Origin of Digital Image Processing

→ News paper Industry → Image or picture
→ Were sent by Submarine Cable
→ london & New york.

→ Bartlane Cable - 1920s - reduced trans-
-mission time - Image to 3 hours from
more than one week - Atlantic.

- reproduced by telegraph printer
with Special face.
- This method was aboundend - 1921

- New technique - photographic repro-
-duction made from tapes.

- Early Bartlane s/ms - Coding image
in 5 distinct gray level.
- Capability - increased to 15 distinct
levels of gray - 1929

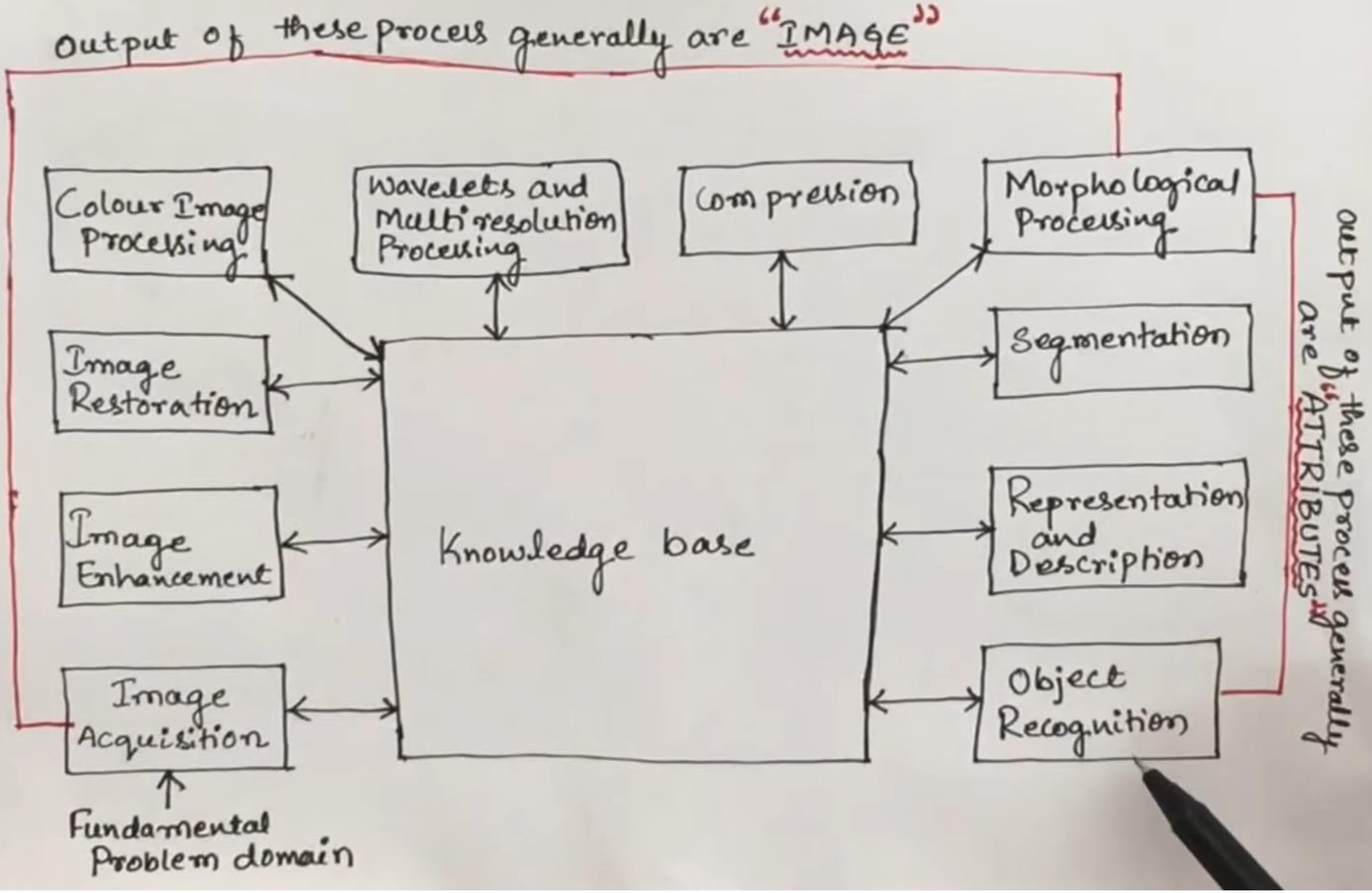
- Idea - Modern digital computers - 1940s
- Jhon Von Neumann - two Key concepts
(i) Memory (ii) Conditional branching
- Foundation of CPU

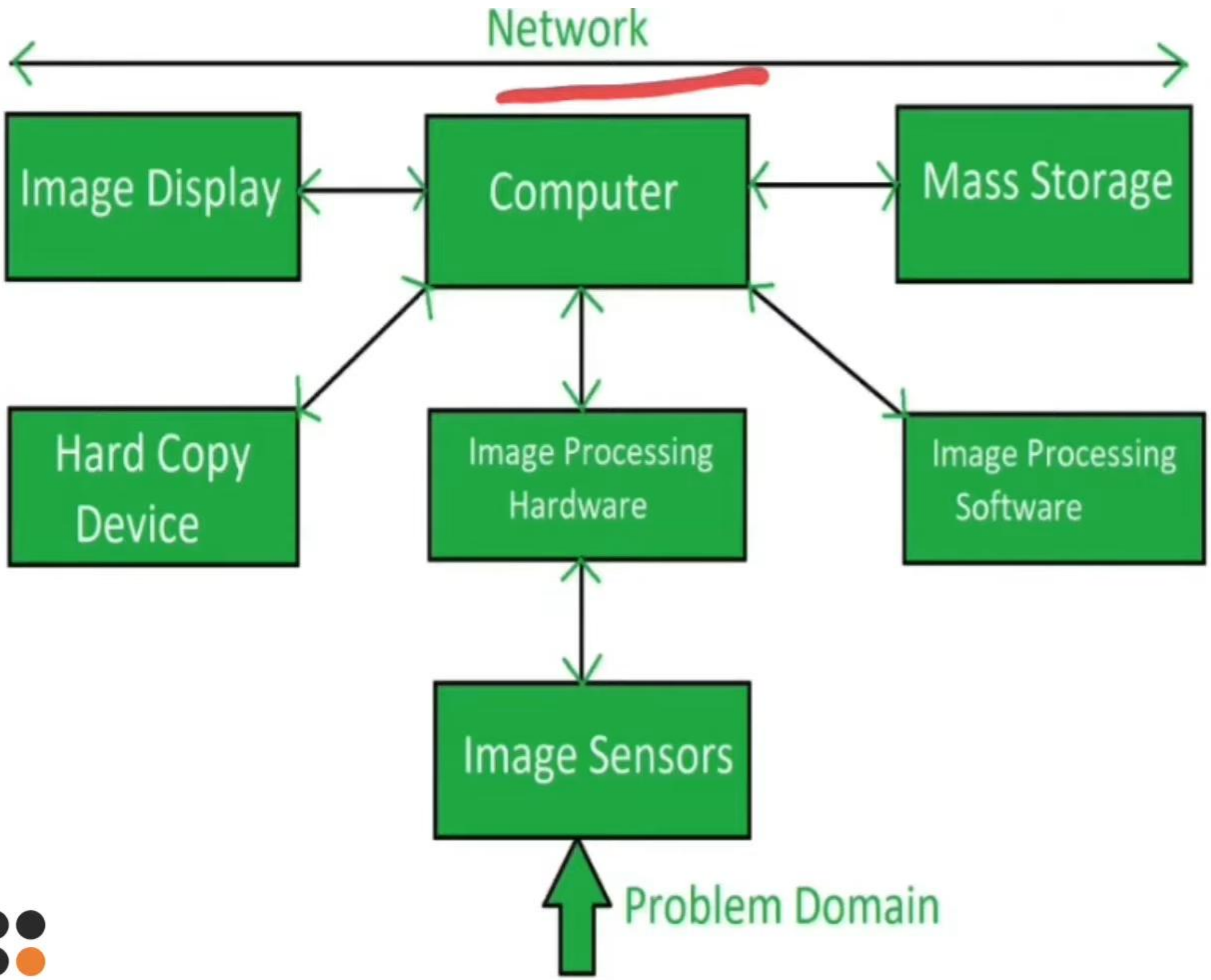
- Key advances - computer - powerful
- Digital image processing.

* Transistors - Bell labs - 1948
* high level programming language
* IC - Texas Instruments - 1958
* OS - early 1960s
* MP - Intel - 1970s
* PC - IBM - 1981
* large scale ICs - late 1970s
* VLSI - 1980s [ULSI - Present]
* IC technology, Mass storage & display s/ms.

