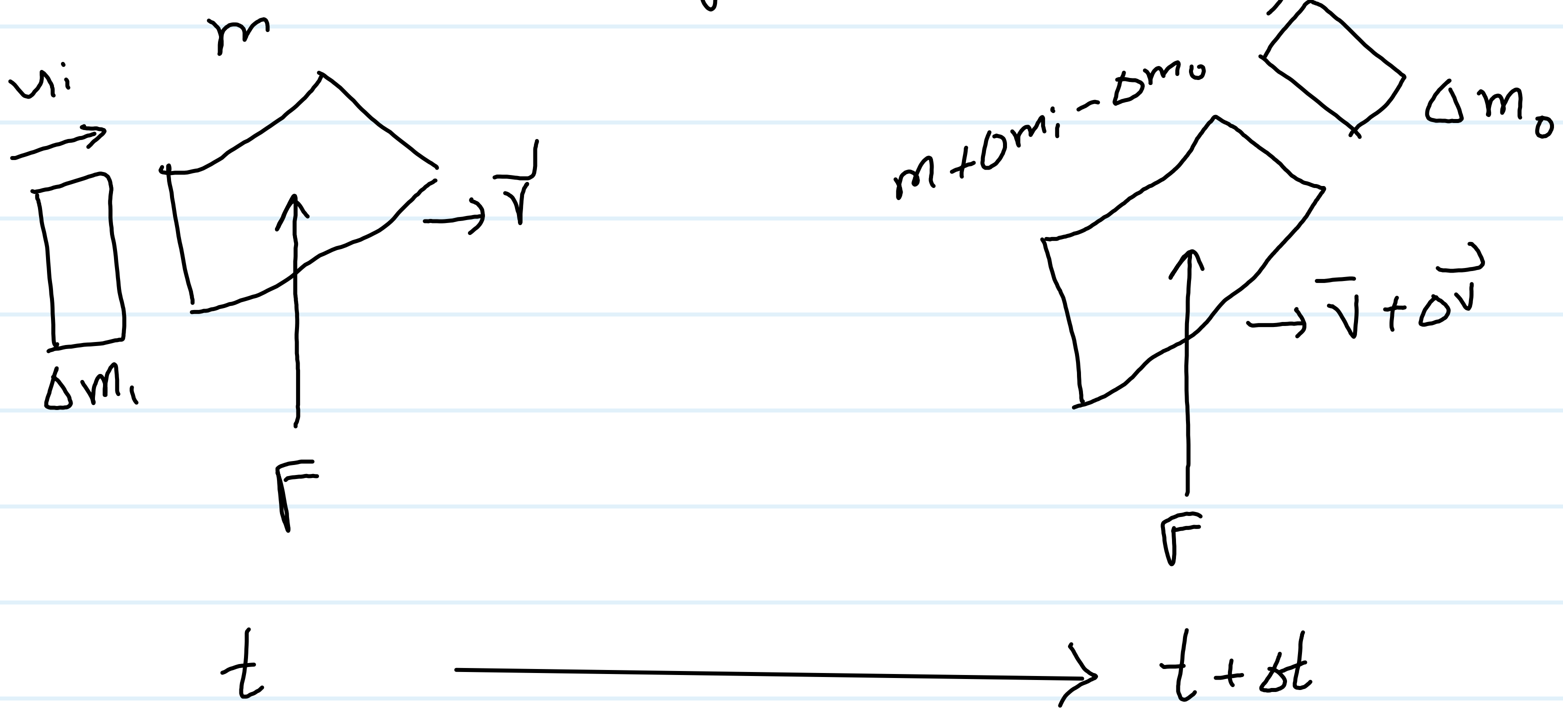
Mass flow:



$\vec{u}_1, \vec{u}_0, \vec{v} \Rightarrow$ absolute velocities

Newton's second law:

$$\lim_{\Delta t \to 0} \frac{\Delta \vec{G}}{\Delta t} = \vec{F}$$

$$\Delta \vec{G} = (m + \Delta m_1 - \Delta m_0)(\vec{v} + d\vec{v}) + \Delta m_0 \vec{u}_0 - (m\vec{v} + \Delta m_i \vec{u}_1)$$

$$\Delta \vec{G} = \cancel{m\vec{v}} + m\,d\vec{v} + (\Delta m_1 - \Delta m_0)\vec{v} + \Delta m_0 \vec{v}_0 - \cancel{m\vec{v}} - \Delta m_1 \vec{u}_1$$

$$\lim_{\Delta t \to 0}\left(m\,d\vec{v} + \Delta m_0(\vec{v}_0 - \vec{v}) + \Delta m_i(\vec{v} - \vec{u}_1)\right)\frac{1}{\Delta t} = \vec{F}$$

$$m\vec{a} \qquad \dot{m}_0(\vec{u}_0 - \vec{v}) \quad - \quad \dot{m}_i(\vec{u}_i - \vec{v})$$

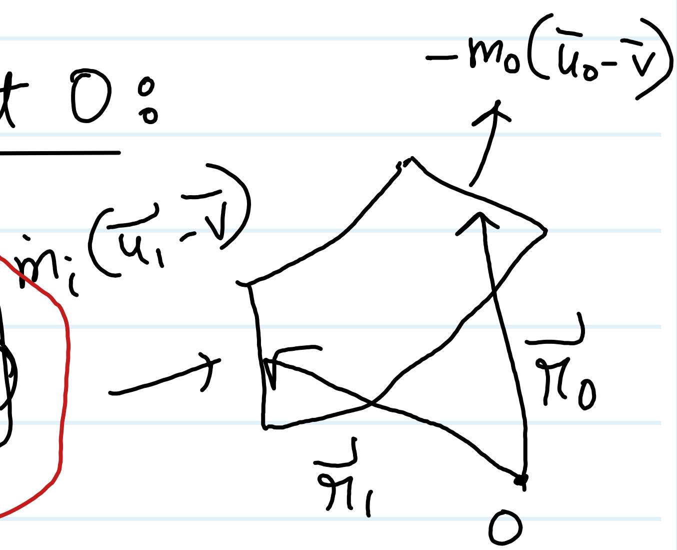
$$\boxed{m\vec{a} - \dot{m}_i(\vec{u}_i - \vec{v}) + \dot{m}_o(\vec{u}_o - \vec{v}) = \vec{F}} \quad ☆$$

$$\vec{a} = \frac{d\vec{v}}{dt} \qquad \dot{m}_i = \frac{dm_i}{dt} \qquad \dot{m}_0 = \frac{dm_0}{dt}$$

$$m\vec{a} = \underbrace{\vec{F}}_{\text{Non-flow forces}} + \underbrace{\underbrace{\dot{m}_i(\vec{u}_1 - \vec{v})}_{\text{inflow}} - \underbrace{\dot{m}_0(\vec{u}_0 - \vec{v})}_{\text{outflow}}}_{\text{Force due to mass flow}}$$

Moment due to flow about O:

$$M_0 = \left[\vec{r}_1 \times \dot{m}_i(\vec{u}_1 - \vec{v})\right] + \left[-\left(\vec{r}_0 \times \dot{m}_0(\vec{u}_0 - \vec{v})\right)\right]$$

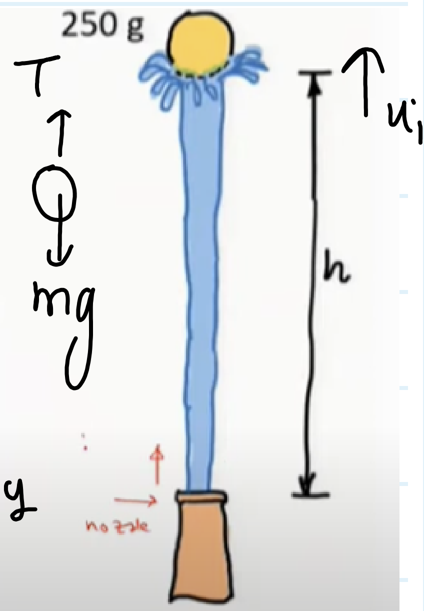


$$m\vec{a} - \dot{m}_i(\vec{u}_i - \vec{v}) + \dot{m}_o(\vec{u}_o - \vec{v}) = \vec{F}$$

Mass conservation.

$$\dot{m} = \dot{m}_i - \dot{m}_o$$

A 250 g ball is supported by the vertical stream of fresh water which issues from a 12 mm diameter nozzle with a velocity 10 m/s. Calculate the height h of the ball above the nozzle. Assume that the stream remains intact and there is no energy lost in the jet stream.



