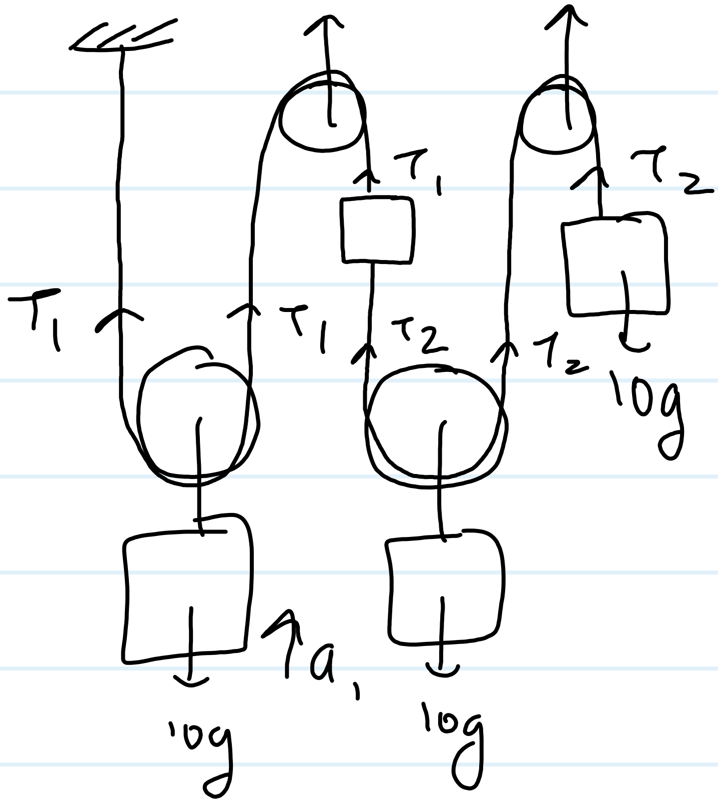
1 2 3 4 5 6 7 8

9 10 11 12 13

2)



$2T_1 = 500$

$\boxed{T_1 = 250}$

$2T_2 = 250$

$\boxed{T_2 = 125}$

$2T_1 - 10g = 10a_1$

$\boxed{a_1 = 40} \uparrow$

$T_1 - T_2 - 10g = 10a_2$

$\boxed{2.5 = a_2} \uparrow$

$\boxed{15 = a_3 \uparrow}$

$2\ 5 = a_4$

$g = 10 \quad a_4 = 15\,ms^{-2}$

$g = 9\ 8 = 18\ 19\,ms^{-2}$

3) $\Delta E = \cancel{W}_{a-b}^{\,NC^{\,0}}$

$0 \; \Delta E = 0$

$\cancel{T_1} + V_1 = T_2 + V_2$

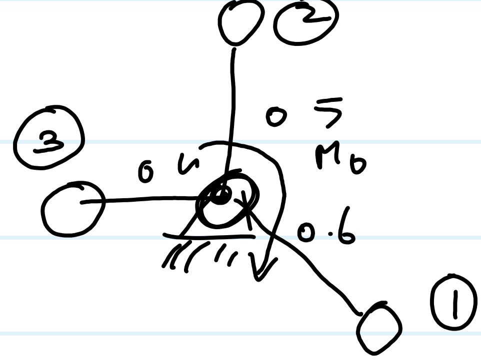
$$mg\frac{b}{\sqrt{2}} + 2mg\frac{b}{\sqrt{2}} = 0 + \frac{1}{2}3mv^2$$

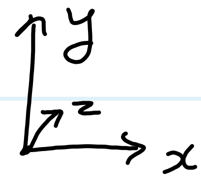
$$\frac{\cancel{3}\cancel{m}gb}{\sqrt{2}} = \frac{\cancel{3}}{2}\cancel{m}v^2$$

$$\boxed{\sqrt{\sqrt{2}gb} = v}$$

L

4)





$$\Delta H_0 = H_2 - H_1 = \int_{t_1}^{t_2} M_0\,dt$$

$$H = \sum \bar{\rho}_i \times m_i \dot{\bar{\rho}}_i \qquad \dot{\bar{\rho}}_i = \frac{\partial \bar{\rho}_i}{\partial t} + (\bar{\omega} \times \bar{\rho}_i)$$

$$= \sum \bar{\rho}_i \times m_i(\bar{\omega} \times \rho_i)$$

i

$\hat{k}$ $\quad j$

(2)

$$\Big[(0.5\hat{j}) \times (3)\underbrace{(\omega_0 \hat{k} \times 0.5\hat{j})}^{-1.5\omega_0 \hat{i}}\Big] +$$