- First Computer - Image Processing - 1960s
- First pic. of moon - US space craft
Ranger7 - July 31 1964 at 9:09 AM.
- In late 60s & early 70s - Image
Processing Applications.
* Medical Imaging * Remote earth resource & Astronomy.
- From - 60s to present - IP - broad
range of Applications.
* Constrast Enhancement
* Image Enhancement & Restoration.

Fundamental Steps in DIP:

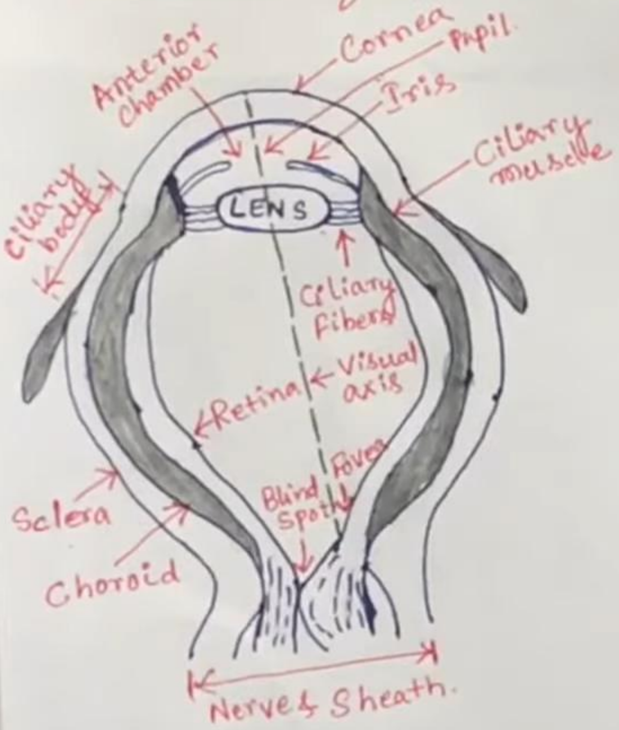




Ec A

compnents of DIP system

Structure of Human Eye :-



→ shape → Sphere, diameter → 20mm
→ Enclosed → Various Membranes
Cornea & Sclera → outer Cover.

→ Cornea → Convex outerior portion.
→ It covers Iris & pupil

→ Sclera → tough white fibrous
Cover entire eyball except Cornea.

→ Choroid → lies below Sclera. → n/w of blood Vessels
→ divided → Ciliary body & Iris diaphram
→ Iris → Contracts or Expand → control light.
↳ Central opening → PUPIL → 2 to 8 mm.
Front → visible pigment, Back → black pigment.

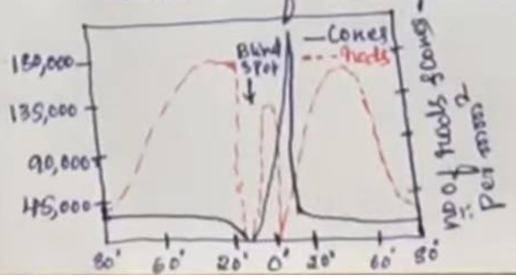
→ Lens → Concentric layers of fibrous cells → Suspended by Fibers.
→ 60% to 70% water, 6% fat, Protein.
→ Coloured → Slightly yellow pigment. → Obsorbs 8% visible light.

→ Retina → Innermost mebrane of eye → Object → Imaged
↳ discrete light receptors → CONES & RODS
↳ patter vision.

→ Cones → 6 to 7 million → Located → Central portion of retina → Fovea. → Vision → Photopic or bright light vision

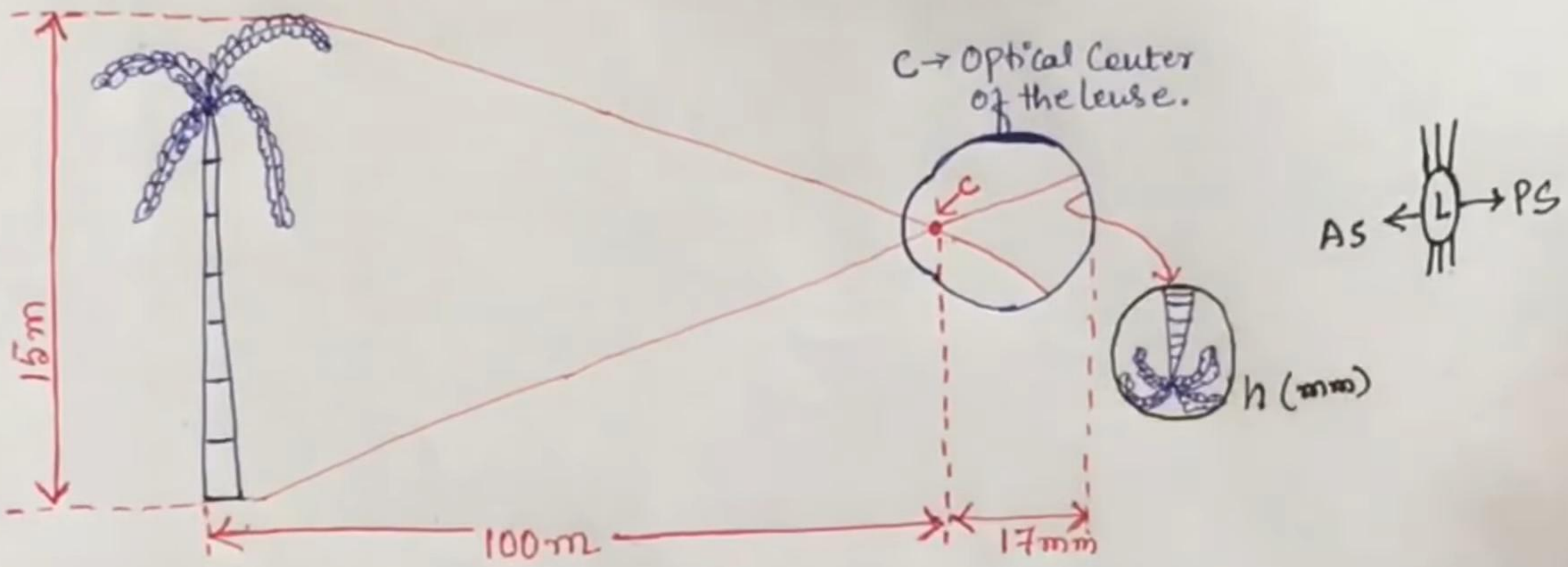
→ Rods → 75 to 150 million → distributed → Surface of Retina.
→ not involved in Colour Vision.

→ Absense of receptors → BLIND SPOT



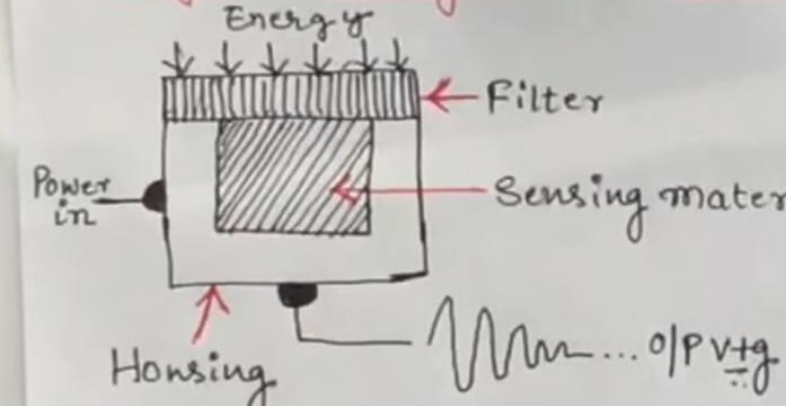
→ Fovea → 1.5mm × 1.5mm
density 150,000/mm²
highest vision → 3,37,000/mm²

Image Formation in an eye:

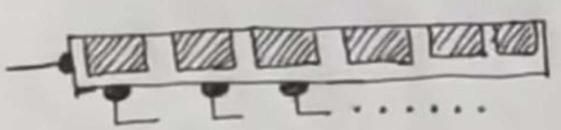


$$\frac{15}{100} = \frac{h}{17} \Rightarrow \boxed{h = 255\,mm}$$

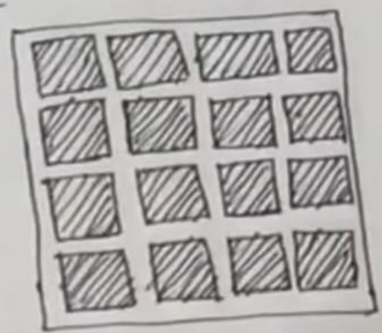
Image Sensing and Acquisition



(a) Single Image sensor



(b) Line Sensor



(c) Array of Sensor

Image acquisition using Single Sensor

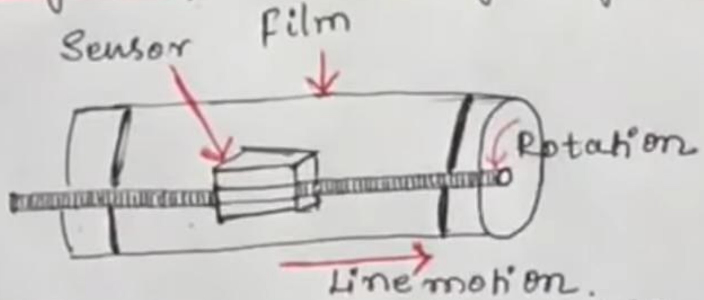
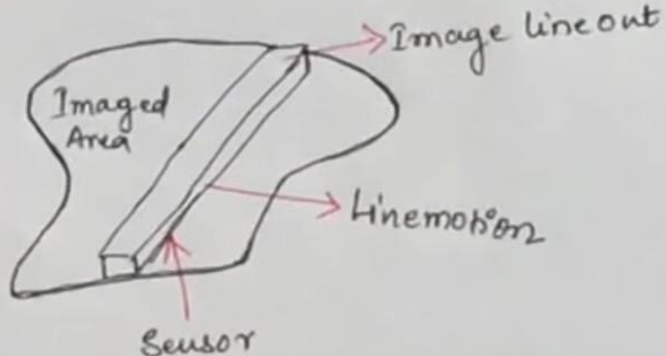
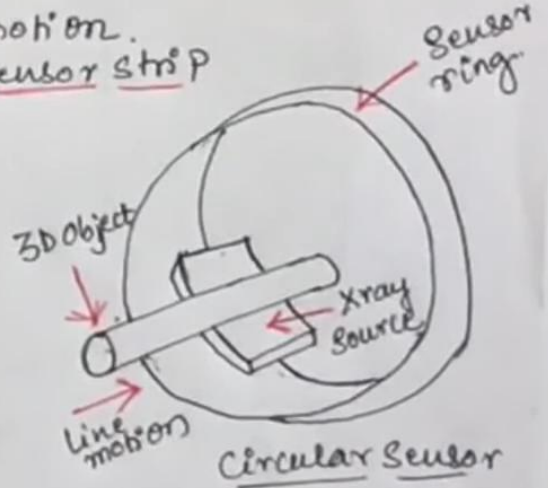


Image acquisition using Sensor strip



Line Sensor (or) Linear Sensor



Circular Sensor

Image acquisition using Array sensor

Image Sampling & Quantization:

→ O/P sensor → Continous Vtg

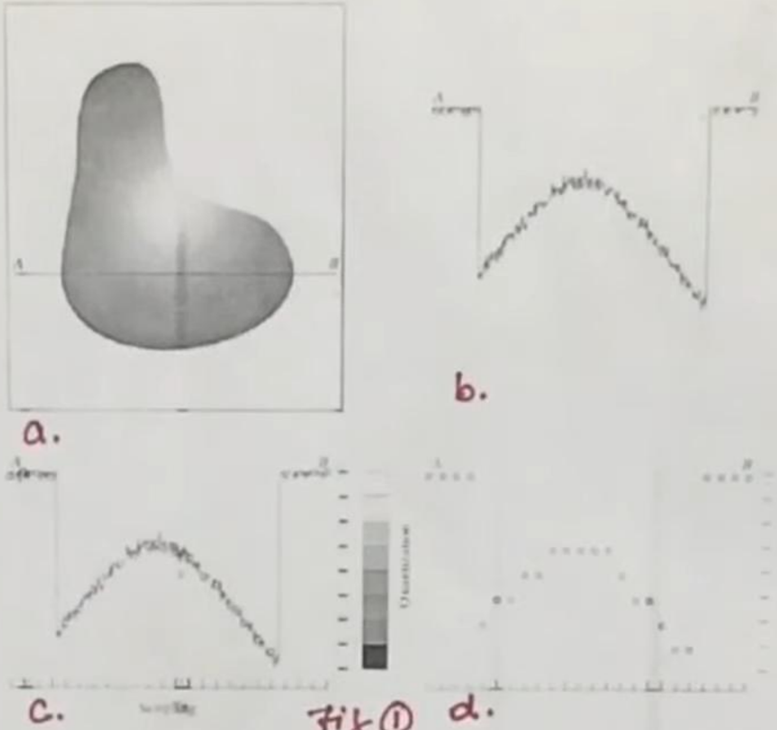
→ Convert → continous sensed data
↓
digital form.

→ Image → Continous → x & y
└→ Amplitude.

→ Digital form → Sample → x & y
└→ Amplitude.

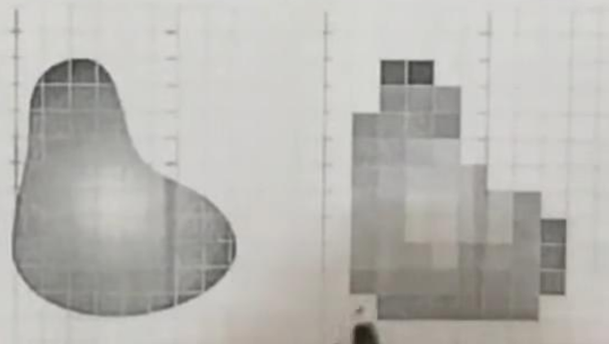
→ Digitizing Co-ordinate Values
⇓
"SAMPLING"

→ Digitizing Amplitude values.
⇓
"QUANTIZATION"



a. b. c. d.

Fig ①



a. b.

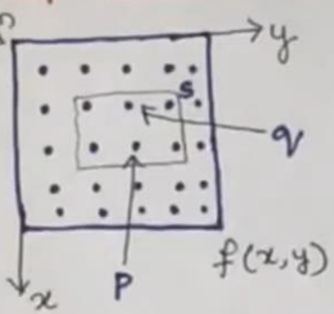
Fig ②

Basic Relationship between Pixels.

Image → $f(x,y)$

S → Subset of Pixels

q, P → Particular pixels.



4- Neighbors [$N_4(P)$]

2 Horizontal
$(x,y-1),(x,y+1)$

2 vertical
$(x-1,y),(x+1,y)$

	$(x-1,y)$	
$(x,y-1)$	P(x,y)	$(x,y+1)$
	$(x+1,y)$	

$\therefore N_4(P) = (x,y-1),(x,y+1),(x-1,y),(x+1,y)$

Diagonal Neighbours [$N_D(P)$]

$(x-1,y-1)$		$(x-1,y+1)$
	P(x,y)	
$(x+1,y-1)$		$(x+1,y+1)$

$N_D(P) = (x+1,y-1),(x-1,y-1),(x-1,y+1),(x+1,y+1)$

8 neighbour [$N_8(P)$]

$N_8(P) = N_4(P) + N_D(P)$

$(x-1,y-1)$	$(x-1,y)$	$(x-1,y+1)$
$(x,y-1)$	P(x,y)	$(x,y+1)$
$(x+1,y-1)$	$(x+1,y)$	$(x+1,y+1)$

Connectivity | Adjacent

1. 4-adjacency
2. 8-adjacency
3. m-adjacency [mixed-adjacency]

Binary Image

$V=\{1\}$

0	1	0	1
0	0	1	0
0	0	1	0
1	0	0	0

8-adj

4-adj

Gray scale Image

8-bit [0-255] $V=\{1,2,3\cdots10\}$

54	10	100	5
81	150	2	34
201	200	3	45
7	70	147	56

8-adj

4-adj

not connected

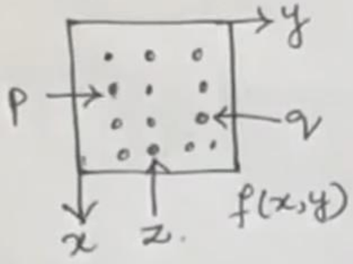
0	1	1
0	1	0
0	0	1

0	1	1
0	1	0
0	0	1

DISTANCE MEASURE:

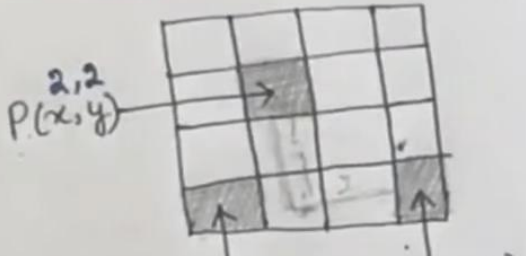
Image $\rightarrow f(x, y)$

$P, q, z \rightarrow$ particular pixels.



