A Hybrid Fault-Tolerant Routing based on Gaussian Network for Wireless Sensor Network

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Abstract—In this paper, we have proposed a hybrid fault-tolerant routing to solve fault-tolerant issue in wireless sensor networks (WSNs) based on hierarchical topology. The hierarchical topology is a combination of clustering and the labeling of sensor nodes as Gaussian integers. Accordingly, the network area is divided into small square grids, the cluster head of each grid is represented by a Gaussian integer. These cluster heads are connected together to create a Gaussian network. Through node symmetry, and the shortest distance in the Gaussian network, as well as the advantages of multi-path routing, this paper proposes a hybrid fault-tolerant clustering routing protocol based on Gaussian network for wireless sensor network (FCGW). The purpose of FCGW is to improve fault tolerance, increase data reliability and reduce energy consumption for wireless sensor networks. The experimental results of the proposed scheme show that FCGW protocol has high data reliability. In addition, the FCGW protocol consumes about 48% of the energy in the network, while other protocols consume 70% more energy.

Index Terms—Clustering, fault-tolerance, Gaussian network, multi-path routing, wireless sensor network.

I. INTRODUCTION

IN wireless sensor network (WSN), the metrics such as data reliability, optimal energy consumption, memory limit, and data latency are major challenges in effective implementation of network [1]–[3]. In particular, in WSN, due to limited resource of sensor nodes, as well as harsh communication environments such as rain, wind, snow and water, always leads to faulty connections as in [4]. Therefore, improving fault tolerance in the network will improve the quality of service and availability of the WSN. Since then, addressing fault tolerance issues has been an important requirement in WSN design.

Currently, there are several fault-tolerant techniques that have been proposed to improve fault tolerance. These fault-tolerant techniques will be classified into three main categories: redundancy based mechanisms, clustering based mechanisms, and deployment-based mechanisms as in [5]. Specifi-

cally, each mechanism has different advantages and disadvantages:

- Redundancy based mechanisms: The fault tolerance is improved by redundancy factors such as redundant routing paths, redundancy time, redundant data [6]. In this mechanism, the main fault recovery approach includes [7] active replication and passive replication. In active replication, all fault recovery requests will be processed by all replicas. On the other hand, in passive replication, a fault recovery requests will be processed by a single replica. If this replica fails, another replica will be continued. Therefore, this mechanism increases data recovery capacity and the data reliability. However, the cost of deployment and maintenance of the network is very large.

- Clustering based mechanisms: This mechanism will create small clusters, making it easy to manage and expand the network [8], [9]. Therefore, this mechanism avoids the random failures of nodes as well as improves energy efficiency through the cluster head node (CH). However, when a CH fails, it will affect its cluster. So, improving the fault tolerance efficiency of CH is essential. Based on the redundancy of CH or re-election of CH after a period, several approaches have been proposed for CH fault tolerance, lead to more energy consumption and increased data latency.

- Deployment based mechanisms: WSNs are self-organizing networks that are randomly deployed or pre-located. Hence, it is crucial to evade connection failure to maintain a strong network topology which is essential to avoid connection failure [10]. Network topology optimizations based on virtual backbone is an example to avoid connection failure using the connected dominating set (CDS) [11]. This mechanism optimizes data transmission by a virtual backbone, avoids information interference and reduces energy loss. Even though optimizing the network structure can solve the NP-hard problems, this will increase the complexity of the solution.

Through the summary above, it has been shown that the optimizing the network topology and clustering efficiency have more advantages for fault tolerance in the wireless sensor networks. In addition, our previous research [12] has proposed a modeling of WSN based on a Gaussian network to optimize the management of network topology as well as improve energy consumption by the shortest path routing protocol. Therefore, this paper continues to expand previous research and propose a solution using the Gaussian network model to solve fault tolerance issue in WSN. The main fault tolerance idea in this proposal is to exploit the efficiency in clustering routing and symmetry of Gaussian network based on multi-path routing.

Our main contributions are as follows:

- The random sensors will be divided into virtual grids (clusters) using geographic adaptive fidelity (GAF) algorithm [13]. Each cluster will select one active node as a CH node at a time to improve consumption energy efficiency and prolong network lifetime.
- Each CH node will be represented as a Gaussian integer and connected together to form a Gaussian network, so WSN-management will be optimized as a Gaussian network.
- This paper proposes a fault-tolerance mechanism for CH nodes. Based on the symmetric link in the Gaussian network, if the main routing path fails, it is easily replaced by the redundant routing paths, thus increasing the data reliability as well as optimized energy consumption.

The rest of this paper is organized as follows: Section II presents the related work. Section III presents a wireless sensor network model based on a Gaussian network. Section IV section presents the fault tolerance mechanism. The simulation and analysis results are explained in Section V and the conclusion is in Section VI.

II. RELATED WORK

A. Comprehend Fault Tolerance

Fault tolerance is one of the important characteristics of wireless sensor networks. Accordingly, fault tolerance is improved to ensure the reliability and availability of the network in the harsh environment and limited energy of the nodes. Therefore, in order to classify and understand fault tolerance mechanisms, some related concepts have been defined in the previous literature [4], [7], [14], [15], [16], as follows:

- **A Fault:** A fault is an unexpected change or abnormal condition in a system. It is any kind of defect that leads to an error. A fault in any component of a system can lead to a system failure.
- **Fault detection:** To provide any countermeasures, the first step a system must perform is to detect that specific faults occurred in the network.
- **Fault diagnosis:** Is the process of determining the factors affecting the faults and diagnosing the types of causes that result in faults.
- **Fault recovery:** After the system has detected a fault, the next step is to prevent further faults or recover from the detected fault. The main technique to achieve this goal is to replicate components when an error occurs in the system.

