

The VIX® Index Decomposition

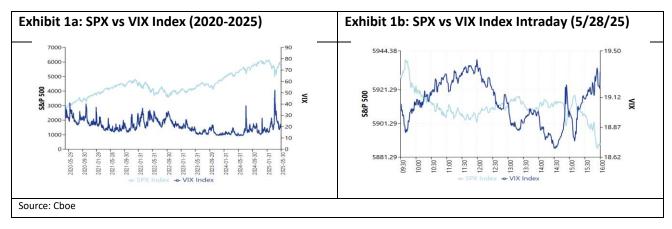
A Heuristic Framework to Unravel Unexpected Behaviors in the VIX Index

Introduction

Prior to the US market open on August 5, 2024, the S&P 500® Index had fallen nearly 5% as traders labored to process the 12% overnight decline in the Nikkei and the ensuing implications of the Yen-Carry Unwind for US equities. In response to this market uncertainty, the Cboe Volatility Index® (the VIX® Index) jumped from a prior day close of 23 and registered an intraday preopen high of 65 before closing near 40 at the end of the day's trading session. The 3x intraday surge in the VIX Index befuddled market commentators with many expressing surprise at the magnitude of the VIX Index move and with some even speculating that the VIX Index was somehow "broken". Although Cboe quickly verified that the VIX Index priced exactly in accordance with its stated methodology, this reinforced a need for additional tools to help investors better understand the drivers of the VIX index. To that end, Cboe is introducing the first iteration of a VIX Index decomposition framework where we have distilled the movements of the VIX index into 6 principal components commonly used by volatility traders to describe the evolution of the volatility surface, including the "expected" VIX index change, the premia due to the supply and demand for optionality, as well as changes due to bearish vs. bullish positioning in the underlying SPX options market.

General Expectations for the VIX Index Behavior

In general, many financial professionals think of the VIX Index as a "fear gauge" and thus expect the VIX Index to have an inverse relationship with respect to US broad market equities, or more specifically, the S&P 500 Index (SPX). On the whole, that relationship does in fact bear out both intraday as well as over time. To illustrate, Exhibits 1a and 1b below show the stereotypical relationship of the SPX Index and the VIX Index on a daily basis from May 2020 to May 2025 and throughout the entire trading day on May 28, 2025 from 9:30am to 4pm. In both cases, one observes that the VIX Index tends to rise when the SPX Index declines and vice-versa. In fact, hedgers rely upon this relationship in using VIX Index instruments in the construction of hedges for large-cap US equity portfolios.





Counterintuitive VIX Index Behaviors

Occasionally, however, the VIX Index behaves counter to typical investor expectations and can be segmented into two effects: 1) a counter-intuitive move in terms of directionality and 2) a counter-intuitive move in terms of magnitude.

<u>Directionality</u>: The counter-intuitive directional move is a scenario in which VIX Index moves in the same direction as the SPX Index. This scenario is often referred to by volatility traders as "spot down, vol down" (i.e., the VIX Index declines as the SPX Index falls) or conversely "spot up, vol up" (i.e., the VIX Index rises as the SPX Index advances). Since the VIX Index inception in 1990, the VIX Index and the SPX Index have exhibited daily positive co-movement roughly 20% of the time with the SPX Index and VIX Index advances congruently 10.5% of the time and falling congruently 10.3% of time. (See Exhibit 2)

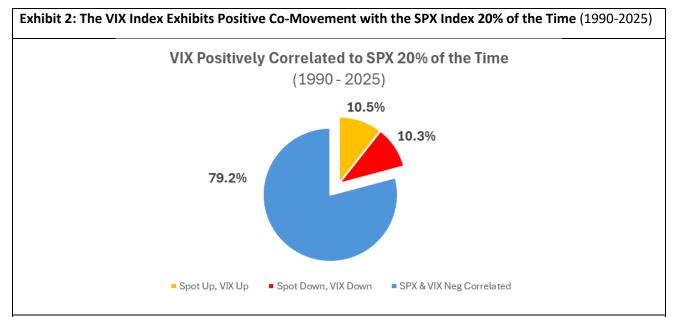


Exhibit 2a: Notable Examples of Positive Co-Movement: SPX Down, VIX Index Down (1990-2025)

Date	SPX	VIX	SPX 1D Chg	VIX Implied 1D % Chg	VIX 1D pt Chg
3/20/2020	2304.92	66.04	-4.34%	-4.16%	-5.96
3/23/2020	2237.40	61.59	-2.93%	-3.88%	-4.45
8/25/2015	1867.61	36.02	-1.35%	-2.27%	-4.72
4/3/2020	2488.65	46.80	-1.51%	-2.95%	-4.11
3/31/2020	2584.59	53.54	-1.60%	-3.37%	-3.54
12/18/2008	885.28	47.34	-2.12%	-2.98%	-2.50
7/20/1990	361.61	15.63	-1.02%	-0.98%	-2.93
6/27/2016	2000.54	23.85	-1.81%	-1.50%	-1.91
2/26/1991	362.81	20.38	-1.21%	-1.28%	-2.48
8/13/2002	884.21	35.82	-2.17%	-2.26%	-1.23



Date	SPX	VIX	SPX 1D Chg	VIX Implied 1D % Chg	VIX 1D pt Chg
5/5/1997	830.24	20.13	2.12%	1.27%	2.62
3/17/2003	862.79	31.75	3.54%	2.00%	0.77
11/27/2002	938.87	27.25	2.80%	1.72%	1.28
3/8/1993	454.71	16.22	1.93%	1.02%	2.14
12/30/1991	415.14	17.63	2.14%	1.11%	1.93
7/15/2009	932.68	25.89	2.96%	1.63%	0.87
6/1/2009	942.87	30.04	2.58%	1.89%	1.12
5/11/1990	352.00	19.81	2.38%	1.25%	1.22
12/12/2022	3990.56	25.00	1.43%	1.57%	2.17
3/25/2020	2475.56	63.95	1.15%	4.03%	2.28

<u>Magnitude</u>: A counter-intuitive move in terms of magnitude refers to scenarios in which the VIX Index either over-reacts or under-reacts relative to the typical expectations of casual observers. (The VIX Index reaction to the Aug 5, 2025 Yen-Carry Unwind described in the introduction is a recent example of the former scenario.)

The "VIX is Broken" Narrative

Source: Cboe, Bloomberg

In aggregate, both instances of unexpected moves in the VIX Index have sometimes fueled a "VIX is broken" narrative, despite the VIX Index performing in accordance to its published methodology. This underscores the need for greater understanding of the factors that drive the VIX index.





The VIX Index Methodology

Exhibit 4 shows the formula for the VIX Index which involves hundreds and sometimes thousands of component calculations. Conceptually, it calculates the weighted average implied volatility across the active strike spectrum of two S&P-500 option contract expiries.

