2022

EGR 550 Mechatronics

PROJECT – 3 | MIMO LAB

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EGR - 550 : Project 3 (MIMO LAB)

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Aim : To derive a nonlinear transient model for two heaters and compare model predictions using the Arduino TC lab.

Procedure :

1. Connect the TC lab kit to the monitor through USB and open MATLAB.

2. Download the TClab - mimo files and run the test_models.m script.

3. The script to describe the dynamic temperature response of two temperature sensors given the heat inputs uses the following energy balance equations.

$$mC_p \frac{dT_1}{dt} = UA(T_\infty - T_1) + \epsilon\sigma A(T_\infty^4 - T_1^4) + Q_{C12} + Q_{R12} + \alpha_1 Q_1$$

$$mC_p \frac{dT_2}{dt} = UA(T_\infty - T_2) + \epsilon\sigma A(T_\infty^4 - T_2^4) + Q_{C12} + Q_{R12} + \alpha_2 Q_2$$

* Output :

* Observational

* The heater are operated for ten minute

* Initially heater was turned on and this had effect on temperature even with the absence of heater 2.

* At 300sec, heater 2 was turned on & the increase in the temperature values can be observed.

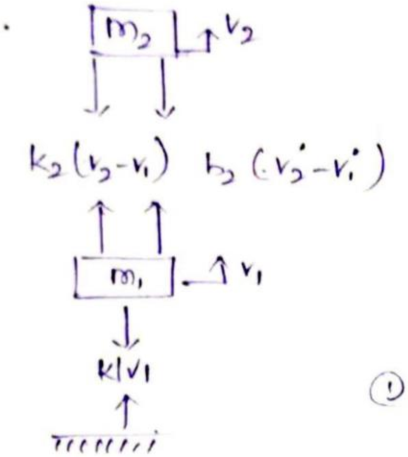
* As this system is a first order system, there is a time lag b/w the input and the result.

* It is clear that due to the disturbances experienced by the sensors the output response curve is not smooth.

* The accuracy of the model can be improved by tuning both the internal and external parameters.

2.26 Drawing the free body diagram of the given system

a.



Lets assume, v_2 = output

$u(t)$ = input $\quad G(s) = v_2(s)/u(s)$

For mass 1, using law of conservation

$$\Sigma \vec{F} = m\vec{a}$$

$$\Rightarrow k_2(v_2-v_1)+b_2(\dot{v}_2-\dot{v}_1)-k_1v_1-m_1\ddot{v}_1 = u(t)$$

$$① \Rightarrow u(t) = k_2v_2-k_2v_1+b_2\dot{v}_2-b_2\dot{v}_1-k_1v_1-m_1\ddot{v}_1$$

For mass 2, $k_2(v_2-v_1)+b_2(\dot{v}_2-\dot{v}_1)+k_1v_2-m_2\ddot{v}_2 = 0$

$$\Rightarrow k_2v_2-k_2v_1+b_2\dot{v}_2-b_2\dot{v}_1+k_1v_2-m_2\ddot{v}_2 = 0 - ②$$

writing eq ① & ② in matrix form, $[m]\ddot{v} = [b]v + [k]v = u(t)$

$$③ \begin{bmatrix} u(t) \\ v_2 \end{bmatrix} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{v}_1 \\ \ddot{v}_2 \end{bmatrix} + \begin{bmatrix} b_2 & -b_2 \\ -b_2 & b_2 \end{bmatrix}\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_1+k_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

c. Constitutive equations;

$$v_1 = \frac{1}{m_1}\int F_1' dt \text{ (or) } \dot{v}_1 = \frac{1}{m_1}F_1'$$

$$v_2 = \frac{1}{m_2}\int F_2' dt \text{ (or) } \dot{v}_2 = \frac{1}{m_2}F_2'$$

$F_1 = k_1 \int v_1' dt$ (or) $\dot{F}_1 = k_1 v_1$

$F_2 = k_2 \cdot \int v_2' dt$ (or) $\dot{F}_2 = k_2 v_2'$

$-v_1' = -v(t)+v_2$ (or) $v_1' = v(t) - v_2$

$-v_2' = -v_2 + v_1$ (or) $v_2' = v_2 - v_1$

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$F_2' = F_1 + F_1''$ $\therefore \dot{v}_1 = \frac{1}{m_1}\left[F_2 + F_1 - b_1(v_2 - v_1)\right)$

$F_1' = F_2 - F_1$ $v_2' = \frac{1}{m_2}\left[F_1 + b_1(v_2 - v_1)\right]$

$F_1'' = b_1 v_1'$ $F_1 = K_1(v(t) - v_2)$

$F_2 = K_2(v_2 - v_1)$

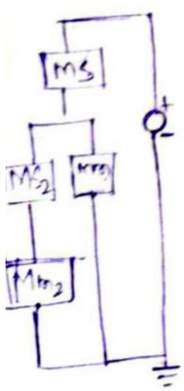
writing as $Ax = B$, $A = \begin{bmatrix} -b_1/m_2 & b_1/m_2 & 1/m_2 & 0 \\ b_1/m_1 & -b_1/m_1 & -1/m_1 & 1/m_1 \\ -K_2 & K_1 & 0 & 0 \\ 0 & -K_1 & 0 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_1 \end{bmatrix}$ $\therefore x = \left[v_1\, v_2\, F_1\, F_2\right]^T$

The system is a second order system.

