MAE - 547 MODELING AND CONTROL OF ROBOTS

HOME-WORK-2

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Problem 1.1 :

Given $r = \begin{bmatrix} 0.8814 & -0.2362 & 0.4091 \end{bmatrix}^T$, $\theta = 1.1598$ rad.

The rotation matrix corresponding to the following angle-axis representation

$$R(\theta, r) = \begin{bmatrix} r_x^2(1-c\theta)+c\theta & r_x r_y(1-c\theta)-r_z s\theta & r_x r_z(1-c\theta)+r_y s\theta \\ r_x r_y[1-c\theta]+r_z s\theta & r_y^2(1-c\theta)+c\theta & r_y r_z[1-c\theta]-r_x s\theta \\ r_x r_z[1-c\theta]-r_y s\theta & r_y r_z(1-c\theta]+r_x s\theta & r_z^2[1-c\theta]+c\theta \end{bmatrix}$$

$\theta = 1.1598$

$\cos\theta = 0.3995$

$\sin\theta = 0.91672$

$r_x = 0.8814$

$r_y = -0.2362$

$r_z = 0.4091$

$\because R_{11} = r_x^2[1-c\theta]+c\theta$

$= (0.8814)^2[1-0.3995]+0.3995$

$R_{11} = 0.8660$

$R_{12} = r_x r_y[1-c\theta]-r_z s\theta$

$= (0.8814)(-0.2362)[1-0.3995] - 0.4091[0.91672]$

$R_{12} = -0.500$

$R_{13} = r_x r_z[1-c\theta]+r_y s\theta$

$= 0.8814 \times 0.4091[1-0.3995]+(-0.2362)(0.91672)$

$R_{13} = -5.2963 \times 10^{-7} \simeq -0.00$

$c\theta$

$R_{21} = r_x r_y (1-c\theta) + r_z s\theta$

$= [0.8814][-0.2362][1-0.3995] + 0.4091\ [0.91672]$

$R_{21} = 0.250$

$R_{22} = r_y^2 [1-c\theta] + c\theta$

$= [-0.2362]^2 [1-0.3995] + 0.3995$

$R_{22} = 0.4330$

$R_{23} = r_y r_z (1-c\theta] - r_x s\theta$

$= [0.2362) [0.4091] (1-0.3995] - 0.8814\ [0.91672]$

$R_{23} = -0.8660$

$R_{31} = r_x r_z (1-c\theta] - r_y s\theta$

$= [0.8814] [0.4091] [1-0.3995] - [-0.2362] [0.91672]$

$R_{31} = 0.4330$

$R_{32} = r_y r_z [1-c\theta] + r_x s\theta$

$= [0.2362] [0.4091] [1-0.3995] + 0.8814\ [0.91672]$

$R_{32} = 0.7499$

$$R_{33} = r_2^2 [1 - c\theta] + c\theta$$

$$= [0.4091]^2 [1 - 0.3995] + 0.3995.$$

$$R_{33} = 0.500$$

$$R[\theta, r] = \begin{bmatrix} 0.866 & -0.500 & 0 \\ 0.250 & 0.432 & -0.866 \\ 0.4330 & 0.7499 & 0.500 \end{bmatrix}$$

Problem 1.2:

$$\text{Given } R = \begin{bmatrix} 0.7071 & 0.6124 & 0.3536 \\ 0 & 0.5 & -0.866 \\ -0.7071 & 0.6124 & 0.3536 \end{bmatrix}$$

$\Rightarrow$ Inverse formulas for angle-axis $\because \theta = a\cos\left[\dfrac{9_{11} + 9_{12} + 9_{33} - 1}{2}\right]$

$$\theta = a\cos\left[\frac{0.7071 + 0.5 + 0.3536 - 1}{2}\right]$$

$\because \boxed{\theta = 73.712^\circ}$ $\quad 9_x = \dfrac{1}{2\sin\theta}[9_{32} - 9_{23}]$

