Irrotational flow:

$$\boxed{\nabla \times \bar{V} = 0}$$

$$\boxed{\vec{\omega} = \frac{1}{2}(\nabla \times \bar{V})}$$

$$\vec{V} = -\nabla \phi$$

$\phi$ - velocity potential

$$u = -\frac{\partial \phi}{\partial x} \qquad v = -\frac{\partial \phi}{\partial y} \qquad w = -\frac{\partial \phi}{\partial z}$$

$\phi =$ constant $\rightarrow$ on equipotential line

---

$$d\phi = 0$$

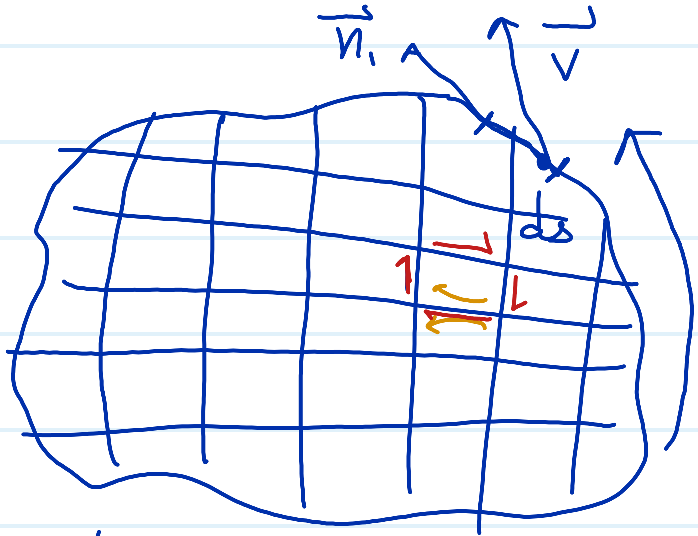
$$\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$$

$\boxed{u\,dx + v\,dy = 0}$ Equipotential line

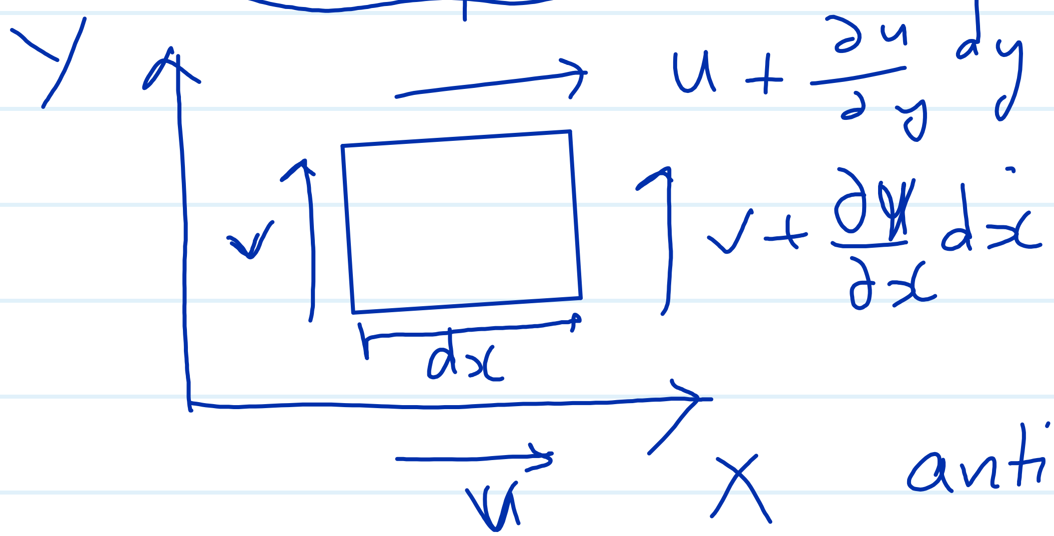
$\boxed{v\,dx - u\,dy = 0}$ Streamline

# Circulation:

$$\boxed{\oint \vec{V}\cdot\vec{n}_1\, ds = \Gamma}$$




circulation around the countor




anticlockwise = +ve

$$= \cancel{u\,dx} + \left(\cancel{v} + \frac{\partial v}{\partial x} dx\right) dy - \cancel{v\,dy} - \left(\cancel{u} + \frac{\partial u}{\partial y} dy\right) dx$$