d. solving 1, & 2 eq, recounting as matrix form ③, we get.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{v}_1 \\ \dot{v}_2 \end{bmatrix} + \begin{bmatrix} b_2 & -b_2 \\ -b_2 & b_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_1+k_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u(t) \\ v_2 \end{bmatrix}$$



$$z = \frac{1}{mm_1} + \frac{1}{ms_2 + m_{m_2}}, \quad v' = \left[\frac{1/z}{ms + 1/z}\right] v = \left[\frac{1}{msz+1}\right] v$$

$$v_m = \left[\frac{mm_2}{ms_2 + mm_2}\right] \cdot v_1' \; ; \; T_m = \frac{v_m}{v} = \left[\frac{1}{1 + msz}\right]\left[\frac{mm_2}{ms_2 + mm_2}\right]$$

$$\therefore T_m = \left[\frac{z_s}{z + z_s}\right]\left[\frac{mm_2}{ms_2 + mm_2}\right] \left(\text{here } z = \frac{1}{ms_2 + mm_2} + zm_1\right)$$

$$\therefore T_m = \left[\left(\frac{1}{zm_2\left[\frac{1}{zm_1 + z_s} + \frac{1}{zs_2}\right]}\right) + 1\right]\left[\frac{z_s}{zm_1 + z_s}\right]$$

This is a first order system.

2.28 From the given mechanical model, $J_m \dot{w}_m = T_m - b_m w_m - T_k$

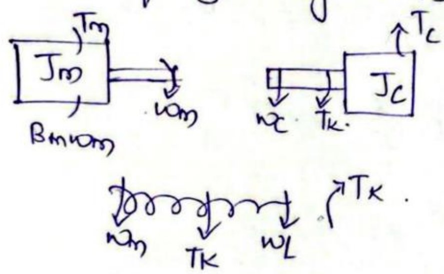
$$\dot{T}_k = K(w_m - w_c) \Rightarrow J_m \dot{w}_m = T_m - b_m w_m - K(w_m - w_c)$$

$$\Rightarrow J_c \dot{w}_c = T_k - b_c w_c - c|w_c|w_c$$

$$\Rightarrow J_c \dot{w}_c = K(w_m - w_c) - b_c w_c - c|w_c|w_c$$

$$T_c = c|w_c|w_c$$

The FBD of given system is as shown.



State space equations
$$\frac{dw_m}{dt} = \frac{-b_m}{J_m} w_m - \frac{1}{J_m} T_K + \frac{1}{J_m} T_m$$

$$\frac{dw_c}{dt} = \frac{-c}{J_c} |w_c| w_c + \frac{1}{J_c} T_c + \frac{1}{J_c} T_c.$$

$$\frac{dT_K}{dt} = K_c w_m - K_c w_c.$$

Assuming of $\cdot c \frac{1}{K_c} T$, also $y = w_m - w_c$ using linearization $w_c = w_m = c\bar{\omega}$
$\dot{w}_m = \dot{\omega}_c = 0$
$T_m = \bar{T}_m$. $w_m - w_c = \Delta\bar{\omega}$

$$\therefore T_m - b_m \bar{\omega} = K_c \Delta\bar{\omega} = 0.$$

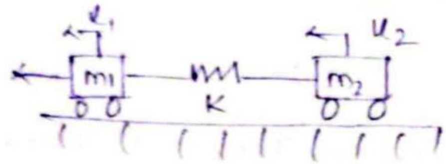
$$K_c \Delta\bar{\omega} - (\bar{\omega})^2 - T_c = 0$$

Let $q_1 \rightarrow$ variation of w_m steady state value.
$q_2 \rightarrow$ variation of w_c steady state value. Applying Taylor series,

$$J_m \dot{\bar{\omega}} + J_m \ddot{q}_1 = T_m + u + b_m \bar{\omega} - b_m \dot{q}_1 - K_c \Delta\bar{\omega} - K_c (q_1 - q_2)$$

$$J_c \dot{\bar{\omega}} + J_c \ddot{q}_2 = K_c \Delta\bar{\omega} + K_c (q_1 - q_2) - c\bar{\omega}^2 - 2c\bar{\omega}\dot{q}_2 - T$$

$$\therefore \boxed{\begin{aligned} J_m \ddot{q}_1 &= u - b_m \dot{q}_1 - K_c (q_1 - q_2) \\ J_c \ddot{q}_2 &= K_c (q_1 - q_2) - 2 w \dot{q}_2 \end{aligned}} \rightarrow \text{System linear model.}$$

P2.73 $F(t)$ 

a) $F(s)$ for the given system:

$$F(s) = m_1 s^2 x_1(s) + K[x_1(s) - x_2(s)] \rightarrow ①$$

$$0 = m_2 s^2 x_2(s) + K[x_2(s) - x_1(s)] \rightarrow ②$$

$$\frac{x_2(s)}{x_1(s)} = \frac{k}{m_2 s^2 + k} \Rightarrow x_2(s) = \frac{K x_1(s)}{K + m_2 s^2} \rightarrow ③$$

Substituting eq ③ in eq ①

$$\frac{x_1(s)}{F(s)} = \frac{m_2 s^2 + k}{(m_1 s^2 + k)(m_2 s^2 + k) - k^2}$$

② If T.F is zero, m_1 will be motionless, $\therefore m_2 s^2 + k = 0$

$$s = j\sqrt{\frac{K}{m_2}} = j\omega$$

$$\therefore \boxed{\omega = \sqrt{\frac{K}{m_2}}}$$

Output: (Question -1)