Manmeet Kaur sidhu
102103478
COE17

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Q1 To find maximum likelihood estimates of the parameters θ_1 (mean) & θ_2 (variance) for a normal distribution, we use likelihood function & then maximize it.

Given that $x_1, x_2, \ldots x_n$ is a random sample from a normal distribution with mean θ_1 & variance θ_2 the likelihood function is :-

$$L(\theta_1, \theta_2 \mid x_1 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi v_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

taking log on both sides

$$\ln L(\theta_1, \theta_2 \mid x_1, \ldots x_n) =$$

$$-\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

to find MLE, we will differentiate the log-likelihood with respect to θ_1 & θ_2, set derivative equal to zero.

(i) for θ_1:

$$\frac{\partial}{\partial\theta_1} \ln\{L(\theta_1, \theta_2 \mid x_1, \ldots x_n) =$$

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

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setting this equal to zero:

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0 \Rightarrow \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0$$

$$\therefore \hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

so, the MLE for θ_1 is the sample mean.

(ii) for θ_2:

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 \mid x_1 \dots x_n) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

setting this eq to 0:

$$\frac{-n}{2\hat{\theta}_2} + \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2 = 0$$

$$\Rightarrow \frac{n}{2\hat{\theta}_2} - \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

$$\Rightarrow \hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

so, MLE for θ_2 is the sample variance

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Q2

To find the MLE of θ for a random sample $x_1, \ldots x_n$ from a Bernoulli distribution with parameter θ & a known m, the liklihood for this scenario is:

$$L(\theta | x_1 .. x_n) = \prod_{i=1}^{n} P(X_i = x_i / \theta)$$

since X_i follow a Bernoulli dist,

$$P(X_i = x_i / \theta) = \theta^{x_i} (1-\theta)^{m - x_i} \text{ for each } i$$

Taking log on both sides:

$$\ln L(\theta / x_1 .. x_n) = \sum_{i=1}^{n} \ln(\theta^{x_i}(1-\theta)^{m-x_i})$$

$$= \sum_{i=1}^{n} (x_i \ln \theta + (m - x_i) \ln (1-\theta))$$

Now differentiate with respect to θ and set to zero

$$\frac{d}{d\theta}(\ln L(\theta / x_1 \ldots x_n)) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right) = 0$$

$$\therefore \theta = \sum_{i=1}^{n} \frac{x_i}{n.m}$$

So, max likelihood estimate for θ is:

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}.$$

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