$u = 10\,ms^{-1}$

$\rho A V = \dot{m}$ ← A $\phi = 12mm$ y

$$\cancel{m\vec{a}}^{\,0} - \dot{m}_i(\vec{u}_i - \cancel{\vec{v}}^{\,0}) + \dot{m}_o(\cancel{\vec{u}_o}^{\,0} - \cancel{\vec{v}}^{\,0}) = \vec{F} = -mg\hat{j}$$

$\bar{a}, \bar{v} \rightarrow$ ball

outlet velocity zero as the water stop after hitting ball

$$u_1 = \sqrt{u^2 - 2gh} = \sqrt{100 - 2(9.81)h}$$

$$\dot{m}_1 = \rho A u = 1000\frac{\pi}{4}(0.012)^2(10)\ kg/s \quad \text{(at nozzle)}$$

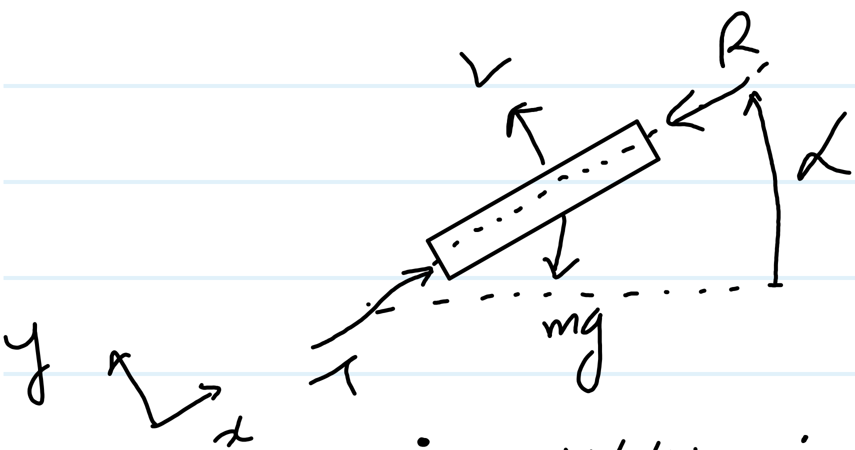
$$+\dot{m}_i\bar{u}_1 = +mg\hat{j}$$

$$\rho A u (u_i \hat{j}) = mg\hat{j}$$

$$1000\left(\frac{\pi}{4}\right)(0.012)^2(10)\sqrt{100-2(9.81)h} = 0.25(9.81)$$

$$\boxed{h = 4.86m}$$

The jet aircraft has a mass of 4.6 Mg and drag of 32 kN at a speed of 1000 km/h. The aircraft consumes air at a rate or 106 kg/s through its intake scoop and uses fuel at a rate of 4 kg/s. If the exhaust has a rearward velocity of 680 m/s relative to the aircraft, determine the maximum angle of elevation α at which the aircraft can fly with the constant speed of 1000 km/h.



$\dot{m}_i = 106 \, kg/s \quad \dot{m} = -4$

$V = 1000 \, km/h = \frac{2500}{9} \, m/s$

$R = 32000 \, N$

$m = 4600 \, kg$

relative exhaust vel = 680

α_{max}

$$\overset{0}{\cancel{m\vec{a}}} - m_i(\overset{0}{\cancel{\hat{u}_i}} - \bar{v}) + m_0(\bar{u}_0 - \bar{v}) = \bar{F}$$

$$= -R\hat{i} + L\hat{j} - mg(\sin\alpha\,\hat{i} + \cos\alpha\,\hat{j})$$

$\dot{m}_i \neq \dot{m}_0$

$\dot{m}_0 = \dot{m}_i - \dot{m} = 110\,kg/s$

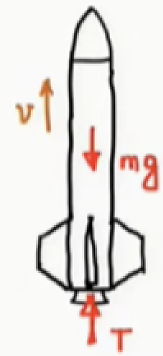
$\bar{v} = \dfrac{2500}{a}\hat{i}\,m/s \qquad \vec{u}_i = 0^{(\text{air})} \qquad \bar{u}_0 - \bar{v} = -680\hat{i}$

