Assignment - 3

MAE - 547

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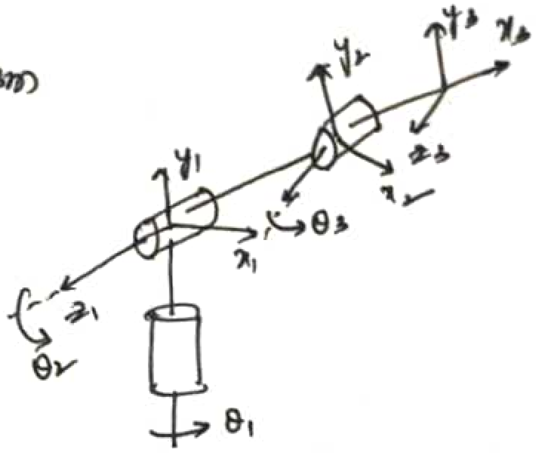
Q.1 Anthromorphic arm manipulator

Given $\theta_1 = 45^\circ$, $\theta_2 = 30^\circ$, $\theta_3 = 30^\circ$, $a_2 = 2m$, $a_3 = 3m$

It is a 3-DOF : Two link planar arm + addition rotation.



To find the Homogeneous matrix table

$$T_3^0 = A_1^0(\theta_1)\, A_2^1(\theta_2)\, A_3^2(\theta_3)$$

Using D.H. table $\underbrace{\text{w.r.t } x_i}$ $\underbrace{\text{w.r.t } z_{i-1}}$

Frame link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

- substituting given value in the D.H table.

frame link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	0	45°
2	2m	0	0	30°
3	3m	0	0	30°

$$A_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\because \theta = 45^\circ$, substituting in A_1^0

$$A_1^0 = \begin{bmatrix} \cos 45^\circ & 0 & \sin 45^\circ & 0 \\ \sin 45^\circ & 0 & -\cos 45^\circ & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_1^0 = \begin{bmatrix} 0.7071 & 0 & 0.7071 & 0 \\ 0.7071 & 0 & -0.7071 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $A_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\because \theta_2 = 30$, $a_2 = 2m$

$$A_2^1 = \begin{bmatrix} \cos 30 & -\sin 30 & 0 & 2 \times \cos 30 \\ \sin 30 & \cos 30 & 0 & 2 \times \sin 30 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 & 1.732 \\ 0.5 & 0.866 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\because \theta_3 = 30^\circ$, $a_3 = 3m$

$$A_3^2 = \begin{bmatrix} \cos 30 & -\sin 30 & 0 & 3 \times \cos 30 \\ \sin 30 & \cos 30 & 0 & 3 \times \sin 30 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 & 2.598 \\ 0.5 & 0.866 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

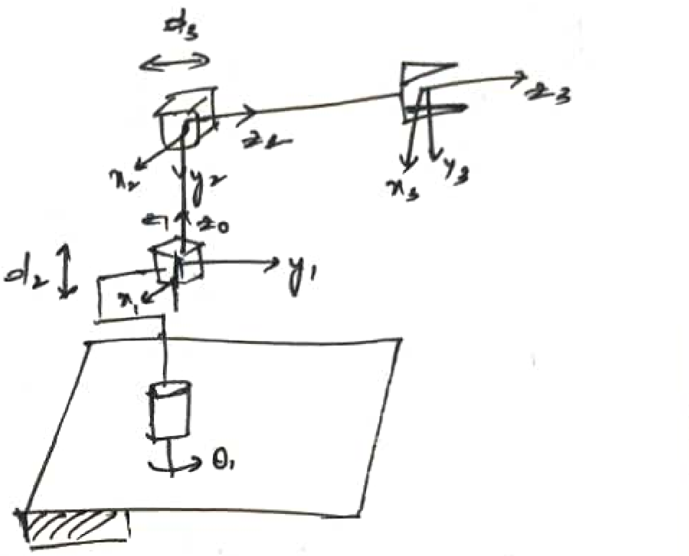
$$T_3^0 = A_1^0(\theta_1)\, A_2^1(\theta_2)\, A_3^2$$

$$T_3^0 = \begin{bmatrix} 0.7071 & 0 & 0.7071 & 0 \\ 0.7071 & 0 & -0.7071 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.5 & 0 & 1.732 \\ 0.5000 & 0.866 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.5 & 0 & 2.598 \\ 0.5 & 0.866 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} 0.3535 & -0.6123 & 0.7071 & 2.2853 \\ 0.3535 & -0.6123 & -0.7071 & 2.2853 \\ 0.8660 & 0.500 & 0 & 3.5980 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}$$ – using Matlab. answers

