MAE 547 - MODELLING AND CONTROL OF ROBOTS   H.W-1.

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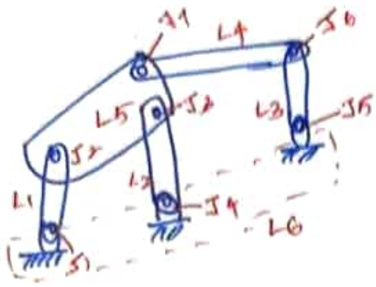
Stephenson six-bar linkages.

Q1.



Degree of freedom → the "smallest" number of real valued cordinates

$$DoF = m(N-1-J) + \sum_{i=1}^{J} f_i$$

for planar mechanism $m=3$

N - no. of links including ground

J - Joints.

$m=3$

$N=6$

$J=7$ (Revolute Joints)

D.O.F = 1

$f_i = 1$ ($i = 1,2,3,4,5,6,7$) - Rev.

$$DoF = 3(6-1-7) + 1 \cdot 7$$

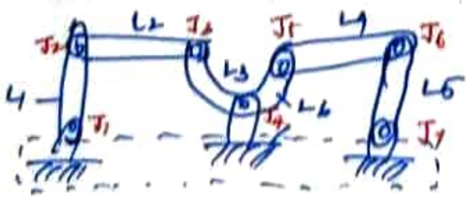
$$= 3(-2) + 7$$

$$= -6 + 7$$

$$\boxed{D.O.F = 1}$$

Watt-six-bar linkage

Q1(b)



it is a planar mechanism

$\therefore m=3$

$$D.O.F = m(N-1-J) + \sum_{i=1}^{J} F_i$$

no. of links $N = 6$. , $f_i = 1$

no. of Joints $= 7$ (Revolute Joints $\because$ D.o.f $= 1$)

$$D.o.f = m(N-1-J) + \sum_{i=1}^{J=7} f_i$$

$$= 3(6-1-7) + 1.7$$

$$= 3(-2) + 7$$

$$= -6 + 7$$

$$\boxed{D.o.F = 1}$$

Q.1.iii parallel manipulator.

since, it is a spatial mechanism $m = 6$.

$\Rightarrow$ No. of links $N = 2 \times 3 + 1 + 1$ (including grounds)

$N = 8$

No. of Joints $J = 9$

$$f_i = \begin{cases} 2 & (i = 1, 2, 3, 4, 5, 6 \text{ ; Universal joint}) \\ 1 & (i = 7, 8, 9) - \text{prismatic joint} \end{cases}$$

$$\because D.O.F = m(N-1-J) + \sum_{i=1}^{J=9} F_i$$

$$= 6(8-1-9) + 6(2) + 3(1)$$

$$= 6(-2) + 12 + 3$$

$$\boxed{D.O.F = 3}$$

Q.1 (4) ⇒ Parallel manipulator.

Sol^n :- Since, it is a spatial mechanism $m = 6$

No. of links (including ground) $N = 2 \times 6 + 1 + 1$

$N = 14$

No. of Joints $J = 18$ $\left[\because \begin{array}{l} J_1 - 6 \text{ Revolute Joints} \\ J_2 - 6 \text{ universal Joints} \\ J_3 - 6 \text{ Spechirical Joints} \end{array} \right]$

$$F_i = \begin{cases} 1 & (i = (1,2,3,4,5,6) - \text{Revolute Joints}) \\ 2 & (i = 7,8,9,10,11,12) - \text{Universal Joints}) \\ 3 & (i = 13,14,15,16,17,18) - \text{Spherical Joints}) \end{cases}$$

$$D.O.F = m(N - 1 - J) + \sum_{i=1}^{J=18} F_i$$

$$= 6(14 - 1 - 18) + 6(3) + 6(2) + 6(1)$$

$$= 6(-5) + 6(6)$$

$$= -30 + 36$$

$$\boxed{D.O.F = 6}$$

Q.2 Rotation matrix.

