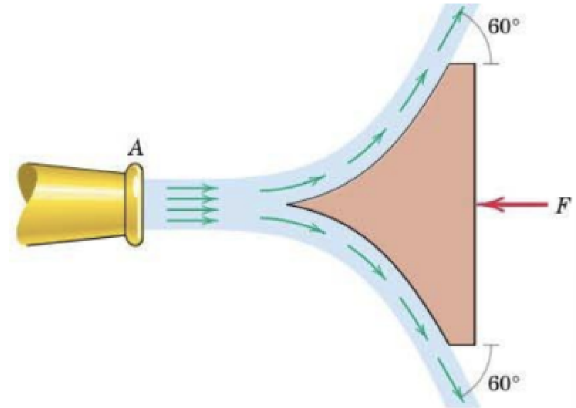
Fresh water issues from the nozzle with a velocity of 30 m/s at the rate of 0.05 m^3/s and is split into two equal streams by the fixed vane and deflected through 60° as shown. Calculate the force F required to hold the vane in place. The density of water is 1000 kg/m^3.

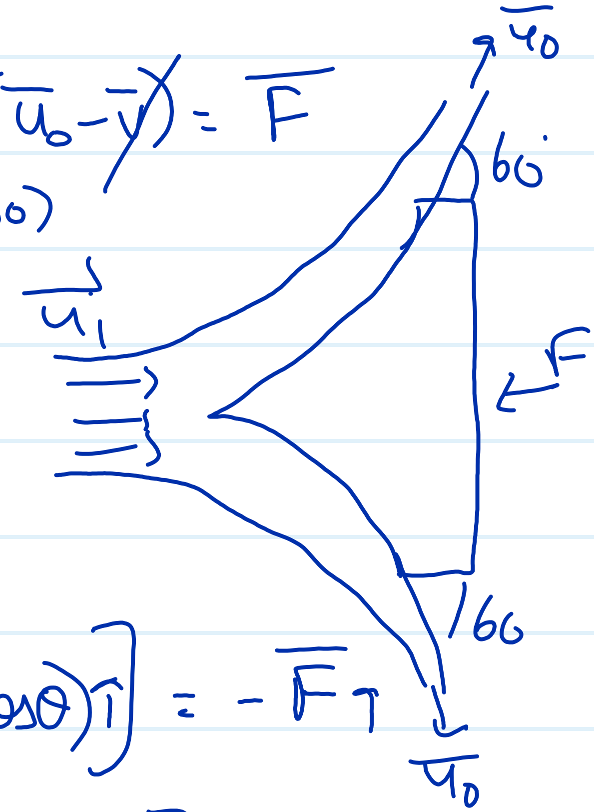
Ans. $F = 750$ N



Sol.

$u_i = 30 \, m s^{-1} \qquad Q = 0.05 \, m^3/s$

$\cancel{m\vec{a}}^{\,0} - \dot{m}_i(\bar{u}_1 - \cancel{\bar{v}}) + \dot{m}_o(\bar{u}_o - \cancel{\bar{v}}) = \bar{F}$



$\bar{m}_1 = \bar{m}_o = m = (\overset{(1000)}{0}.05)$

$= 50 \, kg/s$

$-50[30\hat{i}] + 50[(30\cos\theta)\hat{i}] = -\bar{F}\hat{i}$

$-50[30\hat{i}] + 50[(15)\hat{i}] = -F\hat{i}$

$50(15 - 30)$

-75×50

$\boxed{-750 = F}$

The fire tug discharges a stream of salt water (density 1030 kg/m^3) with a nozzle velocity of 40 m/s at the rate of 0.080 m^3/s. Calculate the propeller thrust T which must be developed by the tug to maintain a fixed position while pumping.

