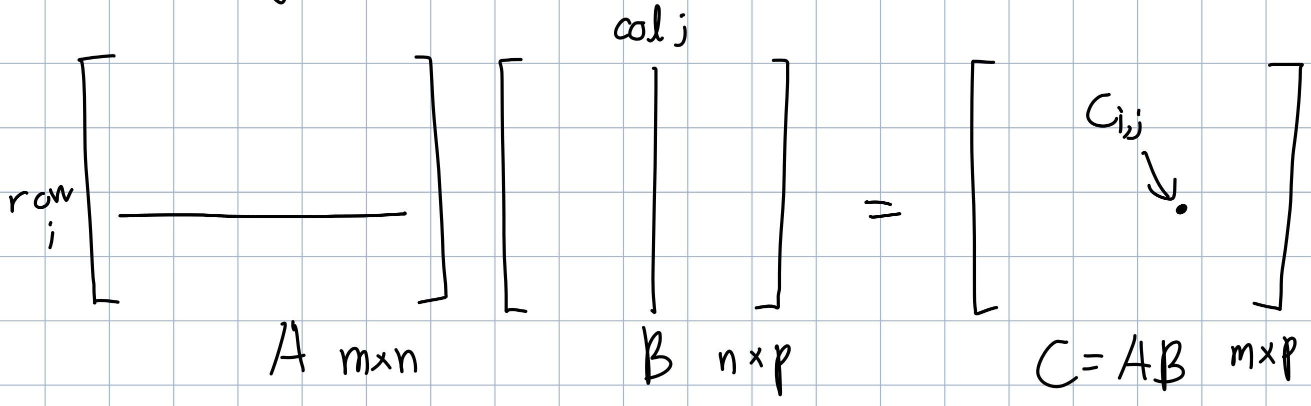
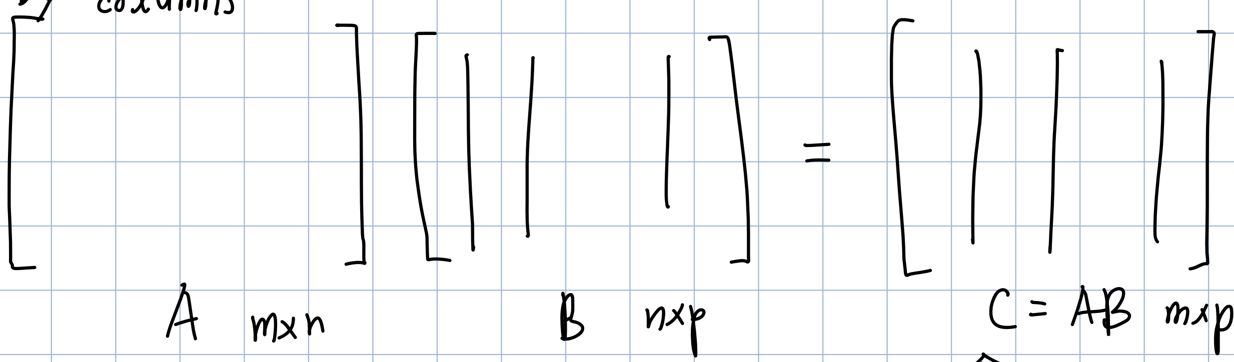
multiplication

① multiplying a matrix by a vector



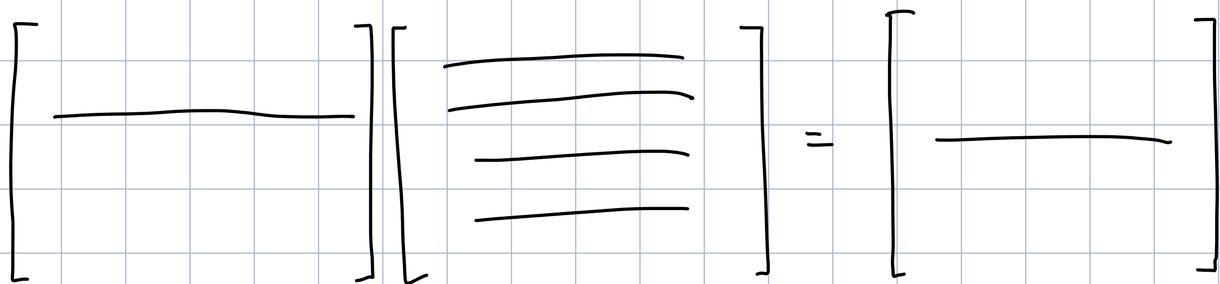
C_{ij} = (row i of A) · (col j of B)

② by columns



Columns of C are combinations of columns of A.

③ by rows



rows of C are combinations of rows of B

④ column of A × row of B

$m \times 1$ $\quad 1 \times p$

ex) $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 3 & 18 \\ 4 & 24 \end{bmatrix}$

AB = sum of (cols of A) × (rows of B)

ex) $\begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix}$

⑤ Block multiplication

$A_1B_1 + A_2B_3$

$$\underset{A}{\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}} \underset{B}{\begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Inverses (square matrices)

$A^{-1} A = I = A A^{-1}$

if this matrix exists

invertible, nonsingular

Singular Case
(No inverse)

$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

You can find a vector x $(x \neq 0)$ with $Ax = 0$.

$Ax = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Then A is singular case

because $A^{-1}Ax = x = 0 = A^{-1} \cdot 0$

false

Gauss-Jordan (solve 2 equations at once)

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\begin{array}{cc} A & I \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\begin{array}{cc} I & A^{-1} \end{array}$$