MAE 547 – Homework 4

Due by 11:59 pm on November 16

Problem 1 (10 pts)

One solution overcoming the problem of inverting differential kinematics in the neighborhood of a singularity is provided by the so-called damped least-squares (DLS) inverse

$$\boldsymbol{J}^{*} = \boldsymbol{J}^{\boldsymbol{T}}(\boldsymbol{J}\boldsymbol{J}^{\boldsymbol{T}} + k^{2}\boldsymbol{I})^{-1}$$

where k is a damping factor that renders the inversion better conditioned from a numerical viewpoint. The problem is that to invert differential kinematics $\boldsymbol{v_e} = \boldsymbol{J}\dot{\boldsymbol{q}}$ by tolerating a finite error $\boldsymbol{\epsilon}$, i.e., $\boldsymbol{v_e} - \boldsymbol{J}\dot{\boldsymbol{q}} = \boldsymbol{\epsilon}$. Find the solution by minimizing the cost functional

$$g''(\dot{\boldsymbol{q}}, \boldsymbol{\epsilon}) = \frac{1}{2}k^{2}\dot{\boldsymbol{q}}^{T}\dot{\boldsymbol{q}} + \frac{1}{2}\boldsymbol{\epsilon}^{T}\boldsymbol{\epsilon}$$

(Hint: You can simplify the above differential kinematics constraint and cost functional with

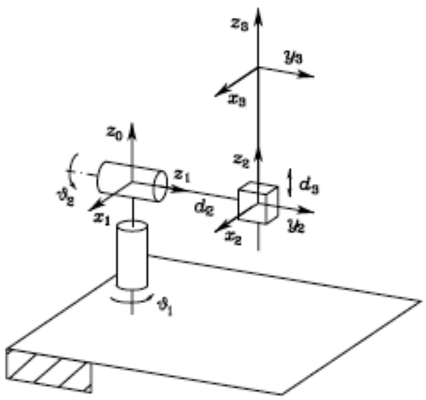
$$\boldsymbol{w} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{\epsilon} \end{bmatrix}, \qquad \boldsymbol{A} = [\boldsymbol{J} \quad \boldsymbol{I}]$$

Problem 2 (10 pts)

With reference to $\boldsymbol{\omega_e} = \boldsymbol{T}(\boldsymbol{\phi_e})\dot{\boldsymbol{\phi}}_{\boldsymbol{e}}$, find the transformation matrix $\boldsymbol{T}(\boldsymbol{\phi_e})$ in the case of Z(φ)Y(θ)X(ψ) rotations in the current frame.

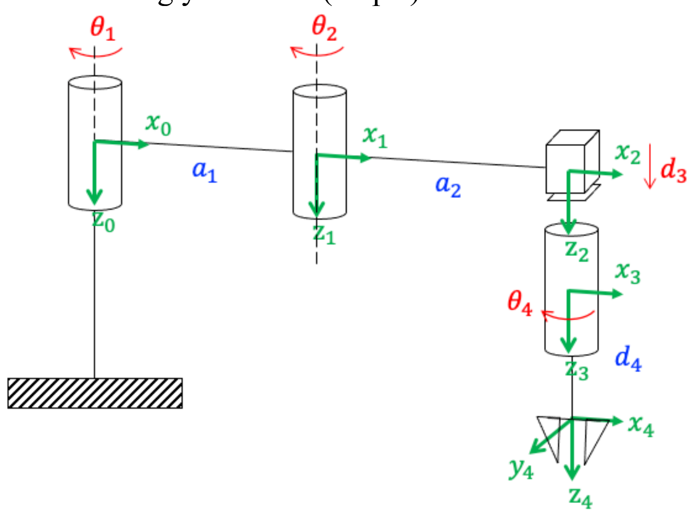
Problem 3 (10 pts)

When only end-effector linear velocity is considered for the spherical arm shown below, calculate the analytical Jacobian.



Problem 4 (25 pts)

1) Use MATLAB to solve inverse kinematics problem for the SCARA robot (shown above) using inverse Jacobian. Submit i) plots comparing the desired end-effector pose and the actual end-effector pose for $0 \le t \le 5$ s (10 pts); and ii) MATLAB codes showing your work (10 pts).



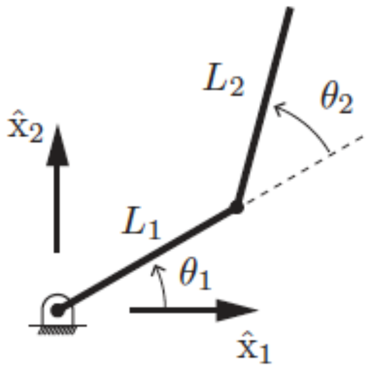
- Link parameters
  $a_1 = 0.5$ m, $a_2 = 0.3$ m, $d_4 = 0.2$ m

- Desired end-effector pose trajectory
  $\boldsymbol{p}_{\mathbf{d}} = [0.4 * \cos\left(t * \frac{\pi}{10}\right),\ 0.4 * \sin\left(t * \frac{\pi}{10}\right),\ 0.1 * (1 + \sin(t))]^T$ $(0 \le t \le 5 \text{ s})$
  $\boldsymbol{\phi}_{\mathbf{d}} = t * \frac{\pi}{10} + \frac{5\pi}{12}$ $\qquad (0 \le t \le 5 \text{ s})$

- Initial end-effector pose
  $\boldsymbol{q}_0 = [\frac{\pi}{4}\ rad,\ \frac{\pi}{4}\ rad,\ 0.3\ m, 0\ rad]^T$

2) Try different **K** values in your inverse kinematics algorithm and discuss how the selection of **K** influences overall performance (5 pts).

Problem 5 (10 pts)



For the 2R robot arm with $L_1 = 1$ and $L_2 = 1$, draw manipulability ellipses for the following two configurations:

1) $\theta_1 = 0\, rad, \theta_2 = \frac{\pi}{4} rad$

2) $\theta_1 = 0\, rad, \theta_2 = \frac{3\pi}{4} rad$

Please use MATLAB for your answers.

Problem 6 (10 pts)

Write and solve the equations necessary to compute the joint trajectory from $q(t=0) = 1$ to $q(t=3) = 5$ with zero initial and final velocities and accelerations.