OR-assignment-3

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ICT-B

Q18

Player B

Player A	B1	B2	B3	B4	Min. row
A1	62	44	55	40	40
A2	60	45	48	51	$\boxed{45}$
A3	40	42	30	40	30
Max. col.	62	$\boxed{45}$	55	51	

Minimax

Maximin

∴ Saddle point = (A2, B2)

Q17

Player Y

Player X	A	B	C	D	Min. row
I	0.25	0.20	0.14	0.30	$\boxed{0.14}$
II	0.27	0.16	0.12	0.14	0.12
III	0.35	0.08	0.15	0.19	0.08
IV	-0.02	0.08	0.13	0.0	-0.02
Max col.	$\boxed{0.35}$	0.2	0.15	0.3	

Minimax

Maximin

∴ No saddle points

Now reducing pay off matrix by dominance

	Player Y	
	B	C
I	0.20	0.14
III	0.08	0.15
	0.01	0.12

Oddments

0.07

0.06

Probability : $\frac{0.07}{0.13} = \frac{7}{13}$ $\frac{0.06}{0.13} = \frac{6}{13}$

$\frac{0.01}{13} = \frac{1}{13}$ $\frac{0.12}{0.13} = \frac{12}{13}$

Value : $\frac{(0.2)(0.07) + (0.08)(0.06)}{(0.07)+(0.06)}$

$\boxed{= 0.145}$

Stratagey for players:

Player X : $\left[\frac{7}{13}, 0, \frac{6}{13}, 0\right]$

Player Y : $\left[0, \frac{1}{13}, \frac{12}{13}, 0\right]$

Q15

		Player B				
		I	II	III	IV	Min row
	I	3	2	4	0	0
	II	3	4	2	4	$\boxed{2}$
Player A	III	4	2	4	0	0
	IV	0	4	0	8	0
	Max Col.	$\boxed{4}$	4	4	8	

$\therefore$ No saddle points.

Now reducing pay off matrix by dominance

		Player B		odd mens
		III	IV	
	III	4	0	8
Player A	IV	0	8	4
		8	4	

$\therefore$ Probability $= \frac{8}{12} = \frac{2}{3}, \quad \frac{4}{12} = \frac{1}{3}$

$\frac{8}{12} = \frac{2}{3}, \quad \frac{4}{12} = \frac{1}{3}$

$\therefore$ Value $= \frac{32+0}{12} = \boxed{\frac{8}{3}}$

Optimal stratagey:

Player A : $[0, 0, 2/3, 1/3]$

Player B : $[0, 0, 2/3, 1/3]$

~~Q13~~ ~~Year~~

~~1~~
~~2~~
~~3~~
~~4~~
~~5~~

Q13

$v = \frac{1}{1+r}$

$= \frac{1}{1+0.05}$

$= 0.9524$

Year	Operating & main. cost R_n	Discounted factor v^{n-1}	Discounted maintainence cost $(R_n \cdot v^{n-1})$
1	0	1	0
2	100	0.9524	95.24
3	200	0.9071	181.42
4	300	0.8639	259.17
5	400	0.8228	329.12

~~Commutative Dis main cost~~ $\cancel{\Sigma(R_n v^{n-1})}$	Dis. total cost	Comm. dis cost	total
	500	1	500
	595.24	1.9524	304.88
	776.66	2.8595	$\boxed{271.61}$
	1035.83	3.7234	278.19
	1364.95	4.5462	300.24

Equipment should be replaced by end of 3rd year.

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$c = 20,000$

Q12

Year	Maintainence Cost	Resale price (s)	Comm. maint. cost (t)	Depreciation cost (c-s)
1	1500	17000	1500	3000
2	1700	15300	3200	4700
3	2000	14000	5200	6000
4	2500	12000	7700	8000
5	3500	8000	11200	12000
6	5500	3000	16700	17000

Total cost	Total avg. Cost
4500	4500
7900	3950
11200	$\boxed{3733.33}$
15700	3925
23200	4640
33700	5616.67

After 3 years, equipment should be replaced.