(3)

$$\Big[(-0.4\hat{i}) \times (4)\overbrace{(\omega_0 \hat{k} \times (-0.4)\hat{i})}^{-0.4\omega_0 \hat{j}}\Big]$$

(1)

$$\left(\frac{0.6}{\sqrt{2}}(\hat{i}+\hat{j}) \times 3\left(\omega_0 \hat{k} \times \frac{0.6}{\sqrt{2}}(\hat{i}+\hat{j})\right)\right)$$

$$0.75\omega_0 \hat{k} + (0.4)^2(4)\omega_0 \hat{k} + 3(0.6)^2 \omega_0 \hat{k}$$

$$\frac{0.6}{\sqrt{2}}(i+j) \times \left(\frac{3\omega_0 0.6}{\sqrt{2}}\hat{j} - \frac{3\omega_0 0.6}{\sqrt{2}}\hat{i}\right)$$

$$\frac{3(0.6)^2 w_0}{2}\hat{k} + \frac{3w_0(0.6)^2}{2}\hat{k}$$

$$2.47(w - w_0)\hat{k} = \int_0^5 -30\hat{k}\,dt$$

$$2.47(w + 20)\hat{k} = -30(5)\hat{k}$$

$$w = -\frac{30(5)}{2.47} - 20$$

$$3(0.5) + 4(0.4) + 3(0.6)$$

$$3(0.5)^2 + 4(0.4)^2 + 3(0.6)^2$$

$$\frac{150 + (2.47)(400)}{2.47}$$

$$= 80.7$$

5)

$$-(10^3 + 20)(1.2) = -10^3 v_1 + 20(1200 - v_1)$$

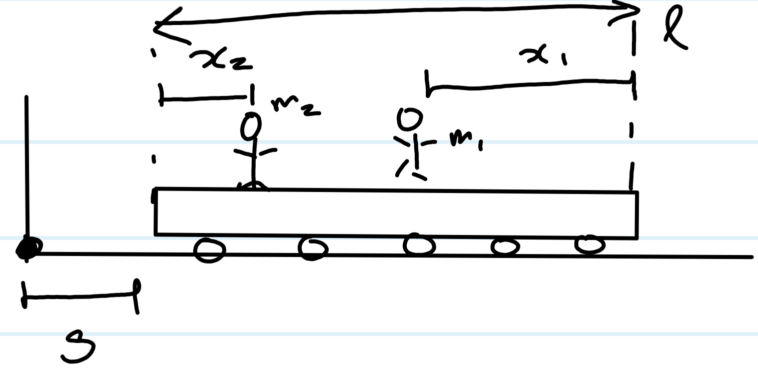
$$-(10^3+20)(1.2) - 20(1200) = -(10^3+20)v_1$$

$$\boxed{\frac{(10^3+20)(1.2) + 20(1200)}{(10^3+20)} = v_1} = 24.729\ \text{m/s}$$

$$V_p = V_M + V_{p/M}$$

$$V_p = -V_1 + 1200$$

6)



Initially

$s = 0$

8)

$$\overline{F} = \Sigma m_1 \overrightarrow{a_G}$$

$$F = 2m\overline{a}_G$$

$$\overline{a}_G = \frac{F}{2m}$$

$$\frac{dH}{dt} = M \qquad \omega = \theta$$

$$H = \Sigma \rho_i \times m(\overline{\omega} \times \overline{\rho_i})$$

$$\left(\frac{L}{2}\hat{j}\right) \times m\left(\dot{\theta}\hat{k} \times \frac{L}{2}\hat{j}\right) + \left(\frac{-L}{2}\hat{j}\right) \times \left(m\left(\dot{\theta}\hat{k} \times \left(\frac{-L}{2}\hat{j}\right)\right)\right)$$

$$\left(\frac{L}{2}\hat{j}\right) \times \left(-\frac{m\dot{\theta}L}{2}\hat{i}\right) \qquad \left(-\frac{L}{2}\hat{j}\right) \times \left(\frac{m\theta L}{2}\hat{i}\right)$$