(i) Rotation about x-axis by 60° CCW

Substituting $\phi = 60^\circ$ in the Reference 'x' matrix

$$\cancel{R_y(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & & \end{bmatrix}}$$

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \longrightarrow \text{eq'n } ①$$

Substituting $\phi = 60^\circ$ in the eqn ① in counter clockwise (+ve)

$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60^\circ & -\sin 60^\circ \\ 0 & \sin 60^\circ & \cos 60^\circ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -0.866 \\ 0 & 0.866 & 0.5 \end{bmatrix}$$

(ii) Rotation about the current y axis by 30° C.C.W

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$\cancel{\phi}$ $\theta = 30^\circ$ in counter clock wise i.e., +ve gives R_2^1

$$R_2^1 = \begin{bmatrix} \cos 30^\circ & 0 & \sin 30^\circ \\ 0 & 1 & 0 \\ -\sin 30^\circ & 0 & \cos 30^\circ \end{bmatrix} = \begin{bmatrix} 0.866 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & 0.866 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & -1 & 0 \end{bmatrix}$$

(iii) Rotation about the Reference z-axis by 60° C.C.W. (+ve)

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 60° & -\sin 60° & 0 \\ \sin 60° & \cos 60° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3^2 =$

∵ With respect to Reference rotation ∵ $R_3^0 = R_3^2 \times R_1^0 \times R_2^1$

premultiplication of R_3^2 w.r.t R_1^0, R_2^1

$$\therefore R_3^0 = \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -0.866 \\ 0 & 0.866 & 0.5 \end{bmatrix} \begin{bmatrix} 0.866 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & 0.86 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & -0.433 & 0.7445 \\ 0.866 & 0.251 & -0.431 \\ 0 & 0.86 & 0.5 \end{bmatrix} \begin{bmatrix} 0.866 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & 0.866 \end{bmatrix}$$

$$\therefore R_3^0 = \begin{bmatrix} 0.05812 & -0.433 & 0.849 \\ 0.9649 & 0.25 & \cdot 0.0580 \\ -0.25 & 0.86 & 0.433 \end{bmatrix}$$

(iv) Rotation about the current x-axis by 45° CW

$$\therefore R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad \because \text{clockwise } R_{x\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta) & -\sin(-\theta) \\ 0 & \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$\therefore R_x\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

Substituting $\theta = 45°$ in the above matrix

$$R_4^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(45°) & \sin 45° \\ 0 & -\sin 45° & \cos 45° \end{bmatrix}$$

$$R_4^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.707 & 0.707 \\ 0 & -0.707 & 0.707 \end{bmatrix}$$

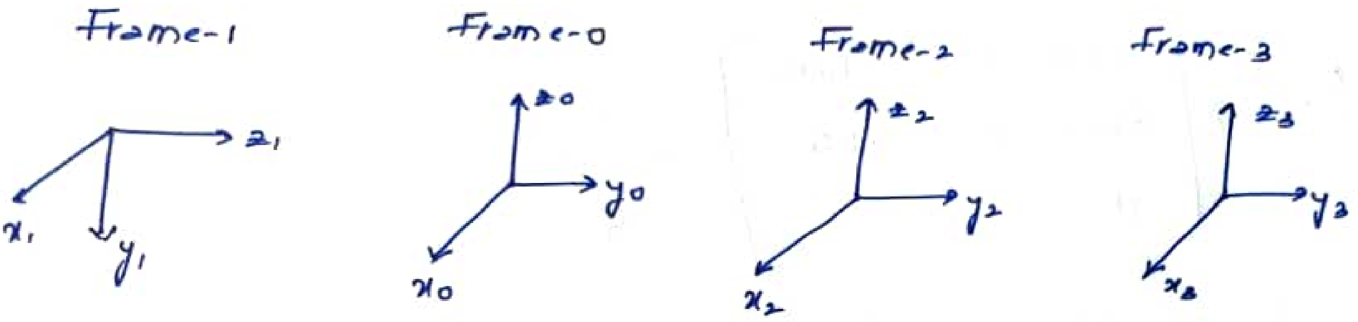
Resultant $\Rightarrow$ $R_4^0 = R_3^0 \times R_4^3$

$$= \begin{bmatrix} 0.0580 & -0.433 & 0.8949 \\ 0.9649 & 0.25 & 0.0580 \\ -0.25 & 0.86 & 0.433 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.707 & 0.707 \\ 0 & -0.707 & 0.707 \end{bmatrix}$$

$$R_4^0 = \begin{bmatrix} 0.0580 & -0.9422 & 0.3299 \\ 0.964 & 0.13491 & 0.2174 \\ -0.25 & 0.306 & 0.9186 \end{bmatrix}$$

