MAE - 547

Home-work - 5

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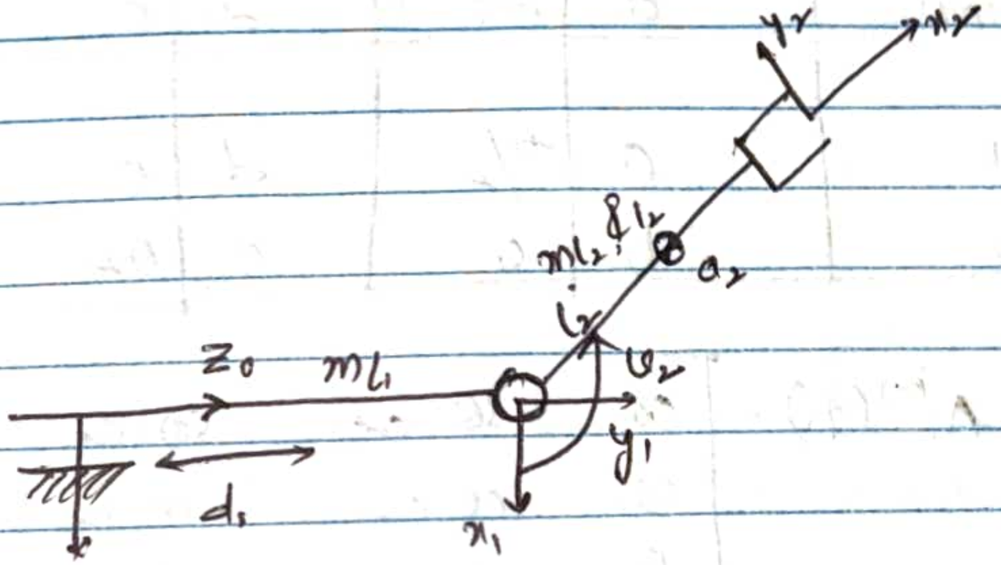
problem-1




Given: $B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$

$l_2$ - Distance from center to revolute joint

$k_{r_1}, k_{r_2}$ — gear ratios

$m_{m_1}, m_{m_2}$ — masses of the rotors of two joints

$I_{m_1}, I_{m_2}$ - moment of inertia

$\tau_1, \tau_2$ - Torques applied to the joint

applying the D-H convention to the two-link planar

| | w.r.t $x_i$ | | w.r.t $z_i$ | |
|---|---|---|---|---|
| Links | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
| 1 | 0 | $\pi/2$ | $d_1$ | 0 |
| 2 | $l_2$ | 0 | 0 | $\theta_2$ |

$$A_i^{i-1}(q_i) = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for the robtoing $T^0_2 = T^0_1 T^1_2$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let us consider

$J_p^{l_1}, J_p^{l_2}$ are the position jacobian of links

$J_p^{m_1}, J_p^{m_2}$ are the position jacobian of motors

$J_O^{l_1}, J_O^{l_2}$ are the orientation jacobian of links

$J_O^{m_1}, J_O^{m_2}$ are the orientation jacobian of motors

$$J_p^{l_1} = \begin{bmatrix} J_p^{l_1} & 0 \end{bmatrix} = \begin{bmatrix} z_0 & 0 \end{bmatrix}$$

$$J_p^{l_2} = \begin{bmatrix} J_{p_1}^{l_1} & J_{p_2}^{l_2} \end{bmatrix} = \begin{bmatrix} z_0 & z_1 (p_{l_2} - p_{l_1}) \end{bmatrix}$$

from the transformation matrix, $z_0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

$p_{l_1} = \begin{bmatrix} 0 & 0 & d_1 \end{bmatrix}^T$, $z_1 = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T$ and

$p_{l_2} = \begin{bmatrix} l_2 c\theta_2 & 0 & l_2 s\theta_2 + d_1 \end{bmatrix}^T$

$$J_p^{l_2} = \begin{bmatrix} z_0 & z_1 (p_{l_2} - p_1) \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} l_2 c\theta_2 - 0 \\ 0 - 0 \\ l_2 s\theta_2 + d_1 - d_1 \end{bmatrix} \end{bmatrix}$$

$$\therefore \cancel{J_p} = J_p^{l_2} = \begin{bmatrix} 0 & -l_2 s\theta_2 \\ 0 & 0 \\ 1 & l_2 c\theta_2 \end{bmatrix}$$

