% Domain Setup

Solving the Heat Eq. using FFT/MATLAB
(Steve Brunton code)

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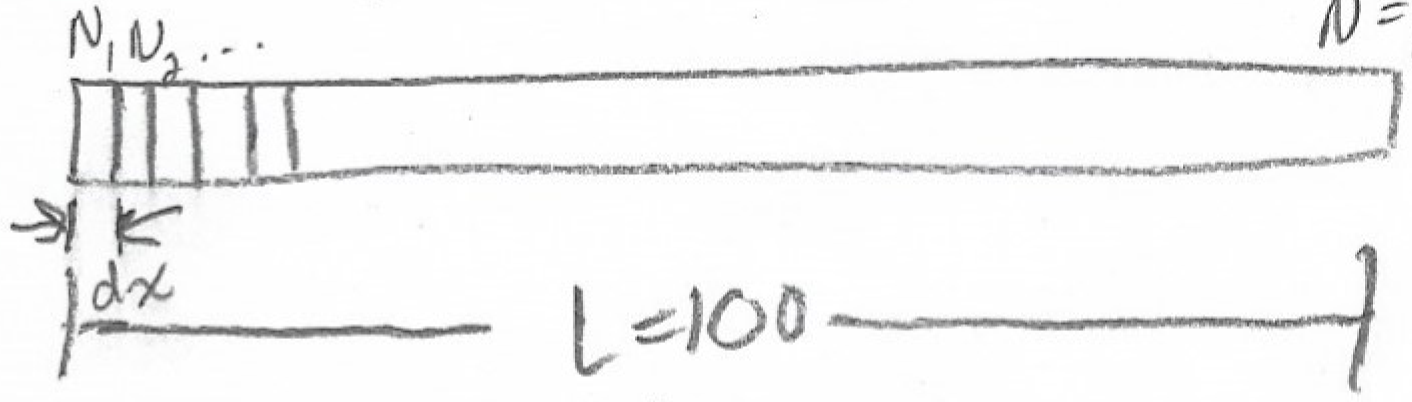

N=1000, # of discretization points

← Domain length

$dx = L/N$, step size

$x = -L/2 : dx : L/2 - dx$, space vector

$= -50 \longrightarrow 50 - 0$ term
increments of dx

Remember: You're decomposing the input function into a function of a bunch series of sine waves.

% Define discrete wave numbers

$$\omega = \frac{2\pi}{L} * [-N/2 : N/2 - 1]$$

$$= \frac{2\pi}{L} * [-500 : 500 - 1]$$

the zero term

= −31.41, −31.35 ... 0 ... 31.35
(1) (2) (501) (1000)

Yes, so this is essentially transforming my input space from a spatial domain to a frequency domain.

How often is the tip of the pencil touching that specific input value?

↳ This is the discrete wave #.

ωShift = 0.0, 0.0628 ... −31.41, −31.35 ... −0.0628
(1) (2) (501) (502) (1000)

I'm not really sure why I had to shift this, but I understand how/to what order it was shifted...

% Initial Condition

u0 = 0*x % Initializing

= 1 x 1000 array, all equal to 0

u0((L/2 - L/10)/dx : (L/2 + L/10)/dx) = 1;

(40/0.1 : 60/0.1) = 1

(400 : 600) = 1 % Elements 400 to 600 have the zero replaced with a value of 1.

↑ Creates a step function.

% Simulate in Fourier frequency domain

t = 0: 0.1 : 20 % time vector

% This is where the simulation (magic) happens

% $\hat{u}_t = -\alpha^2 \omega^2 \hat{u}$

[t, uhat] = ode45(@(t, uhat) rhsHeat(t, uhat, KappaShift, a), t, fft(u0));

↑ ↑

Output arguments

% uhat is the state of the system at each time, i.e. it's the temperature of the rod at each element → ie. for each omega at each time increment.

% t is there because uhat is a function of time, so it only makes sense to have them ~~both~~ both output as pairs.

Recall: $C_n = \int_0^1 f(t) e^{-n \cdot 2\pi i t} dt$

% the output of uhat is 201 x 1000 array

- Row: represents the value of the ODE at each time step

- Column: represents the value of the ODE at each frequency, ω

$$\frac{\partial^2 U_{nh}}{\partial x^2} + \frac{\partial^2 U_{nh}}{\partial y^2} + \frac{\partial^2 U_{nh}}{\partial z^2} + \frac{\partial^2 U_{nh}}{\partial w^2} + \beta^2 U_{nh} = -\beta_0^2 x U_h$$

If we have derivatives:

$$\frac{\partial U_{nh}}{\partial x} \overset{\text{Fourier}}{\Longrightarrow} i\,\text{Kappa}\,\hat{u}$$

$$\frac{\partial^2 U_{nh}}{\partial x^2} \overset{\text{Fourier}}{\Longrightarrow} -\text{Kappa}^2\,\hat{u}$$

$$\Rightarrow \underbrace{-\text{Kappa}_x^2\,\hat{u} - \text{Kappa}_y^2\,\hat{u} + \text{Kappa}_z^2\,\hat{u} - \text{Kappa}_w^2\,\hat{u}}_{-\text{Kappa}_n^2\,\hat{u}} + \beta^2 U_{nh} = -\beta_0^2 x U_h$$

$$\Rightarrow U_{nh} = \frac{+\text{Kappa}_n^2\,\hat{u} - \beta_0^2 x U_h}{\beta^2}$$