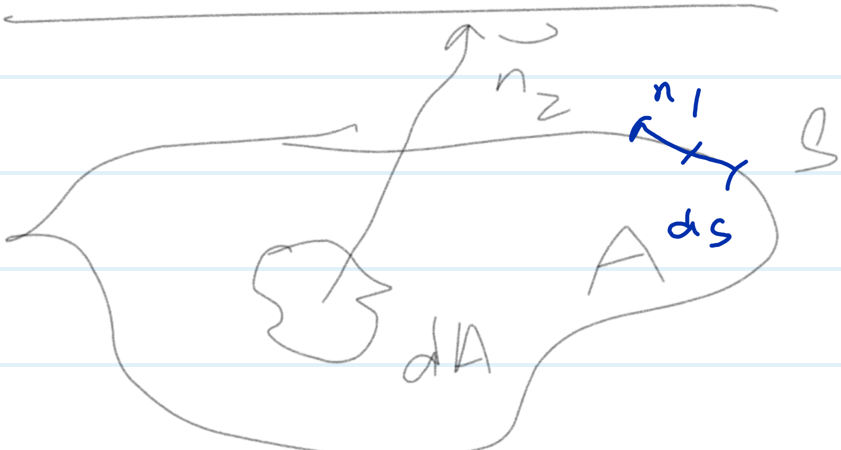
$$\Gamma = dx\,dy \left(\overbrace{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}}^{\overline{\Omega} = 2\times\overline{\omega}_z}\right)$$

$$\Gamma = \Omega_z \, dx \, dy$$

$$\boxed{\frac{\Gamma}{A} = \Omega_z} \quad \star$$

$$\boxed{\text{vorticity} = \frac{\text{Circulation}}{\text{Area}}} \quad \star$$

