End Semester Examination (5th sem), 2021

→ Subject Name :→ Information and Coding Theory.

→ Subject Code :→ IT 3105

→ Date of Examination :→ 10. 12. 2021

→ Name :→ Aniket Majhi

→ Examination Roll Number :→ 510819019.

→ G Suite ID :→ 510819019.aniket@students.iiests.ac.in

→ Number of sheets uploaded :→ 14.

(1)

(a.) Channel capacity :→

The gaussian chanel resulting from the sampling of the signals is a discrete channel.

The channel capacity is given by the gaussian following equation,

$$C = B \log_2 (1 + S/N) \text{ bits/sec}.$$

C = channel capacity.

B = Chanel Bandwidth

S = average signal power.

N = average noise channel bandwidth.

$= N_0 \cdot B$.

two sided Power spectral density.

~~There is an equivalent expression for the signal to noise ration~~,

☑ channel capacity is actually defined as the number of possible signals that can be transmitted reliably.

Shanon Limit :→

Shanon limit can be analyzed from the equation,

$$\frac{C}{B} = \log_2\left(1 + \frac{E_0}{N_0} \cdot \frac{C}{B}\right)$$

If we use the expression,

$$\frac{C}{B} = \frac{C}{B} \frac{E_b}{N_0} \log_2\left(1 + \frac{E_b}{N_0} \cdot \frac{C}{B}\right)^{N_0 B / C E_b}.$$

$$\Rightarrow 1 = \frac{E_b}{N_0} \log_2\left(1 + \frac{E_b}{N_0} \cdot \frac{C}{B}\right)^{N_0 B / C E_b}$$

$$\boxed{\lim_{x \to} (1+x)^{1/x} = e}$$

If $\frac{C}{B} \to 0$, $B \to \infty$

$$\frac{E_b}{N_0} = \frac{1}{\log_2(e)}$$

$$= 0.693$$

$$\therefore \left(\frac{E_b}{N_0}\right)_{dB} = -1.59 \text{ dB}.$$

This value is known as shanon limit.

(b.) Space time Code :→

A Space time Code is a method employed to improve the reliability of data transmission in wireless communication systems using multiple transmit antennas.

The space time codes rely on transmitting multiple, redundant copies of a data stream on the receiver in the hope that at least some of them may survive the physical path between transmission and reception in a good enough state to allow reliable ~~code~~ decoding.

It can be split into two ~~time codes~~ main types —

(i) Space Time Trellis Code :→

(ii) Space Time Block Code :→

(3)

(a) Priory and posteriory entropy :→

The probability of occurance of a given output symbol y_i is $P(y_j)$ which can be calculated by the ~~matrix~~ Probability matrix, P_{ch} of $U \times V$ order

where U = input symbols
V = output symbols.

$$P_{ch} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1V} \\ P_{21} & P_{22} & \cdots & P_{2V} \\ \vdots & \vdots & & \\ P_{U1} & P_{U2} & \cdots & P_{UV} \end{bmatrix}.$$

here, $P(y_j) = \sum_{i=1}^{V} P_{ij}\, P(x_i)$.

However, the actual information transmitted symbol x_i is known, then the related conditional probability of the output symbol becomes $P(y_j/x_i)$. In the same way, the probability of a given input symbol, initially $P(x_i)$, can also be refined if the actual output is known. Thus if the received y_j appears at the output of the chanel. then the related input symbol condition probability becomes $P(x_i/y_j)$.

The Probability $P(x_i)$ is known as priory ~~entropy~~ Probability

That is, it the Probability that characterized the input symbol before at the presence of any output symbol is known.

The Probability $P(xi/y_j)$ is an estimate of the symbol x_i after knowing that a given symbol y_j appeared at the chanel output and is called the posteriori probability.

The definition of the priory entropy is, given by the equation,

$$\boxed{H(X) = \sum_i P(x_i) \log_2 \left[\frac{1}{P(x_i)}\right]}$$

The definition of the posteriory entropy is given by the equation,

$$H(X/y_j) = \sum_i P(x_i/y_j) \log_2 \left[\frac{1}{P(x_i/y_j)}\right]$$

$$i = 1, 2, \ldots U$$

The chanel Matrix,

$$P_{ch} = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) & \cdots & P(y_v/x_1) \\ P(y_1/x_2) & P(y_2/x_2) & \cdots & P(y_v/x_2) \\ \vdots & \vdots & & \\ P(y_1/x_u) & P(y_2/x_u) & \cdots & P(y_v/x_u) \end{bmatrix}$$

$$= \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1v} \\ P_{21} & P_{22} & \cdots & P_{2v} \\ P_{u1} & P_{u2} & \cdots & P_{uv} \end{bmatrix}$$

(b.) Average mutual information of BSC :→

We know that a BSC is constructed with two inputs (x_1, x_2) and two outputs (y_1, y_2) with alphabets over range $\{0, 1\}$.