(2) iii Rotation matrix R_3^0

Given:- With respective to the given question there are 4 frames as below



To find R_3^0 we need to multiply the matrices R_1^0, R_2^1, R_3^2

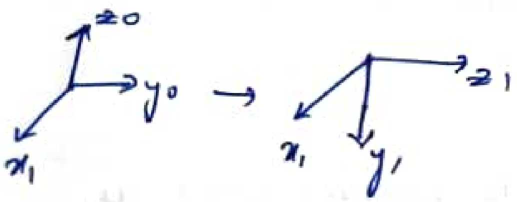
$$\because \quad R_3^0 = R_1^0 \times R_2^1 \times R_3^2$$

Step-1 The frame rotation is done by n to n-1 frames.

Frame-1 & frame-0 are mis aligned by rotating with a compensating matrix. Two axis will be aligned in same. w.r.t to the z_0 axis

$$[z_0][C] = R_1^0$$

C - compensating matrix



$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{w.r.t. } \theta_1$$

$$R_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & -1 & 0 \end{bmatrix}$$

Step-2 Aligning Frame-I w.r.t Frame-2

To rotate frame-1 w.r.t Frame-2 we need to implement a compensate matrix $R_2^1 = [z_1][c]$ $\quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad z_1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

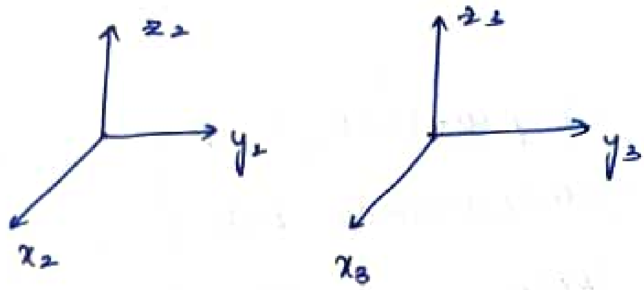
compensating matrix $\qquad$ z_1 is rotating w.r.t θ_2

$$R_2^1 = [z_1][c]$$

$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ \sin\theta_2 & 0 & -\cos\theta_2 \\ 0 & 1 & 0 \end{bmatrix}$$

Step - III



These two frames are same in alignment so, we need to multiply simply with Converse Identity matrix

$$\because \quad R_2^3 = I$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step - IV The Resultant matrix

$$R_3^0 = R_1^0 \times R_2^1 \times R_3^2$$

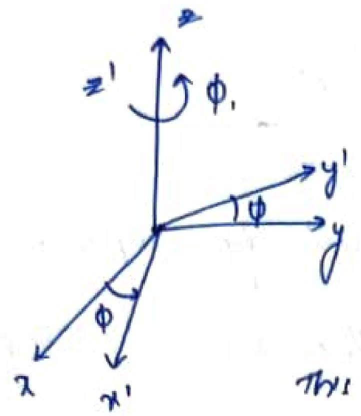
$$R_3^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ \sin\theta_2 & 0 & -\cos\theta_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3^0 = \begin{bmatrix} c\theta_1 c\theta_2 & -s\theta_1 & c\theta_1 s\theta_2 \\ s\theta_1 c\theta_2 & c\theta_1 & s\theta_1 s\theta_2 \\ -s\theta_2 & 0 & c\theta_2 \end{bmatrix}$$

Q.3 ?

Euler angle rotation ZYX

sol

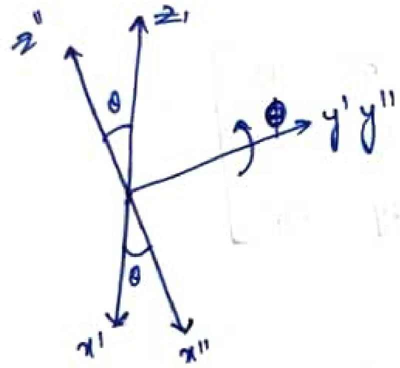


$\Rightarrow$ Rotate the current frame by the angle ϕ about axis z

This rotation is described by

$$R_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$

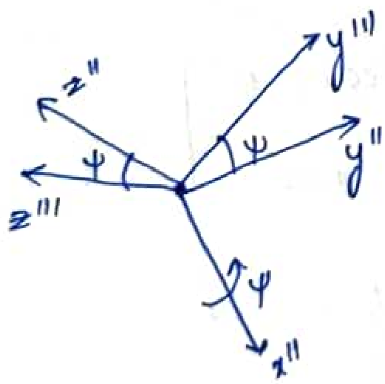


Rotate the current frame by the angle θ about axis y

this rotation is described by

$$R_{y'}(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$\Rightarrow$



Rotate the current frame by the angle ψ about axis x.