## Stokes Theorem

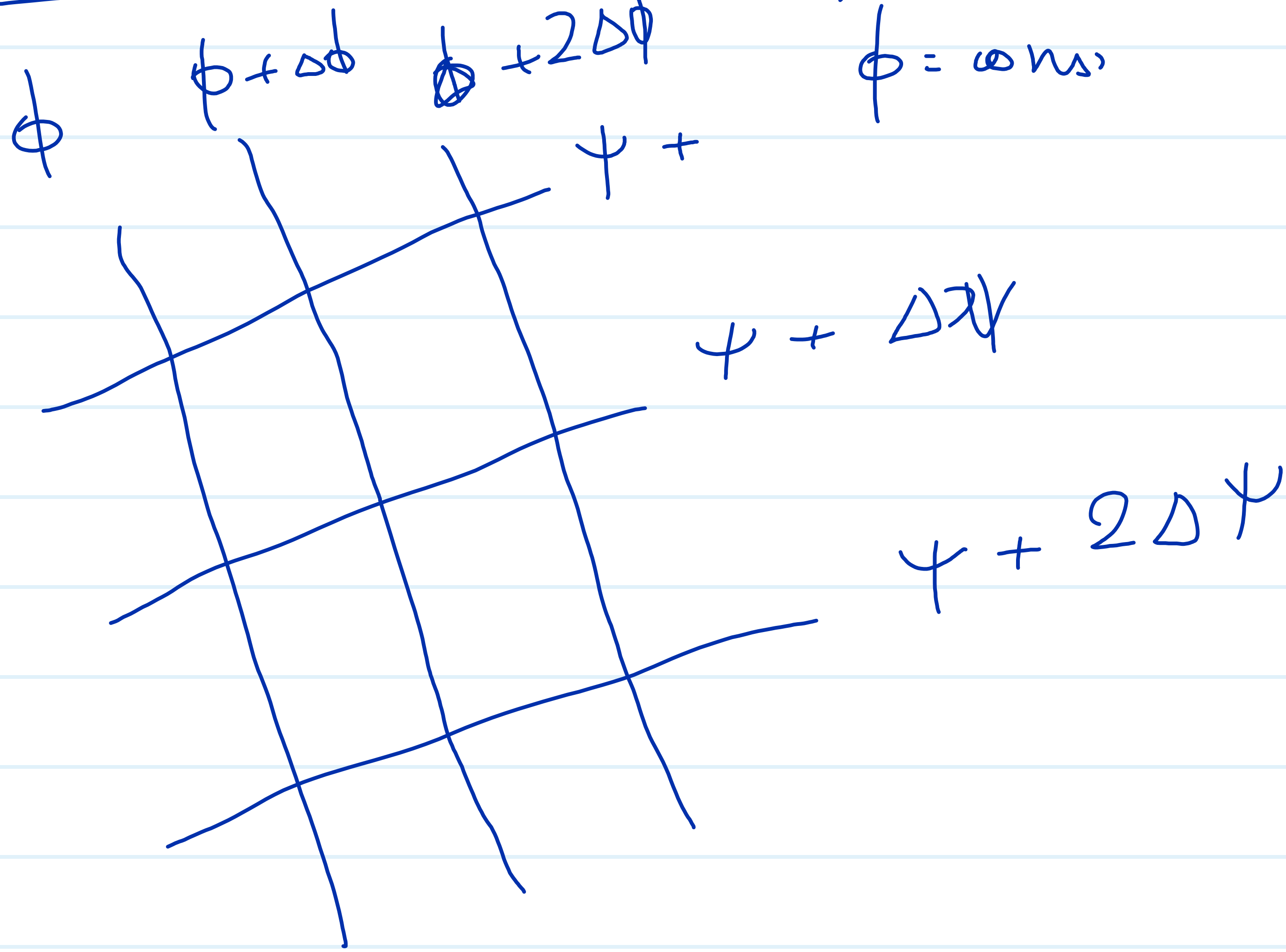



$$\boxed{\iint (\nabla \times \vec{P}) \, \vec{n}_2 \, dA = \oint \vec{P} \; \vec{n}_1 \, ds}$$

# Flow Nets

$\psi = \text{consta}$

$\phi = \text{cons}$




when $\Delta\psi = \Delta\phi = \Delta S$

$\Delta S \to 0$ the squ

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

$$u = -\frac{\partial \phi}{\partial x} \qquad v = -\frac{\partial \phi}{\partial y}$$

velocity $\uparrow$

closeness $\uparrow$

# Incompressible & Irrotational

$\psi = \psi(x, y)$ $\qquad\qquad$ $\phi = \phi(x, y)$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

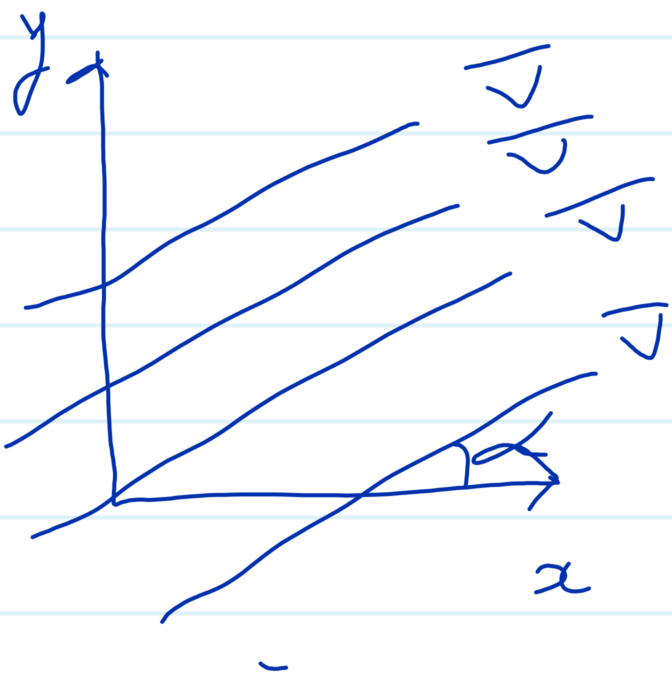
$$u = \frac{\partial \psi}{\partial y} \qquad\qquad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

$$v = -\frac{\partial \psi}{\partial x} \qquad\qquad \boxed{\nabla^2 \psi = 0}$$

$$\boxed{\nabla^2 \phi = 0}$$

# Basic Flows:

## 1) Rectilinear Flows:




$$\vec{V} = u\hat{i} + v\hat{j}$$

$$u = V\cos\alpha$$

$$v = V\sin\alpha$$

$$u = \frac{\partial \psi}{\partial y} \qquad v = \frac{-\partial \psi}{\partial x}$$

$$\psi = uy + f(x) + c_1$$

$$\psi = -vx + f(y) + c_2$$

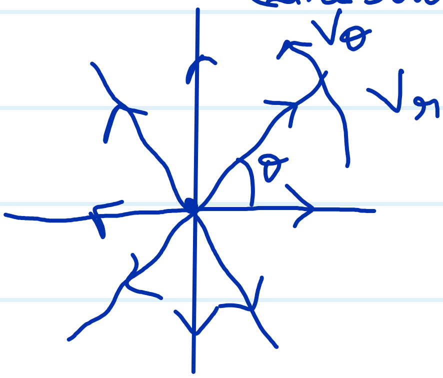
$$\boxed{\psi = uy - vx + c} \quad (1)$$

$$\frac{\partial \phi}{\partial x} = u \qquad \frac{\partial \phi}{\partial y} = v$$

$$\boxed{\phi = ux + vy + c'} \quad (2)$$

# Source & Sink :

(Line Source)