$$106\left(\frac{2500}{9}\right)\hat{i} + 110(-680\hat{i}) = -32000\hat{i} + L\hat{j} - mg(\sin\alpha\,\hat{i} + \cos\alpha\,\hat{j})$$

$$106\left(\frac{2500}{9}\right) - 110(680) + 3200 = -mg\sin$$

$$\boxed{\alpha = 11.72^\circ}$$

A rocket of initial mass M_0 expels exhaust at a constant velocity v_r with respect to the nozzle. The mass of the rocket depletes at a constant rate r to a final mass m_b at burn-out. Calculate the velocity of the rocket with time, and the velocity attained at burn-out. Neglect atmospheric drag, and variation of acceleration due to gravity.



$$m\vec{a} - \dot{m}_i(\vec{u}_i - \vec{v}) + \dot{m}_o(\vec{u}_o - \vec{v}) = \vec{F}$$

Mass conservation.

$$\dot{m} = \dot{m}_i - \dot{m}_o$$

Sol^n:

M_0

$$\vec{u}_0 - \vec{v} = -v_r\hat{j}$$

$$m\bar{a} - \dot{m}_i(\bar{u}_i - \bar{v}) + m_o(\vec{u}_o - \bar{v}) = -mg\hat{j}$$

$$\bar{u}_0 - \bar{v} = -v_r\hat{j}$$

$$m = M_0 - rt$$

$$m(t_b) = m_b$$

$$\boxed{t_b = \frac{M_0 - m_b}{r}}$$

$$(M_0 - rt)\dot{v}\hat{j} - rv_r\hat{j} = -(M_0 - rt)g\hat{j}$$

$$(M_0 - rt)\dot{v} = (rtg - M_0 g) + rV_r$$

$$\int_0^v dv = \int_0^t \frac{g(rt - M_0) + rV_r}{M_0 - rt}\, dt$$

$$v = -g(t) + \int_0^t \frac{rV_r}{M_0 - rt}\, dt$$

$$v = -gt + \frac{rV_r \left(\ln(M_0 - rt)\right)_0^t}{-1}$$

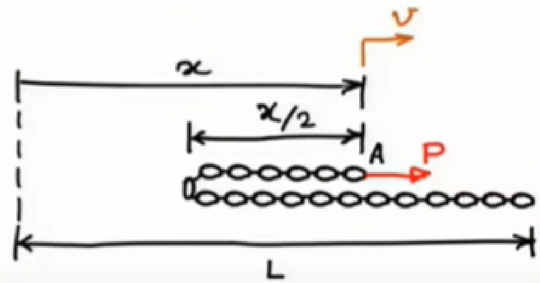
$$\boxed{v = -gt + v_r \ln\left(\frac{M_0}{M_0 - rt}\right)}$$

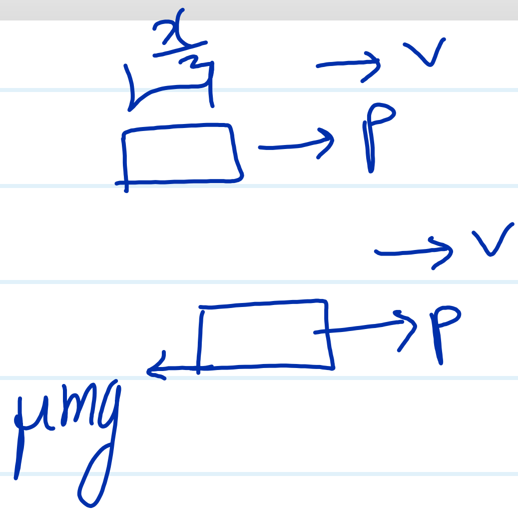
$$t_b = \frac{M_0 - m_b}{r} \qquad \frac{M_0}{M_0 + m_b}$$

V_b

$$\boxed{V_b = -\frac{g}{r}\left[M_0 - m_b\right] + V_r\left[\ln\left[\frac{M_0}{m_B}\right]\right]}$$

