Elimination

$$
\begin{aligned}
x + 2y + z &= 2 \\
3x + 8y + z &= 12 \\
4y + z &= 2
\end{aligned}
$$

$$A \; x = b$$

1st pivot

$$
\underset{A}{\begin{matrix} \boxed{1} & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{matrix}}
\longrightarrow
\begin{matrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 4 & 1 \end{matrix}
$$

$$\downarrow$$

$$
\underset{U}{\begin{matrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 0 & \boxed{5} \end{matrix}}
$$

Back - Substitution

$$
\begin{matrix} \boxed{1} & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \\ & A & & b \end{matrix}
\longrightarrow
\begin{matrix} \boxed{1} & 2 & 1 & 2 \\ 0 & \boxed{2} & -2 & 6 \\ 0 & 4 & 1 & 2 \end{matrix}
\longrightarrow
\begin{matrix} \boxed{1} & 2 & 1 & 2 \\ 0 & \boxed{2} & -2 & 6 \\ 0 & 0 & \boxed{5} & -10 \\ & U & & c \end{matrix}
$$

$$
\begin{aligned}
x &= 2 \\
y &= 1 \\
z &= -2
\end{aligned}
$$

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{matrix} 3\times \text{col } 1 \\ +4\times \text{col } 2 \\ +5\times \text{col } 3 \end{matrix}$$

matrix × column
= column

$$\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \begin{matrix} 1\times \text{row } 1 \\ +2\times \text{row } 2 \\ +7\times \text{row } 3 \end{matrix}$$

Matrices: Subtract 3 × row 1 from row 2

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

E_{21}

Step 2 : Subtract 2× row 2 from row 3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

E_{32}

$$E_{32}(E_{21}A) = U \qquad \text{(associative law)}$$

$$(E_{32}E_{21})A = U$$

Permutation

Exchange rows 1 and 2

$$\underset{P}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

Inverses

$$\underset{E^{-1}}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{E}{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \underset{I}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$