This Rotation is described by

$R_{x''}(\psi) = \begin{bmatrix} \cancel{\cos\psi} & \cancel{-\sin\psi} & \cancel{0} \\ \cancel{\sin\psi} & \cancel{\cos\psi} & \cancel{0} \\ \cdot & & \end{bmatrix}$

$$R_{x''}(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

$\Rightarrow$ the Resulting frame Orientation is obtained by composition of rotations w.r.t to current frames and then it can be computed via post-multiplication of the matrices of elementary Rotation

$$R = R_z(\phi)\, R_{y'}(\theta)\, R_{x''}(\psi)$$

$$R = \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix}$$

$$R = \begin{bmatrix} c\phi c\theta & -s\phi & c\phi s\theta \\ s\phi c\theta & c\phi & s\phi s\theta \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix}$$

$$R = \begin{bmatrix} c\phi\, c\theta & -s\phi\, c\psi + c\phi\, s\theta\, s\psi & s\phi\, s\psi + c\phi\, c\psi\, s\theta \\ s\phi\, c\theta & c\phi\, c\psi + s\phi\, s\theta\, s\psi & -c\phi\, s\psi + c\psi\, s\phi\, s\theta \\ -s\theta & c\theta\, s\psi & c\theta\, c\psi \end{bmatrix}$$

Q.3 :

$$R = \begin{bmatrix} 0.5 & -0.1464 & 0.8536 \\ 0.5 & 0.8536 & -0.1464 \\ -0.707 & 0.5 & 0.5 \end{bmatrix}$$

Comparing above matrix with zyx matrix

$$R = \begin{bmatrix} c\theta c\phi & -s\theta c\psi + c\phi s\theta c\psi & s\theta s\psi + c\phi c\psi s\theta \\ s\theta c\phi & c\phi c\psi + s\phi s\theta s\psi & -c\phi s\psi + c\psi s\psi s\theta \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix}$$

From the equations $\theta = atan2\left(-r_{31} \pm \sqrt{r_{11}^2 + r_{21}^2}\right)$

$$= atan2\left(-[-0.7071] \pm \sqrt{0.5^2 + 0.5^2}\right)$$

$$= atan2\left(0.7071 \pm 0.7071\right)$$

$\therefore \boxed{\theta = 45^\circ}$ $\boxed{\theta = 135^\circ}$

$\Rightarrow$ We know that $\phi = atan2\left[\frac{r_{21}}{\cos\theta}, \frac{r_{11}}{\cos\theta}\right]$

$\phi = atan2\left[\frac{0.5}{\cos 45}, \frac{0.5}{\cos 45}\right]$ $\phi = atan2\left(\frac{0.5}{\cos 135}, \frac{0.5}{\cos 135}\right)$

$\phi = atan2\left(0.7071, 0.7071\right)$ $\phi = atan2\left(-0.7071, -0.7071\right)$

$\boxed{\phi = 45^\circ}$ $\boxed{\phi = -135^\circ}$

$\Rightarrow$ Also $\psi = \text{atan2}\left[\dfrac{r_{32}}{\cos\theta}, \dfrac{r_{33}}{\cos\theta}\right]$

$\psi = \text{atan2}\left[\dfrac{0.5}{\cos 45^\circ}, \dfrac{0.5}{\cos 45}\right)$ $\qquad$ $\psi = \text{atan2}\left(\dfrac{0.5}{\cos 135}, \dfrac{0.5}{\cos 135}\right]$

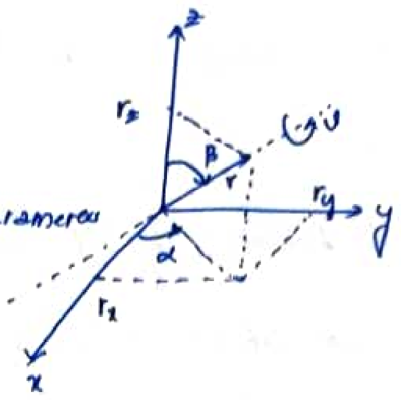
$\psi = \text{atan2}(0.7071, 0.7071)$ $\qquad$ $\psi = \text{atan2}[-0.7071, -0.7071]$