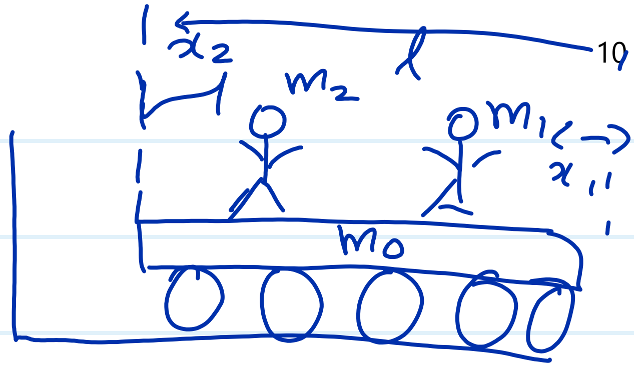
$$\frac{m\dot{\theta}L^2}{4}\hat{k} \qquad \frac{m\theta L^2}{4}\hat{k}$$

$$\frac{mL^2}{2}\dot{\theta}\hat{k}$$

$$\frac{mL^2}{2}\ddot{\theta} = Fb$$

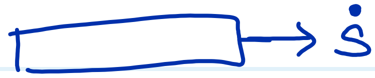
$$\boxed{\dot{\theta} = \frac{2Fb}{mL^2}}$$

6)



$\cancel{G_1} = G_2 \nearrow 0$

$m_0\dot{s} + m_1(\dot{s} - \dot{x}_1) + m_2(\dot{s} + \dot{x}_2) = 0$



$m_0\dot{s} = (m_1 + m_2)(\dot{x}_1 - \dot{s})$

$d = (s + l - x_1)$ $\quad d = x_2 + s$

$v_m = \dot{s} - \dot{x}_1$ $\quad s + x_2$

$$\dot{s} = \frac{(m_1 + m_2)\dot{x}_1}{m_0 + m_1 + m_2}$$

$v_w = \dot{s} + \dot{x}_2$

$d = (s + l - x_1) - (s + x_2)$

$(m_0 + m_1 + m_2)\dot{s} = m_1\dot{x}_1 - m_2\dot{x}_2$

$d = l - (x_1 + x_2)$ $\quad (l - x_2)$

$\boxed{l = x_1 + x_2}$

$$\dot{s} = \frac{m_1\dot{x}_1 - m_2\dot{x}_2}{(m_0 + m_1 + m_2)}$$

$d_m = s + l - x_1$

$d_w = s + x_2$

$l = x_1 + x_2$

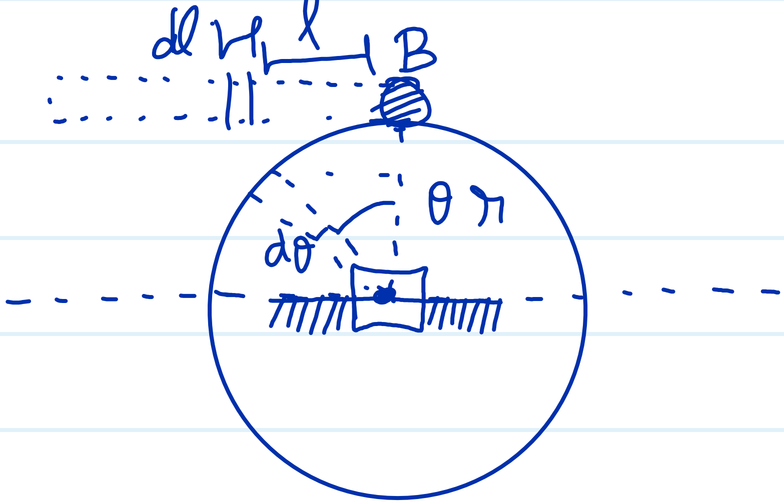
$d_m + x_1 - l = d_w - x_2$

$x_2 = l - x_1$

$(m_0 + m_1 + m_2)\underline{s} = m_1\underline{x_1} - m_2\underline{x_2}$

$$s = \frac{m_1x_1 - m_2x_2}{m_0 + m_1 + m_2} = \frac{m_1x_2 - m_2l + m_1x_1}{m_0 + m_1 + m_2}$$

