Event space → same as probability (P)

information (I) ∝ log (1/P)

$P_i(x_i) = P_i$ if that massage is emitted.

$P(X = x_i) = P_i$

self information → $I_i = \log_2(1/P)$.

equiprobable massages (P) → $\log_2 n$.

$P = 1/n$; $I = k\log_2(1/P)$

$I_i \geq 0$ $\quad 0 \leq P_i \leq 1$

$I_i \to 0$ $\quad$ if $P_i \to 1$

$I_i > I_j$ $\quad$ if $P_i < P_j$.

unit is bits.

for equiprobable →
$P = 1/n$.
$I = \log n (1/P)$ bits.

composite :-

$P_k P_l$: $I_{k,l} = \log_2\left(\frac{1}{P_k P_l}\right)$

$= \log_2(1/P_k) + \log_2(1/P_l)$

$= I_k + I_l$

But not today i clearly bcz i am not able to see

Entropy

$$H(x) = \sum_{i=1}^{M} P_i \log(1/P_i) \text{ bits/msg.}$$

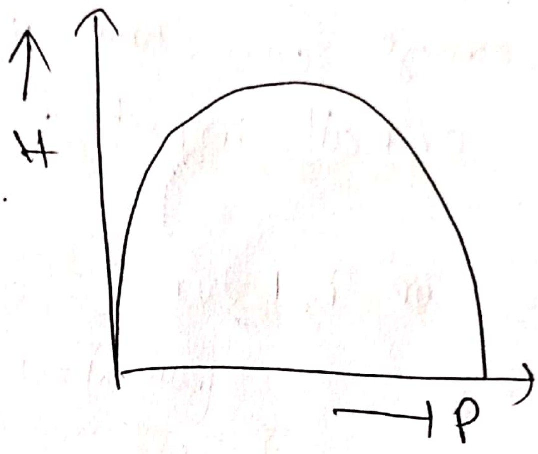
for binary source → (only for two levels 0/1).

$$H = P\log_2(1/P) + (1-P)\log\left(\frac{1}{1-P}\right).$$

for $I = 0$ for $P = 0$.
and $I = 0$ for $P = 1$.

for P betn $P = 0.4$ & 0.6.

I is closed to 1.



EU JAAN

Can we relate entropy of DMS & entropy of the extended DMS with n extension?
$H(x)$ with $H(x^n)$.

BSC

$K > 1$ then code is not properly written,

$\eta = \frac{H(x)}{N} \to$

$N \to$	2.0	1.25	1.875	1.75
$K \to$	1.0	1.5	0.9375	1.0

this are decodable ✓ — not code properly — for best case

$$\boxed{K = \sum_{i=1}^{M} 2^{-N_i} \leq 1}$$

kraft inequality.

x_i	P_i	I	II	III	IV
A	1/2	00	0	01	00
B	1/4	01	1	011	01
C	1/8	10	10	0111	10
D	1/8	11	11	01111	11

Shannon's source coding Theorem

Avg code length $\to N = \sum_{i=0}^{M} P_i N_i$

$\to H(X) \leq N < H(x) + 1$

$\eta = \frac{H(X)}{N} = \frac{R}{r_0} \leq 1$ optimal encoding.

$H(X) = \sum P_i \log \frac{1}{P_i}$

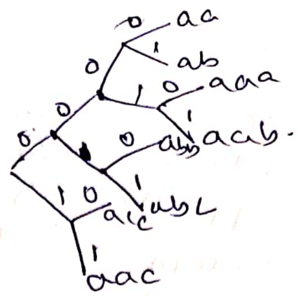
$= \frac{1}{2} \times \log 2 + \frac{1}{4} \cdot 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 3$

$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{3}{8} = \frac{4+4+6}{8}$

$= \frac{14}{8} = \frac{7}{4} = 1.75$

Ex

aa
ab.
a aa
a ab
a bb
a bc
a cc
a aC



Shennon fano coding:

x_i	P_i				
m_1	$1/2$	0			
m_2	$1/8$	1	0	0	
m_3	$1/8$	1	0	1	
m_4	$1/16$	1	1	0	0
m_5	$1/16$	1	1	0	1
m_6	$1/16$	1	1	1	0
m_7	$1/32$	1	1	1	0
m_8	$1/32$	1	1	1	1