In WSN, the fault occurs for a variety of reasons in network due to in hardware, software, communication environment, power limit, timeout, malicious nodes etc. On the other hand, based on fault detection and fault diagnosis approaches [15], [17], the fault tolerance approaches will be divided into three types as below:

- **Centralized approach:** This approach is mainly based on the use of a centralized node to perform fault management. The status of all nodes in the network will be determined by this centralized node by receiving and processing information from all nodes. Therefore, the centralized approach is ineffective in large-scale WSNs. Accordingly, the energy consumption for network management and data transmission is huge, as well as increasing data latency, which greatly affects the efficiency of fault detection and fault recovery in WSNs.
- **Distributed approach:** In this approach, the status of each node will be determined by collecting and analyzing information from neighboring nodes. The distributed approach will reduce data transmission to the central node, which helping to reduce energy consumption, prolong lifetimes and reduce data latency. Therefore, this approach can be effectively applied to large-scale WSNs. However, this approach depends heavily on the density of neighboring nodes. Because over time the node density decreases, which inevitably has a negative effect on fault detection accuracy.
- **Hybrid approach:** The main idea of this approach is to rely on both the centralized node to collect information and the neighbor node to exchange information to determine the state of the nodes. The hybrid approach addresses the large-scale network problem of centralized methods and node density problem in distributed approach by adding additional equipment (super nodes, mobile sink). However, installing additional equipment will increase costs and make it difficult to deploy actual networks.

According to these fault tolerance approaches, there are several fault routing protocols have been proposed to achieve greater fault recovery efficiency such as clustering based protocols; multi-path based protocols; hybrid based protocols. The details of some common fault tolerance routing will be discussed in the next section.

B. The Fault-Tolerant Routing in Literatures

In this section, some fault-tolerant routing protocols will be introduced in detail. These routing protocols will be incorporated in our proposal as well as during the performance evaluation process in Section V.

1) Fault-tolerant Clustering Routing Protocols: In WSN, the clustering protocol has proven to be an energy-efficient protocols as in [18]. In addition, clustering protocols also greatly improve the fault tolerance such as several routing protocols FT-LEACH [20], HEED [21], and FTS [22], etc.

- The FT-LEACH protocol reduces the CH faults in LEACH [19] by randomly selecting the CH nodes based on the residual energy of the nodes. In FT-LEACH, the main idea for fault tolerance is that each common member (CM) node will only send data to CH when its value differs from the value of the previous time. The advantage of this proposal is that it avoids the CH nodes processing repetitive data and prevents the CH node from failing due to energy outage. However, the author does not clearly explain the energy consumption parameters for the data exchange.
- The HEED protocol periodically selects CH nodes based on a hybrid of the residual energy of nodes and a secondary parameter, such as node proximity to its neighbors

or node degree. Each node will calculate a CH_{prob} (CH probability) based on the residual energy of the node to elect as CH nodes. The CH_{prob} helps the HEED protocol to avoid the election of low energy nodes as CH which leads to the network that will quickly lose connection. However, HEED depends on the density of high-energy nodes in the network, so if the fault nodes density is increased, it will lead to low fault tolerance performance.

- The FTS protocol is proposed for fault diagnosis and fault recovery. In FTS, each cluster will elect a spare CH (SCH) node based on the remaining energy of nodes and its distance to the CH node to fault diagnosis of CH node. This algorithm is highly effective in fault tolerance. However, due to FTS dependency on information exchanged between nodes, therefore, over time, reducing the node density will lead to a decrease in routing performance.

2) Fault-tolerant Multi-path Routing Protocols: Improving fault tolerance in wireless sensor networks based on multi-path routing is an important approach. The multi-path routing will build a few routing paths from source node to sink node. After that, the data will be transmitted through one of these paths, leading to, the data reliability can be increased. In case the primary routing path fails, it can be replaced by another path [23]. This is a good approach to fault tolerance. The benefits of multi-path routing in WSN such as fault tolerance, load balancing, quality of service, etc. are also mentioned in the literature [24] to [26].

The fault tolerance based on multi-path routing can be mainly divided into three categories as in [24].

- Alternate path routing: a few multi-path routing have been proposed for alternative path routing as in literature [27], etc. The purpose of these protocols is to ensure that multiple paths are redundant. The multiple paths are built by the sink node to broadcast messages to other nodes in the network. Therefore, these protocols rely heavily on sink nodes resulting in low efficiency and passive construction of alternatives.

- Reliable data transmission: the protocols have proposed to increase the reliability of data by creating redundant packets during data transfer as in [28], [29]. The advantage of these protocols is that it does not maintain routing table. However, the routing costs and the delay time will increase to determine redundancy paths in case of a fault.

- Efficient network resource utilization: the protocols such as HEED-FT [30], I2MR [31], etc., have introduced multi-path protocols for resource efficiency. Accordingly, routing maintains some redundant routing paths in some cases to balance resource efficiency and restore routing path when a disconnection occurs on the primary path.

