

# Second Assignment

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## Task 1 — Big O Notation

Prove that  $O(n^2) = an^2 + bn - c$ , where  $a$  - is first 2 digits of your student code,  $b$  - is 3rd and 4th digit of your student code,  $c$  - last two digits of your student code.

Code 201752IVSB, then  $a = 20$ ,  $b = 17$ ,  $c = 52$ ,  
equation is  $O(n^2) = 20n^2 + 17n - 52$

From theory we know that  $f(n) \leq c \cdot g(n)$ , so we get:

$$20n^2 + 17n - 52 \leq c \cdot n^2$$

To simplify our equation we can choose that  $c = 21$ . Now let's solve this:

$$\begin{aligned} 20n^2 + 17n - 52 &\leq 21n^2 \\ 20n^2 - 21n^2 + 17n - 52 &\leq 0 \\ -n^2 + 17n - 52 &\leq 0 \quad | \cdot -1 \\ n^2 - 17n + 52 &\geq 0 \end{aligned}$$

By taking the derivative of the quadratic formula we get:

$$\begin{aligned} 2n - 17 &\geq 0 \\ 2n &\geq 17 \\ n &= 8.5 \end{aligned}$$

Meaning that the function starts to grow again after the value 8.5 is encountered.

By solving this quadratic formula equation we get the following values:

$$n_1 = 13, n_2 = 4, \text{ so } n < 4 \vee n > 13$$

Meaning that the function is positive only when bigger than 4 or 13.

As a result we get —  $c = 21, n = 14$

## Task 2 — Complexity theory

Give an example of a *search* problem and corresponding *decision* problem, which was not discussed in the lectures.

Is  $x \in \mathbb{Z}$  positive or negative?

Consider the corresponding verification function  $V(x, y)$ :

$$\begin{cases} 1 & \text{where } x = y + z \text{ and } (y \geq 0 \text{ and } z \geq 0) \text{ or } (+y > -|z| \text{ and viceversa)} \\ 0 & \text{if otherwise} \end{cases}$$