$$\eta = \frac{H(x)}{N}$$

$$= \frac{8^{5/16}}{2^{5/16}}$$

.21

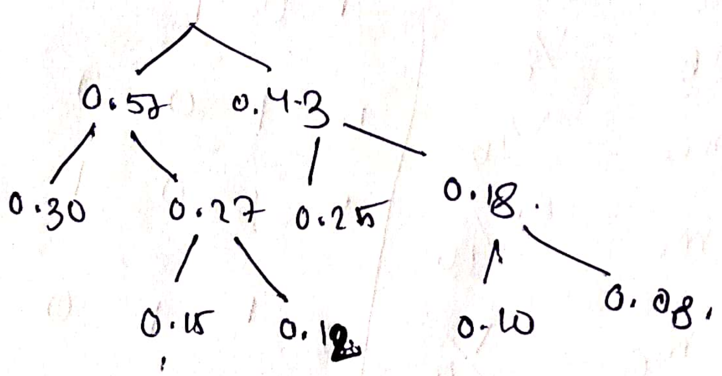
H/W

P_i
0.46
0.15
0.10
0.10
0.07
0.07
0.03
0.02.

Hoffman coding

m_1	0.30 00	0.30 00	0.30 00	0.30	0.43 0	0.57 → 0
m_2	0.25 10	0.25 10	0.27 01	0.43	0.30 0	0.43 → 1
m_3	0.15 010	0.18 11	0.25 10	0.27	0.27 01	
m_4	0.12 011	0.15 010	0.18 11			
m_5	0.10	0.12 011				
m_6	0.08	110				
		111				

Eta ager jeta code hat hobe
setai hobe tar por decode hobe
tar por 0 or 1 ~~hobe~~ hobe



$N = 2.45$ bits / symbol.

$H(x) = 2.418$ bit / symbol

$$\eta = \frac{H(x)}{N}$$

$= 0.976$.

Markoff Statistical Model for Information Source

~~Propose~~

T_s

$\downarrow$

$i \rightarrow x_i \downarrow$

$i+1 \rightarrow x_{i+1}$

$P_{ix} \downarrow$

$x_{i+1} \downarrow$

Statistically model the symbol sequence emitted by the discrete source by a class of random process known as discrete stationary Markov process.

Properties:

1. $1, 2 \dots n,$
$i \rightarrow j \quad P_{ij} \quad \sum_{j=1}^{n} P_{ij} = 1$

2. As the source is emitting symbol transition from i to j the symbol emitted depends on the transition, $i \rightarrow j$

3. $S_1, S_2 \dots S_n, \; X_1, X_2 \dots X_k \dots$
$P(X_k = S_q \mid X_1, X_2 \dots X_{k-1})$.

4. $P(X_k = S_q \mid X_1, X_2 \dots X_{k-1})$
$= P(X_k = S_q / S_k)$

5. $1, 2, \dots n,$
$P_1(1), P_2(1), \dots P_n(1). \quad \sum_{i=1}^{n} P_i(1) = 1.$

6. $P_j(k+1) = \sum_{i=1}^{n} P_i(k) P_{ij}$ $\quad P(k) \rightarrow n \times 1$ column vector

$P_j(n) \quad P \Rightarrow n \times n \rightarrow$ matrix $\quad i^{th} \rightarrow P_i(k)$ entry

$(i, j) \rightarrow P_{ij}$

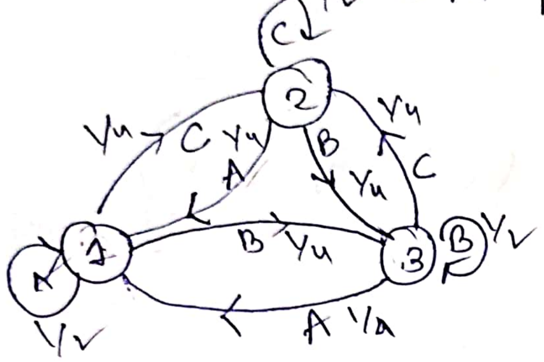
~~$P(K+1) = \phi^T P(K)$~~

ϕ is probability transition matrix.

$P(K+1) = \phi^T P(K)$.

If $\boxed{P(K) = \phi^T P(K)}$ → stationary Markov process.

with $K = 1$



$P_1(1) = 1/3$

$P_2(1) = 1/3$

$P_3(1) = 1/3$

Q what is probability of emission of AB from the source. $P(AB)$

$P_1 \quad 3 \to 1 \to 3$

$P_2 \quad 2 \to 1 \to 3$

$P_3 \quad 1 \to 1 \to 3$

$P(AB)$

$= P_1 + P_2 + P_3$

$= P(S_1=1, S_2=1, S_3=3) + P(S_1=2, S_2=1, S_3=3)$
$+ P(S_1=3, S_2=1, S_3=3)$.

