How Fractal Trees Work

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Talk at CRIBB, November 4 2011

Tokutek

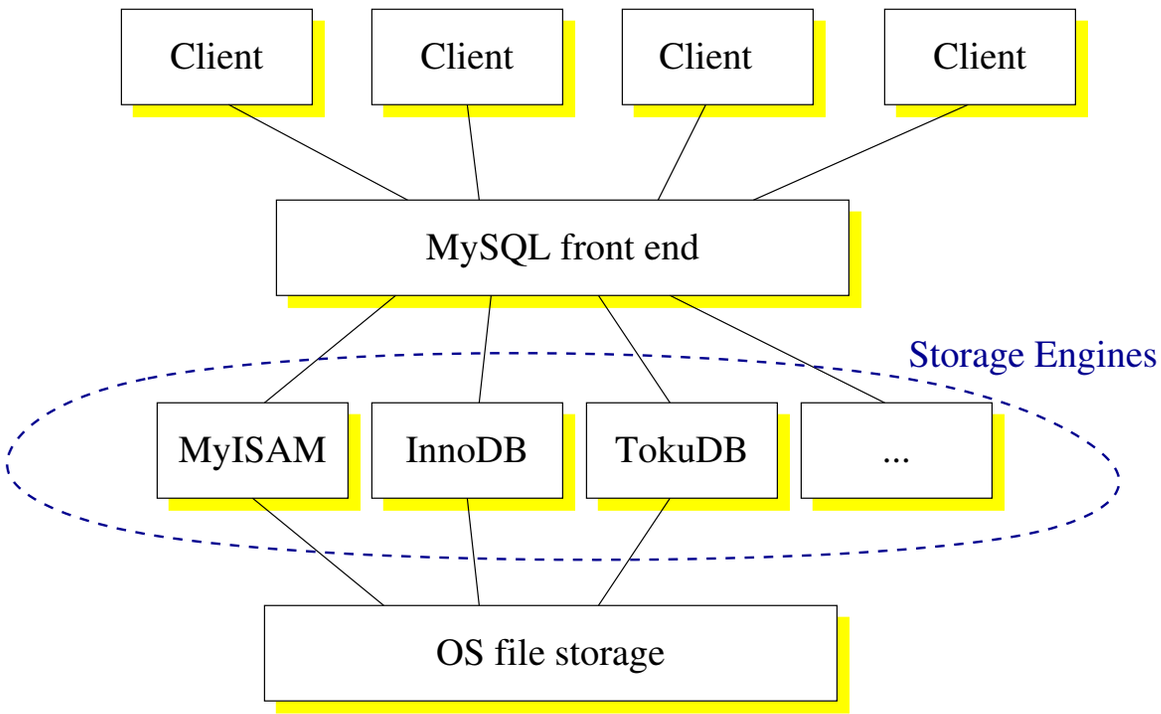
A few years ago I started collaborating with Michael Bender and Martin Farach-Colton on how to store data on disk to achieve high performance.

Mike

Martin

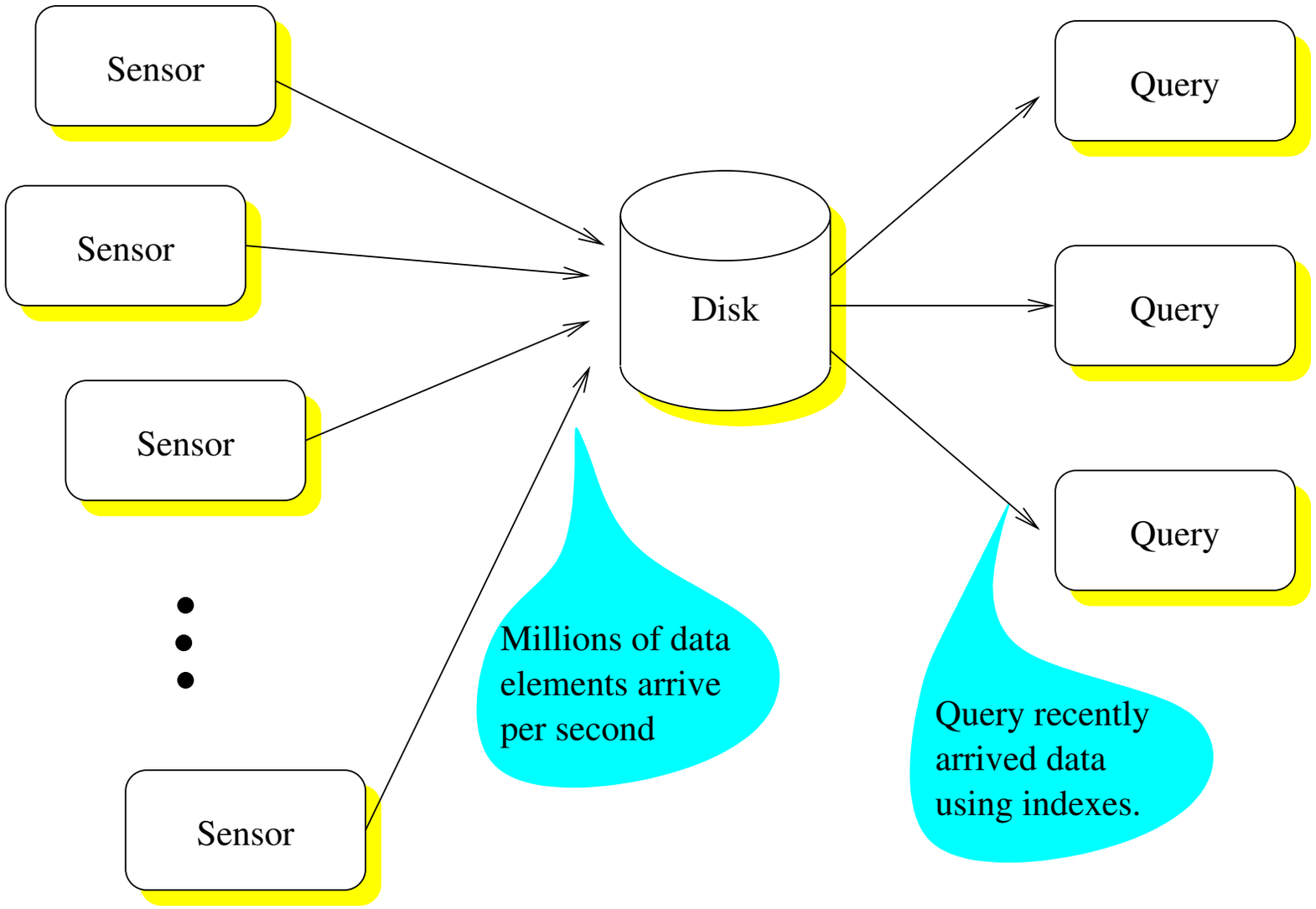
We started Tokutek to commercialize the research.

Storage Engines in MySQL



Tokutek sells TokuDB, a closed-source storage engine for MySQL.

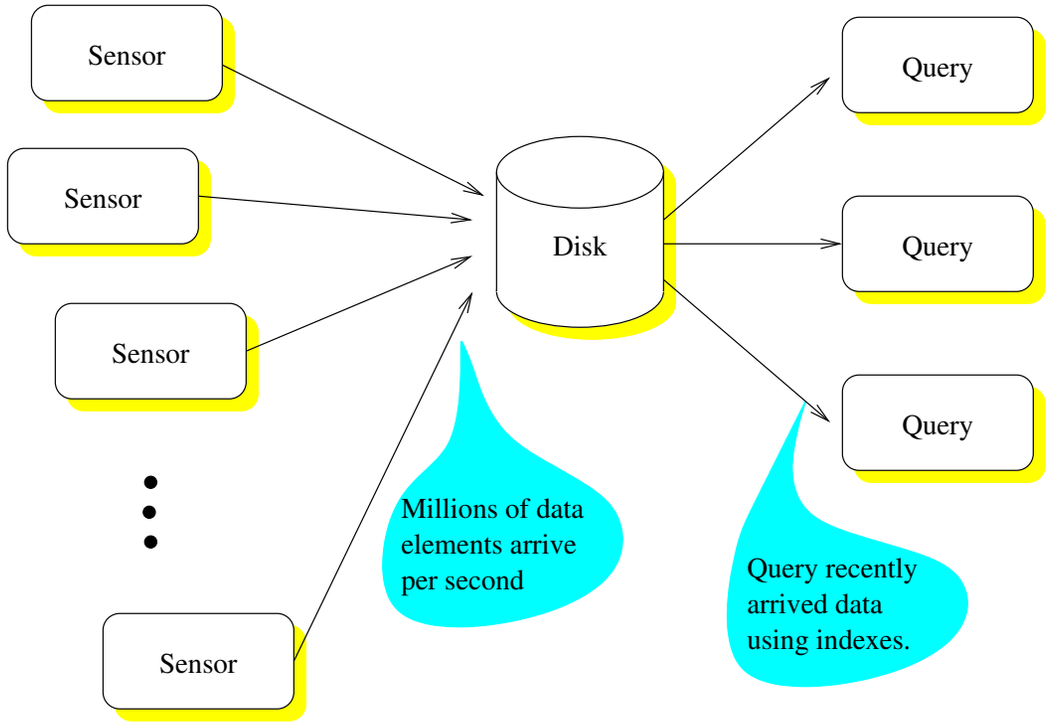
I/O is a Big Bottleneck



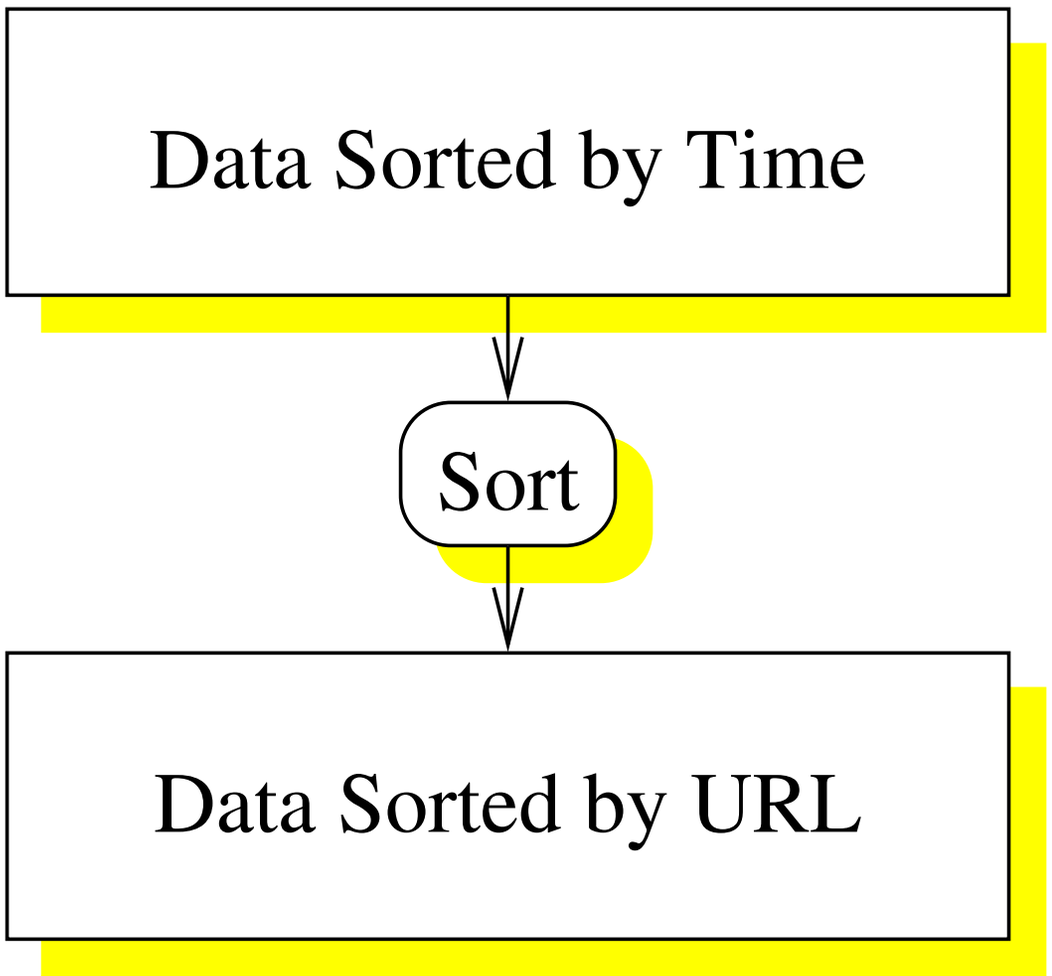
Systems include sensors and storage, and want to perform queries on recent data.

The Data Indexing Problem

- Data arrives in one order (say, sorted by the time of the observation).
- Data is queried in another order (say, by URL or location).



Why Not Simply Sort?