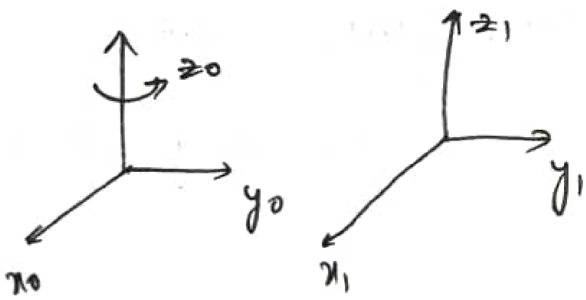
Problem – 4:

Inverse kinematics for the cylindrical arm

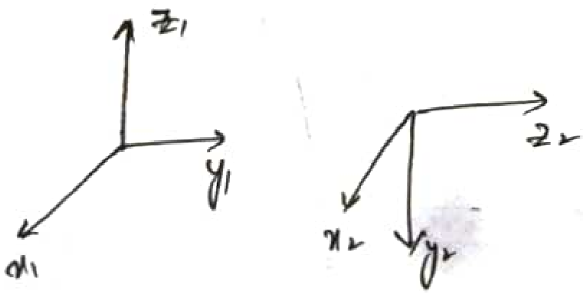


D.H for cylindrical arm in the fig.

frame	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1
2	0	$-\pi/2$	d_2	0
3	0	0	d_3	0

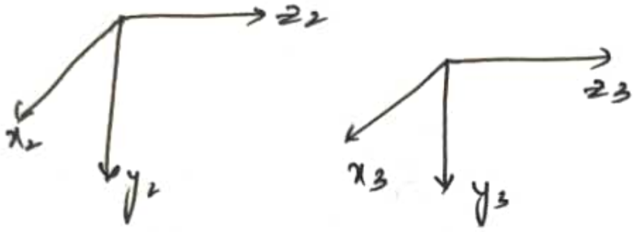


$$A_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$A_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∵ DH formula $A_i^{i-1}(\theta_i) = \begin{bmatrix} c_1 & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$



$$A_3^2(d_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1^0(\theta_1)\, A_2^1(d_2)\, A_3^2(d_3) = \begin{bmatrix} c_1 & 0 & -s_1 & -d_3 s_1 \\ s_1 & 0 & c_1 & d_3 c_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P_x = -d_3 \sin\theta_1, \quad P_y = d_3 \cos\theta_1, \quad P_z = d_2$

$\theta_1 = \operatorname{atan2}(-p_x, p_y) = \operatorname{atan2}(d_3 \sin\theta_1, d_3 \cos\theta_1)$

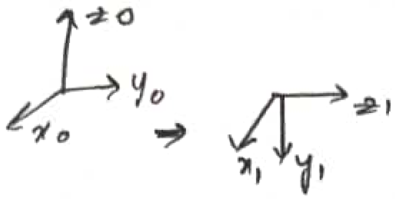
$d_3 = \sqrt{p_x^2 + p_y^2}$

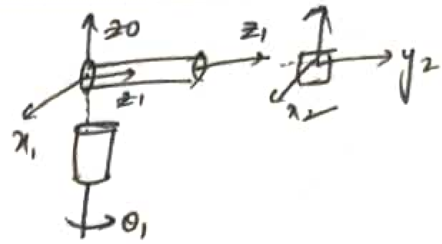
$\boxed{\theta_1 = \operatorname{atan2}[d_3 \sin\theta_1, d_3 \cos\theta_1]}$

