

Kinematics: Reference Frames, Flow Fields, and Reynolds Transport Theorem (RTT)

AA 507 (Incompressible Fluid Mechanics) Winter 2026, UW

Assignment: Homework 2

(Ref file: HW2.pdf)

Author: Emmanuel Rodriguez

emmanueljrodriguez.com

Date: 24JAN2026

- 29JAN2026

Location: Renton, Seattle, WA

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Problem 2. Passive scalar transport from Reynolds Transport Theorem

- [A passive scalar is a scalar quantity that is transported by advection (the transfer of a substance, or matter, by bulk motion of a fluid) and diffusion in a flow but does not influence the velocity field or fluid dynamics.]

1. Time rate of change of dye mass

Fundamentals

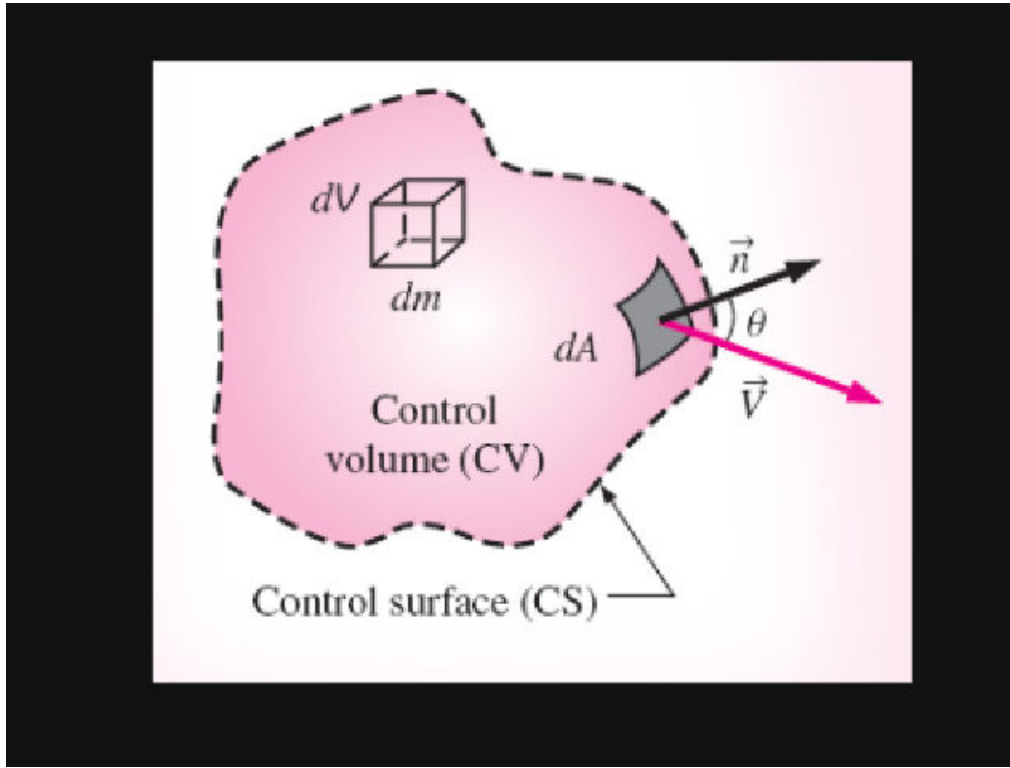
a. Conservation of Mass Principle

$$m_{in} - m_{out} = \Delta_{CV}$$

The net mass transfer to or from a control volume during a time interval Δt is equal to the net change in the total mass within the control volume during Δt . [1]

General Conservation of Mass

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$$\frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA = 0 \quad (2-1)$$

"... the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero."

b. Total Dye Mass:

Since:

$$c = \frac{M_{dye}}{M_{mixture}}$$

then,

$$M_{dye} = cM_{mixture} = c\rho_{mixture}V$$

Generally:

$$M_{dye}(t) = \int_V \rho(x, t)c(x, t) dV \quad (2-2)$$

c. Time Rate of Change of Dye Mass:

The dye is conserved, so expanding on the *general conservation of mass* expression (2-1) above, gives:

$$\frac{d}{dt} \int_V \rho c dV + \oint_{\partial V} (\rho c \vec{u} + \mathbf{j}) \cdot \vec{n} dA = 0 \quad (2-3)$$

- The left integral represents the time rate of change of mass within the CV (i.e., change in a CV).
- The right integral represents the total flux through the boundary surface (i.e., change in a system).
- Where $(\rho c \vec{u}) \cdot \vec{n}$ is the **advective/convective** dye flux and $\mathbf{j} \cdot \vec{n}$ is the **diffusive dye flux**.

2. Reynolds Transport Theorem and Divergence Theorem for the Differential Equation

d. Reynolds Transport Theorem

For the fixed CV, RTT reduces to:

$$\frac{d}{dt} \int_V \rho c dV = \int_V \frac{\partial(\rho c)}{\partial t} dV$$

Since the CV is not moving or deforming with time, the time derivative can be moved inside since it is irrelevant whether we differentiate or integrate first. [1] It is expressed as a partial derivative since density and the dye mass ratio are space-dependent (functions of x and t).

e. Divergence Theorem on the Surface Flux

$$\oint_{\partial V} \mathbf{G} \cdot \mathbf{n} dA = \int_V \nabla \cdot \mathbf{G} dV$$

Let:

$$\mathbf{G} = (\rho c \vec{u} + \mathbf{j})$$

$$\oint_{\partial V} (\rho c \vec{u} + \mathbf{j}) \cdot \vec{n} dA = \int_V \nabla \cdot (\rho c \vec{u} + \mathbf{j}) dV$$

f. Combine Them

$$\frac{\partial(\rho c)}{\partial t} + \nabla \cdot (\rho c \vec{u} - \rho D \nabla c) = 0 \quad (2-4)$$

3. Incompressible Flow and Constant Density and Diffusivity

Incompressible flow:

$$\frac{\partial \rho}{\partial t} = 0 \quad (\text{density does not change with time})$$

and

$$\nabla \cdot \vec{u} = 0$$

Expand $\nabla \cdot (c \vec{u})$

$$\nabla \cdot (c \vec{u}) = \vec{u} \cdot \nabla c + c(\nabla \cdot \vec{u}) = \vec{u} \cdot \nabla c$$

(2-4) becomes:

$$\frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c = D \nabla^2 c \quad (2-5)$$

- This is the simplified passive-scalar transport equation.

Applications

References

[1] Cengel, Y. (2018). *Fluid Mechanics: Fundamentals and Applications* (4th ed.). New York: McGraw-Hill.