$$J_o^{l_1} = \begin{bmatrix} J_{o_1}^{l_1} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad f$$

$$J_o^{l_2} = \begin{bmatrix} J_{o_1}^{l_2} & J_{o_2}^{l_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$J_p^{m_1} = \begin{bmatrix} J_{p_1}^{m_1} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_p^{m_2} = \begin{bmatrix} J_{p_1}^{m_2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$J_o^{m_1} = \begin{bmatrix} J_o^{m_1} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ kr_1 & 0 \end{bmatrix}$$

$$J_O^{m_2} = \left[ J_{O_1}^{m_2} \quad J_{O_2}^{m_2} \right] = \begin{bmatrix} 0 & 0 \\ 0 & -kr_2 \\ 0 & 0 \end{bmatrix}$$

Calculating Inertia matrix

$$R_{m_1} = I$$

$$R_{m_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ 0 & 0 & -1 \\ S\theta_2 & C\theta_2 & 0 \end{bmatrix}$$

$$B(q) = \sum_{i=1}^{2} \Big[ m_{l_1} J_p^{(l_1)T} J_p^{l_1} (R_1 I_{l_1}^1 R_1^T) J_O^{l_1} + m_{m_1} J_p^{(m_1)T} J_p^{m_1} + J_O^{m_1} (R_m I_m^{m_1} R_{m_1}^T J_O^{m_1}) + m_{l_2} J_p^{(l_2)T} J_p^{l_2} + J_O^{(l_2)T} (R_2 I_{l_2}^2 R_2^T) J_O^{l_2} + m_{m_2} J_p^{(m_2)T} J_p^{m_2} + J_O^{(m_2)T} (R_{m_2} I_{m_2}^{m_2} R_{m_2}^T J_O^{m_2})$$

Substituting the Values, in the equation

$$B(q) = \begin{bmatrix} I_{m_1} kr_1^2 + m_{l_1} + m_{l_2} + m_{m_2} & m_{l_2} l_2 c_2 \\ l_2 m_{l_2} c_2 & I_{m_2}(kr_2)^2 + m_{l_2} l^2 + I_{l_2} \end{bmatrix}$$

$$c(q,\dot{q}) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$c_{ij} = \sum_{i=1}^{2} c_{ijk}\, q_k$$

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right]$$

$$c_{111} = \frac{1}{2}\left(\frac{\partial b_{11}}{\partial q_1}\right) = 0 \longrightarrow \text{from matrix 1}$$

$$c_{112} = c_{121} = \frac{1}{2}\frac{\partial b_{11}}{\partial q_2} = 0$$

$$c_{122} = \frac{\partial b_{12}}{\partial q_2} = \frac{1}{2}\left(\frac{\partial b_{22}}{\partial q_1}\right) = -l_2 m l_2 s\theta_2 - 0$$

$$= -l_2 m l_2 s\theta_2$$

$$c_{211} = \frac{1}{2}\left(\frac{2\partial b_{21}}{\partial q_1}\right) - \frac{\partial b_{11}}{\partial q_2}\left(\frac{1}{2}\right) = 0$$

$$c_{212} = c_{221} = \frac{1}{2}\frac{\partial b_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2}\frac{\partial b_{22}}{\partial q_2} = 0$$

$$c(q,\dot{q}) = \begin{bmatrix} 0 & -l_2 m_{l_2} s\theta_2 \dot{\theta}_2 \\ 0 & 0 \end{bmatrix}$$

$$g(q) = \text{gravity} = -\sum_{i=1}^{2} \left( m_{l_i} g_0^T J_{p_i}^{l_j}(q) + m_{m_j} g_0^T J_{p_i}^{m_j}(q) \right)$$

$$g_1 = -\left( m_{l_1} g_0^T J_p^{l_1} + m_{m_1} g_0^T J_{p_1}^{m_1} + m_{m_2} g_0^T J_{p_1}^{m_2} \right)$$

$$g_0^{-1} = [g \; 0 \; 0]^T$$

$$g_1 = -(0+0+0) = 0$$

$$g_2 = -\left( m_{l_1} g_0^T J_{p_2}^{(1,1)} + m_{m_1} g_0^T J_{p_2}^{m_1} + m_{l_2} g_0 J_{p_2}^{l_2} + m_{m_2} g_0^T J_{p_2}^{m_2} \right)$$

$$\therefore g_0^{-1} = [g \; 0 \; 0]$$

$$g_2 = [g l_2 m_{l_2} s\theta_2]$$

$$\therefore \tau = B(q)\ddot{q} + c(q,\dot{q})\dot{q} + g(q)$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} I_{m_1} k_{r_1}^2 + m_{l_1} + m_{l_2} + m_{m_2} & m_{l_2} l_2 c_2 \\ l_2 m_{l_2} c_2 & I_{m_2}(k_{r_2})^2 + m_{l_2} l_2^2 + I_{l_2} \end{bmatrix} \times \begin{bmatrix} \ddot{d}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & -l_2 m \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ g l_2 m_{l_2} s_2 \end{bmatrix}$$

