Designing Speed Control System of DC Motor based on PID control

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ABSTRACT

This study explores the design and analysis of a PID controller for a DC motor system. Beginning with the derivation of the motor's transfer function, the study progresses to PID controller design using time-domain methods. Stability analysis using the root locus method emphasizes the importance of selecting appropriate controller gains. Simulation results demonstrate the superior performance of the PID controller compared to proportional control, showcasing faster response times, reduced overshoot, and improved steady-state accuracy. Furthermore, frequency domain analysis through Bode plot analysis offers insights into the system's frequency response characteristics.

1 INTRODUCTION

DC Motor:
A direct current (DC) motor is a type of electric motor that converts electrical energy into mechanical energy. It operates on the principle of electromagnetism.

Torque:
When current passes through the armature circuit, it interacts with the magnetic field, resulting in a mechanical force known as torque which is directly proportional to the current flowing through its winding's.

$$\tau = k \cdot I$$

Back EMF:
According to Faraday's law of electromagnetic induction, the changing magnetic flux induces a voltage in the motor's armature winding, opposing the applied voltage. This back EMF limits the current flow through the motor, affecting its speed.

$$E_b = k_b \cdot \omega$$

Damping Coefficient:
The damping coefficient(b) in a DC motor represents the resistance to rotational motion caused by friction and other mechanical losses. It influences the motor's ability to accelerate or decelerate and affects its dynamic response to control inputs.

$$b = \frac{F_{\text{damping}}}{\omega}$$

Newton's Law:
Relates torque τ to angular acceleration α:

$$\tau = J \cdot \alpha$$

Kirchhoff's Law:
Voltage law: $\sum V = 0$ in closed loops.
Current law: $\sum I_{\text{in}} = \sum I_{\text{out}}$ at nodes.

PID Control:
Proportional term P, Integral term I, Derivative term D:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}$$

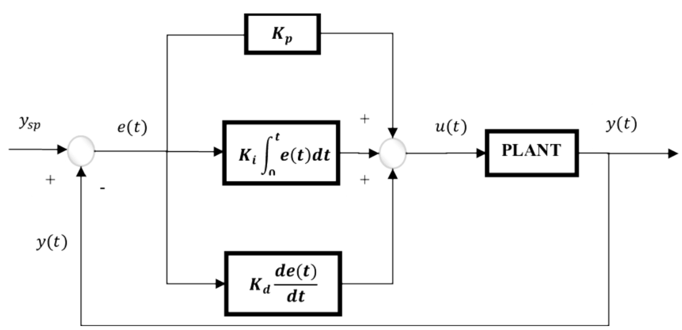


Figure 1: PID

Circuit:
The electrical circuit of a DC motor typically consists of the motor winding's connected to a power supply, along with additional components such as switches, resistors, and capacitors for control and protection. The power supply provides the voltage necessary to drive the motor.

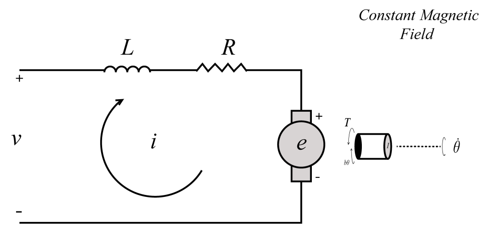


Figure 2: DC motor diagram

2 MATHEMATICAL FORMULATIONS

The torque generated by a DC motor is proportional to the circuit current . Here, we assumed that the magnetic field is constant and that the motor torque is proportional to only the armature current i by a constant factor K_t as shown in the equation below.

To derive the governing equations for the DC motor system, we can start with Newton's second law, which relates the torque applied to a rotational system to its angular acceleration. Then, we can use Kirchhoff's voltage law to describe the electrical behavior of the system.

1. Using the Newton's second law of motion the first equation is.

$$J\ddot{\theta} + b\dot{\theta} = T$$

Here, J is the moment of inertia of the motor and load (rotational mass). $\ddot{\theta}$ is the angular acceleration of the shaft. b is the damping coefficient (friction and other losses). $\dot{\theta}$ is the angular velocity of the shaft. -T is the torque applied to the motor shaft.

2. Kirchhoff's voltage law in the circuit gives.

$$L\frac{di}{dt} + Ri = V - e$$

Here, - L is the inductance of the motor winding. - $\frac{di}{dt}$ is the rate of change of current (induced voltage). - R is the resistance of the motor winding. - i is the current through the motor winding. - V is the applied voltage. - e is the back electromotive force (EMF) generated by the motor.

