

1. Counting Solutions

a)

$$\begin{array}{l} 1) \quad 2x + 3y = 5 \\ \quad \quad x + y = 2 \end{array}$$

(multiply row 2 by -2 and add to row 1)

$$\begin{array}{l} 2) \quad 2x - 2x + 3y - 2y = 5 - 4 \\ \quad \quad x + y = 2 \end{array}$$

(simplify)

$$\begin{array}{l} 3) \quad y = 1 \\ \quad \quad x + y = 2 \end{array}$$

(substitute $y = 1$ to row 2)

$$\begin{array}{l} 4) \quad y = 1 \\ \quad \quad x = 2 - 1 \end{array}$$

(find x)

$$\begin{array}{l} 5) \quad \mathbf{y = 1} \\ \quad \quad \mathbf{x = 1} \end{array} \quad \mathbf{Unique \ solution}$$

b)

$$\begin{aligned} 1) \quad & x + y + z = 3 \\ & 2x + 2y + 2z = 5 \end{aligned}$$

(multiply row 1 by -2 and add to row 2)

$$\begin{aligned} 2) \quad & x + y + z = 3 \\ & 2x - 2x + 2y - 2y + 2z - 2z = 5 - 6 \end{aligned}$$

(simplify)

$$\begin{aligned} 3) \quad & x + y + z = 3 \\ & 0 = -1 \end{aligned}$$

There's **no solution** to this problem because no choice of x, y, z will make $0 = -1$ true.

In other words, $0 = -1$

$$0x + 0y + 0z = -1$$

No matter how we choose x, y, z , this equation is never true.

Thus, **no solution**.

c)

$$\begin{array}{rcl} 1) & -y + 2z & = 1 \\ & 2x + z & = 2 \end{array}$$

(find x and y)

$$\begin{array}{rcl} 2) & y & = 2z - 1 \\ & 2x & = 2 - z \end{array}$$

(divide row 2 by 2)

$$\begin{array}{rcl} 3) & y & = 2z - 1 \\ & x & = 1 - z/2 \end{array}$$

There's an **infinite number of solutions** to this problem. This is because there's an infinite number of ways to choose z.

In other words, we can let z be any real number and we can find a corresponding x and y using the equations above, which will also be a solution. Thus, there's an infinite number of solutions to this problem.

Set of solutions: $(1 - t/2, 2z - 1, t)$

d)

$$\begin{array}{l} 1) \quad x + 2y = 3 \\ \quad 2x - y = 1 \\ \quad 3x + y = 4 \end{array}$$

(add row 2 to row 3)

$$\begin{array}{l} 2) \quad x + 2y = 3 \\ \quad 2x - y = 1 \\ \quad 5x = 5 \end{array}$$

(multiply row 1 by -2 and add to row 2)

$$\begin{array}{l} 3) \quad x + 2y = 3 \\ \quad -y - 4y = 1 - 6 \\ \quad x = 1 \end{array}$$

(simplify)

$$\begin{array}{l} 4) \quad x + 2y = 3 \\ \quad -5y = -5 \\ \quad x = 1 \end{array}$$

$$5) \quad x + 2y = 3$$

$$y = 1$$

$$x = 1$$

(plug in x and y to row 1)

$$6) \quad 3 = 3$$

$$y = 1$$

$$x = 1$$

$$7) \quad \mathbf{x = 1}$$

$$\mathbf{y = 1}$$

Unique solution

e)

$$\begin{array}{l} 1) \quad x + 2y = 3 \\ \quad 2x - y = 1 \\ \quad x - 3y = -5 \end{array}$$

(row 3 minus row 1)

$$\begin{array}{l} 2) \quad x + 2y = 3 \\ \quad 2x - y = 1 \\ \quad x - x - 3y - 2y = -5 - 3 \end{array}$$

(multiply row 1 by -2 and add to row 2)

$$\begin{array}{l} 3) \quad x + 2y = 3 \\ \quad 2x - 2x - y - 4y = 1 - 6 \\ \quad -5y = -8 \end{array}$$

(simplify)

$$\begin{array}{l} 4) \quad x + 2y = 3 \\ \quad -5y = -5 \\ \quad -5y = -8 \end{array}$$

(subtract row 2 from row 3)

5) $x + 2y = 3$

$$y = 1$$

$$0 = -3$$

This is the same situation as (b). There's no values we can choose for x and y to make $0 = -3$ true. Thus, **no solution**.

