Straight lines

(1) Sectional Formula →

$$(x, y) = \left\{ \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right\}$$

(2) Area →

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(3) Slope, $m_1 = m_2 \longrightarrow$ {if lines are parallel}

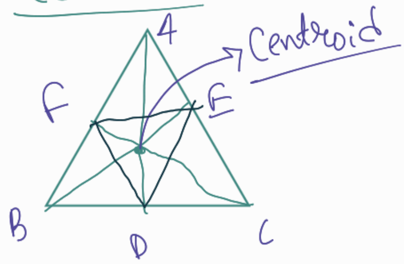
$m_1 \times m_2 = -1 \longrightarrow$ (if lines are $\perp^r$).

(4)

Slope ⇒ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

(*) If lines is equally inclined in a axis then slope $= \pm 1$.

(5) Centroid :-

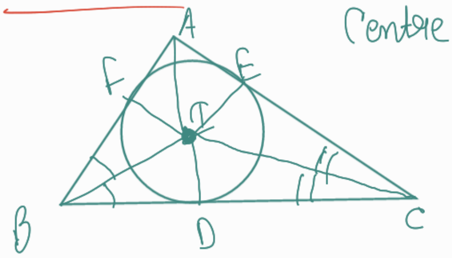


Here, $ar(\Delta DEF) = Ar(\Delta AEF) = Ar(\Delta BDF)$

$= Ar(\Delta CDE) = \frac{1}{4} Ar(ABC)$

Centroid, → $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3} \right)$

(6) Incentre :-



Centre = Incentre, Radius = inradius

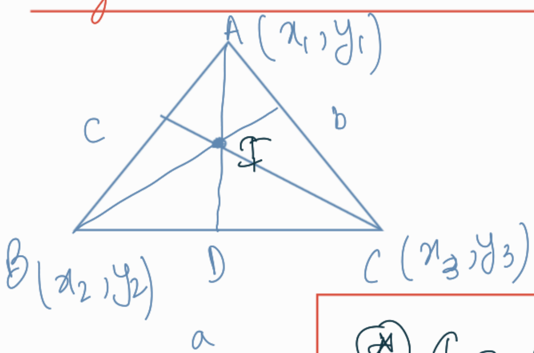
$\Delta IBD \cong \Delta IBF$

So, IA, IB and IC are bisects ∠A, ∠B, ∠C respectively

IF, IE and ID are radius of circle.

$$\boxed{r = \frac{\Delta\, Ar}{S}}$$

(7) Angle bisector theorem :-



$BD : DC = AB : AC$

$= c : b$

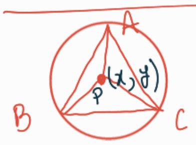
$BD = \dfrac{ac}{b+c}$

$DC = \dfrac{ab}{b+c}$

$AI : ID = (b+c) : a$

$$\boxed{(*)\ I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)}$$

(8) Circumcentre →

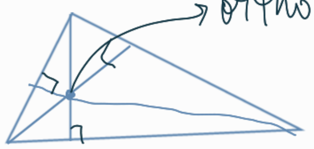


$PA = PB = PC = r$

(*) Use distance formula and equate radius of circle to find $P(x, y)$.

NOTE :- For centroid and incentre, circle is inside Δ.
But for circumcentre, circle can be anywhere.

(9) Orthocentre :-



⊛ use the concept of $\perp^r$ slopes to find co-ordinates of orthocentre

(10) Relation b/w orthogonal, centroid and circumcentre →

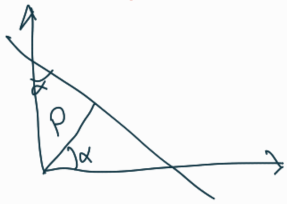
$$\underset{H\,(\text{ortho})}{\bullet}\xrightarrow{\quad 2 \quad}\underset{G\,(\text{centroid})}{\bullet}\xrightarrow{\quad 1 \quad}\underset{P\,(\text{circum})}{\bullet}$$

$\therefore HG : GP = 2:1$

(11) Intercept form of a line →

$y = mx + c$, $y - y_1 = m(x - x_1)$, $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

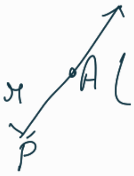
(12) Normal form of line →



Line P → $x\cos\alpha + y\sin\alpha = P$.

