Date
Page

Name :- Manohar Kumar
Branch :- CSE
Roll no. :- 29
B. Roll no. :- 19402060012
Subject :- Engg. mathematics
Semester :- 3rd
Session :- 2019 - 22
Date :- 17/04/21

Q. (a) Most appropriate root of the equation:-

$x^4 - x - 10 = 0$ by bisection method

Soln: $x^4 - x = 10 \Rightarrow f(x) = x^4 - x - 10$

x	0	1	2
$f(x)$	-10	-10	4

Here, $f(1) = -10 < 0$
$f(2) = 4 > 0$

The root lies between 1 and 2.
Now, $f(1) \cdot f(2) = -10 \times 4 = -40$

1st approximation :-
$x_1 = \frac{1+3}{3} = 1.5$

$\therefore f(1.5) = (1.5)^4 - (1.5) - 10$
$= -6.4375 < 0$

$\therefore$ root lies between 1.5 and 2

Generated by CamScanner

Date
Page

2^{nd} approximation :-

$$x_2 = \frac{1.5+2}{2} = \frac{3.5}{2} = 1.75$$

Now,

$$f(1.75) = (1.75)^4 - 1.75 - 10$$
$$= -2.37109 < 0$$

$\therefore$ root lies between 1.75 and 2

3^{rd} approximation :-

$$x_3 = \frac{1.75+2}{2} = \frac{3.75}{2} = 1.875$$

Now,

$$f(1.875) = (1.875)^4 - (1.875) - 10$$
$$= 0.48461 < 0$$

$= \frac{3.625}{2}$

$= 1.8125$

4^{th} approximation :-

$$x_4 = \frac{1.75 + 1.875}{2} = 1.8125$$

Hence, Ans $\rightarrow$ (b) approx Ans

Q. 2.) Most approximate root of the equation :-

$x^3 - 3x + 4 = 0$ by regula false method.

Soln:- Given eq:-

$$f(x) = x^3 - 3x + 4 = 0$$
$$f(x) = x^3 - 3x + 4$$

By regula falsi,

$$x_n = x_1 - \frac{x_{n-1} - x_1}{f(x_{n-1}) - f(x_1)} \times f(x_1)$$

$x_{n-1} = -2, \quad x_1 = -3$

$\therefore f(x_1) = f(-3)$

$f(-3) = (-3)^3 - 3(-3) + 4 = -14$

when, $f(x_{n-1}) = f(-2)$

$f(-2) = (-2)^3 - 3(-2) + 4 = 2$

$\therefore x_n = -3 - \dfrac{(-2+3)\,(-14)}{2+14}$

$\therefore x_n = -2.125$ Ans Hence, ans → (a)

Q.3.) Using Newton's Raphson method:- Solve: $\sqrt[4]{32}$.

Soln:- Given,

$x = \sqrt[4]{32}$

$x^4 = 32$

$x^4 - 32 = 0$

By Newton Raphson method:-

$f(x) = x^4 - 32$

$f'(x) = 4x^3$

x	0	1	2	3
$f(x)$	-32	-31	-16	49

let $x_0 = 3$

$x_1 = x_0 - \dfrac{f(x)}{f'(x)}$

$= 3 - \dfrac{49}{108}$

$= 3 - 0.4537$

$= 2.5463$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.5463 - \frac{10.018}{66.037}$$

$$= 2.5463 - 0.152$$

$$= 2.3943.$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.3943 - \frac{0.8635}{54.90}$$

$$= 2.3943 - 0.0157$$

$$= 2.3786$$

Hence, Ans → (b)