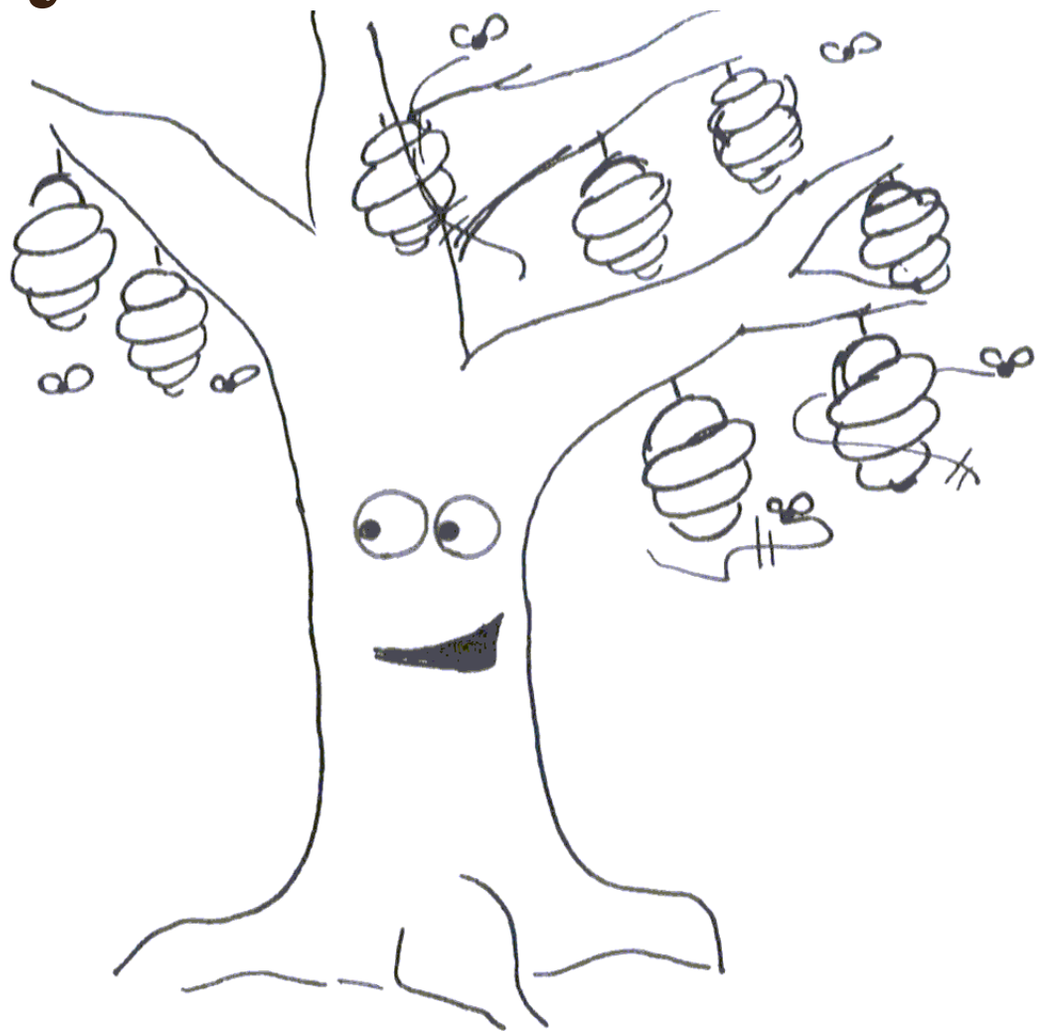


- This is what data warehouses do.
- The problem is that you must wait to sort the data before querying it: typically an overnight delay.

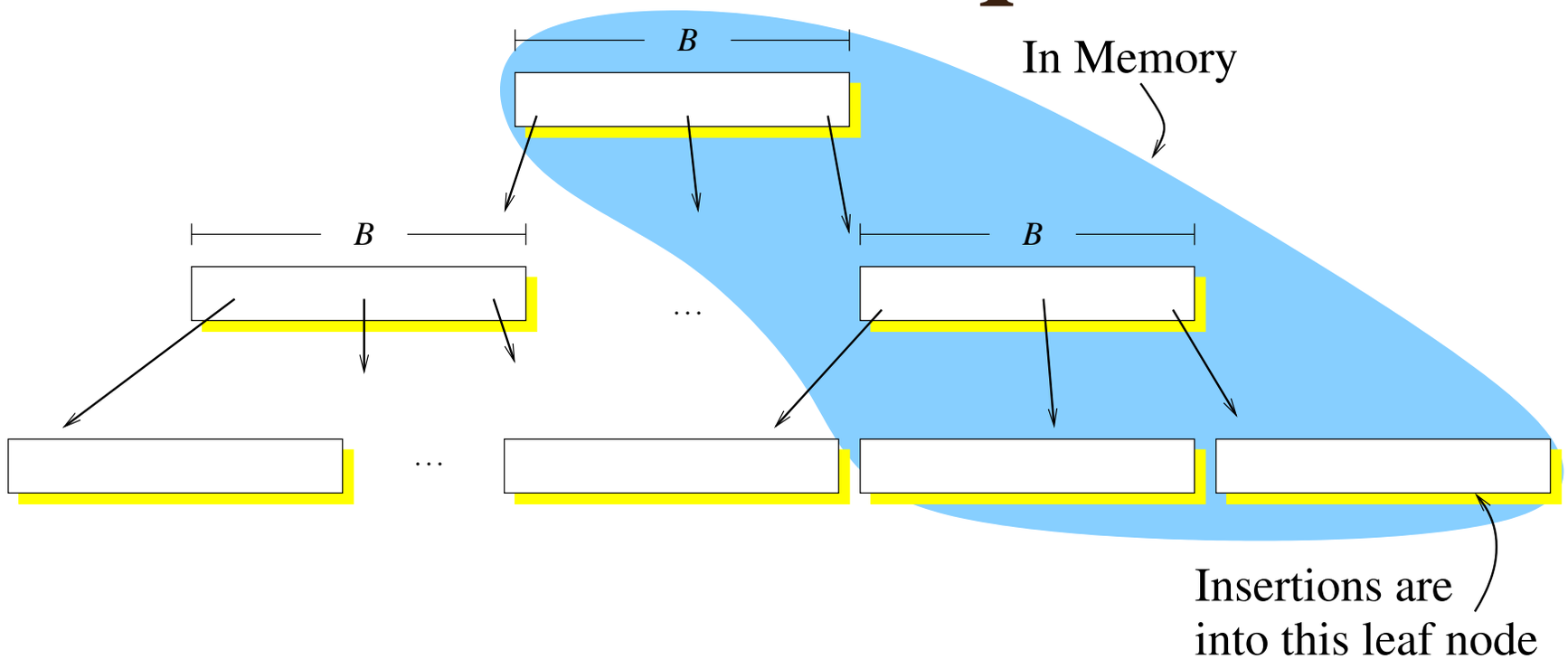
The system must maintain data in (effectively) several sorted orders. This problem is called *maintaining indexes*.

B-Trees are Everywhere

B-Trees show up in database indexes (such as MyISAM and InnoDB), file systems (such as XFS), and many other storage systems.

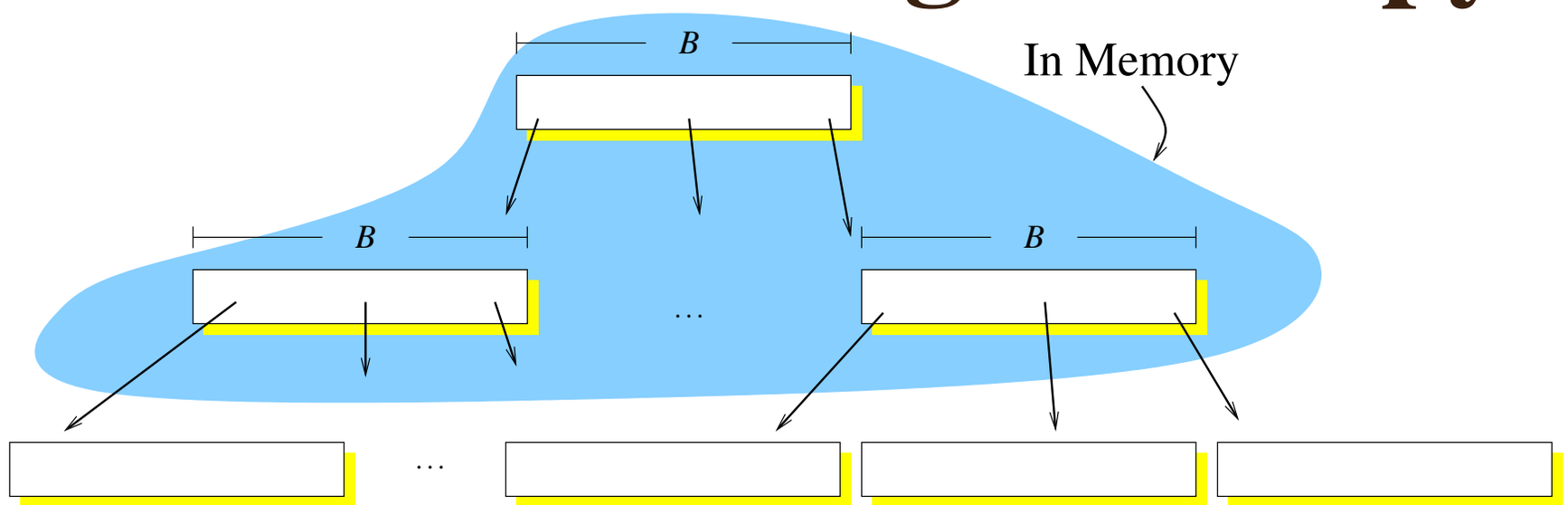


B-Trees are Fast at Sequential Inserts



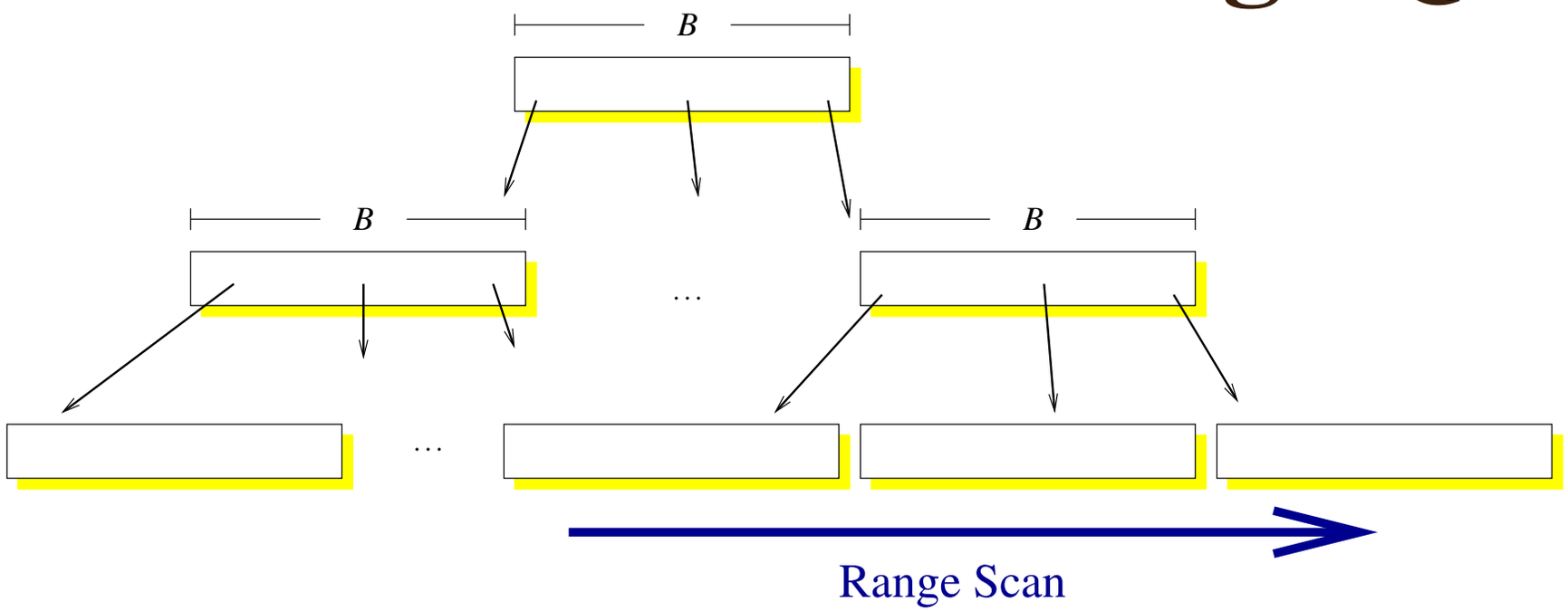
- One disk I/O per leaf (which contains many rows).
- Sequential disk I/O.
- Performance is limited by *disk bandwidth*.

