\text{span}\left\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}}$

for $\lambda = 5$: $\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \sim \begin{bmatrix} -4 & 2 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix}$

$x_1 = \frac{1}{2}x_2$

$2x_1 = x_2$

eigenspace = $\underline{\text{span}\left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\}}$

7. Homework Process

I worked on this homework alone. I did all practice problems on the website, read until Note 9 and was able to do the homework after that.

2. The Dynamics of Romeo and Juliet's Love Affair

a) $A\vec{v}_1 = \lambda\vec{v}_1$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a+b \\ c+d \end{bmatrix} \qquad \text{(Given that } a+b=c+d\text{)}$$

Since $a+b=c+d$, $\vec{v}_1$ is an eigenvector $\left((a+b)\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$

Corresponding eigenvalue $\underline{\lambda_1 = a+b}$ or can be $\lambda_1 = c+d$

$A\vec{v}_2 = \lambda\vec{v}_2$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} b \\ -c \end{bmatrix} = \begin{bmatrix} ab-bc \\ cb-dc \end{bmatrix} = \begin{bmatrix} b(a-c) \\ c(b-d) \end{bmatrix}$$

Since $a+b=c+d \Rightarrow a-c=d-b$

let $u=a-c=d-b$, then

$$\begin{bmatrix} b(a-c) \\ c(b-d) \end{bmatrix} = \begin{bmatrix} bu \\ -cu \end{bmatrix} = u\begin{bmatrix} b \\ -c \end{bmatrix}$$

So, $\vec{v}_2$ is an eigenvector of A

Corresponding eigenvalue $\underline{\lambda_2 = a-c}$ or can be $\lambda_2 = d-b$

for $\lambda_1 = a+b = c+d$:

$A\vec{v}_1 = \lambda_1\vec{v}_1$

$(A-\lambda I)\vec{v}_1 = \vec{0}$

$$\begin{bmatrix} a-a-b & b \\ c & d-c-d \end{bmatrix} \sim \begin{bmatrix} -b & b \\ c & -c \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

eigenspace $=$ $\underline{\text{span}\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}}$

for $\lambda_2 = a-c = d-b$:

$A\vec{v}_2 = \lambda_2\vec{v}_2$

$(A-\lambda_2 I)\vec{v}_2 = \vec{0}$

$$\begin{bmatrix} a-a+c & b \\ c & d-d+b \end{bmatrix} \sim \begin{bmatrix} c & b \\ c & b \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{b}{c} \\ 0 & 0 \end{bmatrix}$$

eigenspace $=$ $\underline{\text{span}\left\{\begin{bmatrix} -\frac{b}{c} \\ 1 \end{bmatrix}\right\}}$

b) Using answers from part (A):

$$\lambda_1 = a + b = 0.75 + 0.25 = 1$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = a - c = 0.75 - 0.25 = 0.5$$

$$\vec{v}_2 = \begin{bmatrix} b \\ -c \end{bmatrix} = \begin{bmatrix} 0.25 \\ -0.25 \end{bmatrix}$$

c) $A\vec{z} = \vec{z}$

That is same as $A\vec{z} = 1 \cdot \vec{z}$

So, the set of all steady states is the eigenspace of $\lambda = 1$

Using answer from part (B): $\boxed{\left\{ \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \alpha \in \mathbb{R} \right\}}$

d) $A = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$ $\quad \vec{z}[0] \in \text{span}\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

Let $\vec{z}[0] = \alpha \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \alpha \in \mathbb{R}$

$$\vec{z}[1] = A\vec{z}[0] = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} \alpha \\ -\alpha \end{bmatrix} = \alpha \begin{bmatrix} 0.75 - 0.25 \\ 0.25 - 0.75 \end{bmatrix} = \alpha \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$$

$$\vec{z}[2] = A\vec{z}[1] = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 0.5\alpha \\ -0.5\alpha \end{bmatrix} = \alpha \begin{bmatrix} 0.5(0.75 - 0.25) \\ -0.5(0.25 - 0.75) \end{bmatrix}$$

$$\vec{z}[2] = \alpha \begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix}$$

This pattern continues and as $n \to \infty$, $\vec{z}[n]$ will approach $\vec{0}$

So, $\boxed{\text{as } n \to \infty \quad \vec{z}[n] \to \vec{0}}$

Means their relationship will come to a neutral stance over time.

