Solving PDEs with the FFT - MATLAB

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- Steve Brunton, Youtube vid.

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① 1D Heat Equation: $\frac{\partial T}{\partial t}(x,t) = \alpha \frac{\partial^2 T}{\partial x^2}(x,t)$, $U_T = \alpha^2 U_{xx}$,

② $\underset{\text{Space}}{u}(x, \overset{\text{time}}{t})$ (1) $\xrightarrow{F} \hat{u}(\underset{\text{Spatial freq.}}{K}, t)$ (2)

Temp.

Fourier Transform

③ Note that (2) is still a function of time.

Also, if we have derivatives:

$u_x \xrightarrow{F} iK\hat{u}$,

$u_{xx} \Rightarrow -K^2\hat{u}$,

④ Note $\hat{u}$ is a vector of Fourier coeff., K and K^2 are vectors of frequencies.

⑤ $-\begin{bmatrix} K_1^2 \hat{u}_1 \\ K_2^2 \hat{u}_2 \\ \vdots \\ K_n^2 \hat{u}_n \end{bmatrix}$

⑥ then, once we take the inverse Fourier transform of this vector, we recover the 2nd derivative U_{xx}

I.e. We're using this formulation to approximate the second derivative.

⑦ Fourier T. of Heat Eq.

$$\hat{u}_t = -\alpha^2 K^2 \hat{u} \quad (3)$$

Note that $\hat{u}_t$ is a function of Kappa and time.

(3) is an ODE, n decoupled ODEs one for each K_j

⑧ Computation plan:

- Use the Fast FT to approx. derivative