Dimensional Analysis :

1) Buckingham π theorem :

→ No of π terms (n) = $K - R$
where the flow phenomenon depends on K-parameters with R-basic dimension

Steps

1) Identify K parameters
2) Express K parameters in terms of $(M, L, T) / (F, L, T)$
3) Identify R value
4) Select R out of K parameters.
   → Combined, these parameters must contain all basic dimensions
   → Each parameter must be independent
5) Select one additional parameter and start forming π terms

Frequently Used Non-dimensional Numbers.

→ Reynolds Number = $\frac{\rho V \ell}{\mu}$

→ Euler Number = $\frac{\Delta p}{\rho v^2}$

→ Froude Number = $\frac{V}{\sqrt{g\ell}}$

→ Weber Number = $\frac{\rho V^2 \ell}{\sigma}$

→ Mach Number = $\frac{V}{c}$

→ Strouhal Number = $\frac{\omega \ell}{V}$ (ω = freq)

→ Drag coeff, lift coeff = $\frac{F}{\frac{1}{2}\rho V^2 \ell^2}$

Similitude and Modelling

$$\boxed{\pi_N^{(p)} = \pi_N^{(m)}}$$

Irrotational $\nabla \times \bar{V} = 0$, Incompressible $\nabla \cdot \bar{V} = 0$

$2\bar{\omega} = \Omega = \nabla \times \bar{V}$

$\frac{\partial \phi}{\partial x} = u \quad \frac{\partial \phi}{\partial y} = v$

$\frac{\partial \psi}{\partial y} = u \quad -\frac{\partial \psi}{\partial x} = v$

$\boxed{-\frac{\partial \psi}{\partial r} = V_\theta} \quad \boxed{\frac{1}{r}\frac{\partial \psi}{\partial \theta} = V_r}$

$\nabla^2 \phi = 0 \quad \nabla^2 \psi = 0$

$d\phi = 0 \qquad u\,dx + v\,dy = 0 \rightarrow$ equipotential line

$v\,dx - u\,dy = 0 \rightarrow$ streamline

Circulation.

$$\boxed{\oint \vec{V} \cdot \vec{ds} = \Gamma}$$

$$\boxed{\Omega_z = \frac{\Gamma}{A}}$$

Basic Flows

1) Rectilinear Flows:

$$\vec{V} = u\hat{i} + v\hat{j}$$

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

$$\boxed{\psi = uy - vx + c} \quad \text{--- (1)}$$

$$\boxed{\phi = ux + vy + c'} \quad \text{--- (2)}$$

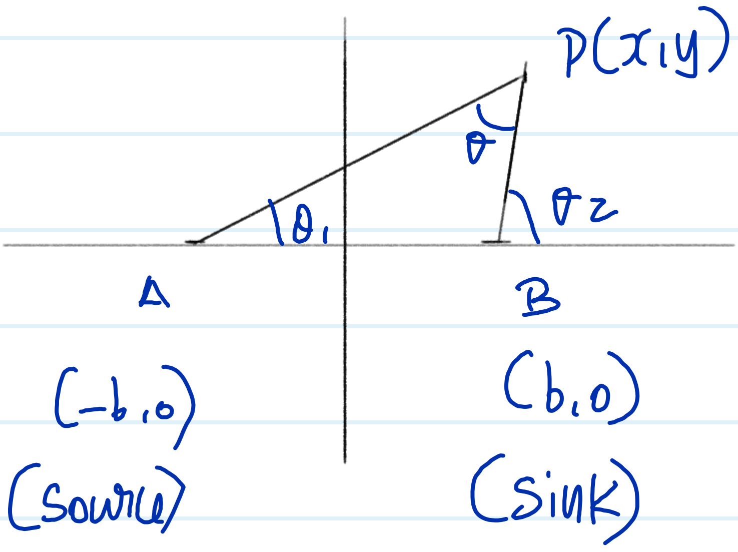
For a Source:-

$\psi = \dfrac{\Lambda}{2\pi}\theta$	$\phi = \dfrac{\Lambda}{2\pi}\ln r$	$V_r = \dfrac{\Lambda}{2\pi r}$

For a Sink:

$\psi = \dfrac{-\Lambda}{2\pi}\theta$	$\phi = \dfrac{\Lambda}{2\pi}\ln r$	$V_r = \dfrac{-\Lambda}{2\pi r}$

Source and Sink:



$$\psi_A = \frac{\Lambda}{2\pi}\theta_1$$

$$\psi_B = \frac{-\Lambda}{2\pi}\theta_2$$

$$\therefore \psi_{A-B} = \frac{\Lambda}{2\pi}(\theta_1 - \theta_2)$$

$$\boxed{\psi_{A-B} = \frac{\Lambda}{2\pi}\tan^{-1}\left(\frac{2by}{x^2+y^2-b^2}\right)}$$

Doublet.