$P(S_1=1, S_2=1, S_3=3) = P(S_1=1)\, P(S_2=1/S_1=1)\, P(S_3=3/S_1=1, S_2=1)$

$= P(S_1=1)\, P(S_2=1/S_1=1)\, P(S_3=3/S_2=1)$

$= 1/3 \times \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{24}$

$P(S_1=2, S_2=1, S_3=3)$

$= P(S_1=2)\, P(S_2=1 / S_1=2)\, P(S_3=3 / S_2=1)$

$= \frac{1}{3} \times \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{48}$

$P(S_1=3, S_2=1, S_3=3)$

$= P(S_1=3)\, P(S_2=1 / S_1=3)\, P(S_3=3 / S_2=1)$

$= \frac{1}{3} \times \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{48}$

$\therefore P(AB) = P_1 + P_2 + P_3$

$= \frac{1}{48} + \frac{1}{48} + \frac{1}{24}$

$= \frac{1}{12}$

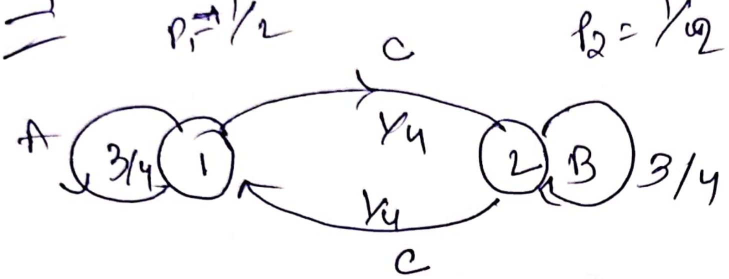
$H_i = \sum_{i=1}^{A} P_{ij} \log_2\left(\frac{1}{P_{ij}}\right)$

$H = \sum_{i=1}^{A} H_i P_i$ bits/symbol

$R = rH$ bps.

$r \rightarrow$ no. of stack transition/sec

Q



$$H_1 = \frac{1}{4}\log_2(4) + \frac{3}{4}\log_2(4/3)$$

$$H_2 = \frac{1}{4}\log_2(4) + \frac{3}{4}\log_2(4/3)$$

$P(AAA) = ?$

$P(AAB) = ?$

Error correction code - linear block code

$d \rightarrow d_1 - d_2 \quad d_k$ data k digits.

$c \rightarrow c_1 c_2 \quad c_n$.

code word - n digit redundant lists.

$n > k$. $(n-k)$

(n, k) check digits.

efficiency of block code $= \frac{k}{n}$

Hamming Bound:

Hamming distance

(i) 101 $\rightarrow$ $d=2$

110

(ii) 101 $\rightarrow$ $d=1$

111

$d_{min} \geq t+1$ error detection code

$d_{min} \geq 2t+1$. error correction code

error correction code $\rightarrow$ linear block

(n, k) linear block code

$n \rightarrow$ code digits

$k \rightarrow$ data digits.

~~tot~~ total data ~~no~~ words $= 2^k$

total code words 2^n,

the number of sequence of n digit that differ from a given sequence by j digits is ^nC_j

Let t errors can occur,
Number of ways it error can occurs,

$$\sum_{j=1}^{t} \binom{n}{j}$$

$$2^k \sum_{j=1}^{t} \binom{n}{j}.$$

$$2^n \geq 2^k + 2^k \sum_{j=1}^{t} \binom{n}{j}$$

$$\geq 2^k \sum_{j=0}^{t} \binom{n}{j}$$

or, $2^{n-k} \geq \sum_{j=0}^{t} \binom{n}{j}$ Hamming Bound

$\Rightarrow 2^m = \sum_{j=0}^{t} {}^nC_j$ perfect code

Binary single error correction code is called Hamming code

$t=1$

$$\boxed{d_{min} \geq 2t+1}$$

single error correcting code

$d_{min} = 3.$

$2^m = {}^nC_0 + {}^nC_1$

$= 1 + n.$

$\Rightarrow 2^m = n+1$

$(6,3) \rightarrow$ given $\qquad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \rightarrow$ given