B-Trees are Slow for High-Entropy Inserts



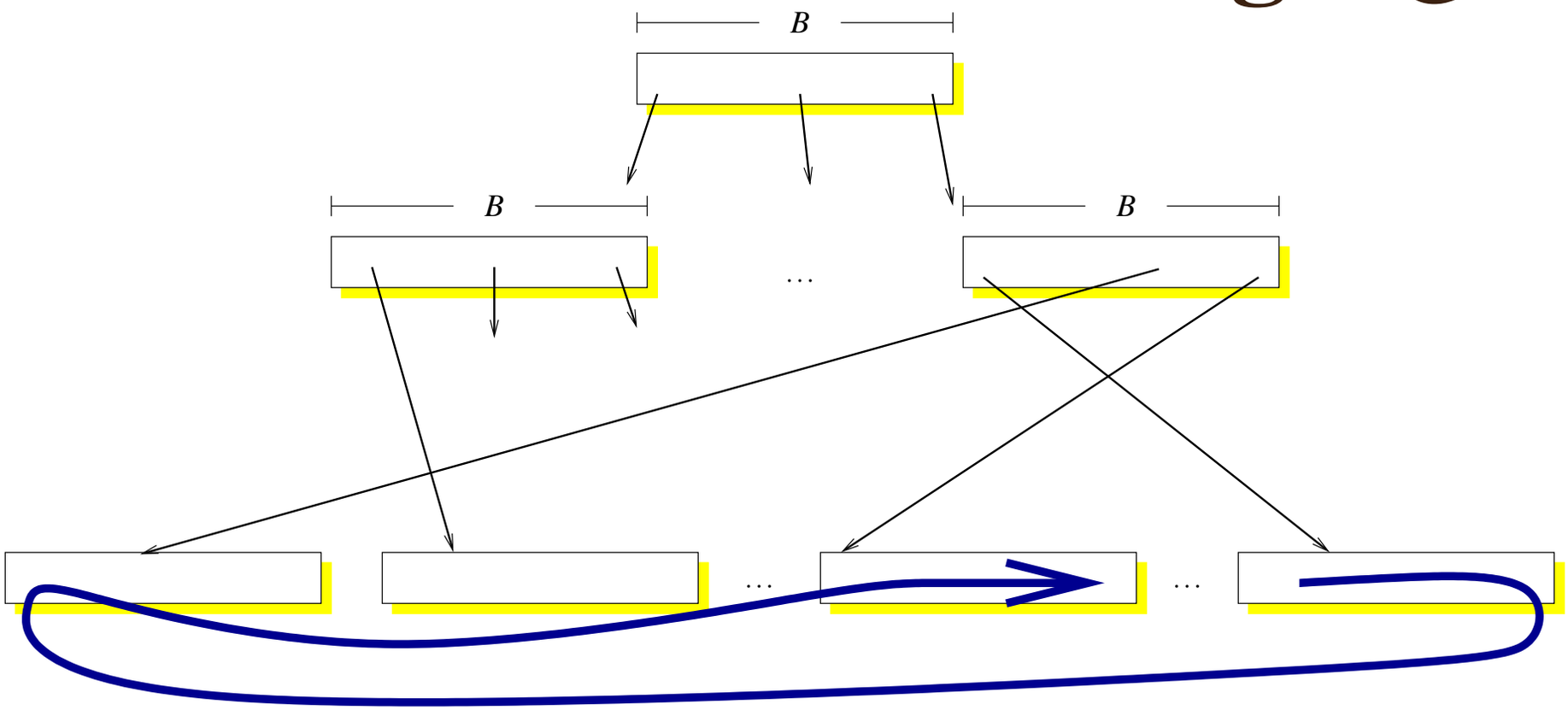
- Most nodes are not in main memory.
- Most insertions require a random disk I/O.
- Performance is limited by *disk head movement*.
- Only 100's of inserts/s/disk ($\leq$ 0.2% of disk bandwidth).

New B-Trees Run Fast Range Queries



- In newly created B-trees, the leaf nodes are often laid out sequentially on disk.
- Can get near 100% of disk bandwidth.
- About 100MB/s per disk.

Aged B-Trees Run Slow Range Queries

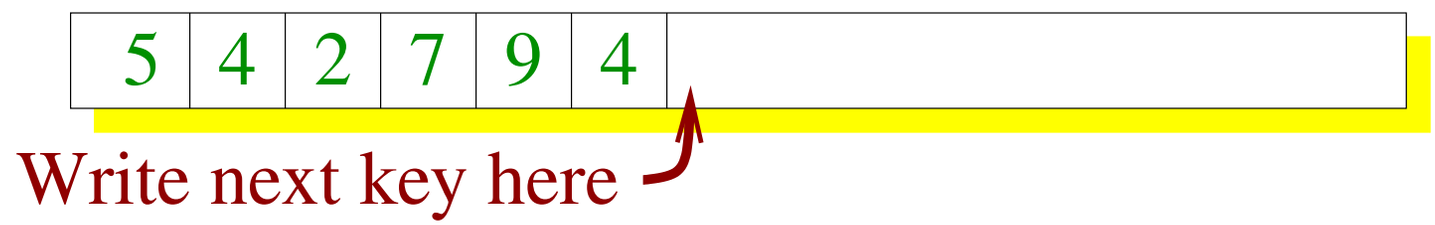


Leaf Blocks Scattered Over Disk

- In aged trees, the leaf blocks end up scattered over disk.
- For 16KB nodes, as little as 1.6% of disk bandwidth.
- About 16KB/s per disk.

Append-to-file Beats B-Trees at Insertions

Here's a data structure that is very fast for insertions:



Write to the end of a file.

Pros:

- Achieve disk bandwidth even for random keys.

Cons:

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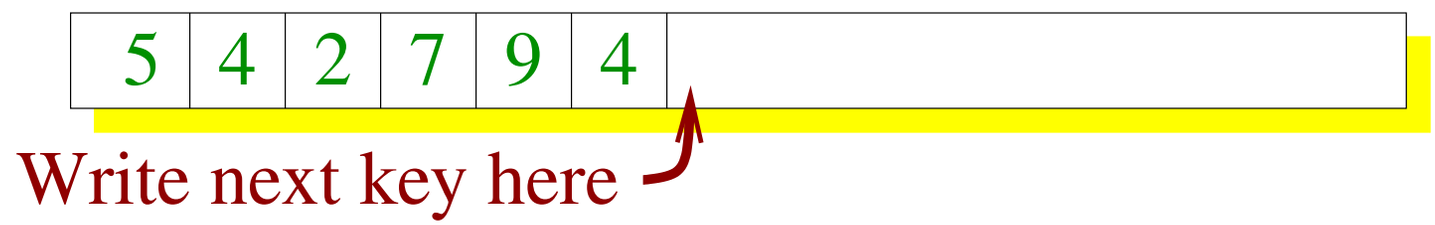

Write to the end of a file.

Pros:

- Achieve disk bandwidth even for random keys.

Cons:

- Looking up anything requires a table scan.

A Performance Tradeoff?

Structure	Inserts	Point Queries	Range Queries
B-Tree	Horrible	Good	Good (young)
Append	Wonderful	Horrible	Horrible

- B-trees are good at lookup, but bad at insert.
- Append-to-file is good at insert, but bad at lookup.
- Is there a data structure that is about as good as a B-tree for lookup, but has insertion performance closer to append?

A Performance Tradeoff?

Structure	Inserts	Point Queries	Range Queries
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Fractal Tree	Good	Good	Good

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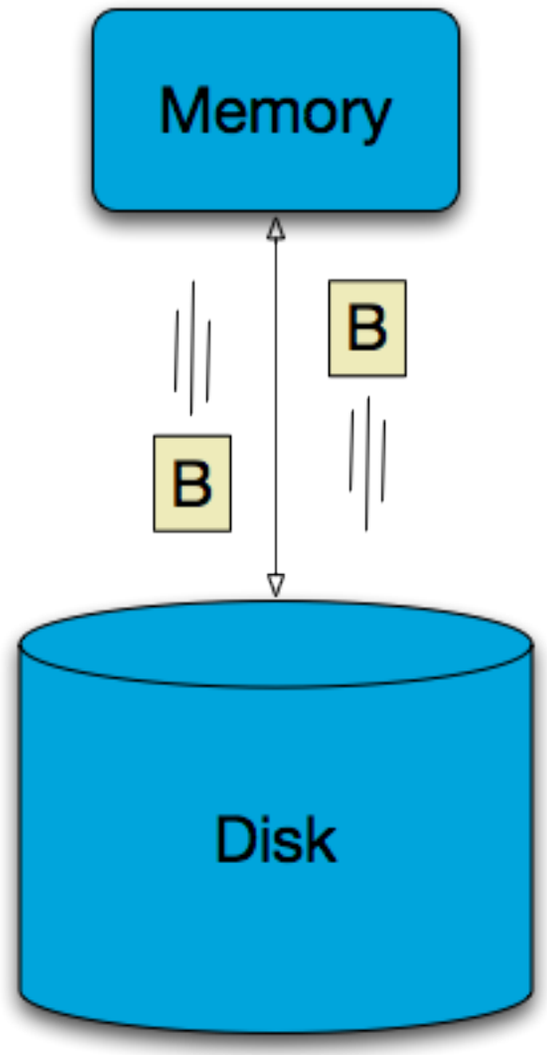
- Is there a data structure that is about as good as a B-tree for lookup, but has insertion performance closer to append?

Yes, Fractal Trees!

An Algorithmic Performance Model

To analyze performance we use the Disk-Access Machine (DAM) model. [Aggrawal, Vitter 88]

- Two levels of memory.
- Two parameters: block size B, and memory size M.
- The game: Minimize the number of block transfers. Don't worry about CPU cycles.



Theoretical Results

Structure	Insert	Point Query
B-Tree	$O\left(\frac{\log N}{\log B}\right)$	$O\left(\frac{\log N}{\log B}\right)$
Append	$O\left(\frac{1}{B}\right)$	$O\left(\frac{N}{B}\right)$
Fractal Tree	$O\left(\frac{\log N}{B^{1-\varepsilon}}\right)$	$O\left(\frac{\log N}{\varepsilon \log B^{1-\varepsilon}}\right)$
Fractal Tree $(\varepsilon = 2)$	$O\left(\frac{\log N}{\sqrt{B}}\right)$	$O\left(\frac{\log N}{\log B}\right)$

Example of Insertion Cost

- 1 billion 128-byte rows. $N = 2^{30}$; $\log(N) = 30$.
- 1MB block holds 8192 rows. $B = 8192$; $\log B = 13$.

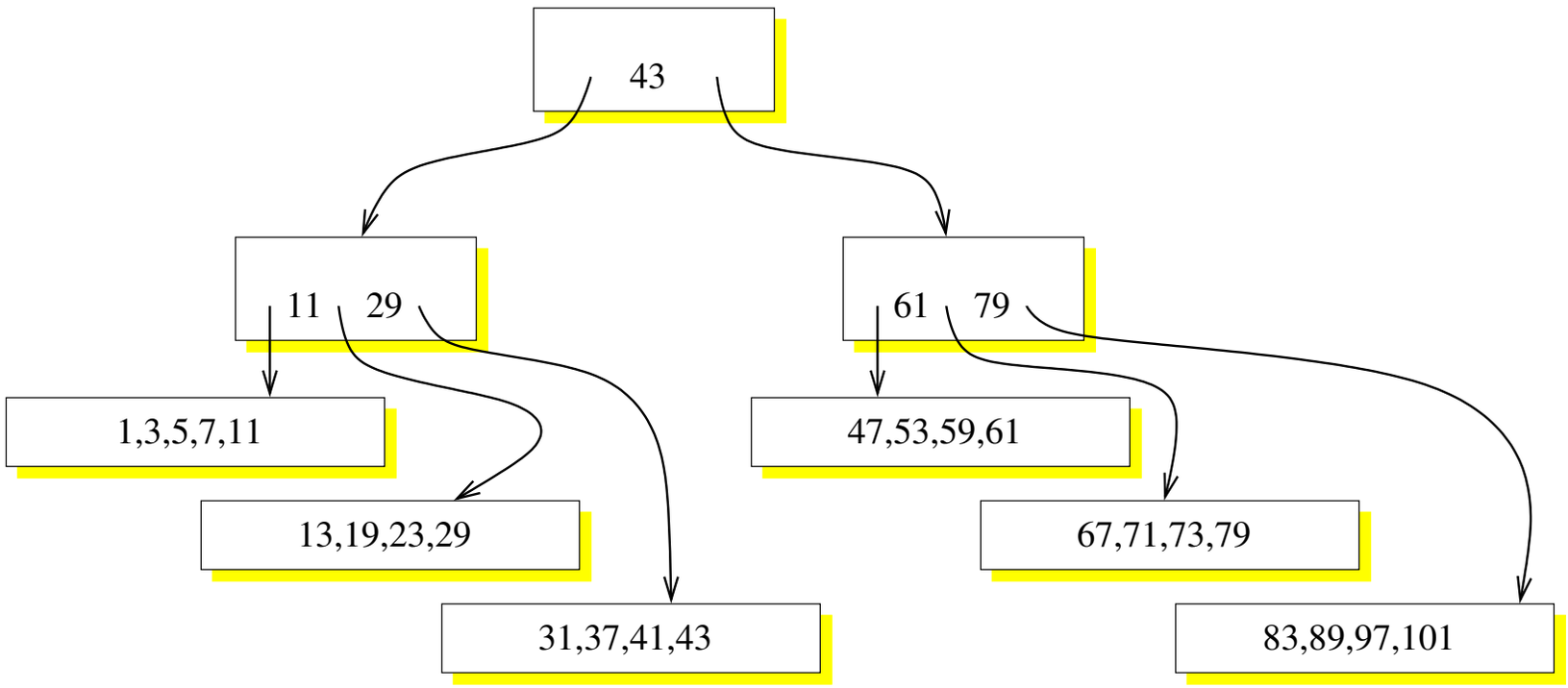
B-Tree: $O\left(\frac{\log N}{\log B}\right) = O\left(\frac{30}{13}\right) \approx 3$

Fractal Tree: $O\left(\frac{\log N}{B}\right) = O\left(\frac{30}{8192}\right) \approx 0.003.$

Fractal Trees use $<<$ 1 disk I/O per insertion.

