First Assignment

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Task 1

Calculate GCD(a, b) and find Bezout's identity for a=2022, b=752.

rem	**val**	**expr**
r_0	2022	a
r_1	752	b
$r_2 = r_0 \text{ mod } r_1$	518	a - 2b
$r_3 = r_1 \text{ mod } r_2$	234	b - (a - 2b) = 3b - a
$r_4 = r_2 \text{ mod } r_3$	50	(a - 2b) - 2(3b - a) = 3a - 8b
$r_5 = r_3 \text{ mod } r_4$	34	(3b - a) - 4(3a - 8b) = 35b - 13a
$r_6 = r_4 \text{ mod } r_5$	16	(3a - 8b) - (35b - 13a) = 16a - 43b
$r_7 = r_5 \text{ mod } r_6$	2	(35b - 13a) - 2(16a - 43b) = 121b - 45a
$r_8 = r_6 \text{ mod } r_7$	0	

The gcd of 2022 and 752 is **2**.
From the final line of the table we can see that Bezout's identity is fulfilled with **-45** for x and **121** for y.
This can be checked with: $(2022 \cdot (-45)) + (752 \cdot 121) = 2$

Task 2

Solve the following congruences:

1) x + 17 = 23 (mod 37) | -17
   x + 17 - 17 = 23 - 17 (mod 37)
   x = 6 (mod 37)
   x = **6**

2) x + 42 = 19 (mod 51) | -42
   x + 42 - 42 = 19 - 42 (mod 51)
   x = -23 (mod 51)
   x = -23 + 51 (mod 51)
   x = 28 (mod 51)
   x = **28**

Task 3

Solve the following congruences:

1) $23^{37} \bmod 40 =$
   $23 \cdot 23^{36} \bmod 40 =$
   $23 \cdot 23^{12 \cdot 3} \bmod 40 =$
   $23 \cdot 23^{4 \cdot 3 \cdot 3} \bmod 40 =$
   $23 \cdot 23^{2 \cdot 2 \cdot 3 \cdot 3} \bmod 40 =$
   $23 \cdot 529^{2 \cdot 3 \cdot 3} \bmod 40 =$
   $23 \cdot (529^{2 \cdot 3 \cdot 3} \bmod 40) \bmod 40 =$
   $23 \cdot 9^{2 \cdot 3 \cdot 3} \bmod 40 =$
   $23 \cdot 81^{9} \bmod 40 =$
   $23 \cdot 1^{9} \bmod 40 =$ **23**

2) $(-133)^{100}$ mod 10 =
(-133 mod 10)^{100} mod 10 =
7^{100} mod 10 =
$7^{4 \cdot 25}$ mod 10 =
2401^{25} mod 10 =
1^{25} mod 10 = **1**

Task 4

Consider the following sequence of operations:

$Plaintext \rightarrow S_1 \rightarrow P_1 \rightarrow S_2 \rightarrow P_2 \rightarrow Ciphertext$

Plaintext is **MOTIVATION**. S_1 is a shift cipher with key $k_{S_1} = 17$. S_2 is a shift cipher with the key $k_{S_2} = 8$. P_1 is a permutation cipher with a key $k_{P_1} = (5,1,3,2,4)$. P_2 is a permutation cipher with a key $k_{P_2} = (3,4,5,1,2)$.

The task: what is the ciphertext?

This task was solved both with code and manually. The shift into Caesar cipher was made by a Python script I wrote in Appendix A. The code has one global variable called alphabet that stores two entire English alphabets. This was made in order to avoid the string index out of range in case the shift makes a number greater than 26, in that case the alphabet restart from a, but not as 0 but 26. The main function is called tocaesar, which takes a plaintext and the desired shift as input and encrypts the plaintext. In order to encrypt every single letter I ran a loop for each in the text, then I found the index of the every inspected letter and assigned it to a temporary variable called position. Once we get the position we need to calculate the new shifted one so I just summed the old position with the input shift. Knowing the new shifted position of the letter, I appended the corresponding alphabet letter with the index of the new position to an initially empty string called result. The transposition part was made by hand.

After running the script for the first time giving input of MOTIVATION and 17 we get the first ciphertext **dfkzmrkzfe**.

Now with the key (5, 1, 3, 2, 4) we need to transpose the current text.

1 2 3 4 5 1 2 3 4 5
d f k z m r k z f e

5 1 3 2 4 5 1 3 2 4
m d k f z e r z k f

After running the script again giving input of mdkfzerzkf and 8 we get the third ciphertext **ulsnhmzhsn**.

Finally we can run the final transposition and get the final result using the following key (3,4,5,1,2).

1 2 3 4 5 1 2 3 4 5
u l s n h m z h s n

3 4 5 1 2 3 4 5 1 2
s n h u l h s n m z

The final ciphertext is: **snhulhsnmz**.

Task 5

Assume that the Affine cipher is implemented Z_{89}, not in Z_{26}.

1. Write down encryption and decryption functions for this modification of Affine cipher.

a, b - together form the key k(a, b)
m - plaintext message
c - ciphertext

The encryption key is: E_m = am + b mod 89
The decryption key is: $D_c = a^{-1} \cdot (c - b)$ mod 89

2. What is the number of possible keys?

The number of possible keys is: $89 \cdot \Theta(89)$ or $89 \cdot 88$. 88 is what we get from the Euler's function, following the property: $\Theta(p) = p - 1$, where p represents a prime number. This means that there is a total of 7832 possible keys.

3. Suppose that modulus p = 89 is public. Malicious Eve intercepts two ciphertexts encrypted with the same key sent from Alice to Bob c_1 = 1 and c_2 = 69. Assume that Eve also managed to find out that the corresponding plaintexts are m_1 = 10 and m_2 = 7. Find out the encryption key and use it to encrypt message m_3 = 13.

