E M I

Physics

Electromagnetic Induction (EMI)

① Magnetic flux, $d\phi = \vec{B}\cdot d\vec{A} = B\cdot dA\cdot\cos\theta$ (θ is angle b/w area and M·F)

or, $\phi = \int \vec{B}\cdot d\vec{A}$ Unit = Weber.

Conditions :→ $\boxed{\text{Surface} \perp \text{M·F} \;/\; \vec{A} \parallel \text{M·F}}$ $\Rightarrow \boxed{\theta = 0° \text{ or } 180°}$

② E.M.F induced $= \left|\frac{d\phi}{dt}\right|$

$i = \frac{|\varepsilon|}{R}$, $\frac{dq}{dt} = \frac{d\phi}{dt}\times\frac{1}{R}$

or, $\Delta Q = \frac{\Delta\phi}{R}$.

$\varepsilon = BA\omega\sin(\omega t)$, ω represents angular velocity.

③ Lenz's Law :→ (only gives direction of induced current)

$\varepsilon = \frac{-d\phi}{dt}$

④ For motional EMF :→

$\varepsilon = -(\vec{v}\times\vec{B})$

$\varepsilon = \int \vec{E}\, d\vec{l}$ (by defination), $\boxed{\varepsilon = \int(\vec{v}\times\vec{B})\,dl.}$

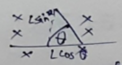
Modifications

(i) if v and B is uniform :→ $\varepsilon = (\vec{v}\times\vec{B})\cdot\vec{L}$

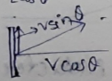
(ii) if $\vec{v}\parallel\vec{B}$ or $\vec{B}\parallel\vec{L}$ or $\vec{v}\parallel\vec{L}$, $\varepsilon = 0$.

(iii) $\varepsilon = BLv$ (all B, L, v should be ⊥ mutually).

(iv) $\varepsilon = BL_\perp v$
$= BL\sin\theta\, v$.



(v) $\varepsilon = BLv_\perp$
$= BLv\sin\theta$.



⑤ Rotating Conductor in a Magnetic field :→

$|\varepsilon| = \left(\frac{B\omega L^2}{2}\right)$; L = distance of the other point from axis.

P.d b/w 2 points in a Rotating rod, of which starting must be axis. $\omega \to$ rad/s

$\omega = \pi\times v$ → v → rps

~~⑥ Conductor on Rails~~

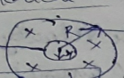
⑥ Time Varying M·F :→

$\varepsilon = \frac{-d\phi}{dt}$, $\varepsilon = -\int \vec{E}\cdot d\vec{r}$ (by defination)

$\boxed{\Rightarrow \int \vec{E}\cdot d\vec{r} = \frac{d\phi}{dt}}$

Derivation

(i) Cylindrical region TVMF



inside (r < R)

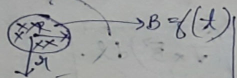
$\int \vec{E}\cdot d\vec{r} = \frac{d\phi}{dt}$

$\Rightarrow E\cdot 2\pi r = \frac{d}{dt}(B\cdot\pi r^2)$

$\Rightarrow \boxed{E = \frac{r}{2}\left(\frac{dB}{dt}\right)}$

$E \propto r$ (linear)

(ii)



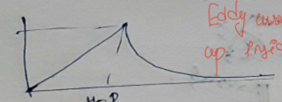
outside, r ≥ R

$\Rightarrow E\cdot 2\pi r = \frac{d}{dt}(B\pi R^2)$

$\Rightarrow \boxed{E = \frac{R^2}{2r}\cdot\frac{dB}{dt}}$

$\therefore E \propto \frac{1}{r}$.

Graph



Eddy currents are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it change

Tesla

$\phi = B\cdot A$ → Area

ϕ = Weber

⑦ Self Induction of inductor ($L \to$ Wb/m → Henry)

$\frac{\phi}{i} = L$

Henry = Weber / Area

(i) $B = \mu_0\left(\frac{N}{l}\right)i$

$\phi = NBA$
$= N\left(\frac{\mu_0 N i}{l}\right)A$

~~$\phi = \frac{\mu_0 N^2 A i}{l}$~~

$\phi = Li$

$\boxed{\therefore L = \frac{\mu_0 N^2 A}{l}}$

$\therefore L = \frac{\mu_0\mu_r N^2 A}{l}$

Dimension :→ $\boxed{[M^{1}T^{-2}L^{2}A^{-2}]}$

⑧ Induced EMF in a coil :→

$\mathcal{E}_s = -L\frac{di}{dt}$.

$L_{series} = L_1 + L_2 + L_3$

⑨ Energy stored in inductors :→

$U = \frac{1}{2}Li^2$

$\frac{1}{L_{||}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

⑩ Energy Density :—

$f = \frac{U}{V}$ → energy, → volume.