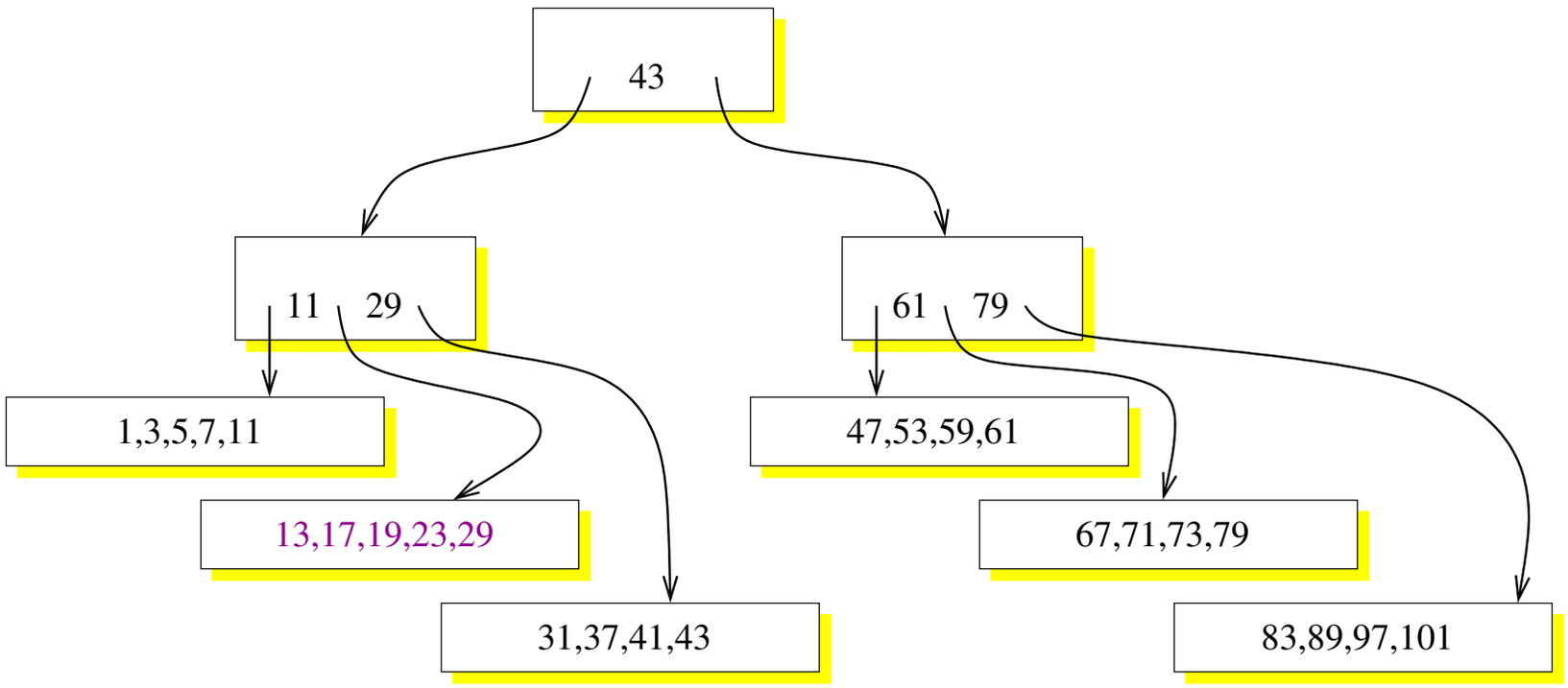
Recipe for a fractal tree

(Similar to a Buffered Repository Tree [Buchsbaum Goldwasser Venkatasubramanian Westbrook '06, Brodal Fagerberg '02].) Start with a B-tree:



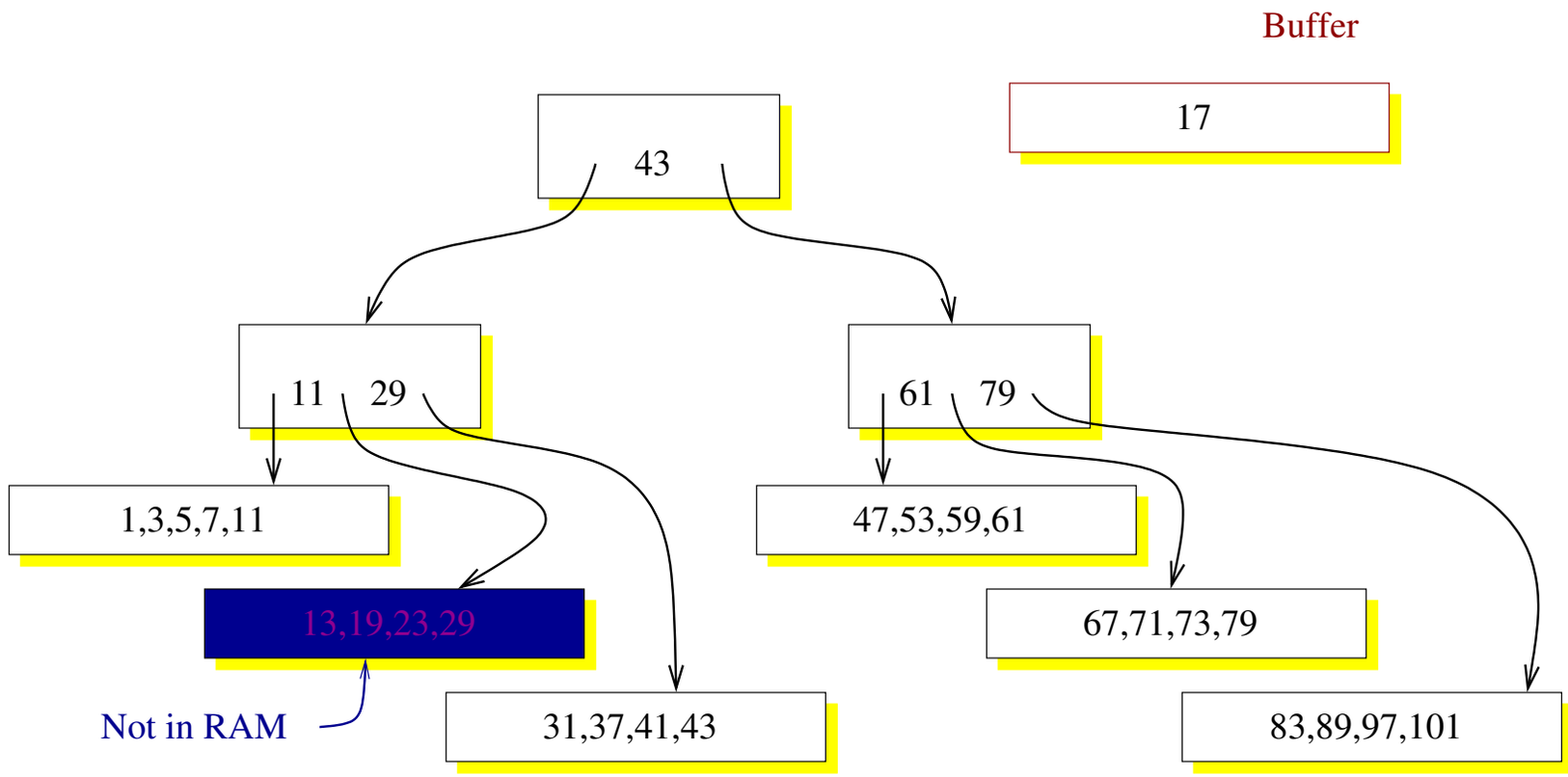
Oops, I forgot 17.

Added 17



Maybe needed several disk I/Os to bring blocks in to store 17.

InnoDB Adds a Buffer

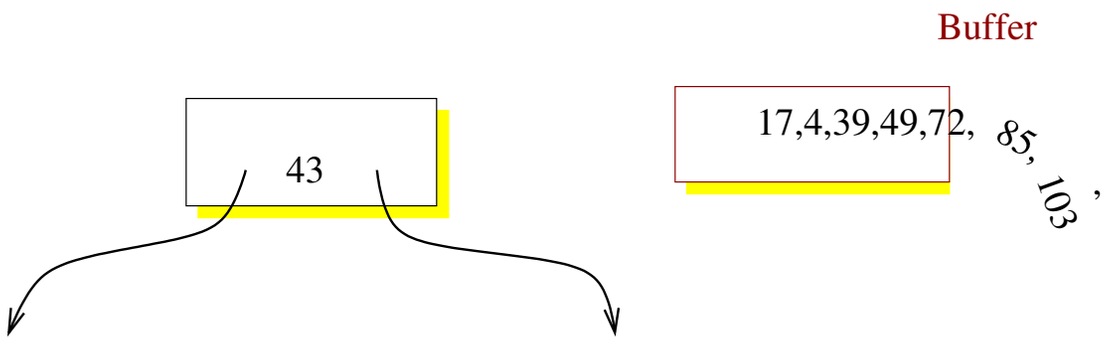


- If a block is on disk (not in RAM) then keep the row in a buffer.
- Later, move rows from the buffer to the tree nodes.

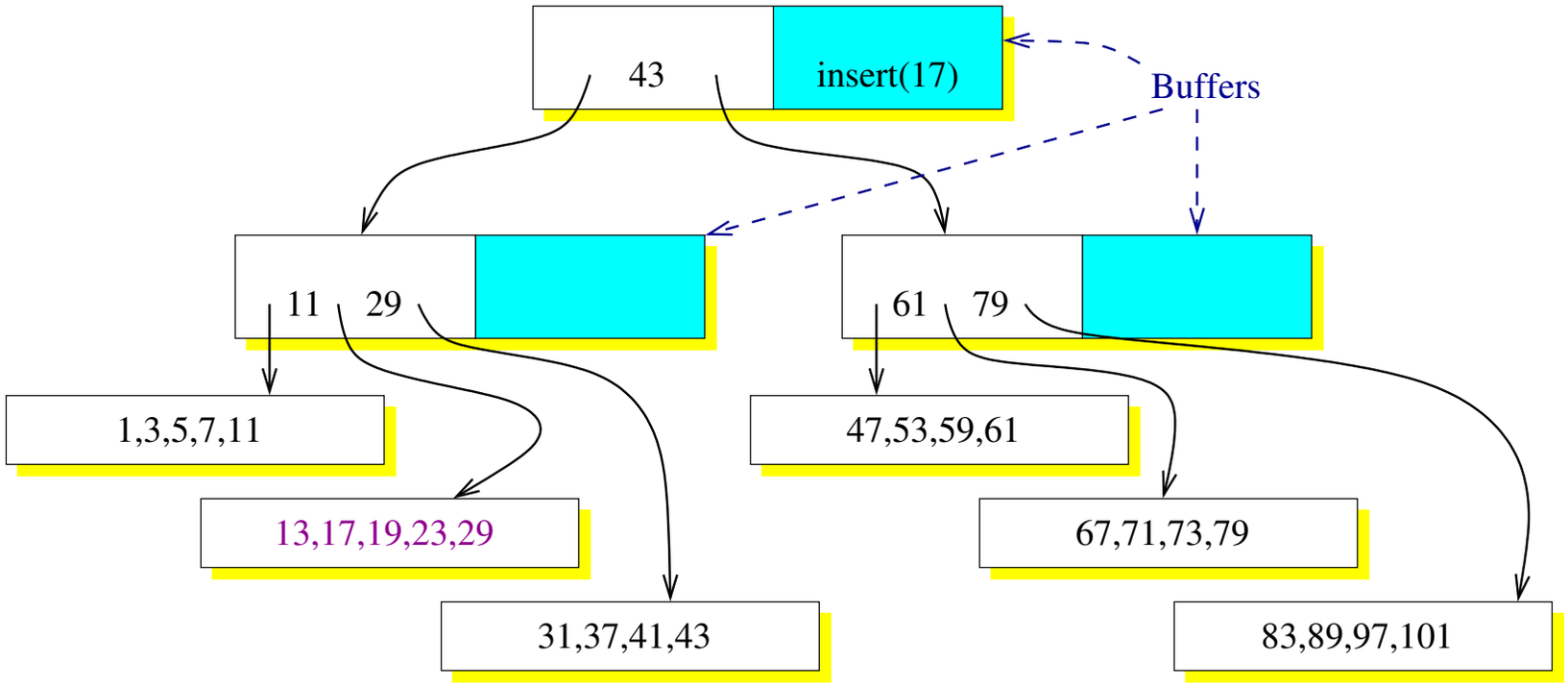
A Buffer Helps (a Little)

Sometimes an InnoDB-style buffer works great: The buffer collects several rows for a single tree node, and for the cost of one I/O, several rows are moved to disk.

Sometimes the buffer fills up, and the system slows down. Even in these situations, an insertion buffer helps (but not by much).