13



$$dm = \rho\, dl$$

$$r\theta = l \qquad r(1-\cos\theta) = h$$

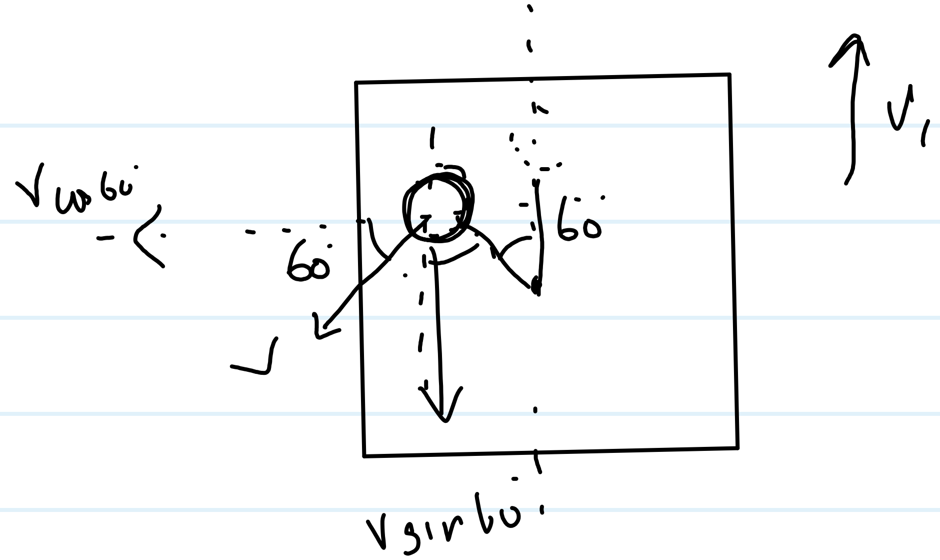
$$r\, d\theta = dl$$

$$(\cancel{T_1}^{\,0} + V_1) - (T_2 + V_2) = \Delta E$$

$$\left((dm)gr\right) - \left(\frac{1}{2}(dm)v^2 + dm\,g\,r(1-\cos\theta)\right) = \Delta E$$

$$(dm)gr\cos\theta - \frac{1}{2}(dm)v^2 = dE$$

9)

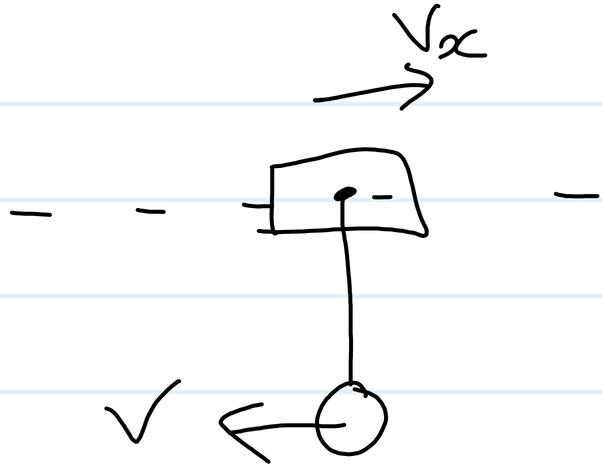


$$(25)(0.6) = 20(v_1) + 5(v - 16\sin 60)$$

$$(20)(0.6) + 5(0.6) = 20(v_1) + 5(v_1 + 16\sin 60$$

$$\frac{15 + 5(16)\sin 60}{25}$$

11) $\Delta E = 0$



$\Delta E = 0$

$$2mgL = -2mgL + \frac{1}{2}(2m)V_x^2 + \frac{1}{2}(2m)(v - v_x)^2$$

momentum

$$0 = 2mV_x + 2m(V_x - V)$$

$$2V_x = V$$

$$4mgL = mV_x^2 + mV_x^2$$

$$\cancel{2}\,\cancel{4}\,\cancel{m}gL = \cancel{2}\cancel{m}V_x^2$$

$$\boxed{V_x = \sqrt{2gL}}$$

$$V = l\theta \qquad l\theta = 2\sqrt{2gL}$$

$$\theta = 2\sqrt{\frac{2g}{2}}$$

12) $12\cos(30)$

Initially

$$(30\cancel{0})\left(\frac{6}{1\cancel{0}}\right) - (40\cancel{0})\left(\frac{3}{1\cancel{0}}\right) + 10\cancel{0}\left(\frac{\overset{6}{\cancel{12}}}{1\cancel{0}}\right)\frac{\sqrt{3}}{\cancel{2}}$$