$n = 6$

$k = 3$

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

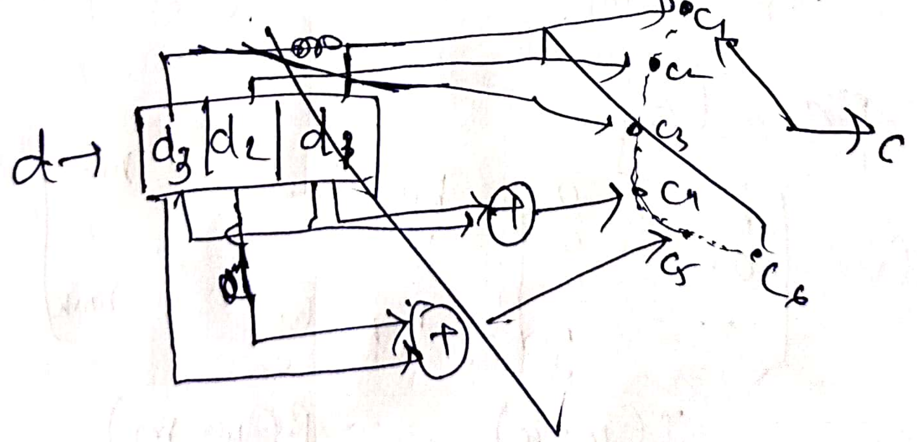
$\qquad I_k \qquad\qquad P$

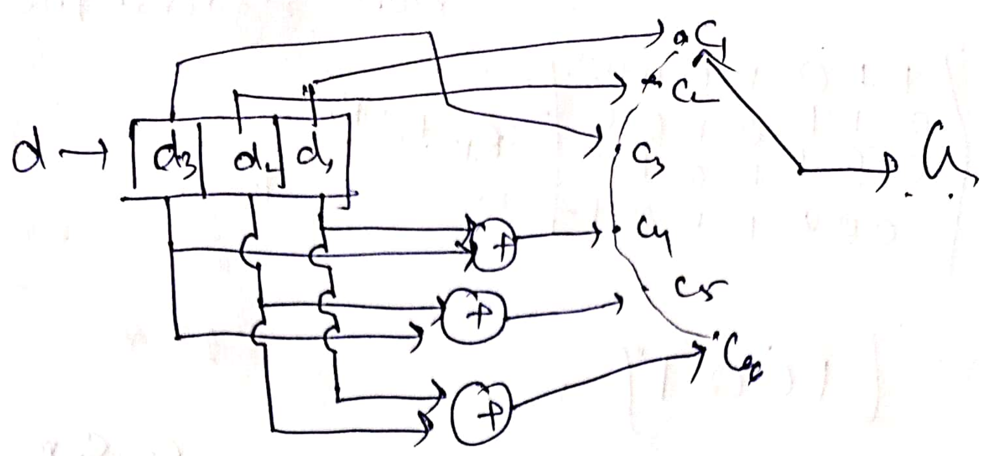
$C = dG$

1+

d	c ($c_1 c_2 c_3$ $c_4 c_5 c_6$)
000	000 000
001	001 110
010	010 011
011	011 101
100	100 101
101	101 011
110	110 110
111	111 000

Encoder circuit for (6,3) systematic code





C_i C_j

$C_i - C_j$

$w(t) \rightarrow$ weight of the code.
$\rightarrow$ no. of non zero elements.

Single tone Bound $\rightarrow$

$$\boxed{d_{min} \leq n-k+1}$$

If $(n,k) \rightarrow C$ is a linear code block then $(n, n-k)$ is $C^{\perp}$ dual of code

when, $C = C^{\perp}$ then it is called self dual.

$C_i \rightarrow 101$

$C_j \rightarrow 110$

011

Non systematic

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$\boxed{R_1 = R_1 \oplus R_2}$

$n = 7$.

$k = 4$

$m = [1 0 0 1]$

~~c = Gm~~

$c = Gm$.

$c = 1100101$.

we can do the operation betn rows for simplify.

$\boxed{G = [P, I]}$.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & \boxed{1} & \boxed{1} & 0 & 0 \\ 0 & 1 & 1 & 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$\boxed{R_1 = R_1 \oplus R_2}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$R \oplus \cdots = R$

$$\begin{bmatrix} 1 & 1 & 0 & \boxed{1} & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$R_4 = R_2 \oplus R_4$

4×3.

$$\begin{bmatrix} 1 & 1 & 0 & \boxed{1} & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & \boxed{1} & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & \boxed{1} \end{bmatrix}$$

$R_3 = R_1 \oplus R_3$

$R_4 = R_1 \oplus R_4$

$$\left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$\underbrace{}_{P}$ $\underbrace{}_{I}$

$H = [I_{n-k}, P^T]$.

$$= \begin{array}{cc} \begin{matrix} c_0 & c_1 & c_2 \end{matrix} & \begin{matrix} c_3 & c_4 & c_5 & c_6 \end{matrix} \\ \left[\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \right. & \left. \begin{matrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{matrix}\right] \\ I_{n-k} & P^T. \end{array}$$

we get linear equation where bit is 1.

