Chapter 2. The Physical Layer

The Physical Layer

- It defines the mechanical, electrical, and timing interfaces to the network.
- Three kinds of transmission media:
  - Guided (copper wire and fiber optics),
  - Wireless (terrestrial radio),
  - Satellite.
- Three examples of communication systems used in practice for wide area computer networks: **the (fixed) telephone system, the mobile phone system, and the cable television system.**

The Theoretical Basis for Data Communication

- Information can be transmitted on wires by varying some physical property such as voltage or current.
- By representing the value of this voltage or current as a single-valued function of time, $f(t)$, we can model the behavior of the signal and analyze it mathematically.

Fourier Analysis

- In the early 19th century, the French mathematician Jean-Baptiste Fourier proved that any reasonably behaved periodic function, g(t) with period T can be constructed as the sum of a (possibly infinite) number of sines and cosines:

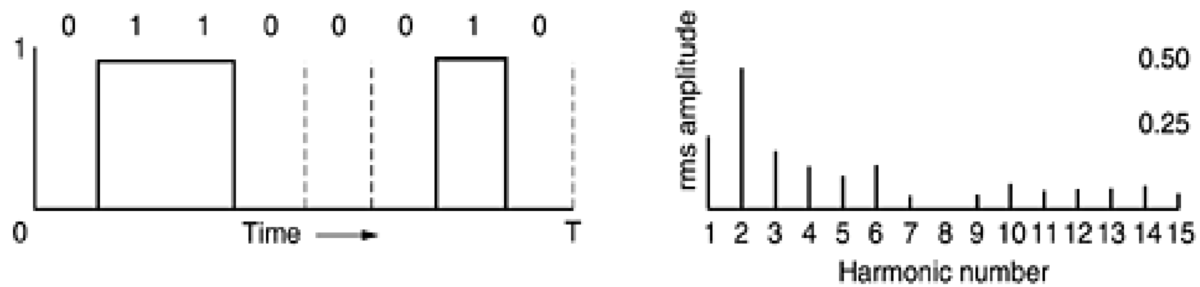
$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

- Where $f = 1/T$ is the fundamental frequency, a_n and b_n are the sine and cosine amplitudes of the n^{th} harmonics (terms), and c is a constant. Such a decomposition is called a Fourier series.

$$a_n = \frac{2}{T}\int_0^T g(t)\sin(2\pi nft)\,dt \qquad b_n = \frac{2}{T}\int_0^T g(t)\cos(2\pi nft)\,dt \qquad c = \frac{2}{T}\int_0^T g(t)\,dt$$

Bandwidth-Limited Signals

- let us consider a specific example: the transmission of the ASCII character ''b'' encoded in an 8-bit byte. The bit pattern that is to be transmitted is 01100010.



(a)