$$9_x = \frac{1}{2\sin[73.712]}[0.6124 - (-0.866]$$

$$9_x = \frac{1}{2\sin 73.712}[0.6124 + 0.866]$$

$$9_x = 0.77001$$

$$q_y = \frac{1}{2\sin\theta}\left[q_{13} - q_{31}\right]$$

$$= \frac{1}{2\sin\theta}\left[0.3536 - (-0.7071)\right]$$

$$= \frac{1}{2\sin\theta}\left[1.0607\right] = \frac{1}{2\sin(73.71)}$$

$$\boxed{q_y = 0.55}$$

$$q_z = \frac{1}{2\sin\theta}\left[q_{21} - q_{12}\right] \Rightarrow \frac{1}{2\sin(73.71)}\left[0 - 0.6124\right]$$

$$q_z \Rightarrow -0.319 \qquad \boxed{q_z = -0.319}$$

∴ The first set of angle-axis representation is given by $[R, \theta]$

$$R_1 = \left[0.7708 \quad 0.55 \quad -0.319\right] \quad \because \theta = 73.71^\circ$$

The second set of angle-axis representation is given by $[-R, -\theta]$

$$R_2 = \left[-0.7708 \quad -0.55 \quad +0.319\right] \qquad \theta = -73.71$$

Problem

1.4 $q_1 \to$ quaternion of R_1 & $q_2 \to$ quaternion of R_2

$$\because q_1 \times q_2 = \{\eta_1\eta_2 - \varepsilon_1^T\varepsilon_2,\ \eta_1\varepsilon_2 + \eta_2\varepsilon_1 + \varepsilon_1 \times \varepsilon_2\}$$

$$\Rightarrow \left\{ (0.9751 \times 0.9676) - \left[[0.0099 \quad 0.09434 \quad 0.19768] \times \begin{bmatrix} -0.0025 \\ 0.049 \\ 0.247 \end{bmatrix} \right] \right.$$

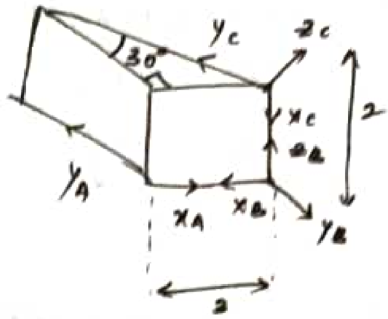
$$\Rightarrow \left. 0.9716 \begin{bmatrix} -0.0025 \\ 0.049 \\ 0.247 \end{bmatrix} + 0.9676 \begin{bmatrix} 0.0099 \\ 0.0993 \\ 0.1976 \end{bmatrix} + \begin{bmatrix} 0.099 \\ 0.0993 \\ 0.1976 \end{bmatrix} \begin{bmatrix} 0.0025 \\ 0.049 \\ 0.247 \end{bmatrix} \right\}$$

$$q_1 \times q_2 = \{ 0.88989, [0.0219 \quad 0.1418 \quad 0.4330] \}$$

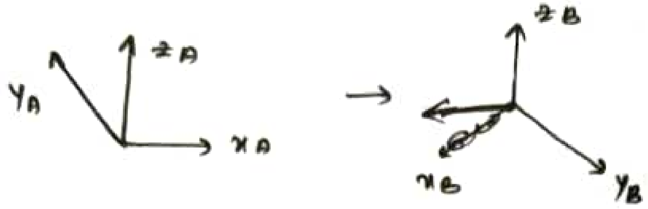
proved in MATLAB

Problem - 2

1. T^A_B



Frame- A



Aligning frame A to frame B and x_A & x_B are of ~~o~~ to 180°.

Rotating z by 180° ccw

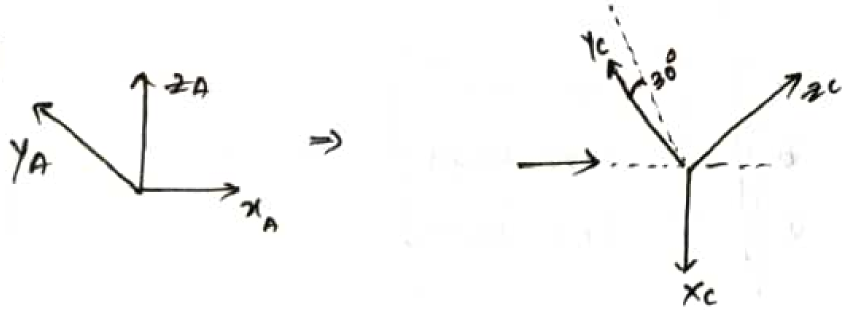
$$T^A_B = \begin{bmatrix} \cos 180 & -\sin 180 & 0 \\ \sin 180 & \cos 180 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now the Origins are 3 units away from the base.