Firstly, we write down the following system of equations:

$$\begin{cases} 69 = 7a + b \text{mod } 89 \\ 1 = 10a + b \text{mod } 89 \end{cases}$$

Now we can solve it and find the key k(a, b).

$$\begin{cases} 69 = 7a + b \text{mod } 89 \mid \cdot (-1) \\ 1 = 10a + b \text{mod } 89 \end{cases}$$

$$\begin{cases} -69 = -7a - b \text{mod } 89 \\ 1 = 10a + b \text{mod } 89 \end{cases}$$

After this we get:

-69 + 1 = -7a + 10a + 0 mod 89
-68 = 3a mod 89
(-68 mod 89) = 3a mod 89
21 = 3a mod 89 | : 3
7 = a mod 89
a = 7 mod 89
a = 7

Now that we have a we can find b by just replacing a's value into one of the equations in the system.

$69 = 7 \cdot 7 + \text{b mod } 89$
$69 = 49 + \text{b mod } 89$
$69 - 49 = \text{b mod } 89$
$20 = \text{b mod } 89$
$\text{b} = 20 \text{ mod } 89$
$\text{b} = 20$

a and b form the encryption key **k(7, 20)**. Knowing that $m_3 = 13$ we can insert it into the Affine encryption function $E_m = \text{am} + \text{b mod } 89$, knowing that a = 7 and b = 20.

The ciphertext we get is:

$7 \cdot 13 + 20 \text{ mod } 89 =$
$91 + 20 \text{ mod } 89 =$
$111 \text{ mod } 89 = 22$

Ciphertext c_3 is **22**.

Task 6

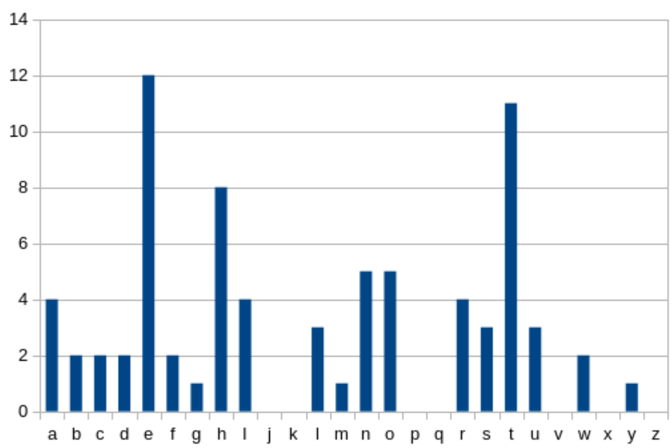
Let the message be

"The most beautiful things in the world cannot be seen or touched, they are felt with the heart." — Antoine de Saint-Exupery, The Little Prince

Make frequency table for that message and compute index of coincidence.

In order to calculate the index of coincidence I wrote a small C program to do the trick. In main I declared three variables called alphabet, an array that contains all of the letters from the english alphabet, counter, an array of 26 0's, and txt that contains the quote from Exupery. After their declaration a function called findIC is called that takes as input these three variables. The function loops through the text and for every letter stops, compares the letter with the ones in the alphabet's array (this is done by the internal loop) and if there is a match the letter's respective counter (each letter has its own counter) increases by one. Also the length of the text is increased by one (I had to do it manually in order to avoid counting spaces) and finally the inner loop breaks. The outer loop only ends when the txt pointer is pointing to the array's terminating 0. Once we have the length of the text and the counters for each of the letters I computed the index of coincidence with its respective formula. To compute the index of coincidence I ran a loop 26 times, one time for each of the letter. In the end I return the IC as a double value and print it.

The frequency table that we get is:



The index of coincidence is: **0.0724324**

APPENDIX A - task 4 program

```
alphabet = ['a', 'b', 'c', 'd', 'e', 'f', 'g', 'h', 'i', 'j', 'k', 'l',
    'n', 'o', 'p', 'q', 'r', 's', 't', 'u', 'v', 'w', 'x', 'y', 'z']

alphabet += ['a', 'b', 'c', 'd', 'e', 'f', 'g', 'h', 'i', 'j', 'k', 'l'
    'n', 'o', 'p', 'q', 'r', 's', 't', 'u', 'v', 'w', 'x', 'y', 'z']


def tocaesar(text, shift):
    result = ""
    for letter in text:
```

```
        position = alphabet.index(letter)
        newPosition = position + shift
        result += alphabet[newPosition]
    return result


text = input("Type in the desired text: ")
shift = int(input("Type in the shift: "))
print(tocaesar(text, shift))
```

APPENDIX B - task 6 program

```
#include <stdio.h>


double findIC(char* txt, char* alphabet, int* counters) {
    int txtLength = 0;
    for (int i = 0; *(txt + i); i++) {
        for (int j = 0;  j < 26; j++) {
            if (*(txt + i) == *(alphabet + j)) {
                *(counters + j) += 1;
                txtLength++;
                break;
            }
        }
    }
    double ic = 0;
    for (int i = 0; i < 26; i++) {
        ic += ((double)counters[i] / txtLength) *
            ((double)(counters[i] - 1) / ( txtLength - 1));
    }
    return ic;
}
```

```
int main() {
    char alphabet[] = "abcdefghijklmnopqrstuvwxyz";
    int counters[26] = {0};
    char txt[] = "The most beautiful things in the world cannot be"
        "seen or touched, they are felt with the heart.";
    double ic = findIC(txt, alphabet, counters);
    printf("The index of coincidence for the text is: %lg\n", ic);
    return 0;
}
```