Exhibit 4: The VIX® Index Formula

The VIX Index Calculation: Step-by-Step

Stock indexes, such as the S&P 500 Index, are calculated using the prices of their component stocks. Each index employs rules that govern the selection of component securities and a formula to calculate index values.

The VIX Index is a volatility index comprised of options rather than stocks, with the price of each option reflecting the market's expectation of future volatility. Like conventional indexes, the VIX Index calculation employs rules for selecting component options and a formula to calculate index values.

The generalized formula used in the VIX Index calculation[§] is:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2$$

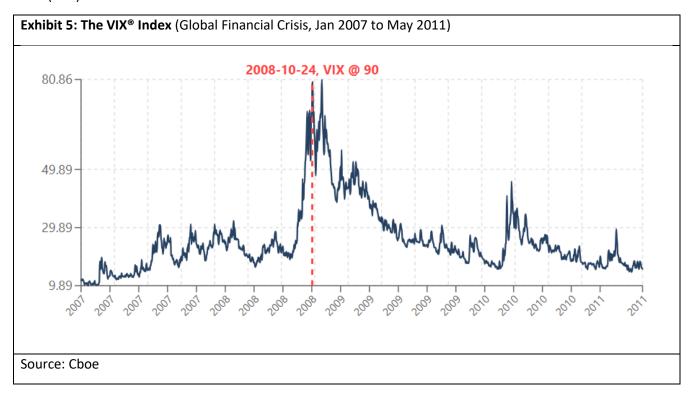
where

σ	$VIX index = \sigma \times 100$	ΔK_i	Interval between strike prices – half the difference between the strike on either side of K_i :
T	Time to expiration (in years)		$K_{i+1} - K_{i-1}$
F	Option-implied forward price		$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$
K_0	First strike equal to or otherwise immediately below the	R	Risk-free interest rate to expiration
for	ward index level, F	T.	Nisk-free lifterest rate to expiration
K_i	Strike price of the j^{th} out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$.	$Q(K_i)$	The midpoint of the bid-ask spread for each option with strike Ki.



SPX Vol Dynamics Underlying the VIX Index Calculation

To better visualize the extremity of the dynamics that underscore the VIX Index calculation, we show in Exhibits 5a-5d, snapshots of the front and back month SPX Index implied volatility skews that define the VIX Index as it evolved into, during, and coming out of one of the most violent episodes of US equity market volatility - the Global Financial Crisis (GFC).



Each graph in Exhibits 5a-5d is depicted in fixed strike space with SPX Index option strikes along the x-axis and the implied volatility of SPX Index options contract along the y-axis. Call side implied volatilities are depicted in orange while the put side implied volatilities are depicted in blue.

Note that we only show implied volatilities for the strikes of the options contracts that were actually used to calculate the VIX Index for the relevant day. Thus, if an SPX Index options contract for a given expiry exists but is not included in the VIX Index calculation (per the methodology) – it will NOT be represented in the skew graphs.

To maintain a fixed point of reference, we show in light blue/light orange, the front and back month SPX Index implied volatility skews as of February 1, 2007, at the onset of the Credit Crisis. Recall that on this date the SPX Index was trading near its high for the year and the VIX Index was priced near a historical low of 10.



0%

330

660

990

Strike Price

1320

Exhibit 5a: S&P 500 Index Option Implied Volatility Skew -Start of the Global Financial Crisis (Oct 10, 2007) Volatility Skew: 2007-02-01 vs 2007-10-10 2007-10-10 (VIX: 16.67) **Front Month Back Month** 120% Call 2007-02-01 Call 2007-02-01 Call 2007-10-10 Call 2007-10-10 Put 2007-02-01 Put 2007-02-01 96% 96% Put 2007-10-10 Put 2007-10-10 Implied Volatility 48% Implied Volatility 48% 24% 24%

Exhibit 5b: S&P 500 Index Option Implied Volatility Skew During Global Financial Crisis (July 23, 2008)

1650

0%

660

990

Strike Price

1320

1650

330

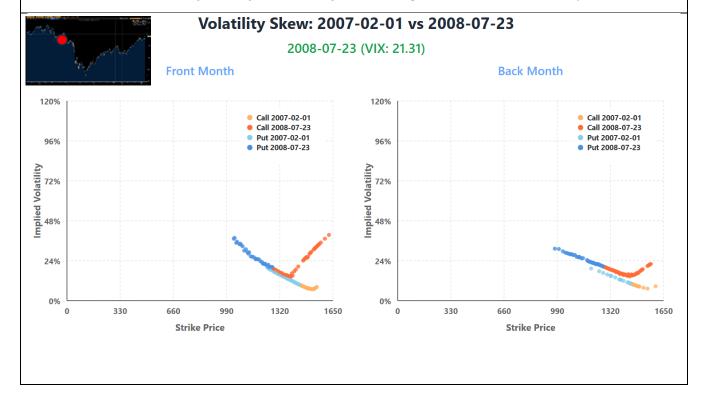
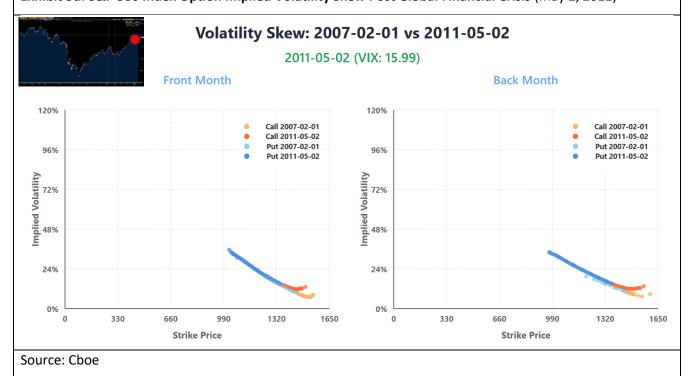




Exhibit 5c: S&P 500 Index Option Implied Volatility Skew During Global Financial Crisis (Oct 24, 2008) Volatility Skew: 2007-02-01 vs 2008-10-24 2008-10-24 (VIX: 79.13) **Front Month Back Month** 120% 120% Call 2007-02-01 Call 2007-02-01 Call 2008-10-24 Call 2008-10-24 Put 2007-02-01 Put 2007-02-01 96% 96% Put 2008-10-24 Put 2008-10-24 Implied Volatility Implied Volatility 72% 48% 48% 24% 24% 0% 0% 330 660 990 1320 1650 0 330 660 990 1320 1650 Strike Price Strike Price

Exhibit 5d: S&P 500 Index Option Implied Volatility Skew Post Global Financial Crisis (May 2, 2011)





Note the varied number of skew dynamics at play as the markets traversed throughout the GFC including:

- Parallel shifts higher and lower of the skew surface
- Expansion and compressions of the skew strike range, and
- Fluctuations of the curvature for both put and call side skew.