→ minimum average cost = 3733.33
→ economic life of machine = 3 years

Q1) C = ₹2,00,000

Year	Running Cost f(t)	Resale value (S)	Depreciation cost (C-S)	~~Total Cost~~
1	30,000	1,00,000	1,00,000	~~1,30,000~~
2	38,000	50,000	1,50,000	~~1,09,000~~
3	46,000	25,000	1,75,000	~~2,89,000~~
4	58,000	12,000	1,88,000	~~3[illegible],60,000~~
5	72,000	8,000	1,92,000	~~2,36[illegible],000~~
6	90,000	8,000	1,92,000	~~2,82,000~~
7	1,10,000	8,000	1,92,000	~~3,02,000~~

~~Total avg. cost~~	Total cost	Total avg. Cost
	1,30,000	130,000
~~1,30,000~~	2,18,000	1,09,000
~~94,000~~	2,89,000	96,333
~~73,333.3~~	3,60,000	90,000
	4,36,000	$\boxed{87,200}$
	5,26,000	87,666
	6,36,000	90,857

Equipment should be replaced after 5th year.

Q10

Year	Maintainance Cost	Comm. run cost	Depreciation Cost (C-S)	Total Cost
1	200	200	11,800	12000
2	500	700	11,800	12500
3	800	1500	11,800	13300
4	1200	2700	11,800	14500
5	1800	4500	11,800	16300
6	2500	7000	11,800	18800
7	3200	10,200	11,800	22000
8	4000	14,200	11,800	26000

Total avg cost	
12000	
6250	Equipment should be replaced after 6th year
4433	
3625	
3260	
$\boxed{3133}$	
3142	
3250	

Q9

$C = ₹8000$

Year	Running Cost	Resale Value	Comm Running Cost	Depreciation Cost (C-S)
1	1000	4000	1000	4000
2	1500	3500	2500	4500
3	2000	3000	4500	5000
4	2500	2500	7000	5500
5	3000	2000	10,000	6000
6	3500	1500	13500	6500
7	4000	1000	17500	7000

Total Cost	Total avg. Cost
5000	5000
7000	3500
9500	3166.8
12500	$\boxed{3125}$
16000	3200
20000	3333.33
24500	3500

Equipment should be replaced after 4^{th} year

Q8 Arrival rate $(\lambda) = 6/hr$

Service rate $(\mu) = 10/hr$

(i) Utilization Factor $(\rho) = \frac{\lambda}{\mu} = \frac{6}{10}$

$$\boxed{\rho = 0.6}$$

(ii) Probability that system is idle $\therefore P_0 = 1 - \frac{\lambda}{\mu}$

$= 1 - 0.6$

$$\boxed{\therefore P_0 = 0.4}$$

(iii) Avg. time that person is free on 10 hr

$= 10 - 10(\rho)$

$= 10 - 10(0.6)$

$\boxed{= 4 \text{ hr}}$

(iv) Probability of one customer in ~~queuing~~ queuing system.

$\therefore P_1 = \frac{\lambda}{\mu}\left(1 - \frac{\lambda}{\mu}\right) = (0.6)(0.4)$

$\boxed{P_1 = 0.24}$

(v) Expected number of customers in shop

= Average number in system

$\therefore L_s = \frac{\lambda}{\mu - \lambda} = \frac{6}{4}$

$\boxed{\therefore L_s = 1.5}$

(vi) Expected number of customers waiting

= Average number waiting

$\therefore L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = 0.9$

$\boxed{\therefore L_q = 0.9}$

(vii) Expected time customer will spend

= Average number in system

$\therefore W_s = \frac{1}{\mu - \lambda} = 0.25 \text{ hrs}$

$\boxed{W_s = 0.25 \text{ hrs.}}$

Q7 ~~from~~ Mean $\frac{1}{\lambda} = 10\text{ min}$ $\therefore \lambda = 6/\text{hr}$

$\frac{1}{\mu} = 6\text{ min}$ $\therefore \mu = 10/\text{hr}$

(i) Probability of arriving customer directly in system is $P_0 + P_1 + P_2$ (max. three)

$$= \left(1 - \frac{\lambda}{\mu}\right) + \frac{\lambda}{\mu}\left(1 - \frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^2\left(1 - \frac{\lambda}{\mu}\right)$$

$$= \left(1 - \frac{\lambda}{\mu}\right)\left(1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2\right)$$

$$\boxed{\therefore P_0 + P_1 + P_2 = P = 0.784}$$

(ii) Probability of arriving customer has to wait. $1 - (P_0 + P_1 + P_2)$

$$= \boxed{0.216}$$

(iii) Expected wait time

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{6}{10(10-6)}$$

$$\boxed{= 0.15\text{ hrs}}$$

Q6 $\mu = 10/\text{min}$

$\lambda = 8/\text{min}$

(i) Utilization factor : $\frac{\lambda}{\mu} = \boxed{0.8}$

(ii) Ideal time for 8 houres $= 8 - \frac{8\lambda}{\mu}$

$\boxed{= 1.6 \text{ hrs}}$

(iii) Number of person waiting in sys.