$\boxed{d_3 = \sqrt{d_3^2 \sin^2\theta_1 + d_3^2 \cos\theta_1}}$

$\boxed{d_2 = P_z}$

Problem - 6





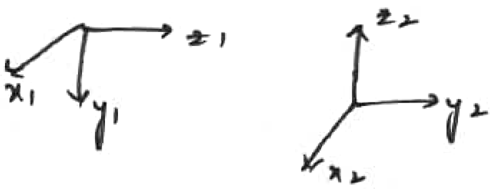
$z_1 \Leftrightarrow y_0$ & $z_0 \Leftrightarrow -y_1$ then matrix is given by $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

$$R_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

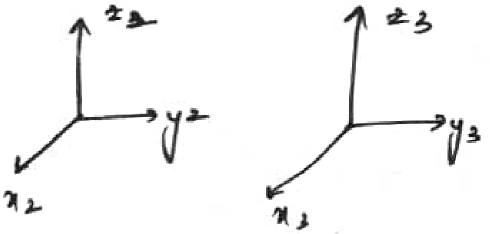
frame 2 & 1

for Reference

∴ $c_1 = \cos\theta_1$
$s_1 = \sin\theta_1$
$c_2 = \cos\theta_2$
$s_2 = \sin\theta_2$
$-s_1 = -\sin\theta_1$
$-c_1 = -\cos\theta_1$



$$\Rightarrow R_2^1 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$= R_3^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^0 = R_1^0 \times R_2^1 = \begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & -s_1d_2 \\ s_1c_2 & c_1 & s_1s_2 & c_1d_2 \\ -s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_3^0 = \begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & c_1s_2d_3 - s_1d_2 \\ s_1c_2 & c_1 & s_1s_2 & s_1s_2d_3 + c_1d_2 \\ -s_2 & 0 & c_2 & c_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} J_{P_1} & J_{P_2} & J_{P_3} \\ J_{O_1} & J_{O_2} & J_{O_3} \end{bmatrix}$$ singularity $J_{P_1}, J_{P_2}, J_{P_3}$ should be considered

$$J_{P_1} = z_0 \times (P_e - P_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} c_1 s_2 d_3 - s_1 d_2 \\ s_1 s_2 d_3 + c_1 d_2 \\ c_2 d_3 \end{bmatrix} = \begin{bmatrix} -s_1 s_2 d_3 - c_1 d_2 \\ c_1 s_2 d_3 - s_1 d_2 \\ 0 \end{bmatrix}$$

$$J_{P_2} = z_1 [P_e - P_1] = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \begin{bmatrix} c_1 s_2 d_3 - s_1 d_2 \\ s_1 s_2 d_3 + c_1 d_2 \\ c_2 d_3 \end{bmatrix} = \begin{bmatrix} c_1 c_2 d_3 \\ s_1 c_2 d_3 \\ -s_2 d_3 \end{bmatrix}$$

$$J_{P_3} = z_2 \,[\text{prismatic}] = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix}$$

$$\therefore J_P = \begin{bmatrix} -s_1 s_2 d_3 - c_1 d_2 & c_1 c_2 d_3 & c_1 s_2 \\ c_1 s_2 d_3 - s_1 d_2 & s_1 c_2 d_3 & s_1 s_2 \\ 0 & -s_2 d_3 & c_2 \end{bmatrix}$$

$$\Rightarrow \det|J_P| \Rightarrow \left[-s_1 s_2 d_3 - c_1 d_2 [s_1 c_2 d_3 c_2 + s_1 s_2 d_3 s_2]\right]$$

$$- \left[c_1 c_2 d_3 \left((c_1 s_2 d_3 - s_1 d_2) c_2 - 0\right]\right. + \left[c_1 s_2 [s_2 d_3 (s_1 d_2 - c_1 s_2 d_3] - 0\right]$$

$$\Rightarrow \left[-s_1 s_2 d_3 - c_1 d_2 s_1 c_2 d_3 c_2 - c_1 d_2 s_1 s_2 d_3\right] - \left[c_1 c_2 d_3 c_1 s_2 d_3 - c_1 c_2 d_3 s_1 d_2 c_3\right] +$$

$$\left[c_1 s_2 s_2 d_2 s_1 d_2 - c_1 s_2 d_3 c_1 s_2 d_3\right]$$

$\Rightarrow \det(J_P) = 0 \quad \therefore d_3 \sin\theta_2 = 0 \Rightarrow \theta = \pi$ or $d_3 = 0 \Rightarrow$ singularity

condition for singularity

Problem-5 To find Jacobian

$$\because J = \left[\frac{J_P}{J_O}\right] = \left[\begin{array}{c|c|c} J_{P_1} & J_{P_2} & J_{P_3} \\ \hline J_{O_1} & J_{O_2} & J_{O_3} \end{array}\right]$$