The VIX Index, in short, attempts to summarize the complexity and interplay of all the above dynamics into a single number.

Note: To view an animated rendering of the evolution of SPX Index implied volatility skews used to calculate the VIX Index across notable tail risk events, we invite you to contact your Cboe sales representative.

The VIX Index Decomposition (Overview)

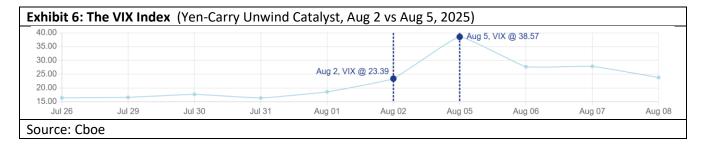
To gain deeper insight into the dynamics that drive a given move in the VIX Index, Cboe has developed a heuristic tool for market observers to disentangle a given VIX® Index move based on a six-factor principal components style framework*, commonly used amongst volatility traders to discuss the supply and demand for optionality along the volatility surface in response to macroeconomic and geopolitical catalysts. (* akin to the 3-factor principal components framework used by fixed income traders to describe yield curve dynamics)

To enhance intuition for the deterministic factors driving the VIX Index, the following sections explaining the decomposition will portray the principal components driving the VIX Index as if the VIX Index were calculated according to a single 30-day fixed strike skew, (i.e., the 30-day skew used in the analysis is interpolated in variance space based on the prevailing front and back week option contracts used to calculate the VIX Index). It then implements a perturbation analysis based on the following six factors:

- 1. Expected move per sticky strike
- 2. Parallel shift of the skew
- 3. Change in the slope of the put skew gradient
- 4. Change in the slope of the call skew gradient
- 5. Demand for downside convexity
- 6. Demand for upside convexity

Yen-Carry Unwind Scenario (Overview)

To intuit the contributions of the various components, we will reference the interpolated EOD fixed-strike SPX Index implied volatility skews as it transitioned from August 2, 2024 into the August 5, 2024 Yen-Carry Trade Unwind which saw the SPX index fall 3% and the VIX index jump from 23 to 38 (close to close). See Exhibit 6.

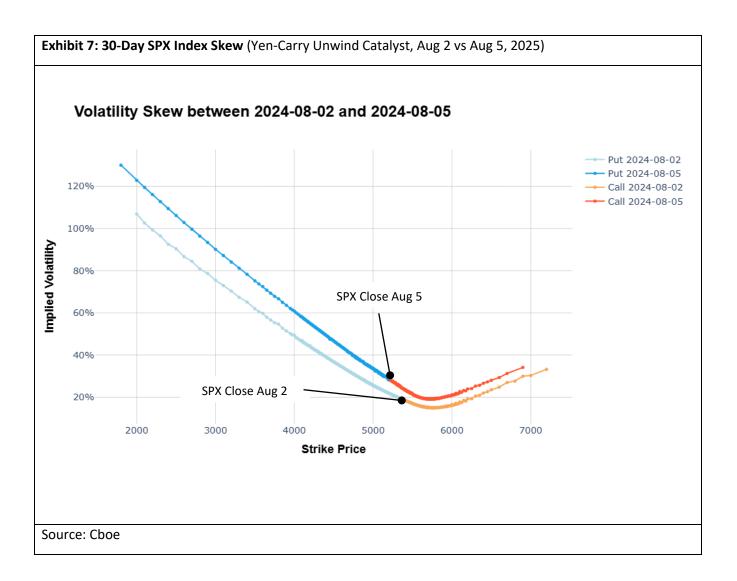




Accordingly, the chart in Exhibit 7 depicts two SPX Index implied volatility skews as of:

- Aug 2nd 2024 (bottom skew in light blue/ light orange) and
- Aug 5th 2024 (top skew in dark blue/ dark orange).

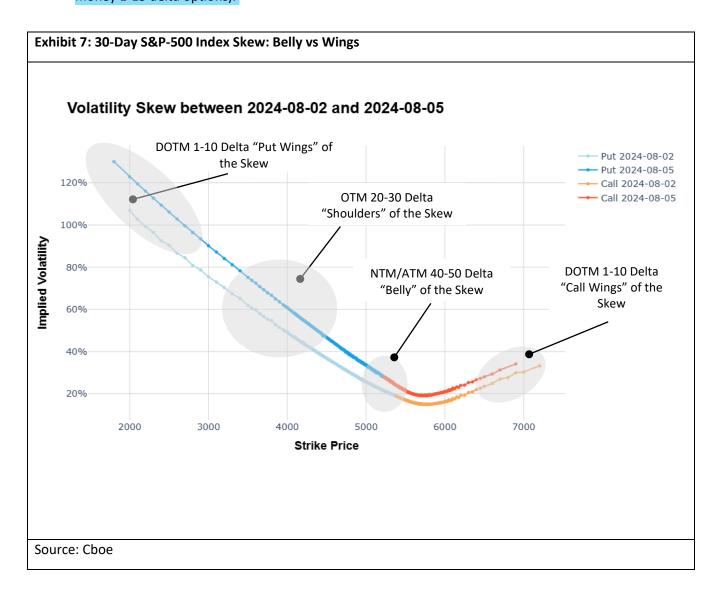
The EOD close for the SPX index (i.e., ATM strike) for Aug 2nd of 5,346 is denoted by the intersection between the light blue and light orange lines along the Aug 2 skew. Likewise, the EOD close for Aug 5th of 5,186 is denoted by the intersection between the dark blue and light orange lines along the Aug 5th skew.





In our discussion of the various VIX Index decomposition factors, we will segment the options strike range which defines the skew into the following sections:

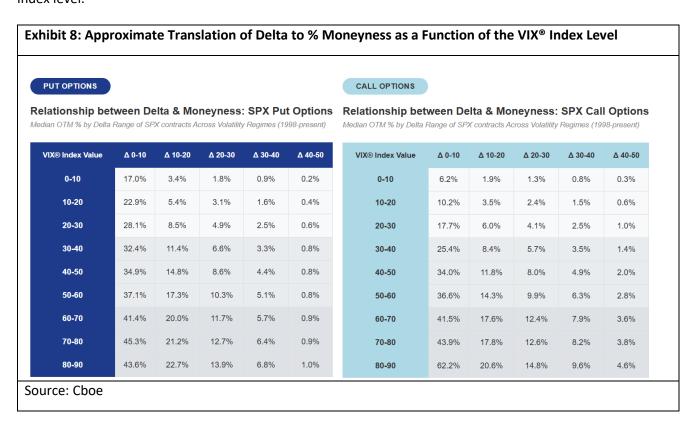
- Sticky strike and parallel shift factors reference the "belly" of the implied volatility skew (i.e., the strike range associated with at-the-money and near-the-money 45-50 delta options)
- put and call and skew gradient factors reference the "shoulders" of the skew (i.e., the strike range associated with the most liquid and active out-of-the-money 20-40 delta options),
- convexity factors reference the "wings" of the skew (i.e., the strike range associate with deep out-of-themoney 1-15 delta options).