let, The symbol probabilities are,

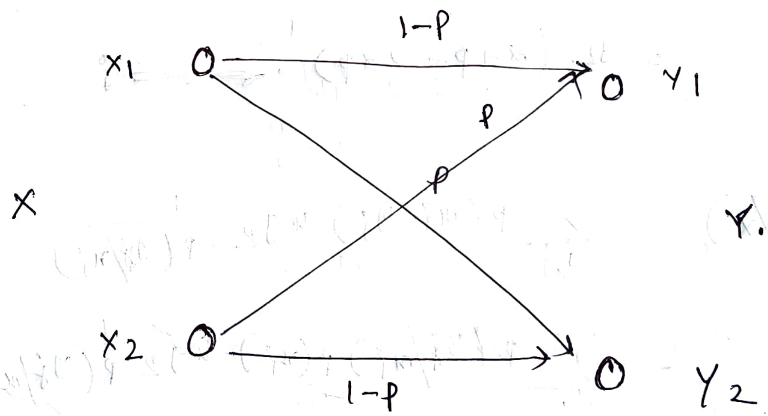
$$P(x_1) = \alpha$$

$$\therefore P(x_2) = 1-\alpha.$$

and the channel ~~errors~~ transition probabilities are,

$$P(y_1/x_2) = P(y_2/x_1) = p.$$

$$P(y_2/x_2) = P(y_1/x_1) = 1-p.$$



The error Probability:

$$P = P(x_1)\cdot P(y_2/x_1) + P(x_2)\cdot P(y_1/x_2)$$

$$= \alpha\cdot p + (1-\alpha)\cdot p$$

$$= p.$$

∴ The mutual information,

$$I(X,Y) = H(Y) - H(Y/X). \quad \text{——} (1)$$

The output Y has two symbols y_1 and y_2.

where, $P(y_2) = 1 - P(y_1)$.

Now,

$$P(y_1) = P(y_1/x_1)\,P(x_1) + P(y_1/x_2)\,P(x_2)$$

$$= (1-p)\,\alpha + p(1-\alpha)$$

$$= \alpha - p\alpha + p - p\alpha$$

$$= \alpha + p - 2\alpha p.$$

Now,

$$H(Y) = P(y_1)\cdot\log_2\frac{1}{P(y_1)} + [1 - P(y_1)]\log_2\frac{1}{(1-P(y_1))}$$

$$= \Omega\,[P(y_1)]$$

$$= \Omega\,(\alpha + p - 2\alpha p).$$

Now,

$$H(Y/X) = \sum_{i,j} P(x_i, y_j)\log_2\frac{1}{P(y_j/x_i)}$$

$$= \sum_{i,j} P(y_j/x_i)\,P(x_i)\log_2\frac{1}{P(y_j/x_i)}$$

$$= \sum_{i} P(x_i)\left[\sum_j P(y_j/x_i)\log_2\frac{1}{P(y_j/x_i)}\right]$$

$$= P(x_1)\left[P(y_2/x_1)\log_2\frac{1}{P(y_2/x_1)} + P(y_1/x_1)\log_2\frac{1}{P(y_1/x_1)}\right] +$$

$$P(x_2)\left[P(y_2/x_2)\log_2\frac{1}{P(y_2/x_2)} + P(y_1/x_2)\log_2\frac{1}{P(y_1/x_2)}\right].$$

$$= \alpha \left[p \log_2 \frac{1}{p} + (1-p) \log_2 \frac{1}{(1-p)} \right] +$$
$$(1-\alpha) \left[(1-p) \log_2 \frac{1}{(1-p)} + p \log_2 \frac{1}{p} \right]$$

$$= \alpha p \log_2 \frac{1}{p} + \alpha (1-p) \log_2 \frac{1}{(1-p)} +$$
$$(1-\alpha)(1-p) \log_2 \frac{1}{(1-p)} + \overset{(1-\alpha)}{p} \log_2 \frac{1}{p}$$

$$= \log_2 \frac{1}{p} (\cancel{\alpha p} + p - \cancel{\alpha p}) + \log_2 \frac{1}{(1-p)} [\cancel{\alpha} - \cancel{\alpha p} + 1 - p - \cancel{\alpha} + \cancel{\alpha p}]$$

$$= p \log_2 \frac{1}{p} + (1-p) \log_2 \frac{1}{(1-p)}$$

$$= \Omega(p)$$

Now,

from (1) we know,

$$\boxed{\begin{aligned} I(X,Y) &= H(Y) - H(Y/X) \\ &= \Omega(\alpha + p - 2\alpha p) - \Omega(p) \end{aligned}}$$

↳ The average mutual information depends on the source probability α and error probability.