$\boxed{\psi = 45^\circ}$ $\qquad$ $\boxed{\psi = -135^\circ}$

$\therefore$ The two sets of the Euler angles are given by

$$\begin{bmatrix}\phi \\ \theta \\ \psi\end{bmatrix} = \begin{bmatrix}45^\circ \\ 45^\circ \\ 45^\circ\end{bmatrix}, \quad \begin{bmatrix}\phi \\ \theta \\ \psi\end{bmatrix} = \begin{bmatrix}-135^\circ \\ 135^\circ \\ -135^\circ\end{bmatrix}$$

Q.4 soln:-



⇒ A nominal representation of orientation can be obtained by restoring to 4-parameters expressing a rotation of a given angle about an axis in space

⇒ O - xyz Reference frame. $\because \boxed{r_x^2 + r_y^2 + r_z^2 = 1} = r$

⇒ R is a unit - vector.

⇒ Aligning r with z, which is obtained as the sequence a rotation of $-\alpha$ ~~axis~~ about z-axis and a rotation of $-\beta$ about y - axis

$$R_y(-\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(-\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Rotate by θ about z - axis.

⇒ Realign with the initial direction of r, which is obtained as the sequence

Rotation by β about y-axis followed by a rotation α about z-axis

$$R_z(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

In sum, the resulting rotation matrix is

$$R(\theta, r] = R_z(\alpha)\, R_y(\beta)\, R_z(\theta)\, R_y(-\beta)\, R_z(-\alpha)$$

$$R[\theta, r] = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_z(\alpha)$ $\quad R[\theta, r] = R_z(\alpha)\, R_y(\beta)\, R_z(\theta)\, R_y(-\beta)\, R_z(-\alpha)$

$$R_z(\alpha)\, R_y(\beta)\, R_z(\alpha) = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\alpha c\beta c\theta - s\alpha s\theta & -c\alpha c\beta s\theta + s\alpha c\theta & c\alpha s\beta \\ s\alpha c\beta c\theta + c\alpha s\theta & -s\alpha c\beta s\theta + c\alpha c\theta & s\alpha s\beta \\ -s\beta c\theta & s\beta c\theta & c\theta \end{bmatrix}$$

$R(\theta, r) =$

$R_z(\alpha)\, R_y(\beta)\, R_z(\theta)\, R_y(-\beta)\, R_z(-\alpha)$

$$\begin{bmatrix} c\alpha c\beta c\theta - s\alpha s\theta & -c\alpha c\beta s\theta + s\alpha c\theta & c\alpha s\beta \\ s\alpha c\beta c\theta + c\alpha s\theta & -s\alpha c\beta s\theta + c\alpha c\theta & s\alpha s\beta \\ -s\beta c\theta & s\beta c\theta & c\theta \end{bmatrix} \begin{bmatrix} c\beta & 0 & -s\beta \\ 0 & 1 & 0 \\ s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} c\alpha & s\alpha & 0 \\ -s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta, r) = \begin{bmatrix} (s^2\alpha + c^2\alpha c\beta)c\theta + c^2\alpha s^2\beta & -c\beta s\theta + s\alpha c\alpha s^2\beta[1-c\theta] & s\alpha s\beta s\theta + c\alpha c\beta s\beta[1-c\theta] \\ c\beta s\theta + s\alpha c\alpha s^2\beta[1-c\theta] & [s^2\alpha c^2\beta + c^2\alpha]c\theta + s^2\alpha s^2\beta & s\alpha s\beta c\beta(1-c\theta) - c\alpha s\beta s\theta \\ c\alpha s\beta c\beta(1-c\theta) - s\alpha s\beta s\theta & s\alpha s\beta c\beta[1-c\theta] + c\alpha s\beta s\theta & s^2\beta c\theta + c^2\beta \end{bmatrix}$$

where, as we have a unit vector i.e., $a_x^2 + a_y^2 + a_z^2 = 1$

$$\boxed{\cos\alpha = \frac{r_x}{\sqrt{r_x^2+r_y^2}}} \rightarrow ①$$

$$\boxed{\cos\beta = r_z} \rightarrow ②$$

$$\boxed{\sin\alpha = \frac{r_y}{\sqrt{r_x^2+r_y^2}}} \rightarrow ③$$

$$\boxed{\sin\beta = \sqrt{r_x^2+r_y^2}} \rightarrow ④$$

Substituting the eq'n ①, eq'n ②, eq'n ③ & eq'n ④ in the elements of the matrix $R(r, \theta)$