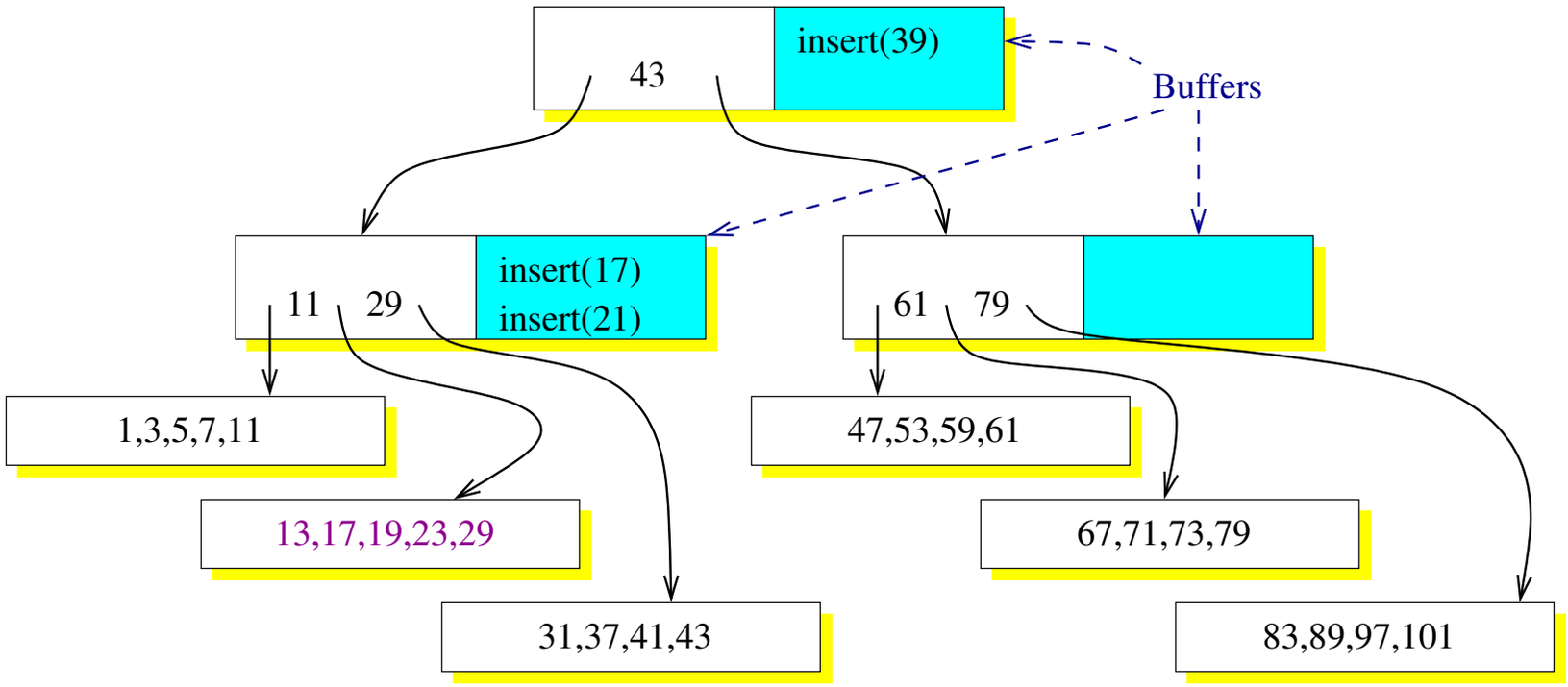


Fractal Trees Indexes Add Many Buffers



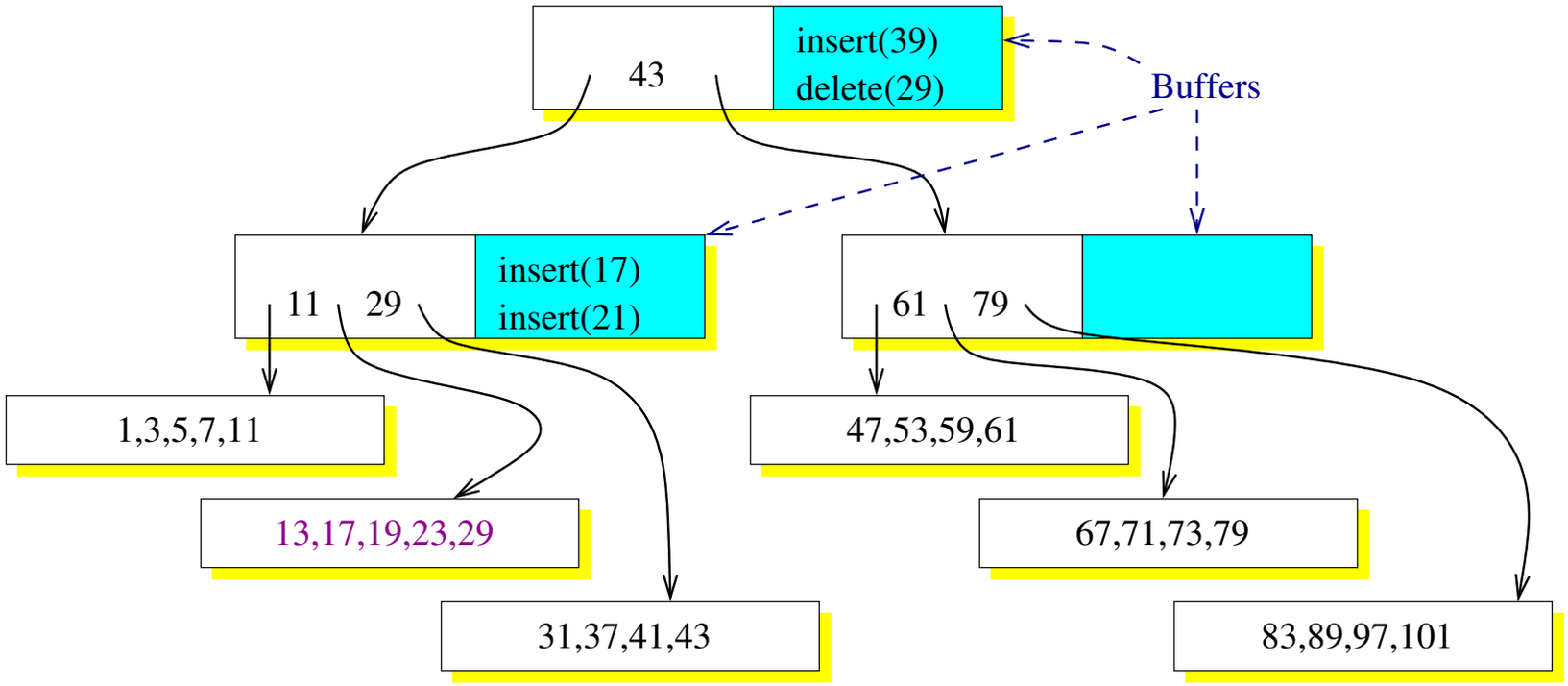
- All the internal nodes of the tree include buffers.
- To insert a row, put a message in the root's buffer.

When a Buffer Overflows

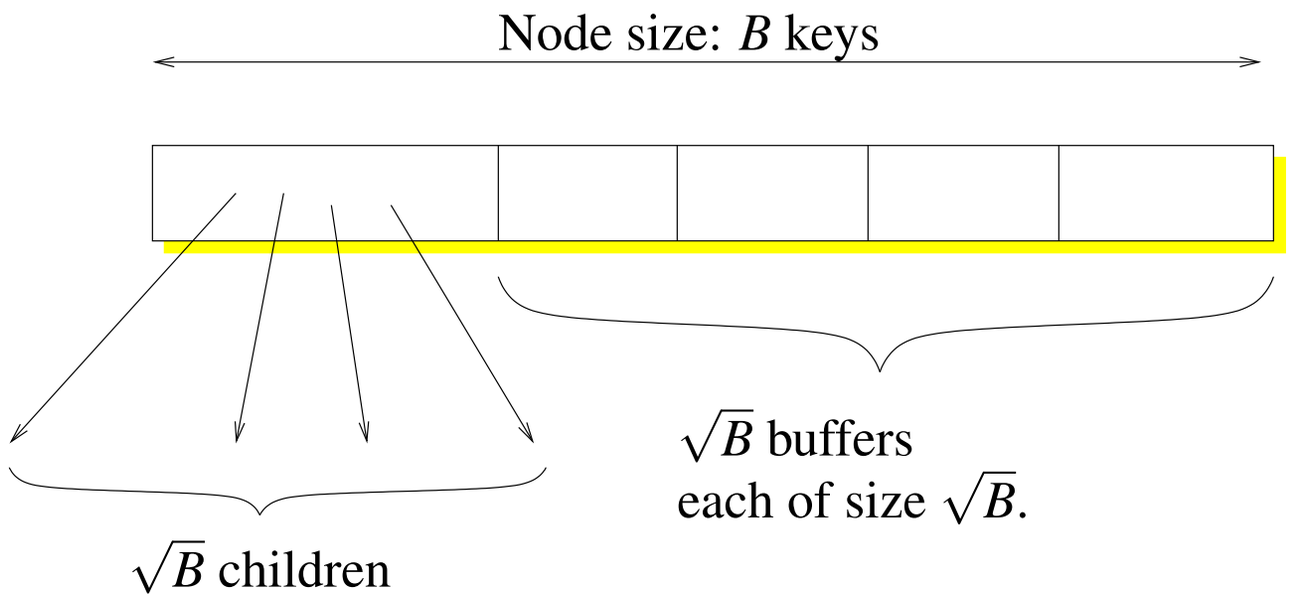


- Move messages down the tree when buffers fill.
- When a message arrives at a leaves, apply it.

Deletes are Also Messages



Sizing the Buffers



So when we do an I/O to move data down, we move $\Omega\sqrt{B}$ elements.
The height of the tree is $O(\log_{\sqrt{B}} N) = O(\log_B N)$.
Lookups cost $O(\log_B N)$ I/Os, matching the B-tree.
No insertion costs more than $O(\log_B)$, matchingB-tree.

Average Cost is Lower

Even for the worst-case insertion pattern, the average insertion cost is lower.
The average insertion cost can be accounted for by looking at

- The number of times an element moves down the tree, $O(\log_B N)$, times
- the cost of the move $O(1/\sqrt{B})$.
- Total cost is

$$O\left(\frac{\log_B N}{\sqrt{B}}\right).$$

Comparing Data Structures

Disk I/Os are the important metric. How many does it take to do an insertion or a query?

Data structure	Insertion cost	Query cost (in practice)
B-tree	Many I/Os	*One I/O*
LSM tree	*Fraction of an I/O*	Many I/Os
Fractal Tree	*Fraction of an I/O*	*One I/O*

- Fractal tree indexes reduce the cost of index maintenance by two to three orders of magnitude.
- So you can ingest data faster and maintain more indexes.

Cache-Aware Data Structures

We treat main memory as a cache for disk. This cache has two important parameters:

- *B*, the block size for transfers from disk to memory, and
- *M*, the total size of main memory.

The B-tree encodes *B* into the data structure (as the fanout of the tree), and so the B-tree is *cache aware*.

Cache-Oblivious Data Structures

A data structure that does not know the cache parameters is *cache oblivious* [Frigo Leiserson Prokop 99].

Surprisingly there are many asymptotically optimal cache oblivious algorithms and data structures, including

- Matrix multiplication,
- FFT,
- Priority queues, and
- B-trees.

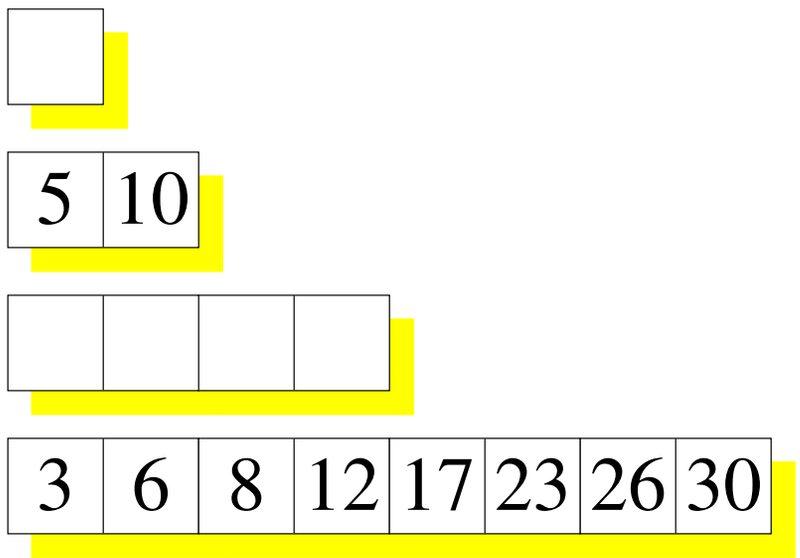
Example: Cache-Oblivious Matrix Multiplication

In matrix multiplication using divide and conquer, as the algorithm recursively attacks smaller matrices, eventually the submatrices fit in main memory.