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

e) Using answers from part (A):

$$\lambda_1 = a + b = 2$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = a - c = 0$$

$$\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

f) If $\vec{3}[0] \in \text{span}\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$,

let $\vec{3}[0] = \alpha \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\alpha \in R$

$$A\vec{3}[0] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha \\ -\alpha \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \vec{0} = \vec{3}[1]$$

Since $\vec{3}[1] = \vec{0}$, $A\vec{3}[1] = \vec{3}[2] = \vec{0}$

So, as $n \to \infty$, $\boxed{\vec{3}[n] = \vec{0}}$

Over time, their relationship will come to neutral stance

g) Given $\vec{3}[0] \in \text{span}\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$, let $\vec{3}[0] = \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\alpha \in R$

$$\vec{3}[1] = A\vec{3}[0] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha \\ \alpha \end{bmatrix} = \begin{bmatrix} 2\alpha \\ 2\alpha \end{bmatrix} = 2\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\vec{3}[2] = A\vec{3}[1] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2\alpha \\ 2\alpha \end{bmatrix} = \begin{bmatrix} 4\alpha \\ 4\alpha \end{bmatrix} = 4\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Since multiplying A by $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ just adds, 1 of 2 can happen

if $\alpha > 0$, then as $n \to \infty$, $\vec{3}[n] \to +\infty$

Over time, they will like each other more

if $\alpha < 0$, then as $n \to \infty$, $\vec{3}[n] \to -\infty$

Over time, they will hate each other more

h) $A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

Using part (A):

$\lambda_1 = a+b = -1$

$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = a-c = 3$

$\vec{v}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

i) Given $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$

let $\vec{s}[0] = \alpha \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\alpha \in R$

$\vec{s}[1] = A\vec{s}[0] = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} \alpha \\ -\alpha \end{bmatrix} = \alpha \begin{bmatrix} 3 \\ -3 \end{bmatrix}$

$\vec{s}[2] = A\vec{s}[1] = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3\alpha \\ -3\alpha \end{bmatrix} = \alpha \begin{bmatrix} 9 \\ -9 \end{bmatrix}$

if $\alpha > 0$, then $R[0] > 0$ and $J[0] < 0$, then as $n \to \infty$, Romeo will like Juliet more and Juliet will have hate Romeo over time.

if $\alpha < 0$, then $R[0] < 0$ and $J[0] > 0$, then as $n \to \infty$, Romeo will hate Juliet more over time ($R[n] \to -\infty$) and Juliet will love Romeo more over time ($J[n] \to +\infty$)

j) let $\vec{s}[0] = \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\alpha \in R$

$\vec{s}[1] = A\vec{s}[0] = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$\vec{s}[2] = A\vec{s}[1] = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \implies \vec{s}[2] = \vec{s}[0]$

It's an oscillation! Over time, they will keep liking and hating each other. So, $s[n]$ is undefined as $n \to \infty$

3. Noisy Images

a) $\vec{z} = H\vec{c} + \vec{w}$

$H\vec{c} = \vec{z} - \vec{w}$

$\vec{c} = \boxed{H^{-1}(\vec{z} - \vec{w})}$

b) Given $\vec{w} = \alpha_1 \vec{b}_1 + \ldots + \alpha_N \vec{b}_N$

$\hat{\vec{w}} = H^{-1}\vec{w} = \lambda \vec{w}$

λ eigenvalue of H^{-1}

Since $\vec{b}_1$ is eigenvector of H^{-1}, it will be scaled by λ_1, and similar for all eigenvectors of H^{-1}.

