# MTL106
# Probability and Stochastic Processes

Assignment 1
**Mind Games**

Deadline : 10th September 2024

Alice and Bob play a game of chess, but they can choose their style of playing each round. In each round, they can choose to be **aggressive**, **balanced** or **defensive**. A win gets +1 point, a draw gets +0.5, and a loss gets 0 points.

The following matrix describes the probabilities $[p_1, p_2, p_3]$, given the current points of Alice and Bob $(n_B)$ where, $p_1$ = probability of Alice winning
$p_2$ = probability of draw
$p_3$ = probability of Bob winning
$n_A$ = current points of Alice
$n_B$ = current points of Bob

| | | Bob | | |
|---|---|---|---|---|
| | | *Attack* | *Balanced* | *Defence* |
| | *Attack* | $(\frac{n_B}{n_A+n_B}, 0, \frac{n_A}{n_A+n_B})$ | $(\frac{7}{10}, 0, \frac{3}{10})$ | $(\frac{5}{11}, 0, \frac{6}{11})$ |
| Alice | *Balanced* | $(\frac{3}{10}, 0, \frac{7}{10})$ | $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ | $(\frac{3}{10}, \frac{1}{2}, \frac{1}{5})$ |
| | *Defence* | $(\frac{6}{11}, 0, \frac{5}{11})$ | $(\frac{1}{5}, \frac{1}{2}, \frac{3}{10})$ | $(\frac{1}{10}, \frac{4}{5}, \frac{1}{10})$ |

Alice wins the first round and loses the second. Determined to outperform Bob, Alice decides to analyze what her optimal strategy should be in response to various strategies that Bob might use.

1. If Alice and Bob choose to attack all the time,

   (a) What is the probability that after $T$ rounds, Alice wins $T_1T_2$ matches and Bob wins $T_3T_4$ matches where $T = T_1T_2 + T_3T_4$ and $T_1, T_2, T_3, T_4$ represent the last 4 digits of your Entry Number?(Replace any 0's in your Entry Number with 9) [1 pt]
   *For ex. if your entry number is 2020AB12345, then $T_1T_2$ and $T_3T_4$ are 23, 45 respectively and $T = 68$.*

   (b) Define a Random Variable $X_i = 1$ if Alice wins round $i$, $X_i = 0$ if it's a draw, $X_i = -1$ if Alice loses. Calculate $\mathbb{E}[\sum_{i=1}^{T} X_i]$ and $\text{Var}(\sum_{i=1}^{T} X_i)$ where $T = T_3T_4$ represents the last 2 digits of your Entry Number (Replace any 0's in your Entry Number with 9). [1 pt + 1 pt]
   *Note : For both of the above parts, it can be shown that the answer can be expressed as an irreducible fraction $\frac{p}{q}$, where $p$ and $q$ are integers and $q \not\equiv 0 (mod\, M)$. Output the integer equal to $p.q^{-1} mod\, M$. In other words, output such an integer $x$ that $0 \leq x < M$ and $x.q \equiv p\, (mod\, M)$. Let $M = 10^9 + 7$.*

2. Bob's form is susceptible to his performance in the previous round.

   - If he won the previous round, then he chooses to play defensively.
   - If the previous round resulted in a draw, then he plays balanced.
   - If he lost the previous round, then he plays aggressively.

   (a) What is the optimal strategy for Alice to maximize her points gained in the current round? Perform a Monte Carlo simulation to demonstrate the advantage. [1 pt]

   (b) Is it optimal for Alice to play the greedy strategy (derived in part (a)) in every round to maximize total points? If not, give a situation where a non-greedy strategy outperforms the greedy strategy. Validate your findings by running a Monte Carlo simulation. [1 pt]

(c) Define a new Random Variable $\tau = \{\text{Number of Rounds for Alice to get } T \text{ wins}\}$. For the greedy strategy, estimate $\mathbb{E}[\tau]$ by performing Monte Carlo simulations where $T = T_3T_4$ represents the last 2 digits of your Entry Number. (Replace any 0's in your Entry Number with 9). [1 pt]
*For ex. if your entry number is 2020AB12345, then $T_3T_4$ is 45.*

3. Bob is now more intelligent, and instead of being predictable, he chooses his play style uniformly at random for each round.

   (a) What should Alice's strategy be to maximize the number of points gained in the current round? Run Monte Carlo to validate your findings. [1 pt]

   (b) What should Alice's strategy be to maximize the expected number of points in $T$ future rounds, where $T = T_3T_4$ represents the last 2 digits of your Entry Number? Also, calculate the expected number of points Alice has at the end of $T$ rounds. (Replace any 0's in your Entry Number with 9) . [1 pt]
   *Note : In this part, it can be shown that the answer can be expressed as an irreducible fraction $\frac{p}{q}$, where $p$ and $q$ are integers and $q \not\equiv 0 (mod\, M)$. Output the integer equal to $p.q^{-1} mod\, M$. In other words, output such an integer $x$ that $0 \leq x < M$ and $x.q \equiv p\, (mod\, M)$. Let $M = 10^9 + 7$.*

## Remarks

- Run your Monte Carlo simulations for only $10^5$ iterations.
- In the questions involving Monte Carlo simulation, implement the strategy and the functions to run the simulation. The solution will be judged by how close your simulated answer is to the actual answer.
- Clearly show all your calculations wherever necessary and comment your code for readability.
- Any instance of copying (theory/code) will be penalized heavily. All code will be checked for plagiarism.
- All your code should run within 20 seconds to pass all the testcases.

## Submission Format

We have provided you with a starter code to help you implement the required functions and perform Monte Carlo simulations. For questions 1, 2a, 2b, 2c, 3a, 3b, implement your logic in respective .py files. Some files contains classes for Alice and Bob. It initialises variables `past_play_styles`, `results`, `opp_play_styles`, `points`. You have to implement 2 methods of the player class for Alice and Bob, `play_move` and `observe_result`.

Additionally, you have to implement the functions `simulate_round` and `monte_carlo` given to run the simulations.

For the derivations for questions 1a, 1b, 2a, 2b, 3a, 3b, upload your solutions by either using Latex or neatly writing them down on a paper with your entry number at the top of the page.

For the coding part, submit your code in a folder named `EntryNumber_mtl106_a1` containing files for all the questions named according to the question number similar to the stater code given to you.