Torque $\tau_1$ $\tau_2$ applied to joints are given by

$$\tau_1 = [(\mathcal{I} m_1 k_{r_1}^2 + m_{l_2} + m_{l_2} + m m_2)\ddot{d}_1 + (m l_2 l_2 c_2)\ddot{\theta}_2 + (-l_2 m l_2 s_2 \dot{\theta}_2^2)]$$

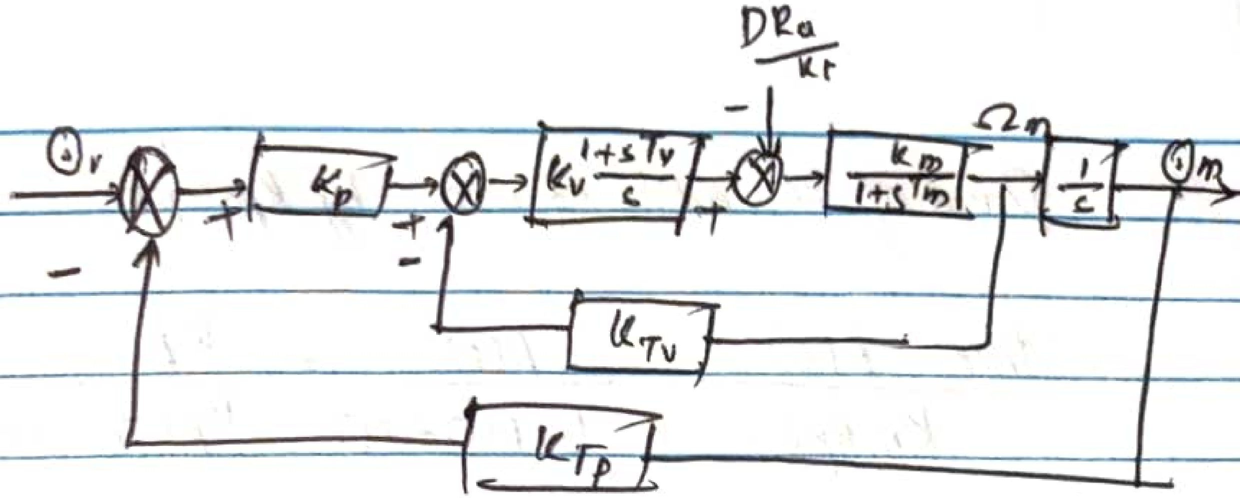
$$\tau_2 = ((l_2 m l_2 c_2)\ddot{d}_1 + (\mathcal{I} m_2 (k_{r_2})^2 + m_{l_2} l_2^2 + \mathcal{I} l_2)\ddot{\theta}_2 + (g l_2 m l_2 s_2)]$$

1. (ii) The man of enhanced connection is varied by a concentrated tip load of mass

$$r_2' = m_1 + m_2$$

Because both $\mathcal{I}_1$ and $\mathcal{I}_2$ are defined with regard to the same point at the tip. and they stay the same.

