(CS1.301) Advanced Algorithm Design (Monsoon 2022) Due: 06.10.2022

Problem Set 2

Instructions:

- Discussions amongst the students are not discouraged, but all writeups must be done individually and must include names of all collaborators.
- Referring sources other than the lecture notes is discouraged as solutions to some of the problems can be found easily via a web search. But if you do use an outside source (eg., text books, other lecture notes, any material available online), do mention the same in your writeup. This will not affect your grades. However dishonesty of any sort when caught shall be heavily penalized.
- Be clear in your arguments. Vague arguments shall not be given full credit.
- Total marks are 30 but they will be normalized.

Question 1 [4 marks]

Let G has n vertices and m edges whose edge weights are all distinct. Give an algorithm to decide whether a given edge e is contained in a minimum spanning tree of G.

Question 2 [8 marks]

Let $G = (V, E)$ be an undirected graph with edge costs $c_e \geqslant 0$ on the edges $e \in E$. Let T be a Minimal Spanning Tree in G. Let a new edge (v, w) with edge weight c, be added to G. Give an efficient algorithm to test if T remains a Minimal Spanning Tree after addition of (v, w) to G (and not to T). If T is no longer the minimal spanning tree, then give an efficient algorithm to update the tree T to the new Minimum Spanning Tree.

Question 3 [8 marks]

Given a list of n natural numbers $d_1, d_2, \ldots, d_n$, show how to decide in polynomial time whether there exists an undirected (simple) graph $G = (V, E)$ whose node degrees are precisely the numbers $d_1, d_2, \ldots, d_n$.

Question 4 [10 marks]

Let $G = (V, E)$ be a graph with n nodes in which each pair of nodes is joined by an edge. There is a positive weight $w_{i,j}$ on each edge (i, j); and we will assume these weights satisfy the triangle inequality $w_{i,k} \leqslant w_{i,j} + w_{j,k}$. For a subset $V' \subseteq V$, we will use $G[V']$ to denote the subgraph (with edge weights) induced on the nodes in V'.

We are given a set $X \subseteq V$ of k terminals that must be connected by edges. We say that a Steiner tree on X is a set Z so that $X \subseteq Z \subseteq V$, together with a spanning subtree T of $G[Z]$. The weight of the Steiner tree is the weight of the tree T. Show that the problem of finding a minimum-weight Steiner tree on X can be solved in time $n^{O(k)}$.