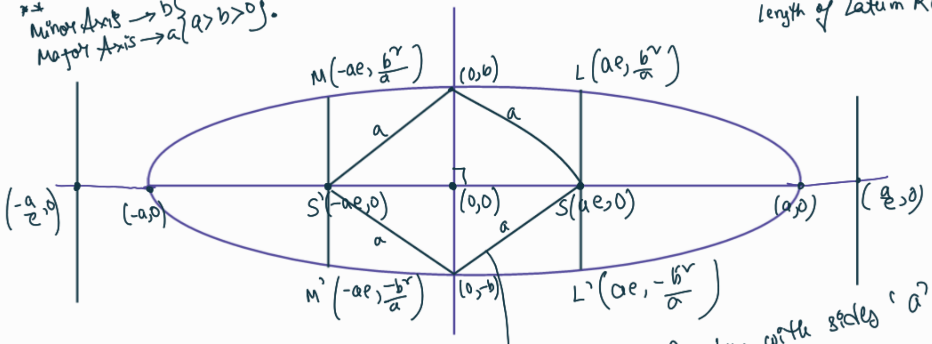
Ellipse

(i) Standard Equⁿ of ellipse →

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \; (a>b>0)$

** Minor Axis → b, Major Axis → a $\{a>b>0\}$.

Length of Latum Rectum $= \frac{2b^2}{a}$, $e=\sqrt{\frac{a^2-b^2}{a^2}}$

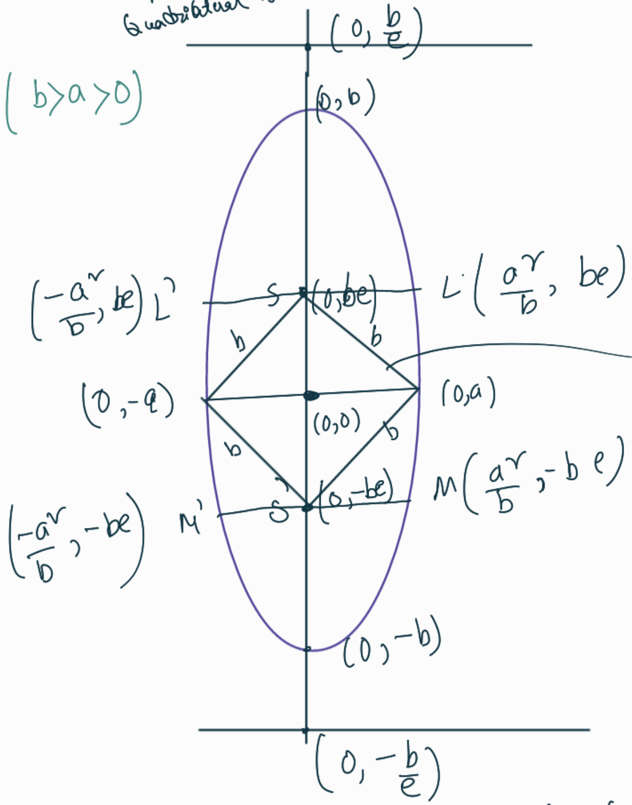


Quadrilateral is a Rhombus with sides 'a'.

(ii) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \; (b>a>0)$

Length of Latus Rectum $= \frac{2a^2}{b}$

$e=\sqrt{\frac{b^2-a^2}{b^2}}$



Quadrilateral is a Rhombus, with sides 'b'.

(*) Use the same concept if center is shifted to (h,k) from origin.

(ii) Auxilary Circle :→

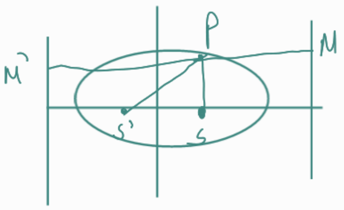
take major axis as diameter and draw a circle

(iii) Parametric form of an ellipse :-

For an ellipse → $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

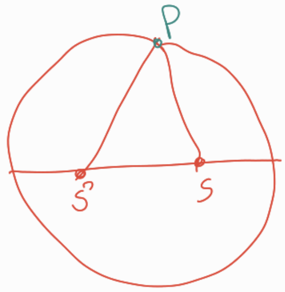
Parametric form : $(a\cos\theta, b\sin\theta)$

(iv) Focal Distance in an ellipse :-



$\boxed{PS + PS' = \text{Length of the major axis}}$

(v)



S and S' are fixed points and P is a variable moving point

$\therefore PS + PS' = \text{const.}$ (Let say K)

$\boxed{K > SS'}$, K = length of major axis.

