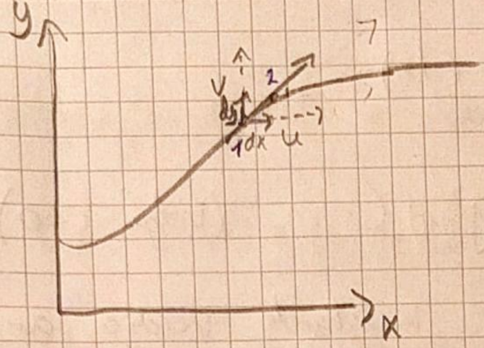
TMR4247 ØVING 1 Jon Magnus Moen

1 a)



partikkelen må forflytte seg dx og dy på lik tid til ny posisjon 2, og derfor

må $\Delta t = \frac{dx}{u} = \frac{dy}{v}$

siden $\Delta t = \frac{\Delta s}{v}$

b) $\frac{dx}{u} = \frac{dy}{v}$ gir $\frac{dx}{x} + \frac{dy}{y} = 0$

$$\int \frac{dx}{x} + \int \frac{dy}{y} = \int 0$$

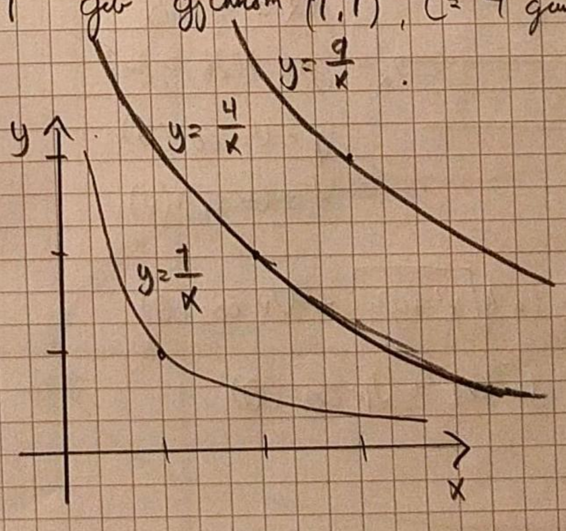
$$\ln x + \ln y = C_1$$

$$\ln(xy) = C_1$$

Dermed er en ligning for strømlinjene $xy = e^{C_1} = C_2$

$$y = \frac{C}{x}$$

$C = 1$ gir gjennom $(1,1)$, $C = 4$ gir gjennom $(2,2)$, og $C = 9$ gir $(3,3)$

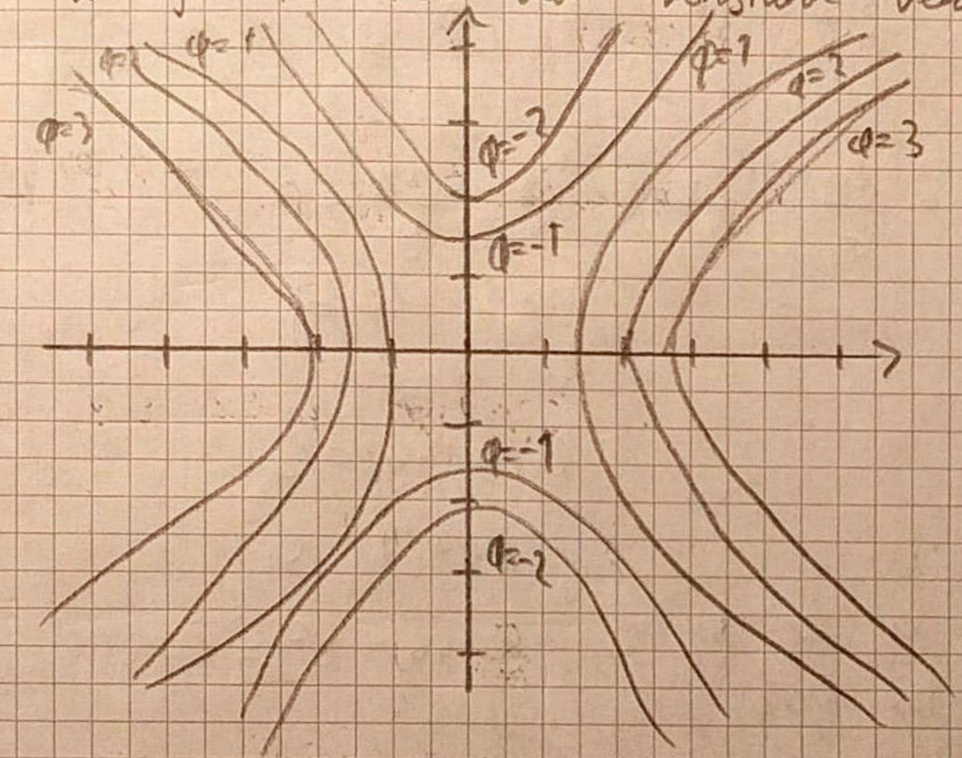


c) $\frac{\partial \varphi}{\partial x} = x \Rightarrow \varphi = \frac{1}{2}x^2 + C(y)$