A open-link chain of length L = 8 m with mass 48 kg is resting on a rough horizontal surface when end A is doubled back on itself by a force P. The coefficient of kinetic friction between the chain and the surface is $\mu_k = 0.4$. (a) Determine the force P as a function of x required to pull the chain with a constant velocity of 1.5 m/s. (b) If the force P is constant, determine the velocity of the chain as a function of x.





a)

$$m\cancel{a}^{\,0} - \dot m_i(\cancel{\bar a_i}^{\,0} - \bar v) + \cancel{\dot m_o(\bar a_o - \bar v)}^{\,0} = \bar F$$

$\frac{\cancel{48}\,8}{\cancel{8}} : 8$

$$-\left[\eta \bar v\left[-\bar v\right]\right] = \bar F$$

$\eta : 8$

$$\eta(1.5)(1.5\hat i) = P\hat i - \eta\mu\frac{x}{2}\hat i$$

$$(1.5)^2\eta = \left(P - \eta\mu\frac{x}{2}\right)$$

$$P = \eta \mu \frac{x}{2} + (1\,5)^2 \eta$$

$$8\left[\cancel{0\mu}^{0.2} \frac{x}{\cancel{2}} + 1^2 25^4\right]$$

$$\boxed{P = [1.6x + 10]}$$

(2)

$$m\bar{a} - m_1(\cancel{\bar{u}_1}^{0} - \bar{v}) + \cancel{m_0}(\bar{u}_0 - \bar{v})^{0} = \bar{F}$$

$$m = \rho \frac{x}{2} \qquad m_i = \rho \frac{v}{2} \qquad \bar{v} = v\hat{i}$$

$$\rho \frac{x}{2} \frac{dv}{dt}\hat{i} - \rho \frac{v}{2}(-v\hat{i}) = \left[P - \frac{1}{2}\mu_k g x \rho\right]\hat{i}$$

$$2\left[\frac{\rho \frac{x}{2}}{\rho x}\dot{v} + \frac{\rho \frac{v^2}{2}}{\rho x}\right] = \frac{2\left[P - \frac{1}{2}\mu g x \rho\right]}{\rho x}$$

$$a + \frac{v^2}{x} = \frac{2P}{\rho x} - \mu_k g$$

$$a = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{v^2}{2}\right)$$

$$\frac{d}{dx}\left(\frac{v^2}{2}\right) + \left(\frac{2}{x}\right)\left(\frac{v^2}{2}\right) = \frac{2P}{\rho x} - \mu_k g$$

$$e^{\int \frac{2}{x} dx} \qquad x^2$$

$$\frac{v^2}{2} x^2 = \int \left[\frac{2P}{\rho} x - \mu_k g x^2 \, dx\right]$$

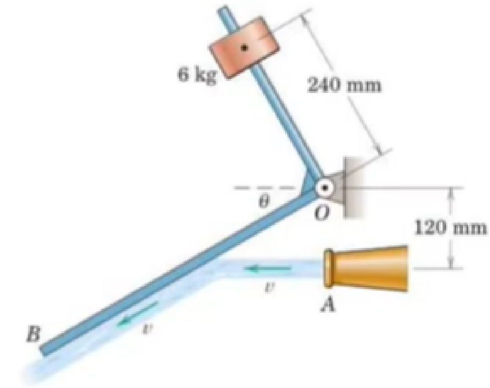
$$\frac{v^2}{2} x^2 = \frac{\cancel{2}P}{\rho}\frac{x^2}{\cancel{2}} - \frac{\mu_k g x^3}{3} + C$$

$$\frac{v^2}{2} = \frac{P}{\rho} - \frac{\mu g x}{3}$$

$$\boxed{v = \sqrt{\frac{2P}{\rho} - \frac{2}{3}\mu g x}}$$

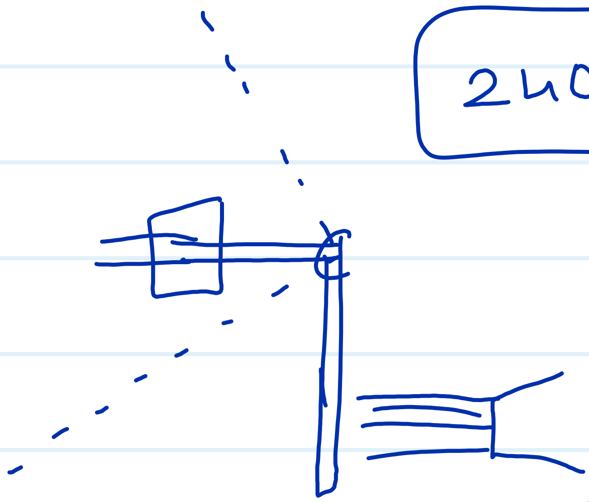
Problem 4:

A high-speed jet of air issues from the 40-mm-diameter nozzle A with a velocity v of 240 m/s and impinges on the vane OB, shown in its edge view. The vane and its right-angle extension have negligible mass compared with the attached 6-kg cylinder and are freely pivoted about a horizontal axis through O. Calculate the angle θ assumed by the vane with the horizontal. The air density under the prevailing conditions is 1.206 kg/m³. State any assumptions.



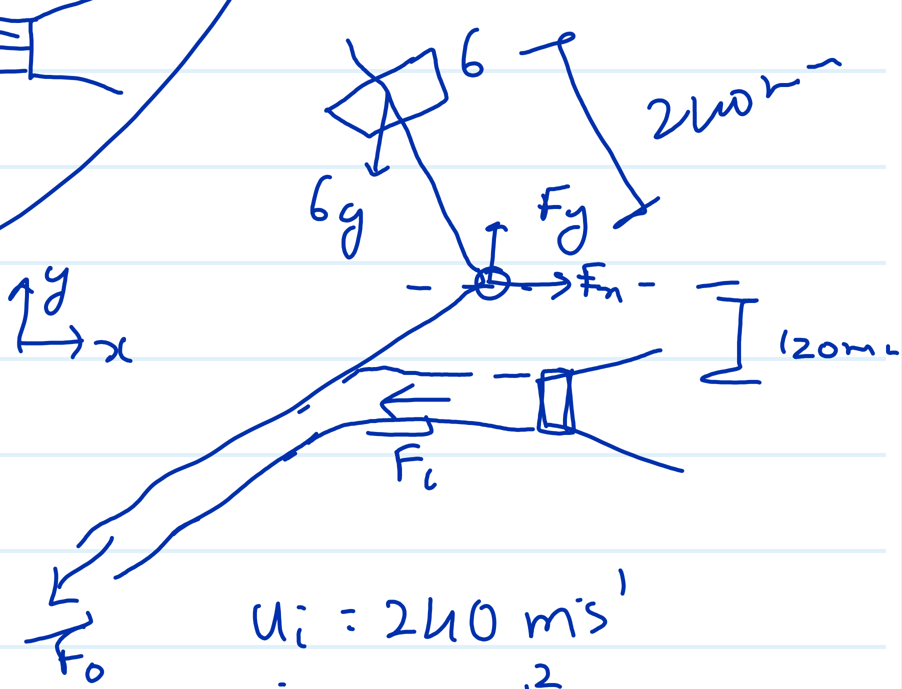
Source: Dynamics, Meriam and Kraige

$\boxed{240 ms^{-1}}$



Moment Equation

FBD



$u_i = 240 ms^{-1}$

$\dot{m}_1 = \rho \frac{\pi}{4} d^2 u.$

$= 0.364 kg/s$

$\theta = ?$

$$\vec{M}_O = \left(\bar{r}_i \times \dot{m}_i (\vec{u}_1 - \bar{v}) \right) - \left(\cancel{\bar{r}_o}^{\,0} \times \dot{m}_o (\bar{u}_o - \bar{v}) \right)$$

$$= -0.12\hat{j} \times (0.364)(-240\hat{i}) = -10.48\hat{k} \text{ Nm}$$

$$M_{6kg} = \bar{r} \times \bar{F} = 240 \left[\cos\theta\hat{j} - \sin\theta\hat{i} \right) \times (-6g\hat{j})$$

$$= 0.24 \sin\theta \, (6g)$$

$$M = 1.44 \, g\sin\theta$$

$$1.44 g\sin\theta + 10.48 = 0$$

$$\sin\theta = \frac{10.48}{1.44g} =$$