$b \to 0$ $\quad \lambda \to \infty$ $\quad 2\lambda b = \text{constant} = \chi$

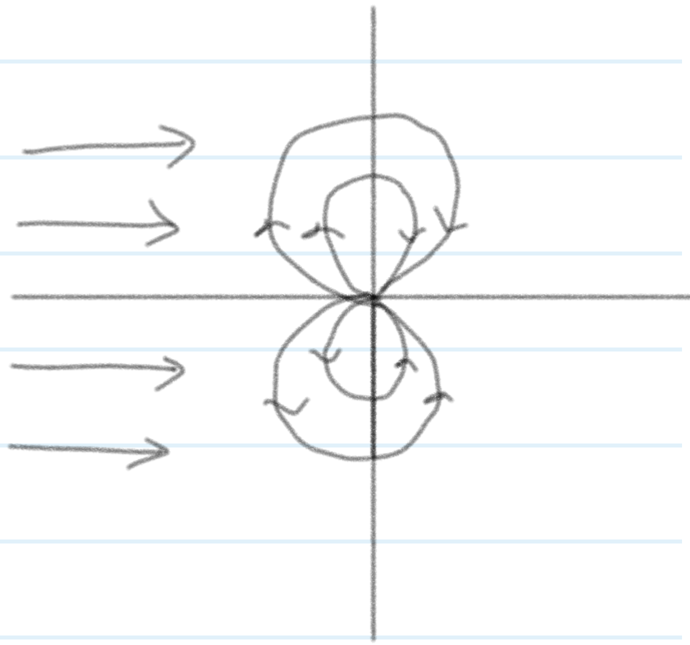
$$\psi = \chi \left(\frac{y}{x^2+y^2} \right) \qquad \psi = \chi \frac{\sin\theta}{r}$$

Rectilinear + doublet

$$\psi = -uy + \frac{\chi \sin\theta}{r}$$

$$\psi = -ur\sin\theta + \frac{\chi \sin\theta}{r}$$

$$\psi = -r\sin\theta \left(u - \frac{\chi}{r^2} \right)$$



D $\psi = 0$ when

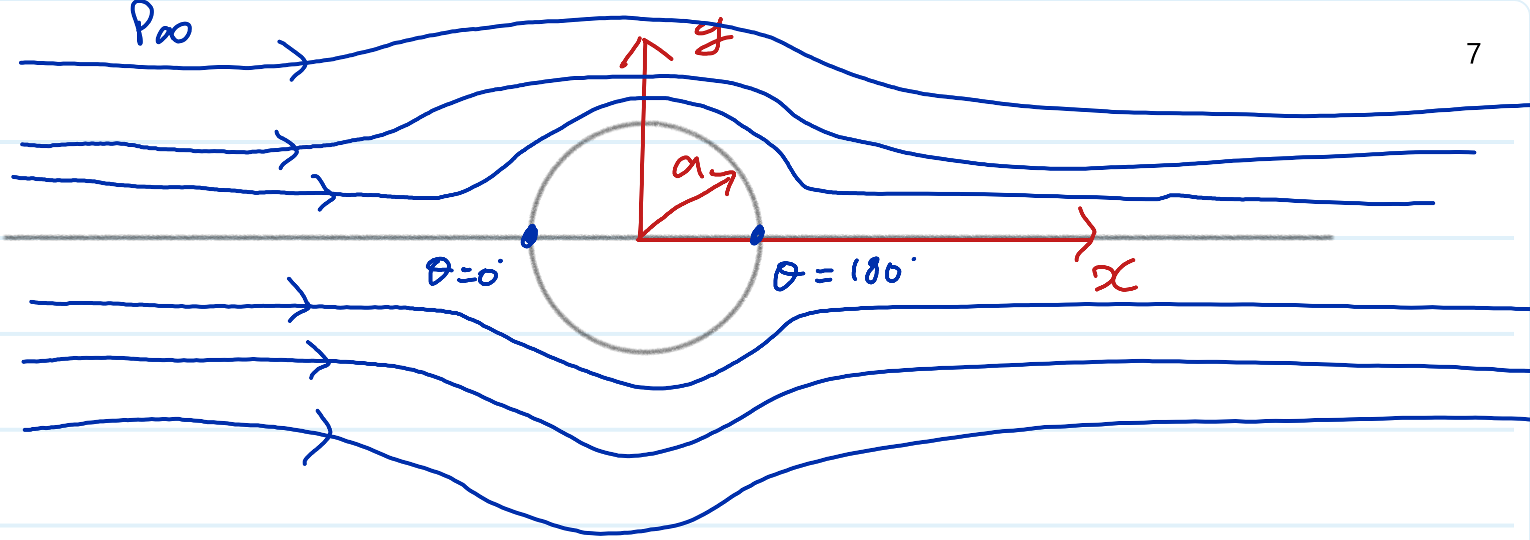
$$\frac{\chi}{r} = ur \qquad \sin\theta \,(0 \text{ or } 180^\circ)$$

