Units and Dimensions

Physical Quantities

Fundamental (7) → Total 7
Mass, Length, time, Temperature, Luminous Intensity, Current, Amount of substance
([cd])

Derived
Volume, Area, Speed, Pressure, Force, etc

System of Units

- MKS → SI unit
- CGS
- FPS

Supplementary Units

→ plane angle, $\theta = \frac{\text{arc}}{\text{radius}}$ → unit is Radian

→ Solid Angle, $\Omega = \frac{\text{Surface Area}}{(\text{Radius})^2}$ → unit is Steradian

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Dimensional Analysis

Q) 1

$1\ J = \_\_\_\_\_\ erg$

$1\ kg\ \frac{m^2}{s^2} = \alpha\ gm\ \frac{cm^2}{s^2}$

$\Rightarrow 1\left(\frac{10^3\ gm}{gm}\right)\left(\frac{100\ cm}{cm}\right)^2 = \alpha$

$\Rightarrow \alpha = 10^7$ (Ans)

Q)2 $g = 10\ m/s^2$ ---- $\frac{cm}{min^2}$

$= 10\left(\frac{100\ cm}{cm}\right) \times \left(\frac{min}{s}\right)^2 = 10^3 \times \left(\frac{60\ s}{s}\right)^2$

$= 10^3 \times 36$

$= 36 \times 10^5 = 3.6 \times 10^6$ (Ans)

Q)3 $V = As^2 + \frac{B}{C+t^2}$, $B = ?$

$\Rightarrow \frac{B}{[T^2]} = [LT^{-1}] \Rightarrow B = [LT]$ (Ans)

Q)4) $M \propto v^x s^y g^z \Rightarrow M = c\, v^x s^y g^z$

$[M] = [LT^{-1}]^x [ML^{-3}]^y [LT^{-2}]^z$

$\Rightarrow [M]^1 = [L]^{x-3y+z} [T]^{-x-2z} [M]^y$

$\Rightarrow$ On comparing $\rightarrow$ $y = 1$ | $x - 3y + z = 0$ | $-x - 2z = 0$

On calculating $\rightarrow$

$x = 6, y = 1, z = -3$

Q)5 $V \propto f^x \mu^y$

$\Rightarrow V = \sqrt{\frac{T}{\mu}} \Rightarrow V = f^{1/2} \mu^{-1/2}$ (Ans)

Q)6 $F = AB \sin(Bt)$ $\rightarrow [MLT^{-2}] = A\,[T]$

$\Rightarrow A = [ML]$ (Ans)

Significant Figures

Rule-1 :- All non-zero digits are significant.

Rule-2 :-

Zero

- **Leading zeros** — never significant
  - 0.00 3
  - 0.02
  - 0.1
- **In b/w zeros** — Always significant
  - 1.003
- **Trailing zero**
  - with decimal — Always significant
    - 3.0 → 2
    - 2.00 → 3
  - without decimal — never significant
    - 100 → 1

Rules for Addition/subtraction

In +/- , we see the digits after decimal.
And the number with least decimal values — will be there in answer.

Rules for Multiplication/Division :-

Count the total significant figures and then put the least significant figures in answer.

Errors

Error = True value - Measured value

- Random Error
- Systematic Error → Error due to machines.

Calculation of Random Error

Relative Error → $\frac{\text{Mean Absolute Error}}{\text{True value}}$

%age Error → $\frac{MAE}{TV} \times 100$

(*)

2.04 →	-0.03
2.05 →	-0.04
2.01 →	0
1.96 →	0.05
1.99 →	0.02
$R_{avg} = 2.01$	0.02

$\therefore$ Error = 2.01 ± 0.02 (Ans)

$R_{avg} = T.V$

(*) Error = $T.V / R_{avg} \pm$ Mean Absolute value

Q) % error in measurment of 20.1 Ω resistance is 4%. Then the resistance →

⇒ % error : $\frac{MAE}{TV} \times 100$

⇒ $4 = \frac{MAE}{20.1} \times 100$

⇒ $MAE = \frac{4 \times 20.1}{100} = \frac{20.1}{250} \approx 0.08$

∴ Representation → 20.1 ± 0.08 (Ans)

Q) $T = (10 \pm 0.2)$ S , %age error = ?