strength $= \dot{m} = \rho\, 2\pi V_r$

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad V_\theta = -\frac{\partial\psi}{\partial r} = 0$$

by definition $V_\theta = 0$ $\qquad \psi = f(\theta)$

$$\frac{1}{\cancel{r}}\frac{\partial\psi}{\partial\theta} = \frac{m}{2\pi\cancel{r}\rho}$$

$$\psi = \frac{m}{2\pi\rho}\theta + c_1$$

strength = mass flow rate $(m) = \rho\, 2\pi r V_r$

$(\Lambda)$ volume flow rate $= \frac{m}{\rho} = 2\pi r V_r$

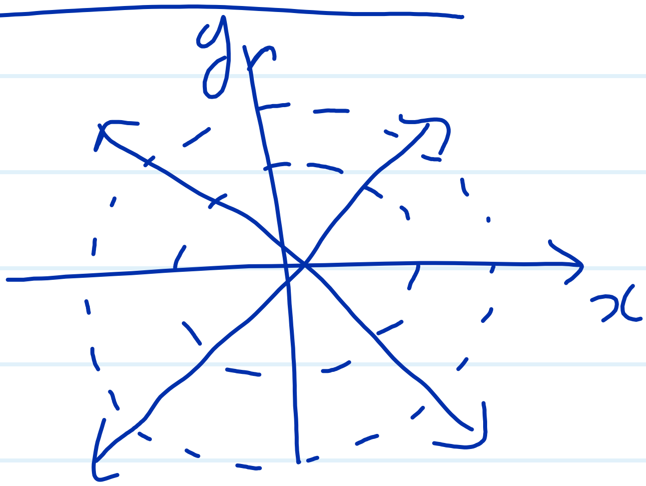
$$\boxed{\psi = \frac{\Lambda}{2\pi}\theta + c_1}$$ ★

$$V_r = \frac{\partial\phi}{\partial r} \qquad \overset{0}{\cancel{V_\theta}} = \frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

$$\frac{\partial \phi}{\partial r} = \frac{\Lambda}{2\pi r}$$

$$\boxed{\phi = \frac{\Lambda}{2\pi} \ln\left(\frac{r}{c_2}\right)}$$ ☆

## For a Source:




$$\psi = \frac{\Lambda}{2\pi}\theta + C,$$

$$\phi = \frac{\Lambda}{2\pi} \ln\left(\frac{r}{c_2}\right)$$

$\phi = \psi = 0 = r = \theta$

$c_1 = 0 \quad c_2 = 0$

$$\boxed{\psi = \frac{\Lambda}{2\pi}\theta}$$

$$\boxed{\phi = \frac{\Lambda}{2\pi} \ln(r)}$$

## For a Sink.

physically same but negative

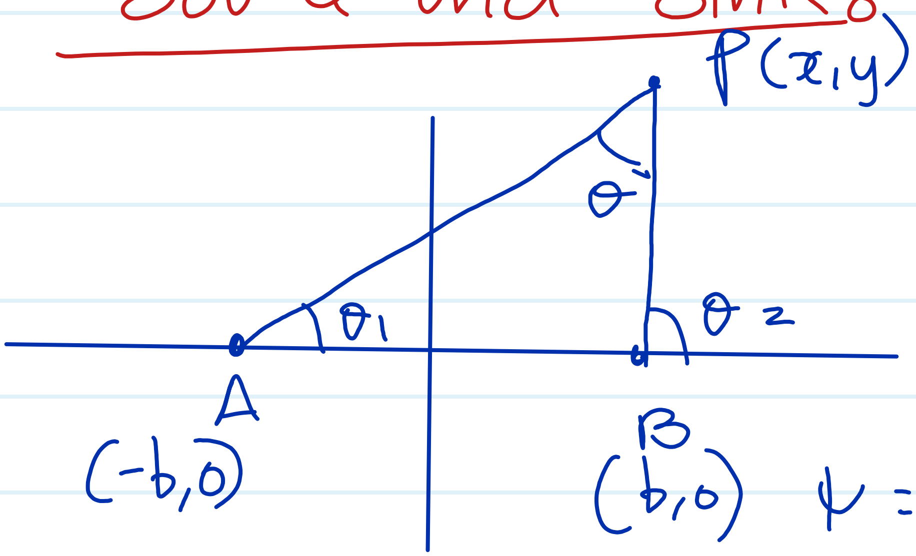
$$\boxed{\psi = -\frac{\Lambda}{2\pi}\theta \quad \Big| \quad \phi = -\frac{\Lambda}{2\pi} \ln r}$$