$L_s = \frac{\lambda}{(\mu - \lambda)} = \frac{8}{2}$ $\boxed{\therefore L_s = 4}$

(iv) Number of person waiting in queue

$L_Q = \frac{\lambda^2}{\mu(\mu-\lambda)}$

$\boxed{\therefore L_Q = 3.2}$

(v) Avg. waiting time in Queue :

$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

$\boxed{W_q = 0.4 \text{ min}}$

(vi) Total time spent : $W_q + W_s$

$= 0.4 + \frac{1}{2}$

$\boxed{= 0.9 \text{ min}}$

Q5 $\frac{1}{\lambda} = 10$ min $\therefore \lambda = 6/hr$

$\frac{1}{\mu} = 6$ min $\therefore \mu = 10/hr$

(i) Avg. length of queue $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{36}{10 \cdot 4}$

$$\boxed{L_q = 0.9}$$

(ii) Avg. time operator spends in system

$$= \frac{1}{\mu-\lambda} = \frac{1}{4}$$

$$\boxed{= 0.25 \text{ hrs}}$$

(iii) Probability of no. of person's waiting ≥ 6

$$\therefore 1 - \left(1-\frac{\lambda}{\mu}\right) - \left(\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right) - \left(\frac{\lambda}{\mu}\right)^2\left(1-\frac{\lambda}{\mu}\right)$$

$$- \left(\frac{\lambda}{\mu}\right)^3\left(1-\frac{\lambda}{\mu}\right) - \left(\frac{\lambda}{\mu}\right)^4\left(1-\frac{\lambda}{\mu}\right) - \left(\frac{\lambda}{\mu}\right)^5\left(1-\frac{\lambda}{\mu}\right)$$

$$\boxed{= 0.0467}$$

Q4 $\lambda = 12/hr$

$\mu = 30/hr$

(i) Utilization power $= \frac{\lambda}{\mu} = \boxed{0.4}$

(ii) Probability that there shall be 4 customers in system $\left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right)$

$= \left(\frac{12}{30}\right)\left(1-\frac{12}{30}\right)$

$\boxed{= 0.01536}$

Q3 $\mu = 10/min$

$\lambda = 8/min$

(i) Utilization factor $= \frac{\lambda}{\mu} = \frac{8}{10} = \boxed{0.8}$

(ii) Idle time for 8 hrs $= 8 - 8(0.8)$

$\boxed{1.6 hrs}$

(iii) No. of person waiting in sys. $= \frac{\lambda}{\mu-\lambda} = \frac{8}{2}$

$= \boxed{4}$

.... Same as 6

Q2 $\frac{1}{\lambda} = 3\text{ min}$ ∴ $\lambda = 20/\text{hrs}$

$\frac{1}{\mu} = 2\text{ min}$ ∴ $\mu = 30/\text{hrs}$

(i) Avg. waiting time in $W_s = \frac{1}{\mu-\lambda} = \frac{1}{10}$

$$\boxed{W_s = 6\text{mins}}$$

~~Avg waiting time in W_s~~

(ii) No. of customers $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{20^2}{30(30-20)}$

$$\boxed{L_q = \frac{4}{3}}$$

(iii) If $W_s = 8\text{ min}$

$$\frac{8}{60} = \frac{1}{\mu-\lambda}$$

$$\boxed{\therefore \lambda = 22.5/\text{hr}}$$

Q1 $\lambda = 10/8 = 5/4$ sets per hr.

$\mu = (1/30)60 = 2$ sets per hr.

Expected idle time/day $= 8\left(\frac{\lambda}{4}\right) = 8 \times \frac{(5/4)}{2}$

$= \boxed{5\text{ hours}}$

Expected no. of tv sets $E(L_s) = \frac{\lambda}{\mu-\lambda} = \frac{5/4}{2-5/4}$

$$\boxed{= 1.67\text{ TV sets}}$$

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→ No. of idle hrs = 8-5 = $\boxed{\text{3hrs}}$