3) Hybrid Fault-tolerant Routing Protocols: Multi-path routing improves data reliability and optimizes fault connections, while clustering ensures high energy efficiency and prolongs network life. Therefore, several fault-tolerant routing protocols have been proposed based on a combination of multi-path routing and cluster routing.

A protocol called robust energy-efficient distributed clustering (REED) for the k-connectivity network has been proposed

in [32]. The benefit of REED is that it finds and replaces the faulty connection quickly and simply. So, that it optimizes the cost of restoring fault connections. However, this algorithm applies only to the CH node, the CM node fault has not been addressed in this proposal. In addition to this proposal depends on the density of neighboring nodes and the high cost of maintaining other routing paths in the network.

The authors in [30] have presented a Fault-tolerant multi-path routing protocol for WSN based on HEED (HEED-FT). The aim of HEED-FT is to increase data reliability and load balancing based on multi-path routing and clustering. In HEED-FT, to improve fault tolerance, each cluster will select a deputy cluster head node (SCH) based on the energy-weighted center of parameters (EWNC, energy-weighted node centrality). Accordingly, the data will be transmitted on one of two paths established based on SCH or CH. However, the maintenance of SCH nodes, as well as highly complex algorithms results in lower energy efficiency over time.

C. The Gaussian Network and Benefits

The authors in [34][35] introduced a Gaussian network (Gaussian graph), in which nodes are labeled using a subset of the complex numbers. The Gaussian network is represented by a set of Gaussian integers $\mathbb{Z}[i] = \{x + yi | x, y \in \mathbb{Z}\}$, where $\mathbb{Z}$ is the set of integers and $i^2 = -1$, that is defined as follows:

Definition: Let $0 \neq \alpha = a + bi \in \mathbb{Z}[i]$, $\mathbb{Z}[i]_\alpha$ *is set of Gaussian integers modulo α, we define the Gaussian network as $G_\alpha(V, E)$ where*:

1) $V = \mathbb{Z}[i]_\alpha$ *is the node set,*

2) $E = \{(\beta, \gamma) \in V \times V | (\beta - \gamma) \equiv \pm 1, \pm i(\mod \alpha)\}$ *is the edge set, where β, γ are two Gaussian integer.*

We call G_α the Gaussian network generated by α.

In a Gaussian network, every node is connected to four adjacent nodes in four different directions: North, East, South, and West. In addition, the adjacent links are undirected and node-symmetric. In this paper, we represent a Gaussian network G_α with $\alpha = a + bi$, $0 \leq a \leq b$ and the greatest common divisor $\gcd(a, b) = d > 1$ as a mesh network in a rectangle. In the case $\gcd(a, b) = 1$ is presented in [34]. Accordingly, the nodes in G_α are arranged in a rectangle of size $(d \times f)$ where $f = (a^2 + b^2)/d$. Node 0 is located at the bottom left of the graph shown as Fig. 1. The connections between adjacent nodes are explained in Theorem 2.2 of the literature [35].

In the Gaussian network $G_{\alpha = a + bi}$, for two nodes $\beta = x_1 + y_1 i$ and $\gamma = x_2 + y_2 i$. The distance between β and γ is given by formula (1). For Example, in Gaussian network G_{4+4i}, and $\beta = 0 + 0i$, $\gamma = 1 + 1i$, then $D(0, 1 + i) = 2$, since $D(0, 1 + i) = \min\{|x| + |y| \ |x + yi = (1 + i) - 0 + \delta(4 + 4i)\}$, when $\delta = 0$, $x + yi = 1 + i$.

$$D_{(\beta, \gamma)} = \min\{|x| + |y| \ |x + yi = \gamma - \beta + \delta\alpha\}, \quad (1)$$

where $\delta \in \mathbb{Z}[i]$ is an arbitrary Gaussian integer.

Therefore, one major advantage of the Gaussian network over 2D-torus network is that for a given number of nodes N, the network diameter and average distance between two nodes of the Gaussian network can be much smaller. This is good
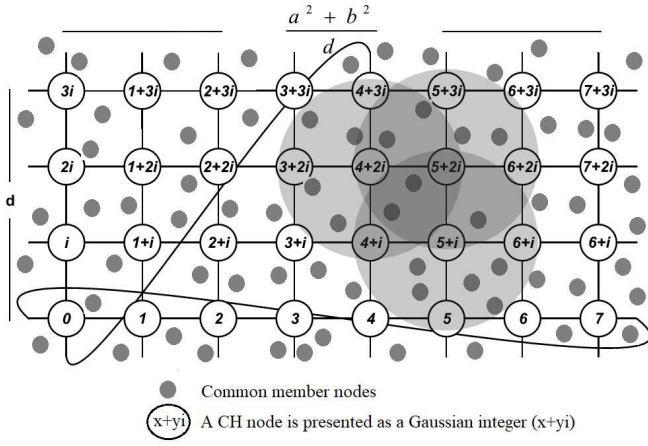
Fig. 1. The CHs are connected as mesh network based on Gaussian network connection model (example $\alpha = 4 + 4i$).

(a). The sensors are randomly allocated.

(b). The sensors after division by GAF algorithm.