$\boxed{\therefore f = \frac{B^2}{2\mu_0}}$

$(*)\ \left(i_1 = \frac{L_2}{L_1+L_2} \times i \right)$

$(*)\ \left(i_2 = \frac{L_1}{L_1+L_2} \times i \right)$

11 Growth of current in RL-circuit

(i) $i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$., $i_0\,(i\ max) = \frac{V}{R}$

at $t=0$	at $t=\infty$ (Steady state)
$i=0$	$i_{max} = i_0 = \frac{V}{R}$.
(L behaves like open circuit)	(L behaves like a short wire).

(ii) Time Constant, $\tau = \frac{L}{R}$, $i = i_0\left(1 - e^{-\frac{t}{\tau}}\right)$.

(iii) Half life / or become $\frac{1}{2}$ of i_0

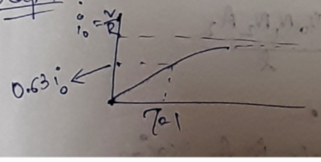
$\boxed{t = \tau \ln 2}$.

$\boxed{e = 2.7,\ \frac{1}{e} = 0.37}$.

(iv) current after 1τ :— $i = i_0 \times 0.63$ i.e 63% of maximum i_0

(v) $t = \tau \ln\left(\frac{i}{i_0 - i}\right)$.

Graph :

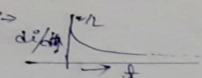


(vi) $i = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$

$\therefore \frac{di}{dt} = \frac{V}{L} e^{-t/\tau}$, at $t=0$, $\frac{di}{dt} = \frac{V}{L}$ (Maximum)

at $t=\infty$, $\frac{di}{dt} = 0$ (Minimum)

Graph :→

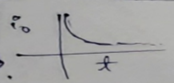


12 Decay in L-R circuit :→

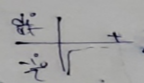
$i = i_0 e^{-t/\tau}$

In 1τ, $i = 0.37 i_0$ i.e 37% of maximum

Half life :→ $\boxed{t = \tau \ln 2}$.



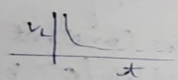
$\frac{di}{dt} = i_0 e^{-t/\tau}\left(\frac{-1}{\tau}\right)$



$V_L = -L\frac{di}{dt}$

$V_L = L\frac{i_0}{\tau} e^{-t/\tau}$

$V_L = i_0 R \cdot e^{-t/\tau}$



(13) Mutual Inductance

(i) ${}_1\phi_2 = N_2 B_1 A_2$

${}_1\phi_2 = N_2 \frac{\mu_0 N_1 i_1}{l} A_2$, $B_1 = \frac{\mu_0 N_1 i}{l}$

$({}_1\phi_2) = \frac{\mu_0 N_1 N_2 A_2}{l} \times i$

$\boxed{{}_1\phi_2 = {}_1M_2\, i}$, ${}_1M_2 = \frac{\mu_0 N_1 N_2 A_2}{l}$

vice versa for ${}_2\phi_1$, B_2, ${}_2M_1$.

Note :→ M.F of coil 2 is only in A_2 region

(i) $M = \frac{\mu_0 N_1 N_2 A}{l}$ → Common Area

(ii) $\phi_2 = M i_1$, (iii) $\phi_1 = M i_2$

$\mathcal{E}_2 = -M \frac{di_1}{dt}$, $\mathcal{E}_1 = -M \frac{di_2}{dt}$

(iv) $\boxed{{}_2M_1 = {}_1M_2}$ → for all.

$M = k\sqrt{L_1 L_2}$

Coefficient of coupling

% of flux Linkage

100% → $k = 1$

50% → $k = 1/2$

Conductor on Rails

Formula's (only)

(i) $\mathcal{E} = Blv$ (ii) $i = \frac{\mathcal{E}}{R}$ (iii) $i = \frac{Blv}{R}$

(iv) $f = ilB$

$= \frac{Blv}{R} \times lB = \frac{B^2 l^2 v}{R}$

ext. force required for constant velocity.

Power Loss →

$P = i^2 R$

$= \left(\frac{Blv}{R}\right)^2 \times R$

$\boxed{P = \frac{B^2 l^2 v^2}{R}}$

or,

$P = F_{ext} \times v$

$= ilB \times v = \frac{Blv}{R} \times lB \times v$

$\boxed{P = \frac{B^2 l^2 v^2}{R}}$

LC Oscillations

(i) $\frac{Q^2_{max}}{2C} = \frac{1}{2} L i^2_{max}$

(ii) $\frac{1}{2} L i^2 + \frac{Q^2}{2C} = \frac{1}{2} L i^2_{max}$

(iii) $\frac{1}{2} L i^2 + \frac{Q^2}{2C} = \frac{Q^2_{max}}{2C}$

U_{total}