Reference

$\therefore c_1 = \cos\theta_1$

$s_1 = \sin\theta_1$

$c_2 = \cos\theta_2$

$s_2 = \sin\theta_2$

$\pm c_1 = \pm\cos\theta_1$

$\pm s_1 = \pm\sin\theta_1$

Prismatic Joint

$J_{Pi} = z_{i-1}$

$J_{Oi} = 0$

Revolute Joints

$J_{Pi} = z_{i-1} \times (P_e - P_{i-1})$

$J_{Oi} = z_{i-1}$

Joint 1 $\Rightarrow$ Revolute

$$J_{P_1} = z_0 \begin{bmatrix} -d_3 s_1 \\ d_3 c_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -d_3 s_1 \\ d_3 c_1 \\ d_2 \end{bmatrix}$$

$$J_{P_1} = \begin{bmatrix} -d_3 c_1 \\ -d_3 s_1 \\ 0 \end{bmatrix}$$

$$J_{O_1} = z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Joint 2 $\Rightarrow$ prismatic

$$J_{P_2} = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \text{3rd column of } R^0_1$$

$$J_{O_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ since it is prismatic}$$

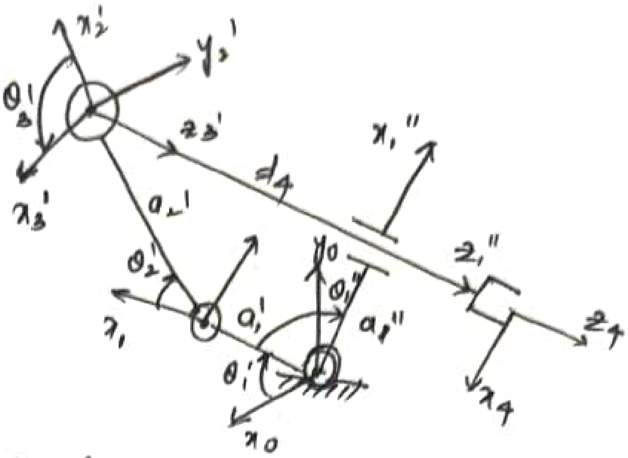
Joint 3 $\rightarrow$ prismatic

$$J_{P_3} = z_2 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \rightarrow \text{3rd column of } R^0_2$$

$$J_{O_3} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{The Jacobian matrix} = \left[\begin{array}{c|c|c} -d_3 c_1 & 0 & -s_1 \\ -d_3 s_1 & 0 & c_1 \\ 0 & 1 & 1 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right]$$

Problem - 3



Four - link closed chain planar arm.

Extracting DH parameters from the frame

Link	a_i (w.r.t x_i)	α_i (w.r.t x_i)	d_i (w.r.t z_{i-1})	θ_i (w.r.t z_{i-1})
$1'$	a_1'	0	0	θ_1'
$2'$	a_2'	0	0	θ_2'
$3'$	0	$\pi/2$	0	θ_3'
$1''$	a_1''	$-\pi/2$	0	θ_1''
4	0	0	d_4	0

The homogeneous transformation matrices for the four joints.