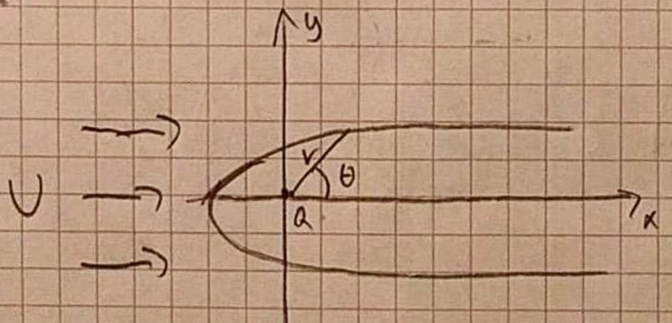
$\frac{\partial \varphi}{\partial y} = -y \Rightarrow \varphi = -\frac{1}{2}y^2 + C(x)$

$\Rightarrow \varphi = \frac{1}{2}x^2 - \frac{1}{2}y^2 + C$ (velger $C=0$)

Potensiallinjer får man ved konstante verdier av φ:



2



$U = 10\, m/s$

$Q = \frac{\pi}{2}\, m^3/s$

$\rho = 1026\, kg/m^3$

$\phi = Ur\cos(\theta)$

$\psi = Ur\sin(\theta)$

$\phi = \varphi_{kilde} + \varphi_{SL} = \frac{Q}{2\pi}\ln\sqrt{x^2+y^2} + Ur\cos(\theta)$

$= \frac{Q}{2\pi}\ln(r) + Ur\cos(\theta)$

$$u_r = \frac{\partial \varphi}{\partial r} = \frac{Q}{2\pi r} + U\cos(\theta)$$

$$u_\theta = \frac{1}{r}\frac{\partial \varphi}{\partial \theta} = -U\sin(\theta)$$

Stagnasjonspunktet er der $u_r = 0 = u_\theta$:

$$u_\theta = -U\sin(\theta) = 0 \Rightarrow \theta = \pi \quad \text{(fra tegning)}$$

$$u_r = \frac{Q}{2\pi r} + U\cos(\pi) = 0$$

$$r = \frac{-Q}{2\pi U\cos(\pi)} = -\frac{\frac{\pi}{2}}{2\pi \cdot 10 \cdot (-1)} = \underline{0{,}025\text{m}}$$

$$r = \frac{Q}{2\pi U}$$

Separasjonsstrømlinjen er strømlinjen gjennom stagnasjonspunktet.

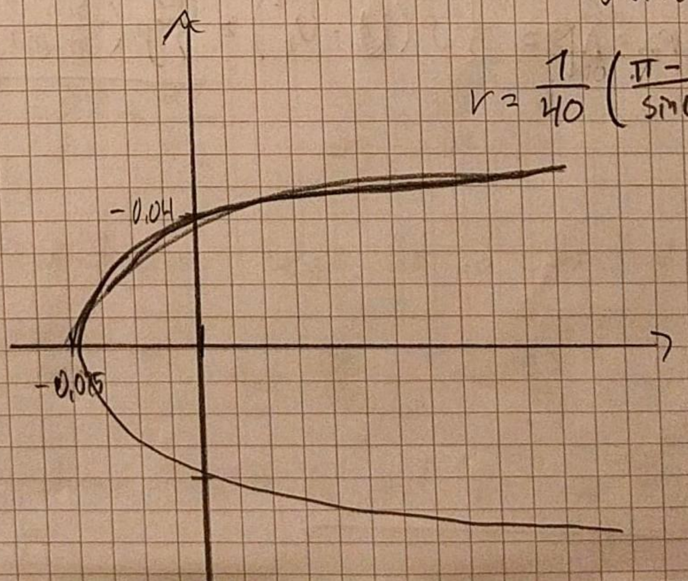
$$\psi = \psi_{\text{kilde}} + \psi_{SL} = \frac{Q}{2\pi}\theta + Ur\sin(\theta)$$

$$\psi\left(\frac{Q}{2\pi U}, \pi\right) = \frac{Q}{2\pi}\cdot\pi + U\frac{Q}{2\pi U}\,0 = \underline{\frac{Q}{2}}$$