# Superimposition of Basic Flows

$\vec{V_1}\ \psi_1 \qquad \vec{V_2}\ \psi_2$

$\vec{V} = \vec{V_1} + \vec{V_2} \qquad \psi = \psi_1 + \psi_2$

## Source and Sink:




$$\psi_A = \frac{\Lambda}{2\pi}\theta_1$$

$$\psi_B = \frac{\Lambda}{2\pi}\theta_2$$

$$\psi = \frac{\Lambda}{2\pi}(\theta_1 - \theta_2)$$

$$\boxed{\psi = \frac{\Lambda}{2\pi}\theta} \quad \star$$

$$\tan\theta_1 = \frac{y}{b+x} \qquad \tan\theta_2 = \frac{y}{x-b}$$

$$\tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$$

$$\tan(\theta_1 - \theta_2) = \frac{2by}{x^2+y^2-b^2}$$

$$\theta = \tan^{-1}\left(\frac{2by}{x^2+y^2-b^2}\right)$$

$$\boxed{\psi = \frac{\Lambda}{2\pi}\tan^{-1}\left(\frac{2by}{x^2+y^2-b^2}\right)}$$

$2b \to 0$ $\qquad$ $\tan^{-1}x = x$

# Doublet

$x \to 0$

$2\Lambda b = \text{const}$ Doublet

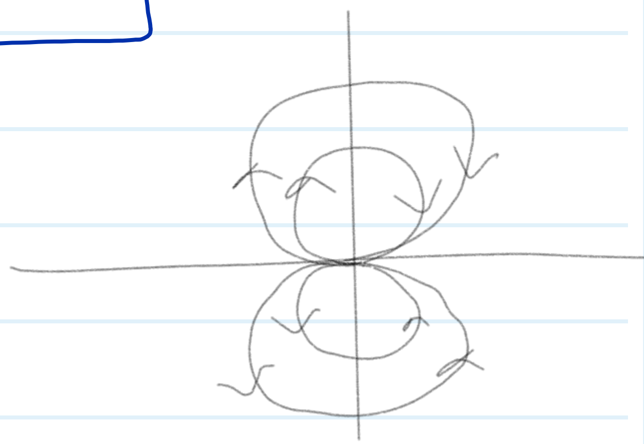
$$\psi = \lim_{b \to 0} \frac{\Lambda}{2\pi} \frac{2by}{x^2+y^2-b^2}$$