$\boxed{CH^T = 0}$ *

$c_0 + c_3 + c_5 + c_6 = 0 \rightarrow c_0 = c_3 + c_5 + c_6$

$c_1 + c_3 + c_4 + c_5 = 0 \rightarrow c_1 = c_3 + c_4 + c_5$

$c_2 + c_4 + c_5 + c_6 = 0 \rightarrow c_2 = c_4 + c_5 + c_6$.

Scanned with CamScanner

Jishu Shit

510819001

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$n = 6$

$k = 3$

$$G = \begin{bmatrix} P & I_k \end{bmatrix}$$

$R_1 = R_1 \oplus R_2$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$R_2 = R_2 \oplus R_1$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$R_3 = R_2 \oplus R_3$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 = R_1 \oplus R_3$

$$\Big[\underbrace{\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{matrix}}_{P} \quad \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_k}\Big]$$

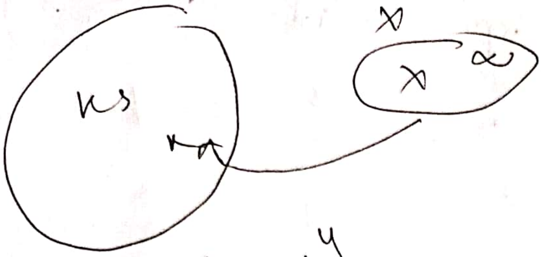
$\therefore$ Generator Matrix : $G = [I_k \; P]$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Data ~~Code~~ word	Code
0 0 0	0 0 0 0 0 0
0 0 1	1 1 0 0 0 1
0 1 0	1 1 1 0 1 0
0 1 1	0 0 1 0 1 1
1 0 0	1 0 1 1 0 0
1 0 1	0 1 1 1 0 1
1 1 0	0 1 0 1 1 0
1 1 1	1 0 0 1 1 1.

Coding Theory



(n, k)

$\longrightarrow$ $d(u)$ $n-k, k$

Cyclic code

$(c_0, c_1, c_2, \ldots c_n)$

1. linearity properties
2. Cyclic properties. $(c_n, c_0, c_1 c_2 \cdots c_{n-1})$

$c_i + c_j = c(x)$ $m = [\underset{0}{1}\ \underset{1}{0}\ \underset{2}{1}\ \underset{3}{0}]$

$m(x) = 1 + x^2$

$\therefore x c(x)$ indeterminate $+ c_{n-1} x^{n-1}$, $x^0\ x^1\ x^2\ x^3$

$+ c_n x^n$ $\downarrow\ \downarrow\ \downarrow\ \downarrow$ $1\ 0\ 1\ 0$

Systematic encoding

Generator Polynomial

$$g(x) = 1 + \sum_{i=1}^{n-k-1} g_i x^i + x^{n-k}$$

$$c(x) = a(x)\, g(x)$$

$a(x) \rightarrow$ polynomial of degree $k-1$,

$0, 1$, $\{b_0 b_1, b_{n-k-1}$ $m_0 m_1, \ldots m_{k-1}\}$

$m(x) = m_0 + m_1 x + m_2 x^2 + \ldots + m_{k-1} x^{k-1}$

$b(x) = b_0 + b_1 x + \ldots + b_{n-k-1} x^{n-k-1}$

$C(x) = b(x) + x^{n-k} m(x)$

$a(x) g(x) = b(x) + x^{n-k} m(x)$

$$\frac{x^{n-k} m(x)}{g(x)} = a(x) + \frac{b(x)}{g(x)}$$

1. $m(x) \cdot x^{n-k}$

2. $\text{Rem}\left[\frac{x^{n-k} m(x)}{g(x)}\right] \to b(x)$

3. $b(x) + x^{n-k} m(x)$

(7,4)

$g(x) = x^3 + x^2 + 1$

$m = 1010$ LSB

$m(x) = x^3 + x$

1. $x^3(x^3 + x) = x^6 + x^4$

2. $\cancel{(x^3 + x^2 + 1)\, x^6 + x^4 (x^3 + x^2 + 1)}$

$x^{n-k}(m(x))$

$$\begin{array}{r|l} x^3 + x^2 + 1 & x^6 + x^4 \quad (x^3 + x^2 + 1 \\ g(x) & x^6 + x^5 + x^3 \\ \hline & x^5 + x^4 + x^3 \\ & x^5 + x^4 + x^2 \\ \hline & x^3 + x^2 \\ & x^3 + x^2 + 1 \\ \hline & 1 \end{array}$$