finally

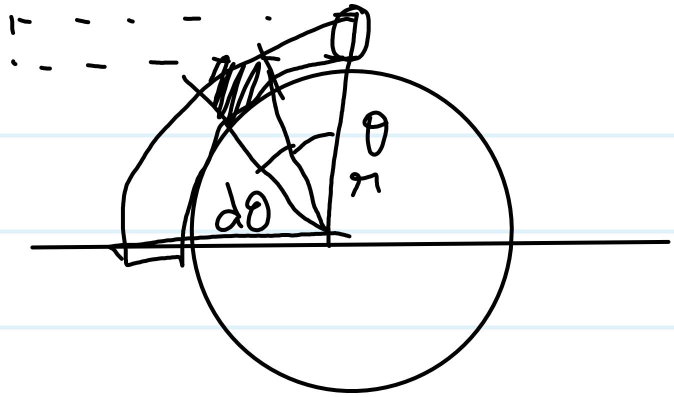
$$(800)(v)$$

$$\frac{180 - 120 + 60\sqrt{3}}{800} = v$$

$$\frac{\overset{3}{\cancel{60}}}{\underset{40}{\cancel{800}}}(1+\sqrt{3}) = v$$

$$\boxed{v = 0.205\ ms^{-1}}$$

13)



$$\frac{\rho \pi r^2}{4} g = V_1$$

$$dl = r\,d\theta$$

$$dm = \rho r\,d\theta$$

$$dh = r\cos\theta$$

$$\rho r^2 \int_0^{90} \cos\theta\, d\theta\, g$$

$$\rho r^2 g[1-0]$$

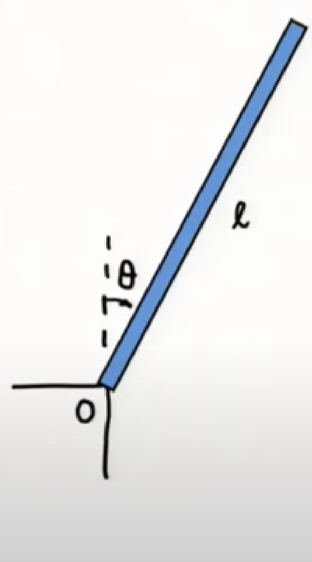
$$\frac{4R}{3\pi}$$

$$\frac{\rho \cancel{\pi} r}{\cancel{4}} g \left(\frac{\cancel{4}R}{3\cancel{\pi}}\right)$$

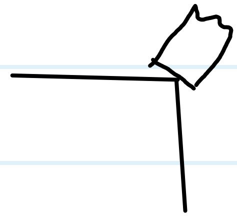
$$\frac{\rho r^2 g}{3}$$

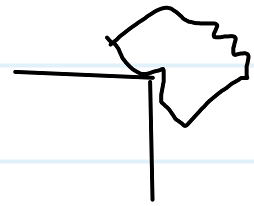
Problem 3:

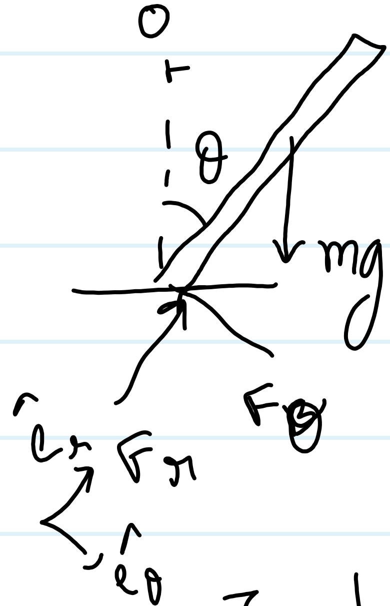
A uniform slender bar of mass m and length l, released from rest in the vertical position, pivots on its short flat end about the corner at O as shown. (a) If the bar is observed to slip when θ=30 deg, find the coefficient of static friction μ_s between the bar and the corner. (b) If the end of the bar is notched (so that it cannot slip), find the angle θ at which contact between the bar and the corner ceases.