$$\psi = \frac{b\Lambda}{\pi} \frac{y}{x^2+y^2}$$

$$\psi = \frac{b\Lambda}{\pi} \frac{r\sin\theta}{r^2}$$

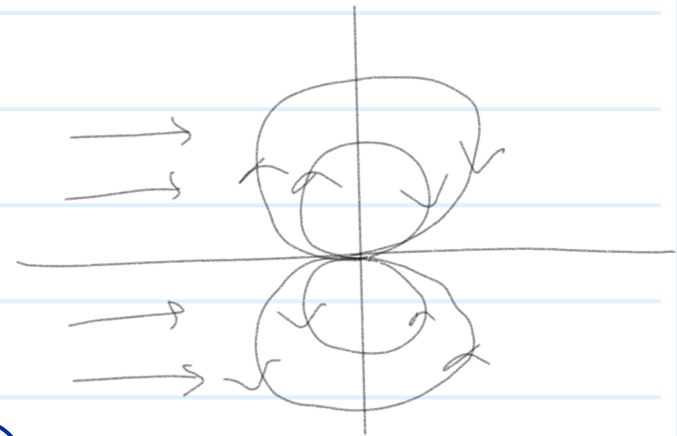
$$c = \frac{b\Lambda}{\pi}$$

$$\boxed{\psi = c\,\frac{\sin\theta}{r}}$$



# Superimposition of Rectilinear flow and doublet parallel to x-axis

$$\psi = -uy + \frac{c\sin\theta}{r}$$



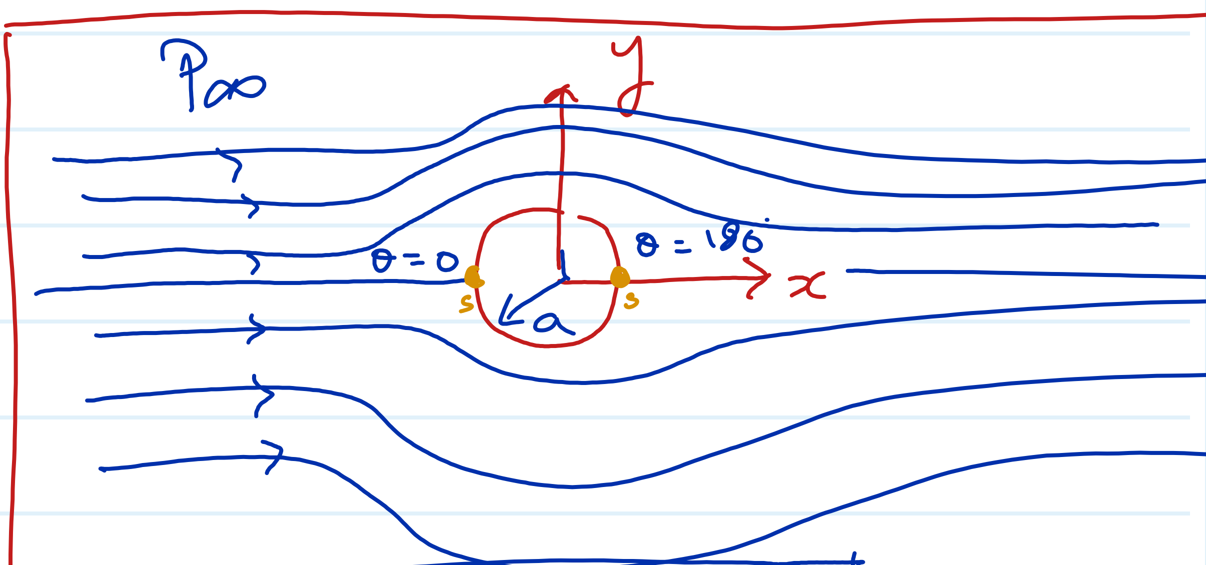
$$\psi = -ur\sin\theta + \frac{c\sin\theta}{r}$$

$\psi = 0$ at $\sin\theta - (0$ or $180)$

$$u r = \frac{c}{r}$$

$$\boxed{r = \sqrt{\frac{c}{u}}}$$

$$\boxed{\psi = -u\sin\theta\left(r - \frac{a^2}{r}\right) = 0}$$




$$\boxed{\psi = -u\sin\theta\left(r - \frac{a^2}{r}\right) = 0}$$

$$V_\theta = \frac{\partial \psi}{\partial r} = -u\sin\theta\left(1+\frac{a^2}{r^2}\right)$$

$$V_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta} = u\cos\theta\left(1-\frac{a^2}{r}\right)$$

at $r=a$,

$$V_r = 0$$

$$V_\theta = -2u\sin\theta$$

$\theta = 0, 180$

Stagnation points

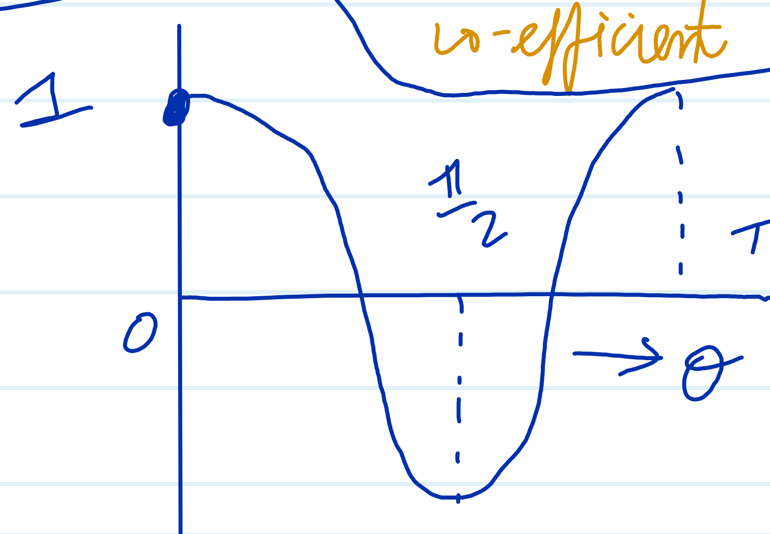
$$P_\infty + \frac{1}{2}\rho u^2 = P + \frac{1}{2}\rho(-2u\sin\theta)^2$$