The Homogeneous Transformation T^A $= \left[\begin{array}{ccc|c} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$ // $\because \left[\begin{array}{c|c} R & d \\ \hline 0 & 1 \end{array}\right]$

(2)

T^A_C



Rotation matrix = R_{90° about y × R_{-30° about x_c.

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & \sin 30 \\ 0 & -\sin 30 & \cos 30 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & 1/2 \\ 0 & -0.5 & 0.866 \end{bmatrix}$$

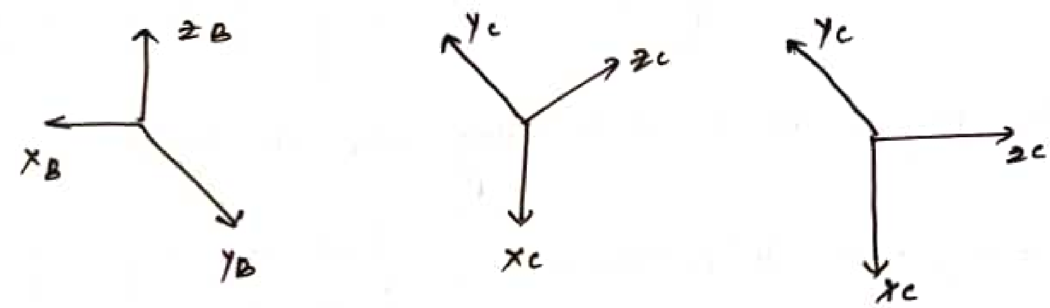
$$= \begin{bmatrix} 0 & -0.5 & 0.866 \\ 0 & 0.866 & 0.5 \\ -1 & 0 & 0 \end{bmatrix}$$

Now the frames are 3 units & 2 units away from origin

The Homogeneous Transformation matrix is

$$T^A_C = \left[\begin{array}{ccc|c} 0 & -0.5 & 0.866 & 3 \\ 0 & 0.866 & 0.5 & 0 \\ -1 & 0 & 0 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

③ T^B_C



Rotation matrix = Allignment of the matrix × R_{-30} about x_C

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & \sin 30 \\ 0 & -\sin 30 & \cos 30 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \sin 30 & -\cos 30 \\ 0 & -\cos 30 & -\sin 30 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -0.866 \\ 0 & -0.866 & -0.5 \\ -1 & 0 & 0 \end{bmatrix}$$

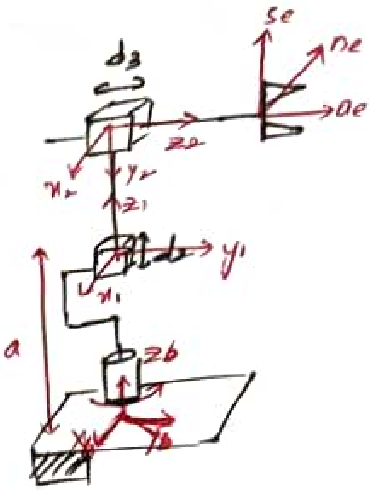
Origins are 2 units away

Homogeneous Transformation

$$T_C^B = \begin{bmatrix} 0 & 0.5 & -0.866 & 0 \\ 0 & -0.866 & -0.5 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

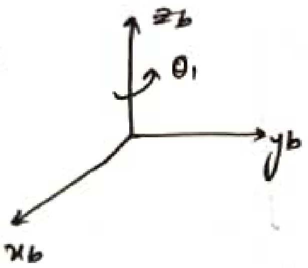
Problem - 4.1

the direct kinematics equation (T_e^b) for the cylindrical arm.