$b(x) = 1$ ✓

$c(x) = \underbrace{1}_{b(x)} + \underbrace{x^4 + x^6}_{x^{n-k} m(x)}$

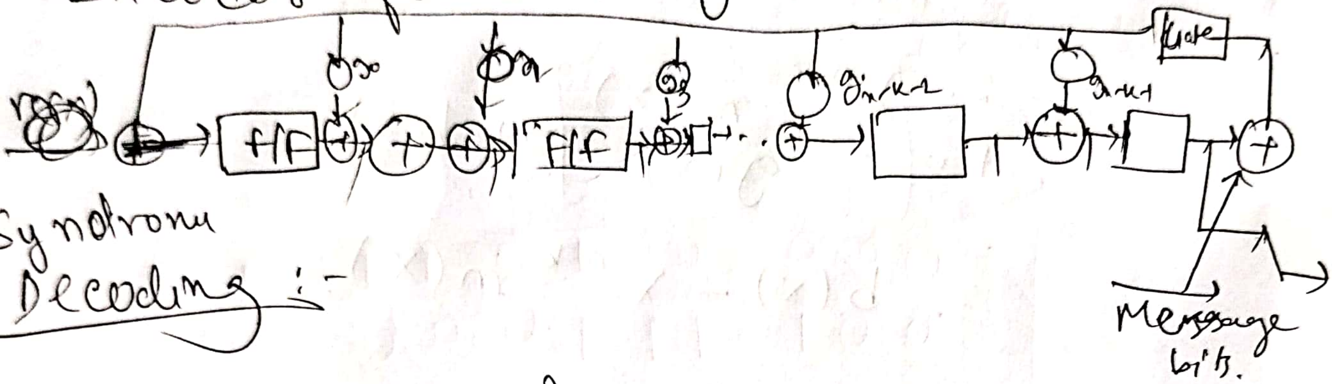
$$m \Rightarrow \begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{matrix} 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{matrix} \end{array}$$

Parity check polynomial.

$$h(X) = 1 + \sum_{i=1}^{k-1} h_i X^i + X^k.$$

$$g(x)\, h(x) = X^n + 1.$$

Encoder for (n,k) cyclic code :-



Syndrome Decoding :-

$\{r_0, r_1, \ldots\; r_{n-1}\}$

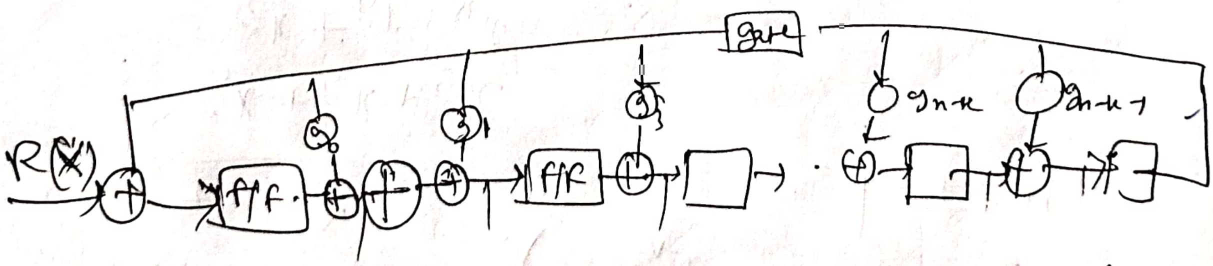
$r(X) = r_0 + r_1 x + r_2 x^2 + \ldots\ldots + r_n X^n.$

$r(x) = q(x) \cdot g(x) + S(x) \rightarrow$ Ok

$\Rightarrow S(x) = \dfrac{q(x)\, g(x)}{r(x)}$

If $S(X)$ is 1 or any value then there is an error

If it is 0 then it has no error.