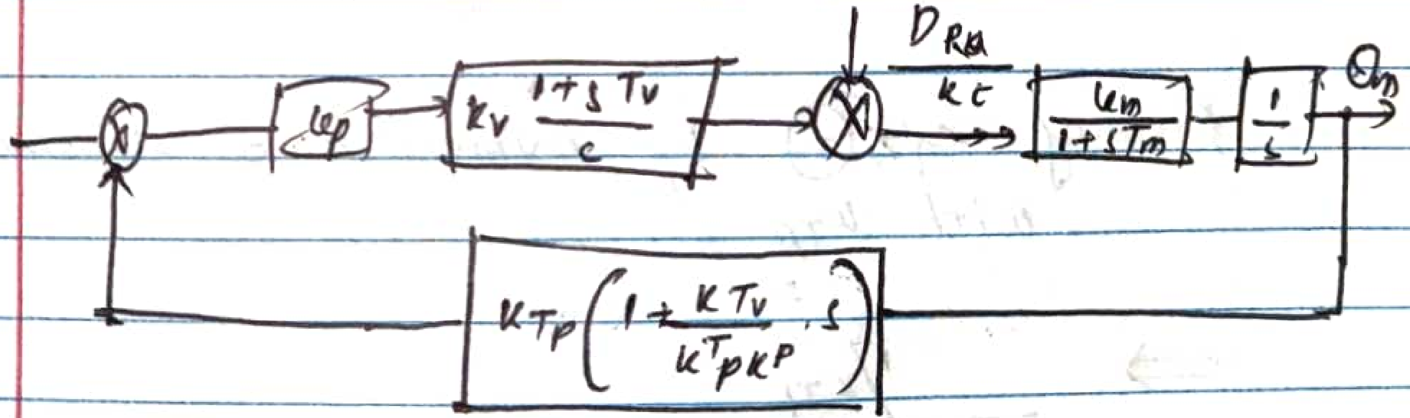
problem-2




① for finding the transfer function $\frac{\Theta_m(s)}{\Theta_r(s)}$

$K_{Tv}$ & $K_{Tp}$ are transducer constant

$$H(s) = K_{Tp}\left(1 + \frac{sK_{Tv}}{K_pK_{Tp}}\right)$$




forward path transfer function $(P(s)) = \frac{K_mK_pK_v}{s} \frac{(1+sT_v)}{(1+sT_m)} \cdot \frac{1}{s}$

$$P(s) = \frac{K_mK_pK_v}{s^2} \quad \{As \cdot T_v = T_m\}$$

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{P(s)}{1+P(s)H(s)} = \frac{K_mK_pK_v}{s^2} \times \left(\frac{K_p s^2}{(K_pK_{Tp}+sK_{Tv})(K_mK_pK_v)+K_ps^2}\right)$$

Dividing the whole equation by $K_m K_p K_v$.

then $\frac{P(s)}{K_m K_p K_v} = \frac{P(s)}{1+P(s)H(s)} = \frac{K_p}{K_p K_{Tp} + sK_v^T + \frac{K_p s^2}{K_m K_p K_v}} \rightarrow ①$

again dividing above equation with $K_p$ to eqn ①

$$= \frac{1}{K_{Tp} + \frac{sK_{TV}}{K_p} + \frac{s^2}{K_p K_v K_m}} \rightarrow ②$$

Dividing eq'n ② by numerator & denominator with $K_{TP}$

$$\Rightarrow \frac{1/K_{TP}}{1 + \frac{sK_{TV}}{K_p K_{TP}} + \frac{s^2}{K_p K_v K_m K_{TP}}}$$

Therefore, we get transfer function

$$\frac{\theta_m(s)}{\theta_r(s)} = \frac{1/K_{Tp}}{1 + \frac{sK_{Tv}}{K_p K_{TP}} + \frac{s^2}{2K_p K_v K_{TP}}} \quad \because K_m = 2$$

(iii) finding the transfer function $\frac{\theta_m(s)}{D(s)}$

$$\theta_m(s) = P(s) = \frac{K_m}{1+sT_m} \times \frac{1}{s}$$

$$H(s) = \frac{K_{Tp}(1+K_{Tv}s)}{K_{Tp}\cdot K_p} \cdot K_pK_v\frac{(1+sT_v)}{s}$$

$$\frac{\theta_m(s)}{D(s)\frac{R_a}{K_t}} = \frac{K_m/s(1+sT_m)}{1+\left(\frac{K_m}{1+sT_m}\right)\frac{1}{s}\cdot K_{Tp}\left(\frac{1+sK_{Tv}}{K_pK_{Tp}}\right)\cdot\frac{K_pK_v(1+sT_v)}{s}}$$

$$\Rightarrow \frac{\frac{K_m}{s(1+sT_m)}}{1+\frac{K_m}{s^2}\cdot(K_pK_{Tp}+sK_{Tv})\cdot K_v}$$

$$\Rightarrow \frac{K_m s^2}{s(1+sT_m)\left(s^2+K_mK_v(K_pK_{Tp}+sK_{Tv})\right)}$$