Ans. $T = 2.85$ kN

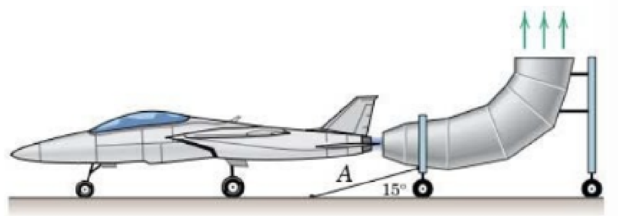
$$\overset{0}{\cancel{m\bar{a}}} - \overset{0}{\cancel{\dot{m}_1}}(\bar{u}_1 - \bar{v}) + \dot{m}_0(\bar{u}_0 - \overset{0}{\cancel{\bar{v}}}) = \vec{F}$$

$$1030(0.080)\left(40(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\right) = -T\,\hat{i}$$

$$\boxed{1030(0.080)\,20\sqrt{3} = T}$$

$T = 2.85\text{kN}$

A jet-engine noise suppressor consists of a movable duct which is secured directly behind the jet exhaust by cable *A* and deflects the blast directly upward. During a ground test, the engine sucks in air at the rate of 43 kg/s and burns fuel at the rate of 0.8 kg/s. The exhaust velocity is 720 m/s. Determine the tension *T* in the cable.



Ans. $T = 32.6$ kN

$$\cancel{m\bar{a}}^{\,0} - \dot{m}_i(\bar{u}_i - \bar{v}) + \dot{m}_o(\bar{u}_o - \bar{v}) = \bar{F}$$

$m_1 = 43\ \text{kg/s}$ $\qquad$ $\dot{m}_o = 43\ 8\ \text{kg/s}$

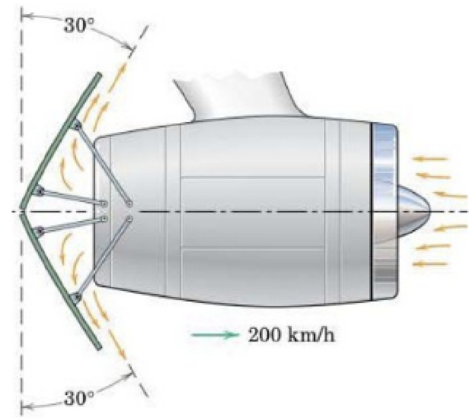
$$\bar{F} = -T\sin 15\,\hat{j}$$

$T\sin 15^\circ$

$$438\,[720\,\hat{j}] = -T\sin 15^\circ$$

A jet-engine thrust reverser to reduce an aircraft speed of 200 km/h after landing employs folding vanes which deflect the exhaust gases in the direction indicated. If the engine is consuming 50 kg of air and 0.65 kg of fuel per second, calculate the braking thrust as a fraction n of the engine thrust without the deflector vanes. The exhaust gases have a velocity of 650 m/s relative to the nozzle.

Ans. $n = 0.638$



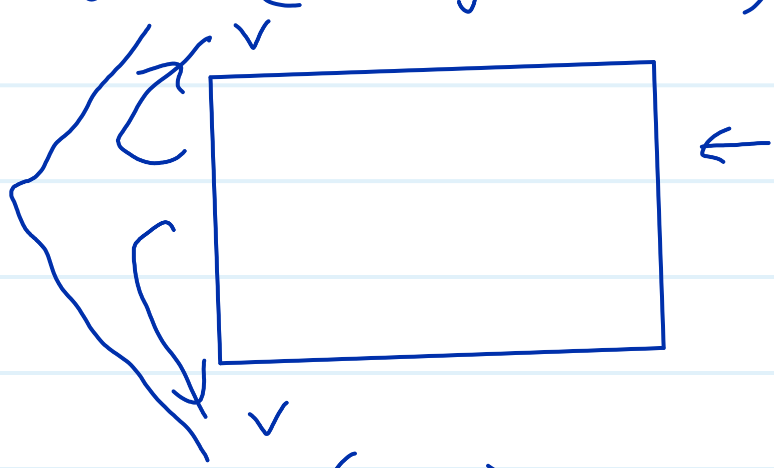
Sol.

