CS461 Homework 3

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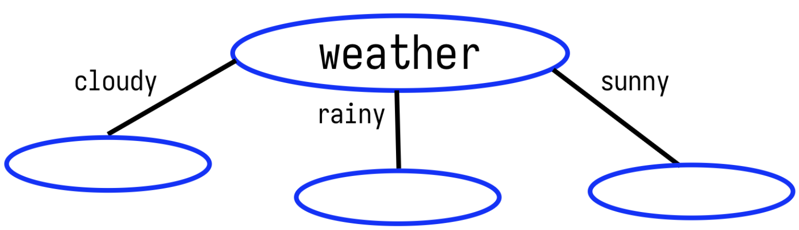
1. Decision Tree

1.1 Information Gain and Root Node Selection

The initial entropy of the Play is: $H(Play) = 1.0000$

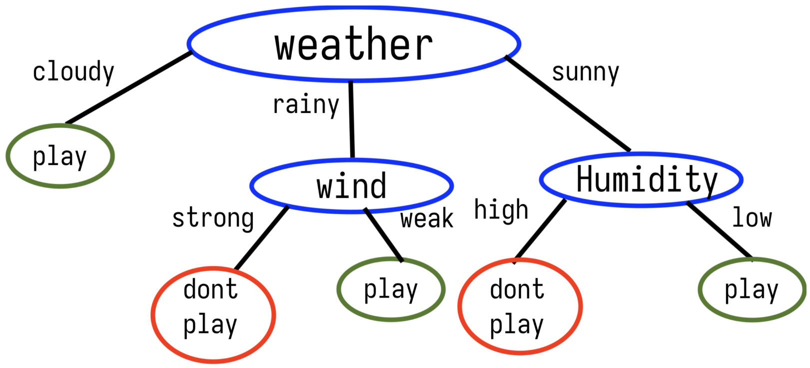
- **Weather:** $IG(\text{Weather}) = 0.4000$
- **Temperature:** $IG(\text{Temperature}) = 0.1145$
- **Humidity:** $IG(\text{Humidity}) = 0.0349$
- **Wind:** $IG(\text{Wind}) = 0.1245$

Since **Weather** has the highest Information Gain 0.4000, it is selected as the root node of the decision tree.



1.2 Constructing the Decision Tree

The decision tree is shown below:



1.3 Pruning (Extra Points)

$$C(T) = \sum_{\tau \in T'} Q(\tau) + \lambda \cdot \text{num of leaves in } T'$$

For the initial tree:

$$C(T) = 0 + \lambda \cdot 5 = 5\lambda$$

When pruning the **Humidity sub-tree**:

$$C(T) = -\left(\frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\log_2\frac{2}{3}\right) + \lambda \cdot 4$$

$$C(T) = 0.918 + \lambda \cdot 4$$

When pruning the **Wind sub-tree**:

$$C(T) = -\left(\frac{1}{4}\log_2\frac{27}{256}\right) + \lambda \cdot 4$$

$$C(T) = 0.811 + \lambda \cdot 4$$

Pruning to the **root only**:

$$C(T) = -\left(\frac{5}{10}\log_2\frac{5}{10} + \frac{5}{10}\log_2\frac{5}{10}\right) + \lambda \cdot 1$$

$$C(T) = 1 + \lambda$$

Observation: As λ increases, the penalty for the number of leaves grows. At $\lambda = 0$, the unpruned tree minimizes cost. For $\lambda = 0.25$, pruning to the root achieves the lowest cost.

2. Perceptron

2.1 Single Data Point

With a step size of 1, it will always take *one* iteration to classify the data point.

2.2 Random Initialization

Given a randomly initialized weight vector, if w_0 correctly classifies the point, then 0 iterations will be required, otherwise, we must consider another case:

Let $w_0 = (a_1, a_2) \neq (0, 0)$. If $a_1x_1 + a_2x_2 > 0$, then the point is classified correctly without requiring any updates. If $a_1x_1 + a_2x_2 \leq 0$, first define $x = (x_1, x_2)$.

