

# Waves

(i)  $v = f \lambda$       (ii)  $\frac{1}{\lambda} = \text{Wave no.}$       (iii)  $k = \frac{2\pi}{\lambda}$  Angular Wave number.  
 (iv)  $\omega = 2\pi f$       (v)  $\frac{\omega}{k} = v = f \lambda$

## Wave Equn

$$y(x, t) = A \sin(\omega t - kx + \phi)$$

$$\Delta \phi = \frac{2\pi}{\lambda} (\Delta x) \quad , \quad v_p = -v_{\omega} \text{ slope}$$

## Checking whether a given eqn is a wave eqn

differentiate 2 times w.r.t  $x$  and 2 times w.r.t  $t$ , then divide if const. value comes, then the eqn is a wave eqn.

$$\frac{d^2 y}{dt^2} = v_w^2 \times \frac{d^2 y}{dx^2}$$

## Wave Speed on string

$$v = \sqrt{\frac{T}{\mu}}$$

$\uparrow$  Tension  
 $\mu$  Mass/unit length

## variable mass

$$\lambda = \sqrt{\frac{4L}{g}}$$

## Energy

(i)  $\frac{dK}{dx} = \frac{1}{2} \mu A^2 \omega^2 \cos^2(\omega t - kx + \phi)$

(ii)  $\frac{dU}{dx} = \frac{1}{2} \mu A^2 \omega^2 \cos^2(\omega t - kx + \phi)$

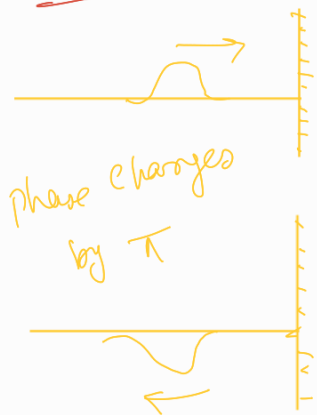
(iii) Total energy,  $\frac{dE}{dx} = \frac{dK}{dx} + \frac{dU}{dx} = \mu A^2 \omega^2 \cos^2(\omega t - kx + \phi)$

(iv) Power,  $P = \frac{dE}{dx} \times v = v \mu A^2 \omega^2 \cos^2(\omega t - kx + \phi)$

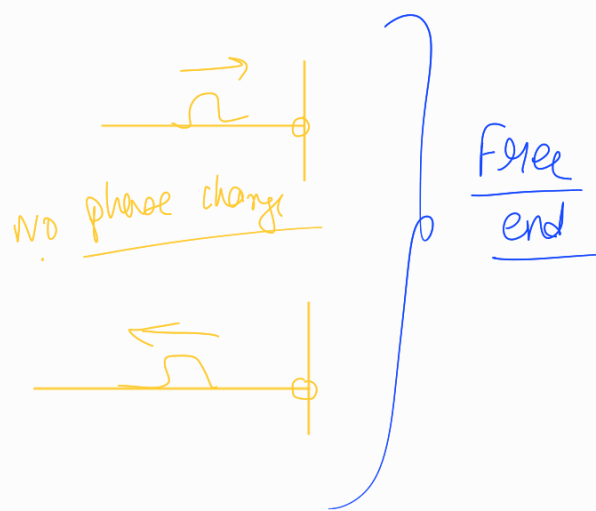
(v) Avg,  $\langle P \rangle = 2\pi^2 A^2 f^2 \mu v$

(vi) Intensity,  $I = \frac{P}{A} = 2\pi^2 A^2 f^2 \rho v$

## Reflection of wave



fixed end



Free end

## Equation of standing waves :-

$$y_{\text{net}} = 2A \sin(\omega t) \cos(kx)$$

$$\begin{array}{l} 2A \cos = \sin - \\ 2A \sin = \sin - \\ 2A \cos = \cos - \end{array}$$