● Cluster head nodes (CHs)
● Common Member nodes (CM)
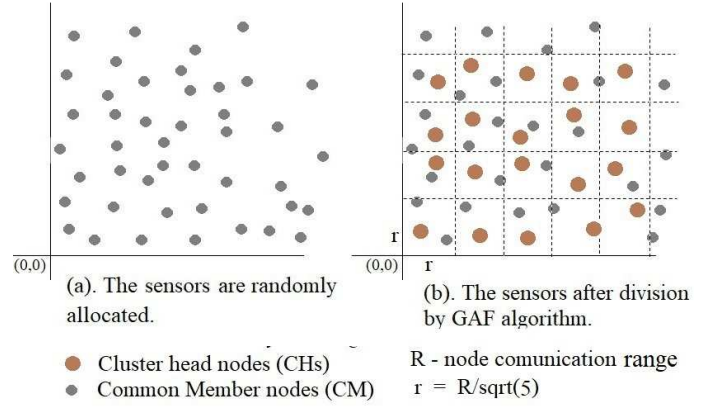
R - node comunication range
r = R/sqrt(5)

Fig. 2. The network are is divided into several square virtual grids by GAF algorithm.

Fig. 3. Each cluster (represented by one CH) is represented as a Gaussian integer in our network model.

approach for transmitting data with the shortest path distance in torus networks [36], optical macro-chip interconnect [37], higher dimensional networks [38] or for network reconfiguration in Network-on-Chip [39].

According to the Gaussian network benefits, we have proposed the wireless sensor network model as shown in [12]. In this work, we extend our previous research to improve fault tolerance for wireless sensor networks.

III. THE WIRELESS SENSOR NETWORK MODEL

A. The Proposed Hierarchical Network Model

1) WSN Model: In this paper, we consider a WSN model that consists of a base station node (BS) or a sink node and n identical sensor nodes that are randomly allocated over the rectangular area $S = X \times Y$. Here, based on the geographical position of the nodes, the network area will be divided into some virtual square grids. Accordingly, each grid will choose an active node as the CH node to aggregate and transmit data as in Fig. 2 by GAF algorithm (geographic adaptive fidelity) [13].

The GAF enhanced versions as well as GAF benefits are also introduced in [40]. The GAF algorithm divides the network into virtual square grids with unit $r = R/\sqrt{5}$, where R is the sensor communication range. The nodes in each grid will receive one of three statuses, namely: discovery, sleep or active at a time. Initially, all nodes start with the discovery state and set a predefined time T_d (time period for discovery state). When a node finish T_d time, it broadcasts a message to all nodes in grid. A node will be switch to active state if the energy level is greater than a threshold and it does not receive any other discovery message and set a time of operation T_a, else it will be switch to sleep state and set a time of sleep T_s, details as in [13]. Each virtual grid has only one active node at a time. The active node is selected as a CH of cluster (grid), which will collect the packets in intra-clusters and transmits packets to BS through single-hop or multi-hops.

In our approach, after each cluster has selected a CH node, the Gaussian network (G_α) of CH nodes will be built through
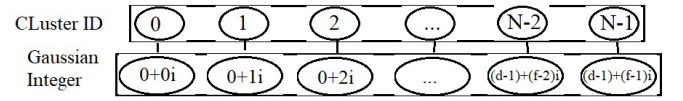
connections between CH nodes [12]. Accordingly, each grid will only have a CH node and it is represented by a Gaussian integer ($\beta = x + yi$). The G_α of CH nodes (CH-Gaussian network) as shown in Fig. 1, which will be discussed in detail in the next section.

2) The CH-Gaussian Network: Our previous work has proposed a hierarchical topology based on Gaussian network connection properties to efficiency of WSN topology [12]. In that work, we presented three connection models corresponding to the four adjacent links of each Gaussian node in the wireless sensor network. Accordingly, based on the WSN model in this work, we have a Gaussian network G_α of CH nodes generated by a Gaussian integer $\alpha = a + bi$, with $0 \le a \le b$, and the largest common divisor $\gcd(a, b) = d > 1$. Thus, the total number of Gaussian nodes in the G_α is N as in formula (2). After the WSN divided into a set of virtual square grids (clusters) $\Gamma_{(G)} = N$, the CH node in each cluster is connected together to form a Gaussian network of CH nodes in the network area $S = X \times Y$ as in formula (3). Hence, each CH node in each cluster is equivalent to a node in the Gaussian network as in Fig. 3.

$$N = \{d \times f \mid f = (a^2 + b^2)/d\} \qquad (2)$$

$$S = \{X \times Y \mid X = d \times r, Y = f \times r\} \qquad (3)$$

According to the set of clusters, each cluster will be assigned with an ID. since, corresponding to each cluster ID ($Gr \subset \Gamma_{(G)}$) will only have one CH node and only one Gaussian integer representation $\beta = x + yi$, which the relationship between Gr and β is shown in formula (4). Therefore, in our proposal, we based on the cluster ID to

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Algorithm 1: Get Gaussian integer from cluster ID

Input: Gr, f, d
Output: $\beta = x + yi$ //where $x, y \in \mathbb{Z}$
$y = 0;$
while $y \leq d$ **do**
  $\quad x = Gr - yf$ //as in formula (4);
  $\quad$ **if** *($x \geq 0$) and ($x < f$)* **then**
  $\quad\quad$ | return $x, y;$
  $\quad$ **else**
  $\quad\quad$ | $y = y + 1;$
  $\quad$ **end**
end

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calculate the corresponding Gaussian integer as **algorithm 1**.