Delta vs Moneyness

Going forward, we will use the term "delta" to refer to the moneyness of an option. The term "delta" in this context is often interpreted as the probability the market estimates for a particular option to expire in the money. In Exhibit 8 below, we provide a rough translation of delta into moneyness space for a 30-day option as a function of the VIX Index level.



Expected Move per Sticky Strike (1st Factor)

<u>Intuition</u>: The expected VIX Index move per sticky strike quantifies the change in the VIX Index that has already been priced into the volatility markets given a move in the SPX Index.

<u>Details</u>: The implied volatility skew is essentially a mapping of where volatility traders expect at the money implied volatility to be given a specific SPX Index price. "Sticky strike" is a term coined by Emmanuel Derman in his seminal paper "Regimes of Volatility" in 1999. It refers to the scenario in which at-the-money implied volatility evolves exactly according to a pre-established fixed strike skew. In other words, under a sticky strike regime, once one observes a particular move in SPX Index spot, one can then use the implied volatility skew to determine volatility traders' expectations for ATM implied volatility.



Empirically, sticky strike does in fact do a good job describing the evolution of volatility over short time periods in low to moderate vol of vol market regimes. To demonstrate, in Exhibit 9 below we show the contribution due to the "expected move per sticky strike" as a percentage of the daily VIX Index move conditional on:

- the overall VIX Index level and
- the magnitude of the VIX index move (in absolute points)

The intersection of "VIX < 15" and "<1 pt", for example, is 88.1% which means that when the VIX Index is below 15 and experiences a move of less than one point, "the expected move per sticky strike" component is responsible for 88.1% of the entire VIX Index move and the average size of that contribution is 0.35 VIX points.

Thematically, the analysis reveals that in "typical" volatility regimes (VIX <20) with subdued levels of vol-of-vol (daily VIX move < 1 VIX pts), daily VIX changes are almost entirely explained by the expected move per sticky strike. Even in cases where VIX is elevated but vol-of-vol is low to moderate (daily VIX move= 1 to 3pts), sticky strike remains the dominant component in explaining VIX index moves. Sticky strike therefore forms the basis for how market participants generally expect the VIX Index to move.

Exhibit 9: Average % Contribution of Expected Move per Sticky Strike to Explain VIX® Index Movement

		VIX® Index Move						
		<1 pt	1-3pts	3-5pts	5-10 pts	10+ pts		
	<15	86.9% (0.35)	46.1% (1.39)	44.4% (3.18)	No data	No data		
VIX ® Index Value	15-20	85.4% (0.43)	55.7% (1.64)	34.3% (3.72)	36.5% (5.72)	28.5% (11.75)		
	20-30	87.2% (0.49)	50.6% (1.68)	48.9% (3.75)	35.7% (6.84)	16.9% (15.18)		
	30+	85.7% (0.54)	51.5% (1.79)	55.2% (3.16)	31.6% (6.37)	31.3% (14.95)		

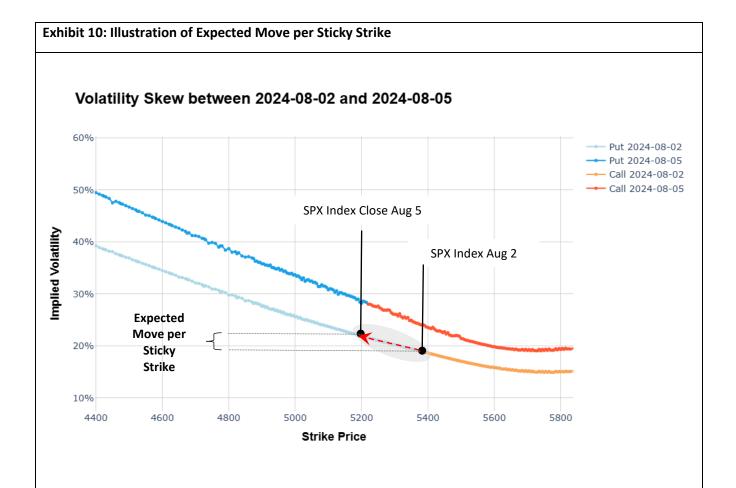
Values represent average percentage contribution of Expected Move per Sticky Strike factor to explain the VIX® Index movement.

Numbers in parentheses show average VIX® Index point movement for each element.

Source: Cboe

Note however, that the expected move per sticky strike does not wholly explain the move in the VIX Index and that the contribution of this component decreases as the intensity of the shock relative to the a priori level of the VIX Index increases. In fact, it is these incidents when the other five decomposition factors overwhelm the expected move per sticky strike in which the VIX Index is accused of being "broken".





Example: Yen-Carry Unwind Scenario (Decomposition Factor: Sticky Strike)

To illustrate the expected move in the VIX Index due to sticky strike we reference as a proxy, the SPX Index skew as of Aug 2nd 2024 and calculate the change in the level of ATM implied volatility as it rides up the Aug 2nd implied volatility skew using the following methodology:

- SPX Index close Aug 2 = 5,346, ATM implied vol = 18.99
- SPX Index close Aug 5 = 5,186
- Implied volatility for 5,186 strike using Aug 2 skew = 21.55
- Expected change in SPX Index option implied volatility due to 3% SPX Index decline from 5,346 to 5,186 = 21.55 18.99 = 2.57



Parallel Shift of the Volatility Skew (2nd Factor)

<u>Intuition</u>: The contribution of an upward parallel shift of the volatility skew represents a wholesale bid for optionality across the entire strike range -- usually in response to a major macroeconomic or geopolitical downside catalyst. This upward parallel shift is followed by a series of offsetting of downward shifts as the catalyst dissipates.

<u>Parallel Upshift</u>: A parallel upward shift of the volatility skew occurs when volatility traders are faced with a new and unexpected (~2+ standard deviation) market dislocation. As a result, volatility traders react to this high level of uncertainty by repricing the cost of optionality for both puts AND calls higher across the entire volatility surface to avoid selling optionality too cheaply in face of the new catalyst. In cases where the underlying fundamentals of the markets have completely changed due to the shock, the bid for volatility represented by the parallel upshift may remain for weeks, months, and even years going forward. Volatility traders refer to this persistence of volatility premium as a "volatility regime change".

