Series

Question 1 2013 BC Q90

Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \frac{(-1)^n}{\sqrt{n} + (-1)^n}$?

I. The series is alternating.

II. $|a_{n+1}| \le |a_n|$ for all $n \ge 2$

III. $\lim_{n \to \infty} a_n = 0$

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

Question 2 2014 BC Q6

The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \ge 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) The series diverges.

Question 3 2014 BC Q10

What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

(A) $\frac{-2}{e^2 - 2e}$

(B) $\frac{-2}{e^2 + 2e}$

(C) $\frac{-2}{e + 2}$

(D) $\frac{e}{e + 2}$

(E) The series diverges.

Question 4 *2014 BC Q24*

Which of the following series converge?

I. $1 + (-1) + 1 + \cdots + (-1)^{n-1} + \cdots$

II. $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$

III. $1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-1}} + \cdots$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

Question 5 *2014 Q87*

If $0 < b_n < a_n$ for $n \geq 1$, which of the following must be true?

(A) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} b_n = 0$.

(C) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.

(D) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

Question 6 *2015 BC Q10*

Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{3n}{n+2}$

(B) $\sum_{n=1}^{\infty} \frac{3n}{n^2+2}$

(C) $\sum_{n=1}^{\infty} \frac{3n}{n^2+2n}$

(D) $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+2n}$

(E) $\sum_{n=1}^{\infty} \frac{3n^2}{n^4+2n}$

Question 7 *2015 BC Q79*

The function f has derivatives of all orders for all real numbers with $f(0) = 3,\ f'(0) = -4,\ f''(0) = 2,$ and $f'''(0) = 1.$ Let g be the function given by $g(x) = \int_0^x f(t)\,dt.$ What is the third-degree Taylor polynomial for g about $x = 0$?

(A) $-4x + 2x^2 + \frac{1}{3}x^3$

(B) $-4x + x^2 + \frac{1}{6}x^3$

(C) $3x - 2x^2 + \frac{1}{3}x^3$

(D) $3x - 2x^2 + \frac{2}{3}x^3$

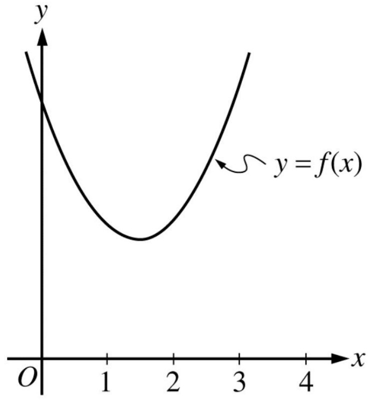
(E) $3 - 4x + x^2 + \frac{1}{6}x^3$

Question 8 *2015 BC Q83*

The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = \frac{n}{3n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) The series diverges.

Question 9 *2016 BC Q14*



The figure above shows the graph of a function f. Which of the following could be the second-degree Taylor polynomial for f about $x = 2$?

(A) $2 - x - x^2$

(B) $2 + x - x^2$

(C) $2 - (x - 2) + (x - 2)^2$

(D) $2 + (x - 2) - (x - 2)^2$

(E) $2 + (x - 2) + (x - 2)^2$

Question 10 *2016 BC Q18*

Let P be the second-degree Taylor polynomial for e^{-2x} about $x = 3$. What is the slope of the line tangent to the graph of P at $x = 3$?

(A) $-2e^{-6}$ (B) e^{-6} (C) $2e^{-6}$ (D) $4e^{-6}$ (E) $10e^{-6}$

Question 11 *2016 BC Q24*

Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

II. $\sum_{n=1}^{\infty} \frac{1}{3^n}$

III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

Question 12 2016 BC Q92

Let f be a positive, continuous, decreasing function. If $\int_1^{\infty} f(x)\,dx = 5,$ which of the following statements about the series $\sum_{n=1}^{\infty} f(n)$ must be true?

(A) $\sum_{n=1}^{\infty} f(n) = 0$

(B) $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) < 5.$

(C) $\sum_{n=1}^{\infty} f(n) = 5$

(D) $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) > 5.$

(E) $\sum_{n=1}^{\infty} f(n)$ diverges.

Question 13 2017 BC Q5

To what number does the series $\sum_{k=0}^{\infty} \left(\frac{-e}{\pi}\right)^k$ converge?

(A) 0 (B) $\frac{-e}{\pi + e}$ (C) $\frac{\pi}{\pi + e}$ (D) The series does not converge.

Question 14 2016 BC Q21

The power series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{2^n n^2}$ has radius of convergence 2. At which of the following values of x can the alternating series test be used with this series to verify convergence at x ?

(A) 6 (B) 4 (C) 2 (D) 0 (E) -1

Question 15 2017 BC Q29

Which of the following is a power series expansion of $\dfrac{e^x + e^{-x}}{2}$?

(A) $1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!} + \cdots + \dfrac{x^{2n}}{(2n)!} + \cdots$

(B) $1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots + (-1)^n \dfrac{x^{2n}}{(2n)!} + \cdots$

(C) $x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \dfrac{x^7}{7!} + \cdots + \dfrac{x^{2n+1}}{(2n+1)!} + \cdots$

(D) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots + (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} + \cdots$

Question 16 2014 BC FR Q6

The Taylor series for a function f about $x = 1$ is given by $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}(x-1)^n$ and converges to $f(x)$ for $|x-1| < R$, where R is the radius of convergence of the Taylor series.

(a) Find the value of R.

(b) Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about $x = 1$.

(c) The Taylor series for f' about $x = 1$, found in part (b), is a geometric series. Find the function f' to which the series converges for $|x-1| < R$. Use this function to determine f for $|x-1| < R$.

Question 17 2013 BC FR Q6

A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about $x = 0$.

(a) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.

(b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.

(c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.

Question 18 2012 BC FR Q6

The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.