13 Parametric form of a line →

slope = $\tan\theta$, $AP = r$



$\therefore P(x_1 \pm r\cos\theta,\ y_1 \pm r\sin\theta)$

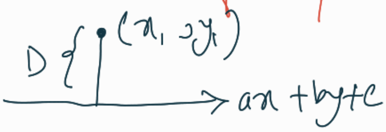
14 Angle b/w 2 lines →

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

if, m_1 → defined and m_2 → not defined then,

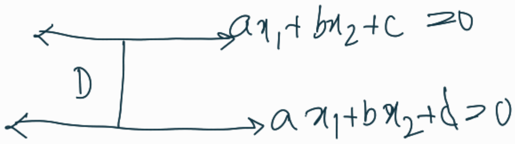
$$\tan\theta = \left|\frac{1}{m_1}\right|.$$

(15) Distance of a point from a line →



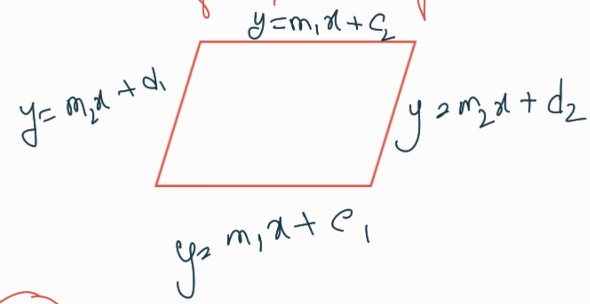
$$\Rightarrow D = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

(16) Distance b/w two parallel lines →



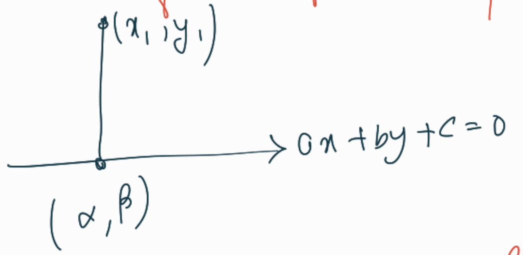
$$D = \left|\frac{c - d}{\sqrt{a^2 + b^2}}\right|$$

(17) Area of parallelogram whose side is given →



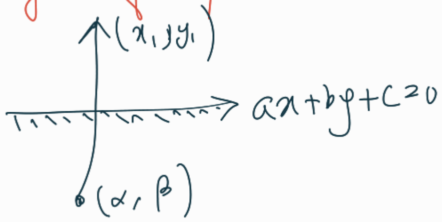
$$\text{Area} = \left| \frac{(c_1 - c_2)(d_1 - d_2)}{(m_1 - m_2)} \right|$$

(18) Foot of ⊥ʳ from a point to a line →



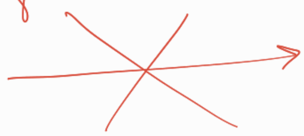
$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

19 Image of a point in a line →



$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

20 Condition of Concurrency of 3 lines →



lines → $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$a_3x + b_3y + c_3 = 0$

Lines to be concurrent →

**

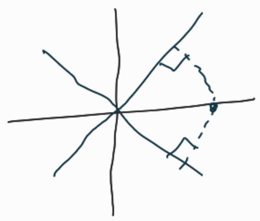
$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

NOTE :- Lines may be concurrent with same slope.

(21) Eqⁿ of Angle bisector :-

$L_1 \rightarrow a_1x + b_1y + c_1 = 0$

$L_2 \Rightarrow a_2x + b_2y + c_2 = 0$



∴ Angle bisector →

$$\left| \frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} \right| = \left| \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}} \right|$$

⊛ Acute and Obtuse Angle bisector :-

Check if $a_1 a_2 + b_1 b_2 > 0$ then bisector with $\oplus$ sign bisects the obtuse angle

if $a_1 a_2 + b_1 b_2 < 0$ then, bisector with $\ominus$ sign bisects obtuse angle

Properties of angle bisector →

Read from class Copy.

Circles

(1) Eqn of circle :-

$x^2 + y^2 = r^2$ (if circle is at origin)

else, $(x-\alpha)^2 + (y-\beta)^2 = r^2$