$\hat{\vec{w}} = H^{-1}\vec{w} = \lambda(\alpha_1 \vec{b}_1 + \ldots + \alpha_N \vec{b}_N)$

$\hat{\vec{w}} = \lambda_1 \alpha_1 \vec{b}_1 + \ldots + \lambda_N \alpha_N \vec{b}_N$

For large eigenvalues, noise will be amplified and it'll be attenuated for small eigenvalues.

c) H_1 performs the best because it has the largest eigenvalue. H_1 is an identity matrix.
Regarding the eigenvalues, $\lambda_{H_1} > \lambda_{H_2} > \lambda_{H_3}$.
Small eigenvalues on the noise results in larger noise on the image. This is due to the fact that small eigenvalues of H turns into large eigenvalues of H^{-1}.

d) $H\vec{v} = \lambda \vec{v}$

$H^{-1}(H\vec{v}) = H^{-1}(\lambda \vec{v})$

$\vec{v} = H^{-1}\lambda \vec{v}$

$\frac{1}{\lambda}\vec{v} = \frac{1}{\lambda}(H^{-1}\lambda \vec{v})$

$\frac{1}{\lambda}\vec{v} = \frac{1}{\lambda} \cdot \lambda H^{-1}\vec{v}$

$\boxed{\frac{1}{\lambda}\vec{v} = H^{-1}\vec{v}}$ $\frac{1}{\lambda}$ is an eigenvalue of H^{-1}.

4. Cubic Polynomials

a) Prove that $S = \{p_0 + p_1 t + p_2 t^2 + p_3 t^3 \mid p_0, p_1, p_2, p_3 \in R\}$ is a vector space.

1) let $\vec{u}, \vec{v} \in S$, then

$\vec{u} = u_0 + u_1 t + u_2 t^2 + u_3 t^3$

$\vec{v} = v_0 + v_1 t + v_2 t^2 + v_3 t^3$

$\vec{u} + \vec{v} = u_0 + v_0 + u_1 t + v_1 t + u_2 t^2 + v_2 t^2 + u_3 t^3 + v_3 t^3$

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$

Commutative

2) If $p_0 = p_1 = p_2 = p_3 = 0$, then

$0 + 0 \cdot t + 0 \cdot t^2 + 0 \cdot t^3 = \vec{0}$

$\vec{0}$ exists in S

3) let $\vec{u}, \vec{v} \in S$

$\vec{u} = u_0 + u_1 t + u_2 t^2 + u_3 t^3$

$\vec{v} = v_0 + v_1 t + v_2 t^2 + v_3 t^3$

$\vec{u} + \vec{v} = u_0 + u_1 t + u_2 t^2 + u_3 t^3 + v_0 + v_1 t + v_2 t^2 + v_3 t^3$

$\vec{u} + \vec{v} = (u_0 + v_0) + (u_1 + v_1) t + (u_2 + v_2) t^2 + (u_3 + v_3) t^3$

$\vec{u} + \vec{v} \in S$

Closed under vector addition

4) let $\vec{u}, \vec{v}, \vec{w} \in S$,

$\vec{u} = u_0 + u_1 t + u_2 t^2 + u_3 t^3$

$\vec{v} = v_0 + v_1 t + v_2 t^2 + v_3 t^3$

$\vec{w} = w_0 + w_1 t + w_2 t^2 + w_3 t^3$

$\vec{u} + (\vec{v} + \vec{w}) = \vec{u} + (v_0 + v_1 t + v_2 t^2 + v_3 t^3 + w_0 + w_1 t + w_2 t^2 + w_3 t^3)$

$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + v_0 + v_1 t + v_2 t^2 + v_3 t^3) + w_0 + w_1 t + w_2 t^2 + w_3 t^3$

$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$

Associative

5) let $\vec{v} \in S$ and we proved $\vec{0} \in S$

Since we also proved closure under vector addition

$\vec{v} + \vec{0} \in S$

Additive Identity

6) let $\vec{u} \in S$ and $\alpha \in R$

$\alpha \vec{u} = \alpha u_0 + \alpha u_1 t + \alpha u_2 t^2 + \alpha u_3 t^3$

$\alpha u_0, \alpha u_1, \alpha u_2, \alpha u_3 \in R$

Closed under scalar multiplication

7) let $\vec{v} \in S$

If $\alpha = -1$, $\alpha \vec{v} = (-1)\vec{v} = -\vec{v} \in S$

$\vec{v} + (-\vec{v}) = v_0 + v_1 t + v_2 t^2 + v_3 t^3 - v_0 - v_1 t - v_2 t^2 + v_3 t^3$

$\vec{v} + (-\vec{v}) = \vec{0}$

Additive Inverse

8) let $\vec{u}, \vec{v} \in S$, $\alpha \in R$

$\alpha(\vec{u} + \vec{v}) = \alpha(u_0 + u_1 t + u_2 t^2 + u_3 t^3 + v_0 + v_1 t + v_2 t^2 + v_3 t^3)$