Sol.

a) 

b) 



$$\vec{a}_r = (\cancel{\ddot r} - r\dot\theta^2)\hat e_r + (r\ddot\theta + \cancel{2\dot r\dot\theta})\hat e_\theta$$

$$\vec{a}_G = \left(-\frac{l}{2}\dot\theta^2\right)\hat e_r + \left(\frac{l}{2}\ddot\theta\right)\hat e_\theta$$

$$I_O = \frac{1}{3}ml^2$$

$$m\vec{a}_G = \vec{F}$$

$$-m\frac{l}{2}\dot\theta^2 = F_r - mg\cos\theta$$

$$\frac{ml}{2}\ddot\theta = -F_\theta + mg\sin\theta$$

$$\frac{1}{3}l\ddot\theta = \frac{g}{2}\sin\theta$$

$$\boxed{\ddot\theta = \frac{3}{2}\frac{g}{l}\sin\theta}$$

$$\frac{m\cancel{l}}{2}\frac{1}{6}\frac{g}{\cancel{l}}\sin\theta - mg\sin\theta = -F_\theta$$

$$\boxed{\frac{11}{12}mg\sin\theta = F_\theta}$$

$$\dot\theta^2 = \frac{3g}{2}(1-\cos\theta)$$

$$-\frac{3mg}{4}(1-\cos\theta) + mg\cos\theta = F_r$$

$$F_r = \frac{3mg}{4}$$

$$\boxed{F_r = \frac{1}{2} mg \left[5\cos\theta - 3 \right]}$$

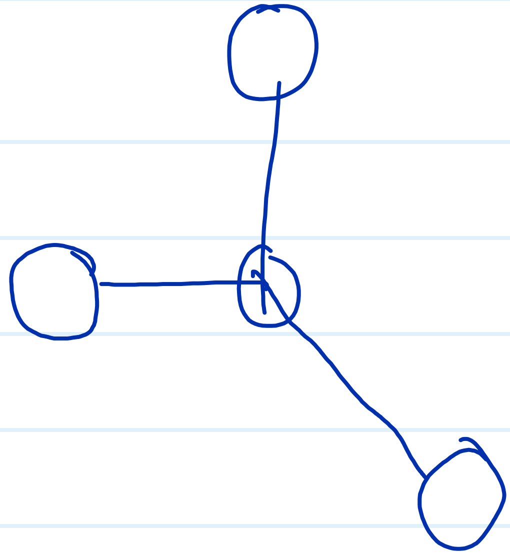
$$\frac{F_\theta}{F_r} = \mu_s$$

$$\mu_s = \left. \frac{\sin\theta}{2(5\cos\theta - 3)} \right|_{\theta = 30^\circ} = 0.188$$

b) $F_r = 0$

$$\theta = \cos^{-1}\left(\frac{3}{5}\right)$$

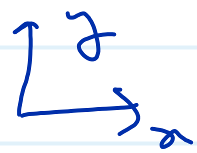
4



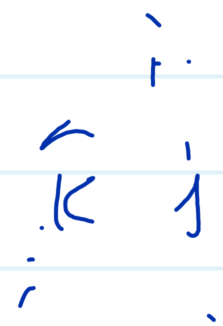
$$H = \sum \rho_1 \times m_i \dot{\bar{\rho}}_1$$

$$\dot{\bar{\rho}}_i = \cancel{\frac{\partial \rho}{\partial t}}^{\to 0} + (\bar{\omega} \times \bar{\rho}_i)$$

$$\dot{\bar{\rho}}_1 = (\bar{\omega} \times \bar{\rho}_1)$$



$$\sum \bar{\rho}_1 \times m_i \bar{\rho}_i$$



$$\bar{H} = (0.5\hat{j}) \times \left[3(\omega \hat{k} \times 0.5\hat{j})\right]$$

$$+ (-0.4\hat{i}) \times \left[4(\omega \hat{k} \times (-0.4\hat{i}))\right]$$

$$+ \frac{0.6}{\sqrt{2}}(\hat{i} - \hat{j}) \times \left[3\left(\omega \hat{k} \times \left(\frac{0.6}{\sqrt{2}}(\hat{i} - \hat{j})\right)\right)\right]$$

$$\Delta H = \int_{t_i}^{t_2} M_o\,dt = \int_0^5 -30\,dt$$

$$= -30(5-0) = -150$$

$$= 3(0\ 5)^2 \omega \hat{k} + (0\ 4)^2 4\omega \hat{k}$$

$$\frac{0\ 6}{\sqrt{2}}(\hat{i}-\hat{j}) \times \left(3\omega\left(\frac{0\ 6}{\sqrt{2}}\right)\hat{j} + 3\omega\left(\frac{0\ 6}{\sqrt{2}}\right)\hat{i}\right)$$