$(7,4) \to$ n, k

$n=7 \quad k=3.$ $\qquad g(x)\,h(x) = x^n + 1$

$$x^7 + 1 = \underline{(1+x)(1+x^2+x^3)}\,\underline{(1+x+x^3)}.$$

$$g(x) = 1 + x + x^3$$

$$h(x) = (1+x)(1+x^2+x^3)$$

Let $m\,[1\,0\,0\,1]$ (indices $3\;2\;1\;0$).

$$m(x) = 1 + x^3$$

$$\underline{x^{n-k}m(x)} = x^3(1+x^3) = x^3 + x^6$$

$$b(x) \to \quad 1+x+x^3 \,\big)\, x^3 + x^6 \,\big(\, x^3 + x + 1$$

$$\begin{array}{l} x^6 + x^4 + x^3 \\ \hline \quad x^4 \\ \quad x^1 + x^2 + x^4 \\ \hline \quad x^2 + x \to b(x) \end{array}$$

$$g(x) = 1 + x + x^3$$

$G \to$

$$x\,g(x) = x + x^2 + x^4$$

$$x^2 g(x) = x^2 + x^3 + x^5$$

$$x^3 g(x) = x^3 + x^4 + x^6$$

$$\begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \to g(x) \\ \to x\,g(x) \\ \to x^2 g(x) \\ \to x^3 g(x) \end{array}$$

$\underline{k \times n}$

$h(x) = 1 + x^2 + \cancel{x^3} + x + \cancel{x^3} + x^4$ $\rightarrow 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

1st row

$x \cdot x^{n-k} = 1 + x + x^2 + x^4$.

$\rightarrow x^4 h(x) = x^4 + x^5 + x^6 + x^8$. $x^k h(x)$ as $x^{n'} f_i$

2nd row

$x^2 \cdot x^{n-k}$

$\rightarrow x^5 h(x) \rightarrow 3^{rd}$ row $= x^5 + x^6 + x^7 + x^9$.

~~$\rightarrow x^6 h(x) \rightarrow 4^{th}$ row $= x^6 + x^7 + x^8 + x^{10}$~~ → 3rd row.

$$
\begin{array}{ccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\end{array}
$$

$$
\begin{bmatrix}
1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1
\end{bmatrix}
$$

as here max power of x is greater then n

so, $h(x^{-1})$. $(n-k) \times n$.

$x^4 h(x^{-1}) = x^4 + x^3 + x^2 + 1$

$x^5 h(x^{-1}) = x^5 + x^4 + x^3 + x$

$x^6 h(x^{-1}) = x^6 + x^5 + x^4 + x^2$

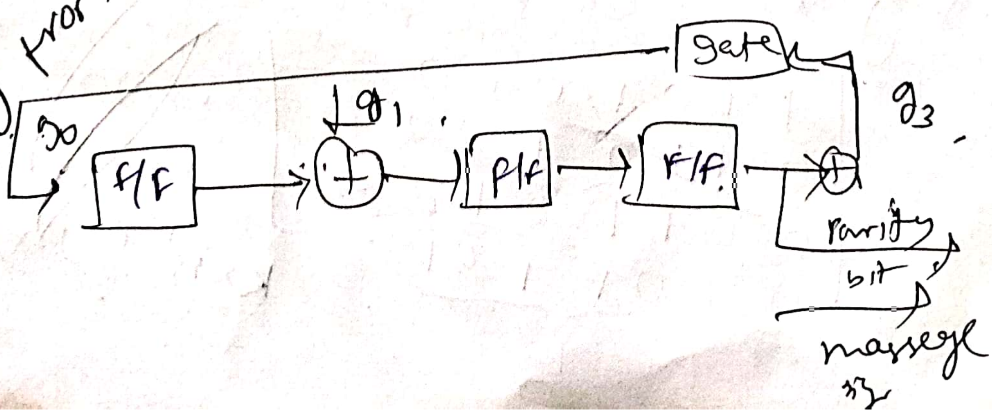
$$
\begin{array}{c|ccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
x^4 h(x^{-1}) \rightarrow & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\
x^5 h(x^{-1}) \rightarrow & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
x^6 h(x^{-1}) \rightarrow & 0 & 0 & 1 & 0 & 1 & 1 & 1
\end{array}
$$

$n-k$

$3 \times 7 \rightarrow n$.

$$h(x) = 1 + \sum_{i=1}^{k-1} h_i x^i + x^k.$$

this is coming from $g(x) = 1 + x + x^3$



o/p code

CRC BCH & RS code

$$\boxed{C(x) = R_{g(x)}[x^{n-k} \cdot m(x)] + x^{n-k} \cdot m(x)}$$

Nonsystematic

$c(x) = m(x)\,g(x)$.

$g(x) = (1+x)(1+x+x^3)$

$= 1+x^2+x^3+x^4$.