$$\Downarrow$$

$$\boxed{a = \sqrt{\frac{\chi}{u}}}$$

$$\psi = -u\sin\theta \left(r - \frac{\chi}{ur} \right)$$

$$\boxed{\psi = -u\sin\theta \left(r - \frac{a^2}{r} \right)}$$



Stagnation point at $\theta = 0, 180$, $r = a$

$V_\theta = -u\sin\theta\left(1+\frac{a^2}{r^2}\right)$ $V_r = u\cos\theta\left(r - \frac{a^2}{r}\right)$

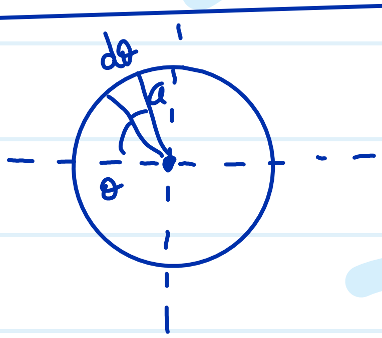
$r = a$ $V_r = 0$

$\boxed{V_\theta = -2u\sin\theta}$ $\theta = 0, 180$ stagnat.

$P_\infty + \frac{1}{2}\rho u^2 = P + \frac{1}{2}\rho(-2u\sin\theta)^2$

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho u^2} = 1 - 4\sin^2\theta$$

pressure co-eff



$F_x = \int_0^{2\pi} P\cos\theta \, a\,d\theta = 0$

$F_y = -\int_0^{2\pi} P\sin\theta \, a\,d\theta = 0$

Vortex Flow

$V_r = 0 \qquad V_\theta = \frac{C_1}{r}$

$\omega_z = 0 \qquad \Omega_{r\theta} = 0$

$$\Gamma = \oint_0^{2\pi} V_\theta \, r \, d\theta = \frac{c}{\cancel{r}} \cancel{r}\, 2\pi$$

$$\boxed{c = \frac{\Gamma}{2\pi}} \qquad \boxed{V_\theta = \frac{\Gamma}{2\pi r}}$$

$$V_\theta = \frac{\partial \psi}{\partial r}$$

$$\boxed{\psi = \frac{\Gamma}{2\pi} \ln(r)}$$

Rectilinear, Doublet, Vortex.

$$\psi = -u\sin\theta\, r + \frac{ua^2}{r} + \frac{\Gamma}{2\pi}\ln r$$

$$\frac{\partial\psi}{\partial r} \quad \boxed{V_\theta = -u\sin\theta\left(1+\frac{a^2}{r^2}\right) + \frac{\Gamma}{2\pi r}}$$

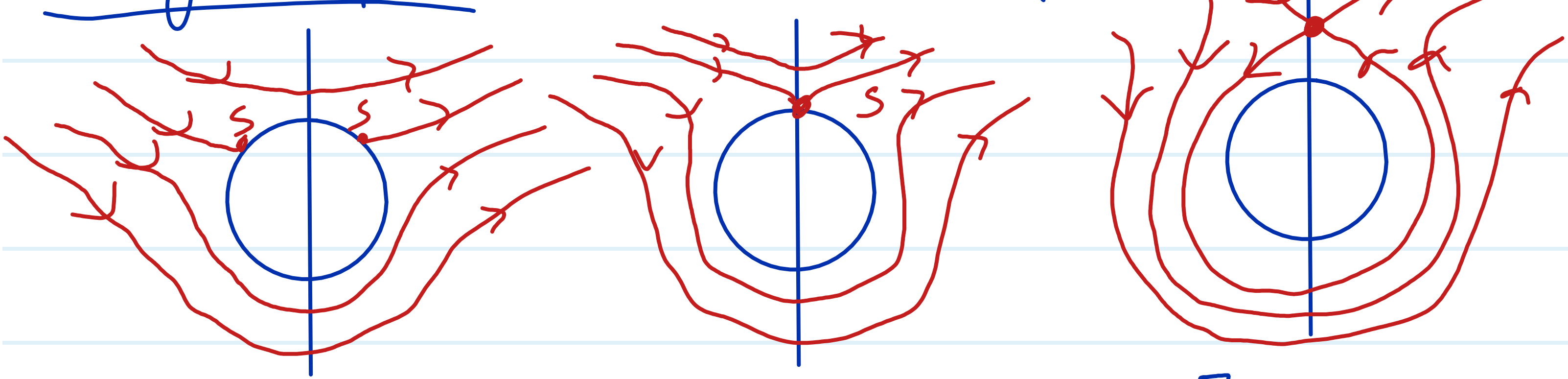
$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} \quad \boxed{V_r = u\cos\theta\left(1-\frac{a^2}{r^2}\right)}$$