$\alpha(\vec{u} + \vec{v}) = \alpha u_0 + \alpha u_1 t + \alpha u_2 t^2 + \alpha u_3 t^3 + \alpha v_0 + \alpha v_1 t + \alpha v_2 t^2 + \alpha v_3 t^3$

$\alpha(\vec{u} + \vec{v}) = \alpha \vec{u} + \alpha \vec{v}$

Distributive in vector addition

9) let any $\vec{v} \in S$

$1 \cdot \vec{v} = 1 \cdot v_0 + 1 \cdot v_1 t + 1 \cdot v_2 t^2 + 1 \cdot v_3 t^3$

$1 \cdot \vec{v} = \vec{v}$

Multiplicative Identity

10) let $\alpha, \beta \in R$ and $\vec{v} \in S$

$\alpha(\beta \vec{v}) = \alpha(\beta v_0 + \beta v_1 t + \beta v_2 t^2 + \beta v_3 t^3)$

$\alpha(\beta \vec{v}) = \alpha\beta v_0 + \alpha\beta v_1 t + \alpha\beta v_2 t^2 + \alpha\beta v_3 t^3$

$\alpha(\beta \vec{v}) = (\alpha\beta)\vec{v}$

Associative for scalars

Thus, S is a vector space.

Dimension = $\boxed{4}$

b) $p(t) = p_0 + p_1 t + p_2 t^2 + p_3 t^3$, $\quad p_0, p_1, p_2, p_3 \in R$

$$= \begin{bmatrix} p_0 & p_1 & p_2 & p_3 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}$$

let $p_0 = c_0$, $p_1 = c_1$, $p_2 = c_2$, $p_3 = c_3$ and

$\vec{c} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$ $\quad \vec{\varphi}(t) = \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}$.

Then, $p(t) = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}$

$$\boxed{p(t) = \vec{c}^{\,T} \vec{\varphi}(t)}$$

c) For vector space $S = \{p_0 + p_1 t + p_2 t^2 + p_3 t^3 \mid p_0, p_1, p_2, p_3 \in R\}$ let $\vec{p} \in S$, then the set $\{p_0, p_1 t, p_2 t^2, p_3 t^3\}$ is linearly independent.
And the basis would be span$\{1, t, t^2, t^3\}$
So, the monomials $\varphi_k(t) = t^k$, for $t = 0,1,2,3$ is a basis for the cubic polynomials.

d) $\varphi_3(t) = 1 + t + t^2 + t^3$

$\frac{d}{dt} \varphi_3(t) = 0 + 1 + 2t + 3t^2$

$$\frac{d}{dt} \varphi_3(t) = 1 \cdot \varphi_0(t) + 2 \cdot \varphi_1(t) + 3 \varphi_2(t)$$

e) $\frac{d}{dt} p(t) = (D\vec{c})^T \vec{\varphi}(t)$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{D} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} c_2 \\ c_3 \\ c_4 \\ 0 \end{bmatrix}^T \Rightarrow \begin{bmatrix} c_2 & c_3 & c_4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix} = \frac{d}{dt} p(t)$$

I came up with the values for D through observation and verified that it was correct.

EECS16A: Homework 5

Problem 3: Noisy Images

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

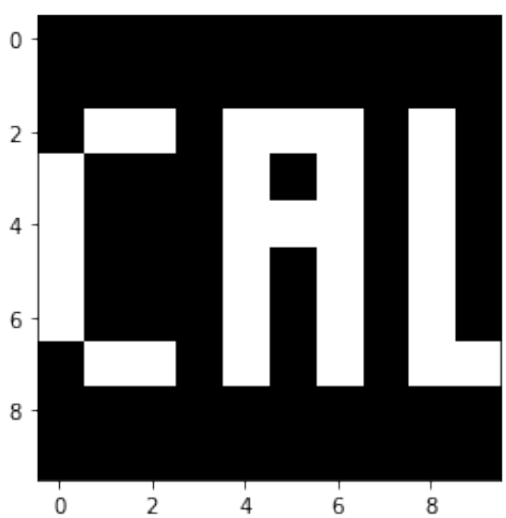
Let's load some data to start off with.