$\Rightarrow$ Separasjonsstrømlinjen: $\frac{Q}{2\pi}\theta + Ur\sin(\theta) = \frac{Q}{2}$

$$r = \frac{\frac{Q}{2}\left(1 - \frac{\theta}{\pi}\right)}{U\sin(\theta)}$$

$$r = \frac{1}{40}\left(\frac{\pi - \theta}{\sin(\theta)}\right)$$



b) Velger punkt en langt borte fra foilen, og punkt to på overflaten.

$$\frac{p_\infty}{\rho} + \frac{1}{2}U_1^2 = \frac{p_2}{\rho} + \frac{1}{2}U_2^2$$

$$U_2^2 = u_r^2 + u_\theta^2 = \left(\frac{Q}{2\pi r} + U\cos(\theta)\right)^2 + \left(-U\sin(\theta)\right)^2$$

$$= \frac{Q^2}{4\pi^2 r^2} + \frac{Q}{\pi r}U\cos\theta + U^2\cos^2(\theta) + U^2\sin^2(\theta)$$

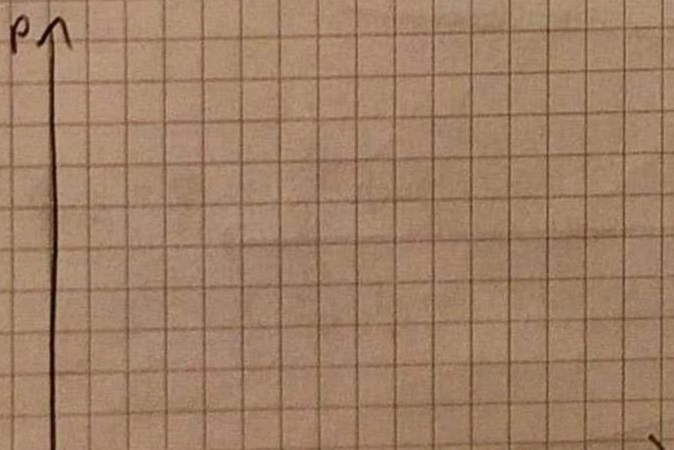
$$= \frac{Q^2}{4\pi^2 \frac{Q^2}{4U^2\sin^2\theta}\left(1-\frac{\theta}{\pi}\right)^2} + \frac{Q\,U\cos\theta}{\pi \frac{Q}{2U\sin\theta}\left(1-\frac{\theta}{\pi}\right)} + U^2$$

$$= \frac{U^2\sin^2\theta}{\pi^2\left(1-\frac{\theta}{\pi}\right)^2} + \frac{U^2\cos\theta\sin\theta}{\pi\left(1-\frac{\theta}{\pi}\right)} + U^2$$

$$= \frac{U^2\sin^2\theta}{(\pi-\theta)^2} + \frac{U^2\sin(2\theta)}{(\pi-\theta)} + U^2$$

$$= U^2\left(1 + \frac{\sin^2\theta}{(\pi-\theta)^2} + \frac{\sin(2\theta)}{\pi-\theta}\right)$$

$$\Rightarrow p_2 - p_\infty = \Delta p = \frac{1}{2}\rho\left(U_1^2 - U_2^2\right) = \underline{\frac{1}{2}\rho\left(-\frac{\sin^2\theta}{(\pi-\theta)^2} - \frac{\sin(2\theta)}{\pi-\theta}\right)}$$



c) Kavitasjon skjer i områder med lavest trykk. (Der det går under damptrykket).

3 a)

- Strømfunksjonen er en funksjon for å beskrive strømningen visuelt, og er definert slik:

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

Setter vi ψ = en konstant, får vi strømlinjene.

