Dimensional Analysis

→ Simple method of predicting physical phenomena

for instance,

$$F_D = f(L, V, \mu, \rho)$$

Collect these 5 parameters into a smaller number non-dimensional terms

→ Dimensionless groups are called π - terms

If we have "N" number of π terms

$$\boxed{\pi_N = f(\pi_1, \pi_2, \pi_3, \ldots, \pi_{N-1})}$$ ☆

Buckingham π theorem

If a flow phenomenon depends on K terms with R basic dimensions

$$\boxed{N = K - R}$$ ✰

How to Solve:

1) Identify K parameters
2) Express K parameters in terms of M, L, T
3) Identify R value
4) Select R out of K parameters
   - ✰ combined R parameters must contain all basic dimensions
   - ✰ Each parameter must be independent
5) Select one additional parameter and use $(R+1)$ terms to form π terms
6) Repeat 5 $(K-R-1)$ times

Common Non-Dimensional Numbers in Fluid Mechan

Reynolds Number $= \dfrac{\rho V l}{\mu} = \dfrac{\text{inertia force}}{\text{viscous force}}$

Euler Number $= \dfrac{\Delta p}{\rho v^2} = \dfrac{\text{pressure force}}{\text{inertia force}}$

Froude Number $= \dfrac{V}{\sqrt{g l}} = \dfrac{\text{inertia force}}{\text{gravitational force}}$

Weber Number $= \dfrac{\rho v^2 l}{\sigma} = \dfrac{\text{inertia force}}{\text{surface tension}}$

Mach Number $= \dfrac{V}{c} = \dfrac{\text{inertia force}}{\text{compressibility fo}}$

$\boxed{\omega \text{ - freq}}$

Strouhal Number $= \dfrac{\omega l}{V} = \dfrac{\text{local inertia}}{\text{convective inertia}}$

Drag coefficient $= \dfrac{F_D}{\frac{1}{2}\rho v^2 l^2} = \dfrac{\text{drag}}{\text{inertia}}$

Lift coeff. $= \dfrac{F_L}{\frac{1}{2}\rho V^2 l^2} = \dfrac{\text{lift}}{\text{inertia}}$

Problems

① $F_D = f(l, V, \mu, \rho)$

Sol:

1) $F_D = f(l, V, \mu, \rho)$

2) $F_D = [MLT^{-2}]$

$l = [L]$

$V = [LT^{-1}]$

$\mu = [ML^{-1}T^{-1}]$

$\rho = [ML^{-3}]$

3) $M, L, T \quad (R = 3)$

4) Select 3 parameter (ρ, l, V)

5) $\pi_1 \rightarrow (F_D, \rho, l, V) \rightarrow$ first time choose LHS

6) $\pi_2 \rightarrow (\mu, \rho, l, V)$

$$\pi_1 \rightarrow F_D \, \rho^a l^b v^c$$

$$[MLT^{-2}][ML^{-3}]^a[L]^b[LT^{-1}]^c = M^0L^0T^0$$

$$M^{1+a} \; L^{1+b-3a+c} \; T^{-2-c} = M^0L^0T^0$$

$1+a=0 \qquad a=-1$

$1+b-3a+c=0 \Rightarrow 1+b+3-2=0 \qquad b=-2$

$-2-c=0 \qquad c=-2$

$$\boxed{\pi_1 = \frac{F_D}{\rho l^2 v^2}}$$

$$\pi_2 \rightarrow (\mu, \rho, l, v)$$

Reylonds number $= \dfrac{\rho v l}{\mu}$

$$\boxed{\frac{F_D}{\rho l^2 v^2} = fn(Re)}$$

② Turbulent flow in a pipe,

Δp is a fn of $D, V, l, \rho, \mu, \left(\frac{\varepsilon}{D}\right)$ roughness

Find suitable π terms

Sol.

1) $\Delta p = f(D, v, l, \rho, \mu, \varepsilon/D)$

2) $\Delta p = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

$D = [L]$

$l = [L]$

$\rho = [ML^{-3}]$

ε is length $\quad \mu = [ML^{-1}T^{-1}]$

$\frac{\varepsilon}{D} = [0]$

3) $R = 3$

4) $\cancel{(D, V, l)}$ $\quad (D, v, \rho)$

1) Combination - all M, L, T, should be included

2) Dont repeat same dimensions

5) $\Pi_1 (\Delta p, D, V, \rho) \Rightarrow \dfrac{\Delta p}{\rho V^2}$ (Euler's number)