We claim that for all $n \in \mathbb{N}$, the weight vector after n iterations is given by $w_n = w_0 + nx$. To prove this, note that for $n = 1$, we have $w_1 = w_0 + x$ by the definition of the Perceptron update rule. For a fixed $n = k$, assume $w_k = w_0 + kx$. Then, for the next iteration:

$$w_{k+1} = w_k + x = (w_0 + kx) + x = w_0 + (k + 1)x.$$

By induction, $w_n = w_0 + nx$ holds for all $n \in \mathbb{N}$.

For the classifier to correctly classify the point x, we require $w_n \cdot x > 0$, where:

$$w_n \cdot x = (w_0 + nx) \cdot x = w_0 \cdot x + n\|x\|^2.$$

Since $\|x\|^2 = x_1^2 + x_2^2 > 0$ and $w_0 \cdot x = a_1x_1 + a_2x_2 \leq 0$ by assumption, we have:

$$w_n \cdot x > 0 \implies n > -\frac{w_0 \cdot x}{\|x\|^2}.$$

Thus, choose $n \in \mathbb{N}$ such that $n > -\frac{w_0 \cdot x}{\|x\|^2}$. This is the number of iterations required for the Perceptron to correctly classify the data point if it was initially misclassified.

2.3 Iterative Updates

Iteration	**w**
0	$w_0 = (0, 0)$
1	$w_1 = (0, 0) + (0, 1) = (0, 1)$
2	$w_2 = (0, 1) - (1, 0.5) = (-1, -0.5)$
3	$w_3 = (-1, -0.5) + (1, 1) = (0, 1.5)$
4	$w_4 = (0, 1.5) - (1, 0.5) = (-1, 1)$
5	$w_5 = (-1, 1) + (1, 1) = (0, 2)$
6	$w_6 = (0, 2) - (1, 0.5) = (-1, 1.5)$

3. Gaussian Discriminant Analysis (GDA)

3.1 Estimating Parameters

Class	Mean (μ)	Variance (σ^2)
Class +1	-0.0722	1.3031
Class -1	0.9402	1.9426

Table 1: Mean and variance for each class.

3.2 Test Accuracy

The test accuracy is 61%

3.3 Improving the Classifier

As currently constructed, we use MLE estimation. MLE only looks at likelihood and ignores prior probabilities. Using MAP rule, we can increase our effectiveness. With MAP, accuracy went up to 90%

3.4 2D GDA Statistics

Table is on the next page

Class	Mean (μ)	Covariance (Σ)
Class +1	$[0.0130754, 0.06295251]$	$\begin{bmatrix} 0.98285498 & 0.00612046 \\ 0.00612046 & 1.05782804 \end{bmatrix}$
Class -1	$[-0.02313942, -0.02114952]$	$\begin{bmatrix} 1.00329037 & -0.01142356 \\ -0.01142356 & 4.97693356 \end{bmatrix}$

Table 2: 3.4 - Mean and covariance for each class.

3.5 2D GDA Predictions

The test accuracy is 84%

3.6 Density-Based Accuracy (Extra Points)

The test accuracy is 85% which shows us that GDA is suitable. GDA provides a reasonable framework for classification because it balances robustness, simplicity, and efficiency, achieving near-optimal accuracy even when the data deviates slightly from Gaussian assumptions. A more complex mixture model wasn't that much of an improvement.

4. Logistic Regression

4.1 Data Preprocessing

TF-IDF is a weighted metric that finds words that best describe one and only one document from a corpus of documents. It finds the frequency of the word in the document, then weights frequency outside of the target document negatively, making sure there is a degree of uniqueness to the word and the document.

4.2 Dimensionality Reduction

file is attached

4.3 Gradient Descent

file is attached

4.4 Train and Test Accuracy

Train Accuracy: 97.03% Test Accuracy: 96%

4.5 Extra Points: Email Classification