$$\therefore \; A_i^{i-1}(\theta_i) = \begin{bmatrix} c_i & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{1'}^{0} = \begin{bmatrix} c_1' & s_1' & 0 & a_1' c_1' \\ s_1' & c_1' & 0 & a_1' s_1' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{1'}_{2'} = \begin{bmatrix} c_2' & s_2' & 0 & a_2' c_2' \\ s_1' & c_1' & 0 & a_1' s_2' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A^{2'}_{3'} = \begin{bmatrix} c_3' & 0 & s_3' & 0 \\ s_3' & 0 & -c_3' & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^0_{3'}(t') = A^0_{1'} A^{1'}_{2'} A^{2'}_{3'} = \begin{bmatrix} c_{1'2'3'} & 0 & s_{1'2'3'} & a_1' c_1' + a_2' c_{1'2'} \\ s_{1'2'3'} & 0 & -c_{1'2'3'} & a_1' s_1' + a_2' s_{1'2'} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $t' = [\theta_1', \theta_2', \theta_3']^T$, and

$$A^0_{1''}[t''] = \begin{bmatrix} c_1'' & 0 & -s_1'' & a_1'' c_1'' \\ s_1'' & 0 & c_1'' & a_1'' s_1'' \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\because t'' = \theta_1'' \rightarrow$ the homogeneous transformation for the last link

$$A^{3'}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orientation constraints are $(\theta_{3'1''} = \pi)$, $z^0_{3'}(t') = z^0_{1''}(t'')$

$x^{0\,T}_{3'}(t')\, x^0_{1''}(t'') = -1,$

$\therefore s_{1'2'3'} = -s_1''$

$c_{1'2'3'} = -c_1''$

$\theta_2' + \theta_3' = \pi - \theta_1' + \theta_1''$

$$\begin{bmatrix} x_{3'}^{0\,T}(t') \\ y_{3'}^{0\,T}(t') \end{bmatrix} \left(p_{3'}^{0}(q') - p_{1''}^{0}(t'') \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \cancel{a_1' c[\theta_2' + \theta_3'] + a_2' \cos\theta}$$