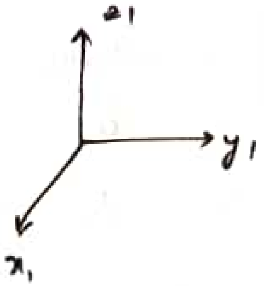


Base to effector method

Base vector



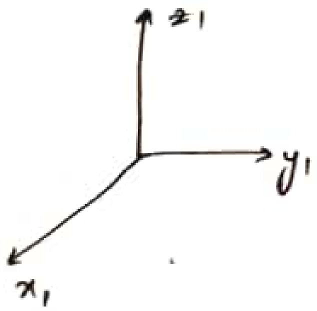
$\Rightarrow$



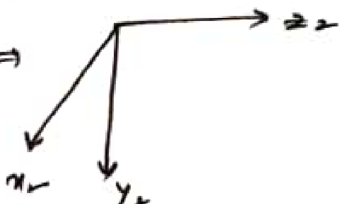
$$T_1^b = \left[\begin{array}{ccc|c} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & a \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Homogeneous matrix

\# Distance between the Base axes and frame-1 is a

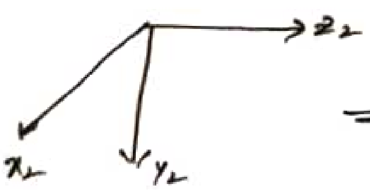


$\Rightarrow$



$$T_2^1 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

\# Distance between frame 1 & frame 2 is d_2



$\Longrightarrow$

$$T_e^2 = \left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

\# Distance between frame 2 & frame 'e' is d_3

$$\boxed{T_e^b = T_1^b \times T_2^1 \times T_e^2}$$

$$T^b_e = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By performing matrix multiplicat
we get final T^b_e.

$$T^b_e = \left[\begin{array}{ccc|c} -\cos\theta_1 & 0 & -\sin\theta_1 & -d_3\sin\theta_1 \\ -\sin\theta_1 & 0 & \cos\theta_1 & d_3\cos\theta_1 \\ 0 & 1 & 0 & a+d_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Final Transformation matrix from frame b to frame e.

Problem 4.2

DH table for a system with 2 revolute joints and 1 prismatic joint

	w.r.t x_i		w.r.t z_{i-1}	
Link	a_i	α_i	d_i	θ_i
1	a_2	0	a_1	θ_1
2	a_3	π	a_5	θ_2
3	0	0	d_3+a_4	0

a_i - ~~angle~~ distance between O_i and O_{i-1}

$\alpha_i \to$ angle between z_{i-1} & z_i

d_i - coordinate of O_i' along z_{i-1}

$\theta_i \to$ angle between x_{i-1} & x_i

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% STUDENT NAME : MANOHAR AKULA
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% ROBOTICS AND AUTONOMOUS SYSTEMS (EE)

%PROBLEM - 1.3
%Finding the quaternion representation q1 and q2 for the two
%rotation matrices R1 and R2

R1 = [0.9021 -0.3836 0.1977; 0.3875 0.9216 0.0198; -0.1898 0.0587 0.9801]

R2 = [0.8729 -0.4785 0.0954; 0.4780 0.8779 0.0295; -0.0978 0.0198 0.9950]

% Finding Unit UnitQuaternions q1 and q2 of Rotation matrices R1 and R2
q1 = UnitQuaternion(R1)

q2 = UnitQuaternion(R2)

R1 =

    0.9021   -0.3836    0.1977
    0.3875    0.9216    0.0198
   -0.1898    0.0587    0.9801


R2 =

    0.8729   -0.4785    0.0954
    0.4780    0.8779    0.0295
   -0.0978    0.0198    0.9950