$\dot{m}_i = 50\,\text{kg/s}$ $\quad \dot{m}_o = 50.65\,\text{kg/s}$ $\quad$ y x

$\bar{u}_i - \bar{v} = -200\hat{i}$ $\quad \bar{v} = 200\hat{i}$

$\vec{u}_o = v(\cos 30\hat{j} + \sin 30\hat{i})$

$\cancel{m\bar{a}}^{0} - \dot{m}_i(\bar{u}_i - \bar{v}) + \dot{m}_o(\bar{u}_o - \bar{v}) = F$



$-(50)(-200\hat{i}) + (50.65)\left(\frac{650}{2}\right) = T_R$

x $\quad$ y

$$T = -(50)(-200T) + (50.65)(650)$$

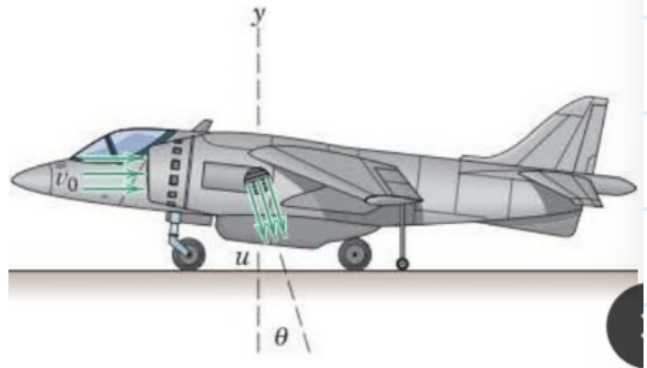
$$\frac{T_R}{T} \approx 0.6$$

$$\frac{x + \frac{y}{2}}{x+y} \qquad \frac{x}{x+y} + 1$$

$$\frac{2x + y}{2(x+y)}$$

$$\frac{x}{2(x+y)} + \frac{1}{2}$$

The VTOL (vertical takeoff and landing) military aircraft is capable of rising vertically under the action of its jet exhaust, which can be "vectored" from $\theta \cong 0$ for takeoff and hovering to $\theta = 90°$ for forward flight. The loaded aircraft has a mass of 8600 kg. At full takeoff power, its turbo-fan engine consumes air at the rate of 90 kg/s and has an air–fuel ratio of 18. Exhaust-gas velocity is 1020 m/s with essentially atmospheric pressure across the exhaust nozzles. Air with a density of 1.206 kg/m^3 is sucked into the intake scoops at a pressure of -2 kPa (gage) over the total inlet area of 1.10 m^2. Determine the angle θ for vertical takeoff and the corresponding vertical acceleration a_y of the aircraft. *Ans.* $\theta = 2.31^0, a_y = 1.45\ \mathrm{m/s^2}$



$m = 8600\,\text{kg}$

$\bar{m}_1 = 90\,\text{kg/s}$

$\boxed{\dot{m}_f = \frac{90}{18}}\ \text{kg/s}$

$\bar{u}_0 = 1020$ $\qquad m_0 \;\; 90\left[\frac{19}{18}\right]$

$v = 0$

$$\bar{u}_1 = \frac{90}{(1.206)(1.10)} = 67.842$$

$$ma - 90(67.842\,\hat{i}) + 90\left(\frac{19}{18}\right)\left[\begin{matrix}-1020\cos\theta\,\hat{j} \\ +1020\sin\theta\,\hat{i}\end{matrix}\right]$$

$$= -mg\,\hat{j} - 2000(1.10)\,\hat{i}$$

$$90\left(\frac{19}{18}\right)(1020)\sin\theta - 90(67.842) = -2000(1.10)$$

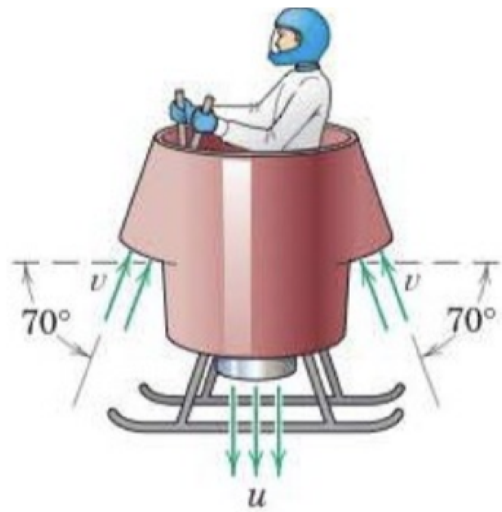
$$\sin\theta = \frac{90(67.842) - 2000(1.10)}{90\left(\frac{19}{18}\right)(1020)} \Rightarrow \theta = 2.31^{0}$$

$$m\bar{a} - 90\left(\frac{19}{18}\right](1020)$$

6)