$$r_{11} = \left[S_\alpha^2 + C_\alpha^2 C_\beta^2 \right] C\theta + C_\alpha^2 S_\beta^2 = \left[1 - r_x^2\right] C\theta + r_x^2$$

$$\because S^2\alpha + C^2\alpha C^2\beta = 1 - r_x^2 \times \frac{r_x^2}{\cancel{r_x^2+r_y^2}} \left(\sqrt{\cancel{r_x^2+r_y^2}}\right)^2$$

$$\boxed{r_{11} = r_x^2 \left(1 - C\theta\right] + C\theta}$$

$$r_{12} = -C\beta S\theta + S\alpha C\alpha S^2\beta \left[1 - C\theta\right].$$

$$= \frac{r_y}{\sqrt{r_x^2+r_y^2}} \times \frac{r_x}{\sqrt{r_x^2+r_y^2}} \times \sqrt{r_x^2+r_y^2} \left(1 - C\theta\right] - r_z S\theta$$

$$\boxed{r_{12} = r_y r_x \left(1 - C\theta\right] - r_z S\theta}$$

$$r_{13} = c\alpha\, s\beta\, c\beta [1 - c\theta] + s\alpha\, s\beta\, s\theta.$$

$$= \frac{r_x}{\cancel{\sqrt{r_x^2+r_y^2}}} \cancel{\sqrt{r_x^2+r_y^2}} \cdot r_z [1 - c\theta] + \frac{r_y}{\cancel{\sqrt{r_x^2+r_y^2}}} \cancel{\sqrt{r_x^2+r_y^2}}\, s\theta$$

$$\boxed{r_{13} = r_x r_z [1 - c\theta] + r_y s\theta}$$

$$r_{21} = s\alpha\, c\alpha\, s^2\beta [1 - c\theta] + c\beta\, s\theta$$

$$= \frac{r_y}{\cancel{\sqrt{r_x^2+r_y^2}}} \frac{r_x}{\cancel{\sqrt{r_x^2+r_y^2}}} \left(\cancel{\sqrt{r_x^2+r_y^2}}\right)^2 [1 - c\theta] + r_z s\theta$$

$$\boxed{r_{21} = r_x r_y (1 - c\theta] + r_z s\theta}$$

$$r_{22} = [s^2\alpha\, c\beta^2 + c\alpha^2]\, c\theta + s^2\alpha\, s^2\beta$$

$$\left\{ \frac{r_y^2}{\left(\sqrt{r_x^2+r_y^2}\right)^2} \times r_z^2 + \frac{r_x^2}{\left(\sqrt{r_y^2+r_x^2}\right)^2} \right\} c\theta + \frac{r_y^2}{\left(\sqrt{r_x^2+r_y^2}\right)^2} \times [r_x^2 + r_y^2]$$

$$r_{22} = (1 - r_y^2)\, c\theta + r_y^2 \qquad [\because s^2\alpha\, c\beta^2 + c\alpha^2 = 1 - r_y^2]$$

$$\boxed{r_{22} = r_y^2 [1 - c\theta] + c\theta}$$

$$r_{23} = c\alpha s\beta c_\beta (1 - c\theta) - s\alpha s\beta c\theta$$

$$= \frac{r_x}{\cancel{\sqrt{r_x^2+r_y^2}}} \times \cancel{\sqrt{r_x^2+r_y^2}} \times r_z (1 - c\theta) - \frac{r_y}{\cancel{\sqrt{r_x^2+r_y^2}}} \times \cancel{\sqrt{r_x^2+r_y^2}} \; s\theta$$

$$\boxed{r_{23} = r_x r_z [1 - c\theta] - r_y s\theta}$$

$$r_{31} = c\alpha s\beta c\beta (1 - c\theta) - s\alpha s\beta c\theta$$

$$= \frac{r_x}{\cancel{\sqrt{r_x^2+r_y^2}}} \times \cancel{\sqrt{r_x^2+r_y^2}} \times r_z [1 - c\theta] - \frac{r_y}{\cancel{\sqrt{r_x^2+r_y^2}}} \times \cancel{\sqrt{r_x^2+r_y^2}} \; s\theta$$