q1 =

0.97517 < 0.0099755, 0.099328, 0.1977 >


q2 =

0.9677 < -0.00249, 0.049904, 0.24709 >

>>
```

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>>
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% STUDENT NAME : MANOHAR AKULA
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% ROBOTICS AND AUTONOMOUS SYSTEMS (EE)

%PROBLEM - 1.4
%Finding q1 and q2 quaternions corresponding to R1R2 = q1*q2

R1 = [0.9021 -0.3836 0.1977; 0.3875 0.9216 0.0198; -0.1898 0.0587 0.9801]

R2 = [0.8729 -0.4785 0.0954; 0.4780 0.8779 0.0295; -0.0978 0.0198 0.9950]

% Finding Unit UnitQuaternions q1 and q2 of Rotation matrices R1 and R2

q1 = UnitQuaternion(R1)

q2 = UnitQuaternion(R2)

% Calculating quaternion corresponding to R1R2 - q1*q2

q1*q2

R1 =

    0.9021   -0.3836    0.1977
    0.3875    0.9216    0.0198
   -0.1898    0.0587    0.9801


R2 =

    0.8729   -0.4785    0.0954
    0.4780    0.8779    0.0295
   -0.0978    0.0198    0.9950


q1 =

0.97517 < 0.0099755, 0.099328, 0.1977 >


q2 =

0.9677 < -0.00249, 0.049904, 0.24709 >


ans =

0.88989 < 0.021902, 0.14183, 0.43301 >

>>
```

```
>> % HOME WORK - 2, MAE 547 MODELING AND CONTROL OF ROBOTS
% STUDENT NAME : MANOHAR AKULA
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% ROBOTICS AND AUTONOMOUS SYSTEMS (EE)

%PROBLEM - 3.1 - Calculation of Homogeneous Transformation matrix Tab when
%user enters ZYX Euler angles

syms a b c x y z

% where a = phi = 20 deg
% b = theta = 30 deg
% c = si = 45 deg

R1 = [cosd(a) -sind(a) 0; sind(a) cosd(a) 0; 0 0 1]

R2 = [cosd(b) 0 sind(b); 0 1 0; -sind(b)  0 cosd(b)]

R3 = [1 0 0;0 cosd(c) -sind(c); 0 sind(c) cosd(c)]

simplify (R1*R2*R3) % Output of the Rotation matrix w.r.t to zyx.

%For Homogeneous Transformation Matrix adding a newcolumn and newrow to the ↙
Rotation
%matrix.

newcolumn = [x;y;z]

newrow = [0 0 0 1]

ans = [ans newcolumn]

ans = [ans; newrow]

%Substituting the values of phi, theta, si, x, y, and z.



a = 20

b = 30

c = 45

x = 2

y = 1

z = 3

subs(ans)
```

```
R1 =

[cos((pi*a)/180), -sin((pi*a)/180), 0]
[sin((pi*a)/180),  cos((pi*a)/180), 0]
[               0,                 0, 1]


R2 =

[ cos((pi*b)/180), 0, sin((pi*b)/180)]
[               0, 1,               0]