- $\vec{V} = \nabla \phi$ slik at $u = \frac{\partial \phi}{\partial x}$, $v = \frac{\partial \phi}{\partial y}$

og $u = \frac{\partial \psi}{\partial y}$, $v = \frac{\partial \psi}{\partial x}$

$$\Rightarrow \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \qquad \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

- The streamfunction equal to a constant is a streamline.
- $\vec{V} = \vec{\nabla}\phi \Rightarrow u = \frac{\partial \phi}{\partial x}$, $v = \frac{\partial \phi}{\partial y}$, $w = \frac{\partial \phi}{\partial z}$
- The stream function is not defined for three dimensions so no.

b)

$$\varphi = x^2 - y^2$$

- Kont: $\nabla \vec{U} = \nabla(\nabla\varphi) = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 2 - 2 = \underline{0}$

- $u = 2x \quad , \quad v = -2y$

$u = \frac{\partial\psi}{\partial y} \Rightarrow \psi = 2xy + C(y)$

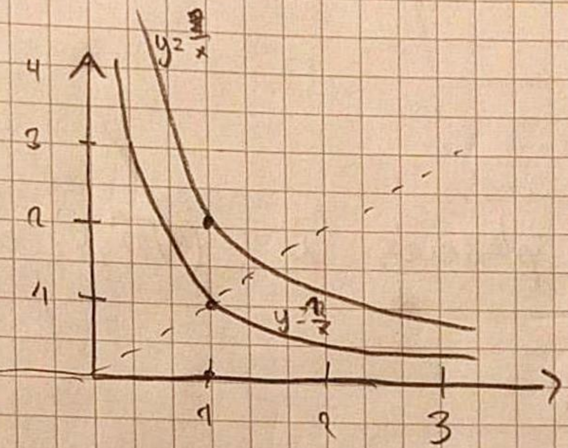
$v = -\frac{\partial\psi}{\partial x} \Rightarrow \psi = 2xy + C(x)$

$\Rightarrow \underline{\psi = 2xy}$

- $(1,0) = 2\cdot 1\cdot 0 = 0 \Rightarrow \psi = 2xy = 0 \qquad y = 0$

$(1,1) = 2\cdot 1\cdot 1 = 2 \Rightarrow \psi = 2xy = 2 \qquad y = \frac{1}{x}$

$(1,2) = 2\cdot 1\cdot 2 = 4 \Rightarrow \psi = 2xy = 4 \qquad y = \frac{2}{x}$



- Strømningen er differansen mellom to strømlinjer:

$\dot{A} = 4 - 2 = \underline{2\,m^2/s}$

- $\varphi = 1^2 - 1^2 = 0 \Rightarrow \varphi = x^2 - y^2 = 0 \qquad y_2 = x$

$\frac{dy_1}{dx} = \frac{d\,1/x}{dx} = -1x^{-2}$ i $(1,1) = -1$

$\frac{dy_2}{dx} = 1 \qquad \frac{dy_2}{dx}(1,1)\cdot\frac{dy_1}{dx}(1,1) = -1 \Rightarrow \underline{y_2 \perp y_1}$

c) Må plassert i $(-3,0)$:

$$\phi = \frac{Q}{2\pi} \ln \sqrt{(x-3)^2 + y^2}$$

$$u = \frac{\partial \phi}{\partial x} = \frac{Q}{2\pi} \frac{2(x-3)}{(x-3)^2 + y^2} \Rightarrow u(0,2) = \frac{Q(-3)}{2\pi(9+4)} = \underline{\frac{3Q}{26\pi}}$$

$$v = \frac{\partial \phi}{\partial y} = \frac{Q}{2\pi} \frac{y}{(x-3)^2 + y^2} \Rightarrow v(0,2) = \frac{\cancel{2}Q}{\cancel{2}\pi(4+9)} = \underline{\frac{Q}{13\pi}}$$

På x-aksen:

$$v = \frac{-2Q}{2\pi} \frac{y}{x^2 + 2^2}$$, må være lik $-\frac{Q}{13\pi}$ i $(0,2)$.

$$\frac{-2Q}{\cancel{2\pi}} \frac{\cancel{2}}{x^2 + 2^2} = -\frac{Q}{13\pi}$$

$$x^2 + 4 = 26$$

$x = \pm\sqrt{22}$, plasseres i $(\sqrt{22}, 0)$ eller $(-\sqrt{22}, 0)$

d)

$$\phi = \frac{Q}{2\pi} \ln \sqrt{(x-3)^2 + (y-2)^2}$$

$$u = \frac{\partial \phi}{\partial x} = \frac{Q}{2\pi} \frac{x-3}{(x-3)^2 + (y-2)^2} \qquad u(1,1) = \frac{\cancel{Q}}{\cancel{2\pi}} \frac{(-2)}{4+1} = \underline{-0,4}$$

$$v = \frac{\partial \phi}{\partial x} = \frac{Q}{2\pi} \frac{y-2}{(x-3)^2 + (y-2)^2} \qquad v(1,1) = \frac{\cancel{Q}}{\cancel{2\pi}} \frac{-1}{5} = \underline{-0,2}$$