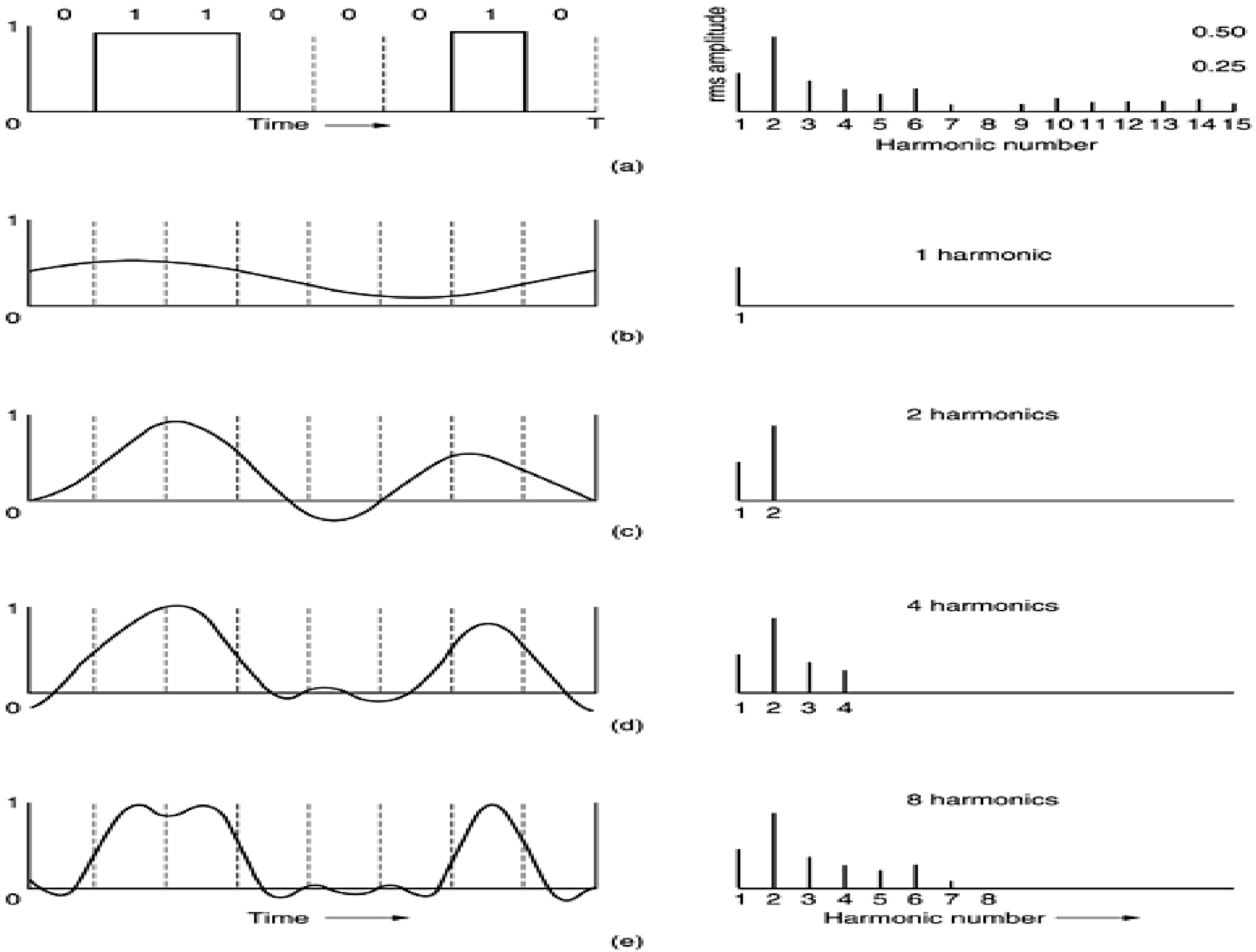
(a) A binary signal and its root-mean-square Fourier amplitudes.

$$a_n = \frac{1}{\pi n}[\cos(\pi n/4) - \cos(3\pi n/4) + \cos(6\pi n/4) - \cos(7\pi n/4)]$$

$$b_n = \frac{1}{\pi n}[\sin(3\pi n/4) - \sin(\pi n/4) + \sin(7\pi n/4) - \sin(6\pi n/4)]$$

$$c = 3/4$$

(b)-(e) Successive approximations to the original signal.



- No transmission facility can transmit signals without losing some power in the process.
- If all the Fourier components were equally diminished, the resulting signal would be reduced in amplitude but not distorted.
- Unfortunately, all transmission facilities diminish different Fourier components by different amounts, thus introducing distortion.
- Usually, the amplitudes are transmitted undiminished from 0 up to some frequency fc [measured in cycles/sec or Hertz (Hz)] with all frequencies above this cutoff frequency attenuated.
- **The range of frequencies transmitted without being strongly attenuated is called the bandwidth.**
- In practice, the cutoff is not really sharp, so often the quoted bandwidth is from 0 to the frequency at which half the power gets through.
- **The bandwidth is a physical property of the transmission medium and usually depends on the construction, thickness, and length of the medium.**
- In some cases a filter is introduced into the circuit to limit the amount of bandwidth available to each customer.
- Figure 2-1(b) shows the signal that results from a channel that allows only the first harmonic (the fundamental, f) to pass through. Similarly, Fig. (c)-(e) show the spectra and reconstructed functions for higher-bandwidth channels.

Relation between data rate and harmonics

Example: Assume you want to send 8 bits at 9600 bps over an ordinary phone line, BW = 3000Hz

- b=9600 bps

The time to send 8 bits is T= 8/b=8/9600 = 0.83 msec.