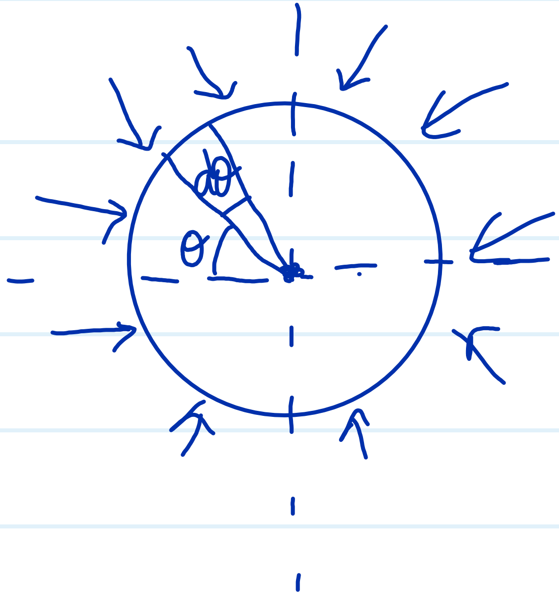
$$P = P_\infty + \frac{1}{2}u^2 - 2u^2\sin^2\theta \,.$$

$$\boxed{\frac{(P-P_\infty)}{\frac{1}{2}\rho u^2} = 1-4\sin^2\theta = C_p}$$

$\downarrow$

pressure co-efficient





$$F_x = \int_0^{2\pi} P a \cos\theta \, d\theta = 0$$

$$F_y = -\int_0^{2\pi} p a \sin\theta \, d\theta = 0$$

# Vortex Flow

$$V_\theta = \frac{C_1}{r} \qquad \omega_z = 0$$

$$\Omega_{r\theta} = 0$$

$$V_r = 0$$

$$\Gamma = \int_0^{2\pi} V_\theta \, r \, d\theta = C_1 \, 2\pi$$

$$\boxed{V_\theta = \frac{\Gamma}{2\pi r}} \qquad \boxed{C_1 = \frac{\Gamma}{2\pi}}$$

$$v_\theta = \frac{c_1}{r}$$

$$v_\theta = \frac{\partial \psi}{\partial r}$$

$$\frac{\partial \psi}{c_1} = \frac{\partial r}{r}$$

$$\boxed{\psi = c_1 \ln\left(\frac{r}{r_0}\right)}$$

$$\boxed{\psi = \frac{\Gamma}{2\pi} \ln\left(\frac{r}{r_0}\right)}$$

# Rectilinear Flow ∥ to x axis + Doublet + Irrotational vortex

$$\psi = -u\sin\theta\left(r - \frac{a^2}{r}\right) + \frac{\Gamma}{2\pi}\ln\left(\frac{r}{r_0}\right)$$

$$V_\theta = \frac{\partial\psi}{\partial r} = -u\sin\theta\left(1 + \frac{a^2}{r^2}\right) + \frac{\Gamma}{2\pi r}$$