Exhibit 11: Frequency (%) Parallel Shifts Materially Exacerbates Up Moves in the VIX Index

Material Parallel Shift Contribution - VIX® Index Move Up

		VIX® Index Move						
		<1 pt	1-3pts	3-5pts	5-10 pts	10+ pts		
	<15	1.5% (0.40)	4.3% (1.41)	100.0% (3.18)	No data	No data		
Index Value	15-20	No data	13.6% (1.77)	100.0% (3.72)	100.0% (5.72)	100.0% (11.75)		
VIX ® Inc	20-30	3.4 % (0.48)	32.1% (1.81)	50.0% (3.97)	100.0% (7.39)	100.0% (15.18)		
	30+	No data	25.0% (1.61)	No data	100.0% (6.23)	100.0% (15.29)		

Values represent percentage of cases where parallel shift has a material contribution (\geq 50% Expected Move per Sticky Strike and the contribution is >1 VIX® Index point).

Numbers in parentheses show the average VIX® Index move for each bucket.

In Exhibit 11 above, we demonstrate the increasing importance of the parallel shift component as a function of the initial VIX Index level and the size of the VIX Index move. Specifically, the table shows the % frequency in which parallel shifts materially exacerbates the expected move of the VIX Index where "material" is defined as a contribution in excess of 50% of the expected move per sticky strike. The average contribution of the parallel shift component in the VIX Index points is denoted in light gray. To demonstrate, the intersection of {VIX index value < 15 and VIX move of 3-5 pts} on the top center row of the table means that in cases where the VIX Index rises in excess of 3 to 5 pts off an initial value of <15, the parallel shift



component is responsible for at least half of the move in the VIX Index 100% of the time. The amount of the move in VIX pts under these conditions is on average 3.18 VIX pts. Taken as a whole, Exhibit 11 implies that parallel shifts of the volatility skew become an increasingly influential determinant of the VIX Index when the size of the VIX Index move is unusually large relative to expectations per the prior VIX Index level. Ofttimes it is a parallel upshift of the skew that causes the VIX Index to seemingly overreact to an SPX Index decline. In fact, we believe that the size of the parallel upshifts of the implied skew in reaction to a market decline is directly proportional to the degree of uncertainty introduced by the new catalyst.

<u>Parallel Downshift</u>: Parallel upshifts of the volatility skew are in many cases transient and is eventually countered by a series of small parallel downshifts of the volatility skew as traders gain a better understanding of implications of the 2+ standard deviation shock that incited the original upshift.

Note that parallel shifts are rarely a material component in driving the VIX Index lower. As shown in Exhibit 12, there are virtually no occurrences in which parallel shifts of the volatility surface constituted a material portion of the VIX Index move when the VIX Index was below 20. The cases in which parallel downshifts of the volatility surface are material in accentuating the VIX Index declines occur when the VIX Index mean reverts lower in a high volatility regime.

Exhibit 12: Frequency (%) Parallel Shifts Materially Exacerbates Down Moves in the VIX Index

Material Parallel Shift Contribution - VIX® Index Move Down

		VIX® Index Move						
		<1 pt	1-3pts	3-5pts	5-10 pts	10+ pts		
	<15	No data	No data	No data	No data	No data		
Index Value	15-20	No data	5.1% (-1.59)	No data	No data	No data		
VIX ® Inde	20-30	2.0% (-0.55)	18.0% (-1.64)	50.0% (-3.61)	100.0% (-5.73)	No data		
>	30+	20.0% (-0.60)	50.0% (-1.97)	33.3% (-3.16)	100.0% (-6.67)	100.0% (-14.79)		

Values represent percentage of cases where parallel shift has a material contribution (≥50% Expected Move per Sticky Strike and the contribution is >1 VIX® Index point).

Numbers in parentheses show the average VIX® Index move for each bucket.



Positive Spot/Vol Co-Movement: Note, however, there are infrequent cases in which traders will parallel shift the entire volatility surface in concert with the directional move in the SPX index level. The more common case is when traders induce an upward parallel shift of the volatility surface as the market rallies (aka spot up, vol up). This phenomenon usually occurs when the market is grinding higher but yet traders feel that despite the market advance, the inherent risks to the markets remain unchanged. Less common is the case when traders induce a downward parallel shift of the volatility surface as the market declines (aka spot down, vol down). This typically occurs as the volatility markets gain a better understanding of the implications of a tail event and vol-of-vol begins to settle down despite the fact that the tail risk event has not yet fully resolved. Positive co-movement of the VIX Index and SPX Index manifests in a volatility regime that Derman refers to as "sticky delta" which means that when the SPX Index rises (falls), the level of ATM implied volatility remains static instead of sliding down the skew to a lower (higher) level as per sticky strike.

Exhibit 13 quantifies the role of parallel shifts when SPX spot and the VIX Index experience positive co-movements. The table reveals that:

- positive co-movements of the SPX Index and the VIX Index only occur when the VIX Index moves are relatively modest (<3 VIX Index pts)
- Nearly all cases of positive co-movements of the SPX index and the VIX Index result from a parallel shift of the volatility skew

Exhibit 13: Frequency (%) Parallel Shift Contribution Has Positive Co-movement with the SPX Index

Frequency at which Parallel Shift exhibits Positive Co-movement (Spot Up, Vol Up)

		VIX® Index Move						
		<1 pt	<1 pt 1-3pts 3-5pts 5-10 pts 10+ pts					
<u>o</u>	<15	95.3% (n=43)	100.0% (n=1)	No data	No data	No data		
VIX ® Index Value	15-20	91.7% (n=24)	No data	No data	No data	No data		
	20-30	100.0% (n=13)	100.0% (n=3)	No data	No data	No data		
	30+	No data	No data	No data	No data	No data		

Values represent percentage of cases where parallel shift shows positive co-movement with SPX® Index.

Numbers in parentheses show the count of observations in each bucket.



Frequency at which Parallel Shift exhibits Positive Co-movement (Spot Down, Vol Down)

		VIX® Index Move						
		<1 pt	1-3pts	3-5pts	5-10 pts	10+ pts		
	<15	96.4% (n=28)	No data	No data	No data	No data		
VIX ® Index Value	15-20	90.9% (n=33)	100.0% (n=2)	No data	No data	No data		
	20-30	96.7% (n=30)	100.0% (n=8)	No data	No data	No data		
	30+	83.3% (n=6)	100.0% (n=3)	No data	No data	No data		

Values represent percentage of cases where parallel shift shows positive co-movement with SPX® Index.

Numbers in parentheses show the count of observations in each bucket.

