Shaurya Kalra and Manan Shukla: Cooking with Calculus - Scandinavian Coffee Cake Answer Sheet

1) $\int_2^\infty x^{-2}\,dx = \lim_{b\to\infty} -\frac{1}{x}\Big|_2^b$

$\lim_{b\to\infty} -\frac{1}{b} + \frac{1}{2}$

$-\frac{1}{\infty} + \frac{1}{2}$

$0 + \frac{1}{2}$

$\boxed{1/2}$

2) (calculator)

$y' = -2\sin 2x$

$\int_0^{\pi/4} \sqrt{1+(-2\sin 2x)^2}\,dx = 1.31759\ldots$ $\boxed{1}$

3) $y = \cos 3x - 3/4$ Critical values when $y'=0$ and endpoints

$y' = -3\sin 3x$ $x = 0, \frac{\pi}{3}, \frac{\pi}{2}$

$-3\sin 3x = 0$

$\sin 3x = 0$

$3x = 0 \quad 3x = \pi$

$x = 0 \quad x = \frac{\pi}{3}$

x	y
0	1/4
$\pi/3$	-7/4
$\pi/2$	-3/4

absolute max is 1/4

4) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

$\left|\frac{(-1)^{n+1}x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n x^n}\right| < 1$ $\lim_{n\to\infty} \left|\frac{n^2}{(n+1)^2}\right| \cdot |x| < 1$

$-1 < x < 1$

radius $= 1$

$3 - 1 = \boxed{3}$

5) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}$ $r = 1/2$

$-1 < 1/2 < 1$

converges by AST ✓

Converges to $\frac{a_1}{1-r}$

$\frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = \boxed{2}$

6) $\lim_{x\to\infty} \frac{1/2x^4+3x}{3/4x^4+1738} \to \frac{\infty}{\infty}$

$\frac{2x^3+3}{3x^3} \to \frac{\infty}{\infty}$

$\frac{6x}{9x} \to \frac{\infty}{\infty}$

$\frac{6}{9} = \boxed{2/3}$

7) $20\pi \left[\frac{1}{\frac{\pi}{4}-0}\int_0^{\pi/4} \sec^2 x\,dx\right]$

$\frac{20\pi \cdot 4}{\pi} \tan x\Big|_0^{\pi/4}$

$80\left[\tan\pi/4 - \tan 0\right]$

$80(1-0)$

$\boxed{80}$

8) $\int_0^{\sqrt[3]{4}} 2\cdot 3x^2\,dx$

$6\int_0^{\sqrt[3]{4}} x^2\,dx$ $6\left[\frac{1}{3}x^3\Big|_0^{\sqrt[3]{4}}\right]$

$6\left(\frac{4}{3} - 0\right)$

$\boxed{8}$

9) Speed: $\sqrt{(x')^2+(y')^2}$

velocity: $\langle 5t+1, \sqrt{21+t}\rangle$

$\sqrt{(5t+1)^2 + 21+t}$ @ $t=2$

$\sqrt{11^2+23}$

$\sqrt{144}$

$\boxed{12}$

10) $5/4x^2 + 3/x + 1738$

$5/2x + 34$

$\frac{5}{2}\left(\frac{22}{5}\right) + 34$

$11 + 34$

$\boxed{45}$

11) $-1125\int_0^{\pi/2}\cos(3x)\,dx$

$\frac{1}{3}\sin 3x\Big|_0^{\pi/2}$

$\frac{1}{3}\sin\frac{3\pi}{2} - \frac{1}{3}\sin 0$

$-\frac{1}{3} - 0$

$-1125\left[-\frac{1}{3}\right]$

$\boxed{375}$

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12) $3x^4+9x^2+40$

$12x^3+18x$

$36x^2+18$

$36\left(\sqrt{\tfrac{1}{3}}\right)^2+18$

$\frac{36}{3}+18$

$12+18$

$\boxed{30}$