Distance function D

Properties of D



(i) $D(P, q) \geq 0$

(ii) $D(P, q) = 0$ if $P = q$

$z(u,v)$ $q(s,t)$ 4,4

(iii) $D(P, q) = D(q, P)$

(iv) $D(P, z) \leq D(P, q) + D(q, z)$

Distance measure

(i) Euclidean : $D_E(P, q) = \left[(x-s)^2 + (y-t)^2\right]^{1/2}$

(ii) City Block : $D_4(P, q) = |x-s| + |y-t|$

(iii) Chess board : $D_8(P, q) = \max\{|x-s|, |y-t|\}$

Ex:-

ⓐ $D_E(P, q) = \left[(2-4)^2 + (2-4)^2\right]^{1/2}$
$= [8]^{1/2}$

ⓑ $D_4(P, q) = |2-4| + |2-4| = 2+2$
$= \underline{4}$

4		2		4
	2	1	2	
2	1	0	1	2
	2	1	2	
4		2		4

ⓒ $D_8(P, q) = \max\{|2-4|, |2-4|\}$
$= \max\{2, 2\}$
$= \underline{2}$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

$\max\{2, 1\}$
$= 2$

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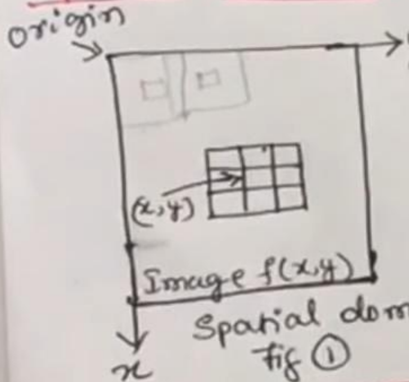
Introduction to Image Enhancement using Spatial domain.

Image Enhancement - Process → Improves the Quality of an Image.
→ To highlight the Important details
→ To remove noise → Image → more appealing.

Methods:

1. Spatial domain :- Manipulation of pixel values.
2. Frequency domain :- Modifying the F.T. of Image.
3. Combination method :- Combination of first two method.

Spatial domain: → Image plane itself



Spatial domain
fig ①

→ Direct manipulation of pixels.

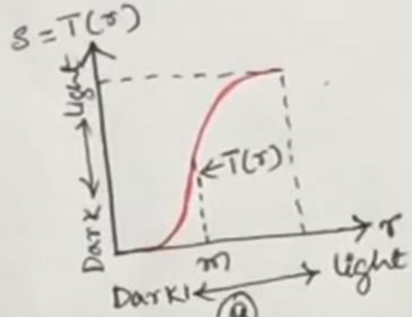
ⓐ Intensity Transformation.
ⓑ Spatial Filtering

$\boxed{g(x,y) = T\{f(x,y)\}}$ T → operator

Simplest form → neighbourhood - size 1×1
T → grey level transformation [intensity (or) mapping].

$\boxed{s = T(r)}$

s → o/p Image pixel value
r → i/p Image pixel value



Contrast Streaching

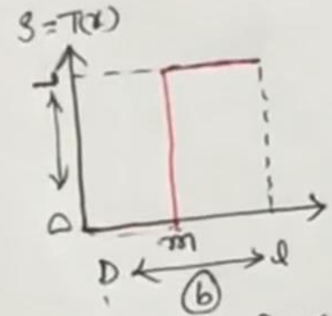


Fig ②

Point processing

mask → Small [3×3] 2D array
↳ [Filters, Kernels, Tamplets (or) window]
→ Mask processing (or) Filtering.

Basic Intensity transformation function:

Gray level transformation.

$\boxed{s = T(r)}$

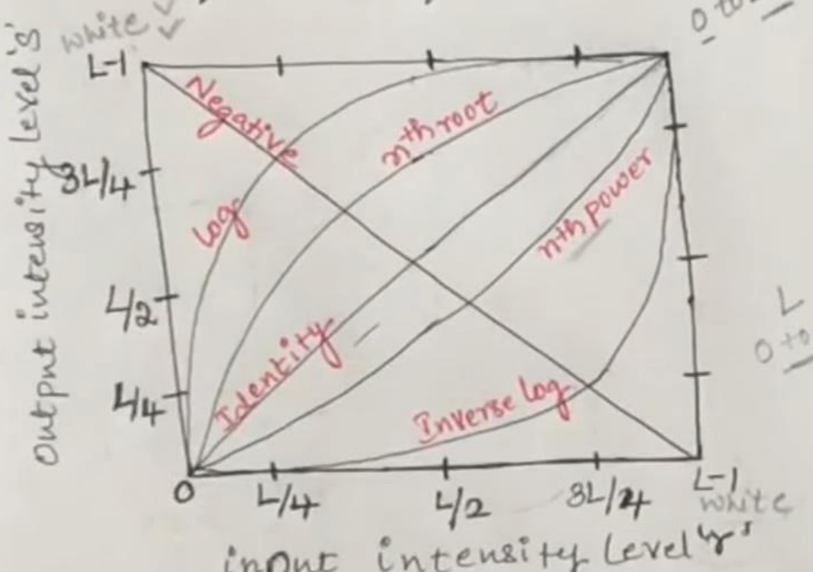
$r \rightarrow$ Value of pixel before processing

$s \rightarrow$ Value of pixel after processing.

$T \rightarrow$ Transformation function.

$2^8 = 256$

0 to 255



L

0 to L-1

Some Basic transformation fun

(a) Linear [Negative & Identity]

(b) Logarithmic [Log and Inverse log.]

(c) Power law [n^{th} power & n^{th} root]

(a) Image Negative

Intensity level $\rightarrow [0, L-1]$

$\boxed{s = L - 1 - r}$

Ex:- $r = 0$ ✓

$\boxed{s = L-1}$

$r = L-1$ ✓

$\boxed{s = 0}$ ✓

(b) Log Transformation.

$\boxed{s = c \log(1+r)}$

$c \rightarrow$ Constant

$r \geq 0$

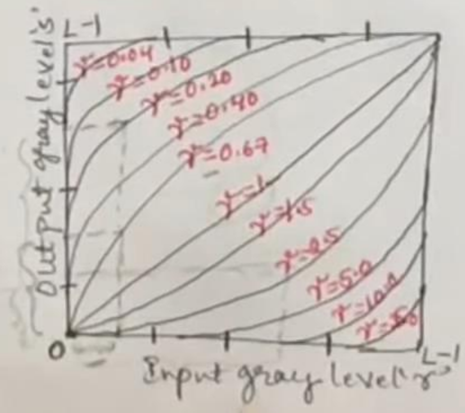
$\log 0 = 0$

(c) Power law [gamma correction]

$\boxed{s = c r^{\gamma}}$

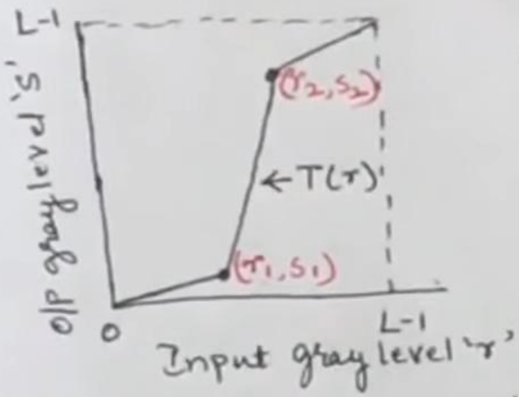
$\gamma > 1 \rightarrow n^{th}$ power

$\gamma < 1 \rightarrow n^{th}$ root

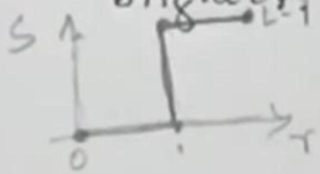


Piecewise - Linear Transformation fun:

① Contrast Stretching:



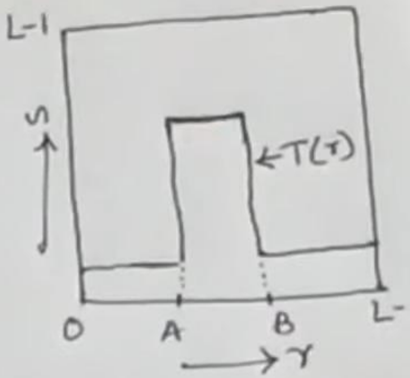
make
→ Dark portion, darker
→ bright portion, brighter



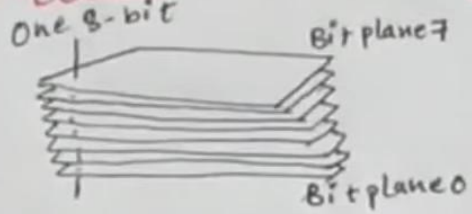
1. $r_1 = s_1$ & $r_2 = s_2 \rightarrow$ Linear Transformation
2. $r_1 = r_2$ & $s_1 = 0$, $s_2 = L-1 \rightarrow$ Thresholding
3. Intermediate values (r_1, s_1) & $(r_2, s_2) \rightarrow$ Various degrees of spread in grey levels
4. Generally, $r_1 \le r_2$ & $s_1 \le s_2 \rightarrow$ Single valued & monotonically increasing

② Gray level slicing:

→ Highlight specific range of graylevels



③ Bit Plane slicing:



→ Highlights → contribution → specific bits
→ 8 bit Image → 8 1-bits
→ Top 4 bits → Majority of visually significant data.
→ Useful → Analyzing the relative importance → each bit.
→ Image compression.

