n linear equations, n unknowns
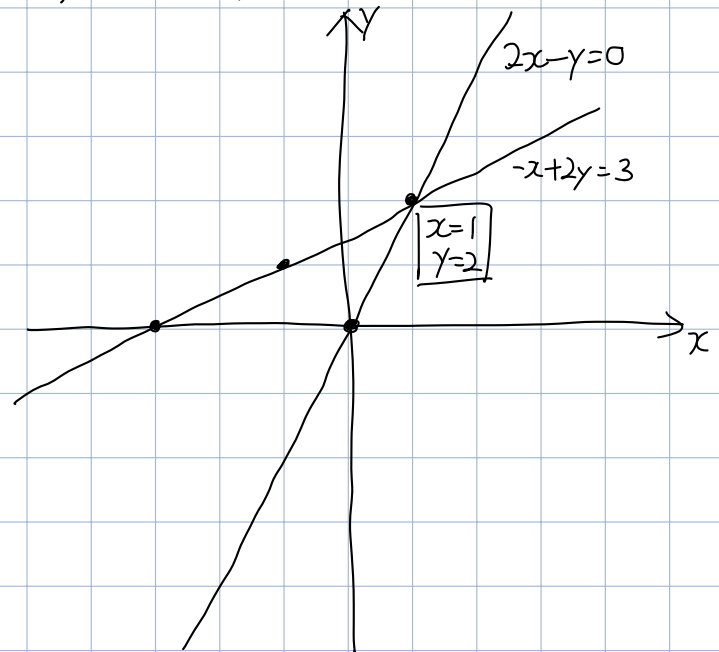
Row Picture
Column Picture
Matrix form

ex) $2x - y = 0$
$-x + 2y = 3$

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$
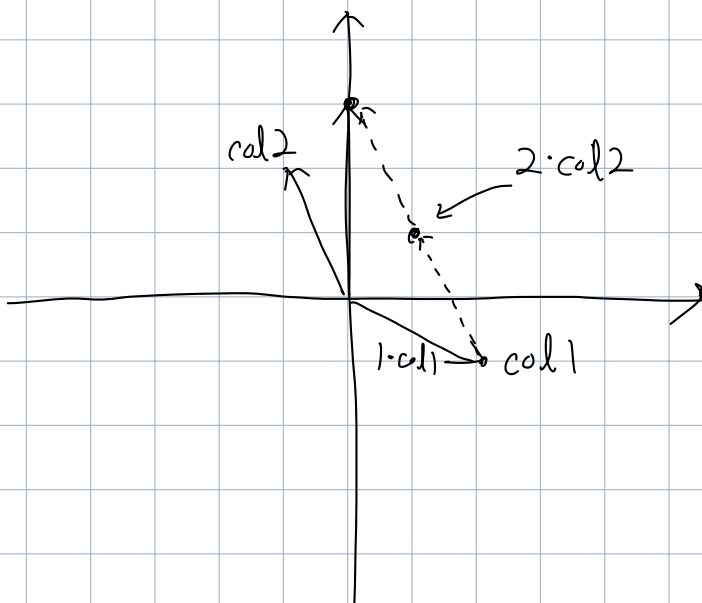
$$A \qquad X = b$$

Row Picture

$2x - y = 0$

$-x + 2y = 3$

$\boxed{\begin{aligned} x &= 1 \\ y &= 2 \end{aligned}}$

Column Picture

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

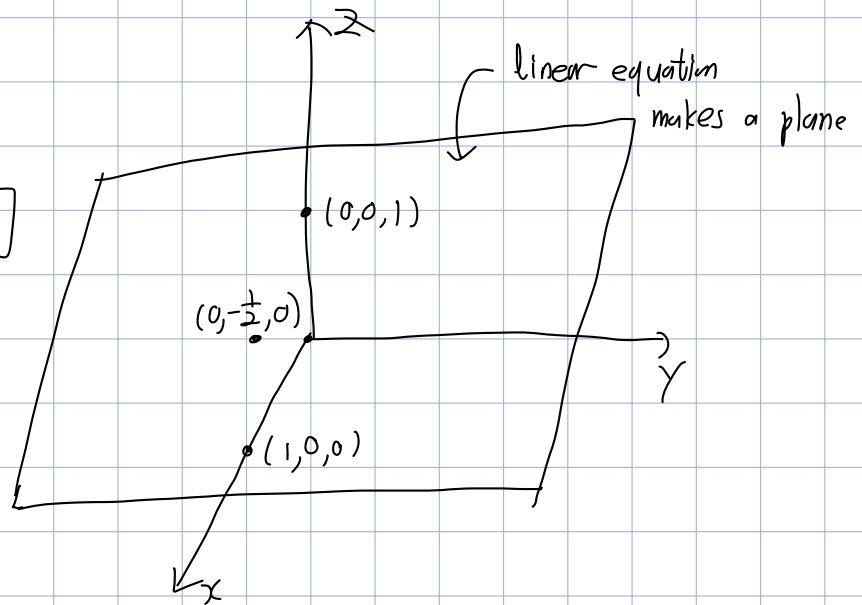linear combination of columns

col2

$2 \cdot col2$

$1 \cdot col1 \to col1$

ex) $2x - y = 0$

$-x + 2y - z = -1$

$-3y + 4z = 4$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

Row Picture

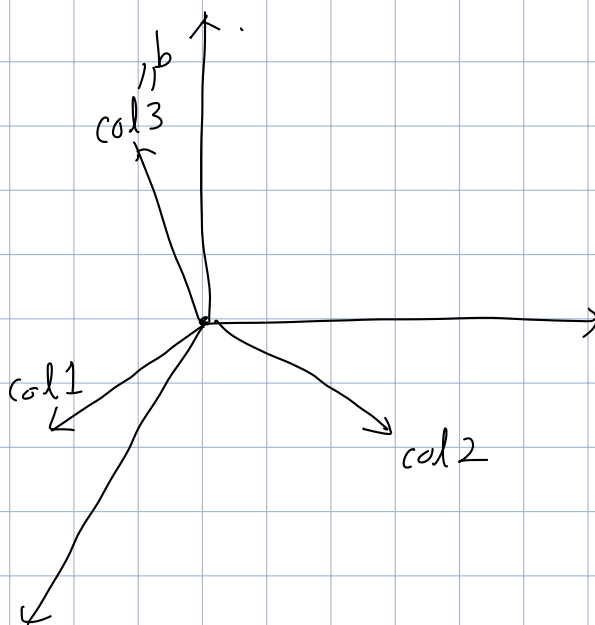linear equation makes a plane

three planes meet in one plane, and that's the solution

Column picture

$$x \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

$x = 0,$
$y = 0,$
$z = 1$

col3

b

col1

col2

Can I solve $Ax = b$ for every b?

$=$

Do the linear combinations of the columns fill 3D space?

For this matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix}$, answer is YES.

Matrix Multiplication

$Ax = b$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

Ax is a combination of columns of A.