Exhibit 14: Illustration of Change in the VIX Index Due to Parallel Shift of the Volatility Skew







Yen-Carry Unwind Scenario (Decomposition Factor: Parallel Shift)

To determine the expected move in the VIX Index due to parallel shift of the skew we conduct the following:

- 1. Reference the ATM strike for Aug 5 (SPX Index = 5,185).
- 2. Calculate the difference between the implied volatilities associated with 5,185 strike from the Aug 5 skew (28.85) and the 5,185 strike from the Aug 2 skew (21.55).
- 3. The difference (28.85 21.55 = 7.29) quantifies the parallel upshift from Aug 2 to Aug 5

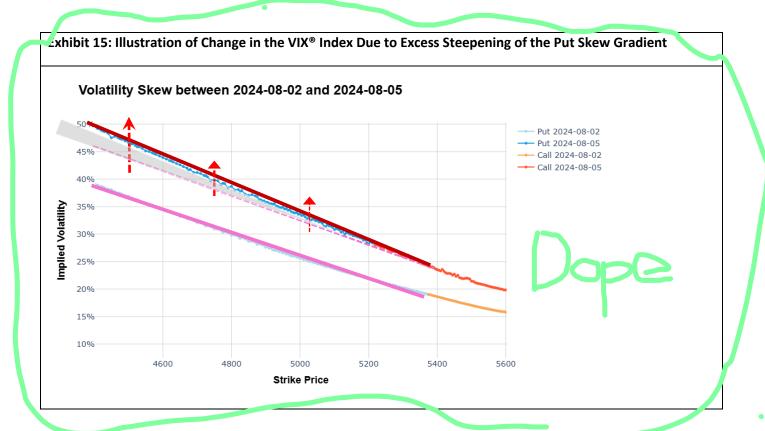


Change in the Slope of the Put & Call Skew Gradients (3rd and 4th Factors)

<u>Intuition</u>: The contribution to the VIX Index due to changes in the put/call skew gradient quantifies the marginal change in the VIX Index stemming from the marginal demand for the most liquid and actively traded protective put options and upside call options along the shoulder of the implied volatility skew.

<u>Details</u>: For a variety of reasons, portfolio managers typically trade slightly OTM puts and calls to hedge and to express directional views. Amongst practitioners, the "sweet spot" commonly targeted to strike OTM puts and calls is roughly the strike equivalent of a 30-delta option. Additionally, to cushion both the cost of premium expenditure and time decay associated with holding a long options position, portfolio managers often fund a long NTM options position by selling either a deeper OTM option of the same type resulting in a put-spread/ call-spread or a risk-reversal (long put, short call). As a result, industry conventions typically define skew as a single parameter which quantifies skew as the normalized difference between two points along the shoulder of the skew. (e.g., [30 delta put impvol – 30 delta call impvol]/ 50 delta impvol)

To allow for an additional degree of freedom, our decomposition methodology disentangles the contribution of OTM puts and OTM calls into two distinct factors and measures the change in the VIX Index by calculating the weighted average of all options along the shoulders of the implied volatility skew within the 15-45 delta strike range *in excess* of the parallel shift contribution described above. The segmentation of skew into a put skew and call skew component thus allows us to better explain counter-intuitive moves in the VIX index in which the index increases as the market rallies (aka spot up, vol up) solely due to a bid for upside optionality.







Yen-Carry Unwind Scenario (Decomposition Factor: Put & Call Skew Gradients)

During the Aug 5th 2024 Yen-Carry Unwind, we observed the more typical case in which a VIX[®] Index increase is due to a lift (steepening) in OTM put skew. Note that we are quantifying the excess bid for put implied volatilities after taking into account the change in implied volatility due to a parallel shift of the volatility surface.

We will describe an abbreviated example of the calculation for contribution to the VIX Index stemming from a change in the put skew gradient using the 30 delta put to represent the options along the put skew gradient:

- 30 delta SPX Index put strike (Aug 5) = 4,960
- implied volatility for SPX Index 4 960 strike put (Aug 5) = 36.22
- implied volatility for SPX Index 4 960 strike put (Aug 2) = 27.27
- Implied volatility change for SPX Index 4 960 strike put (Aug 5 Aug 2) = 8.95
- Parallel shift Aug 5 Aug 2 = +7.29
- Excess bid for 30 delta SPX Index put after controlling for parallel shift = 8.95 7.29 = 1.66

The contribution of the 30 delta put to the VIX Index is thus the marginal price impact of increasing the 30 delta put by 1.66 pts of implied vol. The total contribution of the put skew gradient is the re-computation of the VIX Index after adjusting prices for all puts along the 15-45 delta strike range.





Demand for Up and Downside Convexity (5th and 6th Factors)

<u>Intuition</u>: The contribution to the VIX Index due to changes in the put/call convexity quantifies the marginal change in the VIX® Index stemming from the marginal demand for tail risk put options and levered upside call options.

<u>Details</u>: "Convexity" in this context refers to low probability, low cost positions that have the potential to deliver outsized performance that is often orders of magnitude higher than the initial investment. In the context of the equity volatility markets these low probability strike options which we define as 1 to 15 delta options have a different supply and dynamic than the OTM options along the shoulder of the skew. These demand dynamics are often prompted by:

- A bid for downside tail risk protection and
- A bid for levered upside exposure or performance chasing

While the supply often results from a desire to

- fund higher cost, high probability positions in NTM options by selling the lower probability options
- create yield enhancement by earning the volatility risk premium by selling DOTM options

As a result, while the volatility skew along the 15-50 delta range of options strike tends to be relatively linear in nature, options along the 1-15 delta strikes sometimes embodies a materially higher degree of curvature.

In our decomposition, changes from convexity can arise from 2 sources:

- A change in the implied volatilities of 1-15 delta strike options in excess of the volatility move previously
 accounted for by the parallel shift and change in put/skew gradient
- The activation of lower strike put options and higher strike call options that were not included in the prior day's VIX Index calculation. (this causes the "accordion" effect shown in Exhibit 5b in which the length of the skew lengthens and contracts as the VIX Index evolves through different market risk scenarios).



Exhibit 16: Marginal Increase in the VIX Index Due to Demand for Up and Downside Convexity



Yen-Carry Unwind Scenario (Decomposition Factor: Upside & Downside Convexity)

During the Aug 5th Yen-Carry Unwind, we observed an excess bid for downside convexity (i.e., DOTM put implied volatilities) after taking into account both the change in implied volatility due to a parallel shift of the volatility surface and 2) the steepening of the 15-45 delta put skew gradient.

We will describe an abbreviated example of the calculation for contribution to the VIX Index stemming from a change in the put skew gradient using the 10 delta put to represent the options along the downside convexity strike range:

- 10 delta SPX Index put strike (Aug 5) = 4,365
- implied volatility for SPX Index 4,365 strike put (Aug 5) = 50.43
- implied volatility for SPX Index 4,365 strike put (Aug 2) = 39.71
- Implied volatility change for SPX Index 4,365 strike put (Aug 5 Aug 2) = 10.72
- Parallel shift Aug 5 Aug 2 = +7.29
- Excess bid for SPX Index 4,365 strike put after controlling for parallel shift = 10.72 7.29 = 3.43
- VIX Index attribution to put skew gradient = 2.77
- Excess bid for 10 delta SPX Index put after controlling for parallel shift = 3.43 2.77 = 0.66

The contribution of the 10 delta SPX Index put to the VIX Index is thus the marginal price impact of increasing the 10 delta put by 0.66pts of implied vol. The total contribution of the downside convexity component is the recomputation of the VIX Index after adjusting prices for all puts along the 1-15 delta strike range.