Dividing the numerator & Denominator by $K_mK_vK_pK_{Tp}$

$$\frac{\theta_m(s)}{D(s)} = \frac{\frac{s}{K_vK_pK_{Tp}(1+sT_m)}}{1+\frac{s^2}{K_mK_vK_pK_{Tp}}+\frac{s\cdot K_{Tv}}{K_pK_{Tp}}}$$

$$\boxed{\frac{\Theta_m(s)}{D(s)} = \frac{\dfrac{-s \cdot R_a}{K_t K_v K_p K_{tp}(1+sT_m)}}{1 + \dfrac{s^2}{K_m K_v K_p K_{TP}} + \dfrac{s \cdot K_{TV}}{K_p K_{TP}}}}$$

(iii) finding the parameters of the controller with position and velocity feedback.

$\because \xi \geq 0.4$ and $\omega_n = 20$ rad/s.

# $K_v K_{TV} = \dfrac{2 J \omega_n}{K_m}$ $\qquad \therefore K_{TV} = \dfrac{2 J \omega_n}{K_m \cdot K_v} \longrightarrow (1)$

$K_p K_{Tp} K_v = \dfrac{\omega_n^2}{K_m}$ $\qquad K_p K_{Tp} = \dfrac{\omega_n^2}{K_m K_v} \longrightarrow (2)$

$K_{Tp} = 1$ $\quad K_{TV} = 1$, $\quad K_m = 2$

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{1/K_{Tp}}{1 + \dfrac{s K_{TV}}{K_p K_{Tp}} + \dfrac{s^2}{K_p K_v K_m K_{Tp}}}$$

$$= \frac{\dfrac{1}{K_{TP}}}{1 + \dfrac{s \cdot 2 \cdot J \omega_n}{K_m K_v} \times \dfrac{K_m K_v}{\omega_n^2} + \dfrac{s^2}{\omega_n^2}}$$

$$= \frac{1/K_{Tp}}{1 + \dfrac{2 T_s}{\omega_n} + \dfrac{s^2}{\omega_n^2}}$$

∴ from The equations ① & ②

$$K_v = \frac{\cancel{2} \times 0.4 \times 20}{\cancel{2} \times 1 \times 1} = 8$$

$$\boxed{K_v = 8}$$

$$K_p = \frac{\omega_n^2}{K_m K_v K_{Tp}} = \frac{20^2}{2 \times 1 \times 8} = \frac{400}{16} = 25$$

$$\boxed{K_p = 25}$$

Problem-3

$$q = -K_p q - K_p \dot{q} + r \qquad \cdot q = \dot{q}$$

$$\ddot{q} + K_p \dot{q} + K_p q = r \qquad \ddot{\tilde{q}} + K_p \dot{\tilde{q}} + K_p \tilde{q} = \phi$$

Therefore $r = \ddot{q}_d + K_p \dot{q}_d + K_p q_d$

$K_p$ and $K_p$ are determined by the given joint error dynamics

$\therefore$ Joint 1 : $T_s = \dfrac{4}{\zeta \omega_n} = 0.5$

$$\zeta = \frac{-\ln(\%05/100)}{\sqrt{\pi^2 + \ln^2(\%05/100)}} = 0.591$$

$$\omega_n = \frac{8}{\zeta} = \frac{8}{0.591} = 13.54$$

$$2\zeta\omega_n = 2(0.591) \cdot 13.54 = 16.00$$

$$\omega_n^2 = 183.33$$

$$T_s = \frac{4}{\zeta\omega_n} = 1$$


$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \pi$$

$$\cos\theta = \zeta \qquad \therefore \zeta = \frac{4}{\sqrt{1+16}} = 0.97$$

$$\omega_n = \frac{4}{\zeta} = \frac{4}{0.591} = 4.12$$

$$K_{d-22} = 2\zeta\omega_n = 2\times 0.97\times 4.12 = 7.99$$

$$K_{p-22} = \omega_n^2 = (4.12)^2 = 16.97$$

$$\therefore \ddot{q} = -K_p q - K_p\dot{q} + \ddot{q}_d + K_p\dot{q}_d + K_p q_d$$

$$= \ddot{q}_d + K_p\dot{\tilde{q}} + K_p\tilde{q}$$

$$\boxed{\therefore K_p = \begin{bmatrix} 183.42 & 0 \\ 0 & 16.912 \end{bmatrix}}$$

16.912

$$\therefore \boxed{kd = \begin{bmatrix} 15.992 & 0 \\ 0 & 8.0012 \end{bmatrix}}$$