AA/ME 507 Winter 2026
Homework 1
January 16, 2026
Due Date: January 23, 2026 (11:59 PM)
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Note:

All problems are to be solved symbolically first. Only plug in values at the end to report numerical answers.

Problem 1. Dimensional analysis: oscillating Stokes layer A large, flat plate oscillates sinusoidally in its own plane with velocity

$$U(t) = U_0 \cos(\omega t),$$

beneath an effectively infinite layer of a Newtonian fluid (density ρ, dynamic viscosity μ). Far from the plate the fluid is at rest. Assume the flow is laminar and two-dimensional, and neglect compressibility and gravity.

Engineers define a characteristic thickness δ of the oscillatory boundary layer as the distance from the wall where the velocity amplitude has decayed to $1/e$ of its wall value.

1. **Dimensional analysis for δ.** Assume

$$\delta = f(\omega, \rho, \mu).$$

   (a) Use the Buckingham–Π theorem to obtain a minimal set of independent dimensionless groups.

   (b) Write your result in the form
   $$\frac{\delta}{L_*} = \Phi(\Pi),$$
   choosing a convenient length scale L_* constructed from the given variables.

2. **Identify the controlling parameter.** Let $\nu = \mu/\rho$ be the kinematic viscosity. Rewrite your dimensionless group(s) using ν, and interpret in one sentence what physical processes the parameter compares.

Bonus (optional). If the plate velocity amplitude U_0 is retained, repeat the dimensional analysis assuming

$$\delta = f(U_0, \omega, \rho, \mu),$$

and express the result in terms of an appropriate nondimensional velocity group.

Problem 2. Dimensional analysis: laminar liquid jet breakup A steady, round liquid jet of diameter D issues from a nozzle into a quiescent gas. The liquid has density ρ and dynamic viscosity μ, and the liquid–gas surface tension is σ. The jet exit speed is U. Assume gravity is negligible over the breakup length and the gas effects can be ignored except through σ.

Define the breakup length L_b as the mean distance from the nozzle exit to the point where the continuous liquid column first pinches off into droplets.

Definitions. The Weber number and capillary number are

$$\mathrm{We} \equiv \frac{\rho U^2 D}{\sigma}, \qquad \mathrm{Ca} \equiv \frac{\mu U}{\sigma}.$$

1. **Dimensional analysis.** Assume

$$L_b = f(U, D, \rho, \mu, \sigma).$$

   (a) Use Buckingham–Π to obtain a minimal set of independent dimensionless groups.

   (b) Write the result in the form

$$\frac{L_b}{D} = \Phi(\Pi_1, \Pi_2),$$

   and identify Π_1, Π_2 with standard fluid-mechanics numbers.

2. **Inertial–capillary regime.** Experiments indicate that for sufficiently large Reynolds number and negligible viscosity effects, the breakup length scales like

$$\frac{L_b}{D} \propto \mathrm{We}^{1/2}.$$

   Using your dimensionless groups, convert this into a scaling law for L_b in terms of (U, D, ρ, σ).

3. **Viscous–capillary regime.** In a different regime, experiments suggest

$$\frac{L_b}{D} \propto \mathrm{Ca}^{-1},$$

   where Ca is the capillary number. Using your dimensionless groups, write the implied scaling for L_b in terms of (U, D, μ, σ) and briefly state what physical balance this corresponds to.

4. **Similarity.** You want to design a scaled experiment with a different liquid (different ρ, μ, σ) but the same nozzle diameter D to reproduce the same nondimensional breakup length L_b/D at a different speed. Which dimensionless group(s) must be matched in each of the two regimes above?

Problem 3. Vector calculus warm-up: solid-body rotation Consider the velocity field

$$\mathbf{u}(x, y, z) = (-\Omega y,\ \Omega x,\ 0), \qquad \Omega > 0.$$

1. Compute the divergence $\nabla \cdot \mathbf{u}$. Is the flow incompressible?

2. Compute the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$.

3. Let C be the circle $x^2 + y^2 = R^2$ in the plane $z = 0$, oriented counterclockwise (as seen from $+z$). Compute the circulation

$$\Gamma = \oint_C \mathbf{u} \cdot d\boldsymbol{\ell}.$$

4. Let S be the disk bounded by C. Verify Stokes' theorem by showing

$$\Gamma = \int_S (\nabla \times \mathbf{u}) \cdot \mathbf{n}\, dS,$$

where $\mathbf{n}$ is the unit normal to S.

5. Give a one-sentence physical interpretation of the vorticity field you found.

Problem 4. Vector calculus warm-up: planar stagnation point flow Consider the velocity field

$$\mathbf{u}(x, y, z) = (ax,\ -ay,\ 0), \qquad a > 0.$$

1. Compute $\nabla \cdot \mathbf{u}$ and $\nabla \times \mathbf{u}$.

2. Find the streamlines in the plane $z = 0$ by solving

$$\frac{dy}{dx} = \frac{v}{u}.$$

3. Compute the net outward flux across the boundary of the square $0 \le x \le L$, $0 \le y \le L$ in the plane $z = 0$,

$$\Phi = \oint_{\partial A} \mathbf{u} \cdot \mathbf{n}\, ds,$$

where ∂A is the square boundary oriented counterclockwise and $\mathbf{n}$ denotes the outward unit normal in the plane.

4. Explain briefly why your flux result is consistent with your value of $\nabla \cdot \mathbf{u}$.