Case Study: Yen-Carry Unwind vs Liberation Day

To illustrate the interpretation of the VIX Index decomposition, we will perform a case study comparing the surge in the VIX® Index values following the Aug 5, 2024 Yen-Carry Unwind and the Apr 4, 2025 Liberation Day reaction. The reason why we chose these two particular dates is twofold. Firstly, they are two of the most jarring episodes of post-Covid market volatility. Secondly, they embody a number of similarities:

- In both cases, volatility was triggered by a 2+ standard deviation market sell-off of 3-6%
- In both cases, the VIX Index started near slightly elevated levels in the low 20s going into the shock and doubled to levels in the low to mid 40s in response to the shock.

As we examine the decomposition of the VIX Index in response to the Yen-Carry Unwind (Exhibit 17, left bar chart), we observe what is best described as the "classic" reaction of option traders to tail shock namely:

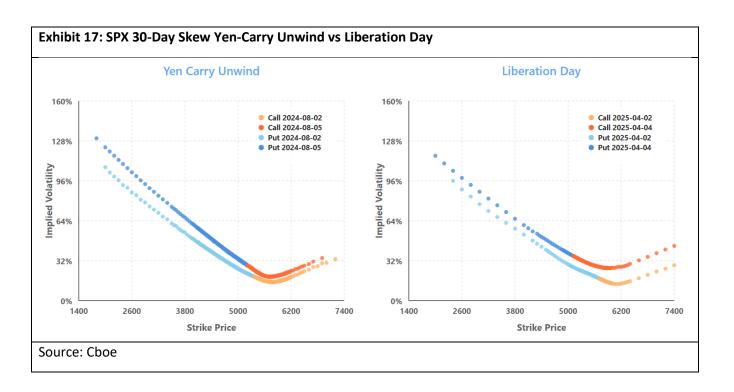
- A small contribution to higher VIX Index due to the "expected move per sticky strike" indicating that volatility traders had accorded a very low volatility risk premium to the possibility of a -3% shock during the August summer doldrums.
- A sizeable positive contribution to higher VIX Index stemming from a significant upwards parallel shift of the entire volatility surface as volatility traders repriced optionality higher in response to an "unknown"
- A moderate positive contribution to higher VIX Index stemming from a steepening in the put skew gradient as portfolio managers rushed to buy higher probability OTM puts
- A sizeable positive contribution to higher VIX Index stemming from a bid for downside convexity as portfolio
 managers bought DOTM puts as a hedge against the tail risk that the markets continue to spiral downwards.
- At the same time, the positive contributions of the three aforementioned elements were counterbalanced to some extent by small negative contributions to the VIX Index from both the call skew gradient and upside convexity. This action suggests that portfolio managers either discounted or were willing to risk the possibility of a near term rally and were thus willing to sell both OTM and DOTM calls in order to fund the purchase of downside protection.

Turning now to the decomposition of the VIX Index in response to Liberation Day (Exhibit 17, right bar chart), we note a slightly different dynamic.

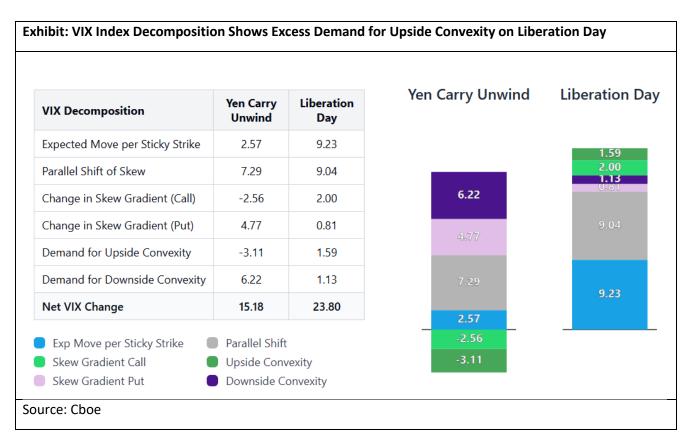
- In contrast to the Yen-Carry Unwind, volatility traders had priced in a much higher increase in volatility in response to the 6% decline triggered by the Liberation Day catalyst and thus a sizeable positive contribution to the VIX Index was explained by "the expected move per sticky strike".
- Nonetheless, the high level of uncertainty regarding the market implications of Liberation Day forced volatility traders to recalibrate their understanding of the market risk environment and thus a sizeable

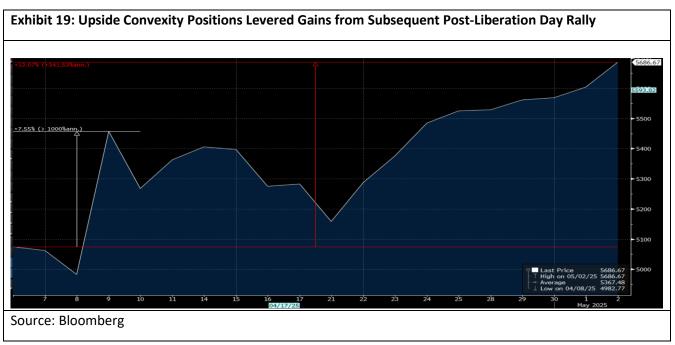


- positive contribution to higher VIX Index stemmed from a significant upwards parallel shift of the entire volatility surface.
- It is the interplay in the put and call positioning, however, where the Liberation Day decomposition differs markedly from the Aug 5th Yen-Carry Unwind. Specifically, while a material portion of the VIX Index increase on Aug 5th stemmed from a bearish bid for downside convexity, a similar sized increase in VIX Index on Liberation Day was due instead to a moderately bullish bid for upside call skew as well as an extremely bullish bid for upside convexity. The implication is that the VIX Index increase was driven in part by traders positioned to lever into a massive rally. Interestingly, as shown in Exhibit 14, the equity market did in fact rally +7% on the following 2 days and +14% over the following 30 days.













Concluding Remarks

Expectations of the VIX Index behavior is often framed in the context of "sticky strike" in which the VIX Index is expected to move in opposition to the direction of SPX Index changes. However, a number of other factors including shifts in the volatility surface, changes in the skew gradient, and varying demand for up and downside convexity can result in either directionally counterintuitive moves, or seemingly under/over reactions of the VIX Index relative to the changes in SPX Index spot. Our VIX Index decomposition model is meant to provide additional color to help investors better understand the effects of investor positioning that sometimes overwhelm expected VIX Index dynamics and to help investors extract the underlying derivatives market sentiment to allow them to make more informed decisions across different volatility regimes.