Q.4.) Solve by using Gauses ellimination method in The correct value of the equation:-

$$3x - y - z = 4 \quad \text{--- (i)}$$

$$x + y - 6z = -12 \quad \text{--- (ii)}$$

$$x + 4y - z = -5 \quad \text{--- (iii)}$$

Soln:- Multiply eq (3) by (3) and rule from eq (1),

$$\cancel{3x} - 4y \cancel{-y} - z = 4$$

$$\cancel{3x} \ominus 12y \ominus 3z = \ominus 15$$

$$- \quad + \quad + \quad +$$

$$13y + 2z = 19 \quad \text{--- (4)}$$

Multiply eq (2) by (3) and solve from eq (1)

$$\cancel{3x} - y - z = 4$$

$$\cancel{3x} \oplus 3y \ominus 18z = \ominus 36$$

$$- \quad - \quad + \quad +$$

$$-4y + 17z = 40$$

Date
Page

Q.5-) The value of $\int_0^{\pi/2} \cos^8 x \, dx$

Soln:- $I = \int_0^{\pi/2} \cos^8 x \, dx$ — (i)

$= \int_0^{\pi/2} \cos^8\left(\frac{\pi}{2} - 0\right) dx$

$= \int_0^{\pi/2} \sin^8 x \, dx$ — (ii)

Adding eq (i) and (ii) :-

$I + I = \int_0^{\pi/2} \cos^8 x \, dx + \int_0^{\pi/2} \sin^8 x \, dx$

$2I = \int_0^{\pi/2} (\cos^8 x + \sin^8 x) \, dx$

$2I = \int_0^{\pi/2} (\cos^4 x + \sin^4 x)^2 \, dx$

$2I = \int_0^{\pi/2} \{(\cos^4 x + \sin^4 x)^2 - (2\sin^4 x \cdot \cos^4 x)\} \, dx$

$2I = \int_0^{\pi/2} [\{(\cos^2 x + \sin^2 x)^2 - (2\sin^2 x \cdot \cos^2 x)\}$
$- 2\sin^4 x \cdot \cos^4 x] \, dx$

$2I = \int_0^{\pi/2} (1 + 2\sin^4 x \cdot \cos^4 x - 4\sin^2 x \cdot \cos^2 x) \, dx$

$2I = \int_0^{\pi/2} 1 + \frac{2}{(2)^4} (2 \cdot \sin x \cdot \cos x)^4 - \{2(\sin x \cdot \cos x)\}^2 \, dx$

$2I = \int_0^{\pi/2} \left\{1 + \frac{1}{(2)^5} \cdot \sin^4 2x - \sin^2 2x\right\} dx$

Generated by CamScanner

$\because \cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$$\therefore 2I = \int_0^{\pi/2} 1 * \frac{1}{(2)^3}\left(\frac{1-\cos 4x}{2}\right)^2 \left(\frac{1-\cos 4x}{2}\right) dx$$

$$2I = \int_0^{\pi/2} \frac{1}{2} * \frac{1}{(2)^5}\left(1+\cos^2 4x - 2\cos 4x\right) + \frac{\cos 4x}{2} dx$$

$$2I = \int_0^{\pi/4} \frac{1}{2} * \frac{1}{(2)^5}\left(1 + \frac{1+\cos 8x}{2} - 2\cos 4x\right) + \frac{\cos 4x}{2}$$

$$2I = \int_0^{\pi/2} \left\{\frac{1}{2} + \frac{1}{(2)^5} + \frac{1}{(2)^6} + \frac{\cos 8x}{2} - \frac{\cos 4x}{(2)^4} + \frac{\cos 4x}{2}\right\}$$

$$2I = \int_0^{\pi/2} \frac{(3)^5}{(2)^6} + \frac{\cos 8x}{(2)^6} + \cos 4x\left(\frac{1}{2} - \frac{1}{(2)^4}\right) dx$$

$$2I = \int_0^{\pi/2} \left(\frac{(3)^5}{(2)^6} + \frac{\cos 8x}{(2)^6} + \cos 4x \cdot \frac{7}{(2)^4}\right\} dx$$

$$2I = \int_0^{\pi/2} \left(\frac{(3)^5}{(2)^6} + \frac{\sin 8x}{(2)^6 \cdot 8} + \frac{\sin 4x}{4} \cdot \frac{7}{(2)^4}\right) dx$$

$$2I = \left[\frac{35\pi}{(2)^7} + 0 - 0 - 0\right]$$

$$I = \frac{35\pi}{256} \quad \underline{\text{Ans}}$$

Hence, Ans $\rightarrow$ (d)