$$Gr = |x| + |f \times y| \tag{4}$$

B. Shortest Path Routing Protocol

In our network model, the network is divided into clusters, which will optimize the energy efficiency of the wireless sensor network. However, to increase routing efficiency more, in [12] based on the shortest distance in the Gaussian network, we proposed the shortest path routing protocol. Accordingly, the length of the path from $CH(\gamma = x_s + y_s i)$ to $BS(\beta = x_d + y_d i)$ is the shortest distance between γ and β, $D(\beta, \gamma) = |X_{\min}| + |Y_{\min}|$ as in formula (1). Therefore, the data transmission path $P_{(\beta, \gamma)}$ from CH node to BS node consists passing through $X_{\min}$ steps in the real axis (x-axis) and $Y_{\min}$ steps in the imaginary axis (y-axis). A simple presentation of a routing path as shown in formula (5), in which the packets pass through nodes on the x-axis, then the nodes on the y-axis. In addition, the CH nodes are represented by a Gaussian integer, so during routing, nodes will rely on the Gaussian integer to forward the packet without performing any additional work such as flooding to find the next node, which results in reducing costs for routing. This routing algorithm in the network is divided into 3 steps as in **algorithm 2**. Since, in this paper, through improving the shortest path routing, we have proposed a fault-tolerant approach for wireless sensor networks. The details of the fault tolerance approach will be presented in the next section.

$$P_{(\beta, \gamma)} = \{ \sum_{m=0}^{|X_{\min}|} ((x_s \pm m) + y_s i) \cap \sum_{n=0}^{|Y_{\min}|} (x_s + (y_s \pm n)i) \} \tag{5}$$

IV. FAULT TOLERANCE MECHANISM

According to the hierarchical topology based on a Gaussian network of WSN, this paper propose a hybrid fault-tolerant routing protocol based on a combination of clustering and multi-path routing. Our proposal is called fault tolerance clustering based on a Gaussian network for wireless sensor network protocol (FCGW). In FCGW, the fault tolerance mechanism focuses on fault detection and fault recovery for CH nodes. Therefore, our fault tolerance mechanism includes

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Algorithm 2: Shortest path routing in our WSN model

Input: WSN information, BS, source node (CH)
Output: Routing from CH node to BS
Step 1: Initialize;
- Calculate r virtual grids unit: $r = R/\sqrt{5}$
- Select CH for each grid /*Using GAF algorithm*/
- Get $\alpha = a + bi$ /*Using formula (2),(3)*/
Step 2: Get Gaussian integers (β, γ) from CH and BS
- Calculate β from BS /* Using algorithm 1*/
- Calculate γ from CH /* Using algorithm 1*/
Step 3: Data Forwarding
- Calculate $Y_{\min}$, $X_{\min}$ from $D(\beta, \gamma)$ /*Using formula (1)*/
while *($X_{\min} > 0$) and ($Y_{\min} > 0$)* **do**
  $\quad$ forwarding data to BS;
  $\quad X_{\min} = X_{\min} - 1$ /* if forward to the x-axis */
  $\quad$ or $Y_{\min} = Y_{\min} - 1$ /*if forward to the y-axis*/
end

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two phases: fault detection phase and fault recovery phase. Accordingly, based on the symmetric links of the Gaussian network and the shortest path routing as in formula (5), the fault recovery process will be optimized as multiple path routing.

A. Fault Detection Phase

In wireless sensor networks, there are many causes for faults such as software failure, hardware failure, natural factors, harsh device deployment environment, resulting in network connection failures. In this paper, we propose a fault detection mechanism for CH nodes based on clustering and self-diagnostics by following the information exchanged between neighbors CH. Accordingly, after selecting CHs, the CHs periodically broadcast the grid-ID (grid index) and CH-ID (the CH index) for the CHs in four adjacent clusters. Therefore, each CH node will update a CH neighbor table ($NeigTable_{CH}$), which includes adjacent grid-ID, CH-ID, and CH states of adjacent clusters (Fig. 4). In a given period of information exchange time $T_{heathCH}$ ($T_{heathCH} < T_a$), if a CH does not receive information from the adjacent CHs then it will update the state of the corresponding CH in the CH neighbor table as "false". According to the CH status in the CH neighbor table, our proposal will diagnose CH node fault or not.

B. Fault Recovery Phase

In our network model, the faults may occur at the CH nodes or CM nodes, which would cause the link between the CH nodes (extra-connections) or between CH and CMs (intra-connections) to be lost.

- In case of the CMs fault, CM nodes are automatically removed from the cluster, and
- In case of the CHs fault, the fault recovery mechanism is executed until another CH is re-selected in the cluster.

In fault recovery mechanism, one CH before forwarding the packet, it will get the next CH state from $NeigTable_{CH}$. If
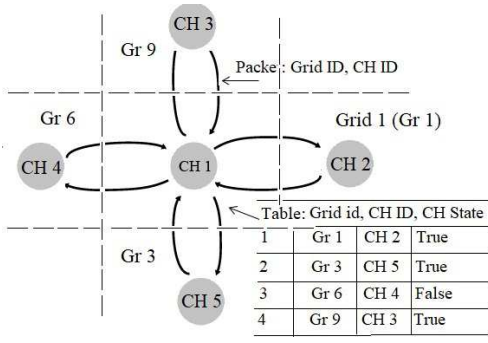
Fig. 4. Definition of the CH neighbor table ($NeigTable_{CH}$) and an example of CH neighbor table of CH 1.
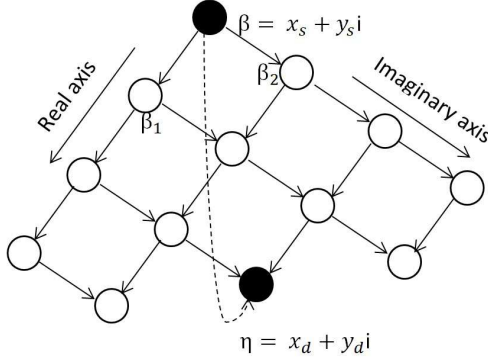

Fig. 5. Multi-path routing in fault-recovery, for example, packets at node β can be transmitted to β_1 or β_2.

this next CH fails, the packets will be automatically forwarded to other next CH nodes as in **algorithm 3**.

