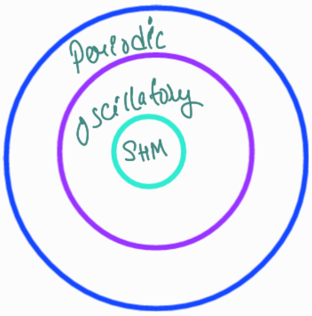
# Simple Harmonic Motion




$F \propto x$

$F = -\boxed{K}x$ → SHM constant

(*) SHM possible in stable equilibrium

$F = -Kx \Rightarrow a = -\left(\frac{K}{m}\right)x$

$\therefore \; a = -\omega^2 x \;, \; \omega = \sqrt{\frac{K}{m}}$

(*) $x(t) = A\sin(\omega t + \phi)$

$\Rightarrow v(t) = A\omega\cos(\omega t + \phi)$

$\Rightarrow a(t) = -A\omega^2\sin(\omega t + \phi) = -\omega^2 x.$

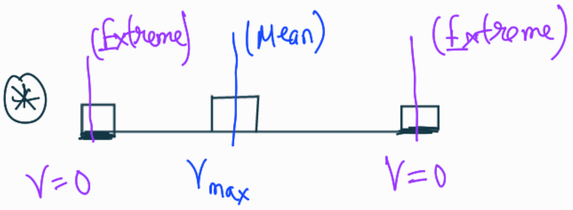
(*) velocity at any point →

$v = \omega\sqrt{A^2 - x^2}$

also,

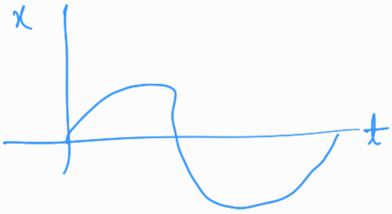
$$\boxed{\frac{v^2}{\omega^2 A^2} + \frac{a^2}{\omega^4 A^2} = 1}$$ → represents eqn of ellipse

(*)

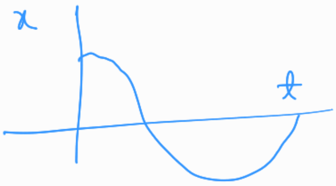



## (*) Graphs

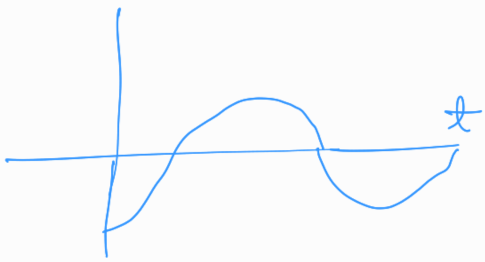
Displacement →




velocity →




acceleration →

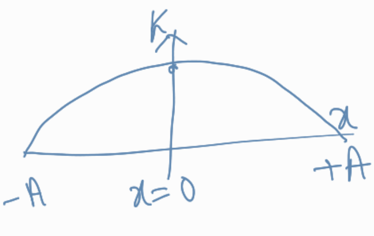



## Energy

$K.E = \frac{1}{2}K(A^2 - x^2)$

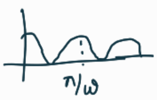
$P.E = \frac{1}{2}Kx^2$

$T.E = \frac{1}{2}KA^2$



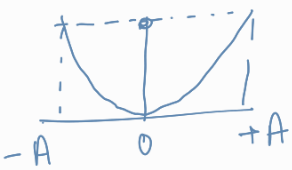

K.E

$$\boxed{\begin{array}{l} K.E_{max} = P.E_{max} = \frac{1}{2}KA^2 \end{array}}$$

P.E =  , K.E = 



* frequency of K.E of a body of S.H.M = 2f, if frequency of S.H.M = f.




P.E

* Time period on a spring :- $\boxed{T = 2\pi\sqrt{\frac{M}{K}}}$ , $\frac{M}{K}$ doesn't depend upon Amplitude

* $K$

Series

$$\frac{1}{K_s} = \frac{1}{K_1} + \frac{1}{K_2}$$

parallel

$$K_p = K_1 + K_2$$

* Spring Pulley Arrangement
* phasors
* Energy Method

Read from class notes

* Angular S.H.M

$$T = 2\pi\sqrt{\frac{l}{g}}$$

, $\theta = \theta_0 \sin(\omega t + \phi)$

$$\underbrace{\omega}_{\text{Angular velocity}} = \frac{d\theta}{dt} = \theta_0 \underbrace{\omega}_{\text{Angular frequency}} \cos(\omega t + \phi)$$

* physical Pendulum :

$$T = 2\pi\sqrt{\frac{I_0}{MgD}}$$

* Torsinal Pendulum :-

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$C \rightarrow$ Torsinal constant

Calculations on S.H.M —

(i) $A_{net} = \sqrt{(A_1^2 + A_2^2 + 2A_1A_2\cos\phi)}$ , $\phi$ = phase difference

$X_{net} = A_{net} \sin(\omega t + \phi)$ , $\tan\alpha = \frac{A_1 \sin\phi}{A_1 + A_2 \cos\phi}$

Damped Oscillation :

$$F_{net} = -kx - bv \Rightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\boxed{A = A_0 e^{-bt/2m}}$$

$b \rightarrow$ drag const., $m$ = mass

$$\omega' = \sqrt{\left(\frac{k}{m}\right) - \left(\frac{b}{2m}\right)^2}$$