[-sin((pi*b)/180), 0, cos((pi*b)/180)]


R3 =

[1,                0,                0]
[0, cos((pi*c)/180), -sin((pi*c)/180)]
[0, sin((pi*c)/180),  cos((pi*c)/180)]


ans =

[cos((pi*a)/180)*cos((pi*b)/180), cos((pi*a)/180)*sin((pi*b)/180)*sin((pi*c)/180) - ↙
cos((pi*c)/180)*sin((pi*a)/180), sin((pi*a)/180)*sin((pi*c)/180) + cos((pi*a)/180) ↙
*cos((pi*c)/180)*sin((pi*b)/180)]
[cos((pi*b)/180)*sin((pi*a)/180), cos((pi*a)/180)*cos((pi*c)/180) + sin((pi*a)/180) ↙
*sin((pi*b)/180)*sin((pi*c)/180), cos((pi*c)/180)*sin((pi*a)/180)*sin((pi*b)/180) - ↙
cos((pi*a)/180)*sin((pi*c)/180)]
[               -sin((pi*b)/180), ↙
cos((pi*b)/180)*sin((pi*c)/180), ↙
cos((pi*b)/180)*cos((pi*c)/180)]


newcolumn =

x
y
z


newrow =

     0     0     0     1


ans =

[cos((pi*a)/180)*cos((pi*b)/180), cos((pi*a)/180)*sin((pi*b)/180)*sin((pi*c)/180) - ↙
cos((pi*c)/180)*sin((pi*a)/180), sin((pi*a)/180)*sin((pi*c)/180) + cos((pi*a)/180) ↙
*cos((pi*c)/180)*sin((pi*b)/180), x]
[cos((pi*b)/180)*sin((pi*a)/180), cos((pi*a)/180)*cos((pi*c)/180) + sin((pi*a)/180) ↙
*sin((pi*b)/180)*sin((pi*c)/180), cos((pi*c)/180)*sin((pi*a)/180)*sin((pi*b)/180) - ↙
```

```
cos((pi*a)/180)*sin((pi*c)/180), y]
[                -sin((pi*b)/180),↙
cos((pi*b)/180)*sin((pi*c)/180),↙
cos((pi*b)/180)*cos((pi*c)/180), z]


ans =

[cos((pi*a)/180)*cos((pi*b)/180), cos((pi*a)/180)*sin((pi*b)/180)*sin((pi*c)/180) -↙
cos((pi*c)/180)*sin((pi*a)/180), sin((pi*a)/180)*sin((pi*c)/180) + cos((pi*a)/180)↙
*cos((pi*c)/180)*sin((pi*b)/180), x]
[cos((pi*b)/180)*sin((pi*a)/180), cos((pi*a)/180)*cos((pi*c)/180) + sin((pi*a)/180)↙
*sin((pi*b)/180)*sin((pi*c)/180), cos((pi*c)/180)*sin((pi*a)/180)*sin((pi*b)/180) -↙
cos((pi*a)/180)*sin((pi*c)/180), y]
[                -sin((pi*b)/180),↙
cos((pi*b)/180)*sin((pi*c)/180),↙
cos((pi*b)/180)*cos((pi*c)/180), z]
[                               0,↙
0,↙
0, 1]


a =

    20


b =

    30


c =

    45


x =

     2


y =

     1


z =

     3


ans =
```

```
[(3^(1/2)*cos(pi/9))/2, (2^(1/2)*cos(pi/9))/4 - (2^(1/2)*sin(pi/9))/2, (2^(1/2)*cos↙
(pi/9))/4 + (2^(1/2)*sin(pi/9))/2, 2]
[(3^(1/2)*sin(pi/9))/2, (2^(1/2)*cos(pi/9))/2 + (2^(1/2)*sin(pi/9))/4, (2^(1/2)*sin↙
(pi/9))/4 - (2^(1/2)*cos(pi/9))/2, 1]