Example:

$$f(x,y) = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 3 & 4 & 5 & 4 & 3 \\ 3 & 5 & 5 & 5 & 3 \\ 3 & 4 & 5 & 4 & 3 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix} \xrightarrow{H.E.} \begin{bmatrix} 6 & 6 & 6 & 6 & 6 \\ 2 & 6 & 7 & 6 & 2 \\ 2 & 7 & 7 & 7 & 2 \\ 2 & 6 & 7 & 6 & 2 \\ 6 & 6 & 6 & 6 & 6 \end{bmatrix}$$

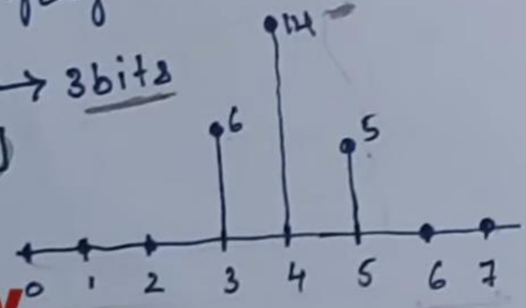
i/p Image   o/p HE

Graylevels	0	1	2	3	4	5	6	7
no of pixels n_K	0	0	0	6	14	5	0	0

highest gray value = 5

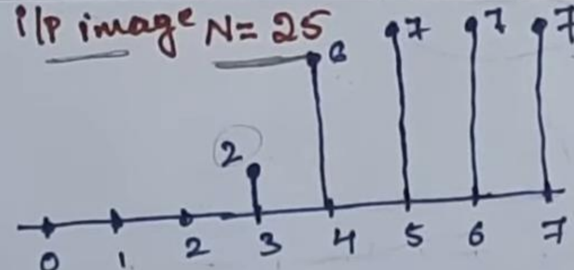
$2^3 = 8 \rightarrow$ 3bits

[0 to 7]



Gray level	no. of pixels n_K	PDF = n_K/sum	CDF = S_K	$S_K \times 7$	Histogram equal. level
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	6	6/25 = 0.24	0.24	1.68	2
4	14	$\frac{14}{25}$ = 0.56	0.8	5.6	6
5	5	5/25 = 0.2	1.0	7	7
6	0	0	1.0	7	7
7	0	0	1.0	7	7.

i/p image N = 25   o/p



HISTOGRAM EQUALIZATION → Image Enhancement.

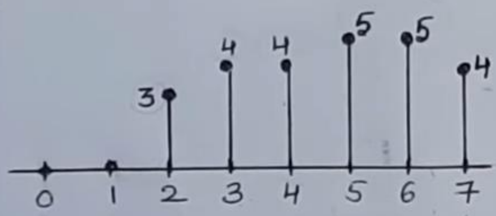
→ Graphical Representation → Data.

→ Image Processing → Data related to the Digital Image.

→ Representation → Frequency of occurance of Various gray levels.

6	6	7	7	6
5	2	2	3	4
3	3	4	4	5
5	7	3	6	2
7	6	5	5	4

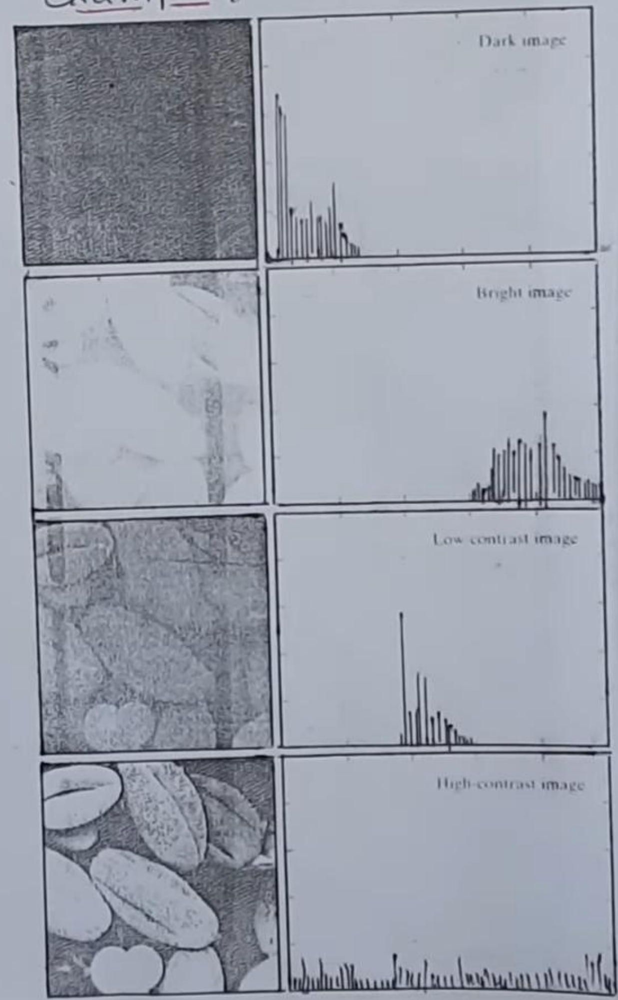
0 → 0
1 → 0
2 → 3
3 → 4
4 → 4
5 → 5
6 → 5
7 → 4



→ Used → Manipulating Contrast & Brightness.

→ Quality → Normalizing → Histogram → Flat profile

Example:



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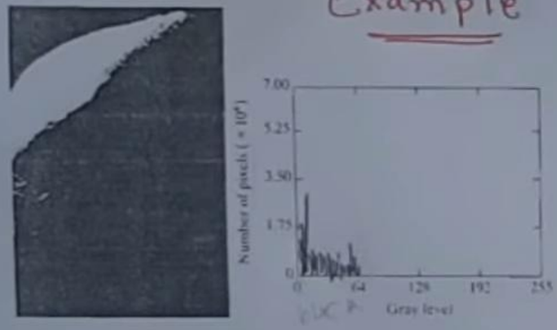
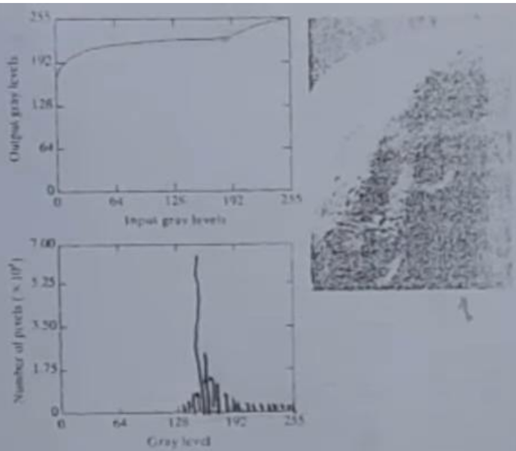
Example



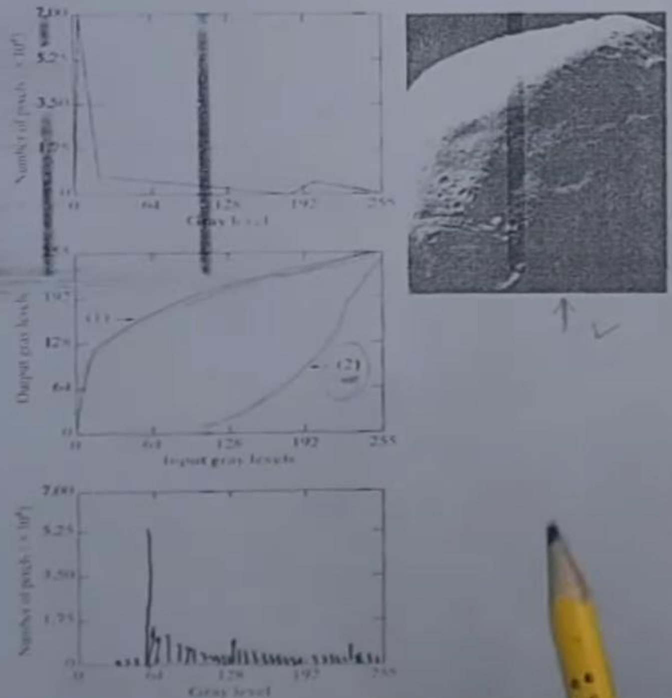
a b

FIGURE 3.20 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)



a b
c

FIGURE 3.21 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).



a c
b
d

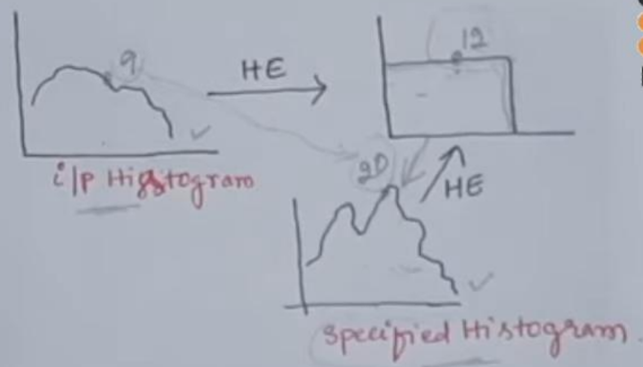
FIGURE 3.22 (a) Specified histogram. (b) Curve (1) is from Eq. (3.3.14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3.17). (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).