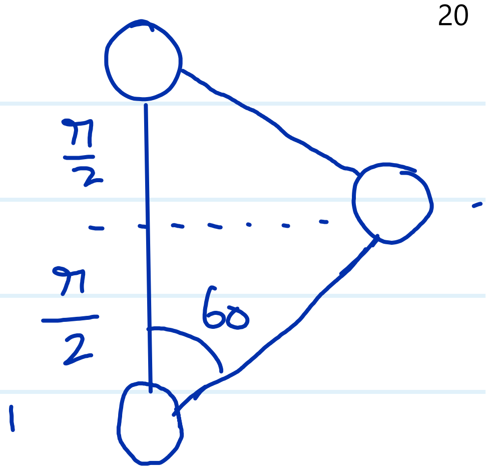
$$6\omega\left(\frac{0.6}{\sqrt{2}}\right)^2 \hat{k}$$

$$= \left(3(0\ 5)^2 \omega + 4(0\ 4)^2 \omega + 6\left(\frac{0\ 6}{\sqrt{2}}\right)^2 \omega\right)\hat{k}$$

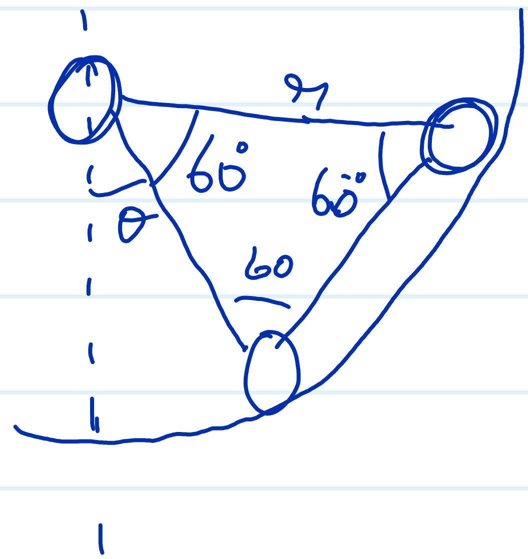
$$\left(3(0\ 5)^2 + 4(0\ 4)^2 + 6\left(\frac{0\ 6}{\sqrt{2}}\right)^2\right)(\omega - \omega_0) = -150$$

1) $\Delta E = W_{a-b}$

$(\cancel{T_2} + V_2) - (\cancel{T_1} + V_1) = W_{a-b}$



$\left(mgr + mg\frac{r}{2}\right) = V_1$



$mg(r) + mg(r(1-\cos\theta))$

$+ mg(r(1-\cos(60° + \theta)))$

$\cos A \cos B - \sin A \sin B$

$\frac{\cos\theta}{2} - \frac{\sqrt{3}\sin\theta}{2}$

$$\left\{ \cancel{\frac{6}{2}mgr} - mgr\cos\theta - \frac{mgr\cos\theta}{2} + \frac{\sqrt{3}mgr\sin\theta}{2} \right\}$$

$-\cancel{\frac{3}{2}mgr}$

$$\frac{3}{2} mgr - mgr\cos\theta - \frac{1}{2} mgr\cos\theta + \frac{\sqrt{3}}{2} mgr\sin\theta$$

$$= \int_0^{60^\circ} \frac{Pr}{2}\, d\theta$$

$$\frac{3}{2} mgr - \frac{mgr}{2} - \frac{1}{4} mgr + \frac{3}{4} mgr$$

$$= \frac{Pr}{2} \frac{\pi}{3}$$

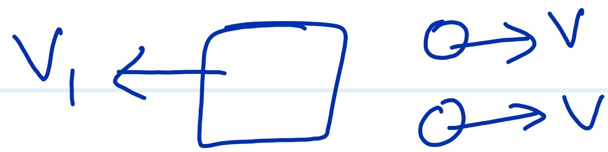
$$\frac{1}{\cancel{2}} mgr = \frac{Pr\pi}{\cancel{6}_{3}}$$

$$\frac{3mg}{\pi} = P_{min}$$

5) No external force

$G_1 = G_2$

$\boxed{G_1 = 0}$ (10^3 kg)



$$10^3 v_1 = 20 v_1 - 24000$$

$$(10^3 - 20) v_1 = -24000$$

$$\boxed{v_1 = -\frac{2400}{9980}}$$

$$\bar{F} = m_1 \bar{a}_1 + m_2 \bar{a}_2$$