Indian Institute of Engineering Science and Technology, Shibpur

B. Tech. Information Technology, 5ᵗʰ Semester End Semester Examinations, December 2021

Algorithms (IT-3104)

Full Marks: 50   Answer any five questions   Time: 1 Hour 30 Minutes

1. a) Why does not Dijkstra algorithm for single-source shortest path work with negative weights? Can you explain with an example?   [5]
    b) Suppose H_1 and H_2 are two Fibonacci Trees. The root list of H_1 contains 8 nodes and that of H_2 contains 3 nodes. How will you unite these Fibonacci heaps? Explain each operation i.e., comparison, assignment etc. for this process.
    Explain with an example how will you insert a node in a Fibonacci Tree.   [5]

2. a) Under what condition in decrease key operation of Fibonacci Tree, there will be modification of only one node? How will you combine three Fibonacci Heaps $H_1, H_2,$ and H_3 and how many pointers have to be modified for this process?   [5]
    b) Differentiate between a Binary Search Tree and a Red-Black Tree. Describe any two properties of a Red-Black tree.   [5]

3. a) What is the black height of a node? What is the black-height of a Red-Black tree? Given the same height of two Red-Black trees, H_1 and H_2, can their black-heights be different? Explain.   [5]
    b) What can be the maximum height of a Red-Black tree with n internal nodes? Prove it.   [5]

4. a) What do you understand by the branching factor of a B-Tree? Does it necessarily mean that a large branching factor will lead to reduced height of a B-Tree? Why the root node of a B-Tree should be kept permanently in the main memory?   [5]
    b) Insert 12 in the following B-Tree.

    Then insert 25 and show the resulting tree.   [5]

5. a) What is an independent set? Let V= (G, E) be a graph, then S is an independent set if and only if its complement V-S is a vertex cover – prove it.   [5]
    b) Prove that Vertex Cover $\leq_P$ Independent Set.   [5]

6. a) What do you understand by Y is polynomial time reducible to X? Why such reducibility is important in the context of intractability of problems?   [5]
    b) Prove that Independent Set $\leq_P$ Set Packing problem.   [5]

7. a) What is a Euler tour? What is Hamiltonian Cycle? Can you relate between the two?   [5]
    b) What are the properties of an efficient certifier? Explain what can be an efficient certifier for a set cover problem?   [5]

8. a) What is the randomized quicksort algorithm? Compare quicksort, heapsort, and mergesort with respect to locality of reference.   [5]
    b) Define a binary heap. Explain with an example. What is a complete binary tree?   [5]