$$\Rightarrow a_1' c(\theta_2' + \theta_3') + a_2' c\theta_2' - a_1'' c(\theta_1' + \theta_2' + \theta_3' - \theta_1'') = 0$$

```
>> % HOME WORK - 3, MAE 547 MODELING AND CONTROL OF ROBOTS
% STUDENT NAME : MANOHAR AKULA
% ASU ID: 1223335191
% ROBOTICS AND AUTONOMOUS SYSTEMS (EE)
%PROBLEM - 1b

theta1 = deg2rad(45);
theta2 = deg2rad(30);
theta3 = deg2rad(30);


Q = [theta1, theta2, theta3];

l(1) = Link('d', 0, 'a', 0, 'alpha', pi/2);
l(2) = Link('d', 0, 'a', 2, 'alpha', 0);
l(3) = Link('d', 0, 'a', 3, 'alpha', 0);

problem1b = SerialLink(l);
problem1b.plot(Q)

problem1b.A(1:3, Q)


ans =
    0.3536   -0.6124    0.7071     2.285
    0.3536   -0.6124   -0.7071     2.285
    0.8660    0.5000         0     3.598
         0         0         0         1
>>
>>
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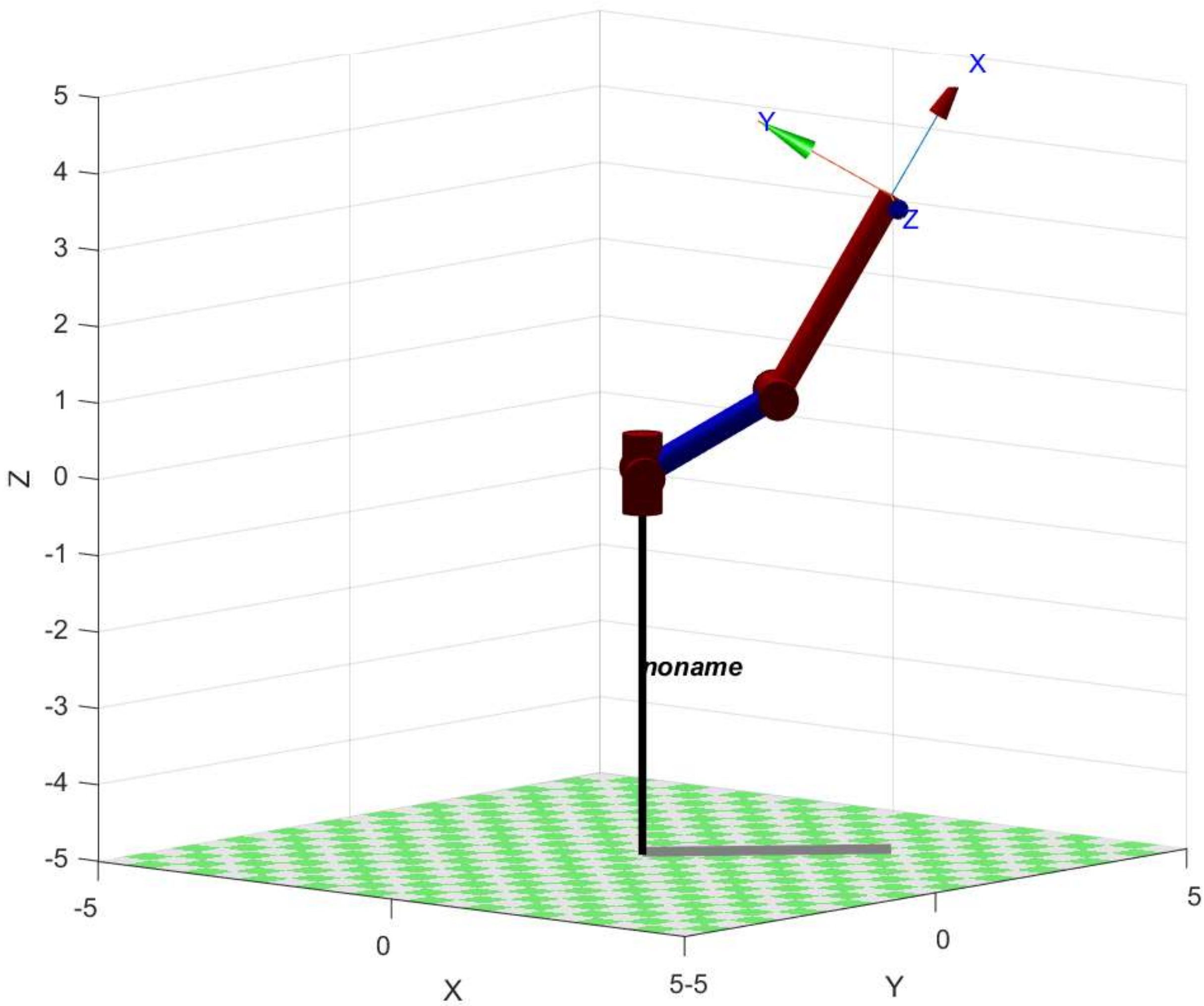


X
Y
Z
5
4
3
2
1
0
-1
-2
-3
-4
-5
noname
-5
0
5-5
0
5
X
Y
Z

```
>> % HOME WORK - 3, MAE 547 MODELING AND CONTROL OF ROBOTS
% STUDENT NAME : MANOHAR AKULA
% ASU ID: 1223335191
% ROBOTICS AND AUTONOMOUS SYSTEMS (EE)
%PROBLEM - 2

close all
link1Min= -pi/3 % link 1 minimum possible theta1 value
link1Max= pi/3% link 1 maxmimum possible theta1 value
link2Min= -2*pi/3 % link 2 minimum possible theta1 value
link2Max= 2*pi/3 % link 2 maxmimum possible theta1 value
link3Min= -pi/2 % link 3 minimum possible theta1 value
link3Max= pi/2 % link 3 maxmimum possible theta1 value
samples = 0.5
a1=0.3
a2=0.5
a3=0.2