The feasibility of a one-passenger VTOL (vertical takeoff and landing) craft is under review. The preliminary design calls for a small engine with a high power-to-weight ratio driving an air pump that draws in air through the 70° ducts with an inlet velocity $v = 40$ m/s at a static gage pressure of -1.8 kPa across the inlet areas totaling $0.1320\ m^2$. The air is exhausted vertically down with a velocity $u = 420$ m/s. For a 90-kg passenger, calculate the maximum net mass m of the machine for which it can take off and hover. (See Table D/1 for air density.)

Ans. $m = 184.3$ kg



$$\cancel{m\bar{a}}^{\,0} - \dot{m}_i(\bar{u}_i - \bar{v}) + \dot{m}_o(\bar{u}_o - \bar{v}) = F$$

y, x

$$\bar{u}_1 = 40\left[-\cos 70°\,\hat{i} + \sin 70°\,\hat{j}\right] ms^{-1}$$

$$\bar{v} = 0$$

$$\bar{u}_o = -420\,\hat{j}\ ms^{-1}$$

$u\sin 70°$

$u\cos 70°$

$$\dot{m}_o = 6.336\ kg/s$$

$$\dot{m}_1 = 6.336\ kg/s.$$

3.168

$$-6.336\left[40\sin 70°\right] + 6.336\left[-420\hat{j}\right]$$

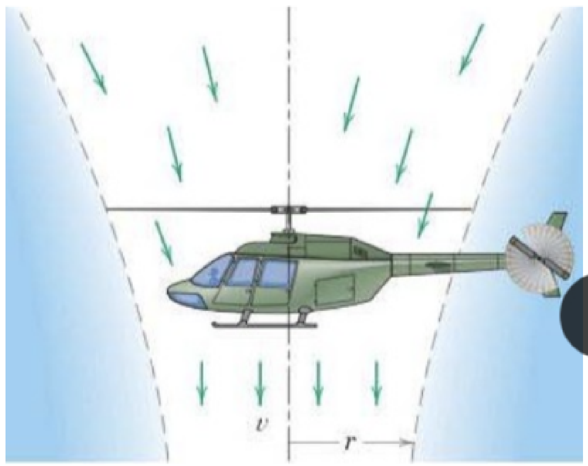
$$= -(90+m)g\,\hat{j} + (PA\sin 70°)\,\hat{j}$$

$$(90+m) = \frac{PA\sin 70^\circ + 6.336\left[40\sin 70 + 420\right]}{g}$$

$$m \simeq 184.3\,kg$$

The helicopter shown has a mass m and hovers in position by imparting downward momentum to a column of air defined by the slipstream boundary shown. Find the downward velocity v given to the air by the rotor at a section in the stream below the rotor, where the pressure is atmospheric and the stream radius is r. Also find the power P required of the engine. Neglect the rotational energy of the air, any temperature rise due to air friction, and any change in air density ρ.

Ans. $v = \frac{1}{r}\sqrt{\frac{mg}{\pi\rho}}$, $P = \frac{mg}{2r}\sqrt{\frac{mg}{\pi\rho}}$



$$\cancel{ma}^{0} - \dot m_1(\bar u_1 - \cancel{v}^{0}) + \dot m_0(\bar u_0 - \cancel{v}^{0}) = -mg\hat j$$

