3D Geometry

DC $l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$

$$l^2 + m^2 + n^2 = 1 \quad **$$

DR $\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \lambda$

D.R = a, b, c.

$a = \lambda l,\ b = \lambda m,\ c = \lambda n,\ \lambda \in R - \{0\}$

D.R $\propto$ (a, b, c)

(*) $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k} \Rightarrow |\vec{r}| = \sqrt{a^2+b^2+c^2}$

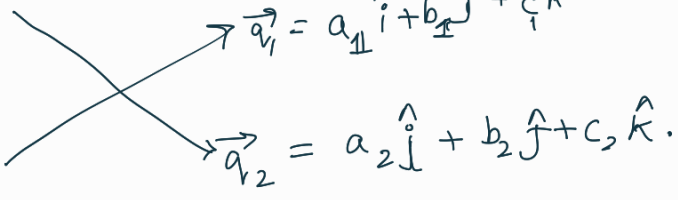
$$l = \cos\alpha = \frac{a}{|\vec{r}|},\quad m = \cos\beta = \frac{b}{|\vec{r}|},\quad n = \cos\gamma = \frac{c}{|\vec{r}|}$$

a, b, c are D.R.

Whereas; l, m, n are D.C.

(*) **Angle between 2 lines :→**

$$\cos\theta = \frac{\vec{a_1}\cdot\vec{a_2}}{|a_1||a_2|}.$$

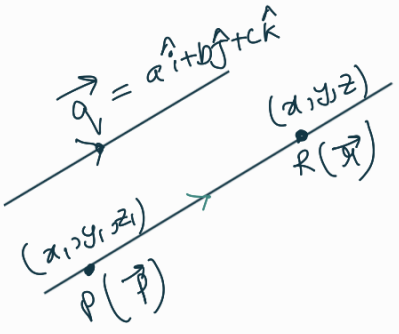


(*) **Straight line in 3-D**

$\vec{PR} = \lambda\vec{q}$

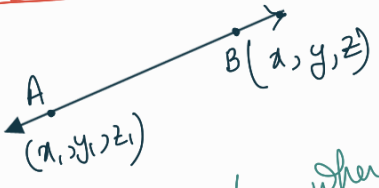
$\vec{r} - \vec{p} = \lambda\vec{q}$

$$\vec{r} = \vec{p} + \lambda\vec{q} \quad **$$



(*) **Equn of line in cartesian form :→**

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$



a, b, c are D.R and x_1, y_1, z_1 are the point of the lines, from where it passed.

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

* Point of intersection

E.g. $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ | $\frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4}$

$\Rightarrow P(3\lambda+5, -\lambda+7, \lambda-2)$

put here

$\Rightarrow \frac{3\lambda+8}{-36} = \frac{-\lambda+4}{2} = \frac{\lambda-8}{4}$

solve and get λ

Put λ and get the intersecting point.

* Shortest distance b/w two lines

(i)

$\vec{r_1} = \vec{a_1} + \lambda(\vec{b})$

$\vec{r_2} = \vec{a_2} + \lambda(\vec{b})$,

$\vec{d} = \left| \frac{(a_2 - a_1) \times \vec{b}}{|\vec{b}|} \right|$

(ii) For skew lines :-

$$d = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

* NOTE :→

if $[(\vec{a_2} - \vec{a_1}) \ \vec{b_1} \ \vec{b_2}] = 0$, then coplanar

(*) If lines are intersecting, parallel then $d = 0$

** PLANES is not in JEE mains