When $n \approx \sqrt{M}$, several $n \times n$ submatrices fit in main memory and the algorithm performs n^3 arithmetic operations for only n^2/B cache misses.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} E & F \\ G & H \end{pmatrix}$$

A Cache-Oblivious Fractal Tree



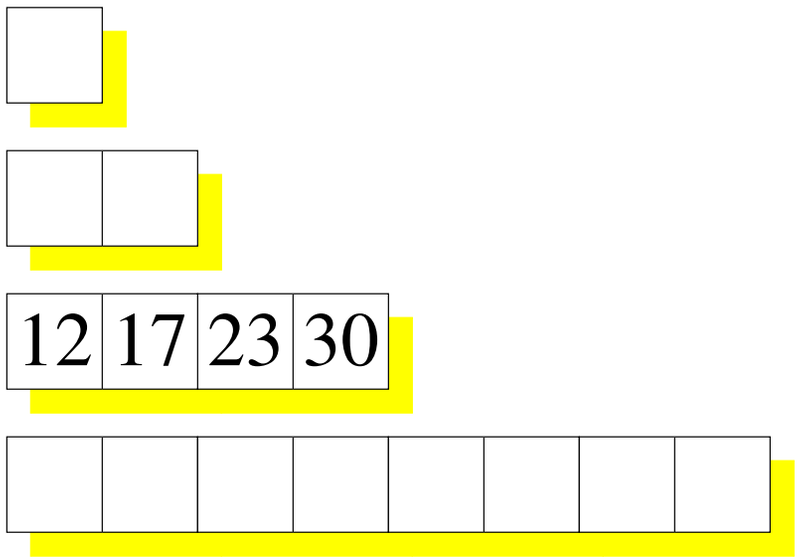
(Based on COLA [BFK07])

- $\log N$ arrays, one array for each power of two.
- Each array is completely full or empty.
- Each array is sorted.

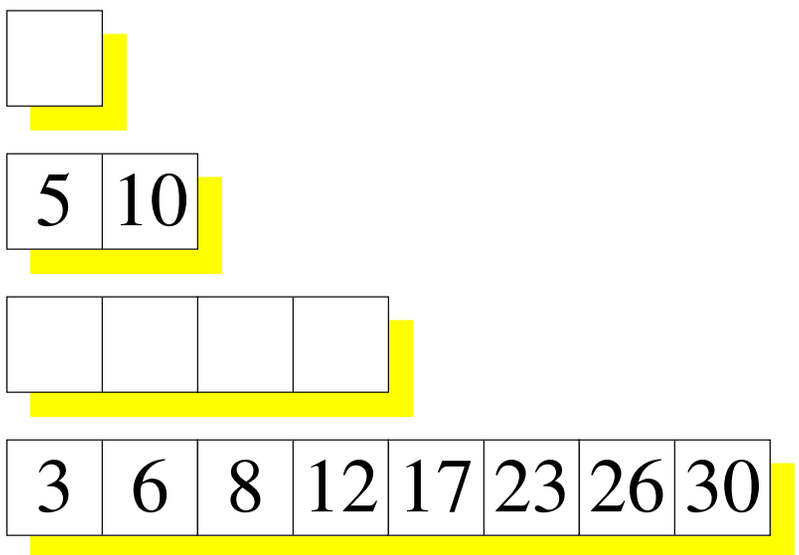
This data structure is *cache oblivious*. The block size B does not appear in the data structure, and yet it achieves similar performance.

Example (4 elements)

If there are 4 elements in our fractal tree, the structure looks like this:



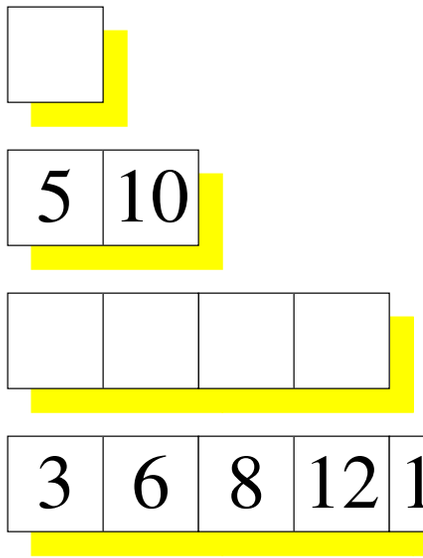
If there are 10 elements in our fractal tree, the structure might look like this:



But there is some freedom.

- Each array is full or empty, so the 2-array and the 8-array must be full.
- However, which elements go where isn't completely specified.

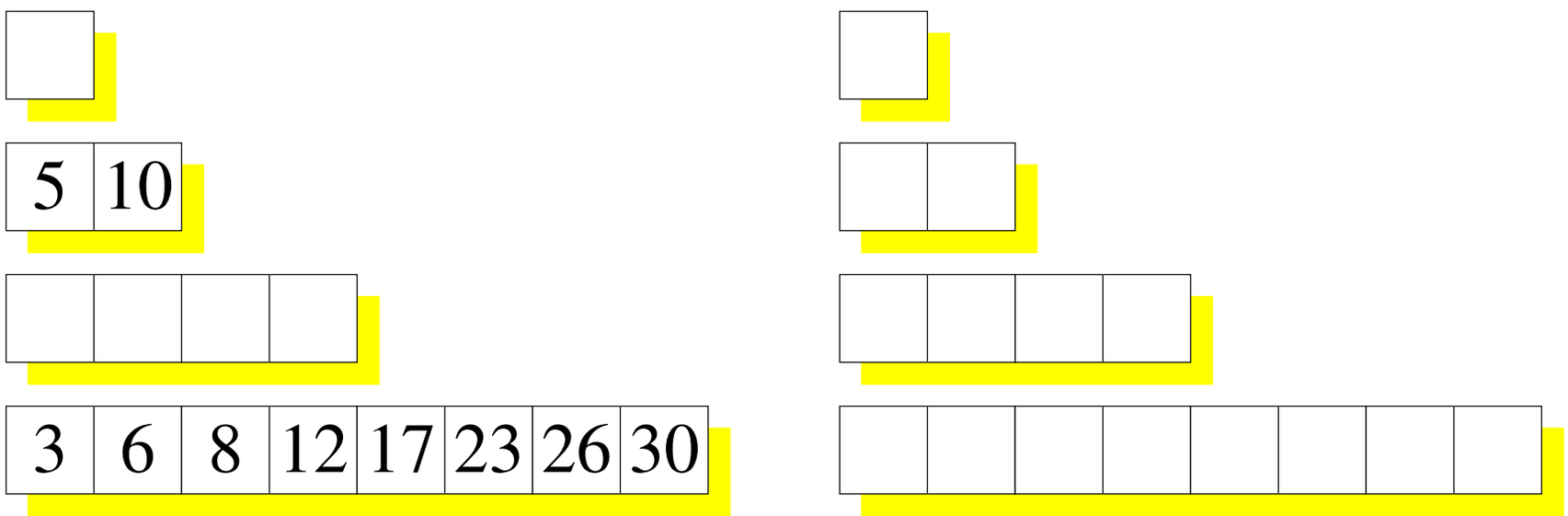
Searching in a Simplified Fractal Tree



- Idea: Perform a binary search in each array.
- Pros: It works. It's faster than a table scan.
- Cons: It's slower than a B-tree at $O(\log^2 N)$ block transfers.

Let's put search aside, and consider insert.

Inserting in a Simplified Fractal Tree

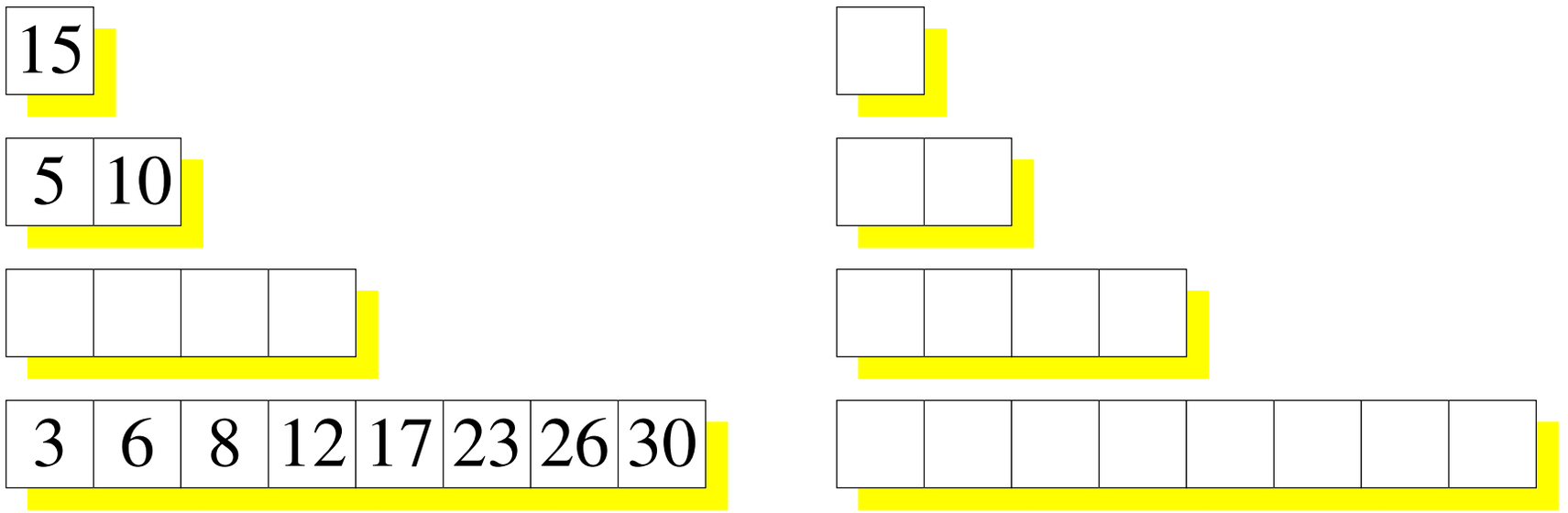


Add another array of each size for temporary storage.

At the beginning of each step, the temporary arrays are empty.

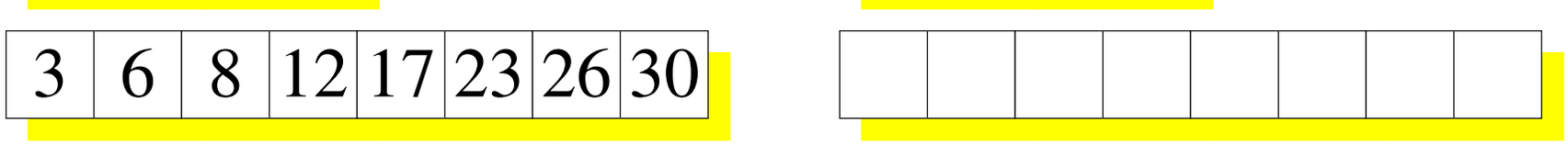
Insert 15

To insert 15, there is only one place to put it: In the 1-array.