% functions to calculate the x and y coordinates for four links with variable ↙
angles

for i = -pi/3:samples:pi/3
    for j= -2*pi/3:samples:pi/3
        for k = -2*pi/2:samples:pi/2
            x = a1*cos(i)+a2*cos(i+j)+a3*cos(i+j+k); % compute x coordinates
            y = a1*sin(i)+a2*sin(i+j)+a3*sin(i+j+k); % compute y coordinates

            plot(x,y,'k')
            hold on

        end
    end
end



link1Min =

   -1.0472


link1Max =

    1.0472


link2Min =

   -2.0944


link2Max =
```

```
    2.0944


link3Min =

   -1.5708


link3Max =

    1.5708


samples =

    0.5000


a1 =

    0.3000

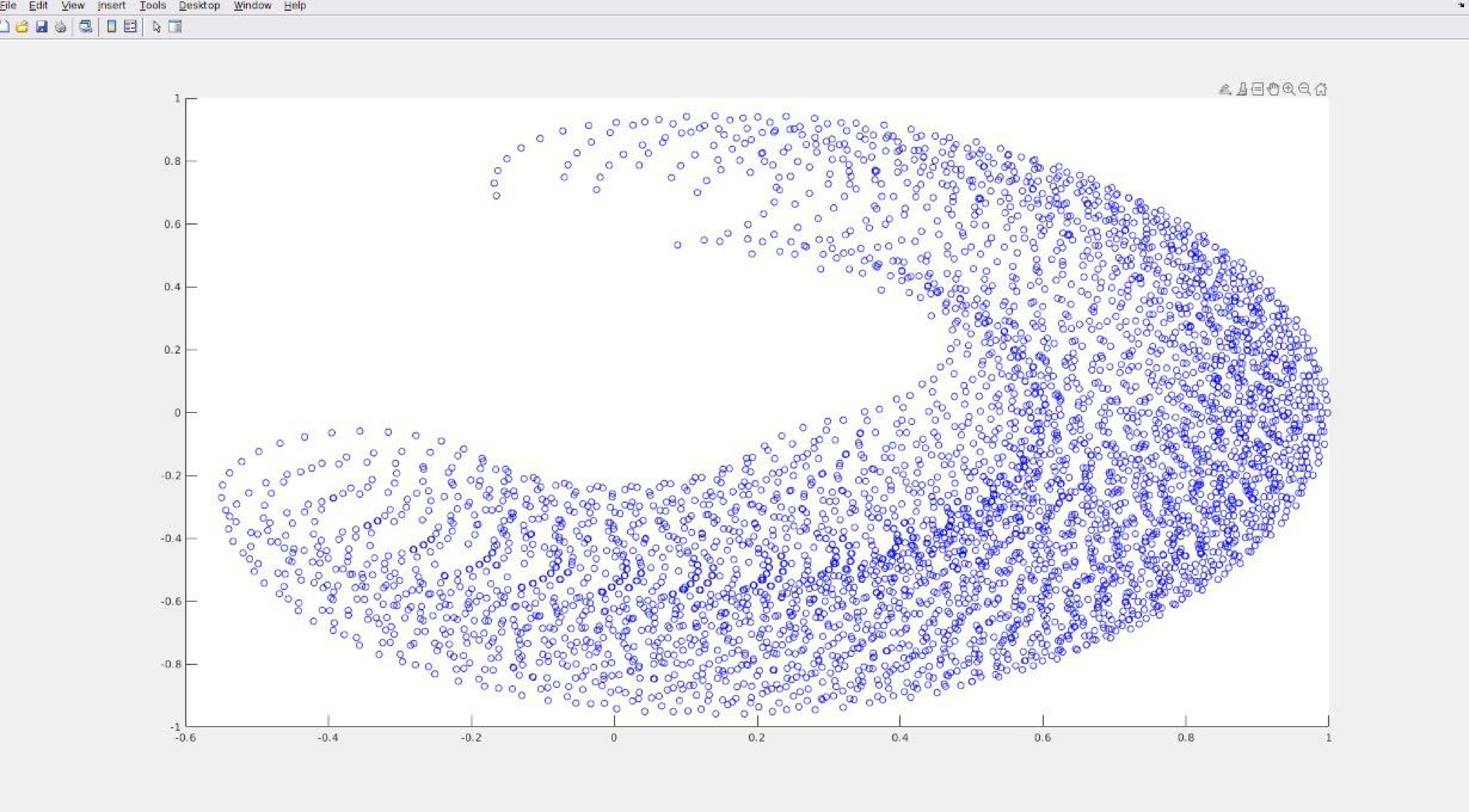

a2 =

    0.5000


a3 =

    0.2000

>>
```


File Edit View Insert Tools Desktop Window Help
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1