The back EMF (e) in the circuit is as shown in the equation:

$$e = K_e\dot{\theta}$$

Here, K_e represents the back EMF constant, which is typically equal to the motor torque constant (K_t) in SI units. So, for simplicity, often both K_e and K_t are represented by a single constant K. From the given information, we know that $T = K_t i$ and $e = K_e\dot{\theta}$. Since $K_t = K_e = K$, we can substitute these relationships into the equations above:

$$J\ddot{\theta} + b\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta}$$

These equations describe the dynamic behaviour of the DC motor system, where the first equation relates the mechanical aspects (torque and inertia), and the second relates the electrical aspects (voltage, current, and back EMF).

3 CONTROL LAW DESIGN

3.1 Transfer Function

The open-loop transfer function $P(s)$ represents the relationship between the output, which is the angular velocity $\dot{\Theta}(s)$, and the input, which is the armature voltage $V(s)$, in the Laplace domain. To derive this transfer function, we eliminate $I(s)$ between equations (5) and (6) and express $\dot{\Theta}(s)$ in terms of $V(s)$.

Starting from equations (5) and (6):

$$s(Js + b)\Theta(s) = KI(s) \quad (5)$$

$$(Ls + R)I(s) = V(s) - Ks\Theta(s) \quad (6)$$

We can solve equation (6) for $I(s)$:

$$I(s) = \frac{V(s)}{Ls + R} - \frac{Ks\Theta(s)}{Ls + R}$$

Now, substitute $I(s)$ into equation (5):

$$s(Js + b)\Theta(s) = K\left(\frac{V(s)}{Ls + R} - \frac{Ks\Theta(s)}{Ls + R}\right)$$

Expand and rearrange terms:

$$s(Js + b)\Theta(s) = \frac{KV(s)}{Ls + R} - \frac{K^2 s\Theta(s)}{Ls + R}$$

Combine terms involving $\Theta(s)$ on one side:

$$s(Js + b + \frac{K^2}{Ls + R})\Theta(s) = \frac{KV(s)}{Ls + R}$$

Divide both sides by $V(s)$ to express the transfer function $G_P(s)$:

$$G_P(s) = \frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js + b)(Ls + R) + K^2}$$

So, the open-loop transfer function $G_P(s)$ is given by equation (7):

$$G_P(s) = \frac{K}{(Js + b)(Ls + R) + K^2}$$

This transfer function describes the dynamic relationship between the input voltage $V(s)$ and the output angular velocity $\dot{\Theta}(s)$ in the Laplace domain.

3.2 Design Requirements with Specification

The specifications for control design is as follows:

- Settling time (ts) <= 5 sec
- Rise time (tr) <= 2 sec
- Maximum overshoot (Mp) <= 10%
- Steady state error for step disturbance (Md) must be zero

3.3 PID Control

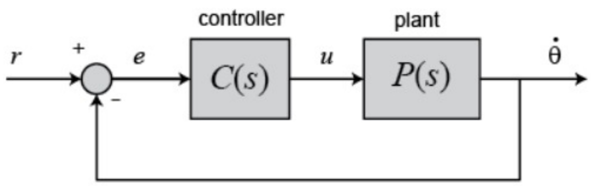


Figure 3: PID Controller

The transfer function of the plant

$$G_p(s) = \frac{K}{(Js+b)(Ls+R)+K^2}$$

Motor Parameters: J=0.1, K=0.02, L=0.5, b=0.1, R=0.5
Also,
Settling time $t_s \leq 5sec$
Rise time $t_r \leq 2sec$
Maximum overshoot $M_p \leq 10\%$
Steady state error for step disturbance M_d must be 0
A PID controller can be designed as a combined PD and PI controller
PD controller: $G_{PD} = K_{PD}(T_{PD}s+1)$
PI controller: $G_{PI} = K_{PI}[(T_{PI}s+1)/s]$
PID controller:

$$G_c(s) = K_p + K_i/s + K_d s$$

First lets handle overshoot constraint,
for $M_p \leq 10\%$, Phase margin $PM \geq 58.35$ to give ourself a small margin,
we choose $PM = 60$ and damping ratio corresponding to it is $\epsilon = 0.61$
From,

$$\omega_g \geq \frac{\pi - \beta}{t_r\sqrt{(1-\epsilon^2)(\sqrt{4\epsilon^4+1}-2\epsilon^2)}}$$

for $\epsilon = 0.61$ and $t_r = 2$,

$$\omega_g \geq 1.9944sec$$

from

$$\omega_g \geq \frac{-ln(0.02\sqrt{1-\epsilon^2})}{\epsilon t_s(\sqrt{\sqrt{4\epsilon^4+1}-2\epsilon^2})}$$

for $\epsilon = 0.61$ and $t_s = 5$,

$$\omega_g \geq 1.9228sec$$

Hence,

$$\omega_g \geq 1.9944sec$$

So to give us small margin $\omega_g = 2rad/sec$.