6) Repeat $k-r-1$ times

$7-3-1 = 3$ times

$\Pi_2 (\ell^1, D^a, V^b, \rho^c) \Rightarrow \dfrac{L}{D}$ $a=-1$ $b=0$ $c=0$

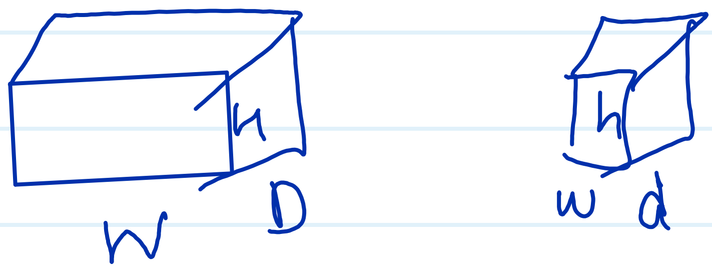
$\Pi_3 (\mu, D, V, \rho) \Rightarrow \dfrac{\rho V D}{\mu}$ (Reynolds)

$\Pi_4 \left(\frac{\epsilon}{D}^1, D^a, V^b, \rho^c\right) \Rightarrow \dfrac{\epsilon}{D}$ $a=0$ $b=0$ $c=0$

7) $$\frac{\Delta p}{\rho V^2} = f\left(\frac{L}{D}, \frac{\epsilon}{D}, \frac{\rho V D}{\mu}\right)$$

Similitude and Modelling

1) Geometric Similarity: $$\frac{B}{b} = \frac{H}{h} = \frac{W}{w}$$



2) Kinematic Similarity.

velocities / accelerations

→ same direction
→ multiplied regardin

3) Dynamic Similarity

force (scaled)

↓

Modelling: predict parameters

Flow around the full scaled body
prototype

$$\pi_1^{(p)} = f(\pi_2^{(p)}, \pi_3^{(p)} \ldots \pi_{N-1}^{(p)}$$

Test a scaled down model

$$\pi_1^{(m)} = f\left(\pi_2^{(m)}, \pi_3^{(m)}, \ldots \pi_{N-1}^{(m)}\right)$$

$$\boxed{\begin{array}{l} \pi_1^{(p)} = \pi_1^{(m)} \\ \pi_{N-1}^{(p)} = \pi_{N-1}^{(m)} \end{array}} \rightarrow \text{Similarity requirements}$$

A $\frac{1}{5}$ scale of a vehicle is to be tested in two tunnels. One in a wind tunnel, second in a water tunnel. Determine the maximum velocity of water and air in the tunnels if the maximum speed of the prototype is 120 mph.

Kinematic

Sol. Reynolds number $= \dfrac{\rho v l}{\mu}$ $\qquad \boxed{\nu = \dfrac{\mu}{\rho}}$

viscosity

$$= \frac{V l}{\nu}$$

$$Re^{(m)} = Re^{(p)}$$

$$\frac{v_m l_m}{\nu_m} = \frac{v_p l_p}{\nu_p}$$

$$V_m = \underset{\substack{\downarrow \\ 120}}{V_p} \left(\frac{l_p}{\underset{\substack{\downarrow \\ 5}}{l_m}}\right)\left(\frac{\nu_m}{\underset{\substack{\downarrow \\ 1}}{\nu_p}}\right) = 600\text{mph}$$

Example: A model with a scale of 1/4 is to be tested to determine the velocity of discharge from a pinhole crack on the side of a pressured tank. The velocity is a function of the pressure in the tank, wall thickness, diameter of the pinhole crack and the viscosity of the fluid in the tank. Take the viscosity scale as 1.4.

a) Find the prediction equation and similarity requirements

b) Find the velocity scale

Sol

1) $V = f(P, t, D, \mu)$

2) $V = [L T^{-1}]$ $t = [L]$ $D = [L]$ $P = [M L^{-1} T^{-2}]$

$\mu = [M L^{-1} T^{-1}]$

3) $R = 3$

4) P, t, μ

5) $\pi_1 = f(V, P, t, \mu) = \dfrac{V \mu}{P d}$

b) $\pi_2 = f(D, \rho, t, \mu) = \frac{D}{t}$

$$\frac{\rho_m^{\ } v^m t^m}{\mu^m} = \frac{\rho_p\, v^p t^p}{\mu^p}$$ prediction equation

$$\frac{D^m}{t^m} = \frac{D^p}{t^p}$$ requirement

$$\frac{v^m}{v^p} = \left(\frac{t^p}{t^m}\right)\left(\frac{\mu^m}{\mu^p}\right)\left(\frac{\rho^p}{\rho^m}\right)$$