$\boxed{r = a}$ $\quad V_r = 0$

$$V_\theta = -2u\sin\theta + \frac{\Gamma}{2\pi a}$$

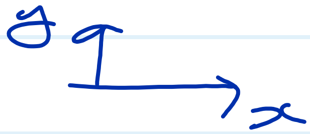
Stagnation points

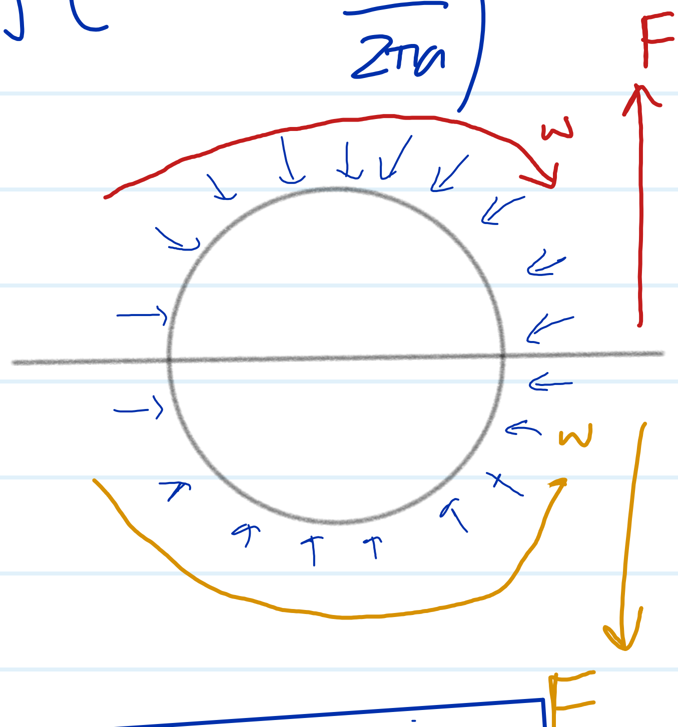
$V_\theta = 0 \quad \sin\theta = \frac{\Gamma}{4\pi a u}$



$\frac{\Gamma}{4\pi a u} < 1$ $\qquad$ $\frac{\Gamma}{4\pi a u} = 1$ $\qquad$ $\frac{\Gamma}{4\pi a u} > 1$

$$P_\infty + \frac{1}{2}\rho u^2 = P + \frac{1}{2}\rho\left(-2u\sin\theta + \frac{\Gamma}{2\pi a}\right)^2$$

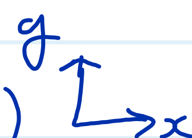




$$F_x = 0$$

$$\boxed{F_y = -\rho u \Gamma}$$

Γ = anticlockwise (neg)



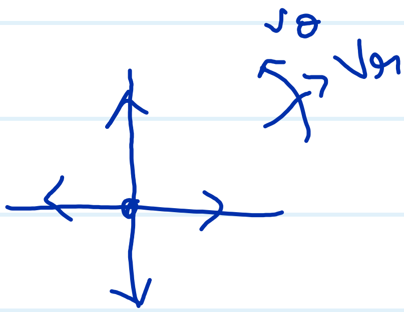
2) Source / Sink

$$\dot{m} = 2\pi r V_r \rho$$

$\dot{m}$ = mass flow rate

Λ = volume flow rate $= \dfrac{\dot{m}}{\rho}$

K = strength $= \dfrac{\dot{m}}{2\pi\rho} = \dfrac{\Lambda}{2\pi}$



$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad \cancel{v_\theta}^{\,0} = \frac{\partial \psi}{\partial r}$$

$$\boxed{\psi = \frac{\dot{m}}{2\pi\rho}\theta + C_1 = \frac{\Lambda}{2\pi}\theta + C_1}$$

$$v_r = \frac{\partial \phi}{\partial r} \qquad \cancel{v_\theta}^{\,0} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

$$\boxed{\phi = \frac{\Lambda}{2\pi}\ln\left(\frac{r}{C_2}\right)}$$