Open loop transfer function,

$$G_o(s) = KG_p(s)G_{PD}(s)G_{PI}(s)$$

First design PD controller, which can provide desired PM
Assuming that the PI part of PID add approximately 5 degree phase lag at $\omega_g = 2rad/sec$
$PM_p = 180 + \angle G_o(j\omega_g)$
$=180 + \angle G_p(j\omega_g) + \angle G_{PD}(j\omega_g) + \angle G_{PI}(j\omega_g)$
$60 = 180 - 143.67 + \angle G_{PD}(j\omega_g) - 5$
Therefore, $\angle G_{PD}(j\omega_g) = 28.67$
Controller gain K can be choosen so that it gives ω_g

$$T_{PD} = \frac{\tan 28.67}{\omega_g} = 0.2734sec$$

Designing PI controller,

$$\frac{1}{T_{PI}} = 0.1\omega_g$$

Therefore, $T_{PI} = 5sec$
Final part of PID design is to choose gain K such that $\omega_g = 2rad/sec$

$$1 = |G_o(j\omega_g)|$$

$$K = \frac{1}{|G_p(2j)||G_{PD}(2j)||G_{PI}(2j)|}$$

$G_{PD}(s) = 0.2734s + 1$
$G_P(j\omega_g) = -0.1496 - 0.11j$
$G_{PI}(s) = \frac{5s+1}{s}$
Therefore, $K = 2.18$
$K_p = K(T_{PD} + T_{PI}) = 11.51$
$K_d = K(T_{PD}T_{PI}) = 2.98$
$K_i = K = 2.18$

4 STABILITY ANALYSIS

We have used the root locus plot to discuss the stability of this system. The main idea of root locus design is to predict the closed-loop response from the root locus plot which depicts possible closed-loop pole locations and is drawn from the open-loop transfer function. Then by adding zeros and/or poles via the controller, the root locus can be modified in order to achieve a desired closed-loop response. The open loop transfer function of the entire system is given by equation :

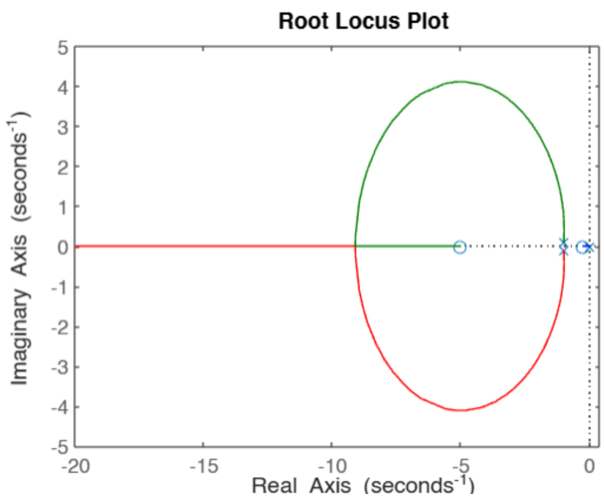
$$G_0(s) = G_P(s)G_C(s)$$

where we get $G_P(s)$ and $G_C(s)$ are derived earlier

After combining the two and substituting values of K_p,K_i and K_d we get the open loop transfer function as:

$$G_0(s) = \frac{K^2K_ps + KK_i + KK_ds^2}{JLs^3 + (JR+Lb)s^2 + (K^2+Rb)s}$$

$$G_0(s) = \frac{4.764s^2 + 25.12s + 6.512}{0.05s^3 + 0.1s^2 + 0.0504s}$$



The System is stable

5 RESULTS AND DISCUSSIONS

We found the followings Transfer Functions for plant and controller

$$G_P(s) = \frac{2.183}{0.05s^2 + 0.1s + 0.0504}$$

$$G_c(s) = K_p + K_i/s + K_d s$$

with $K_p = 11.5$, $K_i = 2.98$, $K_d = 2.18$

5.1 Open and Close loop Transfer function

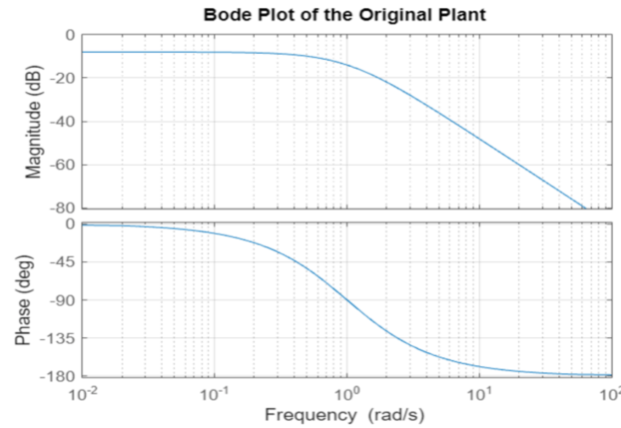
Open and closed loop transfer function of the DC Motor in Laplace Domain