j, i

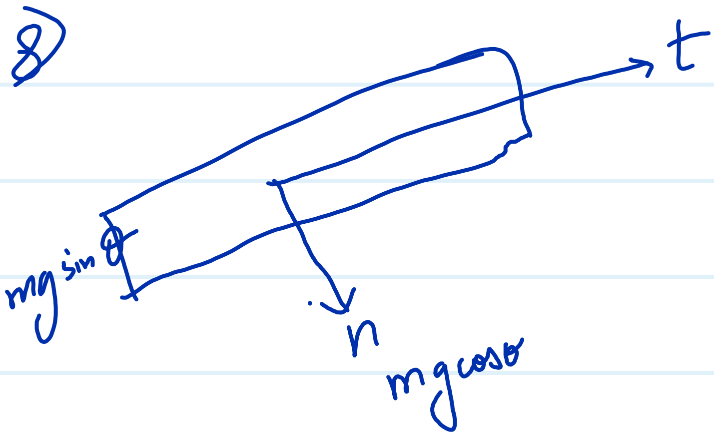
$$-\rho A u_1^2 + \rho A u_0^2 = -mg$$

$$\rho A(u_0^2 - u_1^2) = mg.$$

$$u_0^2 = \frac{mg}{\rho \pi r^2}$$

$$u_0 = \frac{1}{r}\sqrt{\frac{mg}{\rho\pi}}$$

8)



$$m\bar{a} + \dot{m}_0(\bar{u}_0 - \bar{v}) = \bar{F}$$

$$m\bar{a} - \dot{m}_0(600\hat{t}) = \left(\frac{mg\sqrt{3}}{2}\right)\hat{n} - \left(\frac{mg}{2}\right)\hat{t}$$

$$\bar{a} = \left[\frac{\dot{m}_0}{m}(600) - \frac{g}{2}\right]\hat{t} + \left(\frac{g\sqrt{3}}{2}\right)\hat{n}$$

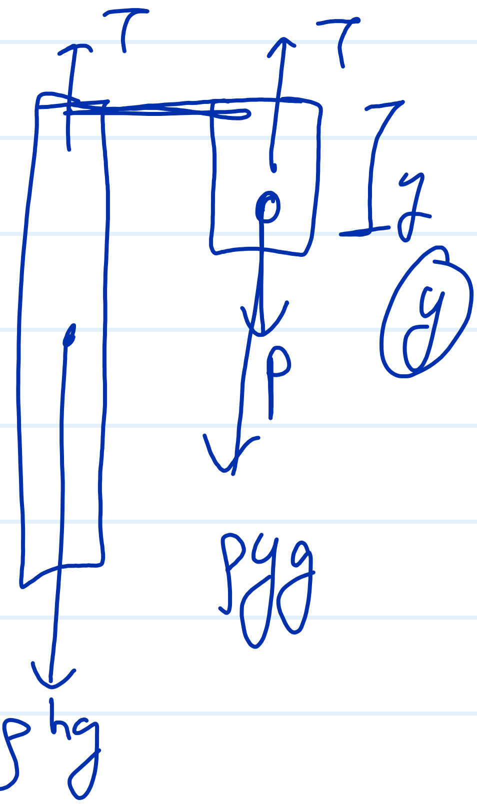
$$\bar{a} = 21.22\hat{t} + 8.31\hat{n}$$

9) $v = \dot y$ ρ 0

$$\boxed{\cancel{m\vec{a}}^{\,0} - \dot m_i(\bar u_i - \bar v) + \cancel{\dot m_o}^{\,\vec 0}(\bar u_o - \bar v) = \bar F}$$

$$\cancel{-\rho\dot h(0 - \dot h\hat j) + \dot p}$$

$$= T\hat j - \rho h g\hat j$$



$$\rho \dot h^2 = T - \rho h g$$

$$\rho v^2 = T - \rho h g$$

$$\boxed{T = \rho v^2 + \rho h g}$$

$$\cancel{T = \rho y g}$$

$$\bar F = 0$$

$$-\rho v(v\hat j + v\hat j)$$

$$-2\rho v^2 = T - P - \rho y g$$

$$P = \tau - \rho y g + 2\rho v^2$$

$$= \rho v^2 + \rho h g + 2\rho v^2 - \rho y g$$

$$P = 3\rho v^2 + \rho(h-y)g$$

$$-\rho v(2v)$$

10)

$$m\bar{a} - \dot{m}_i(\bar{u}_i - \bar{v}) + \cancel{\dot{m}_o(\bar{u}_o - \bar{v})}^{\,0} = \bar{F}$$