$$\boxed{r_{31} = r_x r_z (1 - c\theta] - r_y s\theta}$$

$$r_{32} = s\alpha s\beta c\beta (1 - c\theta] + c\alpha s\beta \, s\theta$$

$$= \frac{r_y}{\cancel{\sqrt{r_x^2+r_y^2}}} \times \cancel{\sqrt{r_x^2+r_y^2}} \; r_z (1 - c\theta] + \frac{r_x}{\cancel{\sqrt{r_x^2+r_y^2}}} \cancel{\sqrt{r_x^2+r_y^2}} \; s\theta$$

$$\boxed{r_{32} = r_y r_z [1 - c\theta] + r_x s\theta}$$

$$r_{33} = s\beta^2 c\theta + c\beta^2$$

$$= [r_x^2 + r_y^2] c\theta + r_z^2 \qquad \because r_x^2 + r_y^2 + r_z^2 = 1$$

$$= (1 - r_z^2] c\theta + r_z^2 \qquad r_z^2 = 1 - r_x^2 + r_y^2$$

$$\boxed{r_{33} = r_z^2[1-c\theta]+c\theta}$$

Rewriting each element i.e., $\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$$R(\theta, r) = \begin{bmatrix} r_x^2[1-c\theta]+c\theta & r_x r_y[1-c\theta]-r_z c\theta & r_x r_z(1-c\theta)+r_y s\theta \\ r_x r_y[1-c\theta]+r_z s\theta & r_y^2[1-c\theta]+c\theta & r_y r_z[1-c\theta]-r_x s\theta \\ r_x r_z[1-c\theta]-r_y s\theta & r_y r_z[1-c\theta]+r_x c\theta & r_z^2[1-c\theta]+c\theta \end{bmatrix}$$

```
>> % HOME WORK - 1, MAE 547 MODELING AND CONTROL OF ROBOTS
% STUDENT NAME : MANOHAR AKULA
% ASU ID: 1223335191
% ROBOTICS AND AUTONOMOUS SYSTEMS (EE)
>>
>> R01 = rotx(60,'deg')% Rotation about x-axis by 60deg ccw

R01 =

    1.0000         0         0
         0    0.5000   -0.8660
         0    0.8660    0.5000

>> R12 = roty(30,'deg')% Rotation about the current y-axis by 30 deg ccw

R12 =

    0.8660         0    0.5000
         0    1.0000         0
   -0.5000         0    0.8660

>> R02 = R01 * R12

R02 =

    0.8660         0    0.5000
    0.4330    0.5000   -0.7500
   -0.2500    0.8660    0.4330

>> R23 = rotz(60,'deg')

R23 =

    0.5000   -0.8660         0
    0.8660    0.5000         0
         0         0    1.0000

>> R03 = R23 * R01 * R12 % Rotation about the reference z-axis by 60deg ccw

R03 =

    0.0580   -0.4330    0.8995
    0.9665    0.2500    0.0580
   -0.2500    0.8660    0.4330

>> R34 = rotx(-45,'deg')

R34 =

    1.0000         0         0
         0    0.7071    0.7071
         0   -0.7071    0.7071
```

```
>> R04 = R03 * R34 % Rotation about the x-axis by 45 cw

R04 =

    0.0580   -0.9422    0.3299
    0.9665    0.1358    0.2178
   -0.2500    0.3062    0.9186

>>
```

```
>>
% HOME WORK - 1, MAE 547 MODELING AND CONTROL OF ROBOTS
% STUDENT NAME : MANOHAR AKULA
% ASU ID: 1223335191
% ROBOTICS AND AUTONOMOUS SYSTEMS (EE)
>>
>> % Problem 3(3)
>>
>> M = [[0.5 -0.1464 0.8536];[0.5 0.8536 -0.1464];[-0.7071 0.5 0.5]] % Loaded given↙
matrix in a variable M

M =

    0.5000   -0.1464    0.8536
    0.5000    0.8536   -0.1464
   -0.7071    0.5000    0.5000

>>  % Using Trignometry function rotating matrix to ZYX format
>>
>> rpy = tr2rpy(M,'zyx','deg')

rpy =

   45.0000   44.9997   45.0000

>>
```