Node (A=0)

$$x = \frac{n\lambda}{2}$$

- \* All particles has different 'A'
- \* All are in same phase

Antinode (A<sub>max</sub>)

$$x = \frac{n\lambda}{4} (2n+1)$$

\* The phase difference of both sides of nodes  $\rightarrow \pi$

## Standing wave on string

(i) fixed end  $\rightarrow$

$$f_0 = \frac{v}{2L} \quad (\text{fundamental frequency})$$

or,

$$f_0 = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$$

$$\therefore \text{Second Harmonic} \Rightarrow f = 2f_0$$

(1st Overtone)  $\Rightarrow f = 2 \left( \frac{v}{2L} \right)$

Third Harmonic  $\Rightarrow f = 3f_0 = 3 \left( \frac{v}{2L} \right)$

(2nd Overtone)

(ii) Free End :-

$$f_0 = \frac{v}{4L} \text{ (fundamental frequency)}$$

3rd Harmonic :-  $f = 3f_0 = 3\left(\frac{v}{4L}\right)$   
(1st Overtone)

5th Harmonic :-  $f = 5f_0 = 5\left(\frac{v}{4L}\right)$   
(2nd Overtone)

Longitudinal Waves :-

(\*)  $S = S_0 \sin\left(\omega\left(t - \frac{x}{v}\right) + \phi\right)$

(\*)  $\Delta v = A \cos$

(\*) Bulk Modulus =  $\frac{\Delta P}{(-\Delta v/v)}$

(\*)  $\Delta P = BKS_0 \cos(\omega t - kx + \phi)$

(\*)  $\Delta P_0 = BKS_0$   
Pressure Amplitude

Some Imp results

velocity  $\leftarrow v = \sqrt{\frac{\gamma}{\rho}}$ ,  $v = \sqrt{\frac{\beta}{\rho}}$   $\rightarrow$  velocity

Newton

$v_{\text{air}} = \sqrt{\frac{P}{\rho}} \approx 100 \sqrt{10}$   
 $\downarrow$   
Volume

Laplace Correction :-

$$v = \sqrt{\frac{\Delta P}{\rho}} \approx 332 \text{ m/s}$$

also,

$$v = \sqrt{\frac{\gamma R T}{M_0}}$$

$\rightarrow$  always in kg/mol

(\*)  $\langle I \rangle = \frac{(\Delta P_0)^2}{2\rho v}$

(\*)  $\langle I \rangle \propto (\Delta P_0)^2$

## Loudness

$$\beta = 10 \log_{10} \left( \frac{I}{I_0} \right)$$

$\downarrow$   
decibel

Intensity level  
Reference level ( $10^{-12}$ )  
 $I_0 = 10^{-2}$

## Standing Waves

(i) Open organ Pipe (similar to fix end)

$$f_0 = \frac{v}{2L}$$

2nd Harmonic  $= 2f_0 = 2\left(\frac{v}{2L}\right)$   
(1st overtone)

Similar  
3rd Harmonic  $= 3f_0 = 3\left(\frac{v}{2L}\right)$   
(2nd overtone)

(ii) closed organ Pipe

$$f_0 = \frac{v}{4L} \text{ (fundamental frequency)}$$

$\therefore$  3rd harmonic and 1st overtone  $= 3f_0 = 3\left(\frac{v}{4L}\right)$

also, 5th harmonic and 2nd overtone  $= 5f_0 = 5\left(\frac{v}{4L}\right)$

## End Corrections

$$\frac{3L}{4} = L + e$$

$$e = 0.6 \times \text{radius of tube}$$

## Resonance Tube

$$v = 2f(l_1 - l_0)$$

$\downarrow$   
Speed of sound in air

(\*) It will be valid for successive

Beats

$$\Delta f = \frac{1}{f_2 - f_1}$$

also,  $f = f_2 - f_1$

$$\text{no. of max/sec} = f_2 - f_1$$

Doppler's Effect (not in syllabus)

$$f' = f_0 \left( \frac{v \pm v_o}{v \pm v_s} \right)$$