

$$\begin{array}{c}
\frac{a, \perp, a, \neg a \rightarrow b, a \rightarrow \neg b \vdash b, b, \perp}{\perp, a, \neg a \rightarrow b, a \rightarrow \neg b \vdash a \rightarrow b, b, \perp} \quad \frac{\perp, \perp, a, \neg a \rightarrow b, a \rightarrow \neg b \vdash b, \perp}{\perp, a, \neg a \rightarrow b, a \rightarrow \neg b \vdash b, \perp} \\
\frac{a, \neg a \rightarrow b, a \rightarrow \neg b \vdash a \rightarrow b, b, \perp}{\perp, a, \neg a \rightarrow b, a \rightarrow \neg b \vdash b, \perp} \\
\frac{a, \neg a \rightarrow b, a \rightarrow \neg b \vdash b, \perp}{\neg a \rightarrow b, a \rightarrow \neg b \vdash a \rightarrow b, \perp} \quad \frac{\neg a \rightarrow b, a \rightarrow \neg b \vdash \perp}{a \rightarrow \neg b \vdash \neg a \rightarrow b} \quad \frac{\perp, \neg a \rightarrow b, a \rightarrow \neg b \vdash \perp}{\vdash a \rightarrow \neg b \rightarrow \neg a \rightarrow b}
\end{array}$$