Accordingly, the routing paths of packets from $CH(\beta = x_s + y_s i)$ node to $BS(\eta = x_d + y_d i)$ node include $D(\eta, \beta) = |X_{\min}| + |Y_{\min}|$ hops (1). In this case, the packet can be transmitted in real dimension (real axis) or imaginary dimension (imaginary axis), this means that at node $\beta = x_s + y_s i$, the packets can be transmitted to next nodes $\beta_1 = (x_s + 1) + y_s i$ or $\beta_2 = x_s + (y_s + 1)i$ (Fig. 5) as in **algorithm 2**. Therefore, in our fault recovery mechanism, the number of redundant routing paths to send data from the source node to the destination node is N_{path} as formula (6), where N_{path} is a 2-combination of a set $D(\eta, \beta)$. Therefore, during the routing process, if the next CH fault on the real axis (β_1 is faulty), it will be replaced by the next CH node in the imaginary axis (β_2), similarly, if the CH fault on the imaginary axis (β_2 is faulty) will be replaced by β_1 as shown in formula (7). In **algorithm 3**, we introduced in detail the fault detection and fault recovery mechanism in new WSN network model, it is simple and easy to implement. The **algorithm 3** will be applied in **algorithm 2** step 3.

$$N_{\text{path}} = \frac{D(\eta, \beta)!}{2!(D(\eta, \beta) - 2)!} \tag{6}$$

$$P_{(\beta, \eta)} \to \begin{cases} formula(5), & \text{if there is no fault,} \\ \beta_{1(x_s \pm 1, y_s)}, & \text{if fault in imaginary axis,} \\ \beta_{2(x_s, y_s \pm 1)}, & \text{if fault in real axis} \end{cases} \tag{7}$$

Algorithm 3: Fault-recovery algorithm, at $(\gamma = x_s + y_s i)$

Input: $BS(\beta = x_d + y_d i)$, CH$(\gamma = x_s + y_s i)$
$D(\beta, \gamma) = X_{\min} + Y_{\min}$ /*the number of hops in routing path*/
Output: next CH node $(\gamma = x_s + y_s i)$
if $(D(\beta, \gamma) > 0))$ **then**
  if $(X_{\min} > 0)$ **then**
    $\eta = (x_s + 1) + y_s i$; /*temporary next Gaussian integer*/
    $X_{\min} = X_{\min} - 1$;
    Get Grid id of η (Gr_η) /*Using formula (4)*/
    if $(NeigTable_\gamma(Gr_\eta).State)$ is false **then**
      $\eta = x_s + y_{s+1} i$; /*forward to the y-axis*/
      $Y_{\min} = Y_{\min} - 1$; $X_{\min} = X_{\min} + 1$;
    end
    $\gamma = \eta$;
    forwarding $(data, X_{\min}, Y_{\min})$ to γ;
    break;
  end
  if $(Y_{\min} > 0)$ **then**
    $\eta = x_s + (y_s + 1)i$; /*temporary next Gaussian integer*/
    $Y_{\min} = Y_{\min} - 1$;
    Get Grid id of η (Gr_η) /*Using formula (4)*/
    if $(NeigTable_\gamma(Gr_\eta).State)$ is false **then**
      $\eta = (x_s + 1) + y_s i$; /*forward to the x-axis*/
      $X_{\min} = X_{\min} - 1$; $Y_{\min} = Y_{\min} + 1$;
    end
    $\gamma = \eta$;
    forwarding $(data, X_{\min}, Y_{\min})$ to γ;
    break;
  end
**end**

V. SIMULATION AND ANALYSIS

In order to increase the efficiency of comparing routing protocols in WSN with hundreds of sensors, as well as without losing generality, we will evaluate the performance through MATLAB. The MATLAB has also been used effectively as in [41]. In this simulation, to facilitate the installation and evaluation, we have set up the network with parameters as shown in Table I. The sensors are deployed randomly in a rectangular area $S = X$ x Y. We evaluated the performance of our proposal (FCGW) against the existing HEED, FT-LEACH, and PSO-UFC protocols [33]. The PSO-UFC is the new fault-tolerant routing basis on Particle Swarm Optimization approach with a number of particles $N_P = 20$. The protocols were compared in two cases, one when BS is at the center of the network, and the second one is when the BS is located at $(X/2, Y + 50)$, outside the network. The results of comparison and evaluation are as follows.