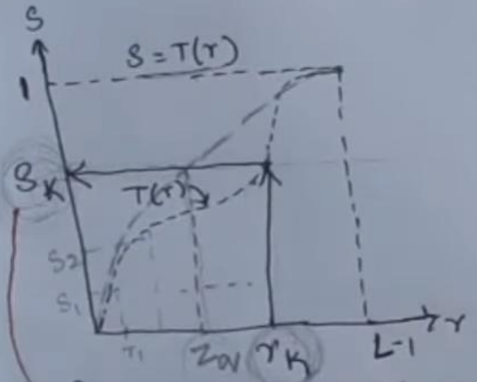
Histogram Matching:

→ Histogram Equalization → not applicable
→ Some applications.

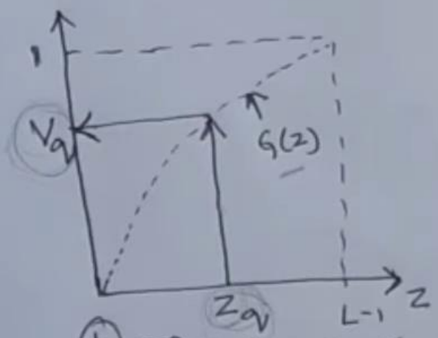
→ Histogram Mating (or) Histogram Specification

→ Method → Generate processed image → Specified histogram. H1 = H2

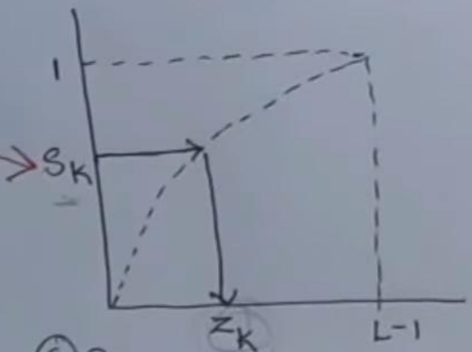




ⓐ mapping from r_K to s_K

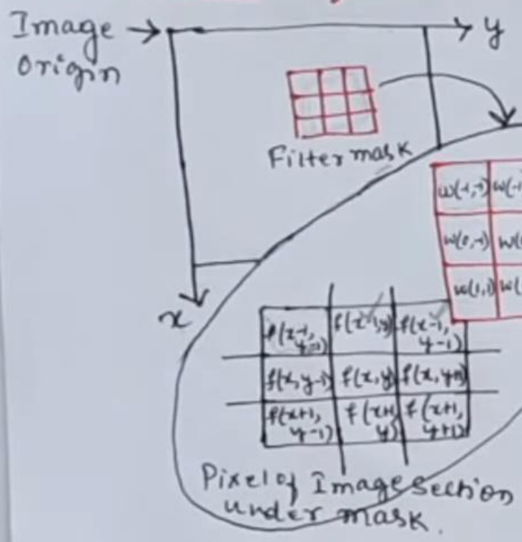


ⓑ mapping of z_q to V_q



ⓒ Inverse mapping of s_K to z_K

Fundamentals of Spatial Filtering:



→ Response of linear Filter

$$R = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \cdots\cdots w(0,0)f(x,y) + \cdots\cdots + w(1,0)f(x+1,y) + w(1,1)f(x+1,y+1) \longrightarrow (1)$$

→ mask $m \times n \Rightarrow m = 2a+1$ & $n = 2b+1$ $\quad a=1,\ b=1$

3×3

Linear Filtering of Image of size $M\times N$ and mask size $m\times n$

$$g(x,y) = \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t) \rightarrow (2)$$

where $a = \frac{m-1}{2}$ & $b = \frac{n-1}{2}$

eqn (2) ⇒ Convolution mask (or) Convolution Kernel.

by simplifying

$$R = w_1 z_1 + w_2 z_2 + \cdots\cdots + w_{mn} z_{mn}$$

$w \rightarrow$ mask Co-efficients

$z \rightarrow$ the values of the image gray level.

$mn \rightarrow$ total no of Co-efficients.

$$R = \sum_{i=1}^{mn} w_i z_i \longrightarrow (3)$$

For 3×3 general mask

$$R = w_1 z_1 + w_2 z_2 + \cdots\cdots + w_9 z_9$$

$$R = \sum_{i=1}^{9} w_i z_i \longrightarrow (4)$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Sub image.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

mask Co-efficients

Smoothing Spatial Filters:

→ Used for Blurring & Noise reduction

→ Blurring: Removal of small details from an Image. Prior to object extraction.

→ Noise reduction: blurring with a linear (or) Non linear filters.

SMOOTHING LINEAR FILTERS:

→ O/P → average of pixels contained in the neighborhood → Filter mask.

→ Averaging Filters (or) low pass Filters

Ex:-

(a)
$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(b)
$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

3x3 Smoothing Filter mask

→ Replacing → each pixel → by avg of gray levels.

→ Application → Noise reduction.

→ side effect → blur edges.

→ Fig (a) → Standard average of Pixel Values

→ m×n mask → $(1/m\times n)$

→ Box Filters

→ Fig (b) → weighted avg

→ pixel at the center of mask → more importance

→ This is to reduce blurring during Smoothing process

→ general Implementation for Image → M×N & mask → m×n

$$\boxed{g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}}$$

$x = 0, 1, 2, \ldots M-1$ & $y = 0, 1, 2 \ldots N-1$

ORDER-STATISTICS Filters:

→ non linear Spatial Filters
→ response → ordering [Ranking] the pixels → Image
→ Replacing the Center pixel value with value determined by ranking result.

1. Median Filter:

→ Replaces the value of a pixel by the median of the gray level.
→ Most popular → excellent noise reduction
→ less blurring
→ Effective for Impulse noise
↓
Salt and pepper noise

Ex:
⇒

10	20	20
20	15	20
20	25	100

⇒

10	20	20
20	**20**	20
20	25	100

10, 15, 20, 20, 20, 20, 20, 25, 100
↑

2. Max filter:

→ Finding the brightest point.

$$R = \max \{ Z_k \mid \underbrace{k = 1, 2, 3 \ldots 9}_{3\times 3} \}$$

3. Min filter:

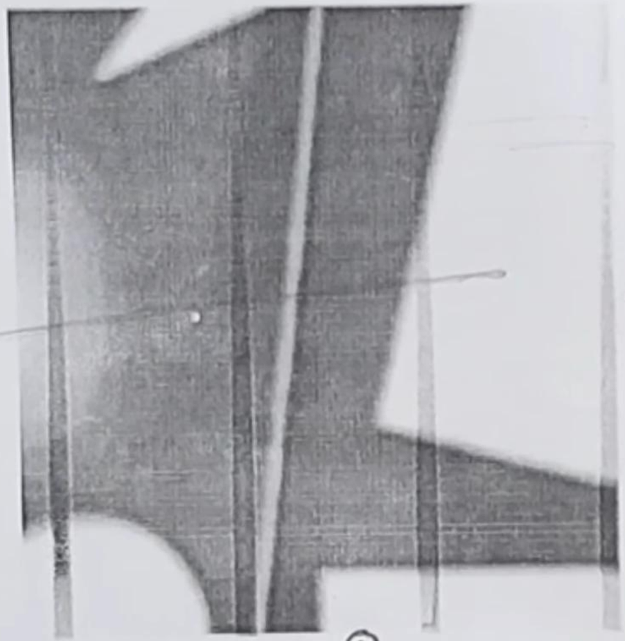
→ Finding the darkest point.

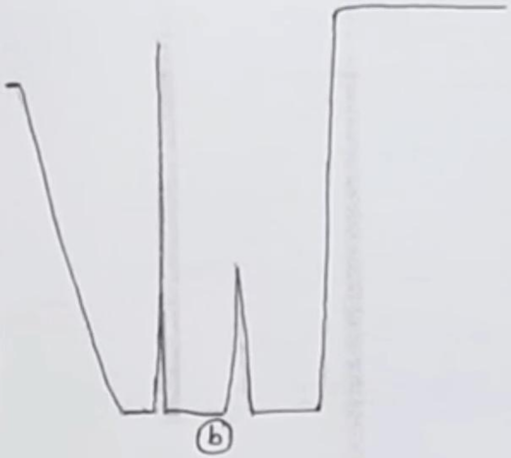
$$R = \min \{ Z_k \mid k = 1, 2, 3 \ldots 9 \}$$

Ex:
max. value : 100 – brightest point.
min. value : 10 – darkest point.

a b
c

FIGURE 3.38
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).