$$G_0(s) = \frac{4.764s^2 + 25.12s + 6.512}{0.05s^3 + 0.1s^2 + 0.0504s}$$

$$G_{cl}(s) = \frac{4.764s^2 + 25.12s + 6.512}{0.05s^3 + 4.864s^2 + 25.17s + 6.512}$$

5.2 Bode plot

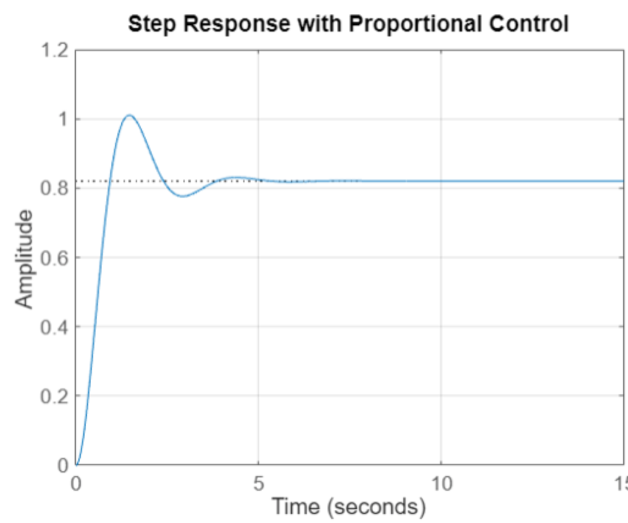
The main idea of frequency-based design is to use the Bode plot of the open-loop transfer function to estimate the closed-loop response. Adding a controller to the system changes the open-loop Bode plot, thereby changing the closed-loop response. It is our goal to design the controller to shape the open-loop Bode plot in such a way that the closed-loop system behaves in a desired manner



5.3 Proportional control

Let’s first try employing a proportional controller with a gain of $K_p = 11.5$, that is, $G_C(s) = 11.5$.

Now let’s examine the closed-loop step response

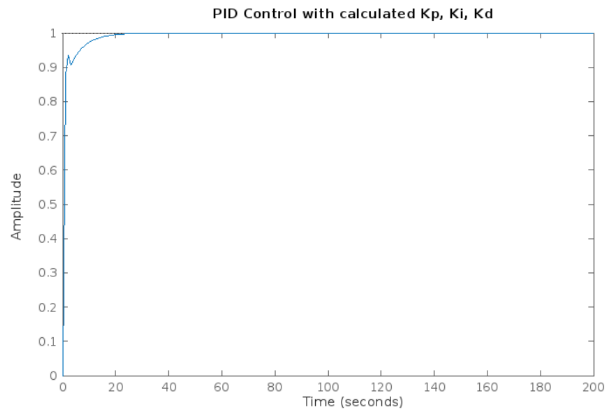


From the plot above we see that both the steady-state error and the overshoot are too large
increasing the proportional gain K_p will reduce the steady-state error. However, increasing K_p often results in increased overshoot, therefore, it appears that not all of the design requirements can be met with a simple proportional controller. This fact was verified by experimenting with different values of K_p.
A proportional controller is insufficient for meeting the given design requirements; derivative and/or integral terms must be added to the controller.

5.4 PID control

Adding an integral term will eliminate the steady-state error to a step reference and a derivative term will often reduce the overshoot.

From the PID controller designed previously we can plot the step response.



6 CONCLUSION

In this study, we thoroughly investigated the design and analysis of a PID controller for a DC motor system. Starting with the derivation of the motor's transfer function using fundamental laws, we proceeded to design the PID controller using time-domain methods. We highlighted the necessity of incorporating integral and derivative action alongside proportional control by showcasing the limitations of using only proportional control.

Stability analysis was performed using the root locus method, emphasizing the importance of selecting appropriate controller gains for system stability. Simulation results demonstrated the superior performance of the PID controller compared to proportional control, showcasing faster response times, reduced overshoot, and improved steady-state accuracy.

Additionally, frequency domain analysis was conducted through Bode plot analysis of the open-loop transfer function, offering insights into the system's frequency response characteristics.

Overall, this study provides a comprehensive framework for designing and analyzing PID control in DC motor systems, contributing to the advancement of control theory in practical engineering applications.

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