Limits

L-Hospital Rule →

$$\lim_{x \to 0} \frac{f(x)}{g(x)}$$

use this rule only when $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right)$ form

Methods of evaluating limits

(i) $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

(ii) $\lim_{x \to \pi/2} \frac{\tan^{-1} x}{\tan x} = 0$

(iii) $\lim_{x \to 0} x \times \sin \frac{1}{x} = 0$

or

$$\lim_{x \to 0} \frac{\sin(1/x)}{(1/x)} = 0$$

(iv) $\lim_{x \to \infty} \frac{a_0 x^m + a_1 x^{m-1} + a_2 x^{m-2} + \dots}{b_0 x^n + b_1 x^{n-1} + \dots} = \frac{a_0}{b_0}$ (only if $m = n$)

$= 0 \quad (m < n)$

$= \begin{cases} \infty, & a_0 b_0 > 0 \\ -\infty, & a_0 b_0 < 0 \end{cases}$ if $(m > n)$

NOTE :-

$$\lim_{x \to \infty} a^x = \begin{cases} \infty, & a > 1 \\ 1, & a = 1 \\ 0, & 0 \le a < 1 \end{cases}$$

$$\lim_{x \to -\infty} f(x) = \lim_{y \to \infty} f(-y)$$

$$\lim_{x \to \infty} f(x) = \lim_{y \to 0} f(1/y)$$

Some standard limits

(i) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(ii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(iii) $\lim_{x \to 0} \left[\frac{\sin x}{x}\right] = 0$

(iv) $\lim_{x \to 0} \left[\frac{\tan x}{x}\right] = 1$

(v) $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1$

(vi) $\lim_{x \to 0} \frac{\tan^{-1} x}{x} = 1$

(vii) $\lim_{x\to 0} \dfrac{\ln(1 \pm x)}{x} = \pm 1.$

(viii) $\lim_{x\to 0} \dfrac{e^{x}-1}{x} = 1$

(ix) $\lim_{x\to 0} \dfrac{a^{x}-1}{x} = \ln a, \boxed{a>0}$

(x) $\lim_{x\to 0} \dfrac{1-\cos x}{x^{2}} = \dfrac{1}{2}$

(xi) $\lim_{x\to 0} \left(\dfrac{x^{n}-a^{n}}{x-a}\right) = na^{n-1}$

(xii) $\lim_{x\to 0} \dfrac{x^{m}-a^{m}}{x^{n}-a^{n}} = \dfrac{m}{n} a^{m-n}$

NOTE (i) $\lim_{x\to a} f(x) = \lim_{h\to 0} f(a+h)$

(xiii) $\lim_{x\to a} f(x)^{g(x)} = e^{(f(x)-1)\,g(x)}$ } Here, form should always be 1^{∞}.

Limit using expansion

(i) $\sin x = x - \dfrac{x^{3}}{3!} + \dfrac{x^{5}}{5!} - - - - - ;$

(ii) $\cos x = 1 - \dfrac{x^{2}}{2!} + \dfrac{x^{4}}{4!} - - - ;$

(iii) $\tan x = x + \dfrac{x^{3}}{3} + \dfrac{2x^{5}}{15} + - - ;$

(iv) $e^{x} = 1 + x + \dfrac{x^{2}}{2!} + \dfrac{x^{3}}{3!} + - - ;$

(v) $\ln(1+x) = x - \dfrac{x^{2}}{2} + \dfrac{x^{3}}{3} - - -$

(vi) $\sin^{-1}x = x + \dfrac{1}{2}\cdot\dfrac{x^{3}}{3} + \dfrac{1\cdot 3}{2\cdot 4}\cdot\dfrac{x^{5}}{5} + \dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\times\dfrac{x^{7}}{7} + - - -$

(vii) $\tan^{-1}x = x - \dfrac{x^{3}}{3} + \dfrac{x^{5}}{5} - \dfrac{x^{7}}{7} + - - -$

(viii) $(1+x)^{1/x} = e\left(1 - \dfrac{x}{2} + \dfrac{11}{24}x^{2} - -\right)$

(ix) $\left(1+\dfrac{1}{x}\right)^{1/x} = e\left(1 - \dfrac{x}{2} + \dfrac{11}{24}x^{2} + - -\right)$

Some standard limits

(i) $$\lim_{f(x) \to 0^+} f(x)^{f(x)} = 1.$$