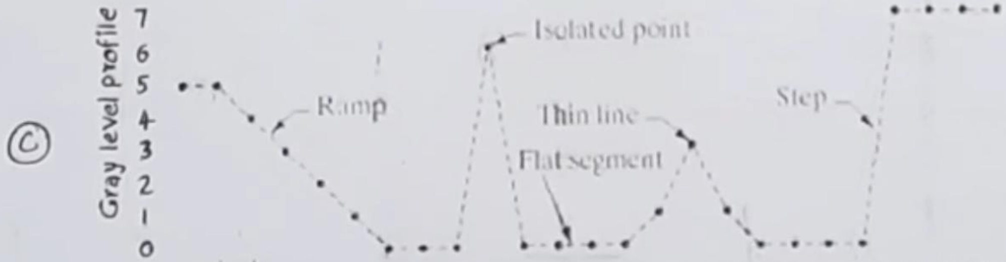


Image strip: 5 5 4 3 2 1 0 0 0 6 0 0 0 0 1 3 1 0 0 0 0 7 7 7 7 . .

First Derivative: -1 -1 -1 -1 -1 0 0 6 -6 0 0 0 1 2 -2 -1 0 0 0 7 0 0 0

Second Derivative: -1 0 0 0 0 1 0 6 -12 6 0 0 1 1 -4 1 1 0 0 7 -7 0 0

$f(x+1) - f(x)$

$f(x+1) + f(x-1) - 2f(x)$

Conclusions

→ 1st → thicker edges
→ 2nd → Fine details.
→ 1st → gray level step.
→ 2nd → double response at step changes

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Sharpening Spatial Filters:

→ highlight the finate detail (or) to enhance details

→ Applications → Electronic printing, Medical imaging, Industrial Inspections and Autonomous Guidance in Military s/ms.

→ Image blurring → pixel averaging.
↳ Integration

→ Sharpening → "Spatial differentiation".

→ Image differentiation → enhances edges and noise & deemphasizes areas with slowly varying gray-level values.

Foundation:

→ First order and second order derivatives.

→ Derivatives → defined interms of differences

→ definition for First derivative.

→ (a) must be zero in flat segments.

(b) must be nonzero at onset of a gray level step (or) ramp

(c) must be non zero along ramp.

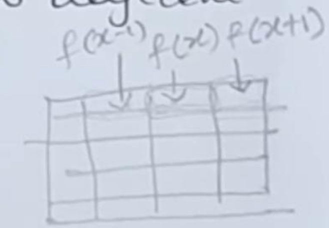
→ Similarly, definition for Second derivative

→ (a) must be zero in flat area.

(b) must be non zero at the onset & end of gray-level step (or) ramp.

(c) must be zero along ramps of constant slope.

→ The shortest distance over which change can occur is b/w adjecent pixels.



The basic definition.

1st order derivative

$$\frac{\delta f}{\delta x} = f(x+1) - f(x)$$

2nd order derivative

$$\frac{\delta^2 f}{\delta x^2} = f(x+1) + f(x-1) - 2f(x)$$

Use of Second order derivatives for Enhancement – The Laplacian

→ Laplacian Filter → x & y

Definition

$$\Delta^2 f = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}$$

$$\frac{\delta^2 f}{\delta x^2} = f(x+1) + f(x-1) - 2f(x)$$

In x-direction

$$\frac{\delta^2 f}{\delta x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

In y-direction

$$\frac{\delta^2 f}{\delta y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\therefore \Delta^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

Filter:

f(x-1,y-1)	f(x,y-1)	f(x+1,y-1)
f(x-1,y)	f(x,y)	f(x+1,y)
f(x-1,y+1)	f(x,y+1)	f(x+1,y+1)

0	1	0
1	-4	1
0	1	0

Different filters:

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

Image Enhancement in frequency domain.

→ From spatial domain to Frequency domain then processed

→ Inverse transform is applied to bring back into spatial domain.

→ Filters → Smoothening & Sharpening
→ removing high & low frq

→ Change → whole image.

Types of Filters →

(a) Low Pass Filter
↳ ~~Sharpening~~
Smoothens

(b) High Pass Filter.
↳ ~~Smoothens the image.~~
Sharpening.

Ideal.
Butterworth
Gaussian.

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Fourier Transform:

→ Relation b/w Spatial & frq domain.

→ Image Enhancement in frq domain.

1D Discrete Fourier Transform

$$x(n) \xleftrightarrow{DFT} X(K)$$

Spatial domain — frq domain.

$N \rightarrow$ no of samples.

$$\therefore X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn} \; ; \; 0 \le K \le N-1$$

$$X(K) \xleftrightarrow{IDFT} x(n)$$

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}Kn} \; ; \; 0 \le n \le N-1$$

Fourier Spectrum

$$|F(u)| = \left[R^2(u) + I^2(u)\right]^{1/2}$$

Phase Angle

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right]$$

Power Spectrum

$$P(u) = |F(u)|^2 = R^2(u) + I^2(u)$$

2D Discrete Fourier Transform [2D DFT]

$$f(x,y) \xleftrightarrow{2D\ DFT} F(u,v)$$

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$F(u,v) \xleftrightarrow{2D\ IDFT} f(x,y)$$

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$u \,\&\, v \rightarrow$ tansform (or) frq variables

$x \,\&\, y \rightarrow$ Spatial (or) Image Variables

Fourier Spectrum

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

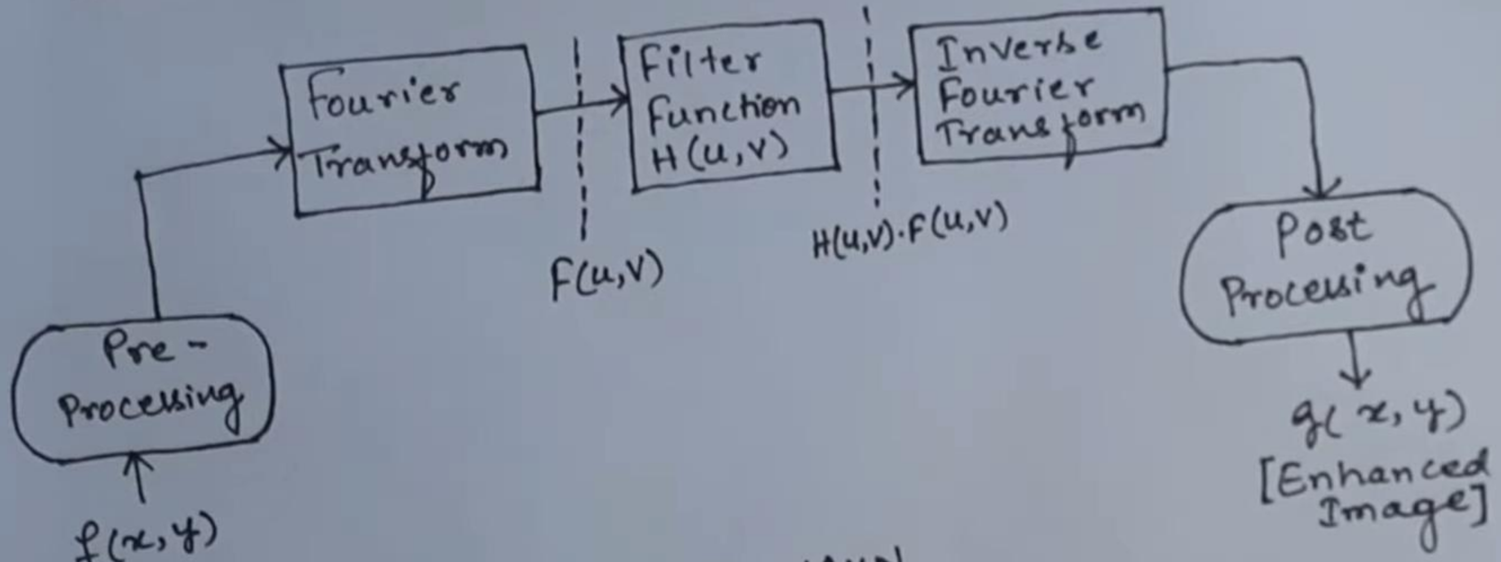
Phase Angle

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

Power Spectrum

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

Steps for Filtering in frequency domain:



1. Image $f(x,y)$ of size $M \times N$
2. $f(x,y)\,(-1)^{x+y}$
3. $\underbrace{F(u,v)}_{\rightarrow \text{DFT}} \longrightarrow$ F.T. of $f(x,y)$
4. $H(u,v) \longrightarrow$ Filter in freq domain.

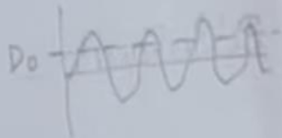
   $\underbrace{G(u,v)}_{\text{IFT}} = H(u,v)\cdot F(u,v)$.
5. $g(x,y) = \mathcal{F}^{-1}[G(u,v)]\,(-1)^{x+y}$

Image Smoothening & Sharpening using Frequency domain Filters:

(a) Low pass Filters

→ Smoothen

(i) Ideal Low pass Filter



$$H(u,v) = \begin{cases} 1 \; ; D(u,v) \le D_0 \\ 0 \; ; D(u,v) > D_0 \end{cases}$$

D_0 → non negative quantity.