While this paper lays out the framework for our VIX Index decomposition model, Cboe plans to launch a web tool this Fall that will allow investors to keep track of the decomposition on an ongoing daily basis. We'll also be publishing historical decompositions breaking down the largest VIX index moves over the last 25 years. To stay abreast of future VIX index decomposition analysis, please subscribe to our Derivatives Market Intelligence distribution list.

Acknowledgements:

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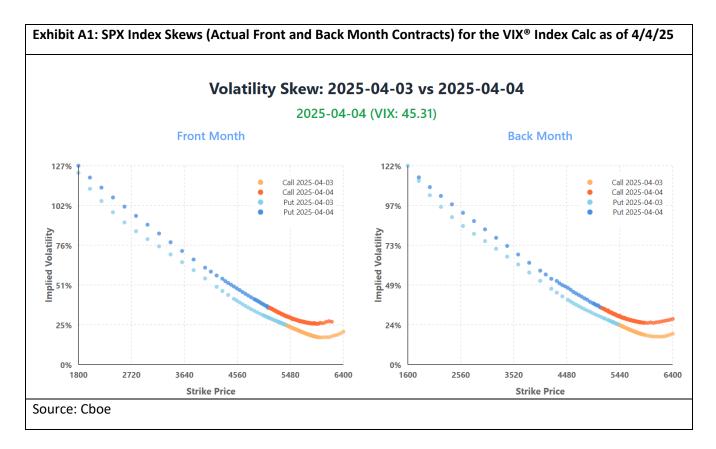
References:

The VIX Index Methodology: Volatility Index Methodology Cboe Volatility Index.pdf



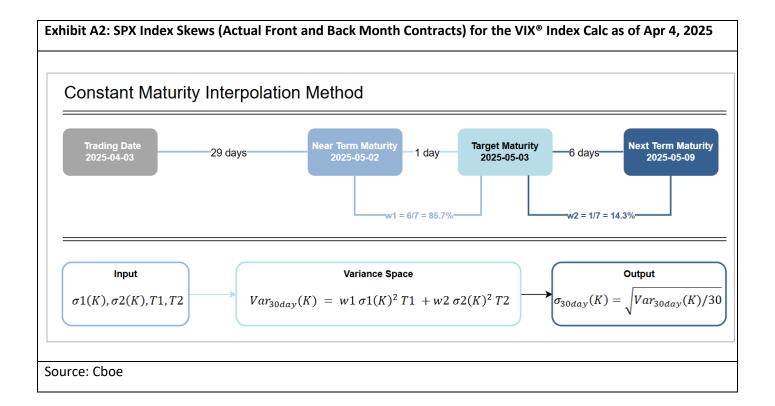
Appendix A: Interpolation of 30-Day Skew

To enhance intuition for the deterministic factors driving the VIX® Index, the decomposition portrays the principal components driving the VIX Index as if the VIX Index were calculated according to a single 30-day fixed strike skew. (i.e., the 30-day skew used in the analysis is interpolated in variance space based on the prevailing front and back month contracts used to calculate the VIX Index.) In the below example we detail the interpolation of the front and back month SPX index skews used to calculate the VIX Index as of Apr 4, 2025.



Specifically, the skew on the left in Exhibit A1 representing the near term expiry is the weekly SPX index options contract expiring 2025-05-02 (29 days to expiry) while the skew on the right in Exhibit A1 representing the back term expiry is the weekly SPX options contract expiring 2025-05-09 (36 days to expiry). We employ an inverted weight method to develop a single synthetic skew for each trading date that creates the interpolated skew with exactly 30 calendar days maturity. (Bear in mind that the distance in calendar between the synthetic expiry 2025-05-03 and the near term expiry 2025-05-02 is 1 day and the next term expiry 2025-05-09 is 6 days.)





Calculate time weights:

T1 = near term days to expiry = 29 days

T2 = next term days to expiry = 36 days

 $T_{target} = 30 \ days$

$$w1 = \frac{\left(T2 - T_{target}\right)}{T2 - T1} = \frac{36 - 30}{36 - 29} = 0.86$$

$$w2 = \frac{\left(T_{target} - T1\right)}{T2 - T1} = \frac{30 - 29}{36 - 29} = 0.14$$

Convert volatilities to variance space:

For each strike K:

$$Var_1(K) = \sigma_1(K)^2 \times T1$$

$$Var_2(K) = \sigma_2(K)^2 \times T2$$



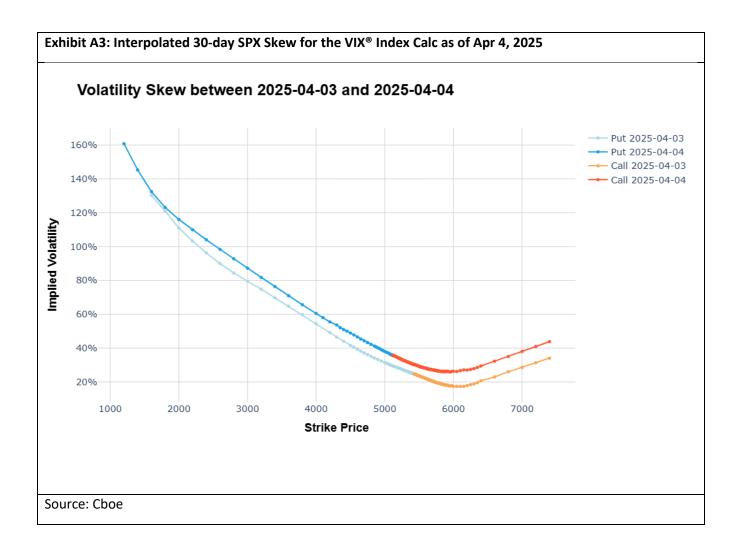
Interpolate in variance space:

$$Var_{30day}(K) = w1 \sigma 1(K)^2 T1 + w2 \sigma 2(K)^2 T2$$

Convert back to 30-day implied volatility:

$$\sigma_{30day}(K) = \sqrt{\frac{Var_{30day}(K)}{30}}$$

• The result is a single interpolated 30-day skew as depicted in Exhibit xx





Cboe Derivatives Market Intelligence

Mandy Xu

VP, Head of Derivatives Market Intelligence +1 646 856 8799 mxu@cboe.com

Henry Schwartz

VP, Derivatives Market Intelligence +1 646 856 8766 hschwartz@cboe.com

Ed Tom

Sr. Dir, Derivatives Market Intelligence +1 212 378 4517 etom@cboe.com

Wei Liao

Dir, Derivatives Market Intelligence +852 2632 9572 wliao@cboe.com

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