Waves

(i) $v = f \times \lambda$ (ii) $\frac{1}{\lambda}$ = Wave no. (iii) $k = \frac{2\pi}{\lambda}$ → Angular wave number.

(iv) $\omega = 2\pi f$ (v) $\frac{\omega}{k} = v = f \times \lambda$

Wave Equ^n

$$y(x,t) = A\sin(\omega t - kx + \phi)$$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x), \quad V_p = -V_\omega \text{ slope}$$

Checking whether a given equ^n is a wave equ^n

differentiate 2 time w.r.t x and 2 times w.r.t t, then divide if const. value comes, then the equ^n is a wave equ^n.

$$\boxed{\frac{d^2y}{dt^2} = V_\omega \times \frac{d^2y}{dx^2}}$$

Wave Speed on string

$$V = \sqrt{\frac{T}{\mu}}$$

T → Tension, μ → Mass/unit length

Variable Mass

$$t = \sqrt{\frac{4L}{g}}$$

Energy

(i) $\frac{dK}{dx} = \frac{1}{2}\mu A^2\omega^2\cos^2(\omega t - kx + \phi)$

(ii) $\frac{dU}{dx} = \frac{1}{2}\mu A^2\omega^2\cos^2(\omega t - kx + \phi)$

(iii) Total energy, $\frac{dE}{dx} = \frac{dK}{dx} + \frac{dU}{dx} = \mu A^2\omega^2\cos^2(\omega t - kx + \phi)$

(iv) Power, $P = \frac{dE}{dx} \times V = V\mu A^2\omega^2\cos^2(\omega t - kx + \phi)$

(v) P_{avg}, $\langle P \rangle = 2\pi^2 A^2 f^2 \mu v$.

(vi) Intensity, $I = \frac{P}{A} = 2\pi^2 A^2 f^2 \rho v$

Reflection of Wave

Fixed End

Free end

(*) Equation of standing waves :-

$$y_{net} = 2A \sin(\omega t) \cos(kx)$$

$2A \cos - \sin -$

$2A \sin - \sin -$

$2A \cos - \cos -$

(*) All particles has different 'A'

(*) All are in same phase

Node $(A = 0)$

$$x = \frac{n\lambda}{2}$$

Antinode (A_{max})

$$x = \frac{\lambda}{4}(2n+1)$$

(*) The phase difference of both sides of nodes $\rightarrow \pi$

Standing Wave on string

(i) Fixed End :-

$$f_0 = \frac{V}{2L} \quad \text{(fundamental frequency)}$$

or,

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

∴ Second Harmonic $\Rightarrow f = 2f_0$

(1st Overtone) $\Rightarrow f = 2\left(\frac{V}{2L}\right)$

Third Harmonic $\Rightarrow f = 3f_0 = 3\left(\frac{V}{2L}\right)$

(2nd Overtone)

(vi) Free End :-

$$f_0 = \frac{v}{4L}$$ (fundamental frequency)

∴ 3rd Harmonic :- $f = 3f_0 = 3\left(\frac{v}{4L}\right)$

(1st Overtone)

5th Harmonic :- $f = 5f_0 = 5\left(\frac{v}{4L}\right)$

(2nd Overtone)

Longitudnal Waves :-

* $S = S_0 \sin\left(\omega\left(t - \frac{x}{v}\right) + \phi\right)$
* $\Delta V = A\, ds.$

* Bulk Modulus $= \frac{\Delta P}{(-\Delta V/V)}$
* $\Delta P = BKS_0 \cos(\omega t - kx + \phi)$
* $\Delta P_0 = BKS_0$ → Pressure Amplitude

Some imp results

$V = \sqrt{\frac{Y}{\rho}}$ → velocity , $V = \sqrt{\frac{\beta}{\rho}}$ → velocity

Newton

$V_{gas} = \sqrt{\frac{P}{\rho}} \approx 100\sqrt{10}$

↓

Volume

Laplace Correction :-

$V = \sqrt{\frac{\gamma P}{\rho}} \approx 332\ m/s$

also, $V = \sqrt{\frac{\gamma RT}{M_0}}$ → always in kg/mol

* $\langle I \rangle = \frac{(\Delta P_0)^2}{2\rho v}$, * $\langle I \rangle \propto (\Delta P_0)^2$

Loudness

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

$I \rightarrow$ Intensity level

$I_0 \rightarrow$ Reference level (10^{-12})

$\beta \rightarrow$ decibel

$I_0 = 10^{-2}$

Standing Waves

(i) Open organ pipe (similar to fix end)

$$f_0 = \frac{v}{2L}$$

2nd Harmonic $= 2f_0 = 2\left(\frac{v}{2L}\right)$
(1st overtone)

Similar
3rd Harmonic $= 3f_0 = 3\left(\frac{v}{2L}\right)$
(2nd overtone)

(ii) Closed organ pipe

$$f_0 = \frac{v}{4L} \quad \text{(fundamental frequency)}$$

$\therefore$ 3rd harmonic and 1st overtone $= 3f_0 = 3\left(\frac{v}{4L}\right)$

also, 5th harmonic and 2nd overtone $= 5f_0 = 5\left(\frac{v}{4L}\right)$

End Corrections

$$\boxed{\frac{3\lambda}{4} = l + e}$$

; $e = 0.6 \times$ radius of tube

Resonance Tube

$$v = 2f(l_1 - l_0)$$

$v \rightarrow$ speed of sound in air

(It will valid for successive)

Beats

$\Delta t = \frac{1}{f_2 - f_1}$, also, $f = f_2 - f_1$

$\boxed{\text{no. of max/sec} = f_2 - f_1}$

Doppler's Effect (not in syllabus)

$$f' = f_0 \left(\frac{v \pm v_0}{v \pm v_s} \right)$$