(vi) Position of a point w.r.to ellipse

$P(x_1, y_1) \rightarrow$ put in the equⁿ of ellipse and get S.

if, $S > 0$ (outside ellipse)

$S = 0$ (lies in the ellipse)

$S < 0$ (inside ellipse)

(vii) Equⁿ of tangent –

For tangent, do $\rightarrow T = 0$

if, slope m is given, then $y = mx \pm \sqrt{a^2m^2 + b^2}$ is the tangent

(viii) Equⁿ of director circle $\rightarrow$

$$\boxed{x^2 + y^2 = a^2 + b^2}$$

(ix) Equⁿ of tangent on parametric form $\rightarrow$

$P(x_1, y_1) = (a\cos\theta, b\sin\theta)$

$\therefore$ Equⁿ $\rightarrow T = 0$

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \Rightarrow \boxed{\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1}$$

(x) Equⁿ of Normal $\rightarrow$

(i) $\boxed{\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2}$

(ii) On parametric form $\rightarrow$

Simply put $\rightarrow (x_1, y_1) \rightarrow (a\cos\theta, b\sin\theta)$

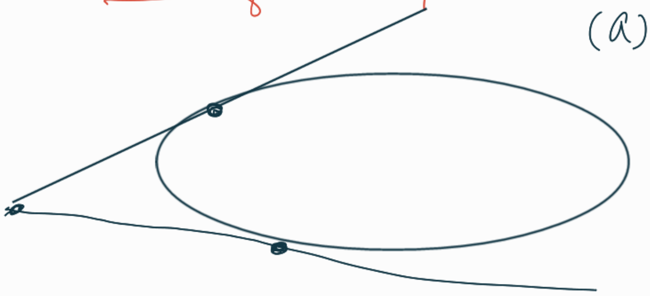
$$\frac{a^2x}{a\cos\theta} - \frac{b^2y}{b\sin\theta} = a^2 - b^2$$

$$\boxed{\Rightarrow \frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2}$$

** Result — Length of major axis $= 2a$, Length of minor axis $= 2b$

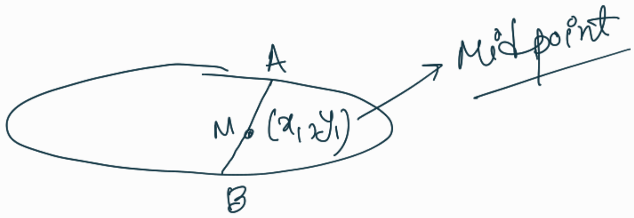
Then, distance of normal from center $\leq (a-b)$.

(xi) Chord of an ellipse :→



(a) Equⁿ of C.O.C → $(T=0)$

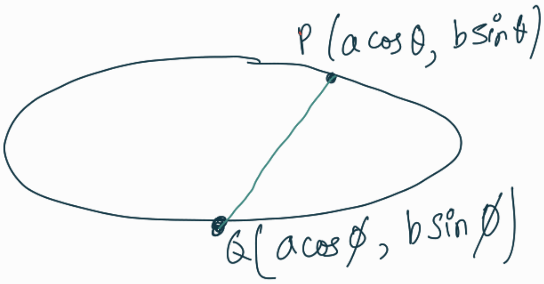
$$\frac{xx_1}{a} + \frac{yy_1}{b} = 1.$$



(b) $T = S_1$

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b} - 1$$

(c)



Then equⁿ of Chord becomes, (PQ)

$$\frac{x}{a}\cos\left(\frac{\theta+\phi}{2}\right) + \frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$$

Result — If Chord passes through $S(ae, 0)$ →

then, $$\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{e-1}{e+1}$$

If chord passes through $S(-ae, 0)$ →

$$\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{e+1}{e-1}.$$

* Some Properties of ellipse :-

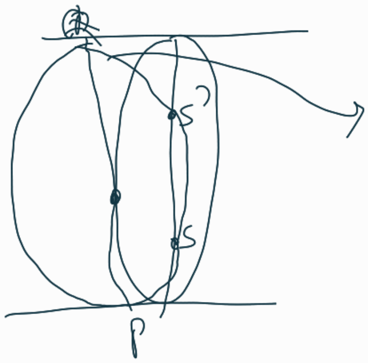
(i)

Here, $P_1 \times P_2 = (\text{semi-major axis})^2$.

(ii) Feet of ⊥ʳ drawn from either focus meets on the director circle

(iii) Reflection Property :->

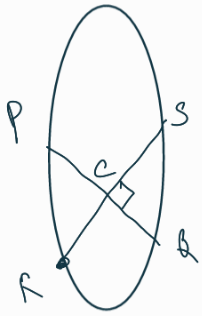
(iv)



Circle on diameter PQ.

A circle is drawn PQ as diameter, then the circle also passes through the focii of the ellipse.

(v)



$$\Rightarrow \frac{1}{(CP)^2} + \frac{1}{(CR)^2} = \frac{1}{a^2} + \frac{1}{b^2}$$