$D(u,v)$ → distance from point (u,v)

$f(x,y) \rightarrow M \times N \quad D(u,v) = \left[u - \frac{M}{2}\right]^2 + \left[v - \frac{N}{2}\right]^2$

(ii) Butter worth LPF

$$H(u,v) = \frac{1}{1 + \left[D(u,v)/D_0\right]^{2n}}.$$

Advantage: useful in defining the edges.

(iii) Gaussian LPF

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

removes low frq noise

(b) High pass Filters

→ Sharpen

(i) Ideal High Pass Filter

$$H(u,v) = \begin{cases} 0 \; ; D(u,v) \le D_0 \\ 1 \; ; D(u,v) > D_0 \end{cases}$$

Disadvantage blurred edges.

(ii) Butterworth HPF

$$H(u,v) = \frac{1}{1 + \left[D_0/D(u,v)\right]^{2n}}$$

(iii) Gaussian HPF

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

removes high frq noise

Image Restoration:

→ Reconstruct (or) Recover an Image
→ degraded → using Prior Knowledge

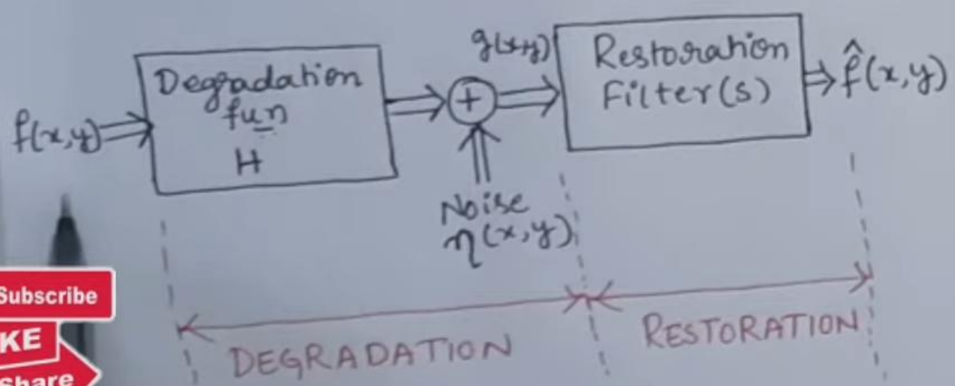
→ Restoration → modeling the degradation
→ Applying Inverse process → recover

→ Restoration techniques
(i) Spatial domain
· Additive Noise
(ii) Frequency domain.
Image blur.

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y) \rightarrow (1)$$

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v) \rightarrow (2)$$

A Model of Image Degradation | Restoration Process:



Periodic Noise

→ Electrical (or) electromechanical interference → Image Acquisition.

→ Spatially dependent noise.

→ Reduced ⇛ Frequency domain Filtering.

Estimation

→ Inspection of Fourier spectrum.

→ Automated Analysis → Knowledge → general location of freq components

→ If Image s/ms are available → Study chr's of s/m noise → Capture a set of images → "Flat"

→ Use the data from Image strips → Calculating → mean & Variance of gray level.

→ Strip (subimage) → S

$$\mu = \sum_{z_i \in S} z_i \, p(z_i) \rightarrow (i)$$

$$\& \quad \sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 p(z_i) \rightarrow (ii)$$

z_i → gray level values of pixels 'S'

$p(z_i)$ → normalized histogram values.

→ Histogram shape → closest PDF match.

→ If the shape is gaussian → mean & Variance.

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Restoration in Presence of Noise only - Spatial Filtering:

Degradation → only due to noise.

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$\& \quad G(u,v) = F(u,v) + N(u,v)$$

I. MEAN FILTERS:

(i) Arithmetic mean Filter:

→ Simplest mean Filter.

$$\boxed{\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)} \longrightarrow (i)$$

S_{xy} → set of co-ordinates in a rectangular subimage window of size $m \times n$. Centered at point (x,y)

→ Avg of carrupted image $g(x,y)$ in the area defined by S_{xy}.

→ Smoothens local variation

→ Noise is reduced due to blurring.

(ii) Geometric mean Filter:

$$\boxed{\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}} \longrightarrow (ii)$$

→ Product of pixels in the subimage window → raised to power $\frac{1}{mn}$

→ Smoothing simillar to arithmetic mean Filter.

→ lose less image details.

(iii) Harmonic mean Filter:

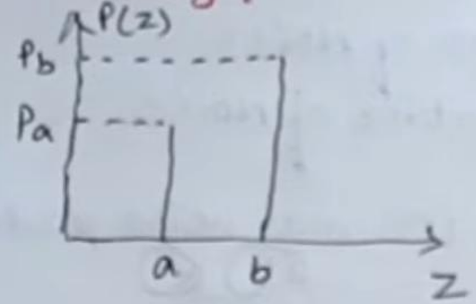
$$\boxed{\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}} \longrightarrow (iii)$$

→ Works well for Salt noise & Fails for Pepper noise

→ also works well for Gaussian noise.

(vi) Salt & Pepper noise [Impulse noise]:

$$P(z) = \begin{cases} P_a & ; \ z = a \\ P_b & ; \ z = b \\ 0 & ; \ \text{otherwise} \end{cases}$$



→ $b > a \Rightarrow$ gray level b → light dot
gray level a → dark dot

→ $P_a = 0$ or $P_b = 0$ → unipolar

→ $P_a \cong P_b$ → salt & pepper granules.

shot & spike noise.

−ve impulses → black (pepper) point
+ve impulses → white (salt) point.

8-bit ⇒ $a = 0$ (black)
$b = 255$ (white)

Important Noise Probability Density fun: [PDF]

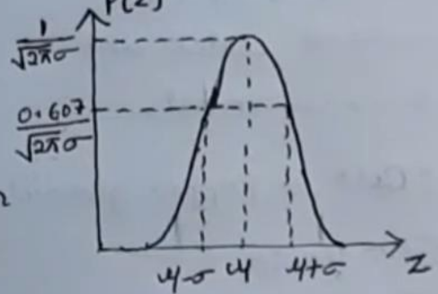
Noise → Image acquisition and/or transmission.

→ Spatial properties of Noise.

→ Frequency properties of Noise.

(i) Gaussian Noise: [Normal noise model]

PDF, $P(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(z-\mu)^2/2\sigma^2}$



z → graylevel

μ → mean of avg of z

σ → Standard deviation

σ^2 → Variance

70% of Values → $[(\mu-\sigma), (\mu+\sigma)]$

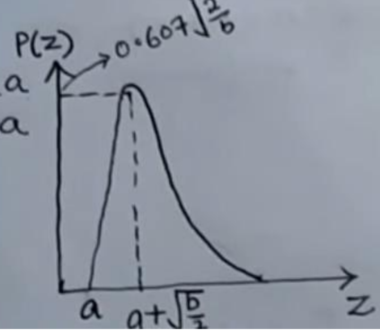
95% of Values → $[(\mu-2\sigma), (\mu+2\sigma)]$

(ii) Rayleigh noise:

$$P(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-(z-a)^2/b} & ; z \geq a \\ 0 & ; z < a \end{cases}$$

mean: $\mu = a + \sqrt{\pi b/4}$

Variance: $\sigma^2 = \frac{b(4+\pi)}{4}$

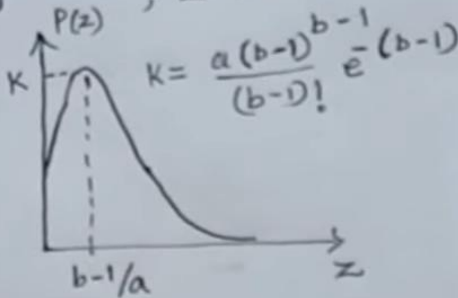


(iii) Erlang [Gamma] noise:

$$P(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & ; z \geq 0 \\ 0 & ; z < 0 \end{cases}$$

mean: $\mu = \frac{b}{a}$

Variance: $\sigma^2 = \frac{b}{a^2}$

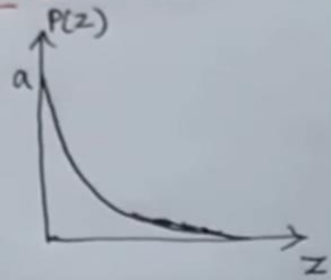


(iv) Exponential noise:

$$P(z) = \begin{cases} a \cdot e^{-az} & ; z \geq 0 \\ 0 & ; z < 0 \end{cases}$$

$\mu = \frac{1}{a}$

$\sigma^2 = \frac{1}{a^2}$



(v) Uniform noise:

$$P(z) = \begin{cases} \frac{1}{b-a} & ; a \leq z \leq b \\ 0 & ; \text{otherwise} \end{cases}$$

$\mu = \frac{a+b}{2}$

$\sigma^2 = \frac{(b+a)^2}{12}$