$$V_r = u\cos\theta\left(1 - \frac{a^2}{r^2}\right) + 0$$

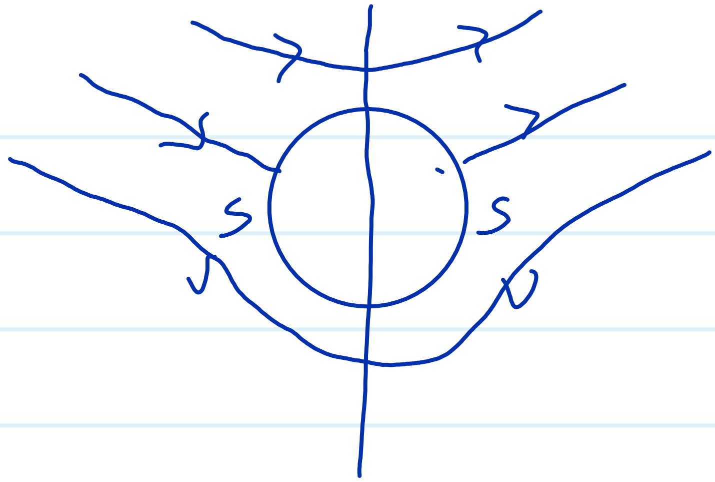
at $r = a$ $\quad V_r = 0$

$$\boxed{(V_\theta)_{r=a} = -2u\sin\theta + \frac{\Gamma}{2\pi a}} \quad ☆$$

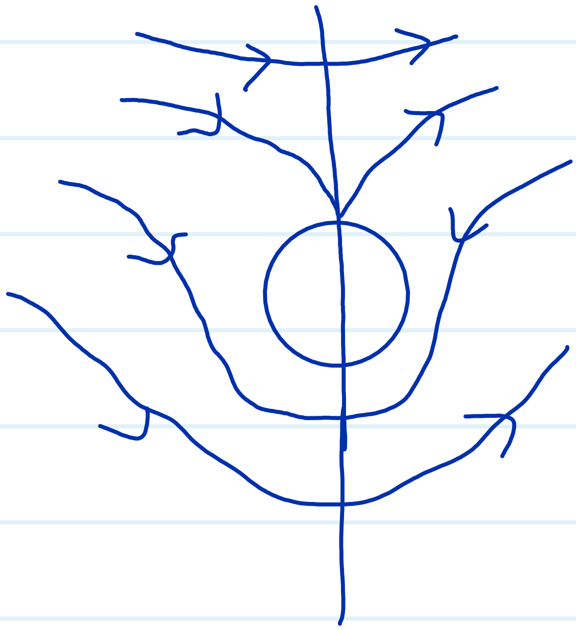
$V_\theta = 0$

$$\boxed{\sin\theta = \frac{\Gamma}{4\pi a u}} \quad ☆$$

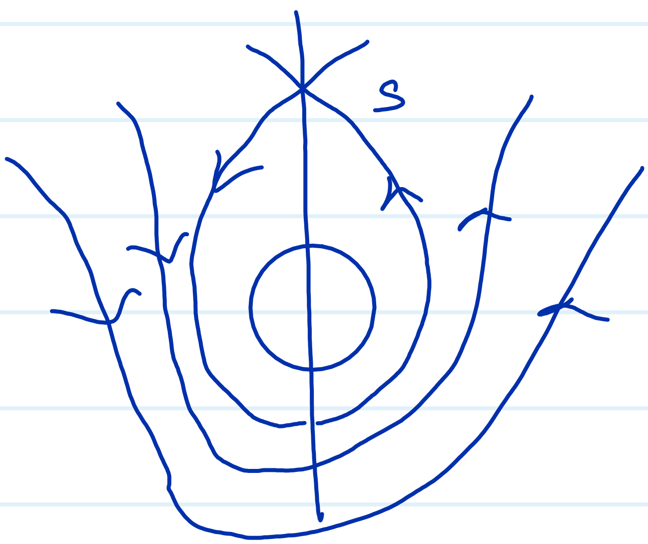
$$\frac{\Gamma}{4\pi a u} < 1$$




$$\frac{\Gamma}{4\pi a u} = 1$$



$$\frac{\Gamma}{4\pi a u} > 1$$



$$P_\infty + \frac{1}{2}\rho u^2 = P + \frac{1}{2}\rho\left(-2u\sin\theta + \frac{\Gamma}{2\pi a}\right)^2$$

$$P_\infty + \frac{1}{2}\rho u^2 \quad P + \frac{1}{2}\rho\left(4u^2\sin^2\theta - \frac{2u\sin\theta\,\Gamma}{\pi a} + \frac{\Gamma^2}{4\pi^2 a^2}\right)$$

$$\boxed{P = \text{constant} - 2\rho u^2\sin^2\theta - \frac{\rho u\Gamma}{\pi a}\sin\theta}$$

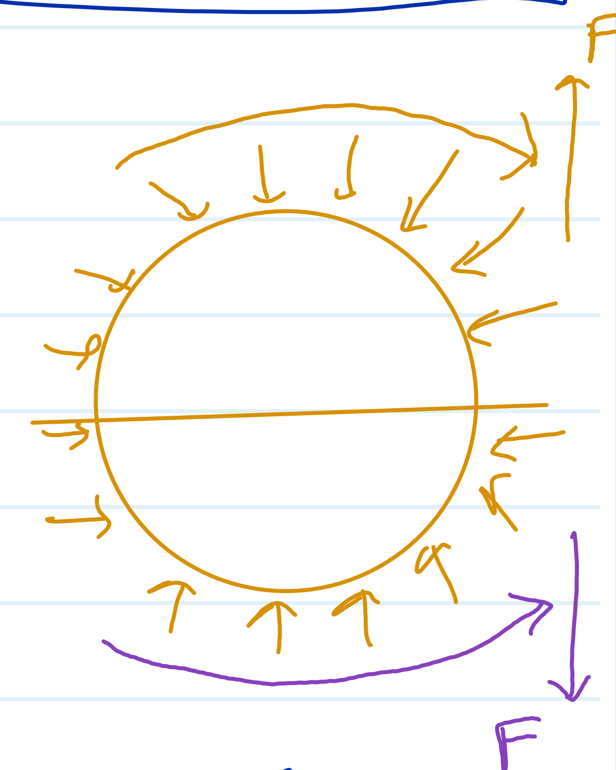
$$F_x = \int_0^{2\pi} pa\cos\theta\, d\theta = 0$$

$$F_y = -\int_0^{2\pi} pa\sin\theta\, d\theta$$

$$F_y = -\frac{\rho u\Gamma}{\pi}\int_0^{2\pi}\sin^2\theta\, d\theta$$

$$F_y = -\rho u\Gamma$$

$$\boxed{F_y = -\rho u\Gamma}$$




Magnus Effect

$F = \rho u\Gamma$

any body

Lift force