[                  -1/2,                              (2^(1/2)*3^(1/2))/4,↙
(2^(1/2)*3^(1/2))/4, 3]
[                     0,                                                    0,↙
0, 1]

>>
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>> % HOME WORK - 2, MAE 547 MODELING AND CONTROL OF ROBOTS
% STUDENT NAME : MANOHAR AKULA
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% ROBOTICS AND AUTONOMOUS SYSTEMS (EE)

%PROBLEM - 3.2 - Calculation of Homogeneous Transformation matrix from cordinate a ↙
to cordinate b when
%user enters ZYX Euler angles.

syms a b c x y z

% where a = phi = 0 deg

% b = theta = 30 deg

% c = si = 0 deg

R1 = [cosd(a) -sind(a) 0; sind(a) cosd(a) 0; 0 0 1]

R2 = [cosd(b) 0 sind(b); 0 1 0; -sind(b) 0 cosd(b)]

R3 = [1 0 0;0 cosd(c) -sind(c); 0 sind(c) cosd(c)]

simplify (R1*R2*R3) % Output of the Rotation matrix w.r.t to zyx.
%For Homogeneous Transformation Matrix adding a newcolumn and newrow to the
%Rotation matrix.

newcolumn = [x;y;z]

newrow = [0 0 0 1]

ans = [ans newcolumn]

ans = [ans; newrow]

%Substituting the values of phi, theta, si, x, y, and z.

a = 0
b = 30
c = 0
x = 3
y = 1
z = 1

subs(ans)

R4 = [1;0;0;1] % Homogeneous multiplication from cordinate a to b.

ans = ans*R4

point1 = [0,0,0]
point2 = [3,1,1]
point3 = [3.86,1,0.5]
```
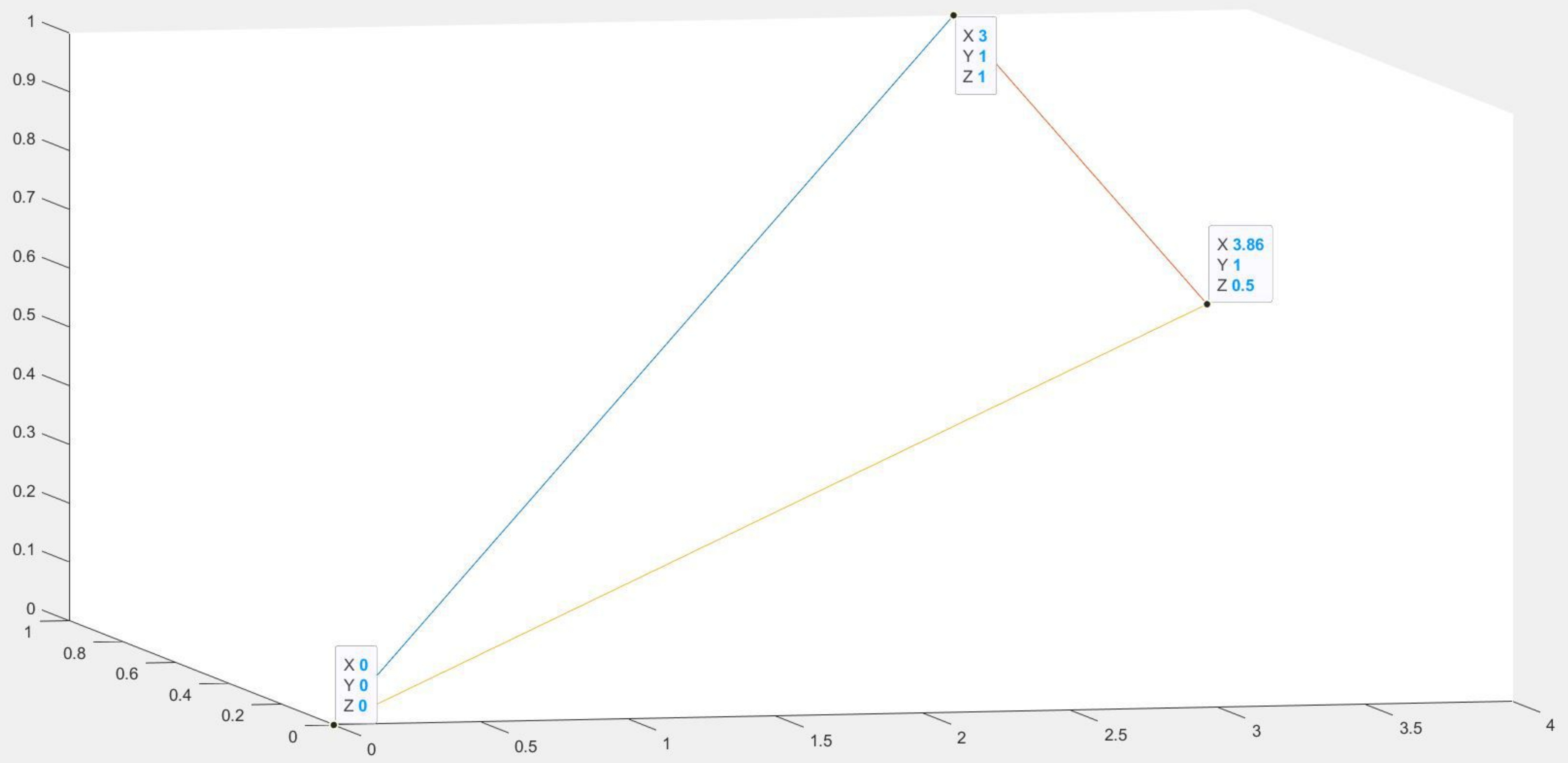

```
vector1 = [point1; point2]

vector2 = [point2; point3]

vector3 = [point3; point1]

plot3(vector1(:,1), vector1(:,2), vector1(:,3))
hold on
plot3(vector2(:,1), vector2(:,2), vector2(:,3))
hold on
plot3(vector3(:,1), vector3(:,2), vector3(:,3))




R1 =

[cos((pi*a)/180), -sin((pi*a)/180), 0]
[sin((pi*a)/180),  cos((pi*a)/180), 0]
[              0,                0, 1]


R2 =

[ cos((pi*b)/180), 0, sin((pi*b)/180)]
[               0, 1,               0]
[-sin((pi*b)/180), 0, cos((pi*b)/180)]


R3 =

[1,               0,                0]
[0, cos((pi*c)/180), -sin((pi*c)/180)]
[0, sin((pi*c)/180),  cos((pi*c)/180)]


ans =

[cos((pi*a)/180)*cos((pi*b)/180), cos((pi*a)/180)*sin((pi*b)/180)*sin((pi*c)/180) - ↙
cos((pi*c)/180)*sin((pi*a)/180), sin((pi*a)/180)*sin((pi*c)/180) + cos((pi*a)/180) ↙
*cos((pi*c)/180)*sin((pi*b)/180)]
[cos((pi*b)/180)*sin((pi*a)/180), cos((pi*a)/180)*cos((pi*c)/180) + sin((pi*a)/180) ↙