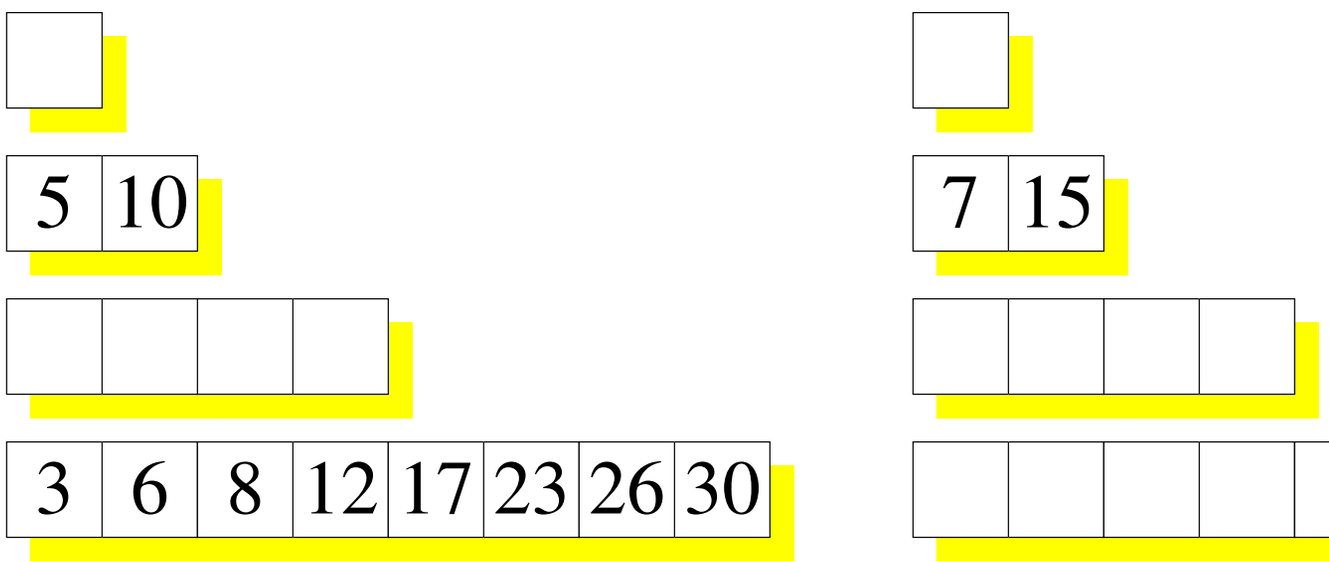


Insert 7

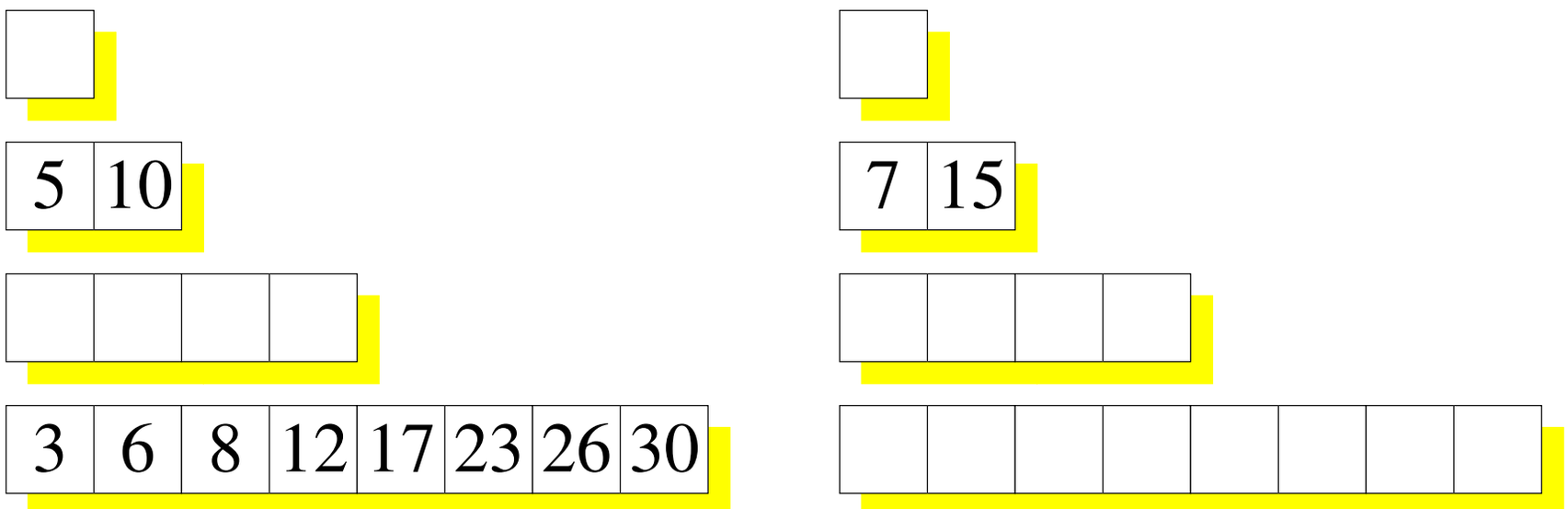
To insert 7, no space in the 1-array. Put it in the temp 1-array.



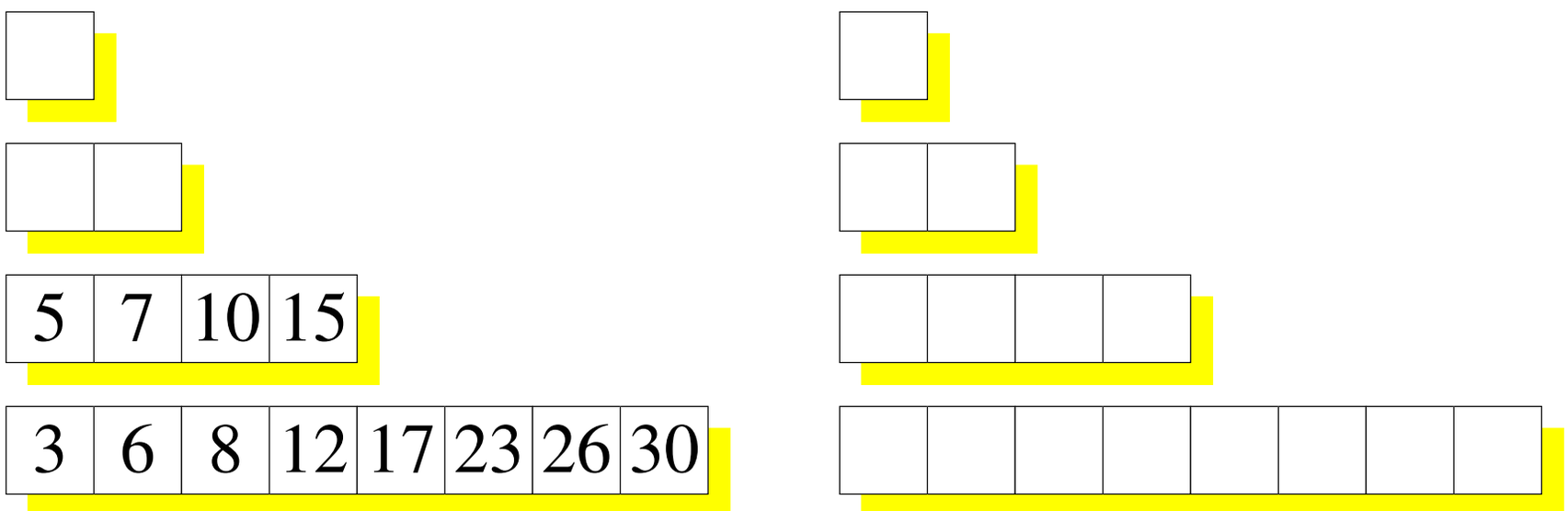
Then merge the two 1-arrays to make a new 2-array.



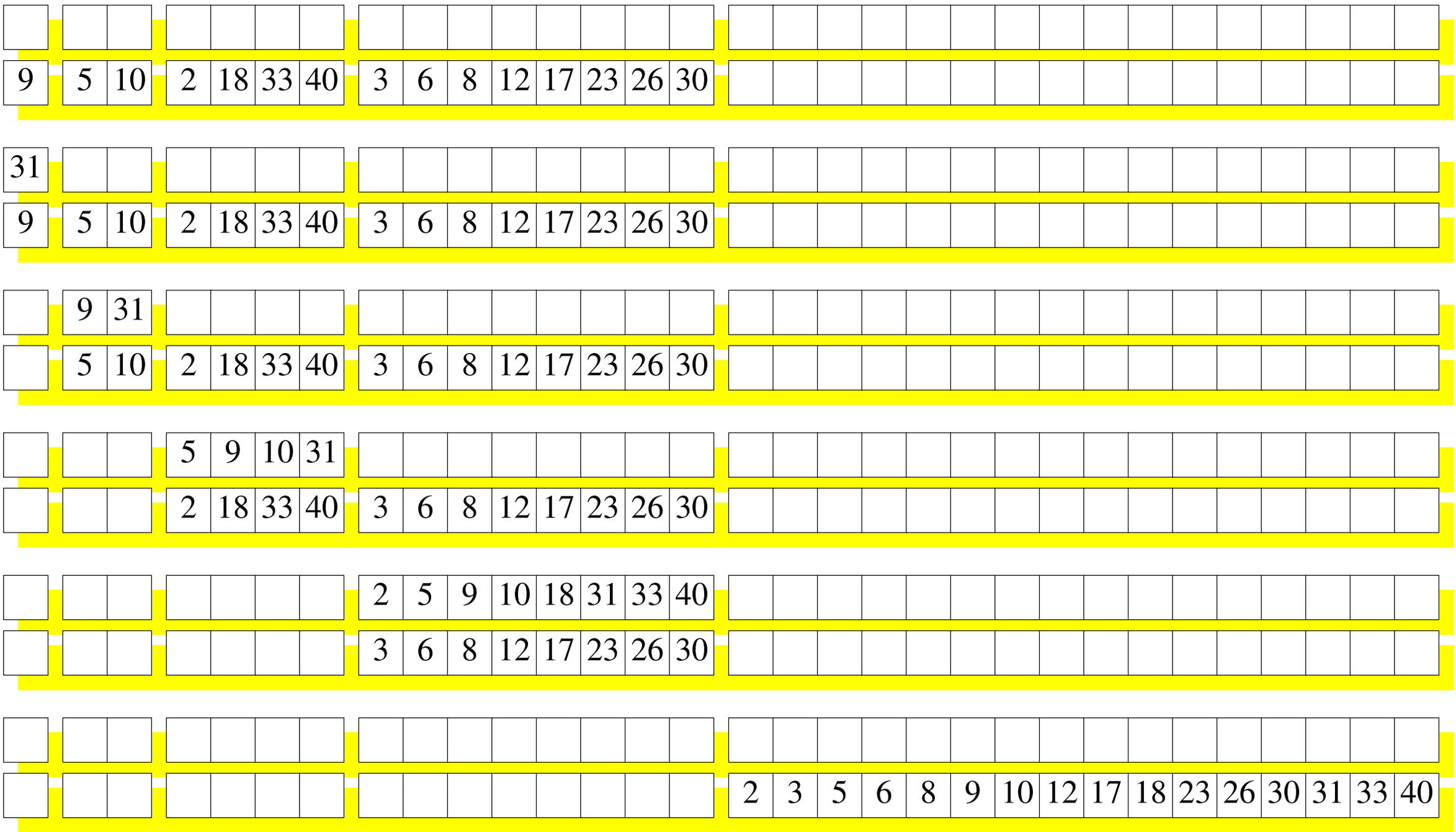
Not done inserting 7



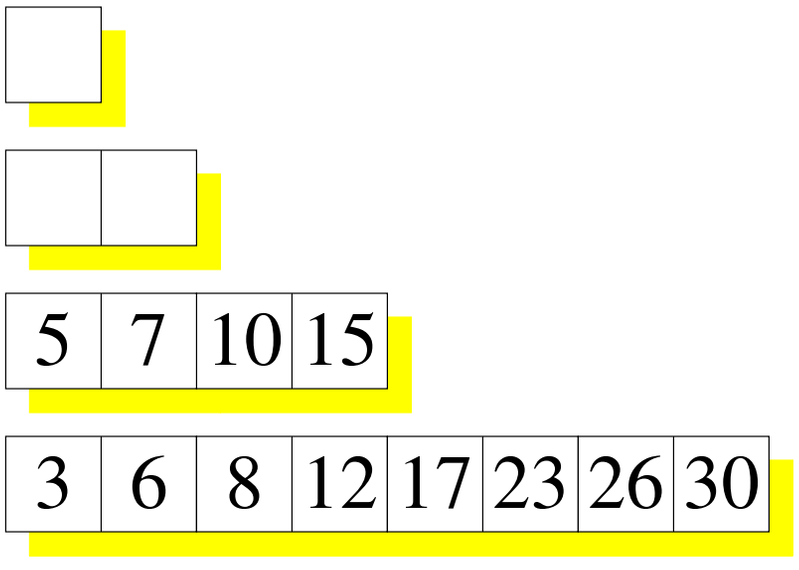
Must merge the 2-arrays to make a 4-array.



An Insert Can Cause Many Merges



Analysis of Insertion into Simplified Fractal Tree



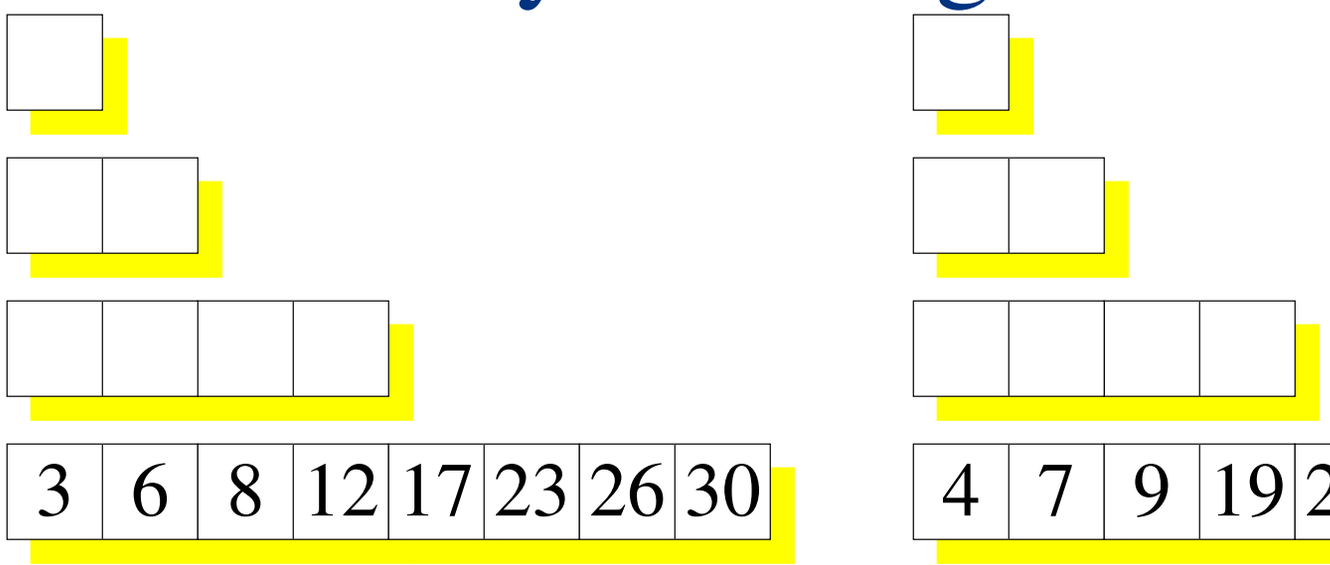
- Cost to merge 2 arrays of size X is $O(X/B)$ block I/Os.

  Merge is very I/O efficient.

- Cost per element to merge is $O(1/B)$ since $O(X)$ elements were merged.
- Max # of times each element is merged is $O(\log N)$.
- Average insert cost is $O\left(\frac{\log N}{B}\right)$.