A. Energy Efficiency and Dead Nodes

In the WSN, energy efficiency can be evaluated through parameters such as energy consumption, average residual

	Parameter	Value
1	Network area	1000 (m) x 500 (m)
2	Initial energy	0.3J
3	Simulation rounds	500 (rounds)
4	Communication range	200 (m)
5	E_{ele}	$50 * 10^{-9}$ (J/bit)
6	E_{mp}	$0.0013 * 10^{-12}$ (J/bit/m^4)
7	E_{fs}	$10 * 10^{-12}$ (J/bit/m^2)
8	CH packet size	4000 Kb
9	CM packet size	200 Kb
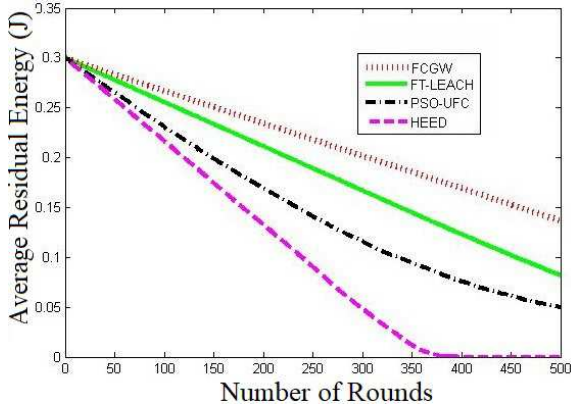
(a) BS position at the network center
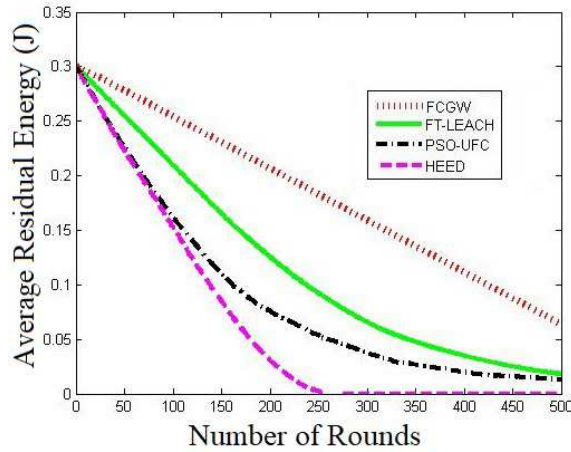
(b) BS position at the network outside

Fig. 6. The average residual energy per round in the network with 150 sensors.

energy, network lifetime, number of dead nodes, etc. The energy consumed at a node for transmitting and receiving data depends on the number of bits l and the distance d. Accordingly, the energy consumption model to transmit and receive respectively as in formula (8) and formula (9) [42].

$$E_{TX}(l,d) = \begin{cases} l \times E_{elec} + l \times E_{fs} \times d^2, & (d < d_0) \\ l \times E_{elec} + l \times E_{mp} \times d^4, & (d \geq d_0) \end{cases} \quad (8)$$

$$E_{RX}(l,d) = l \times E_{elec}, \quad (9)$$

where, E_{elec} is the energy is consumed per bit, E_{fs} is the amplification energy for free space model, E_{mp} is the ampli-

fication energy for multi-path model, and $d_0 = E_{fs}/E_{mp}$ is the threshold transmission distance.

In this energy efficiency evaluation, we simulated the network with 150 sensors. The results are shown by the average residual energy as in Figs. 6(a) and 6(b) and the number of dead nodes as in Figs. 7(a) and 7(b). In Fig. 6, the results have shown that the average residual energy of the FCGW protocol is always better than other protocols in both cases of the BS position. As shown in Fig. 6(a), the average residual energy of HEED is 0, FT-LEACH and PSO-UFC are less than 30% (energy consumption>70%), while the average residual energy of FCGW is about 48% (energy consumption <52%). In Fig. 6(b), the average residual energy of FT-LEACH and PSO-UFC is less than 8%, while for FCGW protocol it is greater than 20%. According to results shown in in Fig. 7, the number of dead nodes of FCGW is the smallest. In Fig. 7(a), at round 400, in HEED all the nodes are dead, PSO-UFC and FT-LEACH have dead nodes greater than 20, while FCGW starts to have dead nodes. In the case when the BS node is outside, the total number of dead nodes has been more clearly assessed with FT-LEACH and FCGW, as shown in Fig. 7(b). From the evaluation results show that the energy efficiency of FCGW protocol is the best, because FCGW is a cluster based protocol for energy optimization. In addition, FCGW based on Gaussian integers to describe cluster heads, resulting in a significant improvement in node search, transmission, and cluster management costs.

B. Data Reliability

In the WSN application, the link quality of the sensors is always affected by some factors such as harsh environment; low transmission power and fault factors. This leads to, in fact, the successful packet delivery ratio is always less than 100%, resulting in reduced packet delivery ratio, data reliability and quality of service. In this evaluation, without loss of generality, assumes that the probability of successful packet transmission P for each link to evaluate the data reliability from the CH to BS. According to multi-path in the FCGW protocol, it leads to increase the probability of successful packet transmission ($P_{(FCGW)}$) as formula (10) than the direct transmission (one-hop) protocols such as FT-LEACH, HEED or PSO-UFC.