- The frequency of the first harmonic is 1 / (8/b) = b/8=9600/8 =1/0.83 msec= 1200 Hz (periods per second)
- Ordinary phone lines have an artificial cut-off bandwidth of 3000Hz.

BW = 3000Hz

- Thus the highest harmonic passed through is = BW / (b/8) = 8 BW/ b = 2.5 => highest harmonic is 2
- The signal received would be tricky to reconstruct

=> limiting the bandwidth limits the data rate

Example: Assume you want to send 8 bits at 300 bps over an ordinary phone line. Find out the highest harmonic ?

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- b=300 bps

The time to send 8 bits is 8/b=8/300 = 26.67 msec.

- The frequency of the first harmonic is 1 / (8/b) = b/8=300/8 =1/26.67 msec= 37.5 Hz (periods per second)
- Ordinary phone lines have an artificial cut-off bandwidth of 3000Hz.

BW = 3000Hz

- Thus the highest harmonic passed through is = BW / (b/8) = 8 BW/ b = 8*3000/300 => highest harmonic is 80
- The signal received would be easy to reconstruct

Example: Assume you want to send 8 bits at 38400 bps over an ordinary phone line

- b=38400 bps

The time to send 8 bits is 8/b=8/38400 = 0.21 msec.

- The frequency of the first harmonic is 1 / (8/b) = b/8=38400/8 =1/0.21 msec= 4800 Hz (periods per second)
- Ordinary phone lines have an artificial cut-off bandwidth of 3000Hz.

BW = 3000Hz

- Thus the highest harmonic passed through is = BW / (b/8) = 8 BW/ b = 8*3000/38400 => highest harmonic is 0
- The signal received not possible to reconstruct

=> limiting the bandwidth limits the data rate

(Relation between data rate and harmonics for send a constant of 8 bits over a 3KHz channel.)

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

The Maximum Data Rate of a Channel

- As early as 1924, an AT&T engineer, Henry Nyquist, realized that even a perfect channel has a finite transmission capacity.
- He derived an equation expressing the maximum data rate for a **finite bandwidth noiseless channel.**
- Nyquist proved that if an arbitrary signal has been run through a low-pass filter of bandwidth **H,** the filtered signal can be completely reconstructed by making only **2H** (exact) samples per second.
- Sampling the line faster than 2H times per second is pointless because the higher frequency components that such sampling could recover have already been filtered out.
- If the signal consists of V discrete levels, **Nyquist's theorem** states:

$$\text{maximum data rate} = 2H \log_2 V \text{ bits/sec}$$

- **For example, a noiseless 3-kHz channel cannot transmit binary (i.e., two-level) signals at a rate exceeding 6000 bps.**

- **What is the maximum data rate in a noiseless 6-kHz channel transmitting 16 bit signals .**
- Answer:

$$\text{maximum data rate} = 2H \log_2 V \text{ bits/sec}$$

- H=6 kHz= 6000, V= 16
- Maximum data rate =2* 6000*4=48000 bps

- So far we have considered only noiseless channels. If random noise is present, the situation deteriorates rapidly. And there is always random (thermal) noise present due to the motion of the molecules in the system.
- In 1948, **Claude Shannon** carried Nyquist's work further and extended it to the case of a channel subject to random (that is, thermodynamic) noise.
- The amount of thermal noise present is measured by the ratio of the signal power to the noise power, called the **signal-to-noise ratio**.
- If we denote the signal power by S and the noise power by N, the signal-to-noise ratio is **S/N.**
- signal-to-noise ratio is S/N is also known as **SNR.**
- Usually, the ratio itself is not quoted; instead, the quantity $10 \log_{10} (S/N)$ is given. These units are called decibels (dB).
- **SNR in dB = $10 \log_{10} (S/N)$ dB**
- **<u>Example:</u>**

  - An S/N ratio of 10 is 10 dB,
- The manufacturers of stereo amplifiers often characterize the bandwidth (frequency range) over which their product is linear by giving the 3-dB frequency on each end.
- These are the points at which the amplification factor has been approximately halved (because $\log_{10}3 = 0.5$).

Calculate the SNR in dB

- Signal to noise ratio of 100,
- Signal to noise ratio of 1000.