Improving Worst-Case Insertion

Although the *average* cost of a merge is low, occasionally we merge a *lot* of stuff.

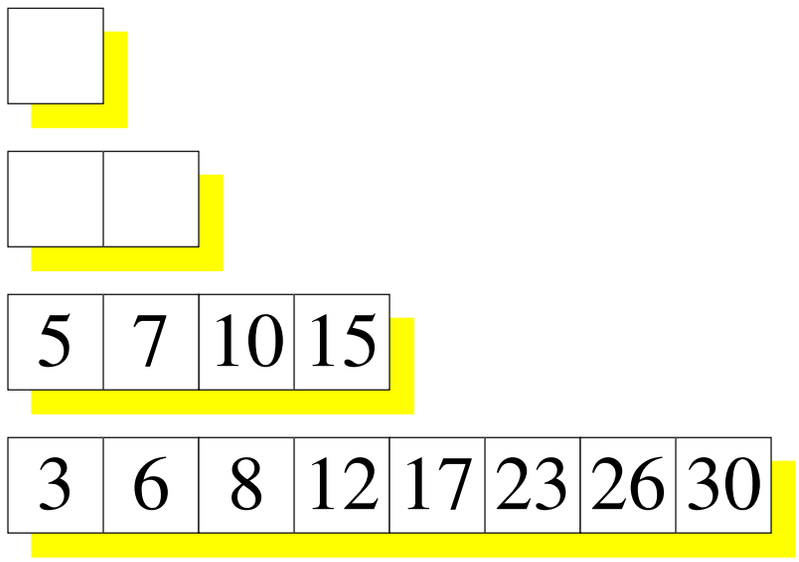


Idea: A separate thread merges arrays. An insert returns quickly.

Lemma: As long as we merge $\Omega(\log N)$ elements for every insertion, the merge thread won't fall behind.

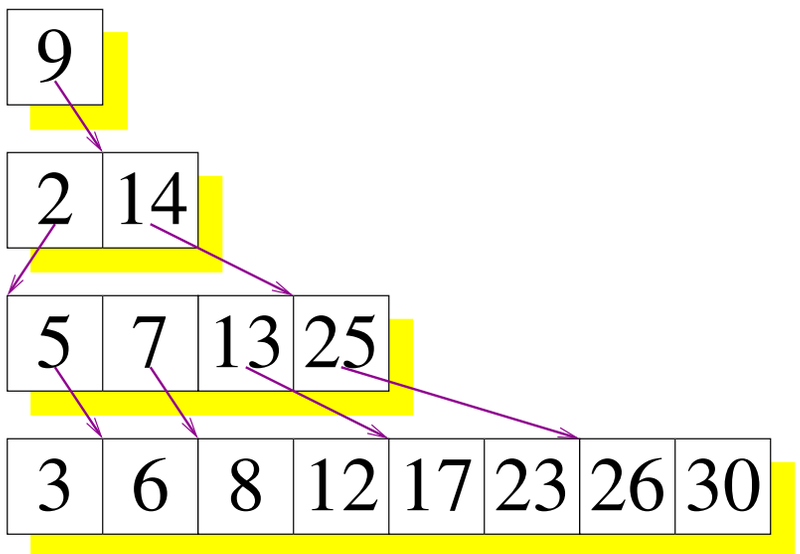
Speeding up Search

At $\log^2 N$, search is too expensive.
Now let's shave a factor of $\log N$.



The idea: Having searched an array for a row, we know where that row would belong in the array. We can gain information about where the row belongs in the next array

Forward Pointers



Each element gets a forward pointer to where that element goes in the next array using *Fractional Cascading*. [Chazelle, Guibas 1986]

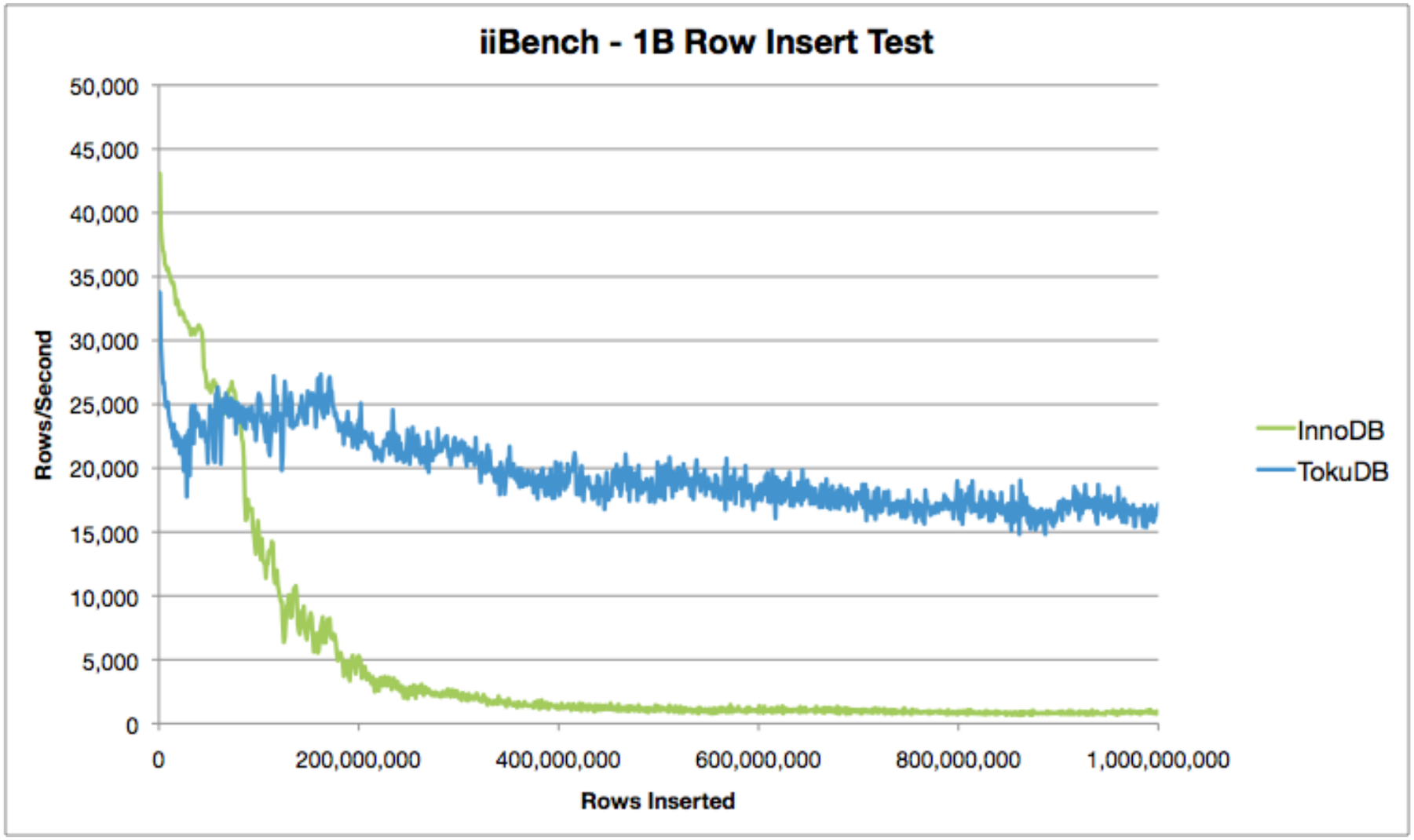
If you are careful, you can arrange for forward pointers to land frequently (separated by at most a constant). Search becomes $O(\log N)$ levels, each looking at a constant number of elements, for $O(\log N)$ I/Os.

Industrial-Grade Fractal Trees

A real implementation, like TokuDB, must deal with

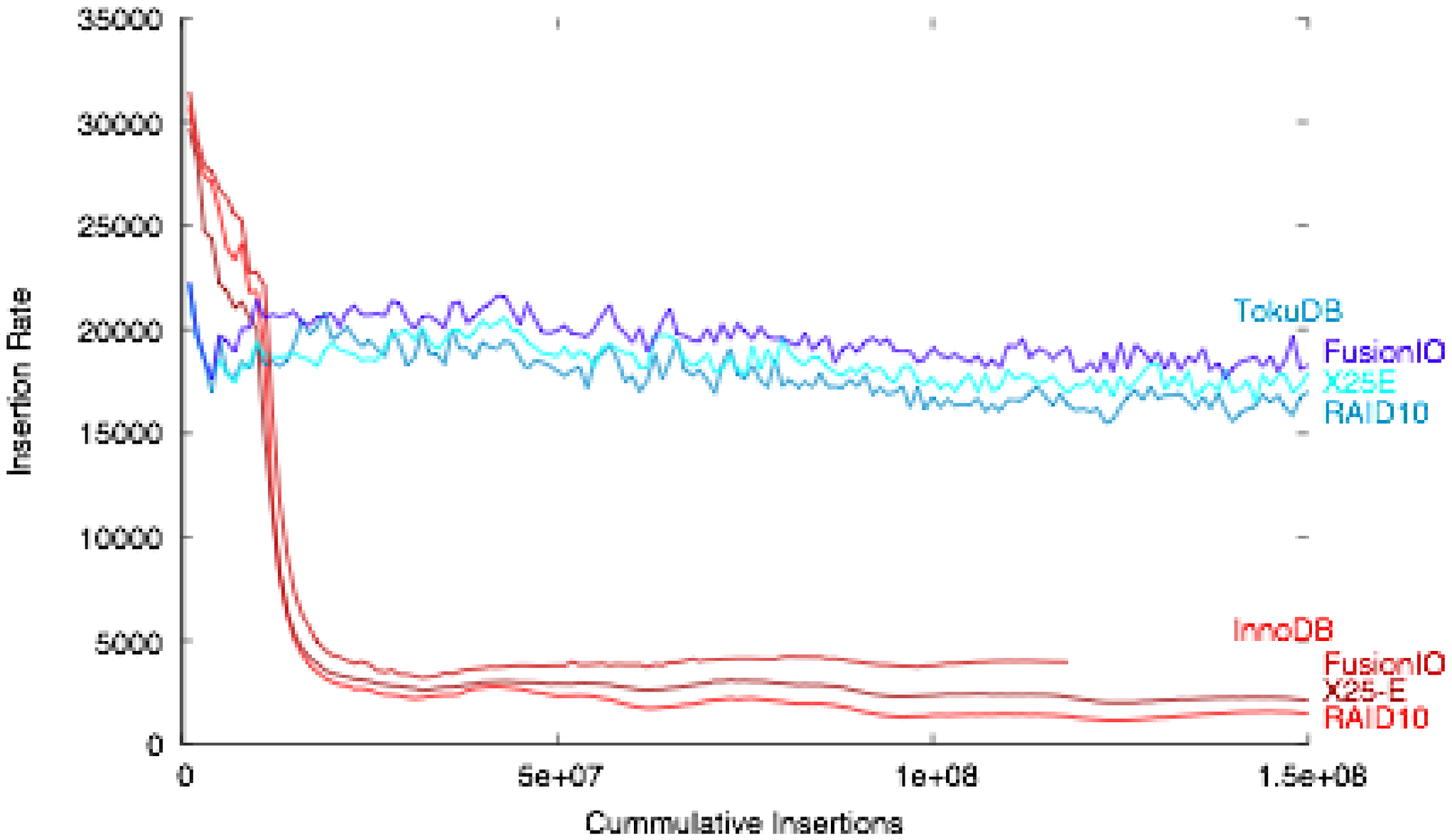
- Variable-sized rows;
- Deletions as well as insertions;
- Transactions, logging, and ACID-compliant crash recovery;
- Must optimize sequential inserts more;
- Better search cost: $O(\log_B N)$, not $O(\log_2 N)$;
- Compression; and
- Multithreading.

iiBench Insert Benchmark



iiBench was developed by us and Mark Callaghan to measure insert performance.
Percona took these measurements about a year ago.

iiBench on SSD



TokuDB on rotating disk beats InnoDB on SSD.

Disk Size and Price Technology Trends

- SSD is getting cheaper.
- Rotating disk is getting cheaper faster. Seagate indicates that 67TB drives will be here in 2017.
- Moore's law for silicon lithography is slower over the next decade than Moore's law for rotating disks.

Conclusion: big data stored on disk isn't going away any time soon.

Fractal Tree indexes are good on disk.

One cannot simply indexes in main memory. One must use disk efficiently.

Speed Trends

- Bandwidth off a rotating disk will hit about 500MB/s at 67TB.
- Seek time will not change much.

Conclusion: Scaling with bandwidth is good. Scaling with seek time is bad.

Fractal Tree indexes scale with bandwidth.

Unlike B-trees, Fractal Tree indexes can consume many CPU cycles.

Power Trends

- Big disks are much more power efficient per byte stored than little disks.
- Making good use of disk bandwidth offers further power savings.

Fractal Tree indexes can use 1/100th the power of B-trees.

CPU Trends

- CPU power will grow dramatically inside servers over the next few years. 100-core machines are around the corner. 1000-core machines are on the horizon.
- Memory bandwidth will also increase.
- I/O bus bandwidth will also grow.

Conclusion: Scale-up machines will be impressive.

Fractal Tree indexes will make good use of cores.

The Future

- Fractal Tree indexes dominate B-trees theoretically.
- Fractal Tree indexes ride the right technology trends.
- In the future, all storage systems will use Fractal Tree indexes.