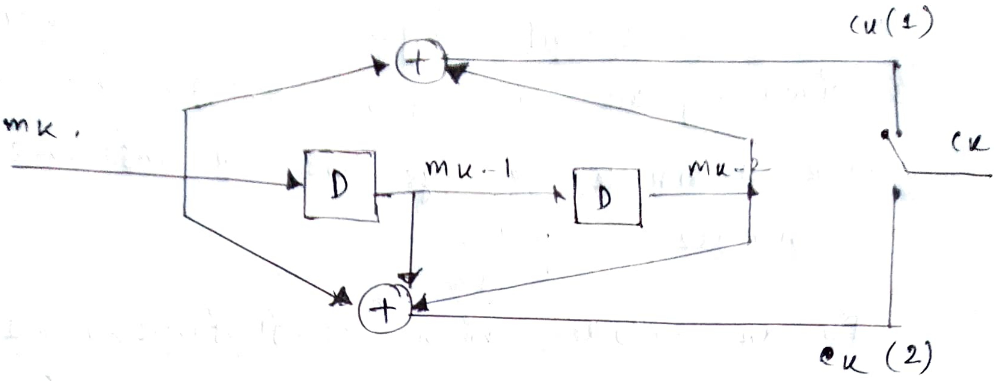
- **Maximum:→** When the channel error probability is p is very small, i.e., $(p = 0)$

$$I(X,Y) \approx \Omega(\alpha) - \Omega(0) \approx H(X).$$

- **Minimum:→** When $p \approx 1/2$, that is its maximum value, then,

$$I(X,Y) = \Omega(\cancel{\alpha} + \tfrac{1}{2} - \cancel{\alpha}) - \Omega(1/2) = 0$$

(5)



(a)

As there are two encoded paths, so 2 bits are produced at unit time. i.e, $n = 2$.

And no of flip flops used is $= 2$.

So memory $= 2$.

The code rate of a convolutional encoder with M number of shift register (flip flops) is given by, $r = \frac{L}{n(L+M)}$ bits/symbol.

where, $L =$ length of message signal.

Assuming, $L >> M$

$$\therefore \quad r = \frac{\cancel{L}}{n \times \cancel{L}}$$

$$\therefore \boxed{r = \frac{1}{n}}$$

So, coderate of the given encoder is,

$\boxed{r = \frac{1}{2}}$ as, $n = 2$.

☒ Constraint length :→

Constraint length of the encoder is given by the number of shifts over which a single message bit can influence the encoder output.

In an encoder with M-flipflops, M + 1 shifts are required for a message bit to enter the shift register and finally come out.

Hence, constraint length, $K = m + 1$

for this case, as $m = 2$.

$\therefore K = 2 + 1$

$\boxed{K = 3}$

(b.)

Given input stream,

$m = [1\ 1\ 0\ 0\ 1\ 0\ 1]$

Coded sequence,

Transfer function matrix,

$G = [1 + x^2,\ 1 + x + x^2]$

$\therefore g^{(1)}(x) = 1 + x^2$

$\Rightarrow G_1 = 101$

$\therefore g^{(2)}(x) = 1 + x + x^2$

$\Rightarrow G_2 = 111$

Message Polynomial,

$$m(x) = 1 + x + x^4 + x^6.$$

Now, output Polynomial for path 1,

$$c^{(1)}(x) = g^{(1)}(x) \cdot m(x)$$

$$= (1 + x^2)(1 + x + x^4 + x^6)$$

$$= 1 + x + x^4 + \cancel{x^6} + x^2 + x^3 + \cancel{x^6} + x^8$$

[Modulo 2 Addition,

$$= 1 + x + x^4 + x^2 + x^3 + x^8$$

$$= 1 + x + x^2 + x^3 + x^4 + x^8$$

→ Output Polynomial for path 2,

$$c^{(2)}(x) = g^{(2)}(x) \cdot m(x)$$

$$= (1 + x + x^2)(1 + x + x^4 + x^6)$$

$$= 1 + \cancel{x} + x^4 + \cancel{x^6} + \cancel{x} + \cancel{x^2} + x^5 + x^7 + \cancel{x^2} + x^3 + \cancel{x^6} + x^8$$

[using modular 2 additi

$$= 1 + x^3 + x^4 + x^5 + x^7 + x^8.$$

So, output sequence of path 1 & 2 are,

c_1 = 1 1 1 1 1 0 0 0 1

c_2 = 1 0 0 1 1 1 0 1 1

Therefore, encoded sequence by taking one bit from both c_1 & c_2 at a time is,

$$c = (11, 10, 10, 11, 11, 01, 00, 01, 11)$$

(e) Trellis Diagram :—

Here number of states = 2^m

$= 2^2$

$= 4.$

State is represented by bits m_1 & m_2.

Let states be a, b, c, d.

So,

m_1	m_2	State
0	0	a
0	1	b
1	0	c
1	1	d

Input bit (m)	State bits m_1	State bits m_2	State	Output bits x_1	Output bits x_2	Next state m_1'	Next state m_2'	
0	0	0	a	0	0	0	0	a
1	0	0	a	1	1	1	0	c
0	0	1	b	1	1	0	0	a
1	0	1	b	0	0	1	0	c
0	1	0	c	0	1	0	1	b
1	1	0	c	1	0	1	1	d
0	1	1	d	1	0	0	1	b
1	1	1	d	0	1	1	1	d.

☒ Trellis diagram :-