Answer:

SNR in dB = 10 $\log_{10}$ (100) dB = 10 * 2 =20 dB

SNR in dB = 10 $\log_{10}$ (1000) dB = 10* 3 =30 dB

- **Shannon's** major result is that the **maximum data rate of a noisy channel** whose bandwidth is **H** Hz, and whose signal-to-noise ratio is **S/N**, is given by

$$\text{maximum number of bits/sec} = H \log_2 (1 + S/N)$$

- **For example,** a channel of 3000-Hz bandwidth with a signal to thermal noise ratio of 30 dB (typical parameters of the analog part of the telephone system), S/N=1000, can never transmit much more than 30,000 bps, no matter how many or how few signal levels are used and no matter how often or how infrequently samples are taken.
- Shannon's result was derived from information-theory arguments and applies to any channel subject to thermal noise.

2. A noiseless 4-kHz channel is sampled every 1 msec. What is the maximum data rate?

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ANS:

A noiseless channel can carry an arbitrary large amount of information, no matter how often it is sampled.

Just send a lot of data per sample.

For 4-KHz channel, make 1000 samples/sec.

If each sample is 16 bits, the channel can send 16 Kbps.

If each sample is 1024 bits, the channel can send

1000 samples/sec * 1024 bits = 1024 Mbps

The key word here is "noiseless". With a normal 4 KHz channel, Shannon limit would not allow this.

For the 4 KHz channel we can make 8000 samples/sec.

In this case if each sample is 1024 bits this channel can send 8.2 Mbps.

3. Television channels are 6 MHz wide. How many bits/sec can be sent if four-level digital signals are used? Assume a noiseless channel.

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- Using the **Nyquist theorem**,
- Max. data rate = 2H log_2V bits/sec",
  we can sample = 2 (6MHz) log_2 (4) = **24 Mbps.**

4. If a binary signal is sent over a 3-kHz channel whose signal-to-noise ratio is 20 dB, what is the maximum achievable data rate?

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$$10\log_{10} S/N = 20dB$$

$$S/N = 100$$

Using Shannon theorem,

Maximum number of bits/sec $= H * \log_2(1 + S/N) = 3KHz * \log_2(1 + 100)$

$$\Rightarrow \log_2(101) \neq 6{,}658 \Rightarrow$$

$$= 3 * 6{,}658 = 19{,}975 Kbps$$

Using Nyquist theorem,

Maximum data rate in bits/sec $= 2 * H * \log_2 V = 2 * 3 * \log_2 2 = 6Kbp$

The bottleneck is therefore the Nyquist limit, giving a maximum channel capacity of 6 Kbps.

1. Compute the Fourier coefficients for the function $f(t) = t$ $(0 \leq t \leq 1)$.

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ANS:

$$a_n = \frac{2}{T}\int_0^T g(t)\sin(2\pi nft)dt = 2\int_0^1 t\sin(2\pi nft)dt \Rightarrow \text{ Assume that } 2\pi nt = a \Rightarrow$$

$$2\int_0^1 \frac{a}{2\pi n}\sin(a)\frac{da}{2\pi n} = \frac{1}{2\pi^2 n^2}\int_0^1 a\sin a da = \frac{1}{2\pi^2 n^2}\int_0^1 x\sin x dx$$

$$x = u \Rightarrow dx = du$$

$$\sin x dx = dv \Rightarrow -\cos x = v$$

$$\frac{1}{2\pi^2 n^2}\int_0^1 (x(-\cos x) - \int_0^1 -\cos x dx)dx = \frac{1}{2\pi^2 n^2}\int_0^1 (-\cos x * x + \sin x)dx$$

$$= \frac{1}{2\pi^2 n^2}\int_0^1 (-\cos(2\pi nt) * 2\pi nt + \sin(2\pi nt))dt = \frac{-2\pi n}{2\pi^2 n^2} t \,|_0^1 = -\frac{1}{\pi n}$$

$$b_n = \frac{2}{T}\int_0^T g(t)\cos(2\pi nft)dt = 2\int_0^1 t\cos(2\pi nft)dt = 0$$

$$c = \frac{2}{T}\int_0^T g(t)dt = 2\int_0^1 tdt = 2\frac{t^2}{2}\,|_0^1 = 1$$