$$(m+\rho x)\bar{a} - \rho v(-v\hat{i}) = 0$$

$$(m+\rho x)\, v\frac{dv}{dx} + \rho v^2 = 0$$

$$(m+\rho x)\frac{dv}{dx} + \rho v = 0$$

$$\frac{dv}{dx} = -\frac{\cancel{2}\rho}{(m+\rho x)}\, v$$

$$\frac{dv}{v} = -\frac{\rho}{m}\frac{dx}{\left(1+\frac{\rho x}{m}\right)}$$

$$\Big[\ln v\Big]_{v_0}^{v} = \left[-\frac{\rho \ln\left(1+\frac{\rho x}{m}\right)}{m\rho}\right]_0^{L}$$

$$\ln \frac{v}{v_0} = -\left[\ln\left[1 + \frac{\rho L}{m}\right]\right]$$

$$v = \frac{v_0}{\left[1 + \frac{\rho L}{m}\right]}$$

11)

$$-\dot{m}_1(\bar{v}_i - \cancel{v}) + \dot{m}_0(\bar{u}_0 - \cancel{v}) = -P_i A_1 \hat{j} + P A_0 \hat{j} - mg\hat{j}$$

$$-\rho A_i v_1(-\bar{u}_1 \hat{j}) + \rho A_0 v_0(-v_0 \hat{j}) = (P A_0 - P_i A_1 - mg)$$

$\rho = 1.206$ $\quad A_1 = \pi(1)^2$ $\quad A_0 = \pi(3)^2$

$V_i = 45\ ms^{-1}$ $\quad V_0 = 5ms^{-1}$

$m = 2200kg$ $\quad g = 9.8ms^{-2}$

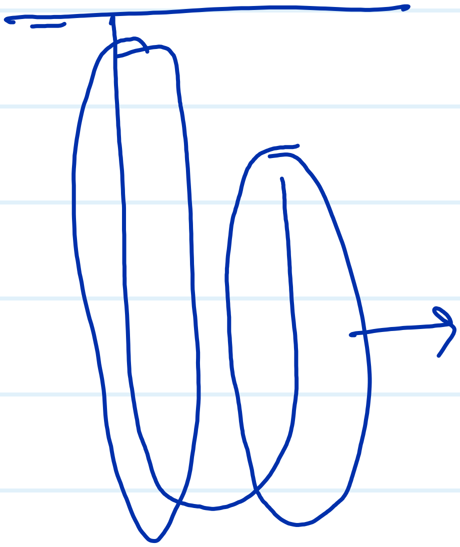
$$+1.206(\pi)(45)^2 - 1.206(\pi)(15)^2$$

$$= (P\,9\pi - 2200(9.8))$$

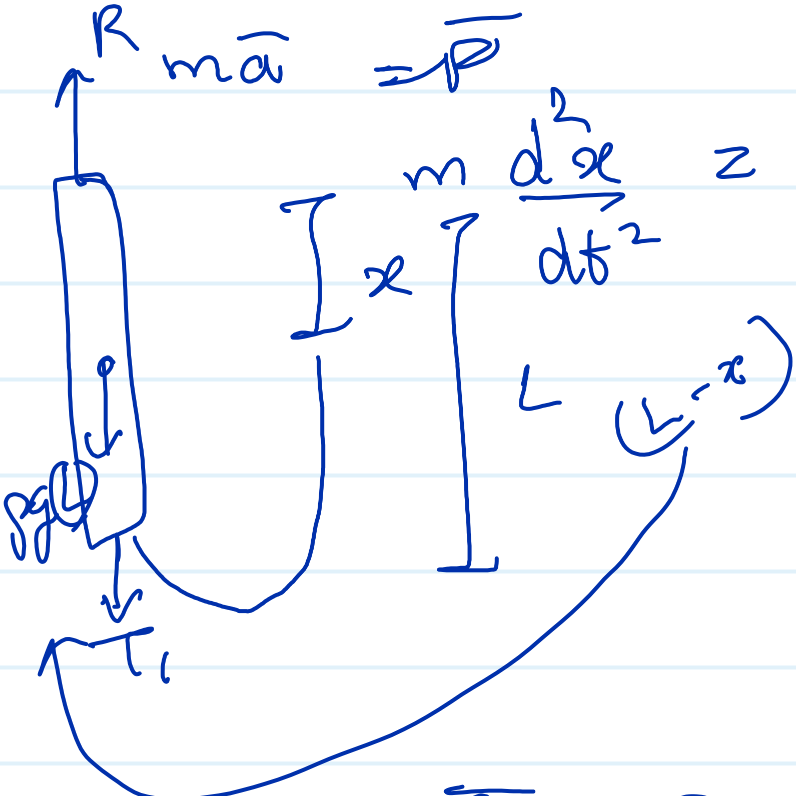
$$\frac{1.206\pi((30)(60)) + 2200(9.8)}{9\pi} = P$$