*sin((pi*b)/180)*sin((pi*c)/180), cos((pi*c)/180)*sin((pi*a)/180)*sin((pi*b)/180) - ↙
cos((pi*a)/180)*sin((pi*c)/180)]
[               -sin((pi*b)/180),↙
cos((pi*b)/180)*sin((pi*c)/180),↙
cos((pi*b)/180)*cos((pi*c)/180)]


newcolumn =

x
```

```
y
z


newrow =

     0     0     0     1


ans =
 
[cos((pi*a)/180)*cos((pi*b)/180), cos((pi*a)/180)*sin((pi*b)/180)*sin((pi*c)/180) - ↙
cos((pi*c)/180)*sin((pi*a)/180), sin((pi*a)/180)*sin((pi*c)/180) + cos((pi*a)/180)↙
*cos((pi*c)/180)*sin((pi*b)/180), x]
[cos((pi*b)/180)*sin((pi*a)/180), cos((pi*a)/180)*cos((pi*c)/180) + sin((pi*a)/180)↙
*sin((pi*b)/180)*sin((pi*c)/180), cos((pi*c)/180)*sin((pi*a)/180)*sin((pi*b)/180) - ↙
cos((pi*a)/180)*sin((pi*c)/180), y]
[                -sin((pi*b)/180),↙
cos((pi*b)/180)*sin((pi*c)/180),↙
cos((pi*b)/180)*cos((pi*c)/180), z]
 

ans =
 
[cos((pi*a)/180)*cos((pi*b)/180), cos((pi*a)/180)*sin((pi*b)/180)*sin((pi*c)/180) - ↙
cos((pi*c)/180)*sin((pi*a)/180), sin((pi*a)/180)*sin((pi*c)/180) + cos((pi*a)/180)↙
*cos((pi*c)/180)*sin((pi*b)/180), x]
[cos((pi*b)/180)*sin((pi*a)/180), cos((pi*a)/180)*cos((pi*c)/180) + sin((pi*a)/180)↙
*sin((pi*b)/180)*sin((pi*c)/180), cos((pi*c)/180)*sin((pi*a)/180)*sin((pi*b)/180) - ↙
cos((pi*a)/180)*sin((pi*c)/180), y]
[                -sin((pi*b)/180),↙
cos((pi*b)/180)*sin((pi*c)/180),↙
cos((pi*b)/180)*cos((pi*c)/180), z]
[                               0,↙
0,↙
0, 1]
 

a =

     0


b =

    30


c =

     0
```

```
x =

     3


y =

     1


z =

     1


ans =

[3^(1/2)/2, 0,       1/2, 3]
[        0, 1,         0, 1]
[     -1/2, 0, 3^(1/2)/2, 1]
[        0, 0,         0, 1]


R4 =

     1
     0
     0
     1


ans =

3^(1/2)/2 + 3
            1
          1/2
            1


point1 =

     0     0     0


point2 =

     3     1     1


point3 =

    3.8600    1.0000    0.5000
```

```
vector1 =

     0     0     0
     3     1     1


vector2 =

    3.0000    1.0000    1.0000
    3.8600    1.0000    0.5000


vector3 =

    3.8600    1.0000    0.5000
         0         0         0

>>
```


X 3
Y 1
Z 1
X 3.86
Y 1
Z 0.5
X 0
Y 0
Z 0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0 0.2 0.4 0.6 0.8 1
0 0.5 1 1.5 2 2.5 3 3.5 4