Name of exam - 5th Sem Midsem 2021

Subj - Information and Coding Theory

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1a) In certain circumstances it is useful to consider information as grouped into blocks of symbols. This is generally done in binary format. For a memoryless source that takes values, in the range $\{x_1, x_2 \ldots x_m\}$ and where P_i is the probability that the symbol x_i is emitted, the order n extension of the range of a source has M^n symbols $\{y_1, y_2 \ldots y_{M^n}\}$. The symbol y_i is constituted from a sequence of n symbols x_{ij}. The probability $P(Y=y_i)$ is the probability of the corresponding sequence $x_{i1}, x_{i2}, \ldots x_{in}$.

$$P(Y=y_i) = P_{i1}, P_{i2} \ldots P_{in}$$

where y_i is the symbol of the extended source that corresponds to the sequence $x_{i1}, x_{i2} \ldots x_{in}$.

Then,

$$H(X^n) = \sum_{y=x^n} P(y_i) \log_2 \frac{1}{P(y_i)}$$

1b) $P(X=x_1) = 1/2$ order $= 2$

$P(x=x_2) = P(x=x_3) = 1/8$

$P(X=x_4) = 1/4$

The entropy of this extended source is equal to

$$H(X^2) = \sum_{i=1}^{M^2} P_i \log_2 \left(\frac{1}{P_i}\right)$$

$$= 0.25 \log_2(4) + 2 \times 0.125 \log_2(8) + 0.0625 \log_2(16)$$
$$+ 4 \times 0.03125 \log_2(32) + 4 \times 0.015625 \log_2(64) = 3.5$$

bits per symbol

ans) 3.5 bits per symbol (ans)

Symbol	Probability	Symbol	Probability	Symbol	Probability	Symbol	Probability
$x_1 x_1$	0.25	$x_2 x_1$	0.0625	$x_3 x_1$	0.0625	$x_4 x_1$	0.125
$x_1 x_2$	0.0625	$x_2 x_2$	0.015625	$x_3 x_2$	0.015625	$x_4 x_2$	0.03125
$x_1 x_3$	0.0625	$x_2 x_3$	0.015625	$x_3 x_3$	0.015625	$x_4 x_3$	0.03125
$x_1 x_4$	0.125	$x_2 x_4$	0.03125	$x_3 x_4$	0.03125	$x_4 x_4$	0.0625

Symbols of the order 2 extended source and their probabilities

The order 2 extended source has an entropy twice that of the entropy of the original. i.e $H(X^n) = n\,H(X)$

2) For (7,4) Hamming Code with generator

$$G' = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

~~Here~~ we perform the following matrix operations to get the generator matrix G in $G = [I\,P]$ form.

(i) $R_1 = R_1 + R_3$

(ii) $R_3 = R_3 + R_4$

$$G' = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(iii) $R_2 = R_2 + R_3$

$$G' = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{I} & \underbrace{\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{matrix}}_{P} \end{bmatrix} = G$$

b) Parity check matrix, $H = [P^T I]$

$$= \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{matrix}}_{P^T} & \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} \end{bmatrix}$$

2c) $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

If msg signal is (m_0, m_1, m_2, m_3)

$\therefore K_0 = m_0 + m_2 + m_3$

$K_1 = m_0 + m_1 + m_2$

$K_2 = m_1 + m_2 + m_3$

Parity bits for msg signal of 1

$\therefore$ (7,4) encoder circuit will be