$P = 1034.68$

12)



$$m\bar{a} - \dot{m}_i(\bar{u}_i - \bar{v}) + \dot{m}_0(\bar{u}_0 - \bar{v}) = \bar{F}$$



$$m\bar{a} = \bar{F}$$

$$m\frac{d^2x}{dt^2} = -\rho(L-x)g -$$

$$\bar{F} = R\hat{j} - T_1\hat{j} - \rho g(2L-x)\hat{j}$$

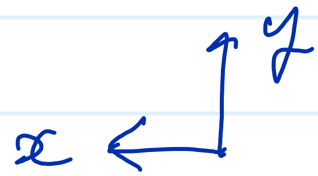
$$m\bar{a} - \rho\dot{x}(0+\dot{x}\hat{j}) -$$

$$-ma\hat{j} - \rho(\dot{x})^2\hat{j} = R - T - \rho g(2L-x)$$

13)

$$\dot{m}_0 = \frac{3}{8}$$

$$\bar{v} = +\frac{700}{9}\ ms^{-1}\ \hat{i}$$



$$m = 16.4 \times 10^{3}\ kg$$

$$\cancel{280} \times \frac{\cancel{1000}}{\cancel{3600}} \qquad \frac{700}{9}$$

$$m\bar{a} - \dot{m}_0\left(0 - \bar{v}\right) = \frac{2238 \times \cancel{1000}}{7\cancel{00}}\ 9$$

$$m\bar{a} = -\dot{m}_0\bar{v} + 2877.428$$

$$\bar{a} = -\frac{\frac{4.5}{12} \times 1000 \left(\frac{700}{9}\right)}{16400} + \frac{2877.428}{16400}$$

$$= -1.778 + 0.1754$$

$$\bar{a} = -1.6025\ ms^{-2}$$

14)

$r = 20 \times 10^{-3} m$

$v = 240 ms^{-1}$

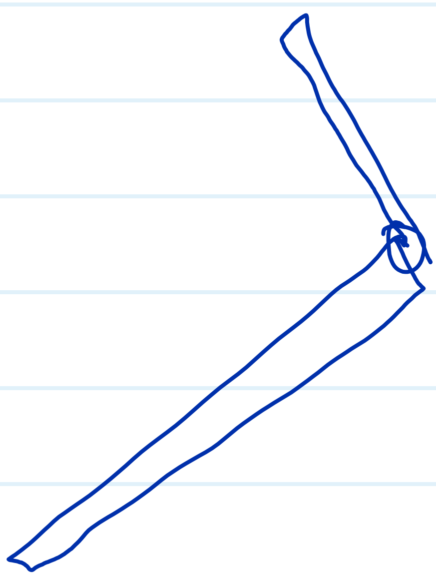
$\rho = 1.206$

$\dot{m}_f = \rho A v$

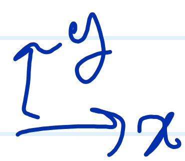
$= 1.206 \left(\pi (20 \times 10^{-3})^2\right) \times 240$

$= 0.363$

$\hat{i}\ \hat{j}$



O



Moment about O

$M = (\bar{r}_i \times \dot{m}_i(\bar{u}_i - \bar{v})) - (\cancel{\bar{r}_o \times \dot{m}_o(\bar{u}_o - \bar{v})})^{0}$

$= (-0.12\hat{j} \times 0.363(-240\hat{i}))$

$\bar{M} = -(0.12)(0.363)(240)\hat{k}$

$$\bar{M} = 6g\,(0.24\sin\theta)$$

$$\sin\theta = \frac{(\cancel{0.12})(0.363)(\cancel{240})}{(\cancel{0.24})(\cancel{6})(9.8)}$$

$\cancel{40}$ 20

2 1

$=$