$$P_{(FCGW)} = 1 - [1 - P]^2 \quad (10)$$

In this data reliability simulation, we perform simulation in case of low density to high density of sensors in the network (40, 100, 150, and 200 nodes). The simulation results in both cases of BS position inside and outside the network shown in Figs. 8(a) and 8(b). In which solid lines indicate the total number of packets sent and dashed lines indicate the total number of packets received by the protocols. The data reliability depends on the number of packets sent and received. That depends on the number of CHs in the network, as well as packet transfer methods and fault factors. Because, CH in FT-LEACH and FCGW are larger than the other protocols, so FT-LEACH and FCGW are best, while CHs in PSO-UFC depend on a number of particles. In addition, the networks have developed low energy sensors to create dead
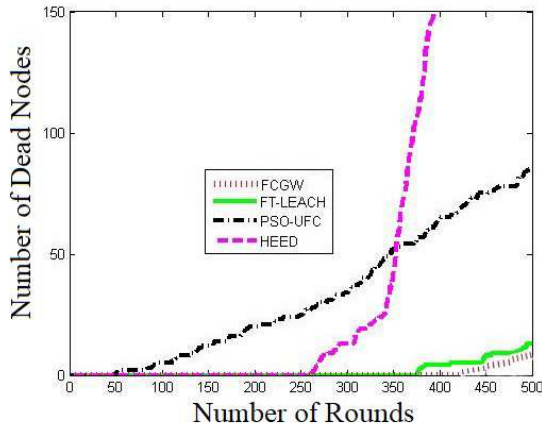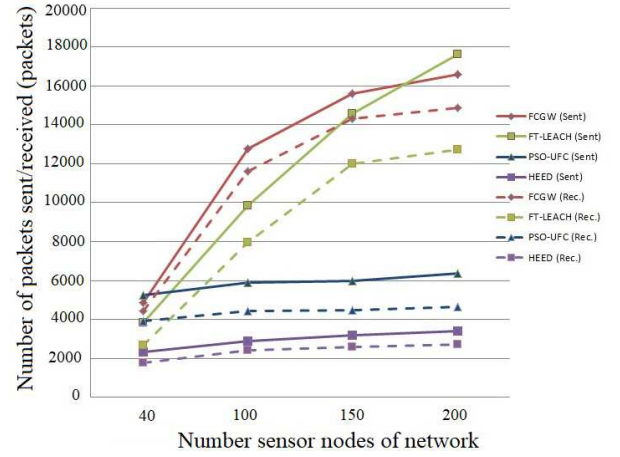
(a) BS at the network center
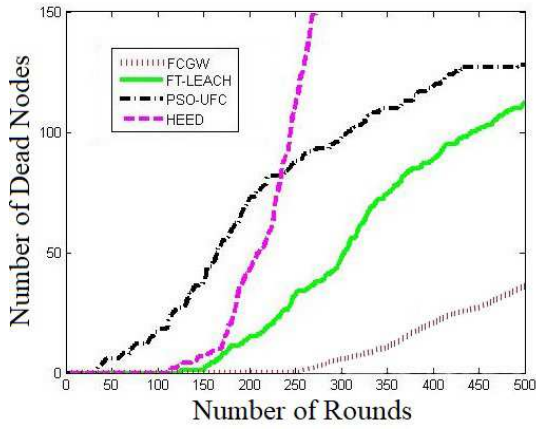
(b) BS at the network outside

Fig. 7. The total number of dead nodes per round in the network with 150 sensors.

(a) The BS position at inside network

(b) The BS position at outside network

Fig. 8. The total number packets are sent and received per round in two case of BS position of networks.
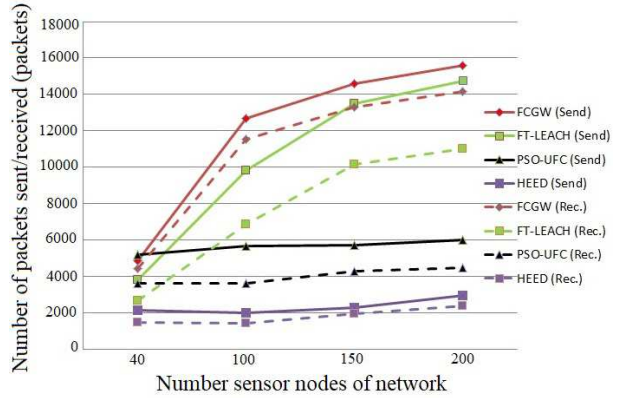
nodes (fault nodes) to evaluate the quality of fault-tolerant routing. Accordingly, FCGW has the least number of dead nodes, so the number of packets sent and received is always larger. In Figs. 8(a) and 8(b), in case of different node density of networks, the number of packets received by FCGW is always better than FT-LEACH. For ex., in the network with 200 nodes, BS position outside as in Fig. 8(b), the total number of received packets of FCGW is 14,000 (packets), while FT-LEACH is 11,000 (packets). The simulation results have shown that the network quality, as well as the data reliability of the FCGW protocol has increased greatly.

VI. CONCLUSION

In this paper, we have constructed a hierarchical topology for wireless sensor network using the Gaussian network connection properties and clustering routing. According to our approach, the sensors are randomly distributed in a rectangular area and clustered into several square grids. The CH nodes are connected together to form a Gaussian network, so this approach has improved fault tolerance through symmetric links and multi-path routing. In addition, the CH nodes are represented as Gaussian integers, which is used in the routing protocol to reduce the complexity of routing algorithms.

Results in the new proposal significantly reduces the cost of routing and maintenance of multi-path routing. However, the connecting long-distance sensor nodes as a Gaussian network in the wireless environment is a major challenge that increases packet delay. In the future, we will continue to address broadcast issues in wireless sensor networks using the Gaussian network properties and its Hamiltonian cycles.

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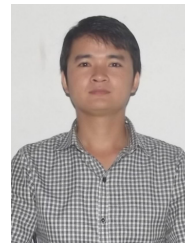
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