2. Filtering out the Troll

$$a) \begin{cases} m_1 = \cos(45^\circ) \vec{a} + \cos(-30^\circ) \vec{b} \\ m_2 = \sin(45^\circ) \vec{a} + \sin(-30^\circ) \vec{b} \end{cases}$$

$$b) \begin{cases} \frac{\sqrt{2}}{2} \vec{a} + \frac{\sqrt{3}}{2} \vec{b} = m_1 \\ \frac{\sqrt{2}}{2} \vec{a} - \frac{1}{2} \vec{b} = m_2 \end{cases}$$

$$\begin{cases} \frac{\sqrt{3}}{2} \vec{b} + \frac{1}{2} \vec{b} = m_1 - m_2 \\ \frac{\sqrt{2}}{2} \vec{a} - \frac{1}{2} \vec{b} = m_2 \end{cases}$$

$$\begin{cases} \frac{\sqrt{3}+1}{2} \vec{b} = m_1 - m_2 \\ \frac{\sqrt{2}}{2} \vec{a} = m_2 + \frac{1}{2} \vec{b} \end{cases}$$

$$\begin{cases} \vec{b} = \frac{2}{\sqrt{3}+1} (m_1 - m_2) \\ \frac{\sqrt{2}}{2} \vec{a} = m_2 + \frac{1}{2} \left(\frac{2}{\sqrt{3}+1} \right) (m_1 - m_2) \end{cases}$$

$$\vec{a} = \sqrt{2} \left[m_2 + \frac{1}{\sqrt{3}+1} (m_1 - m_2) \right]$$

$$\vec{a} = \sqrt{2} \left(\frac{\sqrt{3}+1}{\sqrt{3}+1} m_2 - \frac{1}{\sqrt{3}+1} m_2 + \frac{1}{\sqrt{3}+1} m_1 \right)$$

$$\vec{a} = \sqrt{2} \left(\frac{\sqrt{3}}{\sqrt{3}+1} m_2 + \frac{1}{\sqrt{3}+1} m_1 \right)$$

$$\vec{a} = \boxed{\frac{\sqrt{2}}{\sqrt{3}+1} m_1 + \frac{\sqrt{6}}{\sqrt{3}+1} m_2}$$

\uparrow \uparrow
 u v

c) All human beings are born free and equal in dignity and rights.

Optional: Taken from: Universal Declaration of Human Rights.

3. Homework Process and Study Group

Who else did you work with on this assignment?

I worked on this homework assignment alone.

How did you work on this assignment?

I first read Note 0 and Note 1, then I went through the lab presentation to learn about jupyter notebook. Then, I was able to understand the problems and solve them.

EECS16A: Homework 1

Problem 2: Filtering Out The Troll

```
In [10]: import warnings
import wave as wav

import matplotlib.pyplot as plt
import numpy as np
import scipy
import scipy.io.wavfile
from IPython.display import Audio
from scipy import io
from scipy.io.wavfile import read

# For this to work make sure to download m1.wav and m2.wav to the same l
warnings.filterwarnings("ignore")
sound_file_1 = "m1.wav"
sound_file_2 = "m2.wav"
```

Let's listen to the recording of the first microphone (it can take some time to load the sound file). Run the cell below, then press the play button to listen.

```
In [11]: Audio(url="m1.wav", autoplay=False)
```

```
Out[11]: -0:10
```

And this is the recording of the second microphone (it can take some time to load the sound file). Run the cell below, then press the play button to listen.

```
In [12]: Audio(url="m2.wav", autoplay=False)
```

```
Out[12]: -0:10
```

We read the first recording to the variable `corrupt1` and the second recording to `corrupt2`. Treat `corrupt1` and `corrupt2` as the two sound recordings picked up by microphone 1 and microphone 2 respectively.

```
In [13]: rate1, corrupt1 = scipy.io.wavfile.read("m1.wav")
         rate2, corrupt2 = scipy.io.wavfile.read("m2.wav")
```

Enter the weights of the two recordings to get the clean speech.

Note: The square root of a number a can be written as `np.sqrt(a)` in IPython.

```
In [14]: # enter the weights u (recording 1) and v (recording 2)
         u = 0.5176380902
         v = 0.8965754722
```

Weighted combination of the two recordings:

```
In [15]: a = u * corrupt1 + v * corrupt2
```

Let's listen to the resulting sound file (make sure your speaker's volume is not very high, the sound may be loud if things go wrong).

```
In [17]: Audio(data=a, rate=rate1)
```

```
Out[17]: -0:10
```

```
In [ ]:
```