In [2]:
```
H3 = np.loadtxt("cond_10e6.txt", delimiter=',').reshape(100,100)
H2 = np.loadtxt("cond_1e3.txt", delimiter=',').reshape(100,100)
H1 = np.eye(100)
img = np.loadtxt("image.txt", delimiter=',').reshape(10,10)
```

The code below displays the image.

In [3]:
```
plt.figure(0)
plt.imshow(img,cmap='gray')
```

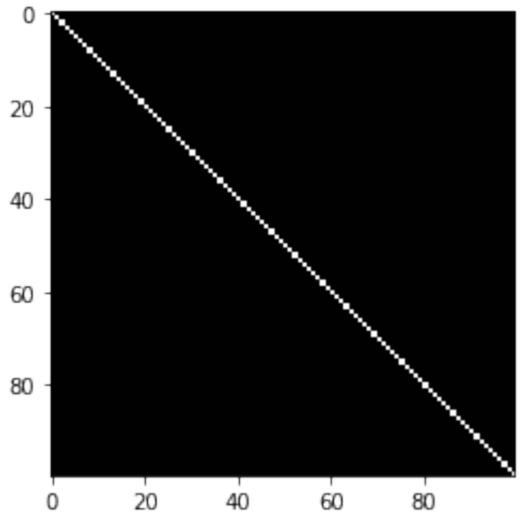
Out[3]: <matplotlib.image.AxesImage at 0x11e57f6d8>

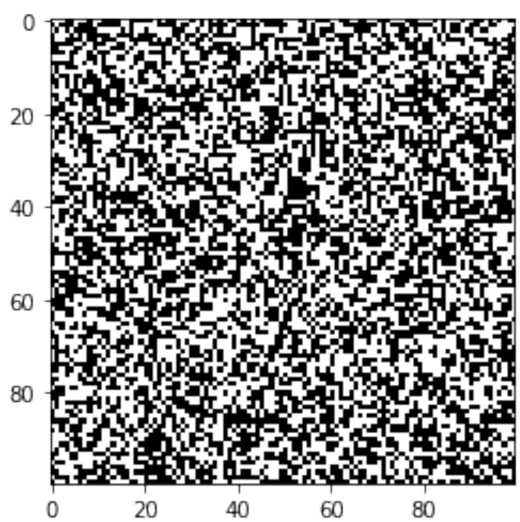


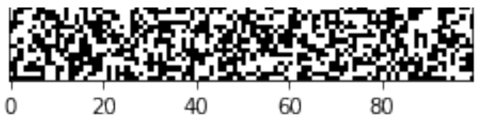
Then, lets display the set of masks

In [4]:
```
plt.figure(1)
plt.imshow(H1,cmap='gray')
plt.figure(2)
plt.imshow(H2,cmap='gray')
plt.figure(3)
plt.imshow(H3,cmap='gray')
```

Out[4]: <matplotlib.image.AxesImage at 0x11e6fe278>







We'll use numpy.random to make some noise.

In [5]:
```
noise = np.random.normal(0.5,0.1)
```

Lets compute the $\vec{b}$ vector for each matrix and add some noise to the $\vec{b}$ vector.

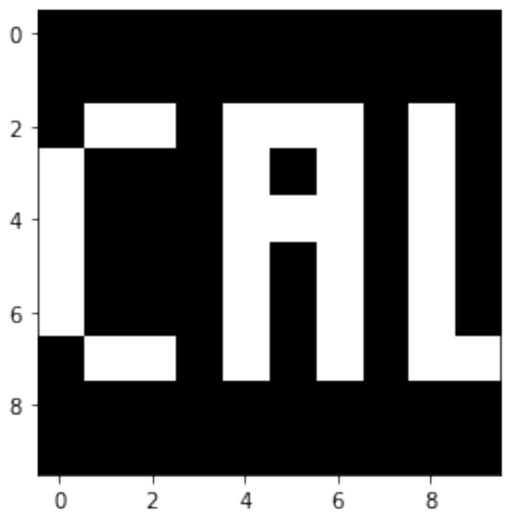
In [6]:
```
b1 = H1.dot(img.reshape(100)) + noise
b2 = H2.dot(img.reshape(100)) + noise
b3 = H3.dot(img.reshape(100)) + noise
```

First, let's compute $\vec{x}_1$ after adding noise and find the minimum eigenvalue of $\mathbf{H_1}$.