⇒ % error = $\frac{0.2}{10} \times 100 = 2\%$ (Ans)

Propagation of Error

→ Addⁿ / Subtraction

→ Multiply or divide

(1) Addⁿ / Subtraction

$A = a \pm \Delta a$

$B = b \pm \Delta b$

$A+B = (a+b) \pm (\Delta a + \Delta b)$

$A-B = (a-b) \pm (\Delta a - \Delta b)$

(2) Multiplication / Division

$A = l \times b$

$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$

(Q) $A = 2.5 \text{ m/s} \pm 0.5 \text{ m/s}$

$B = 0.10 \text{ s} \pm 0.01 \text{ s}$

$C = AB = ? \quad 2.5 \times 0.1 = 0.25$

$\frac{\Delta C}{C} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

$\Rightarrow \Delta C = \left(\frac{0.5}{2.5} + \frac{0.01}{0.10} \right) \times 0.25$

$= \left(\frac{50}{25} + \frac{1}{10} \right) 0.25$

$= \left(\frac{1}{5} + \frac{1}{10} \right) \times 0.25 = 0.08 \text{ m}$ (Ans)

(*) $z = \frac{x^a y^b}{r^c}$

$\Rightarrow \log z = a \log x + b \log y - c \log r$

$\Rightarrow \frac{dz}{z} = \frac{a}{x} dx + \frac{b}{y} dy - c \frac{dr}{r}$

$\Rightarrow \frac{dz}{z} \times 100 = \left(\frac{a}{x} dx \right) 100 + \left(\frac{b}{y} \right) dy \times 100 + c \frac{dr}{r} \times 100$

$\Rightarrow \%z = a(\%x) + b(\%y) + c(\%r)$

(Q) $\Delta r\% = 2\%$

error in volume ?

$\Rightarrow V = \frac{4}{3} \pi r^3$

$\therefore \Delta V\% = 3(\Delta r\%)$

$= 3 \times 2\% = 6\%$ (Ans)

(Q) $T = 2\pi \sqrt{l/g}$

$\Delta l\% = 2\% \quad \Delta g\% = 4\%$

$\Rightarrow \frac{1}{2} \times 2 + \frac{1}{2} \times 4 = 3\%$ (Ans)

(Q) $R = \frac{V}{I}, \quad V = 100 \pm 5$

$I = 10 \pm 0.2$

$\Rightarrow \left(\frac{\Delta R}{R} \right) \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

$= \frac{5}{100} \times 100 + \frac{0.2}{100} \times 100$

$= 7\%$ (Ans)

JEE Q (Errors)

(1) $T = 2\pi\sqrt{l/g} \Rightarrow T^2 = 4\pi^2 l/g \Rightarrow g = 4\pi^2 \frac{l}{T^2}$

Given:
Time period for 20 oscillation of the pendulum is found to be 100 second using a clock of 1s resolution

$$T = \frac{100 \pm 1}{20}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

$$\Rightarrow \%g = \%l + 2(\%T)$$

$$\Rightarrow \%g = \frac{0.1}{10} \times 100 + 2\left(\frac{\Delta T}{T} \times 100\right)$$

$$= 1 + 2\left(\frac{1/200}{100/200} \times 100\right)$$

$$= 3\% \text{ (Ans)}$$

Q/2 $T = 2\pi\sqrt{l/g} \Rightarrow T^2 = 4\pi^2 l/g \Rightarrow g = 4\pi^2 \frac{l}{T^2}$

Measured value of L is 1 m from meter scale having minimum division of 1mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01s resolution.

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = \left(\frac{\Delta l}{l} \times 100\right) + 2\left(\frac{\Delta T}{T} \times 100\right)$$

$$= \left(\frac{10^{-3}}{1} \times 100\right) + 2\left(\frac{0.01}{1.95} \times 100\right)$$

$$= 0.1 + \frac{100}{195}$$

$$= 0.1 + 1.02 = 1.12\%$$

error in g (Ans)

Q/3 $Z = \frac{a^2 b^{2/3}}{c^{1/2} d^3}$

$$\Rightarrow \%\text{ error in } Z = 2(\%a) + \frac{2}{3}(\%b) + \frac{1}{2} \times (\%c) + 3(\%d)$$

$$= 2(2) + \frac{2}{3} \times \frac{3}{2} + \frac{1}{2} \times 4 + 3 \times \frac{2.5}{2}$$

$$= 4 + 1 + 2 + 7.5$$

$$= 14.5\% \text{ (Ans)}$$