$\overset{n}{\uparrow}\ \overset{k}{\uparrow}$
$(7,3)$

data	$m(x)$	$c(x)$
000	0	$m(x)/g(x)$.
001	1	
010	x	
	$1+x$	
	x^2	

Ex

$m(x) =$ $g(x) = x^{16} + x^{15} + x^2 + 1$ $\rightarrow n-k$,

Q $d_0 = 39$ $d_1 = 109$

$\downarrow$ $\downarrow$

$d_0 = (27)_{16}$ $d_1 = 6D_{16}$.

$2|13$, 1
$2|6$, 0
$2|3$, 1
$2|1$
1

$\underbrace{01101101}_{d_1}\ \underbrace{00100111}_{d_0}$

$x^{14} + x^{13} + x^{11} + x^{10} + x^8 + x^5 + x^2 + x + 1$

$S(x) = R_{g(x)}[r(x)]$
$= R_{g(x)}[c(x) + e(x)] = R_{g(x)} e(x)$ → error polynomial

$g(x) \to (1+x)$ → even ~~value~~ parity.

1, 2, ...

$e(x) = x^i \quad 1 < i \leq n-1.$

$(n-k+1)$ → error.

$e(x) = x^i (1 + e_n + \dots + e_{n-k+1} x^{n-k+1} + x^{n-k})$

$g(x)$ → non-zero remainder

Singleton Bound :-

Galois field :-

- $\langle Z_p, +, \cdot \rangle$ forms a field when p is prime, it is called finite field.
- The other finite fields are obtained as extension fields Z_p, using an irreducible ~~polynomial~~ polynomial in $Z_p[x]$

Primitive element of GF(9)

$GF(9) = \{0, 1, 2, \dots 9-1\}$

$GF(2) = \{0, 1\}$

$GF(3) = \{0, 1, 2\}$ → Primitive elements of GF(3)

$2^0 = 1$

3) 4 (1
3
(1)

Coding Theory

Process Synchonization

Decoding of linear block code

(n, K)

$C = d\, C_p$

$= d\,[I_K\, P]$

$= [dI, dP_{n-}]$

$= [d\,;\, C_p]$

$\boxed{C_p = dP}$

$C_p \oplus dP = 0$

or

$[d, C_p]\begin{bmatrix} P \\ I_m \end{bmatrix} = 0$

$CH^T = 0$ (6, 3)

(n, k)

Que $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$r = 100011$

$r = d \oplus c$

Syndrome

$S = rH^T$

$= [c \oplus e]H^T$

$= cH^T \oplus eH^T$

$\boxed{S = eH^T}$

2^K,

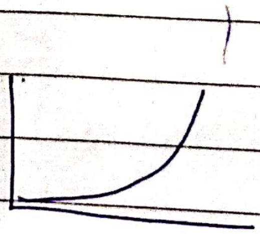
maximum likely hood rule

$P(r/c_i) > P(r/c_K)$

for all $K \neq i$

$P(r/c_i) = P_e(1-P_e)^{n-}$

BSC $P_e < 0$



$e \rightarrow$ minimum no. of 1's

Ex (n, K)

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$S = r H^T$$
$$= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

$$H^T = \begin{array}{c} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \rightarrow P^T \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} I_n \end{matrix}$$

$r = 1\ 0\ 0\ 0\ 1\ 1$

~~[illegible]~~

e (error vector)	S
0 0 0 0 0 0	0 0 0
1 0 0 0 0 0	1 0 1
0 1 0 0 0 0	0 1 1
0 0 1 0 0 0	1 1 0
0 0 0 1 0 0	1 0 0

1 0 0 0 1 0 Syndrome decoding table

$e = 0\ 0\ 1\ 0\ 0\ 0$

$r = 1\ 0\ 0\ 0\ 1\ 1$

$C = r \oplus e = 1\ 0\ 1\ 0\ 1\ 1$

Scanned with CamScanner

Standard array decoding

(7,3) $\quad r = 0011011$

0000000	0111000	1011010	1100100
1000000	1111000	0011010	0100100
0100000	0011000	1111010	1000100
⋮			
1100000			
1010000			

Ans)

$$G = \begin{bmatrix} 0111 & 100 \\ 1011 & 010 \\ 1101 & 001 \end{bmatrix}$$

$r = 0011011$

find error correct code

$$H = \begin{bmatrix} 1000011 \\ 0100101 \\ 0001111 \end{bmatrix}$$

$$rH^T = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \\ 0111 \\ 1011 \\ 1001 \end{bmatrix}$$

$$\boxed{S = rH^T}$$

$$= \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}$$
$$\begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$$

e	S
000000	0000
100000	1000
010000	0100
001000	
000100	

$e \longrightarrow 0101000 \longrightarrow 0101$