In [7]:
```
x1 = np.linalg.inv(H1).dot(b1)
eigenvalues1 = np.linalg.eig(H1)[0]
print("Is the matrix invertible?", abs(np.linalg.det(H1)) > 0.5)
print("The smallest eigenvalue is:", min(np.absolute(eigenvalues1)))
print("Number of eigenvectors:", len(eigenvalues1))
plt.imshow(x1.reshape(10,10), cmap='gray')
```

```
Is the matrix invertible? True
The smallest eigenvalue is: 1.0
Number of eigenvectors: 100
```

Out[7]: <matplotlib.image.AxesImage at 0x11ea2d860>

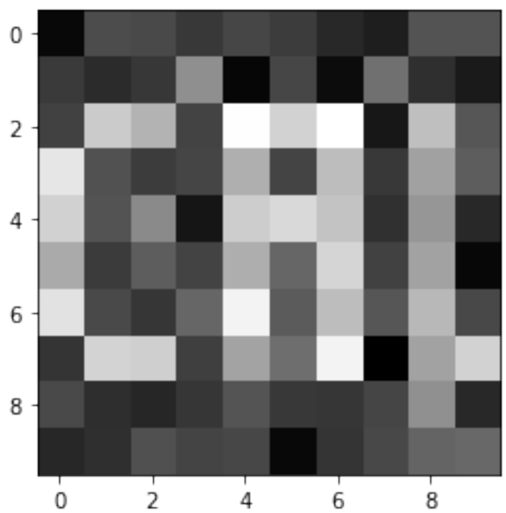


Now let's compute $\vec{x}_2$ and find the minimum eigenvalue of $\mathbf{H}_2$.

In [8]:
```
x2 = np.linalg.inv(H2).dot(b2)
eigenvalues2 = np.linalg.eig(H2)[0]
print("Is the matrix invertible?", abs(np.linalg.det(H2)) > 0.5)
print("The smallest eigenvalue is:", min(np.absolute(eigenvalues2)))
print("Number of eigenvectors:", len(eigenvalues2))
plt.imshow(x2.reshape(10,10), cmap='gray')
```

```
Is the matrix invertible? True
The smallest eigenvalue is: 0.295163633086302
Number of eigenvectors: 100
```

Out[8]:
```
<matplotlib.image.AxesImage at 0x11eb87cf8>
```

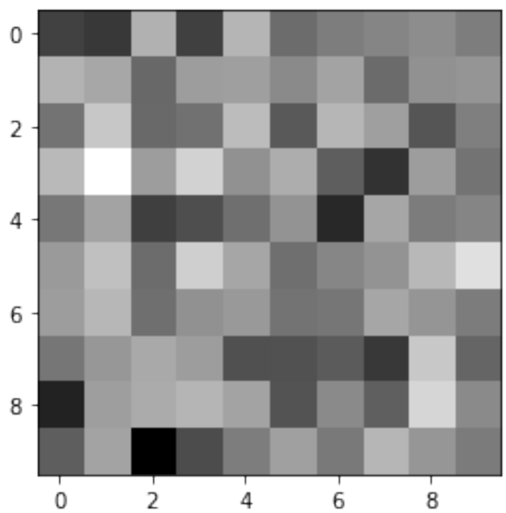


Now let's compute $\vec{x}_3$ and find the minimum eigenvalue of $\mathbf{H}_3$.

In [9]:
```
x3 = np.linalg.inv(H3).dot(b3)
eigenvalues3 = np.linalg.eig(H3)[0]
print("Is the matrix invertible?", abs(np.linalg.det(H3)) > 0.5)
print("The smallest eigenvalue is:", min(np.absolute(eigenvalues3)))
print("Number of eigenvectors:", len(eigenvalues3))
plt.imshow(x3.reshape(10,10), cmap='gray')
```

```
Is the matrix invertible? True
The smallest eigenvalue is: 1.2184217509463857e-05
Number of eigenvectors: 100
```

Out[9]: <matplotlib.image.AxesImage at 0x11f5131d0>



Problem 5: Page Rank

In [1]:
```
# Though it is not required you may use iPython for your calculations in
```

In [ ]:

In [ ]:

In [ ]: