Elimination

$$x + 2y + z = 2$$
$$3x + 8y + z = 12$$
$$4y + z = 2$$

$$A \; x = b$$

1st pivot

$$\underset{A}{\begin{bmatrix} \boxed{1} & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}} \longrightarrow \begin{bmatrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\underset{U}{\begin{bmatrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 0 & \boxed{5} \end{bmatrix}}$$

Back-Substitution

$$\begin{matrix} \boxed{1} & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \\ & A & & b \end{matrix} \longrightarrow \begin{matrix} \boxed{1} & 2 & 1 & 2 \\ 0 & \boxed{2} & -2 & 6 \\ 0 & 4 & 1 & 2 \end{matrix} \longrightarrow \begin{matrix} \boxed{1} & 2 & 1 & 2 \\ 0 & \boxed{2} & -2 & 6 \\ 0 & 0 & \boxed{5} & -10 \\ & U & & c \end{matrix}$$

$x = 2$
$y = 1$
$z = -2$

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{matrix} 3\times \text{col } 1 \\ +4\times \text{col } 2 \\ +5\times \text{col } 3 \end{matrix}$$

matrix × column
= column

$$\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \quad \begin{matrix} 1\times \text{row } 1 \\ +2\times \text{row } 2 \\ +7\times \text{row } 3 \end{matrix}$$

Matrices : Subtract 3 × row 1 from row 2

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_{21}}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

Step 2 : Subtract 2× row 2 from row 3

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}}_{E_{32}}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

$$E_{32}(E_{21}A) = U$$

$$(E_{32}E_{21})A = U$$

(associative law)

Permutation

Exchange rows 1 and 2

$$\underset{P}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

Inverses

$$\underset{E^{-1}}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{E}{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \underset{I}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$